5 SIMPLE HARMONIC MOTION

OBJECTIVES

At the end of this topic, students should be able to:

- define simple harmonic motion;
- give some examples of simple harmonic motion;
- state and define some terms used in describing simple harmonic motion;
- show the link between uniform circular motion and simple harmonic motion;
- show the relationship between linear and angular motion.

Simple harmonic motion (shm)

Simple harmonic motion is the motion of a particle whose acceleration is directly proportional to its displacement from the equilibrium position and is always pointing towards the equilibrium position

Any motion which repeats the same pattern of motion at intervals, is a **periodic motion.** A body vibrates or oscillates when it moves periodically (to and fro, up and down or forth and back) about an equilibrium position. A special form of vibratory or oscillatory motion is called **simple harmonic motion** (shm). *A **body performs simple harmonic motion when it repeats the same pattern of to and fro motion at regular interval such that its energy remains constant***.

The following examples illustrate simple harmonic motion.

â€¢ A simple pendulum swinging to and fro about an equilibrium position;

â€¢ A weight moving up and down at one end of a spiral spring;

â€¢ The motion of the prongs of a sounding tuning fork;

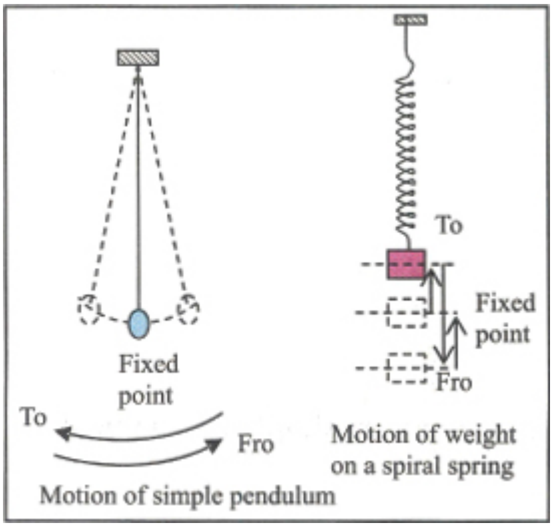
â€¢ The vibration of a wire or string fixed at both ends when it is plucked;

â€¢ The up and down motion of a loaded test tube when it is pressed down a little in a liquid and released;

â€¢ Motion of a marble or ball on a concave surface;

â€¢ Vibrations of the atoms of a solid about their equilibrium position;

â€¢ The forth and back movement of the balance wheel of a watch or clock. Vibration of electric and magnetic waves in electromagnetic waves;



â€¢ The vibration of electrons in a radiating and receiving antennas;
â€¢ The beating of the heart;
â€¢ The pulse of blood pressure flowing in the vein.

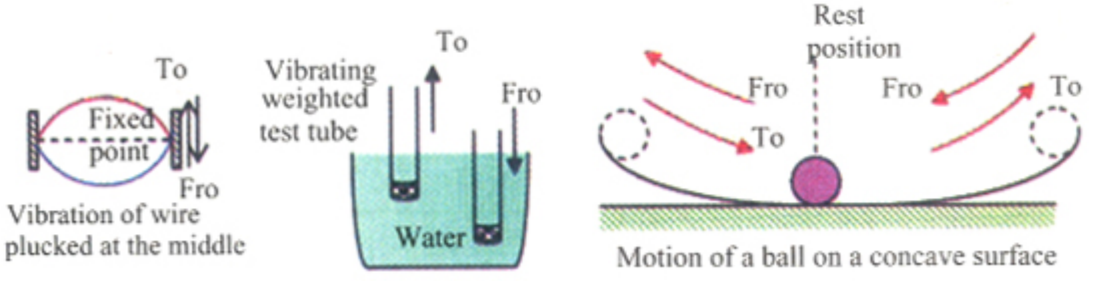


Figure 5.1: Examples of simple harmonic motion

Terms used in describing simple harmonic motion

1. **Cycle or vibration or oscillation:** A cycle or vibration is one complete back and forth movement of a vibrating body.
2. **Period:** Period of oscillation of a vibrating body is the time it takes the body to complete one cycle or vibration. The unit of period is second (s).
3. **Frequency:** Frequency is the number of cycles or vibrations completed in one second. The unit of frequency is hertz (Hz). Frequency of oscillation of a vibrating body is related to the period of oscillation by:

$$Frequency = \frac{1}{period}$$

$$f = \frac{1}{T} \quad or \quad T = \frac{1}{f}$$

4. **Amplitude:** Amplitude is the maximum vertical displacement from the equilibrium or rest position. The unit of amplitude is metre (m).

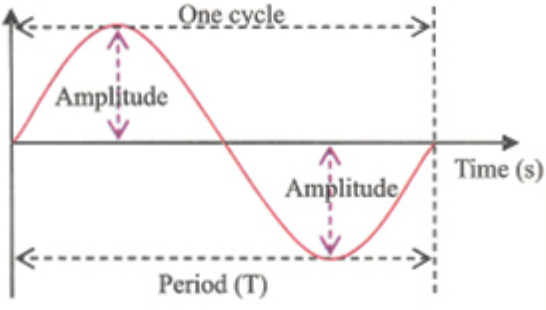


Figure 5.2: Graphical representation of simple harmonic motion

Link between simple harmonic motion and uniform circular motion

Figure 5.3 shows the link between simple harmonic motion and uniform circular motion. The point P moves on a circle of radius (r) with a constant speed (v). The point N is the image or projection of the point P on the y – axis. As the point P moves on the circumference of the circle from W to R with a constant speed (v), its projection (N) on the y – axis moves forth and back (oscillates) along the y – axis on the diameter WR of the circle. The motion of the point P on the circumference of the circle from W to R is **circular motion** while the image or projection of P on the y -axis is **oscillatory motion** (shm).

