Chapter 9

Chapter 9

Longitude and Latitude

OBJECTIVES

At the end of this chapter, students should be able to:

1. describe the earth as a sphere.
2. identify and appreciate the following:
   (a) North and South Poles
   (b) Longitude
   (c) Latitude
   (d) Meridian and equator
   (e) Parallel of latitude
   (f) Radius of parallel of latitude
   (g) Radius of the earth
3. recall and state the formulas on arc length of a curve, and solve problems on them.
4. Solve problems on longitude and latitude.

I. Earth as a Sphere

A sphere is a solid object that has a round shape. It is in the form of a football or an orange.

An orange when cut horizontally through the centre results into two hemispheres (see Figure 9.1). In each of the hemispheres in Figure 9.1, there is a fl at circular face and a curved surface.

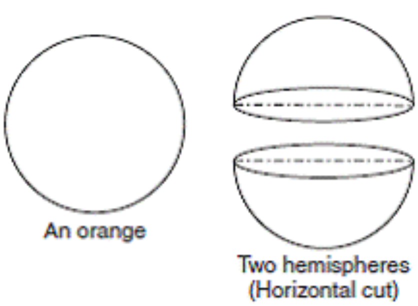

Figure 9.1

Observe that the distance from the centre of the fl at circular face to any point on the surface of the sphere is always equal and it is called the **radius** of the circle. See the Figure 9.2.

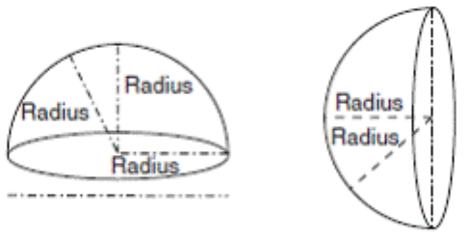

Figure 9.2

An orange when cut with a circular cross-section but not through the centre of the orange, will definitely have a smaller radius which is not the same as the radius of the hemispheres, see Figure 9.3.

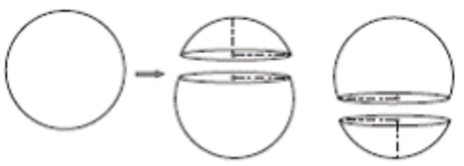

Figure 9.3

II. Identification of North and South Poles

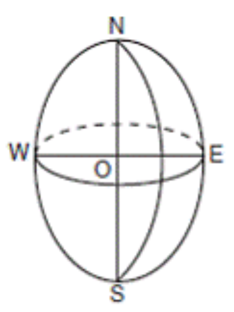

Figure 9.4

From Figure 9.4, the axis of the earth is line NS, where N stands for the North Pole and S for the South Pole. Another axis of the earth is line WE, where W stands for the West Pole and E the East Pole. O is the centre of the earth and the intersecting point of the axes.

III. Identification of Longitudes

The line NOS in Figure 9.4 is called the polar axis through which the earth oscillates.
In Figure 9.4, the section of the earth's surface cut by plane through

$\overline{NS}$ produces great circles called the **meridian**.

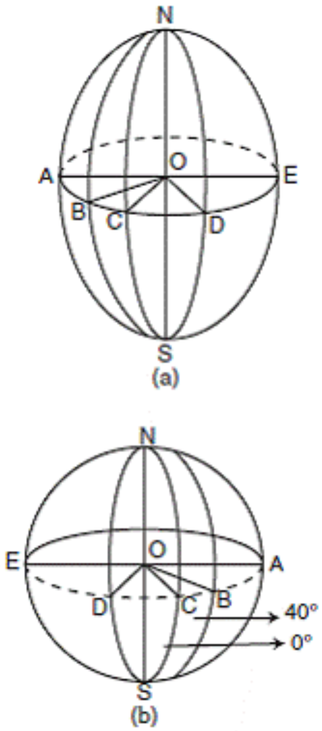

Figure 9.5

In Figure 9.5, NAS, NBS, NCS, NDS and NES are the meridians. Meridian NCS is a representative meridian that passes through the Greenwich in Britain.

NCS is the Greenwich Meridian with longitude 0°. Other meridians lie on the west or east of the Greenwich Meridian and they take their locations from it.

The lowest longitude is 0° and the highest is 180°. Hence, the longitude ranges from 180° W to 180° E.

The longitude of a point on the meridian is the angle that is formed by the meridian and the Greenwich Meridian.

In Figure 9.5(b), the longitude of D is the angle that is formed by NDS and NCS, which is COD.

IV. Radius of the Parallel of Latitude

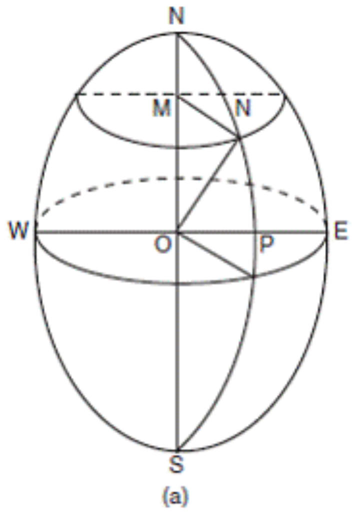

Figure 9.6

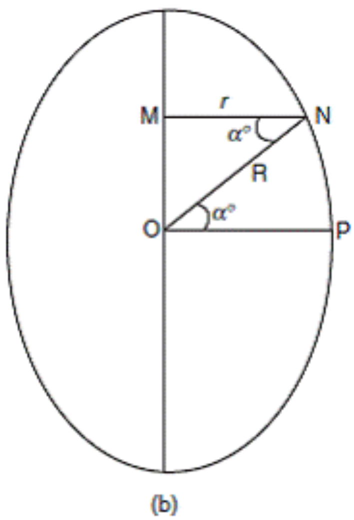

Figure 9.6 Continued

Let the latitude of place N be α° while M is the centre of the latitude where N rests. Let *r* be the radius of the parallel of latitude through N.
In Figure 9.6(b),

$\overline{MN} // \overline{OP}$

So, ∠MNO = ∠PON = α°(alternate angles)
From right-angled triangle MNO,

$$\cos \alpha° = \frac{MN}{ON} = \frac{r}{R}$$

So, $r = R \cos \alpha°$.

Worked Example 1

Two points M and N on the surface of the earth are given by their latitude as M (50° S, 15° E) and N (50° S, 75° E). Calculate the radius of the parallel of latitude on which M and N lie. (Take the radius of the earth as 6 400 km).

SOLUTION

The latitude on which M (50° S, 15° E) and N (50° S, 75° E) lie is 50° S. The formula for calculating the radius of the parallel of latitude is

$$r = R\cos\alpha$$

where r is the radius of the parallel of latitude, R is the radius of the earth which is 6 400 km and α is the latitude.

