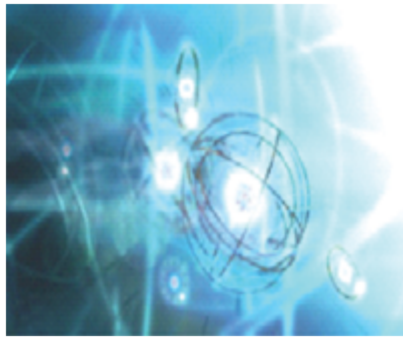


7

ALTERNATING CURRENT CIRCUIT

OBJECTIVES

At the end of the topic, students should be able to:

- explain the peak and root mean square (r.m.s.)values of current and voltage;
- establish the phase relation between current and voltage in an a.c. circuit;
- explain reactance and impedance;
- determine current in circuits containing:
  - resistance and inductance;
  - resistance and capacitance;
  - resistance, inductance and capacitance;
- determine power in an a.c. circuit.

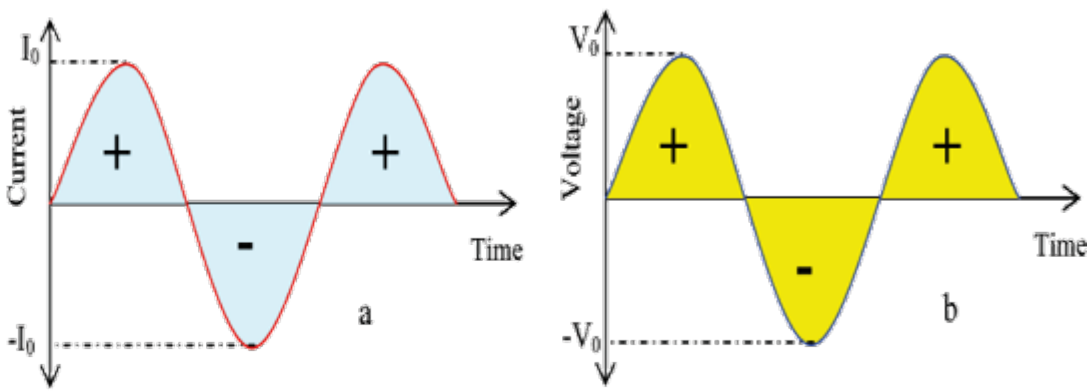


Figure 7.1: Graphical representation of alternating current/voltage

Current/voltage which varies in both magnitude and direction, are called alternating current. The term alternating current (a.c.) is used generally for signals which vary in magnitude and direction. The simplest form of an alternating current/voltage is represented by a sine or a cosine curve. The graphical representation of an alternating current/voltage is shown in Figure 7.1.

Any signal, which can be represented by a sine or cosine curve, is said to be **sinusoidal.** The magnitude of a sinusoidal function at any instant is mathematically represented by:

$$I = I_0 \sin\omega t \text{ or } I = I_0 \sin(2\pi\, ft)$$
$$V = V_0 \sin\omega t \text{ or } V = V_0 \sin(2\pi ft)$$

I/V = are the values of current/voltage at a given time
I_0/V_0 = are the peak or maximum value of current/voltage
Ï‰t = phase angle, **Ï‰** = angular frequency and
f = linear frequency of the a.c. source.

Peak, average and root mean square values of an alternating current (a.c.)

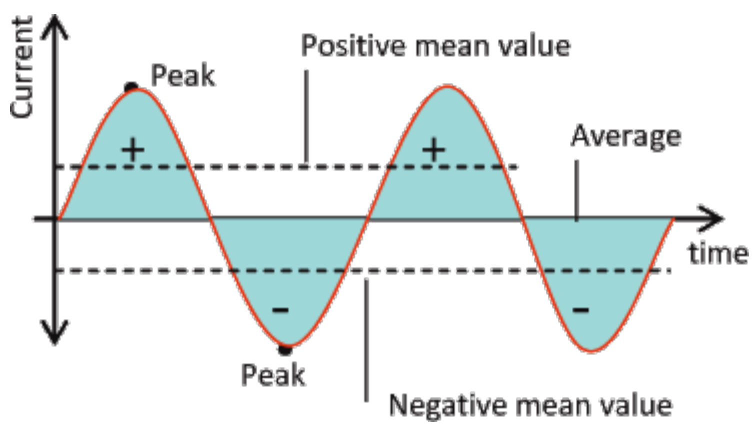


Figure 7.2 : Alternating current/voltage

- The peak value of an alternating current is the maximum value or the amplitude of the a.c. current or voltage.
- The average value of an alternating current is the sum of the currents/voltages above the zero line. The average of a.c. current/ voltage is zero because it changes between maximum positive to maximum negative as shown in Figure 7.2.

The root mean square (r.m.s.) value

The root mean square (r.m.s.) of an alternating current is the value of the a.c. current which produces heat in a resistor at the same rate as the value of the d.c. current.

A d.c. current of 1A, flowing through a resistor of 2 Î© for 10 seconds, will produce 20 joules of heat energy. When an a.c. current passes through the same resistor, its value varies. The value of the a.c. current flowing through a 2 Î© resistor for 10 seconds which produces 20 joules of heat is called the **effective value** of the a.c. current. The effective value of a.c. current is called the **root mean square (r.m.s.)** of the alternating current.

The root mean square (r.m.s.) or the effective value of an a.c. current is related to the peak value of the a.c. current by the equation:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.7071 I_0$$

$$V_{r.m.s.} = \frac{V_0}{\sqrt{2}} = 0.7071 V_0$$

Proof of the root mean square value

The power dissipated in a resistor when a steady d.c. current flows through it is given by:

$$P = I^2 R$$

The same power is dissipated in the resistor by the r.m.s. of an a.c. current.

$$P = I^2_{r.m.s.} R$$

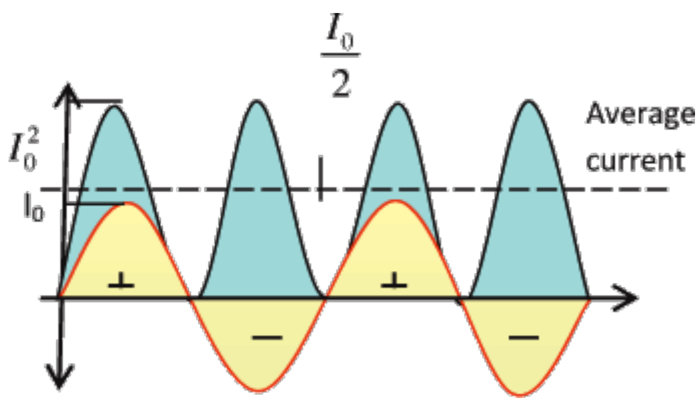


Figure 7.3 : Alternating current

Alternating current varies in magnitude and direction; its average is zero (0) but the mean of its square is not zero. This is shown in Figure 7.3 The average of the mean of the squares of the a.c. current is given by:

$$I^2_{ave.} = \frac{0 + I_0^2}{2} = \frac{I_0^2}{2}$$

Power supplied by the a.c. current to the resistor is given by:

$$P = I^2_{ave.} R = \frac{1}{2} I_0^2 R$$

$$\therefore I^2_{r.m.s.} R = \frac{1}{2} I_0^2 R$$

$$\therefore I^2_{r.m.s.} = \frac{I_0^2}{2}$$

$$I_{r.m.s.} = \frac{I_0}{\sqrt{2}}$$

Worked examples

1.A sinusoidal a.c. current has a peak value of 5 A.Find the:

(i)root mean square current;

(ii)current at an instant when Ï‰t = 60Â° and 150Â°.

