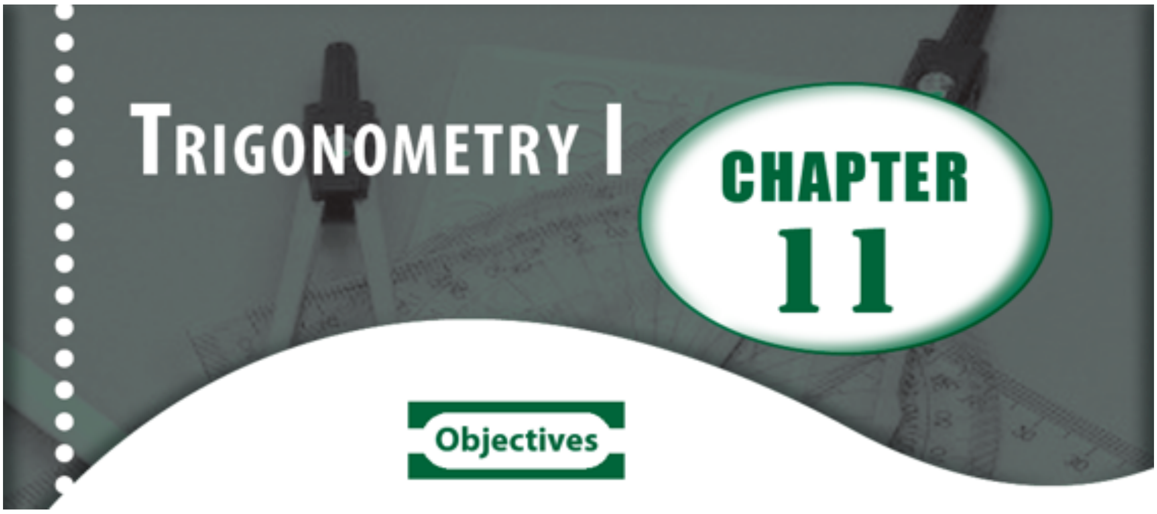
# CHAPTER 11




At the end of the chapter, students should be able to:

1. Solve problems involving the use of sine, cosine and tangent in right-angled triangles.
2. Derive trigonometric ratios of special angles: 30°, 45° and 60°.
3. Apply trigonometric ratios of 30°, 45° and 60° to solving problems without the use of calculating aids.
4. Relate sine and cosine ratios to the unit circle.

## I. Sine, Cosine and Tangent of Right-angled Triangles

Recall, given a right-angled triangle ABC as shown in Fig. 11.1,

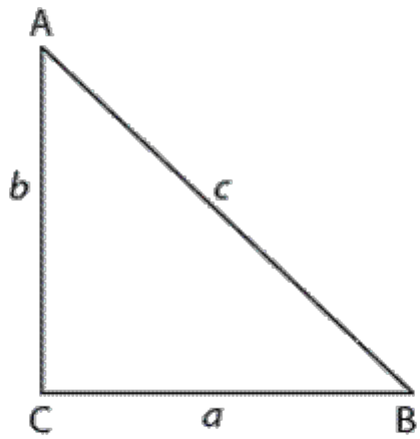



**Fig. 11.1**

with the aid of the mnemonic SOH CAH TOA and giving the ratios from the angles that:

$$\sin B = \frac{\text{Opp}}{\text{Hyp}} = \frac{AC}{AB} = \frac{b}{c}$$

$$\cos B = \frac{\text{Adj}}{\text{Hyp}} = \frac{BC}{AB} = \frac{a}{c}$$

$$\tan B = \frac{\text{Opp}}{\text{Adj}} = \frac{AC}{BC} = \frac{b}{a}$$

Therefore, it can also be observed that:

$$\frac{\sin B}{\cos B} = \frac{b}{c} \div \frac{a}{c}$$

$$= \frac{b}{c} \times \frac{c}{a}$$

$$= \frac{b}{a}$$

Therefore, $\dfrac{\sin B}{\cos B} = \dfrac{b}{a} = \tan B$

$$\sec B = \frac{\text{Hyp}}{\text{Adj}} = \frac{AB}{BC} = \frac{c}{a}$$

$$\text{cosec } B = \frac{\text{Adj}}{\text{Opp}} = \frac{BC}{AC} = \frac{a}{b}$$

**Note:** In $\Delta ABC$, in Fig. 11.2,

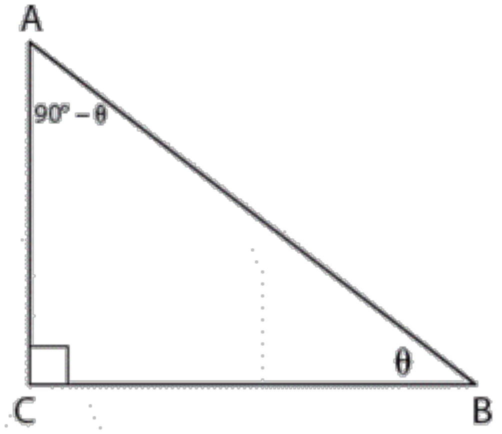



**Fig. 11.2**

angles A and B are complementary angles since $\angle A + \angle B = 90°$.

$\Rightarrow \angle A = 90° - \angle B$ and $\angle B = 90° - \angle A$.

Therefore, $\sin\theta = \cos(90° - \theta) = \frac{AC}{AB}$,

$\cos\theta = \sin(90° - \theta) = \frac{BC}{AB}$ and

$\tan\theta = \cot(90° - \theta) = \frac{AC}{BC}$.

While giving the angles from the ratio are:

$$B = \sin^{-1}\frac{Opp}{Hyp}$$

$$B = \cos^{-1}\frac{Adj}{Hyp}$$

$$B = \tan^{-1}\frac{Opp}{Adj}$$

## Worked Example 1

Find the sine, cosine and tangent of the following angles using tables:

(a) $35°$

(b) $43.6°$

(c) $56.28°$

(d) $63.49°$

## Solution

(a) $\sin 35° = 0.5736$

$\cos 35° = 0.8192$

$\tan 35° = 0.7002$

(b) $\sin 43.6° = 0.6896$

$\cos 43.6° = 0.7242$

$\tan 43.6° = 0.9523$

(c) $\sin 56.28° = 0.8318$

$\cos 56.28° = 0.5551$

$\tan 56.28° = 1.498$

(d) $\sin 63.49° = 0.8949$

$\cos 63.49° = 0.4464$

$\tan 63.49° = 2.005$

## Worked Example 2

Using tables, identify the angle whose

(a) sine is 0.8941

(b) cosine is 0.7891

(c) tangent is 0.9431

## Solution

(a) Given that $\sin\theta = 0.8941$

$\Rightarrow \theta = \sin^{-1} 0.8941°$

Therefore, $\theta = 63.39°$

(b) Given that $\cos\theta = 0.7891$

$\Rightarrow \theta = \cos^{-1} 0.7891$

Therefore, $\theta = 37.90°$

## Worked Example 3

Solve the following equations using the identities in Fig. 11.2:

