CHAPTER 10 Permutation and Combination

This topic presents possible ways that a group of things (living or nonliving) can be organized

Permutation

One can view permutation as a careful young man who cares **so much** about details. If he is told that four students led the class, he will want to know the **order** in which the positions are taken. Moreover, he tries to look at all the possible arrangements of the first four students in the first, second, third and fourth positions. Also, imagine that Mr Permutation names the four students as student 1, student 2, student 3 and student 4. Find below the list of his possible arrangements of students 1, 2, 3 and 4.

1234; 1243; 1324; 1342; 1423; 1432; 2134; 2143; 2314; 2341; 2431; 2413; 3124; 3142; 3214; 3241; 3412; 3421; 4123; 4132; 4213; 4231; 4312 and 4321.

In all, the possible number of arrangements of students 1, 2, 3 and 4 is 24 arrangements. However, imagine how many sheets one will need to write the possible arrangements for 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. For this reason, the knowledge of permutation is inevitable in solving problems like this. Instead of having to list out all the possible arrangements of 1, 2, 3 and 4, as listed above, one can use the permutation method of arranging 4 different things, and this is expressed as ${}^4P_4$ (Pronounced as 4 permutation 4). Evaluating ${}^4P_4$ will give the number of ways of arranging the four numbers, using all the four numbers in all the arrangements.

$6! = 6 \times 5! = 6 \times 5 \times 4! = 6 \times 5 \times 4 \times 3!$ *and so on; thus,* $n! = n(n-1)! = n(n-1)(n-2)!$ *and so on. Also note that* $0! =$ *and* $1! = 1$

$${}^nP_r = \frac{n!}{(n-r)!}; \text{ therefore, } {}^4P_4 = \frac{4!}{(4-4)!}$$

$$= \frac{4 \times 3 \times 2 \times 1}{0!} = \frac{4 \times 3 \times 2 \times 1}{1} = 24.$$

Combination

Unlike Mr. Permutation, Mr. Combination is a care free person; all he cares to know is that four students led the class; he is not interested in who comes first, or who has the fourth position. Mr. Combination sees all the 24 meticulous arrangements of Mr. Permutation as just a single selection, since it's still the same set of 4 students that are being arranged and rearranged. Therefore combination organizes things by selecting them, and one will observe that selecting students 1 2 3 4 and selecting students 4 3 2 1 is still the same one selection (and **not** two selections). Since it is the same set of 4 students that are still being selected. Therefore, all the 24 arrangements using permutation is viewed as one selection by combination, because it is the same one set of 4 students that is being arranged and rearranged over and over.

The number of ways of selecting (choosing) n things, picking r things out of the total n at a time is expressed as ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$. Hence for this reason, the number of ways of selecting 4 students, picking the 4 at a time is ${}^{4}C_{4} = \frac{4!}{(4-4)!4!}$ $= \frac{4!}{0!4!} = \frac{4!}{14!} = \frac{4!}{4!} = 1.$

Permutation (arrangement) of people (or things) in a row or line

The number of possible arrangements of 8 people in a row is the number of ways of arranging 8 people, picking the whole 8 for every arrangement =

$${}^{8}P_{8} \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 8!!$$

And so, the number of ways of arranging 10 people in a row is expressed as $n!$.

Permutation (arrangement) of people (or things) around a circular table

If 10 people are to be seated around a circular table, one of them has to sit down first before the other 9 can be arranged around the first person serving as a reference point. This is because unlike the arrangements in a row, a round table does **not** have a point (or position) that can be called the beginning (or head) of the table; thus, a reference point has to be established before others are arranged about this reference point. Therefore, the number of possible arrangements of 10 people around a circular table is

$${}^{9}P_{9} = \frac{9!}{(9-9)!} = \frac{9!}{0!} = \frac{9!}{1} = 9!..$$

Thus, in other words, the number of possible arrangements of 10 people around a circular table is expressed as $(10 - 1)! = 9!$. For this reason, as a general rule, the number of ways of arranging n people around a circular table is $(n - 1)!$.

There are truck loads of problems to solve with these tools, you can get to work then.

1. Given that ${}^{n}C_{6} = {}^{n+1}C_{5}$, find the value of n.
(WAEC)

Workshop

${}^{n}C_{6} = \frac{n!}{(n-6)!6!}$; ${}^{n+1}C_{5} = \frac{(n+1)!}{((n+1)-5)!\,5!}$.

${}^{n}C_{6} = {}^{n+1}C$. Therefore,

$$\frac{n!}{(n-6)!6!} = \frac{(n+1)!}{((n+1)-5)!5!} \quad \frac{(n+1)!}{(n+1-5)!5!};$$

$$\frac{n!}{(n-6)!6!} = \frac{(n+1)!}{(n-4)!5!}.$$

Recall that, $6! = 6 \times 5! = 6 \times 5 \times 4! = 6 \times 5 \times 4 \times 3!$ *and so on*; thus, $n! = n(n-1)! = n(n-2)!$ *and son no.*

Hence, $(n-4)! = (n-4)(n-5)! = (n-5)(n-6)!$, and

$(n+1)! = (n+1)\ (n+1)-1)! = (n+1)(n+1-1)! = (n+1)n!$.

Therefore, $\frac{n!}{(n-6)!6 \times 5!} = \frac{(n+1)n!}{(n-4)(n-5)(n-6)!5!}$.

Multiply both sides of the equation by $\frac{(n-6)!5!}{n!}$ to get

$$\frac{\cancel{n!}}{\cancel{(n-6)!}6 \times \cancel{5!}} \times \frac{\cancel{(n-6)!5!}}{\cancel{n!}} = \frac{(n+1)\ \cancel{n!}}{(n-4)(n-5)\cancel{(n-6)!5!}} \times \frac{\cancel{(n-6)!5!}}{\cancel{n!}};$$

$\frac{1}{6} = \frac{n+1}{(n-4)(n-5)}$; cross-multiplying, we get

$(n-4)(n-5) = (n+1)6$; $n^2 - 9n + 20 = 6n + 6$; $n^2 - 15n + 14 = 0$;

$n^2 - 14n - n + 14 = 0$; $n(n-14) - 1\ (n-14) = 0$; $(n-14)(n-1) = 0$;

$(n-14) = 0$ *or* $(n-1) = 0$; $n = 14$ or $n = 1$.

Since n cannot be less than 6, (i.e $^1C_6$ cannot be evaluated), then, $n = 14$.

