CHAPTER 9

CONSTRUCTIONS

CHAPTER 9

Objectives

At the end of the chapter, students should be able to:
1. Bisect a given line segment.
2. Bisect a given angle.
3. Construct a triangle with given sides and angles.
4. Construct four-sided plane figures.
5. Construct locus of moving points equidistant from 2 points, 2 lines and a fixed point etc.

I. Revision

(i) Bisection of angles

Figure 9.1 shows some basic bisections using ruler, pencil and compasses.

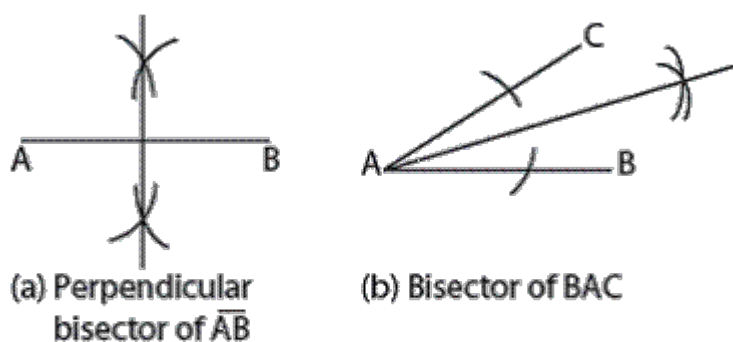(a) Perpendicular bisector of $\overline{AB}$

(b) Bisector of BAC

Fig. 9.1

Note: Using a ruler and a pair of compasses only, we can do the following basic constructions. The use of sharp pencil should be encouraged so as to ensure neatness, accuracy of results and to minimize errors.

Worked Example 1

Construct angle 90°.
Step 1: Draw line

$\overline{BC}$.
Step 2: With B as centre of convenient radius, draw a semi-circle to meet

BC produced at two different points, i.e., D and E.

Step 3: Using the same radius as in step 2, and with D and E as centre respectively, draw arcs to cut the semi-circle at two different points, i.e., F and G.

Step 4: With F and G as centre respectively, draw arcs to meet at A.

Step 5: Join
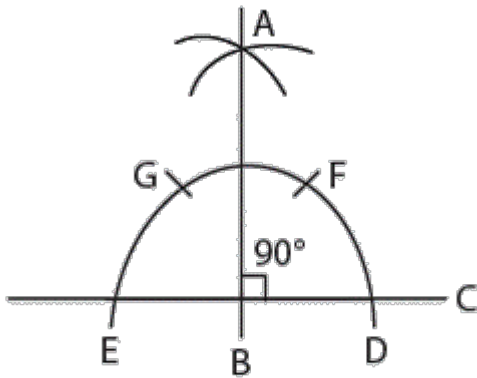
$\overline{AB}$ to give angle 90°.

Fig. 9.2

Worked Example 2

Bisect angle 90°.

Step 1: Construct angle 90° using the same procedure as in Example 1.

Step 2: With D and H as centres respectively, draw arcs to meet at I.

Step 3: Join
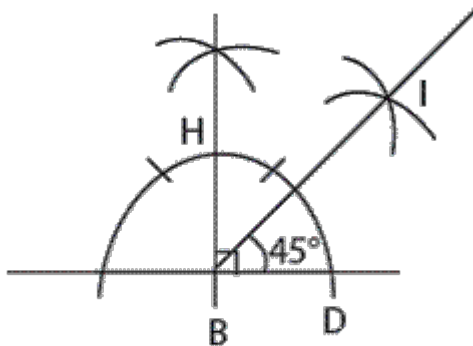
$\overline{BI}$ to give angle 45°.

Fig. 9.3

Construct angle 60°.

Step 1: Draw line $\overline{AB}$.

Step 2: With A as a centre of convenient radius, draw an arc to meet $\overline{AB}$ at C.

Step 3: Using the same radius as in step 2, with C as centre, draw an arc to cut the first arc at D.
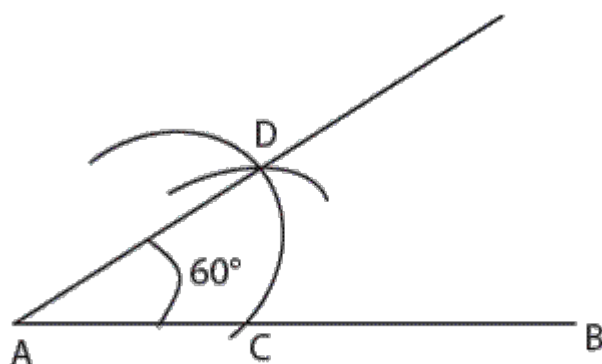
Step 4: Join $\overline{AD}$ to give angle 60°.

Fig. 9.4

Worked Example 4

Bisect angle 60°.

Step 1: Construct angle 60° using the same procedure as in Example 3.