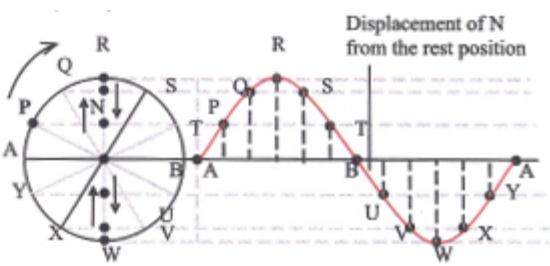


Figure 5.3: Links between simple harmonic motion and uniform circular motion

The period of simple harmonic motion is given by

$$T = \frac{2\pi}{\omega}$$

*2Ï€ = distance covered as the particle moves round the circle once. It is the circumference of the circle measured in radians and **Ï‰** is the angular velocity in rad s^{-1}.*

Angular and linear motion compared

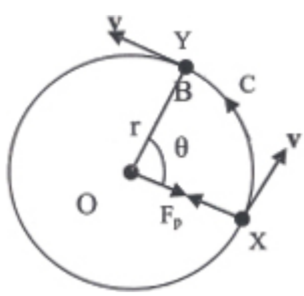


Figure 5.4: Analysis of a moving particle

A particle moving round a circle of radius *(r)* at a constant speed *(v)* covers both linear distance *(s)* and angular distance *(Î¸).* The linear distance *(s)* is covered as the particle moves on the circumference of the circle from X to Y. To move from X to Y, the particle turns through an angle *(Î¸).* The linear distance is related to the angular distance or angle turned by

$$\theta = \frac{s}{t} \quad or \quad s = r\theta$$

Î¸ = angle turned through or angular distance,
s = linear distance or length of arc XY.
The unit of angular distance or angle turned (Î¸) is radian (rad.). Radian as the unit of angle is related to the degree as follows:
2Ï€ radians = 360Â° degrees

$$\pi \text{ radians} = 180^o \text{ degrees}$$

$$\frac{\pi}{2} radians = 90^o \text{ degrees}$$

$$\frac{\pi}{4} radians = 45^o \text{ degrees}$$

$$\frac{\pi}{8} radians = 22.5^o \text{degrees}$$

$$1\ radian = \frac{180}{\pi} = 57.3^o$$

$$1\ degree = \frac{\pi}{180} = 0.0175\ radians$$

Angular velocity

Angular velocity is the angle turned through per second or time rate of change of angular displacement

$$\omega = \frac{\theta}{t} \quad or \quad \theta = \omega t$$

Ï‰ = angular velocity or angle turned per second, Î¸ = angular displacement, t = time. The unit of angular velocity is radian per second (rad s^{-1}). The angular velocity is related to the linear velocity *(v)* by:

$$v = \omega r \quad or \quad \omega = \frac{v}{r}$$

Proof:
s = r Î¸
Divide both sides of the equation above by time (t).

$$\frac{s}{t} = r\frac{\theta}{t} \text{ But } v = \frac{s}{t} \ and\, \omega = \frac{\theta}{t} \therefore v = \omega r$$

Angular acceleration

Angular acceleration is the angle turned through per second squared or time rate of change of angular velocity.

The angular acceleration can be resolved into two perpendicular parts namely centripetal or radial acceleration and tangential acceleration. The part along the radius of the circle pointing towards its centre is called the centripetal or radial acceleration. This component of the acceleration is responsible for simple harmonic motion performed by particles. The other component along the tangent on the circle pointing in the direction of velocity is called the tangential acceleration. The tangential acceleration is given by:*a* = Angular acceleration or angle turned per second per second. The unit of angular acceleration is radian per squared second (rad s^{-2}). The radial or centripetal acceleration is given by

$$\alpha = \frac{\omega}{t} \quad or \quad \omega = \alpha t$$

a = Angular acceleration or angle turned per second per second.

The unit of angular acceleration is radian per squared second (rad S^{-2}).

The radial or centripetal acceleration is given by

$$a = \frac{v^2}{r} \text{ but } v = \omega r \Rightarrow a = \frac{(\omega r)^2}{r} = \omega^2 r$$

$$a = \omega^2 r$$

Linear and angular motions compared

S/N	Linear motion	Angular motion
1	$s = v \times t$	$\theta = \omega \times t$
2	$s = ½ (v + u) t$	$\theta = ½ (\omega + \omega_o) t$
3	$v = u + at$	$\omega = \omega_o + \alpha t$
4	$v^2 = u^2 + 2as$	$\omega^2 = \omega^2_o + 2\alpha\theta$
5	$s = ut + ½ at^2$	$\theta = \omega_o t + ½ \alpha t$
6	$F = ma$	$F = m\omega^2 r$
7	$E_k = ½ mv^2$	$E_k = ½ m(\omega r)^2$

Displacement, velocity and acceleration of a simple harmonic motion

(1) Displacement of simple harmonic motion: The displacement of particle performing simple harmonic motion from its rest position is represented by a sine or cosine curve with constant amplitude. Figure 5.5 is a sine curve representing the displacement of a particle moving in a circle.

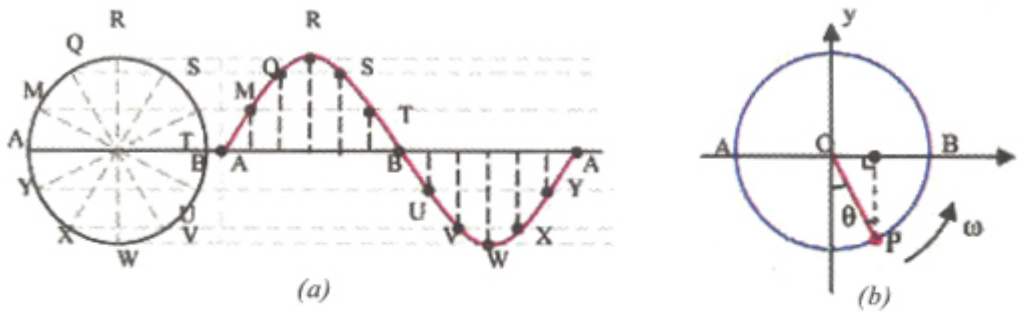


Figure 5.5: Displacements – time graph of a particle performing shm between A and B

