CHAPTER 11: Circle Theorem

OBJECTIVES

At the end of the chapter, students should be able to:

2. Prove that the angle which an arc subtends at the centre is twice the angle it subtends at the circumference.
3. Solve practical problems on the theorem correctly.
4. Prove that the angles in the same segment of a circle are equal.
5. Prove that the angle in a semicircle is a right angle.
6. Prove that the opposite angles in a cyclic quadrilateral are supplementary.
7. Prove that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
8. Prove that the riders are tangents to a circle.
9. Solve practical problems on circle theorem.

I. Circle Theorem

A theorem is a statement that has been proved. It starts with certain assumptions that will help establish the result and ends with a conclusion. Specifically, while writing the proof of a theorem, like â€˜circle theoremâ€™, there are necessary and sufficient procedures that must be adopted.

Step1: State what is given.
Step2: State what is required to prove.
Step3: State the construction (if any).
Step4: Sketch the diagram with the aid of the information given in the first three steps.
Step5: State the proof with all necessary reasons and with appropriate abbreviations.
Step6: Conclusion.

Theorem 1

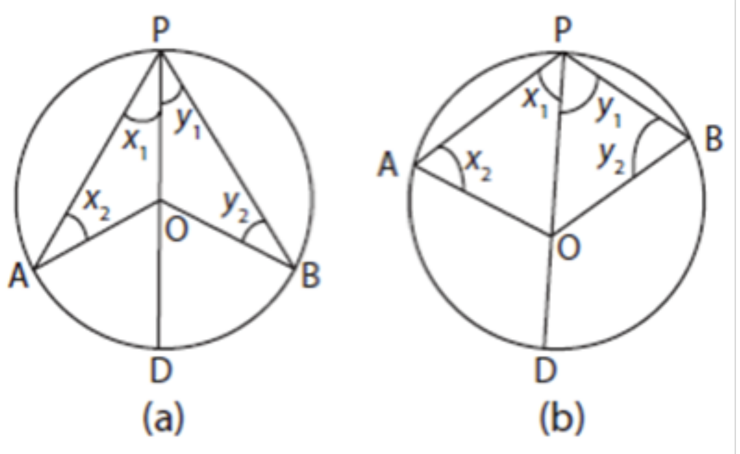
The angle which an arc (or a chord) of a circle subtends at the centre of the circle is twice the angle which it subtends at any point on the remaining part of the circumference. We shall be using the three diagrams as shown in Figure 11.1.

Given: An arc AB of a circle with centre â€˜Oâ€™ and a point â€˜Pâ€™ on the circumference (see Figure 11.1).

Required to prove: AOË† B = 2APË† B

Construction: Join and produce the line to a point D.

Sketch:



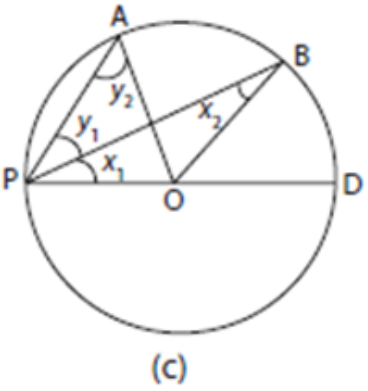


Figure 11.1

Proof: **Using letters in Figure 11.1**
Since /AO/ = /OP/ (radii in the same circle)
x_1 = x_2 (base angles of isosceles AOË† P)
â‡’ AOË† D = x_1 + x_2 (exterior angle of AOË† P)
â‡’ AOË† D = 2x_1 (since x_1 = x_2)
Similarly, BOË† D = 2y_1
In Figure 11.1(a), acute/obtuse AOË† B =
AOË† D + BOË† D.
In Figure 11.1(b), refl ex AOË† B = AOË† D + BOË† D
= 2x_1 + 2y_1
= 2(x_1 + y_1)
= 2APË† B
In Figure 11.1(c), AOË† B = AOË† D âˆ' BOË† D
= 2y_1 âˆ' 2x_1
= 2(y_1 âˆ' x_1)
= 2APË† B
âˆ´ AOË† B = 2APË† B (in all cases).

Worked Example 1

Find the lettered angles in the diagram in Figure 11.2.

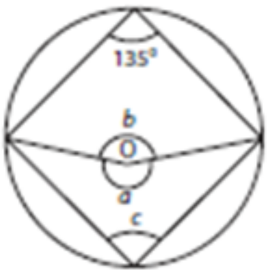


Figure 11.2

SOLUTION

$\frac{a}{2}$ = 135Â° (angle at the centre is twice the angle at the circumference)
®‡'a = 2 Ã— 135Â°
®ˆ× a = 270Â°
b = 360Â° âˆ' a (angle at a point)
®‡' b = 360 âˆ' 270
®ˆ× b = 90Â°

c = $\frac{b}{2}$ (angle at the centre is twice the angle at the circumference)

®‡' c = $\frac{90}{2}$ Â°
®ˆ× c = 45Â°

Worked Example 2

Given a circle with centre O while A, B and C are points on the circumference, find ®ˆ€ABC, if the obtuse ®ˆ€AOC = 125.

SOLUTION

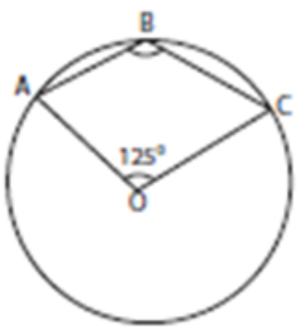


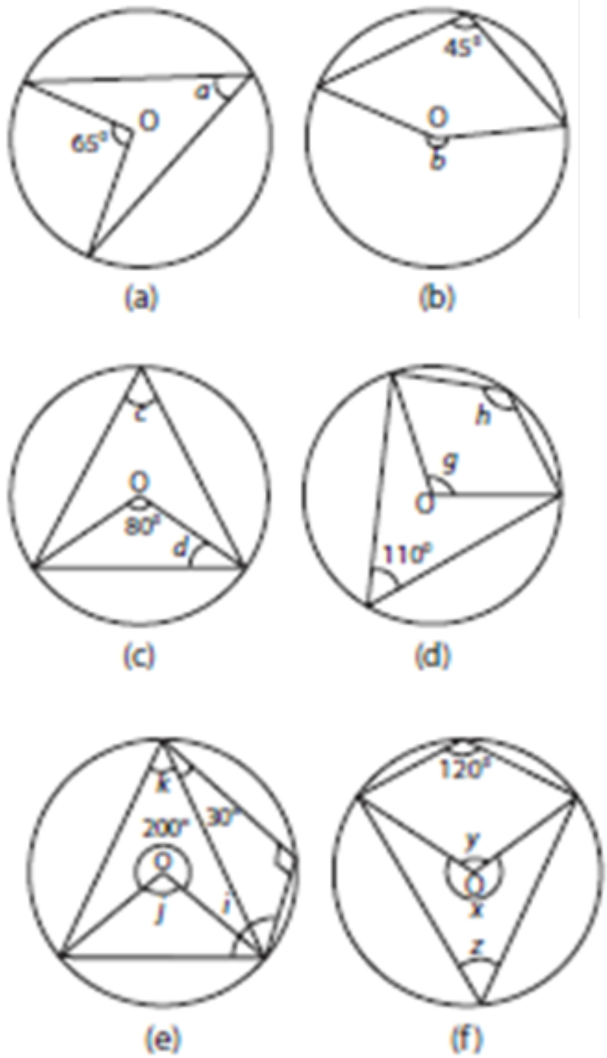
Figure 11.3

Reflex AOË† C = 360Â° âˆ’ 125Â° (angle at a point) = 235Â°

®ˆ× ABC = $\frac{235}{2}$ **Â° (angle at the centre is twice the angle at the circumference)**
= 117.5Â°

Exercise 1

1. Find the lettered angles in each of the fi gures below:



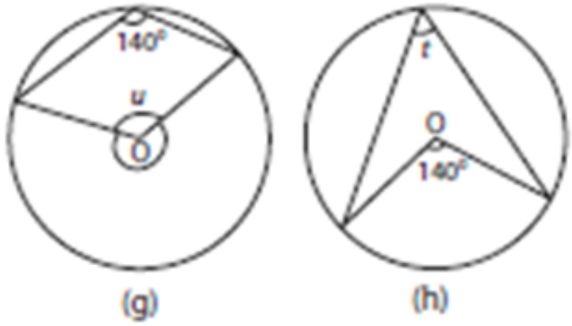


Figure 11.4

2.

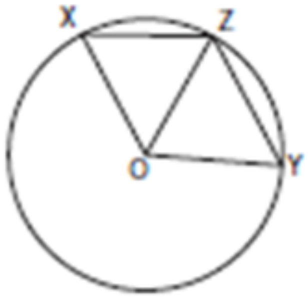


Figure 11.5

In Figure 11.5, O is the centre of the circle, and X, Y and Z are points on the circumference. Prove that reflex ®ˆ€XOY = 2XZY.

3. Calculate ®ˆ€ABC in Figure 11.6, if the circle with centre O passes through the points A, B and C with ®ˆ€BAC = 47ℜ° and ®ˆ€ACB = 64ℜ°.

