Chapter 5: Simultaneous Linear and Quadratic Equations

OBJECTIVES

At the end of the chapter, students should be able to:

1. Solve simultaneous linear equations using elimination, substitution and graphical methods.
2. Draw the graphs of simultaneous linear and quadratic equations.
3. Use the simultaneous linear and quadratic equations to solve other related equations.
4. Use graphs to solve other related equations.
5. Solve word problems involving simultaneous equations.

I. Revision of Some Terms in Simultaneous Linear Equations

(i) Constants and unknowns

Any unchanging or fixed quantity in any given equation is termed constant. However, there are some constants that are natural. Examples include: days of the week (7), seconds in a minute (60), hours in a day (24), months in a year (12), etc.

Numbers such as these are said to be constants. Constants can be whole numbers, fractions or decimal fractions and irrational numbers (surds).

Unknown terms in an equation are called variables. Their values are not fixed. They can be represented by any letter of the alphabet except â€˜Oâ€™ which can be misconstrued with zero.

Multiplication of a constant with any variable gives the coefficient of that variable.

For example, 4 multiplied by w gives 4w. In this case, 4 is the coefficient of w while w is the literal coefficient of 4.

For instance, in âˆ’2yâˆ’7, âˆ’7 is a constant and y is a variable. The coefficient of y in this case is âˆ’2.

Exercise 1

Write down the constant terms in the following expressions:

1. 6x âˆ’ 2
2. 9 âˆ’ 2x + 6y
3. _25_ âˆ’ _ 2r_ + 20p
4. 57 + y + 5x
5. 60 âˆ’ 4x + 7y
6. p + 2h + 25
7. 3y + x âˆ’ 20
8. 5d + 3 âˆ’ 7e
9. 573 + 29 âˆ’ 4c
10. 3x + 5y + 276

State the coefficients of the variables in the following:

11. âˆ’7c
12. 26b
13. 2x
14. 0.83b
15. a âˆ’ b + 3c + 15
16. 0.25y + 7 âˆ’ 5x
17. 20 + 5b âˆ’ 14x
18. 52x + 21 âˆ’ 0.5y + 0.75c
19. State the variables in Questions 1–18.
20. State the literal coefficients in Questions 11–15.

(ii) Term and expression

The product of a constant and any variable is called a term or simply called an algebraic term. For

example: 2ab means 2 x a x b where 2 is the coefficient of â€˜aâ€™ and â€˜bâ€™ which are variables. A constant can also be a term.
Expression is a mathematical statement without an equality sign or which is simply equated to a phrase in English Grammar.
Expressions that contain one term are called monomial. Examples are 2x, 4r, $4r_2$, $6p_3$, 3xy, $11abx_2$, pqx_2, etc.
Expressions which contain two terms are said to be binomial. The two terms are connected by â€˜+â€˜ sign or â€˜âˆ’â€™ sign. Examples are 4x + 2, x âˆ’ $5x_2$, a + 4b, 5a + 2abc, etc.
Expressions that contain three terms are said to be trinomial. The terms are connected by subtraction or addition. Examples are 4a + $2x_2$ âˆ’ 6xyc, x_3 + 2x âˆ’ 1, 2abc + py âˆ’ 2xyz, etc.

Exercise 2

1. Write down the terms with their signs that make up the following expressions.
(a) 2x âˆ’ 3y + $5x_2yz$ âˆ’ 9z + 20
(b) âˆ’xyz âˆ’ 2bc + 3ab âˆ’ z âˆ’ 27
(c) $4a_2$ + $5b_3$ + 2a âˆ’ 9
(d) 10x âˆ’ 9y âˆ’ 5p âˆ’ 3c + z + 4
(e) âˆ’35 âˆ’ 2xy + 4zy âˆ’ 3xz + x âˆ’ y + 6x
2. Identify the constant terms in the Question above .
3. Identify the numerical coefficients in Question 1 (a–e).
4. Classify the following expressions as monomial, binomial, trinomial and polynomial.
(a) 2a âˆ’ 5b âˆ’ 8
(b) 4p + $5q_2$
(c) 4 abcd
(d) 7
(e) 2x âˆ’ y
(f) 10 âˆ’ a + b + 5ab
(g) 2x âˆ’ 3y âˆ’ 42
(h) 5 x_2yz p âˆ’ $2x_2$ âˆ’ $3y_2$ âˆ’ 96
(i) $8abc_2$ âˆ’ 7yx + $5z_2p$ âˆ’ 1 âˆ’ q
(j) Z
5. Specify the constant terms in Question 4 and also specify where there are none.
6. Specify the numerical coefficients in Question 4, and where there is no numerical coefficient, say â€˜noneâ€™.
7. Identify the constant terms in the following:
(a) 6p âˆ’ Î² âˆ’ 4
(b) 4x âˆ’ z
(c) 0.8x âˆ’ 0.75pq + 11.45
(d) 205x + 75x âˆ’ 45

(e) $\frac{x}{7}$ âˆ’ $\frac{3}{5}$ + y
8. Which of the following are trinomials?
(a) 1 âˆ’ 2xy + pq âˆ’ bc
(b) 4p
(c) 2n âˆ’ 4m + 8
(d) $3x_2$ + 5x + 7 âˆ’ k
(e) $^{22}/_{7}r_2$ + h
(f) Ï€r_2h

(g) $\frac{1}{f}+\frac{1}{y}+\frac{1}{r}+24$
(h) 6x + pt + 2
(i) 2l + 2b
(j) $3x_2$ + 5x + 10

9. Identify the polynomials in Question 8.
10. Specify the numerical coefficients in Question 7.

(iii) Degree of terms and expressions

The degree of a term is the addition of all powers of the variables/unknowns in the term. It is also called the order of the term. For example, 3abc, âˆ’ $4xy_2p_4$, 6ax, xy_5z_2 are of degree or order 3, 7, 2, 8, respectively.
The term 16 is a constant term and of zero degree.

The expression $\frac{-36p^4q^7}{z^{10}} = -36p^4q^7z^{-10}$ is of 1st order since (4 + 7 âˆ’ 10 = 1).

The expression $\frac{7x^7y^5z^8}{p^{24}} = 7x^7y^5z^8p^{-24}$ is of degree âˆ’4 (since 7 + 5 + 8 âˆ’ 24 = âˆ’ 4).
If all the terms of an expression are of the same degree, such expression is called a homogenous expression. For example, $5x^3y$ + $2bc^3$ âˆ’ abcd is a homogenous expression of degree 4 in x, y, a, b, c and d. If the terms of an expression are of different degrees, the degree of the expression is the degree of term that has the highest degree in the given expression. For example, the expression:
1 + x^6 + $7xy^4$ + 5x âˆ’ xy^5 + x âˆ’ y is of degree 6.

