Chapter 4

OBJECTIVES

At the end of this chapter, students should be able to:

1. calculate the simple interest, given the principal, rate and time.
2. calculate the compound interest using formula.
3. determine the depreciation value of an item.
4. compute the annuities of a given problem.
5. compute the amortisation in a given problem.
6. solve further problems in capital market using logarithm table.

I. Simple Interest (Revision)

Interest is a fee paid on borrowed assets. It is the price paid for the use of borrowed money, or money earned by deposited funds. It is often expressed as a percentage of the money deposited or borrowed, which is called the principal. The sum of the principal and the interest is called *the amount*. Simple interest is calculated only on the original principal. Accumulated interest from prior periods is not used in calculations for the following periods. The following is the formula for finding simple interest:

$$I = \frac{PRT}{100}$$

where I is the interest, P is the principal, R the rate percent and T the number of periods

Worked Example 1

If a student invests N100 at a 5% annual rate for 1 year, calculate the simple interest.

SOLUTION

$P = ₦100, R = 5\%, T = 1$ year

$$I = \frac{PRT}{100} = \frac{100 \times 5 \times 1}{100} = ₦5.00$$

Hence, the interest is ₦5.00.

Worked Example 2

When Asiru bought his car, he borrowed N50 000 at 5% simple interest. The total interest payable over the period of the loan is N500. How long will it take him to repay the loan?

SOLUTION

$I = ₦500, P = ₦50\,000$ and $R = 5\%$

$$I = \frac{PRT}{100}$$

$$\Rightarrow T = \frac{100I}{PR} = \frac{100 \times 500}{50\,000 \times 5} = \frac{1 \times 5}{5 \times 5}$$

$$= \frac{1}{5} \times 12 \cong 2 \text{ months } 12 \text{ days}$$

Therefore, Asiru will take 2 months and 12 days to repay his loan.

Worked Example 3

Find the simple interest on N5 000.00 for 3 months at $1\frac{1}{2}\%$ per annum.

SOLUTION

$I = ?, P = ₦5\,000.00, T = 3$ months $= 0.25$ years, $R = 1\frac{1}{2}\% = \frac{3}{2}\%$

$$I = \frac{PRT}{100} = \frac{5\,000 \times \frac{3}{2} \times 0.25}{100}$$

$$= \frac{5\,000 \times 1.5 \times 0.25}{100}$$

$$= \frac{1\,875}{100} = ₦18.75$$

Hence, the interest is ₦18.75.

Exercise 1

1. Find the simple interest on ₦10 000 for 3 years at 10% per annum.

2. Tella saved ₦10 000 in a bank. If the simple interest is paid yearly at 4% per annum, find the amount which Tella has in the bank at the end of 4 years.

3. To buy a computer, Abubakar borrowed ₦3 000 at 9% simple interest, calculated yearly. If he will be making monthly payments for 4 years, calculate:
   (a) The amount of interest to be paid.
   (b) The total amount to be paid back.
   (c) The monthly payment amount.

4. A civil servant took a car refurbishing loan of ₦6 500. He is expected to pay back the loan over 5 years at a simple interest of 4% per annum.
   (a) Calculate the simple interest on the loan for 5 years.
   (b) Find the total amount he must pay.
   (c) If the total amount is to be paid back in monthly installments over 5 years, how much does he pay each month?
   (WAEC)

5. A Nigerian visiting India converted ₦702.00 to rupees at the rate of Naira to Rs. 35. He spent

Rs. 2 240.00 and invested the remaining amount in the State Bank of India at $4\frac{1}{2}\%$ simple interest per annum. At the end of 8 months, he transferred the capital and interest to his account in the New Nigerian Bank at the rate of Rs. 21 Rupees to ₦2. What was the amount in Naira, to the nearest kobo, that has been credited to his account?

6. ₦600 is invested for 8 years at 15% simple interest. How much interest will it earn? What will be the amount after 8 years?
7. ₦6 000 was invested for 1 year and 6 months. If the simple interest gained was ₦1 000, what was the rate of interest?
8. Rabiu deposited ₦150 in the bank. At the end of 5 years, the simple interest on the principal was ₦55. Find the rate of interest per annum on the interest paid?

