CHAPTER 4 Matrices and Determinants

A matrix is a rectangular arrangement of numbers or letters in rows and columns, and this arrangement is always in a big bracket.

$$\begin{array}{cc} & \text{row} \\ \text{column} & \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \end{array}$$

Fig. 4.1

As shown in Figure 4.1, the horizontal arrangements are the rows, while the vertical arrangements are the columns. Figure 4.1 shows a matrix with 3 rows and 3 columns; therefore, the order of the matrix is 3 by 3 (3 × 3). Matrix A below has two rows and three columns, and so the order of matrix A is 2 × 3 (pronounced 2 by 3), so A is a 2 × 3 matrix. Matrix B has three rows and two columns which makes its order

$$3 \times 2.\ A = \begin{pmatrix} 1 & 4 & -9 \\ 3 & 15 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 5 & 6 \\ 7 & 10 \\ 3 & 21 \end{pmatrix}.$$

> *Note that the number of rows comes first in defining the order of a matrix. For instance a matrix of order $m \times n$ has m rows and n columns.*

Equality of a Matrix

Two matrices A and B are said to be equal if the following conditions are met:

(i) The two matrices must be of the same order, and

(ii) Corresponding elements of the two matrices must be equal.

For instance, given that $H = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *and* $J = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$

Matrices H and J are of the same order (which is 2 × 2), and for the second condition to be satisfied for matrices H and J to be equal, a **must** be equal to e; b **must** be equal to f; c **must** be equal to g, and d **must** be equal to h.

Addition and Subtraction of Matrices

These two operations (addition and subtraction of matrices) can **only** be done on two matrices that are of the same order, and these two operations are done between corresponding elements of the two matrices. For instance,

$$H+J=\begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}.$$

Scalar Multiplication of Matrices

A matrix *P* can be multiplied by a constant (a scalar) to get another matrix, and all the elements of the resulting matrix will be a multiple of the constant. For instance, if the

$$\text{matrix } P=\begin{pmatrix} u & v \\ w & x \end{pmatrix};\ 4(P)=4\times\begin{pmatrix} u & v \\ w & x \end{pmatrix}=\begin{pmatrix} 4u & 4v \\ 4w & 4x \end{pmatrix}.$$

For this reason, the matrix $\begin{pmatrix} 10 & 5 & -75 \\ 45 & -30 & 65 \\ 90 & 20 & -80 \end{pmatrix}$ Therefore, if there is a factor that is common to **ALL** the elements (numbers) of a matrix, the matrix can be factorized as explained.

Multiplication of Two Matrices

Two matrices *A* and *B* can be multiplied as *AB* **only if** the number of columns of the first matrix (in this case matrix *A*) is equal to the number of rows of the second matrix (that is matrix *B*). Moreover, the multiplication *BA* can be done **only if** the number of columns of *B* (because *B* is coming first in this case) is equal to the number of rows of *A*.

$$A=\begin{pmatrix} c & d \\ e & f \\ g & h \end{pmatrix},\ B=\begin{pmatrix} i & j & k & l \\ m & n & o & p \end{pmatrix}$$

Matrix A above has 2 columns and matrix B has two rows, therefore, A and B can be multiplied as AB. Is the multiplication BA possible? The number of columns of B is 4 while the number of rows of A is 3; the number of columns of B is **not** equal to the number of rows of A, so the operation BA **cannot** be done.

> *Note that in matrices, given matrices P and Q, PQ is* ***not equal to*** *QP* ***except****, if $Q = P^{-1}$.*
> *That is, in matrices, PQ = QP* ***if and only if*** *P and Q are inverse of each other.*

Given that matrix $Q=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ and matrix $R=\begin{pmatrix} j & k & l \\ m & n & o \\ p & s & t \end{pmatrix}$ the product *QR* can be calculated as follows:

$$QR=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} j & k & l \\ m & n & o \\ p & s & t \end{pmatrix}=\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$

First, one must confirm that the number of columns of *Q* is equal to the number of rows of *R*. Yes, the two are equal, so the multiplication of both matrices is possible. The expected answers are represented as elements *x*11 to x_{33}, and x_{11} refers to the element in row 1 column 1, x_{12} is the element in row1 column 2, x_{21} is the element in row 2 column 1, x_{23} is the element in row 2 column 3. The locations of all these elements can be confirmed by carefully looking at the matrix and relating the subscript of each element with its position.

