

Chapter 14: Measures of Central Tendency

OBJECTIVES

At the end of the chapter, students should be able to:

1. State the meaning of (i) mean (ii) median and (iii) mode of ungrouped data.
2. Compute the (i) mean (ii) median and (iii) mode of ungrouped data.

I. Measures of Central

Tendency

Measures of central tendency also known as *measures of location* are measures that give us those data that are found among sets of data. The most common ones are mean, median and mode.

(i) Mean

The *mean* is the most common measure of central tendency. It is the sum of all the items in a set of data divided by the number of items. This is also known as *average*. It is denoted by \bar{x} (read "x-bar"). Therefore, the mean of a set of data such as $x_1, x_2, x_3, \dots, x_n$ is expressed as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$= \frac{\sum_{i=1}^n x_i}{n}$$

The symbol Σ (sigma) represents the summation (addition) of all the items in the set of data and n represents the number of items in the data.

Worked Example 1

Find the mean of 10 students whose

Mathematics scores in a test are: 15, 13, 16, 12, 26, 14, 22, 23, 23, 27.

SOLUTION

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{15+13+16+12+26+14+22+23+23+27}{10}$$

$$= \frac{191}{10} = 19.1$$

Worked Example 2

If the mean of the following set of numbers 3, y, 3, 5, 1, 2, 3, 1, 2, 3, 3, 3, 2y, 2, 3, 5, is 3. Find the value of y.

SOLUTION

$$\frac{3+3+3+3+3+3+3+3+3+3+3+3+2y+2+3+5}{16} = 3$$

$$\hat{+} \frac{39+3y}{16} = 3 \text{ (collect like terms)}$$

$$\hat{+} 39 + 3y = 16 \times 3 \text{ (clear the fraction)}$$

$$\hat{+} 39 + 3y = 48$$

$$\hat{+} 3y = 48 - 39 \text{ (subtract 39 from both sides)}$$

$$\hat{+} 3y = 9$$

$$\hat{+} y = \frac{9}{3} \text{ (solve for y)}$$

$$\hat{+} y = 3.$$

Worked Example 3

Table 14.1 shows the scores in a biology test.

Table 14.1

Scores (x)	11	12	13	14	15	16
Frequency (f)	8	10	15	7	5	2

Find the mean score.

SOLUTION

Scores (x)	Frequency (f)	fx
11	8	$8 \times 11 = 88$
12	10	$10 \times 12 = 120$
13	15	$15 \times 13 = 195$
14	7	$7 \times 14 = 98$
15	5	$5 \times 15 = 75$
16	2	$2 \times 16 = 32$
Total number of students = 47		Total score = 608

$$x = \frac{\sum fx}{\sum f} = \frac{608}{47}$$

$$= 12.9362$$

$$= 12.94 \text{ (2 dp)}$$

Worked Example 4

Table 14.2 represents the amount of rainfall in a town for the first six months.

Table 14.2

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.
Amount of rainfall (cm)	5	15	25	32	30	36

Calculate the mean rainfall.

SOLUTION

$$x = \frac{5+15+25+32+30+36}{6}$$

$$= \frac{143}{6}$$

$$= 23.8333$$

$$= 23.83 \text{ cm (2 dp)}$$

Worked Example 5

Table 14.3 shows the mark distribution of an English language test in which the mean mark is 3.

Find the value of y .

