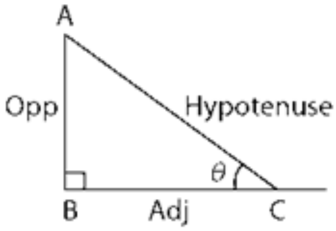
CHAPTER 13: Bearing

OBJECTIVES

At the end of the chapter, students should be able to:

1. Solve problems on trigonometric ratios and angles of elevation and depression.
2. Identify 4, 8 and 16 cardinal points.
3. State the two bearing notations and solve problems involving bearing.
4. Solve practical problems on bearing.

Trigonometric Ratios



Note: The hypotenuse is the longest side of a right-angled triangle, the side opposite the right angle.
Using SOH CAH TOA,

$$\sin\theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{BC}{AC}$$

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{AB}{BC}$$

Reciprocal of sine, cosine and tangent

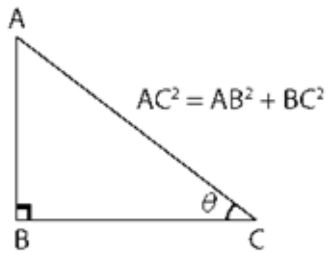
$$\frac{1}{\sin\theta} = \frac{1}{\frac{\text{Opp}}{\text{Hyp}}} = \frac{\text{Hyp}}{\text{Opp}} = \text{cosecant}\,\theta$$

$$\frac{1}{\cos\theta} = \frac{1}{\frac{\text{Adj}}{\text{Hyp}}} = \frac{\text{Hyp}}{\text{Adj}} = \text{secant}\,\theta$$

$$\frac{1}{\tan\theta} = \frac{1}{\frac{\text{Opp}}{\text{Adj}}} = \frac{\text{Adj}}{\text{Opp}} = \text{cotangent}\,\theta$$

Pythagoras' Theorem

The theorem states that in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



Quadrant Equation

2nd	1st
S	A
3rd	4th
T	C

First Quadrant Equation

In the first quadrant, the angles from 0Â° to 90Â° are positive, and the trigonometric functions, sine, cosine and tangent are positive.
sin(90Â° âˆ’ *q*) = cos *q*
cos(90Â° âˆ’ *q*) = sin *q*
tan(90Â° âˆ’ *q*) = cot *q*
sin 70Â° = cos 20Â°
cos 35Â° = sin 55Â°
tan 34Â° = cot 56Â°

Second Quadrant Equation

The second quadrant consists of angles from 90Â° to 180Â°. Here, only sine is positive while cosine and tangent are negative.
sin(180Â° âˆ’ *q*) = sin *q*
sin 124Â° = sin 56Â°
cos(180Â° âˆ’ *q*) = âˆ’cos *q*
cos 110Â° = âˆ’cos 70Â°
tan(180Â° âˆ’ *q*) = âˆ’tan *q*
tan 150Â° = âˆ’tan 30Â°

Third Quadrant Equation

The third quadrant consists of angles from 180Â° to 270Â°. Here, only tangent is positive while cosine and sine are negative.
tan(*q* + 180Â°) = tan *q*
tan 240Â° = tan 60Â°
sin(*q* + 180Â°) = âˆ’sin *q*
sin 240Â° = âˆ’sin 60Â°
cos(*q* + 180Â°) = âˆ’cos *q*
cos 240Â° = âˆ’cos 60Â°

Fourth Quadrant Equation

The fourth quadrant consists of angles from 270Â° to 360Â°. Here, only cosine is positive while sine and tangent are negative.
cos(360Â° âˆ’ *q*) = cos *q*
cos 300Â° = cos 60Â°
sin(360Â° âˆ’ *q*) = âˆ’sin *q*
sin 300Â° = âˆ’sin 60Â°
tan(360Â° âˆ’ *q*) = âˆ’tan *q*
tan 310Â° = âˆ’tan 50Â°

I. Trigonometric Ratios and Angles of Elevation and Depression

(i) Trigonometric ratios

Given a right-angled triangle ABC as in Figure 13.1.

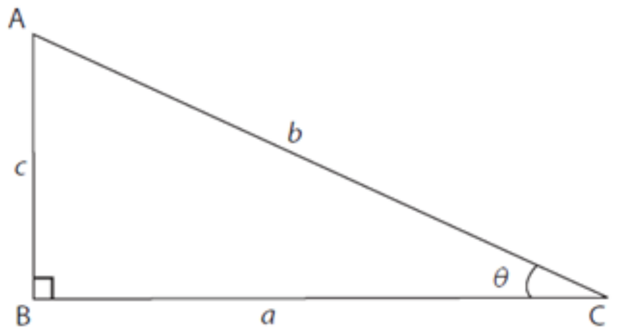


Figure 13.1

The side facing the right angle is the *hypotenuse*. Let angle ACB be *q* as indicated in Figure 13.1.
We can state the trigonometric ratios as follows:

$$\text{sine}\,\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{c}{b}$$

$$\text{cosine}\,\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{a}{b}$$

$$\text{tangent}\,\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{c}{a}$$

$$\text{cosecant}\,\theta = \frac{1}{\text{sine}\,\theta} = \frac{1}{\frac{\text{Opposite}}{\text{Hypotenuse}}}$$

$$= \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{b}{c}$$

$$\text{secant}\,\theta = \frac{1}{\text{cosine}\,\theta} = \frac{1}{\frac{\text{Adjacent}}{\text{Hypotenuse}}}$$

$$= \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{b}{a}$$

$$\text{cotangent}\,\theta = \frac{1}{\text{tangent}} = \frac{1}{\frac{\text{Opposite}}{\text{Adjacent}}}$$

$$= \frac{\text{Adjacent}}{\text{Opposite}} = \frac{a}{c}$$

(ii) Relations between the trigonometric ratios

We have:

(a) $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{c}{b}}{\frac{a}{b}} = \dfrac{c}{b} \div \dfrac{a}{b} = \dfrac{c}{b} \times \dfrac{b}{a} = \dfrac{c}{a} = \tan\theta$

$\therefore \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

(b) $\dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{a}{b}}{\frac{c}{b}} = \dfrac{a}{b} \div \dfrac{c}{b} = \dfrac{a}{b} \times \dfrac{b}{c} = \dfrac{a}{c} = \cot\theta$

$\therefore \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

(c) From Pythagoras' theorem,

$$b^2 = c^2 + a^2$$

Divide through by b^2

$$1 = \frac{c^2}{b^2} + \frac{a^2}{b^2} = \left(\frac{c}{b}\right)^2 + \left(\frac{a}{b}\right)^2$$

$$1 = (\sin\theta)^2 + (\cos\theta)^2$$

$$\therefore 1 = \cos^2\theta + \sin^2\theta$$

Worked Example 1

Show that sin2 *q* = 1 âˆ’ cos2 *q*.

SOLUTION

Since cos2 *q* + sin2 *q* = 1

âˆ´ sin2 *q* = 1 âˆ’ cos2 *q*

Worked Example 2

0Â° < *q* < 90Â°

If $\cos\theta = \dfrac{5}{13}$, calculate the values of:

(a) sin *q*

(b) tan *q*

(c) cos2 + sin2 *q*

SOLUTION

Given that $\cos\theta = \frac{5}{13}$

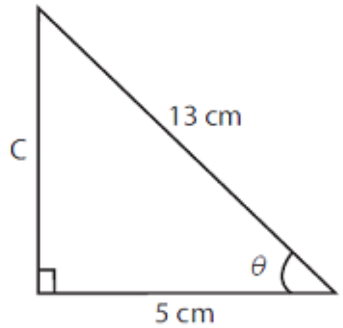


Figure 13.2

Using Figure 13.2:

$C^2 = 13^2 - 5^2$ (Pythagoras' theorem)

$C^2 = 169 - 25$

$C^2 = 144$

$C = \sqrt{144}$

$\therefore C = 12$

(a) $\sin\theta = \frac{12}{13}$

(b) $\tan\theta = \frac{12}{5}$

(c) $\cos^2\theta + \sin^2\theta = \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2$

$$= \frac{25}{169} + \frac{144}{169}$$

$$= \frac{25 + 144}{169}$$

$$= \frac{169}{169} = 1$$

(iii) Complementary angles

When the sum of two angles *A* and *B* is 90Â°, the two angles are said to be *complementary*.
Given a right-angled Î”ABC, there exists a relationship between the two complementary angles as shown in Figure 13.3.

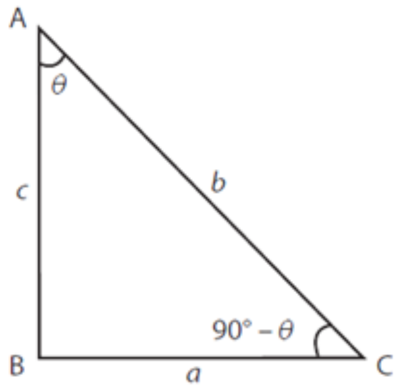


Figure 13.3

We observe from Figure 13.3 that:

$$\sin\theta = \frac{a}{b}, \quad \cos(90^\circ - \theta) = \frac{a}{b}$$

$$\boxed{\sin\theta = \cos(90^\circ - \theta)}$$

$$\cos\theta = \frac{c}{b}, \quad \sin(90^\circ - \theta) = \frac{c}{b}$$

$$\boxed{\cos\theta = \sin(90^\circ - \theta)}$$

Worked Example 3

Solve the following equations:

(a) sin *q* = cos 15Â°

(b) cos *x* = sin (*x* + 33Â°)

(c) sin 2*x* = cos 7*x*

SOLUTION

(a) $\sin\theta = \cos 15^\circ$ (given)

But $\sin\theta = \cos(90^\circ - \theta)$

$\therefore \cos(90^\circ - \theta) = \cos 15^\circ$

$\Rightarrow 90^\circ - \theta = 15^\circ$

$\Rightarrow -\theta = 15^\circ - 90^\circ$

$\Rightarrow -\theta = -75^\circ$

$\therefore \theta = 75^\circ$

(b) $\cos x = \sin(x + 33°)$

Since $\cos x = \sin(90° - x)$

$\Rightarrow \sin(90° - x) = \sin(x + 33°)$

$\Rightarrow 90° - x = x + 33°$

$\Rightarrow -x - x = 33° - 90°$

$\Rightarrow -2x = -57°$

$\Rightarrow 2x = 57°$

$\Rightarrow x = \dfrac{57}{2}$

$\therefore x = 28.5°$

(c) $\sin 2x = \cos 7x$ (given

$\Rightarrow$ But $\sin x = \cos(90° - x)$

$\sin 2x = \cos(90° - 2x)$

$\therefore \cos(90° - 2x) = \cos 7x$

$\Rightarrow 90° - 2x = 7x$

$\Rightarrow -2x - 7x = -90°$

$\Rightarrow -9x = -90°$

$\Rightarrow 9x = 90°$

$\Rightarrow x = \dfrac{90°}{9}$

$\therefore x = 10°$

(iv) Trigonometric ratios of special angles

Given the Î”ABC in Figure 13.4 (a) and (b), we have Table 13.1.

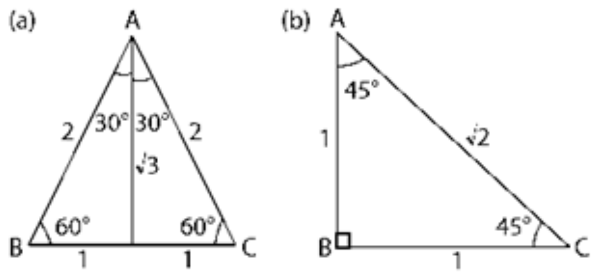


Figure 13.4

Table 13.1

θ	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Worked Example 4

Find the values of *a*, *b* and *c* in Figure 13.5, leaving your answers in surd form.

