Chapter 7: Logical Reasoning (Revision)

OBJECTIVES

At the end of the chapter, students should be able to:

1. State the meaning of simple and compound statements.
2. List the five logical operations and their symbols.
3. Write the truth value of a compound statement involving any of the five logical operations.
4. Use truth table to prove that:

(i) A contrapositive is equivalent to the conditional statement.
(ii) A converse is equivalent to the inverse of the conditional statement.

I. Simple Statement and Compound Statement

(i) Simple statement

A mathematical statement is a declarative sentence that is either true or false, but not both. Mathematical statements are often represented using capital letters. For example, we denote the statement "Mathematics is a science subject" by P. This is also written as P: Mathematics is a science subject.

Worked Example 1

Check whether each of the following sentences is a statement or not. State also whether each is true or false.

(a) A: Nigeria got her independence in the year 1960.
(b) B: Oh, my goodness!
(c) C: Where are you going?
(d) D: Donâ€™t run.
(e) E: There is no rain without clouds.
(f) F: The house is ugly.
(g) G: Every set is a finite set.
(h) H: Men are more intelligent than women.

SOLUTION

Answers are given in Table 7.1, where Y stands for Yes and N stands for No.

Table 7.1

Expression	A statement?	True or false
(a)	Y	True
(b)	N	Not applicable(an exclamation)
(c)	N	Not applicable(a question)
(d)	N	Not applicable(a command)
(e)	Y	True
(f)	N	Impossible to say
(g)	Y	False
(h)	N	Impossible to say

Note: Sentences involving variable time such as â€˜todayâ€™, â€˜tomorrowâ€™ or â€˜yesterdayâ€™ are not statements. This is because it is not known what time is referred to here. The same argument holds for sentences with pronouns unless a particular person is referred to and for variable places such as â€˜hereâ€™, â€˜thereâ€™, etc.

In Table 7.1, some statements are either true or false. We call these closed statements. Closed statements are about well-defined situations. Other statements, such as (f) and (h), do not involve well-defined situations. We do not know whether they are true or false, hence they are called open statements.

Worked Example 2

For each of the following, state if it is a closed or an open statement or neither. Also say whether it is true or false.

(a) A: How old are you?
(b) B: x is an odd number.
(c) C: Chemistry is a science subject.
(d) D: 4 is an even number.
(e) E: Physics is a diffi cult sub ject.
(f) F: Chair and table.

SOLUTION

Table 7.2

Expression	A statement?			True or false
	Closed?	Open?	Neither?	
(a)			√	Not applicable
(b)		√		Impossible to say
(c)	√			True
(d)	√			True
(e)		√		Impossible to say
(f)			√	Not applicable

(ii) Negation of a simple statement

The opposite of a statement is called the negation of the statement. Let us consider the following statement:

P: Ibadan is a city.

The negation of this statement is; *It is not the case that Ibadan is a city.*

This can also be written as *It is false that Ibadan is a city.*

This can simply be expressed as; *Ibadan is not a city.*

If P is a statement, then the negation of P is also a statement and is denoted by **âˆ¼**P, and read as â€˜not Pâ€™.

Note: While forming the negation of a statement, phrases like â€˜It is not the caseâ€™ or â€˜It is false thatâ€™ are also used.

Worked Example 3

Write the negation of the following statements.

(a) A: Both the diagonals of a rectangle have the same length.
(b) B: âˆš7 is a rational number.
(c) C: Everyone in Nigeria speaks English language.
(d) D: David is a lecturer.
(e) E: x = 10
(f) F: Africa is a continent.
(g) G: There does not exist a quadrilateral which has all its sides equal.

SOLUTION

The negations of these statements are as follows:

(a) âˆ¼A: It is false that both the diagonals of a rectangle have the same length.
(b) âˆ¼B: It is not the case that âˆš7 is a rational number or âˆš7 is not a rational number.
(c) âˆ¼C: Not everyone in Nigeria speaks English language.
(d) âˆ¼D: David is not a lecturer.
(e) âˆ¼E: x â‰ 10
(f) âˆ¼F: It is false that Africa is a continent or Africa is not a continent.
(g) âˆ¼G: It is not the case that there does not exist a quadrilateral which has all its sides equal.

(iii) Compound statements

Many mathematical statements are obtained by combining one or more statements using some connecting words like â€˜andâ€™, â€˜orâ€™, etc. as shown below.

