CHAPTER 9 Mechanics

Many years ago, when I was younger, I liked throwing stones. I liked stoning lizards, goats and sometimes, I just take pleasure in throwing stones up and trying to catch it. Mechanics helps to explain the motion of this object (and any other moving object) in relation to the energy causing this motion.

A stone thrown upwards moves with velocity, *v*, on its way up, and as it moves upwards, its speed reduces (deceleration) until it reaches a velocity of zero at its maximum height, then it starts coming down. A force must have pushed the stone up and this force is as a result of the energy released from the hand that threw the stone. From this short explanation, physical quantities like velocity, acceleration, distance (also called height in some cases), force and energy all come to mind. The time of travel is also a crucial quantity in mechanics and in sciences in general. Other useful quantities in mechanics are momentum, moment, impulse, just to name a few. A brief explanation on these quantities is provided below.

Distance measures how far away two positions are from each other. For example, the distance between Ibadan and Lagos is $150km$. Displacement is distance measured in a given direction. So, if Bola runs 150 metres in the direction 050^o, this means Bola runs a distance of 150 meters, while his displacement is **150 metres in the direction 050^o**.

Speed is a measure of how quick a body (a car, for example) is moving. In other words, speed can be defined as the rate of change of distance with time: examples of speed are $100km/h$, $40m/s$, e.t.c. Velocity is speed measured in a specified direction. A car travelling at $75km/h$ in the northeast direction is said to have a speed of $75km/h$, while it has a velocity of **$75km/h$ north-east**.

The acceleration of a body is a measure of how its velocity is changing (increasing or decreasing) with time. Again, acceleration is the rate of change of velocity with time. Imagine that a body's velocity changes from $2m/s$ to $5m/s$ in 5 seconds, then the body's acceleration will be the change in speed divided by the time taken, and is expressed as

$$\frac{(5-2)metre/second}{5\ seconds} = 0.6\ metre/second^2.$$

Acceleration is a vector quantity and it is always in the direction of the force causing it; recall that $F = m \times a = ma$.

Force is normally felt as a push or pull. Mathematically, force is the product of the mass of a body and its acceleration. Sometimes, one observes that mango fruits drop on their own from the mango tree without any breeze blowing, or anyone throwing sticks at them. This is due to the force of gravity from the centre of the earththat is pulling the orange. It is this same force that pulls one back to the ground (the earth's surface) when one jumps up.

Also, energy is the ability to do work. It takes energy to get any work done; however, the availability of energy does not mean work is being done. Imagine a man, idly sitting down after obtaining chemical energy from a sumptuous meal of rice and plantain. Although he has energy to work, he is not using the energy to get any work done.

There are other quantities in mechanics that you will come across while you are working at the workshop; you will definitely have a smooth sailing, as they are well explained at the workshop. But, before we set out for the workshop, let us quickly revise the equations of motion and other fundamental formulas and rules in mechanics.

The following formula hold for a body moving with uniform acceleration:

$v = u + at$ ....(i)

$v^2 = u^2 + 2as$ ....(ii)

$s = ut + \frac{1}{2}at^2$ .... (iii)

Where v is the final velocity of the body, u is its initial velocity, a is the uniform acceleration, s is the distance, and t is the time of travel.

Moment

It may come as a surprise to realise that many times while playing, you have experienced equilibrium with the object being played with. A good example is depicted by balancing a long ruler on a stretched out finger. At that instant when the ruler balances on the finger, so that it does NOT tilt to either side, the ruler is said to be in equilibrium. This important concept of equilibrium is the basic principle of solving moment related problems.

Moment is a turning force about a pivot (in the case of the example above, the pivot is the outstretched finger). The effect of this turning force is dependent on the perpendicular distance between the line of action of the force and the pivot.

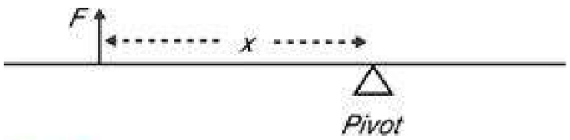

Fig. 9.1

Moment is mathematically expressed as the product of the turning force and the perpendicular distance between the line of action of the force and the pivot. *Moment* = $F \times x$.

1. A body P is allowed to fall from the top of a tower 60*metres* high. At the same time, another body Q is projected vertically upwards from the foot of the tower with a velocity of 24 $ms{-}1$. The bodies collide at a point h metres above the foot of the tower.

(a) Calculate:

(i) the height h.

(ii) the time of collision of the two bodies.

(iii) the velocities of the bodies at the moment of collision.

(b) If P and Q are of masses 3kg and 1kg respectively and after collision the two bodies coalesce and move together, calculate the common velocity with which they reach the foot of the tower. **(WAEC)**

Workshop

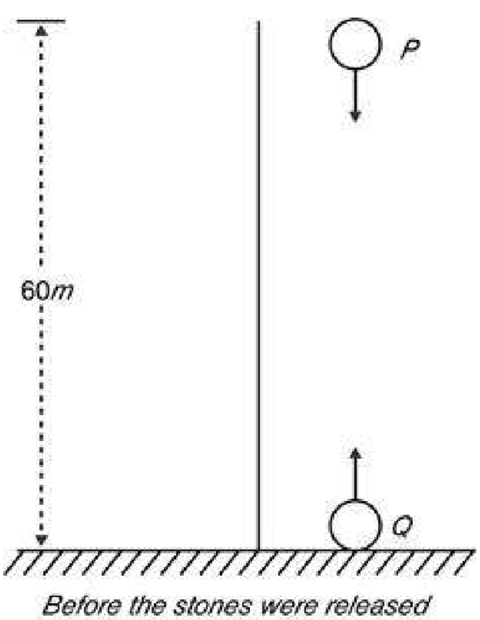

Fig. 9.2(a)

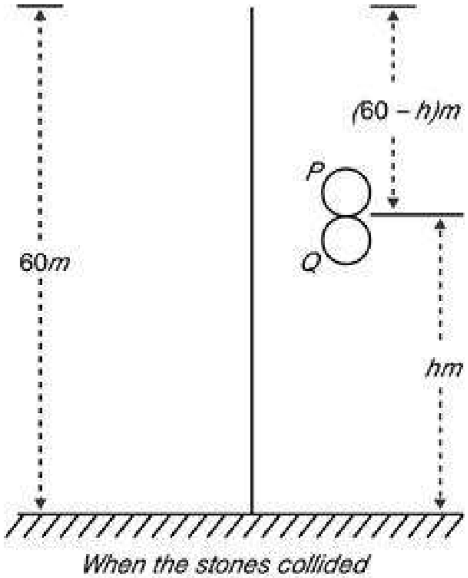

Fig. 9.2(b)

(a) Since the two bodies were set in motion at the same time, then it would take time t before the two bodies would collide at height h, above the foot of the tower, as shown in Figures 9.2a and b. By the equation

tion of motion $s = ut + \frac{1}{2}at^2$, the distance, S_Q, travelled by body Q in t seconds $= S_Q = u_Q t + \frac{1}{2} a_Q t^2$, where u_Q is the initial velocity of body Q, and a_Q is its acceleration. The distance, S_P, travelled by body P in t seconds $= S_P = u_P t + \frac{1}{2} a_P t^2$.

Know that $t_Q = t_P = t$, because the bodies were released at the same time, hence, the two bodies collided after time t.

Where u_P is the initial velocity of body P, and a_P is its acceleration.

The following parameters were given in the question: $u_P = 0$,
(because P is falling freely from a height (rest)), $u_Q = 24ms^{-1}$ $a_P = 10ms^{-2}$, $a_Q = -10ms^{-2}$ *(acceleration against gravity is always negative)*.

Note that a body falling ***vertically*** *downward from a height will fall with acceleration $10ms^{-2}$, while a body shot (projected)* ***vertically*** *upwards, will move up with acceleration $-10ms^{-2}$.*

From Figures 9.2a and b, $S_Q = h$ and $S_P = 60 - h$; putting all these parameters in the above equations gives $S_Q = h = 24t + \frac{1}{2}(-10)t^2$; $h = 24t - 5t^2$ ....(i)

$S_P = 60 - h = 0(t) + \frac{1}{2}(10)t^2 = 5t^2$;
$h = 60 - 5t^2$ ....(ii)

Subtracting (ii) from (i) we get
$h - h = 24t - 5t^2 - (60 - 5t^2)$;

$0 = 24t - 5t^2 - 60 + 5t^2$; $0 = 24t - 60$;

$24t = 60$; $t = \frac{60}{24} = 2.5$ *seconds*.

From equation (ii) $h = 60\ 5t^2 = 60 - 5(2.5)^2 = 60 - 31.25 = 28.75m$.

(i) Therefore, the height, *h*, above the foot of the tower, where the two bodies collide, is 28.75*m* and

(ii) The time taken for the bodies to collide, after the bodies were released, is 2.5 *seconds*.

(iii) By the formula of motion

$v^2 = u^2 + 2as$;

$v_Q^2 = u_Q^2 + 2a_Q s_Q = 24^2 + 2(-10)(28.75)$;

$v_Q^2 = 576 - 575 = 1$; $v_Q = \sqrt{1} = 1ms^{-1}$

$v_P^2 = u_P^2 + 2a_P s_P = 0^2 + 2(10)$
$(60 - 28.75)$ (*Recall that* $s_P = 60 - h$);

$v_P^2 = 20(31.25) = 625$;

$v_P = \sqrt{625} = 25ms^{-1}$

The velocities of the bodies, at the height 28.75, just before they collided, are $1ms^{-1}$ and

25 ms^{-1}, for *Q* and *P* respectively.

(b) $m_Q = 1kg$ *velocity of Q just before collision is* $v_Q = 1ms^{-1}$ $m_P = 3kg$, *velocity of P just before collision* $v_P = 25ms^{-1}$. As the two bodies coalesce (joined together) after collision, the two bodies will have a common final velocity, *w*, after collision. Momentum is always conserved in every collision, therefore, *Total Momentum before collision = Total Momentum after collision.*

Recall that, Momentum, $P = mv$ so that

$$(m_p \times v_p) + (m_Q \times v_Q) = (m_p \times w) + (m_Q \times w);$$

$$m_p v_p + m_Q v_Q = w(m_p + m_Q);$$

$$w = \frac{m_p v_p + m_Q v_Q}{m_p + m_Q}.$$

Recall that velocities are vectors, and these two bodies are travelling opposite to each other. So, for this problem, we will allot positive sign to the downward-facing velocities, while we will give negative sign to the upward-facing velocities; therefore, $v_p = +25ms^{-1}$ and $v_Q = -1ms^{-1}$, so,

$$w = \frac{m_p v_p + m_Q v_Q}{m_p + m_Q} = \frac{(3 \times (+25)) + (1 \times -1)}{3 + 1}$$

$$= \frac{75 + (-1)}{4} = \frac{75 - 1}{4} = \frac{74}{4} = +18.5ms^{-1}.$$

Moreover, recall that we allotted positive signs to the downward facing velocities; therefore, $w = +18.5ms^{-1}$ is facing the downward direction. Hence, the two bodies coalesce, to move downwards, with a velocity of $18.5ms^{-1}$.