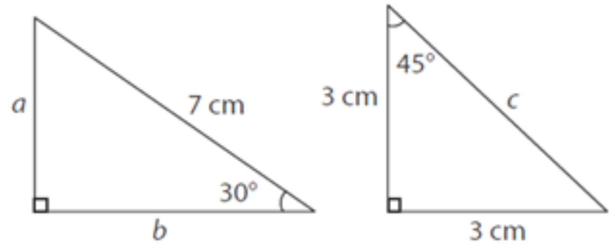


Figure 13.5

SOLUTION

$$\sin 30^\circ = \frac{a}{7}$$

$$\sin 30^\circ = \frac{1}{2} \text{ (using special angles)}$$

$$\Rightarrow \frac{a}{7} = \frac{1}{2} \text{ (using special angles)}$$

$$\Rightarrow a = \frac{7 \times 1}{2} = \frac{7}{2} = 3.5 \text{ cm}$$

$$a = 3.5 \text{ cm}$$

$$\cos 30^\circ = \frac{b}{7}$$

$$\text{but } \cos 30^\circ = \frac{\sqrt{3}}{2} \text{ (using special angles)}$$

$$\Rightarrow \frac{b}{7} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow b = 7\frac{\sqrt{3}}{2}$$

$$\cos 45^\circ = \frac{3}{c}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \text{ (using special angles)}$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{3}{c}$$

$$\Rightarrow c = 3\sqrt{2}$$

Alternatively:

c 2 = 32 + 32
c 2 = 9 + 9
c 2 = 18

$$c = \sqrt{2 \times 3 \times 3}$$

$$c = \sqrt{2 \times 3^2}$$

$$c = 3\sqrt{2}$$

(v) Reading the sine, cosine and tangent tables

The table of values of sine, cosine and tangent of acute angles can be obtained in a four-figure table.

Worked Example 5

Use the table to fi nd

(a) sin 38.27Â°
(b) cos 12.32Â°
(c) tan 43.63Â°

SOLUTION

(a) sin 38.27

Using the sine table:

â— Under the *x* column, look for 38Â°.

â— Move along the 38Â° row under 0.2 which reads 0.6184.
â— Go to the difference column, trace from 7 down to where it intersects with the 38Â° row, which reads add 10 written as 0.0010.
sin 38.27Â° = 0.6184 + 0.0010 = 0.6194
(b) cos 12.32Â°
From the cosine table,
â— Under the *x* column, look for 12Â°.
Move along the 12Â° row under 0.3 which reads 0.9770.
â— Go to the difference column, trace from 3 down to where it intersects with the 12Â° row which reads subtract 1, accurately written as 0.0001.
cos 12.32Â° = 0.9770 âˆ' 0.0001
= 0.9769
(c) tan 43.63Â°
From the tangent table,
â— Under the *x* column, look for 43Â°.
â— Move along 43Â° under 0.6 which
reads 0.9523.
â— Go to the difference column, trace from 3 down to where it intersects with the 43Â° row which reads add 10, written as 0.0010.
âˆ´tan 43.63Â° = 0.9523 + 0.0010
= 0.9533

Exercise 1

1. If $\sin\theta = \frac{12}{13}$, what is the value of $\tan\theta$ for $0° < \theta < 90°$?
2. Given that $\cos x = \frac{3}{5}$, $0° < x < 90°$, calculate $\frac{3\sin x - 2\cos x}{5\tan x}$.
3. If $\cos\theta = \frac{5}{13}$, what is $\tan\theta$ for $0° < \theta < 90°$?
4. Given that $\tan x = \frac{5}{12}$, what is the value of $\sin x + \cos x$, if x is an acute angle.
5. Given that $\sin p = \frac{5}{13}$, where p is acute, find the value of $\cos p - \tan p$.

6. If sin *x* = cos 50Â°, fi nd the value of *x*.
7. Without using tables, fi nd the value
of $\frac{\sin 20°}{\cos 70°} + \frac{\sin 25°}{\cos 65°}$.

8. If sin (*x* + 30Â°) = cos 40Â°, fi nd *x*.
9. If cos (*x* + 25Â°) = sin 45Â°, find the value of *x*.
10. Evaluate cos 45Â° cos 30Â° âˆ' sin 45Â° sin 30Â° leaving the answer in surd form.
11. Given that cos *x* = 0.7431 for 0Â° < *x*
< 90Â°, use tables to find the value of
(a) 3 sin *x* + 2 cos *x*
(b) $\tan\frac{x}{2}$

II. Angles of Elevation and Depression
It is one of the topics where the knowledge and application of trigonometric ratios are needed. Here, we shall calculate the vertical, horizontal height or angular distances of objects from a given point.
(i) Angle of elevation

When an object Q, at a height above the horizontal ground, is viewed from a point *P* on the ground or from the normal eye level, the angle formed between the horizontal ground and the line of sight is called *angle of elevation* as shown in Figure 13.6.

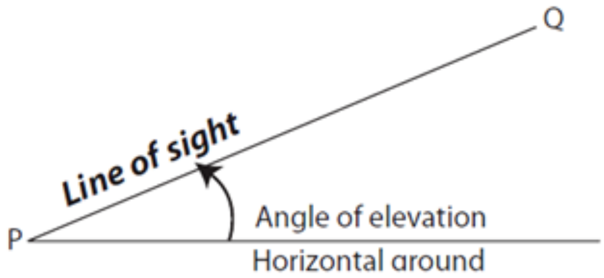


Figure 13.6 Angle of elevation

(ii) Angle of depression

When an object D on the horizontal ground is in sight from a point T at a height above the ground, the angle formed between the line of sight and the horizontal plane from T is called *angle of depression* (see Figure 13.7).

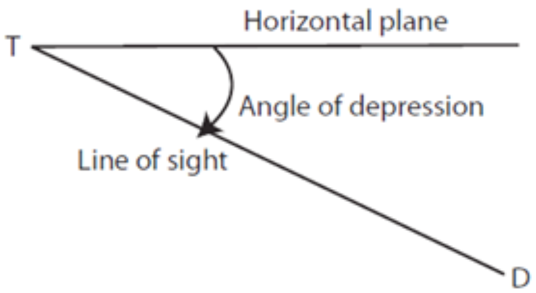


Figure 13.7 Angle of depression

(iii) Relationship between angles of elevation and depression

In Figure 13.8, the angle of elevation of B from C is equal to angle of depression of C from B (recall that alternate angles are equal).

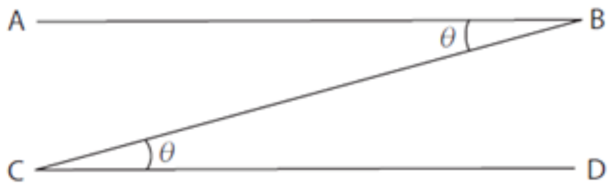


Figure 13.8

Worked Example 6

Figure 13.9 is a Î”ABD, right angled at B.

If |AB| = 3 cm, |AD| = 5 cm, *ACË† B* = 61Â° and *DAË† C* = *x*Â°, calculate, correct to one decimal place, the value of *x*.

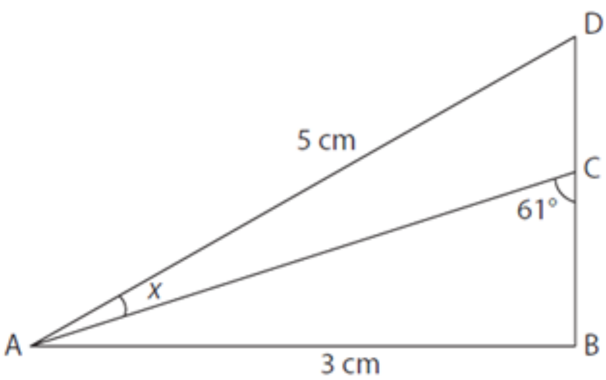


Figure 13.9

SOLUTION

BAË† C = 180Â° âˆ’ (90Â° + 61Â°)

= 180Â° âˆ’ 151Â°

= 29Â°

cos *BAË† D* = 3/5 = 0.6

â‡’ *BAË† D* = cosâˆ’10.6

Hence, *BAË† D* = 53.1301Â°

x = *BAË† D* âˆ’ *BAË† C*
x = 53.1301Â° âˆ’ 29Â°
x = 24.1301
x = 24.1Â° (1 d. p.)

Worked Example 7
The pilot of an aircraft 2000 m above sea level observes at an instant that the angles of depression of two boats which are on a straight line are 58Â° and
72Â°. Find, correct to the nearest metre, the distance between the two boats.
SOLUTION
Sketch the diagram that represents the information.

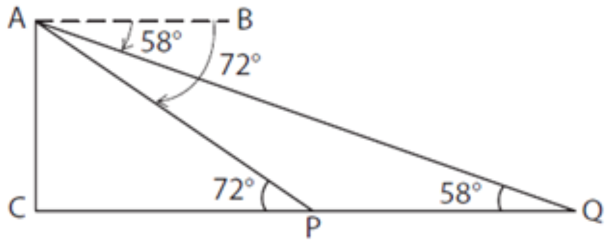


Figure 13.10

Using Figure 13.10,
BAË† Q = *AQË† C* (alternate angles)
and *BAË† P* = *APË† C* (alternate angles)#

$$\text{Hence, } \tan 72^\circ = \frac{2\,000}{|CP|}$$

$$\Rightarrow |CP| = \frac{2\,000}{\tan 72^\circ}$$

$$= \frac{2\,000}{3.0777} = 649.8359 \text{ m}$$

$$\text{and } \tan 58^\circ = \frac{2\,000}{|CQ|}$$

$$\Rightarrow |CQ| = \frac{2\,000}{\tan 58^\circ} = \frac{2\,000}{1.6003}$$

$$= 1\,249.7657 \text{ m}$$

$$|PQ| = |CQ| - |CP|$$

$$= 1\,249.7657 - 649.8359 \text{ m}$$

$$= 599.9298 \text{ m}$$

$$|PQ| = 600 \text{ m (to the nearest metre)}$$

Worked Example 8
Two students of the same height 1.65 m stand at two diff erent points A and B, 45 m apart. They observe that the angles of elevation of a bird from the left are 50Â° and 65Â°, respectively. Find the height of the bird from the horizontal ground.
Express your answer in 3 s.f.
SOLUTION
Sketch the diagram that represents the information

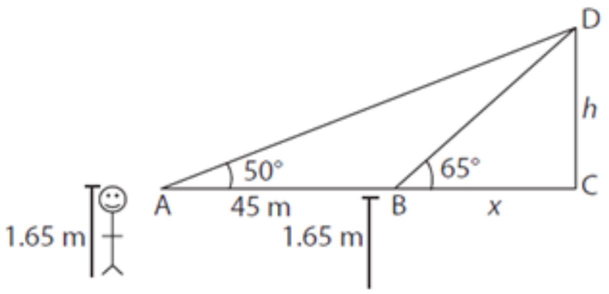


Figure 13.11

Let |*BC*| = *x* and let the height of the bird from the horizontal ground *CD* be *h*.
From Î”BCD,

$$\tan 65^\circ = \frac{h}{x}$$

â‡’ *h* = *x* tan 65Â°...(i)
From Î”ACD,

$$\tan 50^\circ = \frac{|DC|}{|AC|} = \frac{h}{x+45}$$

â‡’ *h* = (*x* + 45) tan 50Â°................................(ii)
By equating (i) and (ii), *x* tan 65Â° = (*x* + 45Â°) tan 50Â°
â‡’ x tan 65Â° = *x* tan 50 + 45Â° tan 50Â°
â‡’ *x* tan 65Â° âˆ’ *x* tan 50Â° = 45Â° tan 50
â‡’ *x*(tan 65Â° âˆ’ tan 50Â°) = 45Â° tan 50Â°

$$\Rightarrow x = \frac{45^\circ \tan 50^\circ}{\tan 65^\circ - \tan 50^\circ}$$

$$\Rightarrow x = \frac{45^\circ \times 1.1918}{2.1445 - 1.1918}$$

$$x = \frac{53.631}{0.9527}$$

$$x = 56.2937$$

From equation (i), *h* = *x* tan 65Â°
â‡’ *h* = 56.2937 Ã— tan 65Â°
= 56.2937 Ã— 2.1445 = 120.7218 m
The height of the spot where the bird is from the horizontal ground is:

$$\begin{array}{r} 120.7218 \\ \underline{+\,1.6500} \\ \underline{122.3718} \end{array}$$ = 122 m (3 s.f.)

