10 EXPANSION OF GASES

Evidence of gas expansion

- â€¢ A narrow capillary tube and a test tube are used to enclose dry air and a small mercury index is used to trap dry air in a narrow capillary tube. The narrow capillary tube used is to ensure that small changes in gas volume are easily noticed.
- â€¢ The test tube is immersed in a beaker of warm water. As the gas expands, it pushes the mercury index upwards.

From the observation in the experiment above, we conclude that gases expand when they are hot. Gases in fact expand faster than solids and liquids for the same rise in temperature. This is because intermolecular forces in gases are weaker than those in solids and liquids. A gas has neither definite shape nor volume, when it is compressed, the volume changes. Pressure changes affect the expansion of a gas. A gas expands (increase in volume) if:

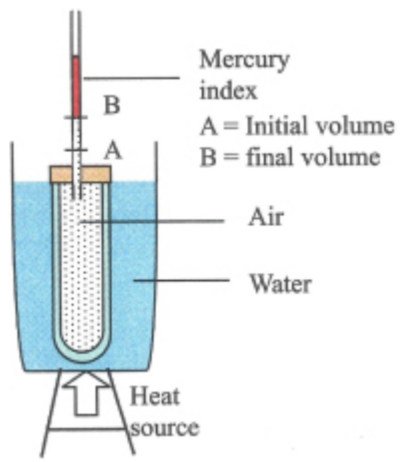


Figure 10.1: Expansion of a gas

- â€¢ the temperature of the gas rises;
- â€¢ the pressure of the gas is reduced.

The way a gas behaves depends on its **volume, temperature** and **pressure**. Although, we can measure the changes in the three quantities at the same time, the relationship between these quantities is best studied if one of these quantities is kept constant.

Measurement of gas pressure and Boyleâ€™s law

OBJECTIVES

At the end of this topic, students should be able to:

- identify and use the instrument for measuring gas pressure;
- state and explain Boyleâ€™s law;
- describe the experiment to verify Boyleâ€™s law;
- solve simple problems on pressure and Boyleâ€™s law;

Measurement of pressure

Pressure is important in life. Doctors and nurses measure blood pressure to know the patientâ€™s state of health. Car mechanics measure pressure of a tyre, submarines and aircrafts use pressure gauge as an altimeter to know the depth of the submarine below the sea and height of the aircraft above the sea level respectively. Changes in weather can be predicted by measuring changes in air or atmospheric pressure. Boiling and freezing points of a liquid are affected by pressure; therefore, the knowledge of pressure is essential. How do we measure pressure? Pressure is measured using the fact that it supports a known height of a liquid. Air or gas pressure can be measured by one of the following pressure measuring instrument:

The liquid manometer

The bourdon gauge

The simple barometer

The Fortinâ€™s barometer

The aneroid barometer and altimeter

The liquid manometer

A simple way to measure the pressure of a gas is by using the U-tube manometer. The manometer consists of a V-tube with a liquid inside. When the pressure of a gas enters through (P), it pushes the liquid up the open side of the manometer labelled. The level of the liquid in the open arm B is steady when the incoming gas pressure B is equal to pressure of the liquid above the level AB. The height difference (h) between the liquid is the pressure of the gas.

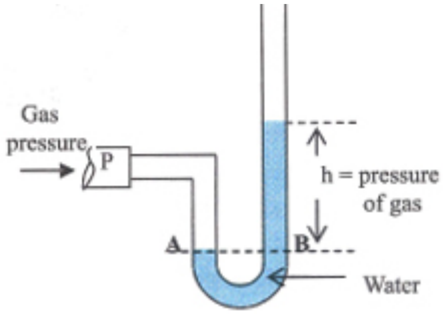


Figure 10.2: Liquid manometer

The rise of liquid in one arm is called the **head of liquid**. In Figure 10.2, the head of water is hand the pressure due to the head of water is given by:

Pressure = head of water Ã— density Ã— gravity

$$P = hpg$$

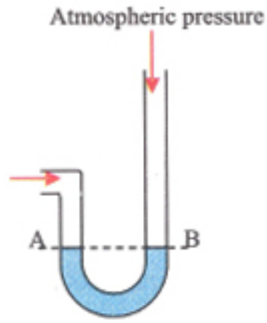


Figure 10.3a

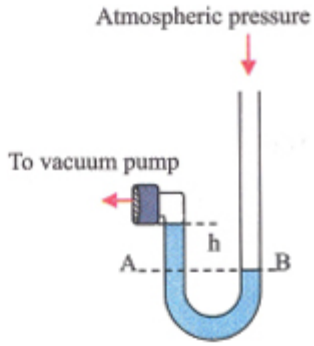


Figure 10.3b

When both arms of the manometer are open to the atmosphere, the level of liquid in both arms are equal as shown in Figure 9.3a. The pressures on both ends of the liquid are equal to atmospheric pressure (H). If one end is connected to a gas source, the pressure of the gas makes the liquid to rise on the opposite arm as shown in Figure 9.2. The total pressure recorded by the manometer is the sum of atmospheric pressure and pressure due to the head of the liquid.

$$P_T = H + hpg \text{ or } P_T = (H + h)\ cm\ Hg$$

If the vacuum pump, pumps out air from the closed end of the manometer, the atmospheric pressure makes the liquid rise as shown in Figure 9.3b. The pressure of the gas inside the closed end is less than the atmospheric pressure. It is given by:

$$P_T = H - hpg \text{ or } P_T = (H - h)\ cm\ Hg$$

The Bourdon pressure gauge

The bourdon pressure gauge is an instrument that measures the pressure of a gas directly on a calibrated scale. It is made of a flexible copper tube which unwinds as the gas pressure increases. A system of levers is used to magnify the movement of the flexible tube many times. A pointer attached to the lever system moves over a calibrated scale to give the pressure of the gas. Bourdon pressure gauge is used as a car oilpressure gauge, in gas cylinders to gauge the pressure of the gas inside the cylinder, and by car mechanics to gauge the air pressure inside the tyre.

Figure 10.4: The Bourdon pressure gauge

The barometer

Air or atmospheric pressure is measured using a **barometer**. A simple barometer consists of a glass tube of uniform cross section containing mercury and inverted in a trough ofmercury. The column of mercury *(h)* supported by the air pressure is about 760 mm at the sea level. Measurement of atmospheric pressure everyday shows that the atmospheric pressure at a location varies depending on weather conditions.

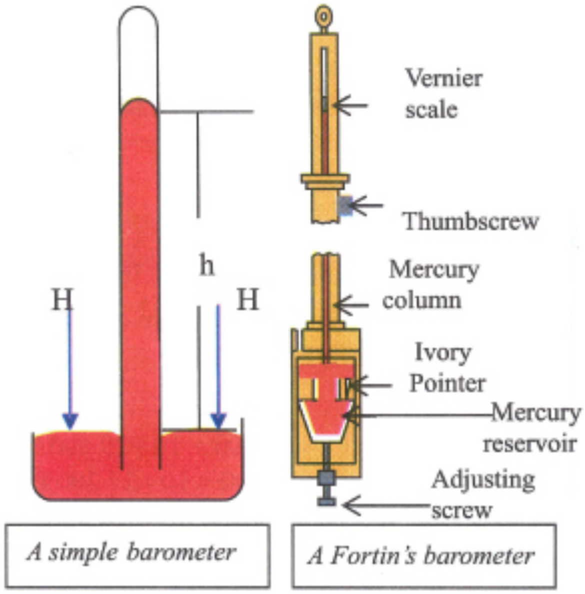


Figure 10.5: Barometers

The Fortinâ€™s barometer

The Fortinâ€™s barometer is an accurate method ofmeasuring the air or atmospheric pressure. It consists of the following:

- â€¢ A barometric tube of length 1m with a uniform cross section dipped into a reservoir of mercury.
- â€¢ An adjustable leather bag which serves as a reservoir of mercury.
- â€¢ Adjusting screw at the base to raise or lower the mercury level in the bag.

â€¢ An ivory pointer which indicates the zero level of the barometer.

â€¢ A vernier scale for accurate determination of the level of mercury meniscus in the tube.

