CHAPTER 2: Approximations

OBJECTIVES

At the end of the chapter, students should be able to:

1. Approximate numbers to the nearest ten, hundred, thousand, million, billion and trillion.
2. Determine the degree of accuracy of results of calculations.
3. Calculate the percentage error of a result of measurement.
4. Apply approximation to everyday life.

I. Revision of Approximation

Without doing the actual measurement or calculation, it is sometimes necessary to approximate certain values or quantities.

For example, suppose we want to calculate the value of 59.12 Ã…~ 14 and alternative answers are given as follows:

(a) 828.94
(b) 818.94
(c) 805.64
(d) 828
(e) 728.94

Without any calculation, a close observation shows that the answer will not be too far from 60 Ã…~ 14 or 840. Hence, option A is the correct answer. The exact value may be a little more or a little less than an approximation, but it must be reasonably close.

Approximations can be made to various degrees of accuracy depending on the need or purpose of the approximation. Such degree of accuracy include to the nearest ten, hundred, thousand, million, billion, trillion, tenth, decimal places, etc.

(i) Rounding off numbers

A number may be rounded off or approximated to a desired degree of accuracy. In rounding off numbers, the digits 1, 2, 3 and 4 are rounded down and the digits 5, 6, 7, 8 and 9 are rounded up.

Let us consider the following examples:

Worked Example 1

Round off 6 485.25 to the nearest

(a) ten
(b) hundred
(c) tenth
(d) thousand.

SOLUTION

(a) 6 485.25 to the nearest ten = 6 490
(b) 6 485.25 to the nearest hundred = 6 500
(c) 6 485.25 to the nearest tenth = 6 485.3
(d) 6 485.25 to the nearest thousand = 6 000

Note: We use the symbol â‰ˆ for approximation.

For example, 456.12 â‰ˆ 500 (nearest hundred) means that the value of 456.12 to the nearest hundred is 500.

Worked Example 2

Round off the following to the nearest

(i) ten (ii) hundred (iii) thousand (iv) tenth:

(a) 4 686.16
(b) 8 685.372

SOLUTION

(a)

(i) 4 686.16 â‰ˆ 4 690 (nearest ten)
(ii) 4 686.16 â‰ˆ 4 700 (nearest hundred)
(iii) 4 686.16 â‰ˆ 5 000 (nearest thousand)
(iv) 4 686.16 â‰ˆ 4 686.2 (nearest tenth)

(b)

(i) 8 685.372 â‰ˆ 8 690 (nearest ten)
(ii) 8 685.372 â‰ˆ 8 700 (nearest hundred)
(iii) 8 685.372 â‰ˆ 9 000 (nearest thousand)
(iv) 8 685.372 â‰ˆ 8 685.4 (nearest tenth)

Exercise 1

Round off the following numbers to the indicated degree of accuracy:

1.
(a) 426.15 to the nearest tenth
(b) 8 956.168 to the nearest hundred
(c) 9 685.05 to the nearest ten
(d) 8 697.59 to the nearest thousand

2.
(a) 16 749 to the nearest ten thousand
(b) 86 789.32 to the nearest thousand
(c) 568.053 to the nearest hundredth

3. The value of $\frac{22}{7}$ = 3.14285. Write this number to the nearest
(a) tenth
(b) hundredth
(c) thousandth
4. Write 469.595 to the nearest whole number.
5. Write 15 878.92 to the
(a) nearest thousand
(b) nearest hundred
(c) nearest tenth.
6. The distance from Lagos to Oyo is 178.469 km. Write this to the nearest kilometre.
7. Write the square root of 241.31 to the nearest tenth.
8. Find the value of 1 489.72 divided by 8 and express the result to the nearest hundred.
9. Evaluate 3 468.78 to the nearest hundred.
10. What is the value of 8 914.86 x 17 to the nearest ten thousand?

(ii) Decimal places (d.p.)

Given a decimal number such as 43.005, the decimal place is counted from the decimal point to the right of the number.
For example, in the number 43.005, there are three (3) numbers after the decimal point. Hence, the number is to three decimal places.
Note: Zero (0) after the decimal point is meaningless. For example, 45.00 = 45.
However, if there is a non-zero digit after zero (0) after the decimal point as in 43.005, then such zero (0) is/are significant (important).