In Figure 5.5b, the particle P moves on the circle and at the instant shown, the line joining the particle to the centre O of the circle makes an angle Î¸ with the y – axis. OP is the radius of the circle. Applying trigonometry to the Î”OPN, we obtain:

$$\frac{ON}{OP} = Sin\,\theta \Rightarrow ON = OPSin\,\theta \quad \text{But è = ùt}$$

$$x = \text{r sin ωt}$$

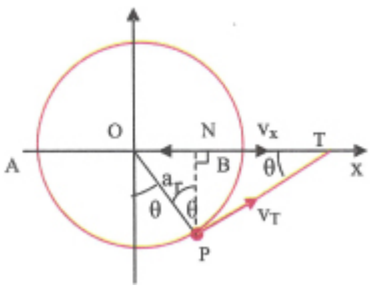
x = displacement of the particle from the equilibrium or rest position

r = amplitude or the maximum displacement of the particle from the equilibrium position

Ï‰ = angular speed or frequency

t = time measured in seconds

(2) Velocity of simple harmonic motion



The constant speed of a particle moving in a circle is always perpendicular to the radius of the circle (that is, it is tangential to the circle at the point of contact). The tangential velocity v_T has two components, one along the *x* - axis and the other along the *y* - axis. The projection (N) of P along the *x* -axis oscillates between A and B with the velocity *v*Ã— the component of v_T along the *x* - axis. Applying trigonometry to Î”PNT, we obtain:

$$\frac{v_X}{v_T} = \cos\theta \quad \Rightarrow v_X = v_T\cos\theta$$

But Î¸ = Ï‰t and v_T = Ï‰r

$$V_x = \text{Ï‰rcos Ï‰t}$$

The velocity – time graph of a particle performing simple harmonic motion is a cosine curve as shown in Figure 5.7.

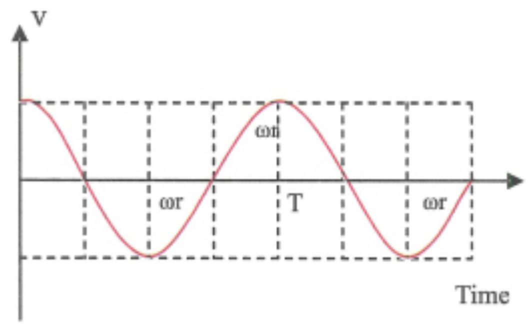


Figure 5.7: Velocity – time graph for a body performing simple harmonic motion

The velocity equation

The displacement of a simple harmonic motion is given by: *x = rsin Ï‰t*

$$\Rightarrow \sin \omega t = \frac{x}{r}$$

The velocity of a simple harmonic motion is given by: v_x = Ï‰rcos Ï‰t

$$\Rightarrow \cos \omega t = \frac{v}{\omega r}$$

Using the identity sin^2 Ï‰t + cos^2 Ï‰t = 1

$$\therefore \left(\frac{x}{r}\right)^2 + \left(\frac{v}{\omega r}\right)^2 = 1$$

$$\therefore \left(\frac{v}{\omega r}\right)^2 = 1 - \left(\frac{x}{r}\right)^2 = \frac{r^2 - x^2}{r^2}$$

$$v^2 = \omega^2\left(r^2 - x^2\right)$$

$$\therefore v = \pm\omega\sqrt{r^2 - x^2}$$

The velocity is maximum when the displacement *x* = 0, therefore, v_{max} = Ï‰r. The symbols have their usual meanings.

Acceleration of simple harmonic motion: The acceleration of the projection N of the particle on the on the *x* – axis is:

- **â€¢** proportional to the displacement *(x)* of the particle from the equilibrium position.
- **â€¢** always pointing towards the equilibrium position. (Displacement is always in the opposite direction to the acceleration).

$$a = -\omega^2 x\text{, but } x = r\sin \omega t$$

$$\therefore a = -\omega^2 r \sin \omega t$$

Ï‰ is a constant linking acceleration to the displacement of the particle from the equilibrium position. The acceleration is assigned a negative value because its direction is opposed to the displacement. It also means that the acceleration is pointing towards the equilibrium position. The acceleration – time graph is an inverted displacement –

time graph as shown in Figure 5.8.

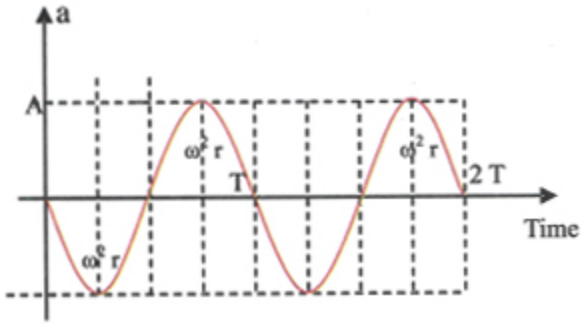


Figure 5.8: Displacement – time graph of a particle performing shm between A and B

Maximum acceleration of a_{max} = - *Ï‰²r* occurs when the displacement is maximum or when the velocity is zero.

Worked examples

1. The displacement *(x)* of a particle vibrating with a simple harmonic motion is given by x = 20sin 100Ï€ t

   Where *x* is millimetre and *t* in seconds; calculate:

   (a) amplitude of oscillation;

   (b) angular velocity;

   (c) velocity at *t* = 0 seconds;

   (d) maximum velocity and acceleration.

Solution

Comparing the given equation with the equation of displacement for a body performing shm; *x* = 20sin 100Ï€t and *x* = *r* sin Ï‰t

(a) Amplitude *(r)* = 20 mm or 0.02 m.

(b) $\omega t = 100\pi t \Rightarrow \omega = 100\pi\, rads^{-1}$

(c) v_x = Ï‰rcos Ï‰t

= (100Ï€) (20) cos 100Ï€t

= 2000Ï€cosÎ¸

= 2000Ï€ mms⁻¹ or 2Ï€ ms⁻¹

2. A particle undergoes simple harmonic motion with an amplitude of 5 cm and an angular velocity of 12Ï€ rad s⁻¹calculate:

   (a) the maximum velocity

   (b) the velocity when it is 2cm from the equilibrium position;

   (c) the maximum acceleration of the particle;

   (d) the period of oscillation. Solution

Solution

(a) v_{max} = Ï‰r = 12Ï€ Ã— 0.05 = 0.6Ï€ ms⁻¹

$v = \pm\omega\sqrt{r^2 - x^2}$

(b) $= \pm 12\pi\sqrt{0.05^2 - 0.02^2} = \pm 0.55\pi\ ms^{-1}$

(c) a_{max} = Ï‰²r = (12Ï€)² Ã— .05 = 7.2Ï€²ms⁻²

(d) $T = \frac{2\pi}{\omega} = \frac{2\pi}{12\pi} = 0.167s$

Summary

â€¢ Simple harmonic motion is the motion of a particle whose acceleration is directly proportional to its displacement from the equilibrium position and is always pointing towards the equilibrium position.