(a) $\sin\theta = \cos 58°$

(b) $\cos 2\theta = \sin 37°$

(c) $\cos 5\theta = \sin 4\theta$

(d) $\sin 2\theta = \cos(2\theta + 30°)$

## Solution

(a) $\sin\theta = \cos 58°$ (given)

But $\sin\theta = \cos(90° - \theta)$

$\Rightarrow \cos(90° - \theta) = \cos 58°$

$\Rightarrow 90° - \theta = 58°$

$\Rightarrow -\theta = 58° - 90°$

$-\theta = -32°$

Therefore, $\theta = 32°$

(b) $\cos 2\theta = \sin 37°$ (given)

But $\cos 2\theta = \sin(90° - 2\theta)$

$\Rightarrow \sin(90° - 2\theta) = \sin 37°$

$$-2\theta = 37° - 90°$$

$$-2\theta = -53°$$

$$2\theta = 53°$$

$$\theta = \frac{53}{2}$$

Therefore, $\theta = 26.5°$

(c) $\cos 5\theta = \sin 4\theta$ (given)

But $\cos 5\theta = \sin(90° - 5\theta)$

$\sin(90° - 5\theta) = \sin 4\theta$

$$90° - 5\theta = 4\theta$$

$$-5\theta - 4\theta = -90°$$

$$9\theta = 90°$$

$$\theta = \frac{90}{9}$$

$$\theta = 10°$$

(d) $\sin 2\theta = \cos(2\theta + 30°)$ (given)

But $\sin 2\theta = \cos(90° - 2\theta)$

$\cos(90° - 2\theta) = \cos(2\theta + 30°)$

$$90° - 2\theta = 2\theta + 30°$$

$$-2\theta - 2\theta = 30° - 90°$$

$$-4\theta = -60°$$

$$\theta = \frac{60}{4}$$

Therefore, $\theta = 15°$

## Worked Example 4

Find the lettered lengths of the triangles in Fig. 11.3.

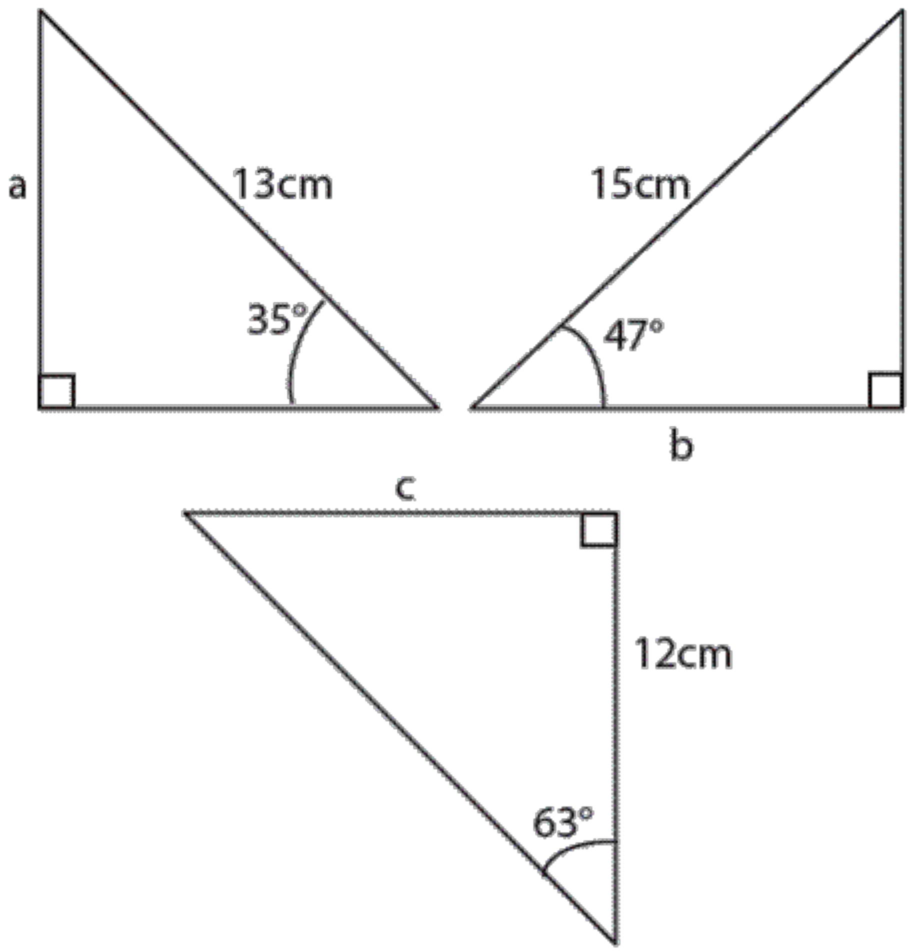



**Fig. 11.3**

## Solution

(a) Recall the trigonometric ratio formula which connects opposite and hypotenuse in sine:

$$\text{Therefore, } \sin 35° = \frac{a}{13 \text{ cm}}$$

$$a = 13 \text{ cm } (0.57358)$$

$$= 7.4565$$

∴ $a$ = 7 cm (to the nearest whole number)

(b) **Note:** cosine connects adjacent and hypotenuse.

$$\text{Therefore, } \cos 47° = \frac{b}{15}$$

$$b = 15 \times \cos 47°$$

$$b = 15 \times 0.6820$$

$$b = 10.23$$

$b$ = 10 cm (to the nearest whole number)

(c) **Note:** tangent connects opposite and adjacent.

$$\text{Therefore, } \tan 63° = \frac{c}{12}$$

$$c = 12 \times \tan 63°$$

$$c = 12 \times 1.9626$$

$$c = 23.55$$

$c$ = 24 cm (to the nearest whole number)

## Worked Example 5

Find the lettered angles of the triangle in Fig. 11.4.

