Chapter 7

Chapter 7
Area of Plane Shapes

OBJECTIVES

At the end of the chapter, students should be able to calculate the area of the following:

1. Rectangle
2. Square
3. Parallelogram
4. Trapezium
5. Rhombus
6. Triangle
7. Circle
8. Sector of a circle
9. Segment of circle

1. Area of a Rectangle (Revision)

The units of measurement for the area of plane shapes including a rectangle are cm^2, m^2 or km^2.

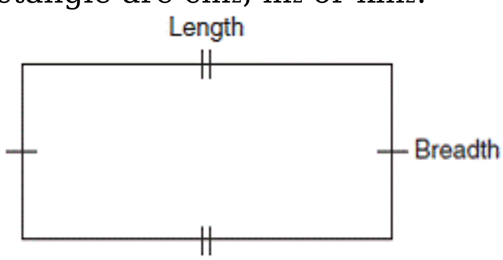


Figure 7.1

Area of rectangle = length × breadth

$$\text{Breadth} = \frac{\text{area}}{\text{length}}$$

and

$$\text{Length} = \frac{\text{area}}{\text{breadth}}$$

Worked Example 1

The following diagram is a rectangle. If its perimeter is 36 m, find the area of the rectangle.

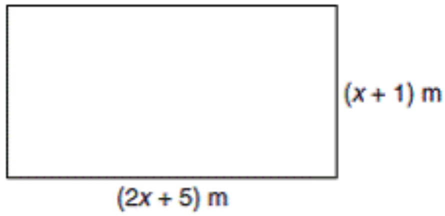


Figure 7.2

SOLUTION

Given: $l = (2x + 5)$ m, $b = (x + 1)$ m $P = 36$ m

$$P = 2(l + b)$$

$$36 = 2[(2x + 5) + (x + 1)]$$

$$\frac{36}{2} = 2x + 5 + x + 1$$

$$18 = 3x + 6$$

$$18 - 6 = 3x$$

$$\frac{12}{3} = x$$

$$\therefore x = 4$$

$$l = (2x + 5)\text{ m} = (2(4) + 5)$$
$$\text{m} = (8 + 5)\text{ m} = 13\text{ m}$$

$$b = (x + 1)\text{ m} = (4 + 1)\text{ m} = 5\text{ m}$$

Hence, $A = l \times b$

$$= 13\text{ m} \times 5\text{ m} = 65\text{ m}^2$$

Worked Example 2

One side of a rectangle is 8 cm and the diagonal is 10 cm. What is the area of the rectangle?

SOLUTION

Given: $l = 8$ cm
$b = ?$

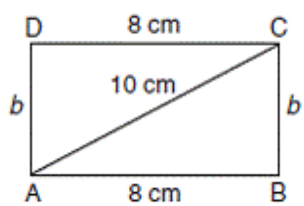


Figure 7.3

Diagonal = 10 cm
Area = ?

$$10^2 = b^2 + 8^2$$
$$10^2 - 8^2 = b^2$$
$$100 - 64 = b^2$$
$$b^2 = \sqrt{36}$$
$$b = 6$$

$$\text{Area of rectangle} = l \times b$$
$$= 8 \text{ cm} \times 6 \text{ cm}$$
$$= 48 \text{ cm}^2$$

Worked Example 3

The difference between the length and width of a rectangle is 6 cm and its area is 135 cm2. What is the length?

SOLUTION

Given: $l - w = 6$ cm, Area $= 135$ cm$^2 = ?$

If $l - w = 6$

$$l = 6 + w \text{ and } w = w$$

But, $A = l \times w$

$$135 = (6 + w)w$$
$$135 = 6w + w^2$$
$$w^2 + 6w - 135 = 0$$
$$w^2 - 9w + 15w - 135 = 0$$
$$w(w - 9) + 15(w - 9) = 0$$
$$(w - 9)(w + 15) = 0$$

either $\quad w - 9 = 0$ or $w + 15 = 0$

$$w = 9 \text{ or } w = -15;$$
$$w = 9, w \neq -15$$

Then $l = 6 + 9 = 15$.

Therefore, the length of the rectangle is 15 cm.

Worked Example 4

A rectangle lawn has an area of 1815 square yards. If its length is 50 m, find its width in metres, given that 1 m equals 1.1 yd.

SOLUTION

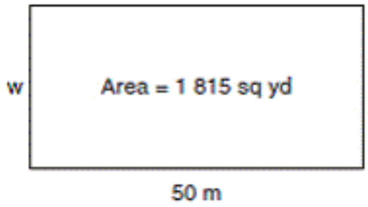


Figure 7.4

Given: $1\text{ m} = 1.1\text{ yd}$

$$l = 50\text{ m} = 50 \times 1.1\text{ yd} = 55\text{ yd}$$

$$b = ?$$

$$A = 1\,815\text{ sq yd}$$

Formula: $A = l \times b$

$$= 1\,815\text{ sq yd} = 55\text{ yd} \times w$$

$$w = \frac{1\,815}{55}\text{ yd}$$

$$w = 33\text{ yds}$$

but $\quad 1.1\text{ yd} = 1\text{m}$

$$1\text{ yd} = \frac{10}{11}\text{ k}$$

Hence, width $= \frac{33}{1} \times \frac{10}{11}\text{ m} = \frac{30}{1}\text{ m}$

$$= 30\text{ m.}$$

Worked Example 5

A rectangular picture 6 cm by 8 cm is enclosed by a frame $\frac{1}{2}$, cm wide. Calculate the area of the frame.

SOLUTION

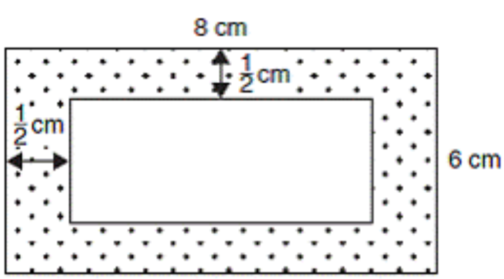


Figure 7.5

Length of the outer rectangle = 8 cm Breadth of the outer rectangle = 6 cm

Area of the outer rectangle

$= 8\text{ cm} \times 6\text{ cm} = 48\text{ cm}^2$

Length of the inner rectangle

$= 8\text{ cm} - \frac{1}{2}\text{ cm} - \frac{1}{2}\text{ cm} = 7\text{ cm}$

Breadth of the inner rectangle

$= 6\text{ cm} - \frac{1}{2}\text{ cm} - \frac{1}{2}\text{ cm} = 5\text{ cm}$

Area of the inner rectangle

$= 7\text{ cm} \times 5\text{ cm} = 35\text{ cm}^2$

Area of the frame = Area of the outer rectangle − Area of the inner rectangle

$= 48\text{ cm}^2 - 35\text{ cm}^2$

$= 13\text{ cm}^2$

Exercise 1

1. The perimeter of a rectangular lawn is 24 *m*. If the area of the lawn is 35 m^2, find the width of the lawn.
2. A rectangular farm 2 m long, 85 cm wide is filled up with humus soil. Find the cost of the humus soil at ₦55 per m^2.
3. The length of a rectangle is 3 times its width. If the perimeter is 72 cm, calculate the width of the rectangle. (WAEC)
4. A rectangle of 8 cm long is equal in area to a square which has a perimeter of 28 cm. Find the width of the rectangle.
5. EFGH is the rectangular floor of a room in which a carpet of 5 m by 3 m is laid leaving a uniform margin of a x round it. If the total area of the margin is 20 m^2, find the value of x. (WAEC)
6. The area of a rectangular floor is 13.5 m^2 and one side is 1.5 m longer than the other.
   (a) Calculate the dimensions of the floor.
   (b) If it costs ₦250.00 per square metre to carpet the floor and only ₦220.00 is available, what area of the floor can be covered with carpet? (WAEC)

7. A rectangular lawn of the length $(x + 5)$ m is $(x - 2)$ m wide. If the diagonal is $(x + 6)$ m, find the area of the lawn. (WAEC)

8. The sides of a rectangular floor are x m and $(x + 7)$ m. If the diagonal is $(x + 8)$ m, calculate the area of the floor in metres. (WAEC)

9. A rectangular playing field is 18 m wide. It is surrounded by a path 6 m wide such that its area is equal to the area of the path. Calculate the length of the field.

10. Calculate the area of the shaded portion in the following diagram.

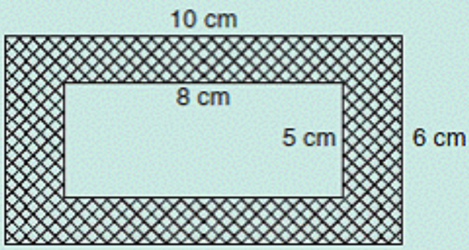


Figure 7.6

II. Area of a Square

A square is a quadrilateral whose sides are equal in length. Hence, the area of a square is calculated as the square of its length of side.

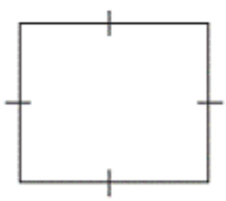


Figure 7.7

Area of the figure in Figure 7.7

$= x \text{ cm} \times x \text{ cm}$

$= x \text{ cm}^2$

$= (\text{length of side})^2$

Square of the length of side = Area of the square

Worked Example 6

In a quadrilateral ABCD, the diagonals AC and BD bisect each other at right angles

$|AC| = 16$ cm and $|BD| = 30$ cm.

(a) Find $|AB|$.

(b) What type of quadrilateral is ABCD?

(c) Calculate the area of the quadrilateral.