Step 2: With C and D as centres respectively, draw arcs to meet at E.

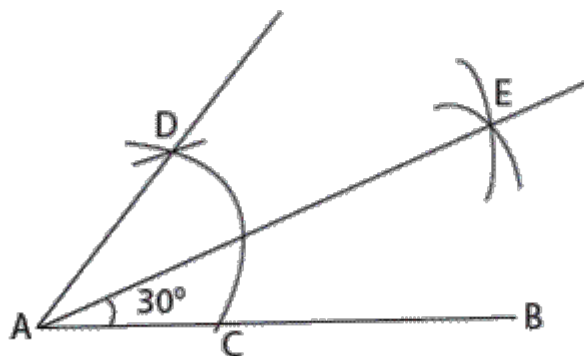**Step 3:** Join $\overline{AE}$ to give angle 30°.

Fig. 9.5

(ii) Construction of triangles with given sides and angles

Worked Example 5

Construct ∆ABC where |AB| = 8 cm, |BC| = 7 cm and |AC| = 6 cm.
Step 1: Draw line |AB| = 8 cm as base line.
Step 2: With B as centre, radius 7 cm, draw an arc.
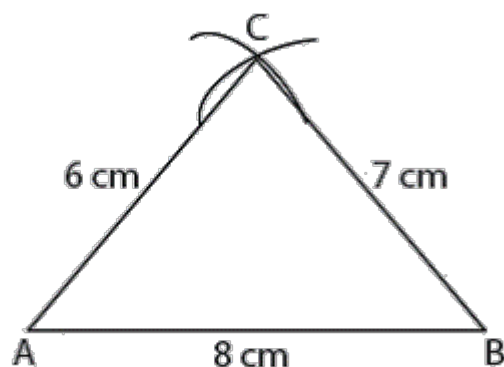**Step 3:** With A as centre, radius 6 cm, draw an arc to cut the first arc at C.

Fig. 9.6

Worked Example 6

Construct ΔABC such that |AC| = 6 cm, |AB| = 4 cm and BÂC = 60°. Measure BC.

Step 1: Draw line |AC| = 6 cm as base line.

Step 2: Construct |BÂC| = 60°.

Step 3: Mark AB off 4 cm to establish the vertex B.

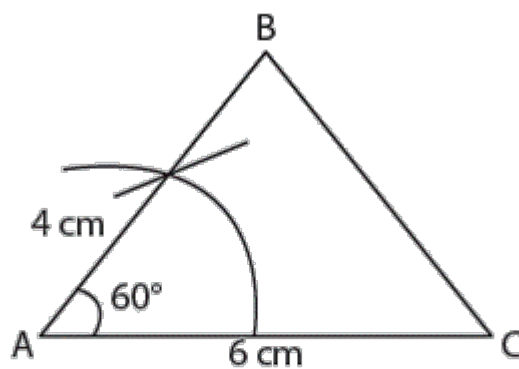**Step 4:** Join BC.

Fig. 9.7

By measurement, |BC| = 5.3 cm.

Exercise 1

1. Using a ruler, pencil and pair of compasses, construct the following angles:
   (a) 15°   (b) 75°   (c) 105°
   (d) 135° (e) 120°

2. Copy and bisect the angles given above.

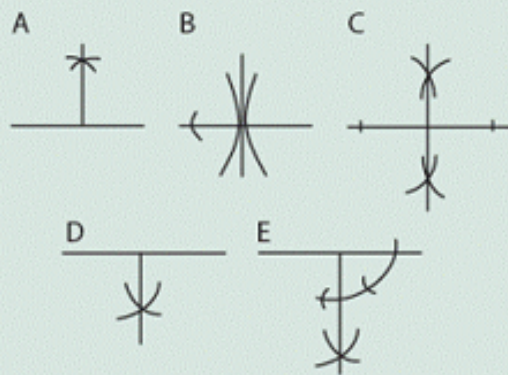3. Which of the following is a correct method of constructing the perpendicular bisector of a line?

Fig. 9.8   (WAEC)
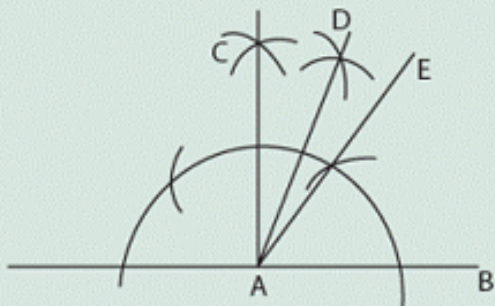
4. What is the value of angle DÂB in the figure below?

Fig. 9.9

5.
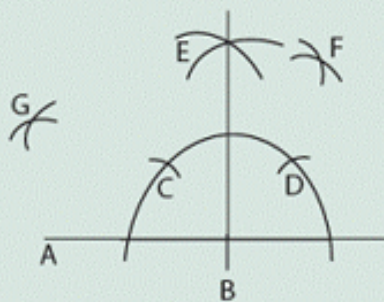

Fig. 9.10

(a) In Fig. 9.10, what is the acute angle formed when the point G is joined to B?