Exercise 3

1. State the degrees of the following terms:
(a) p^6qâˆ’8
(b) $9x^2yz$
(c) –17x
(d) 21

(e) $\frac{3x^2y}{pq}$

(f) $\frac{6m^4n^6}{x^2y^3}$
(g) $3x_2y_4$
(h) 3abc
(i) âˆ’8xyz

(j) $\frac{12p^2q^4}{z^5}$
2. List out the homogenous expressions from the following and state their degrees:
(a) $4r^2$ âˆ’ 6pq + $7q_2$ + $12abc_2$
(b) $14c^3$ âˆ’ 9cde + $9c_2d$ âˆ’ $13d_2f$
(c) $2x^2yz$ âˆ’ $3xyz^2$ + $6xy^2z$
(d) $19x^6$ âˆ’ 4x $^4y^2$ + $3xy^3$ âˆ’ $2x^2$ + 6x âˆ’ 4y âˆ’ 21
(e) $21c^2d^3$ + $7d^2az^2$ âˆ’ $17bxy^3$ âˆ’ xy^4.
3. State the degree of each of the following expressions:
(a) $4x^2$ + 7x âˆ’ 12
(b) $6x^2$ âˆ’ $3x^3$ + 5x âˆ’ 17
(c) $7x^2$ âˆ’ x^7y + $17xy^2$ âˆ’ 21
(d) $35x^2y^2$ âˆ’ 16pqr ^{4} + $6x^2y^3z^4$ âˆ’ 5xyz + 10
(e) âˆ’3abc + $5b^2c^3$ + $4a^3$ âˆ’ bc^4 âˆ’ 16 ab^4c^3 âˆ’ 19
4. Choose the homogeneous expression(s) in Question 3.
5. Indicate the degrees of the following terms:
(a) 8.07 (b) 9x
(c) âˆ’26b (d) $4pq^3r^4s^5$

(e) âˆ’7xy^{2}z^5 (f) 6xy^{4}zp^6

(g) $\frac{24xy^3}{z^5}$

(h) $\frac{7xy^4z^2}{b^4}$

(i) âˆ’8x^{2}b^y

(j) $\frac{3a^2}{4bcdef}$

(iv) Linear expressions and equations

A linear expression contains one or more terms and each of the term has degree one or zero if a constant is involved.

Examples are 3x âˆ’ 5p âˆ’ 6, 3x + 5y âˆ’ 4z + 6p âˆ’ 21, 3x âˆ’ 4y, 3r + 4.

Equations are likened to a complete sentence that has subject, action verb, predicate and object. Unlike an expression that is likened to a phrase, the equality sign (=)makes an equation a complete statement. Examples are 3x **âˆ’** p **âˆ’** 6 = 0, 3x + 4 = **âˆ’**10 and 6x$_2$ + 4x = 4. Equations always have two sides called the left-hand side (LHS) and the right-hand side (RHS).

Exercise 4

Identify the linear and non-linear expressions in the following:

1. $-\frac{4}{a}+\frac{3}{c}-\frac{1}{d}$
2. **âˆ’**2 + ab + y
3. ay^2 + x^{2}p + p**âˆ’**5q^3
4. $\frac{1}{y}+bx+m$
5. $\frac{4ab}{c}+8d$
6. 6a + 4b + 3
7. 2e
8. $\frac{4xy}{c}+2x-10$
9. 3xy **âˆ’** 7 + x^2
10. 2x +y + 4

Which of the following are expressions and which are equations?

11. **âˆ’**2x + mx + c = 0
12. 6x + 7 = 0
13. 2x + 11
14. 2a + b = **âˆ’**6 + y
15. 3x^2 + 6y^{2}x + z + 17 **âˆ’** pz
16. 21 = 2x^2 + 4y^2
17. 25 = 4a^{2}b^2
18. p + 21q **âˆ’** 21
19. y^2 = ab + pq
20. 2a^2 + 6a + 21

21. 6x^2 + 1 = **âˆ’**x
22. 2x = 4
23. 6x âˆ’ 18
24. 2x + z = 20
25. âˆ’p âˆ’ 6q + 14

II. Simultaneous Linear Equations (Revision)

A linear equation in two variables such as ax + by = 0 has an infinite number of solutions. There is no unique solution. Graphically, the equation represents a straight line (hence â€œlinearâ€) and the coordinates (x, y) of any point on the line satisfy the equation. If we are given two such equations at the same time (i.e. simultaneous equations), a unique solution can be obtained unless the equations represent a parallel line.
In that case, there will be no solution. There are three methods of solving simultaneous linear equations, which we learnt in Book1, elimination, substitution and graphical methods.

(i) Elimination method

Worked Example 1

Solve the equations 2x âˆ’ 3y = 24 and 3x + 2y = âˆ’3.
SOLUTION
2x âˆ’ 3y = 24 .... (1)
3x + 2y = âˆ’3 .... (2)
Eliminate x first by multiplying equation (1) by 3 and equation (2) by 2.
That is,
3 (2x âˆ’ 3y) = 24 x 3 = 6x âˆ’ 9y = 72 .... (3)
2 (3x + 2y) = âˆ’3 x 2 = 6x + 4y = âˆ’6 .... (4)
Subtracting (3) from (4) gives
âˆ’9y âˆ’ 4y = 72 + 6
âˆ’13y = 78
y = _âˆ’$^{78}/_{13}$ = âˆ’6
y = âˆ’6
Now, substitute y = âˆ’6 in any of the equations
(1) to (4).
Consider equation (1): 2x âˆ’ 3y = 24.
Substitute y = âˆ’6 in equation (1) and obtain 2x âˆ’ 3(âˆ’6) = 24.
2x + 18 = 24
2x + 18 âˆ’ 18 = 24 âˆ’ 18
2x = 6
2/2x = 6/2
âˆ´ x = 3
âˆ´ x = 3 and y = âˆ’6 is the required solution.

Exercise 5

Solve the following simultaneous linear equations by elimination method:
1. x + y = 1 _12_ ; x âˆ’ y = 2.5 (WAEC)
2. 6y + 5x = 12; 4y âˆ’ 3x = 1 (WAEC)
3. If 3p âˆ’ q = 6 and 2p + 3q = 4, find q. (WAEC)
4. 4x âˆ’ y = 1; 5x + 2y = 4 (WAEC)
5. If 3x âˆ’ 4y = 10 = x âˆ’ 3y, what is the value of x âˆ’ y.
6. x + y = 12; x âˆ’ y = 6
7. 2x + 3y = 11; 4x + y = 12
8. 3(x + y) = 21; 6(x âˆ’ y) = 6
9. 2x + 3y = 4; $3x - 4\frac{1}{2}y = 6$

10. x + y = 21; $\frac{x}{4} + \frac{y}{3} = 6$

11. x âˆ’ y = 4; $\frac{y}{3} + \frac{x}{5} = 5$

12. Solve $x + y = \frac{3}{5};\ x - y = \frac{5}{2}$
13. 4a âˆ’ 1 = âˆ’3b; 4b + 1 = 6a
14. 2x + 3y = 10; 2x + y = 2
15. 4x + 6y = 21; 7x âˆ’ 3y = 3
16. 2x + 5y = 1; 3x âˆ’ 2y = 3
17. 2x âˆ’ y = 8; 3x + y = 17
18. 2m âˆ’ n = 5; 3m + 2n = –24
19. 2a âˆ’ 2 = âˆ’5b; 3a + 2b = 25
20. 3 (r âˆ’ 2s) = 25; 3(s âˆ’ r) = 5
21. 1.2p + q = 0.6; 0.7p + 0.8q = 1

22. p + q = 2; 2p âˆ’ q = $1\frac{1}{2}$
23. 4x âˆ’ 3y = 0; 6x + 5y âˆ’ 13 = 0
24. 2x + 5y = 1; 3x âˆ’ 2y = 30

(ii) Substitution method
Substitution method may also be used to solve Worked Example 1.