â€¢ Angular velocity is the angle turned through per second or time rate of change of angular displacement.

â€¢ Angular acceleration is the angle turned through per second per second or time rate of change of angular velocity.

â€¢ Displacement of simple harmonic motion
x = r sin Ï‰t

â€¢ Velocity of simple harmonic motion
$v_x = \omega r\cos\omega t$ or $v = \pm\omega\sqrt{r^2 - x^2}$
The velocity is maximum when the displacement
x = 0, therefore v_{max} = Ï‰r.

â€¢ The acceleration of a simple harmonic motion is given by *a* = –Ï‰²*r* sin *Ï‰t*. Maximum acceleration of a_{max} = -Ï‰²r occurs when the displacement is maximum or when the velocity is zero.

Practice questions 5a

1. (a) What is simple harmonic motion? Give four examples to illustrate simple harmonic motion.
   (b) Define the terms: amplitude, period and frequency as applied to simple harmonic motion.
   (c) A particle vibrates with simple harmonic motion such that maximum displacement from the equilibrium position is 10 cm. If the acceleration of the particle is

$$a = -9x$$

   where *x* is the displacement (cm) from the equilibrium position, find its:
   (i) angular velocity
   (ii) maximum linear velocity
   (iii) maximum acceleration.
2. A particle moves round a circle of radius 10 cm with a constant velocity of 20 ms^{-1}, calculate the angular velocity.
3. (a) Explain what you understand by simple harmonic motion;
   (b) Explain the link between uniform circular motion and simple harmonic motion.
   (c) A particle undergoing simple harmonic motion has its displacement given by *x* = 0.12sin *4Ï€t* Where *x* is given in

metre and *t* is time in seconds. Find the:

(i) amplitude of oscillation;

(ii) angular velocity;

(iii) maximum linear velocity;

(iv) maximum acceleration;

(v) period of oscillation;

(vi) frequency of oscillation.

4. (a) Define simple harmonic motion.

(b) A particle undergoes simple harmonic motion with an amplitude of 5 cm and an angular velocity of 10Ï€ rad s^{-1} calculate:

(i) the maximum velocity;

(ii) the velocity when it is 2 cm from the equilibrium position;

(iii) the maximum acceleration of the particle;

(iv) the period of oscillation.

Restoring force, period and energy of simple harmonic motion

OBJECTIVES

At the end of the topic, students should be able to:

- explain the role of restoring force in simple harmonic motion;
- state the formula for the period of oscillation for simple pendulum and a mass vibrating at the end of a spiral spring;
- state and explain the energy changes in simple harmonic motion;
- explain forced, free and damped oscillations;
- explain resonance.

Force in simple harmonic motion

A vibrating body has an acceleration which points towards the equilibrium position. The acceleration is produced by a resultant force which acts on the body as it vibrates. This resultant force is called **restoring force** because its direction tends to restore the body to its equilibrium position. (That is, restoring force acts in the opposite direction to the displacement from the equilibrium position).

The restoring force according to Hookeâ€™s law is given by $F = -kx$. Newtonâ€™s second law of motion gives force as $F = ma$ but a = –*Ï‰*^2x.

$$F = -m\omega^2 x$$

The restoring force = $-kx$ = $-m$ *Ï€*^2x.

$$\therefore k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

Ï‰ = angular frequency or velocity, k = force constant of the spiral spring and m = mass of the particle oscillating at the end of the spring.

Vibration of weight on a spiral spring

If a weight $(W = mg)$ is hanged on a spiral spring of force constant (k), it will extend by x_o. The tension or restoring force of the spring is given by $T = kx_o$. This is equal to the weight suspended on the spring.

$$\therefore kx_o = mg ....i$$

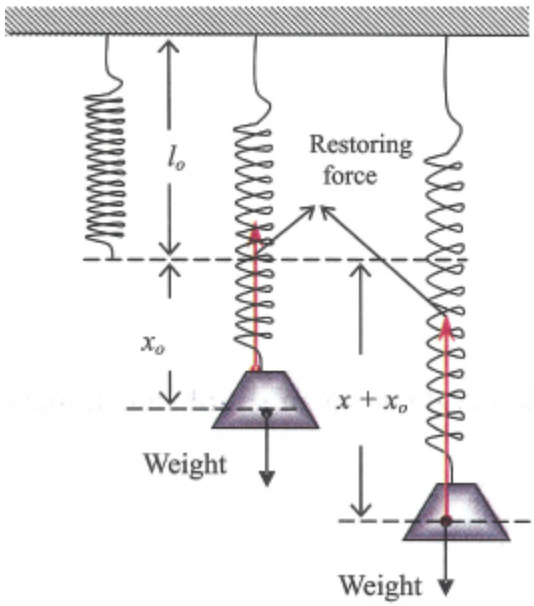


Figure 5.9: Vibration of weight on a spring

If the weight is pulled down a little and released, it vibrates about the equilibrium position. The new tension on the spring is given by $T= k(x + xj)$ where $(x + x_0)$ is the total extension. The restoring force making the weight to vibrate is the resultant of the weight and the tension of the spring.

