CHAPTER 7

At the end of the chapter, students should be able to:
1. Revise factorisation of quadratic expressions.
2. Solve quadratic equations of the form $ab = 0 \Rightarrow a = 0$ or $b = 0$.
3. Form quadratic equations with given roots.
4. Draw quadratic graphs.
5. Obtain roots from a quadratic graph.
6. Solve word problems involving real-life situations.

I. Quadratic Expressions

A quadratic expression is an algebraic expression of the form $ax^2 + bx + c$ where a, b and c are constants and $a \neq 0$. It is also an algebraic expression whose highest power is 2. Examples are $x^2 + 3x + 2$, $2x^2 - 7x + 6$ etc.

Factorisation of quadratic expressions

Worked Example 1

Factorise $x^2 + 3x + 2$.

Solution

In $x^2 + 3x + 2$, the first term is $x2$, the second (middle) term is $+3x$, while the third (last) term is $+2$.

1st method

1st step: Find the product of the first and the last term, that is, $2 \times x_2 = 2x^2$.

2nd step: Find the factors of this product whose sum or difference is equal to the middle term, that is $+3x$. Here the factors of $+2x_2$ are $+x$ and $+2x$ or $-x$ and $-2x$.

The sum: $+x + 2x = +3x$ or $-x - 2x = -3x$

Hence, $+x + 2x = +3x$ (middle term).

Using this in the given expression, we have

$$\begin{aligned} x^2 + 3x + 2 &= x^2 + x + 2x + 2 \\ &= (x^2 + x) + (2x + 2) \\ &= x(x + 1) + 2(x + 1) \\ &= (x + 1)(x + 2) \end{aligned}$$

2nd method

Factorise $x^2 + 3x + 2$.

Solution

Look for two numbers whose sum equals the coefficient of the middle term (coefficient of x) and their product equals the result of the product of the coefficient of x^2 and the constant term. From $x^2 + 3x + 2$, the coefficient of x^2 is 1, while the constant term is +2. Their product is +2. The coefficient of the middle term is +3. Two numbers whose sum equals +3 and product equals +2 are analysed below. For the result of the product of two numbers to be positive, the two numbers must be positive altogether or negative altogether.

Numbers	Products	Sum (+3)
+2 and +1	$+2 \times +1 = +2$	$+2 + (+1) = +3$
−2 and −1	$-2 \times -1 = +2$	$-2 + (-1) = -2 - 1 = -3$

The first set of numbers are the required numbers.

$$\begin{aligned} \text{Hence, } x^2 + 3x + 2 &= x^2 + 2x + 1x + 2 \\ &= (x^2 + 2x) + (1x + 2) \\ &= x(x + 2) + 1(x + 2) \\ &= (x + 2)(x + 1) \end{aligned}$$

Exercise 1

Factorise the following quadratic expressions.

1. $a^2 - 11a + 18$
2. $b^2 + 6b + 8$
3. $c^2 - 7c + 10$
4. $d^2 + 5d + 6$
5. $e^2 - 9e + 14$
6. $f^2 - 9f + 20$
7. $g^2 + 14g + 45$
8. $h^2 + 39h + 38$
9. $k^2 + 8k + 15$
10. $l^2 - 25l + 150$
11. $m^2 + 12m + 35$
12. $n^2 + 10n + 24$
13. $p^2 + 10p + 16$
14. $q^2 + 7q + 6$
15. $r^2 - 2r + 1$

Factorise completely.

16. $a^2 + 2a - 8$
17. $b^2 - b - 30$
18. $c^2 + 3c - 10$
19. $d^2 - 3d - 4$
20. $e^2 - 9e + 20$
21. $f^2 + f - 20$
22. $g^2 - 10g + 25$
23. $h^2 - 12h + 11$
24. $k^2 + 4k - 32$
25. $l^2 - 13l + 42$
26. $m^2 + 6m - 40$
27. $1 + n - 2n^2$
28. $12 - 4p - p^2$
29. $30 - q - q^2$
30. $9 - 6r + r^2$

Worked Example 2

Factorise $x^2 - 6x + 5$.

Solution

Look for two numbers whose sum is −6 (coefficient of the middle term) and the product as +5 (the coefficient of the result of the product of the first term and the constant). The two numbers are analysed below.

Numbers	Product (+5)	Sum (−6)
+5, +1	$+5 \times +1 = +5$	$+5 + (+1) = 5 + 1 = 6$
−5, −1	$-5 \times -1 = +5$	$-5 + (-1) = -5 - 1 = -6$

The second set of numbers are the required numbers, that is, −5 and −1.

$$\begin{aligned} \text{Hence, } x^2 - 6x + 5 &= x^2 - 5x - x + 5 \\ &= (x^2 - 5x) - (x - 5) \\ &= x(x - 5) - 1(x - 5) \\ &= (x - 5)(x - 1) \end{aligned}$$

Worked Example 3

Factorise $2x^2 - 3x - 2$.

Solution

Look for two numbers whose sum is −3 (the coefficient of the middle term) and the product as −4 (the coefficient of the result of the product of the first term and the constant). The two numbers are analysed below: For the result of the product of two numbers to be negative, one of them must be positive, while the other must be negative.