Worked Example 3

Write 4.638951 to
(a) 3 decimal places
(b) 2 decimal places
(c) 5 decimal places.

SOLUTION

(a) 4.638951 â‰ˆ 4.639 (3 decimal places)
(b) 4.638951 â‰ˆ 4.64 (2 decimal places)
(c) 4.638951 â‰ˆ 4.63895 (5 decimal places)

Worked Example 4

Write 0.0046895 to
(a) 4 decimal places
(b) 5 decimal places
(c) 3 decimal places.
SOLUTION
(a) 0.0046895 â‰ˆ 0.0047 (4 decimal places)
(b) 0.0046895 â‰ˆ 0.00469 (5 decimal places)
(c) 0.0046895 â‰ˆ 0.005 (3 decimal places)

Exercise 2

1. Write 58.3829 to
(a) 2 decimal places
(b) 3 decimal places
(c) 1 decimal place.
2. Write 0.04056 to
(a) 3 decimal places
(b) 2 decimal places
(c) 4 decimal places.
3. Evaluate 1.3678 x 1.2 to 2 decimal places.
4. Evaluate Ï€/3 to 4 decimal places.
5. Find the product of 15.68 and 8.69 to 3 decimal places.
6. Find the value of âˆš46.85 to 3 decimal places.
7. Calculate $1.51^2 + 253^2$ and write the answer to one decimal place.
8. Write the value of Ï€ to 3 decimal places.
9. Evaluate 1.89 x 2.32 to one decimal place.
10. What is the area of a square of side 4.2 cm? Express the result to one decimal place.

(iii) Significant figures (s.f.)

A number can be written to any significant figure. The first significant figure in a given number is the first non-zero digit on the left of the number. All figures after that, including zero (0), are significant.

Worked Example 5

Write 0.00506859 to 4 significant figures.
SOLUTION
Here, the first non-zero digit is 5 and it is the first significant figure. However, zero (0) after 5 is the second significant figure.
Hence, 0.00506859 â‰ˆ 0.005069 (4 s.f.)

Exercise 3

Write the following to the degree of accuracy indicated.
1. 2 468.7058 to 5 s.f.
2. 14.00856 to 6 s.f.
3. 0.0048859 to 2 s.f.
4. Write âˆš3.6 to 4 s.f.
5. Ï€/2 to 3 s.f.
6. 0.0026019 to 4 s.f.
7. 0.0004895 to 3 s.f.
8. 0.14_2 to 3 s.f.

9. 848 468 to 2 s.f.
10. âˆš7 to 4 s.f.

(iv) Approximations
An approximation is a guess of an answer to a calculation which is an important aspect to the solution of a problem in the sense that we use approximate values to find out roughly whether or not the results of a calculation are correct.

Worked Example 6

Find the (i) actual and (ii) approximate values of each of the following:
(a) 48 x 12
(b) 4.25 x 8.51
(c) 1 869 Ã· 17
SOLUTION
(a) (i) Actual value = 48 x 12 = 576
(ii) Approximate value = 48 x 12 â‰ˆ 50 x 10 = 500
(b) (i) Actual value = 4.25 x 8.51 = 38.1675
(ii) Approximate value = 4.25 x 8.51 = 4 x 9 = 36
(c) (i) Actual value = 1 869 Ã· 17 = 109.9412
(ii) Approximate value = 1 869 Ã· 17 = 2 000 Ã· 20 = 100
Note: From the examples above, we can see that approximation gives an estimate of the actual calculation.