Worked Example 2
Solve 2x âˆ’ 3y = 24 and 3x + 2y = –3.
SOLUTION
2x âˆ’ 3y = 24 .... (1)
3x + 2y = âˆ’3 .... (2)
Making x the subject in equation (1)
â‡’ 2x = 3y + 24

âˆ´ $x = \frac{3y + 24}{2}$
Substituting the value of x in equation (2)

â‡’ $3\left(\frac{3y + 24}{2}\right) + 2y = -3$ (multiply by 2 to clear the fraction)
â‡’ 3(3y + 24) + 4y = âˆ’6
â‡’ 9y + 72 + 4y = âˆ’6 (open the bracket)
â‡’ 9y + 4y = âˆ’6 âˆ’ 72 (collect like terms)
â‡’ 13y = âˆ’ 78

$y = \frac{-78}{13} = -6$
Now substituting for y = âˆ’6 in equation (i) above,
2x âˆ’ 3(âˆ’6) = 24
2x + 18 = 24
2x = 24 âˆ’ 18 = 6
2x = 6
âˆ´ x = 3
The required result is x = 3 and y = âˆ’6
(as in Worked Example 1).

Worked Example 3

Solve the following pairs of simultaneous equations:

$\frac{3}{r} - \frac{2}{s} = 1$ .... (1)

$\frac{8}{r} + \frac{2}{s} = 10$ .... (2)

SOLUTION

We can rewrite equations (1) and (2) respectively

as $3\left(\frac{1}{r}\right) - 2\left(\frac{1}{s}\right) = 1$ and $8\left(\frac{1}{r}\right) + 2\left(\frac{1}{s}\right) = 10$

Let a = 1__

r and b = 1__

s , then equations (1) and (2) will become

3a âˆ’ 2b = 1 .... (3)

8a + 2b = 10 .... (4)

Making â€˜aâ€™ the subject, we obtain

3a = 2b + 1

$$a = \frac{2b+1}{3}$$

Substituting for â€˜aâ€™ in equation (4)

$8\left(\frac{2b+1}{3}\right)$ + 2b = 10 (multiply by 3 to clear the fraction)

8(2b + 1) + 6b = 30

16b + 8 + 6b = 30

22b = 22

b = $^{22}/_{22}$ = 1

Now substituting for b = 1 in equation (3) above

3a âˆ’ 2(1) = 1

3a âˆ’ 2 = 1

3a = 1 + 2 = 3

3a = 3

a= 1

Solving the two equations simultaneously, we obtain a = 1, and b = 1.

Since a = $\frac{1}{r}$, âˆ´1 = $\frac{1}{r}$ â†’ r = 1

Since b = $\frac{1}{s}$, âˆ´1 = $\frac{1}{s}$ â†’ s = 1

Hence, r = 1 and s = 1.

Worked Example 4

Solve the following pairs of equations:

$3^{x + 2y} = 1$, and $3^{2x+y} = 27$

SOLUTION

Recall that $3^0 = 1$ and that $3^3 = 27$.

âˆ´ $3^{\mathbf{x+2y}} = 3^0$ .... (1)

$3^{\mathbf{2x+y}} = 3^3$ .... (2)

Equating the powers in equations (1) and (2), we obtain
x + 2**y** = 0 .... (i)
2**x** + **y** = 3 ..(ii) [since their bases are equal]
From equation (i), **x** = âˆ’2**y.**
Substituting for **x** in equation (ii), we obtain
2(âˆ’2**y**) + **y** = 3
âˆ’4**y** + **y** = 3
âˆ’3**y** = 3
y = –1
Substituting the value of **y** = âˆ’1 in equation (i),
x + 2(âˆ’1) = 0
x âˆ’ 2 = 0
âˆ´ **x** = 2
The required result is **x** = 2 and **y** = âˆ’1.

Exercise 6

Solve the following simultaneous linear equations by the method of substitution:
1. (**x** + **y**) (**x** + 2) = 28; **x** + **y** = 3 (WAEC)
2. 2**a** + **b** = 0; **a** âˆ’ 2**b** = âˆ’5 (WAEC)
3. **x** + **y** = 5; **x** âˆ’ **y** = 1
4. **x** âˆ’ **y** = 5; **x** + **y** = 7

Solve the following pairs of equations:

5. **x** + **y** = 0; $\frac{27x}{81^{x+2y}} = 9$ (WAEC)
6. 32**p** âˆ’ **q** = 1; $\frac{16^x}{4} = 8^{3p-q}$

7. $\frac{x-1}{5}+\frac{y-1}{3}+\frac{1}{15}=0; 7y+2x+10=0$ (WAEC)

8. $2x+5y=6\frac{1}{2}; 5x-2y=9$ (WAEC)

9. $y=3x; 4y=5x+14$ (WAEC)

Solve the simultaneous equations.

10. $\frac{2}{t}+\frac{3}{u}=9; \frac{1}{t}+\frac{1}{u}=8$

11. $\frac{1}{t}+\frac{2}{r}=3; \frac{1}{t}-\frac{1}{r}=6$

12. $\frac{1}{p}+\frac{1}{q}=5; \frac{2}{p}-\frac{1}{q}=1$

13. $\frac{1}{p}=4-\frac{1}{s}; \frac{1}{p}+2=\frac{1}{s}$

Solve the equations:

14. $\frac{x}{2}+\frac{y}{3}=4, \frac{y}{4}-\frac{x}{3}=\frac{1}{5}$

15. $\frac{2}{x}-\frac{1}{y}=3, \frac{4}{x}+\frac{3}{y}=16$

16. $2x-3y+2=x+2y-5=3x+y$

17. $2a+3b-1=3a+b+7=a+2b$

18. $3y-5x-4=5y+8=2y+x+7$

19. $2^{x+2y}=1; 3^{2x+y}=27$ (WAEC)

20. $\frac{1}{3}(x-3y)=2; \frac{x+y}{4}=\frac{1}{2}$

21. $3(3x+2y)=5-x; 4y+5=2(y-5x)$

22. $3(2x-y)=x+y+5$

23. $5(3x-2y)=2(x-y)+1$

24. $2.32x+1.44y=15.6$
$4.8x-1.92y=2.88$

25. $\frac{4}{3}x+\frac{3}{2}y=4; \frac{x}{2}+\frac{y}{4}+1=0$

26. $\frac{3}{a}-\frac{4}{b}=\frac{1}{3}; \frac{2}{a}-\frac{5}{b}=1$

27. $1.5x-0.7y=0.1; 0.8x+1.1y=2.5$

28. $\frac{5x}{8}-\frac{y}{2}=\frac{1}{4}; \frac{2x}{3}-\frac{3y}{5}=\frac{2}{15}$

29. If 3x âˆ’ 4y = 10 = x âˆ’ 3y, what is the value of x âˆ’ y?

30. Solve the equations $\frac{1}{2}$ (x âˆ’ 2y) = 3(x + 8y); x + 2y = 4

(iii) Graphical method

Recall that the graph of simultaneous linear equations gives two straight lines and where the two lines meet gives the solutions of the equations in (**x**, **y**) coordinates or ordered pairs.
Given a series of values of **x,** the independent variable, the corresponding values of **y** are obtained, that is, the dependent variable, to form a table of values.
After the table of values is obtained, the **x**-values are plotted across from left and right and the

y-values from bottom to top resulting in two straight-line graphs.