Furthermore, to get the value of x_{11} (which is in row 1 column 1 of the product of Q and R) pick the elements in row 1 of matrix Q and multiply them with the corresponding elements in column 1 of matrix R, then add the results as follows: elements of row 1 of matrix Q are $(a\ b\ c)$, while elements of column of R are $\begin{pmatrix} j \\ m \\ p \end{pmatrix}$ So, x_{11} will be evaluated as $x_{11} = (a \times j) + (b \times m) + (c \times p) = aj + bm + cp$.

Also to get the value of another element, like x_{23} (which is in row 2, column 3 of the product of Q and R), select the elements in row 2 of matrix Q and multiply with corresponding elements in column 3 of R, add the result like this: elements in row 2 of matrix Q are $(d\ e\ f)$ and the elements in column 3 of R are $\begin{pmatrix} l \\ o \\ t \end{pmatrix}$ Hence, x_{23} will be $(d \times l) + (e \times o) + (f \times t) = dl + eo + ft$. The other elements of the product of Q and R can also be calculated using same method.

Determinants

The determinant of a matrix $P = \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$ is expressed as follows:

The determinant of a matrix $P = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is expressed as follows:

$$|P| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg).$$

1. Given that $\begin{vmatrix} 5 & 2 & -3 \\ -1 & k & 6 \\ 3 & 9 & (k+2) \end{vmatrix} = 207$, Find the values of the constant k. **(WAEC)**

Workshop

The determinant of a matrix $P = \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$ is expressed as follows:

$$|P| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg),$$

then,

$$|P| = \begin{pmatrix} 5 & 2 & -3 \\ -1 & k & 6 \\ 3 & 9 & (k+2) \end{pmatrix} =$$

$$\begin{bmatrix} 5\langle (k(k+2)) - (6(9)) \rangle - 2\langle (-1(k+2)) - (6(3)) \rangle \\ + (-3)\langle (-1(9)) - (k(3)) \rangle = -207 \end{bmatrix};$$

$5((k^2 + 2k) - 54) - 2[(-k - 2) - 18] - 3[(-9) - 3k] = -207;$

$5(k^2 + 2k - 54) - 2(-k - 2 - 18) - 3(-9 - 3k) = -207;$

$5k^2 + 10k - 270 + 2k + 4 + 36 + 27 + 9k = -207;$

$5k^2 + 21k - 203 = -207;\ 5k^2 + 21k - 203 + 207 = 0;\ 5k^2 + 21k + 4 = 0;$

$5k^2 + 20k + k + 4 = 0;\ 5k(k + 4) + 1(k + 4) = 0;\ (k + 4)(5k + 1) = 0;$

$k + 4 = 0;$ or $5k + 1 = 0;\ k = -4\ or\ 5k = -1\ k = -4\ or\ k = -\frac{1}{5}.$

Therefore, the possible values of the constant k are $-\frac{1}{5}$ *and* -4.

2. Solve the equation $\begin{pmatrix} 10-x & -6 & 2 \\ -6 & 9-x & -4 \\ 2 & -4 & 5-x \end{pmatrix} = 0.$

(WAEC)

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$$\begin{pmatrix} 10-x & -6 & 2 \\ -6 & 9-x & -4 \\ 2 & -4 & 5-x \end{pmatrix} = 0;$$

We are to evaluate this determinant and equate it to zero as

$$\begin{vmatrix} 10-x & -6 & 2 \\ -6 & 9-x & -4 \\ 2 & -4 & 5-x \end{vmatrix} (10 - x)[((9 - x)(5 - x)) - ((-4)(-4))] - (-6)[((-6)(5 - x)) - ((-4)(2))] + (2)[(-(-4) - ((9 - x)(2))] = 0;$$

