



# 10

## ENERGY QUANTIZATION

### ENERGY LEVELS IN ATOMS

#### OBJECTIVES

At the end of the topic, students should be able to:

- explain the concept of energy quantization;
- state the relation between colour and light frequency;
- state and explain the types of spectra;
- solve problems on energy quantization and frequency.

Both **Bohr** and the **wave mechanics** models of atom assign specific energy to an electron when it occupies one of the allowed energy levels. The energy of the electron depends on the number of wavelengths that fits into the circumference of the orbit. The number of wavelengths fitting into the circumference of the orbital level also depends on the principal quantum ( $n$ ). Series of horizontal lines represent the energy levels occupied by electrons. The lowest energy level the electron can occupy is the **ground state energy**. Electrons under normal condition, occupy the ground state energy level where they are **stable**.

When an electron absorbs energy, it moves to a higher energy level (Figure 10.1a). An electron in the higher energy levels is **excited** and **unstable**. An excited electron tends to move down to the ground state energy level where it is stable. When this happens, the electron radiates or gives out its excess energy as light of definite frequency and wavelength. The amount of energy radiated as the electron jumps down to lower energy level or absorbed as the electron moves up to higher energy levels is given by:

$$E_i - E_f = hf$$

The energy of the electron at each energy level is calculated by the equation:

$$E_n = -\frac{E_0}{n^2}$$

$E_n$  = excited energy levels

$E_0$  = - 13.6eV or  $21.76 \text{ Å}^{-1} \text{ J}$  is the ground state energy for hydrogen atom.

$n = 1, 2, 3, \dots$  are the principal quantum numbers.

They represent each energy level.

The **first energy level** or the **ground state energy** is represented by  $n = 1$ . The excited energy levels are denoted by  $n = 2, 3, 4, \dots$ . The attraction of electrons by the nucleus is represented by the minus (-) in the formula. The energy levels and the principal quantum numbers is shown in figure 10.1b

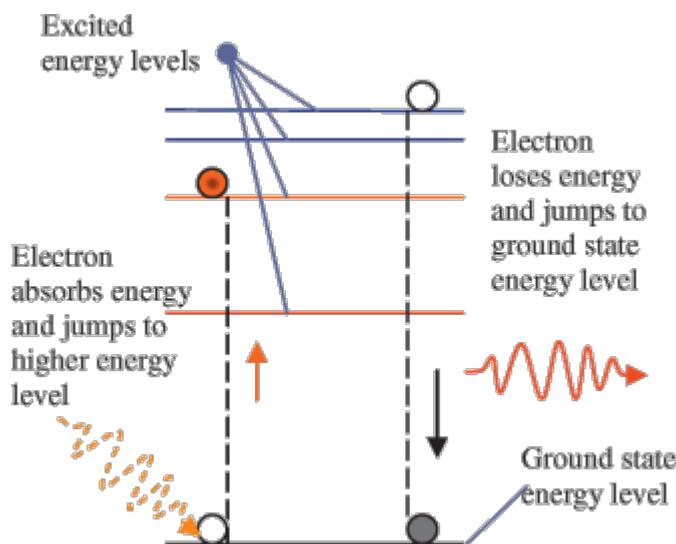


Figure 10.1a: Absorption and radiation of energies

### Excitation and ionization energies

An electron, which absorbs energy, moves up to higher energy levels. ***The energy absorbed by an electron to move to higher energy levels is the excitation energy.*** When an electron absorbs enough energy, it may move completely out of the atom. ***The energy absorbed by an electron to move it completely out of the atom is the ionization energy.***

The energy needed to ionize or remove an electron from an atom depends on initial energy state of the electron. An electron in the ground state needs 13.6 eV to free it from the attractive force of the nucleus. The ionization energy for an electron in the first excited energy level ( $n = 2$ ) is -3.39eV.

$$\text{Ionization energy} = E_{\alpha\gamma} - E_n = 0 - E_n$$

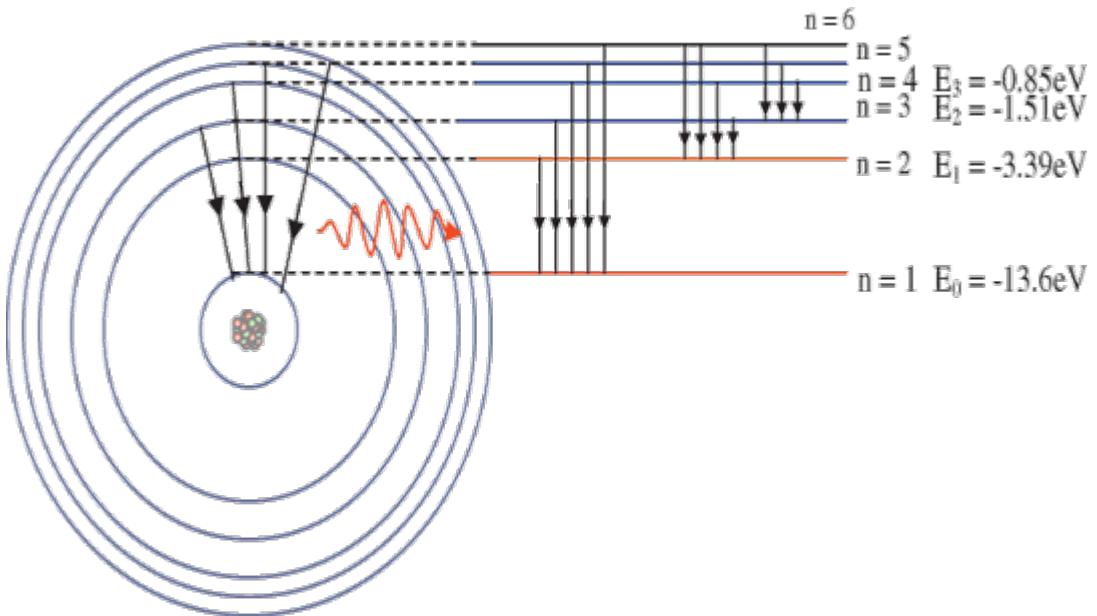


Figure 10.1b Energy levels in an atom

### Colour and light frequency

When an excited electron jumps down to a lower energy level, it gives up its excess energy as radiation. **The bigger the quantum jump, the greater the energy released and the higher the frequency of light emitted.**

The frequency of radiation (light) radiated is given by:

$$f = \frac{\Delta E}{h}$$

The wavelength of light radiated is related to the quantum energy by:

$$\hat{I}'E = E_i - E_f = hf$$

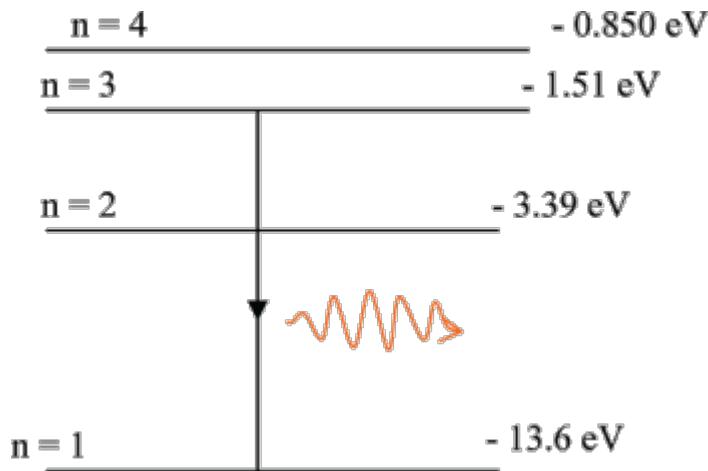
$$E_i - E_f = \frac{hc}{\lambda}$$

**The higher the energy of photon released, the shorter the wavelength.** Wavelength and frequency are related to colours. Red light is a photon of longer wavelength (lower frequency) while violet light is a photon of shorter wavelength (higher frequency).

