CHAPTER 8

Linear Inequalities

OBJECTIVES

At the end of the chapter, students should be able to:
1. Solve inequalities in one variable.
2. Solve problems on inequalities in two variables.
3. Form a component inequality by combining two inequalities.
4. Draw graphs of linear inequalities in two variables.
5. Obtain the required region that satisfies the simultaneous linear inequalities.
6. Find the maximum and minimum values of linear inequalities.
7. Solve word problems on linear inequalities.

I. Linear Inequalities in One Variable (Revision)

The term inequality applies to any statement involving one of the symbols <, >, â‰¤, â‰¥.
Note: Similar to ordinary equations, inequality equations too have solutions.

1. Rules for finding the solutions to inequality equations

(a) Add or subtract the same expression or number to both sides of the inequality and preserve the inequality sign.
(b) Multiply or divide both sides of the inequality by the same positive number and preserve the inequality sign.
(c) Multiply or divide both sides of the inequality by the same negative number and reverse the inequality sign.

The expression 3x âˆ’ 1 > x + 1 is a linear inequality in one variable x. Thus, a linear inequality in x is an inequality in which the highest power of x is one (unity).

Worked Example 1

Solve the following linear inequalities:
(a) 4x + 6 < 2
(b) 5 â€“ x â‰¥ 7
(c) $\frac{x}{3} \le 2 - 6x$

SOLUTION

(a) $4x+6<2$

$4x+6-6<2-6$

(subtracting 6 from both sides)

$4x<-4$

$\frac{4x}{4}<\frac{-4}{4}$

(dividing both sides by 4)

$\therefore x<-1$

(b) $5-x\geq 7$

$5-x-5\geq 7-5$

(subtracting 5 from both sides)

$-x\geq 2$

$-(-x)\geq -(2)$

(multiplying both sides by -1)

$\therefore x\leq -2$

(c) $\frac{x}{3}\leq 2-6x$

$\frac{x}{3}+6x\leq 2-6x+6x$

(adding $6x$ to both sides)

$\frac{19x}{3}\leq 2$

$\frac{3(19x)}{3}\leq 2\times 3$

(multiplying both sides by 3)

$19x\leq 6$

$\frac{19x}{19}<\frac{6}{19}$

(dividing both sides by 19)

$\therefore x<\frac{6}{19}$

(ii) Solution of linear inequalities on the number line

A number line is used to illustrate linear inequalities in one variable. A point $x = a$ divides the number line into 2 parts, $x < a$ and $x > a$ as in Figure 8.1.

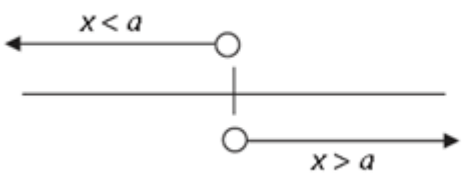

Figure 8.1

When point $x = a$ is included, we have x â‰¤ a and x â‰¥ a which are illustrated in Figure 8.2.

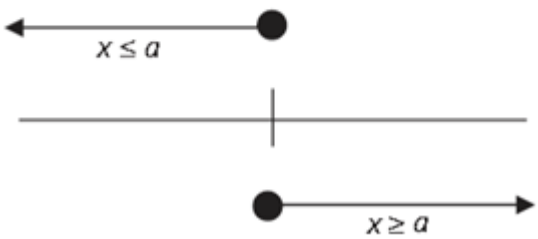

Figure 8.2

Note: at the beginning of the graph shows that point $x = a$ is not included in the inequality, while shows that point $x = a$ is included.

A line segment from a to b is denoted by a â‰¤ x â‰¤ b and is illustrated in Figure 8.3 as

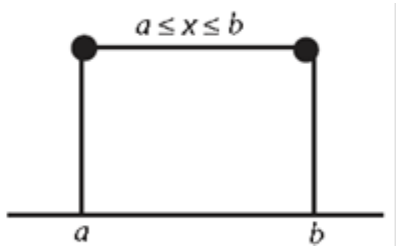

Figure 8.3

Worked Example 2

Solve the inequality and represent the solution on a number line.

$$5-\frac{1}{2}x>-1$$

SOLUTION

$$5-\frac{1}{2}x>-1$$

Subtracting 5 from both sides

$$-\frac{1}{2}x>-6$$

Multiplying both sides by âˆ’2
x < 12

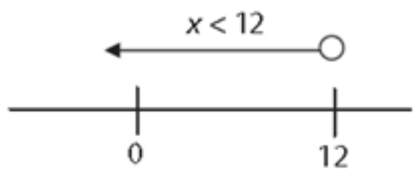

Figure 8.4

Worked Example 3

Solve for x, if âˆ’2 < 4 âˆ’ 2x â‰¤ 4. Represent the solution on a number line.

