CHAPTER 3: Sequences and Series

OBJECTIVES

At the end of the chapter, students should be able to:
1. State the meaning and types of sequence.
2. Define a series.
3. Define arithmetic and geometric progression.
4. Define some other sequences.

I. Meaning and Types of Sequence

Consider the following sets of numbers:
(a)   2, 4, 6, 8, 10, etc.
(b) 1, 3, 5, 7, 9, etc.
We can see that in (a) the first number is 2 = 2 Â°Ã¸ 1, the second number is 4 = 2 Â°Ã¸ 2, the third number is 6 = 2 Â°Ã¸ 3, the fourth number is 8 = 2 Â°Ã¸ 4, etc. This shows that any term of the set is formed by the rule 2n, where n is the number of the term. Similarly in (b) the fi rst number is 1, the second number is 3 = 1 + 2, the third number is 5 = 3 + 2, the fourth number is 7 = 5 + 2, the fi fth number is 9 = 7 + 2, etc.
This shows that the next number is formed by adding 2 to the previous number. If the previous number is denoted by u then the next number will be u + 2. We can see that in (a) and (b), we have a specific rule guiding the formation of numbers. The above two are examples of sequence of numbers.
A sequence is a set of numbers in some definite order. The successive number of the sequence is formed according to some rule. The numbers in a sequence are called terms of the sequence. The terms of a sequence are denoted by U_n. For example, the first term of a sequence is U_1, second term is U_2, third term is U_3, etc. Thus, in general, the n_{th} term of a sequence is U_n. Consider the sequence 1_2, 2_2, 3_2, 4_2, etc. Here, the fi rst term is $U_1 = 1_2$, the second term is $U_2 = 2_2$, the third term is $U_3 = 3_2$, and the n_{th} term will be $U_n = n_2$.

Activity

We should note that the rule defining a sequence is often given in the form of some formula for U_n in terms of n. Hence, for the sequence 1, 2, 3, 4, 5, â€¦, $U_n = n$ and also for the sequence 3, 5, 7, 9, 11, â€¦,
$U_n = 2n + 1$.

Worked Example 1

Find the **n**$_{th}$ term of the following sequences:
(a) 1, _ 14_ , _ 91_ , _ 11_6_ , _ 21_5_ , â€¦
(b) 1, 4, 3, 16, 5, â€¦

SOLUTION

(a) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \cdots$

Here, the first term $U_1 = 1 = \frac{1}{1^2}$

The second term $U_2 = \frac{1}{4} = \frac{1}{2^2}$

The third term $U_3 = \frac{1}{9} = \frac{1}{3^2}$

The fourth term $U_4 = \frac{1}{16} = \frac{1}{4^2}$

The fifth term $U_5 = \frac{1}{25} = \frac{1}{5^2}$

$\therefore$ the n^{th} term $U_n = \frac{1}{n^2}$

(b) 1, 4, 3, 16, 5, ...

Here, the first term $U_1 = 1$ *(odd)*

The second term $U_2 = 4 = 2 \times 2$ *(even)*

The third term $U_3 = 3$ *(odd)*

The fourth term $U_4 = 16 = 4 \times 4$ *(even)*

The fifth term $U_5 = 5$ *(odd)*

From the above, we can see that if n is odd, U_n = n and if n is even, U_n = n2. Hence, the rule guiding the formation of the terms of the sequence is

$$U_n = \begin{cases} n, \text{ if } n \text{ is odd} \\ n^2, \text{ if } n \text{ is even} \end{cases}$$

Worked Example 2

Find $\mathbf{U_n}$ in terms of **n** for the sequence:

(a) 3, 5, 7, 9, â€¦

(b) 1, 2, 6, 24, 120, â€¦

SOLUTION

(a) 3, 5, 7, 9, â€¦

Here, the first term $\mathbf{U_1}$ = 3 = 2 Â°Ã¸ 1 + 1

The second term $\mathbf{U_2}$ = 5 = 2 Â°Ã¸ 2 + 1

The third term $\mathbf{U_3}$ = 7 = 2 Â°Ã¸ 3 + 1

The fourth term $\mathbf{U_4}$ = 9 = 2 Â°Ã¸ 4 + 1

Hence, the **n**th term $\mathbf{U_n}$ = 2**n** + 1

(b) 1, 2, 6, 24, 120, â€¦

Here the first term $\mathbf{U_1}$ = 1 = 1 Â°Ã¸ 1

The second term $\mathbf{U_2}$ = 2 = 2 Â°Ã¸ 1 = 2$\mathbf{U_1}$

The third term $\mathbf{U_3}$ = 6 = 3 Â°Ã¸ 2 = 3$\mathbf{U_2}$

The fourth term $\mathbf{U_4}$ = 24 = 4 Â°Ã¸ 6 = 4$\mathbf{U_3}$

The fifth term $\mathbf{U_5}$ = 120 = 5 Â°Ã¸ 24 = 5$\mathbf{U_4}$

âˆ´ **n**th term $\mathbf{U_n}$ = **nU**nâˆ’1

Worked Example 3

Find the first four terms of the sequence whose **n**th terms ($\mathbf{U_n}$) are:

(a) $U_n = 2n - 1$

(b) $U_n = \left(-\frac{1}{2}\right)^{n-1}$

SOLUTION

(a) $U_n = 2n - 1$

When:

$n = 1, U_1 = 2 \times 1 - 1 = 2 - 1 = 1$

$n = 2, U_2 = 2 \times 2 - 1 = 4 - 1 = 3$

$n = 3, U_3 = 2 \times 3 - 1 = 6 - 1 = 5$

$n = 4, U_4 = 2 \times 4 - 1 = 8 - 1 = 7$

Hence, the first four terms of the sequence are 1, 3, 5, 7, …

(b)

$U_n = \left(-\frac{1}{2}\right)^{n-1}$

When: $n = 1, U_1 = \left(-\frac{1}{2}\right)^{1-1} = \left(-\frac{1}{2}\right)^0 = 1$

$n = 2, U_2 = \left(-\frac{1}{2}\right)^{2-1} = \left(-\frac{1}{2}\right)^1 = -\frac{1}{2}$

$n = 3, U_3 = \left(-\frac{1}{2}\right)^{3-1} = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$n = 4, U_4 = \left(-\frac{1}{2}\right)^{4-1} = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

Hence, the first four terms of the sequence are 1, $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

II. Series

Consider the sequence whose terms are: $\mathbf{U_1}$, $\mathbf{U_2}$, $\mathbf{U_3}$, …, $\mathbf{U_n}$. If each of the term is added together, we have $\mathbf{U_1} + \mathbf{U_2} + \mathbf{U_3} + \ldots + \mathbf{U_n}$. The expression is called a series.