Numbers	Product (-4)	Sum (-3)
$+4, -1$	$+4 \times (-1) = -4$	$+4 + (-1)$ $= 4 - 1 = 3$
$-4, +1$	$-4 \times (+1) = -4$	$-4 + (+1)$ $= -4 + 1 = -3$
$+2, -2$	$+2 \times (-2) = -4$	$+2 + (-2)$ $= 2 - 2 = 0$

The second set of numbers are the required numbers.

$$\begin{aligned} \text{Hence, } 2x^2 - 3x - 2 &= 2x^2 - 4x + 1x - 2 \\ &= (2x^2 - 4x) + (1x - 2) \\ &= 2x(x - 2) + 1(x - 2) \\ &= (x - 2)(2x + 1) \end{aligned}$$

Worked Example 4

Factorise $6 + x - 2x^2$.

Solution

Look for two numbers whose sum is +1 (the coefficient of the middle term) and the product as −12 (the coefficient of the result of the product of the first term and the constant).

The two numbers are analysed below:

Numbers	Product (-12)	Sum $(+1)$
$+12, -1$	$+12 \times -1 = -12$	$+12 + (-1) = 12 - 1 = 11$
$-12, +1$	$-12 \times (+1) = -12$	$-12 + (+1) = -12 + 1 = -11$
$+6, -2$	$+6 \times (-2) = -12$	$+6 + (-2) = 6 - 2 = 4$
$-6, +2$	$-6 \times (+2) = -12$	$-6 + (+2) = -6 + 2 = -4$
$+4, -3$	$+4 \times (-3) = -12$	$+4 + (-3) = 4 - 3 = +1$
$-4, +3$	$-4 \times (+3) = -12$	$-4 + (+3) = -4 + 3 = -1$

The fifth set of numbers are the required numbers.

$$\begin{aligned}\text{Hence, } 6 + x - 2x^2 &= 6 + 4x - 3x - 2x^2 \\ &= (6 + 4x) - (3x + 2x^2) \\ &= 2(3 + 2x) - x(3 + 2x) \\ &= (3 + 2x)(2 - x)\end{aligned}$$

II. Fractorisation of Quadratic Expressions of the Form $ax^2 + bx + c$

Quadratic expressions of the form $ax^2 + bx + c$ can be factorised using the difference of two squares method.

In the quadratic expression $ax^2 + bx + c$, if $b = 0$ and $a \neq 0$, $ax^2 + bx + c$ will become $ax^2 + c$. This kind of quadratic expression can be expressed as a difference of two squares.

Examples of such quadratic algebraic expressions are $25a^2 - 9b^2$, $180 - 5x^2$, $50k^4 - 8b^2$ etc.

Worked Example 5

Factorise $25a^2 - 9b^2$.

Solution

$$\begin{aligned}25a^2 - 9b^2 &= 5^2a^2 - 3^2b^2 \\ &= (5a)^2 - (3b)^2 \\ &= (5a + 3b)(5a - 3b)\end{aligned}$$

Worked Example 6

Factorise $50k^4 - 8b^2$.

Solution

$$\begin{aligned}50k^4 - 8b^2 &= 2[25(k^2)^2 - 4b^2]\\ &= 2[5^2(k^2)^2 - 2^2b^2]\\ &= 2[(5k^2)^2 - (2b)^2]\\ &= 2[(5k^2 + 2b)(5k^2 - 2b)]\end{aligned}$$

Worked Example 7

Factorise $(x^2 + x)^2 - (3x + 3)^2$.

Solution

$$\begin{aligned}(x^2 + x)^2 - (3x + 3)^2 &= [(x^2 + x) + (3x + 3)]\\ &\quad\ [(x^2 + x) - (3x + 3)]\\ &= [x^2 + x + 3x + 3]\\ &\quad\ [x(x + 1) - 3(x + 1)]\\ &= (x^2 + 4x + 3)(x + 1)\\ &\quad\ (x - 3)\end{aligned}$$

Exercise 2

Factorise the following quadratic expressions.

1. $a^2 - 1$
2. $4b^2 - 1$
3. $2c^2 - 32$
4. $100 - 4r^2$
5. $25p^2 - 81q^2$
6. $u^2 - 49v^2$
7. $4m^2 - 9n^2$
8. $2 - 18p^2$
9. $32d^2 - 8e^2$
10. $u^2 - 25v^2$

III. Quadratic Equations

The quadratic expression of the form $ax^2 + bx + c$ that is equated to zero is referred to as the quadratic equation ($ax^2 + bx + c = 0$, where $a \neq 0$, a and b are the coefficients of x^2 and x respectively, and c is a constant).

A quadratic equation is the product of two linear expressions equated to zero. For example, $2x + 1$ and $x + 2$ are linear expressions whose product is $(2x + 1)(x + 2)$. The result of the product is a quadratic expression $2x^2 + 5x + 2$.

For $2x^2 + 5x + 2$ equal to zero to be true, $2x + 1 = 0$ or $x + 2 = 0$.

If $2x + 1 = 0$, $x = -\frac{1}{2}$ and $x + 2 = 0$, $x = -2$.

Hence, the solutions of the equation are $-\frac{1}{2}$ and -2.

The solutions of the equation are otherwise known as the roots of the equation. Quadratic equations can be solved using factorisation, general formula, completing the square and graph methods. Factorisation method and graphical method shall be considered at this level.

(i) Factorisation method

This method is applicable only when the quadratic equation has a factorisable quadratic expression. In solving a quadratic equation by factorisation method, the quadratic expression that the equation contains should be factorised before solving.

