

# Chapter 15: Measures of Dispersion

## OBJECTIVES

At the end of the chapter, students should be able to:

1. State the meaning of dispersion.
2. Compute the range, mean deviation variance and standard deviation of ungrouped frequency table.
3. Identify applications of measures of dispersion in capital market studies, health, population, etc.

## I. Measures of Dispersion

### (i) Introduction

Disperse means to spread, scatter or vary. Hence, measure of dispersion or measure of spread is a statistical measure that shows the extent to which numerical data cluster around a measure of location.

Majorly, measures of dispersion consist of the range, mean deviation, variance and standard deviation.

### (ii) Computation of the range, the mean deviation, the variance and the standard deviation

#### (a) Range

The range of a set of values is calculated by subtracting the lowest value from the highest value. It is the easiest of the measures of dispersion. Range is solved using the two extreme values of the data. It can mislead and hence is not regarded as a good representation of the spread of a distribution

Worked Example 1



Johnson says  
Teacher, please explain how the range of each of the marks below is calculated.  
(a) 18, 16, 5, 6, 28, 38, 32, 32, 54, 48, 10  
(b) 70, 48, 21, 49, 48, 49, 47, 69, 68, 69  
Then choose the one that has more spread.

Teacher replies



Range is the difference between the highest score and the lowest score. In question (a) highest score is 54 while the lowest score is 5. Hence, range is  $54 - 5 = 49$ . In question (b) highest score is 70 while the lowest score is 21. Hence, range is  $70 - 21 = 49$ . It is clear that the range for the sets of marks in both question (a) and (b) are equal but the spread of the marks in question (a) is wider than that of question (b).

Worked Example 2

A cattle rearer recorded the masses to the nearest kg of 20 cattle as follows:

9, 10, 15, 16, 50, 10, 9, 21, 25, 62, 9, 11, 40,  
30, 14, 10, 10, 28, 48, 10

Find the range of the data.

## SOLUTION

Highest mass = 62 kg

Lowest mass = 9 kg

Range =  $62 \text{ kg} - 9 \text{ kg} = 53 \text{ kg}$

Remark: The masses in the above example are widely spread.

### Worked Example 3

Find the range from the following table:

**Table 15.1**

x	25	26	27	28	29	30
y	5	1	8	6	9	2

### SOLUTION

Lowest value = 25

Highest value = 30

Range =  $30 - 25 = 5$

### Exercise 1

- What is the range of the following distribution:

**Table 15.2**

<b>Scores</b>	3	4	5	6	7	8	9	10
<b>Frequency</b>	1	0	7	5	2	3	1	1

(NECO)

- If the range of 3, 5, 8, K, 14 and 17 is 15, find the value of K where K is the least number.
- What is the range of the following scores: 17.1, 50.3, 6.27, 33.2, 15.9, 66.05 and 24? (NECO)
- The mean and the range of the set of numbers: 1.20, 1.00, 0.90, 1.40, 0.80, 0.80, 1.20 and 1.10 are p and q, respectively. What is the value of p + q? (NECO)
- If the mean of 3, 5, 8, K, 14 and 17 is 11, find the range of the set of numbers.
- The following numbers are written according to the time of occurrence: 12.41, 13.52, 14.42, 12.53 and 16.62. Find the sum of the range and the highest 2-digit perfect square.
- Find the sum of the median and range of the following numbers: 16, 13, 10, 11, 26, 9, 8, 38 and 14. (NECO)
- The table below shows the scores of a set of students in a Mathematics test. If x is the range of 2, 4, 1, 3, 0 and y is the first number with three factors. Find the range of the frequency distribution.

**Table 15.3**

<b>Marks</b>	y	x	6	7	8
<b>Frequency</b>	5	9	5	4	2

- Below are the marks of students in social studies in Junior Secondary School 1A and 1B.

**Table 15.4**

Class	Candidates				
	1	2	3	4	5
JSS 1A	40	65	30	42	49
JSS 1B	50	14	61	32	19

If m is the range of marks in JSS 1A and n is that of JSS 1B, find  $\frac{m}{n}$  in decimal fraction.