Table 14.3

Marks	1	2	3	4	5
Frequency (f)	y	3	$y+3$	3	$4-y$

SOLUTION

$$\frac{(y \times 1) + (3 \times 2) + (y + 3)3 + (4 \times 3) + (4 - y)5}{y + 3 + y + 3 + 3 + 4 - y} = 3$$

$$\Rightarrow \frac{y + 6 + 3y + 9 + 12 + 20 - 5y}{y + 3 + y + 3 + 3 + 4 - y} = 3$$

$$\Rightarrow \frac{47 - y}{y + 13} = 3$$

$$\Rightarrow 47 - y = 3(y + 13)$$

$$\Rightarrow 47 - y = 3y + 39$$

$$\Rightarrow -y - 3y = 39 - 47$$

$$\Rightarrow -4y = -8$$

$$\Rightarrow 4y = 8$$

$$\Rightarrow y = \frac{8}{4}$$

$$\therefore y = 2$$

Worked Example 6

Table 14.4 represents the frequency distribution of the ages of staff of a secondary school. If the mean age is 44 years, find the value of y .

Table 14.4

Age	30	35	40	45	50	55
No. of staff	2	3	3	2	y^2	1

SOLUTION

$$\Rightarrow \frac{(2 \times 30) + (3 \times 35) + (3 \times 40) + (2 \times 45) + (y^2 \times 50) + (1 \times 55)}{2 + 3 + 3 + 2 + y^2 + 1} = \frac{44}{1}$$

$$\Rightarrow \frac{60 + 105 + 120 + 90 + 50y^2 + 55}{2 + 3 + 3 + 2 + y^2 + 1} = 44$$

$$\Rightarrow \frac{50y^2 + 430}{y^2 + 11} = 44$$

$$\Rightarrow 50y^2 + 430 = 44(y^2 + 11)$$

$$\Rightarrow 50y^2 + 430 = 44y^2 + 484$$

$$\Rightarrow 50y^2 - 44y^2 = 484 - 430$$

$$\Rightarrow 6y^2 = 54$$

$$\Rightarrow y^2 = \frac{54}{6}$$

$$\Rightarrow y^2 = 9$$

$$\Rightarrow y = \sqrt{9}$$

$$\therefore y = 3$$

Worked Example 7

Form a frequency distribution table and find the mean of the following scores:

Express your answer to the nearest whole number.

1	2	3	1	2	3
3	1	3	4	3	4
1	2	1	2	2	4
3	2	3	3	4	6
3	3	4	4	5	4
4	4	4	5	4	6
4	4	6	4	6	4

Scores (x)	Tally	Frequency (f)	fx
1	###	5	$5 \times 1 = 5$
2	###/	6	$6 \times 2 = 12$
3	######	10	$10 \times 3 = 30$
4	#########	15	$15 \times 4 = 60$
5	//	2	$2 \times 5 = 10$
6	////	4	$4 \times 6 = 24$
Total		$\Sigma f = 42$	$\Sigma fx = 141$

$$x = \frac{\Sigma fx}{\Sigma f} = \frac{141}{42} = 3.36$$

$\cong 3$ (nearest whole number)

Exercise 1

- The following data shows the ages of members of a press-club in a secondary school.

14	13	15	18	19
17	15	18	20	15
12	19	14	17	16

Calculate the mean age of the students.

- Calculate the mean of the following data:

(a) $\hat{a}^{\wedge}3, \hat{a}^{\wedge}4, \hat{a}^{\wedge}9, \hat{a}^{\wedge}15, \hat{a}^{\wedge}8, 10, \hat{a}^{\wedge}2, \hat{a}^{\wedge}13$

(b) 3.42, 4.23, 4.03, 2.7, 15.3, 0.43, 3.73

(c) $3\frac{1}{2}, 5\frac{2}{3}, 2\frac{1}{4}, 16\frac{2}{5}, 13\frac{1}{3}, 1\frac{1}{5}$

- Table 14.5 represents the monthly profit of a fast-food sales girl in the first six months transaction. Find the mean profit.

Table 14.5

Month	Jan.	Feb.	Mar.	Apr.	May	June
Profit (₦)	1 200	3 100	1 100	141.50	4 100	3 010

- Table 14.6 shows the frequency distribution of scores of 90 candidates in a University aptitude test. Find the mean of the scores.

