CHAPTER 5

At the end of the chapter, students should be able to:
1. Define sets.
2. Use set notations.
3. Identify types of sets.
4. Carry out operations on sets.
5. Draw, interpret and use Venn diagrams.
6. Apply the use of Venn diagrams in solving real life problems.

I. Definition of Sets

A set is a collection of clear and well defined objects. The objects can be living things or non-living things such as a boy, fish, dog, table, school, lorry etc. It is not compulsory that the objects in a set must be from the same family, for example, the set consisting of a hen, pencil and stone.
In a classroom, there can be a collection of tables of various sizes, students of different sexes, heights, sizes etc. Each collection of the same item is a set. In some cases, the members of a set may share common properties. Example is the set of triangles such as isosceles, right-angled, equilateral etc. Their common property is that they are polygons with three sides and three angles. A set is denoted by upper case alphabets such as *A, B, C, D,*....

(i) Elements of a set

A member of a set is called an *element* of the set. The elements of a set are always written inside a curly bracket {...} and separated with commas. For example, if *D* is the set that contains the numbers on the faces of a die, *D* = {1, 2, 3, 4, 5, 6}. It means that 1 or 2 or 3 or 4 or 5 or 6 is a member of set *D.* Symbolically, 1 is a member (element) of *D* can be written as $1 \in D$. If on the other hand, 7 is not an element of *D,* we write $7 \notin D$. Where $\in$ means 'member of' and $\notin$ means 'not a member of'.

Worked Example 1

If A = {s, c , h , o , l , a , r}, write down the elements of the following sets.
(a) *B* is a set of vowels in set *A.*
(b) *C* is a set of consonants in set *A.*
(c) *D* is a set of letters in the word shoe in set *A.*

Solution

(a) $B = \{o, a\}$
(b) $C = \{s, c , h , l , r\}$
(c) $D = \{s, h , o\}$

II. Set Notation

Set notation is a way of representing a set using any of the following:
(a) Listing the elements in the set
(b) Word description
(c) Diagram
(d) Rules or set builder notation

(i) Listing elements

Describing or defining a set by listing the elements of the set is a direct way of writing out all the members of the set. This method is called *Roster method*.

Worked Example 2

Define the following sets by listing their elements.
(a) Set A is the set of the unit odd numbers.
(b) Set B is the set of the first five prime numbers.
(c) Set C is the set of vowels in the alphabets.
(d) Set D is the set of all perfect squares.

Solution

(a) $A = \{1, 3, 5, 7, 9\}$
(b) $B = \{2, 3, 5, 7, 11\}$
(c) $C = \{a, e , i , o , u\}$
(d) $D = \{1, 4, 9, 16, 25, 36, \ldots\}$

(ii) Using word description

Words can be used to describe a set. For instance, T = {Tuesday, Thursday} can be described using words as T = {days of the week that begin with letter T}. $B = \{2, 3,5, 7\}$ can be described using words as B = {unit prime numbers}.

(iii) Using diagrams

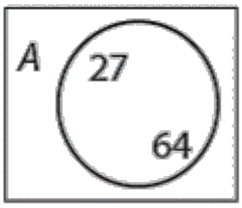
The elements of a set can be enclosed in a circle while the circle itself is the set. The idea of using a diagram to describe a set was introduced by an English Mathematician called Venn Euler. These diagrams were named after him – Venn diagrams.

Worked Example 3

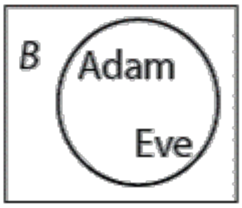
Use Venn diagrams to describe the following:
(a) 2 – digit perfect cubes.
(b) First couple in the garden of Eden.
(c) Arithmetic operational symbols.

Solution

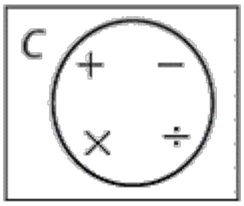
(a) A = {2 – digit perfect cubes}
= {27, 64}



(b) B = {First couple in the Garden of Eden}
= {Adam, Eve}



(c) C = {Arithmetic operational symbols}
= {+, −, ×, ÷}



(iv) Using rules or set builder notation

Sets can be described using set builder notation, for example, T = {x: x is a unit prime factor of 120} The above set is made up of a variable x, a colon (:) and a statement enclosed in curly brackets. The elements of set T stated in the set builder notation are 2, 3 and 5.

Worked Example 4

If M is the set {1, 2, 3,…, 20}, list the elements of the following sets:
A = {multiples of 5 in set M}
B = {multiples of 4 in set M}
C = {perfect squares in set M}
D = {common factors of 20 and 30 in set M}
E = {odd numbers in set M}

Solution

(a) A = {5, 10, 15, 20}
(b) B = {4, 8, 12, 16, 20}
(c) C = {1, 4, 9, 16}
(d) D = {1, 2, 5, 10}

(e) $E = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

Worked Example 5

Given that X, Y and Z are defined as follows:
$X = \{p: 2 \leq p \leq 9, p \text{ is a prime number}\}$
$Y = \{n: 1 \leq n \leq 7, n \text{ is an integer}\}$
$Z = \{m: 2 \leq m \leq 10, m \text{ is a multiple of } 2\}$
List the elements of:
(a) X
(b) Y
(c) Z

Solution

(a) $X = \{2, 3, 5, 7\}$
(b) $Y = \{1, 2, 3, 4, 5, 6, 7\}$
(c) $Z = \{2, 4, 6, 8, 10\}$

Exercise 1

Put $\in$ (member of), $\notin$ (not member of), = (equal to) or $\neq$ (not equal to) in place of the boxes below.