Worked Example 5

Solve graphically the simultaneous linear equations:
2**x** âˆ’ **y** = 0; 3**x** + 2**y** âˆ’ 5 = 0

SOLUTION

Make **y** the subject of the equation in both cases:
2**x** âˆ’ **y** = 0 3**x** + 2**y** âˆ’ 5 = 0
â‡’ âˆ’**y** = âˆ’2**x** 2**y** = 5 âˆ’ 3**x**

âˆ´ **y** = 2**x** âˆ´ $y = \frac{5 - 3x}{2}$

then the two tables of values are as follows: **y** = 2**x**

Table 5.1

x	1	2	3	4	5	6	7
y	2	4	6	8	10	12	14

$y = \frac{5 - 3x}{2}$

Table 5.2

x	−2	−1	1
y	5.5	4	1

In plotting the straight line, three points are enough to have a smooth, perfect straight line.
Next, draw up the lines by plotting the coordinates in Tables 5.1 and 5.2 respectively on the same graph by using a ruler to join the points and read off the point of intersection which gives the solution of the equations.
The lines meet at the point (0.7, 1.4). Therefore, the solution of the equations are x â‰¡ 0.7 and y = 1.4 (as shown in Figure 5.1).

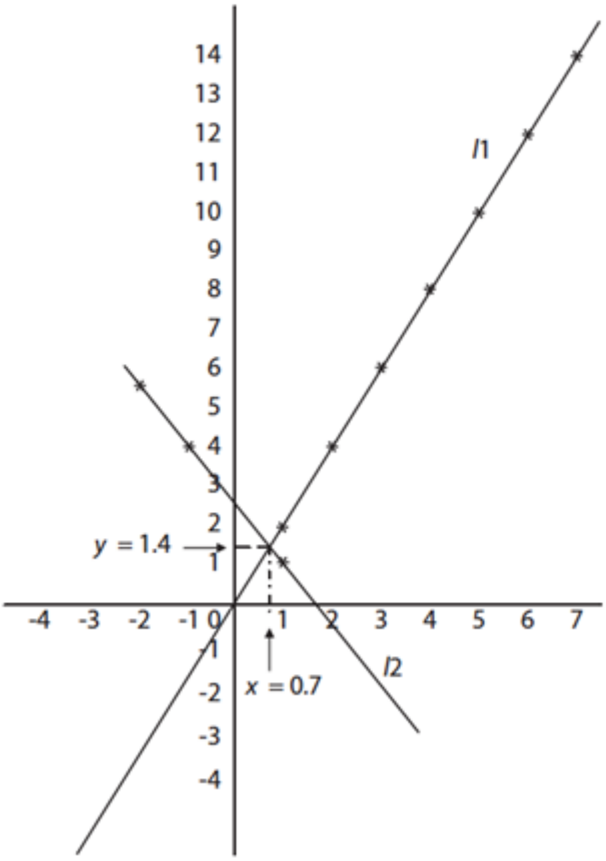


Figure 5.1

Exercise 7

Solve graphically the following simultaneous linear equations of two variables:

1. 6x âˆ’ 4y + 1 = 0; 4x + 3y âˆ’ 5 = 0
2. 2a + 3b + 5 = 0; a + b + 2 = 0
3. 2p âˆ’ 3q âˆ’ 1 = 0; p âˆ’ q = 0
4. 5q âˆ’ 4q + 2 = 0; p + 4q âˆ’ 14 = 0
5. y + x âˆ’ 3 = 0; y âˆ’ 2x + 3 = 0
6. 2x + 3y âˆ’ 19 = 0; 3x âˆ’ 2y âˆ’ 9 = 0
7. 3a âˆ’ 2b + 10 = 0; 6a âˆ’ 4b + 5 = 0
8. a âˆ’ b + 1 = 0; 4a âˆ’ 6b + 5 = 0
9. q + 2p âˆ’ 9 = 0; 2q âˆ’ p âˆ’ 8 = 0
10. y âˆ’ 8x + 11 = 0; 2y âˆ’ 4x + 5 = 0

Determine the point of intersection of the following equations graphically.

11. 4b = 3a + 8; 3b + 4a âˆ’ 6 = 0; 4b + a + 12 = 0
12. a = 0; b âˆ’ 4 = 0; 3b âˆ’ 2a = 0
13. y + 2x âˆ’ 5 = 0; 2y âˆ’ 3x –3 = 0; 3y âˆ’ x = 1
14. 5q + p âˆ’ 16 = 0; q + 6p âˆ’ a = 0; 4q + p = –10
15. 3c âˆ’ 2d = –5; 2c + 5d = 4; 4c âˆ’ 5d + 2 = 0

III. Simultaneous Equations: One Linear and One Quadratic

(i) Quadratic equations

Recall that linear equations are of the form ax + b = 0 or mx + c = 0 or ax + by = 0, while quadratic equations have the general form: $ax^2 + bx + c = 0$.

Note: At least one term of the quadratic equation must be of second order while the other terms first order. Quadratic equations may have one or two variables.

Below are some examples:

$ax_2 + bx + c = 0$

$6x_2 + 5x + 6 = 0$

$18 = 2x_2$ âˆ’ 4x (has one variable)

$2x_2 = 6 + x$

While
$16x^2 = 4xy + 6y^2 - 2x + 7y = 9$
$7xy = 14$ (has two variables)
$10x^2 - 7y^2 = 30$
$6p^2 - 2pq = 14 - 2p + 3q - 2p^2$

Exercise 8

Write Q for quadratic equation and L for linear equation in the following equations:
1. $y^2 + 6y + 5 = 0$
2. $ay + by + 4 = 0$
3. $x^2 = 4xy + 36$
4. $11 - 2x^2 = 15x$
5. $11a - 12 = -13a^2$
6. $13a + b = 31$
7. $14xy = 21$
8. $5y^2 - 4y = -18$
9. $25b^2 = 4a - b$
10. $4x - 21 = 0$
11. $3 + 2 - 5t = 1$
12. $a^2 + 4b^2 = 2$
13. $-x^2 - 2xy + 2y^2 - 5x = 1$
14. $-b^2 - 3a + 3b + 2 = 0$
15. $2a + b = 1$
16. $2\mathbf{p}^2 - 8\mathbf{q} = 0$
17. $3\mathbf{x} + 2\mathbf{y} = 5$
18. $\mathbf{x}^2 - \mathbf{y}^2 - 45 = 0$
19. $2\mathbf{x}^2 + \mathbf{xy} + \mathbf{y}^2 = 14$
20. $60 - \mathbf{a} + \mathbf{b} + \mathbf{y} = \mathbf{k}$

(ii) Solution of simultaneous equations: one linear and one quadratic

Given two equations in two variables, one linear and one quadratic, their solutions are sought for the linear equation first and then the value of **x** or **y** from the linear equation is substituted into the quadratic equation.

