# Measurement induced macroscopic superposition states in cavity optomechanics

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In this project we investigate the feasibility of preparing a macroscopic mechanical oscillator in a Schrödinger cat-like state. To achieve this we utilize a pulsed cavity-enhanced optomechanical interaction which effectively realizes a QND measurement of the mechanical position quadrature. By preparing the optical drive field in an $m$-photon subtracted squeezed state (mPSSS) and performing a phase quadrature measurement of the optical field reflected off the optomechanical cavity, the mechanical oscillator is conditionally prepared in a macroscopic superposition state. To verify the mechanical state we perform subsequent tomographic reconstruction using a squeezed optical probe field.

*Introduction.* Elusive as it is, Schrödinger's cat [1] remain one of the hardest to tame in the quantum world, yet also among the ones most strived for. That due to its quintessential embodiment of the manifestly non-classical properties of quantum mechanics. Successful creation of such coherent superposition states have so far been limited exclusively to isolated microscopic quantum systems, e. g. in ion traps [2, 3] and microwave cavity QED [4, 5], while mesoscopic variants, colloquially termed Schrödinger kittens, have been demonstrated in propagating optical fields [6–8]. However, an intriguing and long standing question is whether also macroscopic objects can be prepared in quantum superpositions of being *here* and *there*?

Within the last decades the field of optomechanics has attracted tremendous theoretical and experimental attention and has developed into a mature research disciplin already contributing a range of key results in quantum physics, including mechanical ground state cooling [9, 10], observation of quantum back-action [11, 12], ponderomotive squeezing [13–15], and recently generation of non-classical mechanical states of motion [16]. The demonstrated technological ability to engineer mechanical oscillators ranging from micro- to macroscopic in size and tailor their interaction with radiation fields places optomechanics among the most promising testbeds for experimental scrutiny of the long debated quantum-classical transition. Experiments of this kind are of utmost importance for understanding the foundations of quantum mechanics and quantum measurement theory [17], and from a technological point of view, engineering and coherent manipulation of mechanical quantum states is paramount for realization of hybrid quantum information processing protocols [18].

A vast number of proposals for optomechanical generation of non-classical mechanical states exist in the literature, covering schemes relying on the nonlinear optomechanical coupling [19, 20], strong single photon interaction [21–23], and pulsed interaction [24]. The latter has been implemented experimentally demonstrating optomechanical entanglement [25]. Unfortunately, existing schemes employing optical single photon resources for generation of non-classical mechanical states are generally of limited practical feasibility because of the insufficient optomechanical interaction strengths currently achievable. One way of enhancing the interaction is to apply an initial displacement to the Fock state, as considered by Sekatski et al. [23]. However, whereas this approach offers mechanical superposition states of distinguishable constituents in phase space, only a modest degree of macroscopicity can be achieved, quickly saturating with the number of displacement photons, as pointed out in [26].

In this Letter, we propose a novel scheme employing displaced photon subtracted squeezed vacuum (PSSV) states [27] in conjunction with a pulsed optomechanical quantum non-demolition (QND) interaction [28, 29] and subsequent optical homodyne detection (Fig. 1). The

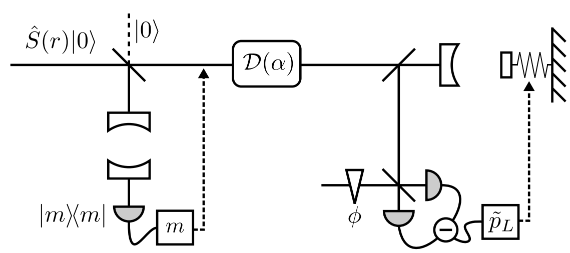


FIG. 1. Employing a displaced PSSV state of light as resource for driving a cavity optomechanical QND interaction, the mechanical oscillator is projected into a highly non-classical state conditioned on the outcome of a subsequent homodyne detection of the optical phase quadrature.

optical input mode is conditionally prepared in a pulsed PSSV state, by photon number resolved detection on a temporally filtered tap-off of a vacuum squeezed state. Before injection into a cavity optomechanical system [30], pre-cooled close to its motional ground state, a displacement operation is applied to enhance the optomechanical interaction strength. For interaction times much shorter than the mechanical evolution time, $\tau \ll \omega_M^{-1}$, an effective optomechanical QND interaction is realized, and grace to that the mechanics can be projected into a highly non-classical quantum state conditioned on a subsequent measurement on the reflected optical field. We model the state preparation analytically using the Wigner function formalism, and the macroscopicity [INTRODUCE CONCEPT] of the prepared mechanical state is assessed by the measure proposed by Lee and Jeong [31].