1. 3 $\square$ [factors of 8]
2. 18 $\square$ [2 – digit prime number]
3. 18 $\square$ [2 – digit multiples of 6]
4. 4 $\square$ [highest common factor of 60 and 36]
5. 4 $\square$ [number of odd factors of 60]
6. 5 $\square$ [perfect squares]
7. -2 $\square$ [square roots of 4]
8. Protractor $\square$ [musical instruments]
9. Olusegun Obasanjo $\square$ [Governors of Nigeria]
10. Yam $\square$ [protein giving foods]

List all the elements of the following sets:

11. $A = \{a: 1 \leq a < 10, a \in Z\}$
12. $B = \{b: 1 < b < 50, b$ is a factor of $50\}$
13. $C = \{c: c > 10, c$ is a factor of $50\}$
14. $D = \{d: d < 100, d$ is a perfect square, $d \in N\}$
15. $E = \{e: 5 < e < 10, e$ is a multiple of $5\}$

16. $F = \{f: f < 27, f \text{ is a perfect cube}\}$
17. $G = \{g: \sqrt{g + 8} = 4, g \in N\}$
18. $H = \{h: h \in 2-\text{digit perfect cubes}\}$
19. $J = \{j: j \in 1-\text{digit numbers}\}$
20. $K = \{k: -4 \leq k \leq 5, k \text{ is a square root of } 4\}$
21. The set of numbers with a factor.
22. The set of prime factors of 90.
23. The set of factors of 60.
24. The set containing the common multiples of 3 and 4.
25. The set of quadrilaterals whose sides are equal.

III. Cardinality of a Set

Cardinality of a set is otherwise known as the number of elements in a set. If A = {months in a year} and $n(A) = 12$, $n(A)$ means the number of elements in set A since the number of months in a year is 12.
From the solution to Example 1, $n(B) = 2$, $n(C) = 5$ and $n(D) = 3$. If M = {m, a , t , h , e , m , a , t , i , c , s}, $n(M) = 11$

Worked Example 6

Consider μ = {prime numbers between 1 and 20}. If P = {even numbers} and Q = {factors of 70}.
(a) List all the elements of μ.
(b) List the members of P.
(c) List the members of Q.
(d) What is $n(P)$?
(e) What is $n(Q)$?

Solution

(a) $\mu = \{2, 3, 5, 7, 11, 13, 17, 19\}$
(b) $P = \{2\}$
(c) $Q = \{2, 5, 7\}$
(d) $n(P) = 1$
(e) $n(Q) = 3$

Exercise 2

Find the number of elements in the following sets:

1. A = {2 – digit prime numbers not more than 50}
2. B = {prime factors of 30}
3. C = {1 – digit numbers which are both even and prime}
4. D = {prime factors of 90}
5. E = {factors of 100 that are multiples of 4}
6. F = {multiples of 3 and 4 not more than 50}
7. $G = \{2, 4, 6, \ldots. 50\}$
8. $H = \{2, 3, 4, 3, 5\}$
9. I = {days of the last three months in a year}
10. J = {e, l, e, p, h, a, n, t}
11. K = {p, r, o, t, r, a, c, t, o, r}
12. L = {even numbers not more than 24}

13. M = {quadrilaterals whose sides are equal in length}

14. N = {2 – digit numbers not more than 40}

15. O = {polygons with at least two lines of symmetry}

16. P = {letters of the alphabet that are used as Roman Numerals}

17. Q = {letters of the alphabet that are consonants}

Answer questions 18–20 below using the results of questions 1–17.

18. $n(A) + n(Q) + n(C)$

19. $n(P) + n(L) + n(K)$

20. $n(B) + n(F) + n(M)$

IV. Types of Sets

(i) Empty set

Empty set can also be called a *null set*. It is a set that contains no element. The number of elements in this type of set is zero. Empty set is denoted by Ø or { }.

Examples of empty sets are:

(a) The set of female students without heads.

(b) The set of natural numbers that are less than zero.

(ii) Singleton set

Singleton set is a set that consists of only one element. It is otherwise known as a *unit set*. For instance, Y = {numbers with only one factor}, that is, $Y = \{1\}$ is a singleton set because it consists of only one element.

(iii) Finite and infinite sets

A finite set is a set that contains definite or specific or countable number of elements. Consider the sets $A = \{1, 2, 3\}$, $B = \{1, 1, 2, 2, 2, 3, 3, 3, 3\}$ and $C = \{2, 1, 3\}$.

Here, the three sets contain three elements 1, 2 and 3. Hence, we say that $A = B = C$. From this, we can see that (i) the number of times an element occurs in a set is not important and (ii) the order of arranging the elements is also not important.

An infinite set is a set that has indefinite or uncountable number of elements. It is a set whose number of elements is unknown. Finite set is the direct opposite of infinite set.

For instance, if $N = \{1, 2, 3, \ldots\}$, then N is an infinite set. Other examples of infinite sets are the set of even numbers, the set of odd numbers, the set of prime numbers etc.

Worked Example 7

Given that $A = \{x: x > 10, x \in N\} = \{11, 12, 13, \ldots\}$ and $B = \{x: x < 10, x \in N\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Which of the two sets is an example of (a) finite set and (b) infinite set?

Solution

(a) Set B is a finite set, because it has finite number of elements.
(b) Set A is an infinite set, because the number of elements in it is unknown.

(iv) Subset of a set

If every element of set $B = \{a, e, i, o, u\}$ is also an element of set $A = \{a, b, c, \ldots, z\}$. Then, set B is a subset of set A. Subset is denoted by $\subset$. Hence, set B is a subset of A which is denoted by $B \subset A$. Notice that every set is a subset of itself. From above, set B is a subset of set B and set A is a subset of set A, which can be written symbolically as $B \subset B$ and $A \subset A$. Also, an empty set is a subset of every set. This is denoted by $\varnothing \subset A$ from above and $\varnothing \subset B$.

(v) Universal set

Consider a set as the set of natural numbers and another as the set of even numbers. For instance: $P = \{1, 2, 3, 4, 5, 6, 7,\ldots\}$ and $Q = \{2, 4, 6, \ldots\}$ Since all the members of set Q are present in set P, set P is the *universal set*. Hence, a universal set is a set that contains all the elements that are to be used in a set theory problem. Universal set is denoted by μ.