### Worked examples

1. An electron drops from energy level – 1.51 eV to – 13.6 eV as shown below. Calculate the:
    - (i)energy of the photon radiated;
    - (ii)frequency and wavelength of the photon radiated.
- $\{h = 6.6 \times 10^{-34} \text{ J s}; c = 3.0 \times 10^8 \text{ m s}^{-1}; 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}\}$

**Solution**



$$(i) \hat{\Delta}E = E_i - E_f$$

$$\hat{\Delta}E = -1.51 - (-13.6) = 12.09 \text{ eV.}$$

$$\hat{\Delta}E = 12.09 \text{ eV} - 1.6 \times 10^{-19} = 1.934 \text{ eV} - 10^{-18} \text{ J}$$

$$(ii) E_i - E_f = hf$$

$$f = \frac{E_i - E_f}{h} = \frac{1.934 \times 10^{-18}}{6.6 \times 10^{34}}$$

$$f = 2.93 \times 10^{15} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{2.93 \times 10^{15}} = 1.02 \times 10^{-7} \text{ m.}$$

2. Calculate the ionization energy of electron in the:

(i) ground state energy level ( $-13.6 \text{ eV}$ );

(ii) excited state ( $-1.51 \text{ eV}$ )

### Solution

$$(i) \hat{\Delta}E = E_i - E_f$$

$$\hat{\Delta}E = 0 - (-13.6) = 13.6 \text{ eV}$$

$$(ii) \hat{\Delta}E = E_i - E_f = 0 - (-1.51) = 1.51 \text{ eV}$$

## Atomic spectra

Every substance radiates light when its excited electrons move down to lower energy levels. The light emitted is characterized by specific wavelength and frequency and is called **line spectra** for gases. Hot solids produce continuous wavelength and frequency. A substance emits light of different frequencies (wavelengths) if its electrons drop from higher energy level to lower energy levels. A series of line spectra are produced depending on the nature of the substance.

## Types of atomic spectra

The two types of atomic spectra are the **emission spectra** and the **absorption spectra**.

### Emission spectra

Electrons jumping from higher energy levels to lower energy levels produce the emission spectra. Three types of emission spectra are:

**1.Bright line spectrum:** Bright line spectrum is produced by excited atoms from burning gases and gases excited by electrical sparks under low pressure. It consists of lines of different colours. Each element has its own characteristics lines, which can be used to identify it. Figure 10.2 represents the bright lines spectra of Balmer series of hydrogen gas.



Figure 10.2: Bright line spectrum of hydrogen gas

**2.Band spectrum:** Oxygen gas produces a band spectrum if excited. A band spectrum consists of many lines formed into a band as shown in Figure 10.3.

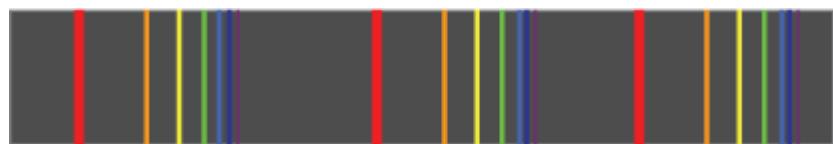


Figure 10.3: Band spectrum of oxygen gas

**3.Continuous spectrum:** Solids heated until they become white-hot, molten metals and gases excited by electrical sparks under high-pressure produce continuous spectrum. This consists of coloured light from red to violet.

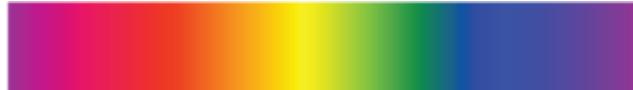


Figure 10.4: A continuous spectrum

### Absorption spectra

When white light is passed through an excited gas and examined with spectroscope, dark lines or bands of dark lines are formed on the continuous spectrum. The dark lines represent those wavelengths or frequencies from the emission spectrum, which has been absorbed. Further study of the absorption spectra reveals that each element absorbs exactly those wavelengths which they emit when they are hot.



Figure 10.5: Absorption spectrum

### Franck and Hertz experiment

Franck and Hertz conducted experiment, which confirmed the existence of energy levels in the atom. Filament

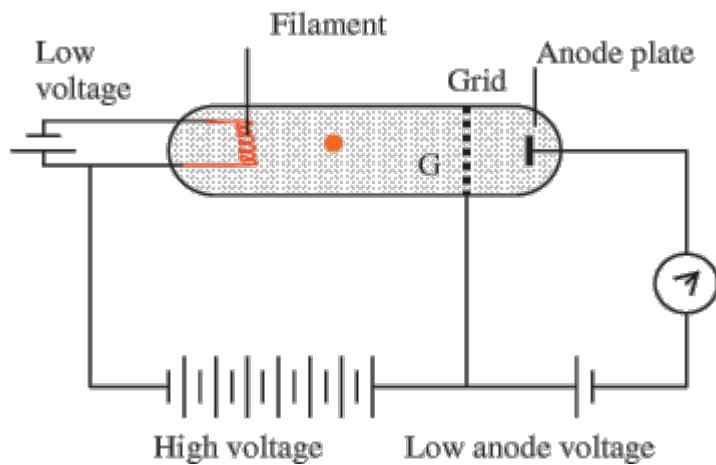


Figure 10.6: Franck and Hertz experiment

Figure 10.6 shows the apparatus used by Franck and Hertz. It consists of:

- a filament which produces electrons when it is hot.
- the accelerating potential connected between the filament and the grid (G).
- anode plate at negative potential to which a current sensitive meter is connected.

Electrons are emitted by heating the filament. These electrons accelerate towards the grid (G) by the high voltage connected between the filament and the grid. The anode at negative potential slows down the electrons, which pass through the grid. If the accelerating potential (high voltage) between the filament and the grid is increased from 0 V, the anode current flows through the meter. At a particular potential (V) corresponding to the excitation energy of the electron, the current flowing in the meter drops to zero. This happens when the collision between the electrons and the atoms of the gas are inelastic. The electrons give up their energies during the collisions, and do not reach the anode; therefore, the anode current drops to zero. Further increase in the accelerating potential will produce zero anode current for higher excitation energies (2 V, 3 V, 4 V etc.).

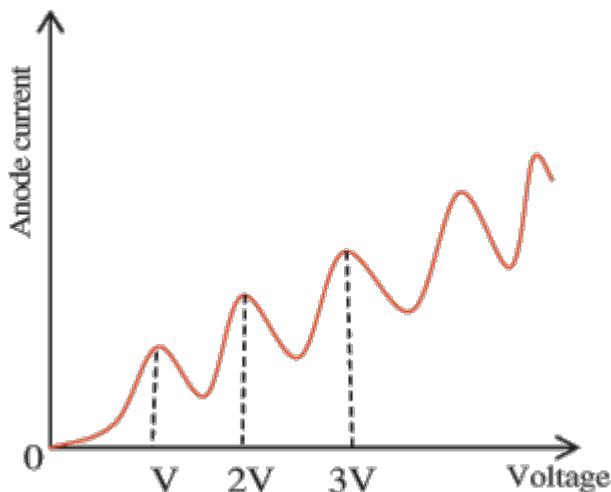


Figure 10.7: Graph of anode current versus accelerating potential

Figure 10.7 shows the graph of anode current against the accelerating potentials. The difference between two successive maxima represents the excitation potential for each

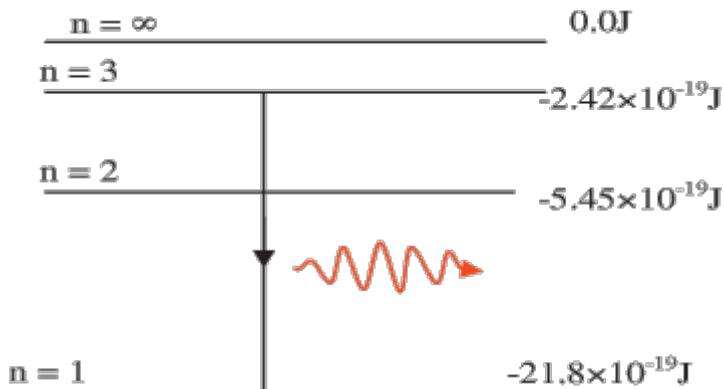
inelastic collision.