(ii) Solving quadratic equations of the form $ax^2 + bx + c = 0$ using factorisation method

Worked Example 8

Solve the equation: $x^2 + 3x + 2 = 0$.

Solution

$$x^2 + 3x + 2 = 0$$
$$x^2 + 2x + 1x + 2 = 0$$
$$(x^2 + 2x) + (1x + 2) = 0$$
$$x(x + 2) + 1(x + 2) = 0$$
$$(x + 2)(x + 1) = 0$$

either $x + 2 = 0$ or $x + 1 = 0$

$x = -2$ or $x = -1$

$x = -2$ or -1

Worked Example 9

Solve $2x^2 - 3x - 2 = 0$.

Solution

$$2x^2 - 3x - 2 = 0$$
$$2x^2 - 4x + 1x - 2 = 0$$
$$(2x^2 - 4x) + (1x - 2) = 0$$
$$2x(x - 2) + 1(x - 2) = 0$$
$$(x - 2)(2x + 1) = 0$$

Either $x - 2 = 0$ or $2x + 1 = 0$

$x = 2$ or $2x + 1 = 0$

$x = 2$ or $\frac{-1}{2}$

Worked Example 10

Solve $3x^2 - 7x - 6 = 0$.

Solution

$$3x^2 - 7x - 6 = 0$$
$$3x^2 - 9x + 2x - 6 = 0$$
$$(3x^2 - 9x) + (2x - 6) = 0$$
$$3x(x - 3) + 2(x - 3) = 0$$
$$(x - 3)(3x + 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad 3x + 2 = 0$$
$$x = 3 \quad \text{or} \quad 3x = -2$$
$$x = 3 \quad \text{or} \quad x = \frac{-2}{3}$$
$$x = 3 \quad \text{or} \quad \frac{-2}{3}$$

Worked Example 11

Solve completely: $8 - 10p - 3p^2 = 0$.

Solution

$$8 - 10p - 3p^2 = 0$$
$$8 - 12p + 2p - 3p^2 = 0$$
$$(8 - 12p) + (2p - 3p^2) = 0$$
$$4(2 - 3p) + p(2 - 3p) = 0$$
$$(2 - 3p)(4 + p) = 0$$

$$\text{either } 2 - 3p = 0 \quad \text{or} \quad 4 + p = 0$$
$$2 = 3p \quad \text{or} \quad p = -4$$
$$p = \frac{2}{3} \quad \text{or} \quad p = -4$$
$$p = \frac{2}{3} \quad \text{or} \quad -4$$

Exercise 3

Solve for the unknown in the following:

1. $a^2 + 6a + 8 = 0$
2. $b^2 + 7b + 6 = 0$
3. $c^2 + 8c = -15$
4. $d^2 = -4d - 3$
5. $10 + 7e + e^2 = 0$
6. $f^2 + 9f + 14 = 0$
7. $g^2 + 12(g + 3) = 0$
8. $h^2 + 7h + 12 = 0$
9. $k^2 + 11k + 24 = 0$
10. $24 + 20l - 24l^2 = 0$
11. $9l^2 + 9l + 2 = 0$
12. $6q^2 - 13q + 6 = 0$
13. $4r^2 + 5r + 1 = 0$
14. $m^2 + 13m + 22 = 0$
15. $56n^2 + 32n + 2 = 0$
16. $m^2 + 2m - 8 = 0$
17. $n^2 - 2n - 15 = 0$
18. $p^2 - 3p - 10 = 0$
19. $q^2 + 5q - 24 = 0$
20. $r^2 + 5r - 6 = 0$
21. $s^2 - 2s - 8 = 0$
22. $t^2 - 2t = 3$
23. $u^2 = 5u + 14$
24. $4v + 32 = v^2$
25. $24 = w^2 - 2w$
26. $x^2 - 3x + 2 = 0$

27. $y^2 - 14y + 40 = 0$

28. $z^2 - 9z + 18 = 0$

29. $m^2 - 10m + 9 = 0$

30. $n^2 + 40 = 13n$

31. $2a^2 - 3a + 1 = 0$

32. $4b^2 - 16b + 15 = 0$

33. $6c - 8c^2 = 1$

34. $16d^2 - 16d + 3 = 0$

35. $2e^2 - 2e - 24 = 0$

36. $6f^2 - 23f + 15 = 0$

37. $6g^2 - 7g + 2 = 0$

38. $20h^2 - 31h + 12 = 0$

39. $36k^2 - 24k - 5 = 0$

40. $5m^2 + 15m - 20 = 0$

41. $6n^2 - 5n - 1 = 0$

42. $12p^2 - 5p - 3 = 0$

43. $2s^2 + 3s - 2 = 0$

44. $25t^2 - 35t + 12 = 0$

45. $6u^2 - u - 1 = 0$

Worked Example 12

Solve the equation $180 - 5x^2 = 0$.

Solution

$$180 - 5x^2 = 0$$
$$5(36 - x^2) = 0$$
$$6^2 - x^2 = 0$$
$$(6 + x)(6 - x) = 0$$

either $6 + x = 0$ or $6 - x = 0$

$$x = -6 \text{ or } x = 6$$
$$x = -6 \text{ or } 6$$

Worked Example 13

Solve the equation $50k^2 - 18 = 0$.