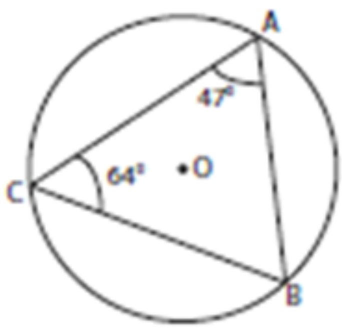


Figure 11.6

4. In Figure 11.7, O is the centre of the circle. If ®ˆ€EOG = 50ℜ° and ®ˆ€EFG = 5x, find the value of x.

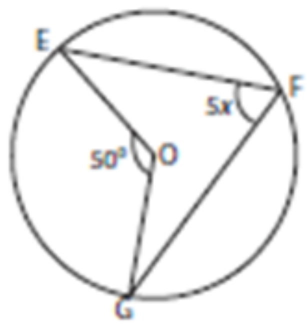


Figure 11.7

5. (a) Prove that the angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.
(b) In the diagram in Figure 11.8, O is the centre of the circle, ®ˆ€OQR = 32ℜ° and ®ˆ€MPQ = 15ℜ°, calculate (i) QPR and (ii) ®ˆ€MQO. (WAEC)

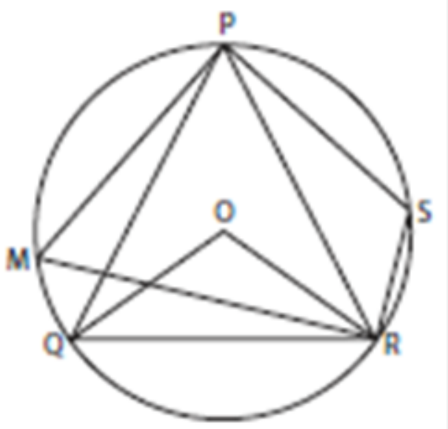


Figure 11.8

6. In the diagram below, O is the centre of the circle ACB. If ®ˆ€CAO = 26ℜ° and ®ˆ€AOB = 130ℜ°, calculate (a) ®ˆ€OBC and (b) ®ˆ€COB. (WASSCE)

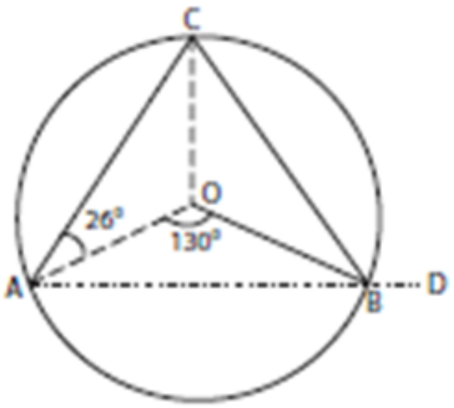


Figure 11.9

Theorem 2

Angles in the same segment of a circle are equal.

Given: **Points A, B and C on the major segment of a circle ABCDE with centre O.**

Required to prove: **®ˆ€EAD =®ˆ€EBD =®ˆ€ECD**

Construction: **Join EO; Join DO**

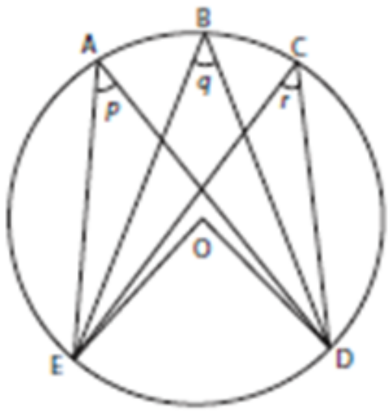


Figure 11.10

Proof:

EOË† D = 2p (angle at the centre = 2 Ã— angle at the circumference)

EOË† D = 2q (angle at the centre = 2 Ã— angle at the circumference)

EOË† D = 2r (angle at the centre = 2 Ã— angle at the circumference)

®‡’ p = q = r

®ˆ× E℘,D = EBË† D = EÄˆD

Worked Example 3

In the diagram below, if ®ˆ€CAD = 3y and ®ˆ€CBD = 6x, express y in terms of x.

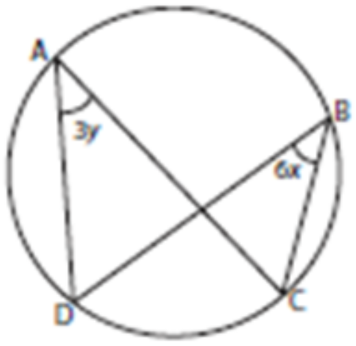


Figure 11.11

SOLUTION

3y = 6x (angle at the same segment)

y = 6/3x

®ˆ× **y = 2x**

Worked Example 4

The diagram below shows a circle ABCD in which ®ˆ€DAC = 55ℜ° and ®ˆ€BCD = 100ℜ°, find BDC.

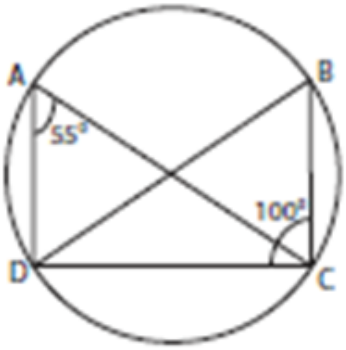


Figure 11.12

SOLUTION

®ˆ€**CAD =** ®ˆ€**CBD = 55ℜ° (angles on the same segment)**

®ˆ× ®ˆ€**BDC +** ®ˆ€**CBD +** ®ˆ€**BCD = 180ℜ° (sum of the angles of a triangle)**

â‡’âˆ BDC + 55 ^{O} + 100ℜ° = 180ℜ°

â‡’ âˆ BDC = 180^{O} âˆ’ 155^{O}

âˆ´ âˆ BDC = 25^{O}

Exercise 2

1. Find the lettered angles in each of the figures below:

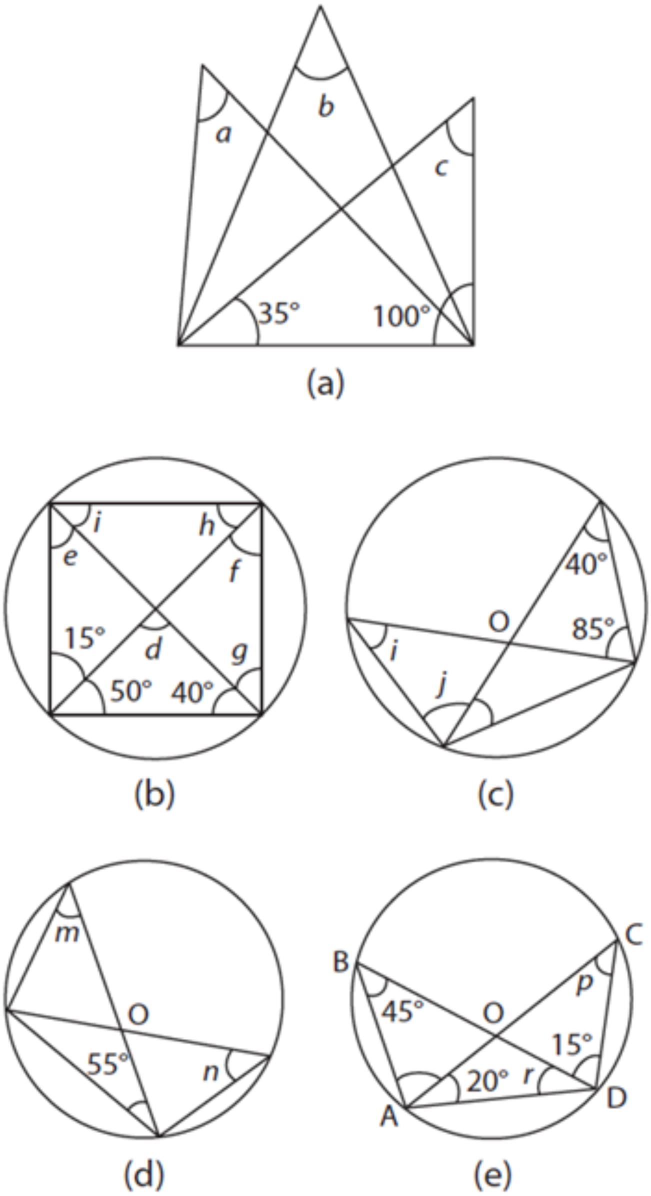


Figure 11.13

2. The diagram below is a circle with its centre at O. Find the value of (a) x and (b) y.

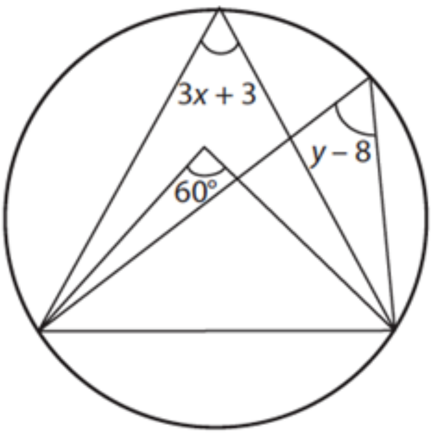


Figure 11.14

3. In the diagram below, PQRS is a circle. If /PT/ = /QT/ and âˆ QPT = 70^{O}, calculate âˆ PRS? WASSCE

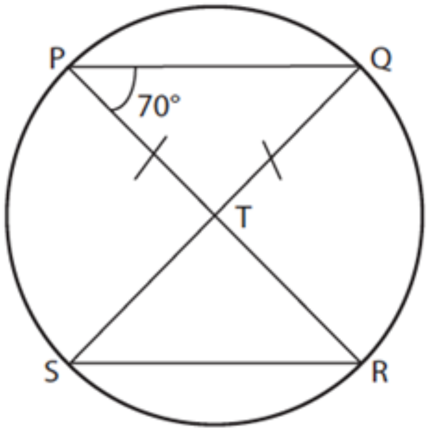


Figure 11.15

4. In Figure 11.16, P, Q, R and S are points on the circle. If âˆ RQS = 30^{O}, âˆ PRS = 50^{O} and âˆ PSQ = 20^{O}, what is the value of x + y?