Restoring force $F = k(x + x_0) - mg$

$F = k(x + x_0) - kx_o$ $\{mg = kx\}$

$F = kx$

The angular frequency or velocity and the period of oscillation of a weight vibrating at the lower end of a spring are given by:

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \times \frac{1}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

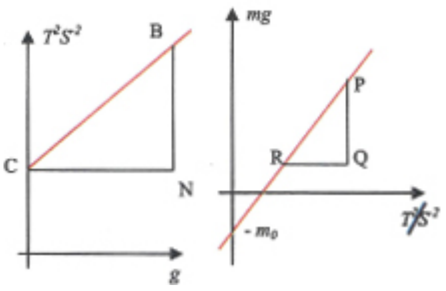
$$T = 2\pi\sqrt{\frac{m}{k}}$$

The equation above implies that the graph of T^2 against m is a straight line through the origin. In practice, the graph intercepts the T^2- axis because of the mass (m_0) of the spring vibrates with the weight. We therefore modify the period of oscillation of an oscillating weight to accommodate the mass of the spiral spring.

$$T = 2\pi\sqrt{\frac{m + m_o}{k}}$$

$$T^2 = \frac{4\pi^2}{k}m + \frac{4\pi^2}{k}m_o \quad \text{or} \quad m = \frac{k}{4\pi^2}T^2 - m_0$$

The slope of the graphs is used to find the force constant of the spring (k).



$$\text{Slope } (s) = \frac{BN}{CN} = \frac{4\pi^2}{k} \quad \text{or} \quad \text{Slope } (s) = \frac{PQ}{RQ} = \frac{k}{4\pi^2}$$

The intercept on the vertical axis is used to find the mass (m_o) of the spiral spring and the force constant (k) of the spring.

$$\text{Intercept (c)} = \frac{4\pi}{k}m_o \quad \text{or} \quad \text{Intercept (c)} = -m_o$$

Worked example

A mass of 0.2 kg suspended at the end of a spiral spring is set into vibration. If it takes 34 seconds to complete 20 oscillations, calculate:

(a) the period of oscillation;

(b) the force constant of the spring, (Ï€ = 3.142)

Solution

(a) $T = \frac{t}{20} = \frac{34}{20} = 1.70 \text{ seconds}$

(b) $T = 2\pi\sqrt{\frac{m}{k}}$

$$1.7 = 2\pi\sqrt{\frac{0.2}{k}} \Rightarrow 2.89 = \frac{4\pi^2}{k}\times 0.2$$

$$k = \frac{4\pi^2}{2.89}\times 0.2 = 2.73\text{kgm}^{-1} \text{ or } 27.3\text{Nm}^{-1}$$

Vibration of simple pendulum

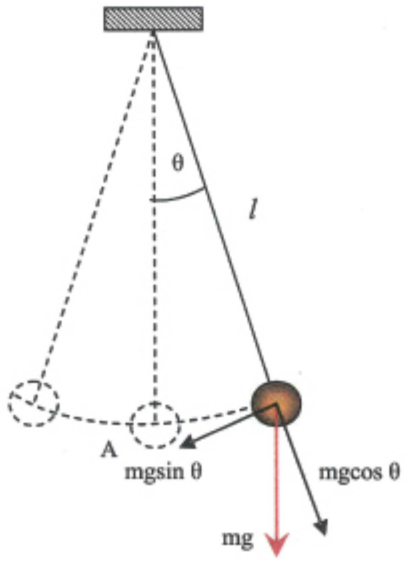


Figure 5.10: Vibration of simple pendulum

The simple pendulum consists of a string which does not extend and a bob of mass *(m)* attached to the lower end. The length of the pendulum is measured from the fixed point to the centre of the bob.

When the pendulum is displaced slightly from the equilibrium through a small angle Î¸ and released, it performs a simple harmonic motion about A. It can be proved that the period of oscillation of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The equation above shows that:

â€¢ period of oscillation of a simple pendulum is directly proportional to the square root of its length; $(T \propto \sqrt{l})$

â€¢ period of oscillation of a simple pendulum is independent of the mass of the bob;

â€¢ the graph of T^2 against l is a straight line through the origin;

â€¢ the slope of the graph = $\frac{4\pi^2}{g}$ and is used to find acceleration due gravity at the locality.

Worked examples

1. The period of oscillation of a simple pendulum is 2.18 seconds. Calculate:

(i) the time to complete 50 oscillations;

(ii) the length of the simple pendulum.

{g= 10 ms^{-2}; Ï€ -3.142}

Solution

(i) $T=\frac{t}{50} \Rightarrow 2.18=\frac{t}{50} \therefore t=2.18\times50$

t = 109 seconds.

(ii) $T=2\pi\sqrt{\frac{l}{g}} \Rightarrow 2.18=2\pi\sqrt{\frac{l}{10}}$

$$4.7524=4\pi^2\frac{l}{10}$$

$$l=4.7524\times10\times\frac{1}{4\pi^2}=1.203\text{m}$$

2. A simple pendulum P completes 20 oscillations in 38.0 seconds. Another simple pendulum Q with the same period of oscillation completes 30 oscillations. Calculate;
   (a) the period of oscillation of P;
   (b) the length of the pendulum P;
   (c) the time it takes Q to complete 30 oscillations, {g = 10 ms^{-2}, Ï€ = 3.142}

Solution

(a) $$T=\frac{\text{Time to complete 20 oscillations}}{\text{number of oscillations}}$$

$$T=\frac{38}{20}=1.90 \text{ seconds}$$

(b) $T=2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2=\frac{4\pi^2}{g}l$

$$l=\frac{g}{4\pi^2}T^2$$

$$=\frac{10}{4\times3.142^2}\times1.90^2=0.9142\ m$$

(c) Time for 30 oscillations = *T*Ã—30

T = 1.9Ã—30 = 57 s.

3. The period of oscillation of a simple pendulum is 2.0 seconds when its length is 1.01 m. What is the length of the pendulum when the period of oscillation is 3.4 seconds?