When a finite number of terms of a sequence are added together, we have a finite series. On the other hand, if we add infinite number of terms of a sequence together, we have an infinite series

Worked Example 4

Which of the following series is finite or infinite?

(a) $3 + 4 + 5 + \ldots + 20$

(b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

(c) $2 + 4 + 6 + 8 + \ldots + 20$

(d) $1 + 4 + 9 + 16 + \ldots$

SOLUTION

(a) $3 + 4 + 5 + \ldots + 20$ (Finite series)

(b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ (Infinite series)

(c) $2 + 4 + 6 + 8 + \ldots + 20$ (Finite series)

(d) $1 + 4 + 9 + 16 + \ldots$ (Infinite series)

Note: The sum of the first n terms of a sequence $U_1, U_2, \ldots, U_n$ is generally denoted by S_n. That is,

$S_n = U_1 + U_2 + \ldots + U_n = \sum_{r=1}^{n} U_r$, where Σ is the summation (addition) of all terms in the series.

For example, $\sum_{r=1}^{n} U_r = 1 + 2 + 3 + \ldots + n.$

In the same way, $\sum_{r=1}^{m} U_r = U_n + U_{n+1} + \ldots + U_m \ (n > m)$

Worked Example 5

Evaluate the following:

(a) $\sum_{r=1}^{5} r^2$

(b) $\sum_{r=2}^{4} (r(r+2r))$

SOLUTION

(a) $\sum_{r=1}^{5} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 1 + 4 + 9 + 16 + 25 = 55$

(b) $\sum_{r=2}^{4} (r(r+2r)) = 2(2+4) + 3(3+6) + 4(4+8)$

$= 12 + 27 + 48 = 87$

Worked Example 6

The sum of the first m terms of a series is given by the formula $S_m = m_2 + 3m$ for all values of m. Find an expression for the n_{th} term of the series.

SOLUTION

If U_n denotes the n_{th} term of the series, it follows that

$S_n = U_1 + U_2 +$ â€¦â€¦â€¦ $+ U_{n\text{âˆ’}1} + U_n$

âˆ´ $S_n = S_{n\text{âˆ’}1} + U_n$

$U_n = S_n – S_{n\text{âˆ’}1}$

But $S_m = m_2 + 3m$

âˆ´ $S_n = n_2 + 3n$ and

$S_{n\text{âˆ’}1}$ = (n âˆ’ 1)$_2$ + 3(n âˆ’ 1)

âˆ´ $U_n = n_2 + 3n$ – [(n âˆ’ 1)$_2$ + 3(n âˆ’ 1)]

$= n_2 + 3n – (n_2 – 2n + 1 + 3n – 3)$

$= n_2 + 3n – (n_2 + n – 2)$

$= n_2 + 3n – n_2 – n + 2$

$U_n = 2n + 2$

Hence, $U_n = 2n + 2$

Exercise 1

1. Find a formula $\mathbf{U_n}$ for the following sequences (a)–(j).

(a) $\frac{1}{2}$, 2, 8, 32, 128, â€¦
(b) 1, **âˆ’**4, 9, **âˆ’**6, 25, â€¦
(c) 1, 8, 27, 64, 125, â€¦
(d) 1, 4, 9, 16, 25, â€¦
(e) 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$
(f) $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$
(g) 4, 2, 0, **âˆ’**2, â€¦
(h) 1, 4, 9, 16, â€¦
(i) 3, 1, **âˆ’**1, **âˆ’**3, â€¦
(j) 2, 5, 8, 11, â€¦

2. Write the next four terms of each of the sequences in (a)–(j) above.

3. Write out the first four terms of the sequence whose general term is given in questions (a)–(e).

(a) $U_n = \frac{1}{n}$

(b) $U_n = \frac{1}{n+1}$

(c) $U_n = \frac{n+1}{n-1}$

(d) $U_n = \frac{1}{3}n^2$

(e) $U_n = n+1$

4. A sequence is defined by the rule **U**$_1$ = 1, **U**$_2$ = 2 and **U**$_\mathbf{r}$ = **U**$_{\text{râˆ’1}}$ + **U**$_{\text{râˆ’2}}$ for **r** â‰¥ 3. Find the first seven terms of this sequence.

5. Find the first six terms of the sequence where **U**$_1$ = 0, **U**$_2$ = 2 and **U**$_\mathbf{r}$ = **U**$_{\text{râˆ’1}}$ – **U**$_{r–2}$ for **r** â‰¥ 2.

Hence evaluate $\sum_{r=1}^{6} U_r$.

6. Evaluate **S**$_8$ for the series 1 + 3 + 6 + 9 + 12 + â€¦

7. Evaluate **S**$_6$ for the series 3 + 9 + 27 + â€¦

8. If **U**$_\mathbf{r}$ = log$_{10}$**r**, show that $\sum_{r=1}^{10} U_r = \sum_{r=1}^{10} \log_{10} r = 3\,628\,300$

III. Arithmetic Sequence or Progression

(i) Definition

If the consecutive terms of a sequence increase or decrease by a constant number, the sequence is said to be an arithmetic sequence or arithmetic progression.

Consider the following sequences:

(a) 2, 4, 6, 8, 10, â€¦

(b) $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, \ldots$

(c) 2, 0, **âˆ’**2, **âˆ’**4, **âˆ’**6, â€¦

(d) 1, 1.2, 1.4, 1.6, â€¦

(e) $3\frac{1}{2}, 4\frac{1}{2}, 5\frac{1}{2}, 6\frac{1}{2}, \ldots$

In (a) the terms differ by 2 hence, the difference is 1.

(b) the terms differ by $^1/_2$ hence, the difference is $^1/_2$

(c) the terms differ by âˆ’2 hence, the difference is âˆ’2.

Now, find the diff erence in terms of the sequences in (d) and (e) above.

