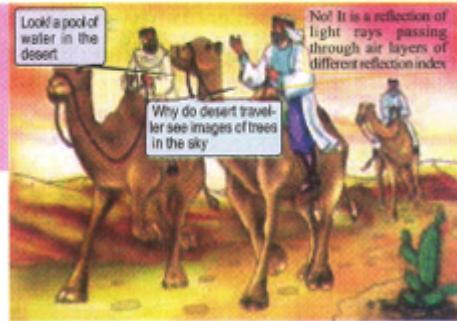


# 13

## REFRACTION OF LIGHT



### Refraction through rectangular and triangular prism

Light travels with the same speed in straight lines between two points in the same medium. The speed of light changes as it enters a new medium. What happens to the direction of light in the new medium depends on how it enters the medium. Light changes direction in the new medium if it enters at an angle, but continues to move in the same direction if light enters normally. The change in speed or direction of light in the new medium is known as **refraction**.

### OBJECTIVES

At the end of the topic, students should be able to:

- explain how the direction of light changes as it travels from one medium to another;
- measure the angles of incidence and refraction for a given pair of media and use it to find refractive index of the second medium;
- state the laws of refraction;
- trace the path of light through a triangular prism and obtain graphically the value of angle of minimum deviation;
- perform some simple calculations on refraction.

### What is refraction?

If a narrow beam of light passes from air to glass, it bends or changes direction at the boundary of air and glass. This is called refraction. Swimming pools look shallower than they really are because of refraction.

***Refraction is the bending or change in direction of light as it passes from one medium to another.***

The light ray moving towards the air-glass boundary is the **incident ray**. The path of the ray in the glass medium is the **refracted ray**. Refracted ray in the glass medium bends towards the normal. The angle the incident ray makes with normal is called the **angle of incidence** ( $i$ ) while the angle the refracted ray makes with the normal is called the **angle of refraction** ( $r$ ).

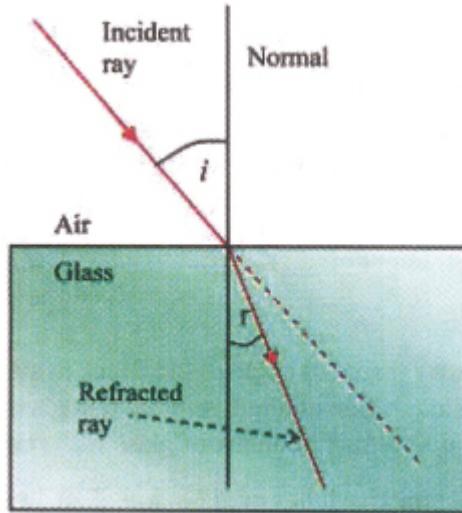


Figure 13.1: Refraction of light rays moving from air to glass

The principle of reversibility of light states that the path of light is easily reversible. If light ray travels in the opposite direction (from glass to air), it bends away from the normal as shown in Figure 13.2.

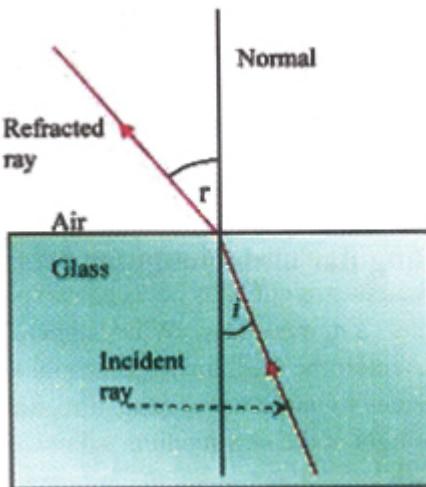


Figure 13.2: Path of light is easily reversible

## The laws of refraction

The discovery of the laws governing refraction of light was not easy. From the time Euclid first observed refraction to the time Willbrod Snell discovered the laws of refraction was about 2000 years. The laws of refraction are:

- 1. The incident and refracted rays are on the opposite sides of the normal and at the point of incidence, all three lie in the same plane.**
- 2. The ratio of sine of angle of incidence to the sine of angle of refraction is a constant for any two pair of media.**

$$\frac{\sin i}{\sin r} = \text{a constant}$$

## Refractive index

The second law of refraction is known as Snellâ€™s law of refraction. The constant of Snellâ€™s law is the **refractive index** of the second medium with respect to the first medium. For light ray moving from air (first medium) to glass (second medium) the refractive index is given by:

$$_a n_g = \frac{\sin i}{\sin r}$$

$\sin i$  = sine of angle of incidence in air

$\sin r$  = sine of angle refraction in glass

$_a n_g$  = refractive index of glass with respect to air

Refractive index of glass  $_a n_g = \frac{3}{2}$  or 1.50 and the refractive index of water  $_a n_w = \frac{4}{3}$  or 1.33. Light rays bend more in a medium with higher refractive index, therefore, light rays bend more towards the normal in glass than water.

A ray of light travelling from glass to air (in the opposite direction), the refractive index is given by:

$$_g n_a = \frac{\text{Sine of angle of incidence in glass}}{\text{Sine of angle refraction in air}}$$

$$_g n_a = \frac{1}{_a n_g}$$

Since the path of light is reversible, it follows that:

**Refractive index of a medium is the ratio of the speed of light in air to the speed of light in the medium.**

For a beam of light travelling from air to glass the refractive index is defined by:

$$_a n_g = \frac{\text{Speed of light in air}}{\text{Speed of light in glass}}$$

## Worked examples

1. A beam of light of light travelling through air at  $3.0 \times 10^8 \text{ ms}^{-1}$  enters a pool of water of refractive index  $\frac{4}{3}$ , what is its speed as it travels through water?

### Solution

$$_a n_w = \frac{\text{Speed of light in air}}{\text{Speed of light in water}}$$

$$\frac{4}{3} = \frac{3.0 \times 10^8}{v}$$

$$v = \frac{3 \times 3.0 \times 10^8}{4} = 2.25 \times 10^8 \text{ m s}^{-1}$$

2. A light ray is incident on glass from air at  $60^\circ$  to the normal. Calculate the angle of refraction if the refractive index of water is  $\frac{4}{3}$ .

### Solution

Angle of incident ( $i$ ) =  $60^\circ$ ,  $n_w$  =, angle of refraction  $r$  =?

$$n_w = \frac{\sin i}{\sin r}$$

$$\frac{4}{3} = \frac{\sin 60^\circ}{\sin r}$$

$$\sin r = \frac{3 \sin 60^\circ}{4} = 0.6495$$

$$\therefore r = \sin^{-1}(0.6495) = 40.5^\circ$$

3. The refractive index of glass is 1.5. A ray of light passing from glass to air has an angle of incidence of  $19.5^\circ$ , what is the angle of refraction in air?

### Solution

$i = 19.5^\circ$ ,  $n_g = 1.5$  and  $r$  =?

$$n_a = \frac{1}{n_g} = \frac{1}{1.5}$$

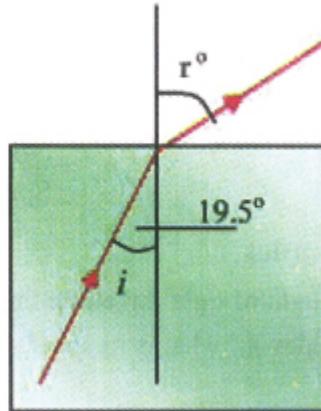
$$n_a = 0.6667$$

$$n_a = \frac{\sin i}{\sin r}$$

$$0.6667 = \frac{\sin 19.5}{\sin r}$$

$$\sin r = \frac{\sin 19.5}{0.6667} = 0.5007$$

$$r = \sin^{-1}(0.5007) = 30^\circ.$$



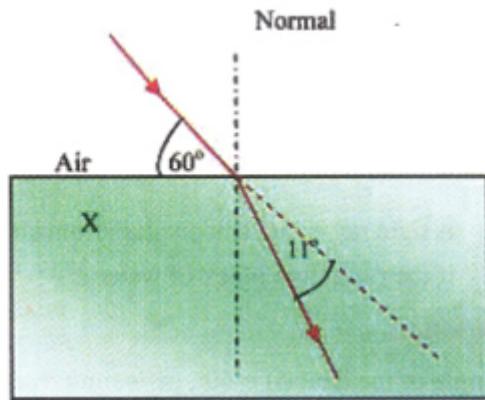
4. In the diagram below, calculate the refractive index of medium X.