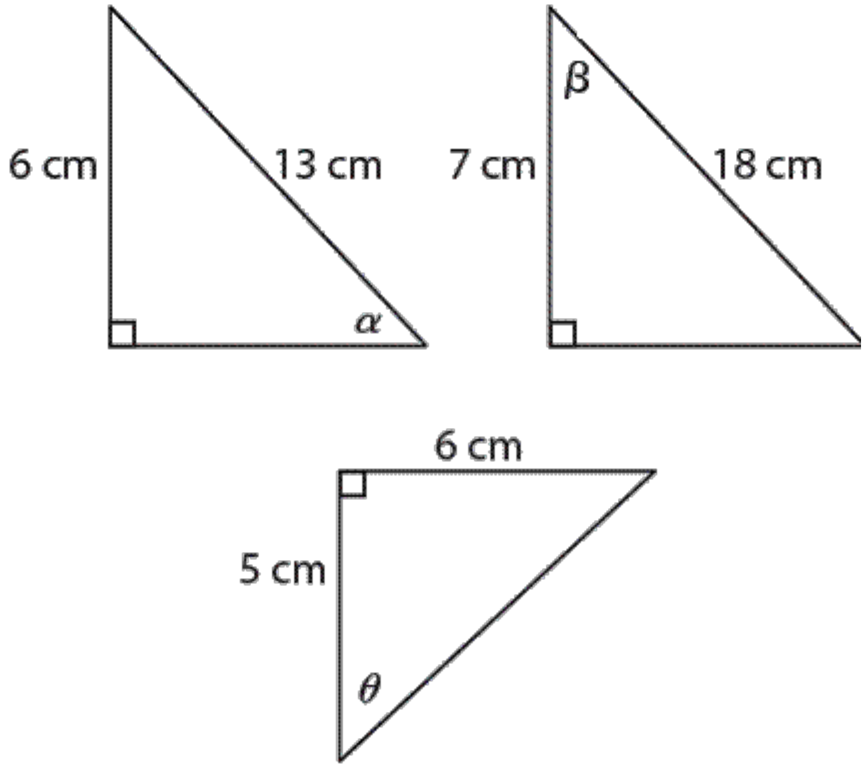



**Fig. 11.4**

## Solution

(a) Sine has the following components namely: opp and hyp.

Therefore, $\sin \alpha = \frac{6}{13} = 0.4615$

$$\alpha = \sin^{-1} 0.4615$$

(Using table or calculator)

$$\alpha = 27.48^\circ$$

Therefore, $\alpha = 27^\circ$ (to the nearest degree)

(b) Cosine has adjacent and hypotenuse.

Therefore, $\cos \beta = \frac{7}{18}$

$$\cos \beta = 0.3889$$

$$\beta = \cos^{-1} 0.3889$$

(Using table or calculator)

$$\beta = 67.11°$$

Therefore, $\beta = 67°$ (to the nearest degree)

(c) Tangent has opposite and adjacent.

Therefore, $\tan \theta = \frac{6}{5}$

$$= 1.2$$

$$\theta = \tan^{-1} 1.2$$

(using table or calculator)

$$\theta = 50.19°$$

Therefore, $\theta = 50°$ (to the nearest degree)

## Exercise 1

1. Find the (a) sine (b) cosine and (c) tangent of the following angles using table:

| | |
|---|---|
| (i) 15° | (ii) 23° |
| (iii) 37° | (iv) 43° |
| (v) 48° | (vi) 56° |
| (vii) 57.14° | (viii) 57.64° |
| (ix) 68.75° | (x) 75.63° |

2. Using tables, find to the nearest degree the angles whose (a) sines and (b) cosines are:

| | |
|---|---|
| (i) 0.1456 | (ii) 0.4361 |
| (iii) 0.4891 | (iv) 0.5789 |
| (v) 0.4976 | (vi) 0.7694 |
| (vii) 0.9416 | (viii) 0.8496 |
| (ix) 0.6946 | (x) 0.0468 |