The two bodies coalesce at height $h = 28.75m$ above the foot of the tower, to attain a common velocity of $18.5ms^{-1}$.

Therefore, the common velocity of the two bodies at height $28.75m$ above the ground, just after collision, is $18.5ms^{-1}$. These two bodies will move down together with acceleration $g = +10ms^{-2}$, from height $28.75m$ above the foot of the tower, and their initial velocity for this downward motion will be $18.5ms^{-1}$ (*because this is the velocity at height* $28.75m$). The two bodies coalesced at height $28.75m$ above the ground, and they are heading downwards, therefore, the distance, *s*, that they will travel before getting to the ground will be $s = 28.75m$.

Recall the equation; $v^2 = u^2 + 2as;$

$$v^2 = 18.5^2 + 2(10 \times 28.75) = 342.25 + 575$$

$$= 917.25 \therefore v = \sqrt{917.25} = 30.3ms^{-1}.$$

Therefore, the common velocity, with which they reach the foot of the tower is $30.3ms^{-1}$.

2. A body of mass 5 *kg* is placed on a smooth plane inclined at an angle 30° to the horizontal. Find the magnitude of the force:

(a) acting parallel to the plane;

(b) required to keep the body in equilibrium. [Take $g = 10\ ms^{-2}$] **(WAEC)**

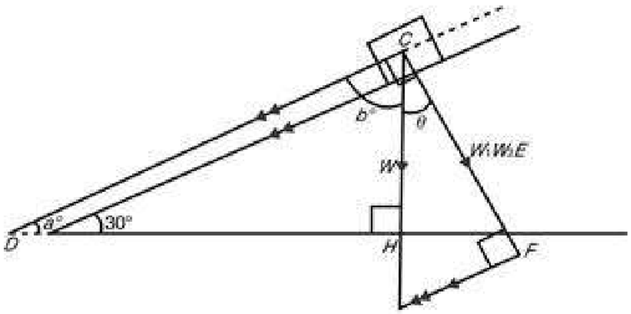

Fig. 9.3

(a) The drawing may seem complicated, but note the explanation below. Recall that the weight W of a body is the force on the body due to gravity that pulls the body towards the centre of the earth. So, weight W, acts vertically downward as shown in Figure 9.3. Because W is acting vertically downward, its line of action will be perpendicular to the horizontal, DH, as represented by angle $C\hat{H}D$ in Figure 9.3. Now, we want to resolve weight (force) W so that one of its component will be parallel to the inclined plane (component W_2), while the other component will be perpendicular to the inclined plane (Component W_1) as shown in Figure 9.3.

Note that the two identical arrows along EF shows that EF is parallel to the inclined plane and line DC.

Triangle CEF in Figure 9.3 is a rightangled triangle with force (weight), W, acting along its hypothenus. On triangle

CEF, $\sin\theta = \frac{OPP}{HYP} = \frac{W_2}{W}$; $\sin\theta = \frac{W_2}{W}$;

$W_2 = W\sin\theta$; $\cos\theta = \frac{ADJ}{HYP} = \frac{W_1}{W}$;

$\cos\theta = \frac{W_1}{W}$; $W_1 = W\cos\theta$.

Therefore, the force acting parallel to the plane is $W_2 = W\sin\theta$.

Recall that $W = mg$ $W_2 = W\sin\theta = mg$ $\sin\theta$. We know that $m = 5\text{kg}$,

$g = 10ms^{-2}$, but we don't know θ. From the diagram, line CD is parallel to the inclined plane and line DH is a straight line cutting CD and the inclined plane. Hence, angle a and angle 30° are corresponding angles. Thus, $a = 30°$ (corresponding angles are equal). Triangle CDH shows that $a + b + 90° = 180°$ (*sum of angles in a triangle is equal to* 180°); $30° + b = 180° - 90°$; $b = 90° - 30° = 60°$. So, angle $b = 60°$. Line CF is perpendicular to the inclined plane and the line CD is parallel to the inclined plane.

Therefore, line CF is also perpendicular to line CD. Therefore, angle $D\hat{C}F = 90°$; $b + \theta = 90°$; $\theta = 90° - b = 90° - 60° = 30°$.

Did you observe that the angle 30° that the inclined plane makes with the horizontal is equal to the angle θ between W1 and W?

Recall that $W2 = W\sin\theta = 5 \times 10$

$\sin\theta = 5 \times 10 \sin 300 = 5 \times 10 \times \frac{1}{2} = 25N.$

Therefore, the force W_2 acting parallel to the plane is $25N$.

*Please, note that we can redraw $W_2 = mg \sin\theta$ as shown in Figure 9.4 below. Recall that vectors are defined by their magnitude and direction. $mg \sin\theta$ is a vector, and it is having the same magnitude and facing the same direction in the diagram below as it did in Figure 9.3. And, really, the $W_2 = mg \sin\theta$ force should ideally be represented as shown in Figure 9.4, as it is the force due to gravity that is pushing the body down the plane. We have represented force $W_2 = mg \sin\theta$, as shown in Figure 9.3, to be able to apply the triangular law of vectors, as it is still having the same magnitude and direction. Also note that, as the plane is smooth, there is **no** frictional force acting on the plane.*

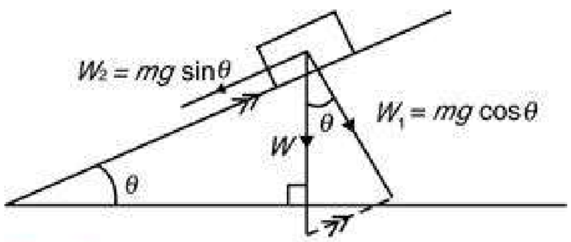

Fig. 9.4

(b) The $mg \cos\theta$ force is already balanced by an equal but opposite force which is the reaction R, acting against the downward acting force $mg \cos\theta$ as shown in Figure 9.4. Recall Newton's third law, "To every action, there is an equal but opposite reaction".

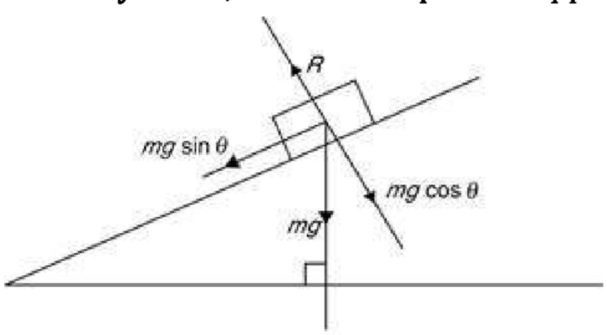

Fig. 9.5

Also note that a body is in equilibrium when it is at rest under the action of a system of forces (two or more forces).

Since the $mg \cos\theta$ force has been balanced by an equal but opposite reaction, R, for the body to be at rest under the action of all the forces in Figure 9.5, (that is, at equilibrium) the $mg \sin\theta$ force must be pulled by a force equal but acting in the opposite direction, given by, $-mg \sin\theta = -25N$.

No mention was made about the action of the frictional force of the plane on the body because the inclined plane is smooth so frictional force is zero.

Moreover, note that the negative sign only shows that this force is opposite in direction to the mg sin$\theta = 25N$ force but they are equal in magnitude. Also note that any one of the two opposite forces can bear the positive sign while the other force bears the negative as these signs are just indicating direction.

Recall that the force required to keep the body in equilibrium is $-25N$.

Then, the **magnitude** of the force required to keep the body in equilibrium will be

$|-25| = \sqrt{(-25)^2} = \sqrt{(-25) \times (-25)} =$

$\sqrt{25 \times 25} = \sqrt{25} \times \sqrt{25} = 5 \times 5 = 25N$

Know that the magnitude of the force is the quantity of the force which is 25N while the negative sign shows that it is in a direction opposite to the force we allotted the positive sign.

3. A body of mass 15 kg is suspended at a point P by two light inextensible Strings $\overrightarrow{XP}$ and $\overrightarrow{YP}$. The strings are inclined at 60° and 40° respectively to the downward vertical. Find correct to two decimal places, the tensions in the strings. [Take = 10 ms^{-2}] **(WAEC)**

Workshop

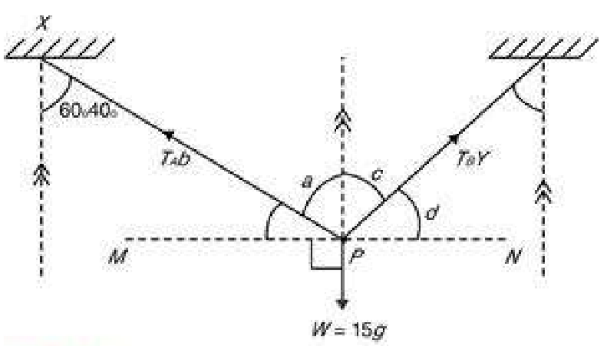

Fig. 9.6

From Figure 9.6, the three broken, vertical lines are parallel to each other, therefore, angle a is alternate to the 60° angle, so, $a = 60°$ (alternate angles are equal). MN is a horizontal line, and the broken line passing through P is a vertical line, thus, $a + b = 90°$; $b = 90° - a = 90° - 60° = 30°$. Angle c is alternate to the 40° angle; therefore, $c = 40°$ (alternate angles are equal). Again, $c + d = 90°$ (sum of angles in a right angle is 90); $d = 90° - c = 90° - 40° = 50°$.

When the body is at equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, where ΣF_x is the sum of the components of these forces in the horizontal direction and ΣF_y is the sum of the components of the forces in the vertical direction.

$\Sigma F_x = T_B \cos d + (-T_A \cos b) + W \cos 90° = 0$, where angle d, b and 90° are the angles that T_B, T_A and W respectively make with the horizontal axis (or x-axis).