(a) Consider the following statements:
P: There is something wrong with the socket or with the wiring.
This statement tells us that there is something wrong with the socket or there is something wrong with the wiring. That means that the given statement is actually made up of two statements:
Q: There is something wrong with the socket.
R: There is something wrong with the wiring.
It is therefore a compound statement connected by â€˜orâ€™
(b) Now, suppose two statements are given as below:
Q: 3 is an odd number.
R: 3 is a prime number.
These two statements can be combined with â€˜and
P: 3 is both an odd and a prime number.
This is a compound statement.
This leads us to the following definition: A compound statement is a statement which is made up of two or more statements. Each of the statements in a compound statement is called a component statement.

Worked Example 4

Find the component statements of the following compound statements.
(a) A: Number 3 is prime or it is odd.
(b) B: All integers are positive or negative.
(c) C: 100 is divisible by 3, 11 and 5.
(d) D: A point occupies a position and its location can be determined.
(e) E: 42 is divisible by 5, 6 and 7.

SOLUTION

(a) P: Number 3 is prime, Q: Number is 3 odd. Both statements are true.
(b) P: All integers are positive, Q: All integers are negative. All the statements are false.
(c) P: 100 is divisible by 3, Q: 100 is divisible by 11, R : 100 is divisible by 5. P and Q are false, R is true.
(d) P: A point occupies a position, Q: A points location can be determined. In both statements, it is impossible to say their truth values.
(e) P: 42 is divisible by 5, Q: 42 is divisible by 6, R : 42 is divisible by 7. P is false, Q and R are true.

Exercise 1

1. Check whether the following sentences are statements. Also say whether it is true or false. Provide
your answers in a tabular form as in Table 7.1.
(a) A: Two times two equals four.
(b) B: The sum of two positive numbers is positive.
(c) C: All prime numbers are odd numbers.
(d) D: The sum of x and y is greater than 0.
(e) E: How beautiful!
(f) F: Open the window.
(g) G: $(3 + 4)_2 = 3_2 + 4_2$

(h) H: 3x + 4 = 10
2. Which of the following sentences are statements? If it is a statement, say if it is a closed or an open statement or neither. Also check whether it is true or false. Provide your answers in a tabular form as in
Table 7.2.
(a) A: There are 30 days in a month.
(b) B: The sum of 6 and 8 is greater than 10.
(c) C: The square of a number is an even number.
(d) D: The sides of a quadrilateral have equal length.
(e) E: Answer this question.
(f) F: The product of (âˆ’1) and 8 is âˆ’8.
(g) G: The sum of all interior angle of a triangle is 180Â°.
(h) H: Today is a sunny day.
(i) I: All real numbers are complex numbers.
3. Give five examples of sentences which are not statements and five which are statements. Give reasons for the answers.
4. Write the negation of the following statements:
(a) He is a handsome man.
(b) John is my friend.
(c) x is not an even number.
(d) The bag is white.
(e) It is hot in Sokoto.
(f) Moses is older than John.
(g) The river is fl owing.
(h) The fi gure is a circle.
(i) The school is open.
(j) She studies Biology.
5. Determine whether B is the negation of A in the following statements. If B is not, then write a correct negation of A .
(a) **A**: Isah is a good boy. **B**: Isah is a bad boy.
(b) **A**: This is an easy problem. **B:** This is a difficult problem.
(c) **A**: y is an odd number. **B**: y is an even number.
(d) **A:** She earns more than 100 naira. **B**: She earns less than 100 naira.
(e) A: Queen is older than Maryam. B: Queen is younger than Maryam.
6. Find the component statements of the following compound statements and check whether they are true or false.
(a) Number 7 is prime or it is odd.
(b) The sky is blue and the grass is green
(c) It is raining and it is cold.
(d) 0 is a positive number or a negative number.

II. Logical Operations and the Truth Table

(i) Conjunction
Given two statements P and Q, the compound statement P and Q is called the conjunction. It is denoted by P âˆ§ Q and is defined by the following truth table:

Table 7.3

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

Note: The conjunction P âˆ§ Q is true only when both P and Q are true.

Worked Example 5

Find the component statements of the following and say whether the conjunction is true or false.
(a) A square is a quadrilateral and its four sides are equal.
(b) Lagos is the capital of Nigeria and Ibadan is the capital of Oyo.
(c) Olympic is organised by UEFA and World Cup is organised by FIFA.
(d) Abuja is the capital of Nigeria and Niger.
(e) $4 > 5$ and $5 < 3$.
(f) 20 is a multiple of 3, 6 and 7.