Here, R = 6 400 km, α = 50°

r = 6 400 km × 0.6428

= 4 113.92

Worked Example 2

Point A (lat 28° N, long 33° E) and point B (lat 28° N, long 27° W) are two points on the surface of the earth. Calculate the radius of the circle of lat 28° N. (Assume the earth to be a sphere of radius 6 400 km). **(WAEC)**

SOLUTION

Given: R = 6 400 km α = 28°

Formula: $r = R\cos\alpha$

= 6 400 × cos 28°

= 6 400 × 0.8829

= 5 650.56 km

= 5 651 km (nearest km)

Exercise 1

1. If the radius of the parallel of latitude $\alpha°$ is 3 200 km, find α.
2. Two cities A and B are on the parallel of latitude 65° S. Calculate the radius of the parallel of latitude.
3. The radius of the parallel of the latitude 48° S is equal to the radius of the parallel of latitude $\beta°$. Calculate the value of β.
4. L and N are two points on the same parallel of latitude 80° N. L is on long 25° E and N on long 62° E. Assuming the earth to be a sphere of radius 6 400 km and $\pi = 3.142$, calculate, correct to three significant figures, the radius of the parallel of latitude on which L and N lie.
5. Enugu and Port Harcourt are two towns whose positions on the globe are (25° N, 22° E) and (75° N, 22° E), respectively. Find the radius of the parallel of latitude on which Enugu lies.
6. X and Y are two towns on the same parallel of latitude 75° S. Calculate the radius of the parallel of latitude.
7. The latitude and longitude of a point U is 45° N and 28° W and V is 45° N and 32° E, respectively. Find the radius of the parallel of latitude on which the two towns lie.
8. Calculate the radius of a circle whose latitude is 46° N.
9. Two points F and G on the surface of the earth are given by their latitudes and longitudes as F (37° S, 37° E) and G (37° S, 37° W). Calculate the radius of the parallel of latitude on which F and G lie.
10. Find the radius of the parallel of latitude 84° N on the globe.

V. Length of Parallel of Latitude

The calculation of the length of the parallel of latitude is the same as calculating the circumference of the parallel of latitude (a circle).

The formula for calculating the length of the parallel of latitude $\alpha°$ is $2\pi r$, where $r = R \cos \alpha$. Here, R is the radius of the earth and α is the latitude.

Worked Example 3

Two towns K and Q are on the parallel of lat 46° N. The longitude of town K is 130° W and that of town Q is 103° W. A third town P also on lat 46° N is on long 23° E.
Calculate the length of the parallel of lat 46° N, to the nearest 100 km.
Take π = 3.142, radius of the earth = 6 400 km. **(WAEC)**

SOLUTION

Formula: Length of parallel of latitude

$= 2\pi r = 2\pi R \cos \alpha$

$(r = R \cos \alpha)$

Given: R = 6 400 km, α = 46° and π = 3.142

So, length of the parallel of lat 46° N

= 2 × 3.142 × 6 400 km × cos 46°

= 40 217.6 × 0.6947

= 27 939.17 km

= 27 939 km (nearest km)

Exercise 2

Calculate the length of the following parallel of latitudes:

1. 75° S 2. 46° N 3. 78° N 4. 56° S 5. 63° N

Find the length of the parallel of latitude of a circle in which points A and B lie whose longitudes and latitudes are given respectively as follows:

	Point A	Point B
6.	lat 44° N, long 42° E	lat 44° N, long 36° W
7.	lat 38° S, long 29° E	lat 38° S long 50° E
8.	lat 43.5° N, long 34° W	lat 43.5° N long 48° E
9.	lat 50° N, long 68° E	lat 50° N long 28° E
10.	lat 50.8° S, long 22° W	lat 50.8° S long 48° E

VI. Angles between Latitudes

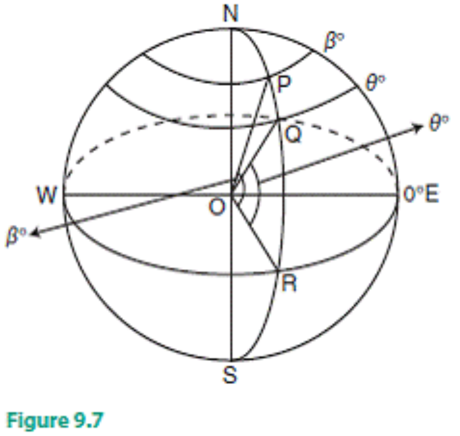

Figure 9.7

In Figure 9.7, places P and Q belong to the Northern part of the equator and on different latitudes. P is on lat θ° N while place Q is on lat β° N. Places P and Q are located on the same longitude PQR. So, the angular difference between

places P and Q is calculated as follows:
$\angle POQ = \angle POR - \angle QOR$
$= \theta° - \beta°$
From the above illustration, the angle between two latitudes that lie on the same side of the equator and on the same longitude is the difference between their latitudes.

Worked Example 4

An aeroplane flies from a town P (lat 40° N, 28° E) to another town Q (lat 15° N, 28° E). Calculate the angular difference of the two towns.

SOLUTION

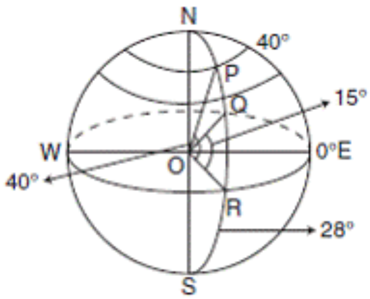

Figure 9.8

Let the angle between the latitudes be $\angle POQ$.
Hence, $\angle POQ = \angle POR - \angle QOR$
$= 40° - 15° = 25°$

Worked Example 5

The longitude on which M (50° S, 50° W) and N (82° S, 50° W) lie is 50° W. Find the angle between the latitudes.

SOLUTION

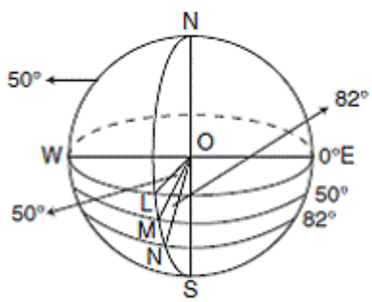

Figure 9.9

Let the angle between the latitude be $\angle MON$.
Hence, $\angle MON = \angle LON - \angle MON$
$= 82° - 50° = 32°$

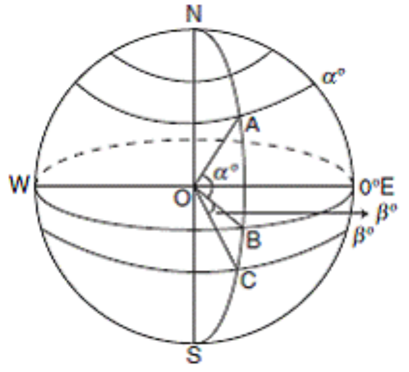

Figure 9.10

In Figure 9.10, place A belongs to the Northern part of the equator while place C belongs to its Southern part. Place A is on lat α° when place C is on lat β°. So the angle between the places A and C is calculated as follows: ∠AOC = ∠AOB + ∠BOC = α° + β°.

From the above illustration, the angle between two latitudes that lie on opposite sides of the equator and on the same longitude is the sum of their latitudes.

Worked Example 6

Point A (lat 28° N, long 28° E) and Point B (lat 28° S, long 28° E) are two points on the earth’s surface. Calculate the angle between the latitudes.

SOLUTION

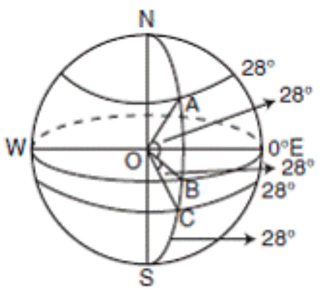

Figure 9.11

Let the angle between the latitudes be ∠AOB.

Hence, ∠AOB = ∠AOX + ∠BOX

= 28° + 28°= 56°

Worked Example 7

X is a point on lat 62° N and long 48° W. Y is on the same longitude as X but lies on parallel 23° S. Find the angle between the latitudes.