We can see that the difference is common to all the terms in each of the sequences hence, the difference is called common difference.

By convention, the first term and the common difference of an AP are denoted by â€˜aâ€™ and â€˜**d**â€™ respectively. The general form of the AP is then a, a+**d**, a+2**d**, â€¦, a + (**nâˆ’**1)**d**, â€¦

(ii) The **n**$_{\text{th}}$ term of an arithmetic sequence (progression)

Consider the sequence, a, a+**d**, a+2**d**, a+3**d**, â€¦

Denote the **n**$_{\text{th}}$ term of the sequence by **U**$_\mathbf{n}$**.**

Hence, first term: **U**$_1$ = **a** = **a** + (1 **âˆ’** 1)**d**

second term: **U**$_2$ = **a** + **d** = **a** + (2 **âˆ’** 1)**d**

third term: **U**$_3$ = **a** + 2**d** = **a** + (3 **âˆ’** 1)**d**

fourth term: **U**$_4$ = **a** + 3**d** = **a** + (4 **âˆ’** 1)**d**

âˆ´ **n**$_{\text{th}}$ term: **U**$_\mathbf{n}$ = **a** + (**n** – 1)**d**

Finally, the **n**$_{\text{th}}$ term of an AP is **U**$_\mathbf{n}$ = **a** + (**n âˆ’** 1)**d**

Note: The above formula gives the general rule for finding the terms of an AP.

Worked Example 7

The 5th term of an AP is **âˆ’**15 and the second term is 0. Find
(i) the first term.
(ii) the common difference.

SOLUTION
Let the first term and the common difference of the AP be a and d.
U_n = a + (n âˆ’ 1)d
5th term = U_5 = a + 4d = âˆ’15 .... (i)
2nd term = U_2 = a + d = 0 .... (ii)
i.e. a + 4d = âˆ’15 .... (i) and
a + d = 0 .... (ii)
We now solve for a and d in (i) and (ii) thus:
From (ii), a = âˆ’d .... (iii)
Put (iii) in (i) to have
âˆ’d + 4d = âˆ’15
3d = âˆ’15, d = âˆ’ $^{15}/_{3}$
âˆ´ d = âˆ’5
From (iii), a = âˆ’ (âˆ’5) = 5.
Thus, the first term of the sequence is 5 and the common difference is âˆ’5.

Worked Example 8
If the **n**th term in Worked Example 7 above is **âˆ’**9.5, find the value of **n.**
SOLUTION
U_n = a + (n âˆ’ 1)d where U_n = âˆ’9.5, a = 5
and d = âˆ’5.
âˆ´U_n = 5 + (n âˆ’ 1) âˆ’5 = âˆ’9.5
â‡’ 5 – 5 (n – 1) = âˆ’9.5
â‡’ [5(1 – (n – 1)] = âˆ’9.5
â‡’ 1 – n + 1 = âˆ’_95_. 5_
âˆ’n = âˆ’1.9
n = 1.9
n âˆ¼ 2

Worked Example 9
The first term of an AP is 3 and the last term is 9. Find the number of terms in the progression, if the common difference is $1^1/_2$.
SOLUTION
First term a = 3 and d = 1 _12_
Last term U_n = a + (n **âˆ’** 1)d = 9
3 + (n **âˆ’** 1) 1 _12_ = 9
3 + (n **âˆ’** 1) _ 23_ = 9
6 + 3 (n **âˆ’** 1) = 18
6 + 3n **âˆ’** 3 = 18
3 + 3n = 18
3n = 15
n = _ 13_5_ = 5
Hence, there are 5 terms in the sequence.

(iii) The arithmetic mean
If x, y and z are three consecutive terms of an AP, then the common difference is y **âˆ’** x or z **âˆ’** y. Hence,
2y = x + z
âˆ´ y = _12_ (z + x)
Thus shows that the arithmetic mean of x and z is y (the middle term) of the first and last terms.

Worked Example 10

Insert 6 arithmetic means between 8 and 22.

SOLUTION

Let $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, $\mathbf{x}_4$, $\mathbf{x}_5$ and $\mathbf{x}_6$ be the means.

Therefore, the number of terms in the sequence will be 8. Hence, the sequence will be

8, $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, $\mathbf{x}_4$, $\mathbf{x}_5$, $\mathbf{x}_6$, 22

Let the common difference be **d,** then the 8th term will be **a** + 7**d** = 22. But **a** = 8,

âˆ´ 8 + 7**d** = 22

7**d** = 22 – 8 = 14

7**d** = 14

âˆ´ **d** = 2

The required AP is 8, 10, 12, 14, 16, 18, 20, 22.

Note: From the above example, if **x** and **y** are two numbers and we want to insert **n** means between them, the sequence will have **n** + 2 terms in which y is the (**n** + 2)th term. If **d** is the common difference, it follows that:

y = **x** + (**n** + 1)**d**

y âˆ’ **x** = (**n** + 1)**d**

âˆ´ **d** = <u>**y** âˆ’ **x**</u>

n + 1

The required means are

$$x+\frac{y-x}{n+1}, x+\frac{2(y-x)}{n+1}, x+\frac{3(y-x)}{n+1}, \ldots, x+\frac{n(y-x)}{n+1}$$

Exercise 2

Find the **n**th term (**U**$_n$) for each of the sequences in questions 1–3:

1. 4, 11, 18, 25, â€¦
2. 0, 9, 18, 27, â€¦
3. 2, 4, 6, 8, 10, â€¦
4. Find the 100th term of an AP in which the first term is 17 and the 30th term is $7^1/_3$.
5. Insert 3 arithmetic means between $19^1/_2$ and $29^1/_2$.

Write down the first 4 terms and the 12th term of the sequences in questions 6–10 whose **n**th terms are:

6. $\left(-\frac{1}{3}\right)^{n-1}$
7. $2^n + n^2$
8. $\frac{2n}{n-1}$
9. $n+1$
10. $\frac{n^2+1}{2}$

11. Find the **n**th term of **x,** _32_ **x**, 2**x**, â€¦
12. The 2nd term of an AP is **âˆ’**9 and the 7th term is 21. Find the 15th term of the sequence.
13. The 4th and 7th terms of an AP are 13 and 22. Find the 100th term of the sequence. (WAEC)
14. If the sum of the 8th and 9th terms of an AP is 72 and the 4th term is **âˆ’**6, find the common difference. (JAMB)
15. If the 6th term of an AP is 11 and the first term is 1, find the common difference. (JAMB)
16. The fi fth term of an AP is 9 and the 8th term is 27. Find the 31st term of the sequence. (WAEC)
17. The 6th term of an AP is 11 and the first term is 1. Find the 20th term of the sequence. (WAEC)
18. The 6th term of an AP is 35 and the 13th term is 77. Find the 20th term of the sequence. (WAEC)
19. Find the arithmetic means between **âˆ’**3 and 18.