Worked Example 6

Solve the following simultaneous linear quadratic equations:
$\mathbf{y} - 2\mathbf{x} = -2$; $\mathbf{x}^2 + \mathbf{y}^2 + 2\mathbf{x} - 3\mathbf{y} = 19$

SOLUTION

Given
$\mathbf{y} - 2\mathbf{x} = -2$ .... (1)
$\mathbf{x}^2 + \mathbf{y}^2 + 2\mathbf{x} - 3\mathbf{y} = 19$ .... (2)
From equation (1), $\mathbf{y} = 2\mathbf{x} - 2$.
Substituting $2\mathbf{x} - 2$ for $\mathbf{y}$ in equation (2), we obtain
$\therefore \mathbf{x}^2 + (2\mathbf{x} - 2)^2 + 2\mathbf{x} - 3(2\mathbf{x} - 2) = 19$
$\therefore \mathbf{x}^2 + 4\mathbf{x}^2 - 8\mathbf{x} + 4 + 2\mathbf{x} - 6\mathbf{x} + 6 = 19$
$5\mathbf{x}^2 - 12\mathbf{x} - 9 = 0$
$(5\mathbf{x} + 3)(\mathbf{x} - 3) = 0$

$\therefore \mathbf{x} = \frac{-3}{5}$ or $\mathbf{x} = 3$

When $x = \frac{-3}{5}$

From equation (1),

$$y = 2\left(\frac{-3}{5}\right) - 2 = \frac{-6}{5} - 2 = \frac{-16}{5} = -3\frac{1}{5}$$

when $x = 3$

$$y = 2(3) - 2 = 6 - 2 = 4$$

Hence, when $x = \frac{-3}{5}, y = -3\frac{1}{5}$ and

when $x = 3, y = 4.$

Worked Example 7

Solve the simultaneous equations
x âˆ’ y = 0, 4x_2 + 3y_2 âˆ’ 5x + y = 3.
SOLUTION
Given
x âˆ’ y = 0 .... (1)
4x_2 + 3y_2 âˆ’ 5x + y = 3 .... (2)
From equation (1), x = y.
Substituting x for y in equation (2), we obtain
4x_2 + 3x_2 âˆ’ 5x + x = 3
7x_2 âˆ’ 4x âˆ’ 3 = 0
(7x + 3) (x âˆ’ 1) = 0

x = $\frac{-3}{7}$ or x = 1
since x = y

when x = $\frac{-3}{7}$, y = $\frac{-3}{7}$
when x = 1, y = 1

Exercise 9

Solving the following simultaneous linear quadratic equations:
1. x + y = 3, x_2 + y_2 âˆ’ 17 = 0
2. y âˆ’ 3x = 1, x_2 + y_2 + 2x âˆ’ 3y = 0
3. 3x + y = 10, 2x_2 + y_2 = 19
4. 3x + 4y = 12, xy = 2
5. 3x âˆ’ y = 3, 9x_2 âˆ’ y_2 = 45
6. 3x + y = 25, xy = 8
7. 2x âˆ’ 5y = 7, xy = 6
8. 3x +4y = 7, 2xy + 3 = 0
9. 3x âˆ’ 4y = 2, xy = 2
10. 3x_2 âˆ’ 4y = âˆ’1, 2x âˆ’ y = 1
11. x_2 + y_2 = +34, x + y = 2
12. 2x_2 + y_2 = 19, x + 3y = 0
13. 9y_2 + 8x = 12, 2x + 3y = 4
14. xy + 5x = 3, 3x + y = 7
15. 3x_2 âˆ’ xy = 0, 2y âˆ’ 5x = 1
16. 2x âˆ’ 3y âˆ’ 5 = 0, 2x_2 + 3xy âˆ’ 3y_2 + 5x âˆ’ 3y + 2 = 0
17. y = 5x + 3, x_2 âˆ’ y_2 + 45 = 0 (WAEC)

18. $25x^2 - 7y^2 = 29$, $5x - 7y = -1$
19. $2a + 3b = 1$, $4a^2 - 9b^2 = -17$
20. $x + y - 2 = 0$, $2x^2 - 3xy + y^2 - x + y = 0$
21. $2x - 3y - 10 = 0$, $x^2 + xy + y^2 + 2x + y - 6 = 0$
22. $x + 2y = 2$, $x^2 + 4y^2 = 2$
23. $2a - 3b = 4$, $a^2 + b^2 = 29$
24. $a + b = 7$, $ab = 12$
25. $5x + 3y - 8 = 0$, $2x^2 - 3xy - y^2 – 3x + y + 2 = 0$

IV. Graphical Solution of Linear and Quadratic Equations

Graphical method can also be used to seek solutions of simultaneous quadratic and linear equations apart from substitution method. Here, we plot the graph of the linear equation which gives a straight line and the graph of the quadratic equation which gives a curve. Thereafter, we read the coordinates of intersection of the line and the curve which gives the required solutions. There are always two sets of solutions corresponding to the two points where a linear graph meets a quadratic graph.

Whenever the straight line meets the curve at a point, it is a tangent to the curve, which means there is one set of solutions to the equations.

At times, the straight line does not meet the curve. This means that the equations have set of solutions not in the real number family but in the family of imaginary number which is beyond the scope of this chapter.

Worked Example 8

Use the graphical method to solve the equations $y = 2x^2 - 3x - 7$ and $y = 2x - 1$ simultaneously.

SOLUTION

By drawing suitable graphs using values of x from $x = -4$ to $+4$ for the quadratic equation. The tables of values shown in Tables 5.3 and 5.4 are used to plot the graphs (Figure 5.2).