SOLUTION

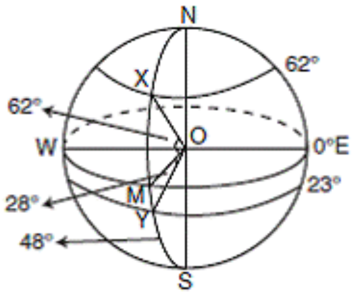

Figure 9.12

Let the angle between the latitude be ∠XOY.
Hence, ∠XOY = ∠XOM + ∠YOM
= 62° + 23°= 85°

VII. Angle between Longitudes

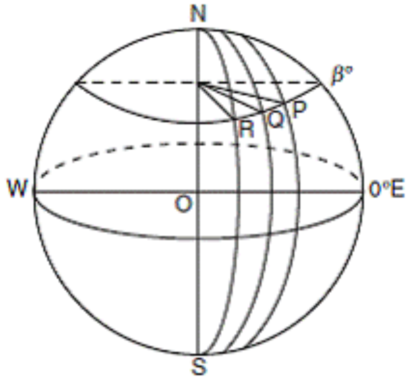

Figure 9.13

In Figure 9.13, places P and Q belong to the Eastern part of the Greenwich Meridian (long 0°) and on diff erent longitudes. Place P is on long θ° E while place Q is on long ϖ° E. Places P and Q are located on the same latitude PQR. Hence, the angle between places P and Q is calculated as follows:

$$\angle PQ = \angle PMR - \angle QMR$$
$$= \theta° - \varpi°$$

From the above illustration, the angle between two longitudes that lie on the same side of the Greenwich Meridian (long 0°) and on the same latitude is the difference between their longitudes.

Worked Example 8

Calculate the angle between two points M (45° N, 50° E) and N (45° N, 38° E).

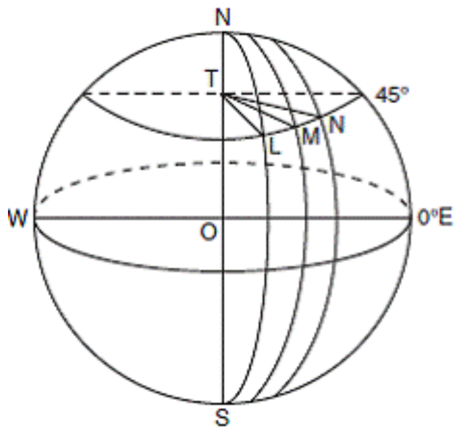

Figure 9.14

SOLUTION

Let the angle between the longitudes be $\angle MTN$.

Hence, $\angle M\bar{T}N = \angle M\bar{T}L - \angle N\bar{T}L$

$= 50° - 38°$

$= 12°$

Worked Example 9

Find the angle between two towns J (62° S, 48° W) and K (62° S, 35° W).

SOLUTION

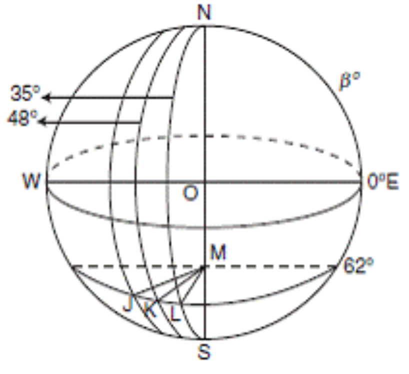

Figure 9.15

Let the angle between the longitudes be $\angle J\bar{M}K$.

Hence, $\angle J\bar{M}K = \angle J\bar{M}L - \angle K\bar{M}L$

$= 48° - 35°$

$= 13°$

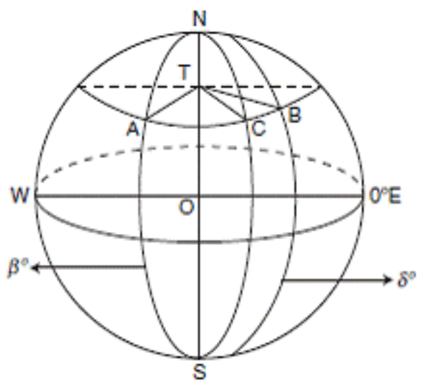

Figure 9.16

In Figure 9.16, place A belongs to the western part of the Greenwich Meridian (long 0°) while place B belongs to its eastern part.
Place A is on long $\beta°$ while place B is on long $\theta°$.
So, the angle between places A and C is calculated as follows:

$$\angle ATB = \angle ATC + \angle BTC$$
$$= \beta° + \theta°$$

From the simple illustration, the angle between the two longitudes that lie on opposite sides on the Greenwich Meridian (Long 0°) and on the same latitude is the sum of the longitudes.

Worked Example 10

The latitude and longitude of a point V are 40° N, 52° W and of another point U are 40° N, 34° E. Calculate the angle between the longitudes.

SOLUTION

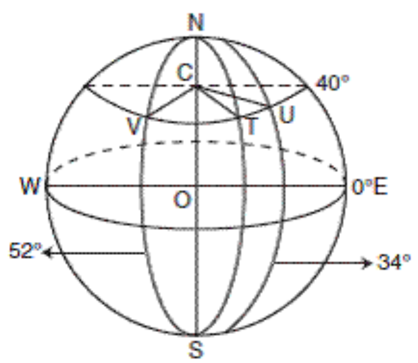

Figure 9.17

Let the angle between the longitudes be

$\angle VCU$.

$$\angle VCU = \angle VCT + \angle UCT$$
$$= 52° + 34°$$
$$= 86°$$

Worked Example 11

Find the angle between the longitudes in A (0° N, 46° E) and B (0° N, 43° W).

SOLUTION

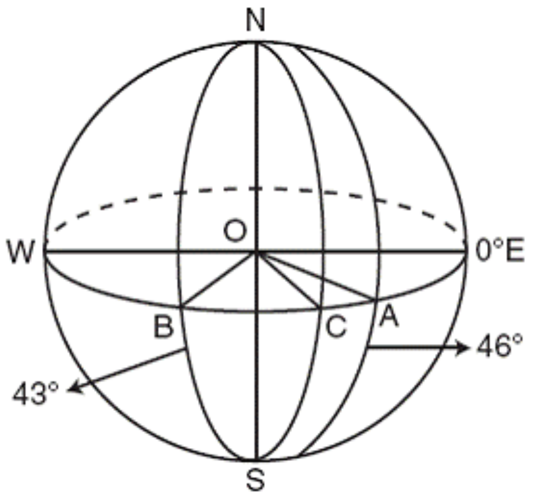

Figure 9.18

Let the angle between the longitudes be ∠AOB.

Hence, ∠AOB = ∠AOC + ∠BOC

= 46° + 43°

= 89°

Exercise 3

Calculate the angular difference between the following points on the earth's surface:

1. A (lat 41.5° N, long 39° E) and B (lat 41.5° N, long 39° W)
2. C (lat θ° N, long 48.5° W) and D (lat θ° N, long 26.5° W)
3. E (lat 56° S, long 65° E) and F (lat 56° S, long 23° E)
4. G (lat 49° S, long 40° E) and H (lat49° S, long 32° W)
5. I (lat 70.8° N, long 70.8° W) and J (lat 70.8° N, long 16.9° W

Calculate the angular difference between two towns A and B with the same lat 62° N and their following respective longitudes:

6. 50° W, 35° E
7. 82° W, 36° W
8. 43° W, 30° E
9. 29° E, 45° W
10. 41° W, 40° E

<u>VII. Distance between Two Places on the Earth</u>

The earth comprises both small circles and great circles. Small circles are the parallels of latitude and the great circles are the lines of longitude and the equators.

(i) Calculation of distance between two places on the earth's surface (a sphere)

Calculation of distance between the two places on a sphere depends on the arc of a circle. Hence, the formula of arc length is required.