SOLUTION

(a) The component statements are
P: A square is a quadrilateral.
Q: A square has all its sides equal.
Both these statements are true. The conjunction is true.
(b) The component statements are
P: Lagos is the capital of Nigeria.
Q: Ibadan is the capital of Oyo.
The first statement is false but the second is true. The conjunction is false.
(c) The component statements are
P: Olympic is organised by UEFA.
Q: World Cup is organised by FIFA.
The first statement is false but the second is true. The conjunction is false.
(d) The component statements are
P: Abuja is the capital of Nigeria.
Q: Abuja is the capital of Niger.
The first statement is true but the second is false. The conjunction is false.
(e) The component statements are
P: $4 > 5$.
Q: $5 < 3$.
Both statements are false. The conjunction is false.
(f) The component statements are
P: 20 is a multiple of 3.
Q: 20 is a multiple of 6.
R: 20 is a multiple of 7.
All the three statements are false. The conjunction is false.

(ii) Disjunction

The statement P or Q is called the disjunction.
It is denoted by P âˆ¨ Q and is defined by the truth table below:

Table 7.4

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Note: P or Q is true, if at least one of the statements is true.

Worked Example 6

Find the component statements of the following and say whether the disjunction is true or false.
(a) All prime numbers are either even or odd.
(b) A person who has taken Physics or Chemistry can go for Chemical Engineering.
(c) $\sqrt{2}$ is a rational number or an irrational number.

(d) 3 is a prime number or 2 is an even number.

SOLUTION

(a) The component statements are

P: All prime numbers are even.

Q: All prime numbers are odd.

Both these statements are false. The disjunction is false.

(b) The component statements are

P: A person who has taken Physics can go for Chemical Engineering.

Q: A person who has taken Chemistry can go for Chemical Engineering.

Both these statements are true. The disjunction is true.

(c) The component statements are

P: $\sqrt{2}$ is a rational number.

Q: $\sqrt{2}$ is an irrational number.

The first statement is false while the second is true. The disjunction is true.

(d) The component statements are

P: 3 is a prime number.

Q: 2 is an even number.

Both statements are true. The disjunction is true.

The two statements P and Q can also be combined using the connective â€˜orâ€™ as in P or Q . This connective has a different meaning in Mathematics from its usage in the English language.

â€˜Today I will go to school or I will play all dayâ€™.

Here, this means that I will do one or the other of these two actions but not both.

The word â€˜orâ€™ used in this sense is called the exclusive â€˜orâ€™.

The sentence, *â€˜Today I will read a book or take a sleepâ€™*, allows for the possibility that I could read a book, or take a sleep, or read a book and take a sleep. The word â€˜orâ€™ used in this way is called the inclusive â€˜orâ€™ and this is the only use of the connective â€˜orâ€™ in Mathematics.

Worked Example 7

For each of the following statements, determine whether an inclusive â€˜orâ€™ or an exclusive â€˜orâ€™ is used. Give reasons for your answer.

(a) To enter a country, you need a passport or a visa.

(b) The school is closed, if it is a holiday or a weekend.

(c) Two lines intersect at a point or are parallel.

(d) Students can take Hausa or Yoruba as their third language.

SOLUTION

(a) Here â€˜orâ€™ is inclusive since a person can have both a passport and a visa to enter a country.

(b) Here â€˜orâ€™ is inclusive since school is closed on a holiday as well as on a weekend.

(c) Here â€˜orâ€™ is exclusive because it is not possible for two lines to intersect and be parallel.

(d) Here â€œorâ€ is exclusive because a student cannot take both Hausa and Yoruba.

(iii) Implication

The statement â€˜If the day is Monday, then John is in schoolâ€™ uses the connective, if â€¦ then, to combine the two statements:

P: the day is Monday and

Q: John is in school.

This type of compound statement is called an implication and is denoted by P â‡’ Q. The truth table for the implication is not as intuitive as the previous truth tables. However, if we consider this statement as a fact, then the only time the fact is broken or the implication is false is if the day is Monday and John is not in school. That is, the only time the statement is false is if P is true and Q is false.

