# tmm final

December 18, 2020

```
[1]: import numpy as np
import matplotlib.pyplot as plt
from tmm_final import *
```

## 0.1 Single Layer Coating

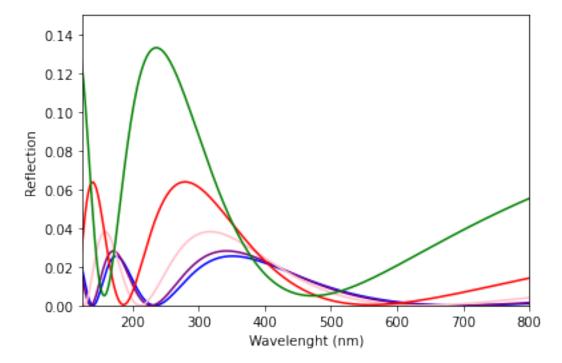
For the case of a single layer of anti-reflecting coating, we want to have a quarter-wave thickness as well as [1]:

$$n_{film} = \sqrt{n_{substrate}}$$

Thus, if we want to eliminate 600nm light and use, for example, a  $MgF_2$  substrate with n = 1.38 our film should have an index of refraction of n = 1.17:

```
[2]: degree = np.pi/180
     lamb = 600
     n_list = [1, 1.17, 1.38]
     d_list = [np.inf, lamb/4, np.inf]
     wl = np.linspace(50, 1000, num=400)
     RO=[]
     R15=[]
     R30=[]
     R45=[]
     R60 = []
     for w in wl:
         RO.append(transfer_matrix_method('s', n_list, d_list, 0, w)['R'])
         R15.append(transfer_matrix_method('s', n_list, d_list, 15 * degree, w)['R'])
         R30.append(transfer_matrix_method('s', n_list, d_list, 30 * degree, w)['R'])
         R45.append(transfer_matrix_method('s', n_list, d_list, 45 * degree, w)['R'])
         R60.append(transfer_matrix_method('s', n_list, d_list, 60 * degree, w)['R'])
```

```
plt.figure()
plt.xlim(125, 800)
plt.ylim(0, 0.15)
plt.plot(wl,R0,'blue',wl,R15,'purple',wl,R30,'pink',wl,R45,'red',wl,R60,'green')
plt.xlabel('Wavelenght (nm)')
plt.ylabel('Reflection')
plt.show()
```



As we can see, the minima shifts with a changing angle of incidence, and light is nearly extinguished around the 600nm mark. We now repeat this process for TM or p-wave radiation:

```
[3]: degree = np.pi/180

lamb = 600

n_list = [1, 1.17, 1.38]

d_list = [np.inf, lamb/4, np.inf]

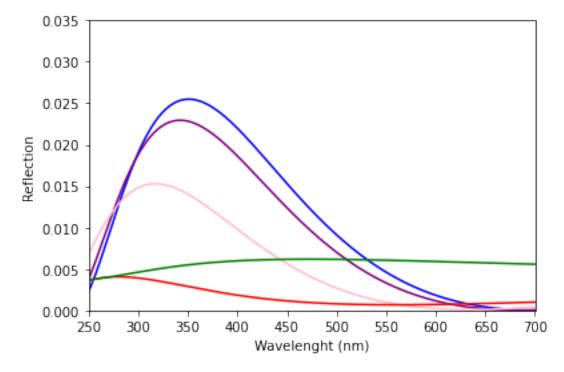
wl = np.linspace(50,1000,num=400)

R0=[]
R15=[]
R30=[]
R45=[]
```

```
for w in wl:

R0.append(transfer_matrix_method('p', n_list, d_list, 0, w)['R'])
R15.append(transfer_matrix_method('p', n_list, d_list, 15 * degree, w)['R'])
R30.append(transfer_matrix_method('p', n_list, d_list, 30 * degree, w)['R'])
R45.append(transfer_matrix_method('p', n_list, d_list, 45 * degree, w)['R'])
R60.append(transfer_matrix_method('p', n_list, d_list, 60 * degree, w)['R'])

plt.figure()
plt.xlim(250, 700)
plt.ylim(0, 0.035)
plt.plot(wl,R0,'blue',wl,R15,'purple',wl,R30,'pink',wl,R45,'red',wl,R60,'green')
plt.xlabel('Wavelenght (nm)')
plt.ylabel('Reflection')
plt.show()
```



Although there is a clear drop in reflectance around 600nm, the overall reflectance is around seven times less than for s-waves.