(b) In Fig. 9.10, what is the obtuse angle formed when F is joined to B?

6. What does the construction mark indicate?
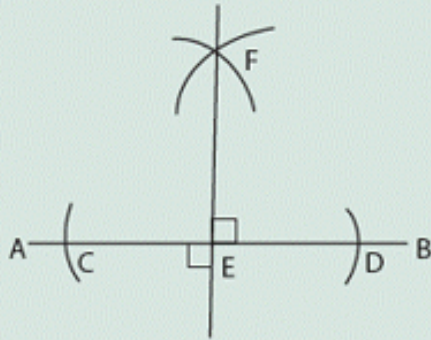

Fig. 9.11

7. Construct $\triangle ABC$ in which $|AB|$ is 9.5 cm, ABC is 105° and $|BC|$ is 8 cm. Measure $\overline{AC}$ and BÂC.

8. Construct $\triangle XYZ$ in which $|XY| = 8$ cm, $|YZ| = 9$ cm and $|XZ| = 10$ cm.

9. Using a ruler and a pair of compasses only, construct $\triangle ABC$ in which $|AB| = 7$ cm, $|BC| = 5$ cm and $\angle ABC = 75°$. Measure $|AC|$.
    (WAEC)

10. Construct $\triangle RST$ in which $|RS| = 8$ cm, $\angle RST = 60°$ and $|ST| = 6.8$ cm. Bisect $\overline{RT}$ and let the bisector meet $\overline{RS}$ at U. Measure $\overline{SU}$.

11. $\triangle XYZ$ has $\angle XYZ = 150°$, $|XY| = 6.4$ cm and $|YZ| = 7$ cm. Construct the triangle. Bisect $\angle YXZ$ and let the bisector meet YZ at P. Measure $\overline{YP}$.

12. Construct $\triangle ABC$ in which $ABC = 30°$, $BCA = 75°$ and $|BC| = 8$ cm. Bisect $\overline{AB}$ and $\overline{AC}$ at D and E respectively. Join DE and measure it.

13. CDF is a triangle in which $|CD| = 5$ cm, $|DF| = 9$ cm and $CDF = 30°$. Bisect $\overline{DF}$ and join E, the midpoint of DF, to C. Measure $\overline{CE}$.

14. (a) Construct $\triangle LMN$ such that $|LM| = 8$ cm, $|MN| = 11$ cm and $|LN| = 10$ cm.

    (b) Mark a point O on $\overline{LM}$ such that $|MO| = 5$ cm. Use a ruler and set square to construct a line through O parallel to $\overline{MN}$ to meet $\overline{LN}$ at P.

    (c) Measure $\overline{OP}$.

15. Construct $\triangle ABC$ where $|AB| = 7$ cm, $|BC| = 8$ cm and $\angle ABC = 75°$. D is a point on $|AC| = 5$ cm from A. Through D, construct $\overline{DE}$ parallel to $\overline{BC}$ to meet $\overline{AB}$ at E. Measure $\overline{AE}$.

II. Construction

(i) Construction of an angle equal to a given angle

Worked Example 7

Construct angle 75°.

Step 1: Draw line $\overline{AB}$.

Step 2: With C as a centre of convenient radius, draw a semi-circle to meet $\overline{AB}$ at D and E.

Step 3: Using the same radius as in step 2, and with D and E as centre respectively, draw arcs to cut the semi-circle at F and G.

Step 4: With F and G respectively, draw arcs to meet at H.

Step 5: Join $\overline{HC}$ as centre.

Step 6: With F and I as centre respectively, draw arcs to meet at J.

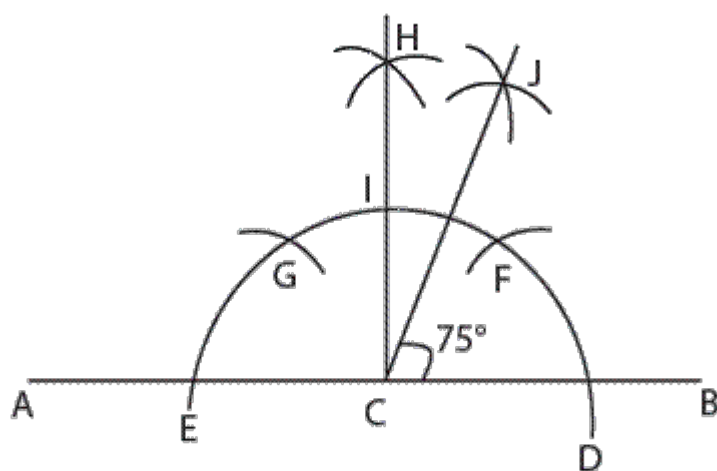**Step 7:** Join $\overline{JC}$ to give angle 75°.

