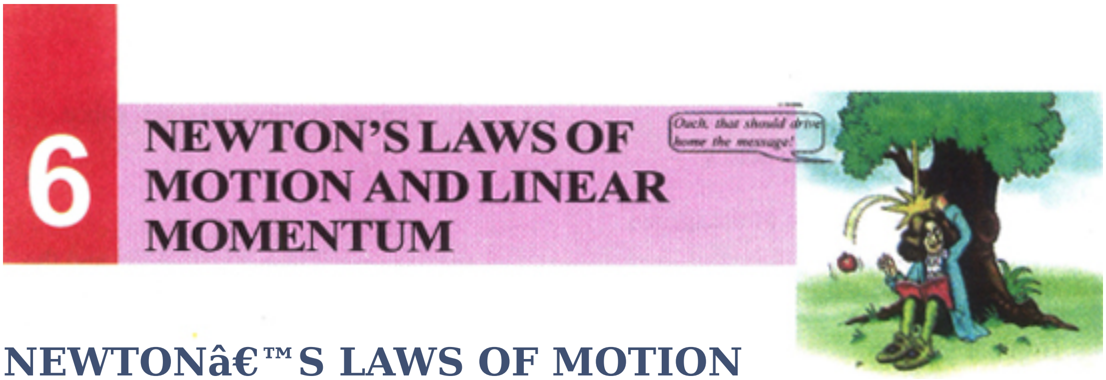


NEWTONâ€™S LAWS OF MOTION

Sir Isaac Newton was an intelligent and a hardworking scientist. As an undergraduate, he studied the works of Galileo, Kepler and other great scientist before him; through his diligence, he laid good foundation for the study of motion. His laws of motion clearly defined the link between force and motion.

OBJECTIVE

At the end of this topic, students should be able to:

- state and explain Newtonâ€™s Laws of motion;
- explain the inertial mass and relationship between mass and weight of a body;
- show graphically the relationship between;
  (i) acceleration and force for a constant mass;
  (ii) acceleration and mass for a constant force;
  (iii) show that $F = ma$ if mass is constant.
- solve simple problems based on Newtonâ€™s laws of motion.

Newtonâ€™s first law of motion

Every object continues at rest or contnue to move with constant velocity in a straight line if no external resultant force acts on it

Experience shows that a body at rest continues to remain at rest if no force is impressed on it. A moving body will come to rest after sometime if no external force acts on it. Many people because of these common experiences tend to believe that we need force to keep a body in uniform motion in a straight line. *The truth is that force is not required to keep a body in uniform motion (moving with constant velocity) but to change its velocity or motion. If no resultant force acts on a body, it either remains at rest or moves with constant velocity in a straight line.*

link between force and motion

Force is needed to change the velocity of a body in motion. Anytime a resultant force acts on a body in the same direction it is moving, the velocity increases. If the resultant force acts in the opposite direction to the motion of the body, the velocity decreases. *Every object in* *motion will always come to rest because a resultant force acts on it to oppose its motion.* This opposing resultant force is called **friction.** If friction is eliminated, all moving bodies will continue their motion in a straight line indefinitely. When a body rides on a layer of air, friction is reduced close to zero. If this is done, a moving object continues to move with constant velocity for a long time. The following are evidences in support of Newtonâ€™s first law of motion.

Aero-train or hovercraft can run at high constant velocity for a long time by riding on a layer of compressed air.

â€¢ Magnetic levitation or Maglev is a train running on a layer of air above the rail with a constant velocity. The Maglev is suspended above the rail by a powerful magnetic repulsion between it and the rail.

â€¢ Rockets and space shuttles once they escape from the influence of the earthâ€™s gravitational attraction continue to travel in straight line with a constant velocity.

All these prove that force is not needed to keep a body moving with a constant velocity.

Inertia

Inertia is the reluctance of a body to change its state of rest or motion in a straight line. Alternatively, inertia is the tendency of a body to continue at rest or constant velocity in a straight line.

The **mass** of a body determines its inertia. Bigger masses are difficult to move when they are at rest or to stop if they are moving. The mass of a body, which tends to keep it in motion or at rest, is its **inertial mass.**

Newtonâ€™s first law and inertia

Newtonâ€™s first law of motion has two parts. The first part of the law states that ***a body continues at rest if no resultant force acts on it.*** The natural tendency for a body at rest is to continue to rest. When a resultant force acts on it to change its state of rest, the body tends to resist it. The tendency of the body to remain at rest is its inertia. The second part of the law ***states that a body continues to move with a constant velocity in a straight line if no resultant force acts on it.*** If an external force acts on the body to change its uniform motion or velocity, the body resists the change. The resistance of a body to change its state of motion or velocity is its inertia. Newtonâ€™s first law of motion therefore is a statement of inertia.

Consequences of inertia

(i) Passengers sitting on the seat in a stationary bus are pushed backward when the bus moves forward suddenly.

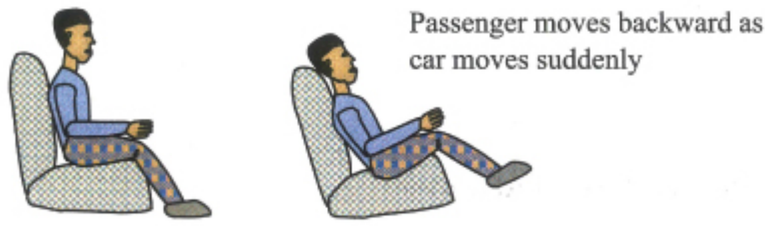


Figure 6.1a: Consequences of inertia

The sudden forward motion of the bus moves the passengerâ€™s seat forward but because the passenger tends to remain at rest, he falls backward to resist the forward motion of the bus. To avoid injuring their necks, passengers are provided with headrest to hold their heads if the bus or car moves forward suddenly.

(ii) Passengers sitting inside an aeroplane are required to fasten their seat belts at landing and taking off. In addition, passengers seating in fast moving cars should fasten their seat belts.

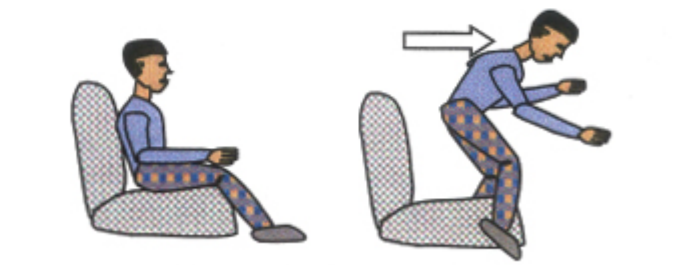


Figure 6.1b: Consequences of inertia

This is because the passengers are moved forward when the car or aeroplane is stopped suddenly. Relative to the ground the passengers are moving very fast, when the aeroplane or the car is stopped suddenly, their inertia tends to keep them moving forward, therefore they are pushed forward to continue their forward motion in a straight line.

Figure 6.1c: Consequences of inertia

***(iii) A coin resting on a thick square paper placed over the mouth of a cup as shown in Figure 6.1d drops into the cup* if the paper is flicked away quickly from the cup.**

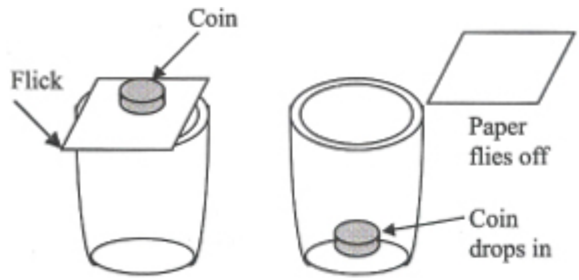


Figure 6.1d: Consequences of inertia

The coin drops into the cup because its inertia tends to keep it at rest. Some people use this trick to pull tablecloth quickly from the table without disturbing the items set on the table.

Momentum

Momentum is a quantity that measures the motion of a body.

Momentum is the product of mass and velocity of a body.

Momentum = Mass Ã— velocity

$$P = \mathrm{m}\, v$$

Momentum is a vector quantity. The unit of momentum is kilogram metre per second (Kg ms^{-1}) or Newton second (N s).

