

11

WAVE-PARTICLE PARADOX

OBJECTIVES

At the end of this topic, students should be able to:

- identify phenomena which are only satisfactorily explained by assuming that matter behaves like (a) waves (b) particles;
- explain the uncertainty principle in general terms and give an example.

Wave nature of light

Light was thought to be a particle as proposed by Newton. Newton's corpuscular theory of light could explain reflection and refraction of light. The particle concept of light was later dropped when Thomas Young conclusively proved that light is a wave. Young showed by experiment that light as a wave could produce interference pattern similar to other known waves. Treating light as a wave, we can explain other properties of waves like diffraction, interference and polarization. Maxwell, using mathematics as a tool, was able to establish the wave nature of light by proving that light belongs to a family of waves called electromagnetic waves.

Particle or corpuscular nature of light

The wave nature of light could not explain some properties of light like the emission of electrons by a metal surface when it is irradiated by light of high frequency (photoelectric emission), radiation and absorption of light by surfaces. Photoelectric effect and radiation of light by hot objects could only be explained by treating light as a particle. Light is a wave as we have already established, but sometimes it acts like a particle.

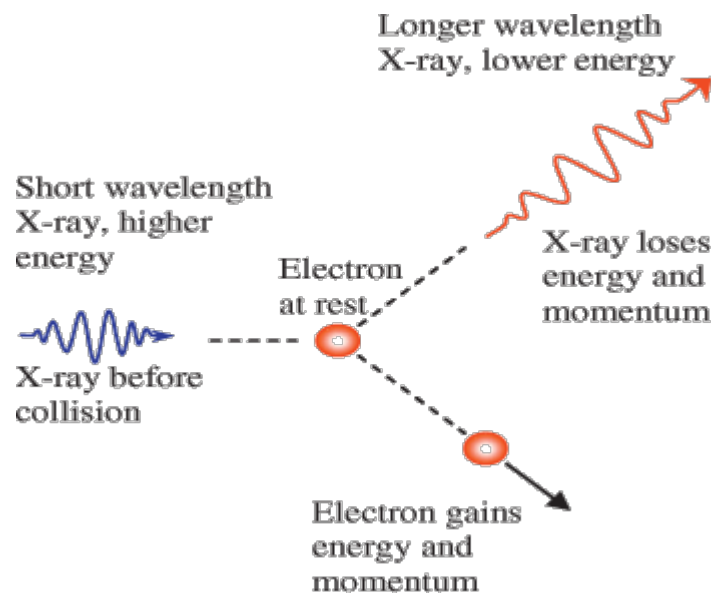


Figure 11.1: Compton effect

Evidence of particle nature of light

(a) Compton effect:

Arthur Compton in 1923 showed that X-ray (light) which is a wave has momentum; a particle property. Compton allowed X-ray of known wavelength to collide with a stationary electron and discovered that the X-ray transfers **energy** and **momentum** to the electron. Treating the X-ray as a particle and applying the laws of momentum and energy, Compton showed that both energy and momentum were conserved during the collision. The energy lost by the X-ray is equal to the energy gained by the electron. The loss of energy by the X-ray means that its wavelength increases after collision.

(b) Photoelectric emission:

Emission of electrons from a metal surface by light of sufficient frequency can be explained by treating light as a particle carrying specific energy. According to Einstein, each light photon has energy, which is proportional to its frequency. When light photon hits a surface, it gives up its energy to the electrons on the surface. If the energy of the incident photon is sufficient to free the electrons from the electrostatic forces binding them to the surface, electrons are removed while the extra energy moves them away from the surface.

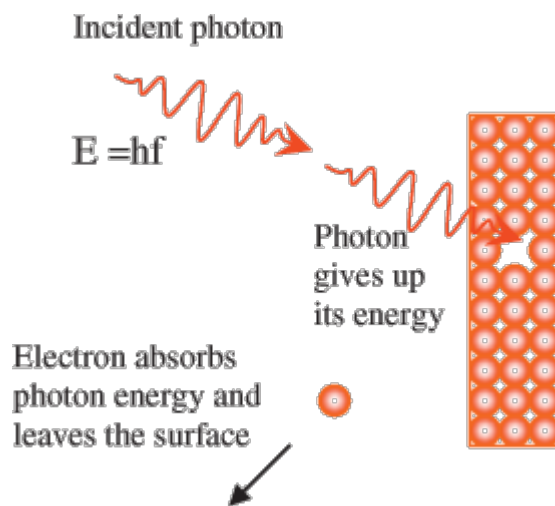


Figure 11.2: emission of electron by incident light photon

(c) Radiation of light by hot objects:

Light is emitted from hot object as a burst of energy. Each burst carries energy, which is proportional to frequency.