Solution

$$50k^2 - 18 = 0$$
$$2(25k^2 - 9) = 0$$
$$5^2k^2 - 3^2 = 0$$
$$[(5k)^2 - 3^2] = 0$$
$$(5k + 3)(5k - 3) = 0$$

either $5k + 3 = 0$ or $5k - 3 = 0$

$$k = \frac{-3}{5} \text{ or } k = \frac{3}{5}$$
$$k = \frac{-3}{5} \text{ or } \frac{3}{5}$$

Worked Example 14

Solve $64m^2 - \frac{1}{16} = 0$.

Solution

$$64m^2 - \frac{1}{16} = 0$$

$$8^2 m^2 - \left(\frac{1}{4}\right)^2 = 0$$

$$(8m)^2 - \left(\frac{1}{4}\right)^2 = 0$$

$$\left(8m + \frac{1}{4}\right)\left(8m - \frac{1}{4}\right) = 0$$

either $8m + \frac{1}{4} = 0$ or $8m - \frac{1}{4} = 0$

$$8m = -\frac{1}{4} \quad \text{or} \quad 8m = \frac{1}{4}$$

$$m = -\frac{1}{4} \times \frac{1}{8} \quad \text{or} \quad m = \frac{1}{4} \times \frac{1}{8}$$

$$m = -\frac{1}{32} \quad \text{or} \quad m = \frac{1}{32}$$

$$m = -\frac{1}{32} \quad \text{or} \quad \frac{1}{32}$$

Exercise 4

Solve

1. $100r^2 - \frac{1}{4} = 0$
2. $4p^2 - 24 = 1$
3. $\frac{v^2}{4} - 49 = 0$
4. $6 - 8m^2 = 16^{\frac{1}{2}}$
5. $100k^2 - 36 = 0$
6. $9d^2 = \frac{d^0}{4}$
7. $36t^2 - t^0 = 0$
8. $u^2 = u^0 + 9^{\frac{1}{2}}$
9. $\frac{1}{4} f^2 = \frac{1}{25}$
10. $12g^2 - 75 = 0$

Forming quadratic equations with given roots

Roots are the solutions of a solved quadratic equation. The reverse of finding the roots of a quadratic equation is to form a quadratic equation from the given roots. If the given roots of a quadratic equation are a and b and it is required that the quadratic equation should be formed, then the procedure below should be followed. Let the two roots be identified as $x = a$ or $x = b$

Then, $x - a = 0$ or $x - b = 0$ [taking $(x - a)$ or $(x - b)$ as the factors of the quadratic expression in the equation]

$$(x - a)(x - b) = 0$$
$$x(x - b) - a(x - b) = 0$$
$$x^2 - bx - ax + ab = 0$$
$$x^2 - (a + b)x + ab = 0$$

Hence, the required equation is $x^2 -$ (sum of the roots) $x +$ product of the roots $= 0$.

Worked Example 15

Form a quadratic equation whose roots are

(i) -6 and $\frac{1}{2}$ (ii) 5 and -3

Solution

Method 1

(i) Given $a = -6, b = \frac{1}{2}$

$$x - a = 0,\ x - b = 0$$
$$x - (-6) = 0,\ x - (\tfrac{1}{2}) = 0$$
$$x + 6 = 0,\ x - \frac{1}{2} = 0$$
$$(x + 6)(x - \frac{1}{2}) = 0$$
$$x(x - \frac{1}{2}) + 6(x - \frac{1}{2}) = 0$$
$$x^2 - \frac{1}{2}x + 6x - 3 = 0$$
$$x^2 + 5\frac{1}{2}x - 3 = 0$$
$$x^2 + \frac{11}{2}x - 3 = 0$$

multiply through by 2

$$2x^2 + 11x - 6 = 0$$

Method 2

Given $a = -6, b = \frac{1}{2}$

$$a + b = -6 + \frac{1}{2} = -5\frac{1}{2} = -\frac{11}{2}$$

$$ab = -6 \times \frac{1}{2} = -3$$

hence, $x^2 - (a + b)x + \text{ab} = 0$

$$x^2 - \left(-\frac{11}{2}\right)x + (-3) = 0$$

$$x^2 + \left(\frac{11}{2}\right)x - 3 = 0$$

multiply through by 2 to have

$$2x^2 + 11x - 6 = 0$$

Method 1

(ii) Given $a = 5, b = -3$

$$x - 5 = 0, x - (-3) = 0$$

$$x - 5 = 0, x + 3 = 0$$

$$(x - 5)(x + 3) = 0$$

$$x(x + 3) - 5(x + 3) = 0$$

$$x^2 + 3x - 5x - 15 = 0$$

$$x^2 - 2x - 15 = 0$$

Method 2

Given $a = 5, b = -3$

$$x^2 - [5 + (-3)]x + 5 \times (-3) = 0$$

$$x^2 - (5 - 3)x + (-15) = 0$$

$$x^2 - 2x - 15 = 0$$

Worked Example 16

Find the sum and product of the roots of $x^2 + 6x - 5 = 0$.

Solution

From $x^2 + 6x - 5 = 0$

$a = 1, b = +6$ and $c = -5$

the sum of the roots $= -\frac{b}{a} = -\frac{6}{1} = -6$

while the product of the roots

$$= \frac{c}{a} = \frac{-5}{1} = -5$$

Worked Example 17

Calculate the sum and product of the roots of $8 - 3x - \frac{1}{4}x^2 = 0$.