### Summary

- Electrons moving round the nucleus exist in different energy states; the lowest energy level of the electron is called the **ground state energy**.
- An electron becomes **excited** and **unstable** when it absorbs energy and moves to a higher energy level. The amount of energy radiated as the electron jumps down or absorbed when it is excited to higher energy level is given by:
$$E_i - E_f = hf$$
- Excitation energy is the energy absorbed by an electron to move up to higher energy levels.
- Ionization energy is the energy absorbed by an electron to move it completely out the atom.
- **Emission spectra** and the **absorption spectra** are the two types of atomic spectra.
- **A bright line spectrum, a band spectrum and continuous spectrum** are examples of emission spectra.
- Absorption spectrum is the dark lines on the continuous spectrum. Every element absorbs exactly those wavelengths which it emits when they are hot.

### Practice questions 10a

1. (a) Explain why an excited atom emits electromagnetic radiation.  
(b) The diagram below represents the energy levels of hydrogen atom. **Stable** electrons occupy the **ground state energy**.



- (i) Explain the words **stable** and **ground state energy**.  
(ii) Calculate the ionization energy of hydrogen atom.  
(iii) Calculate the frequency and wavelength of photon radiated for the transition shown in the diagram.  $\{h = 6.6 \text{ Å}—10^{-34} \text{ J s}; c = 3.0 \text{ Å}—10^8 \text{ m s}^{-1}\}$
2. (a) Explain the terms **ionization energy** and **excited energy**.

- (b) The energy levels of hydrogen atom are given by the equation:

$$E_n = \frac{-13.6}{n^2} eV ; n=1,2,3..$$

Calculate:

- (i) the ionization energy of the atom;
  - (ii) the wavelength of radiation emitted when the electron jumps from  $n = 4$  to  $n = 1$ .  $\{h = 6.6 \text{ Å} - 10^{-34} \text{ J s}; c = 3.0 \text{ Å} - 10^8 \text{ m s}^{-1}; 1 \text{ eV} = 1.6 \text{ Å} - 10^{-19} \text{ J}\}$
3. (a) What is an **atomic spectrum?** Give three examples of **emission spectra.**
- (b) Describe the Franck and Hertz experiment and explain how it proves the existence of energy levels in an atom.
- (c) The spectrum of atomic hydrogen emits a red light of wavelength  $6.60 \text{ Å} - 10^{-7} \text{ m}$ . What is the energy of the photon with this energy?  
 $\{h = 6.6 \text{ Å} - 10^{-34} \text{ J s}; c = 3.0 \text{ Å} - 10^8 \text{ m s}^{-1}\}$

## PHOTOELECTRIC EFFECT

### OBJECTIVES

At the end of the topic, students should be able to:

- explain photoelectric effect;
- use the photon concept to explain the ejection of electrons in photoelectric effect;
- explain work function, threshold frequency and Einstein photoelectric equation;
- solve problems on photoelectric equation;
- describe the working of photocell and state some uses of photocell.

### Photoemission

If light of sufficient frequency (energy) shines on a metal surface, electrons are given out (ejected) from the metal immediately. The emission of electrons from metal surfaces by light is called **photoelectric effect**.

*Photoelectric effect is the emission of electrons from a metal surface when light of sufficient frequency (energy) shines on it.*

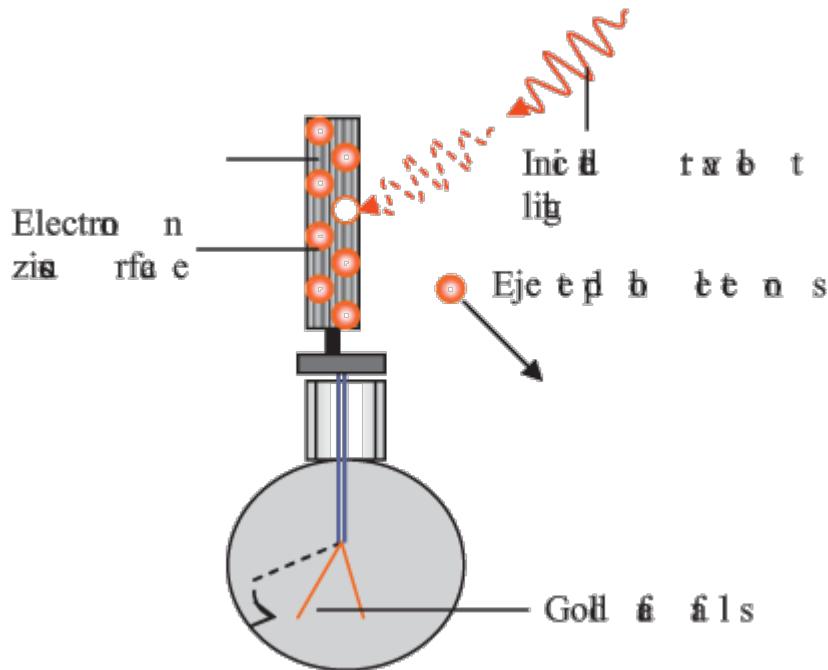


Figure 10.8: Photoelectric emission

Figure 10.8 shows an ultraviolet light falling on a zinc surface. Electrons are ejected immediately from the surface making the zinc metal positively charged. The zinc attracts electrons from the negatively charged electroscope; therefore, the gold leaf falls. The fall of the gold leaf of the electroscope shows that electrons are ejected by ultraviolet light striking the zinc surface. The ejected electrons are called **photoelectrons**.

#### Factors affecting the emission of photoelectrons

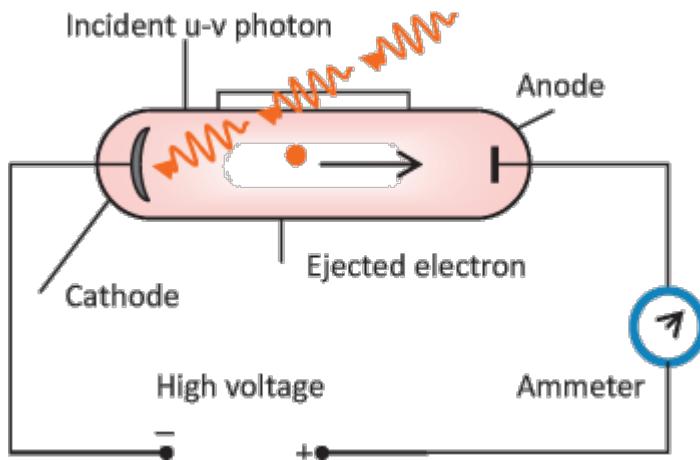


Figure 10.9: An investigating photoelectrons emission

Figure 10.9 is used to investigate the factors, which determines the emission of photoelectrons. When an ultraviolet light shines on the zinc cathode, electrons are emitted and attracted by the anode at positive potential. The electrons reaching the anode cause a small anode current to flow in the circuit. Measurement of the anode current reveals the following surprising results:

1. The maximum kinetic energy of the photoelectrons depends on the frequency or

wavelength of incident light only.

The higher the frequency the faster the electrons are removed from metal surface. The graph of kinetic energy represented by the anode current against frequency is shown in Figure 10.10. The graph reveals that:

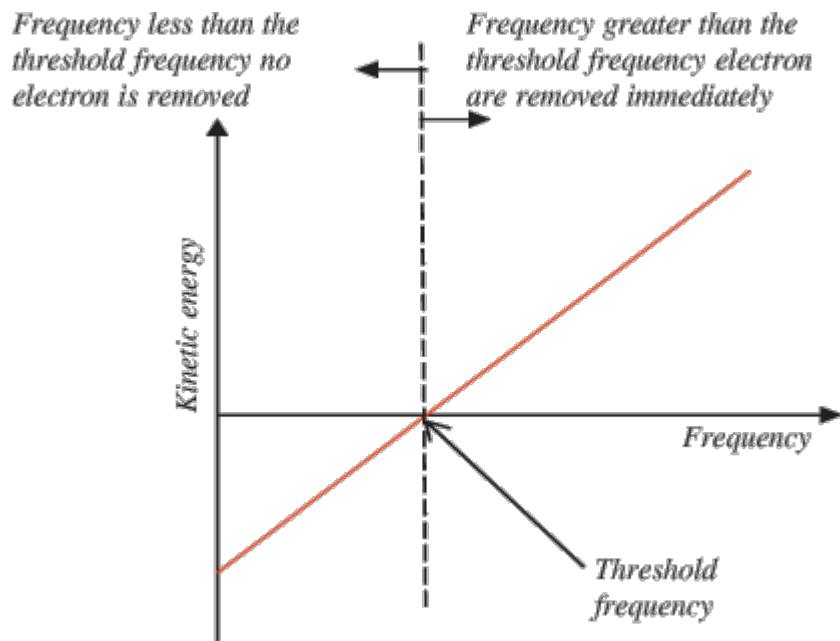


Figure 10.10 : Anode current against frequency

- there is a minimum frequency the incident light must have before electrons will be given out from the surface. This minimum frequency is called **threshold frequency** ( $f_0$ ).