3. What have you observed from the values of the angles?

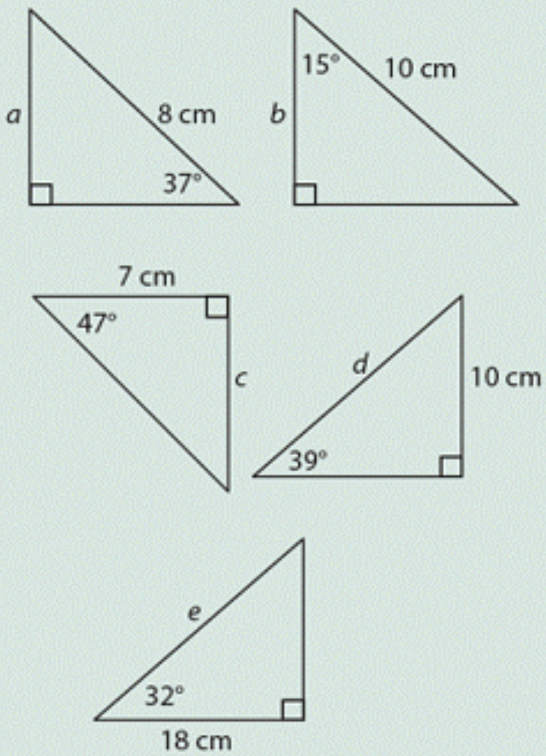



**Fig. 11.5**

4. What are the values of the lettered angles in the triangles in Fig. 11.6.

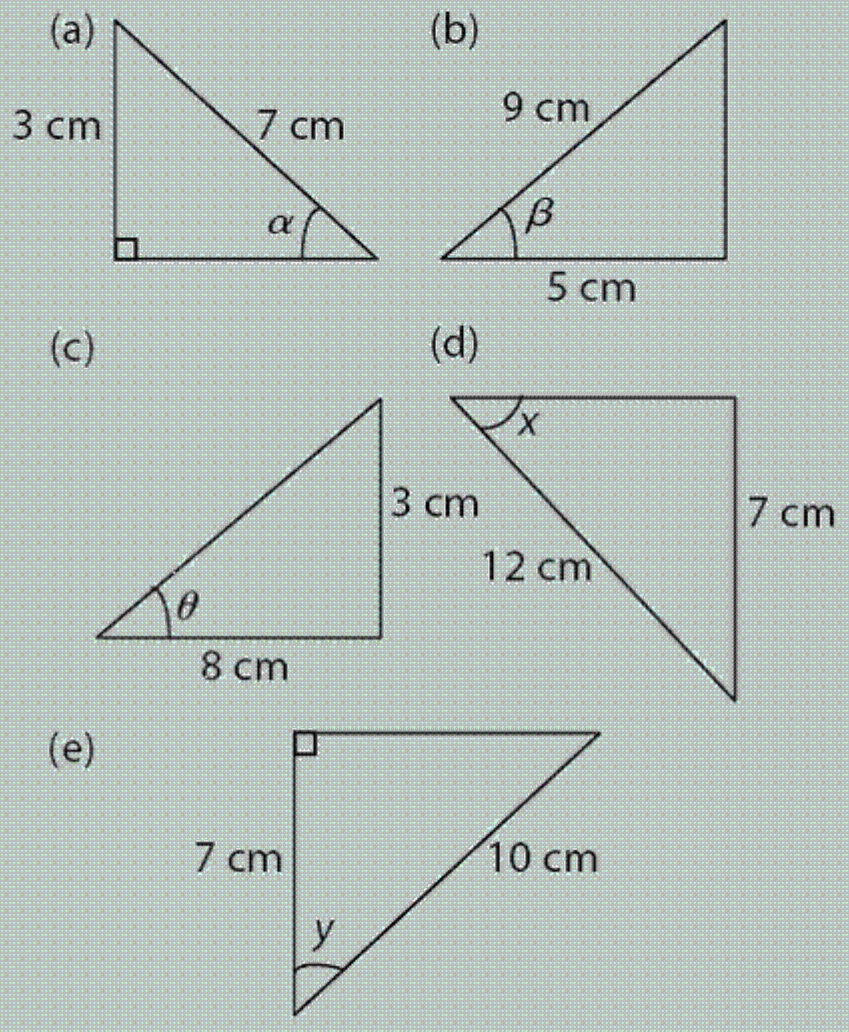



**Fig. 11.6**

5. Solve the following trigonometric equations:

   (a) $\sin\theta = \cos 3\theta$

   (b) $\cos\theta = \sin 5\theta$

   (c) $\cos 3\theta = \sin 2\theta$

   (d) $\sin a = \cos(2a - 35°)$

   (e) $\cos(2\theta + 15°) = \sin\theta$

   (f) $\sin 2\theta - \cos\theta = 0$

   (g) $\cos 4\theta - \sin 2\theta = 0$

   (h) $\cos(5x - 45) = \sin 55°$

   (i) $\sin(\theta + 15°) = \cos(5\theta + 8)$

   (j) $\sin(x + 30°) = \cos 40°$

6. A man walks 150 m on a bearing of 35°. How far north is he from his starting point?

7. If the bearing of A from B is 150°, what is the bearing of B from A?

8. The angle of elevation of the top of a tower from a point on the horizontal ground 50 m away from the foot of the tower is 60°. Calculate the height of the tower to three significant figures.

9. A ladder 20 m long rests against a vertical wall so that the foot of the ladder is 9m from the wall. Find
   (a) Correct to the nearest degree, the angle that the ladder makes with the wall.
   (b) Correct to 1 decimal place, the height above the ground at which the upper end of the ladder touches the wall.

   **(WASSCE)**

10. Find the sine, cosine and tangent of angle B in Fig. 11.7.

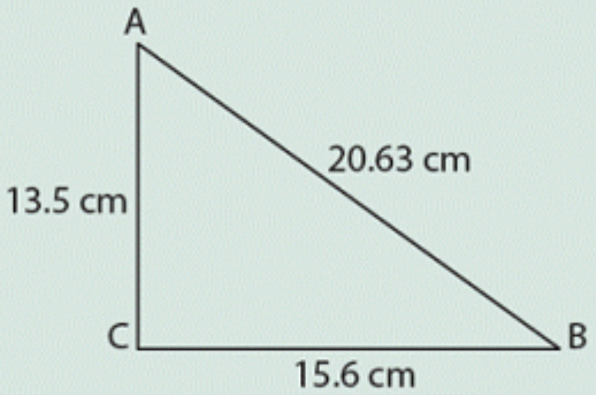



**Fig. 11.7**

# 11. Pythagoras Theorem (Revision)

Recall what we learnt in Junior Secondary School about the following theorem:

Given a right-angled triangle, the square on the hypotenuse is equal in area to the sum of the squares on the other two sides.

Thus: $a^2 + b^2 = c^2$

The theorem is necessary for the calculation of lengths that originate from a right-angled triangle.

## Worked Example 6

Which of these triangles is a right-angled triangle?

(a) 5, 12, 13

(b) 0.2, 0.15, 0.25

(c) 8, 13, 15

## Solution

(a) $a^2 + b^2 = c^2$

$$\begin{aligned} \text{L.H.S} &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \\ \text{R.H.S.} &= 13^2 \\ &= 169 \\ \text{R.H.S.} &= \text{L.H.S.} \end{aligned}$$

∴ It is a right-angled triangle, and 5, 12 and 13 are called *Pythagorean triples*.

(b)

$$\begin{aligned} \text{L.H.S.} &= (0.2)^2 + (0.15)^2 \\ &= 0.04 + 0.0225 \\ &= 0.0625 \\ \text{R.H.S.} &= (0.25)^2 \\ &= 0.0625 \\ \text{L.H.S.} &= \text{R.H.S.} \end{aligned}$$

∴ It is a right-angled triangle and 0.2, 0.15 and 0.25 are Pythagorean triples.

$$\begin{aligned}
\text{(c) L.H.S.} &= 8^2 + 13^2 \\
&= 64 + 169 \\
&= 233 \\
\text{R.H.S.} &= 15^2 \\
&= 225 \\
\text{L.H.S.} &\neq \text{R.H.S}
\end{aligned}$$

$\therefore$ It is not a right-angled triangle, and 8, 13 and 15 are not Pythagorean triples.

## Trigonometry and Pythagoras rule

Recall, when two sides of a right-angled triangle are given, we can identify the third side. Similarly, when any of sine, cosine or tangent is given, we can identify the values of the other two trigonometric ratios since there exist a relationship among them.

### Worked Example 7

If $\sin\theta = \frac{3}{5}$ where $0° < \theta < 90°$, find

(a) $\cos\theta$

(b) $\tan\theta$

(c) $\sin^2\theta + \cos^2\theta$

(d) $\frac{\sin\theta - 1}{\cos\theta}$

### Solution

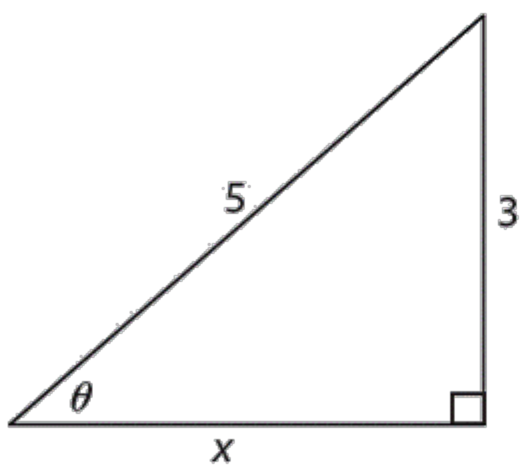



**Fig. 11.8**

In Fig. 11.8, the opposite is 3 while the hypotenuse is 5, then use the Pythagoras theorem to determine the adjacent side.