Solution

From $8 - 3x - \frac{1}{4}x^2$,

$a = -\frac{1}{4}, b = -3$ and $c = 8$

The sum of the roots $= -\frac{b}{a} = -\frac{3}{\frac{1}{4}}$

$$= -\left(3 \div \frac{1}{4}\right) = -\left(\frac{3}{1} \times \frac{4}{1}\right) = \frac{-12}{1}$$

$$= -12$$

The product of the roots $= \frac{c}{a} = \frac{+8}{\frac{-1}{4}}$

$$= -\left(8 \div \frac{1}{4}\right) = -\left(\frac{8}{1} \times \frac{4}{1}\right)$$

$$= -\frac{32}{1} = -32$$

Exercise 5

Form a quadratic equation whose roots are:

1. $\frac{-3}{4}$ and $\frac{1}{5}$
2. $\frac{-5}{4}$ and $\frac{2}{7}$
3. $\frac{-1}{8}$ and $\frac{2}{3}$
4. 2 and $\frac{1}{5}$
5. $\frac{1}{3}$ and $-\frac{1}{3}$
6. $\frac{2}{7}$ and $\frac{5}{7}$
7. 0 and $\frac{1}{4}$
8. $\frac{5}{6}$ and $\frac{7}{12}$
9. $\frac{1}{6}$ and $\frac{3}{4}$
10. $\frac{2}{5}$ and $\frac{-1}{6}$

Calculate the sum and product of the roots of the following quadratic equations.

11. $v^2 - 8v + 20 = 0$
12. $3x^2 - 5x + \frac{1}{6} = 0$
13. $3 - 2y - 8y^2 = 0$
14. $7 - 8k - \frac{5}{6}k^2 = 0$
15. $8x^2 + 3x = 7$
16. $3f = 4f^2 - 6$
17. $3 = x^2 - 4x$
18. $12x^2 - 5x + 8 = 0$
19. $x^2 - 6x + \frac{3}{8} = 0$
20. $7u^2 - 3u - 1 = 0$

(iii) Graphical method

The coefficient of $x^2(a)$ determines the nature of the curve. When a = +ve, we have ∪ and if a = −ve, we have ∩. In solving a quadratic equation graphically, take the following steps:

Step 1: Construct a suitable table of values.

Step 2: Draw the x and y axes using the scales.
Step 3: Plot the points.
Step 4: Join the points with smooth curves (using free hand).
Step 5: Read and interpret the curve.

Worked Example 18

Use the graphical method to solve the equation $x^2 - 6x + 9 = 0$, given that x ranges from -1 to $+4$. Use the scale 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis.

Solution

It should be noted that solving the equation $X^2 - 6x + 9 = 0$ means reading the roots of the equation where $y = 0$ on the graph sheet. Hence, $y = x^2 - 6x + 9$ is the quadratic function to use in constructing a table of values for y for different values of x. Here, the values of x are not given and so suggested values should be given to x. Let x take on the values $-1, 0, 1, 2, 3, 4$.
When