â€¢ Momentum is proportional to the mass of the body if the velocity is constant. The momentum of a basketball is greater than the momentum of a tennis ball if they are moving at the same velocity.

â€¢ Momentum is proportional to the velocity of the body if the mass is constant. A fast moving bullet penetrates a body more than a slow moving bullet of the same mass.

Momentum change (Î”P): We have already established the link between force and motion, that is, velocity of a body changes anytime a resultant force acts on it. The change in velocity means that the momentum also changes.

Î”P = mv – mu

Î”P = change in momentum, *m***v** = final momentum and *mu* = initial momentum.

Impulse

Some forces only act for a short time when they are in contact. Their impact for the time it lasted is called **impulse.**

Impulse of a force is the product of a force and the time during which it acts.

I = F Ã— t

I = impulse of the force, *F* = magnitude of force and *t* = time the force acts.

The unit of impulse is Newton second (N s). The longer the force acts, the more the impulse and the smaller the force.

Newtonâ€™s second law of motion

Force is directly proportional to the time rate of change of linear momentum and it acts in the direction of momentum change.

Newtonâ€™s second law of motion shows the link between force and momentum change. **Force is momentum change per second.** Newtonâ€™s second law of motion is expressed mathematically as:

$$\text{Force} = \frac{\text{Momentum change}}{\text{Time}}$$

$$F = \frac{m\vec{v} - m\vec{u}}{t} = \frac{m(\vec{v} - \vec{u})}{t} = \frac{m\Delta\vec{v}}{t}$$

Impulse and momentum change

The impulse of a force is numerically equal to momentum change. From Newtonâ€™s second law of motion:

Force Ã— Time = Momentum change

Impulse of a force = Momentum change

F Ã— t = *mv* – *mu*

Force, mass and acceleration

Newtonâ€™s second law of motion deals with forces which bring about are increase in velocity of a moving body. The velocity increase per second (unit time) is *acceleration,* therefore, a body accelerates if its velocity increases. The velocity increase measures the size of acceleration produced. Experiments show that:

â€¢ ***Acceleration is proportional to the resultant force if the mass of the body is constant***

$$a \propto F \quad \text{..........(i)}$$

â€¢ ***Acceleration is inversely proportional to the mass if the resultant force on the body is constant***

$$a \propto \frac{1}{m} \quad \text{..........(ii)}$$

Combining equations (i) and (ii) gives:

$$a \propto \frac{F}{m} \Rightarrow F \propto ma \therefore F = kma$$

The constant $k = 1$, therefore $F = ma$.

$$F = ma$$

Newtonâ€™s second law of motion is also stated as:

Force is directly proportional to the product of mass and acceleration of a body and it acts in the direction of acceleration.

Effect of force on acceleration

The effect of force on acceleration is studied using the set up in Figure 6.2;

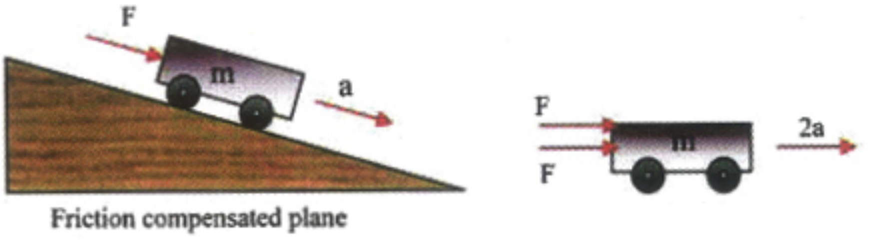


Figure 6.2: Force is proportional to acceleration

(i) A friction compensated runway is used. A runway is friction compensated if it is sloped or inclined to enable the trolley run down its slope at constant speed.

(ii) Tape from ticker timer is attached to the trolley to measure the motion of the trolley as it runs down the runway.

(iii) Constant force is exerted on the trolley by pulling on the elastic band or spring balance attached to the trolley.

(iv) The tape is cut into 10 dots tape chart, which are placed side by side to give the velocity – time graph of the motion.

(v) Repeat the experiment in steps (i) – (iv) with twice the force and three times the force. Conclusion: A constant force produces a constant acceleration. Increasing the force on the trolley increases its acceleration when the force is doubled, the acceleration is also doubled. Acceleration is proportional to the applied force. This is illustrated in Figure 6.3.

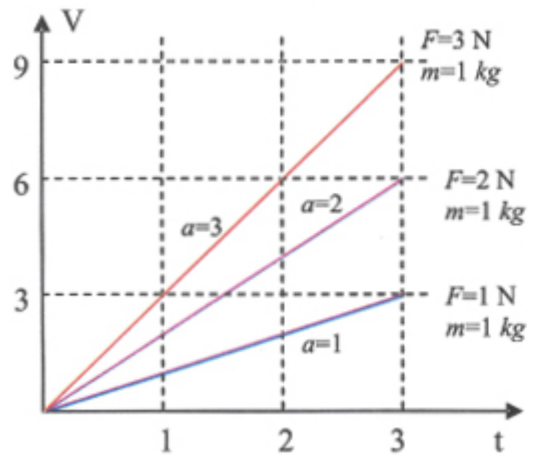


Figure 6.3: Increasing the force on a constant mass increases the acceleration

Effect of mass on acceleration

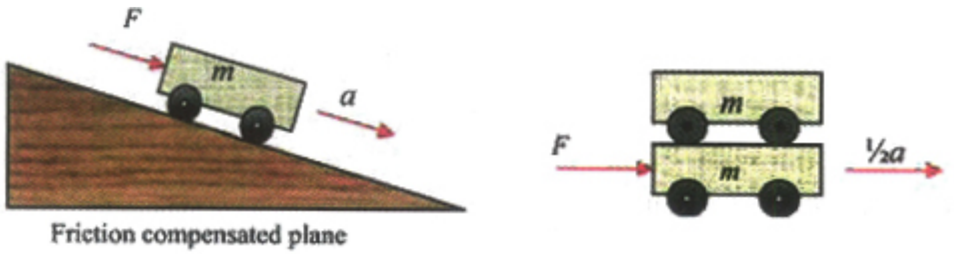


Figure 6.4: Mass is inversely proportional to acceleration

The effect of mass on acceleration is studied by using double decked trolleys as shown in Figure 6.4.

(i) Double the mass of the trolley as in Figure 6.4 and repeat the experiment in steps (i) – (iv) of Figure 6.2.

(ii) Repeat the experiment with three times the mass of the trolley.

Conclusion: Doubling the mass of the trolley reduces the acceleration by half while tripling the mass reduces the acceleration to one -third of its initial value.

Acceleration is inversely proportional to the mass of the trolley if the force is kept constant.

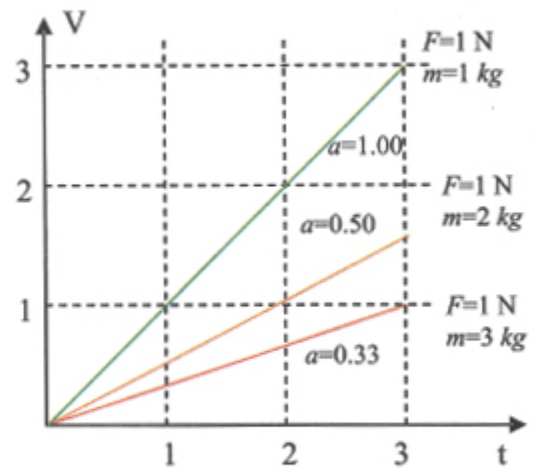


Figure 6.5: Increasing the mass at a constant force decreases the acceleration

Worked examples

1. A body of mass 2.5 kg initially at rest is acted upon by a resultant force of 10 N. Calculate:

(i) the acceleration of the body;

(ii) the velocity of the body after 6 seconds;

(iii) the distance covered in 6 seconds.

Solution

(i) $\text{acceleration} = \frac{\text{force}}{\text{mass}} = \frac{10}{2.5} = 4\ m\,s^{-2}$

(ii) v = u + at = 0 + 4 Ã— 6 = 24ms^{-1}

(iii) s = Â½ (v + u) t = Â½ (0 + 24) = 12 m.