The Fortinâ€™s barometer is designed to overcome the problem of changes in the mercury level in the trough ofa simple barometer. To read a Fortinâ€™s barometer, the adjusting screw at the base of the leather bag is used to raise or lower the mercury level to bring it to the zero level of the barometer. The zero level is reached when the ivory pointer makes contact with the surface of mercury in the bag. The convex meniscus of the mercury surface is read from the vernier scale by adjusting the vernier scale with the thumbscrew until its base rests on the mercury surface.

The aneroid barometer

The aneroid barometer consists of:

â€¢ a thin flexible metal box which is partially evacuated.

â€¢ a system of levers to magnify the small changes in the movement of the partially evacuated box.

â€¢ a pointer, which moves over a scale to record the atmospheric or air pressure.

The scale is graduated in cmHg. When the atmospheric pressure increases, the box contracts or squeezes and expands when the air pressure decreases. The small movement of the flexible box is magnified many times by the levers to move the pointer over a scale.

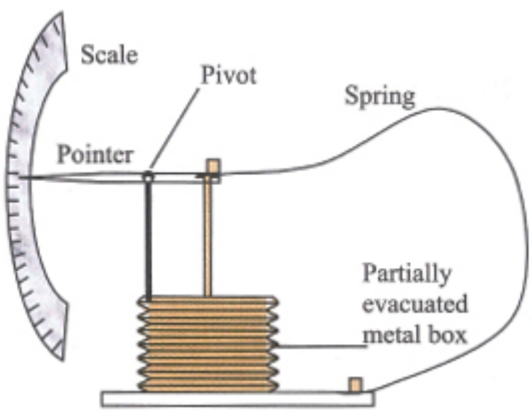


Figure 10.6: The aneroid barometer

Worked examples

1. The normal atmospheric pressure is 760 mm Hg. What is the atmospheric pressure in Pascal?

   {g = 10 m s^{-2}; density of mercury = 13600 kg m^{-3}}

Solution

H = 760 mm Hg = 0.76 m Hg

P= *Hpg*= 0.76 Ã— 13600 Ã— 10 = 103 360 Pa.

2. Convert the following pressures in cmHg to Pascal.

(i) 15 em Hg

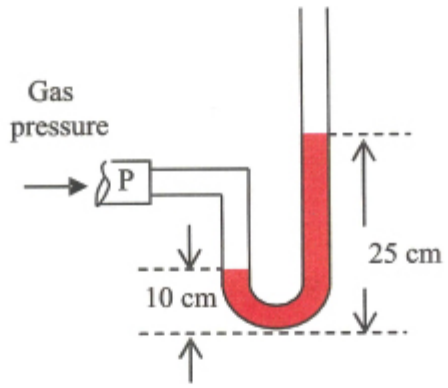
(ii) 20 em Hg

Solution

(i) $P = hpg$ = 0.15 Ã— 13600 Ã— 10 =20 400 Pa

(ii) $P = hpg$ = 0.2 Ã— 13 600 Ã— 10 =27 200 Pa

3. The diagram below shows a manometer used to measure the pressure of a gas, if the atmospheric pressure is 760 mm Hg, find the pressure of the gas. {g = 10 ms^{-2}; density of mercury = 13600 kg m^{-3}}



Solution

Head of mercury h = 25 – 10 = 15 cm Hg= 150 mm Hg.

Total pressure $P_T = H + hpg$

= 760 + 150 = 910 mm Hg

Pressure of gas in Pascal;

Atmospheric pressure H = 103 360 Pa

Pressure due to head of mercury P = hpg

P = hpg

= 0.15 Ã— 13 600 Ã— 10 = 20 400 Pa

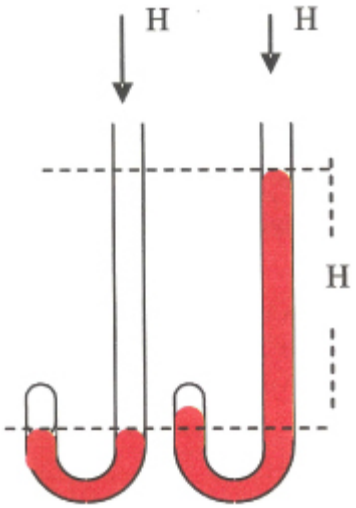
Total pressure $P_T = H + hpg$

= 103 360 + 20 400

= 123 760 Pa

Bolyeâ€™s law

Robert Boyle studied the relation between **pressure** and **volume** of a gas when the temperature is constant. We can demonstrate Boyleâ€™s law as follows:



â€¢ A J-tube is used to trap dry air in the short closed arm. The levels ofmercury in the two arms (the closed and open ends of the J-tube) are adjusted until they are equal. When this happens, the pressure of the air is equal to the atmospheric pressure H.

â€¢ More mercury is added from the open end of the J-tube until the mercury rises by 760 mm above the level of mercury in the shorter arm. That is, the pressure is doubled.

â€¢ The volume of the gas in the closed end of the J-tube is reduced to half.

When the pressure is doubled, the volume of the gas is reduced to half showing that volume of a gas is inversely proportional to its pressure.

The volume of a fixed mass of a gas at constant temperature is inversely proportional to pressure.

Boyleâ€™s law is mathematically stated as:

$$Volume \propto \frac{1}{pressure} \quad \text{Or} \quad V \propto \frac{1}{P}$$

$$V = \frac{k}{P} \quad or \quad PV = k(\text{a constant})$$

Graphs of Boyleâ€™s law

Three graphs are possible from Boyleâ€™s law as illustrated in Figure 9.7.

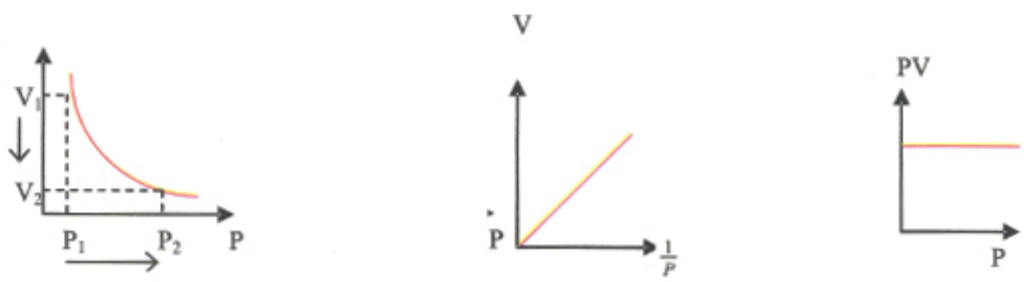


Figure 10.7: Graphs of Boyleâ€™s law

A change in pressure from P_1 to P_2 will cause a change in volume from V_1 to V_2. Increasing the pressure decreases the volume of the gas.
P_1V_1 = a constant and P_2V_2 = a constant
This is the version of Boyleâ€™s law used in solving problems.

$$\therefore P_1V_1 = P_2V_2$$

Worked examples

1. A gas which obeys gas laws, occupies a volume of Vcm³ at a pressure of P mmHg at room temperature. If the volume increases to 3V when the temperature and mass of the gas is fixed calculate the pressure at the new volume.

Solution

$P_1V_1 = P_2V_2$

$P_1 = P,\ V_1 = V,\ V_2 = 3V$ and $P_2 = ?$

$PV = P_2 \times 3V$

$$\therefore P_2 = \frac{PV}{3V} = \frac{P}{3}$$

2. A gas cylinder contains 5 m³ of a gas when the pressure is 2.0 Ã— 10⁶ N m⁻². If the pressure is decreased to 1.5 Ã— 10⁶ N m⁻² at constant temperature, find the new volume of the gas.

Solution

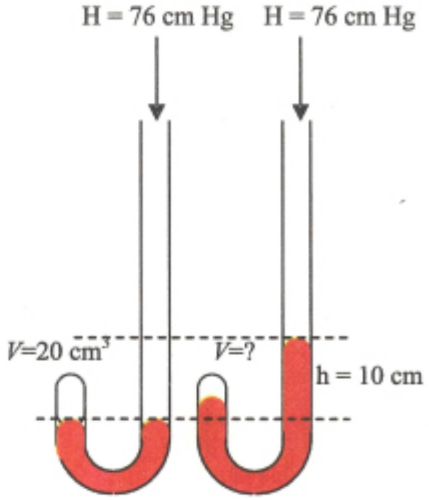
$P_1V_1 = P_2V_2$

P_1 = 2.0 Ã— 106 N m⁻², V_1 = 5 m³, P_2 = 1.5 Ã— 106 N m⁻²and V_2 =?