Solution

(i) $I_{r.m.s.}$= 0.7071I_0

$I_{r.m.s.}$= 0.7071 Ã— 5 = 3.54 A

(ii)I = I_0sin Ï‰t = 5 sin 60Â° = 5 Ã— 0.8660 = 4.33 A

I = I_0sin Ï‰t = 5 sin 150Â° = 5 Ã— 0.5 = 2.5 A

2.A sinusoidal a.c. voltage is given by the equation V = 240sin (100Ï€ t). Find the:

(i)peak voltage;

(ii)r.m.s. value of the voltage;

(iii)frequency of the source.

Solution

(i)Comparing the equations V = V_0sinÏ‰t and V = 240sin (100Ï€t).

Peak value V_0 = 240 V

(ii)$V_{r.m.s}$= 0.7071 Ã— V_0 = 0.7071 Ã— 240 = 169.7 V

(iii)Ï‰ = 100Ï€ but Ï‰ = 2Ï€ f

âˆ´2Ï€ f = 100Ï€

f = 50 Hz.

3.The peak value of alternating current passing through a resistor is $\sqrt{8}$ A. What is the r.m.s. value of the current?

Solution

$$I_{r.m.s.} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{8}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}} = 2A.$$

Alternating current through a pure resistor

A pure resistor has no inductance or capacitance. The a.c. current and voltage flowing through a pure resistor are in phase (they attain their maximum and minimum values at the same time). The value of current and voltage at any time is given by:

I = I_0sin (Ï‰ t) and V = V_0sin (Ï‰ t)

Ohmâ€™s law gives the resistance of the resistor as:

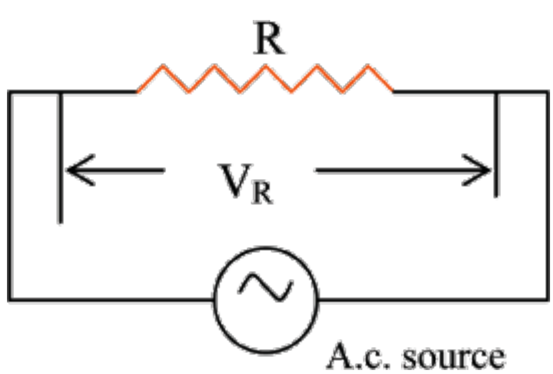


Figure 7.4 : Pure resistor

$$\text{Resistance} = \frac{\text{Voltage}}{\text{Current}}$$

$$R = \frac{V_0 \sin \omega t}{I_0 \sin \omega t}$$

$$R = \frac{V_0}{I_0} = \frac{V_{r.m.s.}}{I_{r.m.s.}}$$

The waveform or phasor diagram representing the variation of current and voltage is shown in Figure 7.5a. The vector diagram is shown in Figure 7.5b. The vector diagram shows the magnitude and the direction of the current and voltage at any given time.

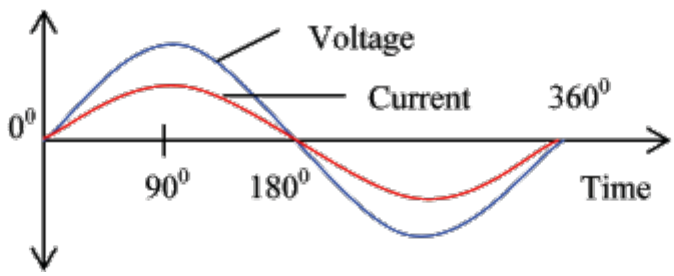


Figure 7.5a : Phasor diagram

Figure 7.5b : Vector diagram

Alternating current through a pure inductor

A pure inductor has no resistance. When an a.c. current flows through an inductor, the changing current induces a back e.m.f. in it. The back e.m.f. opposes the changing current (i.e. a maximum voltage is induced in the coil if the current in it is zero and zero when the current reached its greatest value). The applied voltage is equal to the induced voltage when the current is zero; therefore, current and the applied voltage are out of phase by 90Â° or $\frac{\pi}{2}$ radian. The voltage leads the current by 90Â° or radian.

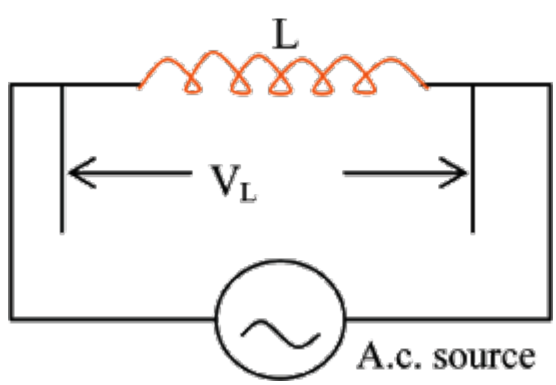


Figure 7.6 : Pure inductor

The phasor and vector diagrams showing how the voltage and the current are related in a pure inductor are shown in Figure 7.7.

Figure 7.7a: A Vector diagram

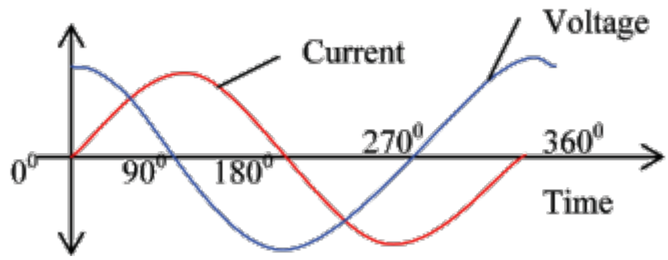


Figure 7.7b : Phase relationship of current and voltage in an inductor

Inductive reactance

Reactance is the opposition of a.c. circuit components to the flow of alternating current. If the resistance to the flow of a.c. current is due to an inductor, it is known as inductive reactance (X_L).

Inductive reactance is the opposition of an inductor to the flow of an a.c. current.

$$\text{Inductive reactance} = \frac{\text{Peak voltage}}{\text{Peak current}}$$

$$X_L = \frac{V_0}{I_0} = \frac{V_{r.m.s.}}{I_{r.m.s.}}$$

The magnitude of inductive reactance (X_L) is given by:

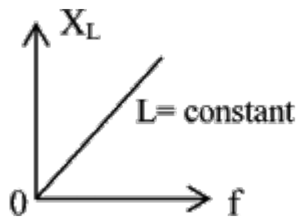
X_L = Ï‰ L = 2Ï€fL

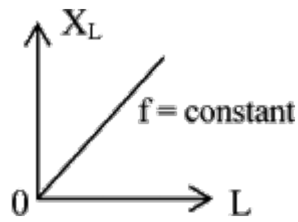
X_L=inductive reactance measured in Ohms (Î©)

f = frequency of the source in Hertz (Hz).

L = self-inductance of the coil in Henry (H).

The reactance of an inductor is directly proportional to the frequency of the source, if the self-inductance is constant. The reactance of an inductor is also directly proportional to the self-inductance, if the frequency of the source is constant.





Alternating current through a pure capacitor

A pure capacitor has no resistance. When a capacitor is connected to an a.c. circuit, current flows and charges are deposited on the plates. The charge deposited is proportional to the voltage applied across the capacitor.

$$Q = I \times t \;........(i)$$
$$Q = CV \;.........(ii)$$
$$\therefore \; I \times t = CV$$
$$I = \frac{CV}{t}$$

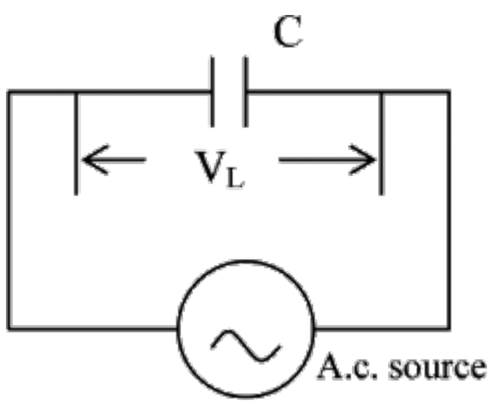


Figure 7.8 : Pure capacitor

Current is proportional to the time rate of change of voltage. As the voltage increases from zero, current decreases from its maximum value to zero. Current and voltage are therefore out of phase by $\frac{\pi}{2}$ radian or 90Â°. We say that current leads the voltage by 90Â° or $\frac{\pi}{2}$ radian.