(vi) Equal Sets

Two sets are equal, if their elements are the same and of equal numbers. The order of arranging the elements in each set does not matter for the equality of the sets. Examples of equal sets are the number of fingers on the right hand of a normal human being which is equal to the fingers on his left hand.
If $P = \{f, i, n, g, e, r\}$ and $Q = \{i, e, f, g, r, n\}$ then $P = Q$.
The symbol for equal sets is the equality sign '='
Remark: Consider the sets $A = \{1, 2, 3\}$, $B = \{1, 1, 2, 2, 2, 3, 3, 3, 3\}$ and $C = \{2, 1, 3\}$
Here, the three sets contain three elements 1, 2 and 3; hence, we say that $A = B = C$. From this, we can see that:
1. The number of times an element occurs in a set is not important.
2. The order of arranging in a set is also not important.

Worked Example 8

Given that $A = \{x: x > 10, x \in N\} = \{11, 12, 13, \ldots\}$ and $B = \{x: x < 10, x \in N\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Which of A or B is a finite set?

Solution

Set B is a finite set because it has specific number of elements. Set A is not a finite set because the number of elements it has is not known.

Exercise 3

Name the type of set in each of the following sets:

1. $A = \{a: -1 < a < 1, a \in N\}$
2. $B = \{b: b > 4, b \in Z\}$
3. $C = \{c: c$ is a factor of 36 and 24$\}$
4. $D = \{d: d$ is a multiple of 3 and 4$\}$
5. $E = \{e: e > 6, e$ is a 2-digit prime number$\}$
6. $F = \{f: 10 \leq f < 100, f \in Z\}$
7. $G = \{g: g^2 = 81\}$
8. $H = \{h: h - 1 = 1 - h, h$ is a prime number$\}$
9. $I = \{i: i + 4 = 4\}$
10. $J = \{j: j$ is a multiple of 5 and 10$\}$
11. $K = \{k: k \leq 15, k \in N\}$
12. $L = \{l: 0 \leq l < 100, l$ is a factor of 11 and also a multiple of 3$\}$
13. $M = \{m: m$ is a 1-digit prime number that is also an odd number$\}$
14. $N = \{n: 5 < n < 10, n$ is the number of fingers on the hands of a human being$\}$
15. $P = \{p: p < 0, p \in N\}$

V. Operations on Sets

(i) Union of sets

The union of two sets A and B is the set of all the elements (numbers) that belong to either A or B or both. Union of sets is symbolically written as $\cup$. Then A union B is written as:

$A \cup B = \{x: x \in A \text{ or } x \in B \text{ or both}\}$

Worked Example 9

If A = {1-digit perfect square less than 20} and B = {1-digit perfect cube less than
20}. Find $A \cup B$.

Solution

A = {1-digit perfect square less than 20}
= {1, 4, 9}
B = {1-digit perfect cube less than 20}
= {1, 8}
$A \cup B = \{1, 4, 8, 9\}$

Worked Example 10

Given: P = { days of the week that begin with letter S}
Q = { days of the week that begin with letter T}
R = { days of the week that has two vowels}
Find $P \cup Q \cup R$.

Solution

P = { days of the week that begin with letter S}
= {Saturday, Sunday}
Q = { days of the week that begin with letter T}
= {Tuesday, Thursday}
R = { days of the week that has two vowels}
= { Monday, Tuesday, Thursday, Friday, Sunday}
$P \cup Q \cup R$ = {Saturday, Sunday, Tuesday, Thursday, Monday, Friday}

(ii) Intersection of sets

The intersection of two sets A and B is the set that contains the list of elements that can be found both in A and B. Intersection of sets is symbolically written as $\cap$. A intersection B is written as $A \cap B$. Thus, $A \cap B = \{x: x \in A \text{ and } x \in B\}$.

Worked Example 11

From Example 10:
(a) $P \cap Q = \{\ \}$ or $\varnothing$
(b) $P \cap R$ = {Sunday}
(c) $P \cap R \cap Q = \{\ \}$
(d) $Q \cap R$ = {Tuesday, Thursday}

Worked Example 12

What is $A \cap B$ from Example 9?

Solution

$A \cap B = \{1\}$

Worked Example 13

If $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 4, 9, 16, 25, 36, \ldots\}$
Find $A \cap B$.

Solution

$A \cap B = \{1, 9\}$

Worked Example 14

The universal set $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ and the subsets
$X = \{$ prime numbers between 1 and 10$\}$
$Y = \{1, 2, 4, 6, 8, 10, 13, 14\}$
Find $X \cap Y$.

Solution

$X = \{2, 3, 5, 7\}$
$X \cap Y = \{2\}$

(iii) Disjoint sets

Two sets are disjoint, if they have no element in common. The number of elements in the intersection of disjoint sets is zero. Examples of disjoint sets are:
(i) The common elements in the set of even numbers and that of odd numbers.
(ii) The common elements in the set of consonants to the set of vowels in the alphabet.

(iv) Complement of a set

The complement of a given set A are those elements in the universal set that are not elements of A.

For instance, the complement of set K, which is the set of consonants is the set V, which is the set of vowels in the letters of the alphabet. The complement of K is written symbolically as K' or K^c.

Note that $K' = K^c = \{x : x \in \mu \text{ and } x \notin k\}$

Worked Example 15

Given that: $\mu = \{-1, 0, 1, 2, 3, 4, 5, 6\}$ where
$P = \{\text{prime numbers}\}$ and
$Q = \{\text{prime factors of } 6\}$ are subsets of μ, find $(P \cap Q)'$.