Exercise 2
1. A ladder 10 m long leans against a vertical wall, making an angle of 65Â° with the horizontal ground.
Calculate, correct to one decimal place, how far the foot of the ladder is from the wall.
2. When an aircraft is 550 m above the horizontal ground, its angle of elevation from a point P on the ground is 30Â°. How far is the aircraft from P by line of sight?
3. The angle of elevation of *X* from *Y* is 35Â°. If /XY/ = 40 m, how high is *X* above *Y*?
4. From the top of a tower 10 m high, the angle of depression of a stone lying on the horizontal ground is
69Â°. Calculate, correct to one decimal place, the distance of the stone from the foot of the tower.
5. A man stands at a distance of 27 m away from the foot of a tower, which is 37 m high. Calculate the angle of elevation of the top of the tower (to the nearest degree).
6. Find the angle of depression of the top of a pole 20 m high from a point on the horizontal ground 62 m away from the foot of the pole.
7. A hunter sighted a prey on top of a palm tree 12 m high. If the hunter is
15 m away from the palm tree, what is the angle of elevation of his eyes, if he is 1.5 m tall?

8. From a point T on the horizontal ground, the angle of elevation of the top R of a tower RS, 38 m high, is
63Â°. Calculate, correct to the nearest metre, the distance between T and S.
9. A ladder of length 4.5 m leans against a vertical wall making an angle of 50Â° with the horizontal. If the bottom of a window is 4 m above the ground, what is the distance between the top of the ladder and the bottom of the window?
(Answer correct to the nearest centimetre). (WAEC)
10. The feet of two vertical poles of height 3 m and 7 m are in line with a point P on the ground, the smaller pole being between the taller pole and P, and at a distance 20 m from P.
The angle of elevation of the top (T) of the taller pole from the top (R) of the smaller pole is 30Â°.
Calculate the:
(a) Distance

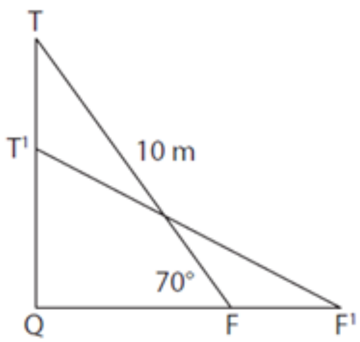
$\overline{RT}$.
(b) Distance of the foot of the taller pole from P, correct to 3 s.f.
(c) Angle of elevation of T from P, correct to the nearest degree.
11. A woman looking out of the window of a building of height 30 m observed that the angle of depression of the top of a flag pole was 44Â°.
If the foot of the pole is 25 m from the foot of the building and on the same horizontal ground, find, correct to the nearest whole number, the:
(a) Angle of depression of the foot of the pole from the woman.
(b) Height of the flag pole.
12.


Figure 13.12

In Figure 13.12, a ladder TF, 10 m long, is placed against a wall at an angle 70Â° to the horizontal.
(a) How high up the wall, correct to the nearest metre, is the ladder?
(b) If the foot (F) of the ladder is shifted to F1 by 1 m,

(i) How far, correct to 2 s.f. , did the top T slide down to T1.

(ii) Calculate, correct to the nearest degree, âˆ QF1TI.

13. A man on the same level ground with a tree stands at a distance 12.82 m away from the foot of the tree. He observes the angle of elevation of the top of the tree to be 52Â°. If the man is 1.24 m tall, calculate, correct to two decimal places, the height of the tree. From two points on opposite sides of a pole 33 m high, the angles of elevation of the top of the pole are 53Â° and 67Â°. If the two points and the base of the pole are on the same horizontal level, calculate, correct to 3 s.f. the distance between the two points.

III. Cardinal points and direction

Directions are traced with the aid of the **c**ardinal points. Hence, it is necessary that we should remind ourselves the 4, 8 and 16 cardinal points.

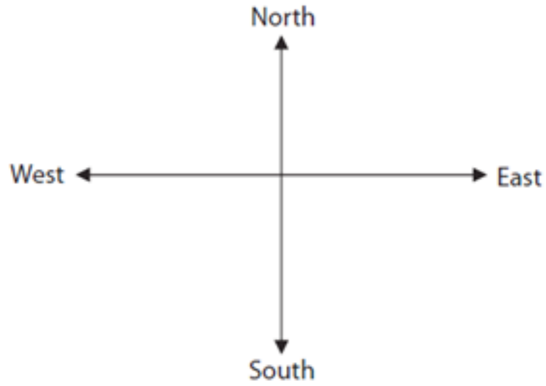


Figure 13.13 Four cardinal points

Figure 13.13 shows the four cardinal points: North, South, East and West.

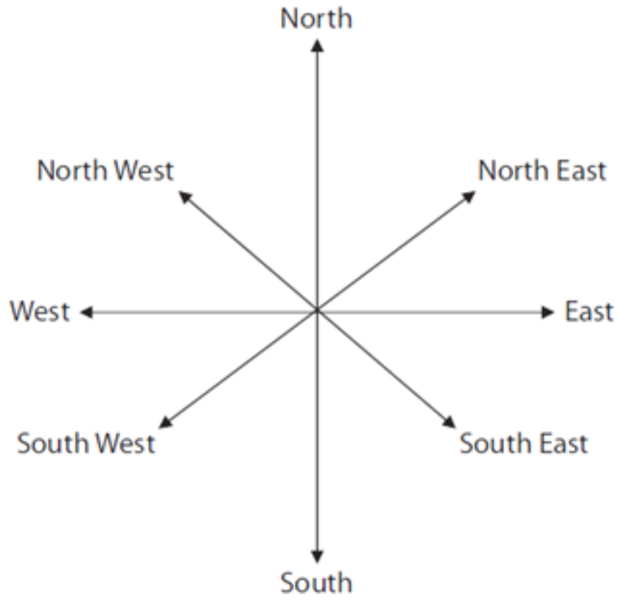


Figure 13.14 Eight cardinal points

The NW, NE, SE and SW directions represent the midway between the North and West, North and East, South and East, and South and West, respectively.

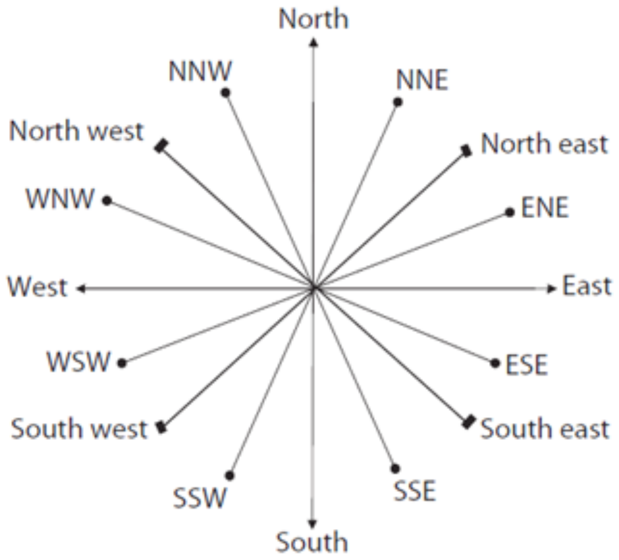


Figure 13.15 Sixteen cardinal points

Directions with the aid of Cardinal Point

Worked Example 9

Figure 13.16 indicates the relative directions of the lettered boxes to one another.

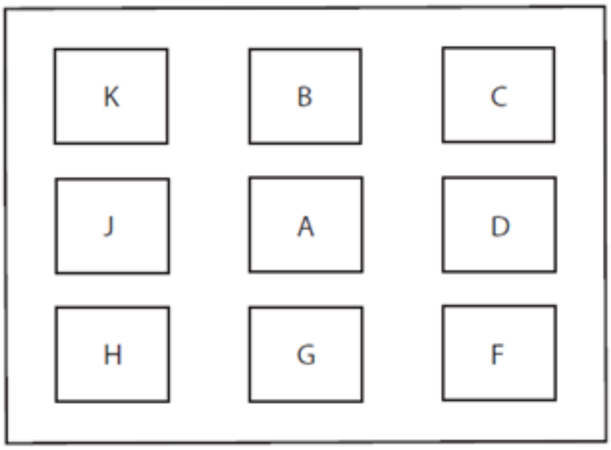


Figure 13.16

SOLUTION

(a) B is due North of A
(b) K is due North of J
(c) C is due North of D
(d) G is due South of A
(e) H is due South of J
(f) F is due South of D
(g) B and C are due East of K
(h) A and D are due East of J
(i) G and F are due East of H
(j) K and B are due West of C
(k) J and A are due west of D
(l) H and G are due West of F
(m) C is North East of A
(n) K is North West of A
(o) H is South West of A
(p) F is South East of A
(q) K is North West of F
(r) H is South West of C
(s) G is South West of D
(t) J is North West of G etc.

Exercise 3

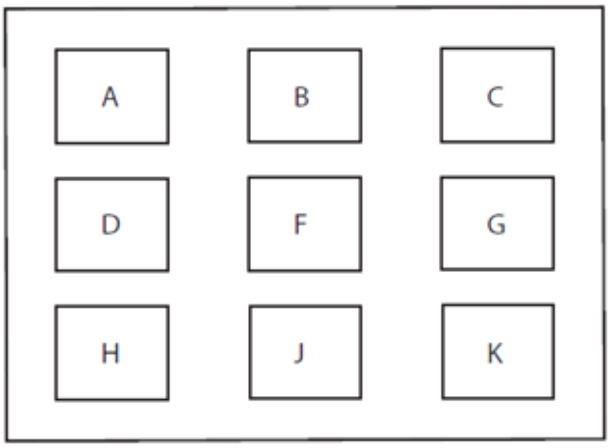


Figure 13.17

With the aid of the lettered boxes in Figure 13.17, determine the directions of:

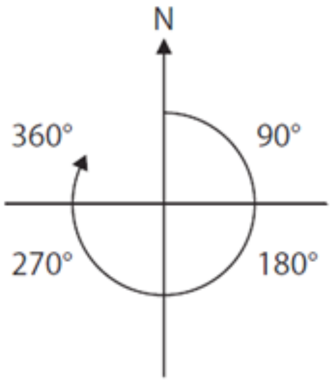
1. A from F
2. B from F
3. C from F
4. D from F
5. G from F
6. H from F
7. J from F
8. K from F
9. A from C
10. A from G
11. A from K
12. K from B
13. K from H
14. D from J

15. C from H 16. B from K
17. J from A 18. B from H
19. B from G 20. F from H

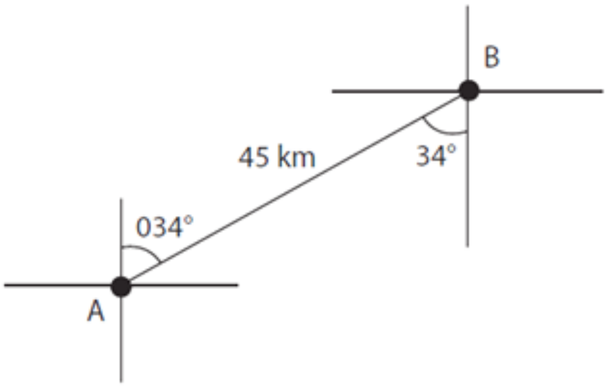
IV. Bearings

In questions involving bearings, the following are necessary:

1. The use of four cardinal points with North as the origin or the starting point. It is collaborated in a clockwise direction.