2.0 Ã— 106 Ã— 5 = 1.5 Ã— 10⁶Ã— V_2

$$V_2 = \frac{2.0\times10^6\times5}{1.5\times10^6} = 4.0\,m^3$$

3. A J-tube with its shorter arm sealed and the longer arm open, is used to trap dry air. When the mercury levels on both arms are equal, the volume of the air is 20 cm³. On pouring in more mercury, the mercury level in the open arm rises by 10 cm. Find the new volume of the air if the atmospheric pressure is 76 cm Hg.



Solution

$P_1V_1 = P_2V_2$

$P_1 = 76\ cm\ Hg,\ V_1 = 20\ cm^3,$

$P_2 = (H+h) = 76+10 = 86$ cm Hg and V_2=?

$76\times20 = 86\times V_2$

$$V_2 = \frac{76\times 20}{86} = 17.67\ cm^3$$

4. A uniform tube closed at one end contains dry air trapped by a thread of mercury 15 cm long. When the tube is held vertically with the open end upward, the column of air trapped by mercury thread is 50 cm. Assuming the atmospheric pressure is 75 cm Hg and the temperature is kept constant, calculate the length of air column if:

   (i) it is held horizontally;

   (ii) it is held vertically with the open end downward.

Solution

(i) When the tube is lying horizontally, the mercury thread does not exert pressure on the air column, so the pressure P_2 is equal to the atmospheric pressure.

$P_2 = H = 75$ cm Hg

$P_1 = H+h = 75+15 = 90$ cm Hg,

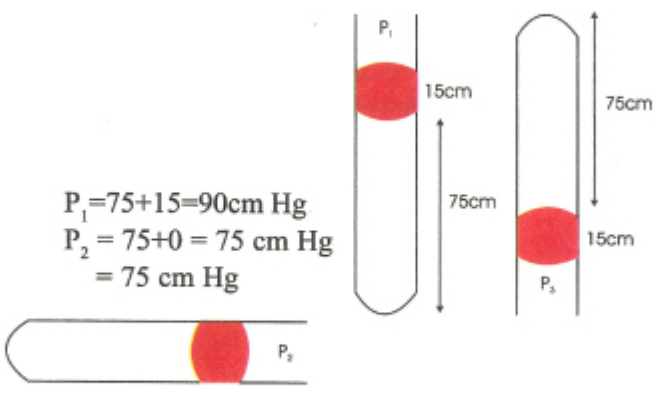
The area of the cross-section of the tube is uniform; therefore, the length of the air column is proportional to the volume of the trapped air.

$V_1 = 50\ cm,\ V_2 = ?$

$P_1V_1 = P_2V_2$

$90 \times 50 = 75\ V_2$

$$V_2 = \frac{90\times 50}{75} = 60\ cm$$



when the tube is held vertically so that the open end is upward.

(ii) When the tube is held vertically so that the open end is downward, the air column is above the mercury thread. The pressure p_3 on the air column is less than the atmospheric pressure.

$P_3 = H - h = 75 - 15 = 60\ cm\ Hg,\ V_3 = ?$

$P_1V_1 = P_3V_3$

$90 \times 50 = 60\ V_3$

$$V_3 = \frac{90 \times 50}{60} = 75 cm$$

Verification of Boyleâ€™s law

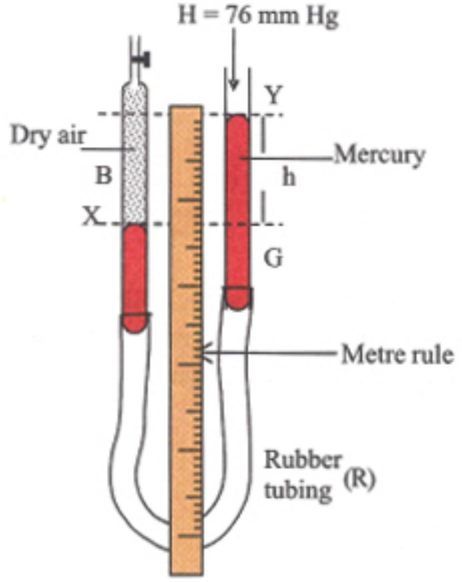


Figure 10.8: Boyleâ€™s law apparatus

Apparatus: Flexible rubber tube (R), burette B, glass tube G, metre rule and mercury.

Method:

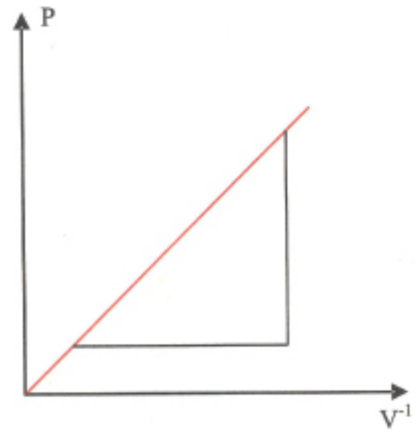
Dry air is trapped inside a burette B connected to a flexible rubber tube R containing mercury. The other end of the flexible tube is connected to a glass tube G. The atmospheric pressure is measured using a good barometer; the initial pressure of the trapped air is measured. This is the difference in height between X and Y.

The pressure on the gas is increased by increasing the height difference h between X and Y. The pressure of the air when this is done is given by $P = H+h$.

To reduce the pressure on the trapped air, the height difference h is decreased by lowering the glass tube. To reduce the pressure on the trapped air below the atmospheric pressure, the height of the mercury Y in the glass tube should be lowered below X. When the level of Y is below X, the pressure is given by $P = H-h$. The length *(l)* of the trapped air is measured as its volume since the burette, the rubber tubing and the glass tube have uniform crass-sectional area. After taking the first measurement, the procedure is repeated five more times and the readings of *h, I, P = HÂ±h* and V^{-1} are tabulated as shown in the table below.

Length of trapped air l cm^{-1}	Height difference between X and Y	Total pressure $P = H \pm h$	Volume of air trapped $V = l^{-1}$

If The pressure is plotted against reciprocal of volume of the air, a straight line through the origin is obtained.



From the graph above;

(i) Pressure of the gas is directly proportional to the reciprocal of the volume;

(ii) The slope of the graph P against V^{-1} is the constant temperature at which the experiment is conducted.

Precautions

1. Air bubbles should be eliminated from the mercury.
2. Only dry air should be used throughout the experiment.
3. All readings ofh and *l* should be taken when the levels of mercury X and Yare constant.
4. A little time is allowed after each adjustment to ensure that the air returns to room temperature before taking the readings of h and 1.

Summary

â€¢ Gases expand when they are warmed. Gases expand more than solids and liquids for the same rise in temperature. The expansion of a gas depends on its **volume**, **temperature** and **pressure**.

â€¢ Air or gas pressure can be measured by one of the following pressure measuring instruments:

- â€¢ The liquid manometer
- â€¢ The Bourdon gauge
- â€¢ The simple barometer

- â€¢ The Fortinâ€™s barometer
- â€¢ The aneroid barometer and altimeter

â€¢ Boyleâ€™s law states that the volume of a fixed mass of a gas at constant temperature is inversely proportional to pressure.

- $V = \frac{k}{P}$ *or* $PV = k$(a constant)

â€¢ The slope of the graph P against V^{-1} is the constant temperature at which the experiment is conducted.

Practice questions 10a

1. Explain why pressure affects the expansion of a gas but does not affect the expansion of: a liquid or solid.
2. How will you show that gas or air expands if it is warmed gradually?
3. Name **four** instruments that can be implored to measure air or gas pressure.
4. Figure 10.9 shows two manometers readings:
   (i) What is the pressure of the gas in each of the diagram?
   (ii) What pressure of gas is required for the mercury to rise by 20 cm in the open arm?