2. How many seven-letter words starting with the letter T, can be formed from the word FURTHER? **(WAEC)**

Workshop

As we are to form seven-letter words in which letter T must always be the first letter from FURTHER, having in mind that FURTHER is a seven letter word, we will have to arrange the other 6 letters at the back of letter T. So we are to arrange the other 6 letters at the back of T picking the 6 letters at a time, and the number of ways of doing this is given by $^6P_6 = 720$*ways.*

However, remember that the word FURTHER contains 2 R's, and in any of the arrangement of the 6 letters at the back of T, the 2 R's **cannot** be distinguished. For example, arrangement T U R_1 F H E R_2 will be the same as T U R_2 F H E R_1 because we cannot distinguish between R_1 and R_2. This means that the 720 ways calculated from $^6P_6$ contains **a pair each or the same word** as explained. Therefore, the number of ways the other 6 letters can be arranged behind T to form

seven letter words will be $\frac{^6P_6}{2} = \frac{6!}{(6-6)!} \div 2 = \frac{6!}{0!} \div 2 = \frac{6!}{2} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2} = 360$ *ways.*

Therefore, there are 360 ways in which the arrangement can be made. This will give rise to 360 *seven-letter words* that start with the letter T, which can be formed from the word FURTHER.

*Note that as ER is not the same as RE, FU is not the same as UF, and so on, we **cannot** use combination for this question. So, permutation was used because the order of arrangement is important. Also, note that, the number of ways n letters, having y identical letters, can be arranged, picking the n letters at a time is $\frac{^nP_n}{y!}$. For example, the number of ways that ABCBAAH can be arranged, picking all the letters at a time, is $\frac{^7P_7}{2!3!}$. We had to divide $^7P_7$ by 2! because there are two Bs in the word, and we also divided $^7P_7$ by 3! because there are three As in the word.*

3. In an election, there are 3 candidates for the post of the chairman and 4 candidates for the post of secretary. An elector is to cast his vote for one candidate only in each of the two posts. In how many ways can he choose his candidates? **(WAEC)**

Workshop

The elector could cast his vote for any one of the three candidates vying for the post of the chairman. For this reason, the number of ways he could choose a candidate for the post of chairman is the number of possible ways of choosing 1 person out of 3 people, which is given by

$$^3C_1 = \frac{3!}{(3-1)!1!} = \frac{3!}{2!} = 3 \text{ ways.}$$

*Note that we use combination because the **order of arrangement** is **not** important; therefore, permutation is not applicable.*

Also, the number of ways of voting for 1 out of 4 contestants for the post of secretary will be

$$^4C_1 = \frac{4!}{(4-1)!1!} = \frac{4!}{3!} = 4 \text{ ways.}$$

For 1 way of selecting a chairman, there are 4 possible ways of selecting a chairman and secretary by an elector. For example, let the contestants for the post of the chairman be C_1, C_2 and C_3, while the contestants for the post of the secretary be S_1, $S2$, $S3$ and $S4$. If an elector chooses $C1$ for the post of the chairman (which is one way of selecting the chairman), he can select any one of $S1$, $S2$, $S3$ or $S4$, to go along with $C1$, hence, the 4 possible ways of selecting a chairman and a secretary will be $\langle C1\ S1\rangle$, $\langle C1\ S2\rangle$, $\langle C1\ S3\rangle$ and $\langle C1\ S4\rangle$. It can now be seen that for 1 way of selecting a chairman, there are 4 possible ways of selecting a chairman and secretary by an elector.

Thus, for 2 ways of selecting a chairman, there will be 4 + 4 = 4 × 2 = 8 possible ways of selecting a chairman and secretary by an elector. Let us try another 1 way of selecting the chairman. But now let us assume that the elector voted for $C2$, then for one way of selecting $C2$, there will be 4 possible ways of selecting $C2$ and any one of $S1$, $S2$, $S3$, $S4$ and these 4 possible selections are $\langle C2\ S1\rangle$, $\langle C2\ S2\rangle$, $\langle C2\ S3\rangle$ and

$\langle C2\ S4\rangle$. Then, considering $\langle C1\ S1\rangle$, $\langle C1\ S2\rangle$, $\langle C1\ S3\rangle$, $\langle C1\ S4\rangle$ and $\langle C2\ S1\rangle$, $\langle C2\ S2\rangle$, $\langle C2\ S3\rangle$, $\langle C2\ S4\rangle$, you can see that for 2 ways of voting for the chairman, (that is voting for $C1$ or $C2$), there are $4 + 4 = 4 \times 2$ possible ways of voting for the chairman and the secretary Then for 3 ways of voting for the chairman, there will be $4 + 4 + 4 = 4 \times 3 = 12$ possible ways of voting for the chairman and the secretary A shorter method can be applied as follows;

$3C1 \times 4C1 = 3 \times 4 = 12.$

Therefore, the elector can choose his candidates in 12 possible ways.

Note that this answer does not mean that the elector casted twelve votes, the elector can only cast a vote each for just one candidate for each of the post. However, the question is asking for the possible number of ways that the elector can cast his vote.

4. Three boys and four girls are to be seated in a row. How many ways can this be done if one of the boys is to sit in the middle? **(WAEC)**

Workshop

The total number of people needed to be seated is $3 + 4 = 7$ people. If one of the boys is to sit in the middle, then, one of the boys had occupied one of the available 7 spaces, so we are left $7 - 1 = 6$ with spaces to arrange $3 - 1 = 2$ boys (*because a boy is already seated*) and 4 girls; this means that we have 6 spaces to place (arrange) 6 students in. The number of ways they can be seated, is the number of ways they can be arranged, and arrangement implies permutation. Hence, the number of ways of arranging 6 students in 6 spaces is given by

$$^6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! = 720 \text{ ways.}$$

Note that we used permutation instead of combination because the order of arrangement is important. For example, the order of arrangement, AB, of boy A and boy B, is different from arrangement BA, so combination is not applicable.