II. Compound Interest

Compound interest is calculated each period on the original principal and all interest accumulated during past periods. Although, the interest may be stated as a yearly rate, the compounding periods can be yearly, semi-annually, quarterly or even continuously. Compound interest can be considered as a series of back-to-back simple interest contracts. The interest earned in each period is added to the principal of the previous period which becomes the principal for the next period.

Worked Example 4

Work out the compound interest on N900 invested for 3 years at 5%.

SOLUTION

1st year: $\frac{5}{100} \times 900 = ₦45$

2nd year: $\frac{5}{100} \times 945 = ₦47.25$

3rd year: $\frac{5}{100} \times 992.25 = ₦49.61$

∴ Compound interest = ₦45 + ₦47.25 + ₦49.61 = ₦141.86

Worked Example 5

Shola borrowed N10 000 for 3 years at 8% interest compounded annually. Find the compound interest.

SOLUTION

1st year: Principal = ₦10 000

$$\text{Interest} = \frac{8}{100} \times 10\,000 = ₦800$$

2nd year: Principal = ₦10 800

$$\text{Interest} = \frac{8}{100} \times 10\,800 = ₦864$$

3rd year: Principal = ₦11 664

$$\text{Interest} = \frac{8}{100} \times 11\,664 = ₦933.12$$

∴ Compound interest = ₦800 + ₦864 + ₦933.12 = ₦2 597.12

Generally, the amount A after T years is given by

$$A = P\left(1 + \frac{R}{100}\right)^T$$

where A is the amount, P the principal, T the time period and R the rate percent.
Therefore, knowing the amount, the total for T years can be obtained from the relation

$$I = A - P$$

where I is the total interest, A the amount and P the principal.

Worked Example 6

A student borrowed N500 at 10% per annum compound interest for 2 years. Calculate the compound interest.

SOLUTION

$P = ₦500$, $R = 10\%$, $T = 2$ years, $A = ?$ and $I = ?$

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$= 500\left(1 + \frac{10}{100}\right)^2$$

$$= ₦605$$

Therefore, the compound interest is

$$I = A - P$$

$$= ₦605 - ₦500$$

$$= ₦105$$

Exercise 2

1. Work out the compound interest on ₦100 invested for 2 years at 6%.
2. Jummai borrowed ₦100 000 for 3 years at 10% interest compound annually. Calculate the compound interest.
3. A trader borrowed ₦6 000 at 12% per annum compound interest. He pays ₦2 000 at the end of each year. How much does he owe at the end of 3 years.
4. The sum of ₦180 is saved in an account which gives $9\frac{1}{2}\%$ per annum compound interest. Find the amount after 2 years, to the nearest kobo.

5. At the beginning of the 1974/75 school year, a man invested ₵2 000.00 in a bank at $12\frac{1}{2}\%$ compound interest. If he withdrew ₵650.00 at the end of each school year to pay his son's school fees, how much was he left with at the beginning of the 1978/79 school year? (WAEC)
6. Two financial institutions ABU and BUK offer 8% and 10% compound interest. A businessman invests ₦125 000 in ABU for 5 years and ₦75 000 in BUK for 2 years. Which of the financial institutions gives a higher interest at the end of the stated period?

   A businessman saved ₦4 000 in a bank which pays compound interest half-yearly at a rate of 10% per annum.

7. What is the amount in the businessman's account at the end of 2 years?
8. What would be the amount in his account, if interest was calculated yearly?
9. A microfinance bank is advertising a loan scheme for farmers. Take ₦5 000 for 5 years at 5% compound interest. If the loan is terminated before a 5 year period, one gets a bonus of ₦1 000. What was the actual amount paid back by someone who terminates the loan in 3 years?
10. A company deposits $240 000 in a bank with compound interest at $9\frac{1}{2}\%$ paid per annum half-yearly. Find the amount in the bank at the end of 1 year.

III. Depreciation

Many items such as vehicles and electrical goods lose value over time. For example, if one buys a television today, then each year the television loses its value (i.e., the cost of the television next year will be less than that of this year). This loss in value is called depreciation and is usually given as a

percentage of the value of the item at the beginning of the year.

Worked Example 7

A car costing $10 000 depreciates by 25% in the 1st year and by 18% in the 2nd year. Calculate its value after 2 years.