Table 14.6

Score (x)	Frequency (f)
12	3
13	5
14	10
15	15
16	20
17	13
18	7
19	15
20	2

5. A group of students took a test and the following frequency table shows the scores:

Table 14.7

Score (x)	0	1	2	3	4	5
Frequency (f)	2	3	4	2	7	2

Find the mean

6. The number of items produced by a company over a five-year period is given below:

Table 14.8

Year	2007	2008	2009	2010	2011
No. of items produced	3 100	1 500	500	800	8 200

What is the average production for the five-year period?

7. Table 14.9 shows the weights to the nearest kilogramme of twelve students in a Further Mathematics class.

Table 14.9

Weight (kg)	55	57	59	61	63
No. of students	2	1	2	4	6

Calculate the mean weight correct to the nearest kilogramme. (WAEC)

8. The mean heights of the three groups of students consisting of 20, 16 and 14 students are 1.67 m, 1.50 m and 1.40 m, respectively. Find the mean height of all the students. (WAEC)
9. Table 14.10 shows the frequency distribution of marks obtained by a group of students in a test.

Table 14.10

Marks	3	4	5	6	7	8
Frequency	5	$x - 1$	x	9	4	1

If the mean is 5m, calculate the value of x .

10. The mean of 7 numbers is 12 and when a number is added, the mean becomes 8. Find the new added number.

(ii) Median

When the number of items is odd, the item that lies in the middle after the set of items (data) have been arranged in order of magnitude either in ascending or descending order is called the *median*. But when the number of items is even, the arithmetic mean of the two middle items gives the median value.

Thus,

Median (odd number of items)

$$= \left(\frac{N+1}{2} \right) \text{th (i.e. } N = \text{odd)}$$

Median (even number of items)

$$= \frac{\left(\frac{N}{2}\right)^{\text{th}} + \left(\frac{N}{2} + 1\right)^{\text{th}}}{2} \quad (\text{i.e. } N = \text{even})$$

Worked Example 8

Find the median of the following numbers: 3, 9, 7, 5, 2, 13, 10.

SOLUTION

Firstly, arrange items in their order of magnitude (either ascending or descending): 2, 3, 5, 7, 9, 10, 13

There are seven items (odd).

$$\begin{aligned} \text{Formula} &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ item} \\ &= \frac{7+1}{2}^{\text{th}} \text{ item} \\ &= \left(\frac{8}{2}\right)^{\text{th}} \text{ item} \\ &= 4^{\text{th}} \text{ item} \end{aligned}$$

∴ The median item is 7.

Worked Example 9

Find the median of the following scores: 14, 2, 9, 7, 18, 15, 13, 8.

SOLUTION

First arrange the scores in either ascending or descending order.

2, 7, 8, 9, 13, 14, 15, 18 (ascending)

There are eight items, i.e. even.

∴ The arithmetic mean of the two items in the middle gives the median.

$$\begin{aligned} \text{Formula} &= \frac{\left(\frac{N}{2}\right)^{\text{th}} + \left(\frac{N}{2} + 1\right)^{\text{th}}}{2} \\ &= \frac{\left(\frac{8}{2}\right)^{\text{th}} + \left(\frac{8}{2} + 1\right)^{\text{th}}}{2} \\ &= \frac{4^{\text{th}} + (4 + 1)^{\text{th}}}{2} \\ &= \frac{4^{\text{th}} + 5^{\text{th}}}{2} \end{aligned}$$

$$\text{Thus, median} = \frac{9 + 13}{2} = \frac{22}{2} = 11$$

Worked Example 10

Use the following distribution to find the median.

Table 14.11(a)

Score (x)	Frequency (f)
1	3
2	10
3	15
4	2
5	1

SOLUTION

Re-arrange the table as shown in

Table 14.11(b).