2. The concept of alternate angles is applied at each point of the bearing.
For example: A man walks 45 km on a bearing of 034Â° to a point B.



3. Sine rule is used when one side and two included angles are given. The third angle is calculated by adding the two given angles and then subtracting from 180Â°. The rule states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ and C are angles where A + B + C = 180Â°.

4. Cosine rule is used when two sides and an included angle are given.
The rule states that

$$\left.\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}\right\} \text{useful when two sides and one angle are given.}$$

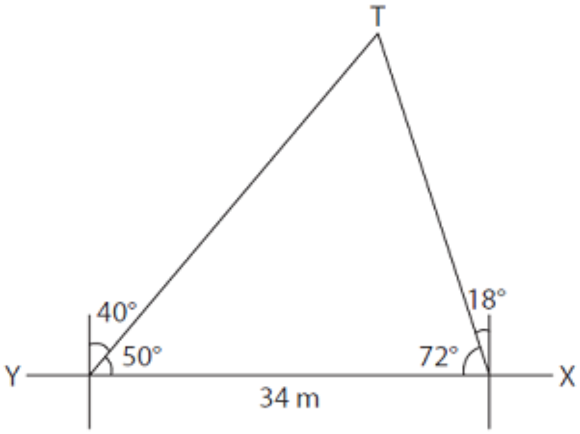
or

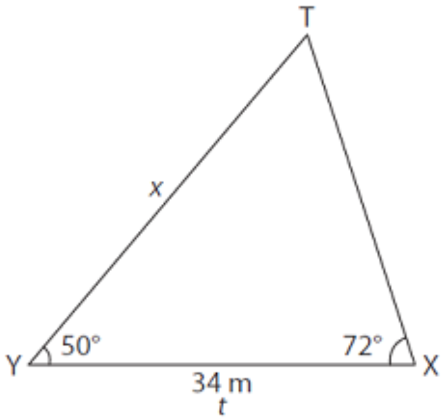
$$\left.\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ \cos B &= \frac{a^2 + c^2 - b^2}{2ac} \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}\right\} \text{useful when all the three sides are given}$$

5. The bearing of B from A refers to angle A and the bearing of C from A refers to angle A .
6. A particular bearing is closed when one point is directly east or west of another point.

Basic Examples on Bearing

Worked Example 10
A point X is 34m due east of a point Y.
The bearing of a flag pole from X and Y are N18Â°W and N40Â°E respectively.
Calculate the distance of the flag pole from Y.
SOLUTION





X + *Y* + *T* = 180Â° (sum of angles in a Î”)
72Â° + 50Â° + *T* = 180Â°
122Â° + *T* = 180Â°
T = 180Â° âˆ’ 122Â°
T = 58Â°
Using sine rule

$$\frac{x}{\sin X} = \frac{t}{\sin T}$$

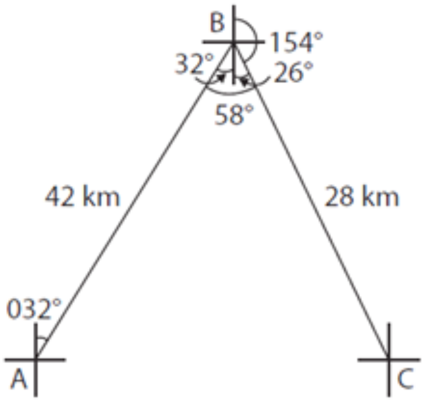
$$\frac{x}{\sin 72^\circ} = \frac{34}{\sin 58^\circ}$$

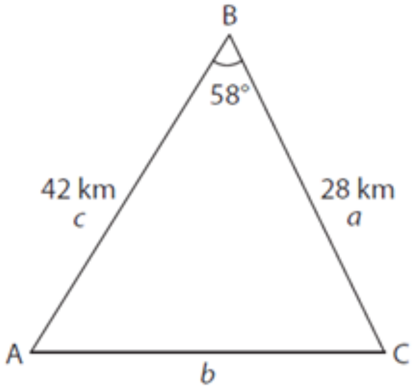
$$x \times \sin 58^\circ = 34 \times \sin 72^\circ$$

$$x = \frac{34 \times \sin 72^\circ}{\sin 58^\circ}$$

$$x = 38.1 \text{ m}$$

Worked Example 11
A man leaves his base camp and drives 42 km on a bearing of 032Â°. He then drives 28 km on a bearing of 154Â°. How far is he then from his base camp and what is his bearing from it?
SOLUTION





Using cosine rule

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 28^2 + 42^2 - 2 \times 28 \times 42 \times \cos 58°$

$b^2 = 784 + 1\,764 - 56 \times 42 \times 0.5299$

$b^2 = 2\,548 - 1\,246.32$

$b^2 = 1\,301.68$

$$b = \sqrt{1\,301.68}$$

$b = 36.079$ km

$b = 36.1$ km

Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{28}{\sin A} = \frac{36.1}{\sin 58°}$$

$\sin A \times 36.1 = 28 \times \sin 58°$

$$\sin A = \frac{28 \times \sin 58°}{36.1}$$

$$= \frac{28 \times 0.8480}{36.1}$$

$\sin A = 0.6577$

$A = \sin^{-1} 0.6577$

$A = 41.12°$

The bearing from the base camp will be

$= 32 + 41.12$

$= 73.12°$

$= 73°$

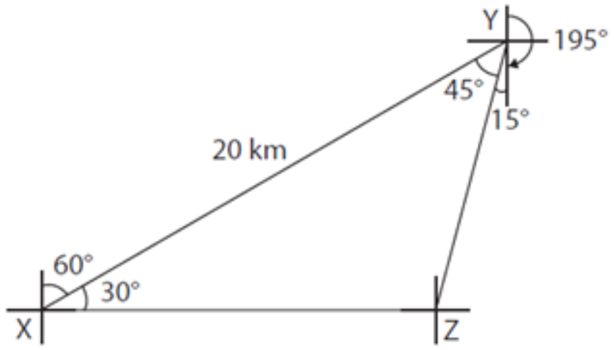
Worked Example 12

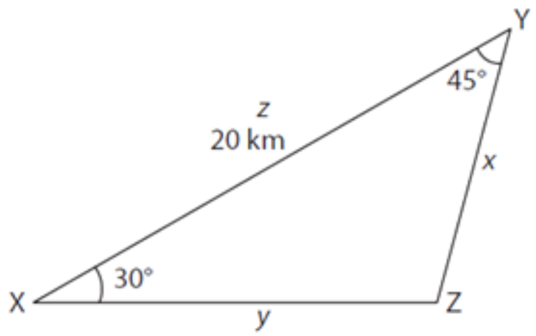
A man travels from a village X on a bearing of 060° to a village *Y* which is 20 km away. From Y he travels to a village Z on a bearing of 195° directly east of X.

Calculate, correct to three significant figures, the distance of (i) Y from Z and

(ii) Z from X.

SOLUTION





X + *Y* + *Z* = 180Â° (sum of angles in a Î”)
30Â° + 45Â° + *Z* = 180Â°
Z = 180Â° âˆ’ 75Â° = 105Â°
Using sine rule

$$\frac{x}{\sin X} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 30^\circ} = \frac{20}{\sin 105^\circ}$$

x Ã— sin 105Â° = 20 Ã— sin 30Â°

$$x = \frac{20 \times \sin 30^\circ}{\sin 105^\circ}$$

$$x = \frac{20 \times 0.5}{\sin 75^\circ} = \frac{10}{0.9659}$$

x = 10.35 km
x = 10.4 km
Using sine rule

$$\frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{y}{\sin 45^\circ} = \frac{20}{\sin 105^\circ}$$

$$y \times \sin 105^\circ = 20 \times \sin 45^\circ$$

$$y = \frac{20 \times \sin 45^\circ}{\sin 105^\circ}$$

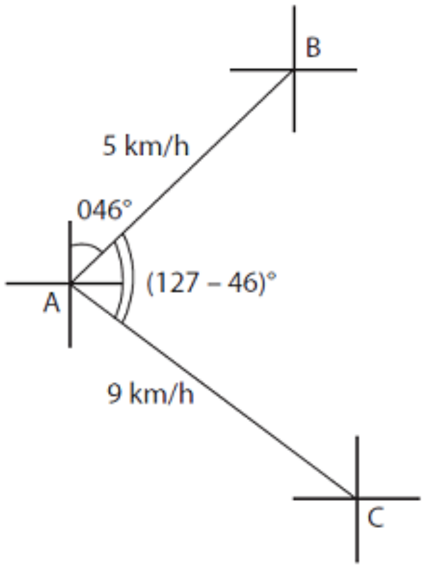
$$y = \frac{20 \times \sin 45^\circ}{\sin 75^\circ}$$

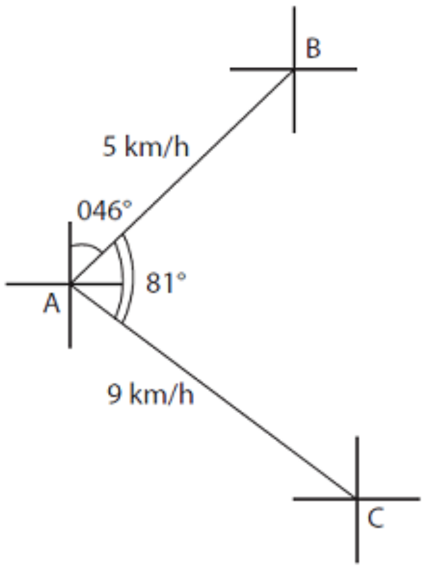
$$y = \frac{20 \times 0.7071}{0.9659}$$

$$y = 14.64 \text{ km}$$

$$y = 14.6 \text{ km}$$

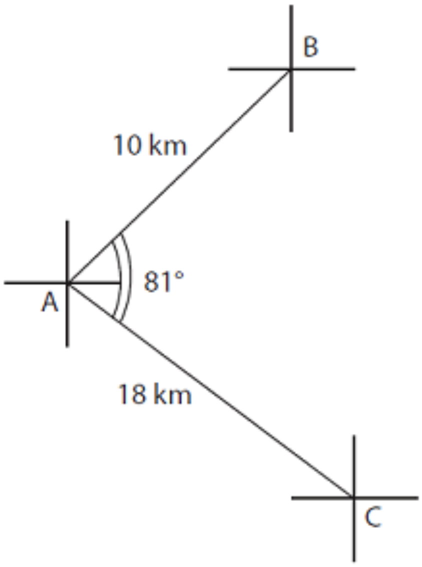
Worked Example 13
Two ships leave port at the same time.
One travels at 5 km/hr on a bearing of 046Â°. The other travels at 9 km/hr on a bearing of 127Â°.
How far apart are the ships after 2 hrs?
SOLUTION