SOLUTION
âˆ’2 < 4 âˆ’ 2x â‰¤ 4
Subtracting 4 from all sides
âˆ’6 < âˆ’2x â‰¤ 0

Dividing both sides by âˆ’2
0 â‰¤ x < 3

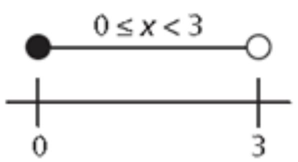

Figure 8.5

Worked Example 4

Solve for x, if $\frac{x+2}{2-x} \le 2.$
SOLUTION
The fraction is undefined when x = 2, then consider the two cases x â‰¤ 2 and x â‰¥ 2 separately.

Case 1: x â‰¤ 2

$$\frac{x+2}{2-x} \leq 2.$$

Multiplying both sides by 2 âˆ’ x, this is a positive number if x â‰¤ 2
x + 2 â‰¤ 4 âˆ’ 2x
Adding 2x to both sides
3x + 2 â‰¤ 4
Subtracting 2 from both sides
3x â‰¤ 2
Dividing both sides by 3

$$x \leq \frac{2}{3}$$

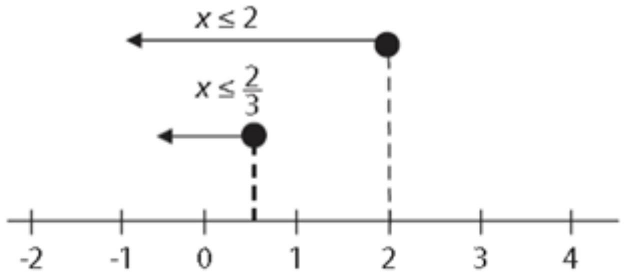

Figure 8.6(a)

As shown in Figure 8.6(a), x satisfies both conditions x â‰¤ 2 and $x \leq \frac{2}{3}$.

Case 2: x â‰¥ 2

$$\frac{x+2}{2-x} \leq 2$$

Multiplying both sides by 2 â€“ x, (which is a negative number of x â‰¥ 2),
x + 2 â‰¥ 4 âˆ’ 2x
Adding 2x to both sides,
3x + 2 â‰¥ 4
Subtracting 2 from both sides,
3x â‰¥ 2
Dividing both sides by 3,

$$x \geq \frac{2}{3}$$

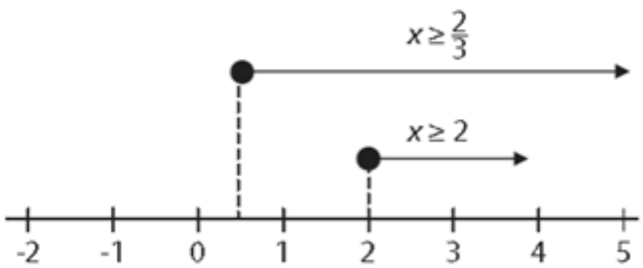

Figure 8.6(b)

As shown in Figure 8.6(b), x satisfies the two conditions x > 2 and which implies that the inequality is satisfied when x > 2.

(iii) Linear inequalities in one variable involving the modulus

The modulus of x written as |x| means the positive value of x. Therefore, if |x| = a, then x = a.

Worked Example 5

Solve the inequality |3 âˆ’ 3x| < 6.

SOLUTION

|3 âˆ’ 3x| < 6

Rewriting the inequality without the modulus sign,

âˆ’6 < 3 âˆ’ 3x < 6

Subtracting 3 from both sides,

âˆ’9 < âˆ’3x < 3

Dividing all the sides by âˆ’3,

âˆ’1 < x < 3

(i.e. x lies between âˆ’1 and 3)

Worked Example 6

Solve the inequality 2y + 4 â‰¤ 2.

SOLUTION

2y + 4 â‰¤ 2

Rewriting the inequality without the modular sign,

âˆ’2 â‰¤ 2y + 4 â‰¤ 2

Subtracting 4 from both sides,

âˆ’6 â‰¤ 2y â‰¤ âˆ’2

Dividing all the sides by 2,

âˆ’3 â‰¤ y â‰¤ âˆ’1

®ˆ× y lies between âˆ’3 and âˆ’1 inclusive.

Exercise 1

Solve the following linear inequalities:

1. 3y + 2 â‰¥ 8
2. 5x âˆ’ 9 â‰¥ 4
3. 6 âˆ’ 3x â‰¤ âˆ’8
4. 3 < 6 âˆ’ 2x
5. $7 - \frac{1}{2}x > 3$
6. 23 âˆ’ 5a â‰¤ 2a + 9
7. 5 âˆ’ 5b < b âˆ’ 4
8. 28 âˆ’ 5x â‰¤ x â€“ 4 (WAEC)
9. 7 (d âˆ’ 1) > 6 (8 âˆ’ 3d)

10. $\frac{2}{3} - 3x \leq 2(1 - x)$ (WAEC)

11. $\frac{x+2}{5} \geq \frac{x-3}{3} + 1$

12. $2x + 6 < 5(x - 3)$ (WAEC)

13. $(x - 3) < \frac{x}{3} + 1$

14. $\frac{1}{3}(2x - 1) < 5$ (WAEC)

15. $x + 6 > 9$

16. $\frac{y}{2} + \frac{1}{6} > \frac{2}{3}y + \frac{5}{2}$

17. $5(a + 2) - 3(a - 5) < 29$

Solve each of the following inequalities and represent the solution on a number line.