Note: If the day is not Monday, John may or may not be in school and the fact about what happens on Monday is not broken. With this reasoning, we make the following definition;
The statement â€œIf P, then Qâ€, called an implication and denoted by P â‡’ Q, is defined by the truth table below.

Table 7.5

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Note: The only time implication is false is when P is true and Q is false.

Worked Example 8

For each of the following compound statements, first identify the corresponding component statements. Then check whether the statement is true or false.
(a) S: If a triangle XYZ is isosceles, then its two sides are equal.
(b) S: If x and y are integers, then xy is a rational number.
SOLUTION
(a) The component statements are given by.
P: Triangle XYZ is isosceles.
Q: Triangle XYZ has two equal sides.
Since an equilateral triangle is isosceles, we infer that the given compound statement is true.
(b) The component statements are given by:
P: x and y are integers.
Q: xy is a rational number.
Since the product of two integers is an integer and therefore a rational number, the compound statement is true.
In the implication S: P â‡’ Q , the sub-statement P is called the antecedent, while the sub-statement Q is called the consequent of P â‡’ Q. The arrow shows that Q follows P. P â‡’ Q is not the same as Q â‡’ P. The statement P â‡’ Q is sometimes called a conditional statement.

Contrapositive and converse
Contrapositive and converse are other statements which can be formed from a given statement with â€˜if–thenâ€™.

Worked Example 9

Write the contrapositive of the following statements:
(a) If the physical environment changes, then the biological environment changes.
(b) If a number is divisible by 4, then it is divisible by 2.
SOLUTION
The contrapositive of these statements are:
(a) If the biological environment does not change, then the physical environment does not change.
(b) If a number is not divisible by 2, then it is not divisible by 4.
Note that both statements convey the same meaning.

Worked Example 10

Write the converse of the following statements:
(a) If a number a is even, then a_2 is even.
(b) If you do all the exercises in the book, then you get an A grade in the class.
(c) If two integers x and y are such that x > y, then x âˆ’ y is always a positive integer.
SOLUTION

The following are the converse of these statements:
(a) If a number a^2 is even, then a is even.
(b) If you get an A grade in the class, then you have done all the exercises of the book.
(c) If two integers x and y are such that x – y is always a positive integer, then x > y.

(iv) Bi-implication
The last connective to consider is the bi-implication statement â€˜P if and only if Qâ€™ as in the statement, I can get a refund if and only if I have my receipt.
The bi-implication â€˜P if and only if Qâ€™ is denoted by P â‡” Q and is defined by the truth table below.

Table 7.6

P	Q	P ⇔ Q
T	T	T
T	F	F
F	T	F
F	F	T

Note: The bi-implication, P â‡” Q , is a true statement only when P and Q have the same truth value. It is sometimes called a bi-conditional statement.

Worked Example 11
Below are two pairs of statements. Combine these two statements using â€˜if and only ifâ€™.
(a) P: If a quadrilateral is a rectangle, then its two opposite sides are equal.
Q: If the two opposite sides of a quadrilateral are equal, then the quadrilateral is a rectangle.
(b) P: If the sum of digits of a number is divisible by 2, then the number is divisible by 2.
Q: If a number is divisible by 2, then the sum of its digits is divisible by 2.
SOLUTION
(a) A quadrilateral is a rectangle if and only if two of its opposite sides are equal.
(b) A number is divisible by 2 if and only if the sum of its digits is divisible by 2.

Exercise 2
1. Find the component statements of the following and say whether the conjunction is true or false.
(a) A line is straight and extends indefinitely in both directions.
(b) 0 is less than every positive integer and greater than every negative integer.
(c) All living things have two fore legs and two eyes.
(d) x = 2 and x = 3 are the roots of the equation $3x^2 – x – 10 = 0$.
(e) All rational numbers are real and all real numbers are complex.
2. State whether the â€˜orâ€™ used in the following statements is exclusive inclusive.
Give reasons for your answer.
(a) Sun rises or moon sets.
(b) To apply for a driver's license, you should have a national identity card or a passport.
(c) All integers are positive or negative.
(d) You are wet when it rains or you are in a river.
(e) 5 is an odd number or a prime number.

3. For each of the following compound statements, first identify the corresponding component statements. Then check whether the compound statement is true or false.
(a) S: If a number is a multiple of 9, then it is a multiple of 3.
(b) S: If you are born in America, then you are a citizen of America.
4. Write the contrapositive and converse of the following statements:
(a) If you are born in Nigeria, then you are a citizen of Nigeria.
(b) If a triangle is equilateral, then all of its sides are of equal length.
(c) If x is a prime number, then x is odd.