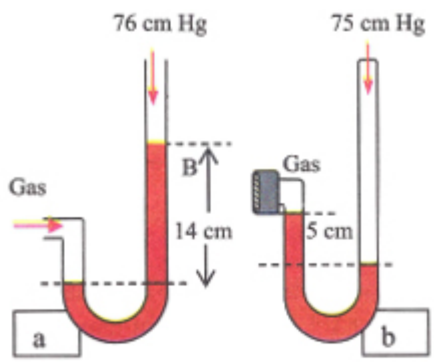


Figure 10.9

5. Name two practical pressure gauges. With a labelled diagram, explain how each of the named instruments work.
6. (a) State Boyleâ€™s law.
   (b) Draw a labelled diagram of the apparatus to verify Boyleâ€™s law. State two precautions you would take to ensure an accurate result.
   (c) Sketch the graphs expected from the experiment.
7. A gas has a volume of 1.2 m^3 at 35Â°C and a pressure of 1.44Ã— 10^3 N m^{-2}. Calculate its volume at the same temperature when the pressure is 1.86Ã— 103 N m^{-2}.
8. (a) Explain what happens to the volume ofa fixed mass ofgas kept at constant temperature.if it is compressed.
   (b) Dry air of mass 10 g is trapped in the shorter arm of a J-tube at

room temperature. When the pressure on the trapped air is 1.03 Ã—10^3N m^{-2} ,the volume of the air at room temperature is 1000 cm^3. Calculate the:

(i) density of the air at room temperature.

(ii) volume and the density of air when the pressure is increased to 1.52 Ã— 10^3 N m^{-2}.

9. A boiling tube 1m long is used to trap dry air at room temperature using a mercury index of length 15 cm. When the tube is held vertically with the open end upwards the length of the trapped air is 30 cm. When the tube is inverted, the length of the trapped air is 45 cm.

(i) Find the atmospheric pressure.

(ii) Find the pressure of the gas in cm Hg in the two positions.

CHARLESâ€™ AND PRESSURE LAWS

OBJECTIVES

At the end of this topic, students should be able to:

- state and explain Chalesâ€™ and pressure laws of expansion of a gas;
- describe the experiment to verify Chalesâ€™ and pressure laws;
- sketh and interpret the graphs of Chalesâ€™ and pressure laws;
- solve simple problems on pressure and Chalesâ€™ and pressure laws;

Charlesâ€™ law

When we study the effect of **temperature** changes on the **volume** of a fixed mass of a gas at a constant **pressure**, it is called Charlesâ€™ law. Charlesâ€™ law states that:

The volume of a fixed mass of gas at constant pressure is dircetly proportional to the absolute temperature.

Charlesâ€™ law is stated mathematically as:

Volume âˆ Absolute temperature

$$V \propto T \quad \text{Or} \quad \frac{V}{T} = constant$$

If the temperature of the gas is increased from T_1 to T_2 , the volume increases from V_1 to V_2. According to Charlesâ€™ law;

$$\frac{V_1}{T_1} = k \text{ and } \frac{V_2}{T_2} = k$$

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Verification of Charlesâ€™ law;

Apparatus: A fine capillary tube, metre rule, thermometer, rubber bands, beaker of water and a source of heat.

Method:

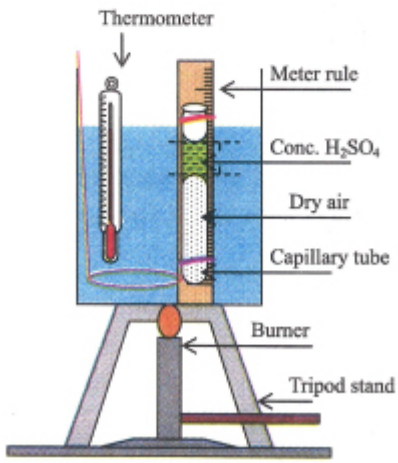


Figure 10.10: Charlesâ€™ law apparatus

(i) The apparatus is arranged as shown in Figure 10.10;

(ii) Concentrated tetraoxosulphate (VI) acid index is used to dry and trap air in a fine capillary tube.

(iii) The pressure of the trapped air is kept constant by keeping the acid index (h) constant. The pressure on the air is always $p = (H+h)$.

(iv) The beaker and its contents are cooled to OÂ°C, the volume and its temperature measured and recorded. The air is warmed indirectly by warming the water in the beaker gently. As the water is wanned, the air becomes hotter and expands; the acid index is pushed upward.

(v) At a time, the heating is stopped; the water is stirred to make the temperature unifonn. The temperature and the length of trapped air are measured and recorded.

(vi) The process is repeated for about five more pairs of temperature and volume and readings tabulated.

(vii) The graph of volume against temperature is plotted as shown in Figure 10.11.

Precautions

1. Only dry air should be used during the experiment.
2. The water in the beaker is stirred continuously to ensure constant or unifonn temperature.
3. Temperatures are taken when the mercury level in the thennometer is steady.
4. Parallax error should be avoided on the metre rule when taking the length of air trapped air.

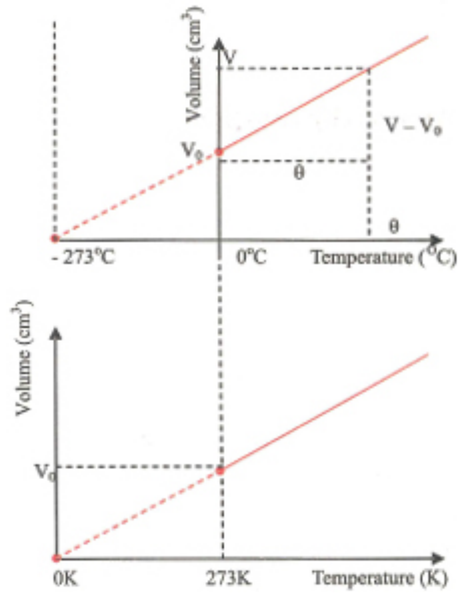


Figure 10.11: Graphs of Charlesâ€™ laws

The intercept on the temperature axis

The intercept on the temperature axis when the graph is produced backward is called the **absolute or Kelvin temperature** (T). The value is about – 273Â°C or 0 K for all types of gases. This is the lowest temperature a gas can attain and is called **absolute zero**. At this temperature, the molecules of gas have their lowest possible energy (i.e. the average velocity of the molecules of the gas is almost zero).

Absolute zero is the temperature at which the molecules of a gas stops moving completely.

The cubic expansivity of a gas at constant pressure

The slope of the graph of volume against temperature is given by:

$$\text{Slope (s)} = \frac{V - V_0}{\theta}$$

The ratio of slope (s) to intercept on the volume axis is a very important constant for all gases. It is called the **cubic expansivity of a gas at constant pressure.**

$$\gamma_p = \frac{\text{Slope}}{\text{Intercept on volume axis}} = \frac{V - V_0 / \theta}{V_0}$$

$$\gamma_p = \frac{V - V_0}{V_0 \theta}$$

Î³$_p$ = cubic expansivity of a gas at constant pressure.

$V – V_0$ = increase in volume of gas from 0Â°.

Î¸ = rise in temperature from 0Â°

Experiment proves that the cubic expansivity of a gas is $\frac{1}{273}$ K^{-1}or 0.00366 K^{-1} for all dry gases. *The cubic expansivity of a gas is* $\frac{1}{273}$ K^{-1} or

0.00366 K^{-1} *means that for every degree Celsius rise in temperature, the volume of a gas increases by* $\frac{1}{273}$ cm^3 *if the pressure is constant. We can now state Charlesâ€™ law as:*

The volume of a fixed mass of a gas at constant pressure increases by $\frac{1}{273}$ ***cm³ of its volume at 0Â°C for each 1Â°C rise in temperature.***

Worked examples

(1) If 5 g of a gas at 27Â°C is compressed from 300 cm^3 at constant pressure, calculate its volume and density when the temperature falls to 27Â°C.

Solution

$T_1 = 273 + 27 = 300$ K; $T_2 = 273 + 7 = 280$ K; $V_1 = 300$ cm³ and V_2 =?