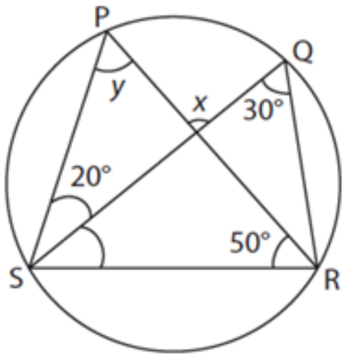


Figure 11.16

5. Figure 11.17 is a circle where AC and BD intersect at E. If âˆ CAD = 50^{O} and âˆ AEB = 80^{O}, calculate âˆ ACB.

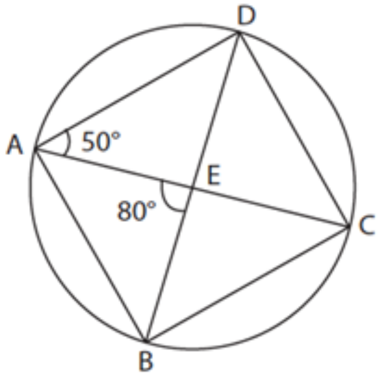


Figure 11.17

Theorem 3

Angle in a semicircle is a right angle.

Given: PQ is the diameter of a circle with centre O and R is any point on the circumference.

Required to prove: PRË‡ Q = 90^{O}

Construction: Join PR ; Join RQ

Sketch:

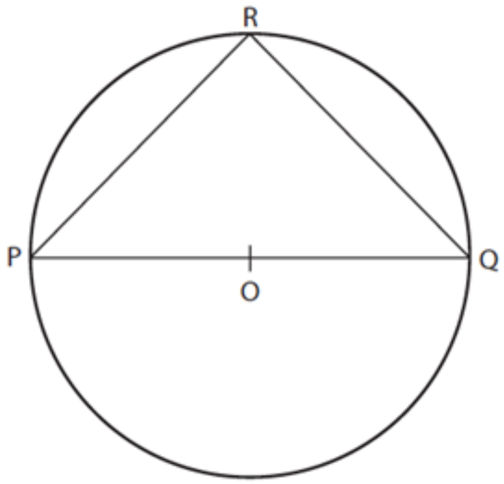


Figure 11.18

Proof: Using letters in Figure 11.18,
âˆ POË† Q = 2PRQ (angle at the centre
is twice the angle at the
circumference)
But POË† Q = 180^{O} (angle on a straight line)
âˆ´ 2PRQ = 180^{O}
â‡’ PRQ = _1_8_0_
2 = 90^{O}
âˆ´ PRQ = 90^{O}

Worked Example 5

In the diagram below, O is the centre of the circle. If âˆ BAC = 55$^{\text{Â}}$°, fi nd the value of âˆ ACB.

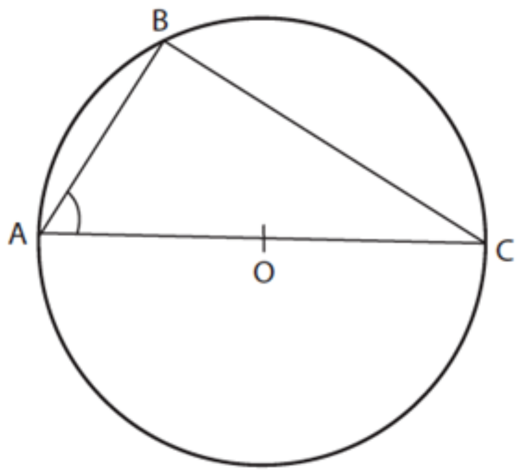


Figure 11.19

SOLUTION
âˆ ABC = 90$^{\text{Â}}$° (angle in a semicircle)
âˆ ABC + âˆ ACB + âˆ BAC = 180$^{\text{Â}}$° (sum of the
three angles of a triangle)
â‡’ 90$^{\text{Â}}$° + âˆ ACB + 55$^{\text{Â}}$° = 180$^{\text{Â}}$°
â‡’ 145$^{\text{Â}}$° + âˆ ACB = 180$^{\text{Â}}$°
â‡’ âˆ ACB = 180$^{\text{Â}}$° âˆ’ 145$^{\text{Â}}$°
âˆ´ âˆ ACB = 35$^{\text{Â}}$°

Worked Example 6

In the diagram below, O is the centre of the circle. If /BC/ = /BD/ and âˆ CBD = 56Ã…Ã£, calculate âˆ ACD.

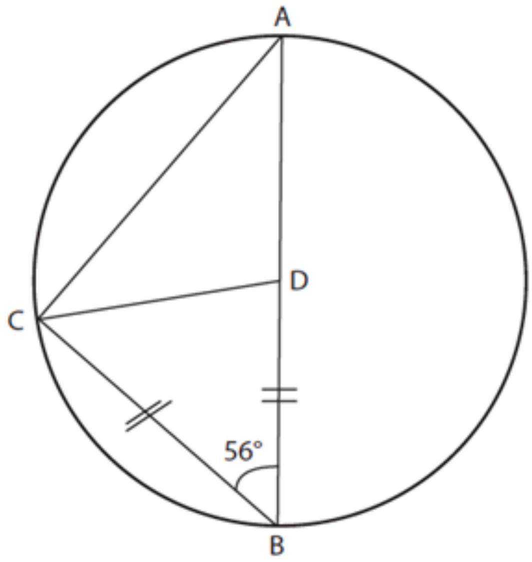


Figure 11.20

SOLUTION

$\angle BCD = \frac{180° - 56°}{2} = \frac{124°}{2} = 62°$

$\Rightarrow BCD = \angle BDC = 62°$ (base angles of an isosceles triangle are equal)

$\angle ACD + \angle BCD = 90°$ (angle in a semicircle)

$$\Rightarrow \angle ACD + 62 = 90°$$

$$\Rightarrow \angle ACD = 90 - 62$$

$$\Rightarrow \angle ACD = 28°$$

Exercise 3

1. Find the values of the lettered angles in the figures below.

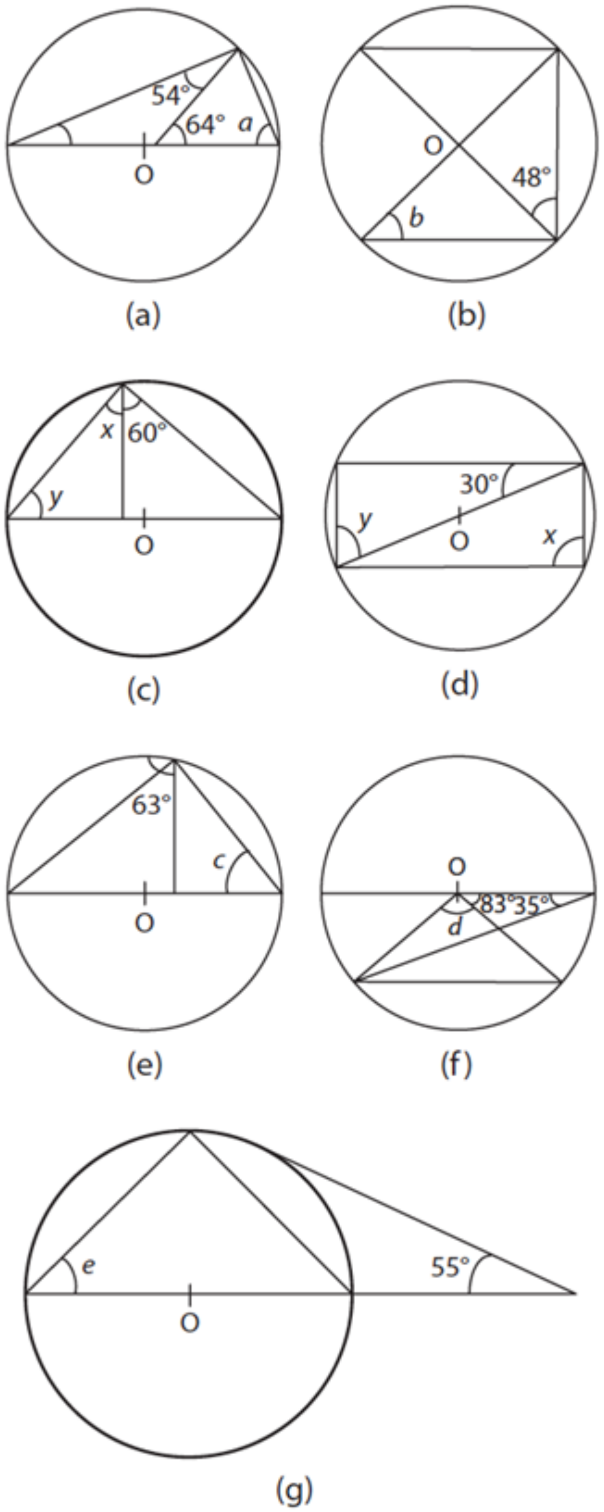


Figure 11.21

2. In the diagram in Figure 11.22, AB is the diameter. $\angle ABC = (5x + 3)°$ and $\angle BAC = (5y + 7)°$. Express y in terms of x.

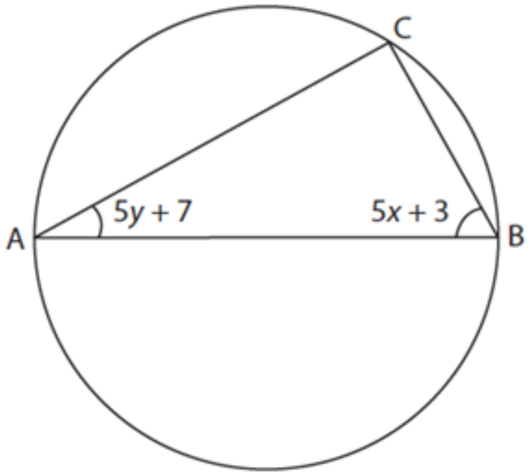


Figure 11.22

3. In Figure 11.23, O is the centre of the circle. QS is the diameter and $\angle QPR$ is 45°. Calculate $\angle SQR$.

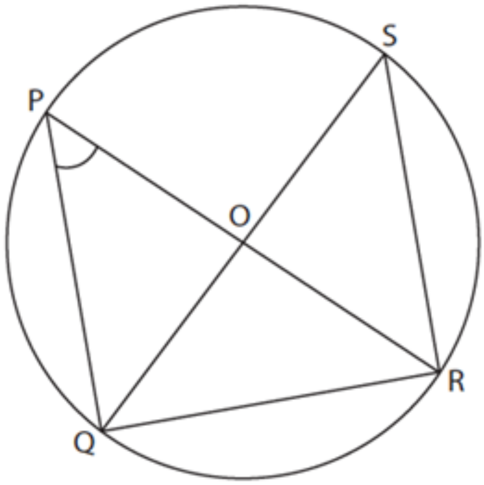


Figure 11.23

4. In Figure 11.24, O is the centre of the circle and MN is the diameter. If $\angle MNQ = 25°$ and $\angle NOR = 95°$ where $\overline{OQ}$ is parallel to $\overline{RS}$, find $\angle ORS$.