(d) If the two lines are not parallel, then they do intersect in the same plane.
(e) Something is hot implies that it has high temperature.
(f) You cannot comprehend geometry, if you do not know how to reason deductively.
(g) y is an even number implies that x is divisible by 2.
5. Given statements in (a) and (b).
Identify the statements given below as contrapositive or converse of each other.
(a) If you live in Moscow, then you have winter clothes.
(i) If you do not have winter clothes, then you do not live in Moscow.
(ii) If you have winter clothes, then you live in Moscow.
(b) If a quadrilateral is a rhombus, then its diagonals bisect each other.
(i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a rhombus.
(ii) If the diagonals of a quadrilateral bisect each other, then it is a rhombus.
6. Write each of the following statements in the form â€˜if–thenâ€™:
(a) You get a job implies that your resume is good.
(b) The banana tree will bloom, if it stays warm for a month.
(c) A quadrilateral is a rhombus, if its diagonals bisect each other.
7. Given below are two pairs of statements. Combine these two statements using â€˜if and only ifâ€™, and check whether the statement is true or false.
(a) P: If a bucket is half empty, then it is half full.
Q: If a bucket is half full, then it is half empty.
(b) P: If a triangle is an equilateral triangle, then all of its three sides are of equal length.
Q: If all the three sides of a triangle are of equal length, then it is an equilateral triangle.
(c) P: If a quadrilateral is a square, then all its four sides are of equal length.
Q: If the four sides of a quadrilateral are of equal length, then it is a square.
(d) P: If you play a game, then your mind is free.
Q: If your mind is free, then you play a game.
(e) P: If a quadrilateral is equiangular, then it is a rectangle.
Q: If a quadrilateral is a rectangle, then it is equiangular.

III. Conditional Statements and Indirect Proofs

In Mathematics, conditional statements and their truth values are very important because many theorems and propositions are stated as conditional statements.
For any two given statements, P and Q, there are four implications that are important to consider.
Suppose we call P â‡’ Q the given statement. Then the four implications are:
Statement: P â‡’ Q
Converse: Q â‡’ P
Contrapositive: âˆ¼Q â‡’ âˆ¼P
Inverse: âˆ¼P â‡’ âˆ¼Q
These four implications are clearly related. In fact, a statement and its contrapositive are equivalent just as the converse and inverse.

Worked Example 12

Use truth table to prove that a statement and its contrapositive are equivalent.
SOLUTION
Given statement P â‡’ Q, the table below verifies that a statement and its contrapositive are equivalent.

P	Q	$P \Rightarrow Q$	$\sim Q \Rightarrow \sim P$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Worked Example 13

Use truth table to prove that converse and inverse of a given statement are equivalent.

SOLUTION

Given statement P â‡’ Q, the table below verifies that the converse and inverse of the given statement are equivalent.

P	Q	~P	~Q	P⇒Q	Q⇒P	~P⇒~Q
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Exercise 3

Use truth table to prove the following:

1. P â‡’ Q â‰¡ âˆ¼P âˆ¨ Q.
2. P â‡” Q â‰¡ [(P â‡’ Q) âˆ§ (Q â‡’ P)].
3. âˆ¼(P âˆ§ Q) â‰¡ âˆ¼P âˆ¨ âˆ¼Q.
4. âˆ¼(P âˆ¨ Q) â‰¡ âˆ¼P âˆ§ âˆ¼Q.
5. âˆ¼(P â‡’ Q) â‰¡ P âˆ§ âˆ¼Q.
6. âˆ¼(P â‡” Q) â‰¡ (P âˆ§ âˆ¼Q) âˆ¨ (Q âˆ§ âˆ¼P).
7. Use a truth table to prove that âˆ¼(P â‡’ Q) â‡” (P âˆ§ âˆ¼Q).

SUMMARY

In this chapter, we have learnt the following:

â– A mathematically acceptable statement is a sentence which is either true or false.

â– Negation of a statement P: If P denotes a statement, then the negation of P is denoted by **âˆ¼**P.

â– Compound statements and their related component statements:

A statement is a compound statement if it is made up of two or more smaller statements.

The smaller statements are called component statements of the compound statement.

â– There are four logical operations: conjunction, disjunction, implication and bi-implication.