The phase relationship between voltage and current and the vector diagram for a capacitive circuit is illustrated in Figure 7.9.

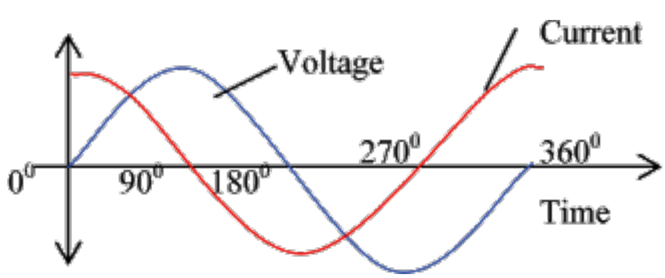


Figure 7.9a Phase relationship of current and voltage in an inductor

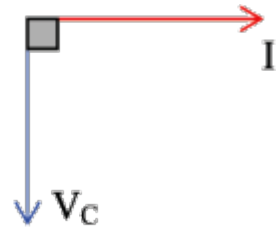


Figure 7.9b :Vector diagram

Capacitive reactance

Capacitive reactance is the opposition of a capacitor to the flow of an a.c. current.

$$\text{Capacitive reactance} = \frac{\text{Peak voltage}}{\text{Peak current}}$$

$$X_C = \frac{V_0}{I_0} = \frac{V_{r.m.s.}}{I_{r.m.s.}}$$

The capacitive reactance (X_C) is given by:

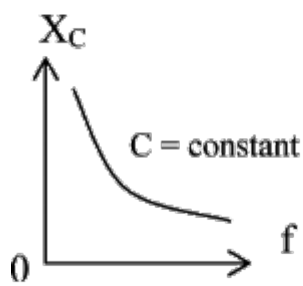
$$X_C = \frac{I}{\omega C} = \frac{I}{2\pi f C}$$

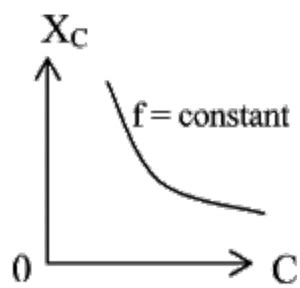
X_C = capacitive reactance in Ohms (Î©)

C = capacitance of capacitor in Farads (F)

f = frequency of the source in Hertz (Hz)

The reactance (X_C) of a capacitor is inversely proportional to the frequency of the source if the capacitance of capacitor is constant and vice versa.





Worked examples

1. An inductor of 0.5 H is connected to an a.c. source of 240 V. Calculate the:

   (i)inductive reactance;

   (ii)current flowing through the circuit;

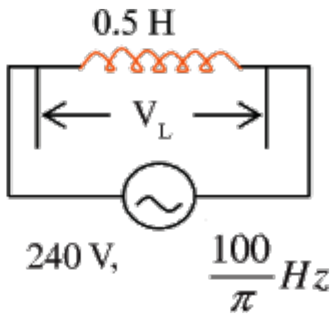
   (iii)energy dissipated in the inductor.

   {frequency of the source is $\frac{100}{\pi} Hz$

Solution

(i) $X_L = 2\pi fL$

$$= 2 \times \pi \times \frac{100}{\pi} Hz \times 0.5 = 100\ \Omega$$

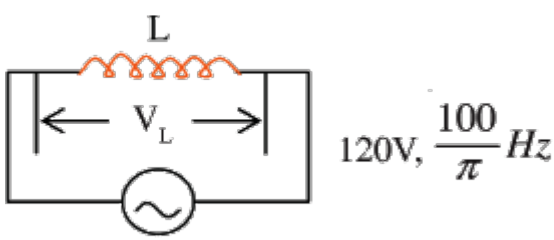


(ii) $I = \frac{V_L}{X_L} = \frac{240}{100} = 2.4A$

(iii) $W = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.5 \times 2.4^2 = 1.44\,J$

2. A current of 1.5 A flows through an inductor (L) when a p.d. of 120 V, $\frac{100}{\pi}$Hz is applied across it. Calculate the:
   (i)inductive reactance;
   (ii)inductance of the inductor.

Solution



(i) $X_L = \frac{V_L}{I} = \frac{120}{1.5} = 80\Omega$

(ii) $L = \frac{X_L}{2\pi f} = \frac{80}{2\pi \times \frac{100}{\pi}} = 0.4\,H$

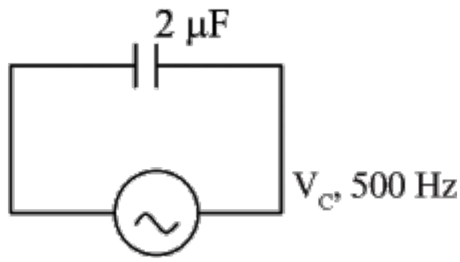
3. A capacitor of capacitance 2 Î¼F is used in a radio circuit where the frequency is 500 Hz. If the current flowing through the circuit is 0.005 A, calculate the:
   (i)reactance of the capacitor;
   (ii)voltage applied across the capacitor.

Solution

(i) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 500 \times 2 \times 10^{-6}}$

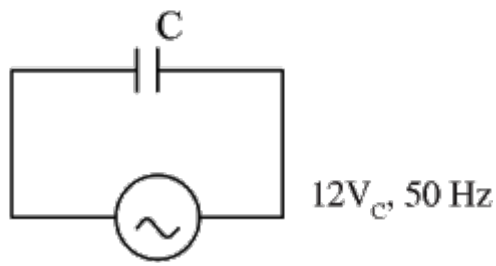
$X_C = 159$Î©.

(ii)$V_C = IX_C = 0.005$ Ã— $159 = 0.8V$.



4. A capacitor of capacitance (C) connected to a 12V; 50Hz source, draws a current of 3.77A. Calculate the:
   (a)capacitive reactance;
   (b)capacitance of the capacitor.

Solution



(a) $X_C = \frac{V_C}{I} = \frac{12}{3.77} = 3.18\Omega$

(b)
$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 3.18}$$
$$C = 1001\mu F$$

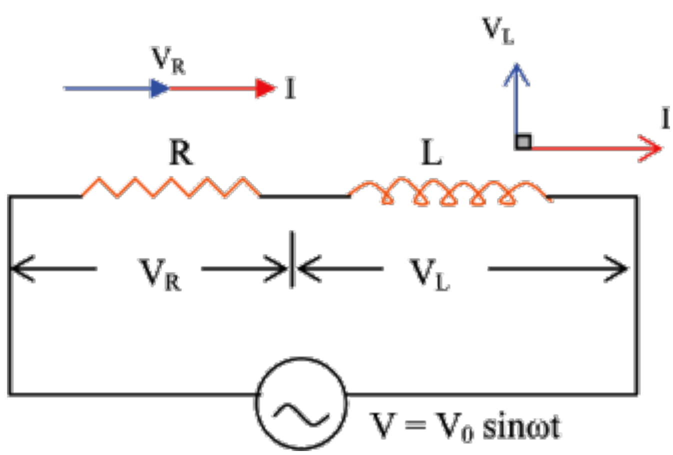


Figure 7.10 : RL circuit

When a resistor and an inductor are connected in series as shown in Figure 7.10; the same current flows through the inductor and resistor. The voltage across the resistor is in the same direction as the current but the voltage across the inductor leads the current by a phase angle of 90Â° or radians. The vector diagram is shown in Figure 7.11.