*Optomechanical interaction.* We consider a single-ended cavity optomechanical system excited by an optical pulse of $N_p$ photons and duration $\tau$, much shorter than the free evolution time of the mechanical oscillator at frequency $\omega_M$. Furthermore, the cavity bandwidth $\kappa$ (HWHM) is assumed to be much broader than that of the optical pulse. Under these conditions, the dynamics of the optical intra-cavity field can be adiabatically eliminated and mechanical damping and noise processes can be neglected during the interaction time. The optical input mode consists of two parts: a quantum fluctuation part described by bosonic operators $a, a^\dagger$ and with temporal mode function $f(t)$, defined by the conditional PSSV preparation scheme [32] and satisfying $\int dt|f(t)|^2 = 1$, and a classical driving field with amplitude $|\alpha(t)| = \sqrt{N_p}f(t)$ matched to the quantum state. To encompass the temporal correlations within the input quantum state, collective quadratures are defined for the entire pulse as $x_L = \int_{-\infty}^{\infty} dt f(t) x_L(t)$ and $p_L = \int_{-\infty}^{\infty} dt f(t) p_L(t)$, obeying $[x_L, p_L] = i$. The mechanical oscillator is similarly described by quadrature operators $x_M = (b + b^\dagger)/\sqrt{2}$, $p_M = -i(b - b^\dagger)/\sqrt{2}$ with $[x_M, p_M] = i$. Assuming resonant interaction, the linearized Hamiltonian takes the form $H = -\hbar g_0 (2|\alpha(t)|x_L(t)x_M + \sqrt{2}|\alpha(t)|^2 x_M)$, where the optomechanical coupling constant is given by $g_0 = x_{zpf}\omega_0/L$ for an optical Fabry-Perot cavity with length $L$, and $x_{zpf} = \sqrt{\hbar/2M\omega_M}$ is the zero-point fluctuation amplitude of the mechanical oscillator with mass $M$. Integrating the corresponding quantum Langevin equations over the entire interaction results in the following input-output transformation of the quadratures

$$x_L^{out} = x_L^{in} \tag{1}$$

$$p_L^{out} = p_L^{in} + \chi x_M^{in} \tag{2}$$

$$x_M^{out} = x_M^{in} \tag{3}$$

$$p_M^{out} = p_M^{in} + \chi x_L^{in} + \Omega, \tag{4}$$

which is of the well-known QND form [33]. In Eqs. (2) and (4) we have introduced the coupling strength $\chi = 4g_0\sqrt{N_p}/\kappa$, weighting the contribution of the measured mechanical position $x_M^{in}$ to the optical output phase quadrature, and the associated back-action momentum transfer $\Omega = \chi\sqrt{N_p/2}$ imposed on the mechanical phase quadrature.

We assume that the initial optical and mechanical states are prepared independently and each characterized by Wigner functions $W_L^{in}$ and $W_M^{in}$, respectively. Through the optomechanical interaction the system degrees of freedom are correlated, the degree of correlation being proportional to the QND coupling strength $\chi$, as evident from Eqs. (1-4). By performing a post interaction homodyne measurement of the reflected optical phase quadrature, the mechanical oscillator can thus be actuated through measurement induced feedback, projecting it into an output state $W_M^{out}$. The actual state prepared is conditioned on the measurement outcome $\tilde{p}_L$. Tracing over the unmeasured optical quadrature the conditionally prepared mechanical state is, up to a normalization factor, given by a convolution of the optical and mechanical input states,

$$W_M^{out}(x_M, p_M) = \int dx'_L W_L^{in}(x'_L, \tilde{p}_L - \chi x_M) \times W_M^{in}(x_M, p_M - \chi x'_L - \Omega). \tag{5}$$

Consequently, by driving the optomechanical interaction with a non-classical optical input, the distinctly quantum properties can be transferred to the mechanical oscillator through measurement induced quantum correlations. As required by the uncertainty principle, the QND measurement of the mechanical position is accompanied by back-action on the conjugate variable, resulting in a measurement strength dependent displacement $\Omega$ of the mechanical state along the momentum quadrature.