$(10 - x)(45 - 14x + x^2 - 16) + 6[-30 + 6x + 8] + 2[24 - 18 + 2x] = 0;$

$(10 - x)(x^2 - 14x + 29) + 6[6x - 22] + 2[2x + 6] = 0;$

$10x^2 + 140x + 290 - x^3 - 14x^2 - 29x + 36x - 132 + 4x + 12 = 0;$

$-x^3 + 10x^2 + 14x^2 - 140x - 29x + 36x + 4x + 290 - 132 + 12 = 0;$

$-x3 + 24x2 - 129x + 170 = 0.$

We can divide through this equation by −1 to get

$x3 - 24x2 + 129x - 170 = 0.$ Let $f(x) = x3 - 24\,x2 + 129x - 170.$

> *Note that, if the question requires drawing or sketching of the curve of* $-x3 + 24x2 - 129x + 170 = 0$, *then, dividing the equation throughout by* -1, *before using the equation to draw the curve,* ***will change the shape of the graph****. But the values of x for which the initial equation is zero will still remain the same. So, for convenience, we were able to divide through the equation by* -1 *because we are* ***not*** *drawing or sketching the curve; we* ***only*** *want to know the value of x when the equation is equal to zero. Know that this also applies to quadratic equations.*

Substitute $x = 2$ into the above equation, to get

$f(2) = 23 - 24(2)2 + 129(2) - 170 = 0;$ $f(2) = 8 - 96 + 258 - 170;$

$f(2) = 266 - 266 = 0;$ $f(2) = 0.$

Recall from your knowledge of polynomials that if $f(x) = ax3 + bx2 + cx - d$, and $f(p) = 0$, then $x - p$ will be a factor of $ax3 + bx2 + cx - d$. Thus, if $ax3 + bx2 + cx - d$ is divided by $x - p$, the remainder will be zero.

Therefore, as $f(2) = 0$ as calculated, $(x - 2)$ will be a factor of $x3 - 24x2 + 129x - 170 = 0$. In other words, if we **divide** the cubic equationby $(x - 2)$, we will have no remainder and we can also get the other factors. The long division can be done as shown below:

$$
\begin{array}{r|l}
 & x2 - 22x + 85 \\
\hline
x - 2 & x3 - 24x2 + 129x - 170 \\
 & -\ x3 - 2x2 \\
\hline
 & \quad - 22x2 + 129x \\
 & -\ - 22x2 + 44x \\
\hline
 & \qquad 85x - 170 \\
 & -\ \ 85x - 170 \\
\hline
 & \qquad 0 \quad 0
\end{array}
$$

There is no remainder from this long division; therefore, $x2 - 22x + 85$ is also a factor of $x3 - 24x2 + 129x - 170$.

Hence, $(x - 2)(x2 - 22x + 85) = x3 - 24x2 + 129x - 170 = 0;$

$(x - 2)(x2 - 22x + 85) = (x - 2)(x2 - 17x - 5x + 85) = 0;$

$(x - 2)(x(x-17) - 5(x-17)) = (x - 2)((x-17)(x-5)) = 0;$

So, $(x - 2)(x - 5) = 0;$ $x - 2 = 0$ or $x - 17 = 0$

or $x - 5 = 0;$

$x = 2$ or $x = 17$ or $x = 5$. Therefore, the valuesof x, for which

$x3 - 24x2 + 129x - 170$, is zero are $x = 2$, $x = 5$and $x = 17$.

Note that you can as well use the quadratic formula to solve $x^2 - 22x + 85$ if you cannot use the factorization method, and you can later fix it back into $(x-2)(x2-22x+85) = 0$ to get $(x-2)(x-5)(x-17) = 0$.

Therefore, the solutions to the equation

$$\begin{vmatrix} 10-x & -6 & 2 \\ -6 & 9-x & -4 \\ 2 & -4 & 5-x \end{vmatrix} = 0 \text{ are } x = 2, x = 5 \text{ and } x = 17$$

3. The matrices P and Q are:

$$P = \begin{pmatrix} 1 & 3 \\ 4 & -1 \end{pmatrix}, Q = \begin{pmatrix} -3 & -K \\ 5 & -2 \end{pmatrix} \text{ where } K \text{ is a constant.}$$

(a) Find |PQ|.

(b) If |PQ/ = 144, find the value of *K*.