Note that the resolution of T_A to the horizontal is facing the negative x-axis direction, hence, $T_A \cos b$ will bear a negative sign.

$T_B \cos d - T_A \cos b + W(0) = 0$; recall that $\cos 90° = 0$.

$T_B \cos 50° - T_A \cos 30° + 0 = 0$; $T_B \cos 50° = T_A \cos 30°$; $0.6428T_B = 0.866T_A$;

$T_B = \dfrac{0.866T_A}{0.6428} = 1.347T_A$. $\Sigma F_y = T_B \sin d + T_A \sin b - W \sin 90° = 0$ (*recall that at equilibrium*, $\Sigma F_y = 0$).

You can also see from Figure 9.6 that the resolution of force W to the vertical points in the negative y-direction, thus, W sin 90° will bear a negative sign.

$T_B \sin 50° + T_A \sin 30° - W \sin 90° = 0$; $T_B (0.7660) + T_A (0.5) = W(1)$;

$0.7660T_B + 0.5T_A = 150$; (*since* $W = mg = 15 \times 10 = 150N$); recall that,

$T_B = 1.347T_A$; therefore, $0.7660(1.347T_A) + 0.5T_A = 150$; $1.032T_A + 0.5T_A = 150$; $1.532T_A = 150$; $T_A = \dfrac{150}{1.532} = 97.91N$; $T_B = 1.347T_A = 1.347(97.91) = 131.88N$. Therefore, the tension T_A in string XP is $97.91N$ while the tension T_B in string YP is $131.88N$.

4. The height h metres, of a ball thrown into the air is $2 + 20t + kt$ 2, after t seconds. If it takes 2 seconds for the ball to reach its highest point, find:
   (a) the value of k;
   (b) its highest point from the point of throw. **(WAEC)**

Workshop

(a) When a ball is thrown vertically upwards, at its maximum (highest) height, the velocity is zero (because the ball will first stop (*velocity* = 0) before returning to the ground). Then, at maximum height, h_{max}, *velocity* = 0. Recall that velocity after t seconds is given by $\frac{ds}{dt}$ but in this case,

$s = height,\ h$ so that $velocity = \frac{dh}{dt}$,

$$\frac{dh}{dt} = \frac{d\,(2 + 20t + kt^2)}{dt} = 20 + 2kt.$$

But it takes $t = 2$ to get to the maximum height, and, at maximum height,

$velocity = \frac{dh}{dt} = 0$; and $t = 2$, hence, $\frac{dh}{dt} = 0$

$= 20 + 20kt = 20 + 2k\,(2)\ 0 = 20 + 2k(2)$;

$0 = 20 + 4k$; $4k = -20$; $k = \frac{-20}{4} = -5$.

Therefore, the value of k in the equation $h = 2 + 20t + kt^2$ is -5.

(b) Now, $h = 2 + 20t + kt^2$; $h = 2 + 20t + (-5)t^2$; $h = 2 + 20t - 5t^2$; (since $k = -5$). It takes 2 seconds to get to the maximum height, h_{max}, so, at maximum height, $t = 2s$. Thus, $h_{max} = 2 + 20t - 5t^2 = 2 + 20(2) - 5(2)^2$; $h_{max} = 2 + 40 - 20 = 22$. Therefore, the highest point h_{max} from the point of throw is 22 *metres*.

5. A ball P moving with velocity $2u\ ms^{-1}$, collides with a similar ball Q, of different mass, which is at rest. After collision, Q moves with velocity $u\ ms^{-1}$ and P with velocity $\frac{1}{2}u\ ms^{-1}$ in the opposite direction. Find the ratio of the masses of P and Q. m**(WAEC)**

Workshop

Let W represent initial velocity while V represent final velocity;

Before collision

Fig. 9.7(a)

After collision

Fig. 9.7(b)

Note that we were told in the question that after collision, ball P went in a direction opposite to the general direction of motion as shown in Figure 9.7(b) . Now, if we allot positive sign to the velocities facing the +x *direction*, we must allot negative to the velocity in the opposite direction (i.e −x *direction*) since velocity is a vector quantity. All these are shown in Figures 9.7(a) and (b) above. The law of conservation of linear momentum, states thus:

$$\begin{pmatrix} Total\ momentum \\ before\ collision \end{pmatrix} = \begin{pmatrix} Total\ momentum \\ after\ collision \end{pmatrix}, \text{ i.e.}$$

$\Sigma P_{(Before\ collision)} = \Sigma P_{(after\ collision)}$, where momentum, $P = mass \times velocity$;

$$\begin{pmatrix} Total\ momentum \\ before\ collision \end{pmatrix} = M_P W_P + M_Q W_Q = M_P W_P + (M_Q \times 0) = M_P W_P;$$

(since $W_Q = 0$). $\begin{pmatrix} Total\ momentum \\ after\ collision \end{pmatrix} = M_P V_P + M_Q V_Q$;

recall that $\Sigma P_{(Before\ collision)} = \Sigma P_{(After\ collision)}$; therefore,

$$M_P W_P = M_P V_P + M_Q V_Q; \; M_P W_P - M_P V_P = M_Q V_Q;$$

$$M_P(W_P - V_P) = M_Q V_Q; \; \frac{M_P(W_P - V_P)}{M_Q} = V_Q; \; \frac{M_P}{M_Q} = \frac{V_Q}{W_P - V_P};$$

$$= \frac{+u}{+2u - \left(-\frac{1}{2}u\right)} \left(Recall\ that\ V_P = -\frac{1}{2}u\ ms^{-1}\right);$$

$$\frac{M_P}{M_Q} = \frac{u}{2u + \frac{1}{2}u} = \frac{u}{\frac{5u}{2}} = u \div \frac{5u}{2} = u \times \frac{2}{5u} = \frac{2u}{5u} = \frac{2}{5}.$$

Therefore, the ratio of mass of P to mass of Q is $\frac{M_P}{M_Q} = \frac{2}{5}$.

6. A uniform plank PQ of length $10m$ and mass $m\ kg$ rests on two supports A and B, where $|PA| = |BQ| = 1m$. A load of mass $8kg$ is placed on the plank at point C such that $|AC| = 3.5m$. If the reaction at B is $100N$, calculate the:

   (a) value of m;

   (b) reaction at A. [Take $g = 10\ ms{-}2$] **(WAEC)**

Workshop

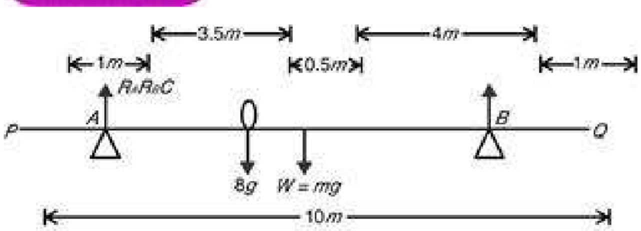

Fig. 9.8

(a) Supports *A* and *B*, in Figure 9.8, will exert an upward reaction (force) *RA* and *RB* respectively against the downward acting weights (forces), *mg* of the plank and 8*g* of the load (Newton's third law). As the plank is **uniform**, its weight *mg* will act at its centre. At equilibrium, the principle of moment holds which states that at equilibrium. (*The sum of clockwise moment*) = (*sum of anticlockwise moment*); recall that moment about point *P* in the diagram below is expressed as

$$Force \times \begin{pmatrix} Perpendicular\ distance \\ between\ point,\ P,\ and\ the \\ line\ of\ action\ of\ the\ force \end{pmatrix} = Force \times x$$

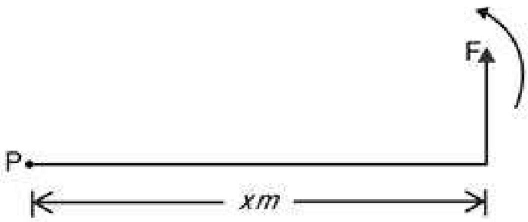

Fig. 9.9(a)

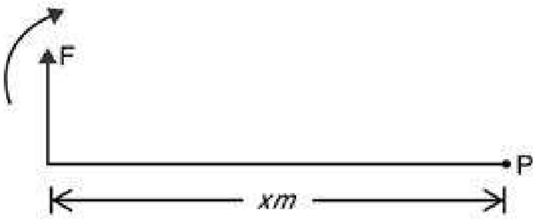

Fig. 9.9(b)

Note that in Figure 9.9(a) above, the force F is moving anticlockwise (a direction opposite that in which the clock hands move) with respect to point P, but in Figure 9.9(b), F is moving in a clockwise direction with respect to point P.

Back to Figure 9.8, if moment is taken about point *A*,

Sum of clockwise moment $= [(8g)\times 3.5] + [(mg)\times 4]$;

Sum of clockwise moment $= R_B \times 8$;

Since we are taking moment about point *A*, moment due to R_A about point *A*

$= R_B \times 0 = 0$. Hence, at equilibrium $[(8g)\times 3.5] + [(mg)\times 4] = R_B \times 8$;

$[(8 \times 10) \times 3.5] + [(m \times 10) \times 4] = 100 \times 8$;
$280 + 40m = 800$;

$40m = 800 - 280 = 520$; $m = \frac{520}{40} = 13kg$.

Also, taking moment about *point B*; *Sum of clockwise moment* $= R_A \times 8$

Sum of clockwise moment $= [(8g)\times 4.5] + [(mg)\times 4]$

At equilibrium, $R_A \times 8$ $[(8g) \times 4.5] + [(mg) \times 4]$;

$8R_A = [8 \times 10 \times 4.5] + [13 \times 10 \times 5]$;

$8R_A = 360 + 520 = 880$; $8R_A = 880$;

$R_A = \frac{880}{8} = 110N$.

Therefore, the value of m is $13kg$ and the reaction at A is $110N$.