5. Ten different books are to be arranged in a row on a bookshelf which can take only seven books. If two of the books are to occupy the ends, how many ways can the books be arranged? **(WAEC)**

Workshop

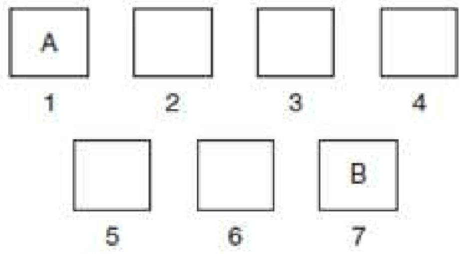

Fig. 10.1

If two books are to occupy the ends of the bookshelf, as represented by the boxes in Fig 10.1, the spaces left can only contain five books; therefore, we are left to arrange (10 – 2) books (8 books) in the (7 – 2) spaces (5 spaces) left. The number of ways of **arranging** 8 books in 5 available spaces is given by

$$^{8}P_{5} = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8\times7\times6\times5\times4\times3!}{3!}$$

$$= 8\times7\times6\times5\times4 = 6{,}720 \text{ ways.}$$

Note that we did not apply the combination formula because the ***order of arrangement*** *of the books is important, since the question demanded that we arrange the books. Also, recall that ABC is different from BAC, so permutation is appropriate. If we had been asked to find the number of ways 5 books can be selected out of 8 books, the solution to this would have been* $^{8}C_{5}$*, since the order of arrangement would not be necessary. This is because selecting ABC and BAC, is one and the same selection while arrangement ABC is different from arrangement BAC.*

6. If $^{16}C_{r} = {}^{16}C_{r+2}$, find r. (WAEC)

Workshop

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}\,;\ ^{16}C_{r} = \frac{16!}{(16-r)!r!};$$

$$^{16}C_{r+12} = \frac{16!}{(16-(r+2)\,!\,(r+2)!)};$$

$$\text{So } ^{16}C_{r} = \frac{16!}{(16-r)!r!} = {}^{16}C_{r+2} = \frac{16!}{(16-r-2)!(r+2)!}$$

$$\frac{16!}{(16-r)!r!} = \frac{16!}{(16-r-2)!(r+2)!}$$

Recall that $6! = 6\ (6-1)! = 6 \times 5! = 6(6-1)$ $(6-2)! = 6 \times 5 \times 4! = 6(6-1)(6-2)(6-3)! =$ $6 \times 5 \times 4 \times 3!$ and so on.

For this reason, $n! = n(n-1)! = n(n-1)(n-2)!$ *and so on.*

And so, $(16-r)! = (16-r)(16-r-1)\,(16-r-2)!$ and $(r+2)! = (r+2)(r+2-1)\ (r+2-2)! =$ $(r+2)(r+1)r!$; therefore,

$$\frac{16!}{(16-r)(16-r-1)(16-r-2)!r!}$$

$$= \frac{16!}{(16-r-2)!(r+2)(r+1)r!}$$

Multiply the equation through, by

$\dfrac{(16-r-2)!r!}{16!}$ to get

$$\frac{16!}{(16-r)(16-r-1)(16-r-2)!r!} \times \frac{(16-r-2)!r!}{16!}$$

$$= \frac{\cancel{16!}}{(\cancel{16-r-2})!(r+2)(r+1)\cancel{r!}} \times \frac{(\cancel{16-r-2})!\cancel{r!}}{\cancel{16!}}$$

7. Three-digit numbers are to be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. If repetition of digits is not allowed, how many

   (a) of such numbers can be formed?

   (b) are divisible by 5?

   (c) are odd? **(WAEC)**

Workshop

Before solving this problem, you should remember that, 123, 132, 231 etc, are not the same. So, the number of three-digit numbers that can be formed from numbers 1 to 9 will be calculated by permutation, since the order of **arrangement** is **important**.

(a) The number of three-digit numbers that can be formed from 1, 2, 3,. . .,9, without repeating digits, can be calculated by picking three digits to be arranged at a time from the 9 digits in question, which is calculated as

$$^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!}$$

$= 504$ *three-digit numbers.*

(b) Numbers divisible by 5, end with either digit zero or 5. Since zero is not in the given set of digits, in the question, then, the 3-digit numbers to be formed, must end with 5 for it to be divisible by 5. Because the three-digit number must end with 5, then, the position of the last digit is already occupied by 5, as explained by Figure 10.2.

Fig. 10.2

From Figure 10.2, 5 must occupy the last box, for the three-digit number to be divisible by 5; so, we are left to arrange the remaining 8 digits, taking 2 at a time, into the two boxes left, without repeating digits. The number of 2-digit numbers that can be arranged into the two boxes, from 8 available numbers, without repeating digits is given by

$${}^{8}P_{2} = \frac{8!}{(8-2)!} = \frac{8 \times 7 \times 6}{6!} = 56$$

three digit numbers.

Therefore, the number of 3 digit numbers, divisible by 5, that can be formed, if repetition of digits is not allowed, is 56.

(b) Recall that, for a number to be an odd number; its last digit must be an odd number. For example; 9 731, 2 883, 4 689 are all odd numbers because they end with an odd digit, while 97134 is an even number, since its last digit is even. From the given digits 1, 2, 3 . . . 9, the odd three-digit numbers, that can be formed, will end with 1, 3, 5, 7 or 9. The number of 3-digit numbers, ending with digit 1 that can be formed is given by $8P2$ below.

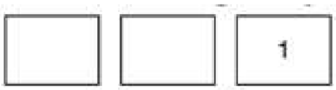

Fig. 10.3

Since 1 must be the last digit of the three-digit number (Figure 10.3), then, we are left to arrange the remaining 8 digits, taking 2 digits at a time, into the two empty boxes. Hence, ${}^{8}P_{2} = \frac{8!}{(8-2)!} = 56$ *three digit numbers.*

By the same argument, the number of three-digit numbers that can be formed with 3, 5, 7 or 9 as the last digit, are 56, 56, 56 and 56 respectively. So, the number of ways of forming odd 3-digit numbers from 1, 2, 3 . . . 9 = 56 + 56 + 56 + 56 + 56 = 280. Therefore, the number of odd three-digit numbers that can be formed, without repetition of digits is 280.

8. If ${}^{8}C_{n} : {}^{6}C_{n-1} = 56 : 15$, find n. (WAEC)

Workshop

$${}^{8}C_{n} : {}^{6}C_{n-1} = 56 : 15; \frac{8!}{(8-n)!n!}$$

$$\div \frac{6!}{(6-[n-1])!(n-1)!} = 56 \div 15;$$

$$\frac{8!}{(8-n)!n!} \times \frac{(6-n+1)!(n-1)!}{6!} = \frac{56}{15};$$

$$\frac{8!}{(8-n)!n!} \times \frac{(7-n)!(n-1)!}{6!} = \frac{56}{15}$$

Recall that, $6! = 6 \times 5! = 6 \times 5 \times 4! = 6 \times 5 \times 4 \times 3!$ and so on, thus, $n! = n(n-1)! = n(n-1)(n-2)!$ and so on; therefore, $n! = n(n-1)!$.