- The marks of a certain number of students in a class are 6, 7, 8, 12, 15, 8, 9, 5, 29, 15, 17, 21. What is the sum of the range and a quarter of the range?
- The table below shows the marks obtained by a set of students in an examination.

**Table 15.5**

<b>Marks (x)</b>	28	29	30	31	32
<b>Frequency (f)</b>	5	4	2	14	20

Calculate the range of the products of f and x given that x represents the marks and f represents the number of students.

12. What is the range of the mean, median and mode of 6, 1, 5, 6, 3, 6, 3, 3, 7, 8, 9, 3?  
 13. What is the range of the distribution of oranges to 30 females in a village in Nigeria?

4	5	7	2	3	6	5	5	8	9
5	4	2	3	7	9	8	7	7	7
3	4	5	5	2	3	6	7	7	2

14. Find the range of the common factors of 72, 48 and 120.  
 15. Find the range of the HCF and the LCM of 20, 40 and 60.

### (b) Mean deviation

Mean deviation is the algebraic sum of the absolute deviations of each observation from the mean divided by the number of observed values. It should be noted that deviations below the mean are negative while those above the mean are positive.

The algebraic sum of the deviation from the mean is zero, i.e.  $\sum f(x - \bar{x}) = 0$  but if the signs of the deviation from the mean in each event is disregarded, then  $\frac{\sum |x - \bar{x}|}{n}$  is called the *mean deviation* from the mean of the distribution

#### Worked Example 4

Find the mean deviation of the set of numbers 4, 8, 10, 6, 2, 6.

#### SOLUTION

$$\bar{x} = \text{mean} = \frac{4 + 8 + 10 + 6 + 2 + 6}{6} = \frac{36}{6} = 6$$

MD = Mean deviation

$$\begin{aligned} &= \frac{|4 - 6| + |8 - 6| + |10 - 6| + |6 - 6| + |2 - 6| + |6 - 6|}{6} \\ &= \frac{2 + 2 + 4 + 0 + 4 + 0}{6} \\ &= \frac{12}{6} = 2 \end{aligned}$$

#### Worked Example 5

The table below shows the number of eggs laid by chickens in a manâ€™s farm in a year.

Table 15.6

Number of eggs per year	Number of chickens
1–5	8
6–10	16
11–15	8
16–20	6
21–25	12

Calculate the (a) mean and (b) mean deviation of the distribution.

#### SOLUTION

**Table 15.7**

Number of eggs per year	Number of chickens	$x$	$fx$	$x - \bar{x}$	$ x - \bar{x} $	$f x - \bar{x} $
1–5	8	3	24	-9.8	9.8	78.4
6–10	16	8	128	-4.8	4.8	76.8
11–15	8	13	104	0.2	0.2	1.6
16–20	6	18	108	5.2	5.2	31.2
21–25	12	23	276	10.2	10.2	122.4
	<u>50</u>		<u>640</u>			<u>310.4</u>

Let  $x$  represent the mean while MD represents the mean deviation:

$$(a) \bar{x} = \text{Mean} = \frac{\sum fx}{\sum f} = \frac{640}{50} = 12.8$$

$$(b) \text{MD} = \frac{\sum f|x - \bar{x}|}{\sum f} = \frac{310.4}{50} = 6.208$$

### Worked Example 6

Find the mean deviation of the following distribution:

**Table 15.8**

<b><math>x</math></b>	3	4	5	6	7	8	9
<b><math>f</math></b>	1	1	3	1	1	2	1

**Table 15.9**

$x$	$f$	$fx$	$x - \bar{x}$	$ x - \bar{x} $	$f x - \bar{x} $
3	1	3	-3	3	3
4	1	4	-2	2	2
5	3	15	-1	1	3
6	1	6	0	0	0
7	1	7	1	1	1
8	2	16	2	2	4
9	<u>1</u>	<u>9</u>	<u>3</u>	<u>3</u>	<u>3</u>
	<u>10</u>	<u>60</u>			<u>16</u>

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{60}{10} = 6$$

$$\text{MD} = \frac{\sum f|x - \bar{x}|}{\sum f} = \frac{16}{10} = 1.6$$

In calculating the mean deviation of a set of observations from the examples provided, it should be noted that some of the results of the deviation of each observation from the mean are negative. These were later made positive when their absolute values were found. The idea of changing the negative value to the positive for mean deviation to be calculated renders mean deviation (as a measure of dispersion) not so reliable, though it is more reliable than the range.