SOLUTION

1st year: Value of car = $ 10 000

$$\text{Depreciated value} = \frac{25}{100} \times 10\,000$$

$$= \$2\,500$$

$$\text{Less: 25\% depreciation} = \underline{-\ 2\,500}$$

$$\$7\,500$$

2nd year: Value of car = $7 500

$$\text{Depreciated value} = \frac{18}{100} \times 7\,500$$

$$= \$1\,350$$

$$\text{Less: 18\% depreciation} = \underline{-1\,350}$$

$$\$6\,150$$

∴ The value of the car after 2 years is $6 150.

Worked Example 8

A computer costs N20 000.00. Its value depreciates by 12 _ 12_ % in the 1st year, 20% in the 2nd year and in the 3rd year. Find its value after 3 years.

SOLUTION

1st year: Value of computer = ₦20 000.00

$$\text{Depreciated value} = \frac{\frac{25}{2}}{100} \times 20\,000 = 2\,500$$

$$\text{Less: } 12\frac{1}{2}\% \text{ depreciation} = \underline{-2\,500.00}$$

₦17 500.00

2nd year: Value of computer = ₦17 500.00

Depreciated value = $\frac{20}{100} \times 17\,500 = 3\,500$

Less: 20% depreciation	– 3 500.00
	₦14 000.00

3rd year: Value of computer = ₦14 000.00

Depreciated value = $\frac{20}{100} \times 14\,000 = 2\,800$

Less: 20% depreciation	– 2 800.00
	₦11 200.00

∴ The value of the computer after 3 years is ₦11 200.00.

Exercise 3

1. The value of a bus depreciates by 20% of its value at the beginning of each year. If the bus costs ₦1 million, find its value after 2 years.
2. The fixed asset of a company is depreciated by 15% each year. What will be the book value at the end of 3 years of a television bought for ₦120 000.00?
3. An aeroplane costs $300 000.00. Its value depreciates by 25% in the 1st year, 20% in the 2nd year and 15% in each of the following years. Find its value after 4 years.
4. An air conditioner depreciates at 6% per annum. What is its book value at the end of 5 years, if it was bought for ₦125 500.00?

5. A radio costs ₦20 025.00. If it depreciates at 3% per annum, what is its value after 7 years?
6. Nigerian Airways bought an aeroplane and insured it with a premium equal to 5% of the value of the aeroplane. If the aeroplane was bought for $1.4 million and its value depreciated by $12\frac{1}{2}\%$ each year, what was the premium paid to the insurance company at the beginning of the 3rd year?
7. A laptop was bought for ₦250 000.00. The owner decides to write down its value by $2\frac{1}{2}$ per annum. What is its book value in 10 years?
8. A new car which costs ₦5 400.00 depreciates in value by 20% at the end of the 1st year. At the end of the subsequent years, the depreciation is 10% of the value at the beginning of the year. Calculate the value of the car at the end of the
   (a) 1st year. (b) 3rd year. (WAEC)
9. The fixed assets of a company are depreciated by $9\frac{1}{2}\%$ each year. What will be the book value at the end of 4 years of an item bought for ₦85 100.00?
10. A fridge is bought by an individual for R50 600. He decides to write down its value by $10\frac{1}{2}\%$ at the end of each year for 5 years. Find the least amount at which he can sell the fridge at this point.

IV. Annuity

An annuity is an amount of money paid to somebody at regular intervals. It is usually paid annually at the end of a year. For example, if a person has an annuity of N1 000.00, it means the person receives N1 000.00 every year. An annuity though often represents only the interest derived from a principal fund, it may also include periodic payments of principal.
The formula for finding the amount of an annuity is as follows:

$$\text{Amount of annuity} = P\sum_{t=0}^{n-1}(1+R)^t$$

where P is the principal, R the rate percent and n the number of years.

Worked Example 9

Find the amount of annuity of N2 000.00 paid yearly for 3 years at 10% per annum.

SOLUTION

Recall, amount of annuity $= P\sum_{t=0}^{n-1}(1+R)^t$

where $P = ₦2\,000.00$, $n = 3$ and $R = \frac{10}{100} = 0.1$

$\therefore$ Amount of annuity

$$= ₦2\,000.00\left[\sum_{t=0}^{2}(1+0.1)^t\right]$$

$$= ₦2\,000.00\left[\sum_{t=0}^{2}(1.1)^t\right]$$

$$= ₦2\,000.00\,[1 + 1.1 + 1.1^2]$$

$$= ₦2\,000.00\,[1 + 1.1 + 1.21]$$

$$= ₦2\,000.00\,[3.31]$$

$$= ₦6\,620.00.$$

Worked Example 10

Compute the terminal value of an annuity of N1 200.00 at 5% per annum for 7 years.