The supply voltage (V) from the vector diagram is given by:
$V^2 = V_R^2 + V_L^2$
But $V = IZ$, $V_R = IR$, and $V_L = IX_L$
â^´ $(IZ)^2 = (IR)^2 + (IX_L)^2$
$Z^2 = R^2 + X^2$

$$Z = \sqrt{R^2 + X_L^2}$$

Z = Impedance of the a.c. circuit. It is measured in Ohm (Î©).

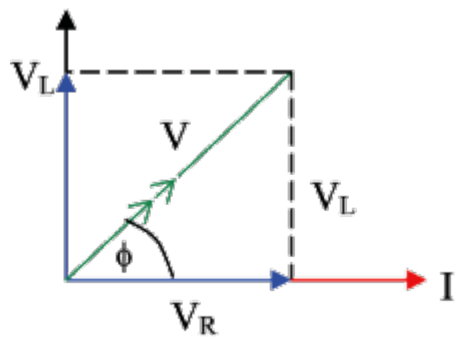


Figure 7.11 : Vector diagram of V_L versus V_R

Impedance (Z) is the total opposition to the flow of alternating current by an a.c. circuit.

Phase angle (Ï†) is the angle between the supply voltage and the current. The vector diagram shows that the supply voltage leads the supply current by Ï†.

In the R-L vector diagram in Figure 7.11, the phase angle can be calculated using the relation

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\tan\phi = \frac{X_L}{R}$$

Worked examples

1. An inductor of 0.05 H and a resistor of 12 Î© are connected in series to an a.c. supply of 65V; $\frac{50}{\pi} Hz$ Calculate:

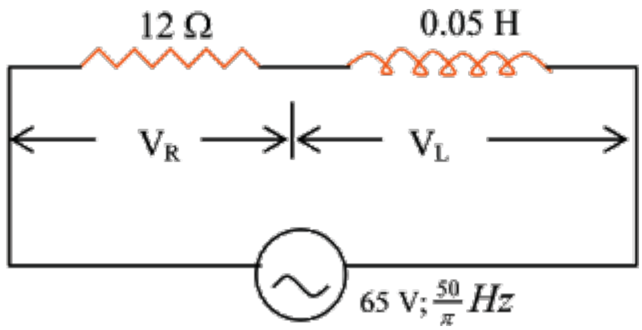
   (a)the impedance of the circuit;

   (b)the current flowing through the circuit;

   (c)voltage across the resistor and the inductor;

   (d)sketch the vector diagram.

Solution



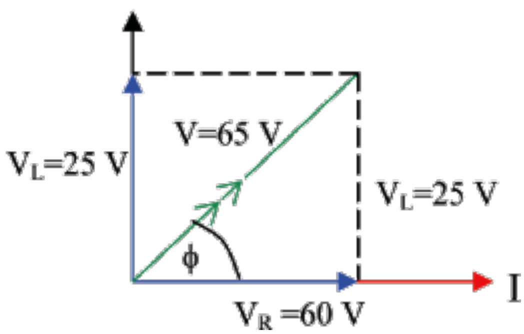
(a) $X_L = 2\pi fL = 2\pi \times \frac{50}{\pi} \times 0.05 = 5\Omega$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 5^2}$$
$$= \sqrt{169} = 13\Omega$$

(b) $I = \frac{V}{Z} = \frac{65}{13} = 5A$

(c) V_L = IX_L = 5 Ã— 5 = 25 V
V_R = IR = 5 Ã— 12 = 60 V

(d)



2. An inductor and a resistance are connected in series to an a. c. supply of V volts. The voltage across the inductor and the resistor are 20V and 15V respectively. Calculate the:

(a)supplied voltage;

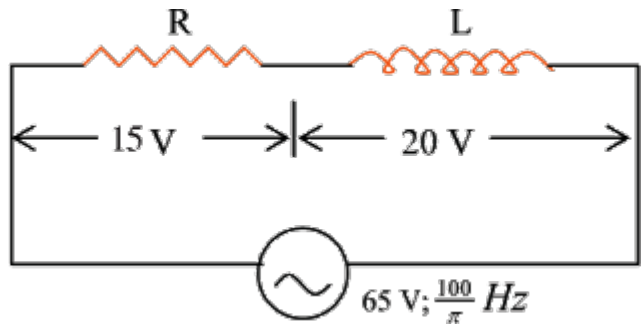
(b)current flowing in the circuit if the impedance is 12.5Î©;

(c)inductance of the coil;

(d)resistance of the coil.

[The frequency of the source is $\frac{100}{\pi} Hz$]

Solution



(a) $V = \sqrt{V_R^2 + V_L^2} = \sqrt{15^2 + 20^2} = \sqrt{625}$

$V = 25$ V.

(b) $I = \frac{V}{Z} = \frac{25}{12.5} = 2A$

(c) $X_L = \frac{V_L}{I} = \frac{20}{2} = 10\Omega$

$$L = \frac{X_L}{2\pi f} = \frac{10}{2 \times \pi \times \frac{100}{\pi\pi}} = \frac{10}{200} = 0.05H$$

(d) $R = \frac{V_R}{I} = \frac{15}{2} = 7.5\Omega$

Resistor and capacitor in series (R-C circuit)

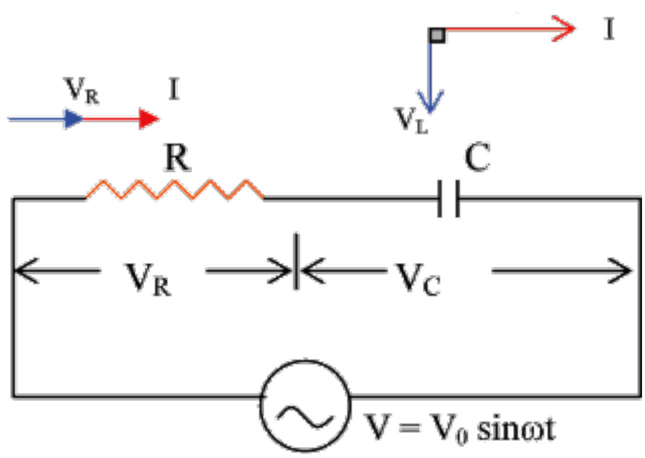


Figure 7.12 : R-C circuit

Figure 7.12 is a series connection of a resistor and a capacitor to an alternating voltage supply. The same current flows through the resistor and the inductor. The current and voltage are in the same direction (phase) in the resistor but the current leads the voltage by

$\frac{\pi}{2}$ radians or 900 in the inductor. The vector diagram is illustrated in Figure 7.13.

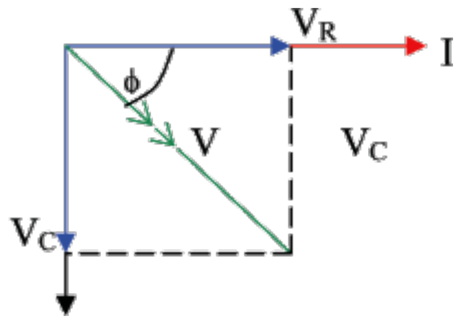


Figure 7.13 : Vector diagram of an R-C circuit

The supply voltage (V) from the vector diagram is given by:

$$V^2 = V_R^{\,2} + V_C^{\,2}$$

But V = IZ, V_R = IR, and $V_C = IX_C$

â^´ $(IZ)^2 = (IR)^2 + (IX_C)^2$

$Z^2 = R^2 + X^2$

$$Z = \sqrt{R^2 + X_C^2}$$

Z = impedance of the of the a.c. circuit. It is measured in Ohms (Î©).

The vector diagram shows that the supplied voltage lags the supplied current by *Ï†*.