$$x^2 = 5^2 - 3^2$$

$$x = \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16}$$

$$= 4$$

Therefore,

(a) $\cos\theta = \frac{4}{5}\left(\frac{\text{Adj}}{\text{Hyp}}\right)$

(b) $\tan\theta = \frac{3}{4}\left(\frac{\text{Opp}}{\text{Adj}}\right)$

(c) $\sin^2\theta + \cos^2\theta$

$$= \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

$$= \frac{9}{25} + \frac{16}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

(d) $\dfrac{\sin\theta - 1}{\cos\theta}$

$$= \frac{\frac{3}{5} - 1}{\frac{4}{5}}$$

$$= \frac{\frac{-2}{5}}{\frac{4}{5}}$$

$$= \frac{-2}{5} \times \frac{5}{4}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

## Exercise 2

1. Given the right-angled triangle in Fig. 11.9.

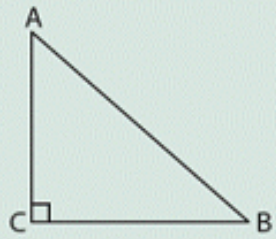



**Fig. 11.9**

   (a) Find |AB|, if |AC| = 3 cm and |BC| = 8 cm.

   (b) Find |AB|, if |AC| = 5 cm and |BC| = 12 cm.

   (c) Find |BC|, if |AB| = 8.3 cm and |AC| = 2.5 cm.

   (d) Find |BC|, if |AB| = 5.3 cm and |AC| = 4.5 cm.

   (e) Find |AC|, if |AB| = 13 cm and |BC| = 5 cm.

2. Triangle PQR is a right-angled at Q. |PQ| = 3*a* cm and |QR| = 4*a* cm. Determine |PR| in terms of *a*. **(WASSCE)**

3. Points X and Y are 12 cm North and 5 cm East respectively of point Z. Calculate |XY|.

4. P, Q and R are points in the same horizontal plane. The bearing of Q from P is 150° and the bearing of R from Q is 160°. If |PQ| = 5 cm and |QR| = 3 m, find the bearing of R from P correct to the nearest degree.

5. In Fig. 11.10, XM is the altitude from X to YZ. $|XY| = 13$ cm, $|XZ| = 15$ cm and $|YM| = 5$ cm. Find the length of YZ.

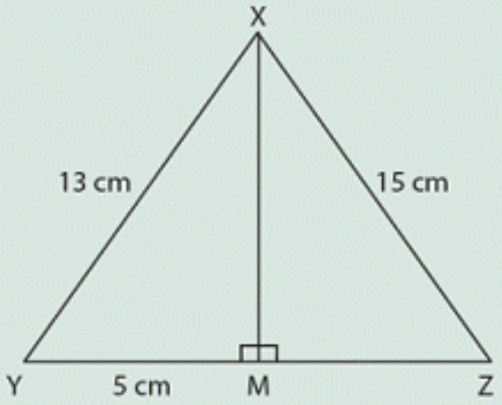



**Fig. 11.10**

6. The diagonal and one side of a square are $x$ and $y$ units respectively. Find an expression for $y$ in terms of $x$.

7. The sides of a right-angled triangle in ascending order of magnitude are 8 cm, $(x - 2)$ cm and $x$ cm. Find $x$.

8. The sides of a rectangular floor are $x$ m and $(x + 7)$ m. The diagonal is $(x + 8)$ m. Calculate in metres (a) the value of $x$ and (b) the area of the floor.

9. The length in cm of the sides of a right-angled triangle are $x$, $(x + 2)$ and $(x + 1)$ where $x > 0$. Find in cm the length of its hypotenuse.

10. Given that $\cos\theta = \frac{5}{13}$, what is the value of $\tan\theta$ for $0° < \theta < 90°$.

11. If $\sin x = \frac{12}{13}$ where $0° < x < 90°$, find the value of $1 - \cos^2 x$.

12. If $\cos x = \frac{3}{8}$, find $\sin x$, leaving your answer in square root.
13. If $\tan x = \frac{2}{3}$, find $\sin x$, leaving your answer in square root.
14. If $\cos \theta = \frac{12}{13}$, find the value of $\frac{1 + \tan \theta}{1 - \cos \theta}$.
15. If $\tan x = \frac{1}{\sqrt{3}}$, find $\cos x - \sin x$ such that $0° < x < 90°$.

## III. Special Angles

Special angles are those angles whose trigonometric ratios can be obtained without necessarily consulting the tables or using a calculator. These angles are 45°, 60°, 30°. However, these are obtained through the following illustration. Consider an isosceles right-angled triangle ABC with $\angle C = 90°$, $|AC| = |BC| = |\text{unit}|$ where $\angle A = \angle B = 45°$ (base angles of an isosceles right-angled triangle) as shown in Fig. 11.11.

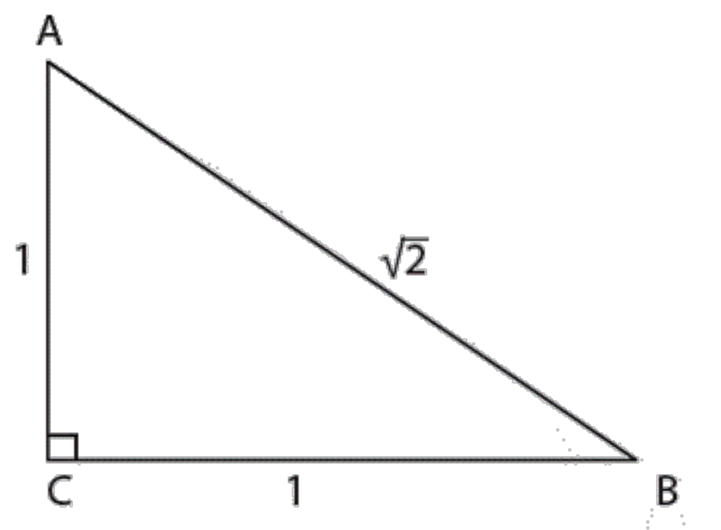



**Fig. 11.11**

Then, $|AB|^2 = |AC|^2 + |BC|^2$ (Pythagoras rule)

$$\Rightarrow |AB|^2 = 1^2 + 1^2$$

$$\Rightarrow |AB|^2 = 2$$

$$AB = \sqrt{2}$$

$$\text{Hence, } \sin 45° = \frac{\text{Opp}}{\text{Hyp}} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{\text{Adj}}{\text{Hyp}} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{\text{Opp}}{\text{Adj}} = \frac{1}{1} = 1$$

$$\text{cosec } 45° = \frac{\text{Hyp}}{\text{Opp}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45° = \frac{\text{Hyp}}{\text{Adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45° = \frac{\text{Adj}}{\text{Opp}} = \frac{1}{1} = 1$$

**Note:** For any triangle with angles 45°, 45° and 90°, the ratio of its sides would be $1:1:\sqrt{2}$.

Given an equilateral triangle ABC with each side 2 units in length. Each of the angles of ABC would be 60°.