SOLUTION

$P = ₦1\,200.00$, $n = 7$ and $R = \frac{5}{100} = 0.05$

$$\text{Amount of annuity} = P\sum_{t=0}^{n-1}(1+R)^t$$

$$= ₦1\,200.00\left[\sum_{t=0}^{6}(1+0.05)^t\right]$$

$$= ₦1\,200.00\,[1 + 1.05 + 1.05^2 + 1.05^3 + 1.05^3 + 1.05^4 + 1.05^6]$$

The series in the bracket is a geometric progression with the first term 1 and a common ratio 1.05.

$$\therefore \text{Amount of annuity} = \frac{1\,200(1.05^7 - 1)}{(1.05 - 1)}$$

$$= \frac{1\,200(1.4071 - 1)}{0.05}$$

$$\text{Amount of annuity} = \frac{1\,200(0.4071)}{0.05}$$

$$= \frac{488.52}{0.05} = ₦9\,770.4$$

Exercise 4

1. Find the amount of an annuity of ₦15 000.00 paid yearly for 4 years at 5% per annum.
2. Compute the terminal value of an annuity of ₦500.00 at 8% per annum for 6 years.
3. An annuity paid for 20 years at 3% interest per annum gave $80 340. What was the annual deposit?

   A bank pays interest at $8\frac{1}{2}\%$ per annum compounded yearly. What is the amount if a person saves
4. ₦500.00 for 2 years.
5. ₦1 000.00 for 3 years.
6. ₦2 000.00 for 4 years.

7. ₦5 000.00 for 5 years.

   An annuity of $500 is paid yearly for 8 years at 3% interest per annum. The total amount received at the end of the 8 years is now paid as an annuity for 2 years at 10% interest per annum:

8. What was the total amount received at the end of 10 years?
9. What was the total interest received?
10. Find the amount arising from an annuity of D24 000.00 per annum payable yearly for 9 years with compound interest at 5% per annum.

V. Amortisation

Amortisation is the distribution of a single lump-sum cash flow into smaller cash flow installments or the process of paying off a debt often from loan or mortgage over time through regular payments. A portion of each payment is for interest while the remaining amount is applied towards the principal balance. The percentage of interest versus principal in each payment is determined in an amortisation schedule. The following is the formula for finding amortization:

$$P = A\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right)$$

where P is the principal amount borrowed, A the periodic payment, r the periodic interest rate divided by 100 and n the total number of payments.

Worked Example 11

Find the amortisation of N1 000 borrowed at 5% interest for 3 years.

SOLUTION

$P = ₦1\,000, A = ?, n = 3 \text{ years}, r = \frac{5}{100} = 0.05$

$$P = A\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right)$$

$$= \frac{1\,000}{\frac{1-\left(\frac{1}{1+0.05}\right)^3}{0.05}}$$

$$= \frac{1\,000}{\frac{1-0.8638377}{0.05}}$$

$$= \frac{1\,000}{\frac{0.1361623}{0.05}} = \frac{1\,000}{2.723246}$$

$A = ₦367.21$

Worked Example 12

A person pays an amortisation of N100 at a mortgage rate of 10% for 1 year. If the payment is monthly, fi nd the principal amount borrowed?

SOLUTION

$P = ?, A = ₦100, r = \frac{0.1}{12} = 0.0083$ (monthly interest rate) and $n = 1 \times 12 = 12$

$$P = A\frac{1-\left(\frac{1}{1+r}\right)^n}{r}$$

$$= ₦100\frac{1-\left(\frac{1}{1+0.0-83}\right)^{12}}{0.083}$$

$$= ₦100 \times \frac{0.0944284}{0.0083}$$

$$= ₦100 \times 11.37691572$$

$$= ₦1\,137.69$$

Exercise 5

1. Find the amortisation of ₦5 000.00 borrowed at 6% interest for 2 years?
2. A mortgage bank lends $200 to a farmer at a mortgage rate of 8% for 2 years. Find the periodic payment, if the payment is monthly?
3. A mortgagor pays ₦200.00 at 4% interest for 3 years. Find the principal amount?
4. A businessman borrowed ₦10 000.00 at 7% for 3 years. If the

payment is in monthly installments, find the amortisation?