**Threshold frequency is the minimum frequency of an incident light needed to remove an electron from a metal surface.**

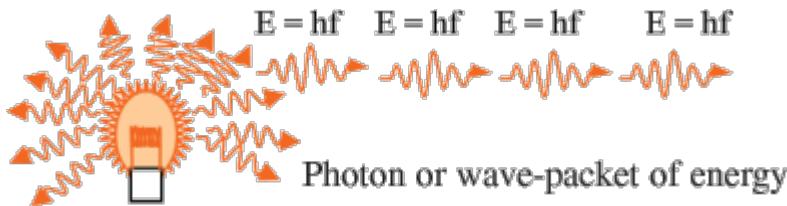
- if the frequency of incident light is more than the threshold frequency ( $f_0$ ), electrons are given out immediately from the surface even with a dim (weak) light.
- below the threshold frequency, no electron is removed from the surface even with a very bright (intense) light.

2. The number of photoelectrons given out from the metal surface by the incident light depends on the brightness or intensity of the incident light.

Once the frequency of the incident light is above the threshold frequency, the number of photoelectrons removed from the metal surface increases as the intensity or brightness of the incident light increases. If the frequency of the incident light is less than the threshold frequency, no photoelectron is produced no matter the brightness of the light.

### Einsteinâ€™s explanation of photoelectric effect

Einstein in 1905 extended Max Planckâ€™s idea of energy quantization to light and used it to explain photoelectric effect. According to Einstein, light is radiated from a source as a **burst of energy** or **wave-packet** called **photon**. *Each wave-packet or photon carries energy, which is proportional to the frequency of the incident light.* The higher the frequency of incident lights the more energy it carries.



The energy of each burst or wave-packet of energy is given by:

$$E = hf \quad \text{or} \quad E = \frac{hc}{\lambda}$$

### Work function ( $w_0$ )

Electrons are bound to the surface of the metal by strong electrostatic force. The incident light must supply enough energy to the electrons to overcome the electrostatic force of the metal. The work done by the incident light to free the electrons is called **work function**.

*Work function is the minimum energy needed to free an electron from the electrostatic force of the surface.*

$$W_0 = hf_0 \quad \text{or} \quad W_0 = \frac{hc}{\lambda_0}$$

$W_0$  = work function,  $f_0$  = threshold frequency,  $\lambda_0$  = threshold wavelength,  $c$  = speed of light and  $h$  = Planck's constant.

When the energy of incident light striking the metal surface is more than its work function, part of it is used to overcome the work function ( $w_0$ ) of the metal. The electrons are now freed from electrostatic force binding them to the surface of the metal. The rest of the energy is transformed to kinetic energy of the photoelectrons. Einstein's explanation of photoelectric effect is summarized by equation:

$$\frac{1}{2}mv^2 = hf - w_0$$

$$\frac{1}{2}mv^2 = \text{maximum kinetic energy of the photoelectrons.}$$

$hf$  = energy of incident photon and  $w_0$  = work function of the metal surface.

If the energy of the incident photon is less than the work function of the metal surface, no electron is removed from the surface. The energy of the incident light warms the surface of the metal to increase its internal energy.

### Stopping potential ( $V_s$ )

Figure 10.9 is also used to measure the maximum kinetic energy of the photoelectrons. Maximum kinetic energy is measured by the value of negative anode potential needed to stop the most energetic electron from reaching the anode. A negative potential is applied to the anode and increased slowly until the anode current flowing in the circuit is zero. *The value of anode negative potential when the anode current is zero is the stopping*

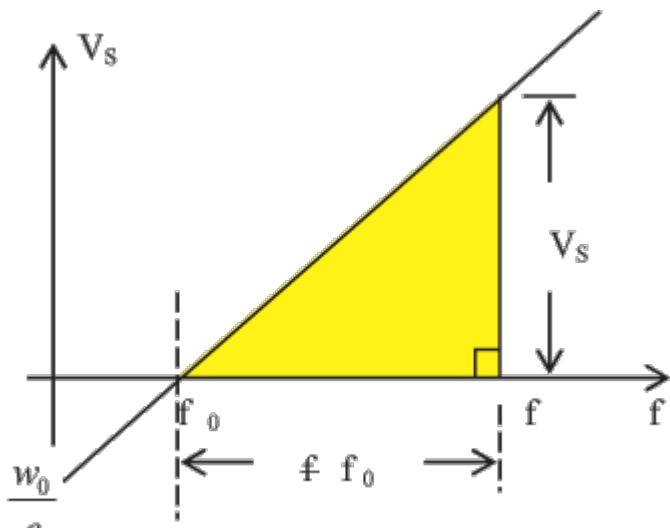
potential ( $V_s$ ).

Maximum kinetic energy of the photoelectrons is the work done by the stopping potential ( $V_s$ ) to prevent the most energetic electrons from reaching the anode. This is mathematically expressed as:

$$\frac{1}{2}mv^2 = eV_s$$

### Determination of Planck's constant (h)

The graph of stopping potential ( $V_s$ ) against frequency is used to find the value of Planck's constant (h).



*Graph of stopping potential  $V_s$  against frequency  $f$*

Einstein's photoelectric effect can be rewritten as:

$$eV_s = hf - w_0$$

$$\therefore V_s = \left(\frac{h}{e}\right)f - \frac{w_0}{e}$$

The slope and the intercept of the equation (s) are  $\frac{h}{e}$  and  $\frac{w_0}{e}$  respectively. The slope (s) of the graph is given by

$$\text{Slope}(s) = \frac{V_s}{f - f_0}$$

Equating the two slopes, we can find the value of Planck's constant.

$$\frac{V_s}{f - f_0} = \frac{h}{e}$$

$$Planck's\ constant(h) = e \times \frac{V_s}{f - f_0}$$

Equating the intercepts, we can find the work function of the metal used. Intercept (I) =  $\frac{w_0}{e}$ . Work function ( $w_0$ ) = e — intercept.

### Worked examples

1. Light of frequency  $6.0 \text{ A}—10^{14} \text{ Hz}$  is incident on a metal surface of work function  $3.0 \text{ A}—10^{-19} \text{ J}$ . Calculate the:
- (i) threshold frequency;
  - (ii) energy of the incident light;
  - (iii) maximum kinetic energy of the photoelectrons;
  - (iv) stopping potential. { $h = 6.6 \text{ A}—10^{-34} \text{ J s}$ ;  $e = 1.6 \text{ A}—10^{-19} \text{ C}$ }

#### Solution

$$(i) f_0 = \frac{w_0}{h} = \frac{3.0 \times 10^{-19}}{6.6 \times 10^{-34}} = 4.5 \times 10^{14} \text{ Hz}$$

$$(ii) E = hf = 6.6 \times 10^{-34} \times 6.0 \times 10^{14} = 3.96 \times 10^{-19} \text{ J}$$

$$(iii) \frac{1}{2}mv^2 = hf - w_0 = 3.96 \times 10^{-19} - 3.0 \times 10^{-19} = 0.96 \times 10^{-19} \text{ J}$$

$$(iv) V_s = \frac{\frac{1}{2}mv^2}{e} = \frac{0.96 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.6V$$

2. Light of wavelength  $2.5 \text{ A}—10^{-7} \text{ m}$  is incident on a metal surface of work function 3.3 eV. Calculate the:
- (a) threshold frequency;
  - (b) energy of the incident light;
  - (c) greatest velocity of the photoelectrons;
  - (d) stopping potential.