### Solution

$$\text{Angle of incidence } i = 90^\circ - 60^\circ = 30^\circ.$$

$$\text{Angle of refraction } r = 30^\circ - 11^\circ = 19^\circ.$$

$$n_X = \frac{\sin i}{\sin r} = \frac{\sin 30^\circ}{\sin 19^\circ} = \frac{0.5000}{0.3256} = 1.54$$



## The cause of refraction

Light is a wave travelling through vacuum or air at a speed of  $3.0 \times 10^8 \text{ ms}^{-1}$ . When light waves pass from air to another medium, its speed changes. The change in speed makes light to bend at the boundary between air and the medium. If light beam enters a glass medium from air, its speed decreases making it bend towards the normal. The speed of light always decreases in a denser medium; light travels through glass with about two-third of its speed. When light beam passes from a denser medium to a less dense medium (e.g. from glass to air) the speed increases making it to bend away from the normal. The presence of the new medium which changes the speed of light is the cause of refraction.

## Refraction through rectangular glass prism

The path of light through a transparent rectangular block is shown in Figure 12.3. A narrow **incident light beam LO** from the ray box strikes the rectangular glass block at **O** and is refracted towards the normal along the path **OU**. The ray **OU** is the **refracted ray**. The refracted ray comes out from the side **DC** of the rectangular block bending away from the normal along the path **UM**. The ray **UM** is the **emergent ray**. The angle the emergent ray makes with the normal is the **angle of emergence** ( $e$ ). The emergent angle ( $e$ ) is always equal to the incident angle ( $i$ ).

The emergent ray is displaced laterally from its original path. The perpendicular distance between the new and old paths is known as **lateral displacement** ( $d$ ).

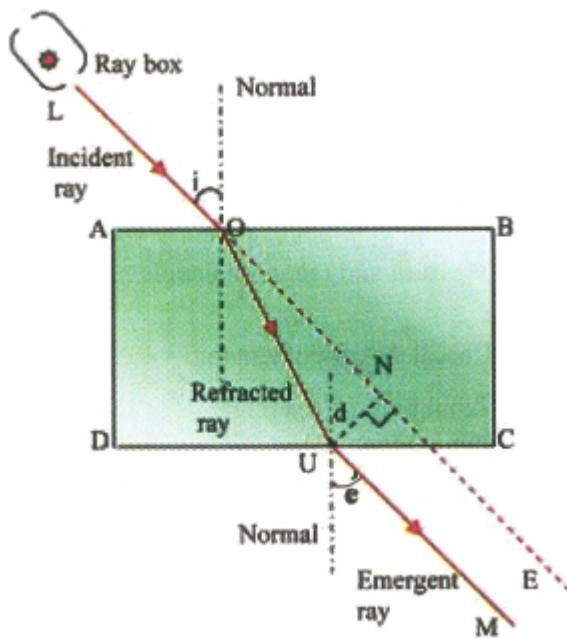


Figure 13.3: Refraction through rectangular block

## Verification of Snellâ€™s law of refraction

**Apparatus:** Rectangular glass block, four optical pins, ruler, sharp pencil and tracing paper.

### Method

- Place the rectangular glass block on the tracing paper and trace the outline ABCD. Remove the glass block and draw a normal NM at O.

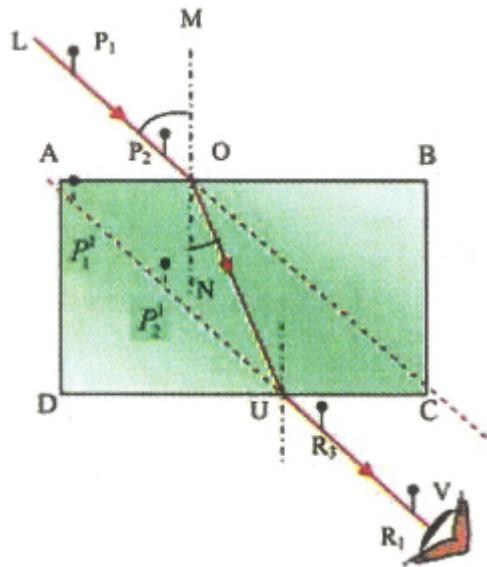


Figure 13.4: Verifying Snellâ€™s law of refraction

- Measure the angle of incidence  $i = 30^\circ$  and draw the line LO to represent the incident ray. Fix two pins  $P_1$  and  $P_2$  on the line LO.
- Return the glass block on the outline exactly. Looking from the side DC of the glass block, fix two other pins  $P_3$  and  $P_4$  such they appear

to be in A straight line with the images  $P'_1$  and  $P'_2$  of pins  $P_1$  and  $P_2$  as seen from the side DC of the glass block.

- (iv) Remove the glass block and join the points  $P_3$  and  $P_4$  to meet the glass block at U. Join also OU.

$$\frac{\sin r}{\sin i}$$

- (v) Measure and record the angle  $r$  and evaluate  $\sin i \sin r$  and  $\frac{\sin r}{\sin i}$ .

- (vi) Repeat the experiment for  $i = 40^\circ, 50^\circ, 60^\circ$  and  $70^\circ$ . In each case evaluate  $\sin i$ ,  $\sin r$  and  $\frac{\sin i}{\sin r}$ .

- (vii) Tabulate your readings.

- (viii) Plot a graph of  $\sin i$  on the vertical axis and  $\sin r$  on the horizontal axis.

- (ix) Find the slope (s) of your graph and compare it with the ratios of  $\frac{\sin i}{\sin r}$ .

$$\frac{\sin i}{\sin r}$$

**Conclusion:** The average of the ratios  $\frac{\sin i}{\sin r}$  is the refractive index of the rectangular glass block. The slope (s) of  $\sin i$  against  $\sin r$  is equal to  $\frac{\sin i}{\sin r}$  the refractive index of the glass block.

**Precautions:** The precautions taken to ensure accurate results are:

- (i) A sharp pencil is used throughout the experiment to ensure neat tracing.
- (ii) The optical pins should be erect and well spaced.
- (iii) Parallax error is avoided while using the protractor.

## Refraction through triangular glass prism

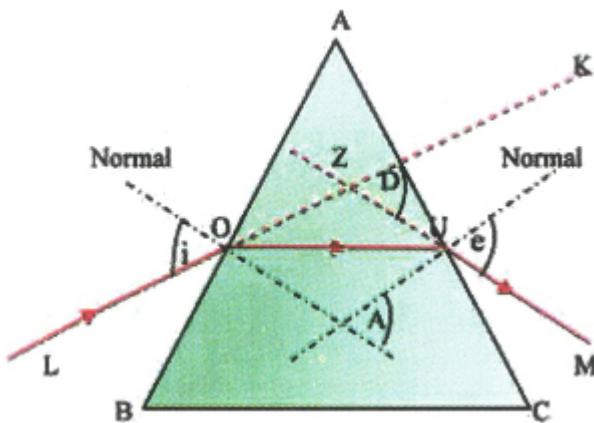


Figure 13.5: Refraction of light through triangular prism

Figure 13.5 shows the path of light through a triangular prism. The ray which emerges from the side AC of the prism, bends toward the base of the prism. The incident ray is deviated by the prism from its original path. Light now travels along the path UM instead of the

original path LK.

The deviation of light is due to double refraction at the faces AB and AC of the triangular prism.

### **Deviation is the change in direction of incident light when it meets a boundary.**

The angle D formed between the incident light and the emergent light is the **angle of deviation**. The amount of deviation produced depends on:

- the refracting angle (A) of the prism;
- refractive index of the material from which the prism is made of;
- angle of incidence (i).

The angle of deviation ( $D$ ) varies as the angle of incidence ( $i$ ) is increased. It decreases to a minimum value ( $D_{Min}$ ) as the angle of incidence increases from zero and then increases to a maximum value as the angle of incidence approaches  $90^\circ$  as shown in Figure 12.6.