Let |AM| be its altitude.

|AM| also bisects ABC as shown in Fig. 11.12.

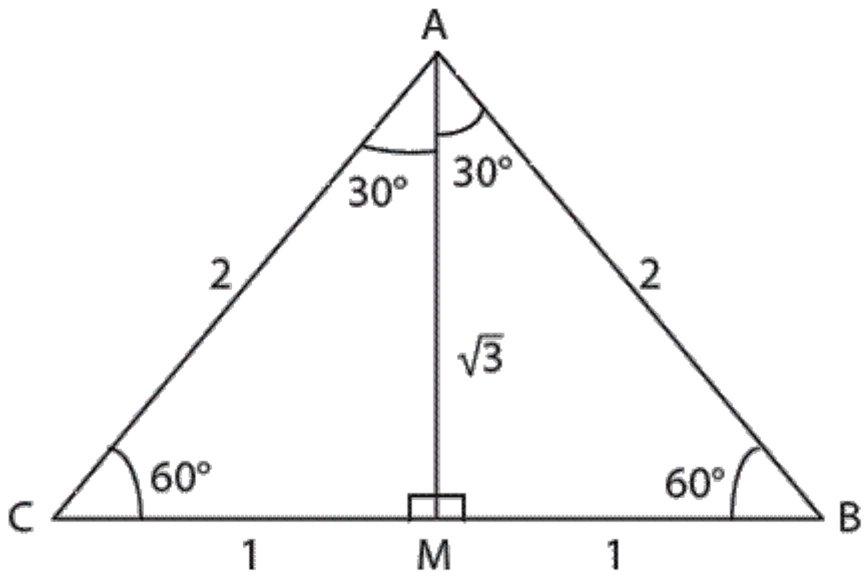



**Fig. 11.12**

Therefore, $|AB| = |AC| = |BC| = 2$ units.

$|BM| = |MC| = 1$ (since $|AM|$ bisects $|BC|$.

$$\begin{aligned}\text{Hence, } |AM|^2 &= |AB|^2 - |BM|^2 \\ &= 2^2 - 1^2 \\ &= 4 - 1 \\ &= 3 \\ |AM| &= \sqrt{3}\end{aligned}$$

$$\sin 60^\circ = \frac{|AM|}{|AB|} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{|BM|}{|AB|} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{|AM|}{|BM|} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\operatorname{cosec} 60^\circ = \frac{|AB|}{|AM|} = \frac{2}{\sqrt{3}}$$

$$\sec 60^\circ = \frac{|AB|}{|BM|} = 2$$

$$\cot 60^\circ = \frac{|BM|}{|AM|} = \frac{1}{\sqrt{3}}$$

Recall, $CAM = BAM = 30^\circ$ ($\overline{AM}$ bisects $\overline{BC}$)

$$\sin 30^\circ = \frac{|BM|}{|AB|} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{|AM|}{|AB|} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{|BM|}{|AM|} = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec} 30^\circ = \frac{|AB|}{|BM|} = 2$$

$$\sec 30^\circ = \frac{|AB|}{|AM|} = \frac{2}{\sqrt{3}}$$

$$\cot 30^\circ = \frac{|AM|}{|BM|} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

## Worked Example 8

Find the length marked $a$ and $b$ in each of the figures. Leave the answers in square root.

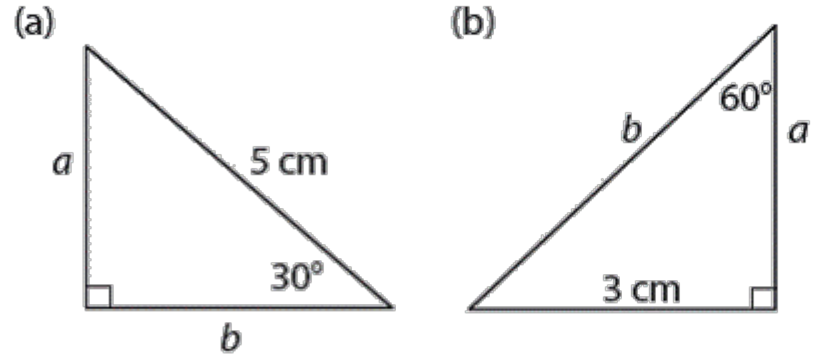



**Fig. 11.13**

(a) $\sin 30° = \frac{a}{5}$

$5 \sin 30° = a$

$a = 5 \times \frac{1}{2}$

Therefore, $a = 2\frac{1}{2}$

$\cos 30° = \frac{b}{5}$

$b = 5 \cos 30°$

$b = 5 \times \frac{\sqrt{3}}{2}$

Therefore, $b = \frac{5\sqrt{3}}{2}$

(b) $\sin 60° = \frac{3}{b}$

$$b = \frac{3}{\sin 60°}$$

$$b = \frac{3}{\frac{\sqrt{3}}{2}}$$

$$b = \frac{3 \times 2}{\sqrt{3}}$$

$$b = \frac{6}{\sqrt{3}}$$

$$\cos 60° = \frac{a}{b} = \frac{a\sqrt{3}}{6}$$

## Worked Example 9

A ladder leans against a vertical wall at an angle 60° to the wall. If the foot of the ladder is 5 m away from the wall, calculate the length of the ladder. (Leave your answer in square root.)