Solution

$\mu = \{-1, 0, 1, 2, 3, 4, 5, 6\}$
$P = \{\text{prime numbers}\} = \{2, 3, 5\}$
$Q = \{\text{prime factors of } 6\} = \{2, 3\}$
$(P \cap Q) = \{2, 3\}$
$(P \cap Q)' = \{-1, 0, 1, 4, 5, 6\}$

Worked Example 16

If $\mu = \{1, 2, 3, \ldots, 10\}$
$A = \{2, 3, 4, 6, 9, 10\}$
$B = \{1, 3, 5, 7, 9\}$
$C = \{4, 5, 6, 7, 8, 9\}$
Find $A' \cap B \cap C'$

Solution

$A' = \{1, 5, 7, 8\}$
$C' = \{1, 2, 3, 10\}$
$A' \cap C' = \{1\}$
$A' \cap B \cap C' = \{1\}$

Worked Example 17

If $\mu = \{k, s, a, p, m, e, c\}$
$P = \{s, p, m, e\}$
$Q = \{k, a, p, c\}$
$R = \{a, p, m, c\}$

Find $Q' \cup (P \cap R)$ **(NECO)**

Solution

$$Q' = \{s, m, e\}$$
$$P \cap R = \{p, m\}$$
$$Q' \cup (P \cap R) = \{s, m, e, p\}$$

Exercise 4

1. If $X = \{\text{all prime factors of } 44\}$ and $Y = \{\text{all prime factors of } 60\}$, find
   (a) $X \cup Y$
   (b) $(X \cup Y) \cap X$
   (c) $(X \cap Y) \cup Y$
2. If $\mu = \{0, 2, 3, 6, 7, 8, 9, 10\}$ is the universal set, $E = \{0, 4, 6, 8, 10\}$ and $F = \{x: x^2 = 2^6, x \text{ is odd}\}$, find $(E \cap F)'$ where $'$ means the complement of a set. **(UME)**
3. If $P = \{x: 2 \leq x \leq 16, x \text{ is prime}\}$ and $Q = \{x: 2 \leq x \leq 13, x \text{ is odd}\}$, evaluate $n(P \cup Q) + n(P \cap Q)$. **(WAEC)**
4. If $P = \{3, 5, 6\}$ and $Q = \{1, 2, 4\}$, find $P \cap Q$.
5. If $\mu = \{y: y \text{ is an integer}, 2 < y < 10\}$, $A = \{y: y \text{ is a prime number}\}$ and $B = \{y: y \text{ is a multiple of } 3\}$,
   (a) Find $A \cup B$.
   (b) List the elements of the set $A' \cap B'$. **(NECO)**
6. If $\mu = \{1, 2, 3, 4, 5, 6\}$, $P = \{3, 4, 5\}$, $Q = \{2, 4, 6\}$ and $R = \{1, 2, 3, 4\}$. List the elements of $(P \cup Q)' \cap R$. **(UME)**

7. Given that the universal set $\mu = \{1, 2, 3, 4, 5, 6\}$ and the sets $P = \{1, 2, 3, 4\}$, $Q = \{3, 4, 5\}$ and $R = \{2, 4, 6\}$. Find $P \cup (Q \cup R)'$. **(UME)**

8. Given that P and Q are subsets of the universal set $\mu = \{-4, -3, -2, -1, 0, 1, 2, 3\}$ where $P = \{-2, -1, 1, 2\}$ and $Q = \{x: x \text{ is an integer}, x^2 < 4\}$. Find $P' \cap Q'$. **(WAEC)**

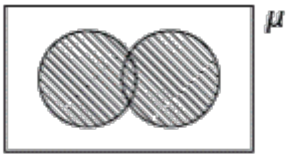
9. If Q = {all perfect squares less than 30} and P = {all odd numbers from 1 to 10}. Find $Q \cap P$. **(WAEC)**

10. If μ = {integers < 20}, P = {multiples of 3} and Q = {multiples of 4}, what are the elements of $P' \cap Q$?

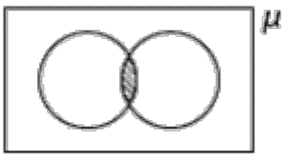
VI. Venn Diagrams

The idea of representing sets in a closed figure diagrammatically was introduced by Venn Euler. Venn diagram is used to represent operations on the union of sets, intersection of sets and complement of sets. The diagrams below show various sets operations.

Venn diagrams involving 2 sets

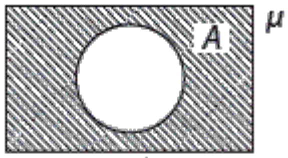
1.


AUB

2.


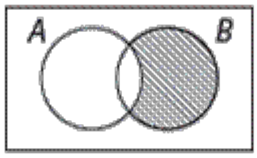
A∩B

3.



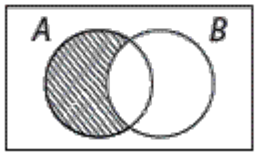
A'

4.



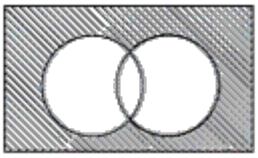
$A' \cap B$

5.



$A \cap B'$

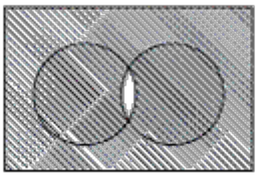
6.



(All shaded parts)

$(A \cup B)'$

7.

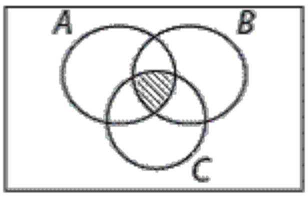


(All shaded parts)

$(A \cap B)'$

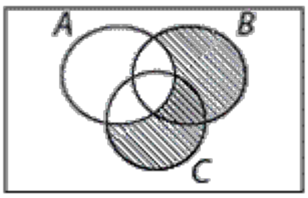
Venn diagrams involving 3 sets

1.