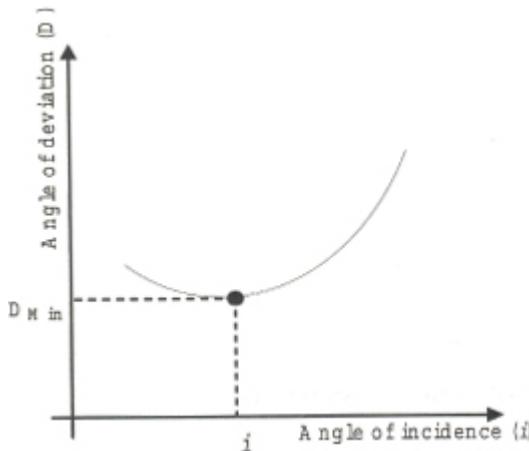


Figure 13.6: Graph of angle of deviation against angle of incidence

Minimum angle of deviation ( $D_{Min}$ ) occurs when:

- (i) the angle of incidence ( $i$ ) is equal to the angle of emergence ( $e$ );
- (ii) the path of the refracted light (OU) inside the triangular prism is parallel to its base BC;
- (iii) the incident light is symmetrical to the emergent light (that is they make the same angle with the faces AB and AC of the prism).

For minimum angle of deviation ( $D_{Min}$ ), the angle of incidence ( $i$ ) is given by:

$$i = \frac{A + D_{Min}}{2}$$

The corresponding angle of refraction (r) is given by:

$$r = \frac{A}{2}.$$

The refractive index of the material of the prism from Snell's law of

refraction is given by:

$$n = \frac{\sin i}{\sin r} \Rightarrow n = \frac{\sin \frac{(A + D_{Min})}{2}}{\sin \frac{A}{2}}$$

## Worked examples

1. The refracting angle of a triangular prism is  $60^\circ$  and the angle of minimum deviation is  $40^\circ$  calculate the:
  - (i) angle of incidence;
  - (ii) refractive index of the prism.

### Solution

$$(i) i = \frac{A + D_{Min}}{2} = \frac{60^\circ + 40^\circ}{2} = 50^\circ$$

$$(ii) n = \frac{\sin \frac{(A + D_{Min})}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{(60^\circ + 40^\circ)}{2}}{\sin \frac{60^\circ}{2}}$$

$$n = \frac{\sin 50^\circ}{\sin 30^\circ} = \frac{0.7660}{0.5000} = 1.532$$

2. The angle of incidence of a narrow beam of light on one face of an equilateral triangular prism is  $48^\circ$ . Calculate the:
  - (i) angle of minimum deviation;
  - (ii) angle of refraction;
  - (iii) refractive index of the material of the prism.

### Solution

$$(i) i = \frac{A + D_{Min}}{2} \Rightarrow 48^\circ = \frac{60^\circ + D_{Min}}{2}$$

$$D_{Min} = 96^\circ - 60^\circ = 36^\circ$$

$$(ii) r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$$

$$(iii) n = \frac{\sin i}{\sin r} = \frac{\sin 48^\circ}{\sin 36^\circ} = \frac{0.7431}{0.5878} = 1.26$$

## How to find the refractive index of a triangular glass prism

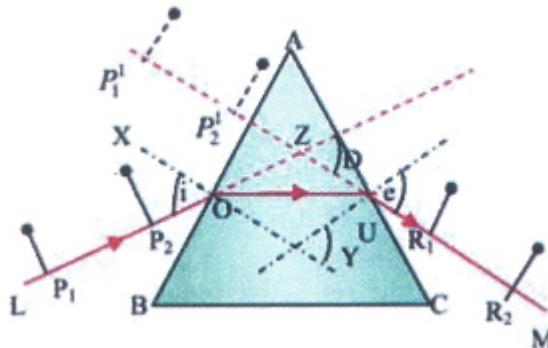


Figure 13.7: Determining the refractive index of a triangular prism

**Apparatus:** Triangular glass prism, four optical pins, ruler, sharp pencil and tracing paper

### Method

- (i) Trace the outline of the triangular glass prism ABC on the tracing paper. Remove the prism and draw a normal XY on the face AB of the prism.
- (ii) Draw a line LO to represent the incident light such that  $i = 40^\circ$ . Fix two pins  $P_1$  and  $P_2$  on the line LO and replace the prism exactly on the outline.
- (iii) From the face AC of the triangular prism fix two other pins  $R_1$  and  $R_2$  such that they appear to be in straight line with the images  $P'_1$  and  $P'_2$  of pins  $P_1$  and  $P_2$ .
- (iv) Remove the triangular prism and join  $R_1$  and  $R_2$  with a line to meet the face AC of the prism at U.
- (v) Join OU, draw a normal at the point U. Measure and record the angle of deviation  $D$ .
- (vi) Repeat the experiment for angle of incidence  $i = 45^\circ$ ,  $50^\circ$ ,  $55^\circ$  and  $60^\circ$ . In each case, measure the angle of deviation  $D$ .
- (vii) Tabulate your readings.
- (viii) Plot a graph of  $D$  on the vertical axis and  $i$  on the horizontal axis. Join your points with a smooth curve.
- (ix) From your curve, determine the angle of minimum deviation  $D_{\text{Min}}$  and the angle of incidence  $i$  for minimum deviation.

$$(x) \quad \text{Evaluate } n = \frac{\sin \frac{(A + D_{\text{Min}})}{2}}{\sin \frac{A}{2}}$$

- (xi) State two precautions taken to ensure accurate results.

### Summary

**â€¢ Refraction** is the bending or change in direction of light as it passes from one medium to another.

**â€¢ Refraction** is caused by a change in the speed of light as it passes from one medium to another.

**â€¢ The laws of refraction states that:**

â€¢ The incident and refracted rays are on the opposite sides of the normal, and at the point of incident, all three lie in the same plane.

â€¢ The ratio of sine of the angle of incidence to the sine of angle refraction is a constant for any pair of media.

$$\bullet \frac{\sin i}{\sin r} = \text{a constant}$$

(Snellâ€™s law of refraction)

**â€¢** The constant of Snellâ€™s law is the **refractive index** ( ${}_{\text{a}}n_g$ ) of the second medium with respect to the first medium.

$${}_{\text{a}}n_g = \frac{\text{Sine of angle of incidence in air}}{\text{Sine of angle of refraction in glass}}$$

**â€¢ Refractive index** of a medium is the ratio of the speed of light in air to the speed of light in glass.

$${}_{\text{a}}n_g = \frac{\text{Speed of light in air}}{\text{Speed of light in glass}}$$

**â€¢** When light is refracted through a triangular block, the path of the emergent ray is always **laterally displaced** from its original path.

**â€¢** The slope of the graph  $\sin i$  against  $\sin r$  is equal to  $\frac{\sin i}{\sin r}$  equal to the refractive index of the glass block.

**â€¢ Deviation** is the change in direction of incident light when it meets a boundary. Minimum angle of deviation occurs if:

â€¢ the angle of incidence ( $i$ ) is equal to the angle of emergence ( $e$ );

â€¢ the path of the refracted light (OU) inside the triangular prism is parallel to its base BC;

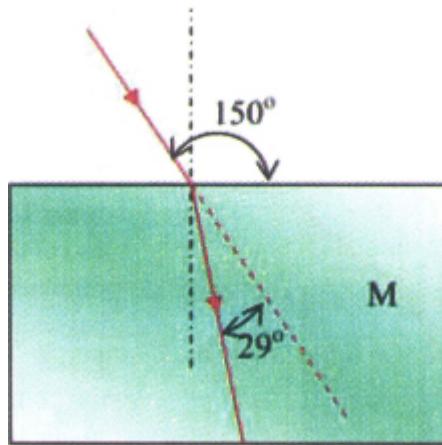
**â€¢** Refractive index for a triangular prism is given by:

$$n = \frac{\sin \frac{(A + D_{\text{Min}})}{2}}{\sin \frac{A}{2}}$$

### Practice Questions 13a

1. (a) Explain the term refraction of light;  
(b) State the laws of refraction of light;  
(c) A ray of light enters a medium M as shown in the diagram

below. Calculate the refractive index of the medium M.