[speed of light (c) =  $3.0 \text{ A}—10^8 \text{ m s}^{-1}$ ;  $h = 6.6 \text{ A}—10^{-34} \text{ J s}$ ;  $1 \text{ eV} = 1.6 \text{ A}—10^{-19} \text{ J}$   
and mass of electron (m) =  $9.0 \text{ A}—10^{-31} \text{ kg}$ ]

#### Solution

(a) Work function ( $w_0$ ) = 3.3 eV =  $3.3 \times 1.6 \times 10^{-19} = 5.28 \times 10^{-19}$  J

$$f_0 = \frac{w_0}{h} = \frac{5.8 \times 10^{-19}}{6.6 \times 10^{-34}} = 0.8 \times 10^{15} \text{ Hz}$$

$$(b) E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{2.5 \times 10^{-34}} = 7.92 \times 10^{-19} \text{ J}$$

$$(c) \frac{1}{2}mv^2 = hf - w_0 = 7.92 \times 10^{-19} - 5.28 \times 10^{-19} = 2.64 \times 10^{-19} \text{ J}$$

$$\frac{1}{2}mv^2 = K.E_{\max}$$

$$v = \sqrt{\frac{2K.E_{\max}}{m}} = \sqrt{\frac{2 \times 2.64 \times 10^{-19}}{9.0 \times 10^{-31}}} = \sqrt{0.5867 \times 10^{12}} = 7.66 \times 10^5 \text{ ms}^{-1}$$

(d)  $eV_s = K.E_{\max}$

$$V_s = \frac{\frac{1}{2}mv^2}{e} = \frac{2.64 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.65 \text{ V}$$

3. The threshold frequency of a certain surface is  $1.0 \times 10^{15}$  Hz. What is the frequency of the incident light that gives the photoelectron a maximum kinetic energy of  $2.0 \times 10^{-19}$  J?

**Solution**

Work function ( $w_0$ ) =  $hf_0 = 6.6 \times 10^{-34} \times 1.0 \times 10^{15} = 6.6 \times 10^{-19}$  J

$$\frac{1}{2}mv^2 = hf - w_0$$

$$f = \frac{\frac{1}{2}mv^2 + w_0}{h} = \frac{2.0 \times 10^{-19} + 6.6 \times 10^{-19}}{6.6 \times 10^{-34}} = \frac{8.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.3 \times 10^{15} \text{ Hz}$$

## Photocell

Photocells are devices, which converts light energy to electrical energy. A photocell consists of:

- cathode (C): a curved metal plate coated with photosensitive material like caesium which has low work function.
- anode (A): a straight rod at high positive potential to attract photoelectrons ejected from the cathode.
- evacuated glass bulb to shield the photoelectrons from the influence of air which may hinder its motion.

When light with frequency above the threshold frequency shines on the cathode, electrons are released immediately. The anode at positive potential attracts the photoelectrons. The electrons reaching the anode cause a small anode current to flow through the ammeter as

shown in figure 10.11. The size of the anode current is proportional to the intensity or brightness of the incident

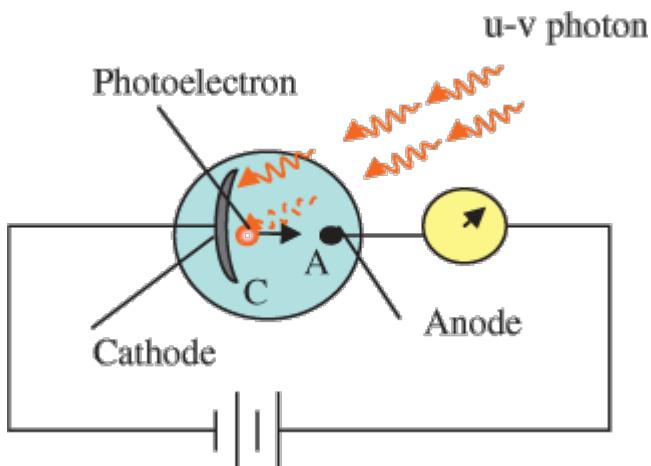


Figure 10.11 Photocell

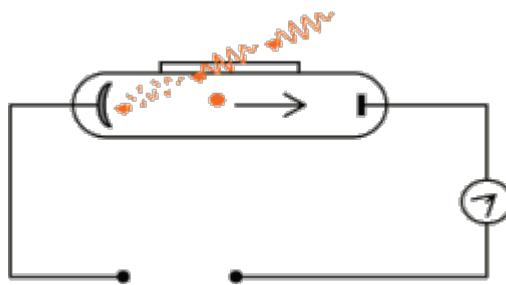
Photocells are used in camera as light meters, electronic ignition circuits to operate or cause doors to open or close when light falling on the photocell is cut off. It is also used to operate loudspeaker and burglar alarm.

### Summary

- Photoelectric effect is the emission of electrons from a metal surface when light of I sufficient frequency (energy) shines on it.
- The maximum kinetic energy of the photoelectrons depends on the frequency or wavelength of incident light only.
- Threshold frequency is the minimum frequency of an incident light needed to remove an electron from a metal surface.
- The number of photoelectrons given out from the metal surface by the incident light depends on the brightness or intensity of the incident light.
- Work function is the minimum energy needed to free an electron from the electrostatic force of the surface.
- Stopping potential ( $V_s$ ) is the voltage applied to the anode to stop the most energetic electrons.
- Photocells are devices, which convert light energy to electrical energy.

### Practice questions 10b

1. (a).



The diagram above shows the apparatus for investigating photoelectric effect in metals.

- (i) Explain why no electron is ejected if the frequency of incident light is less than the threshold frequency.
  - (ii) Explain why more current flows in the circuit if the brightness of the light is increased.
  - (b) What is stopping potential? Explain how it is used to measure the maximum kinetic energy of the photoelectron.
2. (a) What is work function of a metal?
- (b) A photon falling on the surface of a metal is just sufficient to remove electrons from the surface. Calculate the work function if the threshold wavelength of the photon is  $2.0 \text{ Å} = 10^{-7} \text{ m}$ . {Speed of light ( $c$ ) =  $3.0 \text{ Å} = 10^8 \text{ m s}^{-1}$ ;  $h = 6.6 \text{ Å} = 10^{-34} \text{ J s}$ }
3. (a) A photon of energy ( $E = hf$ ) is incident on a metal surface of work function ( $w_0$ ). Explain why electrons are emitted when the photon energy is greater than the work function of the metal.
- (b) Light of wavelength  $2.0 \text{ Å} = 10^{-7} \text{ m}$  eject electrons from a metal with work function 3.2 eV.
  - (i) Find the maximum energy of the ejected photoelectrons.
  - (ii) What is the longest wavelength that will produce photoelectrons?  
(Speed of light ( $c$ ) =  $3.0 \text{ Å} = 10^8 \text{ m s}^{-1}$ ;  $h = 6.6 \text{ Å} = 10^{-34} \text{ J s}$ ;  $1 \text{ eV} = 1.6 \text{ Å} = 10^{-19} \text{ J}$ )
4. Light of frequency  $3.6 \text{ Å} = 10^{14} \text{ Hz}$  is incident on a metal surface of work function  $2.2 \text{ Å} = 10^{-19} \text{ J}$ . Calculate the:
  - (i) threshold frequency;
  - (ii) energy of the incident light;
  - (iii) maximum kinetic energy of the photoelectrons;
  - (iv) stopping potential. { $h = 6.6 \text{ Å} = 10^{-34} \text{ J s}$ ;  $e = 1.6 \text{ Å} = 10^{-19} \text{ C}$ }

## X-RAYS

### OBJECTIVES

At the end of the topic, students should be able to describe X-ray production and state its characteristics, properties and uses.

RÃ¶ntgen discovered X-ray in 1895. He was conducting experiment with a discharge tube when he noticed that an invisible radiation is produced, which could penetrate most materials. He called the unknown radiation X-ray. The origin of x-rays is the transition of electrons from higher energy level to lower energy levels.