5. A building society builds a house for $20 000.00 at an interest rate of 6% for 10 years. Calculate the amortisation?
6. A bank receives an amortization of $20 at 7% interest of the mortgage. Calculate the principal amount borrowed.
7. A mortgagee receives an amortisation of ₦200 at $7\frac{1}{2}\%$ interest of the principal. If the payment is monthly, calculate the principal amount borrowed.
8. A building society received an amortization of $500 at a mortgage rate of 8% of the mortgage, find the principal amount.
9. A farmer borrowed ₦20 000.00 at 9% for 2 years from a bank. Find the amortisation?
10. A bank lends ₦30 000.00 at a mortgage rate of 10% to a mortgagor for 4 years. If the payment is in monthly installments, find the amortisation.

SUMMARY

In this chapter, we have learnt the following:

❖ Simple interest is a fee paid on borrowed assets. The formula for finding simple interest is

$$I = \frac{PRT}{100}$$

where I is the interest, P the principal, R the rate percent and T the number of periods.

❖ Compound interest is calculated each period on the original and all interest accumulated during past periods.

The formula for finding compound interest is

$$I = A - P$$

where I is the total interest, A the amount and P the principal.

The amount A after T years is given by

$$A = P\left[1 + \frac{R}{100}\right]^T$$

where A is the amount, P the principal, T the number of period and R the rate percentage.

❖ Depreciation is a loss in value of anitem over period of time.

❖ An annuity is an amount paid to somebody at regular intervals. The formula for finding amount of an annuity is given as

$$\text{Amount of annuity} = P\sum_{t=0}^{n-1}(1+R)^t$$

where P is the principal, R the rate percentage and n the number of years.

❖ Amortisation is the distribution of a single lump-sum cash flow into smaller cash flow installments or the process.

The formula for finding amortisation is

$$P = A\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right)$$

where P is the principal amount borrowed, A the periodic payment, r the periodic interest rate divided by 100 and n the total number of payments.

GRADUATED EXERCISES

1. Udoh deposited N150.00 in the bank. At the end of 5 years, the simple interest on the principal was N55.00. At what rate percent per annum was the interest paid?
2. A man invests a sum of money at 4% per annum simple interest. After 3 years, the principal amounts to N7 000.00. Find the sum invested.
3. Oke deposited N800.00 in the bank at the rate of $12\frac{1}{2}\%$ per annum simple interest. After some time, the total amount was one-and-half times the principal. For how many years was the money left in the bank? **(JAMB)**
4. A man wishes to keep some money in a savings deposit at 25% compound interest so that after 3 years, he can buy a car for N150 000.00. How much does he need to deposit now? **(JAMB)**
5. A worker's present salary is N24 000.00 per annum. His annual increment is 10% of his basic salary. What would be his annual salary at the beginning of the 3rd year? **(JAMB)**
6. A television is depreciated at $3\frac{1}{5}\%$ per annum. Find its book value at the end of 6 years, if it is bought for N65 250.00?
7. (a) Kalu bought a car for N5 500.00. It depreciated by 15% each year, for 2 years. In the succeeding 2 years, the rate was reduced to 10%. Find the book value of the car, to the nearest Naira, after 4 years.

   (b) He then sold the car for N3 900.00 and deposited the amount for a new car costing N7 800.00. For the balance amount, if he took a loan from the bank at $8\frac{1}{2}\%$ compound interest, how much did he pay for the car correct to the nearest Naira, if he paid back the loan after 2 years. **(WAEC)**
8. An annuity of N1 000.00 is paid for 20 years at 2% interest rate per annum. How much interest was paid at the end of the period?
9. An investor paid an annuity of N1 500.00 for 2 years at 15% interest per annum. He also paid another annuity of N750.00 for 2 years at 30%

interest per annum. Which one of these is the better investment?

10. A businesswoman borrowed N5 000.00 at 5% for 3 years. If the payment is in monthly installments, find the amortisation.