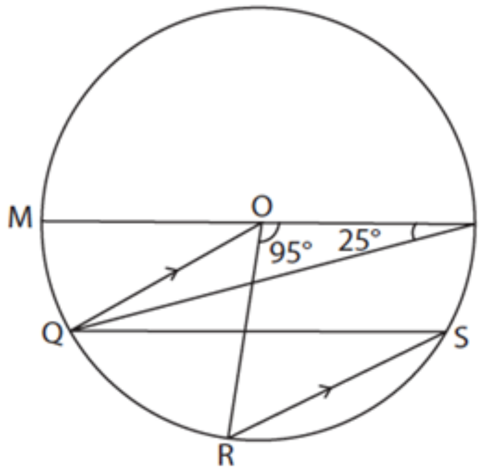


Figure 11.24

5. In Figure 11.25, O is the centre of the circle. Find the angles *a*, *b* and *c*.

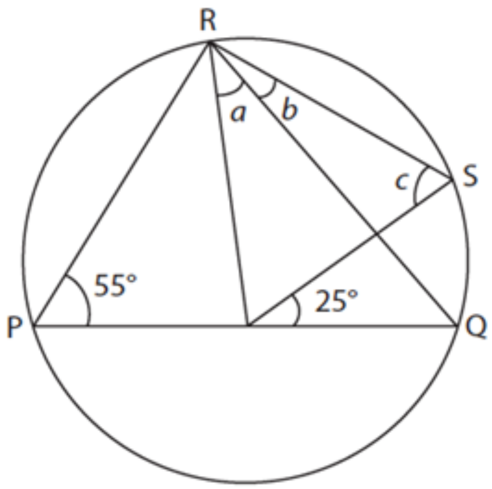


Figure 11.25

II. Cyclic Quadrilateral

(i) Quadrilateral is a four-sided plane shape. Examples of quadrilaterals are: square, rectangle, parallelogram, rhombus and kite.

(ii) A cyclic quadrilateral is a quadrilateral that is enclosed in a circle such that the four vertices touch the circumference of the circle. The four points where the vertices touch are referred to as concyclic points. In Figure 11.26 PQRS is a cyclic quadrilateral.

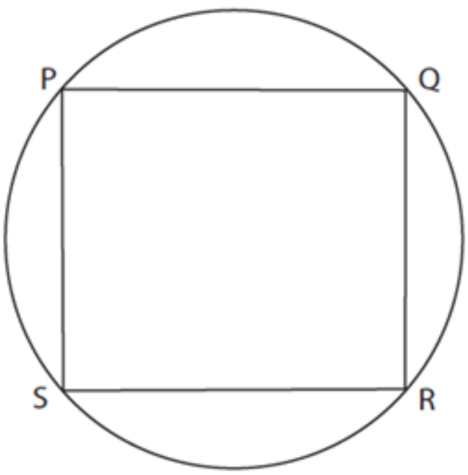


Figure 11.26 Cyclic quadrilateral

(iii) Two angles are said to be complementary, if their sum is 90Ã…Ã£ and supplementary, if their sum is 180Ã…Ã£.

Activity 11.1

(a) With the aid of a pair of compasses and a ruler, draw several cyclic quadrilaterals.
(b) Use a protractor to measure the angles of the quadrilaterals.
(c) Add the two opposite angles. What have you observed?

Theorem 4

The opposite angles in a cyclic quadrilateral are supplementary.
Given: A cyclic quadrilateral ABCD in a circle with centre O.
Required to prove: â^ BAD + â^ BCD = 180Ã…Ã£
Construction: Join OB ; Join OD (see Figure 11.27)
Sketch:

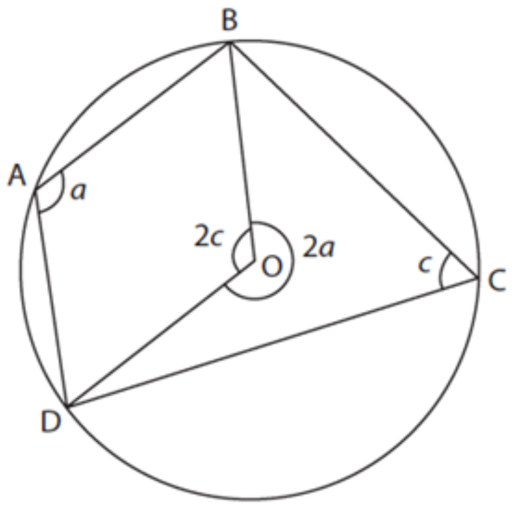


Figure 11.27

Proof: Using letters in Figure 11.27
Let BAD = a
Reflex BOË† D = 2a (angle at the centre is twice the angle at the circumference)
Let BCD = c
Obtuse BOË† D = 2c (angle at the centre is twice the angle at the circumference)
But 2a + 2c = 360Ã…Ã£ (angle at a point)
â‡’ 2(a + c) = 360Ã…Ã£
â‡’ a + c = 360/2Ã…Ã£
â^´ a + c = 180Ã…Ã£
From Theorem 4, Theorem 5 can be established.

Theorem 5 (Corollary)

The exterior angle of a cyclic quadrilateral is equal to interior opposite angles.
Using the letters in Figure 11.28,

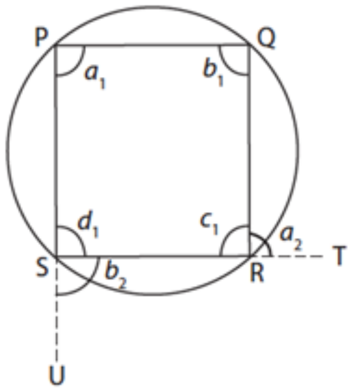


Figure 11.28

Given: A cyclic quadrilateral PQRS (see figure 11.28).
Required to prove: $\mathbf{a}_1 = \mathbf{a}_2$ and $\mathbf{b}_1 = \mathbf{b}_2$

Construction: Produce **SR** to **T** and **PS** to **U.**

Proof:

$\mathbf{a}_1 + \mathbf{c}_1 = 180$Ã…Ã£ (opposite angles of a cyclic quadrilateral) and

$\mathbf{a}_2 + \mathbf{c}_1 = 180$Ã…Ã£ (angles on a straight line)

â‡’ $\mathbf{a}_1 = \mathbf{a}_2$

Similarly,

$\mathbf{b}_1 + \mathbf{d}_1 = 180$Ã…Ã£ (opposite angles of a cyclic quadrilateral) and

$\mathbf{b}_2 + \mathbf{d}_1 = 180$Ã…Ã£ (angles on a straight line)

â‡’ $\mathbf{b}_1 = \mathbf{b}_2$

Worked Example 7

In Figure 11.30, O is the centre of the circle. Which of the following is/are false?

I. **a** = **b**

II. **b** + **c** = 180Ã…Ã£

III. a + **b** = **c**

A. I only

B. II only

C. III only

D. I and II

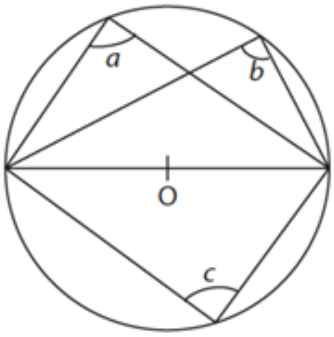


Figure 11.30

SOLUTION

C. Only III

Exercise 4

1. Find the lettered angles in each of the figures below.

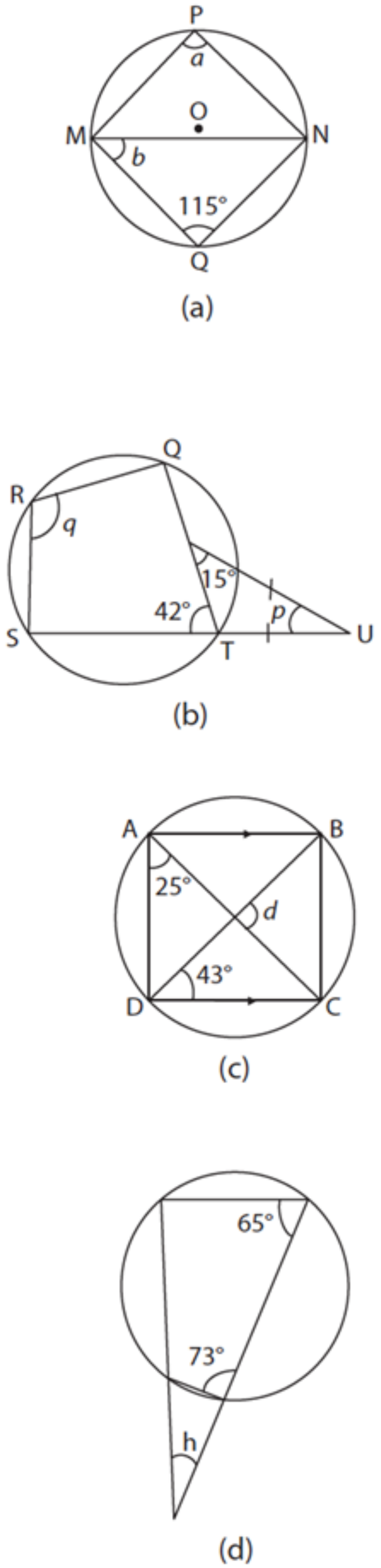


Figure 11.31

2. In Figure 11.32, O is the centre of the circle. **â ˆ** BED = 44**Ã…Ã£** and **â ˆ** ABE = 20**Ã…Ã£**. Calculate **â ˆ** DBE.