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{300}{300} = \frac{V_2}{280}$$

$$V_2 = 280 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{5g}{280\ cm^3} = 0.0179\ gcm^{-3}$$

(2) The length of air trapped at OÂ°C in a uniform glass tube by an index of mercury is 24 cm. When the gas is heated at constant pressure to 100Â°C, the length of air trapped in the tube becomes 32.8 cm. What is the cubic expansivity of the gas?

Solution

Length of trapped air = volume of air since the tube is uniform

$V_0 = l_0 = 24$ cm; $V = l = 32.8$ cm; $\Delta\theta = 100$ °C

$$\gamma_p = \frac{V - V_0}{V_0 \Delta\theta} = \frac{32.8 - 24}{24 \times 100} = 0.003667\ K^{-1}$$

(3) Given that cubic expansivity ofa gas at a constant pressure is K^{-1}, prove that $\frac{V}{T}$ is a constant.

Take V = V_0 (1 + Î³$_p$Î¸)

Solution

$$V = V_0(1 + \gamma_p\theta)$$
$$V = V_0(1 + \tfrac{1}{273}\theta)$$
$$V = \frac{V_0}{273}(273 + \theta)$$
The absolute temperature $T = 273 + \theta$
$$\therefore\ V = \frac{V_0}{273} \times T$$
$$\therefore \frac{V}{T} = \frac{V_0}{273} = \text{a constant.}$$

(4) A fixed mass of a gas occupies 1000 cm^{3}at 0Â°C. If the cubic expansivity of the gas at constant pressure is $\frac{1}{273}$K^{-1} find the increase in volume of the gas at a temperature of 30Â°C.

Solution

$$\gamma_p = \frac{V - V_0}{V_0\theta}$$
$$\therefore V - V_0 = \gamma_p V_0 \theta$$
$$= \tfrac{1}{273} \times 1000 \times 30 = 109.9\ \text{cm}^3.$$
$$\text{Or}\ \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{1000}{273} = \frac{V_2}{303}$$
$$\therefore V_2 = \frac{1000 \times 303}{273} = 1109.89\ cm^3$$
$$V_2 - V_1 = 1109.89 - 1000 = 109.9\ \text{cm}^3$$

Pressure law

The study of changes in the **pressure** of a fixed mass of gas at constant **volume** with **temperature** when the volume is constant is known as pressure law.

The pressure of a fixed mass of a gas at constant volume is directly proportional to the absolute temperature.

Pressure law is stated mathematically as:

Pressure âˆ Absolute temperature

$$P \propto T \quad \text{Or} \quad \frac{P}{T} = \text{constant}$$

If the temperature of the gas is increased from T_1 to T_2, the volume increases from P_1 and P. According to Pressure law;

$$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{P_1}{T_1} = k \text{ and } \frac{P_2}{T_2} = k$$

Verification of pressure law

Apparatus: Flexible rubber tube (R), metre rule thermometer, glass tube G, flask (F), screen, beaker of water and mercury.

Method:

(i) The set up in Figure 10.12 is used to study the variation of pressure with temperature. It is known as **constant gas thermometer.**

(ii) The rubber tubing is connected to the flask (F) filled with dry air cooled to 0Â°C and the other end is fixed to the reservoir of mercury (R).

(iii) The pressure and volume of the gas at 0Â°C are measured and recorded. The mark A on the tube indicates the volume of the gas at 0Â°C. This volume of the gas is kept constant throughout the experiment.

(iv) The gas is warmed indirectly by heating the water in the beaker slowly. After sometime, heating is stopped, the water is stirred to make the temperature uniform, and the mercury reservoir (R) adjusted to bring the mercury level to (A). This ensures that the volume of the gas is constant.

(v) After the adjustment, the difference in height (head of mercury) h is measured and recorded. The pressure of the gas is given by $P = H + h$. Bourdon pressure gauge can be used to measure the pressure directly.

(vi) The process is repeated for another five pairs of temperature and pressure and observations tabulated.

(vii) Graph of pressure against temperature is a straight line as shown in Figure 10.13.

As a gas thermometer, a direct scale inÂ°C is provided so that pressure changes are converted directly to temperature in degree Celsius (Â°C).

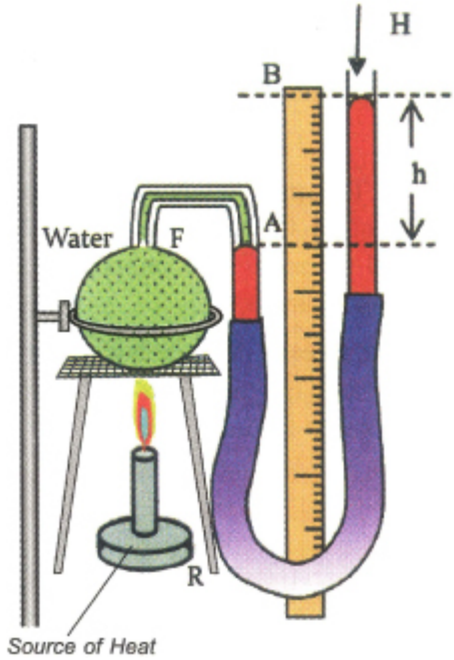


Figure 10.12: Verifying pressure law

Precautions

1. Only dry air should be used during the experiment.
2. The water in the beaker is stirred continuously to ensure constant or uniform temperature.
3. Temperatures are taken when the mercury level in the thermometer is steady.
4. Parallax error should be avoided on the metre rule when taking the length of the trapped air.

Graphs of pressure against temperature

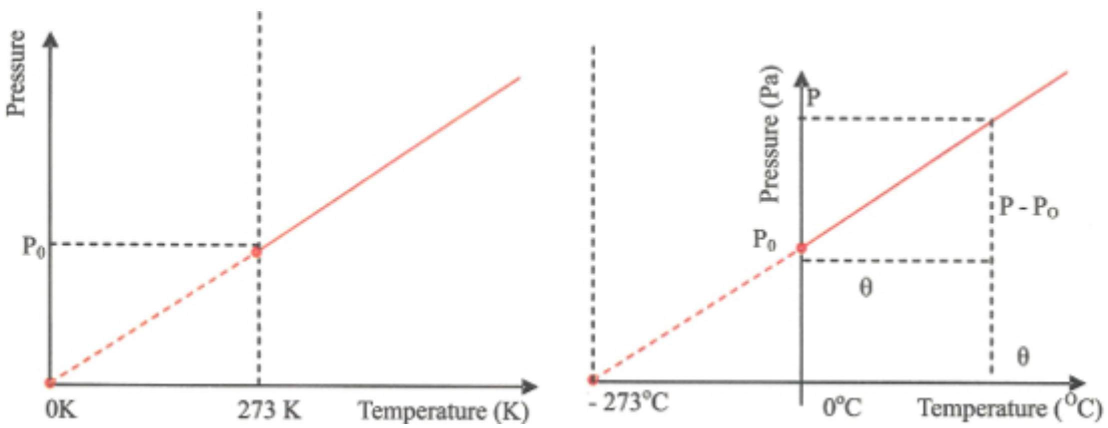


Figure 10.13: Graphs of pressure laws

The graph when produced backwards meets the temperature axis at – 273Â°C, which corresponds to 0K or absolute zero. If it is possible to attain this temperature, the pressure of the gas will be zero. The intercept on the pressure axis is the pressure of the gas when its temperature is 0Â°C.

The pressure expansivity of a gas at constant volume

The ratio of slope (s) to the intercept on the pressure axis P_0 is the pressure expansivity (*Î³*$_v$) of the gas at constant volume.

Pressure expansivity of a gas at constant volume is the increase in pressure of the gas at 0Â°C per unit pressure per 1Â°C rise in temperature.

$$\gamma_V = \frac{\text{Increase in pressure from } 0^\circ\text{C}}{\text{Pressure at } 0^\circ\text{C} \times \text{rise in temperature}}$$

$$\gamma_V = \frac{P - P_0}{P_0 \theta}$$

The result obtained from experiments conducted carefully by scientists shows that for all gases $\gamma_V = \frac{1}{273} K^{-1}$ or $0.00366\ K^{-1}$ or $0.00366\ K^{-1}$. It means that pressure of all real gases increases by $\frac{1}{273}$ *Pa* for each 1Â°C rise in temperature from 0Â°C.