Alternative method

Having evaluated m as $13kg$, also, at equilibrium,

$$\begin{pmatrix} \text{Sum fo up facing} \\ \text{vertical forces} \end{pmatrix} = \begin{pmatrix} \text{Sum of down facing} \\ \text{vertical forces} \end{pmatrix}$$

So, $R_A + R_B = 8g + mg$; $R_A + 100 = (8 \times 10) + (13 \times 10) = 210$; $R_A = 210 - 100 - 110N$

However, note that to use this method, all the forces in question must be vertical forces (that is, all the forces ***must*** *be acting perpendicularly (at 90°) to the plank). In case any of the forces is* ***not*** *vertical, find its vertical component, and use this vertical component as one of the vertical forces in your calculation.*

7. A light inextensible string 8 metres long suspends a ring of mass $5kg$. The two ends of the strings are attached to points P and Q of a ceiling such that $|PQ| = 6.9m$. If the system is in equilibrium, calculate the tension in the string. [Take $g = 10\ ms{-}2$] **(WAEC)**

Workshop

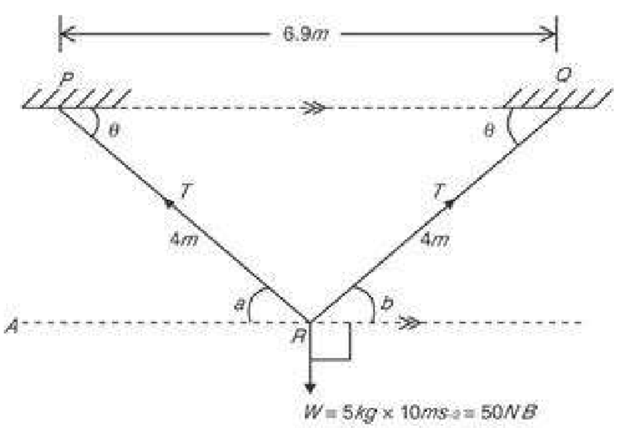

Fig. 9.10

As the ring is hanging freely, it will pull the string down to hang at the centre of the string,

R, dividing the 8 *metre* length of the string into two equal lengths of 4 *metres* each as drawn in Figure 9.10, with equal tensions T on each side of the string. The tension T is the same on the two sides of the rope because the ring is hanging freely, hence, the two equal halves of the rope will support the weight of the ring with equal tension T (*recall Newton's Third Law; 'to every action, there is an equal but opposite reaction*). In this case the weight of the ring (*acting downward*) is the action while the tensions in the string are the reactions (*acting upwards*).

Furthermore, from the diagram, since the length of two sides of the triangle PQR formed are equal, then the triangle PQR is an isosceles triangle. Therefore, the base angles of triangle PQR are equal. The ceiling from which the string is hanging is horizontal, so, the ceiling is parallel to the horizontal broken line AB drawn. Therefore, the angle θ on the left is alternate to angle a. Hence, $a = \theta$ (*alternate angles are equal*). Angle θ on the right is also alternate to angle b, so, $b = \theta$ (*alternate angles are equal*); therefore, $\theta = a = b$. Now, since we know the lengths of all the three sides of triangle PQR, we can calculate the value of angle θ using the cosine rule, which states that: $q2 = p2 + r2 - 2pr \cos Q$; where p, q and r are the lengths of the three sides of a triangle and Q is the angle opposite the side of length q. Then, $42 = 6.9^2 + 4^2 - 2(6.9)(4) \cos\theta$;

$16 = 47.61 + 16 - 55.2 \cos\theta$;

$$16 - 16 - 47.61 = -55.2 \cos\theta;$$

$$\cos\theta = \frac{16 - 16 - 47.61}{-55.2} = \frac{-47.61}{-55.2} = 0.8625$$

$\theta = \cos^{-1} 0.8625 = 30.4°$. Therefore, $\theta = a = b = 30.4°$. When the suspended ring is static (*not moving*), despite the forces acting on it, the system of forces (*in this case W, T and T*) are said to be in equilibrium. At equilibrium $\Sigma f_x = 0$ and $\Sigma f_y = 0$.

Σf_y is the sum of the forces acting parallel to the vertical axis (y-axis). From figure 9.10, since the system of forces is at equilibrium, $\Sigma f_y = 0$; thus,

$\Sigma f_y = T \sin b - W \sin 90° + T \sin a = 0$; where b, 90° and a are the angles that forces T, W and the other T respectively make with the horizontal, AB (x-axis), as drawn in Figure 9.10.

Note that if we resolve the two tensional forces into their respective vertical and horizontal components, the vertical component of one of the tensional force will be +T sin a while the vertical component of the other tensional force will be +T sin b. This is because the resolution of these two tensional forces, to the vertical, point in the positive y-axis direction, with point R as the origin, having coordinates (0, 0). *However, the vertical component of W is −W sin 90° because its vertical component is pointing to the negative y-axis direction with point R as the origin having coordinates* (0, 0).

For example, take a look at Figure 9.11 below.

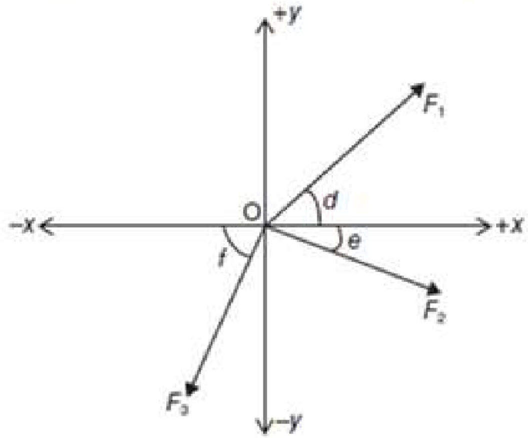

Fig. 9.11

In Figure 9.11, O is the origin with coordinate (0, 0).

The horizontal component of force F_1 is $F_{1x} = +F_1 \cos d$ (since F_1 points in the direction of positive x-axis). The horizontal component of F_2 is $F_{2x} = +F_2 \cos e$ (since F_2 points in the direction of positive x-axis).

The horizontal component of F_3 is $F_{3x} = -F_3 \cos f$ (since F_3 points in the direction of negative x-axis). Then, $\Sigma F_x = +F_1 \cos d + F_2 \cos e + (-F_3 \cos f)$;

$\Sigma F_x = + F_1 \cos d + F_2 \cos e - F_3 \cos f$, where d, e and f are the angles these forces make with the x-axis.

The vertical component of force F_1 is $F_{1y} = + F_1 \sin d$ (since F_1 points in the direction of positive y-axis). The vertical component of force F_2 is $F_{2y} = -F_2 \sin e$ (since F_2 points in the direction of negative y-axis).

The vertical component of force F_3 is $F_{3y} = - F_3 \sin f$ (since F_3 points in the direction of negative y-axis). $\Sigma F_y = +F_1 \sin d + (-F_2 \sin e) + (-F_3 \sin f)$

$= +F_1 \sin d - F_2 \sin e - F_3 \sin f$. Can you see how simple it is? Now let's get back to the problem.

Therefore, $\Sigma F_y = T\sin 30.4 - 50 \sin 90 + T\sin 30.4 = 0$;

$T\sin 30.4° + T\sin 30.4° = 50 \sin 90°$; $\sin 30.4°$

$(T + T) = 50\ (1)$; $\sin 30.4°\ (2T) = 50$;

$$2T = \frac{50}{\sin 30.4°};\ T = \frac{50}{\sin 30.4° \times 2};$$

$$T = \frac{50}{0.5060 \times 2} = \frac{50}{1.012} = 49.4N.$$

Therefore, the tension T in the string when the system is in equilibrium is $49.4N$.

8. The acceleration, in $ms-2$, after time t seconds of a particle moving in a straight line is $12t$.

(a) Find an expression for the displacement x in terms of t.

(b) If $x = 0$ when $t = 0$ and $x = 20$ when $t = 2$, find the displacement and the velocity of the particle, when $t = 5$.

(WAEC)

Workshop

(a) The acceleration after time t seconds of a particle in motion is given by $a = \frac{dv}{dt}$ (where v is the velocity after t seconds).

Thus, if $a = \frac{dv}{dt}$; $dv = a\ dt$; $\int dv = \int a.dt$;

know that $\int dv = \int 1 \times dv = \int v^o \times dv = \int v^o \cdot dv$; (*since* $v^o = 1$);

$\int dv = \int v^o \cdot dv = \frac{v^{0+1}}{0+1} = \frac{v^1}{1} = v$. Therefore, $\int dv = v = \int a \cdot dt$;

$v = \int a \cdot dt = \int 12t \cdot dt$; (*Recall that* $a = 12t$)

$v = \int 12t \cdot dt = 12\frac{t^{1+1}}{1+1} + C = \frac{12}{2}t^2 + C = 6t^2 + C$;

where C is an arbitrary constant. Velocity, v, after t seconds is given by $v = \frac{dx}{dt}$; where x is the displacement of the particle. As $v = \frac{dx}{dt}$; $dx = v \cdot dt$;

$\int dx = \int v \cdot dt$. By the same explanation as for $\int dv$ above, $\int dx = x = \int v \cdot dt$; $x = \int v \cdot dt$ $= \int (6t^2 + C)\,dt = \int (6t^2 + Ct^{\circ}) \cdot dt$;

(Recall that $C = C \times 1 = C \times t^{\circ}$, since $t^{\circ} = 1$).

$x = \int (6t^2 + Ct^{\circ}) \cdot dt = 6\frac{t^{2+1}}{2+1} + C\frac{t^{0+1}}{0+1} + K$,

where K is also an arbitrary constant;

$x = 6\frac{t^3}{3} + C\frac{t^1}{1} + k = 2t^3 + Ct + K.$

Therefore, the expression for displacement x in terms of t is $x = 2t^3 + Ct + K$.

(b) $x = 2t^3 + Ct + K$; when $x = 0$, $t = 0$; so that $0 = 2(0)^3 + C(0) + K$;

$0 = 0 + 0 + K$; $K = 0$. When $x = 20$, $t = 2$; thus, $20 = 2(2)^3 + C(2) + K$; recall that $k = 0$, so $20 = 2(8) + 2C + 0$; $20 - 16 = 2C$; $2C = 4$; $C = 2$.

Then, $x = 2t^3 + Ct + K = 2t^3 + 2t + 0 = 2t^3 + 2t$. when $t = 5$, $x = 2(5)^3 + 2(5) = 250 + 10 = 260$ *metres*. From the solution to 8(*i*) we know $v = 6t^2 + C$. Also, $v = 6t^2 + C = 6(5)^2 + 2 = 150 + 2 = 152\ ms^{-1}$.

The displacement and the velocity of the particle, when $t = 5$ are respectively 260 *metres* and 152 *ms*−1.

Note that we were able to write the unit of the displacement in metres and the unit of velocity in ms^{-1} because we have already been told that the unit of acceleration is in ms^{-2} and time in seconds (s).

