CHAPTER 1

At the end of the chapter, students should be able to:

1. Convert numbers from other bases to base 10.
2. Convert decimal fractions from other bases to base 10.
3. Convert from one base to another base.
4. Perform basic operations on number bases.
5. Apply number base system to computer programming.

I. Conversion of Bases

(i) Conversion from one base to base 10

Recall that a binary number has only two digits, namely 0 and 1. In junior classes, we learnt how to convert base 2 to base 10. In this chapter, we shall learn how to convert other bases to base 10.

In the junior classes, we studied only up to 10 single digits. The digits above 10 are usually represented using letters. A is used to represent 10, B for 11, C for 12, D for 13, E for 14, F for 15, etc. as shown in Table 1.1.

Table 1.1

Base	Digit numbers
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4
6	0, 1, 2, 3, 4, 5
7	0, 1, 2, 3, 4, 5, 6
8	0, 1, 2, 3, 4, 5, 6, 7
9	0, 1, 2, 3, 4, 5, 6, 7, 8
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A

Base	Digit numbers
12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
13	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C
14	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D
15	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E
16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

To change from one base to base 10, we express the digit numbers as the sum of the powers of the base as we have in the following examples:

WORKED EXAMPLE

Worked Example 1

Convert the following numbers to base 10:

(a) 132_4 (b) $8A4C_{13}$ (c) $7D5F_{16}$

Solution

(a) $132_4 = 1 \times 4^2 + 3 \times 4^1 + 2 \times 4^0$
$= 1 \times 16 + 3 \times 4 + 2 \times 1$
$= 16 + 12 + 2$
$= 30_{10}$

(b) $8A4C_{13} = 8 \times 13^3 + A \times 13^2 + 4 \times 13^1 + C \times 13^0$
$= 8 \times 13^3 + 10 \times 13^2 + 4 \times 13^1 + 12 \times 13^0$
$= 8 \times 2\,197 + 10 \times 169 + 4 \times 13 + 12 \times 1$
$= 17\,576 + 1\,690 + 52 + 12$
$= 19\,330_{10}$

(c) $7D5F_{16} = 7 \times 16^3 + D \times 16^2 + 5 \times 16^1 + F \times 16^0$
$= 7 \times 16^3 + 13 \times 16^2 + 5 \times 16^1 + 15 \times 16^0$

$= 7 \times 4\,096 + 13 \times 256 + 5 \times 16 + 15 \times 1$
$= 28\,672 + 3\,328 + 80 + 15$
$= 32\,095_{10}$

Worked Example 2

(a) Find x, if $101_x = 17_{10}$.
(b) Find n given that $231_n = 6_{10}$.

Solution

(a) $101_x = 17_{10}$

$1 \times x^2 + 0 \times x^1 + 1 \times x^0 = 17$

$x^2 + 1 = 17$

$x^2 + 1 - 1 = 17 - 1$

$x^2 = 16$

$x = \pm\sqrt{16}$

$\therefore x = \pm 4$

But $x > 0$

$\therefore x = 4$

(b) $231_n = 6_{10}$

$2 \times n^2 + 3 \times n^1 + 1 \times n^0 = 6$

$2n^2 + 3n + 1 - 6 = 0$

$2n^2 + 3n - 5 = 0$

$2n^2 + 5n - 2n - 5 = 0$

$2n^2 - 2n + 5n - 5 = 0$

$2n(n - 1) + 5(n - 1) = 0$

$(2n + 5)(n - 1) = 0$

$2n + 5 = 0$ or $n - 1 = 0$

$2n = -5$ or $n = 1$

$n = \frac{-5}{2}$ or $n = 1$

But $n > 0$

$\therefore n = 1$

Worked Example 3

Express $5BAB2_{12}$ in base 10.

Solution

$$\begin{aligned}5BAB2_{12} &= 5 \times 12^4 + B \times 12^3 + A \times 12^2 \\ &\quad + B \times 12^1 + 2 \times 12^0 \\ &= 5 \times 12^4 + 11 \times 12^3 + 10 \times 12^2 \\ &\quad + 11 \times 12^1 + 2 \times 12^0 \\ &= 5 \times 20\,736 + 11 \times 1\,728 + 10 \\ &\quad \times 144 + 11 \times 12 + 2 \times 1 \\ &= 103\,680 + 19\,008 + 1\,440 \\ &\quad +132 + 2 \\ &= 124\,262_{10}\end{aligned}$$

(ii) Conversion from base 10 numbers to any base

Conversion of base 10 numbers to any other base is done by repeatedly dividing the base 10 number by the given base till we get zero as the dividend. Continuing the division, the result is obtained by writing all the remainders of the division from bottom to the top.

Worked Example 4

Convert the following base 10 numbers to the indicated bases:
(a) 6 534 to base 9
(b) 312 to base 5
(c) 4 498 to base 16

Solution

(a) Divide 6 534 repeatedly by 9 and keep recording the remainders on the right-hand side:

$$6\,534_{10} = \begin{array}{c|rl} 9 & 6\,534 & \\ \hline 9 & 726 & R\,0 \\ \hline 9 & 80 & R\,6 \\ \hline 9 & 8 & R\,8 \\ \hline & 0 & R\,8 \end{array} \uparrow$$

$$6\,534_{10} = 8860_9$$

(b) $312_{10} =$

$$
\begin{array}{c|rl}
5 & 312 & \\ \hline
5 & 62 & \text{R } 2 \\ \hline
5 & 12 & \text{R } 2 \\ \hline
5 & 2 & \text{R } 2 \\ \hline
 & 0 & \text{R } 2
\end{array}
$$

$312_{10} = 2222_5$

(c) $4\,498_{10} =$

$$
\begin{array}{c|rl}
16 & 4\,498 & \\ \hline
16 & 281 & \text{R } 2 \\ \hline
16 & 17 & \text{R } 9 \\ \hline
16 & 1 & \text{R } 1 \\ \hline
 & 0 & \text{R } 1
\end{array}
$$

$4\,498_{10} = 1192_{16}$

Exercise 1

Express the following numbers in base 10.

1. $56BAB_{13}$
2. 5677_8
3. $4A4C_{12}$
4. 7168_9
5. 6424_8
6. 5442_6
7. $E1B_{16}$
8. 54432_6
9. $B71_{16}$
10. $94AC_{13}$
11. $DF65_{16}$
12. 2684_9
13. Find n given that $145_n = 65_{10}$.
14. Find x, if $266_x = 26_{10}$.
15. Find r, if $2r2_4 = 50_{10}$.

Express the following numbers in base 10:

16. 431_5

17. $4AB1B_{12}$

18. 45324_6

19. 1333_5

20. $17B_{16}$

Convert the following base 10 numbers to the indicated bases:

21. 786 to base 8

22. 322 to base 6

23. 586 to base 9

24. 988 to base 15

25. 6 978 to base 8

26. 6 987 to base 14

27. 892 to base 11

28. 635 to base 7

29. 9 874 to base 7

30. 8 726 to base 9

II. Conversion of Decimal Fractions in One Base to Base 10

Decimal fractions are numbers that consist of two parts, namely the *whole number part* and the *decimal part*. Examples include 17.53, 8.434, 154.165, etc. For instance, in 17.53, 17 is the whole part while 0.53 is the decimal fraction.

Worked Example 5

Convert the following numbers to base 10:
(a) 17.33_5 (b) 154.165_7 (c) $5ACB.4B_{13}$

Solution

(a)
$$\begin{aligned}
17.33_5 &= 1 \times 5^1 + 7 \times 5^0 + 3 \times 5^{-1} \\
&\quad + 3 \times 5^{-2} \\
&= 1 \times 5 + 7 \times 1 + 3 \times \frac{1}{5} \\
&\quad + 3 \times \frac{1}{5^2} \\
&= 5 + 7 + \frac{3}{5} + \frac{3}{25} \\
&= 12 + \frac{3}{5} + \frac{3}{25} \\
&= 12 + \frac{15 + 3}{25} \\
&= 12 + \frac{18}{25} \\
&= 12\frac{18}{25}_{10} \\
&= 12.72_{10}
\end{aligned}$$

$$\therefore 17.33_5 = 12.72_{10}$$

(b) $154.165_7 = 1 \times 7^2 + 5 \times 7^1 + 4 \times 7^0$

$+1 \times 7^{-1} + 6 \times 7^{-2} + 5 \times 7^{-3}$

$= 1 \times 49 + 5 \times 7 + 4 \times 1 + 1$

$\times \frac{1}{7} + 6 \times \frac{1}{49} + 5 \times \frac{1}{343}$

$= 49 + 35 + 4 + \frac{1}{7} + \frac{6}{49} + \frac{5}{343}$

$= 88 + \frac{1}{7} + \frac{6}{49} + \frac{5}{343}$

$= 88 + \frac{49 + 42 + 5}{343}$

$= 88 + \frac{96}{343}$

$= 88 + \frac{96}{343_{10}}$ or 88.2799_{10}

$\therefore 88.2799_{10} = 154.165_7$

(c) $5ACB.4B_{13} = 5 \times 13^3 + A \times 13^2 + C \times 13^1 + B \times 13^0 + 4 \times 13^{-1} + B \times 13^{-2}$

$= 5 \times 2\,197 + A \times 169 + C \times 13 + B \times 1 + 4 \times \frac{1}{13} + B \times \frac{1}{169}$

$= 10\,985 + 10 \times 169 + 12 \times 13 + 11 \times 1 + \frac{4}{13} + 11 \times \frac{1}{169}$

$= 10\,985 + 1\,690 + 156 + 11 + \frac{4}{13} + \frac{11}{169}$

$= 12\,842 + \frac{4}{13} + \frac{11}{169}$

$= 12\,842 + \frac{52 + 11}{169}$

$= 12\,842 + \left(12\,842 + \frac{63}{169}\right)_{10}$

$= 12\,842\,\frac{63}{169_{10}}$ or

$12\,842.373_{10}$

$\therefore 5ACB.4B_{13} = 12\,842.373_{10}$

Exercise 2

Express the following numbers in base 10:

1. 431.3_5
2. 5234.25_6
3. $4ABIB.43_{12}$
4. 45324.22_6
5. 1333.24_5
6. 765.73_8
7. $17B.24_{16}$

8. $ABC6.BA_{13}$

9. $469.BAC_{15}$

10. $489.AAB_{14}$

11. $14.ABA_{16}$

12. $1546.CAC_{16}$

13. 846.14_{9}

14. $74CA.BA_{16}$

15. $CAC7.5A_{14}$

16. $7AAB.A4_{12}$

17. $129.A2_{11}$

18. 4567.22_{8}

19. $CAC24.3B_{13}$

20. $79.DACB_{16}$

III. Conversion of Numbers from One Base to Another Base

To convert any given number from one base to another base, first convert the given number base to base 10, and then divide the result by the required base.

Worked Example 6

Convert the following:
(a) 532_6 to base 9
(b) 4232_9 to base 12

Solution

(a) $$532_6 = 5 \times 6^2 + 3 \times 6^1 + 2 \times 6^0$$
$$= 5 \times 36 + 3 \times 6 + 2 \times 1$$
$$= 180 + 18 + 2$$
$$= 200_{10}$$

Now, convert 200_{10} to a number in base 9 by dividing with 9 and record the remainders.

$$200_{10} = \begin{array}{c|r l} 9 & 200 & \\ \hline 9 & 22 & R\,2 \\ \hline 9 & 2 & R\,4 \\ \hline & 0 & R\,2 \end{array}$$

$$200_{10} = 242_9$$

(b) $$4233_9 = 4 \times 9^3 + 2 \times 9^2 + 3 \times 9^1 + 2 \times 9^0$$
$$= 4 \times 729 + 2 \times 81 + 3 \times 9 + 2 \times 1$$
$$= 2\,916 + 162 + 27 + 2$$
$$= 3\,107_{10}$$

Now to convert $3\,107_{10}$ to a number in base 12, we divide by 12.

$$\begin{array}{c|r l} 12 & 3\,107 & \\ \hline 12 & 258 & R\,11 = B \\ \hline 12 & 21 & R\,6 \\ \hline 12 & 1 & R\,9 \\ \hline & 0 & R\,1 \end{array}$$

$$\therefore 4232_9 = 196B_{12}$$

Worked Example 7

Convert the following:
(a) $6AB4_{12}$ to base 14
(b) $1BC2_{13}$ to base 16

Solution

(a) $$\begin{aligned} 6AB4_{12} &= 6 \times 12^3 + A \times 12^2 + B \\ &\quad \times 12^1 + 4 \times 12^0 \\ &= 6 \times 1\,728 + A \times 144 + B \\ &\quad \times 12 + 4 \times 1 \\ &= 6 \times 1\,728 + 10 \times 144 + 11 \\ &\quad \times 12 + 4 \times 1 \\ &= 10\,368 + 1\,440 + 132 + 4 \\ &= 11\,944_{10} \end{aligned}$$

Now to convert $11\,944_{10}$ to a number in base 14, we divide by 14.

14	11 944	
14	853	R 2
14	60	R 13 = D
14	4	R 4
	0	R 4

$$\therefore 6AB4_{12} = 44D2_{14}$$

(b) $$IBC2_{13} = 1 \times 13^3 + B \times 13^2 + C \times 13^1 + 2 \times 13^0$$

$$= 1 \times 2\,197 + B \times 169 + C \times 13 + 2 \times 1$$

$$= 1 \times 2\,197 + 11 \times 169 + 12 \times 13 + 2 \times 1$$

$$= 2\,197 + 1\,859 + 156 + 2$$

$$= 4\,214_{10}$$

Now to convert 4 214 to a number in base 16, we divide by 16.

16	4 214	
16	263	R 6
16	16	R 7
16	1	R 0
	0	R 1

$$1BC2_{13} = 1\,076_{16}$$

Worked Example 8

(a) Which is the largest of the two numbers, 17474_8 or $1E4A_{16}$?
(b) Which is the largest, 413 in base 7 or 243 in base 9?

Solution

(a) $$\begin{aligned} 17474_8 &= 1 \times 8^4 + 7 \times 8^3 + 4 \times 8^2 \\ &\quad + 7 \times 8^1 + 4 \times 8^0 \\ &= 1 \times 4\,096 + 7 \times 512 + 4 \\ &\quad \times 64 + 7 \times 8 + 4 \times 1 \\ &= 4\,096 + 3\,584 + 256 \\ &\quad + 56 + 4 \\ &= 7\,996_{10} \end{aligned}$$

$$\begin{aligned} 1E4A_{16} &= 1 \times 16^3 + E \times 16^2 + 4 \\ &\quad \times 16^1 + A \times 16^0 \\ &= 1 \times 16^3 + 14 \times 16^2 + 4 \\ &\quad \times 16^1 + 10 \times 16^0 \\ &= 1 \times 4\,096 + 14 \times 256 + 4 \\ &\quad \times 16 + 10 \times 1 \\ &= 4\,096 + 3\,584 + 64 + 10 \\ &= 7\,754_{10} \end{aligned}$$

Since 7 996 is bigger than 7 754, therefore, 17474_8 is bigger than $1E4A_{16}$.

(b) $$\begin{aligned} 413_7 &= 4 \times 7^2 + 1 \times 7^1 + 3 \times 7^0 \\ &= 4 \times 49 + 1 \times 7 + 3 \times 1 \\ &= 196 + 7 + 3 \\ &= 206_{10} \end{aligned}$$

$$\begin{aligned} 243_9 &= 2 \times 9^2 + 4 \times 9^1 + 3 \times 9^0 \\ &= 2 \times 81 + 4 \times 9 + 3 \times 1 \\ &= 162 + 36 + 3 \\ &= 201_{10} \end{aligned}$$

Since 206 is greater than 201, therefore, 513_7 is bigger than 243_9.

Exercise 3

Convert:

1. $A52_{11}$ to base 6
2. 2012_4 to base 7
3. 135_{12} to base 6
4. 8422_{12} to base 4
5. 564_9 to base 16
6. 563_8 to base 6
7. EAE_{16} to base 9
8. 1243_{16} to base 7
9. $1CD4_{16}$ to base 12
10. $4BAC5_{15}$ to base 16
11. $A178_{12}$ to base 4
12. $16AB4_8$ to base 14
13. Which is the largest, $EA16_{16}$ or 4781_9?
14. State which is the largest of the two numbers, $ABC4_{13}$ or $4E4E_{16}$.
15. Express the following:
    (a) $4E6_{16}$ to base 5
    (b) $5E14_{16}$ to base 9

IV. Basic Operations on Number Bases

(i) Addition and subtraction of number bases

We perform operations of addition and subtraction on number bases just as we do in decimal arithmetic. In decimal arithmetic, we at times rename by carrying one whole number to the other number in order to make the addition or subtraction possible when addition or subtraction of number is not possible. In adding or subtracting involving number bases, we also take caution whenever we need to rename taken cognisance of the number base involved in the operation.

Worked Example 9

(a) Add 9642 and 1057 in hexadecimal system.
(b) Add 3452_7 and 1264_7.

Solution

(a)
$$\begin{array}{r} 9642_{16} \\ +1057_{16} \\ \hline A699_{16} \\ \hline \end{array}$$

Note: 2 + 7 = 9 not up to 16
4 + 5 = 9 not up to 16
0 + 6 = 6 not up to 16
9 + 1 = 10 = A not up to 16 but greater than 9.

(b)
$$\begin{array}{r} 3452_{7} \\ +1264_{7} \\ \hline 5046_{7} \\ \hline \end{array}$$

Note: 2 + 4 = 6
5 + 6 = 7 + 4 = 1R4 in base 7
4 + 2 + 1 = 7 + 0 = 1R0 in base 7
3 + 1 + 1 = 5 not up to 7

Worked Example 10

(a) Add $AB45_{12}$ and 9466_{12}.
(b) Add 8DF and 9C8 in hexadecimal system.

Solution

(a)
$$\begin{array}{r} AB45_{12} \\ +9466_{12} \\ \hline 183AB \\ \hline \end{array}$$

Note: 5 + 6 = 11 = B

$4 + 6 = 10 = A$
$B + 4 = 11 + 4 = 15 = 12 + 3 = 1R3$ in base 12.
$A + 9 + 1 = 20 = 1R8$ in base 12.

(b)
$$\begin{array}{r} 8DF_{16} \\ +9C8_{16} \\ \hline 12A7_{16} \\ \hline \end{array}$$

Note: $F + 8 = 15 + 8 = 23 = 16 + 7 = 1R7$
$D + C + 1 = 13 + 12 + 1 = 26 = 16$
$+ 10 = 1RA$
$8 + 9 + 1 = 18 = 16 + 2 = 1R2$

Worked Example 11

(a) Subtract 76CA from D57C in hexadecimal system.
(b) Subtract 377 from 476 in base 8.

Solution

(a)
$$\begin{array}{r} D57C_{16} \\ -76CA_{16} \\ \hline 5EB2_{16} \\ \hline \end{array}$$

Note: $C - A = 12 - 10 = 2$

$7 - C = 7 - 12 = 16 + 7 - 12 = 23$
$- 12 = 11 = B$

$4 - 6 = 16 + 4 - 6 = 20 - 6 = 14 = E$

$D - 1 - 7 = 13 - 1 - 7 = 12 - 7 = 5$

(b) $$\begin{array}{r} 476_8 \\ -377_8 \\ \hline 77_8 \\ \hline \end{array}$$

$6 - 7 = 8 + 6 - 7 = 14 - 7 = 7$

$7 - 1 - 7 = 6 - 7 = 8 + 6 - 7 = 14$

$- 7 = 7$

$3 - 3 = 0$

Worked Example 12

(a) Find the number base x for which $312_4 + 52_x = 96_{10}$.

(b) Given that $23_n - 32_r = Q$, where n and r are number bases, express n in terms of Q and r.

(a) $312_4 = 3 \times 4^2 + 1 \times 4^1 + 2 \times 4^0$

$= 3 \times 16 + 1 \times 4 + 2 \times 1$

$= 48 + 4 + 2$

$= 54_{10}$

$52_x = 5 \times x^1 + 2 \times x^0$

$= 5x + 2 \times 1$

$= (5x + 2)_{10}$

$54 + 5x + 2 = 96$

$54 - 54 + 5x + 2 - 2 = 96 - 54 - 2$

$5x = 40$

$\frac{5x}{5} = \frac{40}{5}$

$\therefore x = 8$

(b) $23n - 32r = Q_{10}$

$$(2 \times n^1 + 3 \times n^0) - (3 \times r^1 + 2 \times r^0) = Q$$

$$(2n + 3 \times 1) - (3r + 2 \times 1) = Q$$

$$(2n + 3) - (3r + 2) = Q$$

$$2n + 3 - 3r - 2 = Q$$

$$2n - 3r + 3 - 2 = Q$$

$$2n - 3r + 1 = Q$$

$$2n = Q + 3r - 1$$

$$\therefore n = \frac{Q + 3r - 1}{2}$$

(ii) Multiplication and division of numbers in different bases

Multiplication and division in decimal arithmetic is not much different from that of numbers in different bases. However, with number bases, special precautions must be taken. For example, if you are multiplying or dividing in octal system, make sure that the digit(s) involved are not greater than 7. On occasions that the digit is more than 7, divide by 8 and write down the remainder. Note that, if you rename any digit when working in base 8, that digit automatically becomes 8 unlike in decimal arithmetic where it becomes 10.

Let's look at a typical example of the multiplication table in base 5 in Table 1.2.

Table 1.2: Multiplication table in base 5

×	1	2	3	4
1	1	2	3	4
2	2	4	11	13
3	3	11	14	22
4	4	13	22	31

Worked Example 13

(a) Multiply 4354_6 by 213_6.

(b) Multiply $765_8 \times 64_8$ in base 8.

Solution

(a)

$$\begin{array}{r} 4354_6 \\ \times \quad 213_6 \\ \hline 21550 \\ 43540 \\ 1315200 \\ \hline 1425130_6 \\ \hline \end{array}$$

1st step

$$\frac{3 \times 4}{6} = \frac{12}{6} = 2R0$$

$$\frac{(3 \times 5) + 2}{6} = \frac{17}{6} = 2R5$$

$$\frac{(3 \times 3) + 2}{6} = \frac{11}{6} = 1R5$$

$$\frac{(3 \times 4) \times 1}{6} = \frac{13}{6} = 2R1$$

2nd step

$1 \times 4 = 4$
$1 \times 5 = 5$
$1 \times 3 = 3$
$1 \times 4 = 4$

3rd step

$$\frac{2 \times 4}{6} = \frac{8}{6} = 1R2$$

$$\frac{(2 \times 5) + 1}{6} = \frac{11}{6} = 1R5$$

$$\frac{(2 \times 3) + 1}{6} = \frac{7}{6} = 1R1$$

$$\frac{(2 \times 4) + 1}{6} = \frac{9}{6} = 1R3$$

4th step (addition)

$$\frac{0+0+0}{6}=\frac{0}{6}=0$$

$$\frac{5+4+0}{6}=\frac{9}{6}=1R3$$

$$\frac{5+5+2+1}{6}=\frac{13}{6}=2R1$$

$$\frac{1+3+5+2}{6}=\frac{11}{6}=1R5$$

$$\frac{2+4+1+1}{6}=\frac{8}{6}=1R2$$

$$\frac{3+1}{6}=\frac{4}{6}=0R4$$

$$\frac{1+0}{6}=\frac{1}{6}=0R1=1$$

(b)

$$\begin{array}{r} 765_8 \\ \times \quad 64_8 \\ \hline 03724 \\ +\ 56760 \\ \hline 62704_8 \\ \hline \end{array}$$

1st step

$$4\times 5=\frac{20}{8}=2R4$$

$$4\times 6+2=\frac{26}{8}=3R2$$

$$4\times 7+3=\frac{31}{8}=3R7$$

2nd step

$$6\times 5=\frac{30}{8}=3R6$$

$$6\times 6+2=\frac{38}{8}=4R6$$

$$6\times 7+4=\frac{46}{8}=5R6$$

3rd step

$$4 + 0 = 4$$

$$2 + 6 = \frac{8}{8} = 1R0$$

$$6 + 7 + 1 = \frac{14}{8} = 1R6$$

$$3 + 6 + 1 = \frac{10}{8} = 1R2 \text{ and } 5 + 0 + 1 = 6$$

Worked Example 14

(a) Divide 343_5 by 24_5.
(b) Divide 132_4 by 101_2.
Convert the numbers to base 10 before division.

$$\begin{aligned} 343_5 &= 3 \times 5^2 + 4 \times 5^1 + 3 \times 5^0 \\ &= 3 \times 25 + 4 \times 5 + 3 \times 1 \\ &= 75 + 20 + 3 = 98_{10} \end{aligned}$$

$$\begin{aligned} 24_5 &= 2 \times 5^1 + 4 \times 5^0 \\ &= 2 \times 5 + 4 \times 1 \\ &= 10 + 4 = 14_{10} \end{aligned}$$

$$\therefore 98_{10} \div 14_{10} = \quad \begin{array}{r|l} & 7_{10} \\ \hline 14 & 98 \\ & \underline{98} \end{array}$$

$$98_{10} \div 14_{10} = 7_{10}$$

To convert 7_{10} to base 5, follow the steps towards division as follows:

$$\begin{array}{c|ll} 5 & 7 & \\ \hline 5 & 1 & R\,2 \\ & 0 & R\,1 \end{array} \quad \uparrow$$

$$\therefore 343_5 \div 24_5 = 98_{10} \div 14_{10} = 7_{10} = 12_5$$

Aliter

$$
\begin{array}{r|l}
 & 12 \\
\hline
24 & 343 \\
 & \underline{24} \\
 & 103 \\
 & \underline{103}
\end{array}
$$

$\therefore 343_5 \div 24_5 = 12_5$

(b) 132_4 by 101_2

$$
\begin{aligned}
132_4 &= 1 \times 4^2 + 3 \times 4^1 + 2 \times 4^0 \\
&= 1 \times 16 + 3 \times 4 + 2 \times 1 \\
&= 16 + 12 + 2 \\
&= 30_{10}
\end{aligned}
$$

$$
\begin{aligned}
101_2 &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
&= 1 \times 4 + 0 \times 2 + 1 \times 1
\end{aligned}
$$

$= 4 + 0 + 1 = 5_{10}$

$\therefore 132_4 \div 101_2 = 30_{10} \div 5_{10} = 6$

Then, convert 6_{10} to base 2 as follows:

$$
\begin{array}{c|cl}
2 & 6 & \\
\hline
2 & 3 & R\,0 \\
\hline
2 & 1 & R\,1 \\
\hline
 & 0 & R\,1
\end{array}
\uparrow
$$

$\therefore 132_4 \div 101_2 = 30_{10} \div 5_{10} = 6_{10} = 110_2$

Note:

(i) When the bases are not the same, conversion is first made to the required base before multiplying or dividing.

(ii) The numbers can either be converted to base 10 and then divided or the division is done within the given base.

Exercise 4

1. Solve the following additions. All numbers being in base 7.

(a) $\begin{array}{r} 654 \\ +324 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} 154 \\ +346 \\ \hline \\ \hline \end{array}$

(c) $\begin{array}{r} 6664 \\ +3543 \\ \hline \\ \hline \end{array}$ (d) $\begin{array}{r} 6054 \\ +1234 \\ \hline \\ \hline \end{array}$

2. Solve the following subtractions in base 9:

(a) $\begin{array}{r} 845 \\ -321 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} 654 \\ -534 \\ \hline \\ \hline \end{array}$

(c) $\begin{array}{r} 8675 \\ -2352 \\ \hline \\ \hline \end{array}$ (d) $\begin{array}{r} 7582 \\ -4758 \\ \hline \\ \hline \end{array}$

3. Simplify each of the following in base 12:
   (a) 8AB + 4BA
   (b) 9B7 + 46A

4. Simplify each of the following in base 16:
   (a) DDE − 76B
   (b) D68D − 76CB
   (c) ECEA − 41AB
   (d) 75DC − 2AB1

5. Simplify the following additions in the duodecimal system:

(a) $\begin{array}{r} \text{AB4} \\ +112 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} \text{A1B} \\ +\text{1B1} \\ \hline \\ \hline \end{array}$

(c) $\begin{array}{r} \text{BBB} \\ +\text{11A} \\ \hline \\ \hline \end{array}$ (d) $\begin{array}{r} \text{A1} \\ +\text{BB} \\ \hline \\ \hline \end{array}$

6. Find the difference between the following numbers:
   (a) 123_5 and 33_5
   (b) $6D4_{16}$ and $4EE_{16}$

7. Simplify the following subtractions in the duodecimal system:

(a) $\begin{array}{r} \text{AAB} \\ -112 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} \text{A11} \\ -\text{11B} \\ \hline \\ \hline \end{array}$

(c) $\begin{array}{r} \text{1AA} \\ -\ \ \text{B} \\ \hline \\ \hline \end{array}$ (d) $\begin{array}{r} \text{AB1} \\ -\text{AAA} \\ \hline \\ \hline \end{array}$

8. If $234_5 + nnnn_5 + 434_5 = 2304_5$, find $nnnn_5$.

9. Find x, if $634_x + 2110_x + 523_x = 3600_x$.

10. Calculate the value of $535_6 + 251_6 - 544_6$.
11. If $52n - 24n = 25n$, calculate the value of n.
12. Find the number base y for which $312_4 + 52_y = 96_{10}$.
13. Given that $42x - 24y = z$, where x and y are number bases, express x in terms of y and z.
14. Find x and y in number bases from the following simultaneous linear equations:

$$34_x + 32_y = 36_{10}$$

$$25_x - 23_y = 6_{10}$$

15. Find x, if $34_x = 22_{10}$.
16. Add 643_7 and 2011_7, leaving the answer in octal.
17. Find x, if $(3)_x + (34)_x - (40)_x = 1$.
18. What is the base of the following addition?

$$222 + 303 + 757 = 1383$$

19. What is the base of the following subtraction?

$$375 - 281 = 84$$

20. Find the quotient of $245_8 \div 41_8$.
21. Find the average of the following: 22_5, 23_5, 24_5 and 32_5. Leave your answer in quinary.
22. Find the missing number, if $xxx_7 \times 46_7 = 44262_7$.
23. What is the product of 254_6 and 635_8 in base 8?

24. If $10_5 \times x_3 = 6_{10}$, what is x?

25. Find $(101_5)^2$, expressing the answer in base 2?

(WAEC)

26. Find the total cost of 253_8 books at 25_8 kendos, if the currency in that country is kendos.

(WAEC)

27. Multiply in base 6.

(a) $4153_6 \times 5_6$

(b) $542_6 \times 32_6$

(c) $532_6 \times 52_6$

(d) $3451_6 \times 234_6$

28. Divide 132_4 by 12_4.

29. Divide 44262_7 by 46_7.

30. Simplify $4ABC_{16}$ by $7A_{16}$.

V. Application of Number Base System to Computer Programming

(i) Introduction

Binary numbering system is widely used in computer programming. Recall that in binary numbering system, which is base 2, there are two digits, namely 0 and 1. The binary numbering applies to computer programming, where 0 is used to represent an off-state and 1 is used to represent an on-state. Recall also that digits 1 and 0 are used to represent an on-off action, that is, a two-way flow chart such as in and out, inside–outside, exit–enter, yes–no, true–false actions etc.

Apart from the number bases 1–9, there is base 16 called hexadecimal. Its digits are from 0 to 15. Digits 10, 11, 12, 13, 14 and 15 are represented by A, B, C, D, E and F respectively. Conversion from decimal to hexadecimal is as in other bases. The last digit of each base usually ends with 1 and 0. These digits are found useful in computer programming since any base can be converted to have 1 and 0. Note that the last digit of each base + 1 = 10.

Base 2 digits 0, 1
$1 + 1 = 1R0$

Base 3 digits 0, 1, 2
$2 + 1 = 1R0$

Base 4 digits 0, 1, 2, 3
$3 + 1 = 1R0$

Base 5 digits 0, 1, 2, 3, 4
$4 + 1 = 1R0$

Base 9 digits 0, 1, 2, 3, 4, 5, 6, 7, 8
$8 + 1 = 1R0$

Binary numbering system is also very useful in storing information and for retrieval purposes as done in teacher functional mathematics basic 9. The teacher should revise with the students on how to code and use punch cards to store information and its interpretation.

(ii) Punch cards

Cards are used to store information about things or events. These make use of a binary number system of a hole punched out (0) or a slot cut out (1). Figure 1.1 shows a card of the student Khaleed Ade. In a survey of subjects passed by students in a class, it is found that Khaleed Ade passed Mathematics and Physics but failed Yoruba and Government.

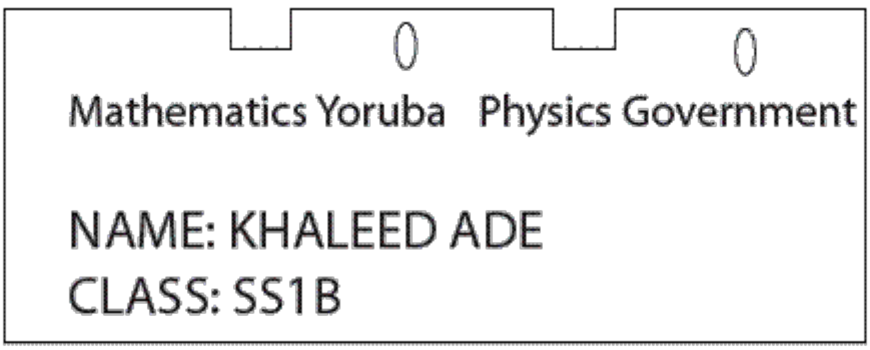

Fig. 1.1

If 1 represents the subject passed and 0 the subject failed, each possible combination of subjects can be represented by a binary number. The number for the subjects passed by Khaleed is 1010.

Worked Example 15

Sketch a punch card for a student who passed Yoruba and Physics only.

Solution

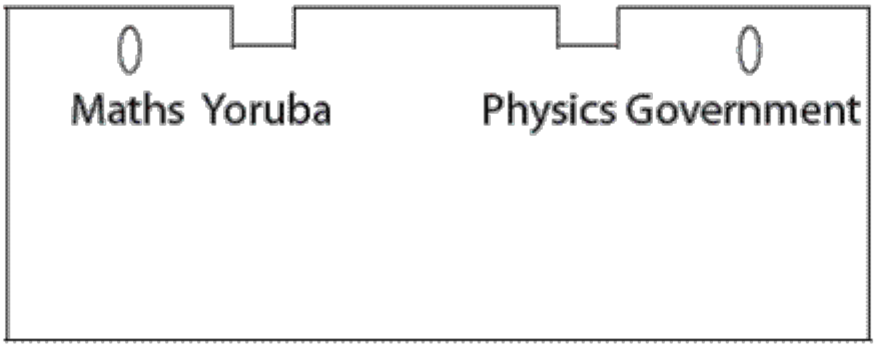

Fig. 1.2

Worked Example 16

What subjects were passed by students whose cards represent the numbers.
(a) 1011 (b) 0010

Solution

(a) Mathematics, Physics and Government
(b) Physics only

(iii) Binary number coding system

The letters of the English alphabet are coded using binary numbers. The coding in a punch tape can be written either by a 'word per line' or 'letter per line' code.

Table 1.3

Letters	Binary numbers	Decimal numbers
A	1	1
B	10	2
C	11	3
D	100	4
E	101	5
F	110	6
G	111	7
H	1000	8
I	1001	9
J	1010	10
K	1011	11
L	1100	12
M	1101	13
N	1110	14
O	1111	15

Letters	Binary numbers	Decimal numbers
P	10000	16
Q	10001	17
R	10010	18
S	10011	19
T	10100	20
U	10101	21
V	10110	22
W	10111	23
X	11000	24
Y	11001	25
Z	11010	26

WORKED EXAMPLE

Worked Example 17

The word 'VERY SMART' is coded thus:

(i) Word-per-line code:

(10110) (101) (10010) (11001) VERY

(10011) (1101) (1) (10010) (10100) SMART

(ii) Letter-per-line code:

10110	V
101	E
10010	R
11001	Y
10011	S
1101	M
1	A
10010	R
10100	T

Worked Example 18

Using a letter-per-line code, write the words represented by the following binary numbers:

(a)		(b)
100		10100
1001		10010
10110		1
1001		10000
100		101
101		11010
1110		1001
100		10101
		1101

Solution

(a) D
I
V
I
D
E
N
D

(b) T
R
A
P
E
Z
I
U
M

Exercise 5

1. On and off indicate ____________ and ____________ in computer programming.
2. Convert base 16 such that it ends with digits 0 and 1.
3. Why does computer programming make use of 0 and 1 digits.

4. Decode ABC11 in alpha numeric form.
5. Convert base 14 such that it ends up having 0 and 1 digits.
6. Decode the punch card below into binary code.

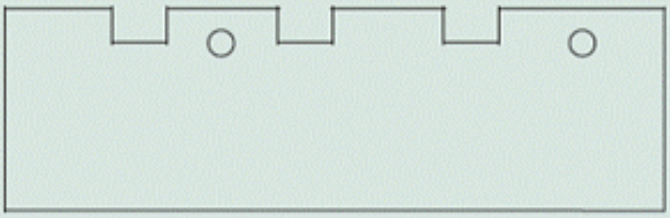

Fig. 1.3

7. Draw a punch card to represent 111001.
8. Represent 13 and 14 on the basis of hexadecimal.
9. How many digits are there in base 14?
10. List out all the digits in base 9.
11. In Fig. 1.1, what subjects were passed by students whose cards represent the following numbers:

    (a) 1101 (b) 1001
12. Sketch a card for a student who passed Yoruba and Government only (see Fig. 1.1).

13. Write the binary code for the name LEONARD SMART in a word-per-line code.

14. Write the binary code for the word 'STAR' in a letter-per-line code.

15. (a) Using a word-per-line code, write the words represented by the following binary numbers:

    (i) (10101) (111) (1100) (11001) (100) (1111) (111)

    (ii) (1110) (101) (1100) (10011) (1111) (110) (1101) (1) (1110) (100) (101) (1100) (1)

    (b) Using a letter-per-line code, write the words represented by the following binary numbers:

(i)	(ii)
100	11
101	1000
1110	1001
1	1011
10010	101
11001	1111
	10
	1001

SUMMARY

In this chapter, we have learnt the following:

◆ Counting is mostly done in tens using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 (zero). This is called the base 10 system (denary).

◆ It is possible to represent numbers using other systems, for example, base 2, base 6, base 9, etc. Base 2 is called the binary system, base 5 is called quinary system, base 8 is called octal system, base 11 is called duodecimal, and base 16 is called hexadecimal.

◆ To convert a number from base 10 to another base, use repeated division as following:

> **(i)** Divide the base 10 number by the new base number.
> **(ii)** Continue dividing until 0 is reached, writing down the track of remainders.
> **(iii)** Start to record the remainders from bottom upwards.

◆ To convert a number from one base to base 10, either expand the given number or use repeated multiplication.

◆ Arithmetic basic operations of addition, subtraction, multiplication and division is possible in any base just as it is done in decimal arithmetic.

◆ Apart from the number bases 1–9, there is base 16 called hexadecimal. The digits are from 0 to 15.

◆ Digits 10, 11, 12, 13, 14 and 15 are represented by A, B, C, D, E and F respectively.

◆ The last digit of each base + 1 = 10, for example, base 7 = digits 0, 1, 2, 3, 4, 5, 6 = 6 + 1 = 10.

◆ Binary number system is important because of its applications to computer programming, punch cards, punch tape and binary code systems.

GRADUATED EXERCISES

1. If $(126)_n = 86_{10}$, find a positive value for *n*. (WAEC)
2. Calculate the value of $(243)_5 + (132)_5$ and leave the answer in base 5. (WAEC)
3. Find the total cost of 263_8 books at 26_8 kendos, if the currency in that country is kendos.
4. What is the total of 12 base 7 numerals whose average is 24_7. (WAEC)
5. Express 534_8 in the binary system. (WAEC)
6. Solve for *x* in the following base 2 equation, leaving your answer in base 2.

$$10_x - \frac{11(1+x)}{10} = 101 \qquad \text{(WAEC)}$$

7. Convert 6 989_{10} to base 16.
8. Express in base 10: $ABC5.4B_{13}$.
9. Convert
   (i) $A64_{11}$ to base 6.
   (ii) 8724_9 to base 16.
10. Which is the largest, $A317_{16}$ or 2468_9?
11. Simplify in base 16: C86D – 67BC.
12. Add in base 12: 42AB + AB76.
13. The sum of three numbers in base 2 is 11101. If two of the numbers are 1011 and 1101, find the third number. (WAEC)
14. Calculate $212_3 \times 201_3$, giving your answer as a number in base 3. (WAEC)
15. Evaluate $202_{12} + 1211_3 - 1021_3$, leaving your answer in base 3.
16. If $110111_2 - y_{10} = 10101_2$, find *y*. (WAEC)
17. What is the base and the missing digit in the following subtraction:

$$\begin{array}{r} 881 \\ -\ 1{*}7 \\ \hline 766 \\ \hline \end{array}$$

18. Express 4020_6 as a number in base 10. (WAEC)

19. Simplify $636_7 + 6444_7 + 545_7$.
20. Find the missing numbers in the following: $757_8 + 567_8 + nnn_8 = 2304_8$
21. If $233_6 – 1Q5_6 = 34_6$, what is Q?
22. What base is x, if $\left(\frac{55}{5}\right) = 1011_2$?
23. Given that $2pQ_7 + 566_7 = 1133_7$, what are the values of p and Q?

24. Find $\sqrt{11001_5}$, expressing the answer in base 2. **(WAEC)**

25. The following addition is done in base 8. Find the missing number. **(WAEC)**

$$\begin{array}{r} 356 \\ 1241 \\ + \quad *** \\ 652 \\ \hline \\ \hline \end{array}$$

26. If $3x = 2(341)_5$, find x in base 10.
27. Convert 82.56_9 to decimal.
28. The sum of two octal numbers is 71_8. One of them is 46_8. Find the other.
29. Simplify
    (a) $8AB_{16} + 2DE_{16}$
    (b) $11011_7 + 265_7$
30. The following simultaneous equations are written in base 5:
    $13x + 4y = 134$
    $x + y = 13$

Solve the equations, leaving your answers in base 5.
(b) A number is written as 14_y. Three times the number is written as 45_y. What is y?