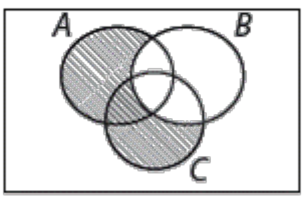
$A \cap B \cap C$

2.



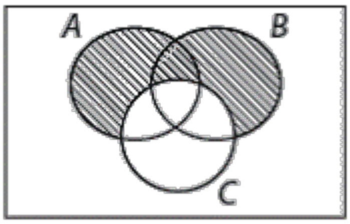
$A' \cap B \cap C$

3.



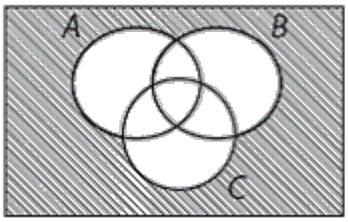
$A \cap B' \cap C$

4.



$A \cap B \cap C'$

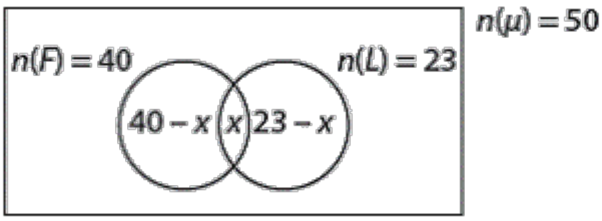
5.



$A' \cap B' \cap C'$

Worked Example 18

In a class of 50 students, 40 can play football and 23 lawn tennis. If all the students can play at least one of the games, how many students play both games?



Solution

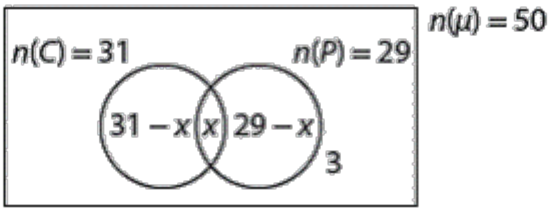
Let x be the number of students that play both games.

$$n(F \cap L') = 40 - x$$
$$n(F \cap L) = x$$
$$n(F' \cap L) = 23 - x$$
$$40 - x + x + 23 - x = 50$$
$$63 - x = 50$$
$$63 - 50 = x$$
$$13 = x$$
$$x = 13$$

∴ The number of students that play both games is 13.

Worked Example 19

In an examination, 31 candidates passed Chemistry, 29 passed Physics and 3 failed both subjects. If 50 candidates sat for the examination, how many of them passed Chemistry only?



Solution

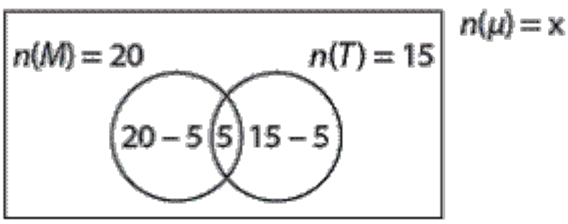
Let x be the number that passed Chemistry and Physics

$$
\begin{aligned}
n(C \cap P') &= 31 - x \\
n(C \cap P) &= x \\
n(C' \cap P) &= 29 - x \\
n(C' \cap P') &= 3 \\
n(\mu) &= 50 \\
31 - x + x + 29 - x + 3 &= 50 \\
63 - x &= 50 \\
63 - 50 &= x \\
13 &= x \\
x &= 13 \\
n(C \cap P') &= 31 - x \\
&= 31 - 13 \\
&= 18
\end{aligned}
$$

Worked Example 20

Every staff in an office owns either a Mercedes and/or Toyota car, 20 own Mercedes, 15 own Toyota and 5 own both. How many staff are there in the office?

Solution



Let x be the total number of staff in the office.

$$\begin{aligned} n(M) &= 20 \\ n(T) &= 15 \\ n(M \cap T') &= 20 - 5 = 15 \\ n(M \cap T) &= 5 \\ n(M' \cap T) &= 15 - 5 = 10 \\ x &= 20 - 5 + 5 + 15 - 5 \\ &= 15 + 5 + 10 \\ &= 30 \end{aligned}$$

The total number of staff in the office is 30.

Worked Example 21

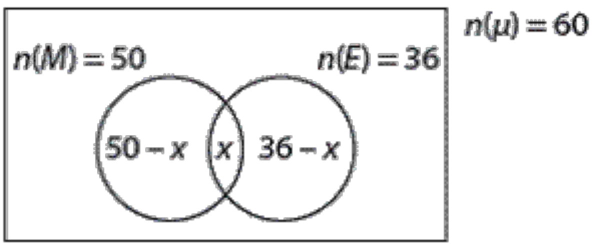
$\frac{5}{6}$ of a class of 60 students study Mathematics and $\frac{3}{5}$ study Economics. Every student studies at least one of these subjects. How many students study both subjects?

Solution

$$\frac{5}{6} = \frac{5 \times 10}{6 \times 10} = \frac{50}{60}$$

$$\frac{3}{5} = \frac{3 \times 12}{5 \times 12} = \frac{36}{60}$$

Let M stand for the number of students that study Mathematics and E for students that study Economics.



Let x stand for the number of students that study both subjects.

$$\begin{aligned} 50 - x + x + 36 - x &= 60 \\ 86 - x &= 60 \\ 86 - 60 &= x \\ 26 &= x \\ x &= 26 \end{aligned}$$

26 students study both subjects.

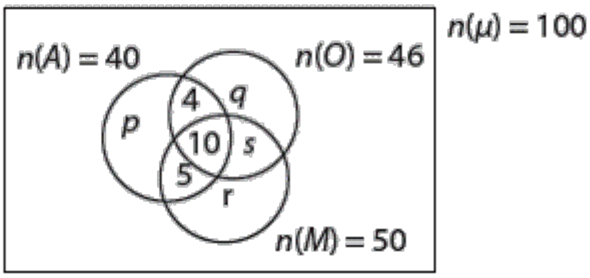
Worked Example 22

In a market survey, 100 traders sell fruits, 40 sell apples, 46 oranges, 50 mangoes, 14 apples and oranges, 15 apples and mangoes and 10 sell the three fruits.
Each of the 100 traders sells at least one of the three fruits.
(i) Represent the information in a Venn diagram.
(ii) Find the number that sell oranges and mangoes only.