9. A non-uniform rod *BC* of length 2 metres and mass 3 kg has its centre of gravity at a point *G*, 0.8 *m* from *B*. The rod rests on two supports at *M* and *N*, which are 0.3 *m* and 1.6 *m* from *B* respectively. If the system is in equilibrium, under the action of these forces, calculate the reactions at *M* and *N*. [Take $g = 10\ ms^{-2}$] **(WAEC)**

Workshop

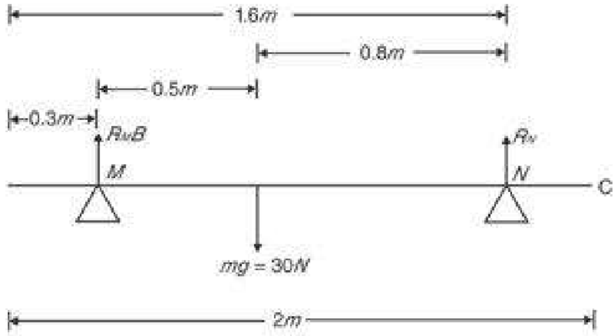

Fig. 9.12

At equilibrium, the sum of the clockwise moment about a point is equal to the sum of the anticlockwise moment about the point. Recall that, Moment about point *A* =

$$(Force)\times\begin{pmatrix}\text{Distance measured, perpendicular to}\\ \text{the direction of action of the force,}\\ \text{between point } A \text{ and the force}\end{pmatrix}$$

Taking moment about support *N*, the clockwise moment about support $N = R_M \times 1.3$.

Anticlockwise moment, about support $N = 30 \times 0.8$.

Since the system is in equilibrium,

$$\begin{pmatrix}\text{The sum of clockwise}\\ \text{moment about a point}\end{pmatrix}=\begin{pmatrix}\text{The sum of anticlock-}\\ \text{wise moment about}\\ \text{the same point}\end{pmatrix}$$

$R_M \times 1.3 = 30 \times 0.8;\ R_M = \frac{30 \times 0.8}{1.3} = 18.46N.$

The moment of R_N about point N is zero because the perpendicular distance between the R_N and point N is zero, and so the moment of R_N about point $N = R_N \times 0 = 0$.

Also, we can take moment about support *M*, the clockwise moment about support *M* is = 30 × 0.5, and the anticlockwise moment about support *M* is = *RN* × 1.3.

Because the system is in equilibrium, about support *M*, *The sum of clockwise moment = The sum of anticlockwise moment*

$30 \times 0.5 = R_N \times 1.3;\ R_N = \frac{30 \times 0.5}{1.3} = 11.54N.$

The moment of R_M about point M is zero because the perpendicular distance between the R_M and point M is zero, so the moment of R_M about point $M = R_M \times 0 = 0$.

Alternative method

Having known the value of R_M, also, at equilibrium,

$$\begin{pmatrix}\text{the sum of upward}\\ \text{vertical forces}\end{pmatrix}=\begin{pmatrix}\text{the sum of downward}\\ \text{vertical forces}\end{pmatrix}$$

Hence, $R_M + R_N = 30N;\ R_M = 18.46,\ so,\ 18.46 + R_N = 30;\ R_N = 30 - 18.46 = 11.54N.$

Therefore, the reactions at *M* and *N* are respectively 18.46*N and* 11.54*N*.

*Note: the upward and the downward vertical forces **must** be **perpendicular** to the rod in question.*

10. A particle moves in a plane such that its displacement(s) from a point O at time t seconds is given by $s = (t^2 + t)i + (3t - 2)j$. Find:

    (a) the velocity,

    (b) the acceleration,

    (c) the speed at $t = 2$ seconds of the particle. **(WAEC)**

Workshop

(a) Recall that displacement, velocity and acceleration are vector quantities, because they have both magnitudes and directions, while distance and speed are scalar quantities, because they have only magnitudes, and **no** direction. Also, recall that velocity is the change in displacement (Δs) per change in time, (Δt) that is,

velocity $= \frac{\Delta s}{\Delta t}$. Thus, velocity at any time

will be $\vec{v} = \frac{d\vec{s}}{dt}$; $s = (t^2 + t)i + (3t - 2)j$,

so that $\vec{v} = \frac{d\vec{s}}{dt}; = \frac{d((t^2 + t)i + (3t - 2)j)}{dt}$

$= \frac{d(t^2 + t)}{dt}i + \frac{d(3t - 2)}{dt}j = (2t + 1)i + 3j.$

(b) Furthermore, recall that acceleration is the change in velocity (Δv) per change in time (Δt), that is, acceleration $= \frac{\Delta v}{\Delta t}$.

Acceleration at any time, $\vec{a} = \frac{d\vec{v}}{dt}$

$= \frac{d((2t + 1)i + 3j)}{dt} = \frac{d(2t + 1)}{dt}i = \frac{d(3)}{dt}j$

$= 2i + 0j = 2i.$

(c) Note that speed is a scalar quantity which is the magnitude of vector $|\vec{v}|$.

$speed = |\vec{v}| = |(2t + 1)i + 3j)|;$

$|\vec{v}| = |(2t + 1)i + 3j)|;$

$= \sqrt{(2t + 1)^2 + 3^2};$

at $t = 2$ $secs$; $|\vec{v}| = \sqrt{(2[2] + 1)^2 + 3^2} = \sqrt{5^2 + 3^2}$

$= \sqrt{34}.$

Therefore, the speed of the particle at time $t = 2$ $secs$ is $\sqrt{34}$ m/s.

11. A force of 15 N acts at an angle of 60° on a body of mass 3kg initially at rest on a smooth horizontal plane. Find its momentum after moving through a distance of 3 metres. **(WAEC)**

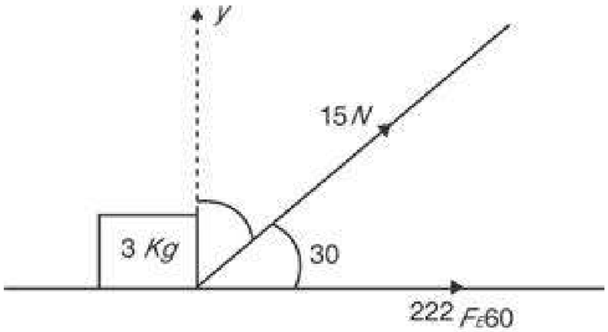

Fig. 9.13

Note that, it was not stated in the question, whether the force makes the angle 60° with the horizontal or vertical axis; directions, such as N60°W, are also not included. Therefore, the angle 60° being referred to, in the question, is measured clockwise from the y-axis (i.e. in the direction the hand of your wall clock moves).

The effective force, moving the body on the smooth horizontal plane, is the force F_E, acting parallel to the direction of movement of the body.

$$F_E = 15 \cos 30° = \frac{15\sqrt{3}}{2} = 12.99N.$$

This is because, from Figure 9.13, $\cos 30° = \frac{Adj}{Hyp} = \frac{F_E}{15}$; $\cos 30° = \frac{F_E}{15}$;
$F_E = 15\cos 30°$. *Also, note that a smooth plane has no friction, so the frictional force exerted on the body by the plane, is zero. To calculate the body's momentum after moving a distance of 3metres, we need to know its velocity after moving a distance of 3metres, then, multiply it by the mass of the body.*

$F_E = ma$; $a = \frac{F_E}{m} = \frac{12.99}{3} = 4.33ms^{-2}$. $a = 4.33ms^{-2}$, $s = 3m$, initial velocity $u = 0$ (*since the body started from rest*). We can find the velocity, after the body moved $3m$, by one of the equations of motion, given as $v^2 = u^2 + 2as$; $v^2 = 0^2 + 2(4.33)3$; $v^2 = 25.98$; $v = \sqrt{25.98} = 5.1ms^{-1}$.

Therefore, the velocity of the body after $3m$ is $5.1ms^{-1}$.

Recall that, momentum, $P = mv$, so

$$\begin{pmatrix} \textit{Momentum of the} \\ \textit{body after moving} \\ \textit{for 3m} \end{pmatrix} = \begin{pmatrix} \textit{mass of} \\ \textit{the body} \end{pmatrix} \times \begin{pmatrix} \textit{Velocity of} \\ \textit{the body after} \\ \textit{moving for 3m} \end{pmatrix}$$

$$\begin{pmatrix} \textit{Momentum of the} \\ \textit{body after moving} \\ \textit{for 3m} \end{pmatrix} = 3kg \times 5.1ms^{-1} = 15.3kgms^{-1}$$

12. A body moving with uniform acceleration covers $19m$ in the first second and $27m$ in the $3rd$ second. Find:

(a) the acceleration;

(b) the initial velocity of the body. **(WAEC)**

Workshop

(a) The equation of motion that shows the relationship between distance, s, acceleration, a, and initial velocity, u, is given by $s = ut + \frac{1}{2}at^2$. Let s_1 be the distance travelled by the body in the first second so that $s_1 = ut_1 + \frac{1}{2}at_1^2$. Note that uniform

acceleration means that acceleration is constant, then acceleration is the same throughout the time of the journey. Also, u is the same for all values of t, since the body had only one initial velocity, u.

In the first second, $t = t_1 = 1sec$; $s_1 = 19$

$= u(1) + \frac{1}{2}a(1^2) = u + \frac{1}{2}a$;

$u + \frac{a}{2} = 19$ .... (i)

The distance travelled in (*within*) the third second, is the difference between the total distance travelled for the first three seconds (3 secs) and the distance travelled for the first two seconds (2secs). That is, distance travelled in the third second $= s_3 - s_2$. From the question, the distance travelled in (*within*) the third second is $27m$. Hence, $27 = s_3 - s_2 = ut_3 + \frac{1}{2}a(t_3)^2 - \left(ut_2 + \frac{1}{2}a(t_2)^2\right)$.

Recall that a is constant, so, $a_1 = a_2 = a_3 = a$, also, u is constant so that, $u_1 = u_2 = u_3 = u$.

The question explained that acceleration is constant, which means that acceleration is the same at any time t. In the third second, t = 3, while in the 2nd second, t = 2, so that

$27 = u(3) + \frac{1}{2}a(3^2) - \left((u(2) + \frac{1}{2}a(2^2)\right);$

$27 = 3u - 2u + \frac{9}{2}a - \frac{4}{2}a;$

$u + \frac{5a}{2} = 27$ .... *(ii)*

subtracting *(ii)* from *(i)* we get $u + \frac{a}{2} -$

$\left(u + \frac{5a}{2}\right) = 19 - 27; u + \frac{a}{2} - u - \frac{5a}{2} = -8;$

$\frac{a}{2} - \frac{5a}{2} = -8; -2a = -8; a = 4ms^{-2}.$

(i) From equation *(ii)* $u = 27$

$-\frac{5}{2}a = 27 - \frac{5}{2}(4) = 27 - 10 = 17ms^{-1}.$

Therefore, the acceleration and initial velocity of the body are $4ms^{-2}$ and $17ms^{-1}$, respectively.