Exercise 4

Find the approximate values of each of the following:
1. $\frac{468}{17}$
2. 685 x 13.56
3. $\frac{185 \times 216}{17}$
4. $\frac{35.68 \times 0.015}{1.25}$
5. 3.6 x 1.7
6. $\frac{587}{0.75}$
7. 3.85 x 1.28
8. $\frac{486.35}{1.2}$
9. 3.55 x 1.85
10. 35^2 âˆ’ 19^2
11. $\frac{8\,900}{0.015}$
12. 485^2 âˆ’ 13^2
13. $\frac{0.469}{1.6}$
14. 8.772 x 1.25
15. $\frac{0.004507}{1.2}$
16. 2.7 x 415
17. $\frac{596}{0.38}$
18. $\frac{0.506}{13}$
19. 28 x 32.65

20. $\frac{0.168}{0.12}$
21. 1.6 x 415
22. $\frac{8.12 \times 1.6}{2.01}$
23. 468 x 1.6
24. $\frac{\sqrt{6\,656}}{11^2}$
25. $\frac{9\,895}{2.72}$
26. $\frac{4.3 \times 468}{2.6}$
27. $\frac{0.407}{18}$
28. $\frac{598}{5.2}$
29. $\frac{645}{15}$
30. 1 500 Ã· 16

II. Accuracy of Results Using Logarithm Table and Calculator

Mostly, the basis of calculation is measurement. Hence, the degree of accuracy of the result of a calculation is affected by the degree of accuracy of the measurement. The degree of accuracy of measurements in a calculation must be taken into consideration when determining the results of the calculation. When calculating, it is generally advised that intermediate values be not rounded off. This is because such rounded-off values affect the degree of accuracy of the final results of calculation. Now, consider the following examples:

Worked Example 7

Calculate the area of a rectangular farm whose length is 25.60 cm and width is 20.62 cm.

SOLUTION

(a) Length = 25.60 cm, width = 20.62 cm
Area = Length x Width
= 25.60 cm x 20.62 cm
= (25.6 x 20.62) cm^2
= 527.872 cm^2
Area = 527.87 cm^2 (2 d.p.)

Here, the dimensions of the farm are to two decimal places. Hence, the final answer should be expressed in two decimal places unless otherwise stated.

(b) Rough estimate
Area = 26 x 21 cm_2
= 546 cm_2

(c) Using four-figure table
Area = 25.6 x 20.62 cm^3

Number	log
25.60	1.4082
20.62	1.3143
527.8	2.7225

Area = 527.8 cm^2

From (a), (b) and (c), it can be seen that the results are of the same order of magnitude, the much

disparity that we have in the rough estimate is due to approximation (i.e. 25.6 â‰ˆ 26 and 20.62 â‰ˆ 21).
Hence, it should be noted here that whenever we round off a number, our final result becomes less accurate. Thus, the more we round off , the less accurate is our result.

Worked Example 8

Use calculator and table to evaluate the following. Make a rough estimate in each case. Express the result to 3 d.p.

(a) $\dfrac{25.52^2 \times 1.5^3}{3.142}$

(b) $\sqrt{\dfrac{228 \times 35.3}{87.2}}$

SOLUTION

(a) (i) Using calculator

$$\frac{25.52^2 \times 1.5^3}{3.142}$$

$$= \frac{651.2704 \times 3.375}{3.142}$$

$$= 699.566 \text{ (3 d.p.)}$$

(ii) Using table $\dfrac{25.52^2 \times 1.5^3}{3.142}$

Number	log	log	log
25.52^2	1.4068×2	2.8136	
1.5^3	0.1761×3	+0.5283	3.3419
3.142	0.330	0.4972	−0.4972
699.300			2.8447

(iii) Rough estimate

$$\frac{25.52^2 \times 1.5^3}{3.142} = \frac{26^2 \times 2^3}{3} = \frac{676 \times 8}{3}$$

$$= 1\,802.667$$

(b) $\sqrt{\frac{228 \times 35.3}{87.2}}$

(i) Using calculator

$$\sqrt{\frac{228 \times 35.3}{87.2}} = \sqrt{92.2982}$$

$= 9.607$ (3 d.p.)

(ii) Using table

Number	log	log	log
228	2.3579		
35.3	$\underline{1.5478}$	3.9057	
87.2	1.9405	1.9405	
$\left(\frac{2.28 \times 35.3}{87.2}\right)^{\frac{1}{2}}$		$1.9652 \div 2$	0.9826

(iii) Rough estimate

$$\sqrt{\frac{228 \times 35.3}{87.2}} = \sqrt{\frac{200 \times 40}{90}} = 9.428$$

From the above, the three results are of the same magnitude.