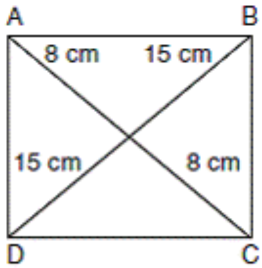


Figure 7.8

SOLUTION

(a) $|AB|^2 = (8 \text{ cm})^2 + (15 \text{ cm})^2$

$= 64 \text{ cm}^2 + 225 \text{ cm}^2$

$= 289 \text{ cm}^2$

$|AB| = \sqrt{289 \text{ cm}^2}$

$= 17$ cm

(b) Quadrilateral *ABCD* is a square since all the sides are equal in length.

(c) Area of the quadrilateral $= (17 \times 17) \text{ cm}^2$.

Worked Example 7

A square picture of length 10 cm is enclosed by a frame $\frac{1}{2}$ cm wide. Calculate the area of the frame.

SOLUTION

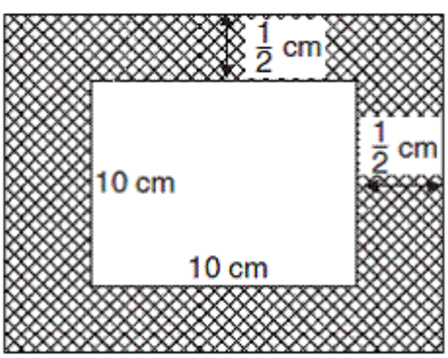


Figure 7.9

Length of small square = 10 cm

Area of small square $= (10 \text{ cm})^2$

$= 10 \text{ cm} \times 10 \text{ cm}$
$= 100 \text{ cm}^2$

Length of big square

$= \frac{1}{2} \text{ cm} + 10 \text{ cm} + \frac{1}{2} \text{ cm}$

$= 11 \text{ cm}$

Area of big square $= 11 \text{ cm} \times 11 \text{ cm}$
$= 121 \text{ cm}^2$

Area of the frame = Area of big square – Area of a small square
$= 121 \text{ cm}^2 - 100 \text{ cm}^2$
$= 21 \text{ cm}^2$

Worked Example 8

Find the length of a square which is equal in area to a rectangle measuring 45 cm by 5 cm.

SOLUTION

Given

Area of a square = Area of a rectangle
$(\text{Length of side})^2 = \text{length} \times \text{breadth}$
$(\text{Length of side})^2 = 45 \text{ cm} \times 5 \text{ cm}$
Length of side $= \sqrt{225}$ cm
Length of side = 15 cm

Worked Example 9

The area of a square is 144 sq cm. Find the length of the diagonal.

SOLUTION

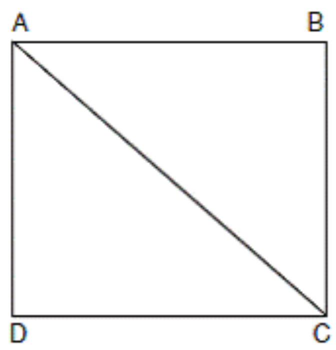


Figure 7.10

Area $= 144 \text{ cm}^2$
$(\text{Length of side})^2$ = Area of a square
Length of side $= \sqrt{144}$ cm
Length of side = 12 cm
Hence $|AC|^2 = |AD|^2 + |CD|^2$
$|AC|^2 = (12 \text{ cm})^2 + (12 \text{ cm})^2$
$|AC|^2 = 144 \text{ cm}^2 + 144 \text{ cm}^2$
$|AC|^2 = 288 \text{ cm}^2$

$|AC| = \sqrt{288}$ cm

$|AC| = 16.97$ cm

The length of the diagonal is 16.97 cm.

Exercise 2

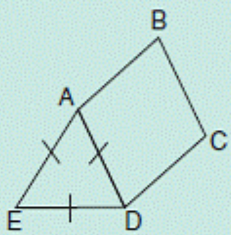
1. If the perimeter of a square room is 28 cm, what is the area of the room?
2. A square board is taped at the perimeter by a piece of ribbon 20 cm long. What is the area of the board? (UME)
3. Find the length of a diagonal of square whose area is 288 cm². (NECO)
4. Calculate the area of a square which is equal in perimeter to that of an equilateral triangle of side 8 cm.
5. If the area of a square is 169 cm² and the length of its side is $(x - 3)$ cm, calculate the value of x.
6. 

Figure 7.11

From the above diagram, if ABCD is a square, calculate the area of ABCD if ED = 4cm.

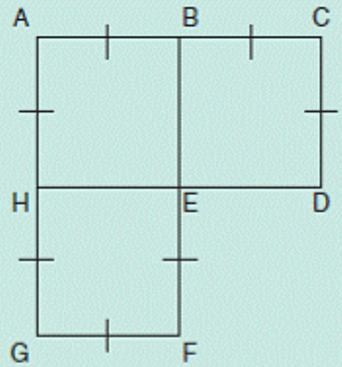
7. 

Figure 7.12

Calculate the total area of ABCDEFGH if GE is $2\sqrt{2}$ cm.

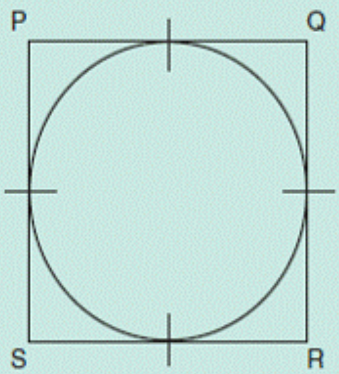
8. 

Figure 7.13

Calculate the area of PQRS in the above diagram, if the circumference of the circle is 88 cm.

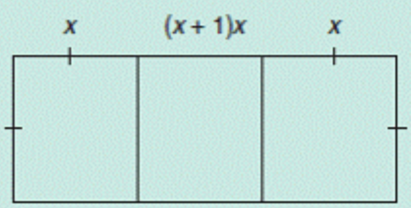
9. 

Figure 7.14

From the diagram above, calculate the value of $(x + 1)$ if the area of the diagram is $30\ cm^2$.

10. Copy and complete the following table of square.

	Length of side	Area
(a)	5 cm	–
(b)	–	$196\ m^2$
(c)	$3\frac{1}{4}$	–
(d)	1.8 m	–
(e)	–	$12\frac{1}{4}$

Table 1

III. Area of a Parallelogram

A parallelogram is a quadrilateral with two pairs of parallel and equal sides.

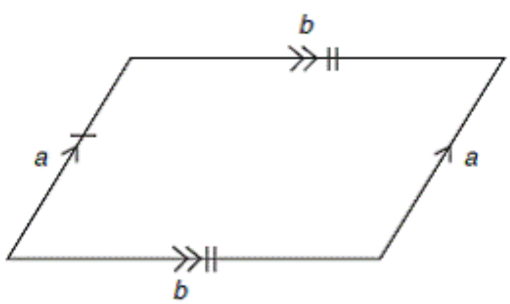


Figure 7.15

The area of a parallelogram is simply the product of its base and perpendicular height.

Hence, area of parallelogram in Figure 7.15 = $b \times h$, where b is the base and h the perpendicular height.

Worked Example 10

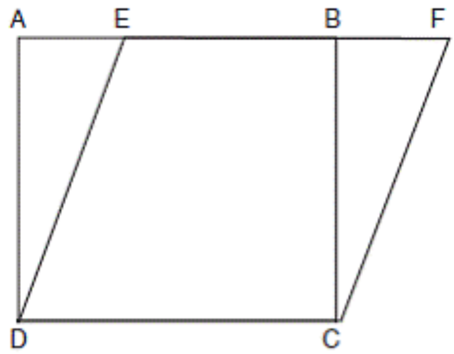


Figure 7.16

In the diagram above, ABCD is a rectangle. E is the mid-point of AB, DE is parallel to CF and AEBF is a straight line. If the area of triangle ADE is 15 cm^2, find the area of the parallelogram EFCD.

SOLUTION

Given: Area of $\Delta ADE = 15\ cm^2$.

Let $AE = x$, $EB = x$.

Then, $DC = x + x = 2x$ *(opposite side of a rectangle)*

But area of $\Delta\ ADE = \frac{1}{2} \times |AE| \times |AD|$

$$15 = \frac{1}{2} \times x \times AD$$

$$\frac{15 \times 2}{x} = AD$$

$$AD = \frac{30}{x}$$

So, area of parallelogram EFCD = base × height.

$= (DC \times AD)\ cm^2$

$= 2x \times \frac{30}{x}\ cm^2$

$= (2 \times x \times \frac{30}{x})\ cm^2$

$= 60\ cm^2$.

Worked Example 11

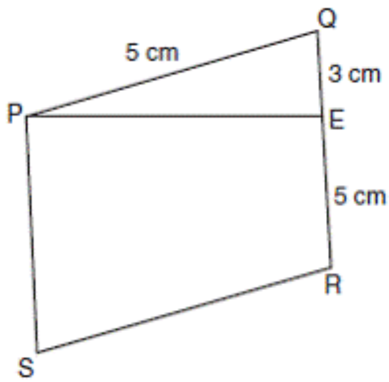


Figure 7.17

In the parallelogram PQRS, PE is perpendicular to QR. Find the area of the parallelogram.

SOLUTION

Given: $|PQ| = 5$ cm

$|QE| = 3$ cm

$|ER| = 5$ cm

From the right-angled triangle EPQ.

$5^2 = 3^2 + |PE|^2$

$25 - 9 = |PE|^2$

$|PE| = 4$ cm

Area of parallelogram PQRS = QR × PE

= (3 + 5) cm × 4 cm

= 8 cm × 4 cm

= 32 cm^2.

Worked Example 12

JKLM is a parallelogram where $\overline{MN} \perp \overline{JK}$. If $|JM| = 13$ cm, $|MN| = 5$ cm and $|NK| = 10$ cm. Find the area of the parallelogram.