Also, $(8-n)! = (8-n)([8-n]-1)! = (8-n)(8-n-1)! = (8-n)(7-n)!$.

For these reasons, the equation we are solving can be rewritten as

$$\frac{8!}{(8-n)(7-n)!n(n-1)!} \times \frac{(7-n)!(n-1)!}{6!} = \frac{56}{15};$$

$$\frac{8 \times 7 \times \cancel{6!}}{(8-n)\cancel{(7-n)!}n\cancel{(n-1)!}} \times \frac{\cancel{(7-n)!}\cancel{(n-1)!}}{\cancel{6!}} = \frac{56}{15};$$

$$\frac{56}{(8-n)n} = \frac{56}{15};\ 56(8-n)n = 15 \times 56;$$

$$(8-n)n = 15;\ 8n - n^2 = 15;$$

$$-n^2 + 8n - 15 = 0; -n^2 + 5n + 3n - 15 = 0;$$

$$-n(n-5) + 3(n-5) = 0;\ (n-5)(-n+3) = 0;$$

$$(n-5) = 0 \text{ or } (-n+3) = 0;$$

$$n = 5 \text{ or } n = 3.$$

Therefore, the values of *n*, satisfying the equation in question, are 3 and 5.

9. In how many ways can a delegation of at least 7 members be selected from 10 officials? **(WAEC)**

Workshop

The phrase "**at least** 7 out of 10" means 7 out of 10 or 8 out of 10 or 9 out of 10 or 10 out of 10, but the least is 7 out of 10.

Let *E* be an event that a delegation of **at least** *7* members will be selected from 10 officials. The following are the possible subsets of event *E*: *A*, a delegation of 7 members will be selected from the 10 officials,

B, a delegation of 8 members will be selected from the 10 officials,

C, a delegation of 9 members will be selected from the 10 officials,

D, a delegation of 10 members will be selected from the 10 officials.

Therefore,

$$\begin{pmatrix} \textit{Number of ways} \\ \textit{of selecting a} \\ \textit{delegation of } \textbf{at} \\ \textbf{least } \textit{7 members} \\ \textit{out of 10 officials} \end{pmatrix} = \begin{pmatrix} \textit{Number of} \\ \textit{ways of} \\ \textit{selecting 7} \\ \textit{delegates out} \\ \textit{of 10 officials} \end{pmatrix} + \begin{pmatrix} \textit{Number} \\ \textit{of ways of} \\ \textit{selecting 8} \\ \textit{delegates out} \\ \textit{of 10 officials} \end{pmatrix}$$

$$+ \begin{pmatrix} \textit{Number of ways} \\ \textit{of selecting 9} \\ \textit{delegates out} \\ \textit{of 10 officials} \end{pmatrix} + \begin{pmatrix} \textit{Number of ways} \\ \textit{of selecting 10} \\ \textit{delegates out of} \\ \textit{10 officials} \end{pmatrix}.$$

From the question, we are not interested in the order in which the members of the delegation are formed. That is, we are not interested in whether a particular member was chosen first, followed by another, and so on. Thus, **combination** is applicable **instead** of permutation. Note the word 'selected' in the question. The number of ways of selecting 7 members of a delegation, out of 10 officials, is given by

$$^{10}C_7 = \frac{10!}{(10-7)!\,7!} = \frac{10 \times 9 \times 8 \times 7!}{3!\,7!}$$

$$= \frac{10 \times 9 \times 8}{3 \times 2} = 120 \text{ ways.}$$

The number of ways of selecting 8 members of a delegation out of 10 officials, is given by

$$^{10}C_8 = \frac{10!}{(10-8)!\,8!} = \frac{10 \times 9 \times 8!}{2!\,8!}$$

$$= \frac{10 \times 9}{2 \times 1} = 45 \text{ ways.}$$

The number of ways of selecting 9 members of a delegation, out of 10 officials, is given by

$$^{10}C_9 = \frac{10!}{(10-9)!\,9!} = \frac{10 \times 9!}{1!\,9!} = \frac{10}{1} = 10 \text{ ways.}$$

The number of ways of selecting 10 members of a delegation, out of 10 officials, is given by

$$^{10}C_{10} = \frac{10!}{(10-10)!\,10!} = \frac{10!}{0!\,10!} = 1 \text{ way.}$$

$$\left(\begin{array}{l}\textit{The number of ways a}\\ \textit{delegation of } \textbf{at least}\\ \textit{7 members will be}\\ \textit{selected from 10 officials}\end{array}\right) \begin{array}{l}= 120 + 45 + 10 + 1\\ = 176 \text{ ways.}\end{array}$$

Note that, if this question had demanded that these delegates be arranged in a particular order, then, we would have used permutation formula instead of combination formula.

10. The digits 1, 2, 3, 4, 5, 6 are written on six identical discs. Three discs are drawn at random and placed in order of drawing. What is the probability that the number formed is less than or equal to 500? **(WAEC)**

Workshop

The order in which each sets of three discs drawn are arranged is important, because number 1 2 3 is different from 1 3 2; 1 3 2 is different from 2 3 1, and so on. Besides, note the phrase "placed in order of drawing" in the question. So, it is obvious that the order of arrangement is important, then, we will use permutation formula to solve this problem. The number of ways, 3 discs can be drawn at random, from 6 discs, and then placed in the order they are drawn, is given by

$$^{6}P_3 = \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120 \text{ ways.}$$

The number of ways, 3 discs can be drawn at random, from 6 discs, is the same as the number of 3-digit numbers, that can be formed on three discs picked at random, out of 6 discs. Hence, the number of 3-

digit numbers that can be formed, is 120.

For the 3-digit numbers formed to be **less than or** equal to 500, the 3-digit numbers must begin with any of 1, 2, 3 or 4. However, if digit 5 is on the first plate (Figure 10.4), and any other digit is on the other two plates, as shown below, the 3 digit numbers that will be formed, will be greater than or equal to 500.