### Exercise 2

- The ages of people in a hospital ward are distributed as follows.

<b>Age (years)</b>	2–6	7–11	12–16
<b>No. of people</b>	3	2	5

Find the mean deviation of the distribution. (NECO)

2. The table below shows the distribution of marks scored by 50 students in a Mathematics test.

<b>Marks</b>	<b>Number of students</b>
1–5	4
6–10	12
11–15	6
16–20	9
21–25	11
26–30	8

Find the mean deviation of the distribution.

3. The dress sizes of 45 women are given in the table below:

<b>Size</b>	20	22	24	26	28	28
<b>Frequency</b>	4	5	6	10	12	8

Calculate the mean deviation for the distribution.

4. Calculate the mean deviation of the frequency table below:

<b>Class</b>	1–5	6–10	11–15	16–20	21–25
<b>Frequency</b>	2	4	6	5	3

5. The marks of 50 pupils in a private school in a mid-term test are grouped as follows:

<b>Percentage</b>	0–9	10–19	20–29	30–39	40–49
<b>Frequency</b>	5	15	13	7	10

Find the mean deviation of the distribution.

6. Construct a frequency distribution for the data below and then calculate the mean deviation of the distribution.

48 50 54 42 51 51 52 49 47 48  
46 52 58 41 40 51 57 49 47 55  
57 50 45 53 45 50 44 43 61 59

7. The table below shows the number of oranges consumed by 40 women during fasting period.

<b>No. of oranges</b>	1	2	3	4	5
<b>Frequency</b>	8	5	9	10	8

8. The distribution of ages of a group of 30 farmers is given in the table below. Find the mean deviation of the distribution.

<b>Age (years)</b>	30	31	32	33	34
<b>Frequency</b>	4	5	7	8	6

9. The table below shows the distribution of 40 students on an excursion in five buses.

<b>Bus</b>	1	2	3	4	5
<b>No. of students</b>	8	7	10	6	9

Calculate the mean deviation of the distribution.

10. The table below shows the distribution of ages of 50 students in a secondary school.

<b>Age (years)</b>	10–12	13–15	16–18	19–24	22–24
<b>Frequency</b>	10	16	8	7	9

Find the mean deviation of the distribution.

**(c) Variance**

Variance is the mean of the squared deviations from the mean. It is an important measure of dispersion. It is denoted by  $S^2$  or  $\bar{f}f^2$ .

Variance of a set of numbers is obtained by finding:

- (i) The mean of the numbers.
- (ii) The deviation of each number from the mean.
- (iii) The square of each of the deviations from the mean.
- (iv) The mean of the squared deviations.

Hence, the formula for calculating variance is:

$$\sigma^2 = S^2 = \frac{\sum d^2}{n}$$

where

$$d = x - \bar{x}$$

or for a frequency distribution,

$$\sigma^2 = S^2 = \frac{\sum f d^2}{\sum f} \quad (f \text{ is the frequency})$$

Similarly, for a grouped frequency distribution:

$$\sigma^2 = S^2 = \frac{\sum f x^2}{\sum f} - \left( \frac{\sum f x}{\sum f} \right)^2$$

$$\sigma^2 = S^2 = \frac{\sum f d^2}{\sum f} - \left( \frac{\sum f d}{\sum f} \right)^2$$

$$\sigma^2 = S^2 = \frac{\sum f u^2}{\sum f} - \left( \frac{\sum f u}{\sum f} \right)^2$$

**Worked Example 7**

Calculate the variance of the set of numbers 4, 5, 6, 7, 8, 9, 10.