2. (a) State the laws of refraction.  
(b) Describe an experiment to determine the refractive index of a rectangular glass block.  
(c) State two precautions you should observe if you were to perform the experiment in the school laboratory.  
(d) Make a sketch to show the path of light through a rectangular prism when the angle of incidence is  $90^\circ$ .
3. (a) Explain what you understand by **refractive index** of a medium.  
(b) Draw a diagram showing the path of light as it passes from air to water at an angle of  $60^\circ$ . Explain why the ray of light bends towards the normal.
4. (a) Explain the meaning of the statement, "the *refractive index of glass is 1.5*".  
(b) State **two** conditions for a minimum angle of deviation to occur.  
(c) A narrow light beam is deviated by a triangular glass prism of refractive index 1.50. Given that the refracting angle of the prism is  $72^\circ$ , Calculate the:
  - (i) angle of minimum deviation;
  - (ii) angle of incidence;
  - (iii) angle of refraction.
5. (a) Describe an experiment to determine the refractive index of a triangular glass prism.  
(b) State **two** precautions you should observe to obtain correct results.

## 1. Effects caused by refraction

### OBJECTIVE

At the end of the topic students should be able to:

- use the change in direction of light as it travels from one medium to another to explain:
  - â€¢ apparent bending of long stick partially immersed in a liquid;
  - â€¢ apparent depth of a pool of water;
  - â€¢ apparent position of the Sun.
- describe the experiment to determine the refractive index of a transparent solid and liquid using apparent depth method.

## 1. Apparent bending of a stick partially immersed in water

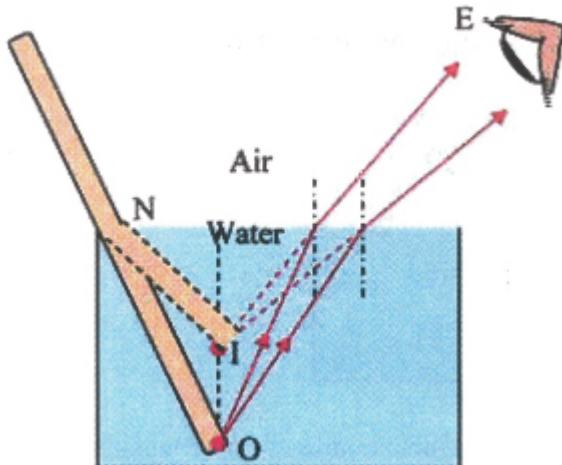


Figure 13.8: Partial bending of the stick is due to refraction

A straight stick partially immersed in water or any other transparent liquid appears bent at the boundary of air and water. Light rays coming from the lower end of the stick O immersed in water is refracted away from the normal as it passes from water to air. The observer E sees the light rays as coming from the point I. The point I is the virtual image of O, therefore the stick appears bent at the boundary of water and air.

## 2. Real and apparent depth

The depth of ponds and swimming pools seem closer than they really are. Letters under a rectangular glass block are closer than other letters outside the glass block. These observations are explained by refraction of light. Light rays coming from a point O under the glass block or water are refracted away from the normal at the water surface.

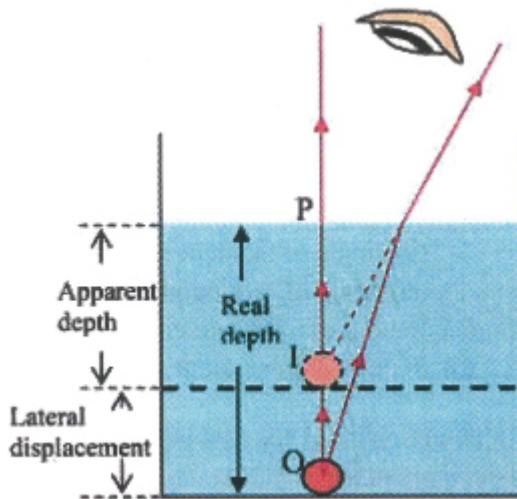


Figure 13.9: Real and apparent depth

The refracted rays enter the observer's eye E as if they are coming from the point I. The point I is the image of the point O at the bottom of the pool or glass block.

The distance PI is the apparent depth of water. It is the distance of the image of the bottom of the pool from the top. The true distance PO of the object from the top of the pool is the real depth.

The refractive index of the medium is given by:

$$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

The refractive index of water is 1.33. A pool of water is 1.33 times deeper as it appears to the observer when it is viewed vertically from the top as shown in Figure 13.9.

Real and apparent depths are used to bring objects hidden in a depth into view. A coin placed at the bottom of cup hidden from the observer can be brought into view by filling the cup with water. Light rays from the coin are refracted away from the normal to the observer's eye. The coin becomes visible without moving the cup or changing the position of the observer.

### 3. Apparent displacement of objects viewed through a prism

Looking at objects through a prism, the image seen by the observer is displaced slightly. Figure 12.11 illustrates this, a ray coming from the object O is refracted twice by the prism. The emergent ray is deviated from the incident ray such that the observer sees the object as coming from the position I.

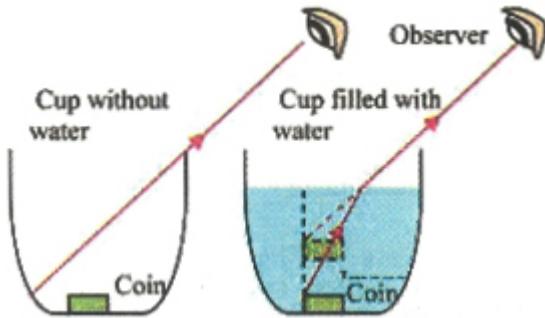


Figure 13.10: Bringing object into view by refraction

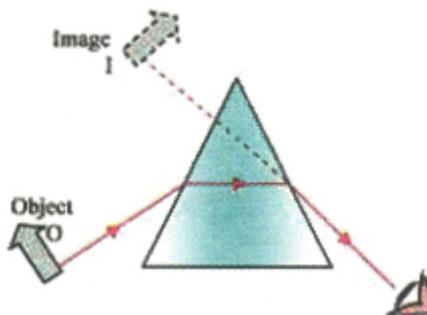


Figure 13.11a: Apparent displacement of object

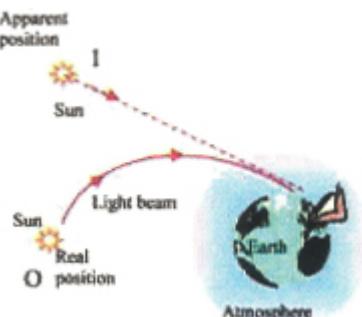


Figure 13.11b: Refraction by earth's atmosphere

The earth's atmosphere acts like a prism refracting beams of light from the sun. The consequence is that sun rises early in the morning and sets late in the evening. Light beams from the sun travelling through vacuum are refracted as they enter the earth's atmosphere. The refracted beam enters the observer's eye as if it is coming from the position I. The observer E sees the sun at the position I before it really comes to that position.

## **Real and apparent depth method of finding the refractive index**

### **(i) Rectangular glass block**

The thickness of the glass block is measured using a metre rule. A pin (P) is placed under the glass block and viewed vertically by the observer E as shown in Figure 13.12. The apparent position of the pin P is found by using a search pin (S) attached to a sliding cork (C). The search pin (S) is moved up and down until no parallax exist between it and the image of the pin (P) seen through the glass. The distance of the

image of the pin (I) is measured from the top of the glass block. Refractive index of the glass block is given by:

$$n_g = \frac{\text{Real thickness of glass block}}{\text{Apparent thickness of glass block}}$$

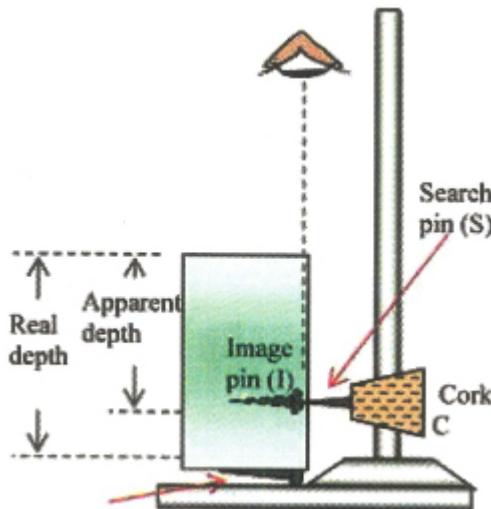


Figure 13.12: Finding the refractive index using real and apparent depth method

## (ii) Liquid

An object pin **O** is dropped at the bottom of a tall measuring cylinder filled with water up to half of its length. The search pin **S** mounted on a cork is moved up and down until the image **I** of the object pin formed by refraction coincides with the search pin **S**. The distance **h** of the search pin from the base of the measuring cylinder is measured and recorded. The experiment is repeated five more times using different lengths of water column.