$$x = -1, \quad y = (-1)^2 - 6(-1) + 9 = 1 + 6 + 9 = 16$$

$$x = 0, \quad y = (0)^2 - 6(0) + 9 = 0 - 0 + 9 = 9$$

$$x = 1, \quad y = (1)^2 - 6(1) + 9 = 1 - 6 + 9 = 4$$

$$x = 2, \quad y = (2)^2 - 6(2) + 9 = 4 - 12 + 9 = 1$$

$$x = 3, \quad y = (3)^2 - 6(3) + 9 = 9 - 18 + 9 = 0$$

$$x = 4, \quad y = (4)^2 - 6(4) + 9 = 16 - 24 + 9 = 1$$

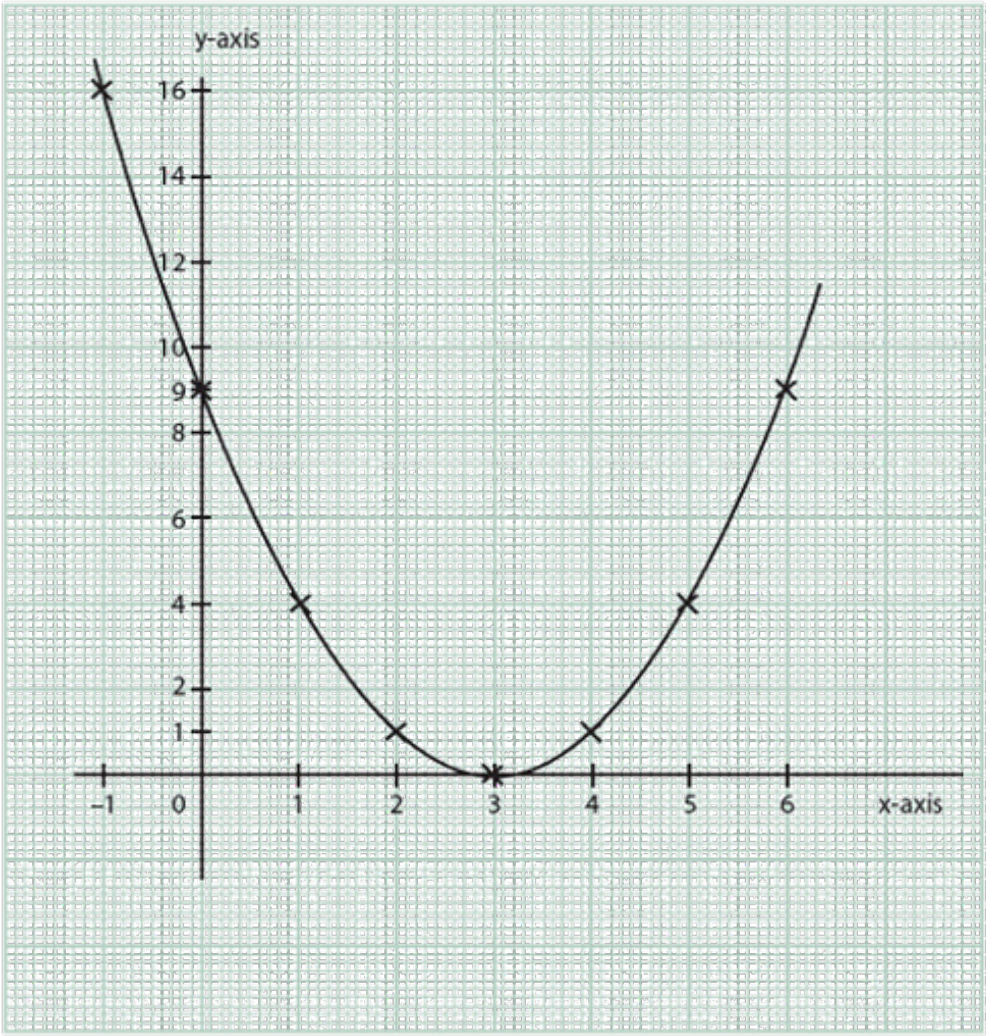
x	-1	0	1	2	3	4	5	6
y	16	9	4	1	0	1	4	9

The scale to use in drawing the x and y-axis depends on the spread of the values of x and will be suitably drawn on graph, if 2 cm represents 1 unit while that of y will be suitably drawn on the graph sheet, if 2 cm represents 2 units.

$$x = 5, y = (5)^2 - 6(5) + 9 = 25 - 30 + 9 = 4$$

$$x = 6, y = (6)^2 - 6(6) + 9 = 36 - 36 + 9 = 9$$

The points are then plotted on the graph sheet by taking a value of x and its corresponding y value. By joining the plotted points, it is observed that the reading of the roots of the equation shows where $y = 0$, $x = 3$. Hence the root of the equation is $x = 3$ (twice). Such roots are called coincident roots.



Worked Example 19

(a) Copy and complete the table below for $y = 3x^2 - 5x - 7$.

x	-3	-2	-1	0	1	2	3	4
$y = 3x^2 - 5x - 7$	35			-7	-9		5	

(b) Using a scale of 2 cm = 1unit along the x-axis and 2 cm = 5units along the y-axis, draw the graph of $y = 3x^2 - 5x - 7$.