Table 14.11(b)

Score	Frequency (f)	Cumulative Frequency (CF)	Position
1	3	3	1st–3rd
2	10	3 + 10 = 13	4th–13th
3	15	3 + 10 + 15 = 28	14th–28th
4	2	3 + 10 + 15 + 2 = 30	29th–30th
5	1	3 + 10 + 15 + 2 + 1 = 31	31st

$N = 31$ (odd)

$$\begin{aligned}\text{Thus, } \left(\frac{N+1}{2}\right)^{\text{th}} &= \left(\frac{31+1}{2}\right)^{\text{th}} \\ &= \left(\frac{31+1}{2}\right)^{\text{th}} \\ &= 16^{\text{th}} \text{ position}\end{aligned}$$

The 16th position score lies between 14th and 28th. It is 3 that lies in this range.

\hat{a}' the median is 3.

Worked Example 11

Table 14.12(a) represents the scores of 40 students in a Mathematics test. Find the median score.

Table 14.12(a)

Score (x)	1	2	3	4	5
Frequency (f)	15	12	8	3	2

SOLUTION

Table 14.12(b)

Score	Frequency (f)	CF	Position
1	15	15	1st–15th
2	12	15 + 12 = 27	16th–27th
3	8	15 + 12 + 8 = 35	28th–35th
4	3	15 + 12 + 8 + 3 = 38	36th–38th
5	2	15 + 12 + 8 + 3 + 2 = 40	39th–40th

$N = 40$ (even)

$$\begin{aligned}\text{Rule: } & \frac{\left(\frac{N}{2}\right)^{\text{th}} + \left(\frac{N}{2} + 1\right)^{\text{th}}}{2} \text{ score} \\ &= \frac{\left(\frac{40}{2}\right)^{\text{th}} + \left(\frac{40}{2} + 1\right)^{\text{th}}}{2} \text{ score} \\ &= \left(\frac{20^{\text{th}} + 21^{\text{st}}}{2}\right) \text{ score}\end{aligned}$$

By observing through the positional column, we see that both the 20th and 21st item is 2

\hat{a}' The median is $\frac{2+2}{2} = \frac{4}{2} = 2$.

Worked Example 12

Table 14.13(a) shows the weights of 50 students in a class to the nearest kilogramme.

Find the median weight.

Table 14.13(a)

Weight (kg)	50	52	54	56	58	60
No. of students	3	10	12	15	7	3

SOLUTION

Table 14.13(b)

Weight (kg)	Frequency (f)	CF	Position
50	3	3	1st–3rd
52	10	3 + 10 = 13	4th–13th
54	12	3 + 10 + 12 = 25	14th–25th
56	15	3 + 10 + 12 + 15 = 40	26th–40th
58	7	3 + 10 + 12 + 15 + 7 = 47	41st–47th
60	3	3 + 10 + 12 + 15 + 7 + 3 = 50	48th–50th

$N = 50$ (even)

$$\begin{aligned}
 \text{Rule: } & \frac{\left(\frac{N}{2}\right)\text{th} + \left(\frac{N}{2} + 1\right)\text{th}}{2} \text{ item} \\
 & = \frac{\left(\frac{50}{2}\right)\text{th} + \left(\frac{50}{2} + 1\right)\text{th}}{2} \text{ item} \\
 & = \frac{25\text{th} + 26\text{th}}{2} \text{ item}
 \end{aligned}$$

Going through the positional column, we see that the 25th item is 54 kg and the 26th item is 56 kg.

$$\hat{a} \hat{'} \text{ The median is } \frac{54 + 56}{2} = \frac{110}{2} = 55\text{kg}$$

Exercise 2

Find the median of the following:

- 3, 8, 10, 15, 13, 7, 5
- 8, 15, 30, 41, 16, 13, 18
- 5, 9, 12, 13, 15, 25
- $\hat{a} \hat{'}^3, \hat{a} \hat{'}^5, \hat{a} \hat{'}^4, \hat{a} \hat{'}^9, \hat{a} \hat{'}^2, \hat{a} \hat{'}^{10}$
- 1, 2, 3, 4, 1, 3, 4, 1, 3, 4, 5, 3, 2, 3, 3, 2, 3, 2
- N20, N10, N20, N20, N50, N50, N10, N30, N10, N40, N60, N60, N60, N40
- $41^\circ, 32^\circ, 23^\circ, 53^\circ, 42^\circ, 15^\circ, 13^\circ, 53^\circ, 43^\circ, 15^\circ$
- 5, 1, 1, 0, 2, 1, 2, 4, 3, 0, 3, 3, 4, 4, 3, 4, 4, 1, 3, 1, 5, 0, 5, 0
- 1, 1, 2, 3, 3, 4, 5, 5, 1, 2, 3, 4, 5
- 1.0, 1.1, 1.2, 1.0, 1.1, 1.0, 1.1, 1.2, 1.1, 1.2, 1.1, 1.4, 3.1, 1.0, 1.1

Calculate the medians of the following frequency distributions:

- Scores of students in an English test.

Table 14.14

Score (x)	1	2	3	4	5
Frequency (f)	15	16	20	11	10

- Weights of students in a particular class.

Table 14.15

Weight (kg)	45	50	55	60
No. of students	14	13	18	6

13. Ages of students in the debating club of a secondary school.

Table 14.16

Age (years)	11	12	13	14	15
No. of students	3	13	43	6	5

14. Marks of candidates in an aptitude test.

Table 14.17

Marks (x)	10	20	30	40	50
Frequency (f)	13	18	34	60	10

15. Shoe sizes of players in a particular football club.

Table 14.18

Shoe sizes (mm)	60	65	70	75	80
No. of students	15	20	35	5	7

(iii) Mode

Mode is the score or item with the highest frequency. That is, the most occurring item.

Worked Example 13

Identify the mode of the following scores:

(a) 3, 10, 15, 4, 9, 6, 4

(b) 9, 8, 2, 10, 16

(c) 5, 7, 10, 7, 15, 7, 3, 10, 4, 10

SOLUTION

(a) The mode is 4 because 4 appear twice while other scores appear only once.

(b) The mode does not exist as none of the scores appear more than once.

(c) Both 7 and 10 appear more than other scores. They appear 3 times each. Therefore, the set is referred to as *bimodal*.

Mode = 7 and 10.

Exercise 3

Find the mode of the following sets of data:

1. 3, 7, 9, 6, 9, 10, 15, 7

2. 9, 10, 4, 6, 15, 20

3. 5, 6, 10, 3, 2, 3, 5, 8

4. \hat{a}^2 , \hat{a}^3 , \hat{a}^1 , \hat{a}^5 , \hat{a}^3 , \hat{a}^{15}

5. 15 kg, 31 kg, 36 kg, 51 kg, 15 kg

6. 34° , 36° , 32° , 42° , 34° , 39° , 50°

7. N143, N189, N76, N94, N13, N76

8. $1\frac{1}{2}$, $2\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{3}$, $5\frac{1}{2}$, $2\frac{1}{2}$, $10\frac{2}{3}$

9.

2	4	4	5	2	4	2
4	5	2	7	5	2	5
2	4	5	5	2	5	2
5	4	2	5	4	4	4
2	5	2	7	2	7	2
5	4	5	7	4	5	4
2	5	2	5	7	7	7

10.