$y = 2x - 1$

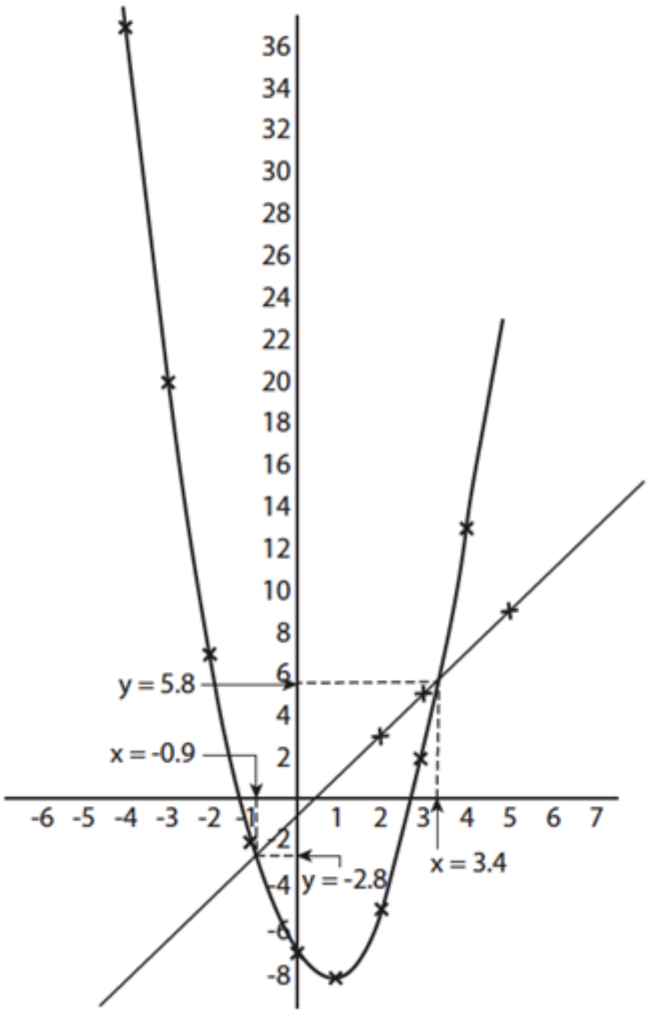
Table 5.3

x	2	3	5
y	3	5	9

$y = 2x^2 - 3x - 7$

Table 5.4

x	–4	–3	–2	–1	0	1	2	3	4
$2x^2$	32	18	+8	+2	0	2	8	18	32
$-3x$	12	9	6	3	0	–3	–6	–9	–12
-7	–7	–7	–7	–7	–7	–7	–7	–7	–7
y	37	20	7	–2	–7	–8	–5	2	13



The solutions are: (3.4, 5.8) and (–0.9, –2.8)

Figure 5.2

Draw the graph of both equations within the same axes. The graphs are drawn from the data in Tables 5.3 and 5.4.

Reading from the graph in Figure 5.2 (see the dotted lines), we conclude that the required solutions are x=3.4 or âˆ’ 0.9, y = 5.8 or âˆ’2.8.

Worked Example 9

Find the solution graphically of the simultaneous equations: y = $2x_2$ âˆ’ 2x âˆ’ 9 and y âˆ’ x âˆ’ 5 = 0.

SOLUTION

Draw the graphs of both equations within the same axes. The graphs are drawn from the data in Tables 5.5 and 5.6.

y = $2x^2$ âˆ’ 2x âˆ’ 9

Table 5.5

x	–4	–3	–2	–1	0	1	2	3	4
$2x^2$	32	18	8	2	0	2	8	18	32
$-2x$	8	6	4	2	0	–2	–4	–6	–8
-9	–9	–9	–9	–9	–9	–9	–9	–9	–9
y	31	15	3	–5	–9	–9	–5	3	15

y = 5 + x

Table 5.6

x	1	2	5
y	6	7	10

Worked Example 10

Use the graph drawn in Worked Example 9 to solve the following equations:

(a) 2**x**$_2$ âˆ’ 2**x** âˆ’ 9 = 8
(b) 2**x**$_2$ âˆ’ 2**x** + 1 = 0
(c) 2**x**$_2$ âˆ’ 2**x** âˆ’ 9 = **x** + 5
(d) 2**x**$_2$ âˆ’ 2**x** âˆ’ 9 = 0

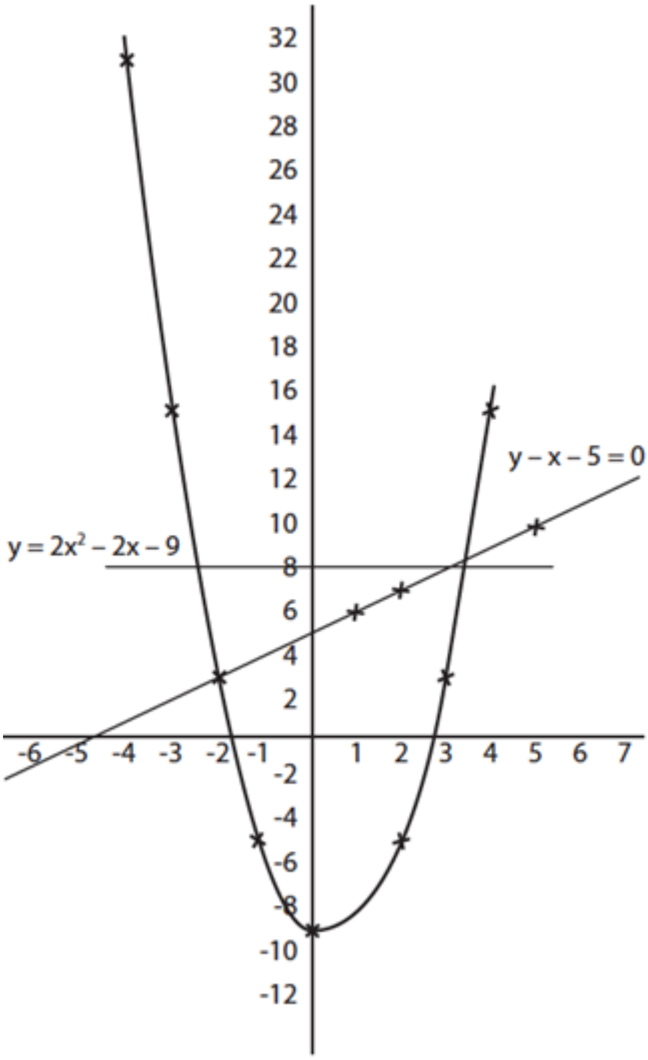


Figure 5.3

SOLUTION

(a) 2x$_2$ âˆ’ 2x âˆ’ 9 = 8 at the points where the line y = 8 cuts the curve. See construction (a) in Figure 5.3.
At these points x = 3.2 or x = âˆ’2.4.
(b) 2x$_2$ âˆ’ 2x + 1 = 0
Subtract 10 from both sides: 2x$_2$ âˆ’ 2x âˆ’ 9 = âˆ’10.
In construction (b) in Figure 5.3, the line y = âˆ’10 cuts the curve at two points. At these points, x = 0 twice.
(c) Construction (c) shows the graph of the line y = x + 5. If y = 2x $_2$– 2x âˆ’ 9 and y = x + 5. Then at the points where the line cuts the curve, 2x$_2$ âˆ’ 2x âˆ’ 9 = x + 5. At these points, x = 3.2 or x = âˆ’2.0.
(d) To solve 2x$_2$ âˆ’ 2x âˆ’ 9 = 0 and y = 0 from the graph, the roots of the equation are the x-coordinates of the points where the straight line y = 0 i.e. x-axis, meets the curve. The required

roots are, therefore, x = 2.8 or âˆ’1.8.

Exercise 10

Solve graphically the following simultaneous equations for values of x between âˆ’3 and 5.

1. $x + y - 1;$ $\quad y + 2 = x(x - 3)$
2. $2x - y + 1 = 0;$ $\quad y = (x - 1)(x + 3)$
3. $2x - 3y + 5 = 0;$ $\quad y = 2x^2 - 2x - 1$
4. $y = 2x^3 - 9x + 4;$ $\quad y = \frac{2x - 7}{3}$
5. $y = 2x^2 + x - 2;$ $\quad y = 4 - 2x$
6. $x + 2y + 1 = 0;$ $\quad 2x^2 + y + x - 1 = 0$
7. $y = 5x + 3;$ $\quad x^2 - y^2 + 45 = 0$
8. $x + 2y - 2;$ $\quad x^2 + 4y^2 = 2$
9. $x + y - 2 = 0;$ $\quad 2x^2 + y^2 - x + y = 0$
10. $xy = 30;$ $\quad 3x + y = 21$
11. Table 5.7 is for the relation

$$y = 2 + x - x^2$$

Table 5.7

x	−2	1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3
y	−4	−1.75	0	1.25	2	2.25	2	1.25	0	−1.75	−4

(a) Using a scale of 2 cm to 1 unit on each axis, draw the graph of the relation in the interval âˆ’2 â‰¤ x â‰¤ 3.
(b) From your graph, find the greatest value of y and the value of x for which they occur.
(c) Using the same scale and axes, draw the graph of y = 1 âˆ’ x.
(d) Use your graph to solve the equation 1 + 2x âˆ’ x_2. (WAEC)

12. Table 5.8 is an incomplete table for the relation

y = 2x_2 âˆ’ 5x + 1.