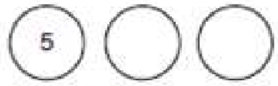

Fig. 10.4

*Note that, digit 500 **cannot be formed**, since there is no zero on any of the discs.*

Fig. 10.5

If digit 1 must be on the first disc to form the 3 digit number (Figure 10.5), then we are left to arrange 2 other discs, out of the remaining 5 discs. Therefore, the number of ways this can be done, is given by

$${}^5P_2 = \frac{5!}{(5-2)!} = \frac{5\times 4\times 3}{3!} = 20$$
ways

If digit 2 must be on the first disc, to form the 3-digit number, then we are left to draw 2 other discs out of the remaining 5 discs. The number of ways this can be done is given by ${}^5P_2 = \frac{5!}{(5-2)!}$ $= \frac{5\times 4\times 3}{3!} = 20$ *ways*. Also, for 3 to be on the

2 discs. Moreover, the number of ways of doing this, for 4 to be on the first disc, is also 20 *ways*. Therefore, the number of ways of having a 3-digit number, less than or equal to 500 formed on the 3 drawn discs is 20 + 20 + 20 +20 = 80 *ways*.

To calculate the **probability** that 'the number formed by the 3 discs drawn is less than, or equal to 500', we need to know the number of elements in the event space, which is the number of 3-digit numbers, less than or equal to 500, that can be formed by drawing 3 discs randomly which is 80 *three-digit numbers*. The sample space is the total number of 3-digit numbers that can be formed from drawing any three of the discs labeled 1 to 6, without any condition, is 120 *three-digit numbers* (*Recall that this was what we first calculated*).

Hence,

$$\Pr\begin{pmatrix} \textit{that the number} \\ \textit{formed by three} \\ \textit{discs drawn} \\ \textit{randomly is less} \\ \textit{than or equal} \\ \textit{to } 500 \end{pmatrix} = \frac{\textit{number of elements}\ \textit{in event sapce}}{\textit{number of elements}\ \textit{in sample sapce}}$$

$$= \frac{\textit{number of 3-digit numbers that is less than or equal to 500 that can be formed}}{\textit{total number of 3-digit numbers that can be formed}}$$

$$= \frac{80}{120} = \frac{2}{3}.$$

11. How many words, taking 6 letters at a time from COUNTRYSIDE, will contain N, T and C? **(WAEC)**

Workshop

As no letter was repeated in the word *COUNTRYSIDE*, the number of words that can be formed, by taking 6 letters at a time, from *COUNTRYSIDE* (*made up of* 11 *letters*), is given by ${}^{11}P_6$. That is, the number of 6-letter words that can be formed from the set of the 11-letter word, *COUNTRYSIDE*, is

$${}^{11}P_6 = \frac{11}{(11-6)}! = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!}$$

$$= 11 \times 10 \times 9 \times 8 \times 7 \times 6$$

= 332, 640 *six – letter words*. The number of 6 letter words, that can be formed, by taking 6 letters at a time, from *COUNTRYSIDE*, that will ***not*** contain *N, T and C*, is given by (11 − 3)*P*6. This is because *N, T and C* will ***not*** be part of the set of 6-letter words; thus, we are left to find the number of 6-letter words that can be formed from the remaining; 11 − 3 = 8 *letters*, which is expressed as

$${}^{8}P_6 = \frac{8!}{(8-6)!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 = 20,160 \textit{ six letter words.}$$

Please note that, in forming words from a set of letters, the order of arrangement of the letters is important. For example, ***can*** *is different from* ***nac****, though they are made up of the same letters; however, the two arrangements give rise to words of different meanings. For this reason, permutation, and* ***not*** *combination, is applicable in solving this problem.*

$$\left(\begin{array}{l}\textit{The total number of}\\ \textit{6 – letter words that}\\ \textit{can be formed from the}\\ \textit{word COUNTRYSIDE}\end{array}\right) = \left(\begin{array}{l}\textit{The number of 6 – letter}\\ \textit{words that can be formed}\\ \textit{from COUNTRYSIDE,}\\ \textit{if N, T and C will be in}\\ \textit{all the 6 – letter words}\\ \textit{formed}\end{array}\right) + \left(\begin{array}{l}\textit{The number of 6 – letter}\\ \textit{words, that can be formed}\\ \textit{from COUNTRYSIDE, if}\\ \textit{N, T and C } \textbf{will not be}\\ \textit{in any of the 6 – letter}\\ \textit{words formed}\end{array}\right)$$

$$\left(\begin{array}{l}\textit{The total number of}\\ \textit{6 – letter words that}\\ \textit{can be formed from the}\\ \textit{word COUNTRYSIDE}\end{array}\right) - \left(\begin{array}{l}\textit{The number of 6 letter}\\ \textit{words that can be formed}\\ \textit{from COUNTRYSIDE, if}\\ \textit{N, T and C } \textbf{will not} \textit{ be}\\ \textit{in any of the 6 – letter}\\ \textit{words formed}\end{array}\right) = \left(\begin{array}{l}\textit{The number of 6 – letter}\\ \textit{words, that can be formed}\\ \textit{from COUNTRYSIDE,}\\ \textit{if N, T and C will be in}\\ \textit{all the 6 – letter words}\\ \textit{formed}\end{array}\right)$$

= 332 640 − 20 160 = 312 480 *six letter words.*

Therefore, the number of words, taking 6 letters at a time from *COUNTRYSIDE*, that will contain *N, T* and *C*, is 312 480 *words.*

12. In how many ways can 2 boys and 5 girls be seated in a row, such that the boys do not sit together? **(WAEC)**

Workshop

The 5 girls can sit any where, thus, they can be arranged in $^5P_5$ *ways* $= \frac{5!}{(5-5)!}$ *ways* $= 5!$ *ways* in a row. That is, arranging 5 people, picking the whole 5, at a time. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ *ways.*

Note that, the possible ways the boys and girls can be seated deal with arrangement, since each boy is different and each girl is different. (For example, if A and B are to be seated, arrangement AB is different from arrangement BA. Hence, permutation is appropriate for this question, and ***not*** *combination.*

Let one, out of the 120 possible arrangements of the five girls in a row, be as in Figure 10.6.

$\square\ \boxed{G_1}\ \square\ \boxed{G_2}\ \square\ \boxed{G_3}\ \square\ \boxed{G_4}\ \square\ \boxed{G_5}\ \square$

Fig. 10.6

Then, one of the two boys can occupy any of the blank positions in the row. Hence, of every **one** arrangement of the girls, one of the boys can occupy any one of the available 6 spaces. This means that for every one arrangement of the girls, one of the boys can sit with the girls, in 6 possible ways.

Now, let us take a second arrangement of thegirls, as in Figure 10.7.

