Chapter 3

Chapter 3

Indices and Logarithms

OBJECTIVES

At the end of this chapter, students should be able to:

1. show the basic laws of logarithms.
2. state and prove the laws of logarithms.
3. apply the laws in solving problems.
4. revise the use of logarithm tables for calculations.

I. Revision on the Basic Rules of Indices

(i) Positive Integer Indices

If x and y are positive integers and a and b are non-zero numbers,

(a) $a^x \times a^y = a^{x+y}$

(b) $a^x \div a^y = a^{x-y}$

(c) $a^x \times a^y \div a^z = a^{x+y-z}$

(d) $(a^x)^y = a^{xy}$

(e) $a^x \times b^x \times c^x = (abc)^x$

(ii) Zero, Negative and Fractional Indices

$$a^x \div a^x = a^{x-x} = a^0$$

$$\text{but } a^x \div a^x = 1$$

$$\therefore \quad a^0 = 1$$

$$\left(\frac{a}{b}\right)^{-x} = \frac{1}{\left(\frac{a}{b}\right)^x} = \left(\frac{b}{a}\right)^x$$

$a^{\frac{x}{y}}$ → Power; } Index; → Root; → Base

$\therefore\ a^{\frac{x}{y}} = (\sqrt[y]{a})^x$ or $a^{\frac{x}{y}} = \left((a)^{\frac{1}{y}}\right)^{\frac{x}{1}}$

For example,

$$16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 2 \times 2 \times 2 = 8$$

Aliter, $16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{\frac{4}{1} \times \frac{3}{4}} = 2^3 = 8$

(iii) Equations Involving Indices

Note that if $a^x = a^y$

$\Rightarrow \quad x = y$

Worked Example 1

If $27^{x+1} = 9^{2x+1}$, find the value of x.

SOLUTION

$$3^{3(x+1)} = 3^{2(2x+1)}$$

$$\Rightarrow \quad 3^{3x+3} = 3^{4x+2}$$

$$\Rightarrow \quad 3x + 3 = 4x + 2$$

$$\Rightarrow \quad 3x - 4x = 2 - 3$$

$$\Rightarrow \quad -x = -1$$

$$\therefore \quad x = 1$$

Worked Example 2

If $\frac{9^{x+1}}{3^{-x}} = \frac{27^{3x-1}}{81^{x-1}}$, find x.

SOLUTION

$$\frac{3^{2(x+1)}}{3^{-x}} = \frac{3^{3(3x-1)}}{3^{4(x-1)}}$$

$$\Rightarrow \quad \frac{3^{2(x+1)}}{3^{-x}} = \frac{3^{9x-1}}{3^{4x-1}}$$

$$\Rightarrow \quad 3^{2x+2-(-x)} = 3^{(9x-3)-(4x-4)}$$

$$\Rightarrow \quad 3^{2x+2+x} = 3^{9x-3-4x-4}$$

$$\Rightarrow \quad 3^{3x+2} = 3^{5x+1}$$

$$\Rightarrow \quad 3x + 2 = 5x + 1 \quad \text{(equating the indices)}$$

$$\Rightarrow \quad 3x - 5x = 1 - 2$$

$$\Rightarrow \quad -2x = -1$$

$$\Rightarrow \quad 2x = 1$$

$$\therefore \quad x = \frac{1}{2}$$

Exponential equations can also be reduced to quadratic form as shown in Worked Examples 3 and 4.

Worked Example 3

Solve the equation $3^{2x} - 8(3^x) - 9 = 0$.

SOLUTION

$$3^{2x} - 8(3^x) - 9 = 0$$

$$\Rightarrow \quad (3^x)^2 - 8(3^x) - 9 = 0$$

Now, let $P = 3x$

$$\Rightarrow \quad P^2 - 8p - 9 = 0$$

$$\Rightarrow \quad (P + 1)\,(P - 9) = 0$$

$$\therefore \quad P = -1 \text{ or } P = 9$$

But $3x = -1$ is impossible

$$\text{and } 3^x = 9$$

$$3x = 3^2$$

$$\therefore \quad x = 2$$

Therefore, the only solution is 2.