Solution

When $x = -2$,

$$y = 3(-2)^2 - 5(2) - 7$$
$$= (3 \times 4) + 10 - 7$$

$$= 12 + 3$$
$$= 15$$

When $x = -1$

$$y = 3(-1)^2 - 5(-1) - 7$$
$$= (3 \times 1) + 5 - 7$$
$$= 3 - 2$$
$$= 1$$

When $x = 2$

$$y = 3(2)^2 - 5(2) - 7$$
$$= (3 \times 4) - 10 - 7$$
$$= 12 - 17$$
$$= -5$$

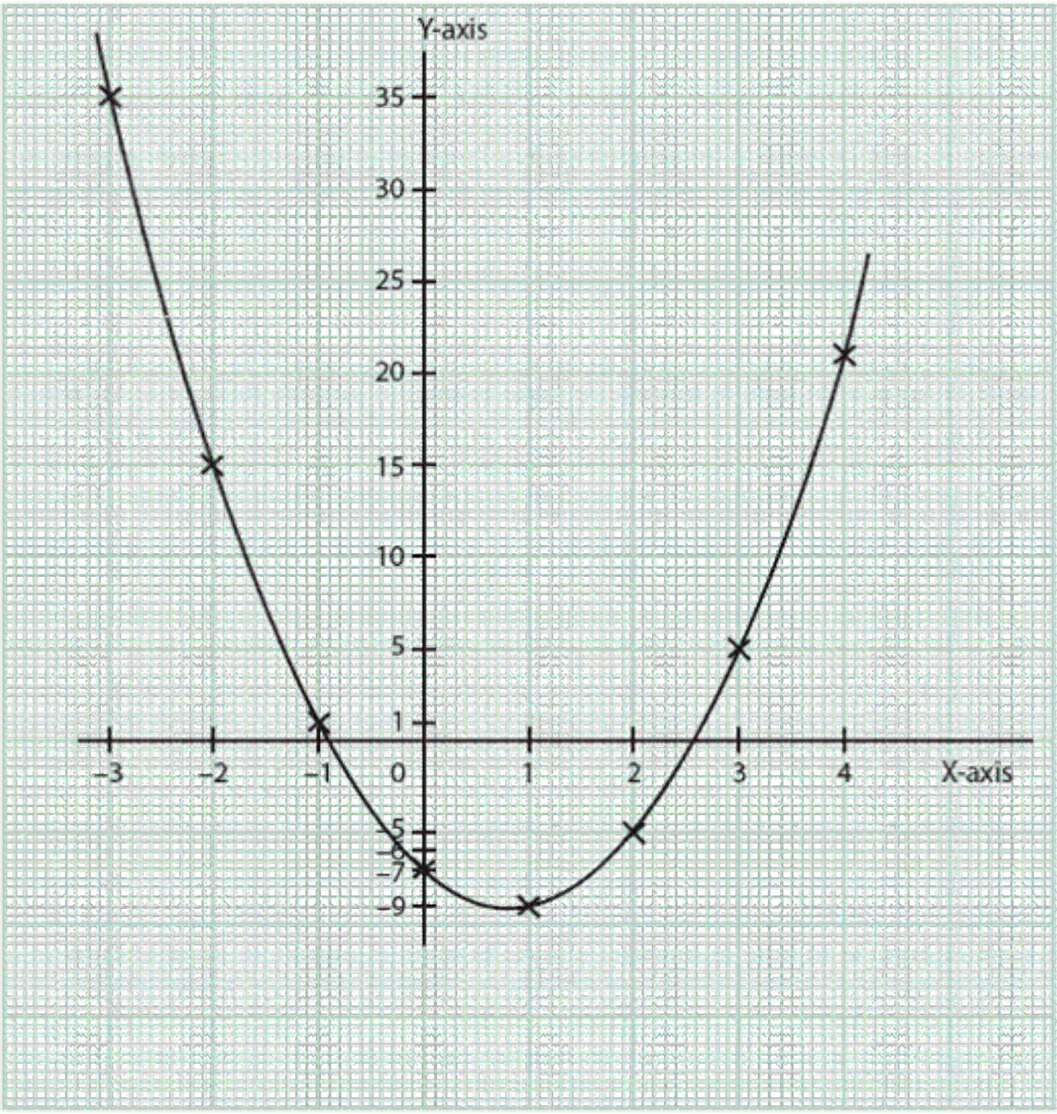
When $x = 4$,

$$y = 3(4)^2 - 5(4) - 7$$
$$= (3 \times 16) - 20 - 7$$
$$= 48 - 27$$
$$= 21$$

The completed table is

x	-3	-2	-1	0	1	2	3	4
$y = 3x^2 - 5x - 7$	35	15	1	-7	-9	-5	5	21

Scale
2 cm = 1 unit on *x*-axis
2 cm = 5 unit on *y*-axis



Worked Example 20

(a) Copy and complete the following table of values for $y = 2x^2 - 9x - 1$.

x	-1	0	1	2	3	4	5	6
y		-1	-8	-11				17

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to Represent 5 units on the y-axis, draw the graph of $y = 2x^2 - 9x - 1$ for the values of x ranging from -1 to 6.