18. $\frac{x}{3} - \frac{x}{5} \geq 4$

19. $2(2y - 3) < 18 - (2y - 5)$

20. $2(a + 1) > \frac{1}{3}(4a + 3)$

21. What is the range of values of x for which 2x âˆ’ 1 > 3 and x âˆ’ 3 < 5 are both satisfied? (WAEC)

22. Solve the linear inequality $\frac{1}{3}(2x + 7) - \frac{1}{9}(1 - 4x) < 4 + x$ (WAEC)

Solve the inequalities:

23. |2x âˆ’ 5| < 5
24. |3y + 4| â‰¥ 2
25. |2 âˆ’ 3y| â‰¥ 6

26. 16x + 11 < 7

II. Linear Inequalities in Two Variables

(i) Solutions to inequalities in two Variables

A linear inequality in two variables x and y is of the form:

ax + by â‰¤ c   ax + by < c
ax + by > c   ax + by â‰¥ c

where a, b and c are constants.
A solution to an inequality is any pair of numbers x and y that satisfies the inequality.

Worked Example 7

Determine the solution set of 5x + 2y â‰¤ 17.

SOLUTION

One solution to 5x + 2y < 17 is x = 2 and y = 3 because 5(2) + 2(3) = 16, which is indeed less than 17. But the pair x = 2 and y = 3 is not the only solution. As a matter of fact, there are infinitely many solutions.
Since we canâ€™t write down all possible solutions to a linear inequality, a good way to describe the set of solutions to any linear inequality is by a graph. If the pair of numbers x and y is a solution, then think of this pair as a point in the plane, so the set of all solutions can be thought of

as a region in the xâ€“y plane.
To illustrate how to determine this region, fi rst express y in terms of x in the inequality:
5x + 2y â‰¤ 17
2y â‰¤ âˆ’5x + 17

$$y \leq -\left(\frac{5}{2}\right)x + \frac{17}{2}$$

Next, draw the line y = on the graph (see Figure 8.7). The set of points (x, y) that lie on this line is the set of all (x, y) such that y is exactly equal to . These points make up a part of the set of solutions of the inequality, but not all. We see that y can also be less than . So all points below the line could also be the solutions. The shaded region in Figure 8.7 shows the solution set.

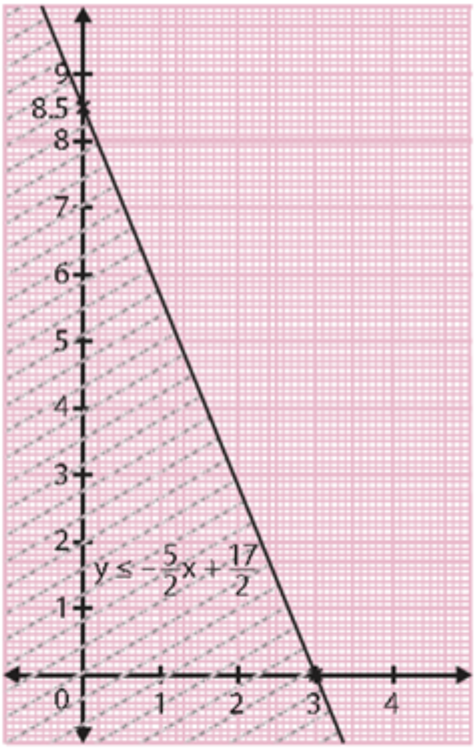

Figure 8.7 W

Worked Example 8

Determine the solution set for the inequality 3x âˆ’ 8y â‰¥ 12.

SOLUTION

Solving for y,
âˆ’8y â‰¥ âˆ’3x + 12

$$y \leq \left(\frac{3}{8}\right)x - \frac{3}{2}$$

Next, graph the line y = ( ) x âˆ’
When x = 0, y =
When y = 0, x = 4

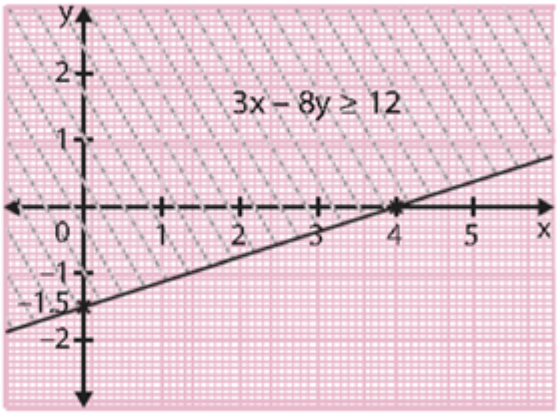

Figure 8.8

Worked Example 9

Graph the solution set for the inequality âˆ’ 10x âˆ’ 2y > 7

SOLUTION

âˆ’10x âˆ’ 2y > 7
Solving for y gives
âˆ’2y > 10x + 7
y < âˆ’5x âˆ’
This example is a little different because there is no equal sign in the inequality.
Graph the line y = âˆ’5x âˆ’ , but draw it as a dashed line. This indicates that the line itself is not a part of the solution set. The actual solution set consists of all points below the dashed line. This is because y must be strictly less than âˆ’5x âˆ’ .