Solution

Solution

Let p represent the number that sell apple only, q represent the number that sell oranges only, r represent the number that sell mangoes only, and s represent the number that sell oranges and mangoes only.



$$p + 5 + 10 + 4 = 40$$
$$p + 19 = 40$$
$$p = 40 - 19 = 21$$
$$q + s + 10 + 4 = 46$$
$$q + s + 14 = 46$$
$$q = 46 - s - 14 = 32 - s$$
$$r + s + 10 + 5 = 50$$
$$r + s + 15 = 50$$
$$r = 50 - s - 15 = 35 - s$$
$$p + q + r + 4 + 5 + s + 10 = 100$$
$$21 + 32 - s + 35 - s + 19 + s = 100$$
$$107 - s = 100$$
$$107 - 100 = s$$
$$s = 7$$

The number that sell oranges and mangoes is 7.

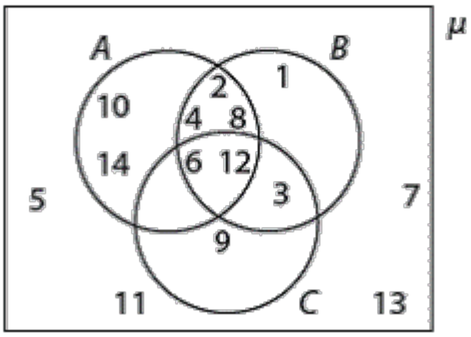
Worked Example 23

Worked Example 23

The sets A, B and C are subsets of the universal set $\mu = \{1, 2, 3, \dots, 14\}$
$A = \{\text{even numbers}\}$, $B = \{\text{factors of } 24\}$ and $C = \{3, 6, 9, 12\}$
(a) List the elements of A and B.
(b) Illustrate the information on a Venn diagram.
(c) Using your Venn diagram, determine
$A' \cap C'$.

Solution

$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$
$A = \{\text{even numbers}\} = \{2, 4, 6, 8, 10, 12, 14\}$
$B = \{\text{factors of } 24\} = \{1, 2, 3, 4, 6, 8, 12$
$C = \{3, 6, 9, 12\}$
$A \cap B = \{2, 4, 6, 8, 12\}$
$B \cap C = \{3, 6, 12\}$
$A \cap C = \{6, 12\}$
$A \cap B \cap C = \{6, 12\}$
$A' = \{1, 3, 5, 7, 9, 11, 13\}$
$B' = \{5, 7, 9, 10, 11, 13, 14\}$
$C' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$
$A' \cap B' \cap C = \{9\}$
$A' \cap B' = \{5, 7, 9, 11, 13\}$
$B' \cap C' = \{5, 7, 10, 11, 13, 14\}$
$A' \cap C' = \{1, 5, 7, 11, 13\}$
$A' \cap B' \cap C' = \{5, 7, 11, 13\}$
$A \cap B' \cap C' = \{10, 14\}$
$A' \cap B \cap C' = \{1\}$
$A \cap B \cap C' = \{2, 4, 8\}$
$A \cap B' \cap C = \{\ \}$
$A' \cap B \cap C = \{3\}$



(c) $A' \cap C' = \{1, 5, 7, 11, 13\}$

Exercise 5

1. A, B and C are subsets of the universal set μ such that:
   $\mu = \{1, 2, 3, \ldots, 20\}$
   $A = \{x: x \text{ is even}\}$
   $B = \{y: y \text{ is odd}\}$
   $C = \{z: z \text{ is prime}\}$

(a) Draw a Venn diagram to illustrate the information given above.

(b) Find

(a) $A \cap B$

(b) $A \cup B' \cap C$

(c) $B \cup C$ **(NECO)**

2. If $\mu = \{y: y \text{ is an integer}, 2 \leq y < 10\}$, $A = \{y: y \text{ is a prime number}\}$ and $B = \{y: y \text{ is a multiple of } 3\}$

(a) Find $A \cup B$.

(b) List the elements of the set $A' \cap B'$. **(NECO)**

3. In a certain examination, 52 candidates offered Biology, 60 History, 96 Mathematics, 21 offered both Biology and History, 22 Mathematics and Biology and 16 Mathematics and History. If 7 candidates offered all three subjects,

(a) How many candidates were there for the examination?

(b) How many candidates offered one subject only?

(c) How many candidates offered two subjects only?

(d) How many candidates offered at least two subjects? **(NECO)**

4. In a quiz competition of 60 competitors, 24 drank Coca-cola, 25 drank Fanta, while 22 drank Maltina, 6 drank Coca Cola and Maltina, 6 drank Coca-cola and Fanta and 5 drank Fanta and Maltina. If a

number of competitors drank the three types of soft drinks and 5 did not drink any of them.

(a) Draw a Venn diagram to illustrate this piece of information.

(b) How many drank

(i) the three types of soft drinks?

(ii) Coca-cola only?

(iii) Fanta only?

(iv) Maltina only? **(NECO)**

5. In a class of 64 students, each student offers either Physics or Mathematics or both. If 50 students offer Mathematics and the number of students offering Mathematics only is twice the number of students offering Physics only, how many students offer both subjects?

(NECO)

6. In a certain school, 45 students are members of at least one of Mathematics, Science and Debating clubs; 20 belong to Mathematics, 24 Science and 20 Debating, 10 belong to Mathematics and Science, 9 belong to Science and Debating, while none belong to Mathematics and Debating clubs.

(a) Represent the above information using a Venn diagram.