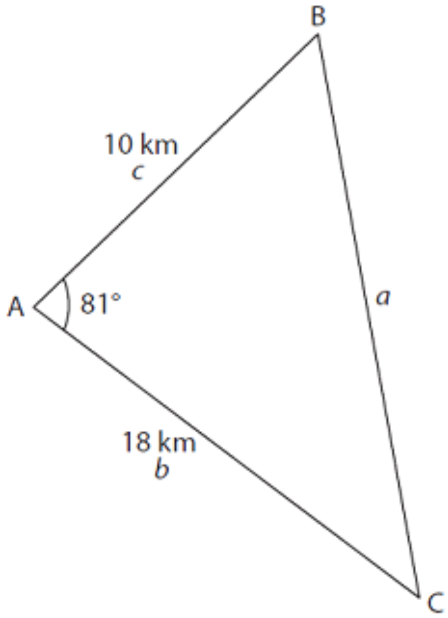




$$\text{Speed} = \frac{D}{T} = \frac{5}{1} = \frac{10}{2} = 10 \text{ km/2 hrs}$$

$$\text{Speed} = \frac{D}{7} \Rightarrow \frac{9}{1} = \frac{18}{2} = 18 \text{ km/2 hrs}$$





Using cosine rule

*a*2 = *b*2 + *c*2 âˆ' 2*bc* cos *A*

*a*2 = 182 + 102 âˆ' 2 Ã— 18 Ã— 10 Ã— cos 81Â°

*a*2 = 324 + 100 âˆ' 360 Ã— 0.1564

*a*2 = 424 âˆ' 56.304

*a*2 = 367.696

$a = \sqrt{367.696} = 19.175$ km

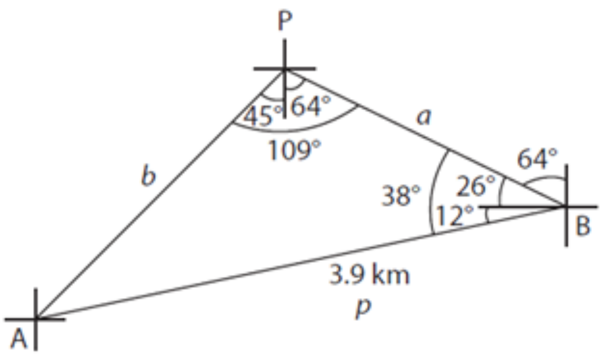
a = 19.2 km

Worked Example 14

The bearings of ship A and B from point P are 225Â° and 116Â° respectively. Ship A is 3.9 km from ship B on a bearing of 258Â°.

Calculate the distance of ship A from P.

SOLUTION



Using sine rule

$$\frac{b}{\sin B} = \frac{p}{\sin P}$$

$$\frac{b}{\sin 38^\circ} = \frac{3.9}{\sin 109^\circ}$$

b Ã— sin 109Â° = 3.9 Ã— sin 38Â°

$$b = \frac{3.9 \times \sin 38^\circ}{\sin 109^\circ}$$

$$b = \frac{3.9 \times \sin 38^\circ}{\sin 71^\circ}$$

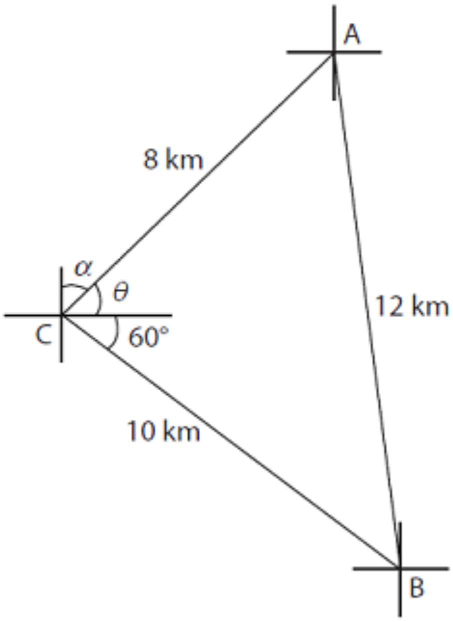
$$b = \frac{3.9 \times 0.6157}{0.9455}$$

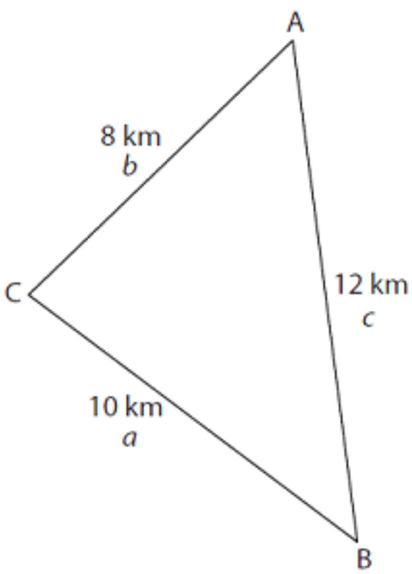
b = 2.54 km
b = 2.5 km

Worked Example 15

Two boats A and B left a port C at the same time along different routes. B travelled on a bearing of 150Â° and A travelled on the north side of B. When A had travelled 8km and B had travelled 10km, the distance between the two boats was found to be 12km. Calculate the bearing of A's route from C.

SOLUTION





Using cosine rule

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{10^2 + 8^2 - 12^2}{2 \times 10 \times 8}$$

$$= \frac{100 + 64 - 144}{160}$$

$$= \frac{20}{160}$$

$$= \frac{1}{8} = 0.1250$$

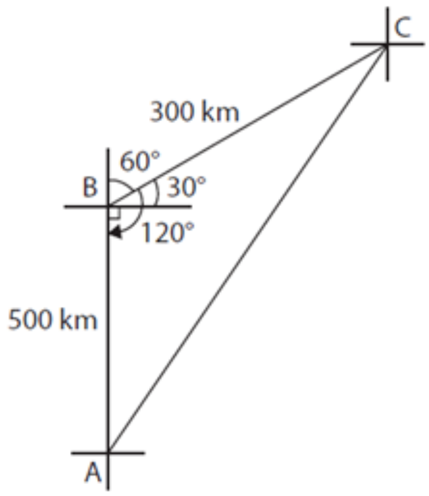
cos *C* = 0.1250
C = cosâˆ'1 0.1250
C = 82.81Â°
C = *q* + 60Â°
82.81Â° = *q* + 60Â°
82.81Â° âˆ' 60Â° = *q*
22.81Â° = *q*
Î± + *q* = 90Â° (complementary angles)
Î± + 22.81Â° = 90Â°
Î± = 90Â° âˆ' 22.81Â°
Î± = 67.19Â°
Î± = 67Â°

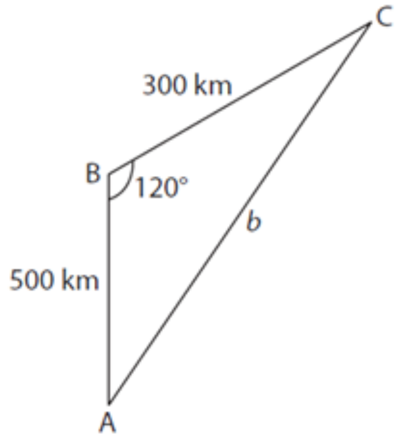
Worked Example 16

An aeroplane flies 500 km due north from Ikeja Airport. It flies on a bearing of 60Â° for a further distance of 300 km before overflying a road junction. Calculate:

(a) The distance of the aeroplane from Ikeja Airport when it was directly above the road junction.
(b) Bearing of the aeroplane from Ikeja Airport at this instant.

SOLUTION





Using cosine rule

$b_2 = a_2 + c_2$ â^' $2ac \cos B$
$b_2 = 300_2 + 500_2$ â^' 2 Ã— 300 Ã— 500 Ã— cos 120
b_2 = 90 000 + 250 000 â^' 600 Ã— 500 Ã— â^' cos 60
b_2 = 340 000 + 600 Ã— 500 Ã— 0.5
b_2 = 340 000 + 150 000
b_2 = 490 000

$$b = \sqrt{490\,000}$$

b = 700 km
Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{300}{\sin A} = \frac{700}{\sin 120^\circ}$$

sin A Ã— 700 = 300 Ã— sin 120Â°

$$\sin A = \frac{300 \times \sin 120^\circ}{700}$$

$$\sin A = \frac{300 \times \sin 60^\circ}{700}$$

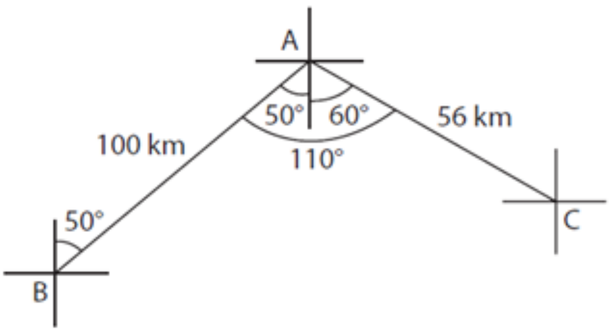
$$\sin A = \frac{300 \times 0.8660}{700}$$

sin A = 0.3711
A = sinâ^'1 0.3711
A = 21.78Â°

Assignment

1. A town A is 100 km from B on N50Â°E.
Town C is 56 km from town A on a bearing of S60Â°E. Sketch a diagram showing the position of the three towns. Calculate:
(a) The distance of town C from town B.
(b) The bearing of town C from town B.

SOLUTION



(BC) = 130.3 km
Bearing of C = 073.82Â°

2. An aeroplane leaves an airport A and flies on a bearing of 035Â° for $1\frac{1}{2}$ hrs at 600 km/hr to an airport B.

It then flies on a bearing of 130Â° for $1\frac{1}{2}$ hrs at 400 km/hr to an airport C.
Calculate:
(a) The distance from C to A, correct to the nearest kilometre.
(b) The bearing of C from A, correct to the nearest degree.

SOLUTION

(a) 1 037 km (b) 670Â°

3. Three towns P, Q and R are such that the distance between P and Q is 50 km and the distance between
P and R is 90 km. If the bearing of Q from P is 075Â° and that of R from P is 310Â°, calculate:
(a) The distance between Q and R.

(b) The bearing of R from Q.

SOLUTION

(a) QR = 125.5 km

(b) 291Â°

4. Tony sets out to travel from X to Z via Y. From X, he travels a distance of 8 km on a bearing of 030Â° to Y.

From Y, he travels a further 6 km due east.

(a) Calculate how far Z is

(i) North of X.

(ii) East of X.

(b) Hence or otherwise, calculate the distance XZ, correct to one decimal place.

SOLUTION

(a) $4\sqrt{3}$□□

(b) 10 km

5. A girl moves from a point P on a bearing of 060Â° to a point *Q*, 40 m away. She then moves from the point Q on a bearing of 120Â° to a point R. The bearing of P from R is 255Â°. Calculate, correct to three significant figures, the distance between P and R .