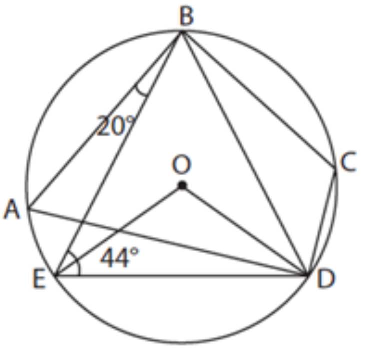


Figure 11.32

3. In Figure 11.33, O is the centre of the circle PQRST. If **âˆ** SPT = 42**Ã…Ã£**, **âˆ** PST = 55**Ã…Ã£** and **âˆ** PSQ = 15**Ã…Ã£**, fi nd **âˆ** QRS.

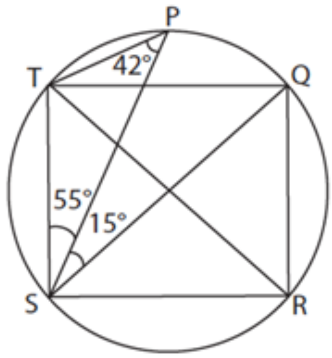


Figure 11.33

4. In Figure 11.34, IJKL is a cyclic quadrilateral given that **âˆ** IJK = 115**Ã…Ã£**, KL is produced to M such that **âˆ** IML = 53**Ã…Ã£**, calculate **âˆ** LIË†M

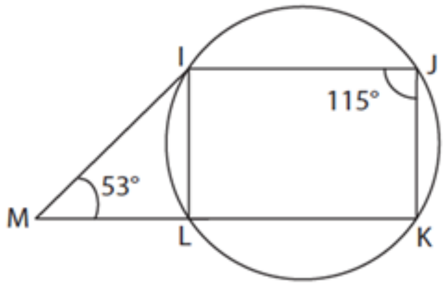


Figure 11.34

5. Figure 11.35 is a cyclic quadrilateral whose points PQRS are on the circle. If PQ_SR, **âˆ** RQS = 65**Ã…Ã£** and **âˆ** PRS = 43**Ã…Ã£**, calculate the angles of the trapezium.

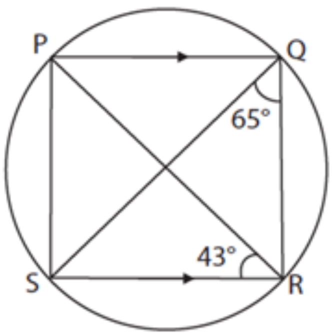


Figure 11.35

III. Tangents to a Circle

A tangent to a circle is a straight line that touches the circle at only one point. The point where the line touches the circle is referred to as the point of contact (Figure 11.36(a)). A secant is a straight line that cuts a given circle into two clear points as shown in Figure 11.36(b).

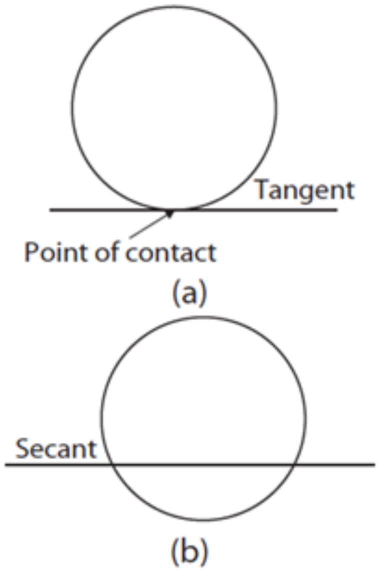


Figure 11.36

Note

1. The tangent to a circle is always perpendicular to the radius drawn to its point of contact.
2. The perpendicular to a tangent at its point of contact passes through the centre of the circle.

These two statements can be further illustrated by the diagram in Figure 11.37.
Given the secant of a circle with centre O, whose two points of contact are B and C.
Join OB and OC .
ABË† O = DCË†O and OBC = BCË†O } since the radii /BO/ and /CO/ are equal.

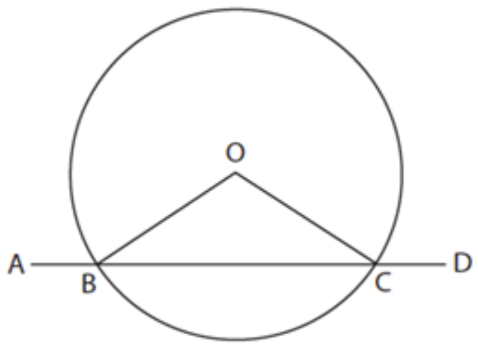


Figure 11.37

When the secant in Figure 11.37 is moved downward, the radii OB and OC will simultaneously coincide to form a radius OP as shown in Figure 11.38.

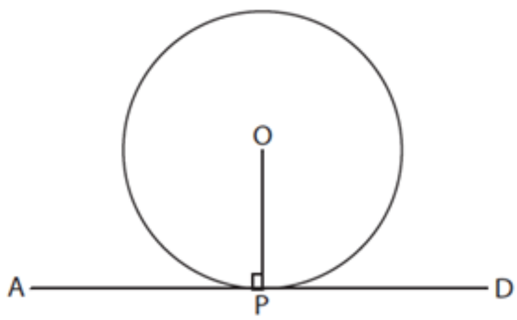


Figure 11.38

Now, since ABË† O = DCË†O (see Figure 11.37), it implies that
APË† O = DPË† O (see Figure 11.38)
But APË† D is a straight line, then, APË† O = DPË† O = _ 1_82_0_ _Ã…Ã£ = 90Ã…Ã£.
Therefore, AD is perpendicular to OP.
Symbolically expressed as AD âŠ¥ OP.

Theorem 6

Two tangents drawn to a circle from an external point are equal in length.
Given: An exterior point T of a circle with centre O. TY and TX are tangents to the circle at

X and Y.

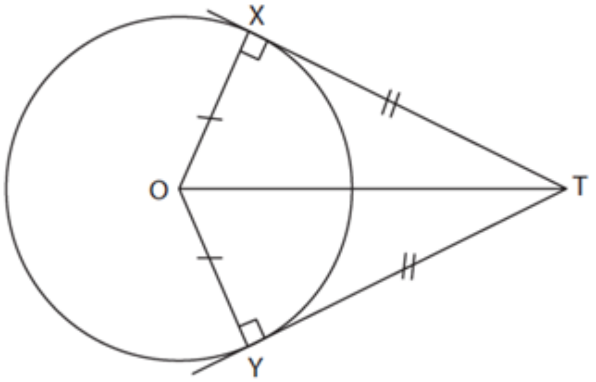


Figure 11.39

Required to prove: /TX/ = /TY/
Construction: Join TX , TO and TY
Proof: In triangles TXOË† and TÅ¶O
T Ë†XO = TÅ¶O = 90Ã…Ã£ (tangent âŠ¥ to radius)
/XO/ = /YO/ (radii)
/TO/ = /TO/ (common)
Î”TXO and Î”TYO are congruent (RHS)
Therefore, / TX/ = / TY/
Note
1. The angle between the tangents is bisected by the line joining the point of intersection of the tangent to the contact.
2. At the same time, the line bisects the angle between the radii drawn to the point of contact.
3. With assertion (1) and (2), the line TO in Figure 11.39 is a line of symmetry.
4. In Figure 11.39, âˆ XOY + âˆ XTY = 180Ã…Ã£.

Worked Example 8

In Figure 11.40, AB and AC are tangents from a point A to a circle centre O. If **âˆ** BAC = 54**Ã…Ã£**, find the value of x.

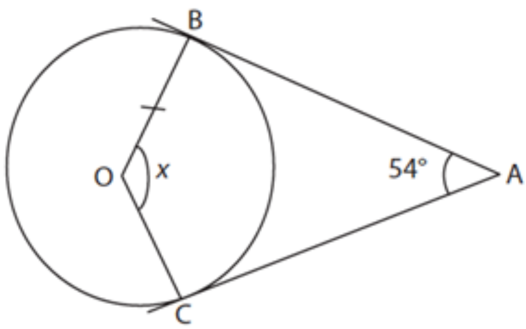


Figure 11.40

SOLUTION
ABË† O = ACË†O (tangents to a circle from an external point are equal)
ABË† O = ACË†O = 90**Ã…Ã£** (tangent **âŠ¥** to radius)
âˆ´ ABË† O + ACË†O + BÃ‚C + x = 360**Ã…Ã£** (sum of angles in a quadrilateral)
â‡’ 90**Ã…Ã£** + 90**Ã…Ã£** + 54**Ã…Ã£** + x**Ã…Ã£** = 360**Ã…Ã£**
â‡’ 234**Ã…Ã£** + x**Ã…Ã£** = 360**Ã…Ã£**
â‡’ x**Ã…Ã£** = 360**Ã…Ã£** âˆ’ 234**Ã…Ã£**
âˆ´ x = 126**Ã…Ã£**

Worked Example 9

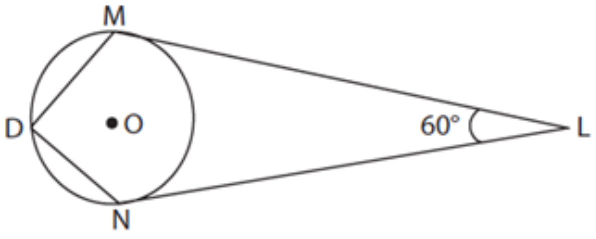


Figure 11.41(a)

In Figure 11.41(a), LM and LN are tangents from an external point L to a circle with centre O. If **âˆ** MLN = 60**Ã…Ã£** and D is a point on the circumference of the circle, find **âˆ** MDN.