2. A car of mass 1500 kg travelling at 10 ms^{-1} brought to rest in 5

seconds. Calculate the:
(i) average acceleration;
(ii) average breaking force;
(iii) total distance covered before coming to rest.

Solution

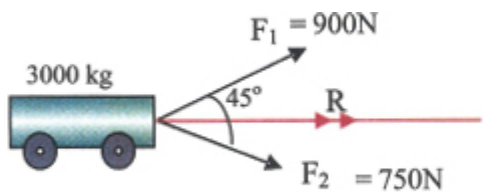
(i) v = u + at
0= 10 +5a
a = – 2 ms^{-2}

(ii) Force = mass Ã— acceleration
= 1500 Ã— (–2) = –3000 N.

(iii) s = Â½*(v + u)t* = Â½(0 + 10)5 = 25 m.

3. Two boys pull a truck of total mass 3000 kg with two ropes inclined at angle of 45Â° to each. If the tensions in the ropes are 750 N and 900 N respectively, calculate the;
(i) magnitude of the resultant force acting on the truck;
(ii) acceleration of the truck.

Solution



(i) Resultant force acting on the truck
(i) Resultant force acting on the truck

$$R^2 = 750^2 + 900^2 - 2\times 750\times 900\cos 135^o$$
$$= 562500 + 810000 + 1350000\cos 45^o$$
$$= 2327094.155$$
$$R = \sqrt{2327094.155} = 1525.5\ N$$

$$\text{acceleration} = \frac{\text{force}}{\text{mass}}$$
(ii) $$= \frac{1525.5}{3000} = 0.51\ m\ s^{-2}$$

4. When a force is impressed on a mass of 25 kg for 5seconds, the velocity changes from 12 ms^{-1} to 15 ms^{-1} calculate the:
(a) initial momentum of the body;
(b) final momentum as a result of the force impressed on it;
(c) magnitude of the force impressed on the body.

Solution

(a) Initial momentum=mass Ã— velocity
= 25 Ã— 12
= 300 kg ms^{-1}.
(b) Final momentum=mass Ã— velocity

$= 25$ Ã— 15

$= 375$ kg ms^{-1}.

(c) Force impressed on the body =

$$\frac{\text{Momentum change}}{\text{time}} = \frac{mv - mu}{t}$$

$$F = \frac{375 - 300}{5} = \frac{75}{5} = 15\ N$$

5. A tennis player hits a tennis ball of mass 0.25 kg with an impact force lasting 0.01 second. The ball leaves the ratchets with a velocity of 25 ms^{-1} calculate:

(a) momentum gain of the ball;

(b) impulse of the force;

(c) impact force on the wall.

Solution

Momentum gained = change in momentum

(a) mv – mu = 0.25 Ã— 25-0 = 6.25 kg ms^{-1}.

(b) Impulse = gain in momentum=6.25 kg ms^{-1}

(c) $\text{Force} = \frac{\text{Momentum gained}}{\text{time}}$

(c) $= \frac{6.25}{0.01} = 625\ N$

6. A space shuttle travelling at a constant velocity of 750 ms^{-1}, burns fuel at the rate of 1500 kg s^{-1}. Calculate the force moving the shuttle forward.

Solution

Force moving the shuttle forward is given

by $F = \frac{mv}{t} = \frac{m}{t}v$ = 1500kg s^{-1} x 750m s^{-1}

$F = 1500 \times 750 = 1125000\ N$ = 1.125 x 10^{6}N

Newtonâ€™s third law of motion

Action and reaction are equal and opposite. Alternatively, to every action there is an equal and opposite reaction.

Newton in his third law observed that forces always act in pairs. The two forces are called **action** and **reaction**. The action force is equal in magnitude to the reaction but acts in the opposite direction. When two objects A and B are in contact, they exert forces on each other. The object A exerts a force F_A on B; the object B in turn impresses an equal but opposite force F_B on A. If F_A is the action, then F_B is the reaction. Figure 6.6 illustrates Newtonâ€™s third law in action.

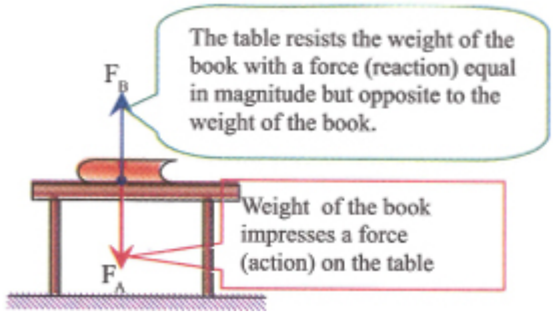


Figure 6.6a: Third law of motion acting on a book

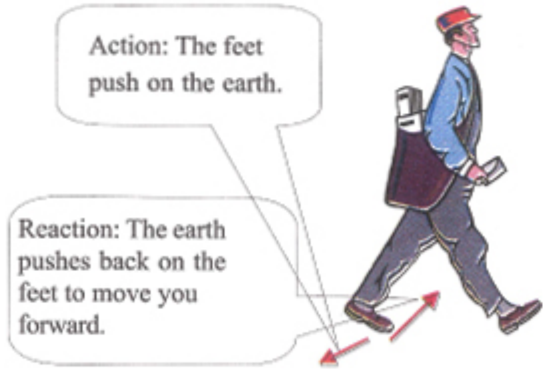


Figure 6.6b: The law of motion is responsible for walking

Walking is possible due to action and reaction between the feet and the earth. To move our body forward, the feet push on the ground, which reacts by pushing us forward with an equal but opposite force.

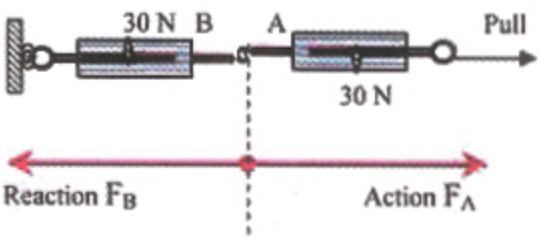


Figure 6c

The pull on the spring balance A, exerts a force F_A on spring balance B. Spring balance B reacts by exerting equal but opposite force F_B on A. Both spring balances reads 30 N because action and reaction are equal and opposite.

Outcomes of Newtonâ€™s laws of motion

(i) Inertia and gravitational mass

Inertia mass is the quantity of matter in a body. It is a measure of its resistance to any force impressed on the body to change its state of rest or motion in a straight line. Inertia mass is independent of gravity and therefore is constant in the universe.

Gravitational mass is called the weight of the body. It is a measure of the force of attraction between the earth and the body in a given location. Gravitational mass or weight is proportional to gravity. It varies from place to place on the earthâ€™s surface.

Weight or gravitational mass = mass Ã— gravity

For details on weight or gravitational mass, check measurement of weight in chapter one of book 1.

(ii) Weight of a body in a lift

Weight measuring instruments like spring balance measures the gravitational mass of the body. Gravitational mass or weight depends on the magnitude and direction of the vertical acceleration of the body. *The body gains weight if the resultant acceleration is upwards and loses weight if the resultant acceleration is downwards. Let us consider three cases.*

(1) Lift stationary or moving with constant velocity:

A body in a lift at rest or moving upward or downward with a constant velocity does not gain or lose weight. This is because the acceleration is zero; therefore, the resultant force acting on the body is zero. The weight of the body impressed on the floor of the lift is equal to the reaction of the lift on the body.