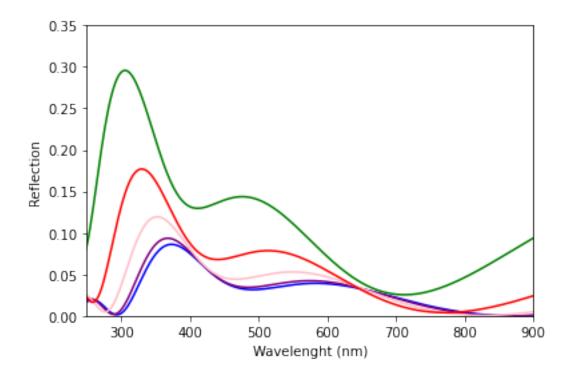
# 0.2 Double Layer Coating

Let us now examine double layer anti-reflection coatings. As before, we will examine quarter-quarter-wave layer thicknesses that also obey:

$$\left(\frac{n_1}{n_2}\right)^2 = \frac{n_{air}}{n_{substrate}}$$

If we use glass with and index of refraction of 1.5 as a substrate, assume that  $n_{air} = 1$ , and re-use  $MgF_2$  a our first layer, the second material must have  $n_2 = 1.69$ . Lets see how this reflects 600nm light:

```
[4]: degree = np.pi/180
     lamb = 600
     n_{list} = [1, 1.38, 1.69, 1.50]
     d_list = [np.inf, lamb/4, lamb/4, np.inf]
     wl = np.linspace(50, 1000, num=400)
     RO=[]
     R15=[]
     R30=[]
     R45=[]
     R60=[]
     for w in wl:
         RO.append(transfer_matrix_method('s', n_list, d_list, 0, w)['R'])
         R15.append(transfer_matrix_method('s', n_list, d_list, 15 * degree, w)['R'])
         R30.append(transfer_matrix_method('s', n_list, d_list, 30 * degree, w)['R'])
         R45.append(transfer_matrix_method('s', n_list, d_list, 45 * degree, w)['R'])
         R60.append(transfer_matrix_method('s', n_list, d_list, 60 * degree, w)['R'])
     plt.figure()
     plt.xlim(250, 900)
     plt.ylim(0, 0.35)
     plt.plot(wl,R0,'blue',wl,R15,'purple',wl,R30,'pink',wl,R45,'red',wl,R60,'green')
     plt.xlabel('Wavelenght (nm)')
     plt.ylabel('Reflection')
     plt.show()
```



For two layers, the reflectance drops to zero around 780nm rather than around the 600nm mark, signifying a shift of around 200nm.

```
[5]: degree = np.pi/180

lamb = 600

n_list = [1, 1.38, 1.69, 1.50]

d_list = [np.inf, lamb/4, lamb/4, np.inf]

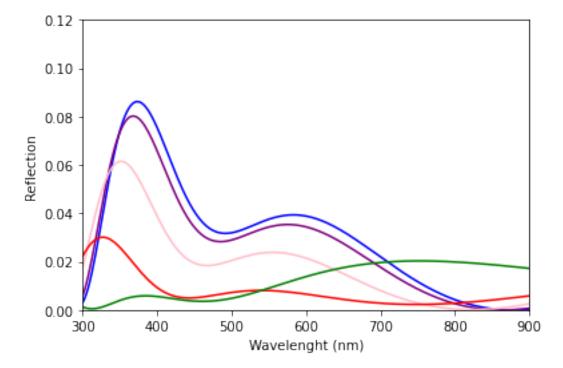
wl = np.linspace(50,1000,num=400)

R0=[]
R15=[]
R30=[]
R45=[]
R60=[]
for w in wl:

R0.append(transfer_matrix_method('p', n_list, d_list, 0, w)['R'])
R15.append(transfer_matrix_method('p', n_list, d_list, 15 * degree, w)['R'])
R30.append(transfer_matrix_method('p', n_list, d_list, 30 * degree, w)['R'])
R45.append(transfer_matrix_method('p', n_list, d_list, 45 * degree, w)['R'])
```

```
R60.append(transfer_matrix_method('p', n_list, d_list, 60 * degree, w)['R'])

plt.figure()
plt.xlim(300, 900)
plt.ylim(0, 0.12)
plt.plot(wl,R0,'blue',wl,R15,'purple',wl,R30,'pink',wl,R45,'red',wl,R60,'green')
plt.xlabel('Wavelenght (nm)')
plt.ylabel('Reflection')
plt.show()
```



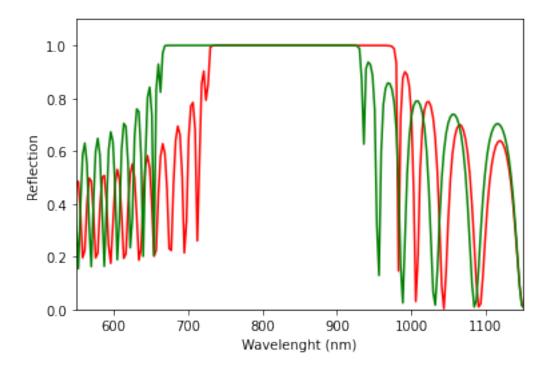
Similar to the single layer coating, the reflectance is much lower for p-waves than for s-wave, though the 200nm shift is still present.