SOLUTION

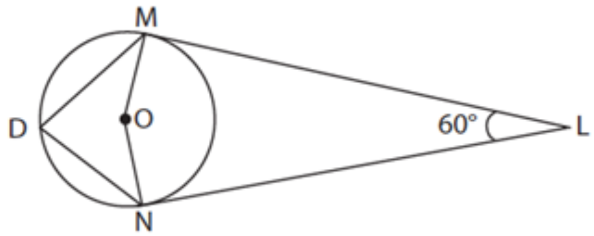


Figure 11.41(b)

Given /LM/ = /LN/ (tangents to a circle from an external point are equal)

Join $\overline{MO}$ and $\overline{NO}$

$\therefore\ L\hat{M}O = L\hat{N}O = 90^\circ$ (tangent $\perp$ to radius)

But $L\hat{M}O + L\hat{N}O + M\hat{O}N + 60^\circ = 360^\circ$ (sum of angles in a quadrilateral)

$$\Rightarrow 90^\circ + 90^\circ + M\hat{O}N + 60^\circ = 360^\circ$$
$$\Rightarrow 240^\circ + M\hat{O}N = 360^\circ$$
$$\Rightarrow M\hat{O}N = 360^\circ - 240^\circ$$
$$\Rightarrow M\hat{O}N = 120^\circ$$

Recall, $M\hat{O}N = 2M\hat{D}N$ (angle at the centre $=$ $2 \times$ angle at the circumference)

$$\Rightarrow 2M\hat{D}N = 120^\circ$$
$$\Rightarrow M\hat{D}N = \frac{120}{2}$$
$$\Rightarrow M\hat{D}N = 60^\circ$$

Alternate Segment

In Figure 11.42, let A be the point of contact of tangent PAN to a circle and AB any chord that divides the circle into two segments ACB and ADB.

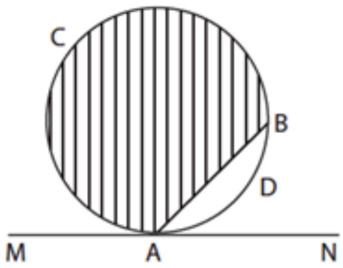


Figure 11.42

Using BAM, the segment ABD is referred to as the alternate segment corresponding to BAP. Similarly, using **â**ˆ BAN, the segment ABC is referred to as the alternate corresponding to **â**ˆ BAN.

Theorem 7

If a straight line touches a circle and from the point of contact a chord is drawn, the angles which the chord makes with the tangent are equal to the angles in the alternate segments of the circle.

Given: A straight line PAN touching a circle at A and a chord AB dividing the circle into two segments ABC and ABQ (see Figure 11.43).

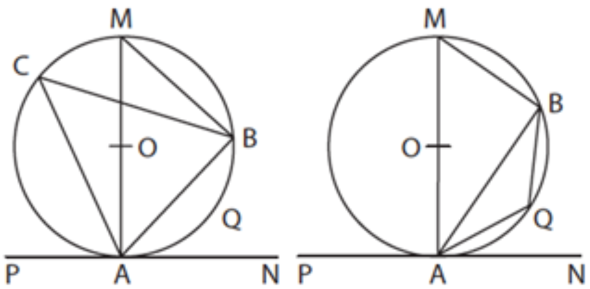


Figure 11.43

Required to prove:

$B\hat{A}N = A\hat{C}B$ in alternate segment $A\hat{C}B$

$B\hat{A}P =$ AQB in alternate segment AQB

Construction: Draw the diameter AM; Join $\overline{MB}$.

Proof: Using the lettering in Figure 11.43(a)

AM is a diameter and PAN is a tangent.

$B\hat{A}P + B\hat{A}N = 180°$ (adjacent angle on a straight line)

Also, $A\hat{B}M = 90°$ (angle in a semicircle)

$AMB + B\hat{A}M = 90°$

Implies that $B\hat{A}M + B\hat{A}N = AMB + B\hat{A}M$

$\therefore B\hat{A}N = AMB$

And $B\hat{A}N = AMB = A\hat{C}B$ (angle in the same segment)

Implies $\angle BAN + \angle ACB$ (angles in the same segment)

$\therefore B\hat{A}N = A\hat{C}B$

(b) ***Construction:*** Join $\overline{AQ}$ and $\overline{BQ}$

Proof: $B\hat{A}P + B\hat{A}N = 180°$ (adjacent angle on a straight line)

And $A\hat{C}B + AQB = 180°$ (sum of the opposite angles of a cyclic quadrilateral)

Implies $BAP + ACB = 180°$ (proved)

$\therefore BAP = AQB$

Worked Example 10

In Figure 11.44 AB and AC are tangents from an external point A to a circle with centre O.

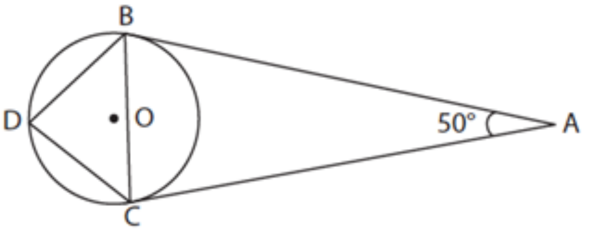


Figure 11.44

If BAË† C = 50Ã…Ã£ and D is a point on the circumference of the circle, calculate âˆ BDC?

SOLUTION

But $\overline{AB}$ and $\overline{AC}$ are tangents from an external point.

$$\Rightarrow /AB/ = /AC/$$

$\therefore$ ΔABC is isosceles

$$\Rightarrow ABC = A\hat{C}B = \frac{180^\circ - 50^\circ}{2}$$

$$= \frac{130^\circ}{2}$$

$$= 65^\circ$$

Since $A\hat{C}B = BDC$ in alternate segment BDC

$$\Rightarrow A\hat{C}B = B\hat{D}C = 65^\circ$$

$$\therefore B\hat{D}C = 65^\circ$$

Worked Example 11

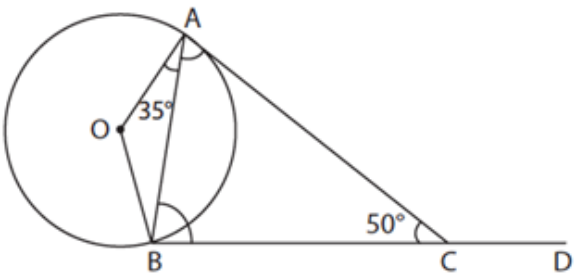


Figure 11.45

Figure 11.45 is a circle with centre O whose tangent $B\hat{C}D$ meets the radius at B.

If BAË† O = 35Ã…Ã£ and ACB = 50Ã…Ã£, fi nd (a) ABË† C and (b) BAË† C.

SOLUTION

(a) /AO/ = /OB/ (radii)

ABO = BAË† O = 35Ã…Ã£ (base angles of an isosceles triangle)

But OË† BC = 90Ã…Ã£ (tangent âŠ¥ to radius)

â‡’ AË† BC = 90Ã…Ã£ âˆ’ 35Ã…Ã£

= 55Ã…Ã£

(b) BÃ‚C = 180Ã…Ã£âˆ’ (55Ã…Ã£ + 50Ã…Ã£)

= 180Ã…Ã£ âˆ’ 105Ã…Ã£ = 75Ã…Ã£

Exercise 5

1. In Figure 11.46, O is the centre of the circle QRT and PTU is a tangent to the circle at T. Calculate the angle **x**.

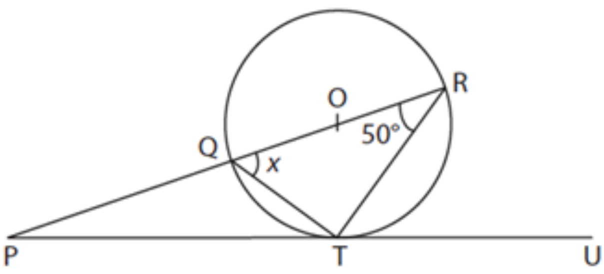


Figure 11.46

2. In Figure 11.47, MNP is a tangent to the circle ABN at N. ABC is a straight line and NC bisects ∠BNP. Find x.

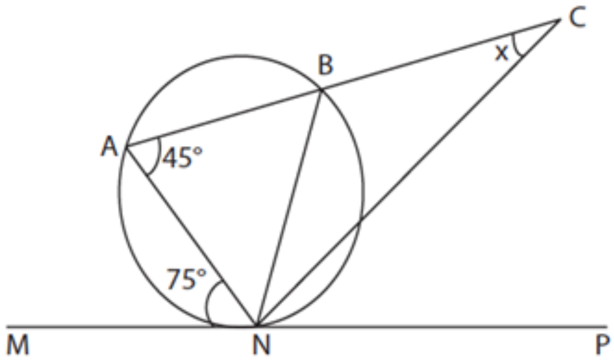


Figure 11.47

3. In Figure 11.48, PMT is a tangent at the point M to the circle with centre O. if $\angle MOQ = 105°$, find $\angle MPO$.

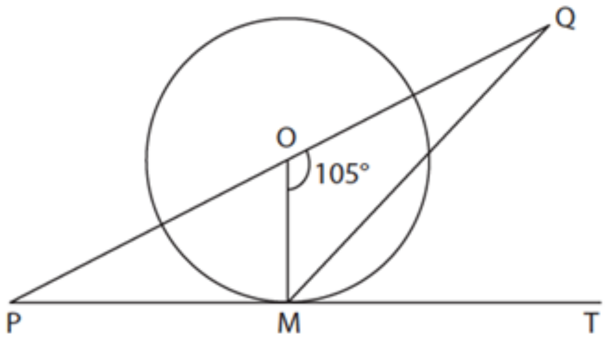


Figure 11.48

4. In Figure 11.49, ABC is a tangent to the circle BTS at B. $\angle BCT = 53°$, and $\angle BTC = 99°$. Find $\angle BST$.