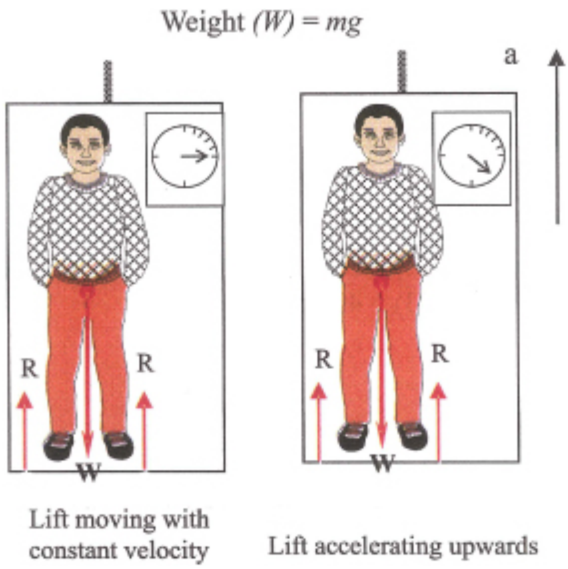


Figure 6.7a

(2) Lift moving upwards with a constant acceleration (a)

A body inside a lift ascending upwards with a constant acceleration gains weight because the reaction (R) of the floor of the lift on the body is greater than the weight of the body on the floor. A resultant upward force acts on the body in the direction of the acceleration. Resultant force $F = R - mg$.

$R - mg = ma$ or $R = ma + mg$

$$R = m(a + g)$$

According to Newtonâ€™s third law of motion, action and reaction are equal and opposite. The weight of the body is equal to the reaction; therefore, weight increases as the body accelerates upwards.

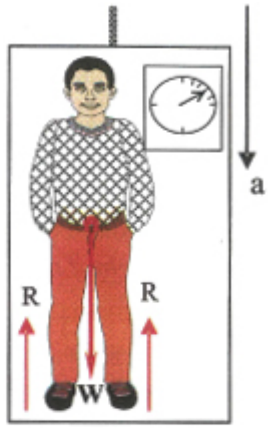
(3) Lift moving downwards with a constant acceleration *(a)*

When a lift descends with acceleration *(a),* the body inside the lift loses weight. The resultant force on the body is downward as the reaction of lift on the body is less than the weight of the body. The resultant force $F = mg - R$.

$mg - R = ma$ or $R = mg - ma$

$R = m(g - a)$

The weight of the body is equal to the reaction of the lift on it; therefore, the body loses weight as the lift accelerates downwards. When a = g, the body inside the lift becomes **weightless** and begins to float about inside the lift. This happens when the lift falls freely under gravity.



Worked example

A man of mass 70 kg is standing on the floor of a lift. Calculate the weight of the man when the lift is moving;

(a) Upwards with a constant velocity;

(b) Upwards with a constant acceleration of 5 ms^{-2};

(c) Downwards with a constant acceleration of 3 ms^{-2}; {g = 10 ms^{-2}}

Solution

(a) R = weight = mg =70 Ã— 10 = 700 N.

(b) R = weight = $m(g+a)$ = 70 (10+5) = 1050 N.

(c) R = weight = $m(g\text{-}a)$ = 70 (10–3) = 490 N.

(iii) Motion along the plane of an inclined plane

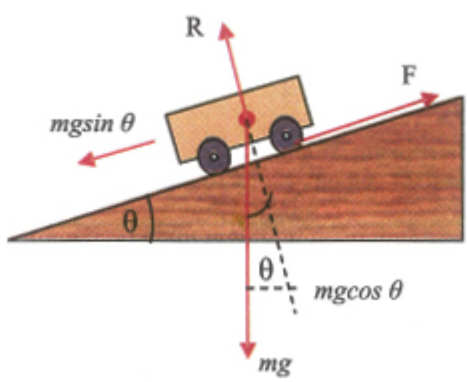


Figure 6.8a: Motion down the inclined plane

A body sliding down an inclined plane and the forces acting on it is illustrated in Figure 6.8. If the body slide with a constant velocity, the resultant force on it is zero, therefore the normal reaction *(R)* is balanced by the part of the weight *(mgcos* Î¸) perpendicular to the plane. The component of the weight along the plane is equal to the frictional force.

R = mgcos Î¸ and F = mgsin Î¸

When the body slides downward with constant acceleration (a), the resultant force on it is given by:

mgsin Î¸ *– F = ma*

If the plane is frictionless, the forces acting on the body along the plane is given by:

mgsin Î¸ = ma

The acceleration of the body down the plane is given by:

*a = gsin*Î¸

The acceleration down the plane is independent of the mass of the sliding body. It depends on the gravity at the location and the angle (Î¸) the inclined plane makes with the horizontal.

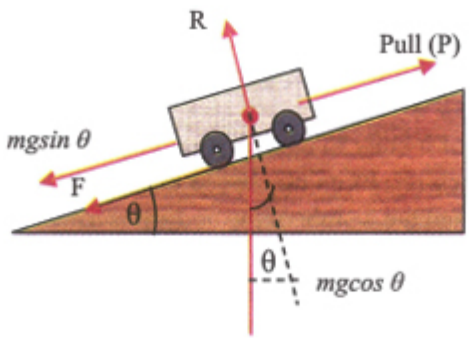


Figure 6.8b: Motion up the inclined plane

When a body is pulled up an inclined plane, the pull (P) is balanced by the friction and *mgsin Î¸* the component of the weight along the plane of the inclined plane. If the body moves up the inclined plane with constant velocity then:

P = mgsin Î¸ + F

If the body accelerates up the plane, the resultant force on it is given by:

P – (mgsin Î¸ + F) = ma

For a frictionless plane F = 0, then the force P needed to keep the body accelerating upwards is given by:

P = mgsin Î¸ + ma

Worked example

A box of mass 50 kg slides down a smooth plane inclined at 15Â° to the level ground. Calculate the:

(a) acceleration of the box as it slides down the plane.

(b) force needed to stop the body from sliding down the plane.

(c) force needed to push the body up the plane with an acceleration of 0.5 ms^{-2}, {g = 10 ms^{-2}}

Solution

(a) a = gsin Î¸ = 10sin 15Â° = 2.588 ms^{-2}.

(b) P = mgsin Î¸ = 50 Ã— 10sin 15Â° = 129.4 N

(c) P = mgsin Î¸ + ma

50 Ã— 10sin 15 + 50Ã—0.5 = 154.4 N.

Summary

Newtonâ€™s laws of motion:

â€¢ Every object continues at rest or moves with constant velocity in a straight line if no external resultant force acts on it.

  â€¢ Force is directly proportional to the time rate of change of linear momentum and it acts in the direction of momentum change.

  â€¢ Action and reaction are equal and opposite.

â€¢ If no resultant force acts on a body, it either remains at rest or moves with a constant velocity in a straight line.

â€¢ Every object in motion will always come to rest because a resultant force (friction) acts on it to oppose its motion.

â€¢ If friction is eliminated, all moving bodies will continue their motion in a straight line indefinitely.

â€¢ Inertia is the resistance of a body to change its state of rest or motion in a straight line.

â€¢ The mass of a body which resists being moved or stopped, is called inertial mass.

â€¢ Gravitational mass is called the weight of the body. It is a measure of the force of attraction between the earth and the body in a given location.

â€¢ Momentum is the product of mass and velocity of a body.

â€¢ Impulse of a force is the product of a force and the time during which it acts.

â€¢ The impulse of a force is numerical equal to momentum change.

â€¢ Force is directly proportional to the product of mass and acceleration of a body and it acts in the direction of acceleration..

â€¢ The body in a moving lift gains weight if the resultant acceleration is upwards and loses weight if the resultant acceleration is downwards.

â€¢ The body does not gain or lose weight if the lift is moving with a constant velocity.