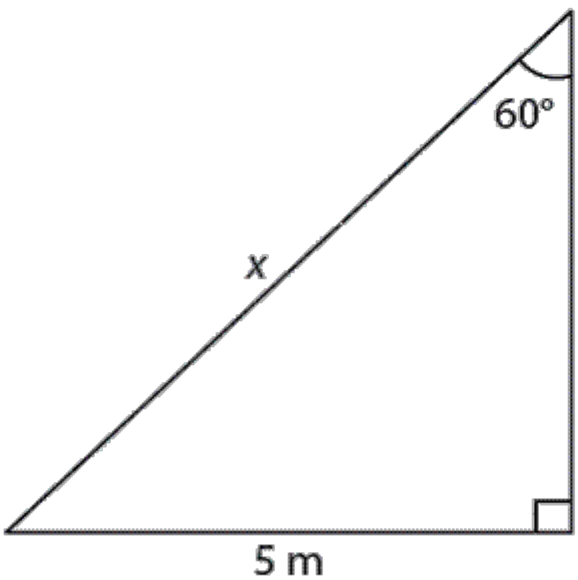



**Fig. 11.14**

$$\sin 60° = \frac{5}{x}$$

$$x = \frac{5}{\sin 60°}$$

$$x = \frac{5}{\frac{\sqrt{3}}{2}}$$

$$x = \frac{10}{\sqrt{3}}\text{ m} = \frac{10 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}\text{ m}$$

$$= \frac{10\sqrt{3}}{3}\text{ m}$$

**Note:** From the above illustration.

## Exercise 3

1. Calculate the length of the side marked with a lower case letter in the figures given below. Leave your answer in square root.

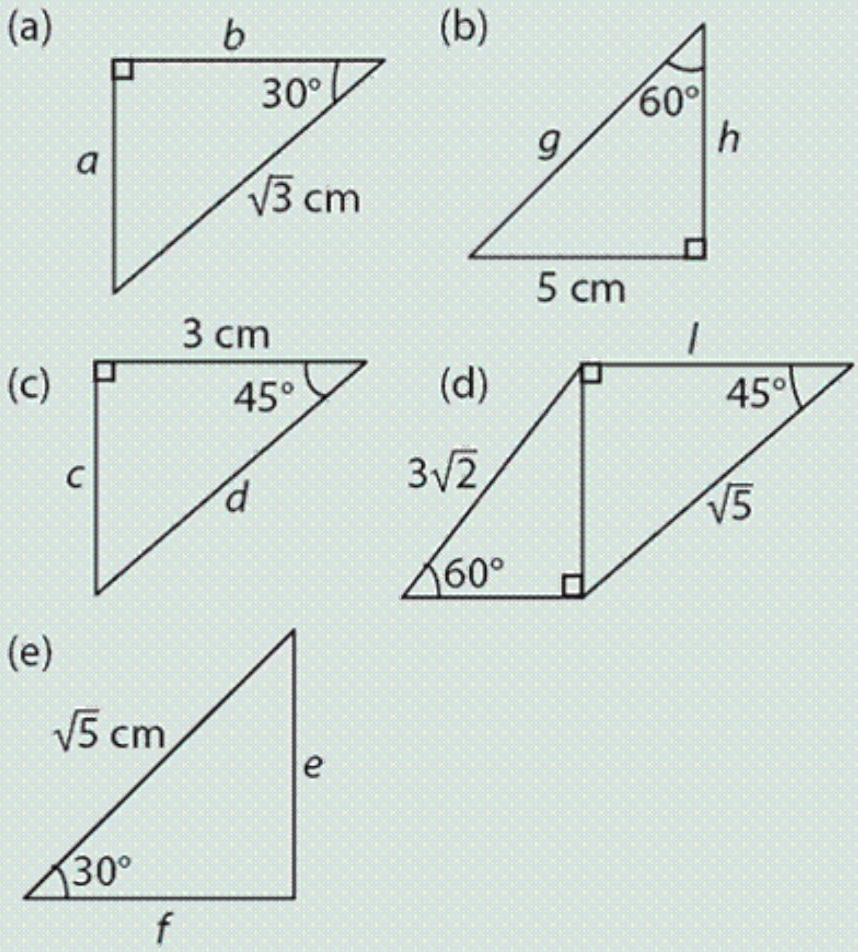



**Fig. 11.15**

2. A boat is on the same horizontal level as the foot of a cliff and the angle of depression of the boat from the top of the cliff is 30°. If the boat is 120 m away from the foot of the cliff, find the height of the cliff, leaving your answer in the square root.

3. The angle of elevation of a point T on a tower from a point U on the horizontal ground is 30°. If TU is 54 m, how high is T above the horizontal ground?

4. In $\Delta PQR$, $\angle PQR$ is a right angle, $|QR| = 2$ cm and $\angle PRQ = 60°$. Find $|PR|$.

5. A ladder 5 m long rests against a wall such that its foot makes an angle 30° with the horizontal. How far is the foot of the ladder from the wall?

6. A pole of length 'L' leans against a vertical wall so that it makes an angle 60° with the horizontal ground and its foot is 2 m away from the base of the wall. Calculate L, leaving your answer in square root.

7. In $\Delta XYZ$, $|YZ| = 6$ cm, YXZ = 60° and XYZ is a right angle. Calculate $|XZ|$ in cm, leaving your answer in square root.

## IV. Relative Sine and Cosine Ratios to the Use of Unit Circles

A unit circle is a circle which has units as its radius. The length of the radius may be 1 cm, 1 m, 1 km, etc. The concept 'unit circle' can be used to find the trigonometric ratio just as we have earlier done in this chapter by using a right-angled triangle.

**Note:** Other scientific units of measurement can be used to produce a unit circle. To determine the trigonometric angles 0° – 90° using the concept of a unit circle, Draw a circle whose radius is 1 cm and centre O. Locate a point 'P' on the circle so that the coordinate of P would be $(x, y)$ i.e. $x$ and $y$ units are horizontal and vertical displacements respectively from the origin O while the angle between OP and the $x$-axis is given by θ and let M be the perpendicular from P to the $x$-axis (see Fig. 11.16).

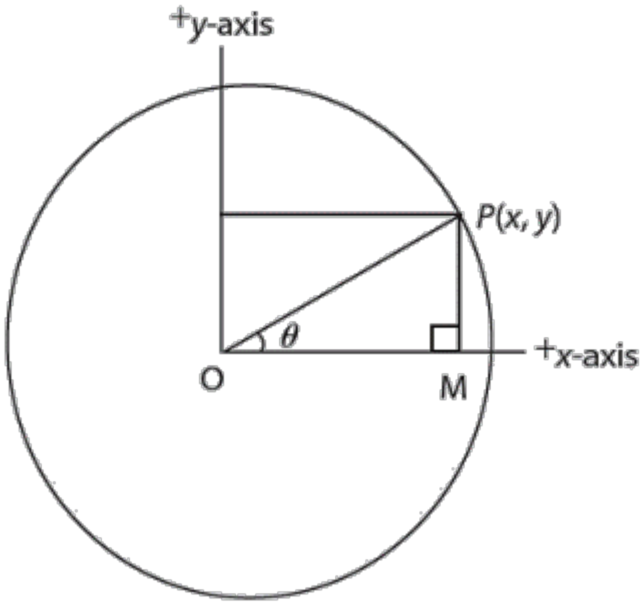



**Fig. 11.16**

Observe that the diagram in Fig. 11.16 strictly adheres to the Pythagoras rule.

