1 preliminaries

Let Σ be a set of bead types, and Σ^* be the set of finite strings of beads. Let $w = a_1, ..., a_n$ be a string of length n for some integer n and bead types $a_1, ..., a_n \in \Sigma$. The *length* of w is expressed by $|w|$. For two indices i, j with $1 \leq i \leq j \leq n$, we let $w[i, j]$ refer to the substring $a_i a_{i+1} \cdots a_{j-1} a_j$. if $i = j$, then we express $w[i, i]$ by $w[i]$.

Oritatami systems operate on the hexagonal lattice. The *gridgraph* of lattice is the graph whose vertexes correspond to the lattice points and connected if the corresponding lattice points are at unit distance hexagonally. For a point p and a bead type $a \in \Sigma$, we call the pair (p,a) an *annotatedpoint*, or simply a *point* if being annotated is clear from context. Two annotated point $(p, a), (q, b)$ are *adjacent* if pq is an edge of the grid graph.

A *path* is a sequence $P = p_1 p_2 \cdots p_n$ of $pairwise{-}distinct$ points $p_1, p_2, \cdots, p_n$ such that $p_i p_{i+1}$ is at unit distance for all $1 \leq i \leq n$. Given a string $w \in \Sigma^*$ of bead types of length n, a *pathannotatedbyw*, or simply $w - path$, is asequence P_w of annotated points $(p_1, w[1]), \cdots, (p_n, w[n])$,

1.1 Oritatami System