Practice questions 6a

1. (a) State Newtonâ€™s first law of motion;
   (b) Give three practical examples in support of Newtonâ€™s first law of motion.
   (c) Explain why moving objects seem not to obey Newtonâ€™s first law of motion on earth.
2. (a) Define inertia of a body and explain how it is related to Newtonâ€™s first law of motion.
(b) Explain the following:
   (i) Modern cars are provided with safety belt and headrests.
   (ii) Passengers inside aeroplanes are requested to put on their safety belts at takeoff and landing.
   (iii) It is difficult to stop a trailer rolling down a hill.
3. (a) State Newtonâ€™s laws of motion;
   (b) Using the first and second laws of motion to explain clearly the link between force and motion.
4. (a) Define the terms;
   (i) linear momentum; and
   (ii) impulse of a force.
   (b) State Newtonâ€™s second law of motion;
   (c) Starting with Newtonâ€™s law of motion, show that impulse of a force is equal to change in momentum.
5. (a) Define *force, impulse* and *momentum.*
   (b) A tennis ball of mass 0.2 kg hits a vertical wall with a velocity of 45 ms^{-1} and rebounds with a velocity of 40 ms^{-1}. Calculate the:
   (i) momentum of the ball before impact;
   (ii) momentum of the ball after impact;
   (iii) impulse of the force on the wall;
   (iv) force if the impact lasted for 0.5 seconds.
6. (a) What is the connection between force and momentum?
   (b) A rocket travelling at a steady speed of 250 ms^{-1} consumes fuel at the rate of 300 kgs^{-1}, calculate force pushing the rocket forward.
7. A bus of mass 1500 kg increases its speed from 10 ms^{-1} to 25 ms^{-1}in 5 seconds. Calculate;
   (a) the acceleration of the bus;
   (b) the force of the engine moving it forward.
8.

The total mass of the car above is 1500 kg. The force produced by the engine is 25,000 N while the resistive force is 10,000 N calculate:

(i) the resultant force moving the car forward;

(ii) the acceleration of the car;

(iii) if extra mass of 250 kg is added to the car, what is the effect of this extra mass on the acceleration of the car?

(iv) work done if the car covers a distance of 250 m.

9. (a) State Newtonâ€™s third law motion and explain why walking is made possible.

(b) Explain why a body resting on the floor of a lift ascending upward with a constant acceleration gains weight.

(c) A boy of mass 50 kg is standing on the floor of a lift moving upward with a constant acceleration of 2 ms^{-2}. Calculate the weight of the boy. {g = 10 ms^{-2}}

Conservation of linear momentum

Collision and Newtonâ€™s third law of motion *Collision occurs anytime two or more objects meet or interact.* During collision or impact, the two colliding objects exert forces on each other called **impact force.** The impact force lasts for a short time called **impact time.** The momentum change produced at collision is the same for the two colliding objects but their directions are oppo site. In Figure 6.9, the impact force of A on B is equal and opposite in direction to the impact force of B on A (Newtonâ€™s third law of motion).

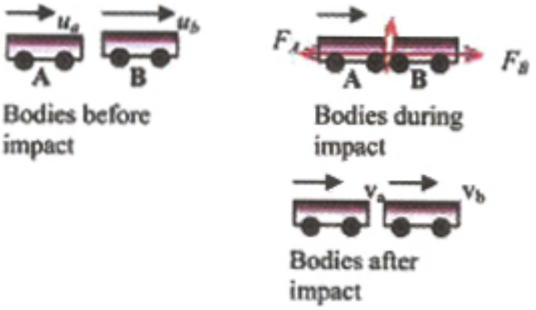


Figure 6.9

At impact $F_A = - F_B$, (Newtonâ€™s third law of motion) but force is momentum change per second according to Newtonâ€™s second law of motion.

$$\therefore \quad \frac{\Delta P_A}{t} = -\frac{\Delta P_B}{t} \text{ (where } \Delta P = \text{change in mementum)}$$

The impact time (t) is the same for both colliding objects and therefore cancels out.

$$\Delta P_A = -\Delta P_B \text{ or } \Delta P_A + \Delta P_B = 0$$

Collision and momentum conservation

The momentum changes are equal and opposite, that is, the gain in momentum of body A is equal to the loss in momentum of body B. This is the conservation of linear momentum.

The principle of conservation of linear momentum states that when two or more bodies collide, the total momentum of the colliding bodies is conserved (remains constant) if no external force is impressed on them.

The principle of conservation of linear momentum can be stated in another way:

The principle of conservation of linear momentum states that when two or more bodies collide, the total momentum of the colliding bodies before collision is equal to their total momentum after collision if no external force is impressed on them.

Mathematical proof of conservation of linear momentum

At impact $F_A = -F_B$, Newton's 3rd law of motion

$\therefore \quad \frac{\Delta P_A}{t} = \frac{\Delta P_B}{t}$ Newton's second law of motion;

$\Delta P_A = -\Delta P_B \; or \; \Delta P_A + \Delta P_B = 0$

$(m_A v_A - m_A u_A) + (m_B v_B - m_B u_B) = 0$

Rearranging the equation above yields

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

During collision or impact, the total momentum before impact is equal to the total momentum after impact, but the total kinetic energy after impact is less than the total kinetic energy before impact. Kinetic energy of the colliding bodies before and after collision is used to differentiate different types of collision.

Elastic and inelastic collision

Elastic collision

Elastic collision is one in which both the momentum and kinetic energy of the colliding bodies are conserved.

â€¢ The total momentum before impact is equal to the total momentum after impact.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

â€¢ The total kinetic energy before impact is equal to the total kinetic energy after impact.

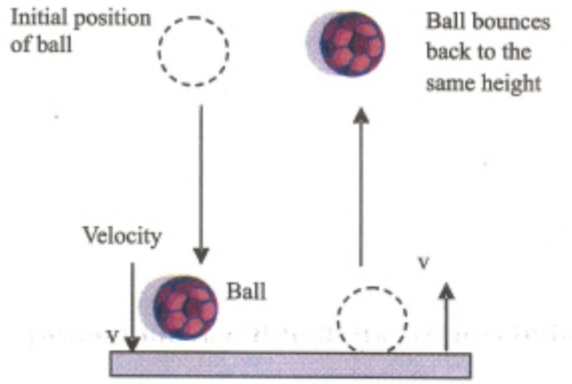
$$\frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

The colliding objects rebound or return with the same velocity since there is no loss of energy during elastic collision. The momentum change is zero (Î”P = 0). Collisions, which can be approximated to an elastic collision, include:

(i) a rubber ball that bounces back to the same height when dropped from a height to a hard floor.

(ii) collision of billiard balls.

(iii) collision of gas molecules.



Inelastic collision

Inelastic collision is one in which the total momentum before and after collision is conserved but the total kinetic energy is not conserved.

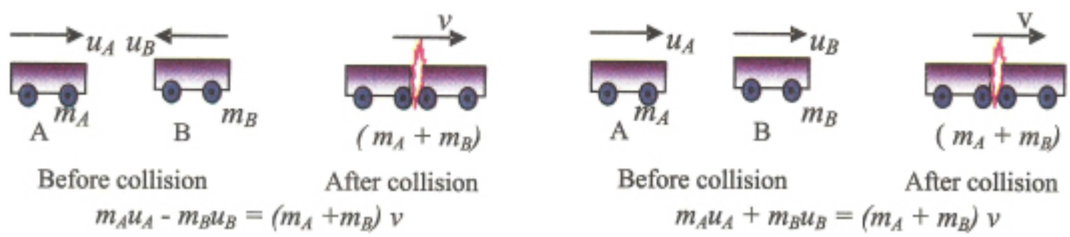


Figure 6.10: Completely inelastic collisions

The total kinetic energy just before collision is more than the total kinetic energy just after collision. The loss in kinetic is converted to heat and sound energies. Most collisions are inelastic because there is always a loss of energy and the bodies separate or move apart after collision. A basketball dropped from a height onto a hard floor rebounds but does not return to the original height because the collision is inelastic.

â€¢ The total momentum before collision is equal to the total momentum after collision.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

â€¢ The total kinetic energy before collision is greater than the total kinetic energy after collision.

$$\frac{1}{2}m_Au_A^2+\frac{1}{2}m_Bu_B^2>\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2$$

In a completely or perfectly inelastic collision, the bodies join or stick together and move with a common velocity *(v).*

Note that momentum is a vector quantity; therefore, its direction is important. If momentum in one direction is positive then momentum in the opposite direction is negative. The kinetic energy before collision is greater than the kinetic energy after collision in both cases above.

$$\therefore \frac{1}{2}m_Au_A^2+\frac{1}{2}m_Bu_B^2>\frac{1}{2}(m_A+m_B)v^2$$

Examples of completely inelastic collision are:

(i) a boy jumps into a moving truck or bike.

(ii) head on collision between two bodies, which coalesce (join) and move off with a common velocity.