(WAEC)

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Note that, in matrices, PQ is not equal to QP ***except if*** *Q is the inverse of P (i.e if $Q = P_{-1}$)*

$$PQ = \begin{pmatrix} 1 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} -3 & -K \\ 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} [(1\times-3)+(3\times 5)] & [(1\times-k)+(3\times-2)] \\ [(4\times-3)+(-1\times 5)] & [(4\times-k)+(-1\times-2)] \end{pmatrix}$$

$$PQ = \begin{pmatrix} (-3+15) & [-k+(-6)] \\ [-12+(-5)] & [-4k+(2)] \end{pmatrix} = \begin{pmatrix} 12 & (-k-6) \\ -17 & (-4k+2) \end{pmatrix}$$

$|PQ|$ is the determinant of matrix PQ, hence,

$$|PQ| = \begin{vmatrix} 12 & (-k-6) \\ -17 & (-4k+2) \end{vmatrix}$$

$$= [12(-4k+2)] - [(-17)(-k-6)];$$

$|PQ| = -48k + 24 - (17k + 102)$

$= -48k - 17k + 24 - 102;$

$|PQ| = -65k - 78$

(b) If $|PQ/ = 144;$

$|PQ| = -65k - 78 - 144; -65k = 144 + 78 = 222;$

4. If $A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 4 \\ -2 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} 4 & 3 \\ -1 & 5 \end{pmatrix}$,

find, a matrix M such that $AC = BM$.

(WAEC)

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$A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -2 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 3 \\ -1 & 5 \end{pmatrix}$ Let the matrix, M, be $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$;

$$AC = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix} \times \begin{pmatrix} 4 & 3 \\ -1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \times 4) + (2 \times (-1)) & (3 \times 3) + (2 \times 5) \\ (-1 \times 4) + (1 \times (-1)) & (-1 \times 3) + (1 \times 5) \end{pmatrix}$$

$$= \begin{pmatrix} 12 + (-2) & 9 + 10 \\ -4 + (-1) & -3 + 5 \end{pmatrix} = \begin{pmatrix} (12 - 2) & 19 \\ (-4 - 1) & 2 \end{pmatrix};$$

$$AC = \begin{pmatrix} 10 & 19 \\ -5 & 2 \end{pmatrix}$$

Note that in matrices, given matrices P and Q, PQ is ***not equal to*** *QP* ***except****, if $Q = P^{-1}$. That is, in matrices, $PQ = QP$* ***if and only if*** *P and Q are inverses of each other.*

$$BM = \begin{pmatrix} 2 & 4 \\ -2 & 0 \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} 2e + 4g & 2f + 4h \\ -2e + 0g & -2f + 0h \end{pmatrix};$$

$BM = \begin{pmatrix} 2e + 4g & 2f + 4h \\ -2e & -2f \end{pmatrix}$. Since $AC = BM$, then,

$$\begin{pmatrix} 10 & 19 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 2e + 4g & 2f + 4h \\ -2e & -2f \end{pmatrix};$$

By comparing elements, $-2e = -5$;

$$e = \frac{-5}{-2} = \frac{5}{2}; -2f = 2; f = \frac{2}{-2} = -1;$$

$$2e + 4g = 10; 2\left(\frac{5}{2}\right) + 4g = 10;$$

$$5 + 4g = 10; 4g = 5; g = \frac{5}{4};$$

$$2f + 4h = 19; 2(-1) + 4h = -2 + 4h = 19;$$

$4h = 21$; $h = \frac{21}{4}$. Therefore, $M = \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} \frac{5}{2} & -1 \\ \frac{5}{4} & \frac{21}{4} \end{pmatrix}$.

5. If $B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$ and I a 2×2 identity matrix, find a matrix A such that $BA = I$.

(WAEC)

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$B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, the 2×2 identity matrix

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

For any square matrix H, $HH^{-1} = I$, where I is an identity matrix (having the same dimension as H) and H^{-1} is the inverse matrix of matrix H. If $BA = I$, we now know that $BB^{-1} = I$ so that

$BA = I = BB^{-1}$, hence $BA = BB^{-1}$, so $A = B^{-1}$. From this explanation, matrix A is the inverse of matrix B.