(b) Use the Venn diagram to find the number of students that belong to:

(a) Science only

(b) Only one club **(NECO)**

7. A survey of 100 students in an institution shows that 80 students speak Hausa and 20 speak Igbo, while only 9 students speak both languages. How many students speak neither Hausa nor Igbo? **(UME)**

8. A group of market women sell at least one of yam, plantain and maize. 12 of them sell maize, 10 sell yam and 14 sell plantain. 5 sell plantain and maize, 4 sell yam and maize, 2 sell yam and plantain only, while 3 sell all the three items. How many women are there in the group?

9. P and Q are 2 sets such that $n(P) = 17$, $n(Q) = 14$ and $n(P \cap Q) = 5$ where $n(X)$ denotes the number of elements in set X. Find $n(P \cup Q)$.

10. A school has 16 boys who serve in the Debating, Science and Mathematics clubs. Of these, 5 serve in the Debating Club only, 4 in the Mathematics Club only, 2 in both Debating and Science clubs only but nobody serves in all the three clubs. All the clubs have equal number of members.

    (a) Represent the information on a Venn diagram.

    (b) Find the number of boys that serve in each club. **(WAEC)**

11. In a class of 45 students, 32 offered Physics (P), 28 offered Government (G) and 12 did not offer any of the two subjects.

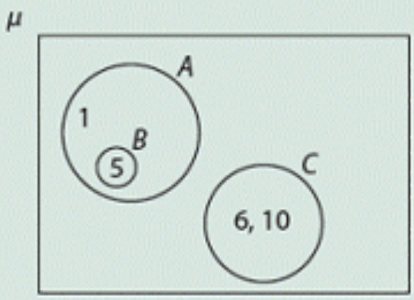
    (a) Draw a Venn diagram to represent the information.

    (b) How many students offered both Physics and Government?

    (c) What is $n(P \cup G)$? **(WAEC)**

12. $\mu = \{1, 2, 3, \ldots, 10\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3, 5\}$ and $C = \{6, 8, 10\}$

    (a) Given that the Venn diagram represents the sets above, copy and fill in the elements.



    (b) Find $A \cap C$.

    (c) Find $A \cap B'$.

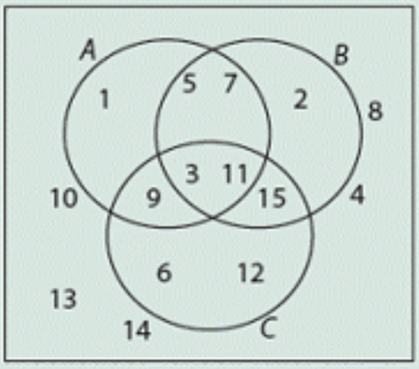
13. Given that X, Y and Z are subsets of the universal set
    $\mu = \{1, 2, 3, 4, \ldots, 10\}$ such that
    $X = \{p: 2 \leq p \leq 9, p \text{ is a prime number}\}$
    $Y = \{n: 1 \leq n \leq 7, n \text{ is an integer}\}$
    $Z = \{m: 2 \leq m \leq 10, m \text{ is a multiple of } 2\}$

    (a) List the elements of:
    (i) X (ii) Y (iii) Z

(b) (i) Draw the Venn diagram to illustrate the information in (a).

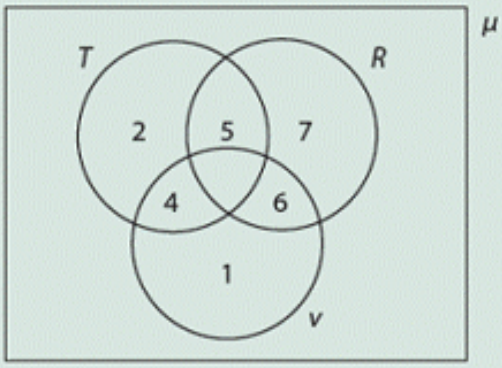
(ii) Find $X' \cap Z' \cap Y$.

14. The sets $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{2, 3, 5, 7, 11, 15\}$ and $C = \{3, 6, 9, 12, 15\}$ are subsets of $\mu = \{1, 2, 3, \ldots, 15\}$



Use your diagram to find: (a) $C \cap A'$ (b) $A' \cap (B \cup C)$.

15.



The Venn diagram shows the number of traders that sell radio (R), television (T) and video (V) in a market.

How many of the traders sell:

(a) Both radio and television?

(b) At least two of the items?

SUMMARY

In this chapter, we have learnt the following:

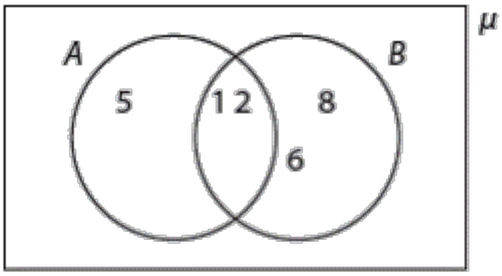
- A set is a collection of clear well-defined objects.
- A member of a set is an element of a set.
- Set notation is a way of representing a set using the listing of elements in a set, word description, diagrams or set builder notation.
- Cardinality of a set simply means the number of elements in a set A which is denoted by $n(A)$.
- The types of sets are empty set (set that contains no element), singleton set (set that contains only one element), finite set (set with countable number of elements), infinite set (set with uncountable number of elements), universal set (set that contains all the elements that are to be used in a given operation) and equal sets (sets with equal elements and equal number of elements).
- Union of two sets X and Y is symbolically represented by $X \cup Y$ and defined as $X \cup Y = \{x: x \in X \text{ or } x \in Y \text{ or } x \in \text{ both } X \text{ and } Y\}$.
- Intersection of two sets X and Y is symbolically represented by $X \cap Y$ and defined as $X \cap Y = \{x: x \in X \text{ and } x \in Y\}$.
- Complement of set M means the set of elements that belong to universal set but not set M. Complement of set M is denoted by M' or M^c and defined as $M^c = \{x: x \in \mu, x \notin M\}$.
- Venn diagram introduced by Venn Euler is simply a way of using pictures or diagrams to represents sets and their relationships.