13. A body of mass $8kg$ placed on a smooth plane inclined at angle 30° to the horizontal, is acted on by an upward force of magnitude $50N$ at angle 45° to the line of greatest slope. Find the:
    (a) acceleration of the body;
    (b) magnitude of the normal reaction.
    [Take g = $10ms–2$] **(WAEC)**

Workshop

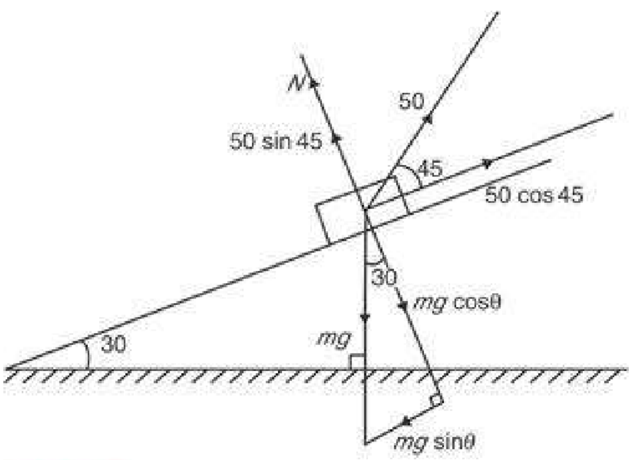

Fig. 9.14

Note from Figure 9.14 that the line that has the greatest slope is the line showing the inclined plane. From the question, the 50N ***upward*** *force makes an angle 45° with the line of greatest slope, which is the inclined plane. But the resolution of the 50N force, parallel to the plane is 50 cos 45°, then, since the 50 cos 45° force is parallel to the plane, the 50N force will also make angle 45° with the line of action of the 50 cos 45° force as shown in the diagram.*

N is the normal reaction to the weight of the body (action).

(a) Forces are vectors, which are represented by magnitude and direction, so, the *mg* sin θ force can be redrawn, having the same magnitude and direction as in Figure 1.14.

Note as the plane is smooth, the frictional force due to the plane is zero.

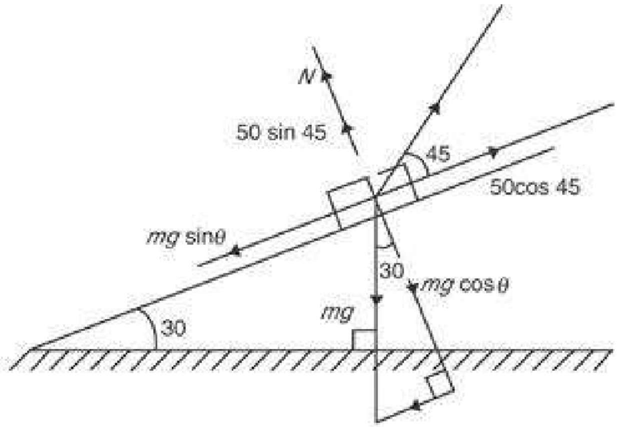

Fig. 9.15

In Figure 9.15, the effective forces acting on the body, parallel to the inclined plane, are *mg* sin θ and 50 cos 45° forces, where θ = 30°. Before the 8*kg* body can accelerate **up** the inclined plane, the 50 cos 45° force, pulling the body up the plane, must overcome the *mg* sin θ force, acting in the opposite direction; the remaining force from the 50 cos 45° force ($F = ma$) will accelerate the body into motion with an acceleration, *a*, up the plane.

Thus, for the body to accelerate up the plane, $50 \cos 45° = mg \sin \theta + ma$;

$50 \cos 45° = (8 \times 10 \sin 30°) + (8 \times a)$;

$50 \times \frac{1}{\sqrt{2}} = 80 \times \frac{1}{2} + (8 \times a)$;

$\frac{50}{\sqrt{2}} = 40 + 8a$; $\frac{50}{\sqrt{2}} - 40 = 8a$;

$35.36 - 40 = 8a$; $8a = -4.64$;

$a = \frac{-4.64}{8} = -0.58ms^{-2}$. The negative sign of the acceleration of the body shows that the body is accelerating down the plane, instead of up the plane as we initially thought (*recall that acceleration is a vector*). This means that $mg \sin 30° > 50 \cos 45°$.

Therefore, the acceleration of the body is $0.58ms^{-2}$ **down the plane**.

(b) From Figure 9.15, for the body to stay on the plane and not move vertically upward (that is against gravity). The sum of the upward forces acting perpendicularly to the inclined plane **must** be **equal** to the sum of the downward forces acting perpendicular to the inclined plane. That is;

$$\left(\begin{array}{l}\textit{The sum of the}\\ \textit{upward forces, acting}\\ \textit{perpendicular to the}\\ \textit{inclined plane}\end{array}\right) = \left(\begin{array}{l}\textit{Sum of the downward}\\ \textit{forces, acting}\\ \textit{perpendicular to the}\\ \textit{inclined plane}\end{array}\right)$$

So, from the Figure 9.15, $N + 50 \sin45° = mg \cos\theta$; $N = mg \cos\theta - 50 \sin45°$;

$$N = (8 \times 10 \cos30°) - \left(50 \times \frac{1}{\sqrt{2}}\right);$$

$$\left(80 \times \frac{\sqrt{3}}{2}\right) - \frac{50}{\sqrt{2}} = 69.28 - 35.36 = 33.92N$$

Therefore the magnitude of the normal reaction, N is $33.92N$.

14. A non-uniform beam *XY* of mass 40 kg and length 8*m* rests horizontally on two supports *A* and *B* (with its centre of gravity at a point 3.5*m* away from *X*). Objects of masses 80 kg and 50 kg are suspended at *X* and *Y* respectively. Two other of masses 70 kg and 30 kg are suspended at distance 3*m* and 5.5*m* from *Y*. If *XA* = 2*m* and *BY* = 1.5*m*, find the reactions at *A* and *B*. [Take *g* = 10*ms*−2] **(WAEC)**

Workshop

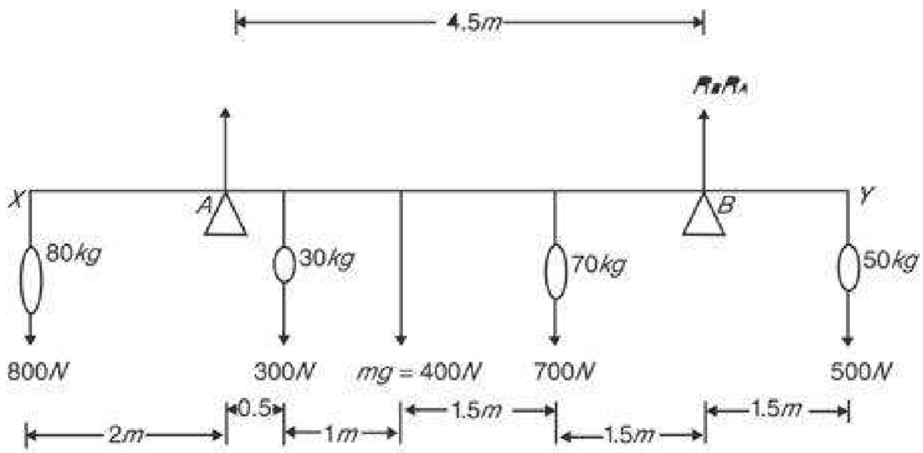

Fig. 9.16

Recall that weight *W* is a force. Therefore, *W* = *ma*, where *a* in this case is acceleration due to gravity, *g*. Thus, *W* = *mg*, so the 40 *kg* mass will exert a force (its weight) of *W* = 40 × *g* = 40 × 10 = 400*N*, as shown in Figure 9.16.

The weights of other masses are also as shown in Figure 9.16. At equilibrium, the sum of the clockwise moment about a point is equal to the sum of the anticlockwise moment about that same point.

Recall that:

$$\text{moment about any point } A = (Force) \times \begin{pmatrix} \text{perpendicular} \\ \text{distance between the} \\ \text{line of action the} \\ \text{force and point } A \end{pmatrix}$$

Let us take moment about point A.

To show how moment can be taken about point A, try to balance your long ruler (meter rule) horizontally on one finger. If weights are hung on both ends of the ruler, so that the ruler is ***still balanced*** *(that is, the ruler does not tilt to any side), then the ruler is at equilibrium. Hence, moment could be taken about your finger (pivot), on which, the ruler and the weights are balanced. Also note that having balanced the ruler on your finger, any weight hung on either side of the ruler will cause the ruler to either move clockwisely, and this will give a clockwise moment about your finger. On the other hand, if the weight causes the ruler to move in the anticlock-wisely, then, this will give an anticlockwise moment about your finger.*

Hence, the clockwise moments about point A are 300×0.5, 400×1.5, 700×3 and 500×6. And the anticlockwise moments about point A are $RB \times 4.5$ and 800×2. Because the system is in equilibrium, then,

$$\begin{pmatrix}\text{The sum of clock-}\\ \text{wise moments}\\ \text{about a point}\end{pmatrix} = \begin{pmatrix}\text{The sum of anticlock-}\\ \text{wise moments about}\\ \text{the same point}\end{pmatrix}$$

Therefore, about point A,

$$(300 \times 0.5 + 400 \times 1.5 + 700 \times 3 + 500 \times 6) = (R_B \times 4.5 + 800 \times 2).$$

To understand this, just assume that A is your finger, on which all these forces are balanced, then you will know which moment is **clockwise** and which, is **anticlockwise**. $\therefore$ $600 + 150 + 2\,100 + 3\,000 = 4.5R_B + 1\,600$; $5\,850 - 1\,600 = 4.5R_B$;

$$4.5R_B = 4\,250; \quad R_B = \frac{4\,250}{4.5} = 944.4\ N.$$

The moment of R_A about point A, is zero because the perpendicular distance between the R_A and point A is zero, hence the moment of R_A about point $A = R_A \times 0 = 0$.

Also, to know R_A, we can take moments about point B.