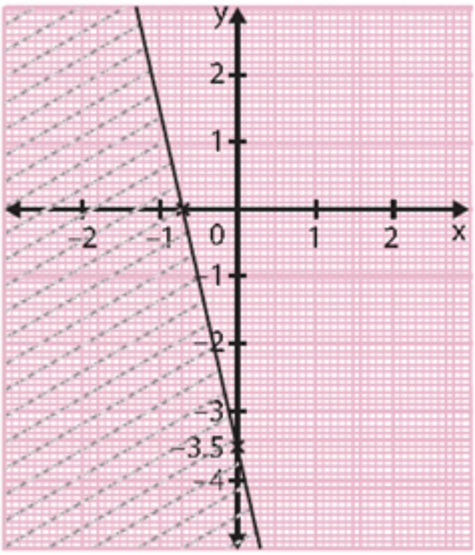

Figure 8.9

(ii) Range of values of combined Inequalities

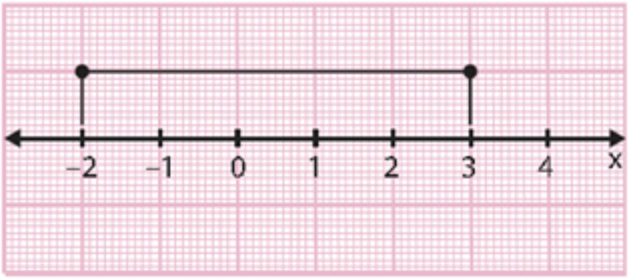

Figure 8.10

In Figure 8.10, x can possess any value between âˆ’2 and +3 inclusive. Hence, x â‰¥ âˆ’2 and x â‰¤ + 3 or âˆ’2 â‰¤ x and x â‰¤ + 3. These two inequalities can be combined as a single inequality. Thus, âˆ’2 â‰¤ x â‰¤ +3

Worked Example 10

What is the range of values of x for which 2x + 6 > 2 and x âˆ’ 4 < 1 are both satisfied?

SOLUTION

2x + 6 > 2

2x > 2 âˆ’ 6 (subtracting 6 from both sides)

2x > âˆ’4

x > âˆ’2 (dividing both sides by 2) and

x âˆ’ 4 < 1

x < 1 + 4 (adding 4 to both sides)

x < 5

Hence, x > âˆ’2 and x < 5 or âˆ’2 < x and x < 5 or âˆ’2 < x < 5.

Both inequalities are satisfied, if âˆ’2 < x < 5.

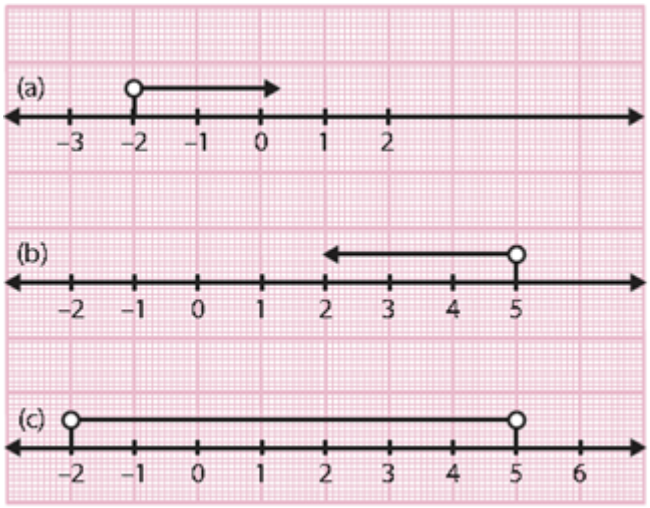

Figure 8.11

Figure 8.11 shows the results of Worked Example 10 in graphical form. The graphs of (a) x > âˆ’2 and (b) x < 5 are combined to give the graph of (c) âˆ’2 < x < 5.

Worked Example 11

State the range of values of x for the graph in Figure 8.12.

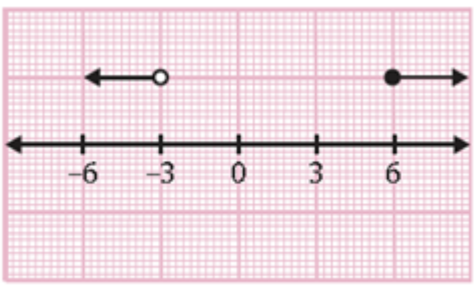

Figure 8.12

SOLUTION

In Figure 8.12, either x < âˆ’3 or x â‰¥ 6.

Note: These inequalities cannot be combined with the word â€˜andâ€™. For example, if x = 8, then x â‰¥ 6 but $x \nless -3$. Either x â‰¥ 4 or x < âˆ’3, but not both.