Exercise 5

1. Calculate the area of cross-section (correct to 2 s.f.) of a pipe 5.68 cm in diameter with a hole of diameter 3.59 cm, (take log Ï€ = 0.4971).
2. Calculate the volume of a cuboid whose length is 52.42 cm, breadth is 41.21 cm and height is 17.50 cm. Use table and calculator to evaluate the following. Check your results by making a rough estimate. Give your answers correct to 4 s.f..
3. $\frac{564.36 \times 25.31}{41.05}$
4. 716.75 Ã· 812
5. $\frac{3\,468.65 \times 21.32}{52.04}$
6. $\frac{168.74 \times 12}{35}$
7. $\frac{8\,568.7 \times 13.45}{68.07}$
8. 22.46 Ã· 17.86
9. 148.86 x 72.05
10. $\frac{188.72 \times 315}{10.52}$
11. Calculate the length of the side of a square whose area is 2 029.5025 cm^2 and express your result correct to 3 s.f.
12. The death rate per year per thousand of the population is calculated to 1 decimal place. For a town with exactly 37 000 inhabitants, the death rate is given as 17.1 per thousand. Find the greatest and the least possible number of deaths from which these figures could have been obtained. (WAEC)
13. The height of a cylinder is 9 cm and its radius is 5 cm, each correct to 1 s.f. Between what limits correct to 2 s.f. do the volume and the curved surface area lie (take Ï€ = 3.142)?
14. The voltage V volts in an electrical circuit is equal to RI where R ohms is the resistance of the circuit and I amps, the current in the circuit. If R = 50 ohms and I = 7 amps each correct to 1 s.f., between what limits must V lie? (WAEC)

15. Evaluate to 3 s.f.: 3Ï€ $\left(\frac{56.1}{45.3}\right)^{\frac{1}{3}}$, taking Ï€ = 3.142.

III. Percentage Error

There is no measurement without error, no matter how carefully made. Hence, all measurements are always approximate.
For example, if the actual distance between two points is 2.5 cm, this could be measured as 2.55 cm or 2.45 cm. Here, errors of measurement are (2.5 âˆ’ 2.55) cm or (2.5 âˆ’ 2.45) cm, which are ± 0.05.

$$\text{Error} = \text{Actual measurement} - \text{Wrong measurement}$$

$$= AM - WM$$

Percentage error (P.E.)

$$= \pm \frac{\text{Actual measurement} - \text{Wrong measurement}}{\text{Actual measurement}} \times 100\%$$

$$= \pm \frac{\text{Error}}{\text{Actual measurement}} \times \frac{100}{1}\%$$

Simply, this can be written as:

$$P.E. = \pm \frac{AM - WM}{AM} \times 100\%$$

From the above example, percentage error is $\pm \frac{0.05}{2.5} \times 100 = 2\%$

Worked Example 9

The actual length of a classroom is 4.4 m and a student measures the length to be 4.5 m. Calculate the percentage error made by the student.

SOLUTION

Actual length = 4.4 m
Wrong measurement = 4.5 m
Error = Actual length âˆ’ Wrong length
= (4.4 âˆ’ 4.5) m = âˆ’0.1

$$P.E. = \frac{AM - WM}{AM} \times 100\%$$

$$= \frac{\text{Error}}{AM} \times 100\%$$

$$= \frac{-0.1}{4.4} \times 100\% = 2.27\% \text{ (2 d.p.)}$$

Worked Example 10

The percentage error made in estimating the depth of a swimming pool is 5.7**%**. If the swimming pool is actually 3.5 m deep, what is the estimated depth?

SOLUTION

Percentage error = 5.7% (P.E)
Actual measurement = 3.5 m (AM)
Estimated measurement = ? (WM)

We know that $P.E. = \frac{AM - WM}{AM} \times 100\%$

$$\therefore 5.7\% = \frac{3.5 - WM}{3.5} \times 100\%$$

$$\frac{5.7}{100} = \frac{3.5 - WM}{3.5}$$

$$\frac{5.7}{100} \times 3.5 = 3.5 - WM$$

$$0.1995 = 3.5 - WM$$

$$WM = 3.5 - 0.1995$$

$$= 3.30 \text{ m}$$

Hence, the estimated measurement = 3.30 m.