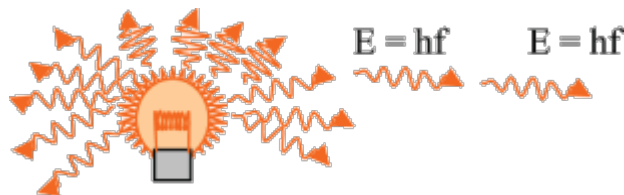


Figure 11.3: Light is emitted as particles

Wave nature of matter

Louis de Broglie reasoned that if light, which is a wave, behaves as a particle, it is possible that a particle like electron could have wave properties. He predicted the wavelength of the wave produced by a particle in motion as:

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

λ = wavelength of the particle wave.

$P = mv$ = momentum of the particle in motion.

h = Planck's universal constant.

Particles like electrons, protons and neutrons have been shown to exhibit wave properties of diffraction and interference. Particles therefore have wave properties when they are moving at a very high speed. The kinetic energy of such particles is related to its momentum by:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \frac{mv^2 \times m}{m} = \frac{P^2}{2m}$$

$$\text{But kinetic energy } \frac{1}{2}mv^2 = eV$$

$$\therefore \frac{P^2}{2m} = eV$$

$$P = \sqrt{2meV}$$

The wavelength of a particle accelerated by a potential difference V using De Broglie equation is:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

Worked example

An electron of mass 9.1×10^{-31} kg is accelerated by a potential difference of 6000 V.

(a) What is the wavelength of the wave associated with the electron?

(b) Calculate the velocity of the electron.

($h = 6.6 \times 10^{-34}$ J s, $m = 9.1 \times 10^{-31}$ Kg, $e = 1.6 \times 10^{-19}$ C)

Solution

$$(a) \lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 6000}}$$

$$\lambda = \frac{6.6 \times 10^{-34}}{4.18 \times 10^{-23}} = 1.58 \times 10^{-11} \text{ m}$$

$$(b) \frac{1}{2}mv^2 = eV = 1.6 \times 10^{-19} \times 6000$$

$$0.5 \times 9.1 \times 10^{-31} v^2 = 9.6 \times 10^{-16}$$

$$v^2 = \frac{9.6 \times 10^{-16}}{4.55 \times 10^{-31}} = 2.11 \times 10^{15}$$

$$v = \sqrt{2.11 \times 10^{15}} = 4.59 \times 10^7 \text{ ms}^{-1}$$

Electron diffraction

Clear evidence in support of the wave nature of matter is the diffraction of electron beam by crystals. G.P. Thomson showed that a beam of electron passed through a crystal produces diffraction pattern similar to the pattern produced by X-rays. A particle like an electron behaves like a wave with wavelength.

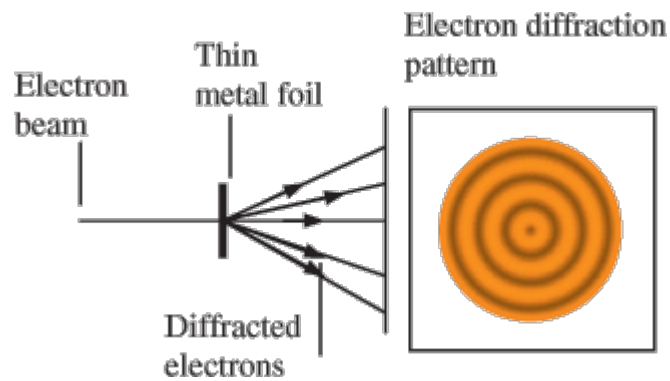


Figure 11.4 Electron diffraction

Wave-particle duality

Matter has two natures. This is called **dual nature of matter or wave-particle duality**. This means that light could have both wave and particle properties depending on the experiment under consideration. In Young's experiment, light acts as a wave but behaves as a particle in photoelectric effect.

On the other hand, particles like electron, proton and neutron exhibit wave properties of diffraction.

Heisenberg uncertainty principle

Particles like electrons, protons and neutrons have momentum and energy. These particles are very tiny that anything introduced to measure its position will change its speed, energy and momentum. Heisenberg stated that the momentum and position of a particle cannot be correctly measured at the same time. The more accurate the position of a particle is measured, the less accurate its momentum is measured at the same time. There is always an uncertainty or error in measuring both momentum and speed of a particle at the same time.

Heisenberg uncertainty principle states that the product of uncertainty in the measurements of the position (Δx) and momentum (Δp) of a particle is equal to or greater than the Planck's constant (h).

This is mathematically stated as:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Δx = uncertainty in the measurement of position.

Δp = uncertainty in the measurement of momentum.

h = Planck's constant.