For instance,

Formula for arc length $= \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended at the centre of a circle,

$\pi = 3.142$ or $\frac{22}{7}$ and r is the radius.

In a sphere, the distance between two places is used instead of the arc length of a circle. So, the length between two places along latitude

$\alpha°$ (small circle) $= \frac{\theta}{360} \times 2\pi r$ where $r = R \cos \alpha$ and R is the radius of the earth $\cong$ 6 400 km

Also, the distance between two places on lat 0° (a great circle)

$= \frac{\theta}{360} \times 2\pi r$ where $r = R \cos \theta°$.

(ii) Calculation of distance between two places along the parallel of latitude α (small circle)

Calculation of distance between two places along the parallel of latitudes α° and latitude 0° (equator) shall be examined here.

Worked Example 12

Q and T are two towns on the same parallel of latitude 50° N. Q is on long 15° E. Assuming the earth to be a sphere of radius 6 400 km and π = 3.142, calculate, correct to three significant figures, the shorter distance QT measured along the parallel of latitude.

SOLUTION

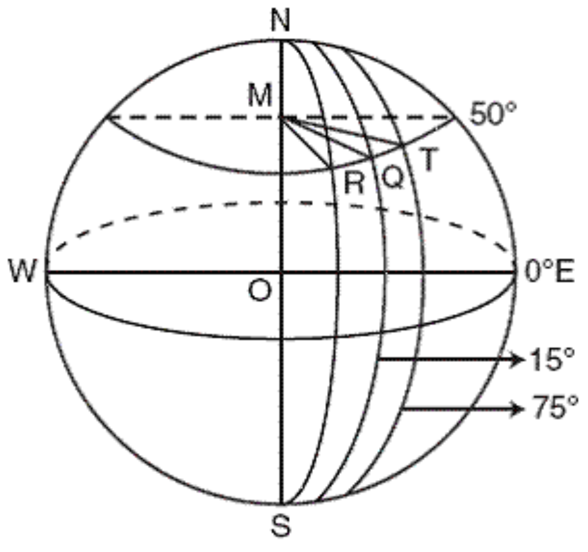

Figure 9.19

Formula:

Distance QT $= \frac{\theta}{360} \times 2\pi r$ but $r = R\cos\theta$

$$\angle QMT = \angle RMT - \angle RMQ$$
$$= 75^\circ - 15^\circ$$
$$= 60^\circ$$
$$r = R\cos 50^\circ \; (\alpha = 50^\circ)$$
$$= (6\,400 \times 0.6428) \text{ km}$$
$$= 4\,113.92 \text{ km}$$

Distance QT $= \frac{60}{360} \times \frac{2}{1} \times 3.142 \times 4\,113.92$ km

$$= 4\,308.646 \text{ km}$$
$$= 4\,310 \text{ km}$$

Worked Example 13

Determine the distance on the earth's surface between two towns P (lat 60° S, long 20° E) and Q (lat 60° S, long 28° W). Radius of the earth is 6 400 km.

SOLUTION

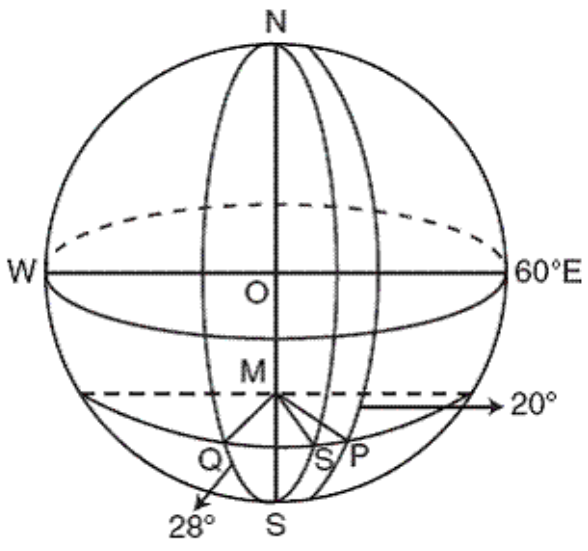

Figure 9.20

Formula: Distance $PQ = \frac{\theta}{360} \times 2\pi r$

where

$r = R \cos \alpha$

$R = 6\,400$, α is the latitude and r the radius

$$\angle(PMQ) = \angle PMS + \angle QMS$$
$$= 20^\circ + 28^\circ$$
$$= 48^\circ$$

$$r = R \cos \alpha$$
$$= 6\,400 \times \cos 60^\circ$$
$$(R = 6\,400 \text{ km}, \alpha = 60^\circ)$$
$$= (6\,400 \times 0.5) \text{ km} = 3\,200 \text{ km}$$

Therefore, distance PQ

$$= \frac{60}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{3\,200}{1} \text{ km}$$
$$= \frac{8\,448\,000}{2\,520} \text{ km}$$
$$= 3\,352.38 \text{ km}$$
$$= 3\,352 \text{ km (nearest km)}$$

Worked Example 14

Two ships on the equator are on longitudes 45° W and 45° E, respectively. Calculate their distance apart, correct to three signifi cant fi gures. (Take the radius of the earth = 6 400 km and π = 22/7)

SOLUTION

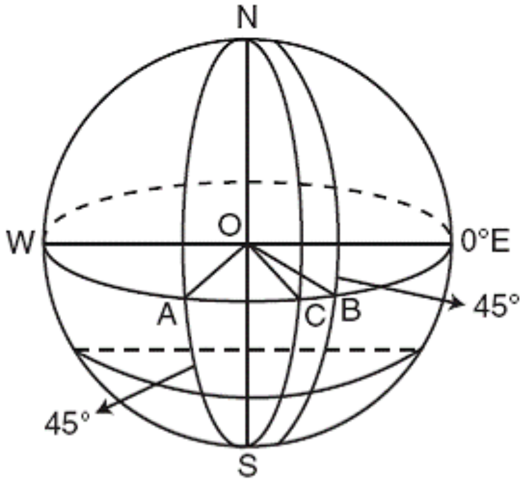

Figure 9.21

$$\text{Distance AB} = \frac{\theta}{360} \times 2\pi r$$

$$(\alpha = 0°)$$

$$\angle AOB = \angle AOC + \angle BOC$$

$$= 45° + 45°$$

$$= 90°$$

$$\text{Distance AB} = \frac{90}{360} \times \frac{2}{1} \times \frac{22}{7} \times 6\,400 \text{ km}$$