(c) Use the graph to find

(i) The roots of the equation $2x^2 - 9x - 1$

(ii) The minimum value of $y = 2x^2 - 9x - 1$

Solution

When $x = 1$,

$$y = 2(-1)^2 - 9(-1) - 1$$
$$= (2 \times 1) + 9 - 1$$
$$= 2 + 8$$
$$= 10$$

When $x = 3$,

$$y = 2(3)^2 - 9(3) - 1$$
$$= (2 \times 9) - 27 - 1$$
$$= 18 - 28$$
$$= -10$$

When $x = 4$,

$$y = 2(4)^2 - 9(4) - 1$$
$$= (2 \times 16) - 36 - 1$$
$$= 32 - 37$$
$$= -5$$

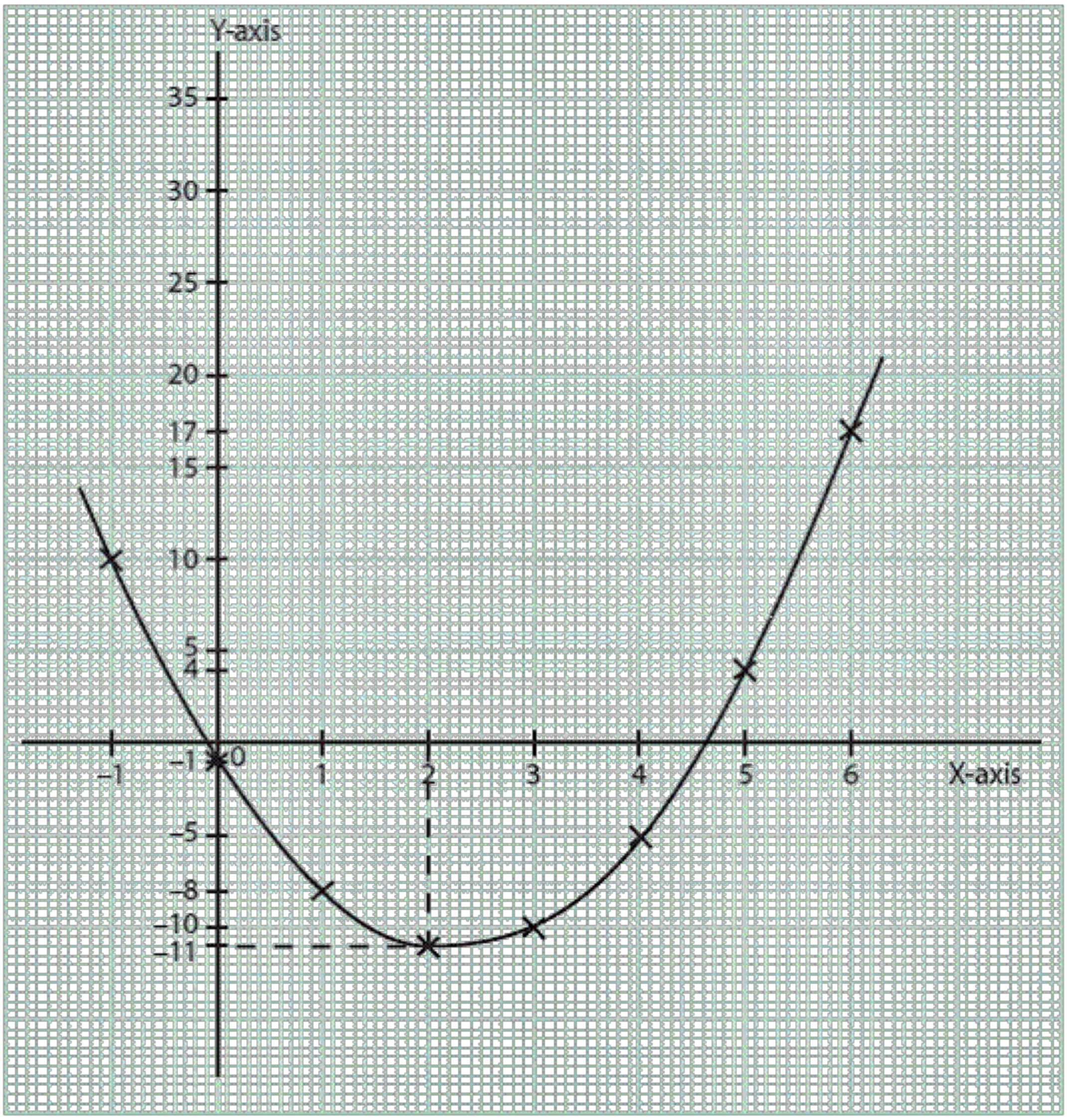
When $x = 5$,

$$y = 2(5)^2 - 9(5) - 1$$
$$= (2 \times 25) - 45 - 1$$
$$= 50 - 46$$
$$= 4$$

The completed table is

x	−1	0	1	2	3	4	5	6
y	10	−1	−8	−11	−10	−5	4	17

Scale
2 cm = 1 unit on x-axis
2 cm = 5 units on y-axis



The minimum value of y is determined at the lowest point on the quadratic curve.

The quadratic curve where minimum is determined must be ∪ shaped or cup shaped. From the curve, the minimum point is $y = -1$ while the corresponding value of x where y is minimum is 2.

SUMMARY

In this chapter, we learnt the following:

- A quadratic expression is an algebraic expression of the form $ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$.
- The highest power of the unknown in an algebraic expression is 2, for example, $x^2 + 3x + 2$, $2x^2 - 7x + 6$, etc.
- A quadratic expression of the form $ax^2 + bx + c$ that is equated to zero is referred to as the quadratic equation.
- A quadratic equation is the product of two linear expressions equated to zero.
- The reverse of finding the roots of a quadratic equation is to form a quadratic equation from given roots.
- The required quadratic equation is given as x^2 − (sum of roots) x + product of roots = 0.
- The nature of curve that a quadratic graph produces is either ∪ shaped or ∩ shaped.
- The coefficient of $x^2(a)$ determines the nature of the curve.
- When a is positive, we have ∪, and when a is negative, we have ∩.

GRADUATED EXERCISES

Factorise

1. $a^2 + 9a + 20$
2. $b^2 - 6b + 5$
3. $c^2 + 11c - 12$
4. $d^2 - 12d + 35$
5. $e^2 + 6e - 16$
6. $2g^2 - 11g + 5$
7. $2h^2 - 2h - 24$
8. $k^2 - 10k + 24$
9. $6m^2 - 5m - 6$
10. $6 + 7m - 3m^2$

Factorise the following quadratic expressions:

11. $4a^2 - 9$

12. $36b^2 - 16c^2$

13. $8d^2 - 2$

14. $5f^2 - 405$

15. $1 - 4m^2$

16. $16 - 25t^2$

17. $4 - 9u^2$

18. $18 - 32y^2$

19. $16x^2 - 36$

20. $6d^2 - 6$

Solve the following quadratic equations using factorisation method:

21. $a^2 - 11a + 28 = 0$

22. $4b^2 + 15b + 9 = 0$

23. $8c^2 - 39c - 5 = 0$

24. $15 - 13d + 2d^2 = 0$

25. $20e^2 - e - 12 = 0$

26. $v^2 - 9v + 20 = 0$

27. $m^2 - 24m = 0$

28. $-40n + n^2 = 0$

29. $4p^2 - 7p = 30$

30. $2q^2 - q - 3 = 0$

Form a quadratic equation in x whose roots are:

31. $\frac{1}{3}$ and $\frac{1}{4}$

32. $\frac{1}{9}$ and $\frac{2}{3}$

33. -5 and $\frac{1}{10}$

34. $\frac{-1}{5}$ and $\frac{-1}{6}$

35. $\frac{-3}{8}$ and 0

36. 2 and -3

37. $\frac{1}{4}$ and $\frac{1}{2}$

38. $\frac{4}{7}$ and 7

39. $\frac{3}{5}$ and $\frac{1}{4}$

40. $\frac{5}{6}$ and $\frac{-3}{4}$

Solve the following quadratic equations graphically:

41. $y = x^2 + 4x + 3$

42. $y = 2x^2 - 11x + 15$

43. $y = 5x^2 - 26x + 5$

44. $y = 3x^2 - 4x - 7$

45. $y = x^2 - 5x - 36$