Heisenberg uncertainty principle is true for simultaneous measurements of energy of a particle and time.

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

Electron diffraction pattern

Worked example

Calculate the uncertainty in the measurement of momentum if the uncertainty in the measurement of its position is 2.2×10^{-10} m.

($h = 6.6 \times 10^{-34}$ Js)

Solution

$$\Delta p \geq \frac{h}{\Delta x} \geq \frac{6.6 \times 10^{-34}}{2.2 \times 10^{-10}} \geq 3.0 \times 10^{-24} \text{ N s}$$

Summary

- Matter has two natures. Light wave sometimes behaves as a particle. In photoelectric effect light acts as a particle.
- Electrons are known particles which act like a wave in electron diffraction.
- In sub-atomic particles, it is not possible to measure correctly the position and momentum of a particle at the same time.

Practice Questions 11

- 1(a) State **two wave properties** of light.
(b) Give two evidences that suggest that light behaves as a particle.
2. Explain how Compton effect proved that X-ray behaves like a particle in the collision with electron.
3. Calculate the de Broglie wavelength of an electron accelerated by a potential difference of 10 KV.
($h = 6.6 \times 10^{-34} \text{ J s}$, $m = 9.1 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$)

Past Questions

1. If the uncertainty in the measurement of the position of a particle is $5.0 \times 10^{-10} \text{ m}$, the uncertainty in the momentum of the particle is
 - A. $3.30 \times 10^{-24} \text{ N s}$.
 - B. $1.32 \times 10^{-24} \text{ N s}$.
 - C. $1.32 \times 10^{-44} \text{ N s}$.
 - D. $3.30 \times 10^{-44} \text{ N s}$.
2. An electron of charge $1.6 \times 10^{-19} \text{ C}$ is accelerated between two plates. If the kinetic energy of the electron is $4.8 \times 10^{-17} \text{ J}$, the potential difference between the plates is
 - A. 400 V
 - B. 300 V
 - C. 30 V
 - D. 40 V

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3. Which of the following observations is a consequence of the wave nature of light?
- A. Black body radiation.
 - B. Existence of photon.
 - C. Photoelectric effect.
 - D. Diffraction.

W.A.S.SC.E.

4. The duality of matter implies that matter
- A. exists as particles of dual composition.
 - B. has momentum and energy.
 - C. has both wave and particle properties.
 - D. is made up of dual materials.

W.A.S.SC.E.

5. Electrons passing through crystals are diffracted because they
- A. are repelled by the atoms in the crystals.
 - B. are attracted by the atoms in the crystals.
 - C. possess wave properties.
 - D. are particles.

W.A.S.SC.E.

6. An evidence of the wave nature of matter is
- A. diffusion.
 - B. Brownian motion.
 - C. diffraction.
 - D. photo electricity.

W.A.S.SC.E.

7. It is not always possible to determine exactly and simultaneously the position and momentum of a particle. This statement is known as the
- A. De Broglie's law.
 - B. Heisenberg uncertainty principle.
 - C. Compton effect.
 - D. Franck-Hertz experimental law.
 - E. Wave-particle paradox.

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8. The particle and wave nature of matter are demonstrated in the equation

A. $\lambda = \frac{h}{p}$

B. $\lambda = \frac{c}{f}$

C. $\lambda = \frac{c}{f}$

D. $\lambda = \frac{hc}{E}$

JAMB

9. (a). State Heisenberg's uncertainty principle.
 (b) Mention **two** phenomena that can be explained in terms of the particulate nature of light.

W.A.S.S.C.E.

10. The mass and wavelength of a moving electron are $9.1 \times 10^{-31} \text{ Kg}$ and $1.0 \times 10^{-10} \text{ m}$ respectively. Calculate the kinetic energy of the electron and hence its velocity.
 ($h = 6.6 \times 10^{-34} \text{ Js}$; $c = 3.0 \times 10^8 \text{ ms}^{-1}$)

W.A.S.S.C.E.

11. (a). What is meant by the wave-particle duality of light?
 (b) Mention **two** phenomena that can be explained in terms of the particulate nature of light.

W.A.S.S.C.E.

12. The uncertainty in the position x of an electron moving through a cathode ray tube is 10^{-10} m . What is the uncertainty in the simultaneous measurement of the velocity v of the electron assuming its mass is 10^{-30} Kg ?
 ($h = 6.6 \times 10^{-34} \text{ J s}$)

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13. (a) Explain what is meant by the duality of matter?
 (b) State **two** examples of observable phenomena to support the concept.

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14. (a). What is meant by the wave-particle duality of light?
 (b) Mention one physical phenomenon, in each case, that can be explained in terms of the wave and particle nature of light.

W.A.S.S.C.E.