$$= \frac{90}{360} \times \frac{44}{7} \times \frac{6\,400}{1} \text{km}$$

$$= \frac{70\,400}{7} \text{ km}$$

$$= 10\,057.1 \text{ km}$$

$$= 10\,100 \text{ km (3 s.f.)}$$

Exercise 4

1. C (lat 38° N, long 38° E) and D (lat 38° N, long 27° W) are two points on the surface of the earth. Calculate the distance between C and D measured along the parallel of lat 38° *N.* (Assume that the earth is a sphere of radius6 400 km)
2. Lagos and Ibadan are two towns on the parallel of the lat 62° N. The longitude of Lagos is 150° W and that of Ibadan is 115° W. Benin is a third town which is also on lat 62° N and on long 32° E. Calculate the following: (Take the radius of the earth = 6 400 km and π = 3.142)
   (a) The distance of Ibadan from Lagos, correct to the nearest 100 km.
   (b) The distance between Ibadan and Benin measured along the parallel of latitude to the nearest 10 km.
3. The longitudes and latitudes of two places on the surface of the earth are given as T (60° S, 18° E) and Q (60° S, 47° E). Calculate the distance TQ measured along their parallel of latitude.
4. P and Q are points on the parallel of lat 68.7° S, their longitudes being 124° W and 56° E, respectively. What is their distance apart measured along the parallel of latitude? (Take R = 6 400 km, π = 3.142)
5. A plane flies from town A (60° N, 40° W) to a town B (60° N, 140° E). The plane then also flies to town C (60° N, 175° E). Calculate, correct to the nearest kilometre, the distance covered by the plane.
6. Three points X (lat 30° N, long 28° E); Y (lat 30° N, long 22° E) and Z (lat 30° N, long 22° W) are on the surface of the earth. Calculate the total distance from points X to Z.
7. P and Q are two points on lat 52° N. Their longitudes are 58° W and 38° E, respectively. Calculate, correct to the nearest kilometer , the distance PQ along the parallel of latitude. (Take the radius of the earth = 6 400 km and π = 22/7)
8. Two points X and Y both on lat 60° S have their long 147° E and 153° W, respectively. Find to the nearest kilometre the distance between X and Y measured along the parallel of latitude. (Take 2πR = 4 × 104 km, where R is the radius of the earth) **(UME)**
9. Determine the distance on the earth's surface between two towns P (lat 60° N, long 20° E) and Q (lat 60° N, long 28° W). (Radius of the earth = 6 400 km) **(UME)**
10. The positions of countries A and B are 51° N, 24° E and 51° N, 43° E, respectively. What is their distance apart?

IX. Distance between Places on the Lines of Longitude and Equator

(i) Calculation of distance between two places along the line of longitude

Two places to be calculated on the line of longitude can be on the same side of the equator or on the opposite sides (NN, SS or NS).

Worked Example 15

Two towns on the same longitude have lat 15° N and 30° S, respectively. Find, in kilometre, their distance apart measured along the longitude, taking the radius of
the earth as 6 400 km. **(WAEC)**

SOLUTION

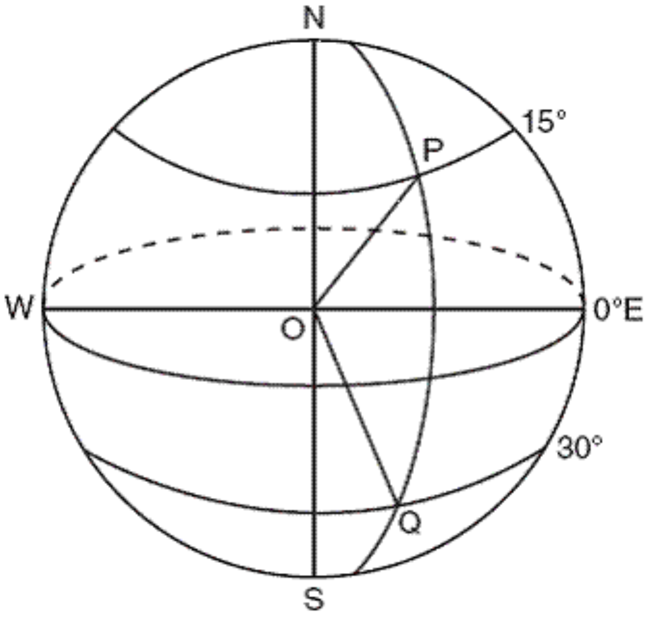

Figure 9.22

(a) The distance between A and B along the parallel of latitude.

(b) The shortest distance between A and B along a great circle.

SOLUTION

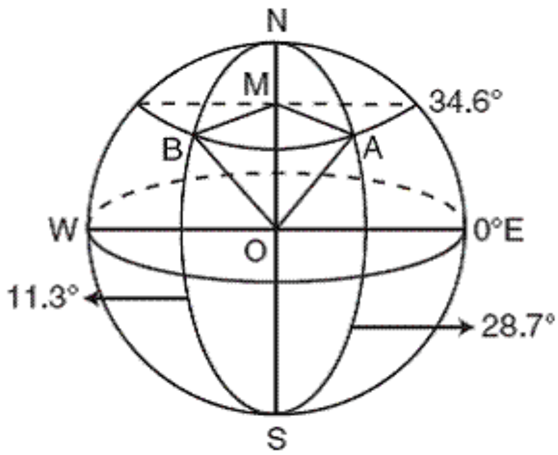

Figure 9.23(a)

(a) Distance AB

$$= \frac{11.3 + 28.7}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{6\,400}{1} \times \text{Cos } 34.6$$

$$= \frac{40.0}{360} \times \frac{44}{7} \times \frac{6\,400}{1} \times \frac{0.8231}{1}$$

$$= \frac{9\,271\,398.4}{2\,520}$$

$$= 3\,679.126 \text{ km}$$

(b) From Figure 9.23(b),

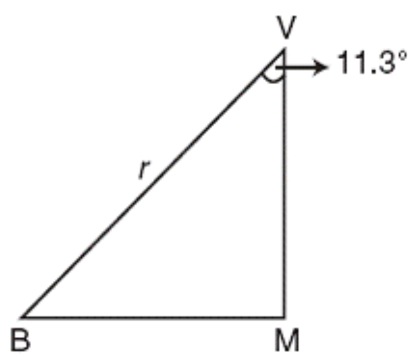

Figure 9.23(b)

$$\sin 11.3 = \frac{BM}{r}$$

$$BM = r \sin 11.3$$

$$\text{But } r = R\cos\alpha = 6\,400 \times \cos 34.6$$

$$= 6\,400 \times 0.8331$$

$$= 5\,331.84$$

$$\text{Hence, } BM = 5\,331.84 \times 0.1959$$

$$= 1\,044.5 \text{ km}$$

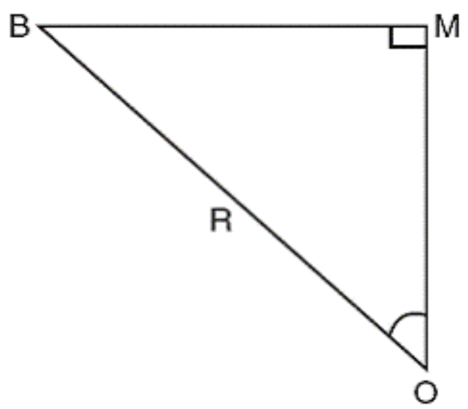

Figure 9.23(c)

$$\sin(\angle BOM) = \frac{BM}{R}$$

$$\sin(\angle BOM) = \frac{1\,044.5}{6\,400}$$

$$\sin(\angle BOM) = 0.1632$$

$$\angle BOM = \sin^{-1} 0.1632$$

$$\angle BOM = 9.4$$

$$\angle BOM = 2 \times 9.4^\circ = 18.8^\circ$$

$$\text{Shortest distance AB} = \frac{\theta}{360} \times 2\pi R$$

$$= \frac{18.8}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{6\,400}{1}\text{ km}$$

$$= \frac{5\,294\,080}{2\,520}\text{ km}$$

$$= 2\,100.825\text{ km}$$

$$= 2\,101\text{ km (nearest km)}$$

(iii) Distance involving three places on the earth's surface

Calculation of distance on the earth's surface is not limited to two places alone but can also extend to three places. The three places may be along the

(i) Same parallel of latitude, (ii) same line of longitude or (iii) parallel of latitude and line of longitude.

Worked Example 18

Two towns K and Q are on the parallel of lat 46° N. The longitude of town K is 130° W and that of town Q is 103° W. A third town P also on lat 46° N is on long 23° E.