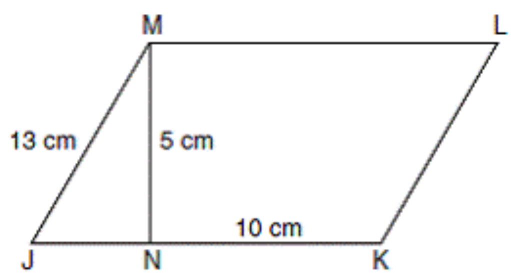


Figure 7.18

SOLUTION

$13^2 = |JN|^2 + 5^2$

$|JN| = 13^2 - 5^2$

$= 169 - 25 = 144$

$|JN| = \sqrt{144} = 12$

Area of parallelogram $= |JK| \times |MN|$

$= (12 \text{ cm} + 10 \text{ cm}) \times 5 \text{ cm}.$

$= 22 \text{ cm} \times 5 \text{ cm}$

$= 110 \text{ cm}^2.$

Exercise 3

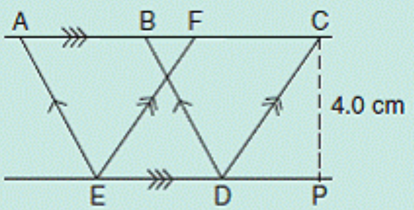
1. 

Figure 7.19

In Figure 7.19, ABDE and FCDE are parallelograms. If $|FC| = 12.2$ cm and the height $|PC| = 4.0$ cm, calculate the area of the parallelogram ABDE. (WAEC)

2. The area of a parallelogram with a height of 19 cm is 513 cm². Calculate the base of the parallelogram. (WAEC)

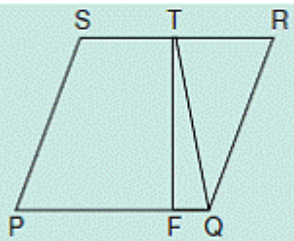
3. 

Figure 7.20

PQRS is a parallelogram with an area 50 sq cm and the side PQ is 10 cm long. T is a point on RS and TF is the altitude of the triangle TFQ. Find |TF|. (WAEC)

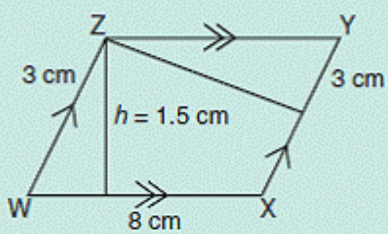
4. 

Figure 7.21

The parallelogram WXYZ is such that $|WZ| = 3$ cm, $|WX| = 8$ cm and a perpendicular from Z to $|WX| = 1.5$ cm (h). What is the length of the perpendicular from Z to XY? (NECO)

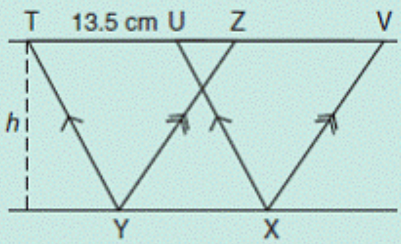
5. 

Figure 7.22

In the diagram above, TUXY and ZVXY are parallelograms. If $|TU| = 13.5$ cm and the area of parallelogram ZVXY is 67.5 cm², calculate its height. (NECO)

6. Calculate the height of a parallelogram whose area and base are 65 cm² and 13 cm, respectively.
7. Find the base of a parallelogram whose height is 10 cm and area equal to 135 cm².

In each of the parallelograms below, calculate the area, the height or the base as applicable.

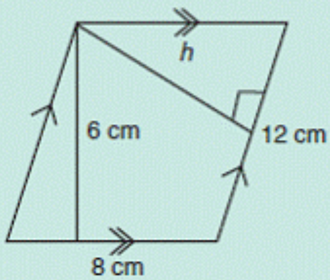
8. 

Figure 7.23

9.

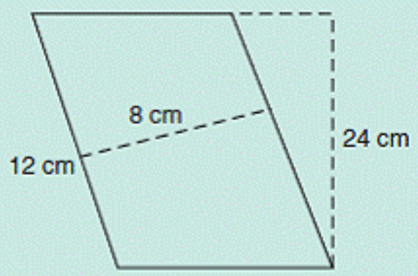


Figure 7.24

10.

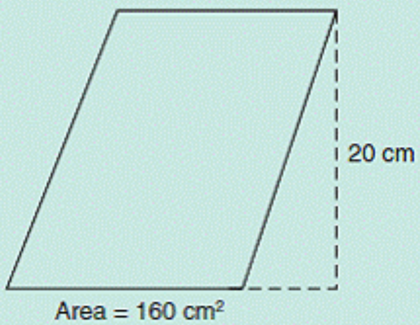


Figure 7.25

IV. Area of a Trapezium

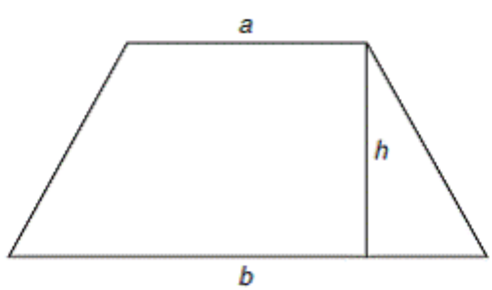


Figure 7.26

Figure 7.26 shows a trapezium with *a* and *b* as parallel sides and *h* the perpendicular height.

Area of trapezium $A = \frac{1}{2}(a + b)h$

Worked Example 13

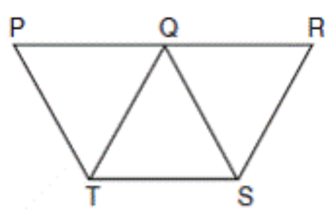


Figure 7.27

PRST is a trapezium with PR _ TS, TS = 14 cm and the distance between the parallel lines is 8 cm. Find the area of the trapezium.

SOLUTION

Given: |TS| = 14 cm, and h = 8 cm

But |PQ| = |TS| = 14 cm and |QR| = TS = 14 cm (opposite sides of a parallelogram)

Therefore, |PR| = |PQ| + |QR| = 14 cm + 14 cm = 28 cm.

$$\text{Area of the trapezium} = \left[\frac{1}{2}(28+14)8\right]^2$$

$$= \frac{1}{2} \times \frac{42}{1} \times \frac{8^2}{1}$$

$$= \frac{168}{1}\text{ cm}^2$$

$$= 168\text{ cm}^2$$

Worked Example 14

In the trapezium XYZM, $\overline{XY} \| \overline{MZ}$ $\angle XYZ$ = 90°. |PZ| = 6 cm. |MZ| = 9 cm and |MX| = 5 cm.

What is the area of the trapezium?

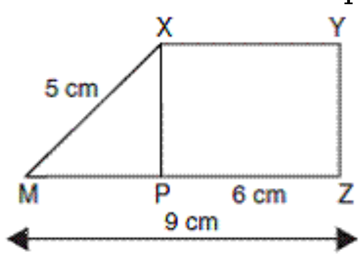


Figure 7.28

SOLUTION

$$|MP| = |\overline{MZ}| - |\overline{PZ}| = 9\text{ cm} - 6\text{ cm}$$

$$= 3\text{ cm}$$

$$(5\text{ cm})^2 = (3\text{ cm})^2 + |PX|^2$$

$$|PX|^2 = 25\text{ cm}^2 - 9\text{ cm}^2$$

$$= 16\text{ cm}^2$$

$$= \sqrt{16^2} = 4\text{ cm}$$

$$|XY| = |PZ| = 6\text{ cm}$$

$$\text{Area of the trapezium} = \frac{1}{2}(a+b)h$$

$$= \left[\frac{1}{2}(6+9) \times 4\right]\text{cm}^2$$

$$= \left(\frac{1}{2} \times 15 \times 4\right)\text{cm}^2$$

$$= 30\text{ cm}^2$$

Worked Example 15

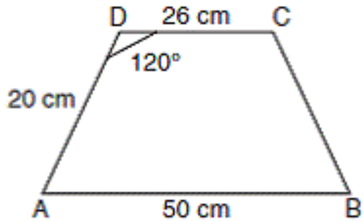


Figure 7.29

In Figure 7.29, ABCD is a trapezium with AB parallel to DC. Given that AB = 50 cm, AD = 20 cm, DC = 26 cm and ∠ADC = 120°, calculate the area, in cm2 to 3 significant figures, of ABCD.

SOLUTION

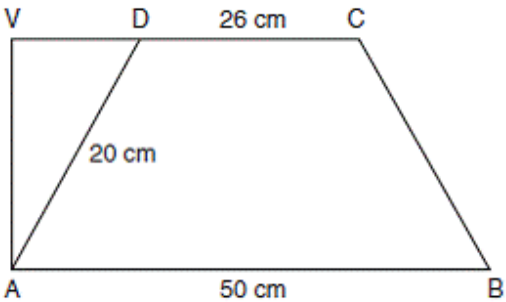


Figure 7.30

$\angle VDA = 180° - 120° = 60°$ (sum of angles on a straight line)

$$\sin VDA = \frac{VA}{20}$$

$$|VA| = 20 \text{ cm} \sin 60° = 20 \text{ cm} \times 0.8660$$

$$|VA| = 17.32 \text{ cm}$$

Area of trapezium ABCD

$$= \left[\frac{1}{2}(26 + 50)\ 17.32\right] \text{cm}^2$$

$$= \left[\frac{1}{2} \times 76 \times 17.32\right] \text{cm}^2$$

$$= 658.16 \text{ cm}^2$$

Worked Example 16

The distance between the parallel sides PQ and SR of a trapezium PQRS is 12 cm. Find SR if PQ is 5 cm and the area of the trapezium is 84 cm2.