### Production of X-rays

X-rays were produced when a beam of fast moving electrons are stopped by a metal target. X-rays are produced in an X-ray tube designed by Coolidge. The structure of an X-ray tube is shown in Figure 10.12.

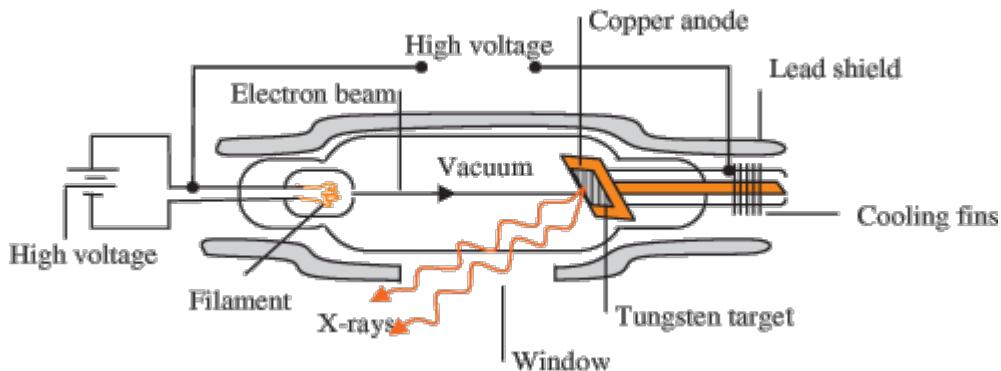


Figure 10.12 : X-ray tube

The filament produces a beam of electron when it is hot. A low voltage heats the filament, which also controls the intensity of the X-ray produced. The high voltage applied between the filament and the anode accelerates the electron beam towards the tungsten target. When the electron beam strikes the tungsten target, their kinetic energy is changed to heat energy and X-rays. Less than 1% of the kinetic energy of the electron beam is converted to X-ray; the rest are transformed to heat energy. Heat produced because of the collision of the electrons with the target is removed by circulating coolant. This prevents the target from melting.

Tungsten is embedded on the copper anode to conduct heat away from the tungsten target. The choice of tungsten as the target is because it has high melting point; therefore can withstand high temperature.

### Intensity of X-rays

The quantity of X-ray produced is referred to as its **intensity**. The intensity of an X-ray is high if the number of electrons hitting the target is increased. The number of electrons striking the target is given by:

$$n = \frac{I}{e}$$

( $I$  = current;  $e$  = electronic charge;  $n$  = number of electrons striking the target per second) It is increased by:

- increasing the filament current. Higher filament current makes the filament hotter and therefore more electrons are emitted.
- using target material with high proton (atomic) number.

## Quality of X-rays

The quality of an X-ray is measured by its ability to penetrate materials. It depends on the wavelength of the X-ray produced. The shorter the wavelength of an X-ray, the higher it penetrates materials and the better the quality. X-rays with high penetrating power (short wavelength) are called **hard** X-rays. **Soft** X-rays have low penetrating power or longer wavelengths. High quality X-rays are produced by increasing the voltage applied between the target and the filament.

## Properties of X-rays

- 1.X-rays are not deflected by electric and magnetic fields.
- 2.X-rays can be diffracted like other waves.
- 3.X-ray travel in straight lines.
- 4.X-rays produces fluorescence in many materials.
- 5.X-rays affects photographic film.
- 6.X-rays penetrates many substances, which are opaque to light but are absorbed by bone, lead, and other material with high density.
- 7.They ionize gases making them to conduct electricity and discharge electroscope.
- 8.They can produce photoemission.

## Uses of X-rays

### A. Medical application of X-rays:

- 1.X-rays are used to reveal things hidden from the ordinary eye. They reveal broken bones, bullets and other dense objects hidden in the body.
- 2.X-ray photographs can reveal obstruction in digestive tract and internal growths.
- 3.Hard X-rays given in a right dose is used to kill cancerous cells and malignant growth in the body.

### B. Industrial and scientific applications of X-rays:

- 1.Hard X-rays are used in the industries to reveal hidden cracks in welded joints and castings.
- 2.X-ray photographs diffracted by crystals gives information on how the atoms of different crystal are arranged.
- 3.X-rays diffraction are used to find the structure of complex organic molecules.
- 4.X-rays are used to reveal covered paintings.

## Worked examples

1. An X-ray tube operating at 100 KV is used to produce X-ray itself. Calculate the:
  - (a)maximum kinetic energy of the X-ray;
  - (b)wavelength of the X-ray produced. { $h = 6.6 \text{ eV} = 6.6 \times 10^{-34} \text{ J s}$ ;  $e = 1.6 \text{ eV} = 1.6 \times 10^{-19} \text{ C}$  =  $3.0 \text{ eV} = 3.0 \times 10^8 \text{ m s}^{-1}$ }

### Solution

$$(a) \text{Maximum kinetic energy} = \frac{1}{2}mv^2 = eV = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-14} \text{ J}$$

$$(b) \lambda = \frac{hc}{E_k} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{1.6 \times 10^{-14}} = 1.24 \times 10^{-11} \text{ m}$$

2. Calculate the minimum voltage applied between the filament and the target of an X-ray tube to produce X-rays of wavelength  $5.0 \text{ \AA} = 10^{-11} \text{ m}$ .

$$\{h = 6.6 \text{ \AA} = 10^{-34} \text{ Js}; e = 1.6 \text{ \AA} = 10^{-19}; c = 3.0 \text{ \AA} = 10^8 \text{ m s}^{-1}\}$$

**Solution**

$$\text{Maximum kinetic energy} = \frac{1}{2}mv^2 = eV = hf$$

$$\therefore eV = \frac{hc}{\lambda}; f = \frac{c}{\lambda}$$

$$\therefore 1.6 \times 10^{-19} \times V = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{5.0 \times 10^{-11}}$$

$$V = \frac{3.96 \times 10^{-15}}{1.6 \times 10^{-19}} = 2.475 \times 10^4 \text{ V}$$

3. An X-ray tube is rated 500W, 100KV. Calculate the:

- (i) current flowing through the tube;
- (ii) number of electrons striking the target per second;
- (iii) speed of the electrons when they strike the target;
- (iv) minimum wavelength of the X-ray produced.

(Mass of electron =  $9.1 \text{ \AA} = 10^{-31} \text{ kg}$ ;  $e = 1.6 \text{ \AA} = 10^{-19}$ ;  $c = 3 \text{ \AA} = 10^8 \text{ m s}^{-1}$  and  $h = 6.6 \text{ \AA} = 10^{-34} \text{ Js}$ )

**Solution**

$$(i) I = \frac{P}{V} = \frac{500}{100000} = 0.005 \text{ A} \quad (ii) n = \frac{I}{e} = \frac{0.005}{1.6 \times 10^{-19}} = 3.125 \times 10^{16}$$

$$(iii) \frac{1}{2}mv^2 = eV$$

$$0.5 \times 9.1 \times 10^{-31} \times v^2 = 1.6 \times 10^{-19} \times 10^5$$

$$v^2 = \frac{1.6 \times 10^{-19} \times 10^5}{0.5 \times 9.1 \times 10^{-31}} = 3.516 \times 10^{16}$$

$$v = 1.875 \times 10^8 \text{ ms}^{-1}$$

$$(iv) \frac{hc}{\lambda} = eV \Rightarrow \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{\lambda} = 1.6 \times 10^{-19} \times 10^5$$

$$\therefore \lambda = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{1.6 \times 10^{-19} \times 10^5} = 1.2375 \times 10^{-11} \text{ m}$$

### Summary

- X-rays are produced when fast moving electrons are stopped by a metal target. The kinetic energy of the electrons are changed to heat energy and X-rays.
- The quantity of X-ray produced is called the intensity of the X-ray. The intensity is controlled by controlling the filament current. Higher filament current using target

- material with high proton (atomic) number produces X-ray of higher intensity.
- The quality of an X-ray is measured by its ability to penetrate materials. It depends on the wavelength of the X-ray produced.