20. Evaluate the **n**th term of the first **n** terms of the arithmetic sequence 3, 7, 11, 15, â€¦

(iv) Sum of n terms of an arithmetic series

Recall that the general form of an AP is

a, **a**+**d**, **a**+2**d**, â€¦, **a**+(**n âˆ’** 1)**d**

Hence, the sum of the first **n** terms of an AP is

a + (**a**+**d**) + (**a**+2**d**) + â€¦ + **a**+(**n âˆ’** 1)**d**

Let S_n denote the sum of the first **n** terms of the series.

âˆ´ S_n = a + (**a**+**d**) + (**a**+2d) + â€¦

+ **a** + (**n âˆ’** 1)**d** .... (i)

Writing (i) in reverse order, we have:

S_n = a + (**n âˆ’** 1)**d** + **a**+(**n âˆ’** 2)**d** + â€¦ + (**a**+**d**) + **a** .... (ii)

Now adding (i) and (ii), we have:

$$2S_n = \frac{2a+(n-1)d+2a+(n-1)d + \ldots\ldots + 2a+(n-1)d}{n \text{ times}}$$

$$\therefore 2S_n = n(2a+(n-1)d)$$

$$\text{Hence, } S_n = \frac{n}{2}(2a+(n-1)d) \text{ .... (iii)}$$

Since the **n**th term of an AP is **U_n** = **a** + (**n âˆ’** 1)**d**.

Now, denote **U_n** = a + (**n âˆ’** 1)**d** by **l.**

Using this in (iii), we have:

S_n = **$^n/_2$**(**a** + **a** + (**n âˆ’** 1) **d**)

= **$^n/_2$**(**a** + **l**) where **l** = **a** + (**n âˆ’** 1)**d**

Finally, the sum of the first n terms of an arithmetic series is

S_n = $^n/_2$[2a + (n âˆ’ 1)d]

S_n = $^n/_2$ (a + l) where l = a + (n âˆ’ 1)d

Worked Example 11

Calculate the sum of the first 22 terms of the series

3 + 7 + 11 + 15 + â€¦

SOLUTION

Here, a = 3, d = 4 and n = 22

S_n = $^n/_2$ (2a + (n âˆ’ 1)d)

âˆ´S_{22} = $^{22}/_2$(2 x 3 + (22 âˆ’ 1)4)

= 11 (6 + 21 x 4)

= 11 x 90 = 990

S_{22} = 990

Hence, the sum of the first 22 terms of the sequence is 990.

Worked Example 12

Obtain a formula for finding the sum of the first n positive integers.

SOLUTION

The sum of the first positive integers is the series:

S_n = 1 + 2 + 3 + â€¦ + n

This is a series in an AP whose first term is 1 and common difference is 1.

S_n = $^n/_2$ (2 x 1 + (n âˆ’ 1)1) = $^n/_2$ (2 + (n âˆ’ 1))

âˆ´ S_n = n(n + 1) / 2

Worked Example 13

Given that the first two terms of an arithmetic series are **âˆ’**2 and 3. How many terms are needed for the sum to equal 306?

SOLUTION

Let the first term and the common difference of the series be a and d respectively.
Hence, the first term a = âˆ’2 .... (i)
The second term a + d = 3 .... (ii)
Using (i) in (ii): âˆ’2 + d = 3
âˆ´ d = 5
S_n = ⁿ/₂(2a + (n âˆ’ 1)d)
âˆ´ S_n = 306 = ⁿ/₂(2(âˆ’ 2) + (n âˆ’ 1)5)
306 = ⁿ/₂ (âˆ’ 4 + 5n âˆ’ 5)
612 = n(5n – 9)
$5n^2$ – 9n – 612 = 0
(5n + 51)(n – 12) = 0
5n = âˆ’51 or n = 12
âˆ´ n = 12 or âˆ’51/5
Since the number of terms cannot be negative, therefore, we take the value n = 12. Hence, to have the sum equal to 306, we need 12 terms of the series.

Worked Example 14

Find three numbers in an AP whose sum is 3 and product is âˆ’15.

SOLUTION

Let a and d denote the first term and the common difference. Since the numbers are in an AP, the three numbers are (a âˆ’ d), a and (a + d).
Sum of the numbers
= (a âˆ’ d) + a + (a + d) = 3
âˆ´ 3a = 3
a = 1
Product of the numbers:
(a – d) x a x (a + d) = âˆ’15
a(a âˆ’ d)(a + d) = âˆ’15
a(a^2 âˆ’ d^2) = âˆ’15
But a = 1: (1 âˆ’ d^2) = âˆ’15
âˆ’d^2 = âˆ’16
d^2 = 16
âˆ´ d = $\underline{+}$4
The numbers are âˆ’3, 1 and 5.

Exercise 3

1. The sum of three numbers in an AP is 18 and the sum of their squares is 206. Find the numbers.
2. Find the sum of the first 18 terms of the series 3 _21_ , 5 _ 12_ , 7 _ 12_ , â€¦
3. The first term of an arithmetic series is 7, the last is 70 and the sum is 385. Find the terms in the series and the common difference.
4. Find the sum of the first n terms of the series: **âˆ’**1 + (**âˆ’**3) + (**âˆ’**5) + (**âˆ’**7) + â€¦
5. The third term of an AP is 18 and the seventh term is 30. Find the sum of the first 30 terms of the progression.
6. Find the sum of the first n terms of the series:
(a) a + 3b + 2a + 6b + 3a + 9b + â€¦
(b) 5 + 11 + 17 + 23 + â€¦
(c) (3a **âˆ’** 2b) + (4a **âˆ’** 4b) + (5a **âˆ’** 6b) + â€¦
7. Find the sum of the first 32 terms of the following AP: 3 + 10 + 17 + â€¦
8. Find the (a) twelfth term and (b) sum of the first 25 terms of the series: **âˆ’**10 + (**âˆ’**8) + (**âˆ’**6) + â€¦ (WAEC)
9. Find the sum of the first 20 consecutive even numbers.
10. Find the sum of the first 22 odd numbers.