SOLUTION

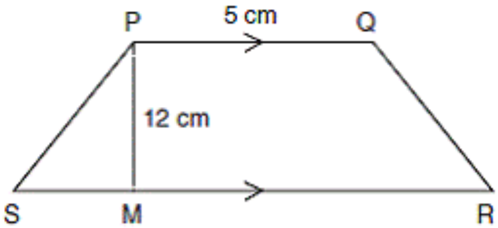


Figure 7.31

$$\text{Area} = 84\ \text{cm}^2$$

$$\text{Area of trap PQRS} = \frac{1}{2}(a+b)h$$

$$84 = \frac{1}{2}(5+\overline{SR})12$$

$$\frac{84\times 2}{12} = 5+\overline{SR}$$

$$7\times 2 = 5 + SR$$

$$14 - 5 = |SR|$$

$$\text{Therefore, } |SR| = 9\ \text{cm}$$

Exercise 4

1.

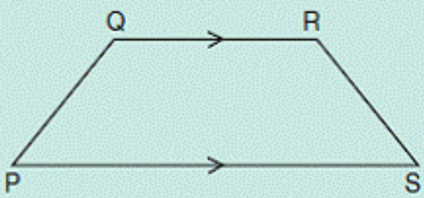


Figure 7.32

In the diagram above, the area of PQRS is 73.5 cm² and its height is 10.5 cm². Find the length of $\overline{PS}$ if $\overline{QR}$ is one-third of $\overline{PS}$. (UME)

2.

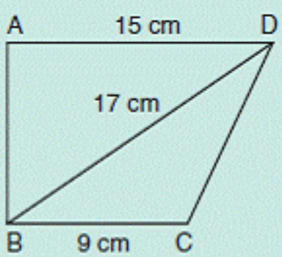


Figure 7.33

In Figure 7.33, ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$ and $\angle ABC$ is a right angle. If $|AD| = 15$ cm, $|BD| = 17$ cm and $|BC| = 9$ cm, calculate the area of the trapezium.

3. Calculate the area of the trapezium given below.

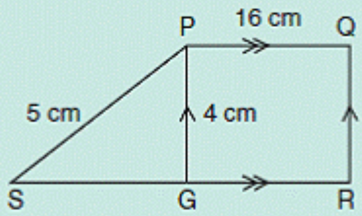


Figure 7.34

4. The lengths of the two parallel sides of a trapezium are 6 cm and 10 cm and the perpendicular distance between them is 5 cm. Find the area of the trapezium.

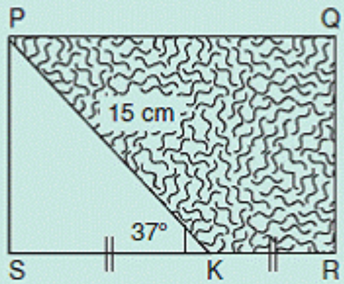
5. 

Figure 7.35

In the diagram, PQRS is a rectangle $|PK| = 15$ cm, $|SK| = |KR|$ and $\angle PKS = 37°$. Calculate the area of the shaded portion, correct to three significant figures. (WAEC)

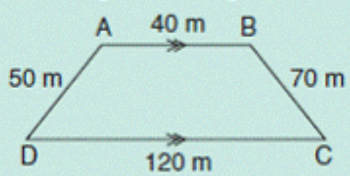
6. 

Figure 7.36

The above diagram represents a playing field in the form of a trapezium. If $|AB| = 40$ m, $|BC| = 70$ m, $|DC| = 120$ m, $|AD| = 50$ m and $AB \parallel DC$, calculate the area of the field correct to three significant figures. (WAEC)

7. The area of the trapezium given below is 48 cm². What is the value of x?

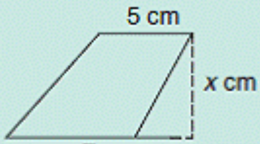


Figure 7.37 (NECO)

8. In the diagram below, EAD is a straight line. ABCD is a parallelogram with base BC and height 4 cm. If $|AD| = 10$ cm and the area of ΔBAE is 30 cm², find the area of the trapezium BCDE. (NECO)

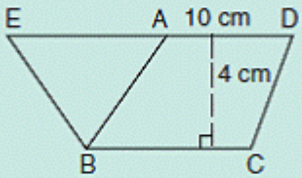


Figure 7.38

V. Area of a Rhombus

A rhombus is a quadrilateral whose sides are equal in length and opposite angles equal in size.

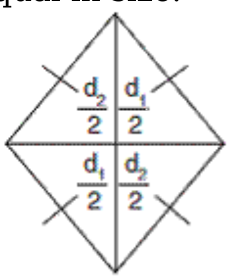


Figure 7.41

A rhombus is sub-divided into four right-angled triangles by its two diagonals. Hence, the area of a rhombus is calculated as 4, multiplied by the area of one of its right-angled triangles.

$$\text{Area of the rhombus} = 4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$$

where d_1 and d_2 are the two diagonals of the rhombus.

Worked Example 17

The length of one of the diagonals of a rhombus is 6 cm long. If the second diagonal is one-and-half times longer than the first diagonal, find the area of the rhombus.

SOLUTION

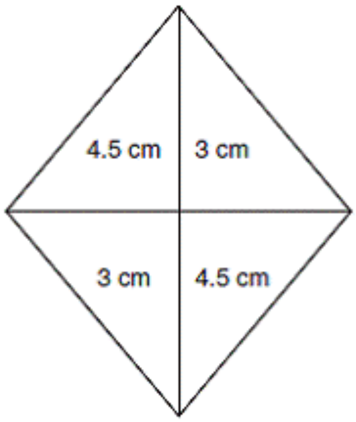


Figure 7.42

Given: $d_1 = 6$ cm

$d_2 = 1\frac{1}{2} \times 6$ cm

$= \frac{3}{2} \times 6$ cm

$= 9$ cm

$$\text{Area of the rhombus} = 4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$$

$$= 4 \times \frac{1}{2} \times \frac{6\text{ cm}}{2} \times \frac{9\text{ cm}}{2}$$

$$= 2 \times 3\text{ cm} \times 4.5\text{ cm}$$

$$= 27.0\text{ cm}^2 = 27\text{ cm}^2$$

Worked Example 18

The area and a diagonal of a rhombus are 60 cm^2 and 12 cm, respectively. Calculate the length of the other diagonal.

SOLUTION

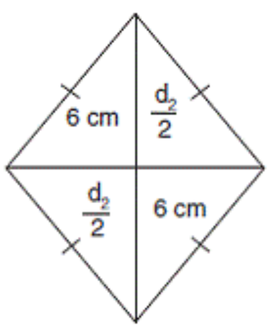


Figure 7.43

Given: $d_1 = 12$ cm, Area $= 60$ cm²

Formula: Area of rhombus

$$= 4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$$

$$60 = 2 \times \frac{12}{2} \times \frac{d_2}{2}$$

$$\frac{60 \times 2 \times 2}{12 \times 2} = d_2$$

$$d_2 = 10 \text{ cm}$$

Length of the other diagonal is 10 cm.

Exercise 5

1. A side of a rhombus is 2 cm in length. An angle of the rhombus is 60°. What is the area of the rhombus?
2. The diagonals PR and QS of a rhombus are 2 cm and 4 cm long, respectively. Calculate the length of side of the rhombus if its area is 100 cm².
3. The area and a diagonal of a rhombus are 120 cm² and 24 cm, respectively. Calculate the length of the other diagonal.
4. Calculate the area of a rhombus whose diagonals are $\sqrt{3}$ cm and $\sqrt{2}$ cm, respectively. Leave your answer in surd form.

Calculate the area of a rhombus whose diagonals are:

5. 18 cm and 9 cm
6. 22 cm and 12 cm
7. 26 cm and 10 cm

Find the area of rhombuses in Questions 8–10 whose given angle and a length of side are respectively:

8. 100° and 5 cm
9. 84° and 7 cm
10. 160° and 12 cm

VI. Area of a Triangle

A triangle is a three-sided closed plane shape whose area is determined based on the nature of the triangle. The first set of triangles to be considered are right angled triangle and isosceles triangle.

The formula to use here is $A = \frac{1}{2}bh$, where A is the area, b is the base and h is the perpendicular height.

Worked Example 19

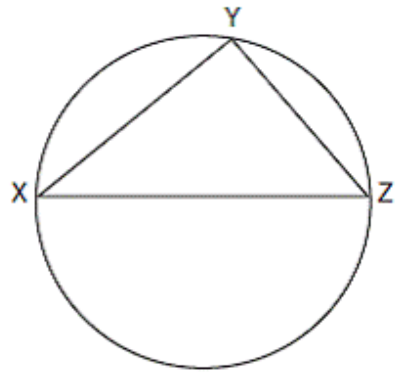


Figure 7.44

In the above diagram XZ is the diameter of a circle of radius $\frac{5}{2}$ cm. If XY is 4 cm, calculate the area of triangle XYZ.

SOLUTION

$$|XY| = 4 \text{ cm (given)}$$

$$\text{Radius} = \frac{5}{2} \text{ cm (given)}$$

Diameter |XY|

$$= 2 \times \text{radius} = 2 \times \frac{5}{2} \text{ cm} = 5 \text{ cm}$$

$$\angle XYZ = 90° \quad \text{(angle in a semi-circle)}$$

$$XZ^2 = XY^2 + YZ^2$$

$$5^2 = 4^2 + YZ^2$$

$$YZ^2 = 25 - 16 = 9$$

$$YZ = \sqrt{9} = 3 \text{ cm}$$

$$\text{Area of } \Delta XYZ = \frac{1}{2}bh$$

$$= \frac{1}{2} \times YZ \times XY$$

$$= \left(\frac{1}{2} \times 3 \times 4\right) \text{cm}^2 = 6 \text{ cm}^2$$

Worked Example 20

In an isosceles triangle PQR, |PQ| = 26 cm, |QR| = 26 cm and |PR| = 20 cm. Calculate its area.