Fig. 9.12

(ii) Construction of four-sided plane figures

Worked Example 8

Construct a quadrilateral ABCD in which $\overline{AB}$ is parallel to $\overline{DC}$. $|AB| = 5$ cm, $|BC| = 6$ cm and $|DC| = 8$ cm and ADC = 120°. Measure the diagonal $\overline{BD}$ and $\overline{AD}$.

Step 1: Draw $\overline{DNC} = 8$ cm with $\overline{DN} = 3$ cm and $\overline{NC} = 5$ cm.

Step 2: Construct $\angle CDM = 120°$.

Step 3: With N as centre and radius $= 6$ cm, draw an arc to cut $\overline{DM}$ at A.

Step 4: With A as centre and a radius of 5 cm, draw an arc. With C as centre and radius = 6 cm, draw a second arc to cut the first arc at B.

Step 5: Join A to B to complete the quadrilateral ABCD.

Fig. 9.13
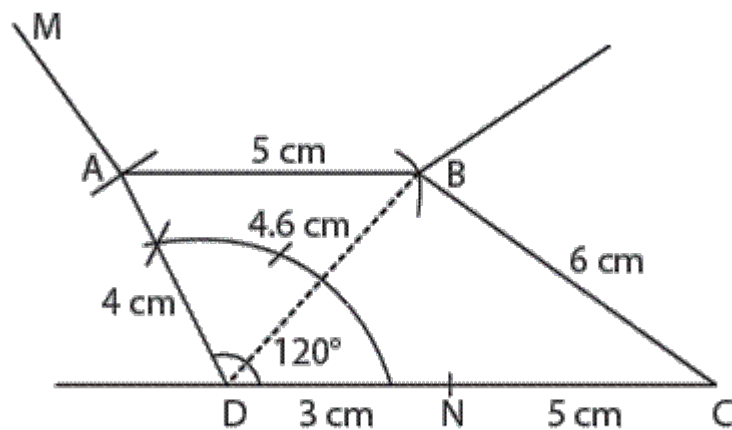
By measurement |BD| = 4.6 cm and |AD| = 4 cm.

Worked Example 9

Construct quadrilateral PQRS with ∠PSR = 60°, |PS| = 5 cm, |SR| = 8 cm, |QR| = 7 cm and |PQ| = 6 cm. Bisect angles S and P and let the bisector meet at X. Measure PX and ∠XPS.

Step 1: Draw |SR| = 8 cm.

Step 2: Construct ∠PSR = 60°.

Step 3: With S as centre and radius = 5 cm, draw an arc to locate P i.e. |PS| = 5 cm.

Step 4: With centres P and R of radii 6 cm and 7 cm respectively, draw arcs to cut each other at Q i.e. |PQ| = 6 cm and |QR| = 7 cm.
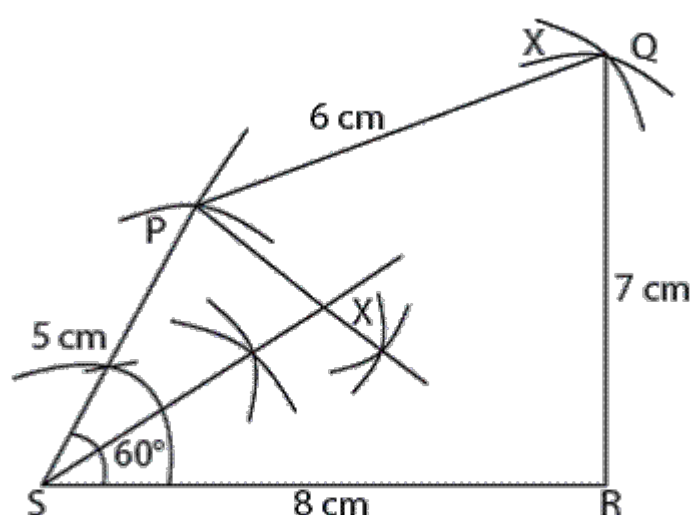
Step 5: Bisect angles S and P to meet at X.

Fig. 9.14

By measurement |PX| = 6 cm and ∠XPS = 73°.

Worked Example 10

Construct the trapezium PQRS in which $\overline{PQ}$ and $\overline{SR}$ are parallel, |PS| = 5 cm and ∠PSR = 30°. The diagonal $\overline{PR}$ is perpendicular to $\overline{PS}$. Find the length of $\overline{PQ}$.

Step 1: Draw any suitable length as base starting from S i.e. $\overline{ST}$.

Step 2: Construct an angle 30° from point S as centre, and with radius 5 cm, draw an arc to locate $\overline{P}$. Join $\overline{SP}$.

Step 3: With P as centre, construct a perpendicular to cut $\overline{ST}$ at R.