In an R-C vector diagram in Figure 7.13, the phase angle (Ï†) can be calculated using the relation

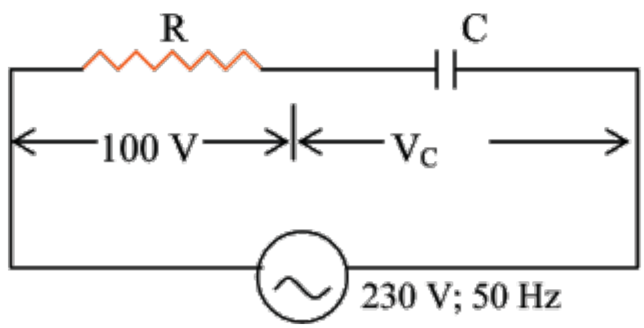
$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\tan\phi = \frac{X_C}{R}$$

Worked examples

1. A metal-filament lamp rated 750 W; 100 V is connected in series with a capacitor to a source of 230 V; 50 Hz. Calculate the:
   (a)voltage across the capacitor;
   (b)capacitive reactance;
   (c)capacitance of the capacitor;
   (d)impedance of the circuit.

Solution



(a) $V^2 = V_R^2 + V_C^2 \quad 230^2 \quad 100^2 \quad V_C^2$

$$V_2 = \sqrt{230^2 - 100^2} = \sqrt{42900} = 207V$$

(b) $I = \frac{P}{V} = \frac{750}{100} = 7.5A$

$$X_C = \frac{V_c}{I} = \frac{207}{7.5} = 27.6\Omega$$

(c) $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 27.6}$

$$= 1.153 \times 10^{-4} F$$

(d) $R = \frac{V_R}{I} = \frac{100}{7.5} = 13.3\Omega$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{13.3^2 + 27.6^2}$$

$$Z = \sqrt{938.65} = 30.6\Omega$$

2. A capacitor of capacitance 5 Î¼F is connected in series with a resistor of 1000 Î© to an a.c. source of 240 V; 50 Hz. Calculate the:
(a)impedance of the circuit;
(b)current flowing in the circuit;
(c)potential difference across the resistor.

Solution

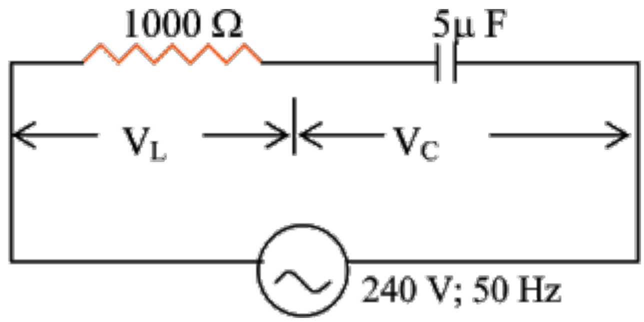
(a) $Xc = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 5 \times 10^{-6}} = 636.9\Omega$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{1000^2 + 636.9^2}$$

$$= \sqrt{1405641.61} = 1185.6\Omega$$

(b) $I = \frac{V}{Z} = \frac{240}{1185.6} = 0.2024A$

(c)V_R = IR = 0.2024 Ã— 1000 = 202.4 V.



Resistor, inductor and capacitor in series (R-L-C circuit)

Figure 7.14 shows a series connection of a resistor, inductor and capacitor to an alternating voltage source (V). The current is in phase with the voltage in the resistor, the voltage across the inductor leads the current by 90Â° while the voltage across the capacitor lags the current by 90Â°. Assuming that the $V_L > V_C$, the vector diagram is as shown in Figure 7.15.

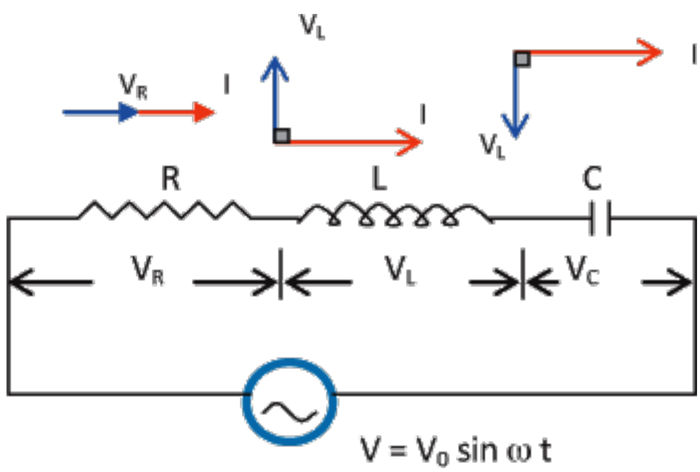


Figure 7.14 : R-L-C circuit

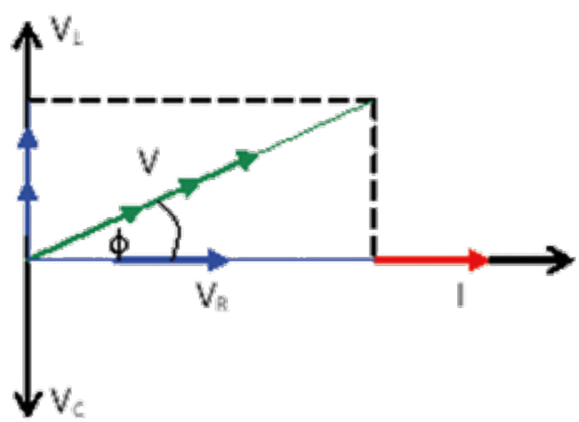


Figure 7.15a : R-L-C vector diagram

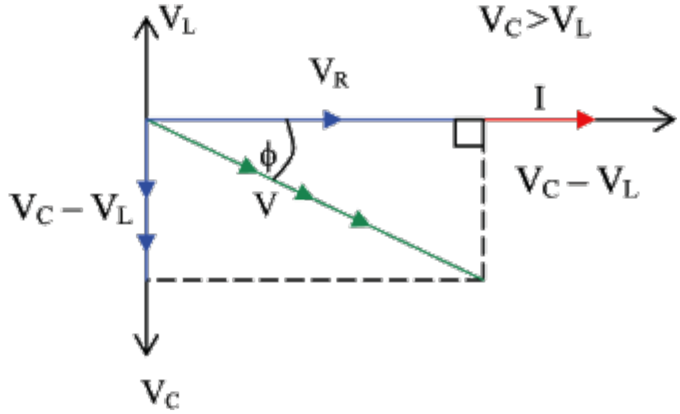


Figure 7.15b : R-L-C vector diagram

If $V_C > V_L$ Figure 7.15b : represents the vector diagram.

Please note that:

- V_L and V_C are anti-phase, therefore their resultant is $V_L - V_C$ if $V_L > V_C$ or $V_C - V_L$ if $V_C > V_L$
- the resultant V is the vector addition of the resultants $V_L - V_C$ or V_C or V_C V_L and V_R.
- V_L or V_C may be greater than the supplied voltage V.

The supplied voltage (V) is given by the vector addition of V_R, V_L and V_C as:

$V^2 = V_R^2 + (V_L - V_c)^2\ (V_L > V_c)$

But V= IZ, $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$

âˆ´ $(IZ)^2 = (IR)^2 + I^2(X_L - X_c)^2$

Cancelling out, the current gives;

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle (Ï†) between the suppied voltage and the current is given by:

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Worked example

A capacitor of 150 Î¼ F, an inductor of 0.1 H and a resistor of 50 Î© are connected in series to a 12 V; 50 Hz power supply. Calculate these:

(a)impedance of the a.c. circuit;

(b)the current flowing through the circuit.