The clockwise moments, about point B, $R_A \times 4.5$ and 500×1.5, while the anticlockwise moments, about point B, are 800×6.5, 300×4, 400×3 and 700×1.5. Because the system is in equilibrium, then,

$$\begin{pmatrix}\text{The sum of clock-}\\ \text{wise moments}\\ \text{about a point}\end{pmatrix} = \begin{pmatrix}\text{The sum of anticlock-}\\ \text{wise moments about}\\ \text{the same point}\end{pmatrix},$$

so that, about point *B*,

($RA \times 4.5 + 500 \times 1.5) = (800 \times 6.5 + 300 \times 4 + 400 \times 3 + 700 \times 1.5$)

$4.5\ RA + 750 = 5\ 200 + 1\ 200 + 1\ 050$;

$4.5\ RA + 750 = 8650$;

$4.5\ R_A = 8\ 650 - 7\ 900;\ R_A = \frac{7\ 900}{4.5} = 1\ 755.6N$

The moment of R_B about point B, is zero because the perpendicular distance between force R_B and point B is zero, hence, the moment of R_B, about point $B = R_B \times 0 = 0$.

Alternative method

Having calculated for R_B as we did above, if the system is in equilibrium,

$$\begin{pmatrix} \textit{sum of upword} \\ \textbf{vertical}\ \textit{forces} \end{pmatrix} = \begin{pmatrix} \textit{sum of downward} \\ \textbf{vertical}\ \textit{forces} \end{pmatrix}.$$

Hence, $R_A + R_B = (800 + 300 + 400 + 700 + 500)$

$N = 2\ 700N$.

Carefully note that this formula is strictly for forces, and ***not*** *moments. Also, know that the forces at equilibrium must be acting at* ***right angle*** *to the horizontal beam. When the beam is at equilibrium, it will not move, since the sum of forces pulling the beam upward is equal to the sum of forces pulling it downward.*

$R_A + R_B = R_A + 944.4 = 2\ 700N$;

$R_A = 2\ 700 - 944.4 = 1755.6N$.

15. The distance, s metres travelled by a car in time t seconds, from the moment the brakes were applied is given by $s = 10\left(t - \frac{t^3}{48}\right)$. Find the:

    (a) time taken for the car to stop;

    (b) distance covered during the period;

    (c) retardation of the car from the moment the brakes were applied.

(WAEC)

(a) The distance covered from the time the brakes were applied until the car stopped, is the **farthest** (*maximum*) distance travelled after the brakes were applied. It will therefore, be the distance s read at the maximum point on the Distance-time curve of $s = 10\left(t - \frac{t^3}{48}\right)$. At this maximum distance the car came to rest, thus, the final velocity, v is zero, i.e $v = 0$. Instantaneous velocity is the velocity at any time t, where

t can be any time in seconds (*t could be 1second, 5seconds, zero second, e.t.c*).

Recall that *instantaneous velocity* $= \frac{ds}{dt}$, so that, when the car came to rest, $v = \frac{ds}{dt} = 0$. Hence, at maximum point on the curve, $\frac{ds}{dt} = 0$; therefore,

$$\frac{ds}{dt} = \frac{d\left(10\left(t - \frac{t^3}{48}\right)\right)}{dt} = \frac{d\left(10t - \frac{10t^3}{48}\right)}{dt}$$

$$= 10 - \frac{10(3t^2)}{48} = 0 \quad 10 - \frac{30t^2}{48} = 0;$$

$\frac{30t^2}{48} = 10$; $30t^2 = 48 \times 10 = 480$;

$t^2 = \frac{480}{30} = 16$; $t = \pm\sqrt{16} = \pm 4$.

Again, recall that at the maximum point on a curve $y = f(x)$, $\frac{d^2y}{dx^2} < 0$, i.e $\frac{d^2y}{dx^2}$ is negative, hence, at the maximum distance s on the curve, $s = 10\left(t - \frac{t^3}{48}\right)$, $\frac{d^2s}{dt^2} < 0$.

$\frac{ds}{dt} = 10 - \frac{30t^2}{48} = 0$;

$$\frac{d^2s}{dt^2} = \frac{d\left(10 - \frac{30t^2}{48}\right)}{dt} = -\frac{60}{48}t;$$

when $t = +4$, $\frac{d^2s}{dt^2} = -\frac{60}{48}t = -\frac{60}{48}(+4)$

$$= -\frac{60}{12} = -5.$$

Because $\frac{d^2s}{dt^2}$ is negative $t = +4$, then, the point where $t = +4$ is a maximum turning

point. As the maximum turning point is the highest point on any cubic curve, then the time taken for the car to stop (*time at maximum distance*) is 4*seconds*.

(b) The distance s covered during the period will be the distance travelled after the brakes were applied until the car came to rest, which is the maximum distance travelled after the brakes had been applied. Recall that the maximum distance travelled after the brakes were applied is the distance at $t = +4$ (*since time at maximum distance is t* = 4*secs*). Therefore, at maximum distance, $t = +4$;

$$s_{max} = 10\left(t - \frac{t^3}{48}\right) = 10\left(4 - \frac{4^3}{48}\right) = 10\left(4 - \frac{64}{48}\right)$$
$$= 10\left(4 - \frac{4}{3}\right) = 10 \times \frac{8}{3} = 26.67m$$

(c) Retardation simply means deceleration, i.e the decrease in velocity of a body with time (*recall that acceleration is the* ***increase*** *in velocity of a body with time*). Thus, retardation is negative acceleration, so retardation can also be represented as a. Therefore, instantaneous retardation,

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}, \text{ since}$$
$$v = \frac{ds}{dt}. \begin{pmatrix} \textit{Retardation at any time from the} \\ \textit{moment the brakes were applied} \end{pmatrix}$$
$$= \left(\frac{d^2s}{dt^2}\right) = \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

Recall that, $\frac{ds}{dt} = 10 - \frac{30t^2}{48}$, therefore,

$$\frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d}{dt}\left(10 - \frac{30t^2}{48}\right) = 0 - \frac{60t}{48}$$
$$= -\frac{5t}{4}. \text{ Hence, } \begin{pmatrix} \textit{retardation} \\ \textit{at any time } t \end{pmatrix} = -\frac{5t}{4}.$$

Since the brakes were applied for 4*seconds* before the car came to rest, then, $t = +4$.

$$\begin{pmatrix} \textit{Retardation in the} \\ \textit{space of 4 seconds} \end{pmatrix} = -\frac{5t}{4} = -\frac{5(4)}{4} = -\frac{20}{4}$$
$$= -5ms^{-2}.$$

Note that the negative sign shows acceleration in a direction opposite to the direction of initial motion of the car (retardation). That is the car decelerated to rest.

16. A body of mass 15*kg* is placed on a smooth plane inclined at angle 30° to the horizontal. If it slides down the plane, calculate:

(a) its acceleration;

(b) the force required to prevent it from sliding. [Take g = 10ms−2] **(WAEC)**

Workshop

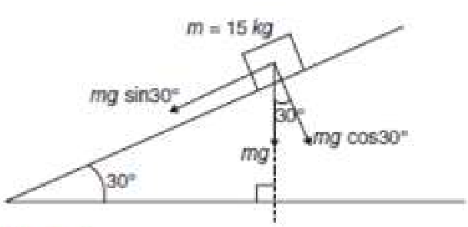

Fig. 9.17

(a) Since the inclined plane on Figure 9.17 is smooth, **no** frictional force, due to the fact that the plane is acting on the body. The effective forces moving the body on the inclined plane will be the forces acting parallel to the plane. The only force acting parallel to the plane is mg sinθ and this is the force that will cause the body to slide down the plane, due to gravity. The force F, with which the body will slide is expressed as follows: $F = mg \sin\theta$; recall that $F = ma$, thus,

$$F = mg\sin\theta = ma;\ a = \frac{mg\sin\theta}{m} = g\sin\theta;$$

$$a = g\sin\theta = 10 \times \sin 30° = 10 \times \frac{1}{2} = 5\ ms^{-2}.$$

Therefore, the acceleration of the body down the plane, is 5 ms^{-2}.

(b) The force required to prevent the body from sliding will **just be equal** to the force causing the body to slide down the plane, so that the body does **not** move up the plane or down the plane. The force required to prevent the body from sliding = *mg* sin θ, but it will act opposite to the downward direction (i.e up the plane). The force *Fup*, that will prevent it from sliding, will be:

$$F_{up} = mg\sin\theta = 15 \times 10 \times \sin 30 = 15 \times 10 \times \frac{1}{2} = 75N \text{ acting up the plane.}$$

Note that writing 75N alone is incomplete; as force is a vector, you need to write the magnitude and direction of this force so as to differentiate it from the force that will cause the body to slide down the plane as they are of the same magnitude.

17. Two forces of magnitudes 12*N* and 8*N* inclined at angle 60° to each other, act on a body of mass 4*kg* which was initially at rest. Calculate the acceleration of the body. **(WAEC)**

Workshop

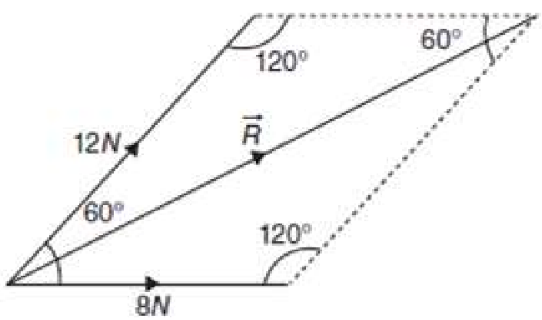

Fig. 9.18(a)

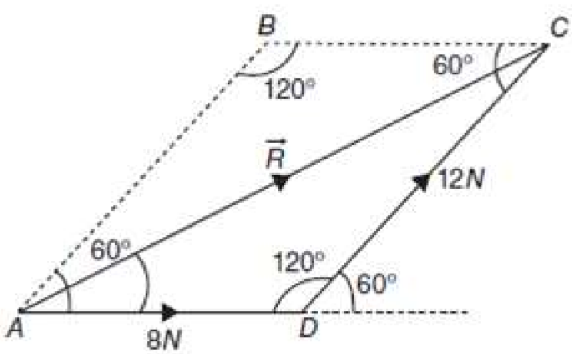

Fig. 9.18(b)

Note that since forces are vectors, which are defined by their magnitude and direction, then the 12N can be represented on either side of the parallelogram. As long as its magnitude is 12N and its direction is 60°, measured from the horizontal, as shown in Figures 9.18(a) and 9.18(b).

Considering triangle ACD in Figure 9.18b, and applying the cosine rule,

$|\vec{R}|^2 = 8^2 + 12^2 - 2(8)(12)\cos 120° = 64 + 144 - 192\cos 120°$.