## Observations

Real depth of water in the cylinder =  $r$  cm;

Distance of image pin **I** from the base =  $h$  cm;

Apparent depth of water =  $a = (r - h)$  cm.

$$\text{Refractive index} = \frac{r}{r - h}$$

## Precautions

- (i) The real depth is read from the lower meniscus of water;
- (ii) Parallax error is avoided when taking the reading from metre rule;
- (iii) Zero error is avoided on the metre rule.

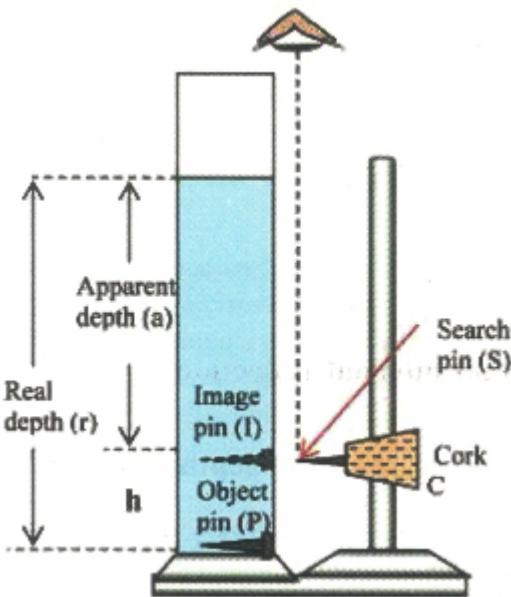


Figure 13.13

## Worked example

- Calculate the real depth of a swimming pool if the apparent depth is 1.2 m. Refractive index of water is  $\lambda = \left[\frac{4}{3}\right]$

### Solution

$$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\frac{4}{3} = \frac{\text{Real depth}}{1.2}$$

$$\text{Real depth} = \frac{4 \times 1.2}{3} = 1.6m$$

## Summary

â€¢ Bending of a long stick partially immersed in a liquid is due to refraction of light.

â€¢ Depth of swimming pools, ponds and alphabets under a glass block appears raised or less deep than they really are due to refraction of light.

â€¢ The refractive index of a material is given by:

$$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

## Practice Questions 13b

- (a) What is refractive index of a medium?  
(b) Explain why light travelling from water to air bends away from the normal.

- (c) Explain why a long stick partially immersed in water appears bent at the boundary of water and air.
2. (a) Using a labelled diagram explain why the bottom of a swimming pool seems closer to an observer looking down vertically into the water.
- (b) The bottom of a swimming pool 10m deep is filled with water of refractive index of  $\left(\frac{4}{3}\right)$ . Calculate the apparent displacement of the bottom of the pool.
3. (a) Describe an experiment to find the refractive index of water.
- (b) State two precautions that must be observed to ensure accurate results.
- (c) In an experiment to find the refractive index of water using a tall measuring cylinder, water is poured into the cylinder until the length of its column is 30 cm. The lateral displacement of the bottom of the cylinder is 7.5 cm. What is the refractive index of water?

## Total internal reflection

### OBJECTIVES

At the end of this topic, students should be able to:

- explain the meaning of critical angle and total internal reflection;
- state the conditions for total internal reflection to occur;
- establish the relationship between critical angle and refractive index;
- solve simple problems on total internal reflection;
- state some uses of total internal reflection.

A semi-circular transparent medium is most suitable for studying total internal reflection. This is because light entering the medium from the curved surface is not deviated until it meets the flat surface.

A narrow beam of light entering the medium at small angle of incidence ( $i$ ) splits into two parts, a faint light reflected back into the medium and bright light refracted away from the normal as it emerges into the air. The angle of refraction ( $r$ ) in air is more than the angle of incidence ( $i$ ) in the medium. See Figure 13.14.

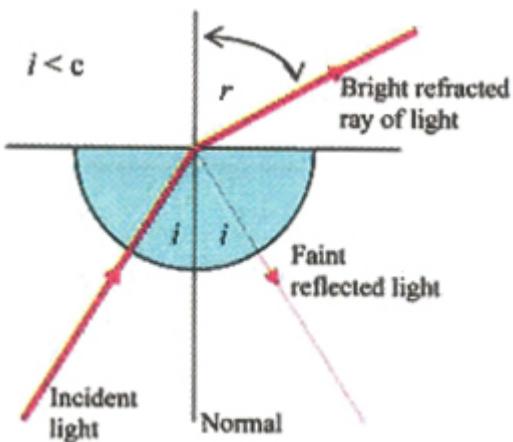


Figure 13.14a: When  $i < c$

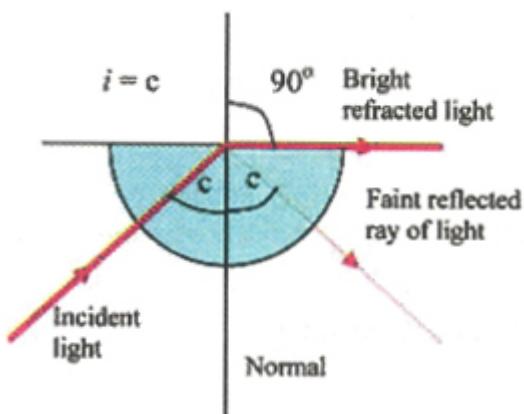


Figure 13.14b:  $i = c$

The refracted ray is bright and the reflected ray is faint. Reflection that occurs is faint. When the angle of incidence ( $i$ ) is increased, the angle of refraction ( $r$ ) in air increases until maximum angle of refraction ( $r = 90^\circ$ ) is obtained. The refracted light grazes the flat surface of the semi circular block (passes exactly along the boundary of air and the medium) as shown in Figure 13.14b.

The angle of incidence when the angle of refraction is  $90^\circ$  is the **critical angle** of the medium.

**Critical angle is the angle of incidence in the transparent medium when the angle of refraction is  $90^\circ$  (or maximum).**

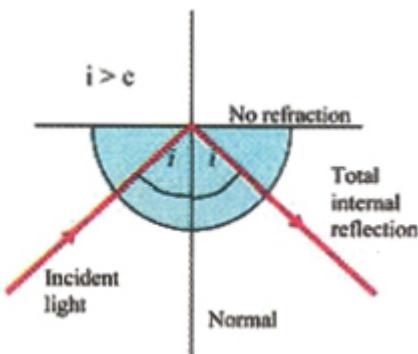


Figure 13.14c: When  $i > c$  total internal reflection occurs

If the angle of incidence ( $i$ ) increases above the critical angle ( $c$ ) of the medium, all the incident light reflects inside the transparent medium and no part of the light is refracted into the air. When this happens **total internal reflection** occurs.

**Total internal reflection is the complete reflection of incident ray inside the denser medium at the boundary of the two media.**

It is called **total internal reflection** because the entire light is reflected inside the denser medium.

### Conditions for total internal reflection

Light must travel from a denser medium (glass) to a less dense medium (air).

The angle of incidence ( $i$ ) in the denser medium must be greater than the critical angle ( $c$ ) of the denser medium.