### Practice questions 10c

1. (a) What is an X-ray? With the aid of labelled diagram, describe how X-rays are produced.
- (b) Explain how the **intensity** and **quality** of X-ray produced can be controlled.
- (c) (i) Explain the terms **hard X-rays** and **soft X-rays**.  
(ii) State one factor which affects the hardness of an X-ray.
2. (a) What are differences between X-rays and sound wave? Outline the energy transformation which occurs during the production of X-rays.
- (b) State;
  - (i) **three** properties of X-rays which suggest that it is a wave.
  - (ii) **two medical** and **two industrial** uses of X-rays.
- (c) An electron of charge  $1.6 \times 10^{-19} \text{ C}$  is accelerated by a p.d. of 60 KV before striking a metal target. What is the wavelength of the X-ray produced?

### Past questions

1. An electron makes a transition from a certain energy level  $E_K$  to the ground state  $E_0$ . If the frequency of emission is  $8.0 \times 10^{14} \text{ Hz}$ , the energy emitted is
  - A.  $5.28 \times 10^{-19} \text{ J}$
  - B.  $8.25 \times 10^{-19} \text{ J}$
  - C.  $8.25 \times 10^{-19} \text{ J}$
  - D.  $5.28 \times 10^{-19} \text{ J}$   
 $[h = 6.60 \times 10^{-34} \text{ Js}]$

JAMB

2. The energy associated with the photon of a radio transmission at  $3 \times 10^5 \text{ Hz}$  is
  - A.  $1.30 \times 10^{-29} \text{ J}$
  - B.  $2.00 \times 10^{-29} \text{ J}$
  - C.  $1.30 \times 10^{-28} \text{ J}$
  - D.  $2.00 \times 10^{-28} \text{ J}$   
 $[h = 6.60 \times 10^{-34} \text{ Js}]$

JAMB

3.  $E_{\infty}(zero)$       Infinite energy level

- E<sub>3</sub> (- 1.6 eV) 3rd level  
 E<sub>2</sub> (-3.7 eV) 2nd level  
 E<sub>1</sub> (-5.5 eV) 1st level  
 E<sub>1</sub> (-10.4 eV) Ground state

The diagram above represents the energy levels of a mercury atom. What is the amount of energy that will be required to excite mercury atoms from the ground state to the second level?

- A. 6.7 eV
- B. 12.6 eV
- C. 15.3 eV
- D. 20.7 eV

WASSCE

4. The minimum energy required to remove an electron from an atom is known as
- A. excitation energy.
  - B. ionization energy.
  - C. binding energy.
  - D. photon energy.

WASSCE

5. In a model of the hydrogen atom, the energy levels W<sub>n</sub> are given by the formula

$$W_n = -\frac{R}{n^2}$$

where n is an integer and R is a constant. Determine the energy released in transition from n = 3 to n = 2.

- A.  $\frac{R}{5}$
- B.  $-\frac{R}{4}$
- C.  $\frac{5R}{36}$
- D.  $-\frac{5R}{36}$

WASSCE

6. The atoms of a gas are excited to produce a discharge which is examined with a spectrometer. The examination will reveal
- A. a totally dark background.
  - B. the colours of the rainbow.
  - C. an emission spectrum.

- D. an absorption spectrum.
- E. a continuous.

WAEC

7. Absorption line spectra exhibited by atoms is a result of
- A. change in kinetic energy of a moving electron.
  - B. instability of the nucleus.
  - C. excitation of an electron in the atom.
  - D. transition of an electron from a higher to a lower energy.

WASSCE

8. A photon of wavelength  $\hat{\lambda}_0$  is emitted when an electron in an atom makes a transition from a level of energy  $2E_K$  to that of energy  $E_K$ . If the electron transits from  $4/3E_K$  to  $E_K$  level, determine the wavelength of the photon that would be emitted.
- A.  $3\hat{\lambda}_0$
  - B.  $2\hat{\lambda}_0$
  - C.  $\frac{3}{4}\hat{\lambda}_0$
  - D.  $\frac{1}{3}\hat{\lambda}_0$

WASSCE

9. An important experimental evidence for existence of line of quantized energy levels in the atom is the existence of
- A. spectral lines.
  - B. massive nucleus.
  - C. ionization energy.
  - D. valence electrons.

WASSCE

10. When light falls on a metallic surface, the number of electrons that may be emitted by the metal would depend solely on the
- A. area of the metallic surface.
  - B. frequency of the light.
  - C. intensity of the light.
  - D. time of exposure of the metallic surface to light.

WASSCE

11. The minimum energy required by a photon incident on a metal surface to liberate electrons from the surface is called the
- A. ionization energy.
  - B. threshold frequency.

- C. work function.
- D. excitation energy.

WASSCE

12. A metal has a work function of 4.375 eV. Calculate its threshold frequency.

[ $h = 6.6 \times 10^{-34} \text{ J s}$ ;  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ]

- A.  $2.01 \times 10^{15} \text{ Hz}$ .
- B.  $1.06 \times 10^{15} \text{ Hz}$
- C.  $6.30 \times 10^{14} \text{ Hz}$
- D.  $1.60 \times 10^{14} \text{ Hz}$

WASSCE

13. The work function of a metal is  $8.6 \times 10^{-19} \text{ J}$ . Calculate the wavelength of its threshold frequency.

[ $h = 6.6 \times 10^{-34} \text{ J s}$ ;  $c = 3.0 \times 10^8 \text{ m s}^{-1}$ ]

- A.  $0.8 \times 10^{-7} \text{ m}$
- B.  $1.0 \times 10^{-7} \text{ m}$
- C.  $2.3 \times 10^{-7} \text{ m}$
- D.  $3.8 \times 10^{-7} \text{ m}$
- E.  $12.4 \times 10^{-7} \text{ m}$

WAEC

14. The maximum kinetic energy of the photoelectrons emitted from a metal surface is 0.34 eV. If the work function of the metal surface is 1.83 eV, find the stopping potential.

- A. 0.34 V
- B. 2.17 V
- C. 1.49 V
- D. 1.09 V

JAMB

15. The kinetic energy of a photoelectron liberated from a metallic surface depends on the

- A. intensity of the incident radiation.
- B. time of duration of the incident radiation.
- C. temperature of the incident radiation.
- D. frequency of the incident radiation.

16. An electron is accelerated from rest through a p.d. of 70 KV in a vacuum. Calculate the maximum speed acquired by the electrons.

[Electronic charge =  $e = 1.6 \times 10^{-19} \text{ C}$ ; mass of an electron =  $9.1 \times 10^{-31} \text{ kg}$ ]

- A.  $3.00 \times 10^8 \text{ m s}^{-1}$ .

- B.  $2.46 \text{ \AA} - 10^8 \text{ m s}^{-1}$ .
- C.  $1.57 \text{ \AA} - 10^8 \text{ m s}^{-1}$ .
- D.  $1.32 \text{ \AA} - 10^8 \text{ m s}^{-1}$ .
- E.  $1.11 \text{ \AA} - 10^8 \text{ m s}^{-1}$ .

WAEC

17. Calculate the energy carried by an X-ray of wavelength  $6.0 \text{ \AA} - 10^{-10} \text{ m}$ .

[ $h = 6.6 \text{ \AA} - 10^{-34} \text{ Js}$ ;  $c = 3.0 \text{ \AA} - 10^8 \text{ m s}^{-1}$ ]

- A.  $3.3 \text{ \AA} - 10^{-12} \text{ J}$ .
- B.  $3.3 \text{ \AA} - 10^{-16} \text{ J}$ .
- C.  $1.1 \text{ \AA} - 10^{-16} \text{ J}$ .
- D.  $3.3 \text{ \AA} - 10^{-14} \text{ J}$ .
- E.  $3.3 \text{ \AA} - 10^{-32} \text{ J}$ .

WAEC

18. X-rays can be used in the study of crystal structures because they

- A. have an extremely short wavelength.
- B. have a very long-reaching wavelength.
- C. are very fast.
- D. are invisible.

JAMB

19. The difference between X-rays and gamma rays is that

- A. X-rays arise from energy changes in the electronic structure of atoms while gamma rays come from the nucleus.
- B. X-rays are electromagnetic radiations while gamma rays are negatively charged radiations.
- C. X-rays have higher frequencies than gamma rays.
- D. X-rays are more penetrating than gamma rays.