$\square\ \boxed{G_2}\ \square\ \boxed{G_1}\ \square\ \boxed{G_3}\ \square\ \boxed{G_4}\ \square\ \boxed{G_5}\ \square$

Fig. 10.7

Also, for this second arrangement of the girls, one of the boys could sit in any of the 6 available spaces. This also means that, for this arrangement of the girls, one of the boys can sit with the girls, in 6 possible ways.

Therefore, for 2 arrangements of the 5 girls, one of the boys can sit with the girls in $(6 + 6) = (6 \times 2)$ *ways.*

Moreover, for 3 arrangements of the 5 girls, one of the boys can sit with the girls in

$(6 + 6 + 6) = (6 \times 3) = 18$ *ways.*

Therefore, for 120 arrangements of the 5 girls, one of the boys can sit with the girls in

$(6 \times 120) = 720$ *ways.*

With one of the boys occupying a position, there are 5 other spaces for the other boy to occupy, without the two boys sitting together, as shown in Figure 10.8.

Fig. 10.8

Looking at Figure 10.8, because the second boy must not sit together with the other boy, of every one way of arranging the five girls and the first boy in a row, there are 5 possible positions (the blank spaces), in which the other boy can occupy, without sitting beside the first boy in the row, as in the diagram. Thus, for every one way of arranging the five girls and the first boy in a row, there are 5 ways of arranging the second boy with **the five girls and the first boy,** without him sitting beside the first boy.

Let us take a look at another arrangement of them5 girls and the first boy as shown in Figure 10.9

Fig. 10.9

For this other arrangement of the 5 girls and the first boy, it can be seen, from Figure 10.9, that there are 5 possible spaces where the second boy can occupy, without sitting beside the first boy. Therefore, for this one way of arranging the 5 girls and the first boy, there are 5 possible ways of arranging the second boy with **the 5 girls and the first boy**.

Hence, for these 2 ways of arranging **the 5 girls and the first boy**, there are $(5 + 5) = 2 \times 5$ possible ways of arranging the second boy with **the 5 girls and the first boy**, without the boys sitting together.

Then, for 720 ways of arranging the five girls and the first boy, there will be (720×5) *ways* of arranging the second boy with **the five girls and the first boy** in the row, such that the boys do not sit together.

Therefore, the number of ways the 2 boys and the 5 girls can be seated in a row, such that the boys do not sit together, is (720×5) *ways* = 3 600 *ways*, or, simply put, $5P5 \times 6 \times 5 = 120 \times 6 \times 5 = 3\,600$ *ways.*

13. How many numbers, greater than 300 can be formed from the digits 0, 2, 3 and 4, if repetition of digits is not allowed?

Workshop

The numbers greater than 300 that can be formed from digits 0, 2, 3, 4, without repetition of digits are 3-digits numbers beginning with 3 or 4 and 4 digit numbers beginning with 2, 3 or 4.

$\boxed{3}\ \boxed{}\ \boxed{}$

Fig. 10.10

From Figure 10.10, if 3 must be the first digit of the 3-digit number, we are left to **arrange** the other digits 0, 2, 4, picking 2 digits at a time out of the three numbers, at the back of digit 3. The number of possible arrangements of three numbers, picking two at a time is given by

$$^{3}P_{2}=\frac{3!}{(3-2)!}=\frac{3!}{1!}=3\times 2\times 1=6 \textit{ arrangements.}$$

Note that, permutation was used, instead of combination, because 20 is different from 02, 42 is different from 24, and so on; since the order of arrangement is important, as you can see, we cannot use combination, instead, we must use permutation.

$\boxed{4}\ \boxed{}\ \boxed{}$

Fig. 10.11

If 4 must be the first digit of the 3-digit number (figure 10.11), we are left to arrange the other digits 0, 2, 3, picking 2 digits at a time out of the 3 numbers. The number of possible arrangements of this is expressed as

$$^{3}P_{2}=\frac{3!}{(3-2)!}=\frac{3!}{1!}=3\times 2\times 1=6 \textit{ arrangements.}$$

Therefore, the number of 3-digit numbers, greater than 300, that can be formed from 0, 2, 3 and 4, if repetition of digits is **not** allowed, is 6 + 6 = 12 *three- digit numbers*.

If the four digit numbers are to begin with 2, we would be left to arrange 0, 3, 4 at the back of digit 2, to form four-digit numbers as shown in Figure 10.12.

$\boxed{2}\ \boxed{}\ \boxed{}\ \boxed{}$

Fig. 10.12

We are to arrange 0, 3, and 4 (3 *digits*) at the back of digit 2, picking the 3 numbers at a time. The number of possible arrangements of 3 numbers, picking the 3 numbers at a time is given by

$$^{3}P_{3}=\frac{3!}{(3-3)!}=\frac{3!}{0!}=\frac{3\times 2\times 1}{1}=6 \textit{ arrangements.}$$

If the four digit numbers are to begin with 3, we would be left to arrange 0, 2, 4 at the back of digit 3, to form four-digit numbers having 3 as shown in Figure 10.13.

$\boxed{3}\ \boxed{}\ \boxed{}\ \boxed{}$

Fig. 10.13

The number of possible arrangements of 3 numbers, picking the 3 numbers at a time, is given by

$$^3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3 \times 2 \times 1}{1} = 6 \ arrangements.$$

By the same explanation, we get 6 arrangements for 4-digit numbers that can be formed from the given numbers, starting with digit 4.

So, the number of 4-digit numbers that can be formed from 0, 2, 3, 4, beginning with digits 2, 3 or 4 is 6 + 6 + 6 = 18 *four − digit numbers.*

Also, by the same explanation, if repetition of digits is **not** allowed, we will have 6 possible arrangements for 4-digit numbers, starting with 0 (zero), and these are written below.

0	2	3	4
0	2	4	3
0	3	2	4
0	3	4	2
0	4	3	2
0	4	2	3

Looking at these sets of numbers also, you will see that the last 4 numbers are also greater than 300; the total number of 4-digit numbers that can be formed from 0, 2, 3, and 4, if repetition of digits is **not** allowed, will be 18 + 4 = 22 *four − digit numbers.*

Therefore, the numbers greater than 300, that can be formed from the digits 0, 2, 3 and 4 if repetition of digits is not allowed = 12 + 22 = 34 *numbers.*

14. A committee of 6 is to be formed from 9 women and 3 men. In how many ways can the members be selected so as to include:

(a) at least one man?