Step 4: Using a convenient radius with S and R as centres respectively, draw arcs to cut $\overline{ST}$ at A and B.

Step 5: With P as centre and radius $\overline{AB}$, draw an arc above B and with B as centre and radius $\overline{PA}$, draw another arc to cut each other at Q.

Step 6: Join $\overline{PQ}$ and $\overline{QR}$ to complete the trapezium PQRS.
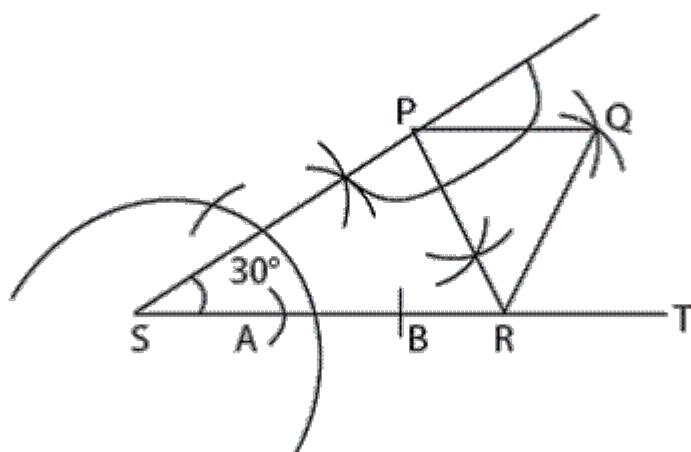

Fig. 9.15

By measurement $|PQ| = 2.5$ cm.

(iii) Locus of moving points

A locus is a set of points which satisfy a given condition or a path traced out by a point under certain given conditions. Plural of locus is loci. If a point is moving, the curve made by the point is its locus. (a) Locus of moving points equidistant from two lines. The locus of a point which is equidistant from two fixed parallel lines is a line parallel to the two fixed parallel lines as shown in Fig. 9.16.
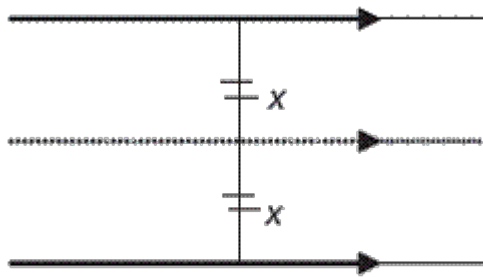
Fig. 9.16

(b) Locus of moving points equidistant from two points The locus of point equidistant from two points A and B is a perpendicular bisector of the line segment

$\overline{AB}$ as shown in Fig. 9.17.

Fig. 9.17

(c) Locus of moving points at constant distance from a point. The locus of points at constant distance from point A is a circle with centre A and radius equal to the constant distance d as shown in Fig. 9.18.
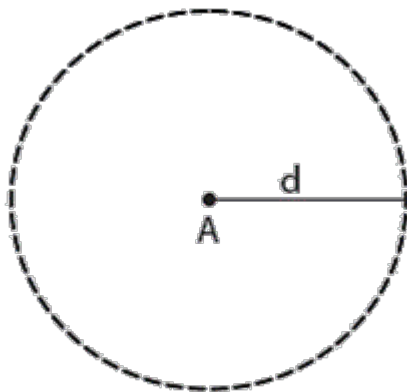

Fig. 9.18

(d) Locus of points at a given distance from a straight line. he locus of points at a given distance from a straight line AB is a pair of parallel lines parallel to the given line and on opposite sides of the given line as shown in Fig. 9.19.

Fig. 9.19

Worked Example 11

Using a ruler and a pair of compasses only:

(a) Construct $\triangle ABC$ such that $|AB| = 6$ cm, $|AC| = 8.5$ cm and $B\hat{A}C = 120°$.

(b) Construct the locus l_1 of points equidistant from point A and B.

(c) Construct the locus l_2 of points equidistant from $\overline{AB}$ and $\overline{AC}$.

(d) Find the points of intersection, P_1 and P_2, of l_1 and l_2 and measure $\overline{P_1P_2}$.

Fig. 9.20

(a) BAC is the required triangle.

(b) l_1 is the perpendicular bisector of $\overline{AB}$.

(c) l_2 is in two parts. $\overline{AP_1}$ is the bisector of BÂC. $\overline{AP_2}$ is perpendicular to $\overline{AP_1}$. Note that points on $\overline{AP_2}$ are equidistant from $\overline{AB}$ and $\overline{CA}$.

(d) By measurement $|P_1P_2| = 6.9$ cm.

Worked Example 12

Using a ruler and a pair of compasses only, construct:

  (i) A triangle XYZ in which $|YZ| = 8\,$cm, $\angle XYZ = 60°$ and $\angle XZY = 75°$. Measure $\overline{XY}$.

 (ii) The locus l_1 of points equidistant from Y and Z.

(iii) The locus l_2 of points equidistant from $\overline{XY}$ and $\overline{YZ}$.