## 0.3 Distributed Bragg Reflectors

Lets now try to reproduce a reflectance plot for a Distributed Bragg Reflector (DBR) using some available data. We will consider 20 layers, each one composed out of two materials such that  $n_1 = 1.86$  and  $n_2 = 1.27$ . Furthermore, we will be using quarter-wave layers on a 600nm beam once again, and a substratue with  $n_{substrate} = 4.1$ :

```
[6]: degree = np.pi/180
lamb = 600
```

```
n_{\text{list}} = [1, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86, 1.27, 1.86
    \rightarrow27,1.86,1.27
                                             ,1.86,1.27,1.86,1.27,1.86,1.27,1.86,1.27,1.86,1.27,1.86,1.27
                                             ,1.86,1.27,1.86,1.27,1.86,1.27,1.86,1.27,1.86,1.27,4.1]
d list = [np.inf, lamb/4, lamb
    \rightarrow4, lamb/4, lamb/4, lamb/4, lamb/4,
                                            lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/
    \rightarrow 4, lamb/4, lamb/4, lamb/4,
                                            lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/4, lamb/
   \rightarrow4, lamb/4, lamb/4, lamb/4,
                                           lamb/4, lamb/4, lamb/4, np.inf]
wl = np.linspace(50, 1500, num=500)
R0= []
R15= []
R30=[]
R45=[]
R60 = []
for w in wl:
                 RO.append(transfer_matrix_method('s', n_list, d_list, 0, w)['R'])
                 R15.append(transfer_matrix_method('s', n_list, d_list, 15 * degree, w)['R'])
                 R30.append(transfer_matrix_method('s', n_list, d_list, 30 * degree, w)['R'])
                 R45.append(transfer_matrix_method('s', n_list, d_list, 45 * degree, w)['R'])
                 R60.append(transfer_matrix_method('s', n_list, d_list, 60 * degree, w)['R'])
plt.figure()
plt.xlim(550, 1150)
plt.ylim(0, 1.1)
plt.plot(w1,R45,'red',w1,R60,'green')
plt.xlabel('Wavelenght (nm)')
plt.ylabel('Reflection')
plt.show()
```



Even if the perfect reflectance region is not centered around 600nm as expected, the plot does exhibit the long flat region that is the hallmark of a DBR.

#### 0.4 Additional DBR Plotting

As a final test of our program, we plot another DBR graph with some experimental data. The layers are not exactly quarter-wave plates, however seeing as how the curves seem to be centered around 200nm and some of the layers are 50nm, we can conclude that this is the target wavelenght.

```
[7]: degree = np.pi/180

n_list = [1, 1.63, 2.74, 1.63, 2.74, 1.63, 2.74, 1.63, 2.74, 1.5]

d_list = [np.inf, 38, 50, 38, 50, 38, 50, np.inf]

wl = np.linspace(50,1000,num=400)

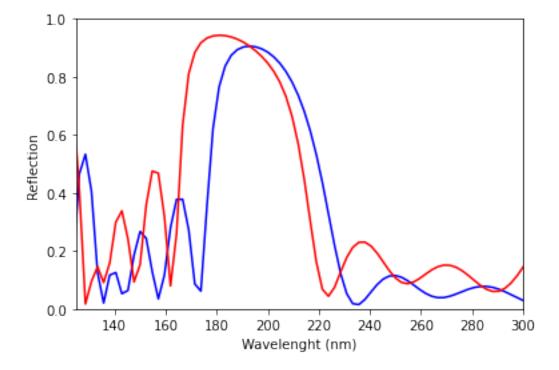
R0=[]
R45=[]
R60=[]

for w in wl:

    R0.append(transfer_matrix_method('s', n_list, d_list, 0, w)['R'])
    R45.append(transfer_matrix_method('s', n_list, d_list, 45 * degree, w)['R'])
```

```
R60.append(transfer_matrix_method('s', n_list, d_list, 60 * degree, w)['R'])

plt.figure()
plt.xlim(125, 300)
plt.ylim(0,1)
plt.plot(wl,R0,'blue',wl,R45,'red')
plt.xlabel('Wavelenght (nm)')
plt.ylabel('Reflection')
plt.show()
```



[]: