4 UNIFORM CIRCULAR MOTION

UNIFORM CIRCULAR MOTION

OBJECTIVES

At the end of the topic, students should be able to:

- identify circular motion;
- use a string tied to a stone to demonstrate circular motion in a horizontal and vertical circle;
- explain centripetal acceleration and centripetal force;
- state some applications of centripetal and centrifugal forces.

Tie a string to a tin of milk and whirl it around your head. The motion of the tin of milk is a circular motion. The force acting towards the centre of the circle is called centripetal force.

Uniform circular motion is the motion of a body moving in a circular path with a constant speed.

It is a special type of motion. The velocity of a body moving round in a circle changes continuously because the direction is always changing but the speed is constant. Figure 4.1 shows a particle of mass (m) moving round a circle of radius (r) with a constant speed (v).
Any particle performing uniform circular motion has the following characteristics:

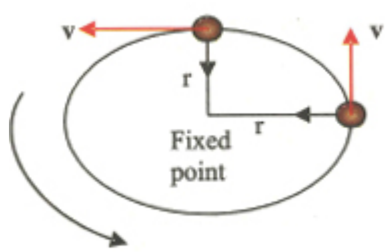


Figure 4.1 Uniform circular motion

- The speed of the body as it moves round the circle is constant but the velocity is continuously changing as the direction changes.
- The velocity at any point of the circle is tangential to the circle and is always perpendicular to the radius of the circle.

- The acceleration of the body is directed towards centre of the circle. This is because the velocity is changing in direction always.
- A force is required to keep the body moving round the circle at constant speed. Examples of circular motion are:
  - motion of planets round the sun
  - electrons revolving round the nucleus
  - the tips blades of an electric fan spinning about its axis

Centripetal acceleration

A body moving in a circle at a constant speed changes its velocity continuously in direction. The directional change in velocity means that the body is accelerating. Acceleration is the velocity change per time. *The acceleration of a body moving round a circle is called centripetal acceleration because it is always pointing towards the centre of the circle.* (Centripetal means centre seeking). The centripetal acceleration is related to the speed of the body (v) and the radius (r) of the circle by:

$$\text{Centripetal acceleration } (a) = \frac{v^2}{r}$$

Centripetal force

A force is needed to change the velocity of a body moving in a circle with a constant speed.

Centripetal force is the force which changes the direction of the velocity of a body moving in a circular path.

Its direction is always towards the centre of the circle, hence, it acts in the same direction as the centripetal acceleration.

$$F = \frac{mv^2}{r}$$

F = centripetal force, Î½ = speed of the body round the circle, r = radius of the circle. The magnitude of the centripetal force increases if:

- mass of the body moving on the circle is increased;
- speed of the body is increased;
- radius of the circle is decreased.

  Centripetal force causes a body to leave its straight-line motion and move in a circular path. The vital question is, what path will the body follow if the centripetal force is removed suddenly? If the tension in the rope is removed in Figure 4.1, the body will fly off tangentially to the circle at the point where the tension is removed and continues to move in a straight line as illustrated in Figure 4.2.

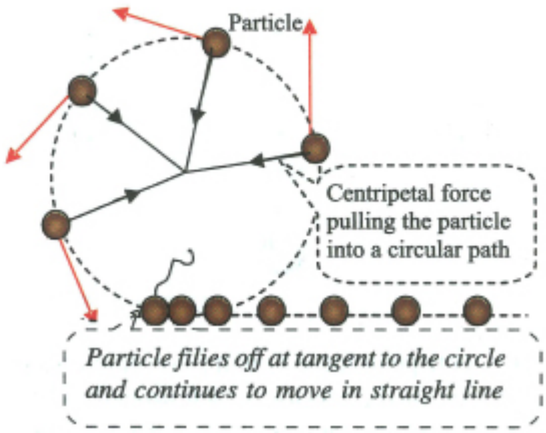


Figure 4.2 Uniform motion in a vertical circle

Centripetal force is called by different names.

- Tension is the centripetal force, which keeps a body attached to a string or rope moving round a circle with a constant speed.
- Gravitational force is the centripetal force responsible for the motion of the planets round the sun.
- Electrostatic force is the centripetal force, which keeps the electrons in orbit round the nucleus.
  If any of these forces is removed, the particle will move off along the path tangential to the circular path. If the gravity of the sun is removed, all the planets will leave their orbital paths in space and move in a straight line indefinitely.

Centrifugal force

When a body moves on a circle with constant speed, the centripetal force pulls the body towards the centre of the circle. To stop the body from falling towards the centre, another force is needed to balance the centripetal force. The force, which balances the centripetal force, is called **centrifugal force**. Centrifugal force acts opposite in direction to centripetal force. Centrifugal force is the kinetic reaction of the centripetal force on the body. The magnitude of centrifugal force is equal to the magnitude of centripetal force but acts in the opposite direction.

Motion in a vertical circle

When a body moves on a vertical circle at constant speed, the centripetal force needed to maintain the motion of the body along the circle comes from the:

- tension in the string or rope;
- weight of the body moving along the circular path.

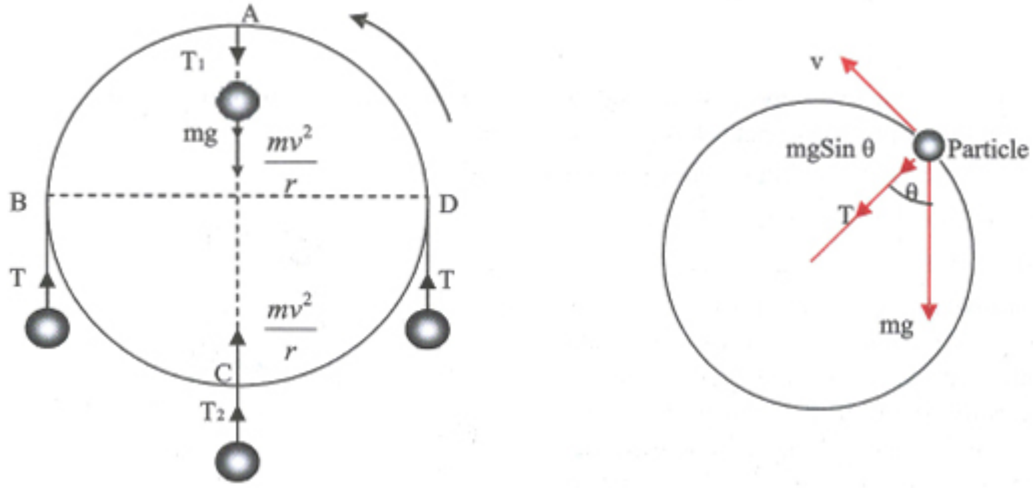


Figure 4.3 Uniform motions in a vertical circle

The centripetal force remains constant throughout the motion but the tension in the string varies. The size of the tension is determined by the size of the component (part) of the weight in the direction of the tension. The centripetal force is given by:

$$\frac{mv^2}{r} = T + W$$

T = Tension in the string, W = Weight of the body

At the top of the circle (A) the tension T_1 and the weight (W= mg) are in the same direction. The centripetal force is given by:

$$\frac{mv^2}{r} = T_1 + mg$$

$$\therefore \quad T_1 = \frac{mv^2}{r} - mg$$

At the bottom of the circle (C) the tension T_2 and the weight (mg) are in the opposite directions. The centripetal force is given by:

$$\frac{mv^2}{r} = T_2 - mg$$

$$\therefore \quad T_2 = \frac{mv^2}{r} + mg$$

The tension is least at A, the top of the circle and greatest at C, the bottom of the circle. The string is under the greatest tension at C and is likely to cut at this point. At B and D, the tension T is equal to the centripetal force.

Application of centripetal and centrifugal forces

1. Moving round a curved path

Centripetal force keeps a body moving along a curved path. *When a car goes round a bent road, the friction between the tyre and the road gives the needed centripetal force to keep the moving car along the curved road without skidding.* The centripetal force pulls the car towards the centre of the curved road. The passengers feel a force pulling them away from the centre of the curved road. This force is centripetal force. Cars some times skid when going round a sharp bend at high speed mostly when the road is wet. The skidding of cars when going round a sharp bend can be avoided by raising the side of the road away from the centre such that it slopes inwards. This is called **banking**. *Banked roads resist the tendency of the car skidding away from the curved road into a straight line path.* Banking of roads increases friction because of the component of the carâ€™s normal reaction is in the direction of the friction.

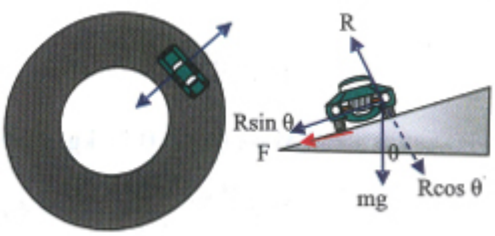


Figure 4.4 Banking of roads prevent skidding

2. Centrifuges

Centrifuges are devices which separate suspended particles in fluid (liquid and gas) into components according to their densities by spinning them in a container at very high speed.

- In this way, milk can be separated from the cream using cream separator.
- Centrifuge spinning at a high speed is used to separate blood particles from blood fluid. The denser particles stay near the open end and are separated.
- Centrifuge dryer, spinning at a great speed, is used to separate water from cloth.

Worked examples

1. A cyclist of total mass 150 kg riding with a speed of 70 km h^{-1}, negotiates a bend in circle of radius 25 m. Calculate the centripetal force which keeps the cyclist along the circular path.

Solution

$$F = \frac{mv^2}{r}$$

Mass (m) = 150 kg, speed (v) = 70km h^{-1} = $\frac{70 \times 1000\, m}{3600\, s} = 19.4\, m\, s^{-1}$,

radius (r) = 25 m

$$F = \frac{150 \times 19.4 \times 19.4}{25} = 2258.2\, N$$

2. A boy swings a small stone tied to a string round and round in a vertical circle of radius 1.5 m. If the mass of the stone is 0.23 kg and it moves at a constant speed of 20 m s^{-1} find:
(i) the centripetal force pulling the stone in a vertical circle.
(ii) the minimum and maximum tension on the string, (g = 10ms^{-1})

Solution

i. $F = \frac{mv^2}{r}$

Mass (m) = 0.23 kg, speed of stone (v) = 20 m s^{-1}, radius (r) = 1.5 m

$$F = \frac{0.23 \times 20 \times 20}{1.5} = 61.3\, N$$

Minimum Tension = Centripetal force – Weight of stone
Maximum Tension = Centripetal force + Weight of stone

(iii) Minimum tension $T_1 = \frac{mv^2}{r} - mg$

= 61.3 N-0.23×10 = 59.0 N

Maximum tension $T_2 = \frac{mv^2}{r} + mg$

= 61.3 N+2.3 N = 63.6 N

Summary

- The motion of a body moving in a circular path or circle with a constant speed is called uniform circular motion.
- A body moving round in a circle with constant speed, accelerates because the velocity is always changing.
- The force, which changes the direction of the velocity of a body moving on circle, is called centripetal force.
- Centrifugal force is the kinetic reaction of the centripetal force on the body.
- Banked roads resist the tendency of the car skidding away from the curved road into a straight line path.

Practice question 4a

1. A car negotiates a bend moving along the arc of a circle.
   (a) What force provides the centripetal force?
   (b) What is the effect on the centripetal force if:
      i. the car travels faster;
      ii. the bend is gradual;
      iii. the car has more passengers.
2. A space shuttle is in orbit round the earth at a certain height.
   (i) What provides the centripetal force to keep the shuttle in orbit?
   (ii) If the shuttle reduces its mass by launching a communication satellite, how does this affect the centripetal force?
   (iii) How is the centripetal force and speed affected if the shuttle moves to a higher orbit?
3. (i) Explain why a force acts on a particle moving in a circle with constant speed.
   (ii) An aircraft of mass 10,000 kg is travelling at a constant speed of 200 m s^{-1} in a horizontal circle of radius 1500 m. Calculate the centripetal force acting on the aircraft.
4. A man spins a mass of 5 kg in a vertical circle of radius 2.5 m, calculate the maximum and minimum tensions in the rope if the speed of the mass is 12 m s^{-1}. [g = 10 ms^{-2}]
5. Why is it necessary to bank a road at sharp bends?

ANGULAR MOTION

OBJECTIVES

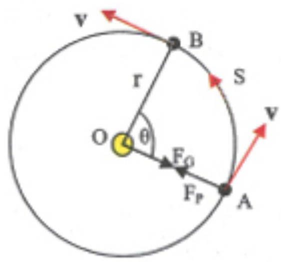
At the end of the topic, students should be able to:

- explain angular distance, angular speed and angular acceleration;
- compare angular and linear motion;
- solve simple problems on linear motion.

When a particle moves in a circle of radius (r) with a constant speed (v), it covers both linear distance (s) and angular distance (Î¸). Suppose the particle goes from point A to another point B along the circumference of the circle, the linear distance (s) covered is the length of the arc AB while the angle turned (Î¸) is the angular distance. The angle turned (angular distance) is given by:

$$\begin{array}{l}\text{Angle turned}\\ \text{(angular distance)}\end{array} = \frac{\text{Length of arc AB}}{\text{Radius of circle}}$$

$$\theta = \frac{S}{r} \Rightarrow S = r\theta$$



The unit of the angle turned or angular distance (Î¸) is radian (rad.). The angle at the centre of a circle in radian is 2Ï€. 360Â° is equivalent to 2Ï€ radian.

Also note the following:

Ï€ radian = 180Â°

$$\frac{\pi}{2} = 90^o \qquad \frac{\pi}{4} = 45^o \quad 1 \text{ radian} = \frac{360}{2\pi} = 57.3^o$$

$$1^o = \frac{2\pi}{360} = \frac{\pi}{180} = 0.0175\, radians$$

Angular velocity (Ï‰)

Angular velocity (w) is the angle turned per second. Alternatively, angular velocity is the time rate of change of angular distance.

$$\text{Angular velocity } (\omega) = \frac{\text{Angular distance}}{\text{Time}}$$

$$\omega = \frac{\theta}{t} \text{ Or } \theta = \omega t$$

The unit of angular velocity (Ï‰) is radian per second (rad s^{-1}).

Linear and angular motion compared

1. The angle turned or angular distance (*Î¸*) is related to the linear distance (S) covered by a particle moving in a circle by: S = r*Î¸*
2. The angular velocity (Ï‰) is related to the linear velocity (v) by: $v = \omega r \text{ or } \omega = \frac{v}{r}$

3. Centripetal acceleration $(a) = \frac{v^2}{r}$ but $v = \omega r$

$$a = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \frac{\omega^2 \text{ x } r^2}{r} = \omega^2 r$$

4. The centripetal force $F = \frac{mv^2}{r}$ but $v = \omega r$ $F = \frac{m(\omega r)^2}{r} = m\omega^2 r$

$$\therefore F = \frac{mv^2}{r} \text{ or } mw^2 r$$

The formula you will use depends on the given parameters.
If the value of angular velocity (w) is given, apply $F = mw^2r$
If the value of the velocity is given, apply

$$F = \frac{mv^2}{r}$$

Worked examples

1. A particle of mass 5 g moves round in a circle of radius 1.2 m completing three revolutions in 2 seconds. Find the:
   i. angular velocity;
   ii. linear velocity;
   iii. the centripetal force pulling it towards the centre of the circle;

Solution

(i) Remember that
One revolution = 2π radians
$\therefore$ Three revolutions = $3 \times 2\pi = 6\pi$ radians.