Explosion

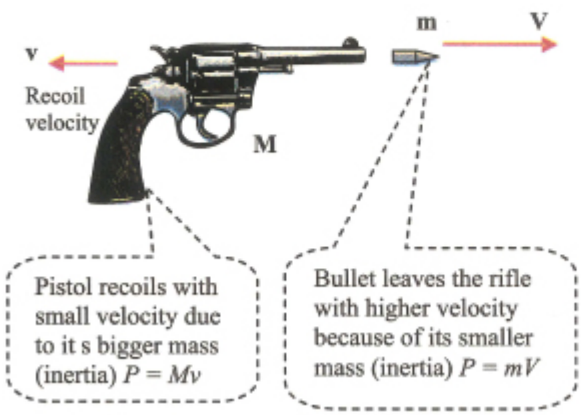
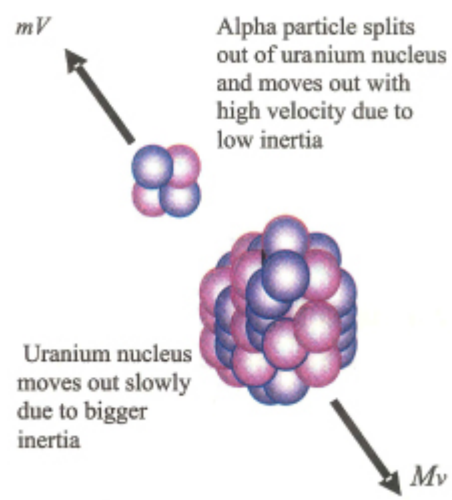


Figure 6.11: Firing a pistol and splitting an atom



Explosion and splitting of atoms

Explosion occurs when an object breaks up and moves apart. The total momentum is conserved but the kinetic energy is increased after the explosion (that is, the total kinetic energy before explosion is less than the total kinetic energy after explosion). Examples of explosion are:

(i) spontaneous disintegration of radioactive nuclides;

(ii) a boy jumps off a moving truck or bike;

(iii) a rifle when fired. The bullet and the rifle separate.

The total momentum of the rifle and the bullet before they move apart is equal to their total momentum after their separation. Before the rifle was fired the total momentum of the rifle and the bullet is zero since they were not moving. When the rifle is fired, the bullet and the rifle move apart in opposite directions such that the sum of their momentum is still zero.

Total momentum of the rifle and bullet before firing = $(M–m)v$ (since u = 0).

Total momentum of the rifle and bullet after firing = Mv – m V

Total momentum of the rifle and bullet after explosion is 0 = *Mv* Ã— *mV*

0 Mv Ã— mV

Application of momentum

1. Rockets and jets propulsion

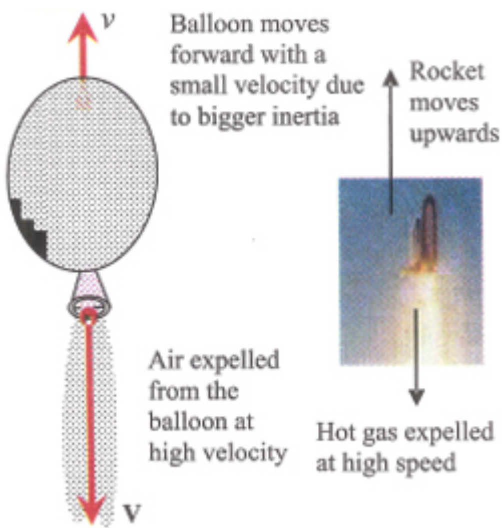


Figure 6.12: Balloon and rocket propulsion

When an inflated balloon is released, compressed air is pushed out from the open neck while balloon flies off in the opposite direction. The momentum of the escaping air is equal and opposite to the momentum of the balloon. This is how rockets and jet engines work; in rocket propulsion, a mixture of compressed air and fuel is burnt to produce a steam of hot gas. The hot gas is expelled at the exhaust with high momentum or velocity. The rocket or jet gains the same amount of momentum to push it forward in the direction opposite to the expelled hot gas.

2. Reduction of damage to automobiles during impact

During collision, the impact force is usually high because the impact time is very short. The high impact force can cause great damage to the vehicle and its occupants if safety precautions are not observed. To reduce the impact force, the impact time should be prolonged; this is achieved by equipping the crumble zones of vehicles with bumpers made of elastic materials to prolong the impact time. Air bags are also provided inside to keep the front occupants from being injured.

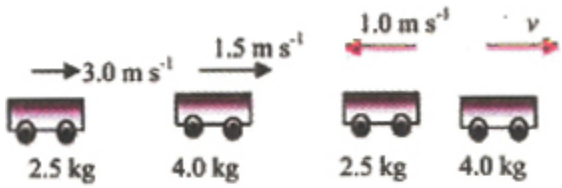
3. In sports

High and long jumpers flex their knees on landing to prolong the impact time and reduce impact forces on their knees and joints. For the same reason cricketers and goalkeepers withdraw their hands when catching a cricket ball or football.

Worked examples

1. A trolley of mass 2.5 kg travelling at 3 ms^{-1} collides with another trolley of mass 4.0 kg travelling at 1.5 ms^{-1} in the same direction. The impact lasted for 0.125 seconds before the 2.5 kg trolley continued to move with a velocity of 2 ms^{-1} in the same direction. Calculate the:
   (a) velocity of 4.0 kg trolley after impact;
   (b) impulse;
   (c) impact force;
   (d) loss of kinetic energy after impact.

Solution



(a) $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

2.5Ã—3 + 4Ã—1.5 = 2.5Ã—2 + 4v_B

13.5 = 4v_B + 5.0

4v_B = 8.5

v_B = 2.125 ms^{-1}.

Impulse of 4.0 kg trolley = $m_B v_B - m_B u_B$

=4Ã—2.125–4Ã—1.5

=2.5 N s

or Impulse of 2.5kg trolley = $m_A v_A - m_A u_A$

= 2.5Ã—2 – 2.5Ã—3

= -2.5 N s

The minus means that the momentum change of 2.5 kg trolley is exactly opposite to momentum change of 4.0 kg trolley.

(b) Impact force $= \frac{\Delta P}{t} = \frac{2.5\ N\ s}{0.125\ s} = 20\ N$

(c) Kinetic energy before impact

$$E_K = \tfrac{1}{2}m_A u_A^2 + \tfrac{1}{2}m_B u_B^2$$

$$E_K = \tfrac{1}{2}\times 2.5\times 3^2 + \tfrac{1}{2}\times 4\times 1.5^2$$

$$11.25\ J + 4.5\ J = 15.75\ J$$

(d) Kinetic energy after impact

$$E_K = \tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2$$

$$E_K = \tfrac{1}{2}\times 2.5\times 2^2 + \tfrac{1}{2}\times 4\times 2.125^2$$

Loss of kinetic energy = 15.75 J – 14.03 J

= 1.72 J

2. A boy of mass 60 kg jumps off a truck of mass 140kg travelling at 7.5 ms^{-1}.If the boy runs towards the truck with a velocity of 2.5 ms^{-1}, calculate:
   (a) The momentum of the truck before the boy jumps off;
   (b) The momentum of the boy on jumping off the truck;
   (c) The velocity of the truck the moment the boy jumped off;
   (d) The loss of kinetic energy the moment the boy jumped off.

Solution

(a) The momentum of the boy and the truck $P = (M_t + M_b)u = (140 + 60)7.5$

$= 1500$ kgms^{-1}

(b) The momentum of the boy on jumping off the truck

$P = m_b v_b = 60$ Ã— $2.5 = 150$ kg ms^{-1}.

(c) $m_b v_b + m_t v_t = (m_b + m_t)u$

$150 + 140 v_t = 1500$

$$v_t = \frac{1350}{140} = 9.64\ m\ s^{-1}$$

(d) Kinetic energy before jumping = E_k

$E_k = ½\ (m_t + m_b)\ u^2 = ½\ (140+60) \times 7.5^2$

$= 5625\ J$

Kinetic energy after jumping

$$E_K = \tfrac{1}{2}m_t v_t^2 + \tfrac{1}{2}m_b v_b^2$$

$E_k = \tfrac{1}{2}\times 140 \times 9.64^2 + \ \times 60\times 2.5^2$

$= 6505 + 187.5 = 6692.5\ J$

Loss of kinetic energy = 6692.5 - 5625

= 1067.5 J

3. A tanker of mass 15000kg travelling at 10ms^{-1} collides and sticks to a bus of mass 5000kg travelling at 20 ms^{-1}in the opposite direction. What is their common velocity after collision?