Solution

$$T_1^2 = \frac{4\pi^2}{g} l_1 \quad and \quad T_2^2 = \frac{4\pi^2}{g} l_2$$

$$\frac{T_2^2}{T_1^2} = \frac{\frac{4\pi^2}{g} l_2}{\frac{4\pi^2}{g} l_1} = \frac{l_2}{l_1}$$

$$\frac{3.4^2}{2.0^2} = \frac{l_2}{1.01} \quad \Rightarrow l_2 = \frac{3.4^2 \times 1.01}{2.0^2} = 2.92\ m.$$

Energy transformation in simple harmonic motion

The energy stored in elastic materials and stretched springs is elastic potential energy. When a spring with a weight at the lower end is pulled down slightly, work is done against the restoring force. Average work done in stretching the spring is given by

$$E_p = ½kx^2$$

When the weight is pulled down and allowed to vibrate, its potential energy is transformed slowly to kinetic energy. At the equilibrium position, all the potential energy is transformed to kinetic energy. The kinetic energy at any position of the particle is given by:

$$E_k = ½\ mv^2$$

But the velocity of vibrating body is given by:

$$v^2 = \omega^2 (r^2 - x^2)$$
$$\text{Then, } Ex = ½\ m\ \omega^2 (r^2 - x^2) = ½\ k\ (r^2 - x^2)$$

Maximum kinetic energy E_k is at the equilibrium position where the displacement *(x)* is zero or the potential energy is E_p is zero. The total energy of the system at the equilibrium position is only kinetic and is given by $E_k = ½kr^2$.

At any other position of the particle, the total energy of the vibrating particle remains constant according to the principle of conservation of energy in a mechanical system. That is, the sum of potential and kinetic energies remain constant.

$$E_k = E_p + E_k = ½\ kr^2$$
$$E = ½\ kx^2 + ½mv^2 = ½\ kr^2$$
$$E = ½\ kx^2 + ½\ k\ (r^2 - x^2) = ½\ kr^2$$

Maximum potential energy occurs when the displacement *(x)* is greatest or when the kinetic energy is zero. When the kinetic energy is greatest at the equilibrium position the potential energy is zero. Figure 5.11 is a graph showing how the potential and kinetic energies vary with the displacement between the two maximum displacements (-*r* and +*r*).

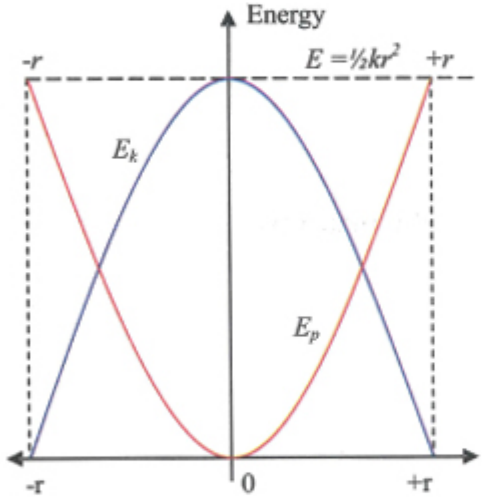


Figure 5.11: Variations of kinetic and potential energies in a mechanical system

Worked examples

1. A body of mass 2 kg vibrates with a frequency of 5 Hz and amplitude of 4 cm. Assuming the oscillation is simple harmonic and not damped, calculate:
   (a) The period of oscillation;
   (b) The maximum kinetic energy;
   (c) The kinetic energy when the particle is 3 cm from the equilibrium position. {Ï€ = 3.142}

Solution

(a) $T = \frac{1}{f} = \frac{1}{5} = 0.2s$

(b) Angular speed$(\omega) = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi\ m\ s^{-1}$ $k = m\ \omega^2 = 2 \times (10\pi)^2 = 200\pi^2$

Maximum kinetic energy = total energy.

$E_k = ½\ kr^2 = ½ \times 200\pi^2 \times 0.04^2 = 1.580J.$

(c) $E_k = ½\ k\ (r^2 - x^2)$

$= ½ \times 200\pi^2\ (0.04^2 - 0.03^2)$

$= 0.691J$

2. A mass of 0.1 kg oscillates in simple harmonic motion with amplitude of 0.2 m and a period of 1.0 s. Calculate:

(i) the force constant of the spring;

(ii) the maximum potential energy of the mass;

(iii) the potential energy if the mass is 0.12m from the equilibrium position. {Ï€ = 3.142}

Solution

(i) $T^2 = \frac{4\pi^2}{k} m$

$$\therefore k = \frac{4\pi^2}{T^2} = \frac{4(3.142)^2}{1.0^2} \times 0.1$$

$$= 3.95\ N\ m^{-1}.$$

(ii) $E_p = ½\ kr^2 = ½ \times 3.95 \times 0.2^2 = 0.079J.$

(iii) $E_p = ½\ kx^2 = ½ \times 3.95 \times 0.12^2$

$= 0.0284J.$

Free, damped and forced vibrations

Free vibration

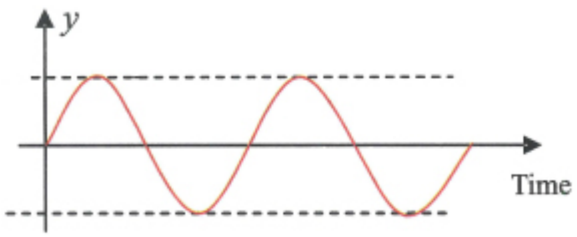


Figure 5.12: Constant energy and amplitude of free vibration

Vibration is said to be free if the total energy of a vibrating object is constant. No energy is lost by the object, therefore the amplitude and period of oscillation remain the same as long as the vibration lasts. Free vibration is difficult to attain because energy is always lost due to air resistance opposing the motion of the object. The vibration of simple pendulum and molecules of a substance can be approximated to free vibration.

Damped vibration

Vibration is damped if energy is continuously lost as the body vibrates. The loss of energy by the body makes its amplitude and period of oscillation to decrease until the body stops vibrating.

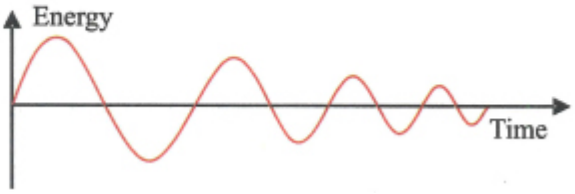


Figure 5.13: Damped vibrations energy is lost continuously

Two types of damped vibrations are **natural and artificial damping.**

Natural damping: Energy is lost constantly through natural means by the vibrating body. Air resistance damps the motion of simple pendulum forcing it to a stop after some time.