i.e. $x^2 + y^2 = 1^2 = 1$

Then trigonometrically,

$$\sin\theta = \frac{MP}{OP} = \frac{y}{1}$$

$$y = \sin\theta$$

Similarly,

$$\cos\theta = \frac{OM}{OP} = \frac{x}{1}$$

$$x = \cos\theta$$

$$\text{And } \tan\theta = \frac{MP}{OM} = \frac{y}{x}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Now, using the above definition of trigonometric ratios, we can determine the values of angles from 0° to 90°. In Fig. 11.16, as θ becomes smaller, $x$ increases and $y$ decreases. If P coincides with the $x$-axis, then θ = 0°.

$$\text{Therefore, } \sin 0° = \frac{0}{1} = 0$$

$$\cos 0° = \frac{1}{1} = 1 \text{ and}$$

$$\tan 0° = \frac{0}{1} = 0$$

In Fig. 11.16, as θ increases, $x$ decreases and $y$ increases. P coincides with the $y$-axis then $\theta = 90°$; $x = 0$ and $y = 1$.

$$\text{Therefore, } \sin 90° = \frac{1}{1} = 1$$

$$\cos 90° = \frac{0}{1} = 0$$

$$\tan 90° = \frac{1}{0}$$

(tan 90° tends to infinite)

## Worked Example 10

(a) If $\sin \theta = 0.3$ and $\cos \theta = 0.2$, find $\tan \theta$.
(b) If $\sin \theta = 0.8$ and $\tan \theta = 0.5$, find $\cos \theta$.

### Solution

(a) $\tan \theta = \dfrac{\sin \theta}{\cos \theta} = \dfrac{0.3}{0.2} = 1.5$

(b) $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

$$\cos \theta = \frac{\sin \theta}{\tan \theta} = \frac{0.8}{0.5} = 1.6$$

## Exercise 4

1. With the aid of a unit circle drawn on a graph sheet, determine the value of the angle whose coordinates are as follows by calculation:

   (a) (0.2, 0.5)

   (b) (0.3, 0.2)

   (c) (0.5, 0.8)

2. Draw the unit circle on a graph sheet with its coordinate of P as shown below and then measure the angles with a protractor.

   (a) (0.2, 0.7)

   (b) (0.8, 0.3)

   (c) (0.5, 0.8)

## SUMMARY

In this chapter, we have learnt the following:

- What is meant by special angles.
- How we can get right-angled isosceles and equilateral triangles to determine the trigonometrical ratio values of special angles without consulting the tables or operating calculators. Thus:

$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \sqrt{3}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = 1$$

- That unit circle can also be used to find the trigonometrical ratios.
- How to use the special angles to solve mathematical problems.
- How to solve trigonometrical equations made up of acute angles.
- The relationship between trigonometry and Pythagoras rules.
- That trigonometrical ratios are also used in solving problems in our everyday life.

## GRADUATED EXERCISES

1. Solve the following trigonometrical equations:
   (a) $\cos 40^\circ = \sin x$
   (b) $\cos 45^\circ = \sin 3x$
   (c) $\sin 10^\circ = \cos 5x$
   (d) $\cos 50^\circ = \sin 2y - 10^\circ$
   (e) $\cos 55^\circ = \sin (50 - 3y)$
2. If $\tan \theta = \frac{5}{12}$, where $= 0^\circ < \theta < 90^\circ$, find $\left(\frac{\sin \theta - 3}{\cos \theta}\right)^2$.

3. Find the sine, cosine and tangent of the following angles using tables.

(a) $43°$

(b) $53.4°$

(c) $64.13°$

4. Using tables, find the angle whose

(a) sine is 0.9336

(b) cosine is 0.3007

(c) tangent is 1.2345

5. The angle of elevation of a boat from the top of a cliff is 60°. If the boat is 10 m from the base of the cliff, find the height of the cliff.

6. If the angle of depression of a point on the horizontal of a tower is 45°, and the tower is 15 m high, what is the length of the point from the top of the tower measuring along the line of sight?

7. A ladder 6 m long rests against a vertical wall so that the foot of the ladder is 3.5 m from the wall. Find, correct to the nearest degree, the angle that the ladder makes with the wall.

Fig. 11.17 is a triangle ABC, such that $\overline{AM}$ is the altitude from A to BC. $|AB| = 5$ cm, $|AC| = 10$ cm and $|BM| = 3$ cm. Use the figure to answer questions 8, 9 and 10.

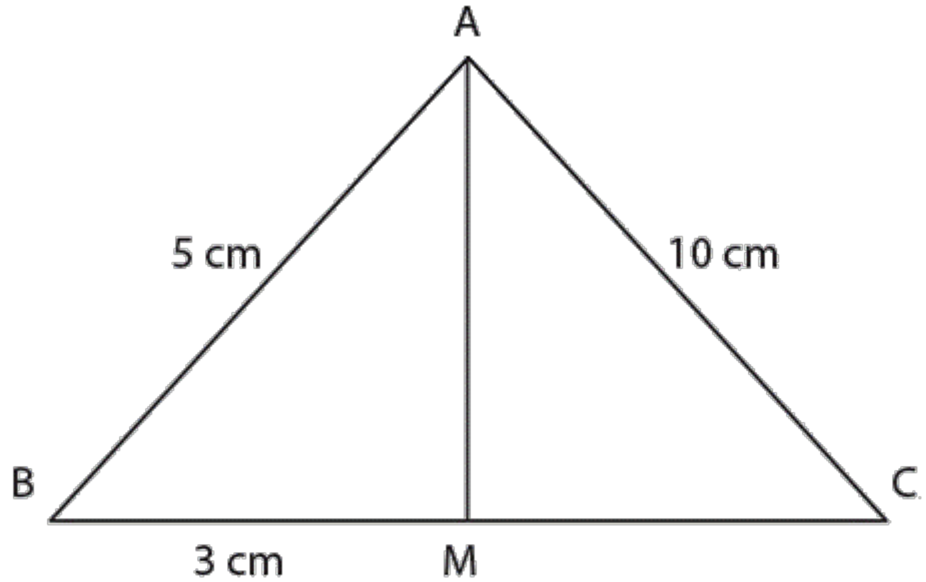



**Fig. 11.17**

8. Find the length BC correct to 2 decimal places.

9. What is the value of the angle CAM (to the nearest degree)?
10. What is the area of the triangle ABC?