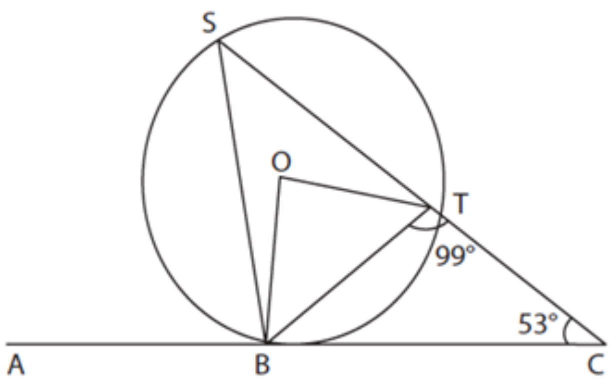


Figure 11.49

5. In Figure 11.50, O is the centre of the circle. PAT is a tangent to the circle at A. **â^** AOB = 126**Ã…Ã£** and **â^** ABC = 27**Ã…Ã£**. Calculate:
(a) PAË† B
(b) Show that OA is parallel to BC. (WASSCE)

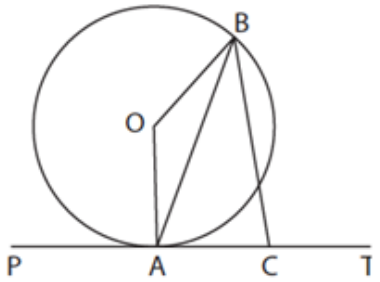


Figure 11.50

SUMMARY

In this chapter, we have learnt the following:

â– Theorem is a statement that has been proved. It starts with a certain assumption that will help establish the result and ends with a conclusion.

â– While writing out the proof of a theorem e.g. circle theorem, one should adopt the following steps:

Step 1: State what is given.

Step 2: State what is required to prove.

Step 3: Construct.

Step 4: Sketch the diagram with the aid of information given in the fi rst three steps.

Step 5: State proof.

Step 6: Conclude.

â– The following theorems were proved and used to solve some exercises:

(a) An angle which an arc (or a chord) of a circle subtends at the centre of the circle is twice the angle which it subtends at any point on the remaining part of the circumference.

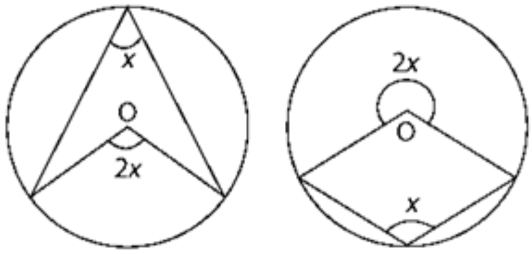


Figure 11.51

(b) Angles in the same segment of a circle are equal.

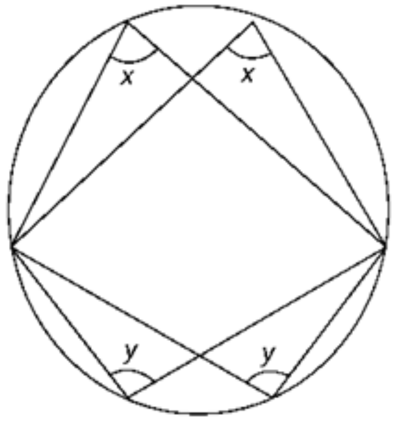


Figure 11.52

(c) Angle in a semicircle is a right angle.

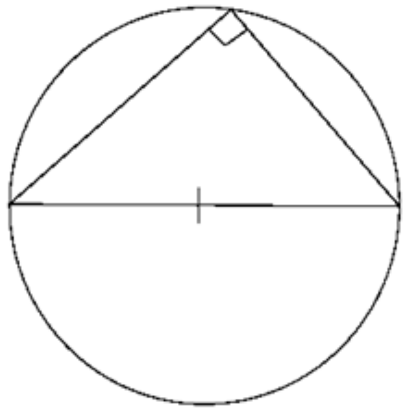


Figure 11.53

(d) The opposite angles in a cyclic quadrilateral are supplementary.

$\angle a + \angle b = 180°$

$\angle c + \angle d = 180°$

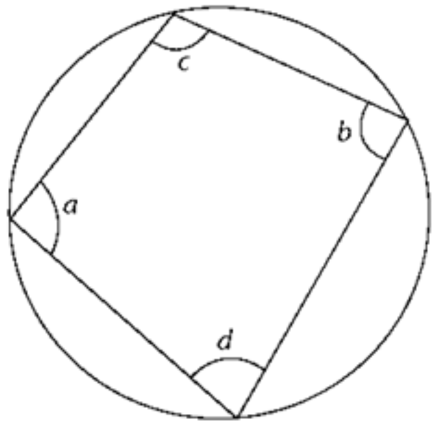


Figure 11.54(a)

(e) The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

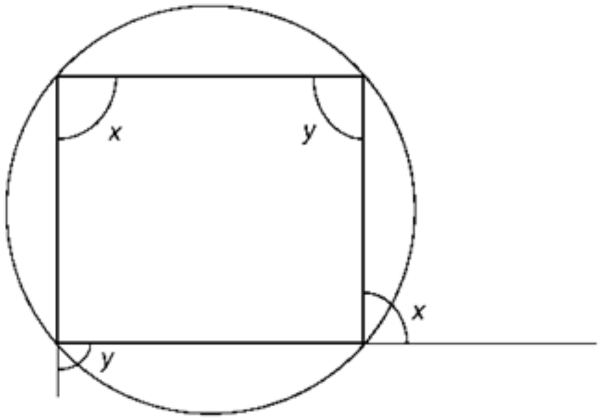


Figure 11.54(b)

(f) Two tangents drawn to a circle from an external point are equal in length.

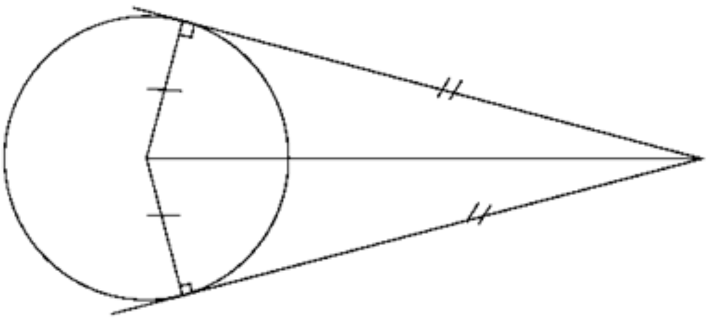


Figure 11.55(a)

(g) If a straight line touches a circle and from the point of contact a chord is drawn, the angle which the chord makes with the tangent is equal to the angle in the alternate segment of the circle.

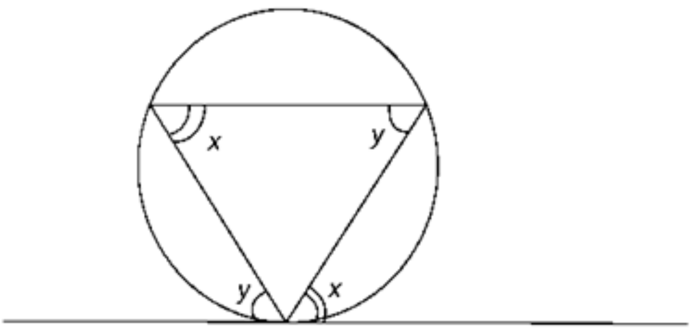


Figure 11.55(b)