III. Solutions of Simultaneous Linear Inequalities

Solutions of simultaneous linear inequalities are the same as solving simultaneous linear equations but with caution. This is because in simultaneous linear inequalities, the solution is usually a range of values while for the simultaneous equation, it is usually a unique value.

Worked Example 12

Find the ranges of values of x and y, if $2x + y < 9$ and $x + 2y > 6$.

SOLUTION

$2x + y < 9$ .... (1)
$x + 2y > 6$ .... (2)

Multiplying equation (1) by â€“1
âˆ’2x â€“ y > âˆ’9 .... (3)

multiplying equation (3) by 2:
âˆ’4x âˆ’ 2y > âˆ’18 .... (4)

Adding equations (2) and (4)
âˆ’3x > âˆ’12
$x < 4$

From equation (2)
$x + 2y > 6$
$4 + 2y > 6$ (substituting $x = 4$)
$2y > 2$
$y > 1$
Hence, $x < 4$ and $y > 1$ are the ranges.

Worked Example 13

Solve the simultaneous linear inequalities
4x + y < 9 and 2x â€“ y < 5.

SOLUTION

$4x + y < 9$ .... (1)
2x â€“ y < 5 .... (2)

Adding equations (1) and (2)
$6x < 14$
x < or $x < 2$

Substituting x =   into equation (2)

2() âˆ’ y < 5

 âˆ’ y < 5
âˆ’3y < 1
y > âˆ’
Hence, x < and y > âˆ’ are the ranges.

Worked Example 14

Solve graphically the simultaneous linear inequalities 2x + 3y < 6, y âˆ’ 4x â‰¤ 2, x â‰¥ 0.

SOLUTION

Consider the first inequality $2x + 3y < 6$. It may be written as
3y < 6 âˆ’ 2x
y < (6 âˆ’ 2x)

®ˆ× y = (6 âˆ’ 2x) is a boundary line.
When x = 0, y = 2
When y = 0, x = 3

Also, y âˆ’ 4x â‰¤ 2 may be written as y â‰¤ 2 + 4x
®ˆ× y = 2 + 4x is a boundary line.
When x = 0, y = 2
When y = 0, x =

In Figure 8.13, points below the broken line through (0, 2) and (3, 0) satisfy the inequality 2x + 3y < 6. Similarly, points on and below the solid line through (0, 2) and (, 0) satisfy the inequality y âˆ’ 4x â‰¤ 2.
Likewise, points on and above the y-axis (i.e. the line x = 0) satisfy the inequality x â‰¥ 0. The solutions are contained in the shaded region.

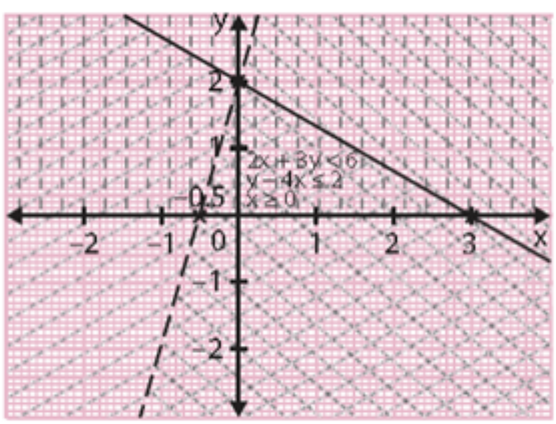

Figure 8.13

Exercise 2

Draw the graph of each inequality.

1. y â‰¥ x + 4
2. y < 4
3. 3x âˆ’ 2y < 6
4. 3x â€“ y â‰¥ 0
5. y âˆ’ 3x + 15 â‰¥ 0
6. y âˆ’ 5x + 5 > 0
7. 6y + 5 â‰¥ 11 âˆ’ x
8. 2x + 3y â‰¥ 2y + 1
9. x + 2y â‰¤ 2x + 2
10. y + 4x + 10 > 0

Draw number lines to show the following ranges of values of x.

11. 0 < x < +4
12. âˆ’1 â‰¤ x â‰¤ + 6
13. âˆ’5 < x â‰¤ âˆ’3
14. âˆ’3 â‰¤ x â‰¤ + 1

15. +3 < x < + 7
16. x â‰¤ âˆ’3 or x > 2
17. x â‰¥ 6 or x < âˆ’4

IV. Application of Linear Inequalities in Real Life

Word problems leading to linear inequalities

Worked Example 15

An employer employs x men and y women. He can afford to employ not more than 20 people. Due to some heavy work to be done, he needs more than 6 men. But some precise work can be done better by women so he needs at least 8 women.

(a) Write down three inequalities involving x and y.
(b) Draw a graph to show the three inequalities.
(c) Use the graph to fi nd out the maximum number of men and women he can employ.
(d) What is the minimum number of people he can employ?