### Critical angle and refractive index

The angle of incidence in the denser medium when the angle of refraction in air is  $90^\circ$  is called the **critical angle**. Applying Snellâ€™s law of refraction to Figure 13.14b gives:

$$_g n_a = \frac{\sin c}{\sin r} = \frac{\sin c}{\sin 90^\circ}$$

$$\therefore _g n_a = \sin c$$

Refractive index of the material of the medium (glass) is given by:

$$\sin c = \frac{1}{_a n_g} \text{ or } {}_a n_g = \frac{1}{\sin c}$$

### Worked examples

- What is the critical angle of water if the refractive index of water is  $\frac{4}{3}$ ?

### Solution

$$\sin c = \frac{1}{_a n_w} = \frac{1}{\frac{4}{3}} = \frac{3}{4} = 0.7500 \quad c = \sin^{-1}(0.7500) = 48.6^\circ \approx 49^\circ$$

- Calculate the critical angle of glass given that the refractive index of glass 1.5.

### Solution

$$\sin c = \frac{1}{_a n_g} = \frac{1}{1.5} = 0.6667$$

$$c = \sin^{-1}(0.6667) = 41.8^\circ \approx 42^\circ$$

The critical angle of glass is approximately  $42^\circ$ . If light travelling from glass to air meets the boundary of air and glass at an angle

greater than  $42^\circ$ , it will be totally reflected inside the glass material.

## Applications of total internal reflection

### 1. Totally reflecting prisms:

A right-angled isosceles triangle prism ( $45^\circ - 90^\circ$ ) has the ability of reflecting the entire incident light inside the prism. It is used to turn the path of light through  $90^\circ$  or  $180^\circ$ . This is because a ray of light entering the prism normally on any face meets any other face at an angle of incidence ( $i = 45^\circ$ ) greater than the critical angle of glass ( $c = 42^\circ$ ) and is totally internally reflected.

#### (a) Bending light through $90^\circ$

A narrow beam of light enters normally through the side AC of the prism and meets the face AB at an angle of incidence ( $i = 45^\circ$ ). This angle is greater than the critical angle of glass ( $42^\circ$ ). The face AB acting like a mirror reflects the entire incident light turning it through an angle of  $90^\circ$ .

#### (b) Bending light through $180^\circ$

The incident light beam is totally internally reflected twice inside the prism at the faces AC and BC. Light that emerges from the prism is turned through  $180^\circ$ . When the isosceles right-angled triangular prism is used in this way, it is called an **erecting prism**. An erecting prism is used in prism binoculars and cameras to produce erect images.

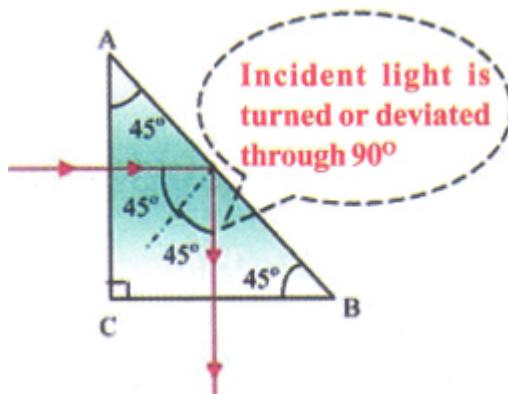


Figure 13.15a: Using a right-angled triangular prism to bend light through  $90^\circ$ .

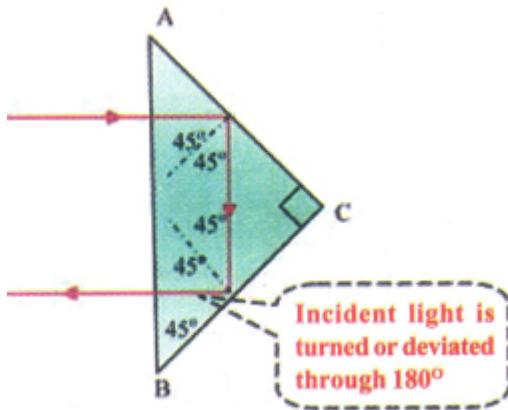


Figure 13.15b: Using a right-angled triangular prism to bend light through  $180^\circ$

## 2. Prism periscope

Prism periscope is produced by arranging two isosceles right-angled triangular prisms as shown in Figure 12.16.

Light from the object is turned by the top prism through  $90^\circ$  and emerges from the side BC of the top prism. The bottom prism turns the light again through  $90^\circ$ , the observer E sees an image of the object at the position I.

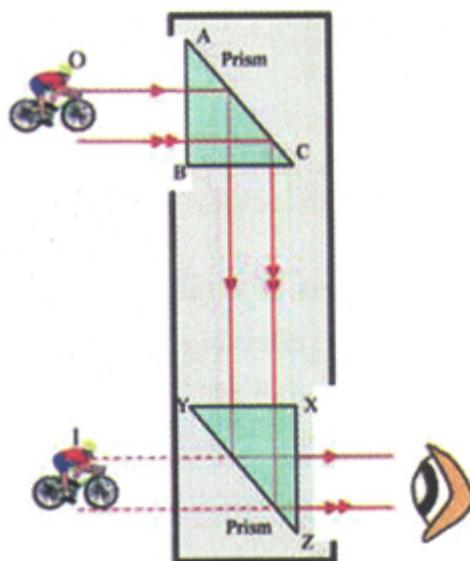


Figure 13.16: Prism periscope

## Advantages of using prism over mirror in optical instrument

Prism periscopes are better than mirror periscopes for the following reasons.

1. The totally reflecting prism produces only one image but a mirror produces multiple images due to multiple reflections caused by the thickness of the mirror.
2. A totally reflecting prism reflects more light while a mirror reflects

less light.

3. The coating at the back of the mirror is easily damaged.

## How mirrors produce multiple images

The mirror made from thick glass produces multiple number of images due to multiple reflections as shown in Figure 13.17. Each reflection produces a faint image called **ghost image** except the second image, which is bright and clear. Ghost images can cause confusion in optical instruments like telescopes, cameras and periscopes.

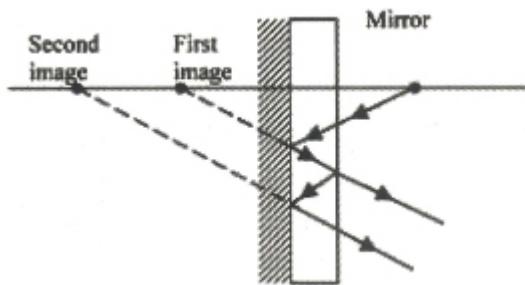


Figure 13.17: How a mirror produce images

## 3. The fish eye-view

A fish or swimmer under water can see every object outside the water surface inside a cone with an angle of  $98^\circ$ , by refraction of light. The water-air boundary outside the cone of  $98^\circ$  acts like a mirror reflecting light from objects inside the water. Fishes see objects inside water and the seabed (outside the cone of  $98^\circ$ ) by total internal reflection at the water-air boundary.

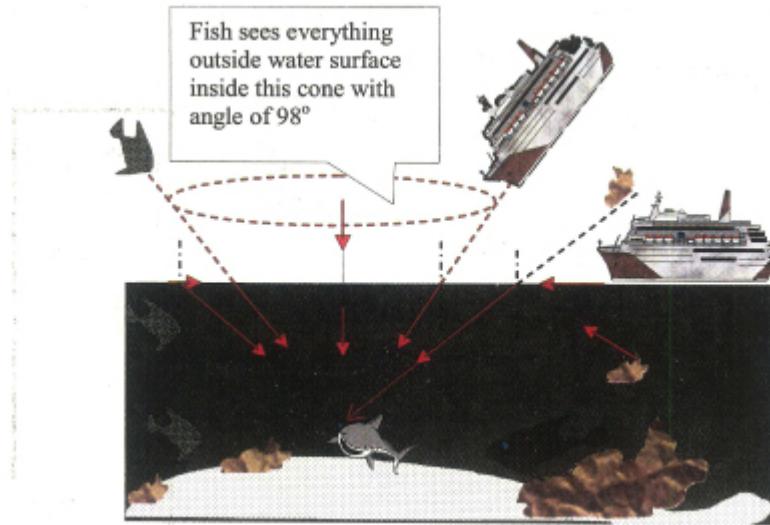


Figure 13.18: Fish eye-view

The critical angle of water is  $49^\circ$ . A ray moving along the surface of the water is refracted into the water such that the angle of refraction is  $49^\circ$  (critical angle of water). Every object on the surface of water is squeezed into a cone of  $98^\circ$ .