Pressure law can now be stated as:

The pressure of a fixed mass of a gas at constant volume increases by $\frac{1}{273}$ Pa of its pressure at 0Â°C for each lÂ°C rise in temperature.

Worked example

1. A gas cylinder contains gas at 27Â°C and pressure of 2.5 Ã— 10^5 Pa. Find its pressure when the temperature is 87Â°C.

Solution

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{2.5\times10^5}{300} = \frac{P_2}{360}$$

$$P_2 = \frac{2.5\times10^5\times360}{300} = 3.0\times10^5\ Pa$$

Summary

â€¢ Charlesâ€™ law states that the volume of a fixed mass of gas at constant pressure is directly proportional to its absolute temperature.

â€¢ Absolute zero is the temperature at which the molecules of a gas stop moving completely. It is – 273Â°C or 0 K.

â€¢ At absolute temperature, the molecules of the gas have their lowest possible energy or the average velocity of the molecules of the gas is almost zero.

â€¢ The increase in volume of a gas per unit volume for each 1Â°C rise in temperature is called the cubic expansivity of a gas at constant pressure. Cubic expansivity of gas at constant pressure is $\frac{1}{273}$ K_{-1}or 0.00366 K^{-1} for all dry gases.

â€¢ Pressure law states that the pressure of a fixed mass of a gas at constant volume is directly proportional to the absolute temperature.

â€¢ The pressure of the gas at absolute zero temperature is zero because the molecules are not moving.

â€¢ Pressure expansivity of a gas at constant volume is increase in pressure of the gas from 0Â°C per unit pressure per 1Â°C rise in temperature.

â€¢ Pressure expansivity from experiments conducted carefully by scientists is $\frac{1}{273}$ K^{-1} or 0.00366 K^{-1} for all gases.

Practice questions 10b

1. (a) State Charlesâ€™ law;
   (b) Sketch the graph expected from experiment to verify Charlesâ€™ law. Give a physical significance of the intercept on the volume and temperature axes.
   (c) A gas cylinder contains 25000 cm^3 of gas at 30Â°C. What is the volume when the temperature is increased to 77Â°C?
2. (a) Explain what you understand by the statement, *"the cubic expansivity of a gas at constant pressure is K^{-1}"*
   (b) Describe an experiment to determine the cubic expansivity of a gas at constant pressure.
   (c) State two precautions that should be observed to obtain accurate result.
3. (a) State the pressure law of gas expansion;
   (b) Describe an experiment to determine the pressure expansivity of a gas at constant volume.
   (c) The pressure of air inside a football at 27Â°C is 3.09 Ã— 10^5 Pa. Find the pressure when the temperature is increased to 37Â°C.
4. The following readings were obtained in an experiment to determine the pressure expansivity of a gas at constant volume.

Temp(°C)	10	20	30	40	50
Pressure ($\times 10^5$ Pa)	1.24	1.29	1.33	1.38	1.42

Questions

a. Plot the graph of pressure against temperature.

b. Determine the slope(s) and the intercept on the temperature axis.

c. Evaluate $P = \frac{S}{I}$

d. State with reason what is P?

GENERAL GAS LAW AND KINETIC THEORY

OBJECTIVES

At the end of these topics, students should be able to:

- deduce the general gas law from a given mass of gas which obeys Charlesâ€™ law;
- state the assumptions of kinetic theory of gases,
- explain, using the ideas of the kinetic theory of gases:
  - â€¢ Charlesâ€™ law
  - â€¢ Boyleâ€™s law
  - â€¢ Pressure law;
- identify the limitations of the gas laws,
- solve simple problems on general gas laws.

The general gas law

The general gas law is a combination of any two of the gas laws.

1. **Boyleâ€™s law:** At constant temperature, volume of a fixed mass of gas is inversely proportional to its pressure.

   $V \propto \frac{1}{P}$ {temperature constant}

2. **Charlesâ€™ law:** At constant pressure, volume of a fixed mass of gas is directly proportional to its temperature in Kelvin.

   $V \propto T$ {pressure constant}

3. **Pressure law:** At constant volume, pressure of a fixed mass of gas is directly proportional to its temperature in Kelvin.

   $P \propto T$ {volume constant}

Combining Charles' law and Boyleâ€™s law gives:

$$V \propto \frac{T}{P}$$

Introducing a constant R

$$V = \frac{RT}{P}$$

$$\therefore \frac{PV}{T} = R(\text{constant})$$

A fixed mass of gas with volume V_1 pressure P_1 and absolute temperature T_1 if the temperature is changed from T_1 to T_2, the volume will change from V_1 to V_2 and the pressure from P_1 to P_2.

$$\therefore \frac{P_1V_1}{T_1} = R(\text{constant})$$

$$\therefore \frac{P_2V_2}{T_2} = R(\text{constant})$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

The equation above is known as general gas law or equation. This can be written in the form:

$$PV = nRT$$

n = the number of moles of a gas
R = the universal molar gas constant
The value of R = 8.31 J K^{-1} mol^{-1} and is a constant for all gases. Ideal or perfect gases obey the general gas law perfectly. Such gases do not exist in practice.

(i) $\frac{PV}{T}$ = constant **Ideal gas law**

(ii) PV = constant **Boyle's law**

(iii) $\frac{V}{T}$ = constant **Charles' law**

(iv) $\frac{P}{T}$ =constant **Pressure law**

Kinetic theory of gases

Kinetic theory is a model used to explain the properties of gases using the motions of the molecules which make up the gas. Molecules are tiny and so many in number that only their average velocity or kinetic energy is used to explain the properties of gases. We can explain the gas laws by making the following assumptions:

â€¢ Gases are made up of molecules which are like elastic spheres.
â€¢ Molecules of a gas are in constant motion.
â€¢ The force of attraction between molecules of gases is very small that it can be neglected except when they collide.
â€¢ Collision between gas molecules and the wall of the vessel containing the gas is elastic.
â€¢ The volume occupied by the gas is far greater than the volume occupied by the gas molecules.
â€¢ The time the molecules stay together during collision with each other or with the wall of the container is small when compared with the time between collisions.

Explaining gas laws using kinetic theory of gases

We can explain the gas laws if the motion of the gas molecules can be related to:

(i) **The pressure of the gas:** The pressure of the gas is caused by the collisions of the gas molecules with the wall of the container. The force per square metre due to the collision of gas molecules on the wall of the container is the pressure of the gas.

(ii) ***The volume of gas:*** The volume of a gas is the space available for the molecules to move.

(iii) ***The total number of gas molecules in the container.***

Boyleâ€™s law

If the total number of gas molecules in the cylinder (fixed mass of gas) and the temperature are constant, the number of collisions per second made by the molecules remains constant. When the volume of gas in the cylinder is reduced to half, the molecules make more collisions per second because they now travel shorter distances between the piston and the base of the cylinder. The number of collisions made by the gas molecules or pressure of the gas is doubled.

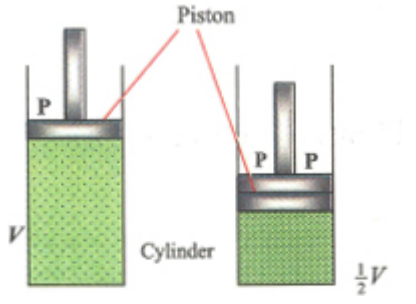


Figure 10.14: Boyles law: Relating prescribed and volumes

Charlesâ€™s law

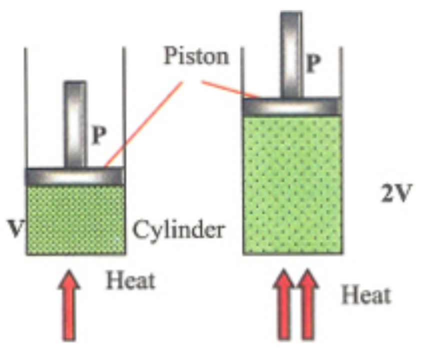


Figure 10.15: Charlesâ€™ law relating volume and temperature

For a fixed number of gas molecules contained in a cylinder at constant pressure, the number of collisions made per second is constant if the volume does not change. When heat is supplied to the gas, the molecules gain kinetic energy and move faster, therefore the number of collisions per second of the molecules with the wall of the cylinder is increased. The volume of the gas increases to keep the pressure constant.