Solution

(a) $$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$

$X_C = 21.2\ \Omega.$

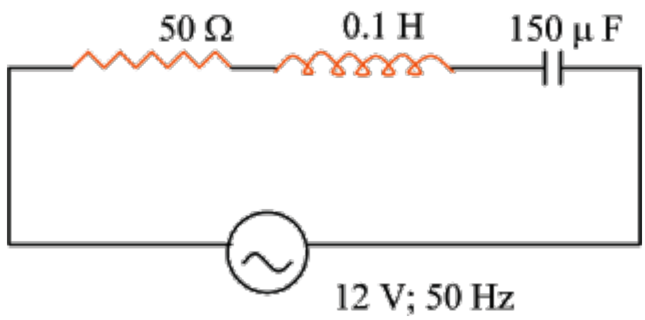
$X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.4\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{50^2 + (31.4 - 21.2)^2}$$

$$Z = \sqrt{2604.04} = 51\Omega$$

(b) $$I = \frac{V}{Z} = \frac{12}{51} = 0.24\,A$$



Power dissipated by a.c. components

Power dissipated by a pure inductor and a pure capacitor when an alternating current flows through them is zero. **The only power dissipated in an a.c. circuit is across the resistor.** Capacitors and inductors are used to limit the current passing through an a.c. circuit.

Power dissipated in an a.c. circuit is the product of current and the component of voltage in the direction of current.

Power (P) = IV_R........(i)

Power (P) = I^2R........(ii)

Power (P) = IVcos(Ï•) ... (iii)

Note that Vcos Ï• = V_R, the component of supplied voltage in the direction of current. The moderating factor cos Ï• is called the **power factor.** The power factor is related to V and V_R by:

$$Power\ factor = Cos\phi = \frac{V_R}{V} = \frac{R}{Z}$$

Worked examples

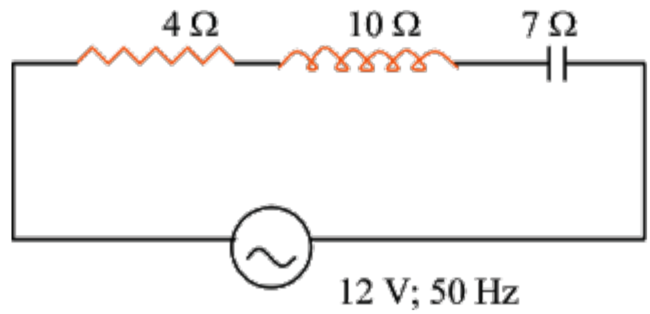
1.In the circuit above, a resistance of 4 Î© is connected in series with an inductor of reactance 10 Î© and capacitor of reactance 7 Î© to a power source of 12 V; 50 Hz.

Calculate:

(a)the impedance of the circuit;

(b)the power dissipated in the circuit;

(c)the inductance of the inductor.



Solution

(a) $$Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$Z=\sqrt{4^2+(10-7)^2}=\sqrt{4^2+3^2}$$
$$=\sqrt{25}=5\Omega$$

(b) Power factor = $\cos\phi = \frac{R}{Z}=\frac{4}{5}=0.8$

$$I=\frac{V}{Z}=\frac{12}{5}=2.4A$$

Power dissipated (P) = $Iv\cos\phi$
$$=2.4\times12\times0.8$$
$$=23.04\ \text{W}$$

(c) $$L=\frac{X_L}{2\pi f}=\frac{10}{2\pi\times50}=0.032H$$

2. A resistor, an inductor and a capacitor are connected in series to an a.c. power supply. A current of 2 A flows in the circuit when the frequency of the applied voltage V is $\frac{200}{\pi}$ Hz. Voltmeters connected across the terminals of the resistor, the inductor and the capacitor reads 100 V, 52 V and 270 V respectively. Find:

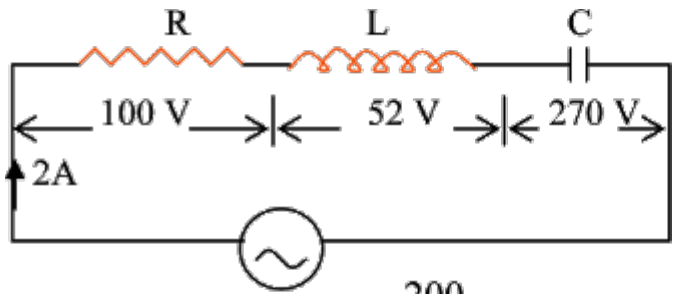
(a)the supplied voltage;

(b)the resistance of the resistor;

(c)the inductance of the inductor;

(d)the capacitance of the capacitance.

Solution



$$V; \frac{200}{\pi} Hz.$$

(a) $V = \sqrt{V_R^2 + (V_C - V_L)^2}$

$$V = \sqrt{100^2 + (270-52)^2} = \sqrt{57\ 524}$$

V = 239.8 V

$$L = \frac{X_L}{2\pi f} = \frac{26}{2\pi \times \frac{200}{\pi}} = 0.065\,H.$$

(b) $R = \frac{V_R}{I} = \frac{100}{2} = 50\Omega$

(c) $X_L = \frac{V_R}{I} = \frac{52}{2} = 26\Omega$

(d) $X_C = \frac{V_C}{I} = \frac{270}{2} = 135\Omega$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times \frac{200}{\pi} \times 135}$$

$$C = 1.85 \times 10^{-5} F = 18.5\mu F$$

Resonance occurs in an a.c. circuit when the inductive reactance is equal to the capacitive reactance. This is possible if the values of the inductor and capacitor are adjusted until a maximum current flow in the circuit. At resonance:

- impedance of the circuit is minimum. The impedance is equal to resistance since capacitive reactance is cancelled by the inductive reactance.
- maximum current flows through the circuit;
- supplied current is in phase with the supplied voltage.

The variation of X_e, X_C, R, and Z with frequency is illustrated in Figure 7.16.

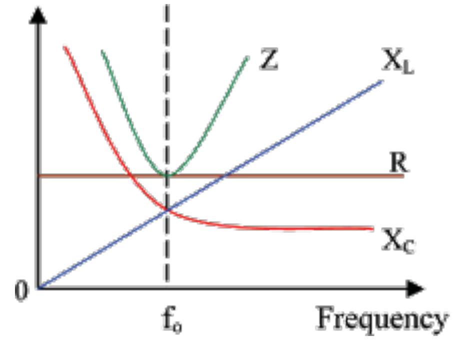


Figure 7.16a: Variation of X_L, x_c R, Z- with frequency

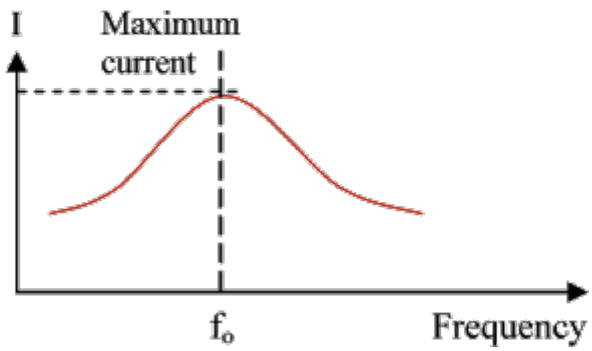


Figure 7.16b: Current against frequency graph

The frequency when maximum current flows through the circuit is called resonant frequency (f_0). This occurs when $X_L = X_C$.

$$\therefore\ X_L = X_C$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$4\pi^2 f_0^2 LC = 1$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The L-C circuit is used in the tuning circuits of radio and television sets. The capacitor and the inductor are varied until the circuit is tuned to resonate at the frequency of the transmitting station.

Worked example

A coil of inductance 0.2 H is connected in series to a capacitor of capacitance 5 Î¼F Calculate the resonant frequency of the circuit.