SOLUTION

(a) From the first two statements,
x + y â‰¤ 20

From the third statement,
$x > 6$
From the last statement,
y â‰¥ 8

(b)

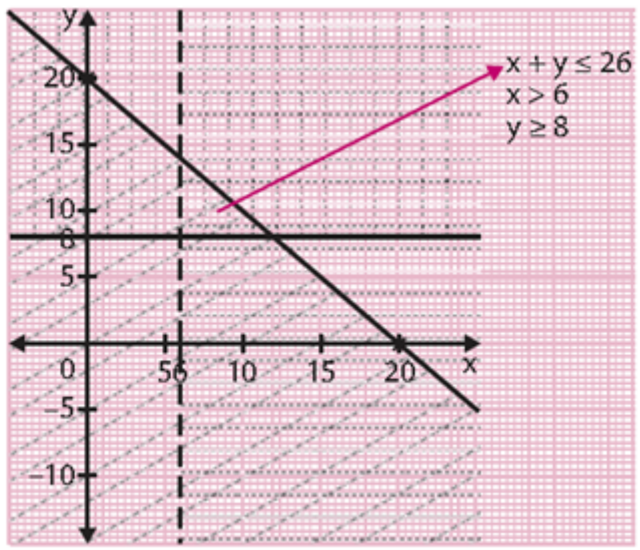

Figure 8.14

(c) From the graph in Figure 8.14: The solution sets are

Men (x)	Women (y)	
7	8	
8	9	
9	10	
10	11	
11	12	
12	13	(the circled are the maximum number of people he can employ)
13	14	
14	15	
15	16	

Thus, maximum number of men is 15 and maximum number of women is 16.

(d) The minimum number of people he can employ is 7 men and 8 women, i.e. 15 people.

Worked Example 16

A transport company needs at least 10 buses and 15 minibuses. The company is not able to run more than 30 vehicles altogether. A bus takes up 3 units of garage space, a minibus takes up 1 unit of garage space and there are only 54 units available. If x and y are the numbers of buses and minibuses, respectively,

(a) Write down four inequalities which represent the constraints on the businessman

(b) Draw the graph that shows a region representing possible values of x and y

SOLUTION

(c) From the fi rst statement,
x â‰¥ 10 and y â‰¥ 15
From the second statement,
x + y â‰¤ 30
From the third statement,
3x + y â‰¤ 54

(d) The unshaded region in Figure 8.15 shows a graph that contains the possible values of x and y.

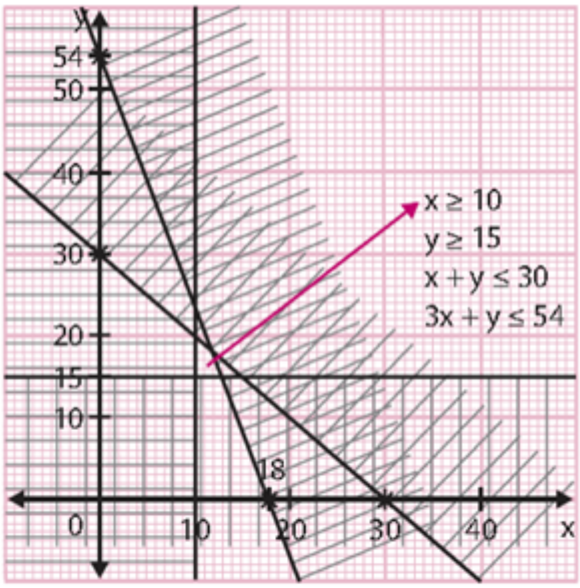

Figure 8.15

V. Introduction to Linear Programming

Linear programming is the process of taking various linear inequalities relating to some situation and
finding the â€˜bestâ€™ value obtainable under those conditions. A typical example would be taking the
limitations of materials and labour, and then determining the â€˜bestâ€™ production levels for maximal profits under those conditions.
In â€˜real lifeâ€™, linear programming is part of a very important area of Mathematics called â€˜optimisation techniqueâ€™. This field of study (or at least the applied results of it) is used every day in the organisation and allocation of resources. These â€˜real lifeâ€™ systems can have dozens or hundreds of variables or more. In algebra, though, youâ€™ll only work with the simple (and graphable) two variable linear case.
The general process of solving linear-programming exercises is to graph the inequalities (called the â€˜constraintsâ€™) to form a walled-off area on the xâ€“y plane (called the â€˜feasibility regionâ€™). Then figure out the coordinates of the corners of this feasibility region (i.e. find the intersection points of the
various pairs of lines), and test these corner points in the formula (called the â€˜optimisation equationâ€™) for which you are trying to find the highest or lowest value.