Solution

Total momentum before collision = Total momentum after collision

Total momentum before collision = Total momentum after collision

$\therefore\ m_T u_T - m_B u_B = (m_T + m_B)\, v$

$15000 \times 10 - 5000 \times 20 = (15000 + 5000)\, v$

$50000 = 20000\, v$

$$v = \frac{50000}{20000} = 2.5\ m\ s^{-1}$$

Summary

â€¢ **Collision** occurs anytime two or more objects meet or interact.

â€¢ The **principle of conservation of linear momentum** states that when two or more bodies collide, the total momentum of the colliding bodies before impact is equal to their total momentum after impact if no external force is impressed on them.

â€¢ **Elastic collision** is one in which both the momentum and kinetic energy of the colliding bodies are conserved.

â€¢ **Inelastic collision** is one in which the total momentum before and after collision is conserved but the total kinetic energy after collision is less than the total kinetic energy before collision.

â€¢ **Explosion** is when an object breaks up and moves apart fragments. The total momentum is conserved but the kinetic energy is increased after the explosion.

Practice questions 6b

1. (a) State the law of conservation of linear momentum.

   (b) A snooker ball of 0.1 kg moving with a speed of 4 ms^{-1} had a head-on collision with another ball of the same mass at rest. If the collision is perfectly elastic and the impart time is 0.005 seconds;

   (i) Explain the meaning of the underlined words;

   (ii) Find the speeds of the balls after collision;

   (iii) The average force the balls exert on each other.

2. (a) What do you understand by *collision* and *perfectly inelastic collision*?

   (b) A saloon car of total mass 2000 kg travelling at 40 ms^{-1} collided head-on with a lorry of total mass 8000 kg travelling in the opposite direction. If both are instantly brought to rest, calculate the speed of the lorry before the collision.

3. (a) Explain why modern cars are equipped with bumpers made of soft materials.

   (b) A car of mass 5000 kg, moving at a speed of 32 ms^{-1} hits a massive tree by the roadside and is brought to rest in 4.0 seconds. Calculate;

   (i) The momentum of the car before collision;

(ii) The impact force;

(iii) Explain what will happen if the time the collision lasted is reduced.

4. (a) Differentiate between;

   (i) explosion and collision;

   (ii) elastic and inelastic collision.

   (b) A boy of mass 75 kg, moving with speed of 5 ms^{-1} jumps into a truck of mass 1925 kg travelling in the same direction at 2.5 ms^{-1}. Find the common speed and the change in kinetic as the boy jumps into the truck.

5. (a) Define *linear momentum* and state the *principle of conservation of linear momentum.* (b) A boy playing a snooker game hits a snooker ball of mass 0.1 kg with a force lasting 0.1 seconds. The ball travels with a constant velocity of 6 ms^{-1} before it collides with another ball of mass 0.2 kg at rest. Calculate:

   (i) Their common velocity if they stick;

   (ii) The velocity of 0.2 kg mass if the 0.1 kg mass rebounds with a velocity of 2 ms^{-1};

   (iii) Force exerted by the 0.1 kg on 0.2 kg during collision;

   (iv) The acceleration of 0.1 kg before it collides with the 0.2 kg mass.

6. (a) Explain the terms *linear momentum* and *explosion*;

   (b) Explain why a rifle recoils when it fires a bullet.

   (c) A rifle mass 3.0 kg fires a bullet of mass 0.05 kg. The bullet leaves the rifle with a velocity of 300 ms^{-1} and is stopped by a block of wood, calculate:

   (i) The momentum of the bullet before it hits the wooden block;

   (ii) The recoil velocity of the rifle;

   (iii) The resistive force of the wood if the bullet is stopped 0.02 seconds after hitting the wood.

7. A tennis ball of mass 0. 3 kg travelling at 20 ms^{-1}, hits a vertical wall at right angle and rebounds with a velocity of 16 ms^{-1}. Calculate:

   (i) momentum of the ball before and after impact with the wall;

   (ii) the momentum change after collision;

   (iii) the impulse of the ball on the wall;

   (iv) the impact force of the ball on the wall if the impact time is 0.01 s.

Past questions

1. A force acts on a body for 0.5 s changing its momentum from 16 kg ms^{-1} to 21 kg ms^{-1}. Calculate the magnitude of the force.

A. 42 N
B. 37 N
C. 32 N
D. 10 N **WASSCE**

2. A constant force of 5 N acts 5 s on a mass of 5 kg initially at rest. Calculate the final momentum of the mass.
A. 125 kg ms^{-1}
B. 75 kg ms^{-1}
C. 25 kg ms^{-1}
D. 20 kg ms^{-1}
E. 5 kg ms^{-1} **NECO**

3. A net force of 15 N acts upon a body of mass 3 kg for 5 s, calculate the change in speed of the body.
A. 25.0 ms^{-1}
B. 9.0 ms^{-1}
C. 2.5 ms^{-1}
D. 1.0 ms^{-1} **WASSCE**

4. The tendency of a body to remain at rest when a force is applied to it is called
A. impulse.
B. momentum.
C. inertia.
D. friction. **WASSCE**

5. A ball of mass 5.0 kg hits a smooth vertical wall normally with a speed of $2ms^{-1}$. Determine the magnitude of the resulting impulse.
A. 20.0 kg ms^{-1}
B. 10.0 kg ms^{-1}
C. 5.0 kg ms^{-1}
D. 2.5 kg ms^{-1} **WASSCE**

6. A body of mass M_1 moving with a velocity of u collides with a stationary body of mass M_2 and both move with a common *v*. If linear momentum is conserved, which expression correctly represents *v*?
A. $\frac{M_1 + M_2}{Mu}$

B. $\frac{M_2 u}{M_1 - M_2}$

C. $\frac{M_1 u}{M_1 - M_2}$

D. $\frac{M_1 u}{M_1 + M_2}$

E. $\frac{M_{21} u}{M_1 + M_2}$

WAEC

7. The time rate of change of momentum is
   A. impulse.
   B. force.
   C. power.
   D. pressure.

WASSCE

8. Which of the following statements about elastic collision is correct?
   A. Momentum is lost due to the sound produced.
   B. Loss of momentum is equal to loss in kinetic energy.
   C. Both kinetic energy and momentum are conserved.
   D. Kinetic energy is lost while momentum is conserved.

WASSCE

9. Which statement is true of a block of stone of mass m, with zero momentum?
   A. The acceleration of the stone is nonuniform.
   B. The stone is stationary.
   C. The stone moves at constant velocity.
   D. The stone moves with a maximum velocity.
   E. The stone moves with a minimum velocity.

NECO

10. A ball of mass 100 g travelling with a velocity of 100 ms^{-1} collides with another ball of mass 400 g moving at 50 ms^{-1} in the same direction. If they stick together, what will be their common velocity?
   A. 25 ms^{-1}
   B. 50 ms^{-1}
   C. 60 ms^{-1}
   D. 75 ms^{-1}
   E. 100 ms^{-1}

NECO

11. A body of mass 10 kg moving with a velocity of 10 ms^{-1}, hits a stationary body and has its direction reversed and the velocity changed to 7.5 ms^{-1} in 5 s. Calculate the magnitude of force of impact.

A. 3.5 N

B. 5.0 N

C. 35.0 N

D. 150.0 N

WASSCE

12. A net force of magnitude 0.6 N acts on a body of mass 40 g initially at rest. Calculate the magnitude of the resulting acceleration.

A. 90 ms^{-2}

B. 00 ms^{-2}

C. 30 ms^{-2}

D. 15 ms^{-2}

WASSCE

13. A trolley of mass 4 kg moving on a smooth horizontal platform with a speed of 1.0 ms^{-1} collides perfectly with a stationary trolley of the same mass on same platform. Calculate the total momentum of the trolleys immediately after collision.