## 4. The mirage

Travellers on highways sometimes see ahead of them what looks like a pool of water on the road. This is called a **mirage**. Mirages are formed when light rays from the sky are gradually refracted away from the normal until total internal reflection occurs close to the surface of the road. This happens because air close to the road is hotter and less dense and therefore has lower refractive index. The refractive index of cool air above road surface is higher than that of the hot air near the ground. When light rays from the sky enter the hot air from the cool air, it is refracted away from the normal until the angle of incidence close to the road is greater than the critical angle of the hot air, total internal reflection therefore occurs. The reflected ray moves upwards until it enters the observer's eye. The observer sees the image of the sky as a pool of water on the road ahead.

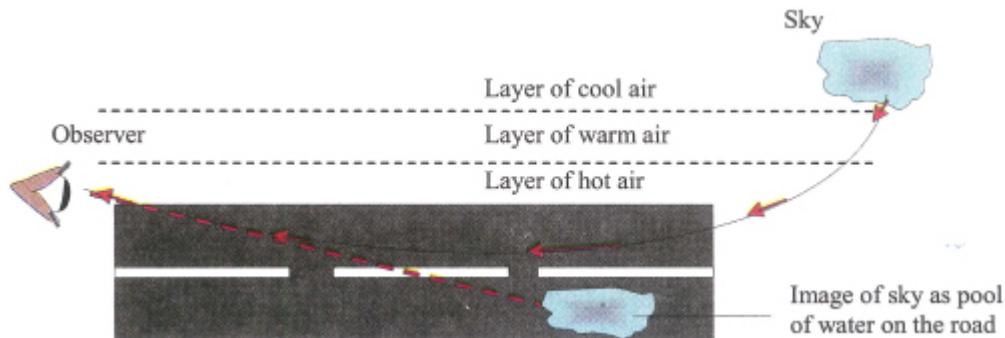


Figure 13.19: Formation of mirage

## 5. Optical fibres

Optical fibres or light guides are used to transmit or guide light through long distances by totally reflecting light inside transparent fibres. The fibres are made of many thousands of fine strands of high quality perspex or glass with high refractive index. Another glass of lower refractive index surrounds this. If light enters the fibres from one end such that the angle of incidence is more than the critical angle of the glass, it undergoes total internal reflection many times until it emerges from the other end with almost the same intensity.

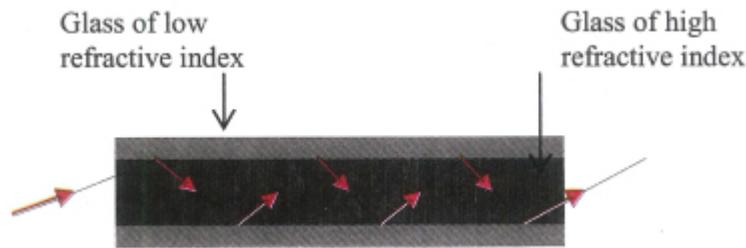


Figure 13.20: Optical fibre

## Uses of optical fibres

1. An endoscope makes use of optical fibres or light guides to view the

internal structures of the lungs and stomach to enable pictures of these hidden parts to be taken.

2. Optical fibres are used in decorative lighting and in some toy cars to give light to the headlamps of these toy cars.
3. Optical fibres are used in television and telecommunications to carry information over a long distance.
4. Optical fibres are now used in place of electricity as indicator lamps on hi-fi equipment, car dashboards and burglary alarm and fire detectors.

## Summary

â€¢ **Critical angle** is the angle of incidence in the transparent medium when the angle of incidence is  $90^\circ$ .

â€¢ **Total internal reflection** occurs when the entire incident light is completely reflected inside the denser medium at the boundary of air and the denser medium.

â€¢ Conditions for total internal reflection to occur are:

â€¢ Light must travel from a denser medium (glass) to a less dense medium (air).

â€¢ The angle of incidence (i) in the denser medium must be greater than the critical angle (c).

â€¢ **Refractive index** of the material of the medium (glass) at critical angle is given by:

$$\sin c = \frac{1}{\text{refractive index}} \text{ or } \text{refractive index} = \frac{1}{\sin c}$$

## Uses of total internal reflection

â€¢ Total internal reflection is used in right-angled isosceles ( $45^\circ - 45^\circ - 90^\circ$ ) prism to turn a narrow beam of light through  $90^\circ$  or  $180^\circ$ .

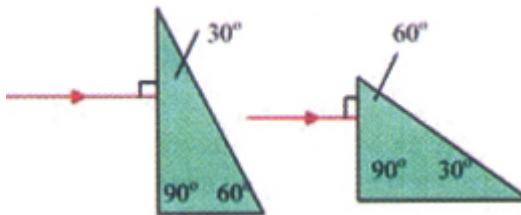
â€¢ Total internal reflection is used in prism periscopes to view objects over an obstacle.

â€¢ Total internal reflection is used in optical fibres to transmit information from one place to another.

## Practice Questions 13

1. (a) What do you understand by: (i) *total internal reflection?* (ii) *critical angle?*  
(b) State the conditions which must be fulfilled for total internal reflection to occur.  
(c) A narrow beam of light enters a right-angled isosceles triangular prism normally. Draw a diagram to show the path of light through the prism and explain why total internal reflection occurs at the hypotenuse side.

2. (a) What is meant by the *critical angle of glass* is  $1.5^\circ$ ?
- (b) Describe how you can find the critical angle of glass. Explain why a semi-circular glass prism is preferred to a rectangular glass in determining the critical angle of glass.
- (c) Show by means of diagrams how  $45^\circ - 45^\circ - 90^\circ$  glass prism can be used to turn a ray of light by (i)  $90^\circ$  (ii)  $180^\circ$ .
3. Explain the following observation:
  - (a) A fish swimming in water sees every object on the surface of water and outside the water in a cone with angle of  $98^\circ$ .
  - (b) The surface of water beyond the cone with angle of  $98^\circ$  looks like a mirror to the fish.
  - (c) Travellers on highway sometimes see what looks like pool of water on the road at a distance ahead.
4. (a) Explain the terms: *refractive index*, *critical angle* and *total internal reflection*.
- (b) The diagrams below shows a ray of light passing normally into a  $30^\circ-60^\circ-90^\circ$ -triangular glass prism of refractive index 1.5.



- (i) Complete the diagrams showing the path of light through each triangular glass prism.
- (ii) Explain why the light does not emerge normally from the prisms.
5. (a) Explain the meaning of *total internal reflection*.
- (b) What is an optical fibre? How does it transmit light over a long distance?
- (c) State **three** applications of optical fibres.
- (d) Give **three** reasons why a totally reflecting triangular prism is better mirror in optical instruments to reflect light.

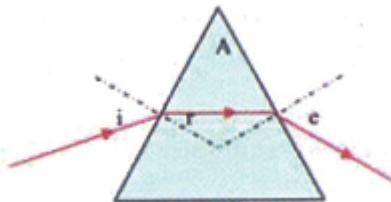
## Past questions

1. A ray of light is incident normally on an air-glass interface. What is its angle of refraction?
  - A.  $90^\circ$ .
  - B.  $60^\circ$ .
  - C.  $30^\circ$ .
  - D.  $0^\circ$ .

**WASSCE**

2. The diagram below shows the path of a ray of light through an

equilateral triangular glass prism. If all the symbols have their usual meanings, which of the following equations is true when there is minimum deviation?