> *Note that the product AB of the two matrices A and B is **not** equal to the product BA, **except** if A is the inverse matrix of matrix B (that is, $A = B^{-1}$)*

$B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, $C \text{ of } B = \begin{pmatrix} \overset{+}{3} & \overset{-}{5} \\ \overset{-}{1} & \overset{+}{2} \end{pmatrix} = \begin{pmatrix} +3 & -1 \\ -1 & +2 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$;

$B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$, $C \text{ of } B = \begin{pmatrix} \overset{+}{3} & \overset{-}{5} \\ \overset{-}{1} & \overset{+}{2} \end{pmatrix} = \begin{pmatrix} +3 & -1 \\ -1 & +2 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$;

$Adj B = (C \text{ of } B)_T = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}^T = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$;

$Adj B = (C \text{ of } B)_T = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}^T = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$;

$B_{-1} = \frac{1}{|B|}(Adj\ B) = \frac{1}{2(3) - 5(1)} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

$= \frac{1}{1} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

$B_{-1} = \frac{1}{|B|}(Adj\ B) = \frac{1}{2(3) - 5(1)} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

$= \frac{1}{1} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

$B_{-1} = A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$. Therefore, the matrix A such

that $BA = I$ is $\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

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that $BA = I$ is $\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

> *Note that for any 2×2 (2 by 2) matrix, $D = \begin{pmatrix} a & b \\ c & e \end{pmatrix}$, $D_{-1} = \frac{1}{|D|}\begin{pmatrix} e & -b \\ -c & a \end{pmatrix}$. We came about this formula for D^{-1} from the formula: $D^{-1} = \frac{1}{|D|}(C \text{ of } D)_T = \frac{1}{|D|}(Adj\ D)$. Also note that the determinant of a matrix D is written as $|D|$. Moreover, this shortcut method is **only** applicable to 2 by 2 matrices.*

6. Using matrix method, solve $-2x + y = 3$; $-x + 4y = 1$. **(WAEC)**

Workshop

$-2x + y = 3$ ....*(i)*

$-x + 4y = 1$ ....*(ii)*

By using matrix method, these simultaneous equations can be written in matrix form as

$\begin{pmatrix}-2 & 1\\-1 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$. Let $\begin{pmatrix}-2 & 1\\-1 & 4\end{pmatrix}=A$,

$\begin{pmatrix}x\\y\end{pmatrix}=X$ *and* $\begin{pmatrix}3\\1\end{pmatrix}=y$ so that $AX=Y$.

Note that X is not the same as x and Y is not the same as y. While x and y are variables, and X and Y are matrices.

$$\begin{array}{cc} & \xrightarrow{\text{row}} \\ \text{column}\uparrow & \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \end{array}$$

Fig. 4.2

Note that in writing matrix A, the coefficient of x in the simultaneous equation must be arranged in the first column as shown above, while the coefficients of y must be arranged in the second column of matrix A as explained above, so that matrix X can be written as $\left(\frac{x}{y}\right)$.

Therefore, the simultaneous equation can be written in matrix form as $AX = Y$. Since we are interested in calculating for X, we can make X the subject of the formula by pre-multiplying through the equation with the inverse matrix of A to get $A^{-1}AX = A^{-1}Y$.

Note that, to pre-multiply a matrix P by another matrix R ***(pre means before)****, you are to multiply P by R in such a way that R comes first in the multiplication as R × P. This is because R and P are matrices, R × P is* ***NOT*** *equal to P × R (except if R is the inverse matrix of P). Now, if R is the inverse of P, that is* $R = P^{-1}$*, then* $RP = P^{-1} = PP^{-1} = I$*, where I is an identity matrix. So,* ***if and only if*** *matrix R is the inverse of matrix P, will PR = RP.*

Also, because we are to pre-multiply through the equation by A^{-1}, then A^{-1}will also come before Y as $A^{-1}Y$. Then, $A^{-1}AX = A-1Y$; Recall that $A^{-1}A = I$; so, $A^{-1}AX = IX = A^{-1}Y$.

Again, if you multiply a square matrix (e.g 2 by 2, 3 by 3, 4 by 4, etc. matrix) by its corresponding identity matrix, I, the result will be that same matrix. For example, IQ = QI = Q.

Hence, $A^{-1}AX = IX = X = A^{-1}Y$ Now we have made X the subject of the formula as $X = A^{-1}Y$.

You can now see that to get X, we find A^{-1} *and multiply it with Y as* $A^{-1}Y$ *and not* YA^{-1} *since the two are matrices. But remember* $AA^{-1} = A_{-1}A = I$.

Now let's calculate for $A-1$. For a 2 × 2 (2 *by* 2) matrix.