Pressure law

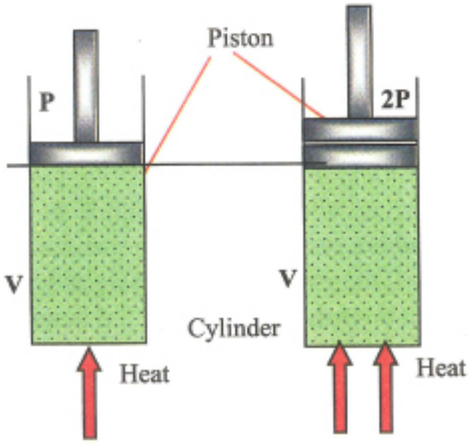


Figure 10.16: Pressure iaw: Relating pressure and temperature

When a fixed number of gas molecules are contained in a cylinder at constant volume, the pressure or the number of collisions on the wall of the cylinder per second is constant. If the temperature of the gas is increased, the gas molecules gain kinetic energy and therefore move faster. The number of collisions per second on the wall of the cylinder is increased leading to increase in pressure.

Limitations of gas laws

Ideal or perfect gases obey gas laws perfectly. No such gas exists as real gases do not obey the gas laws at very high or low pressures and at temperatures close to the temperature at which they liquefy. The reason is that these laws are derived from the assumptions of kinetic theory of gases which is too simple to account for behaviour of real gases. Some assumptions of kinetic theory of gases, which must be corrected to fully explain the behaviour of real gases are:

â€¢ *The force of attraction between molecules is very small that it can be neglected except when they collide*. At very high pressures, molecules are so close to each other that we cannot neglect the intermolecular forces between them.

â€¢ *The volume occupied by the gas is far greater than the volume occupied by the gas molecules*. The volumes of molecules no matter how small influence the behaviour of a gas particularly at high pressures and low temperatures.

â€¢ The energy of ideal gases is only kinetic while real gases have both kinetic and potential energies.

Worked examples

1. The volume of hydrogen gas produced during a chemical reaction when the pressure and temperature are 1.5 Ã— 10^5Pa and 300 K respectively is 20 cm^3. What is the volume of the gas at standard pressure and temperature?

Solution

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$P_1 = 1.50\times 10^5$ Pa; $P_2 = 1.03\times 10^5$ Pa; $T_1 = 300$ K; $T_2 = 273$ K; $V_1 = 20\ cm^3$ and V_2 =?

$$\frac{1.50\times10^5\times20}{300} = \frac{1.03\times10^5\times V_2}{273}$$

$$V_2 = \frac{1.50\times10^5\times20\times273}{300\times1.03\times10^5} = 26.5\ cm^3$$

Solution

2. A fish at the bottom of a river where the temperature is 10Â°C, releases an air bubble of volume 2.0 Ã— $10^{-6}m^3$. The bubble rises to the surface where the temperature is 27Â°C and the pressure of 1.0 atmosphere. If the volume of the bubble at the surface is 8.0 Ã— $10^{-6}m^3$, calculate the depth of the river. {1.0 atmosphere supports 10 m height of water.}

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$P_1 = (H+h) = (10+h)$ m Hg; $P_2 = 10$ m Hg;
$T_1 = 273+10 = 283$ K; $T_2 = 273+27 = 300$K
$V_1 = 2.0\times10^{-6}\ m^3$ and $V_2 = 8.0\times10^{-6}\ m^3$.

$$\frac{(10+h)2.0\times10^{-6}}{283} = \frac{10\times8.0\times10^{-6}}{300} \qquad (10+h) = \frac{8.0\times10^{-6}\times10\times283}{300\times2.0\times10^{-6}} = 37.73\ m$$

$\therefore h = 37.73 - 10 = 27.73$ m

Solution

3. The volume of air inside a pump at 30Â°C is 100 cm^3, when the pressure is 760 mm Hg. When the air is compressed to 25 cm^3, the temperature increases to 45Â°C. Find the new pressure of the air.

Solution

$$\frac{P_1V_1}{T_1} \quad \frac{P_2V_2}{T_2}$$

$P_1 = 760$ mm Hg; P_2 =? $T_1 = 273+30 = 303$ K;
$T_2 = 273+45 = 318$ K; $V_1 = 100\ cm^3$ and
$V_2 = 25\ cm^3$

$$\frac{760\times100}{303} = \frac{P_2\times25}{318}$$

$$P_2 = \frac{760\times100\times318}{303\times25} = 3\,190.5\ mm\ Hg$$

Summary

â€¢ Ideal or perfect gases obey the general gas law perfectly, however real gases do not obey gas laws perfectly.

- The ideal gas equation is given by:
  - $PV = nRT$
  - $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

â€¢ Kinetic theory is a model used to explain the properties of gases using the motions of the molecules which make up the gas.

â€¢ Gas laws have limitations because real gases at very high or low pressures and at j temperatures close the temperature at which they liquefy do not obey them.

â€¢ Using the kinetic theory of gases we can explain the gas laws as follows:

â€¢ **Boyleâ€™s law:** At constant temperature, the number of collisions per second (pressure) increases as the volume is reduced.

â€¢ **Charlesâ€™ law:** At constant pressure, the molecules move faster because of the rise in temperature but the number of collisions per second remains constant due to increase in I volume of the gas.

â€¢ **Pressure law:** At constant volume, the molecules make frequent bombardment (increase in pressure) as a result of increase in temperature.

Practice questions 10c

1. (a) State Charlesâ€™ and Boyleâ€™s laws;

   (b) A bicycle pump has 120 cm^3 of air inside its barrel when the pressure and temperature are 76 cm Hg and 25Â°C respectively. Find the volume of the air in the barrel if the pressure is reduced to 70 cm Hg and the temperature decreased to 20Â°C.

2. (a) State the assumptions of kinetic theory of gases;

   (b) Explain Boyleâ€™s law and pressure law using kinetic theory of gas.

   (c) What modification can be made in the assumptions of kinetic theory to explain the behaviour of real gases?

3. Explain in terms of kinetic theory of gases:

   (i) heating a gas increases its temperature;

   (ii) compressing a gas increases its pressure;

   (iii) warming a gas at constant pressure increases its volume.

4. (a) Explain in terms of kinetic theory of gases why the pressure of the air in a tyre should increase after a long drive;

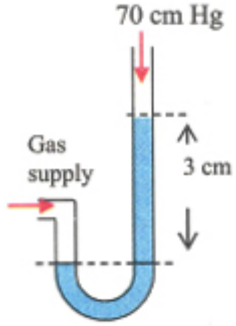
   (b) The volume of air in a tyre of an automobile at a temperature of 27Â°C and pressure of 5 atmospheres is 1600 cm^3. After a long drive, the volume and temperature increases to 1620 cm^3 and 37Â°C respectively, find the pressure of the air in the tyre.

5. State Charlesâ€™ law.

A bubble of air formed at a depth of 10m below the water surface, rises to the surface where the temperature and volume are 27Â°C and 4 cm^3 respectively. If the temperature of the air inside the bubble at 10 m below the surface is 7Â°C, calculate the initial volume of the air bubble. {1.0 atmosphere = 10 m depth of water}

Past questions

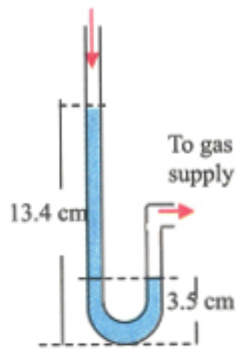
1. The water manometer shown below is measuring the pressure of the gas supply. If the relative density of mercury is 13.6 and the atmospheric pressure is 70 cm Hg, what is the total pressure of the gas supply in cmHg of water?