Calculate the following:

(a) The length of the parallel of lat 46° N, to the nearest 100 km.

(b) The distance between K and Q, correct to the nearest 100 km.

(c) The distance between Q and P measured along the parallel of latitude, to the nearest 10 km.

(Take π = 3.142, radius of the earth = 6 400 km) **(WAEC)**

SOLUTION

(a) Length of the parallel of latitude $= 2\pi r$

$= 2 \times \pi \times R \times \cos\alpha$

$= 2 \times 3.142 \times 6\,400 \times \cos 46^\circ$

$= 40\,217.6 \times 0.6947$ km

$= 27\,939.17$ km

$= 19\,700$ km (nearest 100 km)

(b) Distance KQ $= \frac{130 - 103}{360} \times 2\pi r$

$= \frac{37}{360} \times \frac{37\,939.17}{1}$ km

$= \frac{754\,357.59}{360}$

$= 2\,095.437$ km

$= 2\,100$ km (nearest 100 km)

(c) Distance PQ $= \frac{103 + 23}{360} \times 2\pi r$

$= \frac{126}{360} \times \frac{27\,939.17}{1}$ km

$= \frac{3\,520\,335.42}{360}$ km

$= 9\,778.7095$ km

$= 9\,780$ km (nearest 10 km)

Worked Example 19

An aeroplane fl ies from a town L (lat 50° N, 35° E) to another town M (lat 50° N, 32° W). It later fl ies to a third town N (48° N, 32° W). Calculate the following:

(a) Distance between L and M along their parallel of latitude.

(b) Distance between M and N along the line of longitude.

SOLUTION

(a) Distance LM

$$= \frac{35 + 32}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{6\,400}{1} \times \text{Cos } 50°$$

$$= \frac{37}{360} \times \frac{44}{7} \times \frac{6\,400}{1} \times \frac{0.6428}{1} \text{ km}$$

$$= \frac{12\,127\,836.16}{2\,520} \text{ km}$$

$$= 4\,812.63 \text{ km}$$

$$= 4\,813 \text{ km (nearest km)}$$

(b) Distance MN $= \frac{50 - 48}{360} \times 2\pi R$

$$= \frac{2}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{6\,400}{1} \text{ km}$$

$$= \frac{563\,200}{2\,520} \text{ km}$$

$$= 223.49 \text{ km}$$

$$= 223 \text{ km (nearest km)}$$

Exercise 5

1. Two places on the same meridian have latitudes 25° N and 25° S. What is their distance apart along their meridian?
2. Find the distance between two towns P (45° N, 30° W) and Q (15° S, 30° W) if the radius of the earth is 7 000 km. ($\pi = 22/7$) **(UME)**
3. The positions of two countries P and Q are 15° N, 18° E and 65° N, 18° E, respectively. Calculate their distance apart.
4. Two towns on the same longitude have latitudes 15° N and 30° S, respectively. Find, in kilometre, their distance apart measured along the longitude taking the radius of the earth as 6 400 km. **(WAEC)**
5. Two cities lie on the same meridian and their latitides are 36° Nand 22° S. Find their distance apart in terms of π.
6. J (lat 37° N, long 52° W) and K (lat 25° S, long 52° W) are two places on the earth's surface. Calculate, correct to four significant figures, the distance JK measured along the meridian.
7. Kaduna and Abuja on the same meridian have latitudes 36° N and 57° N. Find

their distance apart measured along the meridian.

8. Determine the distance between towns M (lat 38° S, long 54° W) and N (lat 29° N, long 54° W) along their common longitude. Take *R*, the radius of the earth, as 6 370 km.

9. Two ships on the equator sails on longitudes 52° W and 34° E, respectively. How far apart are they on the equator?

10. Find the distance between V (43° N,36° E) and P (43° S, 36° E) along their line of longitude.

X. Speed and Average Speed

Speed is calculated as the distance covered divided by the time taken. This is written as $s = d/t$. The unit of speed is km/hr or m/sec. The average speed is calculated as the total distance covered divided by the total time taken. This is written as:

$$s = \frac{d_1 - d_2}{t_1 - t_2}.$$

Worked Example 20

Two points X (32° N, 47° W) and Y (32° N, 25° E) are on the earth's surface. If an aeroplane takes 11 hours to fl y from X to Y along the parallel of latitude, calculate its speed, correct to the nearest kilometer per hour. (Take the radius of the earth = 6 400 km and π = 22/7) **(WAEC)**

SOLUTION

Distance XY

$$= \frac{47 + 25}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{6\,400}{1} \times \text{Cos } 32°$$

$$= \frac{72}{360} \times \frac{44}{7} \times \frac{6\,400}{1} \times 0.8480 \text{ km}$$

$$= \frac{17\,193\,369.6}{2\,520} \text{ km}$$

$$= 6\,822.77 \text{ km.}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{\frac{6\,822.77}{11} \text{ km}}{\text{hr}}$$

$$= 620.25 \text{ km/hr}$$

$$= 620 \text{ km/hr (nearest km/hr)}$$

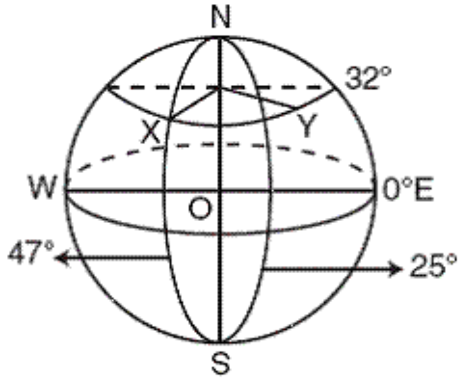

Figure 9.24

Worked Example 21

An aircraft took 2 hours to fl y from P (lat 65° N, long 45° E) to another town T (lat 65° N, long 15° E). The aircraft then changes its course and 9 hours after leaving, T arrived at a third town Q (lat 0°, long 15° E). If the fl ight from P to T was along the line of latitude and that from T to Q was along the meridian, calculate the following, to three signifi cant fi gures:

(a) The total length of the journey.

(b) The average speed of the aircraft. (Take π = 3.142 and the radius of the earth = 6 400 km)

SOLUTION

(a) Distance PT

$$=\frac{45-15}{360}\times 2\times 3.142\times 6\,400\times \text{Cos}\,65^\circ$$

$$=\frac{30}{360}\times\frac{40\,217.6}{1}\times\frac{0.4226}{1}\text{ km}$$

$$=\frac{16\,995.96}{12}$$

$$=1\,416.33\text{ km}$$

$$\text{Distance TQ}=\frac{65}{360}\times\frac{2}{1}\times\frac{3.142}{1}\times\frac{6\,400}{1}\text{ km}$$

$$=\frac{2\,614\,144}{360}\text{ km}$$

$$=7\,261.5\text{ km}$$

Total length of the journey

$$=(1\,416.33+7\,261.5)\text{ km}$$

$$=8\,677.83\text{ km}$$

$$=8\,680\text{ km (3 s.f.)}$$

(b) **Given:**

$$\text{Total time taken}=2\text{ hours}+9\text{ hours}$$

$$=11\text{hours}$$

Therefore, average speed

$$=\frac{\text{Total length of the journey}}{\text{Total time taken}}$$

$$=\frac{8\,677.83}{11}\text{ km/hr}$$

$$=788.89\text{ km/hr}$$

$$=789\text{ km/hr (3 s.f.)}$$

Exercise 6

1. An aeroplane fl ies from a town P (lat 40° N, long 28° E) to another town Q (lat 40° N, long 22° W). It later flies to a third town T (lat 28° N, long 22° W). Calculate the following:
   (a) Distance between P and Q along their parallel of latitude.