(iv) Measure $\overline{QY}$ where Q is the point of intersection of l_1 and l_2.

Fig. 9.21

(a) ΔXYZ is the required triangle.

(b) l_1 is the perpendicular bisector of $\overline{YZ}$.

(c) l_2 is the bisector of $\angle XYZ$.
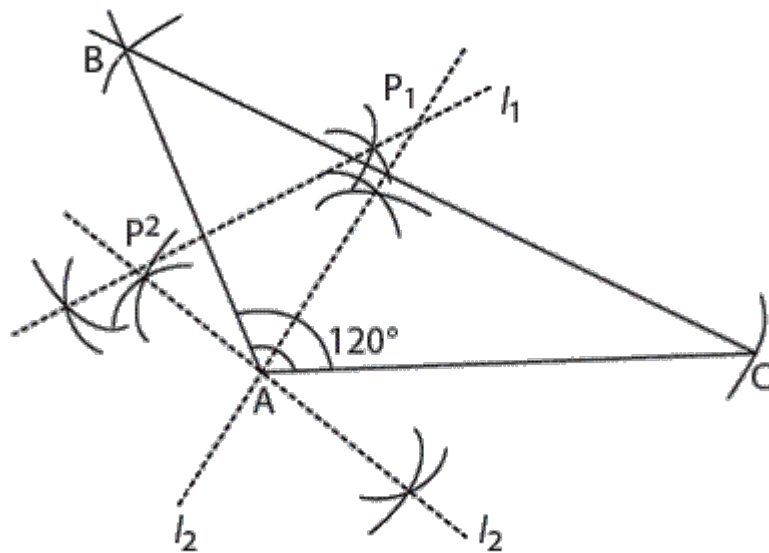
(d) By measurement $|XY| = 11.3$ cm and $|QY| = 4.8$ cm.

Worked Example 13

(a) Using a ruler and a pair of compasses only, construct:

  (i) A triangle ABC such that $|BC| = 6$ cm, $\angle ABC = 120°$ and $\angle BCA = 22.5°$. Measure $\overline{AC}$.

  (ii) The locus π_1 of points equidistant from B and C.

  (iii) The locus π_2 of points 4 cm from B.

(b) Measure $\overline{D_1D_2}$, where D_1 and D_2 are the points of intersection of π_1 and π_2.

Fig. 9.22

(a) ΔACB is the required triangle.

(b) π_1 is the perpendicular bisector of $\overline{BC}$.

(c) π_2 is a circle with centre B and radius 4 cm.

(d) By measurement $|AC| = 8.5$ cm and $|D_1D_2| = 5.3$ cm.

Exercise 2

1. Using a ruler, pencil and a pair of compasses only, construct the following:

   (a) $22\frac{1}{2}°$ (b) $150°$

2. Construct trapezium ABCD in which $\overline{AB}//\overline{DC}$, $|AB| = 8$ cm, $\angle ABC = 60°$, $|BC| = 5.5$ cm and $|BD| = 8.3$ cm.

3. (a) Using a ruler and a pair of compasses only, construct:

   (i) The trapezium ABCD.

   (ii) A rectangle PQCD where P and Q are two points on AB.

   (b) Measure AP and QB. (WAEC)

4. (a) Using a ruler and a pair of compasses only, construct

a parallelogram PQRS with diagonals $|PR| = 9$ cm and $|QS| = 6$ cm intersecting at K and $\angle QKP = 60°$.

(b) Construct a rectangle PABS which is equal in area to PQRS in (a) above and on the same side of PS as PQRS. Measure $\overline{PA}$. **(WAEC)**

5. Construct a parallelogram LMNO such that $|LM| = 5$ cm, $|MN| = 6.4$ cm and $\angle LMN = 60°$. Bisect $\angle MNO$ and let the bisector meet $\overline{LO}$ at P. With $\overline{NP}$ as diameter, draw a circle with centre Q. Join $\overline{QO}$ and measure it. Also, measure $\angle MNP$.

6. Using a ruler and a pair of compasses only, construct a quadrilateral PQRS given that $|PQ| = 9$ cm, $\angle QPS = 105°$, $\angle PQS = 30°$, $|RP| = 10$ cm and $\overline{RS} = \overline{RQ}$. Measure (a) $\overline{RS}$ (b) $\angle QRS$.

7. Using a ruler and a pair of compasses only,

   (a) Construct:

   (i) a $\triangle ABC$ such that $|AB| = 5$ cm, $|AC| = 7.5$ cm and $C\hat{A}B = 120°$.

   (ii) The locus l_1 of points equidistant from A and B.

   (iii) The locus l_2 of points equidistant from $\overline{AB}$ and $\overline{AC}$ which passes through triangle ABC.

   (b) Label the point P where l_1 and l_2 intersect.