A	B	A	B	C	C
C	D	C	A	A	D
D	A	D	D	D	D
C	D	A	B	B	D
A	A	D	C	D	D

Find the modes of the following frequency distributions:

11. Ages of under-18 football club.

Table 14.19

Age (x)	13	14	15	16	17	18
Frequency (f)	3	10	15	21	5	5

12. Weight to the nearest kilogramme of 31 students in a class.

Table 14.20

Weight (kg)	50	55	60	65	70
No. of students	3	10	15	2	1

13. Marks of students in a Mathematics test.

Table 14.21

Marks (x)	1	2	3	4	5	6	7
Frequency (f)	3	10	13	25	10	5	1

14. Ages of 126 out-patients to the nearest ten in a specialist hospital on a particular day.

Table 14.22

Ages	10	20	30	40	50	60	70
No. of patients	5	10	30	45	32	1	3

15. The monthly profit of a G.S.M. credit dealer in the first six months

Table 14.23

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.
Profit (₦)	200	300	500	1 500	300	1 000

SUMMARY

In this chapter, we have learnt the following:

â– Measures of central tendency also known as measures of location describe or identify the middle item of the data set.

â– Mean is the sum of scores divided by the number of items in the set of scores.

â– Median is the item that lies in the middle, if and only if, the items of the set have been arranged in their order of magnitude.

â– Mode is the item/score with the highest frequency. The set is referred to as bimodal, if there exists two items that appear equal number of times and more than others.

â– Computing the mean, median and mode of ungrouped data.

GRADUATED EXERCISES

1. Which of the following is not a measure of central tendency?

(a) Mode (b) Range (c) Mean (d) Median

Table 14.24 shows the frequency distribution of the number of chairs in each of 40 rooms of various houses.

Table 14.24

No. of chairs	1	2	3	4	5	6	7
Frequency	2	7	5	4	9	7	6

Use the above table to answer questions 2â€“4.

2. Find the mean of the distribution.

3. Calculate the median of the distribution.

4. Identify the mode of the distribution.

5. The mean of 1, 2, x , 11, y , 14 is 8 and the median is 9. Find the values of x and y .

6. Table 14.25 shows the distribution of test scores in a class.

Table 14.25

Scores (x)	1	2	3	4	5	6	7	8	9	10
No. of pupils	1	1	5	3	$K^2 + 1$	0	6	2	3	4

If the mean score of the class is 6, find the:

(a) Value of K (b) median score

7. Given that the mean score of 13 students in a Mathematics test is 3 and the mean score of 7 others is 2. Find the mean score of all the students in the same test.

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8. A student bought the following items from a provision store. 7 ball points pen at N20.00 each. 10 pencils at N10.00 each. 13, 30-leaves notebook at N25.00 each.

Find the:

(a) Total amount spent.

(b) Mean price of the items.

9. Below are the scores of 45 students in a Mathematics test marked over ten.

4	5	10	6	4	3	7	6	3
3	9	8	10	10	6	10	7	6
3	6	5	10	2	4	3	7	7
6	10	5	4	3	5	9	3	5
7	10	6	4	4	6	2	3	3

(a) Prepare a frequency table for the data.

(b) Identify the mode.

(c) Find the median.

(d) Calculate the mean.

10. In a Mathematics examination set for 50 students in SSII, 5 of them scored an average of 45%, 23 had an average of 50%, 20 got an average of 65% and 2 scored 75%.

(a) What is the total score?

(b) Calculate the mean score of the 50 students.

11. If the mean weight of 8 boxers is 62 kg, and when a boxer joined them, the mean weight became 65 kg. What is the weight of the new boxer?

12. If the mean of the data in Table 14.26 is 5.4, find y .

Table 14.26

Score (x)	2	4	6	8	9
Frequency (f)	5	4	y	2	4

13. Table 14.27 shows the scores of a group of students in a test. If the average score is 3.5, find the value of x .

Table 14.27

Score (x)	1	2	3	4	5	6
No. of students	1	4	5	6	x	2

(WAEC)

In a Science Senior Model Secondary School that has six streams of SSII classes, the average scores of the 5 core subjects (Math, Eng, Bio, Chem., Phy) were computed as shown in Table 14.28.

Table 14.28

Stream	A	B	C	D	E
No. in class	45	50	46	45	42
Average score	65	63	62	59	45

14. Compute the total score for each stream.
15. Calculate the mean score of all the students in SSII.