SOLUTION

PR = 49 m

(i) Definition of bearing

Bearing is the relative angular relationship between two distant places. It is measured in degrees.

(ii) Notation of bearing

There are two major ways of representing bearings. These are:

(1) Surveyorsâ€™ bearing

(2) Compass bearing

Surveyorsâ€™ bearing

It is also known as three-digit bearing, because its reading is represented in three digits as follows: 007Â°, 069Â°, 314Â°, that is, using zeros to replace empty positions before numbers.

The angular measurement is done in the clockwise direction from the North Pole as shown in Figure 13.18.

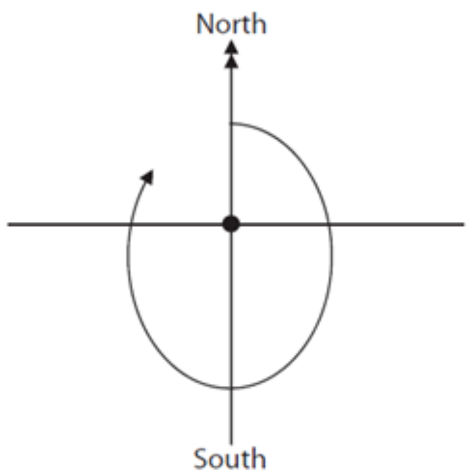


Figure 13.18

Compass bearing

Its angular measurement is done either from N to E or from N to W, if the point is in the northern hemisphere and from

S to E or from S to W, if the point is in the southern hemisphere. That is, direction of North or South is usually stated first, an acute angle follows, and then East or

West is written last (see Figure 13.19).

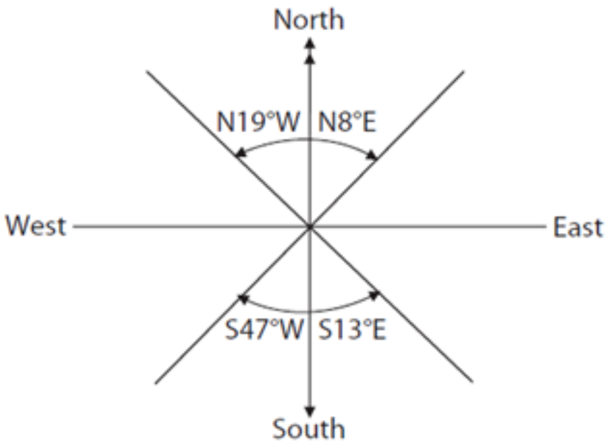
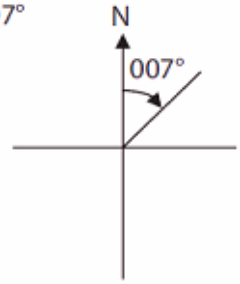
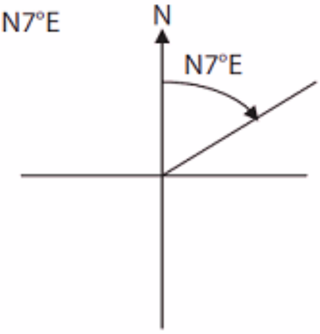
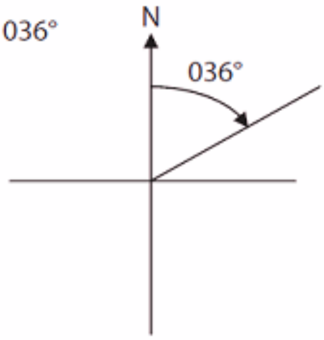
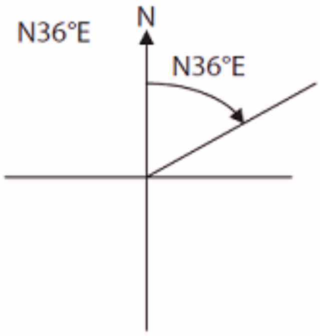
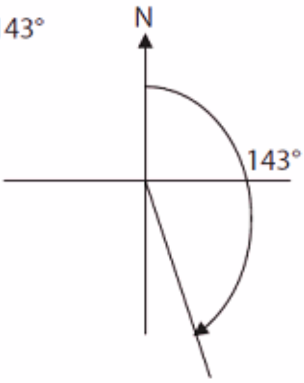
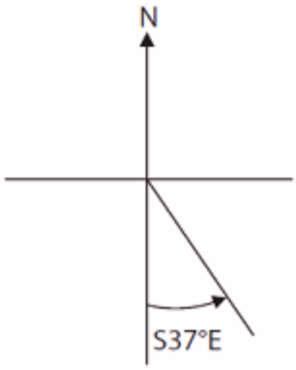
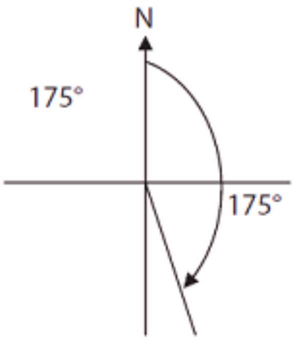
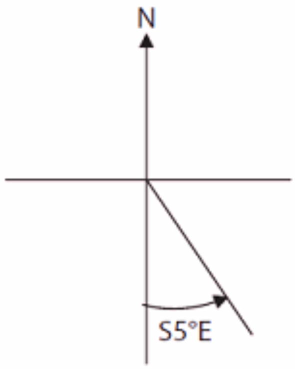


Figure 13.19

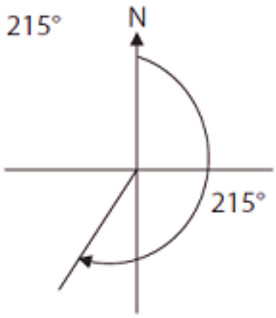
Table 13.2 illustrates some readings in surveyorsâ€™ and their corresponding compass bearings.

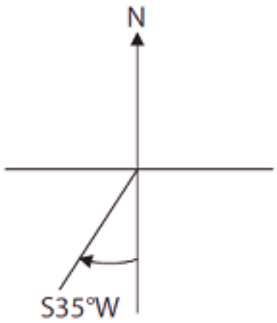
Table 13.2

	Surveyors'	Compass
(i)	007° 	N7°E 
(ii)	036° 	N36°E 
(iii)	143° 	180° – 143° = S37°E 
(iv)		180° – 175° = S5°E 

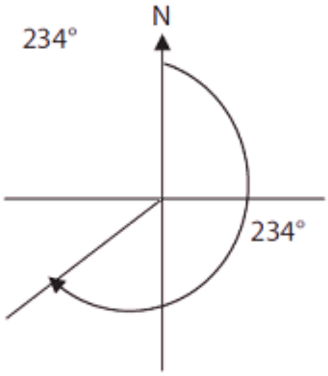
	Surveyors'	Compass

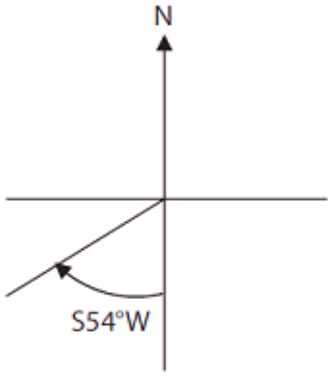
(v) $215^\circ - 180^\circ = S35^\circ W$



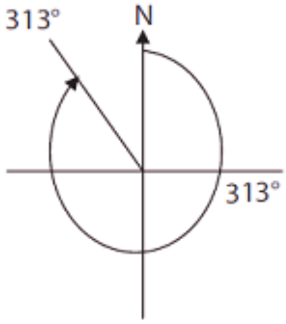


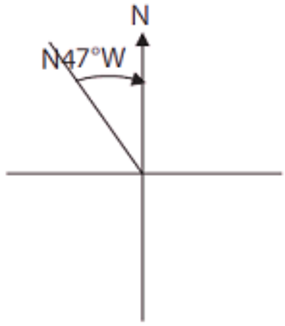
(vi) $234^\circ - 180^\circ = S54^\circ W$



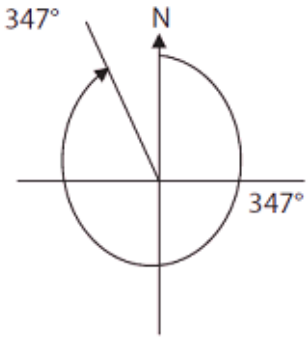


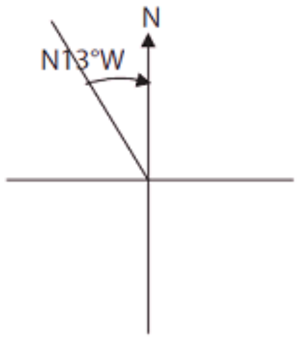
(vii) $360^\circ - 313^\circ = N47^\circ W$





(viii) $360^\circ - 347^\circ = N13^\circ W$





Exercise 4

Copy and complete the table below

Table 13.3

S/No	Surveyors'	Compass
1.	009°	________
2.	________	N8°E
3.	014°	________
4.	________	N39°E
5.	097°	________
6.	________	S16°E
7.	169°	________
8.	________	S19°E
9.	213°	________
10.	________	S3W
11.	237°	________
12.	________	S16°W
13.	343°	________
14.	________	N16°W
15.	354°	________
16.	________	N27°W

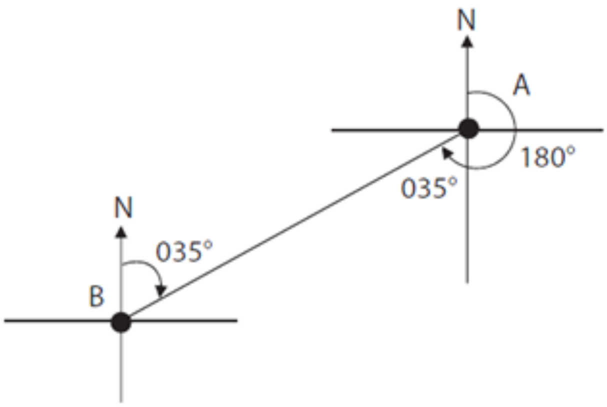
V. Bearings of Objects from a Given Point

Worked Example 17

If the bearing of a point A from a point B is 035Â°, what is the bearing of B from A?

SOLUTION

Two crosses will be drawn. One at A and the other at B. But the first cross shall be drawn at B because of the phrase â€˜From
B is 035Â°â€™.

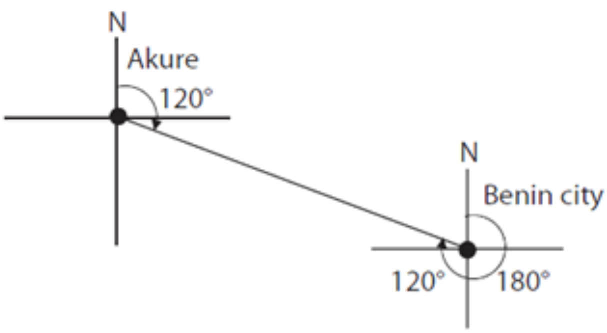


Therefore, the bearing of B from A is 180Â° + 035Â° = 215Â°.

Worked Example 18

If the bearing of Benin City from Akure is 120Â°, what is the bearing of Akure from Benin City?

SOLUTION



180Â° + 120Â° = 300Â°

Worked Example 19

What is the bearing of Yola from Jalingo, if the bearing of Jalingo from Yola is 200Â°?