   (c) Measure $\overline{CP}$. **(WAEC)**

8. (a) Using a ruler and a pair of compasses only, construct:

(i) A triangle PQR such that $|PQ| = 10\,cm$, $|QR| = 7\,cm$ and $PQR = 90°$.

(ii) The locus l_1 of points equidistant from Q and R.

(iii) The locus l_2 of points equidistant from P and Q.

(b) Locate the point O equidistant from P, Q and R.

(c) With O as centre, draw the circumcircle of the triangle PQR.

(d) Measure the radius of the circumcircle.   (WAEC)

9. Using a ruler and a pair of compasses only:

(a) Construct:

(i) $\triangle PQR$ such that $|PQ| = 8\,cm$, $|PR| = 7\,cm$ and $\angle QPR = 105°$.

(ii) Locus l_1 of points equidistant from P and Q.

(iii) Locus l_2 of points equidistant from Q and R.

(b) (i) Label the point T where l_1 and l_2 intersect.

(ii) With T as centre and radius $|TQ|$, construct a circle l_3.

(iii) Complete quadrilateral PQRS such that $|RS| = |QS|$ and $|RQ| = |TS|$.   (WAEC)

10. (a) Using ruler and compasses only, construct $\triangle ABC$ such that $|AC| = 10\,cm$, $|BC| = 8.5\,cm$ and $\angle ACB = 135°$.

(b) Using any geometrical instrument, find a point P within $\triangle ABC$ which is at a distance 2.8 cm from $\overline{AC}$ and 6 cm from B. Measure the length of $\overline{AP}$.

11. (a) Using a ruler and a pair of compasses only, construct $\triangle ABC$ in which $|AB| = 5$ cm, $|BC| = 7.2$ cm and $\angle ABC = 75°$. Measure $|AC|$.

(b) Using the triangle above, locate a point D such that $\overline{CD}$ is parallel to $\overline{AB}$ and D is equidistant from A and C. Measure $\angle BAD$. **(WAEC)**

12. (a) Using a ruler and a pair of compasses only, construct a triangle ABC given that $|BC| = 8.5$ cm, $|AB| = 5.1$ cm and $\angle BAC = 65°$.

(b) (i) Measure $|AC|$.

(ii) Locate a point D on $|AC|$ such that $|AD|:|DC|=3:5$.

(iii) Measure $\angle ABD$ and $\angle CBD$.

13. Using a ruler and a pair of compasses only, construct:

(a) A parallelogram ABCD in which $|AB| = 4.5$ cm, $|BC| = 3.5$ cm and $\angle ABC = 30°$.

(b) The perpendicular from C to AB (meeting AB at F).

(c) The circle which touches AB at F and passes through the point D. Measure the radius of the circle.

14. Using a ruler and a pair of compasses only:

(a) Construct a quadrilateral PQRS such that $|PQ| = 7.5$ cm, $|QR| = 6.5$ cm, $\angle PQR = 120°$ and $|RS| = 8.8$ cm. $\overline{PS}$ is parallel to $\overline{QR}$.

(b) Find a point M on $\overline{QR}$ such that $\overline{RM}:\overline{MQ} = 2:3$.

(c) Find a point N on $\overline{PQ}$ which is equidistant from the lines $\overline{SP}$ and $\overline{SR}$. Measure $\overline{MN}$.(WAEC)

SUMMARY

In this chapter, we have learnt the following:

◆ Revision work on basic construction:
  (a) Construction of special angles (30°, 45°, 60° and 90°).
  (b) Bisection of angles and lines.
  (c) Construction of triangles with given sides.

◆ Construction of four-sided plane figures given certain conditions.

- Locus of moving points:
  - (a) A locus is a set of points which satisfy a given conditions or a path traced out by a point under certain given conditions.
  - (b) Plural of locus is loci.
  - (c) The locus of points which is equidistant from two fixed parallel lines is a line parallel to the two fixed parallel lines.
  - (d) The locus of points equidistant from two points A and B is a perpendicular bisector of the line segment $\overline{AB}$.
  - (e) The locus of points at constant distance from point A is a circle centred at A and of radius equal to the constant distance d.
  - (f) The locus of points at a given distance from a straight line $\overline{AB}$ is a pair of parallel lines parallel to $\overline{AB}$ and on opposite sides of the given line.

GRADUATED EXERCISES

1. Using a ruler, pencil and a pair of compasses only, construct the following:
   (a) $7\frac{1}{2}°$ (b) $165°$

2. Draw the line segment $\overline{AB}$. At A construct $\angle BAC$ to be $60°$. Using $\overline{AC}$ as base, construct $\angle CAD$ to be $45°$. Measure $\angle BAD$.