Artificial damping: Energy is lost by the vibrating body through man made means. Electromagnetic dampings in moving coil instruments and shock absorbers in cars are good examples of artificial damping.

Forced vibration

Most vibrating bodies will stop if left for a long time because they lose energy gradually. To keep such bodies vibrating, energy is given to it at regular intervals to replace the lost energy.

Forced vibration is a vibration in which energy is constantly supplied to the vibrating body by an external periodic force.

If the external force is applied in steps or phases, the amplitude increases until the body vibrates with the largest amplitude. The frequency of the body, when it vibrates with the largest amplitude, is called the **natural frequency.**

Natural frequency is the frequency at which a body tends to vibrate if left undisturbed (without any external force applied).

Examples of forced vibration

â€¢ **A sounding tuning fork when pressed on a tabletop makes the tableâ€™s surface to vibrate:** The vibration of the surface of the table in turn forces the air molecules close to it to vibrate at their natural frequency to produce a loud sound. The sound is not heard if the fork is removed from the tableâ€™s surface. The vibration of the tableâ€™s surface and air molecules are examples of forced vibration.

â€¢ **The vibration of a piston and the connecting rod in an engine:** The piston stops moving if the connecting rod stops vibrating.

Resonance

Resonance occurs if a body is forced to vibrate at its natural frequency by an external periodic force applied to it.

At resonance:

â€¢ the frequency of the applied external force is equal to the natural frequency of the vibrating body;

â€¢ the body vibrates with a maximum amplitude with gained energy replacing lost energy during the same interval.

Examples of resonance

(i) Shattering of a fragile glass by directing sound of high pitch to it: A singer can shatter a glass by sending high-pitched sound waves to it. The glass breaks when the sound waves make it vibrate at its natural frequency.

(ii) Bartonâ€™s pendulum: Bartonâ€™s pendulum, shown in Figure 5.14 is a good example of forced vibration and resonance.

When the heavy pendulum P is set into vibration, it makes the other lighter pendulums to vibrate. The pendulum D with the same length as P, vibrates with the largest amplitude. This is because pendulums P and D have the same natural frequency. The pendulum D is said to resonate with the pendulum P.

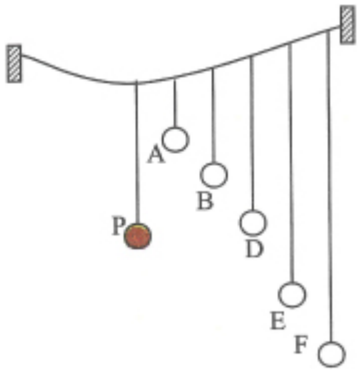


Figure 5.14: Bartonâ€™s pendulum

(iii) Collapse of bridge when it is forced to vibrate at its natural frequency.

For this reason, soldiers do not match in steps or phase across a bridge.

(iv) Bodies and side glasses of old vehicles vibrate strongly when they travel at a particular speed.

(v) Television and radio tuning circuits have capacitors and inductors which resonate at a frequency corresponding to the frequency of the transmitting station.

Summary

â€¢ **Restoring force** is the force which tends to restore a vibrating body to its equilibrium position.

â€¢ **Period of oscillation** is the time to complete one vibration. The period of oscillation for a weight vibrating at the lower end of a spiral spring depends on its mass and the force constant of the spring. For a simple pendulum, the period of oscillation depends on the length of the pendulum only.

â€¢ **The total energy** of a vibrating particle is always constant at any position and is equal to E = Â½kr^2. It is only kinetic at the equilibrium position and potential at the two extreme positions.

â€¢ **Free vibration** occurs when the total energy of a vibrating object is constant.

â€¢ **Damped vibration** occurs when energy is continuously lost as the body vibrates. The two types of damped vibrations are **natural and critical damping.**

â€¢ **Forced vibration** is the vibration in which energy is constantly supplied to the vibrating body by an external periodic force.

â€¢ **Natural frequency** is the frequency at which a body tends to vibrate if left undisturbed (without any external force applied).

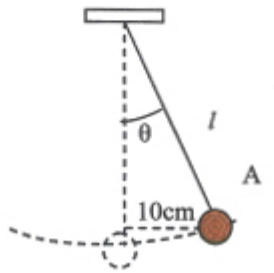
â€¢ **Resonance** occurs if a body is forced to vibrate at its natural frequency by an external periodic force applied to it.

Practice questions 5b

1. (a) Define simple harmonic motion.

   (b) State the energy transformation which occurs during the vibration of simple pendulum.

   (c)



   The pendulum above oscillates with a period of 2.0 seconds when it is displaced through a horizontal distance of 10.0 cm from the equilibrium position and released. Find:

   (i) the length of the pendulum;

   (ii) the angle Î¸ through which it is displaced;

   (iii) the gain in potential energy when the pendulum in position A and the mass of the bob is 50 g. {g=10ms^{-2}}

2. (a) What do you understand by simple harmonic motion?

   (b) Explain the terms: forced vibration, free vibration and damped vibration. Give an example of each.

   (c) A mass of 200 g executes a simple harmonic motion with an amplitude of 0.2 m. The maximum force acting on the mass is 0.064 N. Calculate:

   (i) the maximum velocity;

   (ii) the period of oscillation.

3. (a) State the principles of conservation of energy; use the principle to show that the total energy of an oscillating particle is constant.

   (b) A particle of mass 0.3 kg vibrates with a period of 2.5 seconds. If the amplitude is 0.3 m, calculate the maximum velocity and kinetic energy.

4. (a) Explain the terms: *forced vibration, natural frequency* and *resonance.*

   (b) The simple pendulum has a period of 4.0 seconds. When the length is increased by 1.5 m, the period is 4.7 seconds. Calculate the original length of the simple pendulum.

5. (a) The length of a simple pendulum is 100 cm, what is its period of oscillation?

   (b) Calculate the time it takes the pendulum to complete 30 oscillations.

Past questions

1. The period of oscillatory motion is defined as the
   A. average of the time used in completing different numbers of oscillations.
   B. time to complete a number of oscillations.
   C. time to complete one oscillation.
   D. time taken to move from one extreme position to another.