**SOLUTION**

$$\bar{x} = \frac{4 + 5 + 6 + 7 + 8 + 9 + 10}{7} = \frac{49}{7} = 7$$

$$S^2 = \frac{(4 - 7)^2 + (5 - 7)^2 + (6 - 7)^2 + (7 - 7)^2 + (8 - 7)^2 + (9 - 7)^2 + (10 - 7)^2}{7}$$

$$= \frac{(-3)^2 + (-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2}{7}$$

$$= \frac{9 + 4 + 1 + 0 + 1 + 4 + 9}{7} = 5.4$$

**Worked Example 8**

What is the variance of the following marks scored in English language out of 70 marks?

**Table 15.10**

Range of marks out of 70 marks	45–49	50–54	55–59	60–64	65–69
No. of students	3	1	1	5	10

**SOLUTION**

Table 15.11

No. of Marks	Mid students values	$f_x$	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
45–49	3 47	141	-14.5	210.25	630.75
50–54	1 52	52	-9.5	90.25	90.25
55–59	1 57	57	-4.5	20.25	20.25
60–64	5 62	310	0.5	0.25	1.25
65–69	10 67	670	5.5	30.25	302.5
	<u>20</u>	<u>1 230</u>			<u>1 045</u>

$$\bar{x} = \frac{\sum f_x}{\sum f} = \frac{1 230}{20} = 61.5$$

$$S^2 = \text{Variance} = \frac{\sum f(x - \bar{x})^2}{\sum f}$$

$$= \frac{1 045}{20} = 52.25$$

#### (d) Standard deviation

The *standard deviation*, sometimes referred to as the *root mean squared deviation*, is also measured from the mean.

Standard deviation is the square root of the squared deviation from the mean.

It is also a reliable measure of dispersion because its calculation affects every value in a distribution. Simply, standard deviation is the square root of variance and it is denoted by  $S$  or  $\bar{f}f$ .

##### 1. Real mean

For a set of ungrouped data, the formula is:

$$\sigma = S = \sqrt{\frac{\sum d^2}{n}}$$

where  $d = x - \bar{x}$ .

Worked Example 9

Find the standard deviation of the numbers 4, 7, 3, 2, 8, 6.

**SOLUTION**

$$S = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\bar{x} = \frac{4 + 7 + 3 + 2 + 8 + 6}{6} = \frac{30}{6} = 5$$

$$S = \sqrt{\frac{(4 - 5)^2 + (7 - 5)^2 + (3 - 5)^2 + (2 - 5)^2 + (8 - 5)^2 + (6 - 5)^2}{6}}$$

$$= \sqrt{\frac{(-1)^2 + (2)^2 + (-2)^2 + (-3)^2 + (3)^2 + (1)^2}{6}}$$

$$= \sqrt{\frac{1 + 4 + 4 + 9 + 9 + 1}{6}} = \sqrt{\frac{28}{6}}$$

$$= \sqrt{4.67} = 2.16$$

Worked Example 10

Calculate the standard deviation of the numbers 4.2, 5.6, 3.8, 6.2, 5.4.

**SOLUTION**

$$\bar{x} = \frac{4.2 + 5.6 + 3.8 + 6.2 + 5.4}{5} = \frac{25.2}{5} = 5.04$$

$$\sigma = S = \sqrt{\frac{(4.2 - 5.04)^2 + (5.6 - 5.04)^2 + (3.8 - 5.04)^2 + (6.2 - 5.04)^2 + (5.4 - 5.04)^2}{5}}$$

$$= \sqrt{\frac{0.7056 + 0.3136 + 1.5376 + 1.3456 + 0.1296}{5}}$$

$$= \sqrt{\frac{4.032}{5}}$$

$$= \sqrt{0.8064}$$

$$= 0.8980 = 0.90 \text{ (2 decimal places)}$$

For a frequency distribution, use

$$S = \sqrt{\frac{\sum f d^2}{\sum f}} \text{ where } d = x - \bar{x}$$

$\bar{x}$  is the mean and  $f$  is the frequency.

### Worked Example 11

The following are the ages (in years) of 12 pupils in a given class: 15, 16, 17, 13, 15, 13, 11, 14, 12, 11, 13, 12.

Calculate the standard deviation of the ages.