IV. Geometric Sequence or Progression

(i) Definition

If the consecutive terms of a sequence increase or decrease by a common constant value, the sequence is said to be a geometric sequence or geometric progression (GP). That is, if the consecutive terms of a sequence are all in the same (constant) ratio, the terms are said to form a geometric sequence or a GP. For example, the numbers 2, 4, 8, 16, 32,â€¦ are in a GP and the ratio of any pair of consecutive terms is 2.

Note: The common constant of a GP is often denoted by â€˜râ€™ and the first term is denoted by â€˜aâ€™.

(ii) The nth term of a GP

The general form of a GP is **a, ar**, **ar**2, **ar**3, â€¦ From the above,

â— The first term $\mathbf{U_1} = \mathbf{a} = \mathbf{ar}^{1\text{âˆ’}1}$

â— The second term $\mathbf{U_2} = \mathbf{ar} = \mathbf{ar}^{2\text{âˆ’}1}$

â— The third term $\mathbf{U_3} = \mathbf{ar}^2 = \mathbf{ar}^{3\text{âˆ’}1}$

â— The fourth term $\mathbf{U_4} = \mathbf{ar}^3 = \mathbf{ar}^{4\text{âˆ’}1}$

$\mathbf{n}_{th}$ term $\mathbf{U_n} = \mathbf{ar}^{\mathbf{n}\text{âˆ’}1}$

Thus, the $\mathbf{n}_{th}$ term of a GP is

$\mathbf{U_n} = \mathbf{ar}^{\mathbf{n}\text{âˆ’}1}$

where **a** = first term and **r** = common ratio.

Worked Example 15

The first term of a GP is 5 and the common ratio is 4. Find the $\mathbf{n}_{th}$ term of the sequence. Hence, find the 9_{th} term.

SOLUTION

a= 5, **r** = 4

âˆ´ $\mathbf{U_n} = \mathbf{ar}^{\mathbf{n}\text{âˆ’}1} = 5 \times 4^{\mathbf{n}\text{âˆ’}1}$

The 9_{th} term of the sequence $\mathbf{U_9} = 5 \times 4^{9\text{âˆ’}1}$

$= 5 \times 4^8 = 5 \times 65\ 536$

= 327 680

Worked Example 16

Find the $\mathbf{n}_{th}$ terms of each of the following GP.

(a) 6, 18, 54, 162, â€¦

(b) 156, 52, 17 $^1/_3$, â€¦

(c) 20, âˆ’10, 5, âˆ’2$^1/_2$, 1$^1/_4$.â€¦

SOLUTION

(a) 6, 18, 54, 162, â€¦

$$a = 6, r = \frac{18}{6} = \frac{54}{18} = \frac{162}{54} = 3 \therefore r = 3$$

$\mathbf{U_n}$ = ar$^{\mathbf{n}\text{âˆ’}1}$ = 6 x3$^{\mathbf{n}\text{âˆ’}1}$ = 2 x 3 x 3$^{\mathbf{n}\text{âˆ’}1}$ = 2 x 3$^{\mathbf{n}}$

$\mathbf{n}^{th}$ term = 2 x 3$^{\mathbf{n}}$

(b) 156, 52, 17$^1/_3$, â€¦

$$a = 156, r = \frac{52}{156} = \frac{17\frac{1}{3}}{52} = \frac{1}{3}$$

$$U_n = ar^{n-1} = 156 \times \left(\frac{1}{3}\right)^{n-1}$$

$$= 52 \times 3 \times \left(\frac{1}{3}\right)^{n-1}$$

$$= 52 \times \frac{3}{3^{n-1}} = 52 \times 3^{1-(n-1)}$$

$$= 52 \times 3^{2-n} = 52 \times \frac{3^2}{3^n}$$

$$n^{th} \text{ term} = 52 \times 3^{2-n} \text{ or } 52 \times \frac{3^2}{3^n}$$

(c) $20, -10, 5, -2\frac{1}{2}, 1\frac{1}{4}, \dots$

$$a = 20, r = \frac{-10}{20} = \frac{-5}{10} = -\frac{1}{2}$$

$$U_n = ar^{n-1} = 20 \times \left(-\frac{1}{2}\right)^{n-1}$$

$$= 4 \times 5 \times \left(-\frac{1}{2}\right)^{n-1}$$

$$= 5 \times 2^2 \times \frac{(-1)^{n-1}}{2^{n-1}}$$

$$= 5 \times 2^{2-n+1}(-1)^{n-1}$$

$$= 5 \times 2^{3-n}(-1)^{n-1}$$

$$= (-1)^{n-1} 5 \times \frac{2^3}{2^n}$$

Hence, the n^{th} term $= (-1)^{n-1} \times 5 \times 2^{3-n}$ or

$$= (-1)^{n-1} 5 \times \frac{2^3}{2^n}$$

Worked Example 17

Three numbers are in a G.P such that their sum is 28 and their product is 512. What are the numbers?

SOLUTION

Let the numbers be $^a/_r$, a, ar.

Then, sum : $^a/_r$ + a + ar = 28 .... (i) and

product: $^a/_r$. a . ar = 512 .... (ii)

â^´ a^3 = 512 hence,

a = 3â^š512 = 3â^š8^3 = (8^3) $_{-}^{1}{}_{-}$ ^{3} = 8^3 Â°Ã¸ _ 13_ = 8

â^´ a = 8

Put a = 8 in (i) to have

$^8/_r$ + 8 + 8r = 28

8 + 8r + $8r^2$ = 28r (multiply through by r)

$8r^2$ – 20r + 8 = 0 (collect the like terms)

$2r^2$ – 5r + 2 = 0 (divide through by 4)

$2r^2$ – 4r – r + 2 = 0

2r(r – 2) – 1(r – 2) = 0

(2r – 1)(r – 2) = 0

â^´r = $^1/_2$ or 2

Hence, the numbers are $^8/_2$, 8, 8 x 2, i.e. 4,

8, 16.