3. Which of the following is a correct method of constructing an angle $60°$ at Q?

   (a) I only   (b) II only   (c) III only
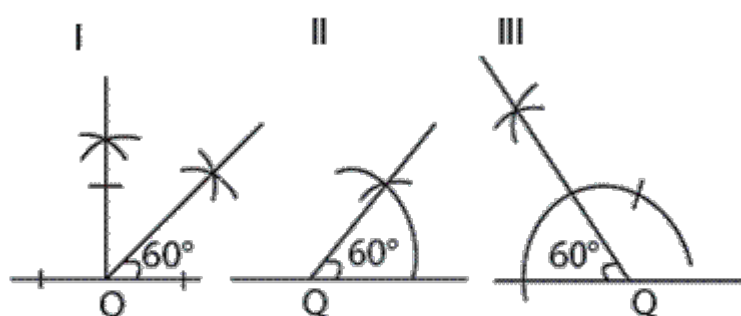   (d) I and II only   (e) II and III only.

Fig. 9.23

4. Which of the sketches below shows the correct method of constructing an angle of 120° at the point P.
   (a)   I only   (b)   II only
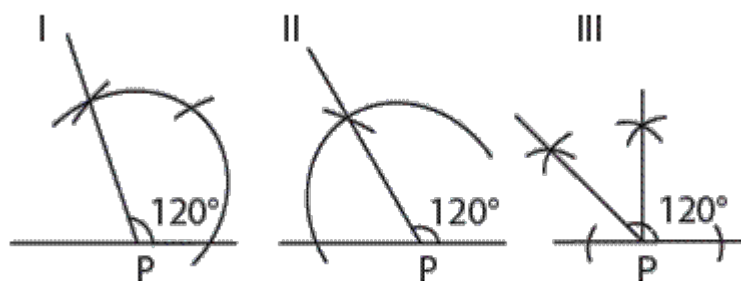   (c)   III only   (d)   I and II only
   (e)   I, II and III.

Fig. 9.24

5. Construct ΔABC in which |BC| = 4 cm, |AC| = 8 cm and ∠BCA = 135°. Measure $\overline{AB}$.

6. Construct ΔXYZ in which |XY| = 7.5 cm, ∠XYZ = 120° and |YZ| = 8.5 cm. Measure $\overline{XZ}$ and ∠YXZ.

7. Use ruler and a pair of compasses to construct ΔLMN in which ∠M = 90°,

   |MN| = 5 cm and |LN| = 10 cm. Measure $\overline{LM}$.

8. Construct ΔRST in which |ST| = 8.3 cm, ∠RST = 60° and ∠STR = 45°. Measure $\overline{TR}$.

9. ΔLMN has ∠LMN = 135°, |LM| = 5 cm and |MN| = 6.5 cm. Bisect ∠LMN and let the bisector meet $\overline{LN}$ at O. Measure $\overline{ON}$.

10. (a) Construct ΔABC in which |AB| = 5 cm, |AC| = 6 cm and |BC| = 9 cm.

   (b) Construct the bisector of B.

   (c) Construct the perpendicular bisector of $\overline{BC}$. The bisectors meet at O. Measure $\overline{OC}$.

11. Construct ΔABC in which |AB| = 5.5 cm, |BC| = 8.5 cm and ∠ABC = 75°. Construct M, the midpoint of $\overline{AC}$. Measure $\overline{BM}$.

12. Using a ruler and a pair of compasses only, construct:

   (a) Triangle QRT with |QR| = 8 cm, |RT| = 6 cm and |QT| = 4.5 cm.

   (b) A quadrilateral QRSP which has a common base $\overline{QR}$ with ΔQRT such that $\overline{QTP}$ is a straight line, $\overline{PQ}//\overline{SR}$, |QP| = 9 cm and |RS| = 4.5 cm.

      (i) Measure |PS|.

      (ii) Find the perpendicular distance between $\overline{RS}$ and $\overline{PQ}$.

      (iii) What is QRSP?

13. Using a ruler and compasses only, construct:

   (a) A parallelogram ABCD with |AB| = 8 cm, |AC| = 12 cm and |BD| = 7.2 cm. Measure BC.

   (b) The perpendicular from B to CD and hence find the area of the parallelogram.   (WAEC)

14. Using a ruler and a pair of compasses only, construct a triangle ABC given that $|AB| = 8.4$ cm, $|BC| = 6.5$ cm and $\angle ABC = 30°$. Construct the locus:

   (a) l_1 of points equidistant from AB and BC, and within $\angle ABC$.

   (b) l_2 of points equidistant from B and C. Locate the point of intersection P of l_1 and l_2. Measure $|AP|$.

   (WAEC)

15. Using a ruler and a pair of compasses only:

   (a) Construct $\triangle XYZ$ such that $|XY| = 8$ cm and $\angle YXZ = \angle ZYX = 45°$.

   (b) Locate a point P inside the triangle equidistant from $\overline{XY}$ and $\overline{XZ}$ and also equidistant from $\overline{YX}$ and $\overline{YZ}$.

   (c) Construct a circle touching the three sides of the triangle.

   (d) Measure the radius of the circle.

   (WAEC)