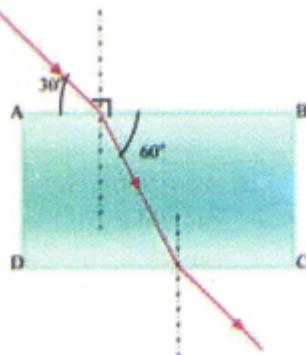
- A.  $i = r$
- B.  $r = e$
- C.  $i = e$
- D.  $i = A.$

**WASSCE**

3. The velocities of light in air and glass are  $3.0 \times 10^8 \text{ ms}^{-1}$  and  $1.8 \times 10^8 \text{ ms}^{-1}$  respectively. Calculate the sine of the angle of incidence that will produce an angle of refraction of  $30^\circ$  for a ray of light incident on glass.
- A. 1.2
  - B. 1.0
  - C. 0.8
  - D. 0.6
  - E. 0.3

**WAEC**

5. The figure below shows the path of light passing through a glass block.



- A. 1.15
- B. 1.50
- C. 1.56
- D. 1.73

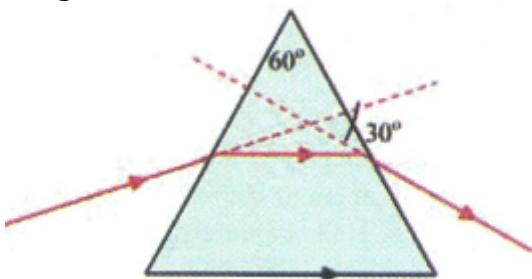
**NECO**

6. A transparent block 5.0 cm thick is placed on a dot. The dot when viewed from above is seen 3.0 cm from the top of the block. Calculate the refractive index of the material of the block.

- A.  $\frac{2}{5}$
- B.  $\frac{3}{5}$
- C.  $\frac{3}{2}$
- D.  $\frac{5}{3}$
- E.  $\frac{5}{2}$

**WAEC**

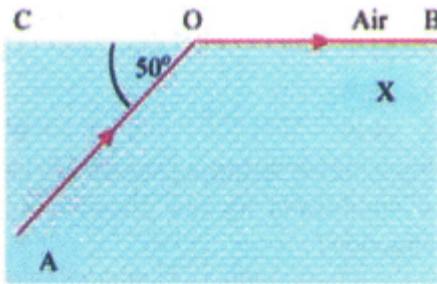
7. Calculate the refractive index of the material for glass prism in the diagram below.



- A.  $\sqrt{2}$
- B.  $\frac{3}{2}$
- C.  $\frac{\sqrt{2}}{2}$
- D.  $\frac{4}{3}$

**JAMB**

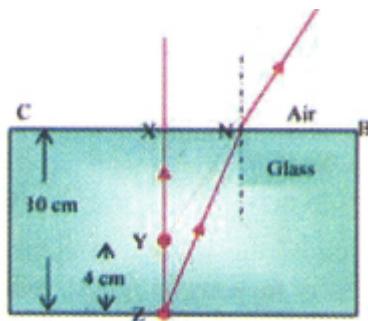
8. The diagram below shows an incident ray AO inclined at an angle of  $50^\circ$  to the interface CB. The refracted ray OB is found to lie along the surface. What is the refractive index of the medium X with respect to air?



- A.  $\frac{\sin 50^\circ}{\sin 40^\circ}$
- C.  $\frac{\sin 90^\circ}{\sin 50^\circ}$
- B.  $\frac{\sin 40^\circ}{\sin 50^\circ}$
- D.  $\frac{\sin 40^\circ}{\sin 90^\circ}$
- E.  $\frac{\sin 90^\circ}{\sin 40^\circ}$

**WAEC**

9. Calculate the refractive index of the material of the glass block shown in the diagram below.



- A. 0.40
- B. 0.60
- C. 1.50
- D. 1.67
- E. 2.50

**WAEC**

10. A ray of light is incident on an angle of  $30^\circ$  on a glass prism of refractive index 1.5. Calculate the angle through which the ray is minimally deviated in the prism. (The medium surrounding the glass is air).

- A.  $10.5^\circ$
- B.  $19.5^\circ$
- C.  $21.1^\circ$
- D.  $38.9^\circ$
- E.  $40.5^\circ$

**WAEC**

11. Which of the following statements **cannot** be explained by the phenomenon of refraction?

- A. A ruler partially and obliquely placed in water appears bent at the air-water interface.
- B. The surface of a road appears wet at a distance on a hot sunny day.
- C. A convex mirror provides a wide field of view as driving mirror.
- D. It may not be possible to catch a fish by throwing a spear at the position where the fish appears to be.

**WASSCE**

12. The horizontal floor of a water reservoir appears to be 1.0 m deep when viewed vertically from above. If the refractive index of water is 1.35, calculate the real depth of the reservoir.

- A. 2.35 m
- B. 1.35 m
- C. 1.00 m
- D. 0.35 m

**WASSCE**

13. Which of the following conditions is necessary for a ray of light travelling from glass to water to be totally internally reflected? The

- A. angle of incidence in glass must be less than the critical angle.
- B. incident ray must lie along the boundary of the two media.
- C. angle of incidence in glass must be greater than the critical angle.
- D. angle of incidence in glass must be equal to the critical angle.

**WASSCE**

14. The refractive index of a medium relative to air is 1.8. Calculate, to the nearest degree, the critical angle for the medium.

- A.  $68^\circ$ .
- B.  $56^\circ$ .
- C.  $34^\circ$ .
- D.  $18^\circ$ .

**WASSCE**

15. Totally reflecting prisms are preferred to mirrors in periscopes because

- I. mirrors give multiple images.
  - II. silvering on mirrors is easily damaged.
  - III. mirrors reflect less light than totally reflecting prism.
- A. I only.
  - B. I and II only.
  - C. I and III only.
  - D. II and III only.
  - E. I, II and III.

**NECO**

16. Explain what is meant by the refractive index of glass.

A piece of stone is placed inside a bucket and water is poured into the bucket to a depth of 20 cm. If the refractive index of water is 1.33, calculate the apparent displacement of the stone.

**WASSCE**

17. (a) Explain the phenomena of:

(i) interference (ii) diffraction.

(b) Show by a ray diagram, how a right-angled glass prism may be used to:

(i) turn a ray through  $90^\circ$ .

(ii) invert a beam of light.

(c) State **TWO** reasons why right-angled glass prisms are preferred to plane mirrors in the construction of periscopes.

(d) A ray of light experiences a minimum deviation when passing symmetrically through an equilateral triangular glass prism.

Calculate the angle of incidence of the ray.

(Refractive index of glass = 1.5)

**NECO**

18. (a) (i) State the laws of refraction of light.

(ii) Describe an experiment to determine the refractive index of a material of an equilateral triangular glass prism using the minimum deviation method.

(b) A triangular glass prism of thickness 12 cm is placed on a mark on a piece of paper resting on a horizontal bench.

(i) Draw a ray diagram to show the apparent position of the mark in the glass prism.

(ii) If the refractive index of the material of the prism is 1.5, calculate the apparent displacement of the mark.

**WASSCE**

19. (a) Water is poured into a jar to a depth of 24 cm. The bottom of the jar appears to be raised by 6cm when viewed vertically. Calculate the refractive index of water.

(b) Can total internal reflection take place when light travels from air to water?

Give a reason for your answer.

**WAEC**

20. (a) The refractive index of light in air is less than that of light in glass. Explain this statement and relate it to:

(i) the relative value of speed of light in air to its speed in glass.

(ii) the path of a ray of light passing from a rectangular glass slab into the surrounding air, when the ray is incident on the glass-air interface at an angle of incidence equal to the

critical angle.

- (b) A ray of light experiences minimum deviation when passing symmetrically through an equilateral glass prism. Calculate the angle of incidence of the ray (refractive index of glass = 1.5).

**WAEC**

21. (a) Explain (i) *refraction of a wave* (ii) *critical angle*.
- (b) State **two** conditions necessary for
- (i) total internal reflection of a wave to occur;
  - (ii) interference wave patterns to be formed.
- (c) The distance between two successive crests of a water wave travelling at  $3.6 \text{ ms}^{-1}$  is 0.45 m, calculate the frequency of the wave.
- (d) A ray of light is incident at an angle of  $30^\circ$  at an air-glass interface.
- (i) Draw a ray diagram to show the deviation of the ray in the glass.
  - (ii) Determine the angle of deviation. [refractive index of glass = 1.50]

**WASSCE**