### SOLUTION

Table 15.12

Age (years)	Tally	Frequency	$fx$	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
11	//	2	22	-2.5	6.25	12.50
12	//	2	24	-1.5	2.25	4.50
13	///	3	39	-0.5	0.25	0.75
14	/	1	14	0.5	0.25	0.25
15	//	2	30	1.5	2.25	4.50
16	/	1	16	2.5	6.25	6.25
17	/	1	17	3.5	12.25	12.25
		<u>12</u>	<u>162</u>			<u>41.00</u>

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{162}{12} = 13.5$$

$$S = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{41}{12}} = \sqrt{3.417}$$

$$= 1.848$$

$$\therefore \sigma = S = 1.85 \text{ (2 decimal places)}$$

### 2. Taking deviations from an assumed mean

In this case, deviations from an assumed mean will be considered, but not deviations from the mean. The formula to use here is:

$$\sigma = S = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

Where  $d = x - \bar{A}$  and  $\bar{A}$  is the assumed mean.

### Worked Example 12

Use the method of an assumed mean to calculate the standard deviation of the data in worked example 11.

### SOLUTION

**Table 15.13**

Age (years)	f	A = 14				
(x)	Frequency	$d = x - A$	fd	$d^2$	$fd^2$	
11	2	-3	-6	9	18	
12	2	-2	-4	4	8	
13	3	-1	-3	1	3	
14	1	0	0	0	0	
15	2	1	2	1	2	
16	1	2	2	4	4	
17	<u>1</u>	3	<u>3</u>	9	<u>9</u>	
	<u>12</u>		<u>-6</u>		<u>44</u>	

$$\begin{aligned}
 S &= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \\
 &= \sqrt{\frac{44}{12} - \frac{1}{4}} = \sqrt{\frac{44 - 3}{12}} \\
 &= \sqrt{\frac{41}{12}} = \sqrt{3.417} \\
 &= 1.848 = 1.85 \text{ (2 decimal places)}
 \end{aligned}$$

### Worked Example 13

Calculate the mean and standard deviation of the distribution shown in the table below.

**Table 15.14**

x	1	2	3	4	5	6
f	8	18	22	26	20	6

### SOLUTION

**Table 15.15**

x	f	A = 4	$d = x - A$	fd	$d^2$	$fd^2$
1	8		-3	-24	9	72
2	18		-2	-36	4	72

3	22	-1	-22	1	22
4	26	0	0	0	0
5	20	1	20	1	20
6	<u>6</u>	2	<u>12</u>	4	<u>24</u>
	<u>100</u>		<u>-50</u>		<u>210</u>

$$\bar{x} = \text{Mean} = A + \frac{\sum fd}{\sum f} = 4 + \left( \frac{-50}{100} \right)$$

$$= 4 - 0.5 = 3.5$$

$$S = \sqrt{\frac{\sum fd^2}{\sum f} - \left( \frac{\sum fd}{\sum f} \right)^2}$$

$$= \sqrt{\frac{210}{100} - \left( \frac{-50}{100} \right)^2}$$

$$= \sqrt{\frac{21}{10} - \left( -\frac{1}{2} \right)^2} = \sqrt{\frac{21}{10} - \frac{1}{4}}$$

$$= \sqrt{\frac{42 - 5}{20}} = \sqrt{\frac{37}{20}} = \sqrt{1.85}$$

$$= 1.36$$

#### Worked Example 14

62 54 53 44 46 55 46 56 68 63  
 59 61 66 54 39 48 47 53 59 57  
 50 35 40 30 46 44 36 49 54 51  
 57 56 45 33 38 41 40 45 53 58  
 51 45 48 34 36 46 43 49 63 52

(a) Using the class intervals of 30–34, 35–39, 40–44, etc., construct the frequency distribution table.