Table 5.8

x	−3	−2	−1	0	1	2	3	4	5
y			8	1		−1		26	

(a) Copy and complete the table.
(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 10 units on the y-axis, draw the graph of
the relation y = 2x_2 âˆ’ 5x + 1 for âˆ’3 â‰¤ x â‰¤ 5.
(c) Using the same scale and axes, draw the graph of y = x + 6.
(d) Estimate from your graph, correct to one decimal place:
(i) The least value of y and the values of x for which this occur.
(ii) The solution of the equation 2x_2 âˆ’ 5x + 1 = x + 6. (WAEC)
13. Draw the graph of y = 3x(4 âˆ’ x) for values of x ranging from –2 to +6.
On the same axes, draw the line y = 5(x âˆ’ 2). Use a scale of 1 cm to 1 unit on the x-axis and 1 cm to 5 units on the y-axis. From your graph:
(a) Find the roots of the equation 3x(4 âˆ’ x) = 0.
(b) Write down the maximum value of y = 3x(4 âˆ’ x).
(c) Deduce the roots of the equation 3x(4 âˆ’ x) = 5(x âˆ’ 2). (WAEC)

14. Draw the graph of y = 4x$_2$ âˆ’ 9x$_2$ âˆ’ 9x âˆ’ 1. Use your graph to solve the following equations.
(a) 4x$_2$ âˆ’ 9x âˆ’ 1 = 7
(b) 4x$_2$ âˆ’ 9x + 3 = 0
(c) 4x$_2$ âˆ’ 9x âˆ’ 1 = _54_x_ + _ 45_
15. Draw the graph of y = 2x$_2$ âˆ’ 9x + 4 from x = âˆ’5 to x = 5. Use your graph to solve the following equations:
(a) 2x$_2$ âˆ’ 9x + 4 = 0
(b) 2x$_2$ âˆ’ 9x + 7 = 0
(c) 2x$_2$ + 5x + 3 = 0
(d) 2x$_2$ âˆ’ 10x âˆ’ 8 = 0

V. Word Problems involving Simultaneous Linear and Quadratic Equations

In solving word problems involving simultaneous linear and quadratic equations, care should be taken to comprehend the problems before translating it to linear equations or quadratic equations, as the case may be, simultaneously.

Worked Example 11

Mr. Salam, a petty trader, bought 3 rulers and 5 erasers for N150 and his co-trader Mr. Bala bought 5 rulers and 2 erasers for N200. If Mr. Bala and Mr. Salam pay the same unit prices for a ruler and an eraser, find these unit prices.

SOLUTION

Let one ruler cost N x and one eraser cost Ny. Therefore, the cost of 3 rulers and 5 erasers will be N(3x + 5y) = N150 and that of 5 rulers and 2 erasers will be N(5x + 2y) = N200. To obtain x and y, we have to solve the two equations simultaneously.
3x + 5y = 150 .... (1)
5x + 2y = 200 .... (2)
Equation (1) x 2 gives
6x + 10y = 300 .... (3)
Equation (2) x 5 gives
25x + 10y = 1 000 .... (4)
Subtracting equation (3) from equation (4)
25x âˆ’ 6x = 1 000 âˆ’ 300
19x = 700

$x = \frac{700}{19}$
x = N36.84k
Substituting x = N36.84 in equation (1)
3(36.84) + 5y = 150
110.52 + 5y = 150
5y = 150 âˆ’ 110.52
5y = 39.48

$y = \frac{39.48}{5}$
âˆ´y = N7.90

Worked Example 12

Find a and b, if 37^a + 45^b = 87^{10} and 32^a **âˆ’** 11^b = 6^{10}.

SOLUTION

37$_a$ + 45$_b$ = (3a + 7) + (4b + 5) = 87 .... (1)
32$_a$ **âˆ’** 11$_b$ = (3a + 2) **âˆ’** (b + 1) = 6 .... (2)
3a + 4b = 87 **âˆ’** 12 = 75 .... (i)
3a **âˆ’** b = 6 **âˆ’** 1 = 5 .... (ii)

3a + 4b = 75 .... (i)
3a **âˆ’** b = 5 .... (ii)
Subtracting equation (i) from (ii), we obtain
4b + b = 75 **âˆ’** 5
5b = 70

$b = \frac{70}{5}$

âˆ´ b = 14
Substituting b = 14 in equation (ii)
3a **âˆ’** b = 5
3a **âˆ’** 14 = 5
3a = 5 + 14
3a = 19
a = 6.3

Worked Example 13

Find two numbers whose difference is 13 and whose product is 90.
SOLUTION
Let the smaller number be x and the bigger number be x + 13.
Their product is x(x + 13).
Hence, x(x + 13) = 90.
â‡’ x^2 + 13x âˆ’ 90 = 0
â‡’ x^2 + 18x âˆ’ 5x âˆ’ 90 = 0
x(x + 18) âˆ’ 5(x + 18) = 0
(x âˆ’ 5) (x + 18) = 0
x = 5 or x = –18
The other number is 5 + 13 or –18 + 13, that is 18 or –5.
The two numbers are 5 and 18 or –18 and âˆ’ 5.

Worked Example 14

A woman is 6 times older than her child. The product of their ages 7 years ago was 180. Find their present ages.
SOLUTION
Let the childâ€™s age be x years.
Then the motherâ€™s age is 6x years.
7 years ago, the childâ€™s age was (x âˆ’ 7) years and the motherâ€™s age was (6xâˆ’ 7) years.
The product of their ages was (x âˆ’ 7) (6x âˆ’ 7).
Hence, (x âˆ’ 7) (6x âˆ’ 7) = 180

$$\Rightarrow x(6x - 7)\,(6x - 7) = 180$$

$$6x^2 - 7x - 42x + 49 = 180$$

$$\Rightarrow 6x^2 - 49x + 49 = 180$$

$$6x^2 - 49x + 49 - 180 = 0$$

$$6x^2 - 49x - 131 = 0$$

$$a = 6,\ b = -49 \text{ and } c = -131$$

$$x = \frac{49 \pm \sqrt{2\,401 + 24}}{12}$$

$$x = \frac{49 \pm \sqrt{2\,425}}{12}$$

$$= \frac{49 \pm 49.24}{12}$$

$$x = 8.19 \text{ or } x = -\,0.02$$

Exercise 11

1. Find the lengths of the sides of a rectangle whose perimeter and area are 26 cm and 42 cm2 respectively.
2. The perimeter of a rectangle is 42 cm and area = 68 cm2. Find the dimensions
3. If the sum of two numbers is 10 and the difference of their squares is also 10, find the numbers?
4. Four knives and six forks cost N136. Six knives and five forks cost N164. Find the cost of (a) a knife and (b) a fork. (WAEC)
5. In a positive number of two digits, the sum of the digits is 15. If the digits are interchanged, the number is increased by 9. Find the number. (WAEC)
6. A number with three digits has the hundred digit three times the unit digit and the sum of the digits is 19. The value of the number is decreased by 594. Find the number.