(b) Distance between Q and T along the line of longitude.
(c) Average speed at which the aeroplane will fl y from P to T via Q if the journey takes 12 hours, correct to three significant figures. (Take the radius of the earth as 6 400 km and π = 3.142)

2. A bird flies from point X (54° N, 45° W) to a point Y (54° N, 37° E). It later changes its course and flies to a third point Z (18° S, 37° E).

Calculate the following:

(a) The distance covered by the bird from X to Y along the parallel of latitude.

(b) The distance covered by the bird from Y to Z along the line of longitude.

(c) The average speed of the bird if the entire journey takes 20 h. Express your answer correct to three significant figures.

(Take π = 3.142)

3. An aeroplane took 3 hours to fl y from town P (lat 65° N, long 45° W) to another town Q (lat 65° N, long 15° W). It then changed its course and 7 hours after leaving Q arrived at a third town R (lat 0°, long 15° W). The flight from P to Q was along the parallel of latitude and that from Q to R was along the meridian of longitude. Calculate the following, correct to three signify cant figures:

(a) Total length of the journey.

(b) Average speed of the aeroplane

(Take π = 3.142 and radius of the earth as 6 400 km). **(WAEC)**

4. An aeroplane flies from A (lat 25° N, long 15° E) due east to B (lat 25° N, long 65° E). If the journey takes 6 hours, calculate the speed of the aeroplane, correct to two significant figures.

5. A speed boat took 5 hours to fly from a town A (lat 56° N, long 54° W) to another town B (lat 56° N, long 35° W). It then changed its course, and 10 hours after leaving B arrived at a third town C (lat 0°, long 35° W). Calculate, correct to three significant figures, the following:

(a) Total length of the journey.

(b) Average speed of the boat

(Take π = 3.142 and radius of the earth = 6 400 km) **(WAEC)**

XI. Time

Time taken is calculated as the distance covered divided by the speed. This is expressed as $t = d/s$ where t is the time, d is the distance and s is the speed. The unit of time is hours, minutes or seconds.

Worked Example 22

P (38.6° N, 28.7° E) and Q (38.6° N, 21.3° W) are two places on the earth's surface. Another place T is on the same meridian as P and its latitude is 29.4° N. Assuming the earth to be a sphere of radius 6 400 km and taking π to be 3.142, calculate the following, correct to three significant figures:
(a) The distance between P and Q along their parallel of latitude.
(b) The shortest distance between P and T on the earth's surface.
(c) The time taken by an aeroplane to fl y from Q to P and then from P to T, if its

average speed is 650 km/hr. **(WAEC)**

SOLUTION

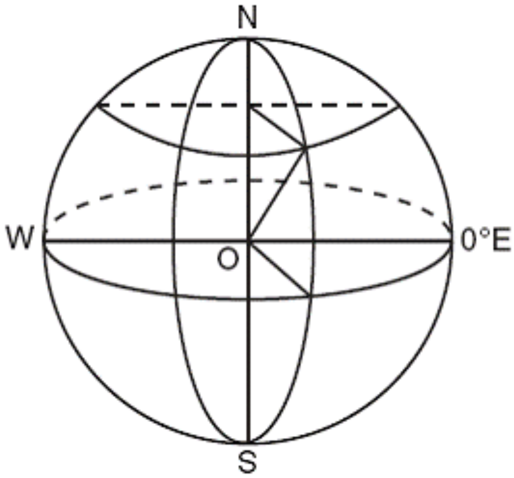

Figure 9.25

(a) Distance PQ

$$= \frac{21.3 + 28.7}{360} \times \frac{2}{1} \times \frac{3.142}{1} \times \frac{6\,400}{1} \times \text{Cos } 38.6°$$

$$= \frac{50}{360} \times \frac{40\,217.6}{1} \times \frac{0.8780}{1}$$

$$= \frac{1\,765\,552.64}{360} \text{ km}$$

$$= 4\,904.313 \text{ km}$$

$$= 4\,900 \text{ km (3 s.f.)}$$

(b) Distance PT

$$= \frac{38.6 - 29.4}{360} \times \frac{2}{1} \times \frac{3.142}{1} \times \frac{6\,400}{1} \text{ km}$$

$$= \frac{9.2}{360} \times \frac{40\,217.6}{1} \text{ km}$$

$$= 1\,027.78 \text{ km}$$

$$= 1\,030 \text{ km (3 s.f.)}$$

(c) **Given:**

Average speed $= 650$ km/hr

Total distance $= (4\,904.312 + 1\,027.78)$ km

$= 5\,932.092$ km

$$\text{Time} = \frac{\text{Total distance}}{\text{Average speed}}$$

$$= \frac{5\,932.092}{650} \text{ hr}$$

$$= 9.126 \text{ hr}$$

$$= 9.13 \text{ hr (3 s.f.)}$$

Exercise 7

1. P (lat 60° N, long 15° E), Q (lat60° N, long 48° W) and R (lat 38° S, long 48° W) are three places on the earth's surface. Calculate the following, correct to three significant figures:
(a) The distance PQ measured along the latitude.
(b) The distance QR measured along the meridian.
(c) The time taken by an aeroplane to cover the distance PQ and QR at an average speed of 800 km/hr. (Take π = _27_2_ and the radius of the earth

= 6 400 km)

2. P (lat 42° N, long 24° W) and Q (lat42° N, long 40° E) are town points on the earth's surface. Assuming that the earth is a sphere of radius 6 400 km, calculate the following, correct to three significant figures:
(a) The radius of lat 42° N .
(b) The distance PQ measured along the parallel of latitude.
(c) The total time taken by an aeroplane to fl y at an average speed of 750 km/hr from P to Q and then along long 40° E to a point S on the equator.
(WAEC)
3. P (lat 42° N, long 24° W) and Q (lat 42° N, Long 40° E) are two points on the earth's surface. Assuming that the earth is a sphere of radius 6 400 km, calculate, correct to three significant figures, the following:
(a) The radius of lat 42°N.
(b) The distance PQ measured along the parallel of latitude.
(c) The total time taken by an aeroplane to fl y at an average speed of 750 km/hr from P to Q and then along long 40°E to a point S on the equator.
(Take π = 3.142) **(WAEC)**
4. A plane flies due east from a point A (lat 53° N, long 25° E) to a point B (lat 53° N, long 85° E) at an average speed of 400 km/hr. The plane then fl ies south from B to a point C 2 000 km away. Calculate the following, correct to the nearest whole number:
(a) The distance between A and B.
(b) The time the plane takes to reach point B . **(WAEC)**
5. A bird fl ies due south from a tree on lat 36° N, long 138° E to a plan on lat 63° S long 138° E. Calculate the following: (Take π = 22/7, R = 6 400 km).
(a) The distance covered, correct to three significant figures.
(b) The time taken, correct to the nearest hour, if the speed of the bird is 800 km/hr.

XII. Longitude and Latitude

In this section, the longitude is calculated when other variables such as distance, time or speed are given. The latitude of places will also be calculated.