â– The compound statement P and Q is called the conjunction and is denoted by P âˆ§ Q .

â– The compound statement P or Q is called the disjunction and is denoted by P âˆ¨ Q .

â– An implication is a statement in the form â€˜if P then Qâ€™ and is written as P â‡’ Q .

A sentence with â€˜if P then Qâ€™ can be written in the following ways:

(a) P implies Q (denoted by P â‡’ Q)

(b) P is a sufficient condition for Q

(c) Q is a necessary condition for P

(d) âˆ¼Q implies âˆ¼P

â– The contrapositive of a statement P â‡’ Q is the statement âˆ¼Q â‡’âˆ¼P. The converse of a statement P â‡’ Q is the statement Q â‡’ P.

â– A bi-implication is a statement in the form â€˜P if and only if Qâ€™. It is denoted by P â‡” Q .

â– The following are the four conditional statements:

(a) Statement: P â‡’ Q

(b) Converse: Q â‡’ P

(c) Contrapositive: âˆ¼Q â‡’ âˆ¼P

(d) Inverse: âˆ¼P â‡’ âˆ¼Q

â– A statement and its contrapositive are equivalent.

â– A converse and inverse of a given statement are equivalent.

GRADUATED EXERCISES

1. What is logical reasoning?
2. What is the difference between simple statements and compound statements?

State which of the following are statements in logical context:

3. Put on the engine.
4. Stop walking.
5. The equator is a great circle.
6. B = {**x**: 2 < **x** < 3, **x** âˆˆ **z**}
7. Oh, what a wonderful lady?
8. y > 6.
9. Hilton hotel at Abuja is a magnificent building. State the truth value of the following statements.
10. 7 is greater than 14
11. The median of the numbers 2, 6, 8, 5, 4, is 5.
12. The volume V cm^3 of a cylinder of radius r cm and height h cm is given by Ï€r^{2}h.
13. The average B of two numbers x and y is given by B = $\frac{1}{2}$ (x + y)
14. There are 36 states in Nigeria.

Write the negation of each of the following statements.

15. Yakubu scored the first goal.
16. Ota Farm is in Ogun State.
17. The diagonals of a rhombus bisect each other.
18. The train is moving fast.
19. C is the shortest boy in the class.
20. List the four connectives you know.
21. How are compound statements formed from simple statements?
22. Given that P, Q and R are statements, express the following compound statements in symbols.

(a) If P and Q then Q and R.
(b) If P implies Q then not Q implies P.
(c) P implies Q if and only if either P or not Q.
(d) P implies that Q implies R.

Form compound statements using â€˜andâ€™. Express the compound statements in symbolic form.

23. P: It is raining heavily, Q: Everywhere is flooded.
24. P: y + 4, Q: y = âˆ’4
25. P: (y + 4)^2 is a perfect square, Q: When y = 0, (y + 2)^2 = 4.
26. P: The liquid contains water, Q: The liquid contains salt.
27. P: He is tall, Q: He is fair

Form compound statements using â€˜orâ€™. Express the compound statements in symbolic form

28. P: I will read Social Science, Q: I will read Art
29. P: Mubarak is fat, Q: Mubarak is slim.
30. P: The solution of **y**^{2} âˆ’ 2**y** âˆ’ 15 = 0 is 5, Q: The solution of **y**^{2} âˆ’ 2**y** âˆ’ 15 = 0 is âˆ’3.
31. P: **x** + 2 = 4, Q: **x** = 2.

Form the compound statements using â€˜if â€¦ thenâ€™.

32. P: Regina is a youth corps member, Q: She has a degree.
33. P: She is frowning, Q: She is angry.
34. P: âˆ’ âˆž < **x** < 10, Q: 100 < **x**^{2} < âˆž.

Write down the converse of each of the following statements:

35. If it is perpendicular to the radius then a line is a tangent to the circle.
36. If the harvest will be bad then it has not rained sufficiently.
37. If it is an equilateral triangle then the three sides are equal.

Write down the contrapositive of each of the following statements:

38. If two sides of a triangle are equal, then it is an isosceles triangle.
39. If two circles have the same centre, then they are concentric circles.
40. If P and Q are two logical statements, copy and complete the following truth table.

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$(P \vee Q) \sim P$	$\sim(P \vee Q) \Rightarrow \sim P$
T	T					
T	F					
F	T					
F	F					