(iii) The geometric mean

Considering the numbers 4, 8 and 16, we can see that 8 = â^š4 x â^š16 = â^š64 . Similarly, for the sequence 24, 18, $^{27}/_2$,

18 = â^š24 x â^š$^{27}/_2$ = â^š12 x â^š27 = â^š324

From the two examples above, if three numbers x, y, z are in a GP, the middle term y is the positive root of the product of the fi rst and the last term. That is, if x, y, z are in a GP then, r (the common ratio) = $^{y}/_{x}$ = $^{z}/_{y}$

â‡’y^2 = xz and

y = $\underline{+}$âˆšxz

Here, y is called the geometric mean of x and z. It follows that the geometric mean of two numbers is the positive square root of the product of the numbers.

Worked Example 18

Find the geometric mean of 4 and 36.

SOLUTION

Geometric mean = âˆš4 x âˆš36 = âˆš144 = âˆš12^2 = 12.

To insert a given number of geometric means n between numbers a and b to have (n + 2) terms GP in which a is the first term and b is the (n + 2)th term, let r be the common ratio then,

b= the (n + 2)th term = ar^{n+1}

i.e. b = ar^{n+1}

âˆ´r^{n+1} = $^{b}/_{a}$ or r = $(^{b}/_{a})^{1/n+1}$ = n+1âˆš$^{b}/_{a}$

Therefore, the required geometric means

are ar, ar^2, ar^3, â€¦, ar^n.

Worked Example 19

Insert 2 geometric means between 54 and 16.

SOLUTION

We are to find 4 terms in a GP in which 54 is the first term and 16 is the last term (4_{th} term).

Hence, the 4_{th} term: U_4 = ar^3

i.e. 16 = $54r^3$

r^3 = $^{16}/_{54}$ = $^{8}/_{27}$ = $(^{2}/_{3})^3$

Hence, the required GP is 54, 54($^{2}/_{3}$), 54$(^{2}/_{3})^2$, 16. That is, 54, 36, 24, 16.

Exercise 4

1. The product of three numbers in a GP is 1. Their sum is $^{âˆ’7}/_{3}$. Find the numbers.
2. Find the 9_{th} and n_{th} term of a GP whose 3_{rd} and 5_{th} terms are âˆ’1 and âˆ’81 respectively.
3. How many terms are there in the GP: 2, **âˆ’**6, 18, â€¦(**âˆ’**486).
4. Find the $\mathbf{n}_{th}$ term of the G.P: 16, 24, 36, 54, 81, â€¦
5. The $\mathbf{p}_{th}$, $\mathbf{q}_{th}$ and $\mathbf{r}_{th}$ terms of an arithmetic sequence are in a GP. Show that the common ratio is $^{\mathbf{q-r}}/_{\mathbf{p-q}}$ or $^{\mathbf{p}}$**âˆ’**$^{\mathbf{q}}/_{\mathbf{q-r}}$
6. Find three geometric means between 5 and 80.
7. Find the $\mathbf{n}_{th}$ term of the sequence $^{1}/_{2,}$ $^{1}/_{3,}$ $^{2}/_{9}$ â€¦.
8. The first and the last terms of a GP are 1 and 81 âˆš3 . Find the common ratio of the sequence given that the third term of the sequence is 3. Find also, the number of terms in the sequence.
9. The 3_{rd} and 6_{th} terms of a GP are 54 and 1 458. Find the 9_{th} term of the sequence.
10. Find the values of **x** and **y,** if the numbers 2, **x, y,** 250 are in a GP.

(iv) The sum of a finite geometric series

Consider a finite geometric sequence whose first term is â€˜**a**â€™ and common ratio â€˜**r**â€™ as shown below:

a, **ar**, **ar**^{2}, â€¦, **ar**$^{\mathbf{n}}$**âˆ’**^{1}

Now, denote by $\mathbf{S_n}$ the sum of the **n** terms of the GP such that

$\mathbf{S_n}$ **= a + ar + ar**^{2} + â€¦. + **ar**$^{\mathbf{n}}$**âˆ’**^{2} + **ar**$^{\mathbf{n}}$**âˆ’**^{1} ....(i)

Now, multiply through by â€œ**r**â€ to have

$rS_n = ar + ar^2 + ar^3 +$ â€¦. $+ ar^{n-1} + ar^n$ ....(ii)

Subtract (ii) from (i):

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r} \ \ldots\ldots\ldots\ldots\ (iii)$$

Here, both numerator and denominator are positive when r < 1. However, they are negative when r > 1. More conveniently, we can write (iii) as $S_n = \frac{a(r^n-1)}{r-1}$. Hence,

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ when } r<1$$

$$\frac{a(r^n-1)}{r-1}, \text{ when } r>1$$

Worked Example 20

Find the sum of the first 10 terms of the sequence 4, 20, 100, 500, â€¦

SOLUTION

4, 20, 100, 500, â€¦

Here, $a = 4, r = \frac{20}{4} = \frac{100}{20} = \frac{500}{100} = 5$

Since $r > 1$, we use $\frac{a(r^n-1)}{r-1}$

$$\therefore S_{10} = \frac{4(5^{10}-1)}{5-1} = \frac{4(5^{10}-1)}{4} = 5^{10} - 1$$

= 9 765 625 âˆ’ 1 = 9 765 624

Worked Example 21

Find the sum of the first eight terms of the series: _2 1_ + _3 1_ + _9 2_ + â€¦ to 2 decimal places.

SOLUTION

$$\frac{1}{2}+\frac{1}{3}+\frac{2}{9}+\dots$$

$a=\frac{1}{2}, r=\frac{2}{3}$. Here, $r<1$ so we use

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$=\frac{\frac{1}{2}\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}}$$

$$=\frac{\frac{1}{2}\left(1-\left(\frac{2}{3}\right)^8\right)}{1-\frac{2}{3}}, n=8$$

$$=\frac{\frac{1}{2}\left(1-\left(\frac{2}{3}\right)^8\right)}{\frac{1}{3}}$$

$$=\frac{1}{2}\times\frac{3}{1}\left(1-\frac{2^8}{3^8}\right)$$

$$=\frac{3}{2}\left(1-\frac{256}{6\,561}\right)$$

$$=\frac{3}{2}\left(\frac{6\,305}{6\,561}\right)=\frac{18\,915}{13\,122}\approx 1.44\ (2.d.p)$$

Worked Example 22

The first and last terms of a geometric series are 2 and 2 048, respectively. The sum of the series is 2 730. Find the number of terms and the common ratio.