Solution

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2\times5\times10^{-6}}} = \frac{1}{0.00628}$$

$$f_0 = 159.2Hz$$

Summary

- **Current/voltage** which varies in both magnitude and direction, are called **alternating current.**
- The **peak** value of an alternating current is the maximum value or the amplitude of the a.c. current or voltage.
- **The root mean square** (r.m.s.) of an alternating current is the value of the a.c. current which produces heat in a resistor at the same rate as that produced by d.c. current.

$$I_{r.m.s.} = \frac{I_0}{\sqrt{2}} \quad V_{r.m.s.} = \frac{V_0}{\sqrt{2}}$$

- The a.c. current and voltage flowing through a pure resistor are in phase (they attain their maximum and minimum values at the same time).
- A **pure inductor** has no resistance. The **voltage leads the current** by 90Â° or radian in an inductor.
- **Inductive reactance** is the opposition of an inductor to the flow of a.c. current.

X_I = Ï‰ L = 2Ï€fL

- A **pure capacitor** has no resistance. Current flowing through a capacitor leads the voltage by radian or 90Â°. Current and voltage are out of phase by radian or 90Â°.
- **Capacitive reactance** is the opposition of a capacitor to the flow of a.c. current.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fc}$$

- **Impedance** (Z) is the total opposition to the flow of alternating current by an a.c. circuit.

$$Z = \sqrt{R^2 + X_L^2}$$

- **Phase angle** (Ï•) is the angle between the supplied voltage and the current.
- **Power** dissipated by a pure inductor and a pure capacitor when an alternating current flows through them is zero (0). **The only power dissipated in an a.c. circuit is across the resistor.**

Power (P) = IV_R........ (i)

Power (P) = I^2R .... (ii)

Power (P) = IVcosÏ• ... (iii)

- Resonance occurs in an a.c. circuit when the inductive reactance is equal to the capacitive reactance.
- **Resonant frequency** (f_0) is the frequency when maximum current flows through the circuit.

Practice Questions 7

1.(a) What is an alternating current?

(b)Explain the following terms as applied to alternating currents (i) *root mean square (r.m.s.) (ii) peak value* of alternating current.

(c)A current generated by a generator is given by I = 220sin 100Ï€t.

(i) Calculate the *root mean square (r.m.s.).*

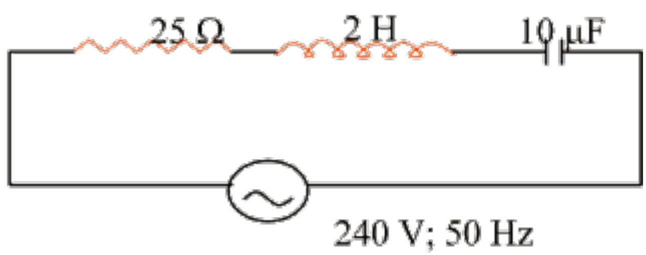
(ii) What is the frequency of the alternating source?

2.(a) Define reactance and impedance.

(b)A capacitor, an inductor and a resistor are connected in series to an alternating source.

(i) Draw a vector diagram showing the relationship between the supply current and voltage.

(ii) Calculate the impedance and supply current for the circuit shown below.



3. (a) What is power factor?

(b)Calculate the power dissipated in the circuit in question 2b (ii).

(c)What is the resonant frequency of the circuit in question 2b (ii).

Past Questions

1. In an a.c. circuit the peak value of the potential difference is 180V. What is the instantaneous p.d. when it has reached $\frac{1}{8}$th of a cycle?

A. 45 V

B. 90 V

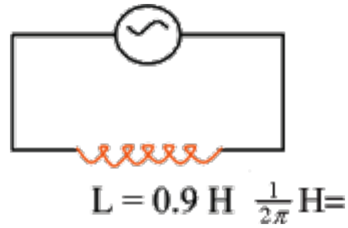
C. $90\sqrt{2}\ V$

D. 180 V

E. $180\sqrt{2}\ V$

WAEC

2. If the frequency of the e.m.f. source in an a.c. circuit illustrated below is $\frac{1}{2\pi}$ Hz, what is the reactance of the inductor?



A. Ï€/450 Î©

B. 0.9 Î©

C. 450/Ï€ Î©

D. 450 Î©

E. 900 Î©

WAEC

3. In a series R-C circuit, the resistance of the resistor is 4 Î© and the capacitive reactance is 3 Î©. Calculate the impedance of the circuit.

A. 1 Î©.

B. 4 Î©.

C. 5 Î©.

D. 7 Î©.

E. 12 Î©.

WAEC

4. The vector diagram shown below represents the resistance, R, the capacitive reactance X_c, the inductive reactance X_L and the impedance, Z in R-L-C circuit. The current in the circuit will be maximum when

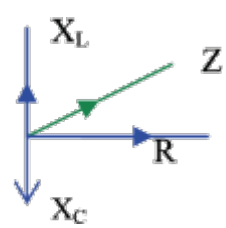
A. $\mathbf{X_C > X_L}$.

B. $\mathbf{X_C = X_L}$.

C. $\mathbf{X_C < X_L}$.

D. R = $\mathbf{X_L}$.

E. R = $\mathbf{X_C}$.



WAEC

5. In a purely resistive a.c. circuit, the current I= I_0sin Ï‰t and the voltage, V = V_0sin Ï‰t. Calculate the instantaneous power dissipated in the circuit in time (t).

A. I_0V_0

B. $2I_0V_0$sin Ï‰t

C. $\frac{I_0V_0}{2}$

D. $I_0V_0\sin^2$ Ï‰t

E. $\frac{I_0^2V_0^2}{2}$

WAEC

6. Calculate the peak voltage of a mains supply of r.m.s. value 220V.

A. 112 V.

B. 150 V.

C. 222 V.

D. 240 V

E. 311 V.

WAEC

7. Calculate the inductance L of the coil in the circuit shown below

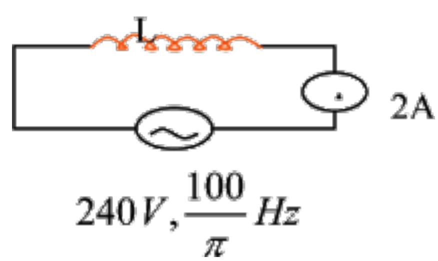
A. 14.4 H

B. 3.8 H

C. 0.6 H

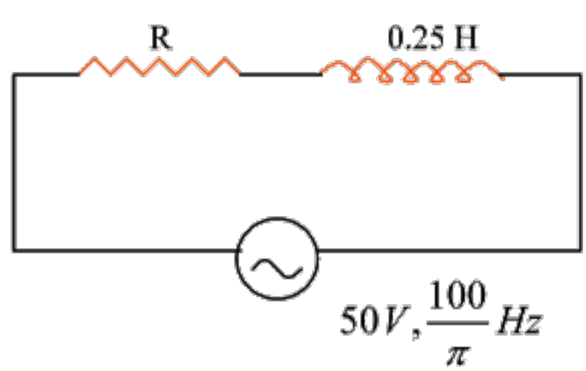
D. 0.4 H

E. 0.2 H



WAEC

8. Calculate the inductive reactance of the circuit shown above.



A. 50.00 Î©

B. 5.00 Î©

C. 0.50 Î©

D. 0.05 Î©

E. 0.02 Î©

WAEC

9. If the current in the resistor R in question 8 is 0.05 A, calculate the p.d. across the inductor.

A. 2.5 V

B. 25.0 V

C. 49.0 V

D. 50.0 V

E. 250.0 V

WAEC

10. A capacitor of capacitance 25Î¼F is connected to an a.c. power source of frequency 200/Ï€ Hz. Calculate the reactance of the capacitor.

A. 0.01 Î©

B. 0.02 Î©

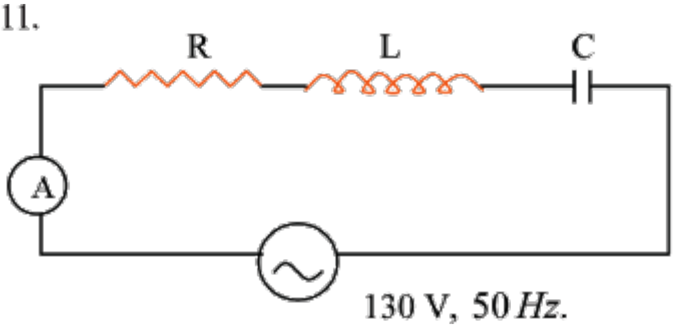
C. 50.00 Î©

D. 100.00 Î©

E. 200.00 Î©

WAEC

11.