Worked Example 4

Solve the equation $7^{2x+1} - 50(7^x) + 7 = 0$

SOLUTION

$$7^{2x+1} - 50(7^x) + 7 = 0$$

$$\Rightarrow \quad (7^x)^2 \times 7 - 50\,(7^x) + 7 = 0$$

Now, let $P = 7^x$

$$\Rightarrow \qquad 7P^2 - 50P + 7 = 0$$

$$\Rightarrow \qquad (7P - 1)(P - 7) = 0$$

$$\therefore \quad 7P - 1 = 0 \text{ or } P - 7 = 0$$

$$7P = 1 \quad P = 7$$

$$P = \frac{1}{7}$$

Then, $7^x = \frac{1}{7}$ or $7^x = 7^1$

$7^x = 7^{-1}$ or $x = 1$

$\therefore \quad x = -1$ or $x = 1$

Exercise 1

1. Simplify $(27^{\frac{1}{3}})^2$.
2. Given that $3^y = 243$. Find the value of y.
3. Simplify $\frac{9^{-\frac{1}{4}}}{27^{\frac{1}{3}}}$.
4. Simplify $125^{-\frac{1}{3}} \times 49^{-\frac{1}{2}} \times 10^0$.
5. If $3^{2x} = 27$, find the value of x.
6. If $9^{2x+1} = \frac{81^{x-2}}{3^x}$, find the value of x.
7. Simplify $\frac{4^{-\frac{1}{2}} \times 16^{\frac{3}{4}}}{4^{\frac{1}{2}}}$.
8. Given that $3 \times 9^{1+x} = 27^{-x}$. Find x.
9. Simplify $56x^{-4} \div 14x^{-8}$.
10. Simplify $\left(\frac{4}{25}\right)^{-\frac{1}{2}} \times 2^4 \div \left(\frac{15}{2}\right)^{-2}$.
11. Simplify $\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \sqrt{\frac{100}{81}}$.
12. Simplify $\frac{1}{3^{5n}} \times 9^{n+1} \times 27^{n-1}$.
13. Solve $\frac{1}{81^{(x-2)}} = 27^{(1+x)}$.
14. Evaluate $\left(\frac{1}{343}\right)^{-\frac{1}{3}} + (64)^{\frac{1}{3}} - \left(\frac{4}{9}\right)^{-\frac{1}{2}}$. (NECO)
15. Given that $9^{2x-1} \times 3^{3x+1} = 27^{x+3}$. Find the value of x.
16. Given that $\frac{8^{x+1}}{2^{-x}} = \frac{16^{2x-1}}{4^{x+1}}$. Find the value of x.

17. If $\frac{125^{x-1}}{25^{3-x}} = \frac{5^{5x-1}}{25^{2x-1}}$, find the value of x.

18. $3^{2x} + 6(3^x) - 27 = 0$

19. $3^{2x+1} - 12(3^x) + 9 = 0$

20. $3^{2y} + 3^{y+1} = 18$

II. Logarithms

(I) Definition of Logarithm

Logarithm of the number '*a*' to the base '*b*', written as $\log_b a$, is the index to which '*b*' is raised to obtain '*a*'. Symbolically, if $\log_b a = c$ (say), it implies $b^c = a$ ('*b*' raised to power of '*c*' is equal to '*a*'). Logarithms can be written to any base. Examples include:

1) $\log_{10}100 = 2$ since $10^2 = 10 \times 10 = 100$
2) $\log_2 64 = 6$ since $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
3) $\log_5 625 = 4$ since $5^4 = 5 \times 5 \times 5 \times 5 = 625$

On the basis of the fact that $\log_b a = c \Rightarrow b^c = a$, we can now deduce the following:

a) $\log_b 1 = 0$ because $b^0 = 1$. (log 1 to any given base '*b*' (say) is zero)
b) $\log_a a = 1$ because $a^1 = a$.
c) $\log_c a = \log_c b$, if and only if $a = b$.

(II) Power Rule of Logarithm

For any positive number *a*, any logarithm base *c* and any real number *x*, $\log_c a^n = n\log_c a$. In particular, $\log_a a^n = n\log_a a = n$

Worked Example 5

Evaluate the following:

(a) $\log_2 32$ (b) $\log_{\frac{1}{4}} \frac{1}{16}$

(c) $\log_{0.5} 0.125$

(d) $\log_a a^3$

(5) $\log_5 5^{2x}$

SOLUTION

(a) Let $\log_2 32 = x$

$\Rightarrow \quad 2x = 32$

$\Rightarrow \quad 2x = 2^5$

$\therefore$ $x = 5$ (equate the indices since the bases are equal)

Aliter,

$$\log_2 32 = \log_2 2^5 = 5\log_2 2 = 5$$

(b) Let $\log_{\frac{1}{4}} \frac{1}{16} = x$

$$\Rightarrow \quad \left(\frac{1}{4}\right)^x = \frac{1}{16}$$

$$\Rightarrow \quad \left(\left(\frac{1}{2}\right)^2\right)^x = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{2x} = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \quad 2x = 4$$

$$\therefore \quad x = 2$$

Aliter,

$$\log_{\frac{1}{4}}\left(\frac{1}{16}\right) = \log_{\frac{1}{4}}\left(\frac{1}{4}\right)^2 = 2\log_{\frac{1}{4}}\left(\frac{1}{4}\right) = 2$$