SOLUTION

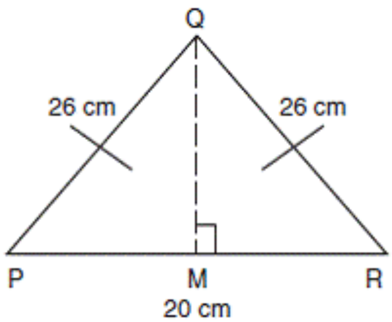


Figure 7.45

Draw a perpendicular line from P to |PR| at M.

$|PM| = |MR|$

Hence $|PM|$

$$= \frac{1}{2}|PR| = \frac{1}{2} \times 20 \text{ cm} = 10 \text{ cm}$$

$$|PQ|^2 = |PM|^2 + |QM|^2$$

$$26^2 = 10^2 + |QM|^2$$

$$|QM|^2 = 26^2 - 10^2$$

$$|QM|^2 = (26 + 10)(26 - 10)$$

$$|QM|^2 = 36 \times 18$$

$$|QM| = \sqrt{576} = 24$$

Area of $\Delta PQR = \frac{1}{2} bh$

$$= \frac{1}{2} \times |PR| \times |QM|$$

$$= \left(\frac{1}{2} \times 20 \times 24\right) \text{cm}^2$$

$$= 240 \text{ cm}^2$$

Worked Example 21

An equilateral triangle has an altitude of 10 cm. Find the area of the triangle.

SOLUTION

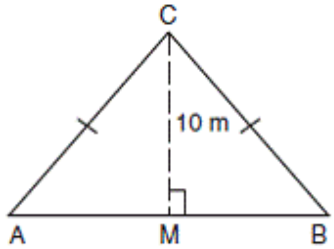


Figure 7.46

Given: Altitude CM = 10 m

$A + B + C = 180°$ (sum of angles in ΔABC)

But $A = B = C = 60°$ (each angle of an equilateral ΔABC)

$$\sin A = \frac{CM}{AC}$$

$$\sin 60° = \frac{10\text{ m}}{AC}$$

$$0.8660 = \frac{10\text{ m}}{AC}$$

$$|AC| = \frac{10\text{ m}}{0.8660}$$

$$|AC| = 11.5\text{ m}$$

$$\text{Hence, } AB = 11.5\text{ m}$$

$$\text{So, area of } \Delta ABC = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 11.5\text{ m} \times 10\text{ m}$$

$$= 57.5\text{ m}^2$$

Worked Example 22

Calculate the area of the following triangle.

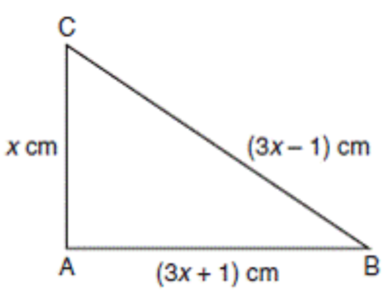


Figure 7.47

SOLUTION

$$(3x+1)^2 = x^2 + (3x-1)$$

$$9x^2 + 6x + 1 = x^2 + 9x^2 - 6x + 1$$

$$0 = x^2 + 9x^2 - 6x + 1 - 9x^2 - 6x - 1$$

$$0 = x^2 - 12x$$

$$x(x-12) = 0$$

either $x = 0$ or $x - 12 = 0$

$$x = 0 \text{ or } x = 12$$

but $x \neq 0, x = 12$

so, $(3x+1)\text{ cm} = [(3 \times 12)1]\text{ cm}$

$$= (36+1)\text{ cm} + 37\text{ cm}$$

$$x\text{ cm} = 12\text{ cm}$$

And $(3x-1)\text{ cm} = [(3 \times 12) + 1]\text{ cm}$

$$= (36-1)\text{ cm} = 35\text{ cm}$$

Hence, Area of $\Delta ABC = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times |AB| \times |AC|$$

$$= \frac{1}{2} \times 37\text{ cm} \times 12\text{ cm}$$

$$= (6 \times 37)\text{ cm}^2$$

$$= 222\text{ cm}^2$$

Another type of triangle is a triangle whose three sides are given. The formula to use here is called Hero's formula and it states that

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$

A is the area of the triangle, *s* is the semi-perimeter and *a, b, c* are the length of sides of the triangle.

SOLUTION

Given: $a = 6\text{ cm}, b = 8\text{ cm}, c = 10\text{ cm}$

$$s = \frac{a+b+c}{2} = \frac{6+8+10}{2}\text{ cm} = \frac{24}{2}\text{ cm}$$

$$= 12\text{ cm}$$

Area of the triangle =

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{12(12-6)(12-8)(12-10)}\text{ cm}^2$$
$$=\sqrt{12\times 6\times 4\times 2}\text{ cm}^2$$
$$=\sqrt{4\times 3\times 3\times 2\times 4\times 2}\text{ cm}^2$$
$$=\sqrt{4\times 9\times 4\times 4}\text{ cm}^2$$
$$=2\times 3\times 2\times 2\text{ cm}^2$$
$$=24\text{ cm}^2$$

Worked Example 24

Find the area of an equilateral triangle whose length of sides is 4 cm.

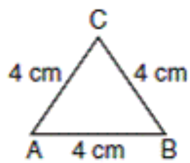


Figure 7.49

SOLUTION

$$s=\frac{4+4+4}{2}\text{ cm}=\frac{12}{2}\text{ cm}$$
$$=6\text{ cm}$$

Area of the triangle

$$=\sqrt{6(6-4)(6-4)(6-4)}^2$$
$$=\sqrt{6\times 2\times 2\times 2}\text{ cm}^2$$
$$=\sqrt{48}\text{ cm}^2$$
$$=6.9\text{ cm}^2$$

The last but not the least type of triangle to be considered is a triangle whose two sides and one included angle are given. The formula to be used depends on the given angle. For instance, the formulas for calculating the areas of the below given triangles are given following the figures.

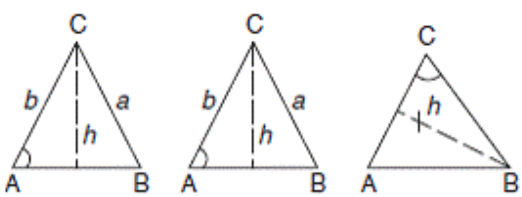


Figure 7.50

Worked Example 25

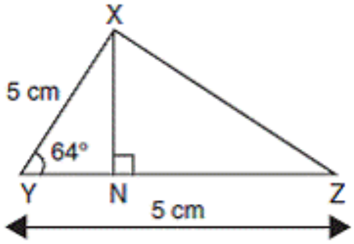


Figure 7.51

In the above diagram, XYZ is a triangle in which |XY| = |YZ| = 5 cm. If $\angle XYZ =$ 64°and XN is perpendicular to YZ, calculate, correct to two decimal places, the area of the triangle.

SOLUTION

Given: Z = 5 cm, X = 5 cm, and Y = 64°

Formula: Area of ΔXYZ

$$= \frac{1}{2} \times 5 \times 5 \times 64°$$

$$= \frac{25}{2} \times 0.8988$$

$$= 11.235 \text{ cm}^2$$

$$= 11.24 \text{ cm}^2 \text{ (2 d.p.)}$$

Worked Example 26

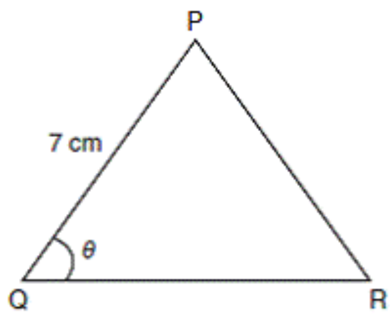


Figure 7.52

If the area of the triangle shown above is $21\sqrt{3}$ cm², what is the value of θ?

SOLUTION

Given: $R = 7$ cm, $P = 12$ cm, $Q = \theta$

Formula: Area $= 21\sqrt{3}$ cm²

$$\text{Area of } \Delta PQR = \frac{1}{2} \times 12 \times 7 \times \theta$$

$$21\sqrt{3} = \frac{1}{2} \times 12 \times 7 \times \theta$$

$$= \frac{2 \times 3 \times \sqrt{3}}{12 \times 7} = \theta$$

$$= \frac{6}{12} \times \sqrt{3}$$

$$= \frac{\sqrt{3}}{2} = \sin\theta$$

$$\sin\theta = 0.8660$$

$$\theta = \sin^{-1} 0.8660$$

$$\theta = 60°$$

Worked Example 27

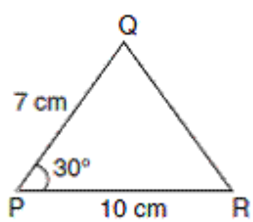


Figure 7.53

In the diagram above, find PQ if the area of triangle PQR is 35 cm2.

SOLUTION

Given: $R = ?$, $Q = 10$ cm, $P = 30°$

Area $= 35$ cm²

Formula: Area of $\Delta PQR = \frac{1}{2}$

$$35 = \frac{1}{2} \times 10 \times 30°$$

$$\frac{35 \times 2}{10 \times 0.5} = R = \frac{70}{5} = 14$$

So, $|PQ| = R = 14$ cm.