Worked Example 17

Find the maximal and minimal values of $Z = 3x + 4y$ subject to the following constraints:

$$\begin{Bmatrix} x + 2y \le 14 \\ 3x - y \ge 0 \\ x - y \le 2 \end{Bmatrix}$$

SOLUTION

The three inequalities in the curly brackets are the constraints. The area of the plane that they mark off will be the feasibility region. The formula â€˜Z = 3x + 4yâ€™ is the optimisation equation. We need to
find the (x, y) corner points of the feasibility region that return the largest and smallest values of Z. The first step is to solve each inequality for the more easily grouped equivalent forms:

$$\begin{Bmatrix} x + 2y \le 14 \\ 3x - y \ge 0 \\ x - y \le 2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} y \le -\frac{1}{2}x + 7 \\ y \le 3x \\ y \ge x - 2 \end{Bmatrix}$$

It is easy to graph the system:

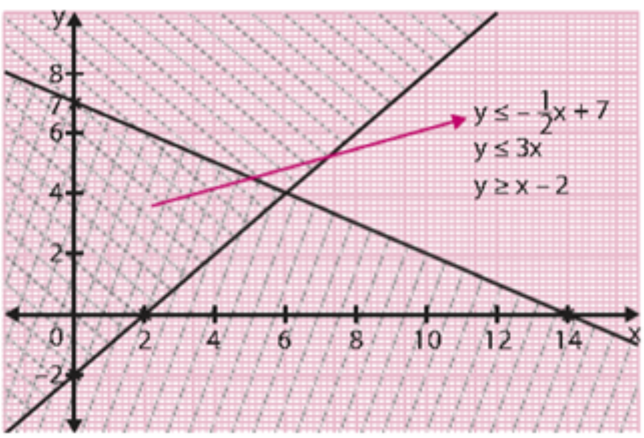

Figure 8.16

To find the corner points, which are not always clear from the graph, pair the lines (thus forming a system of linear equations) and solve:

Table 8.1

$y=-\left(\frac{1}{2}\right)x+7$ $y=3x$	$y=-\left(\frac{1}{2}\right)x+7$ $y=x-2$	$y=3x$ $y=x-2$
$-\left(\frac{1}{2}\right)x+7=3x$ $-x+14=6x$ $14-7x$ $x=2$ $y=3(2)=6$	$-\left(\frac{1}{2}\right)x+7=x-2$ $-x+14=2x-4$ $18-3x$ $x=6$ $y=6-2=4$	$3x=x-2$ $2x=2$ $x=-1$ $y=3(-1)=-3$
Corner point at (2, 6)	Corner point at (6, 4)	Corner point at (–1, –3)

So the corner points are (2, 6), (6, 4) and (âˆ’1, âˆ’3). Therefore, for linear systems like this, the maximum and minimum values of the optimisation equation will always be at the corners of the feasibility region. Hence, to find the solution of this exercise, substitute these three corner points into â€˜Z = 3x + 4y â€™

(2, 6): Z = 3(2) + 4(6) = 6 + 24 = 30

(6, 4): Z = 3(6) + 4(4) = 18 + 16 = 34

(âˆ’1, âˆ’3): Z = 3(âˆ’1) + 4(âˆ’3) = âˆ’3 â€“ 12 = âˆ’15

Then, the maximum of Z = 34 occurs at (6, 4) and the minimum of Z = âˆ’15 occurs at (âˆ’1, âˆ’3).

Worked Example 18

In a certain refinery, the refining process requires the production of at least two gallons of gasoline for each gallon of fuel oil. To meet the anticipated demands of winter, at least 3 million gallons of fuel oil per day will need to be produced. The demand for gasoline, on the other hand, is not more than 6.4 million gallons per day. If gasoline is selling for ₦1.90/gal and fuel for ₦1.50/gal, how much of each should be produced in order to maximize revenue?

SOLUTION

The question asks for the number of gallons to be produced, so variables to represent â€˜gallons producedâ€™ are as follows:
x: gallons of gasoline produced
y: gallons of fuel oil produced

Since this is a â€˜real lifeâ€™ problem, there are no negative production levels hence, the variables cannot be negative. This gives the first two constraints, namely, x â‰¥ 0 and y â‰¥ 0. At least two gallons of gas are produced for every gallon of oil, which implies x â‰¥ 2y. For graphing, use the more manageable form y â‰¤ ()x. The winter demand says that y â‰¥ 3 000 000; note that this constraint eliminates the need for the â€˜y â‰¥ 0â€™ constraint. The gas demand says that x â‰¤ 6 400 000. In order to maximize revenue R, the optimisation equation is R = 1.9x + 1.5y. The model for this word problem is as follows:
R = 1.9x + 1.5y subject to:

x â‰¥ 0

x â‰¤ 6 400 000

y â‰¥ 3 000 000

y â‰¤ () x

Using a scale that counts by millions (so â€˜y = 3â€™ on the graph means â€˜y is 3 millionâ€™), the above system graph is shown as follows:

Figure 8.17

$y = 3$ $y = \left(\frac{1}{2}\right)x$	$y = 3$ $x = 6.4$	$y = \left(\frac{1}{2}\right)x$ $x = 6.4$
$3 = \left(\frac{1}{2}\right)x$ $3 \times 2 = x$ $x = 6, y = 3$	$x = 6.4, y = 3$	$y = \left(\frac{1}{2}\right)6.4$ $y = 3.2$ $x = 6.4, y = 3.2$
Corner point at (6, 3)	Corner point at (6.4, 3)	Corner point at (6.4, 3.2)

So, the corner points are (6, 3), (6.4, 3) and (6.4, 3.2). Therefore;

(6, 3): R = 1.9(6) + 1.5(3) = 11.4 + 4.5 = 15.9

(6.4, 3): R = 1.9(6.4) + 1.5(3) = 12.16 + 4.5 = 16.66

(6.4, 3.2): R = 1.9(6.4) + 1.5(3.2) = 12.16 + 4.8 = 16. 96

Thus, the maximal solution of R = N16.96 at (6.4, 3.2).