Angular velocity $(w) = \frac{\theta}{t} = \frac{6\pi}{2} = 3\pi$ rad s^{-1}

(ii) Linear velocity (v) = $w\,r = 3\pi \times 1.2 = 11.3$ m s^{-1}

(iii) Centripetal force (F) = $m\,w^2 r = 0.005 \times 9\pi^2 \times 1.2 = 0.532$ N

Summary

- Angle turned (angular distance) $= \frac{\text{Length of arcAB}}{\text{Radius of circle}}$
  - Angular velocity (Ï‰) is the angle turned per second, or angular velocity is the time rate of change of angular distance.

Practice questions 4b

1. The earth orbits the sun with the speed of about 3000 ms^{-1} at a distance of 1.5×10^{11}m from the centre of the sun. If the mass of the earth is 6.0 Ã— 10^{24} kg, calculate the:
   (a) Angular velocity (b) Centripetal force.

2. Convert the following angles in radians to degrees $\frac{3\pi}{4}$, $\frac{\pi}{3}$, $\frac{2\pi}{3}$, 4π.

3. A blood centrifuge performs 600 revolutions per minute. If a particle of mass 3.3×10^{-6} kg on the test tube is 1.10 m from the centre of rotation, calculate the speed of the particle and the centripetal force acting on the particle.

Past questions

1. The magnitude of the force required to make an object of mass m move with speed Î½ in a circular path of radius, r is given by the expression:

A. $\frac{mr}{v}$

B. $\frac{mr^2}{v}$

C. $\frac{mv^2}{r}$

D. $\frac{mv}{r^2}$

WASSCE

2. What is the angular velocity of an object in a horizontal circular path of radius 1.5 m with a constant speed of 12 m s^{-1}?

A. 1.6 rad. s^{-1}

B. 4.0 rad. s^{-1}

C. 8.0 rad. s^{-1}

D. 16.0 rad. s^{-1}

E. 32.0 rad. s^{-1}

NECO

3. A body weighing 100 N moves with a speed of 5 m s^{-1} in a horizontal circular path of radius 5m. Calculate the magnitude of the centripetal force acting on the body, [g = 10 m s^{-2}]

A. 10N

B. 50 N

C. 75 N

D. 100 N

WASSCE

4. If the force of attraction between the sun and the planets is removed, the planets will

A. fall towards the sun.

B. scatter and stop moving.
C. continue to move at a tangent to their original orbits.
D. continue to move perpendicular to their original orbits.

WASSCE

5. An object of mass 0.40 kg attached to the end of a string is whirled round a circle of radius 2.0 m with a constant speed of 8 m s^{-1}. Calculate the angular velocity of the object.
A. 0.8 rad. s^{-1}
B. 2.0 rad. s^{-1}
C. 4.0 rad. s^{-1}
D. 8.0 rad. s^{-1}
E. 16.0 rad. s^{-1}

WAEC

6. Which of the following correctly gives the relationship between linear speed Î½ and angular speed Ï‰ of a body moving uniformly in a circle of radius r?
A. Î½ = Ï‰ r
B. Î½ = Ï‰^{2} r
C. v = Ï‰ r^2
D. Î½^2 = Ï‰ r
E. $v = \frac{\omega}{r}$

WAEC

7. A stone tied to a string is made to revolve in a horizontal circle of radius 4 m with an angular speed of 2 rad. s^{-1}. With what tangential velocity will the stone move off the circle if the string cuts?
A. 16 m s^{-1}
B. 8 m s^{-1}
C. 6 m s^{-1}
D. 2 m s^{-1}
E. 0.5 m s^{-1}

WAEC

8. A particle of mass 10^{-2} kg is fixed to the tip of a fan blade which rotates with angular velocity of 100 rad s^{-1}. If the radius of the blade is 0.2 m, the centripetal force is
A. 2 N
B. 20 N
C. 200 N
D. 400 N

JAMB

9. An object moves with uniform speed round a circle. Its acceleration

has
A. constant magnitude and constant direction.
B. constant magnitude and varying direction.
C. varying magnitude but constant direction.
D. varying magnitude and varying direction.

JAMB