(b) no man? **(WAEC)**

Workshop

(a) '**at least** 1 man' means 1 or 2 or all the 3 men must be in the committee; thus,

$$\begin{pmatrix}\textit{The number of ways of}\\ \textit{selecting a committee of}\\ \textit{6 members with } \textbf{at least}\\ \textit{one man}\end{pmatrix} = \begin{pmatrix}\textit{The number of}\\ \textit{ways of selecting 1}\\ \textit{man and 5 women}\end{pmatrix}$$

$$+ \begin{pmatrix}\textit{The number of}\\ \textit{ways of selecting 2}\\ \textit{men and 4 women}\end{pmatrix} + \begin{pmatrix}\textit{The number of}\\ \textit{ways of selecting 3}\\ \textit{men and 3 women}\end{pmatrix}$$

The number of possible ways that 1 man can be selected from 3 men is given by

$^3C_1 = \frac{3!}{(3-1)!1!} = \frac{3 \times 2!}{2!1!} = 3$ ways. The possible number of ways that 5 women can be selected from 9 women is given by

$$^9C_5 = \frac{9!}{(9-5)!5!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{4!5!}$$

$$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126 \text{ ways.}$$

For *one* way of selecting 1 man from the 3 men, there are 126 possible ways of selecting this one man, and selecting 5 women from 9 women. For example, let the 3 men be named *A*, *B* and *C*, and recall there are 126 different possible ways of selecting 5 women from 9 women. Since these 126 possible ways of selecting 5 women from 9 women are different, then selecting man *A* for each of these 126 different ways, will give 126 different possible ways of selecting **man *A* and 5 women from 9 women.** In other words, for 1 (*one*) way of selecting 1 man from the 3 men, there are 126 possible ways of selecting this **one man** and 5 women from 9 women.

126 different possible ways of selecting **man *B* and 5 women from 9 women.**

This means, for 2 (*two*) ways of selecting 1 man from the 3 men (*that is selecting A or B*), there will be 126 + 126 = 126 × 2 = 252 possible ways of selecting a man (that can be either *A* or *B*) and selecting 5 women from 9 women. Recall that there are 3 ways of selecting 1 man from 3 men; therefore, for the 3 ways of selecting 1 man from the 3 men (*that is selecting A or B or C*), there will be 126 + 126 + 126 = 126 × 3 = $^9C_3 \times {}^3C_1$ = 378 *possible ways* of selecting a man (that can be any one of *A*, *B* or *C*) and selecting 5 women from 9 women.

Therefore, the possible number of ways of selecting 1 man from 3 men, **and** 5 women from 9 women to form the 6 member committee = $^3C_1 \times {}^9C_5$ = 378 *ways.*

Note that, we are using combination formula because we are not interested in the order in which the sexes are selected. In other words, we are not interested in whether woman A was selected before woman B, or man B before man C, and so on. Hence, selection AB is still the same as selection BA

Also, going by the explanation above, the number of ways of having 2 men selected from 3 men, and 4 women selected from 9 women, to form the 6-member committee

$$= {}^3C_2 \times {}^9C_4 = \frac{3!}{(3-2)!2!} \times \frac{9!}{(9-4)!4!}$$

$$= \frac{3 \times 2!}{1! \times 2!} \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2 \times 1} = 378 \text{ ways.}$$

The number of ways of having 3 men selected from 3 men, and 3 women selected from 9 women, to form the 6 member committee

$$= {}^{3}C_{3} \times {}^{9}C_{3} = \frac{3!}{(3-3)!3!} \times \frac{9!}{(9-3)!3!}$$

$$= \frac{3!}{0! \times 3!} \times \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1} = 84 \textit{ ways.}$$

$$\begin{pmatrix} \textit{the number of ways of selecting} \\ \textit{a committee of 6 members with} \\ \textit{at } \textbf{at least } 1 \textit{ man} \end{pmatrix}$$

$$= (378 + 378 + 84) \textit{ ways} = 840 \textit{ ways.}$$

(b) If no man is to be on the committee, then we are left to select the 6 committee members from the 9 women. This can be done in ${}^{9}C_{6}$ *ways.*

$$= \frac{9!}{(9-6)!6!} = \frac{9 \times 8 \times 7 \times 6!}{3! \times 6!} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1}$$

$= 84$ *ways.* Therefore, the number of ways the 6 committee members can be selected so as to include **no** man is 84 *ways.*

15. Solve the equation $\frac{{}^{n}P_{5}}{{}^{n}C_{3}} = 12.$ (WAEC)

Workshop

$$\frac{{}^{n}p_{5}}{{}^{n}C_{3}} = {}^{n}p_{5} \div {}^{n}C_{3} = \frac{n!}{(n-5)!} \div \frac{n!}{(n-3)!3!} = 12;$$

$\frac{n!}{(n-5)!} \times \frac{(n-3)!3!}{n!} = 12$; recall that

$$n! = n(n-1)! = n(n-1)(n-2)!$$

$$= n(n-1)(n-2)(n-3) \text{ and so on.}$$

Then, $(n-3)! = [n-3][(n-3)-1][(n-3)-2]!$

$$= (n-3)(n-3-1)(n-3-2)!$$

$$= (n-3)(n-4)(n-5)!\text{. Therefore,}$$

$$\frac{n!}{(n-5)!} \times \frac{(n-3)!3!}{n!}$$

$$= \frac{\cancel{n!}}{\cancel{(n-5)!}} \times \frac{(n-3)(n-4)\cancel{(n-5)!}3!}{\cancel{n!}} = 12;$$

$$(n-3)(n-4)3! = 12; (n^2-7n+12)(3\times2\times1) = 12;$$

$$6(n^2-7n\times 12) = 12;$$

$$n^2-7n+12 = \frac{12}{6} = 2; n^2-7n+12-2 = 0;$$

$$n^2-7n+10 = 0; n^2-5n-2n+10 = 0;$$

$$n(n-5)-2(n-5) = 0; (n-5)(n-2) = 0;$$

$$(n-5) = 0 \; or \; (n-2) = 0; n = 5 \; or \; n = 2$$

${}^2p_5$ and ${}^2p_3$ **cannot** be evaluated; thus, the value of n that satisfies the equation is $n = 5$.

Therefore, the value of n for which $\frac{{}^nP_5}{{}^nC_3} = 12$ is 5.

16. In a factory, 11 people are available for **two** different jobs. How many ways can selection be made if:

(a) 4 are needed for each job?