$D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $D_{-1} = \frac{1}{|D|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. We came about this formula for D_{-1} from the formula

$$D_{-1} = \frac{1}{|D|}(C\ of\ D)_T = \frac{1}{|D|}(Adj\ D).$$

Note that the determinant of a matrix D is written as |D|.

$$A = \begin{pmatrix} -2 & 1 \\ -1 & 4 \end{pmatrix};\ A_{-1} = \frac{1}{\begin{pmatrix} -2 & 1 \\ -1 & 4 \end{pmatrix}}\begin{pmatrix} 4 & -1 \\ -(-1) & -2 \end{pmatrix};$$

$$A_{-1} = \frac{1}{(-2 \times 4) - (-1 \times 1)}\begin{pmatrix} 4 & -1 \\ 1 & -2 \end{pmatrix}$$

$$= \frac{1}{-8+1}\begin{pmatrix} 4 & -1 \\ 1 & -2 \end{pmatrix} = \frac{1}{-7}\begin{pmatrix} 4 & -1 \\ 1 & -2 \end{pmatrix}.$$

Therefore, $A_{-1} = \frac{1}{-7}\begin{pmatrix} 4 & -1 \\ 1 & -2 \end{pmatrix}$ and $Y = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$; thus,

$$X = A_{-1}Y = \frac{1}{-7}\begin{pmatrix} 4 & -1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \frac{-1}{7}\begin{pmatrix} (4 \times 3) + (-1 \times 1) \\ (1 \times 3) + (-2 \times 1) \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} 12 + (-1) \\ 3 + (-2) \end{pmatrix}$$

$$= -\frac{1}{7}\begin{pmatrix} 12 - 1 \\ 3 - 2 \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} 11 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{11}{7} \\ -\frac{1}{7} \end{pmatrix}.$$

So, $X = \begin{pmatrix} -\frac{11}{7} \\ -\frac{1}{7} \end{pmatrix}$. Recall that $X = \begin{pmatrix} x \\ y \end{pmatrix}$, so that

$$X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{11}{7} \\ -\frac{1}{7} \end{pmatrix}$$

Therefore, $x = -\frac{11}{7}$ and $y = -\frac{1}{7}$.

Note that the Determinant method (crammer's rule) for solving equations is different from this matrix method adopted here; thus, do not use the determinant method in place of the matrix method, if you are told to use matrix method to solve a set of simultaneous equations.

7. Use the determinant method to find the area of triangle *ABC* with vertices at $A(1, 6)$, $B(4,2)$ and $C\ (-3, 3)$. **(WAEC)**

Workshop

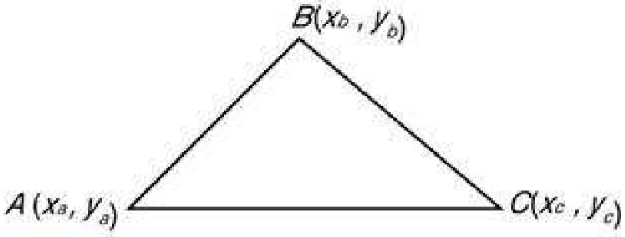

Fig. 4.3

The area of any triangle *ABC* with vertices at $A(x_a, y_a)$ $B(x_b, y_b)$ and $C(x_c, y_c)$, as shown in Figure 4.3, using the determinant method, is expressed as follows:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{vmatrix}$$

$$= \frac{1}{2}\Big[(x_a)[(y_b \times 1) - (y_c \times 1)] - (y_a)[x_b \times 1) - (x_c \times 1)]\Big]$$

$$+ (1)\big[(x_b \times y_c) - (x_c \times y_b)\big].$$

Thus, the area of triangle *ABC*, with verticesat points $A(1, 6)$, $B(4, 2)$ and $C(-3, 3)$ will be expressed as:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 6 & 1 \\ 4 & 2 & 1 \\ -3 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\Big[(1)[(2 \times 1) - (3 \times 1)] - (6)[(4 \times 1) - (-3$$
$$\times 1)] + (1)[(4 \times 3) - (-3 \times 2)]\Big];$$

However, the area of a shape *cannot* be negative, therefore, the area of triangle *ABC* is 12.5 *square units ((units)^2).*