JAMB

20. In an operating X-tube, a high P.d is applied between the cathode and anode for the purpose of

- A. heating the cathode.
- B. producing a stream of electrons.
- C. concentrating electrons on the target.
- D. accelerating the electrons to a very high energy.
- E. heating the anode.

WAEC / NECO

21. Production of X-rays in an X-ray tube begins with

- A. photoelectric emission.
- B. collision of electrons.
- C. thermionic emission.
- D. field emission.

WASSCE

22. The hardness of X-rays is increased by
- A. increasing the filament current.
  - B. widening the distance between cathode and anode.
  - C. increasing the operating voltage of the tube.
  - D. increasing the number of cooling fins.

WASSCE

23. The intensity of X-ray is increased by
- A. using tungsten target.
  - B. increasing the voltage of the tube.
  - C. shortening the distance between anode and cathode.
  - D. increasing the filament current.

WASSCE

24. Which radiation is emitted when a beam of fast-moving cathode rays are stopped by a heavy element?
- A. Alpha rays
  - B. Gamma rays.
  - C. Positive rays.
  - D. Ultraviolet rays.
  - E. X-rays.

NECO

25. (a). (i) Explain the terms: *photoelectric emission* and *threshold frequency*.  
(ii) Einstein's photoelectric equation can be written as  $E = hf - hf_0$

What does each of the symbols used in the equation above represent?

- (b) Calculate the frequency of the photon whose energy is required to eject a surface electron with kinetic energy of  $1.97 \times 10^{-16}$  eV if the work function of the metal is  $1.33 \times 10^{-16}$  eV. [ $h = 6.6 \times 10^{-34}$  J s;  $1 \text{ eV} = 1.6 \times 10^{-19}$  J]
- (c) In a photoelectric cell, no electrons are emitted until the threshold frequency of light is reached. Explain what happens to the energy of the light before emission of electrons begins.
- (d) State one factor that may affect the number of emitted electrons.

WAEC

26. (a) Explain what is meant by **photoelectric emission**.  
 (b) State four applications of photoelectric emission.  
 (c) A metal has a work function of 2.0 eV and is illuminated by monochromatic light of wavelength  $4.5 \text{ \AA} = 10^{-7} \text{ m}$ . Calculate  
 (i) threshold frequency;  
 (ii) maximum energy of the photoelectron.

$[h = 6.6 \text{ \AA} = 10^{-34} \text{ J s}; 1 \text{ eV} = 1.6 \text{ \AA} = 10^{-19} \text{ J and } c = 3.0 \text{ \AA} = 10^8 \text{ m s}^{-1}]$

NECO

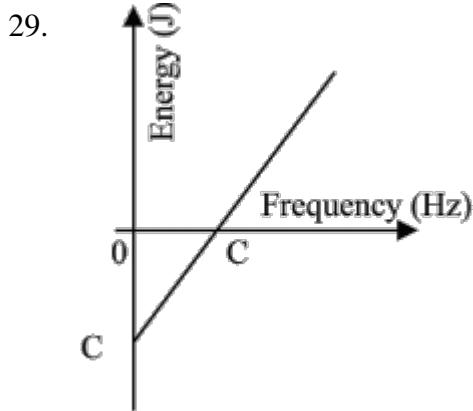
27. (a).  
 (i) Define *work function* and *threshold frequency*.  
 (ii) List three applications of photo electricity.  
 (b) The following observation was made in an experiment on photoelectricity; a green light produces electrons with greater kinetic energy than bright red light. Explain the observation.

WASSCE

28. Light of wavelength  $5.00 \text{ \AA} = 10^{-7} \text{ m}$  is incident on a material of work function 1.90 eV. Calculate the:  
 (i) photon energy;  
 (ii) kinetic energy of the most energetic photoelectron.  
 (iii) stopping potential.

$[h = 6.6 \text{ \AA} = 10^{-34} \text{ J s}; 1 \text{ eV} = 1.6 \text{ \AA} = 10^{-19} \text{ J and } c = 3.0 \text{ \AA} = 10^8 \text{ m s}^{-1}]$

WASSCE



The diagram above represents the graph of the electron energy against the frequency of radiation incident on a metal surface. Interpret the

- (i) slope of the graph;  
 (ii) intercept,  $0C$ ;  
 (iii) intercept,  $0K$ .

WASSCE

30. (a) When a certain photoelectric surface was illuminated with light of different wavelengths, the following stopping potentials were observed;

Wavelength ( $\lambda$ )/ $10^{-10}$ m	3660	4050
Stopping potential ( $V_s$ )	1.48	1.15
4360	4920	5460
0.93	0.62	0.36
		5900
		0.24

Plot a graph of the stopping potential on the vertical axis against the frequency on the horizontal axis.

- (b) From the graph in (a), determine the:

- (i) threshold frequency;
- (ii) work function of the material;
- (iii) value of the Planck's constant.

- (c) (i) Write Einstein's photoelectric equation.

- (ii) identify each term of the equation. [1 eV =  $1.6 \times 10^{-19}$  J;  $c = 3.0 \times 10^8$  m s<sup>-1</sup>.]

WASSCE

31. (a) (i) Explain photoelectric emission.

- (ii) State **four** applications of photoelectric emission.

- (b) Draw and label a diagram showing the structure of a simple type of a photocell and explain its mode of operation.

- (c) In a photocell, no electrons are emitted until a threshold frequency of light is reached.

- (i) Explain what happens to the energy of the light before the emission of electrons begins.

- (ii) State one factor that may affect the number of emitted electrons.

WASSCE

32. (a) (i) By means of labelled diagram, describe the mode of operation of a modern x-ray tube.

- (ii) State the energy transformation which takes place during the operation.

- (b) Explain the terms hardness and intensity as applied to X-rays.

- (c) (i) State **three** uses of X-rays.

- (ii) State **one** hazard of over-exposure to x-rays in a radiological laboratory, indicating two safety precautions.

WAEC

33. (a) (i) Draw a labelled diagram of a modern x-ray tube.

- (ii) State the energy transformation which takes place during its operation.
- (b) Outline any **three** evidences for believing that X-rays are electromagnetic radiation.
- (c) State:(i) **two** uses of x-rays other than in medicine;
- (ii) **two** safety precautions against radiation hazards of X-rays.
- (d) Calculate the minimum wavelength of an x-ray when a voltage of 60 KV is applied to an X-ray tube.  
[ $h = 6.6 \text{ Å} - 10^{-34} \text{ J s}$ ;  $1 \text{ eV} = 1.6 \text{ Å} - 10^{-19} \text{ J}$  and  $c = 3.0 \text{ Å} - 10^8 \text{ m s}^{-1}$ .]

NECO

34. An electron of mass  $9.1 \text{ Å} - 10^{-31} \text{ kg}$  moves with a velocity of  $4.2 \text{ Å} - 10^7 \text{ m s}^{-1}$  between the cathode and anode of an X-ray tube. Calculate the wavelength.  
[ $h = 6.6 \text{ Å} - 10^{-34} \text{ J s}$ ]

WASSCE

35. (a) (i) Describe briefly the production of X-rays.
- (ii) State the energy conservations that take place in an X-ray tube during the production of x-rays.
- (b) State how the intensity and hardness of X-rays produced can be increased.
- (c) The potential difference across an x-ray tube is  $1.5 \text{ Å} - 10^3 \text{ V}$  while the current is  $1.00 \text{ Å} - 10^{-3} \text{ A}$ . Calculate the:
- (i) number of electrons crossing the tube per second;
- (ii) maximum kinetic energy gained by the electron traversing the tube;
- (iii) wavelength of the X-rays produced, if only 0.75% of the kinetic energy of the electrons is converted into X-rays.

(Electronic charge,  $e = 1.602 \text{ Å} - 10^{-19} \text{ C}$ ; Planck's constant,  $h = 6.63 \text{ Å} - 10^{-34} \text{ J s}$ ; Speed of light,  $c = 3.00 \text{ Å} - 10^8 \text{ m s}^{-1}$ )

WASSCE