SOLUTION

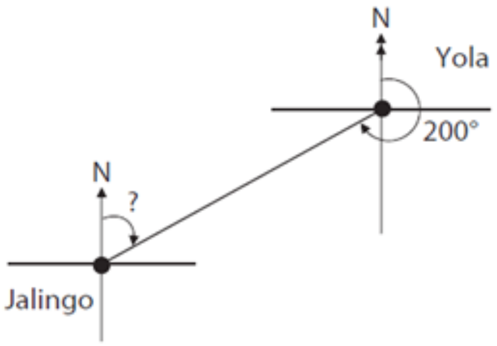


Figure 13.20

âˆ´ The bearing of Yola from Jalingo is 200Â° âˆ’ 180Â° = 020Â°.

Worked \Example 20

If the bearing of Minna from Abuja is 330Â°, what is the bearing of Abuja from Minna?

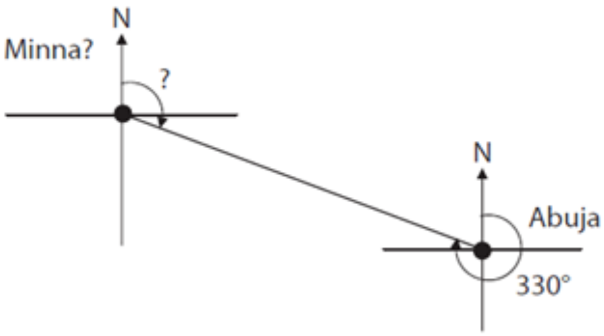


Figure 13.21

âˆ´ The bearing of Abuja from Minna is 330Â° âˆ’ 180Â° = 150Â°.

Worked Example 21

If the bearing of Umuahia from Calabar is N45Â°W, what is the bearing of Calabar from Umuahia?

SOLUTION

Since the question is in compass bearing, the solution must be in compass bearing.

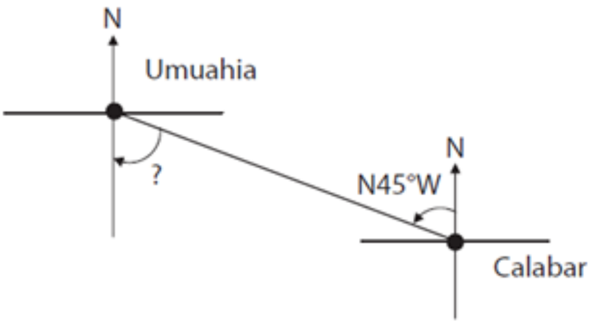


Figure 13.22

âˆ´ The bearing of Calabar from Umuahia is S45Â°E.

Exercise 5

1. If the bearing of Dikwa from Mafa is 065Â°, what is the bearing of Mafa from Dikwa?
2. What is the bearing of Machina from Yusufari, if the bearing of Yusufari from Machina is 115Â°?
3. If the bearing of Kajuru from Ikara is 110Â°, what is the bearing of Ikara from Kajuru?
4. If the bearing of a city A from a city B is
243Â°, what is the bearing of B from A?
5. The bearing of a point X from a point Y is 074Â°. What is the bearing of Y from X? (WAEC)
6. The bearing of a point Q from a point P is 122Â°. What is the bearing of P from Q? (WAEC)
7. A town P is 150 km from a town Q in the direction 050Â°. What is the bearing of Q from P? (WAEC)
8. The bearing of a point P from a point Q is *x*, where 270Â° < *x* < 360Â°. Which among the options is the bearing of Q from P?

(a) (*x* âˆ’ 90)Â°

(b) (*x* âˆ’ 270)Â°

(c) (*x* â^' 135)Â°
(d) (*x* â^' 180)Â° (WAEC)
9. A point X is on the bearing 324Â° from a point Y. What is the bearing of Y from X?
10. In Figure 13.23, P, Q and R are three points in a plane such that the bearing of R from Q is 110Â° and the bearing of Q from P is 050Â°. Find â^ PQR. (WAEC)

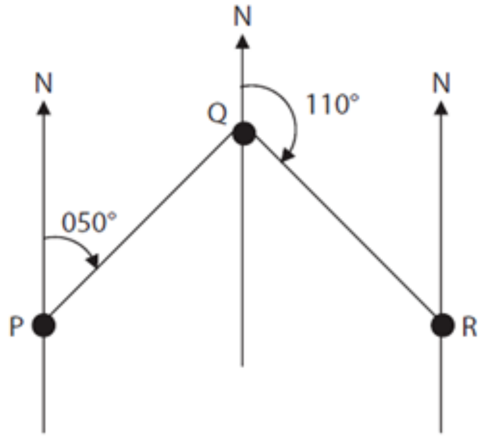


Figure 13.23

VI. Bearings and Distances
Worked Example 22
Points A and B are, respectively, 15 km
North and 10km East of a school.
(a) Calculate the distance from A to B.
(b) What is the bearing of B from A correct to the nearest degree?
SOLUTION
Sketch the diagram

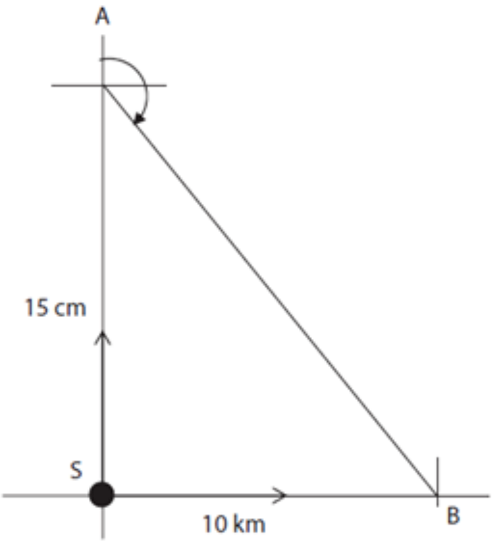


Figure 13.24

Let â€˜Sâ€™ be the school with A located 15km northward and B, 10 km eastward.
(a) Using Pythagorasâ€™ theorem:
|*AB*|2 = |*AS*|2 + |*BS*|2
â‡’ |*AB*|2 = 152 + 102
= 225 + 100 = 325
= 18.0278
â^´ |*AB*| = 18.03 km (2 d .p.)
A cross will be drawn at â€˜Aâ€™ because of the phrase â€˜from Aâ€™ as shown in Figure 13.24. To calculate the bearing of B from A, first of all we have to determine â^ SAB.

tan *SAË†B* =$\frac{10}{15}$= 0.6667
SAË†B = tanâ^'10.667
SAË†B = 33.6914Â°
â^´ The bearing of B from A
= 180Â° â^' 33.6914Â°

= 146.3086Â°
= 146Â°
Worked Example 23
An aeroplane takes off from an airport A at an average speed of 650 km/hr on a bearing of 045Â° for 3 hrs. If it changes course and flies due East at an average speed of 470 km/hr where it lands after $2\frac{1}{2}$hrs.
Calculate the:
(a) Total distance from airport A to airport B.
(b) Average speed of the aeroplane from airport A to B. (Express answers
(a) and (b) to 3 s .f.).
(c) Bearing of airport B from airport A. (Express the answers to the nearest degree.)
SOLUTION
Sketch the diagram that represents the information.

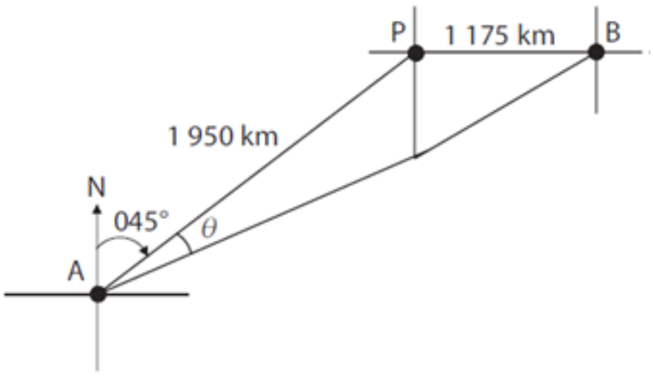


Figure 13.25

The journey from A to B took 3 hrs at
650 km/hr.
âˆ´ |*AP*| = 650 Ã— 3 = 1 950 km
Similarly, from P to B, $2\frac{1}{2}$hrs at 470 km/hr.
âˆ´ |*PB*| = 470 Ã— $2\frac{1}{2}$ = 1 175 km
âˆ APB = 045Â° + 90Â° = 135Â°
(a) By cosine rule
|*AB*|2 = 1 9502 + 1 1752 âˆ’ 2 Ã— 1 950
Ã— 1 175 Ã— cos135Â°
= 3 802 500 + 1 380 625
âˆ’ 4 582 500(âˆ’0.7071)
= 5 183 125 + 3 240 285.75
= 8 423 410.75
|*AB*| = $\sqrt{8423410.75}$
|*AB*| = 2 900 km

Average speed = $\frac{1950+1175}{3+2.5}$
= $\frac{3125}{5.5}$
= 568.1818 km/hr
= 568 km/hr
(c) To find the bearing of airport B from airport A, we must calculate *q* (*BAË†P*).
We shall use sine rule.

$$\frac{\sin\theta}{1\,175}=\frac{\sin 135^\circ}{2\,902.3113}$$

$$\Rightarrow \sin\theta=\frac{1\,175\times\sin 135^\circ}{2\,902.3113}$$

$$\Rightarrow \sin\theta=\frac{1\,175\times 0.7071}{2\,902.3113}$$

$$=\frac{8\,30.8425}{2\,902.3113}$$

$$=0.2863$$

$$\theta=\sin^{-1}0.2863$$

$$\theta=016.6366^\circ$$

$\therefore$ The bearing of B from A

$$=045^\circ+016.6366^\circ$$

$$=061.6366^\circ=062^\circ$$

Worked Example 24

In a school compound, there are 3 mango trees in different positions and named A , B and C . If mango B is 25 m from A on a bearing of S37Â°E and mango C is 43 m from B on a bearing of N56Â°E, calculate:

(a) The distance from mango A to mango C. (Express your answer to 3 s .f.)

(b) The bearing of B from C.

(c) How far South is mango B from A?

(Express your answer to 3 s .f.)

(d) How far East is mango C from B?

(Express your answer to 3 s.f)

SOLUTION

Sketch the diagram that represents the information.

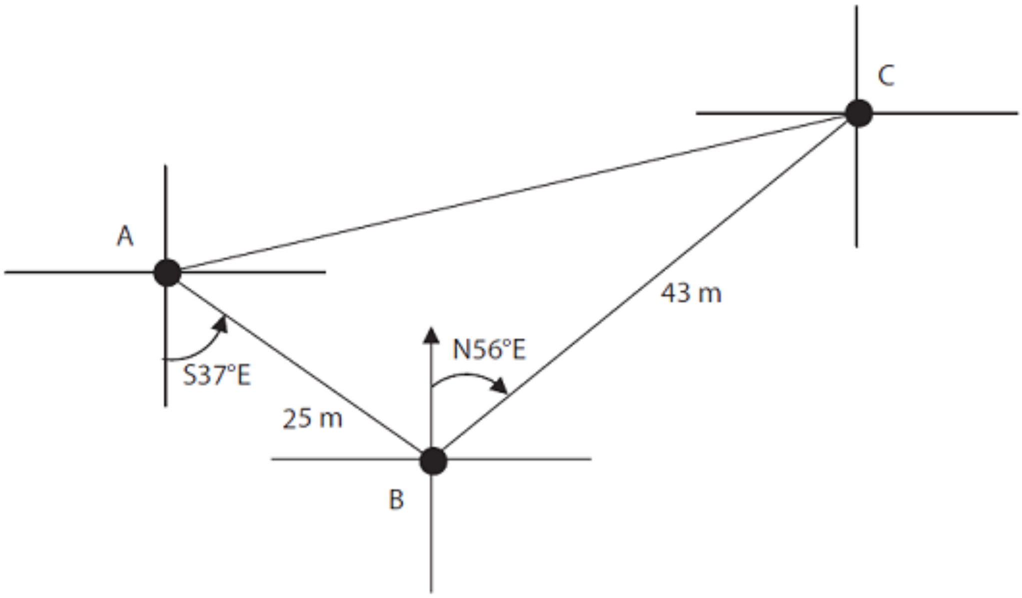


Figure 13.26(a)

(a) âˆ ABC = 037Â° + 056Â° = 093Â°

Using cosine rule

$|AC|_2$ = 25_2 + 43_2 âˆ’ 2 Ã— 25 Ã— 43 Ã— cos 93Â°

= 625 + 1 849 âˆ’ 2 150 (âˆ’0.0523)

= 2 474 âˆ’ 112.445
= 2 361.555
|*AC*| = 48.5958
= 48.6 m (3 s.f.)