$63x + 52y = 44_{10}$

$25x + 64y = 76_{10}$.

7. The sum of the ages of a man and his son is 67 years. Six years ago, the manâ€™s age exceeded 5 times the age of his son by 1. Find their present ages.
8. A two-digit number is such that the sum of the digits is 13 and the product of the digits is 40. Find the number.
9. A two-digit number is less than 6 times the sum of its digits by 1. The difference between the digits is 1. Find the number.
10. A two-digit number exceeds twice the sum of its digits by 7. Find the number, if its units digit also exceeds its tens digit by 7.
11. Find two numbers which differ by 5 and whose product is 50.
12. The width of a dinner table is 6m less than the length. Its area is 216 m2. Find the dimensions of the dinner table.
13. Two numbers have a difference of 7. The sum of their squares is 289. Find the numbers.
14. The base of a triangle is 3 cm longer than its corresponding length. If the area is 44 cm2, find the length of its base.
15. Twice a certain whole number subtracted from 3 times the square of the number gives 133. Find the number.
16. A certain number is subtracted from 18 and 13. The product of the two numbers obtained is 66.

Find the first number.
17. The square of a certain number is 22 less than 13 times the original number. Find the number.
18. Ade buys 20 rulers and 2 pencils for N440 and Uche buys 35 rulers and 4 pencils at the same unit price for N850. Find these unit prices.
19. In a positive number of 2 digits, the sum of the digits is 15. If the digits are interchanged, the number is increased by 9. Find the number.
20. Half the sum of two numbers is 51. One quarter of their difference is 13. Find the number.

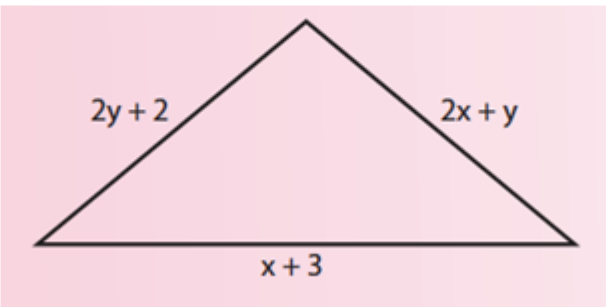
SUMMARY

In this chapter, we have learnt the following:

â– A linear expression consists of linear terms with or without a constant with a connective + or – e.g. 5x + 6, 5x âˆ’ 2y + k.
â– A quadratic expression consists of terms of the second degree or of terms of the first and second degree with or without a constant with a connective âˆ’ or + e.g. 6x^2 + 5x + 6, x^2 + 4xy âˆ’ y^2, 17xy âˆ’ x^2 + y^2 âˆ’ x, 25b^2 âˆ’ 9, x^2 + 5x + 6.
â– An equation has equality of two expressions e.g. 2x^2 + 5x + 6 = 0, 5x^2 + 6x^2y + 7 = 2xy.
â– A linear equation consists of one or no constant and terms of the first degree e.g. 5x + 4 = 0, 3a âˆ’ 2b + 5c = 10.
â– A quadratic equation is of the form ax^2 + bx + c = 0 i.e. it contains one or no constant, several or no linear terms and quadratic terms e.g. 2xy âˆ’ 4 = 0, 4x^2 âˆ’ xy âˆ’ x + y + 6 = 0.
â– Simultaneous equations of two unknowns can be solved by elimination, substitution or graphical methods.
â– Some word problems can lead to quadratic and linear equations. The equation can then be used to solve the problem.

GRADUATED EXERCISES

1. Solve the simultaneous equations: $\frac{2}{t}+\frac{3}{u}=9,\quad \frac{1}{t}+\frac{3}{u}=7$
2. Solve the simultaneous equations: $\frac{1}{t}+\frac{2}{r}=3,\quad \frac{1}{t}-\frac{1}{r}=6$
3. Solve the following simultaneous linear–quadratic equations: **y** âˆ’ 2**x** = âˆ’2; **x**^{2} + **y**^{2} + 2**x** âˆ’ 2**y** = 19
4. Solve the simultaneous equations: **x** âˆ’ **y** = 0; 4**x**^{2} + 3**y**^{2} âˆ’ 5**x** + **y** = 3
5. Solve for **x** and **y** in the equations. **x** + **y** = 3; **x**^{2} + **y**^{2} âˆ’ 17 = 0
6. If 3**p** âˆ’ **q** = 0 and 2**p** + **q** = 4, find **q.** (WAEC)
7. If 2**x** + **y** = 7 and 3**x** âˆ’ 2**y** = 3, by how much is 7**x** greater than 10? (WAEC)
8. A number of 2 digits is increased by 54 when the digits are reversed. The sum of the digits is 12. Find the number. (WAEC)
9. A motorist travels for 3 hours at a certain speed and then increases his speed by 10 km for the next 4 hours. If he travels 355 km altogether, find his speed for the first 3 hours. (WAEC)
10. The sum of the ages of a father and a son is 52 years. 8 years ago the father was eight times as old as the son. How old is the father now.
11. Find the perimeter of the equilateral triangle.



12. The area of a rectangle is 60 cm^2. The length is 1 km more than the width. Find the width.
13. Find two consecutive even numbers whose product is 224.
14. Find two consecutive odd numbers whose product is 195.
15. Solve the equations $3x - 2y = 5$ and $2x + y = 7$.

16. If $x^2 - xy - 6y^2 = 0$, find the values of $\frac{x}{y}$.
17. Solve the equations $x + y = 5$ and $x^2 + y^2 = 13$.
18. The distance from Ibadan to Ilorin is 160 km. If an express train use 16 km/h, it would take 20 mins longer on the journey. Find the average speed of the express train.
19. Solve $x + 3y = 2$ and $x^2 - 2xy + y^2 = 36$.
20. Solve $3x - 2y = 4$ and $6x^2 - xy - 2y^2 + 36 = 0$.

Solve the Following equations:

21. $x + y = 3$; $x^2 - y^2 = 15$
22. $3x - 2y = 10$; $3x^2 + xy - 2y^2 = 50$
23. $2x - y = 8$; $x^2 + 2xy + y^2 = 1$
24. $x - y = 4$; $x^2 - xy = 12$
25. $x + 2y = 1$; $3x^2 + 5xy - 2y^2 = 10$
26. $3x + 2y = 17$; $x^2 - 4xy + 4y^2 = 9$

Solve the following equations graphically:

27. $x + y = 5$; $xy = 6$
28. $x - 2y = 1$; $x^2 + y^2 = 29$
29. $2x + y = 5$; $x^2 - xy = 12$
30. $2x - 3y + 11 = 0$; $2x^2 - xy = 36$