(b) Calculate the

(i) Mean

(ii) Standard deviation (all answers correct to 2 decimal places). (NECO)

#### SOLUTION

Method 1

Table 15.16

Class interval	Tally	f Frequency	Class mark (x)	fx	x <sup>2</sup>	fx <sup>2</sup>
30–34	///	3	32	96	1 024	3 072
35–39	/// /	5	37	185	1 369	6 845
40–44	/// / /	6	42	252	1 764	10 584
45–49	/// / / / /	12	47	564	2 209	26 508
50–54	/// / / /	10	52	520	2 704	27 040
55–59	/// / / /	8	57	456	3 249	25 992
60–64	/// /	4	62	248	3 844	15 376
65–69	//	<u>2</u>	67	<u>134</u>	<u>4 489</u>	<u>8 978</u>
		<u>50</u>		<u>2 455</u>		<u>124 395</u>

$$\sigma = \bar{x} = \text{Mean} = \frac{\sum fx}{\sum f} = \frac{2455}{50} = 49.10 = 49.1$$

$$S = \text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{124395}{50} - \left(\frac{2455}{50}\right)^2}$$

$$= \sqrt{2487.9 - (49.1)^2}$$

$$= \sqrt{2487.9 - 2401.81}$$

$$= \sqrt{86.09}$$

$$= 8.78 \text{ (2 decimal places)}$$

Method 2 (Assumed mean)

Table 15.17

Class interval	Frequency	Class interval	$d = x - A$	$fd$	$d^2$	$fd^2$
30–34	3	32	-15	-45	225	675
35–39	5	37	-10	-50	100	500
40–44	6	42	-5	-30	25	150
45–49	12	47	0	0	0	0
50–54	10	52	5	50	25	250
55–59	8	57	10	80	100	800
60–64	4	62	15	60	225	900
65–69	2	67	20	40	400	800
	<u>50</u>			<u>105</u>		<u>4075</u>

$$\bar{x} = \text{Mean} = A + \frac{\sum fd}{\sum f} = 47 + \frac{105}{50} = 47 + 2.1$$

$$= 49.1$$

$= \sigma = S = \text{Standard deviation}$

$$= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$= \sqrt{\frac{4075}{50} - \left(\frac{105}{50}\right)^2}$$

$$= \sqrt{81.5 - (2.1)^2}$$

$$= \sqrt{81.5 - 4.41} = \sqrt{77.09}$$

$$= 8.78$$

Method 3

Table 15.18

(f)	Class Frequency	mark (x)	A = 47	c = 5	$d = x - A$	$U = \frac{d}{c}$	fu	$u^2$	$fu^2$
30–34	3	32	-15	-3	-9	9	27		
35–39	5	37	-10	-2	-10	4	20		
40–44	6	42	-5	-1	-6	1	6		
45–49	12	47	0	0	0	0	0		
50–54	10	52	5	1	10	1	10		

(f)	Class	$A=47$	$c=5$				
	Frequency mark (x)	$d=x-A$	$U=\frac{d}{c}$	$fu$	$u^2$	$fu^2$	
55–59	8	57	10	2	16	4	32
60–64	4	62	15	3	12	9	36
65–69	2	67	20	4	8	16	32
	<u>50</u>			<u>21</u>		<u>163</u>	

$$\begin{aligned}
 \bar{x} &= \text{Mean} = A + c \frac{\sum fu}{\sum f} \\
 &= 47 + \left( 5 \times \frac{21}{50} \right) \\
 &= 47 + 2.1 = 49.1 \\
 &= \sigma = S = \text{Standard deviation} \\
 &= c \sqrt{\frac{\sum fu^2}{\sum f} - \left( \frac{\sum fu}{\sum f} \right)^2} \\
 &= 5 \sqrt{\frac{163}{50} - \left( \frac{21}{50} \right)^2} = 5 \sqrt{3.26 - (0.42)^2} \\
 &= 5 \times \sqrt{3.26 - 0.1764} = 5 \times \sqrt{3.0836} \\
 &= 5 \tilde{A} - 1.756 \\
 &= 8.78 \text{ (2 decimal places)}
 \end{aligned}$$

### Worked Example 15

The table below shows the masses in kg of 35 students in a particular school.