Worked Example 23

An aeroplane flies due west for 3 hours from P (lat 50° N, long 60° W) to a point Q at an average speed of 600 km/hr. The aeroplane then fl ies due south from Q to a point Y 500 km away. Calculate the following, correct to three significant figures:
(a) The longitude of Q .
(b) The latitude of Y .
(Take the radius of the earth as

6 400 km and $\pi = \frac{22}{7}$)

$$\cos 50^\circ = \frac{r}{R} \Rightarrow r = R \text{ Cos } 50$$

$$r = 6\,400 \times 0.6428 = 4\,113.92 \text{ km}$$

Given:

$$\text{Average speed} = 600 \text{ km/hr}$$

$$\text{Distance PQ} = (3 \times 600) \text{ km}$$

$$= 1\,800 \text{ km}$$

$$\text{But distance PQ} = \frac{\theta}{360} \times 2 \times \pi \times r$$

$$1\,800 = \frac{\theta}{360} \times \frac{2}{1} \times \frac{22}{7} \times 4\,113.92 \text{ km}$$

$$\frac{1\,800 \times 360 \times 7}{44 \times 4\,113.92} = \theta$$

$$\theta = \frac{4\,536\,000}{181\,012.48} = 25.06$$

Note: $\theta = \beta - 60^\circ$ (β = longitude of Q)

$$25.06^\circ = \beta - 60^\circ$$

$$\beta = 60^\circ + 25.06^\circ$$

$$= 85.06^\circ$$

$$= 85.1^\circ \text{ (3 s.f.)}$$

Longitude of Q = 85.1

(b) $\text{Distance QY} = \frac{Q\hat{O}Y}{360} \times 2\pi R$

$$500 = \frac{50 - \alpha}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{6\,400}{1}$$

$$\frac{500 \times 360 \times 7}{2 \times 22 \times 6\,\,400} = 50 - \alpha$$

$$4.474 = \alpha^\circ$$

$$\alpha = 50 - 4.474$$

$$\alpha = 45.526^\circ$$

$$\alpha = 45.5^\circ \text{ (3 s.f.)}$$

The latitude of Y is 45.5° (3 s.f.)

Exercise 8

1. The distance between two points A and B on lat 60° S along theirparallel of latitude is 2 816 km.

(a) Calculate the angular difference in their longitudes.
(b) If A is due west of B and on long 20° W, fi nd the longitude of B.
(Take π = 22/7 and the earth's radius as 6 400 km) **(WAEC)**

2. Two points A and B lie on the parallel of lat 60° N. If A lies on long 20° E and B is 1 500 km due east of A, calculate the following:
(a) Radius of the parallel of latitude on which they lie.
(b) Longitude on which point B lies, correct to the nearest degree. (Take π = 3.142, radius of the earth = 6 400 km) **(WAEC)**

3. K (lat 60° N, long 50° W) is a point on the earth's surface. L is another point due east of K and the third point N is due south of K. The distance KL is 3 520 km and KN is 10 951 km. Calculate the following:
(a) The longitude of L .
(b) The latitude of N .
(Take π = 22/7 and the radius of the earth = 6 400 km) **(WAEC)**

4. P (lat 36° S, long 120° E) is a point on the earth's surface. Q is another point 840 km due north of P. T, a third point, is 840 km due east of P. Calculate, correct to two significant figures, the following:
(a) The radius of latitude of P
(b) The latitude of Q
(c) The longitude of T
(Take π = 3.142 and the radius of the earth = 6 400 km) **(WAEC)**

5. P is the point (60° N, 18° E) on the earth's surface. Q is the point due west of P such that the distance PQ measured along the parallel of latitude is 550 km. Calculate the longitude of Q correct to the nearest degree. (Take the radius of the earth as 6 400 km and π = 22/7) **(WAEC)**

SUMMARY

In this chapter, we have learnt the following:

- The earth as a sphere with an orange shape has its radius being approximated to 6 400 km.

v The earth oscillates on the North–South polar axis.
v The earth comprises small circles and great circles.
v Small circles are the parallel of latitudes while great circles are the equator and the prime meridian.
v Radius of the parallel of latitude is $R\cos\alpha$ where R is the radius of the earth (6 400 km) and α is the latitude.
v The radius of the equator (lat 0°) is R which is approximately 6 400 km.
v The distance between two places on a great circle subtending an angle $\theta°$ at the centre of the earth has the formula $\frac{\theta}{360°} \times 2\pi R$, where R is the radius of the earth (6 400 km) and π = 3.142.
v The distance between two places on the parallel of lat α ° which subtends an angle of $\theta°$ at the centre of the latitude is calculated as $\frac{\theta}{360°} \times 2\pi R$ Cos α, where R is the radius of the earth and π = 3.142 or 22/7
v The length of the parallel of latitude is calculated as $2\pi r$, where $r = R\cos\alpha$, with α being the latitude.

GRADUATED EXERCISES

1. A plane flies due east from A (lat 53° N, long 25° E) to a point B (lat 53° N, long 85° E) at an average speed of 400 km/hr. The plane then flies south from B to a point C 200 km away. Calculate the following correct to the nearest whole number:
(a) The distance between A and B .
(b) The time the plane takes to reach point B .
(c) The latitude of C (Take the radius of the earth as 6 400 km and α = 22/7)
2. A point C is on lat 28° N and long 105° W. Y is another point on the same latitude as X but on long 35° E. Calculate the following, correct to the three significant figures:
(a) The distance between X and Y along lat 28° N.
(b) The distance of X from the equator. (Take π = _27_2_ and the radius of the earth as 6 400 km)
3. P and Q are two towns on the earth's surface on lat 56° N. Their longitudes are 25° E and 95° E, respectively. Find the distance PQ along their parallel of latitude correct to the nearest kilometre. (Take the radius of the earth as 6 400 km and α = _ 27_2_)
4. An aeroplane flies from a town P (40° N, 38° E) to another town Q (40° N, 22°W). It later flies to a third town T (28° N, 22° W). Calculate the following:
5. A moving object takes off from a town P (lat 40° N, long 52° E) and after flying 1 450 km due east, it reaches a town Q. It then flies due north to another town S on lat 60° N. Calculate the following:
(a) Radius of the line of latitude through P .
(b) Longitude of Q correct to the nearest degree .
(c) Distance between Q and S along the parallel of longitude correct to three significant figures. (Take R = 6 400 km, π = 3.142)
6. Two points U and V on the earth's surface have positions lat 78° N, long 18° E and lat 78° N, long 62° W, respectively. Calculate the following:
(a) Distance of arc UV along the parallel of latitude.
(b) Length of the chord UV (Take R = 6 400 km and π = 3.142) **(NECO)**
7. A town X is on long 40° W and lat 50° S. Another town Y is on the same latitude as town X but on long 30° E. Calculate the following:
(a) Circumference of the circle of lat 50° S to the nearest 100 km.
(b) Distance between X and Y measured long their parallel of latitude to the nearest 10 km(Take radius of the earth as 6 400 km and π = 22 / 7) **(NECO)**
8. The points X (32° S, 86° W) and Y (32° S, 14° W) are on the earth's surface.
Calculate the following, correct to three significant figures:
(a) Length of chord XY.
(b) Distance between X and Y along the
    (i) Parallel of latitudes.
    (ii) Great circle (Take R = 6 400 km and π = 3.142) **(NECO)**
9. If two places P and Q are on the same parallel of lat 30° N and they differ

in longitude by 45°, Calculate

(a) Distance between P and Q measured along the parallel of latitude.

(b) Angle subtended at the centre of the earth by PQ.

(c) Shortest distance between P and Q (R = 6 400 km) (**NECO)**

10. Assuming the earth to be a sphere of radius 6 370 km, calculate, correct to three significant figures, the length of an arc of the equator which subtends an angle of 1' at the centre of the earth. (Note that 60’ = 1° and take π = 22/7 or 3.14)