(c) Let $\log_{0.5} 0.125 = x$

$$\Rightarrow \quad (0.5)^x = 0.125$$

$$\Rightarrow \quad \left(\frac{5}{10}\right)^x = \frac{125}{1000}$$

$$\Rightarrow \quad \left(\frac{5}{10}\right)^x = \frac{5^3}{10^3}$$

$$\Rightarrow \quad \left(\frac{5}{10}\right)^x = \left(\frac{5}{10}\right)^3$$

$$\therefore \quad x = 3$$

Aliter,

$$\log_{0.5} 0.125 = \log_{\frac{5}{10}}\left(\frac{125}{1000}\right) = \log_{\frac{5}{10}}\left(\frac{5}{10}\right)^3$$

$$= 3\log_{\frac{5}{10}}\left(\frac{5}{10}\right) = 3$$

(d) Let $\log_a a^3 = x$

$$\Rightarrow \quad a^x = a^3$$

$$\therefore \quad x = 3$$

Hence, $\log_a a^3 = 3$

Aliter,

$$\log_a a^3 = 3\log_a a = 3$$

(e) Let $\log_5 5^{2x} = P$

$$\Rightarrow \quad 5^P = 5^{2x}$$

$$\therefore \quad P = 2x$$

Hence, $\log_5 5^{2x} = 2x$

Aliter,

$$\log_5 5^{2x} = 2x\log_5 5 = 2x$$

Worked Example 6

a) Express the equation $\log_x (5x - 6) = 2$ in index form.

b) Hence, solve for x.

SOLUTION

(a) $\log_x (5x - 6) = 2$

$x^2 = 5x - 6$ (from definition)

(b) $x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

Hence, $x = 2$ or $x = 3$

Exercise 2

Evaluate the following:

1. $\log_2 16$
2. $\log_5 125$
3. $\log_4 1024$
4. $\log_{10} 10000$
5. $\log_3 243$
6. $\log_7 343$
7. $\log_{10} \frac{1}{10}$
8. $\log_2 \frac{1}{8}$
9. $\log_7 \frac{1}{49}$
10. $\log_2 16^{\frac{3}{4}}$
11. $\log_{\frac{2}{5}} \frac{4}{25}$
12. $\log_9 \frac{1}{81}$
13. $\log_4 8$
14. $\log_{16} 0.25$
15. $\log_{1.5} 3.375$
16. $\log_{10} 0.0001$
17. $\log_3 81^{\frac{1}{2}}$

Express the following equations into index form, and solve for x.

18. $\log_x (6 - x) = 2$
19. $\log_x (10x - 25) = 2$
20. $\log_x (5x + 14) = 2$

III. Product Rule of Logarithm

The product rule of logarithm states that *the logarithm of the product of AB (say) is the sum of their logarithm.*

That is, $\log_C (AB) = \log_C (A) + \log_C (B)$

Proof:

Let $\log_C A = P$ and $\log_C B = Q$ where $C_P > 1$ and $C \neq 0$

Then, $C^P = A$ and $C^Q = B$

$\Rightarrow AB = C^P \times C^Q = C^{P+Q}$ (*product of law of indices*)

$\therefore \log_C AB = P + Q$

$= \log_C A + \log_C B$

Worked Example 7

Evaluate $\log_{10} 4 + \log_{10} 25$.

SOLUTION

$$\log_{10} 4 + \log_{10} 25 = \log_{10}(4 \times 25)$$
$$= \log_{10}(4 \times 25)$$
$$= \log_{10} 100 = \log_{10} 10^2 = 2\log_{10} 10$$
$$= 2 \times 1 = 2 \text{ (since } \log_a a = 1)$$

Worked Example 8

Given that $\log_{10} 5 = 0.6990$ and $\log_7 10 = 0.8451$. Find $\log_{10} 35$.

SOLUTION

$$\log_{10} 4 + \log_{10} 25 = \log_{10}(4 \times 25)$$
$$= \log_{10}(4 \times 25)$$
$$= \log_{10} 100 = \log_{10} 10^2 = 2\log_{10} 10$$
$$= 2 \times 1 = 2 \text{ (since } \log_a a = 1)$$

Worked Example 8

Given that $\log_{10} 5 = 0.6990$ and $\log_7 10 = 0.8451$. Find $\log_{10} 35$.