Worked Example 28

If the hypotenuse of a right-angled isosceles triangle is 2 cm, what is the area of the triangle?

SOLUTION

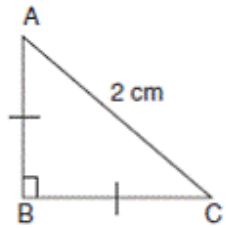


Figure 7.54

Given: $AC = 2$ cm, $AB = BC$

$$|AB|^2 + |BC|^2 = |AC|^2$$
$$|AB|^2 + |AB|^2 = 2^2$$
$$2|AB|^2 = 4$$
$$|AB|^2 = \frac{4}{2} = 2$$
$$|AB| = \sqrt{2}\ \text{cm}^2$$
$$\text{and } |BC| = \sqrt{2}\ \text{cm}^2$$

Hence, area of ΔABC

$$= \left(\frac{1}{2} \times 2 \times 1\right)^2$$
$$= 1\ \text{cm}^2$$

Exercise 6

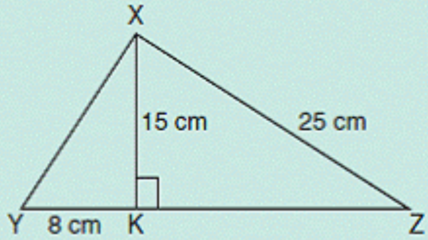
1. 

Figure 7.55

In ΔXZY above, $XKZ = 90°$

$XK = 15$ cm, $XZ = 25$ cm and $YK = 8$ cm. Find the area of ΔXYZ.

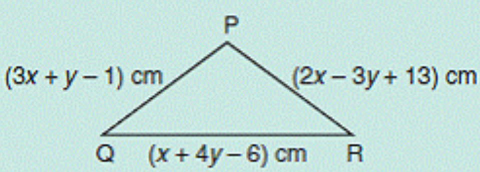
2. Calculate the altitude of a triangle ABC of area 42 cm^2, if the base is 14 cm.
3. Find the area of a triangle whose sides are 5 cm, 6 cm and 7 cm.
4. XYZ is a triangle with $XZ = 9$ cm and $YZ = 10$ cm. The altitude of the triangle when XZ is a base line is 8 cm. Find altitude when the side YZ is the base.
5. PQR is an isosceles triangle with $|PQ| = |PR| = 8$ cm and $\angle QPR = 120°$. Calculate the area of the triangle.
6. ABC is a triangle, right-angled at C. P is the mid-point of AC, $\angle PBC = 37°$ and $|BC| = 5$ cm. Calculate the area of triangle ABC.
7. 

Figure 7.56

PQR is an equilateral triangle. If $|PQ| = (3x + y - 1)$ cm, $|QR| = (x + 4y - 6)$ cm and $|RP| = (2x - 3y + 13)$ cm. Calculate the area of the triangle. (WAEC)

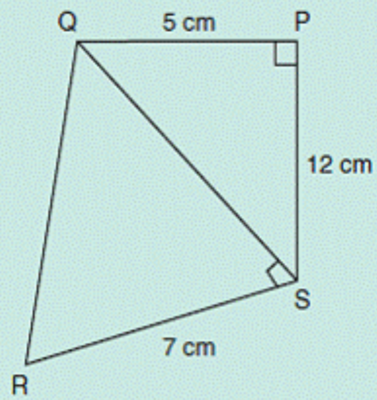
8. 

Figure 7.57

In the diagram, PQRS is a quadrilateral. Given that $\angle QPS = \angle QSR = 90°$, $|PQ| = 5$ cm, $|PS| = 12$ cm and $|RS| = 7$ cm, calculate the area of the quadrilateral correct to one decimal place.

9. The side PQ and PR of ΔPQR are produced to T and S, respectively, such that $\angle TQR = 131°$, $\angle QRS = 98°$ and $|PQ| = 10$ cm. Calculate the area of the triangle.

10. Find the area of a triangle ABC such that $a = 16$ cm, $b = 14$ cm and $c = 12$ cm. Leave your answer in surd form. (NECO)

VII. Area of a Circle

Parts of a circle include semi-circle, quadrants of a circle, sector of a circle and segment of a circle.
Hence, the area of a circle including the area of the above-mentioned parts of a circle shall be calculated.

(i) Formula for the area of a circle

The area of a circle is calculated as the product of the square of its radius and π.

This is written as $A = \pi r^2$, where A is the area of a circle, $\pi = \frac{22}{7}$ or $3\frac{1}{7}$ or 3.142 and
r is the radius of the circle.

Worked Example 29

Find the area of a circle whose perimeter is 22 cm.

SOLUTION

Perimeter of a circle = 22 cm

Area of a circle = A

Perimeter of a circle = $2\pi r = 22$

$$r = \frac{22}{2 \times \pi} = \frac{11}{\pi}$$

$$A = \pi r^2$$

$$A = \pi \times \frac{11}{\pi} \times \frac{11}{\pi}$$

$$= 121 \div \pi$$

$$= 121 \div \frac{22}{7}$$

$$= \frac{121}{1} \times \frac{7}{22} \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2$$

$$= 38.5 \text{ cm}^2$$

Worked Example 30

What is the area between two concrete circles of diameters 26 cm and 20 cm?

SOLUTION

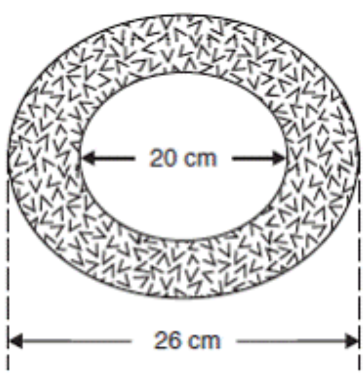


Figure 7.58

Given:

Diameter of the inner circle = 20 cm

Radius of the inner circle = 20 cm ÷ 2 = 10 cm

So, area of the inner circle = πr^2

$= (\pi \times 10 \times 10)$ cm^2

Diameter of the outer circle = 26 cm

Radius of the outer circle = 26 cm ÷ 2 = 13 cm

Area of the outer circle = πr^2

$(\pi \times 13 \times 13)$ cm2 = 169π cm^2

Area between the two concrete circles = Area of the outer circle − Area of the inner circle

= $(169\pi - 100\pi)$ cm^2

= 69π cm^2

Worked Example 31

If the circumference of a circle is equal to two-fifths of the sum of the perimeters of equilateral triangles of length 3 cm and a square of length 4 cm. Find the area of the circle.

SOLUTION

Perimeter of the equilateral triangle = 3 cm + 3 cm + 3 cm = 9 cm

Perimeter of the square = 4 cm + 4 cm + 4 cm + 4 cm = 16 cm

Sum of the perimeters of an equilateral triangle and a square = 9 cm + 16 cm = 25 cm.

Two-fifths of the sum of the perimeters = $\frac{2}{5} \times \frac{25}{1}$ cm = 10 cm

Circumference of a circle = $2\pi r$

But circumference of a circle = 10 cm (given)

Hence, $2\pi r = 10$ cm

$$2 \times \frac{22}{7} \times r = 10 \text{ cm}$$

$$r = \frac{10 \times 7}{44} = \frac{35}{22} \text{ cm}$$

$$\text{Area of the circle} = \frac{22}{7} \times \frac{35}{22} \times \frac{35}{22} \text{ cm}^2$$

$$= \frac{22 \times 35 \times 35}{7 \times 22 \times 22} \text{ cm}^2$$

$$= \frac{175}{22} \text{ cm}^2$$

$$= 7\frac{21}{22} \text{ cm}^2.$$

(ii) Area of a semi-circle and a quadrant of a circle

A semi-circle is half of a circle while the quadrant of a circle is a quarter of a circle. So, the area of a semi-circle is equal to half the area of a circle while that of a quadrant of a circle is one-quarter the area of a circle.

Given:

Area of a circle = πr^2

Area of a semi-circle = $\frac{1}{2}\pi r^2$

Area of a quadrant = $\frac{1}{4}\pi r^2$

Worked Example 32

In the figure below, PQR and RST are semi-circles with diameters 28 cm and 7 cm, respectively. Calculate the area of the figure below.

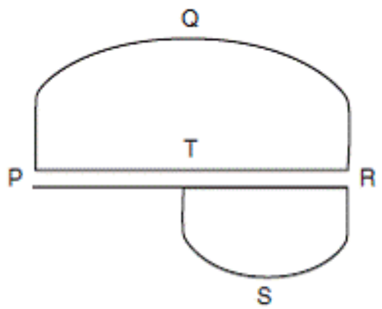


Figure 7.59

SOLUTION

Diameter of semi-circle PQR = 28 cm

Radius of the semi-circle PQR = 28 cm ÷ 2

$= 14$ cm

Area of semi-circle PQR $= \frac{\pi r^2}{2}$

$= \left(\frac{22}{7} \times \frac{14}{1} \times \frac{14}{1} \div \frac{2}{1}\right) \text{cm}^2$

$= \left(\frac{22}{7} \times \frac{14}{1} \times \frac{14}{1} \div \frac{1}{2}\right) \text{cm}^2$

$= 308 \text{ cm}^2$

Diameter of semi-circle RST = 7 cm

Radius of semi-circle RST $= \frac{7}{2}$ cm

Area of semi-circle RST

$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \div \frac{2}{1}\right) \text{cm}^2$

$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \div \frac{1}{2}\right) \text{cm}^2$

$= \frac{77}{4} \text{ cm}^2$

$= 19\frac{1}{4} \text{ cm}^2$

Area of the figure = Area of semi-circle PQR + Area of semi-circle RST

$= 308 \text{ cm}^2 + 19\frac{1}{4} \text{ cm}^2$

$= 327\frac{1}{4} \text{ cm}^2$.