45	43	54	52	57	59	65
50	61	50	48	53	61	66
47	52	48	40	44	60	68
51	47	51	41	50	62	70
58	42	51	49	55	71	60

- Using the above given data, construct a group frequency table with class interval 40–44, 45–49, 50–54, etc.
- From the table, using an assumed mean of 52.0 and correct to 2 decimal places, calculate the
  - Mean
  - Standard deviation (NECO)

### SOLUTION

Table 15.19

Tally	Frequency	Class mark	$x-A$	$fd$	$d^2$	$fd^2$
40-44 //	5	42	-10	-50	100	500
45-49 ////	6	47	-5	-30	25	150
50-54 /////	10	52	0	0	0	0
55-59 ////	4	57	5	20	25	100
60-64 //	5	62	10	50	100	500
65-69 //	3	67	15	45	225	675
70-74 //	2	72	20	40	400	800
	<u>35</u>			<u>75</u>		<u>2725</u>

$$\begin{aligned}\bar{x} &= \text{Mean} = A + \frac{\sum fd}{\sum f} \\ &= 52 + \frac{75}{35} \\ &= 52 + 2.143 \\ &= 54.143 \\ &= 54.14 \text{ (2 decimal places)} \\ \sigma &= S = \text{Standard deviation} = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \\ &= \sqrt{\frac{2725}{35} - \left(\frac{75}{35}\right)^2} \\ &= \sqrt{\frac{545}{7} - \left(\frac{15}{7}\right)^2} \\ &= \sqrt{\frac{3815 - 225}{49}} \\ &= \sqrt{\frac{3590}{49}} \\ &= \sqrt{73.265} \\ &= 8.56 = 8.56 \text{ (2 decimal places)}\end{aligned}$$

**Exercise 3**

- Calculate the standard deviation of the numbers -6, -2, 5, 7, 11. (WAEC)
- The mean of the numbers 3, 7, 1, 6, 0, 4, 9, 2 is 4. Find, correct to 2 significant figures, the standard deviation of the numbers.
- The ages in years of four children in a family are 8, 10, 10 and 12. Find the standard deviation of their ages.  
(WAEC)
- What is the standard deviation, correct to 2 decimal places, of a distribution whose variance is 6.27?
- A group of eight students obtained the following scores in a test: 66, 36, 72, 46, 87, 56, 79, 94. Calculate the standard deviation of the scores.
- Find the positive value of  $y$ , if the standard deviation of 1,  $y + 1$  and  $2y + 1$  is  $\sqrt{6}$   
(NECO)
- using the table below, find the standard deviation.

Marks	3	4	5	6	7	8	9
No. of students	2	3	4	3	1	1	1

(NECO)

8. If 8 is the mean of the following set of numbers 3, 15, 5, 9, 12, 6, 10, 8, 4, calculate the standard deviation.

9. Calculate the standard deviation of the following marks: 2, 3, 6, 2, 5, 0, 4, 2.  
(WAEC)

10. Calculate the variance of the following quantities:  $(x + 2)$ ,  $(x + 1)$ ,  $(x + 2)$  and  $(x + 3)$ .  
(WAEC)

11. In a college, the number of absentees recorded over a period of 30 days was as shown in the frequency distribution table.

No. of absentees	0-4	5-9	10-14	15-19	20-24
No. of days	1	5	10	9	5

Calculate the

- (a) Mean.  
(b) Standard deviation correct to 2 decimal places. (WAEC)

12. Classify the following marks of an English examination in standard deviation. Use class interval 21–30, 31–40, etc. for the classification.

77 62 77 94 92 66 53 70 80 34  
56 52 67 51 43 80 85 81 63 85  
39 60 62 72 62 34 74 51 71 43  
57 47 51 70 40 35 42 66 27 24  
97 42 53 55 72 71 59 51 87 45

## SUMMARY

In this chapter, we have learnt the following:

- â– Dispersion implies spread, scatter or vary and measure of dispersion is the measure of the extent in which numerical data cluster around a measure of location.
- â– Range as a measure of dispersion is the difference between the highest value and the lowest value.
- â– Mean deviation is the average of the algebraic sum of the absolute deviation of each observation from the mean.
- â– Algebraic sum of the deviations from the mean is always zero.
- â– Variance is the mean squared deviation from the mean and it is denoted by  $S^2$  or  $r^2$ .
- â– Standard deviation is the root mean squared deviation from the mean and it is the square root of the variance.
- â– Standard deviation of data can be found using the following methods:
  - (a) Taking deviations from the mean.
  - (b) Taking deviations from the assumed mean.
  - (c) Coding.
- â– Standard deviation is denoted by  $S$  or  $r$ .