$\theta = 120°$ is in the second quadrant, and in this quadrant, $\cos\theta$ is negative, i.e.

$$\cos\theta = -\cos(180° - \theta);$$
$$\cos 120° = -\cos(180° - 120°) = -\cos 60° = -\frac{1}{2};$$
$$\cos 120° = -\frac{1}{2}, \text{ so, } |\vec{R}|^2 = 64 + 144 - 192\left(-\frac{1}{2}\right)$$
$$= 208 + 96 = 304;$$
$$\therefore |\vec{R}| = \sqrt{304} = 17.436N$$

Note that the effective force on the body, due to the two forces, is the resultant of the two forces.

Recall that, $F = ma$, then the magnitude of the effective force on the body will be $= 17.4N = ma$; $a = \frac{17.436}{m} = \frac{17.436}{4} = 4.36\ ms^{-2}$. Therefore, the acceleration of the body is $4.36\ ms^{-2}$, in the direction of the resultant force since a body will accelerate in the direction of the force causing it to accelerate.

It is necessary you indicate the direction of acceleration of the body, just as mentioned above, since acceleration is a vector quantity.

18. A particle, moving in a straight line with a uniform acceleration, has velocities 20 ms^{-1}, 40 ms^{-1} and 80 ms^{-1} at points A, B and C respectively. If | BC | = 100m find:
    (a) the acceleration of the particle;
    (b) | AB |
    (c) the time taken to cover AC. **(WAEC)**

Workshop

(a) $V_A = 20ms^{-1}$; $V_B = 40ms^{-1}$; $V_C = 80ms^{-1}$;
$|BC| = 100m$.

Distance $|BC| = 100m$; distance BC (i.e S_{BC}) means that the particle moved from B to C, then the initial velocity $u = velocity\ at\ B = v_B = 40ms^{-1}$, and the final velocity

will be velocity $v = velocity\ at\ C = v_C = 80ms^{-1}$. To calculate the acceleration between B and C, recall one of the equations of motion

$$v^2 = u^2 + 2aS;\ a = \frac{v^2 - u^2}{2S} = \frac{80^2 - 40^2}{2(100)}$$
$$= \frac{6400 - 1600}{200} = 24ms^{-2}.$$

From the question, acceleration of the particle is uniform (constant). Therefore, the acceleration of the particle from B to C will be its acceleration throughout the journey, which is $24ms^{-2}$.

(b) Recall that $v^2 = u^2 + 2aS$, then, distance $AB = |AB| = S_{AB} = \frac{v^2 - u^2}{2a}$.

Where, for journey AB, initial velocity will be the particle's velocity at A, while the final velocity will be the particle's velocity at B, so that $u = v_A = 20ms^{-1}$; $v = v_B = 40ms^{-1}$; $a = 24ms^{-2}$ *(since acceleration is uniform, it is going to be constant throughout the journey)*.

$$S_{AB} = \frac{40^2 - 20^2}{2(24)} = \frac{1600 - 400}{48} = 25m.$$

Therefore, $|AB| = 25m$.

(c) Since the body is moving in a straight line, from A to C, and its acceleration is constant from A to C, then time taken to cover AC can be calculated from the equation of motion; $v = u + at$; $at = v - u$; $t = \frac{v-u}{a}$, in this case, initial velocity will be the particle's velocity at A, while the final velocity will be the particle's velocity at C, thus, $u = v_A = 20ms^{-1}$, $v = v_C = 80ms^{-1}$, $a = 24ms^{-2}$;

$$\therefore t = \frac{80 - 20}{24} = \frac{60}{24} = 2.5\ seconds.$$

*Always ensure that the units of the quantities are uniform; for example, if velocity is in ms^{-1}, acceleration in ms^{-2}, then time should be in seconds, and **not** hours.*

19. An object of mass 10 *kg*, initially at rest, on a smooth horizontal surface, is acted upon by a horizontal force of 20*N*. After moving for 4 *metres*, the object collides with another object of mass 5*kg* initially at rest. After collision, the two objects continue to move in the same direction with the lighter object having speed of 2*ms*−1. What is the speed of the other object? **(WAEC)**

$mA = 10kg$, $F = 20N$. The surface is smooth, so friction is **zero**. Since friction is zero, the force, F, acting on body A, is given by

$$F = ma;\ a = \frac{F}{m} = \frac{20}{10} = 2ms^{-2}.$$

Note that the 20N, horizontal force is the effective force on the body as the body is moving on a horizontal surface. If the question had said the force was acting at an angle, then the effective force on the body would have been the horizontal component of the force, since the body is moving horizontally.

Distance travelled by body A before collision = $4m$; so, the velocity before collision can be calculated from one of the equations of motion, given as $v^2 = u_2 + 2as$. Let the velocity of body A, **just** before collision, be $v1$. Because body A started from rest $u = 0$, thus, $v1$

$2 = 0^2 + 2as$; $v_1{}^2 = 2as = 2 \times 2 \times 4; = 16 \therefore v1 = \sqrt{16} = 4ms^{-1}$,

so the velocity of body A, just before it collided with the body at rest (body B), is $v1 = 4ms{-}1$. Let the final velocity of body A, **after** collision, be $v2$.

The velocity of body B, before collision, will be $w_1 = 0$, as the body was initially at rest, and $mB = 5kg$. The final velocity of body B (*lighter object*), after collision, is $w2 = 2ms{-}1$. By the law of conservation of momentum which states thus:

$$\begin{pmatrix}\textit{Total momentum}\\ \textit{before collision}\end{pmatrix} = \begin{pmatrix}\textit{Total momentum}\\ \textit{after collision}\end{pmatrix},$$

and recall that momentum, $P = mv$; therefore,

$m_A v_1 + m_B w_1 = m_A v_2 + m_B w_2$;

$(10 \times 4) + (5 \times 0) = (10 \times v_2) + (5 \times 2)$; $40 = 10v_2 + 10$; $10v_2 = 30$; $v_2 = \frac{30}{10} = 3ms^{-1}$.

Therefore, the speed of the heavier object (body A), after collision, $3ms^{-1}$.

20.

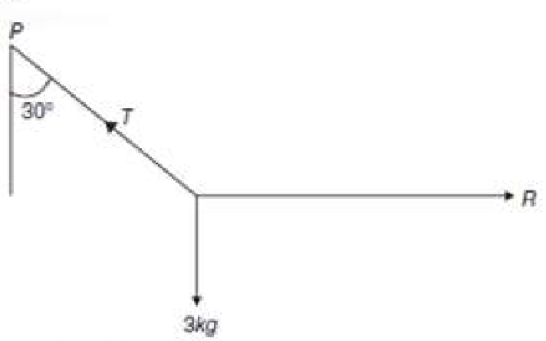

Fig. 9.19

In the diagram, an object of mass 3 kg hanging from a light inextensible string is pulled aside by a horizontal force R. If the string is inclined at 30° to the vertical and the system is in equilibrium, calculate, in Newtons,

(a) the tension *T* in the string;

(b) the force R. [Take $g = 10ms{-}2$] **(WAEC)**

Workshop

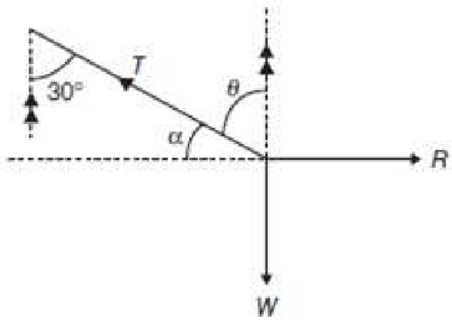

Fig. 9.20

Note that weight W will always act vertically downwards. So, the angle between W and horizontal force, R, will be 90°.

From Figure 9.20, $\theta = 30°$ (alternate angles are equal);

$\alpha + \theta = 90$; $\alpha = 90° - \theta = 90° - 30° = 60°$.

At equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, where ΣF_x is the sum of the horizontal forces, and ΣF_y is the sum of the vertical forces.

If Ali and Charles are to be dragging a shoe between each other, in opposite directions, so that the shoe does not move to either side, it means the shoe is at rest (not moving), though Ali is trying to pull the shoe to himself, while Charles is also trying to do the same. This means the force that Ali is applying is equal to the force that Charles is also Applying. Thus, the shoe is said to be in equilibrium under the forces exerted by Ali and Charles. Since forces are vectors, let us take the force in Ali's direction to be +F, then, the force opposite Ali (in Charlese's direction) will be −F. Hence, ΣF = +F + (−F) = +F − F = 0. Therefore, at equilibrium, ΣF = 0.

The component of tension, *T*, on the vertical axis, is $T \sin \alpha$, while its component on the horizontal axis is $T \cos \alpha$. From figure 9.20, $\Sigma Fx = R \cos 0° - T \cos \alpha = 0$, where α and 0 (*zero*) are the angles that *T* and *R* make with the horizontal axis, respectively.

The reason for the negative sign in front is that, the resolution of force T, on the x–axis, is pointing in the direction of the negative x-axis. Also, note that R makes angle 0° with the horizontal (x-axis).

$R \cos 0° = +T \cos \alpha = + T \cos 60°$;

$R(1) = T \cos 60°$; $R = T \cos 60°$ ... (*);

$\Sigma F_y = T \sin \alpha - W \sin 90° = 0$.

Also, note that force $W \sin 90°$ bears a negative sign, because it is pointing in the negative y direction.

$T \sin \alpha = W \sin 90° = W \times 1 = W$;

$T = \sin \alpha\, W = 3kg \times 10ms^{-2} = 30N$;

$T = \frac{30}{\sin \alpha} = \frac{30}{\sin 60°}$; $T = 30 \div \frac{\sqrt{3}}{2} = 30 \times \frac{2}{\sqrt{3}} = \frac{60}{\sqrt{3}}$.

We can rationalize $\frac{60}{\sqrt{3}}$ to get

$$\frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}.$$

Hence, $T = 20\sqrt{3}N$. Recall from equation (*) that $R = T \cos 60°$; put $T = 20\sqrt{3}$ into equation (*) to get $R = 20\sqrt{3} \cos 60° = 20\sqrt{3} \times \frac{1}{2} = 10\sqrt{3}$.

Therefore, $R = 10\sqrt{3}N$.

(i) The tension T in the string is $20\sqrt{3}N$, while

(ii) The force R is $10\sqrt{3}N$.