SOLUTION

First term a = 2

Last term $U_n = ar^{n-1} = 2\ 048$

i.e. $2r^{n-1} = 2\ 048$

$$\therefore r^{n-1} = 1\,024 \Rightarrow r^n = 1\,024r$$

$$S_n = \frac{a(1-r^n)}{1-r} = 2\,730.$$

$$\therefore 2\,730 = \frac{2(1-1\,024r)}{1-r}$$

$$1\,365 = \frac{(1-1\,024r)}{1-r}$$

$$1 - 1\,024r = 1\,365\,(1-r)$$

$$1 - 1\,024r = 1\,365 - 1\,365r$$

$$1\,365r - 1\,024r = 1\,364$$

$$341r = 1\,364$$

$$\therefore r = 4 = \text{common ratio}$$

But $r^n = 1\,024r\ (r = 4)$

$$4^n = 1\,024 \times 4 = 4^5 \times 4$$

$$4^n = 4^6$$

$$\therefore n = 6$$

Finally, the common ratio of the sequence is 4 (r = 4) and there are 6 terms in the sequence (n = 6).

Exercise 5

1. If 3 âˆ’ x, 6, 7 âˆ’ 5x is a GP. Find two possible values for
   (a) x
   (b) the common ratio
   (c) The sum of the GP
2. The first and second terms of an exponential sequence (GP) are respectively the first and third terms of linear sequence (AP) The fourth term of the linear sequence is 10 and the sum of the first five terms is 60. Find
   (a) The First five terms of the linear sequence and the sum of their First n terms.
   (b) The Sum $\mathbf{S_n}$ of the first **n** terms of the e**x**ponential sequence.
   (c) The limit $\mathbf{S_n}$ find large value of **n.**
3. The sum of the first five terms of a linear sequence (AP) is 26 and that of the ne**x**t five terms is 74. Find the values of
   (a) The First term
   (b) The Common difference
4. The Fifth term of an e**x**ponential sequence (GP) is greater than the Fourth term by $13^1/_2$ and the Fourth term is greater than Third term b**y** 9. Find (i) The Common ratio (ii) The First term
5. How man**y** terms are there in the series 3 + 6 + 12 â€¦â€¦â€¦
6. The Fourth term of an **A.**P 37 and the 6_{th} term is 12 more than the fourth term. Find the first and seventh term.
7. The First and last terms of an AP are 1 and 121 respectivel**y.** Find
   i. The number of terms in **A.**P
   ii. The common difference between term of the sum of its term is
      a. 549
      b. 671
      c. 976
      d. 1281
8. The first second and last terms of a GP are 162, **âˆ’**108, **âˆ’**$21^1/_3$ respectivel**y**. Calculate the number of terms in the GP

9. The Sum to infinity of a GP is 60. If the first term of the series is 12. Find the second term.
10. The 2nd and 5th term of a GP are **âˆ’**7 and 56 respectively. Find the
    (a) The common ratio
    (b) The First term
    (c) The sum of the first five terms
11. The sum of the 6th and 8th terms of AP is 142. If the 4th is 49. Calculate
    (a) the first term
    (b) the common difference
    (c) the sum of the first seven term of progression
12. In AP the first term is 2 the sum of the first and sixth term is $16^1/_2$, what is the 4th term.
13. The sum of the first 9 terms of an AP is 72 and the sum of the next 4 terms 1571, find the AP
14. If the second and fourth terms of a GP are 8 and 32 respectively, what is the sum of the first four terms
15. The third term of a Geometrical progression (GP) is 360 and the sixth term is 1215. Find the
    (a) Common ratio
    (b) The first term
    (c) The sum of the four terms

(v) The infinite geometric series

Consider the geometric series: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{52} +$ â€¦ with common ratio r= $-^1/_2$, the sum of the first n terms of the series is

$$S_n = \frac{1-\left(-\frac{1}{2}\right)^n}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{1-\left(-\frac{1}{2}\right)^n}{\frac{3}{2}}$$

$$= \frac{3}{2}\left[1-\left(-\frac{1}{2}\right)^n\right]$$

Hence, the sum of the first two terms is $^1/_2$. The sum of the first 6 terms of the series is $^{21}/_{32}$. From these results, we can see that the more the number of the terms in the series, the more the sum moves closer to $^3/_2$.

(vi) Sum to infinity of a geometric series

For the general geometric series a + ar + ar^2 + â€¦

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a}{1-r} - \frac{a}{1-r}r^n$$

If **âˆ’**1 < r < 1, r_n decreases as n increases and we say that the limiting value of r $_n$ is zero. Hence, as n increases, S_n approaches the limiting value (denoted by Sâˆ) $\frac{a}{1-r}$.

We say that the series converges to the sum $\frac{a}{1-r}$ which is the sum to infinity of the series.

Worked Example 23

Find the sum to infinity of the series

$$3+2+\frac{4}{3}+\frac{8}{9}+\frac{16}{27}+\dots$$

SOLUTION

a = 3, r = $^2/_3$

$$S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{2}{3}} = 3 \div \frac{1}{3} = 9$$

Worked Example 24

To what sum does the following series converge

$$1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots?$$

SOLUTION

Here a = 1, r =$^{-1}/_3$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{3}\right)} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

Worked Example 25

Express 0.777 recurring as a fraction.

SOLUTION

0.777 recurring = $^7/_{10}$ + $^7/_{100}$ + $^7/_{1000}$ + $^7/_{10000}$ + â€¦ which gives an infinite geometric series whose first term is $^7/_{10}$ with common ratio $^1/_{10}$. Therefore, 0.777 recurring =

$$S_\infty = \frac{\frac{7}{10}}{1-\frac{1}{10}} = \frac{7}{9}$$

V. Some Other Sequences

(i) Harmonic progression (HP)

Numbers p, q, r are said to be in HP, if

$$\frac{p}{r} = \frac{p-q}{q-r}.$$

Note: The reciprocal of numbers in HP are in AP. This is because by the definition of HP, if p, q, r are in HP, it follows that

$$\frac{p}{r} = \frac{p-q}{q-r}$$

$$\Rightarrow p\,(q-r) = r(p-q)$$

$$pq - pr = rp - rq$$

Divide through by pqr to have:

$$\frac{1}{r} - \frac{1}{q} = \frac{1}{q} - \frac{1}{p}$$

which shows that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ is in AP.