Worked Example 33

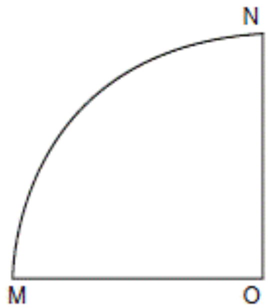


Figure 7.60

Figure 7.60 shows the quadrant of a circle. If the length of arc MN = 48 cm, calculate the area of the quadrant of the circle.

SOLUTION

Circumference of a circle $= 2\pi r$

Length of arc MN $= \frac{2\pi r}{4}$

$$48 = \frac{2\pi r}{4}$$

$$\frac{48 \times 4}{2} = \frac{22}{7} \times r$$

$$\frac{48 \times 2 \times 7}{22} = r$$

$$r = \frac{336}{11}$$

Area of quadrant of a circle

$$= \frac{\pi r^2}{4} = \frac{22}{7} \times \frac{336}{11} \times \frac{336}{11} \div \frac{4}{1}$$

$$= \frac{22}{7} \times \frac{336}{11} \times \frac{336}{11} \div \frac{1}{4}$$

$$= \frac{2 \times 336 \times 84}{7 \times 11}\ \text{cm}^2$$

$$= \frac{8064}{11}\ \text{cm}^2$$

$$= 733\frac{1}{11}\ \text{cm}^2.$$

(iii) Sector of a circle

Sector of a circle is the region between two radii and the circumference of a circle.

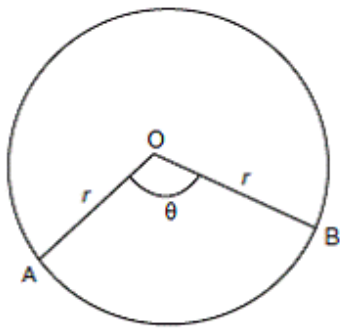


Figure 7.61

Formula:

$$\frac{\text{Area of the sector AOB}}{\text{Area of circle ABC}} = \frac{\text{Angle subtended at centre}}{\text{Sum of angles at a point}}$$

$$\frac{\text{Area of sector AOB}}{\pi r^2} = \frac{\theta}{360^\circ}$$

$$\text{Area of sector AOB} = \frac{\theta}{360^\circ} \times \pi r^2.$$

Worked Example 34

The angle of a sector of a circle of a diameter 8 cm is 135°. Find the area of the sector.

$\left(\text{Take } \pi = \frac{22}{7}\right)$ (WAEC)

SOLUTION

Given: $\theta = 135^\circ$

$d = 8 \text{ cm}, r = \frac{8}{2} \text{ cm} = 4 \text{ cm}$

Area of a sector $= A$

Formula: $A = \frac{\theta}{360^\circ} \times \pi r^2$

$$A = \frac{135}{360} \times \frac{22}{7} \times \frac{4}{1} \times \frac{4}{1} \text{ cm}$$

$$= \frac{132}{7} \text{ cm}^2$$

$$= 18\frac{5}{7} \text{ cm}^2.$$

Worked Example 35

Calculate the area of the sector of a circle whose perimeter of a sector is 43 cm and the angle subtended at the centre is 120°.

SOLUTION

Given: Area of sector $= A$

Perimeter of sector $= 43$ cm

$\theta = 120°$

Formula:

Perimeter of sector $= 2r + \frac{\theta}{360°} \times 2\pi r$

So,

$$43 = 2r + \frac{120}{360} \times 2 \times \frac{22}{7} \times r$$

$$= 2r + \frac{44}{21}r$$

$$43 = \frac{42r + 44r}{21}$$

$$43 \times 21 = 86r$$

$$\frac{43 \times 21}{86} = r$$

$$r = 10.5 \text{ cm}$$

Area of sector of a circle $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{120}{360} \times \frac{22}{7} \times \frac{10.5}{1} \text{ cm}^2$$

$$= (22 \times 1.5 \times 3.5) \text{ cm}^2$$

$$= 115.5 \text{ cm}^2.$$

Worked Example 36

If the length of an arc of a circle is 5.5 cm and the radius is 3.5 cm, calculate the area of sector of the circle.

SOLUTION

Given: Length of arc of a circle $= 5.5$ cm

Radius $= 3.5$ cm

$\theta° = \theta°$

Formula: Length of arc $= \frac{\theta}{360°} \times 2\pi r$

$$5.5 = \frac{\theta}{360°} \times \frac{2}{1} \times \frac{22}{7} \times 3.5$$

$$\frac{5.5 \times 360 \times 7}{2 \times 22 \times 3.5} = \theta$$

$$\theta = 90^\circ$$

Area of sector of a circle

$$= \frac{90}{360} \times \frac{22}{7} \times \frac{3.5}{1} \times \frac{3.5}{1}\ \text{cm}^2$$

$$= \frac{1}{4} \times \frac{22}{1} \times \frac{0.5}{1} \times \frac{3.5}{1}\ \text{cm}^2$$

$$= \frac{19.25}{2}\ \text{cm}^2 = 9.625\ \text{cm}^2.$$

Exercise 7

1. A sector of a circle has an area 55 cm^2. If the radius of the circle is 10 cm, calculate the angle of the sector. (JAMB)

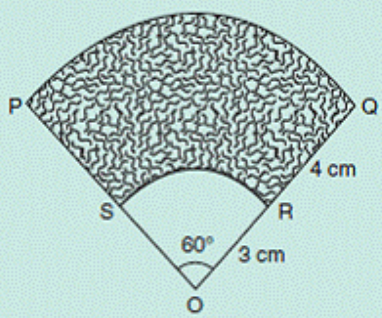
2. 

Figure 7.62

Figure 7.62 is a sector of two concentric circles with centre O, QR = 4 cm, RO = 3 cm and $\angle SOR = 60^\circ$. Calculate the area of the shaded portion. (JAMB)

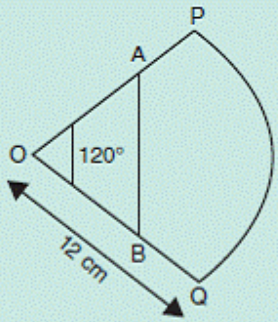
3. 

Figure 7.63

The diagram shows a piece of cardboard in the form of a sector of a circle. The radii PO and OQ are each equal to 12 cm. A and B are the mid-points of OP and OQ, respectively, and POQ = 120°.

Calculate, correct to one decimal place, the area of ABQP. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **(WAEC)**

4. A sector of a circle of radius 9 cm subtends angle 120° at the centre of the circle. Find the area of the sector to the nearest cm^2. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **(WAEC)**
5. A circle has radius x cm. Find the area of a sector of the circle with angle 150°, leaving the answer in terms of x and π. **(WAEC)**
6. The angle of a sector of a circle is 90°. If the perimeter of the sector of the circle is 25 cm, find the area of the sector of the circle.
7. Calculate the area of a sector of a circle of radius 7 cm subtending an angle of 270° at the centre of the circle.

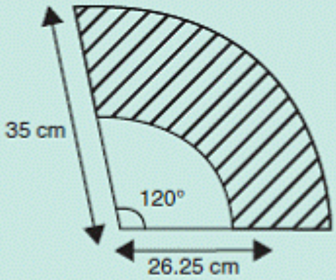
8. 

Figure 7.64

The diagram above represents a fan which is in the shape of a sector of a circle of radius 35 cm. The unshaded part of the fan is of radius 26.25 cm. Calculate the area of shaded part of the fan, correct to three significant figures. $\left(\text{Take } \pi = \frac{22}{7}\right)$ (WAEC)

9. A sector of a circle is bounded by two radii each 7 cm long and an arc of length 6 cm. Find the area of the sector. (UME)

10. An arc of a circle of radius 6 cm is 8 cm long. Find the area of the sector. (UME)

VIII. Area of Segment of a Circle

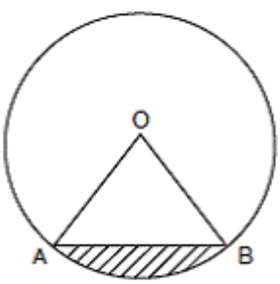


Figure 7.65

The diagram above is a circle with centre O with the shaded region as the segment of the circle.

To calculate the area of the segment of the circle:

(a) Calculate the area of ΔAOB.

(b) Calculate the area of sector AOB.

Then,

Area of the shaded segment = Area of sector AOB − Area of ΔAOB

Worked Example 37

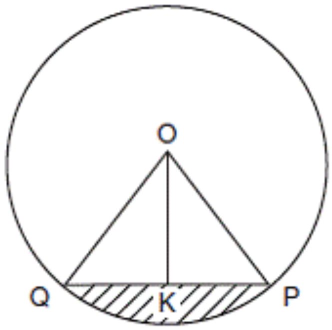


Figure 7.66

In the diagram above, O is the centre of the circle with radius 3.2 cm. If ∠POQ =
84°, calculate, correct to two decimal places, the area of the shaded part.

SOLUTION

Area of minor sector POQ $= \frac{\theta}{360}\pi r^2$

$= \frac{84}{360} \times \frac{22}{7} \times \frac{3.2}{1} \times \frac{3.2}{1}$ cm^2

$= 7.509 \text{ cm}$

$= 7.51 \text{ cm}^2$ (2 d.p.)