(b)Using Figure 13.26(a)

Since the bearing of C from B is N56Â°E, the bearing of B from C is S56Â°W.

(c)

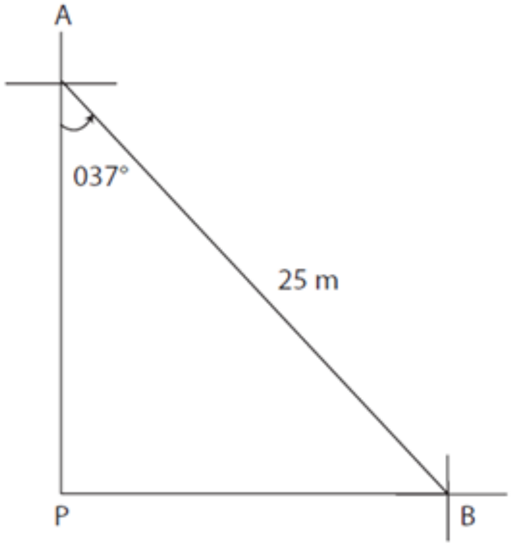


Figure 13.26(b)

$$\cos 37^\circ = \frac{|AP|}{25}$$

â‡’ |*AP*| = 25 cos 37Â°
= 25 Ã— 0.7986 = 19.965
âˆ´ |*AP*| = 20.0 m

(d)

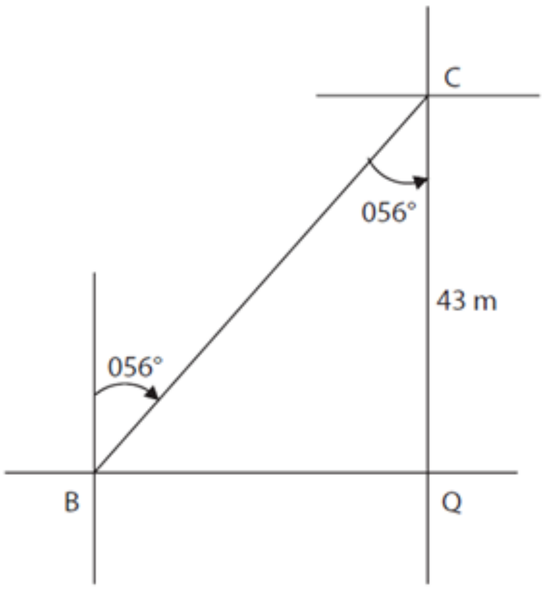


Figure 13.26(c)

$$\tan 56^\circ = \frac{|BQ|}{43}$$

|*BQ*| = 43 tan 56Â° = 43 Ã— 1.483
|*BQ*| = 63.8 m

Exercise 6

1. What is the bearing of Q from P to the nearest whole degree, if points P and Q are, respectively, 34 m North and 10 m East of point T .
2. Points A and B are, respectively, 15 km North and 7 km West of a point P.

(a) Calculate the distance from A to B.
(b) What is the bearing of A from B?
3. The bearing of two points Q and R from a point P are 030Â° and 120Â°, respectively. If |PQ| = 24 m and |PR|
= 7 m. Find the distance | QR |.
(NECO)
4. An aeroplane flies due North from airport P to Q and then flies due East to R. If Q is equidistant from P and R, find the bearing of P from R.
(JAMB)
5. The bearing of two points Y and Z from a point X are 060Â° and 150Â°, respectively, if |XY| = 12 m and
|XZ| = 5 m, find the distance |YZ|.
6. A hunter walked 250 m on a bearing
042Â°. Calculate, correct to the nearest kilometre, the:
(a) Vertical height through which he walked.
(b) Horizontal distance covered.
(NECO)
7. Point Q is on a bearing 060Â° from P while point R is due West of P. If the distance |PQ| is 100 m and |PR| is 120 m, find |QR|.

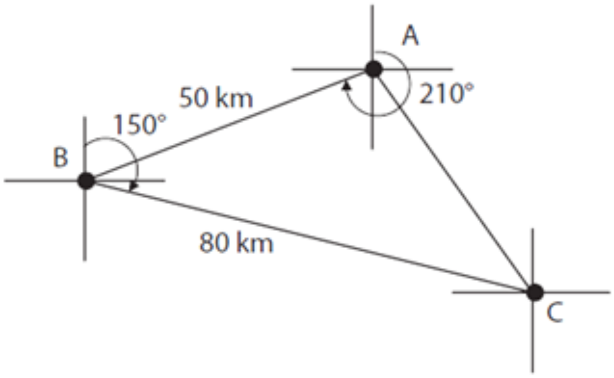


Figure 13.27

(a) In Figure 13.27, A, B and C represent three locations. The bearing of B from A is 210Â° and the bearing of C from B is 150Â°.
Given that |BA| = 50 km and |BC| = 80 km, calculate:
(i) The distance between A and C, correct to the nearest
kilometre.
(ii) The bearing of A from C to the nearest degree.
(b) How far East of B is C.
9. Ayo travels a distance of 24 km from X on a bearing of 060Â° to Y. He then travels a distance of 18 km to a point Z, which is 30 km from X.
(a) Draw the diagram to show the positions of X, Y and Z.
(b) What is the bearing of X from Y?
(c) Calculate the bearing of X from X. (WAEC)
10. A hunter walks from a forest X on a bearing of 080Â° to another forest, which is 40 km away. From there, he went to another forest Z, a distance of 50 km on a bearing of 220Â°, in search of an antelope which he shot.
Calculate the:
(a) Distance of Z from X to the nearest kilometre.
(b) Bearing of Z from X to the nearest degree.
11. A boy travelled from his village P on a bearing of 060Â° to a village Q, which is 20 km away. From Q, he travelled to another village R on a bearing of 135Â°. If R is directly East of P, calculate, correct to 3 s.f., the distance of:
(a) Q from R.
(b) R from P.
12. Town P is on a bearing of 315Â° from town Q, while town R is South of town P and West of town Q. If town R is 60 km away from Q, how far is R from P?
13. The bearing of two points Q and R from a point P are N30Â°E and S60Â°E, respectively. If |PQ| =

12 m and |PR| = 5 m, find the distance QR.
14. A man travels from a village X on a bearing of N60Â°E to a village Y, which is 20 km away. From Y, he travels to a village Z on a bearing of S15Â°W. If Z is directly East of X, calculate, correct to 3 s.f., the distance of:
(a) Y from Z. (b) Z from X.
15. P, Q and R are points in the same horizontal plane. The bearing of Q from P is S30Â°E and the bearing of R from Q is N60Â°E. If $|PQ|$ = 5 cm and $|QR|$ = 3 cm, find the bearing of P, correct to the nearest degree, and let the answer be in compass bearing.
16. An aeroplane flies from a town X on a bearing of N45Â°E to another town Y, a distance of 200 km. It then changes course and flies to another town Z on a bearing of S60Â°E. If Z is directly East of X, calculate, correct to 3 s.f. the distance:
(a) X to Z.
(b) Y to XZ. (WAEC)

SUMMARY

In this chapter, we have learnt the following:

â– When an object at a height above the ground is viewed from a point on the horizontal ground or from the normal eye level, the angle formed between the horizontal ground (eye level) and the line of sight is called â€œangle of elevation.â€

â– When an object on the horizontal ground is sighted from a point at a height above the ground, the angle formed between the line of sight and the horizontal plane is known as the â€œangle of depression.â€

â– The 16 cardinal points.

â– Bearing is the angular relationship between two distant spots.

â– The two bearing notations (compass and surveyorsâ€™ bearing) and how they can be used to solve bearing problems.

â– Pythagorasâ€™ theorem, trigonometric ratios and sine and cosine rules are used to solve triangles, elevation and depression and bearing-related problems.

GRADUATED QUESTIONS

1. A man travels from a village X on a bearing of 060Â° to a village Y, which is 20 km away. From Y, he travels to a village Z on a bearing of 195Â°. If Z is directly East of X, calculate, correct to 3 s .f, the distance of:
(a) Y from Z.
(b) Z from X. (WAEC)
2. A plane flies 120 km on a bearing of 030Â° and then flies 150 km due East. How far East of the starting point is the plane?
3. Three towns, P , Q and R , are such that the distance between P and Q is 50 km and that between P and R is 90 km. If the bearing of Q from P is 075Â° and the bearing of R from P is 310Â°, find the:
(a) Distance between Q and R.
(b) Bearing of R from Q. (WAEC)
4. An aircraft is scheduled to fly from A to B, a distance of 600 km. During bad weather, it is detoured to fly through C where CA makes an angle of 28.15Â° with AB and the distance CA is 500 km. Calculate:
(a) Distance CB.
(b) *AË†CB.*
5. A flag post TP is placed in the middle of a school compound. Student â€˜Aâ€™ placed his school bag 20 m due South of the post and the angle of elevation of the top of the post T from where student â€˜Aâ€™ placed his bag is 035Â°, while student B placed his bag 15 m due West of the post. Calculate:
(a) $|TP|$.
(b) $|AB|$.
(c) The angle of elevation of T from B.
(d) The angle of depression of A from T.
6. A hunter moves from a spot A on a bearing of 059Â° to another spot B, 22 m away. He then moves from B on a bearing of 165Â° to a spot C, 40 m from B. Find the distance between A and C.
7. PQR is an isosceles triangle with $|PQ|$ = $|PR|$ = 8 cm and *QË†PR* = 120Â° . Calculate:

(a) The length of $|QR|$.
(b) The altitude from P to QR.
(WAEC)
8. A dog runs 10 m from a spot A on a bearing of 035Â° and then changes its course and runs 180 m on a bearing of 200Â°.
(a) What is the distance of the dog from spot A?
(b) What is the bearing of the dog from spot A?
9. A man travels from a town X on a bearing 060Â° to a town Y, which is 200 km away.
At Y, he changes course and goes to a town Z on a bearing 195Â°. If Z is directly East of X, calculate, correct to 3 s.f,
(a) How far is Y from Z.
(b) The distance of Z from X. (WAEC)
10. Two boats A and B left a port C at the same time on different routes. B travelled on a bearing of 150Â° and A travelled on the North side of B. When A had travelled
8 km, B travelled 10 km and the distance between the two boats was found to be
12 km. Calculate the bearing of Aâ€™s route from C. (WAEC)