*PSSV state preparation.* Having detailed the optomechanical interaction we now discuss the employed non-classical optical resource states and derive a general analytical expression for the corresponding Wigner functions. Assume that a squeezed vacuum state $\hat{S}(r)|0\rangle$ with squeezing parameter $r$, derived from a pulsed parametric down-conversion source, impinges on a beam splitter with transmittivity $\sqrt{T}$ close to unity (Fig. 1). The reflected field is temporally filtered by a cavity and directed onto a photon number resolving detector, yielding a measurement outcome $m$. Using the standard beam splitter transformation in the Wigner function representation [34] and the corresponding Wigner function for an ideal $m$-photon detection process [35]

$$W_d(x, p) = \frac{1}{\pi} e^{-(x^2+p^2)}(-)^m L_m(2x^2 + 2p^2), \tag{6}$$

where $L_m$ is the $m$'th Laguerre polynomium, the resulting conditional PSSV state of the beam splitter output mode is found to be given by the Wigner function representation:

$$W_L^{in}(x_L,p_L) = \frac{\mathcal{N}}{\pi^2\sqrt{c_1c_2}} e^{-(a^2e^{-2r}+b^2-d_1^2/4c_1)x_L^2}$$
$$\times e^{-(a^2e^{2r}+b^2-d_2^2/4c_2)p_L^2}$$
$$\times \sum_{k=0}^{m}\sum_{l=0}^{k}\Bigg[\frac{2^k(-)^{m-k}}{k!}\frac{m!}{(m-k)!k!}\frac{k!}{(k-l)!l!}$$
$$\times \sum_{i=0}^{l}\frac{(2l)!}{i!(2l-2i)!}\frac{(-2d_1)^{2(l-i)}}{(4c_1)^{2l-i}}x_L^{2(l-i)}$$
$$\times \sum_{j=0}^{k-l}\frac{(2(k-l))!}{j!(2(k-l-j))!}\frac{(-2d_2)^{2(k-l-j)}}{(4c_2)^{2(k-l)-j}}p_L^{2(k-l-j)}\Bigg], \tag{7}$$

Here $\mathcal{N}$ is a normalization constant, $a = \sqrt{T}$, $b = \sqrt{1-T}$, and lumped parameters $c_1(r) = 2e^{-r}(\cosh(r) + T\sinh(r))$, $c_2(r) = c_1(-r)$, $d_1(r) = 4\sqrt{T(1-T)}e^{-r}\sinh(r)$, and $d_2(r) = d_1(-r)$ have been introduced.

*Conditional mechanical state* The mechanical oscillator is initially assumed in thermal equilibrium with a cryogenic bath. Immediately preceding the optomechanical QND interaction, optical feedback cooling is applied and the mechanical mode is precooled close to it motional ground state. In either case, it is described by a thermal state with average phonon occupancy $\bar{n}$ and the corresponding Wigner function is,

$$W_M^{in}(x_M,p_M) = \frac{1}{\pi}\frac{1}{2\bar{n}+1}\exp\left[\frac{-(x_M^2+p_M^2)}{(2\bar{n}+1)}\right]. \tag{8}$$

Near-ground state optical cooling ($\bar{n} < 1$) is necessary in order for the fluctuations in the transferred quantum state to dominate the classical thermal noise, thereby allowing generation of mechanical states with distinctly non-classical properties. Furthermore, cryogenic cooling of the environment is critical for the life time of the generated mechanical state which is related to the re-thermalization rate, proportional to the bath phonon occupation.

Combining the general Eq. (5) for the mechanical output state with the specific input states in Eqs. (7-8) the conditional mechanical output state is found to be

$$W_M^{out}(x_M,p_M) = \frac{\mathcal{N}}{\pi^3\sqrt{c_1c_2}}\frac{1}{2\bar{n}+1}$$
$$\times e^{-(a^2e^{-2r}+b^2-d_2^2/4c_2)p_L^2}$$
$$\times e^{-(x_M^2+(p_M-\Omega)^2)/(2\bar{n}+1)}$$
$$\times \sum_{k=0}^{m}\sum_{l=0}^{k}\Bigg[2^k\frac{(-)^{m-k}}{k!}\frac{m!}{(m-k)!k!}\frac{k!}{(k-l)!l!}$$
$$\times \sum_{i=0}^{l}\gamma(l,i)\Bigg\{\sqrt{\frac{\pi}{\alpha}}e^{\beta^2/4\alpha}$$
$$\times \sum_{s=0}^{l-i}\frac{(2(l-i))!}{s!(2(l-i-s))!}\frac{(2\beta)^{2(l-i-s)}}{(4\alpha)^{2(l-i)-2}}\Bigg\}$$
$$\times \sum_{j=0}^{k-l}\frac{(2(k-l))!}{j!(2(k-l)-2j)!}\frac{(-2d_2)^{2(k-l-j)}}{(4c_2)^{2(k-l)-j}}p_L^{2(k-l-j)}\Bigg], \tag{9}$$

where $\mathcal{N}$ is a normalization constant and

$$\alpha = a^2e^{2r} + b^2 - \frac{d_1^2}{4c_1} + \frac{\chi^2}{2\bar{n}+1},$$
$$\beta = \frac{2\chi(p_M-\Omega)}{2\bar{n}+1}$$
$$\gamma(l,i) = \frac{(2l)!}{i!(2l-2i)!}\frac{(-2d_1)^{2(l-i)}}{(4c_1)^{2l-i}}. \tag{10}$$

The conditional mechanical Wigner function is plotted in Fig. 2 for input PSSV states with up to three photons substracted. As can be seen, the mechanical state progressively approaches that of a Schrödinger cat state for increasing $m$.