A. 67
B. 73
C. 949
D. 952
E. 955

JAMB

2. When one arm of a V-tube manometer is connected to a gas supply, the levels of mercury in the two arms of the V-tube are as shown In the diagram below. If the atmospheric pressure is 76 cm Hg, the gas pressure is



A. 62.6 cm Hg
B. 72.5 cm Hg
C. 79.5 cm Hg
D. 85.9 cm Hg

JAMB

3. Which of the following devices are used to measure pressure?

I Aneroid barometer
II Hydrometer
III Hygrometer
IV manometer

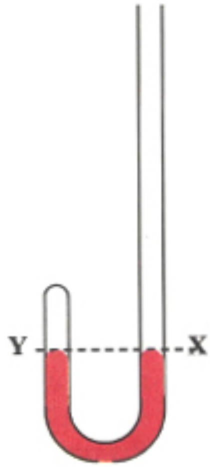
A. I and III
B. II and III
C. III and IV
D. I and IV

JAMB

4. The pressure of gas when cooled at constant volume will decrease because the molecules

A. collide less frequently with the walls of the container.
B. have the same average kinetic energy.
C. break up into smaller molecules.
D. decrease in number.

5. In the J-tube above, **Y** and **X** are on the same horizontal level and 30 cm^3 of air is trapped above **Y** when the atmospheric pressure is 75 cm Hg. Calculate the volume of the trapped air above when 15 cm Hg is now poured into the limb above **X**.



A. 15 cm^3
B. 25 cm^3
C. 35 cm^3
D. 45 cm^3

JAMB

6. When the pressure of a fixed mass of gas is double at constant temperature, the volume of the gas is

A. increased four times.
B. double.
C. unchanged.
D. halved.

WASSCE

7. The volume of a given mass of an ideal gas at 327 K and 9.52 Ã— 10^4 Pa is 40 cm^3. Calculate the volume of the gas at 273 K and 1.034 Ã— 10^4 Pa.

A. 68.23 cm^3

B. 47.91 cm^3

C. 44.60 cm^3

D. 307.5 cm^3

WASSCE

8. An air bubble of volume 4 cm^3 is formed 20 m under water. What will be its volume when it rises to just below the surface of the water, if the atmospheric pressure is equivalent to a height of 10 m of water?

A. 2 cm^3

B. 4 cm^3

C. 6 cm^3

D. 8 cm^3

E. 12 cm^3

9. According to kinetic theory, what does **NOT** occur when the temperature of a constant volume of a gas is increased?

A. a decrease in the mean speed and kinetic energy of the molecules.

B. an increase in the mean speed of the molecules.

C. an increase in the momentum change at each collision.

D. an increase in the number of collisions per second.

E. an increase in the pressure exerted by the gas.

NECO

10. A given mass of an ideal gas has a pressure of 500 N m^{-2}at –13Â°C. If its volume remains constant, calculate its pressure at 247Â°C.

A. 25 N m^{-2}

B. 50 N m^{-2}

C. 500 N m^{-2}

D. l000 N m^{-2}

WASSCE

11. The volume of a certain quantity of gas at 27Â°C is 1200 cm^3. Calculate its volume at 127Â°C if the pressure remains constant.

A. 300 cm^3.

B. 400 cm^3.

C. 1000 cm^3.

D. 1600 cm^3.

WASSCE

12. The volume and pressure of a given mass of gas at 27Â°C are 76 cm^3 and 80 cm Hg respectively. Calculate its volume at s. t. p.

A. 36.2 cm^3

B. 72.8 cm^3

C. 100.0 cm^3

D. 808.9 cm^3

WASSCE

13. A given mass of gas at 30Â°C is trapped in a tube. If its volume is reduced to two-thirds of it initial value by applying a pressure twice the original value, calculate the new temperature of the gas.

A. 404Â°C
B. 313Â°C
C. 131Â°C
D. 101Â°C
E. 40Â°C

WAEC

14. A column of air 10 cm long is trapped in a tube at 27Â°C. What is the length of the volume at 100Â°C?

A. 12.4 cm
B. 13.7 cm
C. 18.5 cm
D. 37.0 cm

JAMB

15. A mass of gas at 7Â°C and 70 cm Hg has a volume of 1200 cm^3. Determine its volume at 27Â°C and pressure of 75 cm Hg of mercury.

A. 1200 cm^3
B. 1378 cm^3
C. 4320 cm^3
D. 4629 cm^3

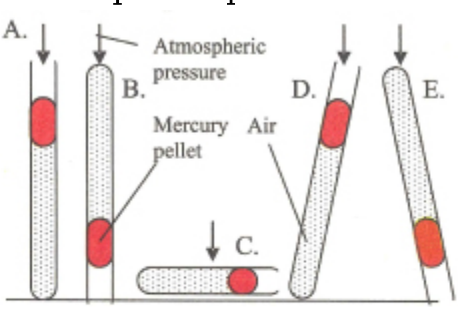
JAMB

16. Dry oxygen is trapped by a pellet of mercury in a uniform capillary tube, which is sealed at one end. The length of the column of the oxygen at 27Â°C is 50 cm. If the pressure of the oxygen is constant, at what temperature will the length be 60 cm?

A. 360.0Â°C
B. 240.6Â°C
C. 237.0Â°C
D. 87.0Â°C
E. 36.0Â°C

WAEC

17. In which of the following diagrams is the pressure of the entrapped air equal to the atmospheric pressure?



WAEC

18. If the volume of a fixed mass of gas is kept constant, the pressure of the gas

A. is inversely proportional to its Celsius temperature.
B. is inversely proportional to its absolute temperature.
C. is directly proportional to its Celsius temperature.
D. is directly proportional to its absolute temperature.

JAMB

E. remains constant also.

19. In a gas experiment, the pressure of the gas is plotted against the reciprocal of the volume of the gas at a constant temperature. The unit of the slope of the resulting curve is

A. force

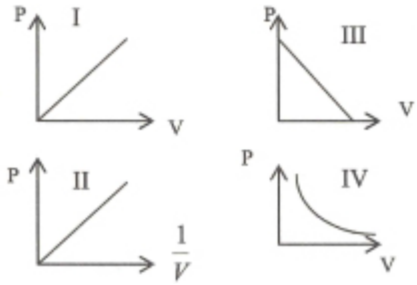
B. force/m

C. work

D. force/m^3

E. energy/m^2 **JAMB**

20. A fixed quantity of gas is subjected to various pressures P and the corresponding volumes V measured at constant temperature. Which of the following graphs best represent the results?



A. I and II

B. II and III

C. II and IV

D. III and IV **JAMB**

21. (a) With the aid of a labelled diagram, describe an experiment to illustrate the relationship between the volume and temperature of a given mass of air at constant pressure.

(b) A uniform capillary tube of negligible expansivity sealed at one end, contains air trapped by a pellet of mercury. The trapped air column is 13.7 cm long at 0Â°C and 18.7 cm long at 100Â°C. Calculate the cubic expansivity of the air at constant pressure.

(c) Using the kinetic theory of gases, explain why the volume of a fixed of a mass of a gas at constant pressure increases with increase in temperature.

WAEC

22. (i) State Boyleâ€™s law.

(ii) With the aid of a labelled diagram, describe an experiment to illustrate the relationship between the volume and pressure of a given mass of gas at constant temperature.

(iii) State two precautions necessary to obtain accurate results.

WASSCE

23. State Boyleâ€™s law.

A thread of mercury of length 15 cm is used to trap some air in a capillary tube with uniform cross-sectional area and closed at one end. With the tube vertical and the open end uppermost, the length of the trapped air is 20 cm. Calculate the length of the air column when the tube is held:

(i) horizontal

(ii) vertical with the open end underneath. {Atmospheric pressure = 76 cm of mercury}

WAEC

24. (i) State Boyleâ€™s law.

(ii) A uniform capillary tube, closed at one end contained dry air trapped by a thread of mercury 8.5 Ã— 10^{-2} m long. When the tube was held horizontally, the length of air column was 5.0xl0^{-2} m, when it was held vertically with the closed end downwards, the length was 4.5 Ã— l0^{-2} m. Determine the value of the atmospheric pressure, [g = 10 ms $^{-2}$, density of mercury = 1.36Ã—10^{4} kg m^{-3}]

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