GRADUATED EXERCISE

1. Given that $A = \{a, u, t, h, o, r, i, t, y\}$, find $n(A)$.
2. What types of sets are:
   (a) B = {odd numbers}
   (b) C = {television that can fly}
   (c) D = {toes on your legs}
   (d) $E = \{e, a, s, t\}$
   (e) $F = \{s, e, a, t\}$
3. State the number of elements in the following sets:
   (a) ϕ
   (b) G = {English alphabets}
   (c) H = {months of the year}
   (d) $J = \{x: 0 < x \leq 20, x \in Z\}$
   (e) $K = \{e, l, e, p, h, a, n, t, i, a, s, i, s\}$
4. Determine the cardinality of the following:
   (a) $L = \{f, e, v, e, r, i, s, h\}$
   (b) $M = \{n, a, t, i, o, n, a, l, i, s, m\}$
   (c) $N = \{\ \}$
   (d) $P = \{(5,6), (7,8), (9,10), (11,12), (13,14)\}$
   (e) $\{i, s, s, u, e\}$
5. If $\mu = \{a, b, c, d, \ldots, z\}$
   A = {vowels},
   $B = \{d, u, t, y\}$ and
   $C = \{f, i, f, t, y\}$
   Find (a) $n(\mu) + n(A) + n(C)$
   (b) $A \cup B$
   (c) $A \cap C$

(d) A'

6.



From the above Venn diagram, find:

(a) $A \cap B$

(b) A'

(c) $A \cap B'$

(d) $A \cup B$

(e) $(A \cap B)'$

7. Represent the following sets in a Venn diagram:

$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{2, 4, 6, 8\}$
$B = \{1, 4, 9\}$
$C = \{1, 2, 4, 8\}$

8. Given $\mu = \{1, 2, 3, \ldots, 10\}$, A = {prime factors of 80}, B = {factors of 9} and C = {multiples of 2 less than 11},

(a) State the number of elements in *A, B* and *C.*

(b) List the members of the following:

(i) $A \cup B \cup C$

(ii) $A \cap B \cap C$

(iii) B'

(iv) $A \cap B' \cap C$

(v) $(A \cup B \cup C)'$

9. Which of the statements below is (are) true?

   (a) $n \in$ {vowels}

   (b) $1 \notin$ {prime numbers}

   (c) Oyo $\in$ {state capital in Nigeria}

   (d) Nail $\in$ {mathematical instrument}

   (e) $1 \in \{x: 1 < x \leq 5\}$

0. If $\mu = \{1, 2, 3, \ldots, 30\}$
   $A =$ {perfect cube}
   $B =$ {perfect square}
   $C =$ {factors of 30}

   (a) List the elements of sets A, B and C.

   (b) Find the number of elements of A, B, C.

   (c) What is $A \cap B \cap C'$?

11. If $y \in \mu$ and $\mu = \{1, 2, 3, \ldots, 20\}$, list the members of the following subsets of μ:

    (a) $\{y: y$ is an even number$\}$

    (b) $\{y: y > 8\}$

    (c) $\{y: y - 4 \leq 6\}$

    (d) $\{y: y + 6 > 12\}$

    (e) $\{y: 4y - 8 \geq 16\}$

12. Given that $A = \{a, b, c, d\}$, $B = \{f, g, b, c\}$ and $C = \{a, b, c, b, d, b\}$,
    then (a) $A = B$
    (b) $n(C) = 6$
    (c) $n(A) = n(C)$
    Which of the above is (are) true?

13. If Z^+ is the set of positive integers and $H = \{x: x \in Z^+, x^2 < 4 \text{ and } x \neq 0\}$

(a) What type of set is (i) Z^+ (ii) H?

(b) State the elements of H.

(c) What is the cardinality of H?

14. If U = {positive numbers less than 30}, P = {multiples of 4}, Q = {multiples of 6},

(a) State the elements of (i) P (ii) Q (iii) $(P')'$.

(b) What is the cardinality of (i) U (ii) P (iii) Q (iv) P'.

(c) Find (i) $P \cup Q$ (ii) $P \cap Q$ (iii) $P \cap Q'$.

15. Given that the universal set $\mu = \{14, 15, 16, 17, 18, 19, 20\}$, $J = \{16, 19, 20\}$ and $K = \{14, 15, 18\}$, find $(J' \cap K')'$.

16. If $\mu = \{k, s, a, p, m, e, c\}$

$P = \{s, p, m, e\}$

$Q = \{k, a, p, c\}$

$R = \{a, p, m, c\}$

Find (a) $Q' \cup (P \cap R)$ (b) $(Q \cap R) \cup P'$ (c) $P \cap Q \cap R'$

17. Let the Universal set μ be the set of integers $\mu = \{x: 0 < x \leq 10\}$. What is the complement of the set $B = \{x: x \in \mu, x \text{ is not divisible by } 3\}$? (NECO)

18. In a school, 220 students offer Biology or Mathematics or both. 125 offer Biology and 110 Mathematics. How many offer Biology but not Mathematics? (WAEC)

19. Out of 400 students in the final year in a Senior Secondary School, 300 are offering Biology and 190 are offering Chemistry.

(a) How many students are offering both Biology and Chemistry, if only 70 students are offering neither Biology nor Chemistry?

(b) How many students are offering at least one of Biology or Chemistry? (WAEC)

20. In a Senior Secondary School class, 22 students offer one or more of Physics, Chemistry and Biology. 12 offer Chemistry, 8 offer Biology and 7 offer Physics. Nobody offers. Chemistry and Physics and 4 students offer Chemistry and Biology.

(a) Draw a Venn diagram to illustrate the information.

(b) Find the number of students that offer

(i) Both physics and Biology?

(ii) Biology only?