*Macroscopicity.* To evaluate the macroscopicity of the prepared mechanical state we use the measure proposed by Lee and Jeong [31] which when transformed to $(x,p)$ phase space takes the form

$$I(\rho) = -\frac{\pi}{2}\iint dxdp\, W(x,p)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial p^2}+2\right)W(x,p). \tag{11}$$

*Practical feasibility.* Target optomechanical system and considerations of laser amplitude fluctuation. Optical feedback cooling (bad-cavity limit)

*Conclusion.* Something...

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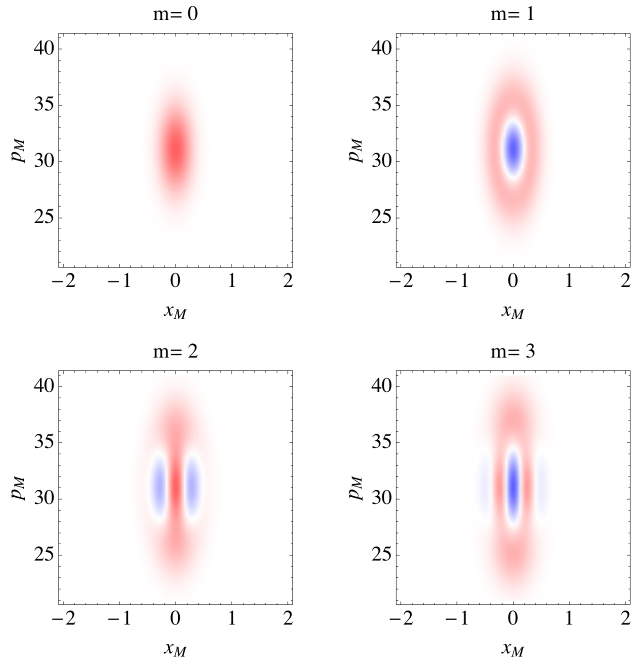


FIG. 2. Wigner function representation $W_M^{out}$ of the conditionally prepared mechanical states for optical PSSV input states with $m = 0$ to $m = 3$ photons subtracted for a phase squeezed vacuum state with $r = 1$. A tap-off beam splitter transmittivity of $T = 0.95$ is assumed. The initial mechanical mode is cooled to $\bar{n} = 0.1$ and the number of photons in the measurement pulse is $N_p = 500$. The optomechanical system is characterized by $g_0/\kappa = 1.1 \cdot 10^{-2}$.

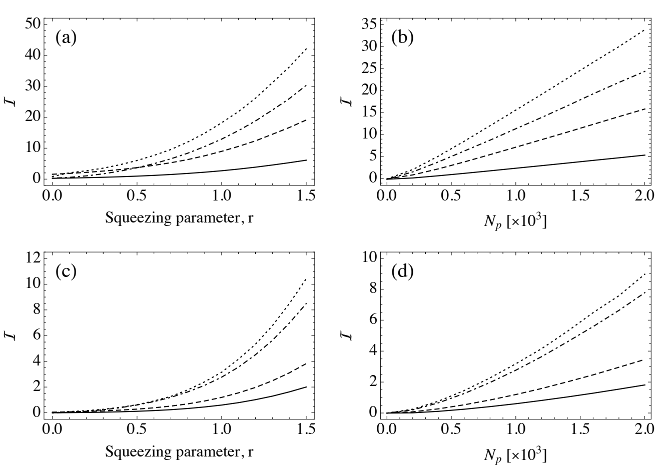


FIG. 3. Macroscopicity of prepared mechanical state as function of squeezing parameter $r$ and pulse photon number $N_p$, starting from a pre-cooled phonon occupancy of $\bar{n} = 0.1$ (subplots a, b) and $\bar{n} = 100$ (subplots c, d). Individual simulation curves correspond to input PSSV states with $m = 0$ (solid), $m = 1$ (dashed), $m = 2$ (dot-dashed), and $m = 3$ (dotted) photons subtracted. Simulations of $\mathcal{I}(r)$ where done for $N_p = 10^3$ while for $\mathcal{I}(N_p)$ a squeezing parameter of $r = 1$ was assumed.

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