A. 0.5 N s

B. l.0 N s

C. 4.0 N s

D. 8.0 N s

WASSCE

14. A 150 kg body is at rest. Calculate the constant force needed to impact a velocity of 8 ms^{-1} to it in 5 s.

A. 600.0 N

B. 300.0 N

C. 240.0 N

D. 180.0 N

E. 60.0 N

WAEC

15. A body of mass 20 kg is set in motion by two forces 3 N and 4 N, acting at right angle to each other. Determine the magnitude of its acceleration.

A. 4.00 ms^{-2}

B. 2.89 ms^{-2}

C. 0.35 ms^{-2}

D. 0.25 ms^{-2}

E. 0.05 ms^{-2}

WAEC

16. A machine gun with mass of 5 kg fires a 50 g bullet at a speed of 100 ms^{-1}. The recoil speed of the machine gun is

A. 0.5 ms^{-1}.

B. 1.5 ms^{-1}.

C. 1.0 ms^{-1}.

JAMB

D. 2.0 ms^{-1}.

E. 4.0 ms^{-1}.

17. A man stands on a scale placed in a lift. The lift descends at a constant velocity. As a result, the scale reads a weight

A. greater than the weight of the man.

B. same as the weight of the man.

C. zero

D. less than the weight of the man.

E. greater than the weight of the man by 1kg.

JAMB

18. A force of 16 N is applied to a 4.0 kg block that is at rest on a smooth, horizontal surface. What is the velocity of the block at t = 5 seconds?

A. 4 ms^{-1}.

B. 10 ms^{-1}.

C. 20 ms^{-1}.

D. 50 ms^{-1}.

E. 80 ms^{-1}.

JAMB

19. A 0.05 kg bullet travelling at 500 ms^{-1} horizontally strikes a thick vertical wall. It stops after penetrating through distance of 0.25 m. What is the magnitude of the average force the wall exerts on the bullet?

A. 25 N.

B. 50 N.

C. 250 N.

D. 5000 N.

E. 25000 N.

JAMB

20. A constant force of magnitude F acts on an object of mass 0.04 kg initially at rest at a point O. If the speed of the object when it has moved 50 m from O is 500 ms^{-1}, what is the value of F?

A. 0.4 N

B. 100 N

C. 250 N

D. 1000 N

JAMB

21. A 1000 kg elevator is descending vertically with an acceleration of 1.0 ms^{-2}. If the acceleration due to gravity is 10.0 ms^{-2}, the tension in the suspending cable is

A. 1.0 N

B. 10.0 N

C. 9000.0 N

D. 11000.0 N

JAMB

22. A jet engine develops a thrust of 270 N when the velocity of the exhaust gases relative to the engine is 300 ms^{-1}, what is the mass of the gas ejected per second?

A. 81.00 kg.

B. 9.00 kg.

C. 0.90 kg.

D. 0.09 kg.

JAMB

23. An elevator of mass 4800 kg is supported by a cable which safely withstands a maximum tension of 60,000 N. The maximum upward acceleration the elevator can have is {g = 10 ms^{-2}}

A. 2.5 ms^{-2}.

B. 5.0 ms^{-2}.

C. 7.5 ms^{-2}.

D. 10.0 ms^{-2}.

JAMB

24. When taking a penalty kick, a footballer applies a force of 30 N for a period of 0.05 s. If the mass of the ball is 0.075 kg, calculate the speed with which the ball moves off.

A. 4.50 ms^{-1}.

B. 11.75 ms^{-1}.

C. 20.00 ms^{-1}.

D. 45.00 ms^{-1}.

JAMB

25. Two bodies have their masses in the ratio 3:1. They experiences forces which impact to them accelerations in the ratio 2:9 respectively. Find the ratio of the forces the masses experience.

A. 1:4

B. 2:1

C. 2:3.

D. 2:5

JAMB

26. (a) Define (i) Linear momentum

(ii) Impulse

(b) State the principle of conservation of linear momentum.

(c) A tractor of mass 5.0 Ã— 10^3 kg is used to tow a car of mass 2.5 Ã— 10^3 kg. The tractor moved with a speed of 3 ms^{-1} just before the towing rope became taut. Calculate the;

(i) speed of the tractor immediately the rope becomes taut;

(ii) loss in kinetic energy of the system just after the car has started moving;

(iii) impulse in the rope when it jerks the car into motion.

WAEC, WASSCE

27. (i) Explain inertia and impulse
(ii) A body of mass 5 kg moving with a speed of 30 $ms^{-1}$is suddenly hit by another body moving in the same direction, thereby changing the speed of the former body to 60 ms^{-1}, calculate the impulse received by the first body.

WASSCE

28. (a) (i) Define the term *linear momentum.*
(ii) State the law of conservation of linear momentum.
(b) A ball **P** of mass 0.25 kg, losses $\frac{1}{2}$ of its velocity when it makes a head on collision with an identical ball **Q** at rest. After the collision, **Q** moves off with a speed of 2 ms^{-1}in the original direction of **P.** Calculate the initial velocity of **P.**
(c) (i) State Newtonâ€™s second law of motion;
(ii) Show that $F = ma$ where **F** is the magnitude of the force acting on the body of mass **m** to give it an acceleration of magnitude **a.**
(d) The engine of a vehicle moves it with a force of 9600 N against a resistive force of 2200 N. If the mass of the body is 3400 kg, calculate the acceleration produced.

WASSCE

29. A bullet of mass 120g is fired horizontally into a fixed wooden block with a speed of 20ms^{-1}. The bullet is brought to rest in the block in 0.1s by a constant resistance. Calculate the:
(i) Magnitude of the resistance;
(ii) The distance moved by the bullet in the wood.

WASSCE

30. State Newtonâ€™s second law of motion.
A body of mass 20 kg resting on a smooth inclined plane at an angle of 30Â° to the horizontal is prevented from sliding by a horizontal force of magnitude P. Calculate the:
(i) value of P;
(ii) reaction of the body on the plane.

F-maths
WASSCE

31. A body of mass 15 kg is placed on a smooth plane inclined at 30Â° to the horizontal. If it slides down the plane, calculate:
(i) its acceleration;
(ii) the force required to prevent it from sliding. {g = 10 ms^{-2}}

F-maths
WASSCE 2002
J

32. A body of mass 80 kg is carried by a lift. Find the reaction exerted by the lift on the body if the lift moves upwards with:

(i) constant velocity;

(ii) an acceleration of 0.25 ms^{-2}. {g = 10 ms^{-2}}

F-maths

WASSCE 2001

N

33. An object of mass 5 kg slides down a smooth plane inclined at an angle of 25Â°. If the object starts from rest, find:

(i) its velocity after 3 m;

(ii) its momentum 3 m from the starting point;

(iii) the force causing it moves. {g = 10 ms^{-2}}

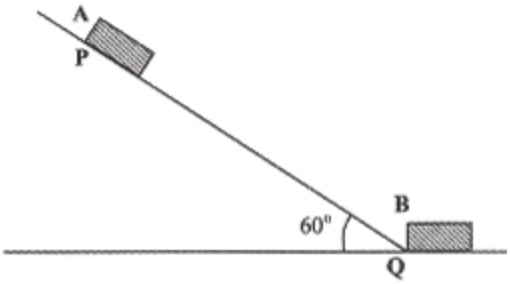
F-maths

WASSCE

34. (a) Explain (i) inertia; (ii) impulse.

(b) State *the law of conservation of linear momentum.*

(c) The diagram below illustrates a block of wood, A, of mass 650 g sliding from rest at point P down a frictionless inclined plane of angle 60Â°. Another block , B, of the same mass as A is placed at the foot of the inclined plane as shown.



(i) Calculate the kinetic energy of A at Q if the distance PQ is 2 m.

(ii) Calculate the horizontal component of the velocity of A at Q.

(iii) If A and B stick together after collision, calculate the velocity with which they move.

[g = 10 ms^{-2}]

WASSCE


Ouch, that should drive home the message!