## GRADUATED EXERCISES

1. Calculate the range of the distribution below.

Length (cm)	10	11	12	13	14	15
Frequency	4	5	6	10	12	8

2. Find the range of the marks scored by 20 students in a biology test in a secondary school.
- 45 57 56 33 38 41 40 53 58 42  
48 51 45 34 36 46 43 63 52 35
3. What is the range of the following numbers: -6, -2, 5, 7, 11, 3, 7, 1, 6, 0, 4, 9, 2, 15.
4. Given that the mean of 4, 6, 9, 15, 18 and  $x$  is 9, find the range of the numbers.
5. Calculate the range of the mean, median and mode of 33, 54, 60, 68, 35, 33, 18 and 19.

6. The table below shows the age distribution of 100 farmers in a village

Age	30–39	40–49	50–59	60–69	70–79	80–89
Frequency	11	5	45	15	10	14

Calculate the mean deviation of the distribution.

7. Calculate the mean deviation of the distribution of marks in the table below.

Marks	0–2	3–5	6–9	9–11	12–14	15–17
No. of students	5	6	4	5	1	1

8. Calculate the mean deviation of the distribution.

Class	60–64	65–69	70–74	75–79	80–84	85–89	90–94
Frequency	4	10	12	12	8	6	2

9. The mean age of the numbers 3, 7, 1, 6, 0, 4, 9, 2 is 4. Find, correct to 2 significant figures, the standard deviation of the numbers.

10. The weights to the nearest kilogramme of a group of 50 students in a College of Technology are given: 65, 70, 60, 46, 51, 55, 59, 63, 68, 53, 47, 53, 72, 53, 67, 62, 64, 70, 57, 56, 73, 56, 48, 51, 58, 63, 65, 62, 49, 64, 53, 59, 63, 50, 48, 72, 67, 56, 61, 64, 66, 52, 49, 62, 71, 58, 53, 69, 63, 59. (a) Prepare a grouped frequency table with class intervals 45–49, 50–54, 55–59 etc. (b) Using an assumed mean of 62. Calculate the mean and standard deviation of the groups data.

11. The table below shows the frequency distribution of the ages of people who attend a village meeting.

Age (years)	21–30	31–40	41–50	51–60	61–70	71–80
Frequency	3	10	14	10	8	5

Calculate the:

(a) Mean.

(b) Standard deviation of the distribution.

12. The table below shows the distribution of the ages, in years, of 120 members of a club.

Age (years)	21–25	26–30	31–35	36–40	41–45	46–50	51–55
Frequency	1	14	42	30	20	10	3

Calculate the

(a) Mean.

(b) Standard deviation of the distribution correct to three significant figures.

(WAEC)

13. The weights, to the nearest kilogramme, of a group of 50 students in a college of technology are given below:

65	70	60	46	51	55	59	63	68	53
47	53	72	53	67	62	64	70	57	56
73	56	48	51	58	63	65	62	49	54
53	59	63	50	48	72	67	56	61	64
66	52	49	62	71	58	53	69	63	59

(a) Prepare a grouped frequency table with class intervals 45–49, 50–54, 55–59, etc

(b) Using an assumed mean of 62 or otherwise, calculate the mean and the standard deviation of the grouped data, correct to 1 decimal place.

14. Many families were asked about the number of children they have. The results are as follows:

No. of children	1	2	3	4	5	6
Frequency	10	15	8	6	4	7

(a) Calculate the:

(I) Mean.

(II) Median.

(b) Find the standard deviation of the distribution correct to 2 decimal places.

(Waec)

15. In a certain village, the distribution of all the males between 1 and 80 years is as follows:

Age (years)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of people	9	13	20	18	15	12	6	5

Calculate the:

(a) Mean.

(b) Standard deviation.