GRADUATED EXERCISES

1. Find the lettered angles.

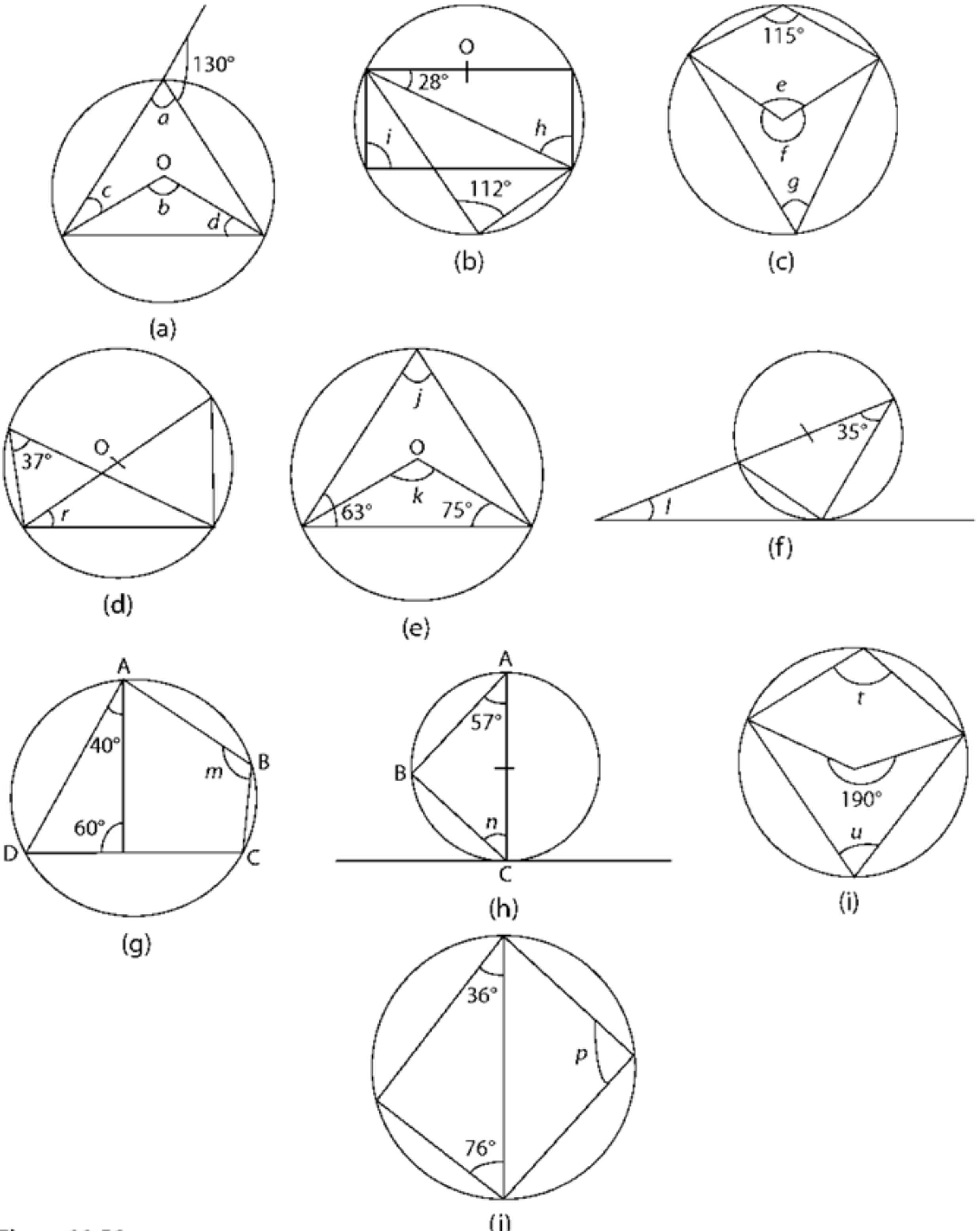

130°
a
O
c
b
d
(a)
O
28°
i
h
112°
(b)
115°
e
f
g
(c)
37°
O
r
(d)
j
O
63°
k
75°
(e)
35°
l
(f)
A
40°
m
B
60°
D
C
(g)
A
57°
B
n
C
(h)
t
190°
u
(i)
36°
p
76°
(j)

Figure 11.56

2. In Figure 11.57, O is the centre of the circle. $\angle OQR = 32°$ and $\angle TPQ = 15°$. Calculate
   (a) $\angle QPR$
   (b) $\angle TQO$ (WASSCE)

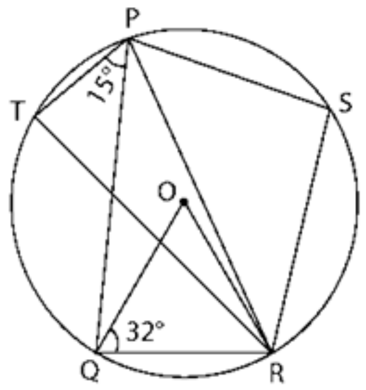


Figure 11.57

3. In Figure 11.58, AB and CB are tangents to the circle. Given that CBA = 54°, calculate $\angle ADC$. (NECO)

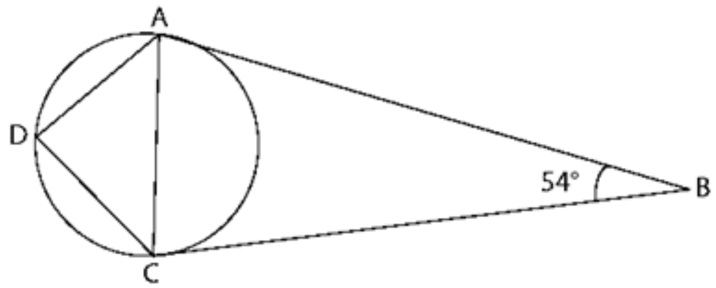


Figure 11.58

4. In Figure 11.59, TP is a tangent to the circle TRQ with centre O. if $\angle TPO = 28°$ and $\angle ORQ = 15°$. Find
   (a) $\angle RQT$
   (b) $\angle QTO$ (NECO)

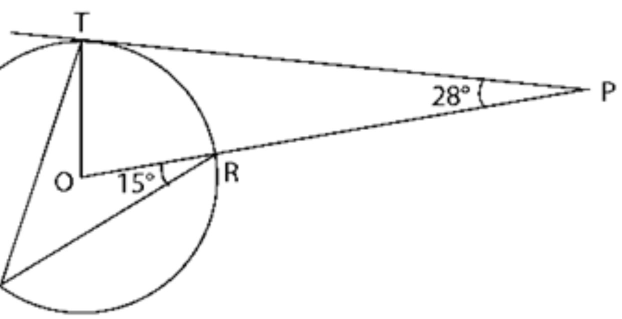


Figure 11.59

5. In Figure 11.60, O is the centre of the circle, and P, Q and R are points on the circumference. Show that $x = 2y$.

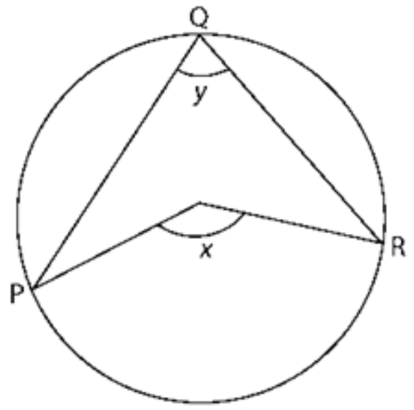


Figure 11.60

6. Figure 11.61 is a circle ABC. Calculate x giving a reason for each step in your answer.

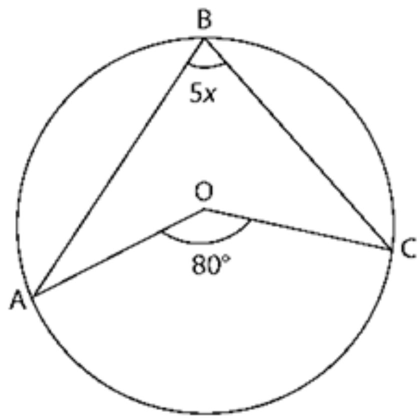


Figure 11.61

7. Figure 11.62 is a circle ABC with centre O. If the exterior angle is 43°, find the unknown angles.

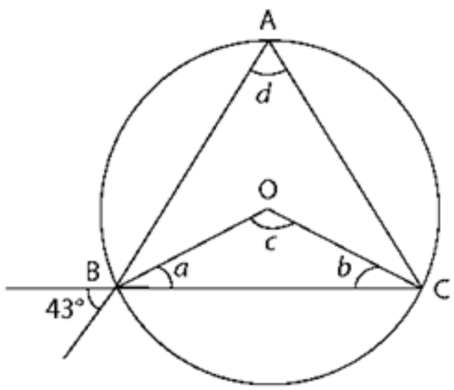


Figure 11.62

8. (a) Prove that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

(b) In Figure 11.63, calculate the value of x giving a reason for each step in your answer. (WASSCE)

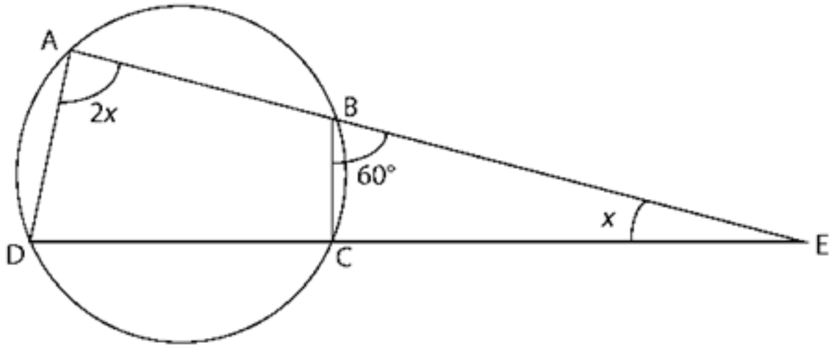


Figure 11.63

9. (a) Prove that the angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.

(b) In Figure 11.64, O is the centre of the circle ACDB. If $\angle CAO = 26^\circ$ and $\angle AOB = 130^\circ$, calculate (a) $\angle OBC$ and (b) $\angle COB$.

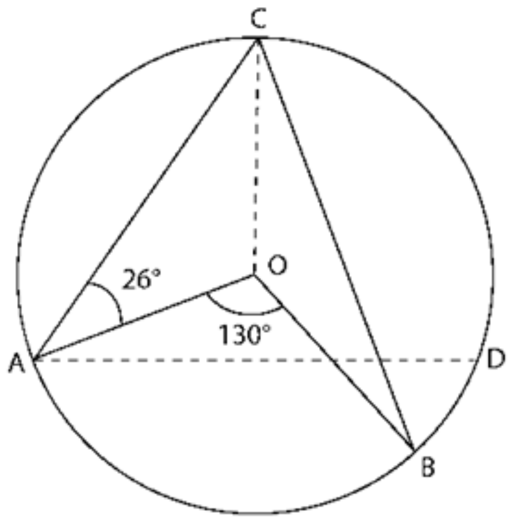


Figure 11.64

10. In Figure 11.65, O is the centre of the circle and ABD is a tangent at B. If BDF = 66° and $D\hat{B}C = 57^\circ$, calculate (a) $E\hat{B}F$ and (b) $B\hat{G}F$.

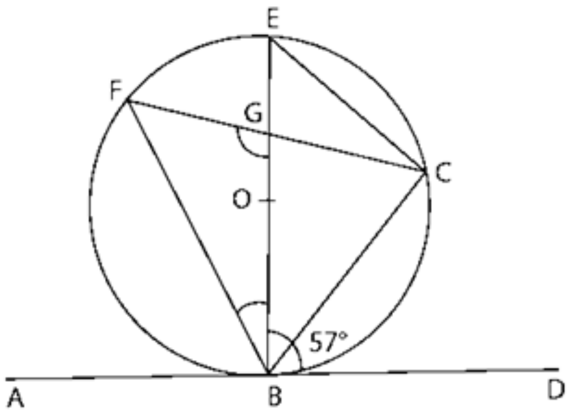


Figure 11.65