Exercise 3

1. Maximise Z = 2x + 3y subject to the constraints:
   3x + 2y â‰¤ 15
   4x â€“ y â‰¥ 0
   x â‰¥ 0
   y â‰¥ 0

2. Maximise Z = 5x + 4y subject to the constraints:

x + 5y â‰¤ 10

4x â€“ y â‰¤ 5

x â‰¥ 0

y â‰¥ 0

3. Maximise Z = 3x + 4y subject to the constraints:

x + y â‰¤ 8

x â‰¥ 2

y â‰¥ 2

4. Maximise Z = 2x + 5y subject to the constraints:

x + y â‰¤ 5

y â€“ x â‰¤ 1

x â‰¥ 0

y â‰¥ 1

5. A goldsmith takes 2 hours to make a wedding ring and 3 hours to make a necklace. He can only work for a maximum of 12 hours in a day. In a day he has an order for at least 2 rings and 2 necklaces. On each wedding ring he makes a profit of N50 and on each necklace he makes a profit of N60. Assuming he makes x rings and y necklaces.

(a) Write down all the inequalities connecting x and y.
(b) Show by shading the region H which satisfies all the inequalities in (a).
(c) How many of the rings and necklaces should the goldsmith make in order to maximum profit.

6. A transporter has a minibus and a stationwagon car. In a trip he calculates that the maintenance cost per passenger of the bus is N2 while the maintenance cost per passenger of the car is N3. He does not want to spend more than N45 on maintenance in a trip. He makes a profi t of N5 per passenger on a bus and a profit N7 per passenger on a car. The garage regulations are such that he cannot carry more than 20 passengers in a trip. If the minibus takes x passengers and car takes y passengers.

(a) Write down all the inequalities connecting x and y.
(b) Show by shading the region G which satisfies the above inequalities.
(c) Find the values of x and y, if he is to maximise bus profit.

7. A shoe-maker makes shoes and sandals from leather. A pair of shoes requires 20 dm^2 of leather while a pair of sandals requires of leather. of leather is available at most. A pair of shoes cost N80 while a pair of sandals cost N60. He has at most N480 to buy leather. He makes N25 profi t on a pair of shoes and N20 on a pair of sandals. Assuming he makes x pairs of shoes and y pairs of sandals.

(a) Write down all the inequalities connection x and y.
(b) Indicate by shading the region which satisfi es the inequalities in x and y.
(c) Determine the values of x and y if the shoemaker wishes to maximise profit.

SUMMARY

In this chapter, we have learnt the following:

- An inequality is any statement involving one of symbols <, >, â‰¤, â‰¥ and .
- The rules for finding the solution of linear inequalities are very similar to that of solving linear equations; there is one important differenceâ€”when multiplying or dividing both sides of an inequality by a negative number, always reverse the inequality.
- Simple linear inequalities can be represented on a number line.
- Graphs of linear inequalities in two variables are then considered as closed or open half-planes with the line of the linear equation arising from the inequality serving as the dividing lines. Thus, the graph of $y > mx + c$ is the closed half-plane above the line $y = mx + c$, while the graph of $y < mx + c$ is the open half-plane below the line $y = mx + c$.
- Solutions of simultaneous linear inequalities are considered as the intersection of the graphs of the individual linear inequalities.
- Linear programming is the process of taking various linear inequalities relating to some situation, and finding the â€˜bestâ€™ value obtainable under those conditions.

GRADUATED EXERCISES

Solve the following inequalities and sketch a number line graph for each solution.

1. 4x âˆ’1 â‰¤ 7 (WAEC)
2. x + 1 â‰¥ 3
3. 3x âˆ’ 5 < 5x âˆ’ 3 (WAEC)
4. (WAEC)
5. Solve (WAEC)
6. Solve

Solve the inequalities

7. | 1 âˆ’ 2x | > 3
8. | 4y âˆ’ 3 | < 4
9. | 3 âˆ’ 5y | â‰¤ 13

10. Solve ; give three highest possible values of x. (WAEC)
11. What is the range of values of y for which 4(1 â€“ y) > 3 and 3(2 + 2y) â‰¥ 0 are both satisfied?
12. x is such that 3x âˆ’ 6 â‰¤ 2x and 2x â‰¤ 5x + 8,
    (a) What range of values of x satisfies both inequalities?
    (b) Express 3x âˆ’ 6 â‰¤ 2x â‰¤ 5x + 8 in the form a â‰¤ x â‰¤ b, where a and b are both integers.