(ii) Harmonic mean

To find the harmonic mean between given numbers, suppose p and q are two numbers and m their harmonic mean, then $^1/_p$, $^1/_m$, $^1/_q$, are in AP.

$$\therefore \frac{1}{m}-\frac{1}{p}=\frac{1}{q}-\frac{1}{m}$$

$$\text{i.e.} \frac{2}{m}=\frac{1}{p}+\frac{1}{q}$$

$$2pq=m(p+q)$$

$$\therefore m=\frac{2pq}{p+q}$$

Note: Let A, G, H denote arithmetic, geometric and harmonic means between p and q. We have shown that A = $\frac{p+q}{2}$,

$G=\sqrt{pq}$ and $H=\frac{2pq}{p+q}$.

$$\therefore A\times H=\frac{p+q}{2}\times\frac{2pq}{p+q}=pq=G^2$$

Worked Example 26

The 15th term of a HP is $^1/_6$ and the 20th term of an AP is $^{23}/_3$. Find the series.

SOLUTION

Let the first term be â€˜**a**â€™ and the common difference of the corresponding AP be **d.**

Then, 6 = the 15th term of the corresponding AP

$6=a+14d$ .... (i) and

$\frac{23}{3}=20^{th}$ term of the AP.

$\frac{23}{3}=a+19d$ .... (ii)

Solving (i) and (ii), it follows that the

AP is $\frac{4}{3},\frac{5}{3},\frac{2}{1},\frac{7}{3},\ldots$ and the

HP is $\frac{3}{4},\frac{3}{5},\frac{1}{2},\frac{2}{7},\ldots$

SUMMARY

In this chapter, we have learnt the following:

ℜ• A sequence is a set of numbers that occur in some definite order.

ℜ• The **n**$_{th}$ term of a sequence is denoted by **U**$_n$.

ℜ• If **U**$_1$, **U**$_2$, **U**$_3$, â€¦, **U**$_n$ are terms of a sequence and if these terms are added together, we have **U**$_1$ + **U**$_2$ + **U**$_3$ + â€¦ +**U**$_n$. This expression is called a series.

ℜ• If the consecutive terms of a sequence increase or decrease by a constant number called the difference, the sequence is said to be an arithmetic sequence or AP.

ℜ• The general form of an AP is **a, a**+**d**, **a**+2**d**, â€¦, **a**+(**n** âˆ’ 1)**d.**

ℜ• The **n**$_{th}$ term of an AP is **U**$_n$ = **a** + (**n** âˆ’ 1)**d.**

ℜ• The sum of **n** terms of an AP is **S**$_n$ = _ **n**2_ (2**a** + (**n** âˆ’ 1)**d**) or **S**$_n$ = _ **n**2_ (**a** + **l**) where **l** = **a** + (**n** âˆ’ 1)**d.**

ℜ• Arithmetic means of **x** and **y** is given as $^{x+y}/_2$

ℜ• To insert a given number of geometric means between two numbers a and b, we use the formula $r = \sqrt[n+1]{\frac{a}{b}}$. Therefore, the required geometric means are ar, ar2, ar3, â€¦, arn.

ℜ• The nth term of a GP is **Un** = **arnâˆ’1**. The sum of the first **n** terms of a GP is

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r<1$$

$$S_n = \frac{a(r^n-1)}{r-1}, \quad r>1$$

ℜ• The sum to infinity of an infinite geometric series is **S**âˆž = **a**/1 âˆ’ **r**

ℜ• If **p, q, r** are in HP, then

$$\frac{p}{r} = \frac{p-q}{q-r}.$$

ℜ• The reciprocal of numbers in HP are in AP.

ℜ• $A = \frac{p+q}{2}, \quad G = \sqrt{pq}, \quad H = \frac{2pq}{p+q}$ and $A \times H = pq = G^2$.

GRADUATED EXERCISES

1. The **n**th term of a sequence is given by (3)2**n** âˆ’ 1. Write down the first three terms of the sequence given that **n** â‰¥ 0. (WAEC)
2. A sequence is given by **Un** = **n**(1 âˆ’ 2**n**).
   (a) Determine the first 3 terms of the sequence.
   (b) If **Un** = âˆ’120, find the value of **n.** (WAEC)
3. The 5th term of an AP is 9 and 8th term is 27. Find the 6th term. (WAEC)
4. Three positive numbers are in AP. The sum of the squares of the three numbers is 155 while the sum of the numbers is 21. If the common difference is positive, find the numbers. (WAEC)
5. The 4th and 9th terms of an AP are âˆ’3 and 12 respectively. Find the
   (a) common difference
   (b) 5th term
   (c) number of terms which will give a sum of 135.
6. If 3, x, y, 18, are in AP, find the values of x and y. (WAEC)

Evaluate the following

7.
   (a) $\sum_{r=1}^{10} r^2$
   (b) $\sum_{r=3}^{11} 2^r$

8.
   (a) $\sum_{r=3}^{n} r(r+3)$
   (b) $\sum_{r=3}^{9} r(r+qr)$

9. The sum of the first n terms of a series is given by S_n = n^2 âˆ’ 2n for all values of n, find the r_{th} term of the series.
10. Find the first six terms of the sequence defined by U1 = 0, U2 = 2 and Ur = Ur âˆ’ 1 – Urâˆ’2 for

r â‰¥ 2. Hence, evaluate $\sum_{r=1}^{6} U_r$.

Evaluate

11. **S**$_7$ for the series 1 + 3 + 6 + 9 + â€¦
12. **S**$_5$ for the series 3 + 9 + 27 + 81 + â€¦
13. Find the three numbers in an AP whose sum is 21 and whose product is 315.
14. Find the sum of the first **n** terms of the series (âˆ’1) + (âˆ’3) +(âˆ’5) + (âˆ’7) + â€¦
15. The third term of an AP is 18 while the 7th term is 30. Find the first sum of the first 33 terms of the sequence.