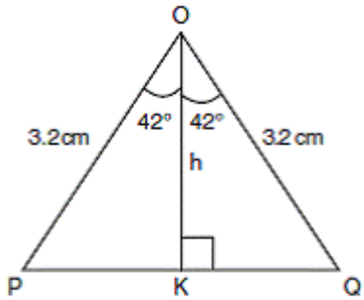


Figure 7.67

Let $OK = h$ cm and

$KQ = x$ cm

$\cos 42° = \frac{h}{3.2}$

$h = 3.2 \cos 42°$

$h = 3.2 \times 0.7431$

$h = 2.3779$

Similarly, $\sin 42° = \frac{x}{3.2}$

$x = 3.2 \sin 42°$

$x = 2.1411$ cm

Therefore,

the length of chord $PQ = 2 \times 2.1411$

$= 4.2822$ cm

Area of $\Delta POQ = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2}(4.28232 \times 2.3779)$

$= \frac{1}{2}(10.1826)$

$= 5.0913$

The required area of shaded segment

= Area of minor sector POQ − Area of ΔPOQ

$= 7.509 \text{ cm}^2 - 5.0913 \text{ cm}^2$

$= 2.4177 \text{ cm}^2$

$= 2.42 \text{ cm}^2$ (2 d.p.)

Exercise 8

1. Calculate the area of shaded segment of the circle shown in the diagram. $\left(\text{Take } \pi = \frac{22}{7}\right)$

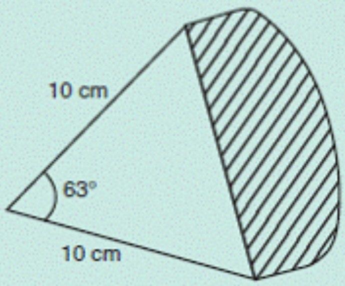


Figure 7.68

2. The diagram below shows the shaded segment of a circle of radius 7 cm. If the area of the triangle OXY is $12\frac{1}{4}\text{cm}^2$, calculate the area of the segment. $\left(\text{Take } \pi = \frac{22}{7}\right)$

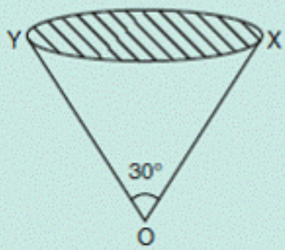


Figure 7.69

3. Find the area of the shaded segment in the figure. Express answer in terms of π and surd form. **(UME)**

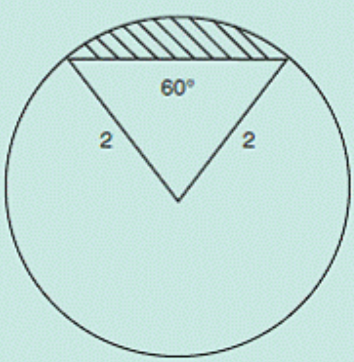


Figure 7.70

4. The diagram is a circle with centre O. Find the area of the shaded portion. Express answer in terms of π.

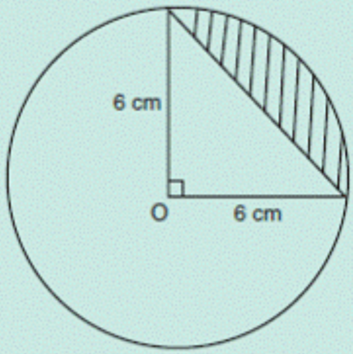


Figure 7.71

5. In the diagram, O is the centre of the circle and PQ is a diameter. Triangle RSO is an equilateral triangle of side 4 cm. Find the area of the shaded region. (WAEC)

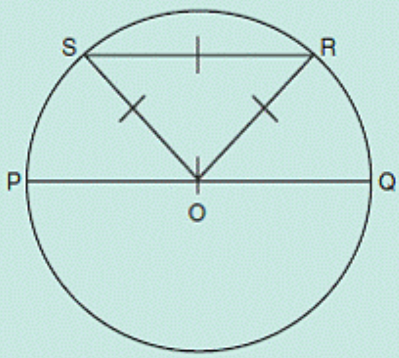


Figure 7.72

6. In the diagram, ABCDEO is two-thirds of a circle with centre O, radius AO of 7 cm and $|AB| = |BC| = |CD| = |DE|$. Calculate, correct to the nearest whole number, the area of the shaded portion. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **(WAEC)**

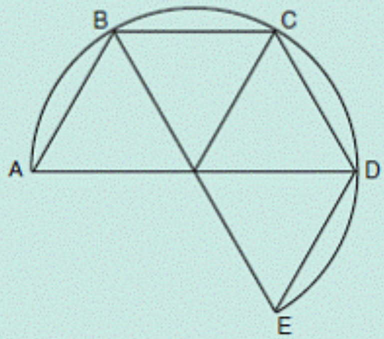


Figure 7.73

7.

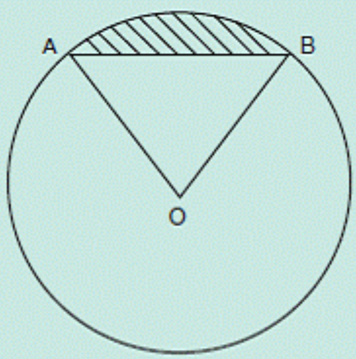


Figure 7.74

The diagram above shows a circle with centre O, with chord AB = $\sqrt{18}$ cm and $\angle AOB = 90°$. Calculate

(a) The radius, in cm, of the circle.

(b) The area of ΔAOB, in cm^2. Taking $\pi = \frac{22}{7}$.

(c) The area of the shaded region, in cm^2 to one decimal place.

SUMMARY

In this chapter, we have learnt the following:

- The international system of unit for measuring the area of plane shape is square centimetre (cm^2), square metre (m^2) or square kilometre (km^2).
- The area of a rectangle is calculated as $A = l \times b$, where l is the length and b the breadth.
- The area of a square is calculated $A = l^2$, where l is the length of side.
- The area of parallelogram is calculated as $A = bh$, where b is the baseand h is the perpendicular height.

- The area of trapezium is calculated as $A = \frac{1}{2}(a + b)h$, where *a* and *b* are the parallel sides and *h* is the perpendicular side.
- The area of a rhombus is calculated as $A = 4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}$, where d_1 and d_2 are the two diagonals of the rhombus.
- The area of a triangle is calculated as $A = \frac{1}{2}bh$, where *b* is the base and *h* is the perpendicular height or $A = \sqrt{s(s-a)(s-b)(s-c)}$ with $s = \frac{a+b+c}{2}$ and *a, b, c* are the sides of the triangle or $A = \frac{a \times b \times \sin\theta}{2}$, where *a* and *b* are adjacent sides and θ is the included angle.
- The area of a circle is calculated as $A = \pi r^2$, where *r* is the radius and $\pi = \frac{22}{7}$ or 3.142.
- The area of the sector of a circle is calculated as $A = \frac{\theta}{360}\pi r^2$ where θ is the angle subtended at the centre of the circle, *r* is the radius of the circle and $\pi = \frac{22}{7}$ or 3.142.
- Area of the segment of a circle = Area of the sector of a circle − Area of the triangle in the sector.

GRADUATED EXERCISES

1. A quadrant of a circle of radius 7 cm is out away from each corner of a rectangle 25 cm long and 18 cm wide. Find the area of the remaining figure.
2. Find the area of a rectangle of length4 cm and whose diagonal is 8 cm. (Leave your answer in surd form) **(WAEC)**
3. The angle of a sector of a circle of diameter 8 cm is 135°. Find the area of the sector. $\left(\text{Take } \pi = \frac{22}{7}\right)$
4. In the diagram below, PXR and PYO are two semi-circles with diameters 14 cm and 7 cm, respectively. Find the area of the enclosed region PXROY correct to the nearest whole number. $\left(\text{Take } \pi = \frac{22}{7}\right)$ (WAEC)
5. In the diagram below, |AD| = 10 cm, |DC| = 8 cm and |CF| = 15 cm. Find the area of quadrilateral ABCD if the area of triangle DCF = 24 cm^2.

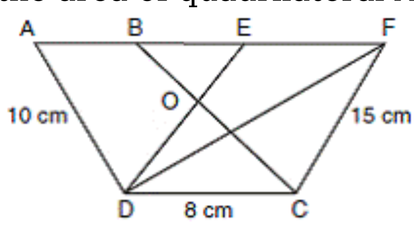
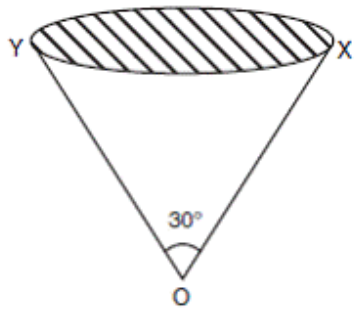


Figure 7.75

6. The diagram below shows the shaded segment of a circle of radius 7 cm. If the area of the triangle OXY is $12\frac{1}{4}$ cm^2. Calculate the area of the



segment. Figure 7.76

7. The diagonals AC and BD of a rhombus ABCD are 16 cm and 12 cm long, respectively. Calculate the area of the rhombus. **(WAEC)**
8. In the diagram below, O is the centre of the circle of radius 3.5 cm and $\angle POQ = 60°$. Find the area of the major sector POQ. $\left(\text{Take } \pi = \frac{22}{7}\right)$

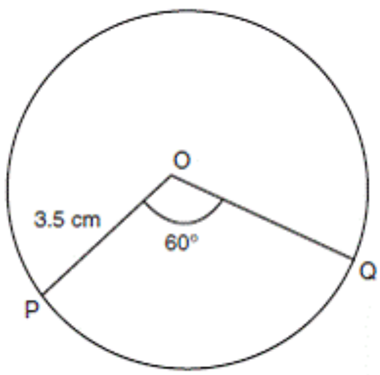


Figure 7.77

9. If the hypotenuse of a right-angled isosceles triangle is 2 cm, calculate the area of the triangle.
10. Calculate the area of a square whose diagonal is of length 5 cm.