(b) 5 are needed for one type and 2 for the other job? **(WAEC)**

Workshop

(a) As 4 people are needed for each job, the factory head will have to select 4, out of the available 11 applicants for the first job, after which he will select the other 4 out of the remaining 11 – 4 = 7 applicants for the second job. The number of ways the first 4 can be selected out of the 11 available applicants is expressed as

$${}^{11}C_4 = \frac{11!}{(11-4)!4!} = \frac{11\times10\times9\times8\times7!}{7!4!}$$

$$= \frac{11\times10\times9\times8}{4\times3\times2\times1} = 330 \; ways.$$

Please, note that, because the order of selection is not important permutation is not applicable to this problem, instead combination is appropriate.

The number of ways the factory head can select the other 4 out of the 7 remaining applicants is given by ${}^7C_4$ = 35 *ways.*

For every **one** way of choosing 4 out of the 11 applicants for the first job, there are 1 × 35 possible ways of selecting 4 out of the 11 applicants for the first job **and** 4 out of the 7 (seven) remaining applicants for the second job.

For example, assuming applicants *A*,*B*, *C* and *D* are the people selected (*from the available* 11 *people*) for the first job (*this is one way of selecting* 4 *people out of* 11 *for the first job*). The 35 possible ways of choosing 4 out of the remaining 7 people for the second job, are 35 **different** ways of choosing 4 out of 7 people. Since these 35 possible ways are different, we can keep choosing *A*, *B*, *C* and *D* for the first job, for each of these 35 different ways of choosing 4 out of 7 people for the second job. So, for every **one** way of choosing 4 out of the 11 applicants for the first job (*that is choosing A, B, C* and *D in this case*) there are 35 possible ways of selecting 4 out of the 11 applicants for the first job **and** 4 out of the 7 (seven) remaining applicants for the second job.

This is because, the 35 ways of choosing 4 out of 7 people are different, then, choosing *A, B, C* and *D* for the first job for each of the 35 different ways of choosing 4 out of 7 people for the second job will make 35 different ways of choosing *A, B, C* and *D* for the first job, and choosing 4 out of 7 people for the second job.

Another one way of choosing 4 out of the 11 people for the first job (*for example choosing applicants E, F, G and H*) shows there are 35 possible ways of choosing 4 out of the 11 people for the first job **and** choosing 4 out of the remaining 7 (seven) people for the second job.

Therefore, for 2 ways of selecting 4 out of the 11 people for the first job, there will be 35 + 35 = 35 × 2 possible ways of choosing 4 out of the 11 people for the first job **and** choosing 4 out of the remaining 7 (seven) people for the second job.

Therefore, for 330 ways of selecting 4 out of the 11 people for the first job, there will be 35 × 330 possible ways of choosing 4 out of the 11 people for the first job **and** choosing 4 out of the remaining 7 (seven) people for the second job. Hence;

$$\begin{pmatrix} \text{The possible number of ways of} \\ \text{selecting 4 out of 11 applicants} \\ \text{and later selecting another 4} \\ \text{from the remaining 7 applicants} \end{pmatrix} = 35 \times 330$$

$$= {}^{7}C_4 \times {}^{11}C_4 = 11{,}550 \text{ ways}$$

Therefore, the number of ways the selection can be made if 4 people are needed for each of the two jobs out of 11 applicants is 11,550 ways.

(b) Let *E* be the event that 5 applicants are needed for one type and 2 for the other job. In the question, we were told that the two jobs were different. Let *M* represent one of the jobs, and *N* the other.

If 5 people are needed for one type and 2 for the other type, and neither of the job is specified, then 5 people may be needed for job *M* and 2 for job *N* **OR** 2 people may be needed for job *M* and 5 for job *N*.

For this reason, the following will be the two possible sub-events of event *E*: *A*, an event that 5 applicants are needed for job *M*, and 2 applicants are needed for job *N*, or *B*, an event that 2 applicants are needed for job *M*, and 5 applicants are needed for job *N*. Therefore,

$$\begin{pmatrix} \text{The number of} \\ \text{possible ways} \\ \text{that events } E \\ \text{can occur} \end{pmatrix} = \begin{pmatrix} \text{The number of} \\ \text{possible ways} \\ \text{that events } A \\ \text{can occur} \end{pmatrix} + \begin{pmatrix} \text{The number of} \\ \text{possible ways} \\ \text{that events } B \\ \text{can occur} \end{pmatrix}$$

The number of ways event *A* could occur is the number of ways of selecting

5 from 11 applicants for the first job **and** later selecting 2 from the remaining

11 – 5 = 6 applicants for the second job.

Going by the same explanation used in solving 16(i) above, the possible number of ways in which this could be done will be expressed as

$${}^{11}C_5 \times {}^{6}C_2 = \frac{11!}{(11-5)!5!} \times \frac{6!}{(6-2)!2!};$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6! \times 5!} \times \frac{6 \times 5 \times 4!}{4! \times 2}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1} = 6{,}930.$$

The number of ways event B could occur, is the number of ways of selecting 2 from 11 applicants for the first job **and** later selecting 5 from the remaining 11 – 2 = 9 applicants for the second job. The number of ways that this could be done will be expressed as

$$^{11}C_2 \times {}^{9}C_5 = \frac{11!}{(11-2)!2!} \times \frac{9!}{(9-5)!5!}$$

$$= \frac{11 \times 10 \times 9!}{9! \times 2!} \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{4! \times 5!}$$

$$= \frac{11 \times 10}{2 \times 1} \times \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 6\,930 \textit{ ways.}$$

$$\text{Hence, } \begin{pmatrix} \textit{The number of} \\ \textit{possible ways that} \\ \textit{events E could occur} \end{pmatrix} \begin{array}{l} = 6\,930 + 6\,930 \\ = 13\,860 \textit{ ways.} \end{array}$$

Therefore, the number of ways the selection could be made from 11 applicants if 5 are needed for one type and 2 for the other job is 13 860 *ways.*

Note that, combination was used for this calculation because from the question, we were not interested in the order in which the applicants are chosen, that is, we are not interested in **arranging** *first, second, third, and so on. But we were rather interested in choosing 4, out of 11 applicants, 4 out of 7 applicants, 5 out of 11 applicants, 2 out of 11 applicants etc* **without considering any order** *of arrangement. Thus, selecting applicants A, B, C, D is the same as selecting B, A, C, D, as the order of selection is not important. Therefore, combination is appropriate to solve this problem and not permutation.*