WASSCE

2. In a simple pendulum experiment, 20 oscillations were completed in 38 s. Calculate the period of oscillation of the pendulum.
   A. 0.03 s
   B. 0.05 s
   C. 0.50 s
   D. 1.90 s

WASSCE

3. What type of motion does the skin of a talking drum perform when it is struck with a drumstick?
   A. Random.
   B. Rotational.
   C. Translational.
   D. Vibratory.

WASSCE

4. Which process will increase the rate of oscillation of a simple pendulum?
   A. Decreasing the amplitude of oscillation of the pendulum.
   B. Decreasing the length of the pendulum.
   C. Decreasing the mass of the pendulum.
   D. Increasing the length of the pendulum.
   E. Increasing the mass of the pendulum.

WAEC; NECO

5. The period of oscillation of a simple pendulum is 2.0 s. Calculate the period if the length of the pendulum is doubled.
   A. 1.0 s
   B. 1.4 s
   C. 2.8 s
   D. 4.0 s

JAMB; WASSCE

6. The amplitude of the motion of a body performing simple harmonic motion decreases with time because

A. frictional forces dissipates the energy of the motion
B. the frequency of oscillation varies with time
C. the period of oscillation varies with time
D. energy is supplied by some external agencies.

WASSCE

7. The amplitude of a particle executing simple harmonic motion is 5 cm while its angular frequency is 10 rad s^{-1}. Calculate the magnitude of the maximum acceleration of the particle.
A. 0.25 ms^{-2}.
B. 0.50 ms^{-2}.
C. 2.00 ms^{-2}.
D. 5.00 ms^{-2}.

WASSCE

8. The period of a body performing simple harmonic motion is 2.0 s. If the amplitude of the motion is 3.5 cm, calculate the maximum speed of the body. $\{\pi = \frac{22}{7}\}$
A. 22.0 cms^{-1}
B. 11.0 cms^{-1}
C. 7.0 cms^{-1}
D. 1.8 cms^{-1}

WASSCE

9. The bob of a simple pendulum of mass 0.025 kg is displaced 0.1 m from its equilibrium position. If the angular frequency is 4 rad s^{-1} and assuming simple harmonic motion, calculate the energy of the system.
A. 5.0 Ã— 10^{-4}J
B. 2.0 Ã— 10^{-3}J
C. 5.0 Ã— 10^{-3}J
D. 7.9 Ã— 10^{-3}J

WASSCE

10. The bob of a simple pendulum takes 0.25s to swing from its equilibrium position to one extreme end. Calculate its period.
A. 0.25 s
B. 0.50 s
C. 0.75 s
D. 1.00 s

WASSCE

11. In a simple pendulum experiment, a boy observed the times for 30 oscillations are 70.0 s, 72.0 s and 67.0 s respectively. Calculate the mean period of oscillation.
A. 0.14 s
B. 0.43 s
C. 2.32 s
D. 2.40 s

NECO

E. 6.97 s

12. A pendulum bob, executing simple harmonic motion, has 2 cm and 12 Hz as amplitude and frequency respectively. Calculate the period of the motion.

A. 2.00 s

B. 0.83 s

C. 0.08 s

D. 0.06 s

WASSCE

13. Two simple pendulums X and Y make 400 and 500 oscillations respectively in equal time. If the period of oscillation of X is 1.5 seconds, what is the period of oscillation of Y?

A. 0.53 s

B. 0.83 s

C. 1.20 s

D. 1.50 s

E. 1.88 s

WAEC

14. Which of the following assumptions is made in a simple pendulum experiment? The

A. suspending string is inextensible.

B. bob has a finite size.

C. bob has a definite mass.

D. initial angle of oscillation must be large.

JAMB

15. A simple pendulum has a period of 17.0 s. When the length is shortened by 1.5 m, its period is 8.5 s. Calculate the original length of the pendulum.

A. 1.5 m

B. 2.0 m

C. 3.0 m

D. 4.0 m

JAMB

16. (a) (i) Define simple harmonic motion.

(ii) State two examples, other than simple pendulum of bodies performing this type of motion.

(b) The period T of an oscillating simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$, where *l*, is the length of the pendulum and g the acceleration due to gravity. Using the equation, describe an experiment to determine the value of g.

(c) The bob of a simple pendulum oscillates in simple harmonic motion with an amplitude of 10cm. If the acceleration of the bob is a = – 4y where y is the displacement, calculate its:

(i) maximum acceleration

(ii) period

(iii) maximum velocity.

WASSCE

17. (a) Define *gravitational field intensity.*

(b) In an experiment to determine the acceleration of free fall due to gravity, g, using a simple pendulum of length *l*, **six** different values of *l* were used to obtain **six** corresponding values of period *T.* If a graph of *l* along the vertical axis is plotted against T^2 on the horizontal axis;

(i) make a sketch to show the nature of the graph;

(ii) write down the equation that relates *T, l and g* hence obtain an expression for the slope of the graph;

(iii) given that the slope of the graph is 0.25, determine the value for g. [Take Ï€ = 3.142]

WASSCE

18. (a) Sketch a diagram of a simple pendulum performing simple harmonic motion and indicate the positions of maximum potential energy and kinetic energy.

(b) A body moving with simple harmonic motion in a straight line has a velocity, v and acceleration, a, when the instantaneous displacement, *x* in cm, from its maximum position is given by *x = 2.5sin 0.4*Ï€*t.* where *t* is in seconds. Determine the magnitude of maximum;

(i) velocity;

(ii) acceleration.

(c) A mass *m* attached to a light spiral spring is caused to perform simple harmonic motion of frequency

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where k

is force constant of the spring.

(i) Explain the physical significance

of $\sqrt{\frac{k}{m}}$

(ii) If *m* = 0.30 kg, *k =30* N m^{-1} and the maximum displacement of the mass from the equilibrium position is 0.015 m, calculate the maximum

(iii) kinetic energy of the system;

(iv) tension in the spring during the motion. [g= 10 ms^{-2}, Ï€ = 3.142]

WASSCE


I rely on this pendulum to save my life from that hungry cat!