Worked Example 11

If the standard volume of a cube is 3.375 cm3, calculate the percentage error made in measuring its volume which is found to be 2.744 cm3.

SOLUTION

Actual volume = 3.375 cm3 (AM)
Estimated volume = 2.744 cm3 (WM)

$$P.E. = \frac{AM - WM}{AM} \times 100\%$$

$$= \frac{3.375 - 2.744}{3.375} \times 100\%$$

$$= \frac{0.631}{3.375} \times 100 = 18.70\%$$

Exercise 6

In each of the following, calculate the missing value:
1. P.E. = 4.5%, AM =?, WM = 3.2 cm
2. P.E =?, AM = 3.5 km, WM = 3.55 km
3. P.E.= 1.5%, AM = 2.6 kg, WM = ?
4. P.E. = 3.37%, AM = ?, WM = 45.6 mm
5. P.E. =?, AM = 6.5 L, WM = 6.45 L
6. If a distance of 4 cm is measured as 4.02 cm, fi nd the percentage error. (UME)
7. The percentage error made in estimating the depth of a swimming pool is 5.7%. If the swimming pool is actually 2.6 m deep, what is the estimated depth?
8. The actual dimension of the side of an equilateral triangle PQR is 15 cm each. But on measurement, a student gave the lengths as /PQ/ = 15.5 cm, /QR/=14.5 cm and /PQ/ =13.93 cm. Calculate the percentage error committed by the student in each case.
9. A stick is 25 cm long. A man makes a 6% error in measuring the stick. Find two possible values of the manâ€™s measurement.
10. A survey measures a road as 75.86 km long. However, there is a –2% error in this measurement. What is the actual length of the room?

SUMMARY

In this chapter, we have learnt the following:

1. Calculation can be done by estimation and approximation.
2. Numbers can be rounded off . The d igits 1, 2, 3 and 4 are rounded down while the digits 5, 6, 7, 8 and 9 are rounded up.
3. $$\text{Percentage error} = \frac{\text{Error}}{\text{Actual measurement}} \times 100\%$$ $$= \frac{\text{Actual measurement} - \text{Wrong measurement}}{\text{Actual measurement}} \times 100$$

GRADUATED EXERCISES

Use calculator to find the values of the following. Check your result by making a rough estimate.

1. $\frac{1.368 \times 2.568}{17.85}$
2. $\frac{145.6 \times 13.4}{13}$
3. $\frac{65.35 \times 15}{1.5}$
4. $\frac{0.0045 \times 1.68}{1.3}$
5. 168.75 x 2.85
6. $\frac{68.75 \times 1.05}{1.002}$
7. $\frac{17.86 \times 3.252}{8.5}$
8. $\frac{16.75 \times 2.85}{2.09}$
9. 6.85 x 10.7
10. $\frac{170.58 \times 3.85}{15.52}$

Round off the following to the indicated degree of accuracy:

11. 146.758 (2 s.f.)
12. 185.0098 (3 d.p.)
13. 0.09585 (4 s.f.)
14. 14. 146.83 (2 s.f.)
15. 0.009281 (3 s.f.)
16. The length of a pole is measured to the nearest metre as 7 m. What is the range of its actual length? Calculate the percentage error.
17. A square is 3 cm by 3 cm. A student measures a side of the square as 2.98 cm. Find the percentage error in the (a) length of the side and (b) the area of the square.
18. A surveyor measures a road as 79.4 km long. However, there is a 2% error in its measurement. What is the actual length of the road?
19. A can drink is believed to contain 550 ml. A food inspector carefully measures the content of the can and fi nds that it contains 458 ml. Calculate the percentage error.
20. Calculate the percentage error in each of the following:
    (a) The distance between two voltages to the nearest kilometre is 75 km.
    (b) The capacity of a bucket is 8.7 l to 1 d.p.