In the diagram above the resistor has a resistance 8 Î© while the reactance of the inductor and capacitor are 10 Î© and 16 Î© respectively. Calculate the current in the circuit.

A. 3.6 A

B. 3.8 A

C. 9.2 A

D. 10.0 A

E. 13.0 A

WAEC

12. A series RLC circuit is said to resonate if the

    A. capacitive reactance is zero.

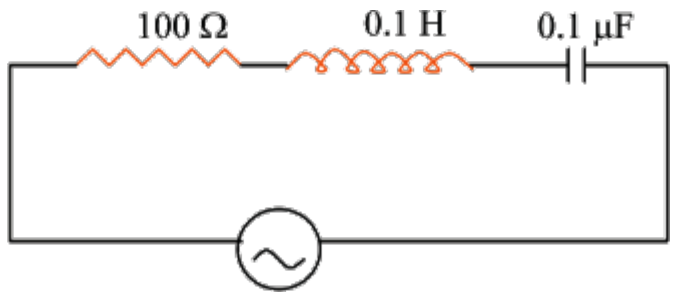
    B. inductive reactance is zero.

    C. current is maximum.

    D. impedance is zero.

WAEC

Use the diagram below to answer questions **13** and **14,** given that the voltage amplitude of the a.c. source is 100 V.



13. Determine the resonant frequency of the source.

    A. 1.6 Hz

    B. 15.9 Hz

    C. 1590.9 Hz

    D. 3182.7 Hz

    E. 10,000.0 Hz

WAEC

14. Calculate the maximum current passing through the resistor.

    A. 2.0 A

    B. 1.0 A

    C. 0.7 A

    D. 0.5 A

    E. 0.1 A

WAEC

15. In an R-L-C circuit, power is mainly dissipated by the

    A. inductive parts.

    B. resistive parts.

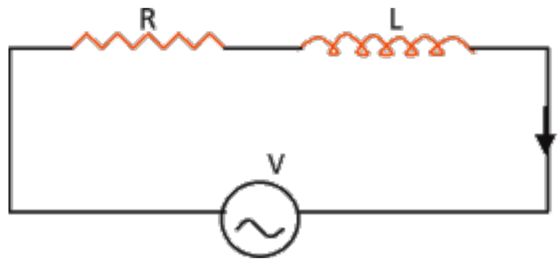
    C. capacitive parts.

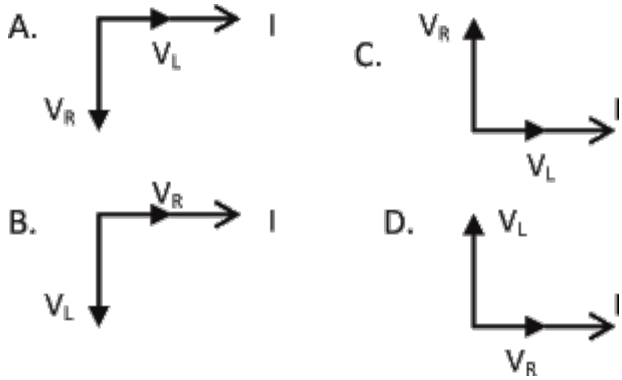
    D. reactive parts.

WASSCE

16. Which of the following graphs shows the correct vector diagram for the circuit above?

JAMB





17. At what frequency would a capacitor of 2.5 Î¼F used in a radio circuit have a reactance of 250 Î©?

A. $\frac{800}{\pi} Hz$

B. 200Ï€ Hz

C. 2000Ï€ Hz

D. $\frac{\pi}{800} Hz$

18. The power dissipated in an a.c. circuit with an r.m.s. current of 5 A, r.m.s. voltage of 10 V and phase angle of 60Â° is

A. 25 W

B. 50 W

C. 120 W

D. 125 W

19. The voltage of the domestic electric supply is represented by the equation

$$V = 311 \sin 314.2\, t.$$

Determine the frequency of the a.c. supply.

A. 50.0 Hz

B. 100.0 Hz

C. 311.0 Hz

D. 314.2 Hz

20. In a purely inductive circuit, the current

A. lags behind the voltage in phase by 90Â°.

B. leads the voltage in phase by 90Â°.

C. is in the same phase with the voltage.

D. leads the voltage by 180Â°.

21. (a). Explain the terms *reactance* and *impedance* in an a.c. circuit.

(b) A series circuit consisting of a 100 Î© resistor, a coil of 0.10 H inductance and a 20 Î¼F capacitor is connected across a 110V, 60 Hz power source.

(i) Draw the circuit of the arrangement. Calculate the:

(ii) inductive reactance;

(iii) capacitive reactance;

(iv) impedance of the circuit;

(v) current in the circuit;

(vi) power loss.

NECO

22. (a) Define: *(i) reactance (ii) impedance.*

(b) *(i)* Explain *resonance frequency* of an R-L-C circuit. *(ii)* Explain the statement the *power supply voltage of a source is 230 V.*

(c) A source of e.m.f. 240 V and frequency 50 Hz is connected to a resistor, an inductor and a capacitor. When the current in the capacitor is 10 A, the potential difference across the resistor is 140 V and that across the inductor is 50 V. Calculate the:

(i) potential difference across the capacitor;

(ii) capacitance of the capacitor;

(iii) inductance of the inductor.

WAEC

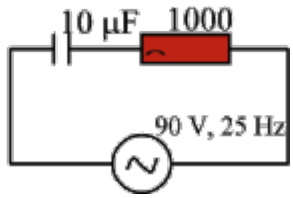
23. An alternating source of 10.0 V and frequency 100 Hz is connected in series with a 5 Î© resistor and 20 mH inductor. Calculate the

(i) reactance of the inductor.

(ii) impedance of the circuit.

(iii) potential difference across the inductor.

WASSCE

24. 

Calculate the following in the series circuit shown above:

(i) reactance of the capacitor;

(ii) impedance of the circuit;

(iii) current through the circuit;

(iv) voltage across the capacitor;

(v) average power used in the circuit.

WAEC

25. The current, I in an a.c. circuit is given by the equation I = 30 sin 100Ï€ t. Where t is the time in seconds; Deduce from the equation

(i) frequency of the current;

(ii) peak value of the current;

(iii) r.m.s. value of the current.

WAEC

26. (a) Explain what is meant by the r.m.s. value of an alternating current.

(i) if the alternating current is represented by I = I_0 sin Ï‰t
State what the symbols I, I Ï‰o and Ï‰t represent.

(ii) Calculate the instantaneous value of such a current, if in a circuit it has r.m.s. value of 15.0 A when the phase angle is 30Â°.

WAEC

27. State, with the aid of a vector diagram, the relationship between the alternating current and the potential difference across each of the following components of an a.c. circuit.

(i) resistor;

(ii) inductor;

(iii) capacitor.

NECO

28. (a) With the aid of a suitable diagram and relevant equation, explain the peak value of an alternating current.

(b) (i) Define the ***root mean square*** of an alternating current.

(ii) Explain the term ***resonant frequency*** as it is applied to an L-C-R series circuit.

(c) A resistor of resistance 50 Î© , a capacitor of capacitance 0.1 Î¼F and an inductor of inductance 0.1 H are connected in series to a 1.50V rms alternating voltage supply.

(i) Draw the circuit diagram.

(ii) Calculate the root mean square current.

(iii) Calculate the resonant frequency.

WASSCE