SOLUTION

$$\log_{10} 35 = \log_{10}(5 \times 7)$$
$$= \log_{10} 5 + \log_{10} 7$$
$$= 0.6990 + 0.8451$$
$$= 1.5441$$

Exercise 3

1. Show that $\log_y PT = \log_y P + \log_y T$.

Simplify the following:

2. $\log_{10} 40 + \log_{10} 25$
3. $\log_{10} \sqrt{25} + \log_{10} \sqrt{16}$
4. $\log_{10} \frac{1}{4} + \log_{10} 8 + \log_{10} \frac{1}{2}$
5. $\log_{10} \frac{16}{5} + \log_{10} \frac{65}{16} + \log_{10} \frac{100}{130}$

Find the value(s) of x;

6. If $\log_{10} x^1 + \log_{10} x^2 + \log_{10} x^3 = 14$.
7. Given that $\log_{10}(3x+1) + \log_{10}(x+5) = \log_{10}(3x^2+4x-19)$.
8. If $\log_7(2x-3) + \log_7(x-4) = 1$.

where $\log_{10} 2 = 0.3010, \log_{10} 3 = 0.4771, \log_{10} 5 = 0.6990, \log_{10} 7 = 0.8451$

Evaluate the following.

9. $\log_{10} 210$
10. $\log 252 + \log 9$
11. $\log_{10} 4\frac{1}{2}$
12. $\log_{10} 1.25 + \log_{10} 0.75$
13. $\log_{10} 125^3$
14. $\log_{10} 14 + \log_{10} 35$
15. $\log_{10} 2\frac{1}{2} - \log_{10} 1\frac{1}{4}$

IV. Quotient Rule of Logarithm

The quotient rule of logarithm states that the logarithm of the quotient of _ AB_ is the subtraction of the logarithm of the denominator from the logarithm of the numerator.

It is written as $\log_C\left(\frac{A}{B}\right) = \log_C A - \log_C B$

where $C > 0$ and $C \neq 0$.

Proof:

Let $\log_C A = P$ and $\log_C B = Q$

$\Rightarrow \quad C^P = A$ and $C^Q = B$

$\Rightarrow \quad \frac{A}{B} = \frac{C^P}{C^Q} = C^{P-Q}$

$\therefore \log_C\left(\frac{A}{B}\right) = P - Q \log_C A - \log_C B$

Worked Example 9

Simplify $\log_3 12 - \log_3 4$.

SOLUTION

$$\log_3\left(\frac{12}{4}\right) = \log_3 3$$
$$= 1$$

Worked Example 10

Express $1 - \log_{10} 5$ as a single logarithm.

SOLUTION

$$\begin{aligned} &1 - \log_{10} 5 \\ &= \log_{10} 10 - \log_{10} 5 \\ &= \log_{10}\left(\frac{10}{5}\right) = \log_{10} 2 \end{aligned}$$

Worked Example 11

Given that $\log_{10}(5x + 1) - \log_{10}(3x - 2) = 1$. Find the value of x.

SOLUTION

$$\begin{aligned} 5x + 1 &= 10(3x - 2) \\ 5x + 1 &= 30x - 20 \\ -25x &= -21 \\ 25x &= 21 \\ x &= \frac{21}{25} \end{aligned}$$

Exercise 4

1. (a) Prove that $\log_c\left(\frac{A}{B}\right) = \log_c A - \log_c B$.

   (b) Express each of the following in terms of sum and differences of log:

   (i) $\log_a \frac{xy^4}{z^3}$

   (ii) $\log_a \sqrt{\frac{a^2b^3}{c^4}}$

   (iii) $\log_c \frac{xy^3}{a^2b^4}$

Simplify the following:

2. $\log_4 8 - \log_4 2$
3. $\log_5 80 - \log_5 8 - \log_5 2$
4. $\log_{10} \frac{4070}{243} - \log_{10} \frac{5}{9^2}$
5. $2\log_3\left(\frac{15}{4}\right) - \log_3 \frac{5}{8} \log_3 \frac{5}{6}$

Find the value(s) of x in the following:

6. $\log_{10} x^5 - \log_{10} x^3 = 2$

7. $\log_2 8^{2x} - \log_2 128 = 0$

8. $\log_{10}(5x+1) - \log_{10}(x-3) = 1$

9. $\log_{10}(2x+1) - \log_{10}(3x-2) = 1$

If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, $\log_{10} 5 = 0.6990$ and $\log_{10} 7 = 0.8451$, find the values of the following without using tables:

10. $2\log_{10} 4\frac{2}{3}$

11. $\log_{10} 75 - \log_{10} 8 - \log_{10} 14$

V. Change of Base of Logarithms

$\log_A P = x$

$\Rightarrow \quad A^x = P$ (from definition of logarithm)

Then,

$\log_C A^x = \log_C P$ (taking the log of both sides)

$\Rightarrow x \log_C A = \log_C P$ (using the power rule)

$\therefore \quad x = \dfrac{\log_C P}{\log_C A}$ (dividing both sides by $\log_C A$)

$\therefore \quad \log_A P = \dfrac{\log_C P}{\log_C A}$ *

where A, C and P are positive numbers.

Also, $\log_C A = \dfrac{\log_A A}{\log_A C}$

$= \dfrac{1}{\log_A C}$ (since $\log_A A = 1$)*

Worked Example 12

Evaluate $\log_3 125 \times \log_5 81$.

SOLUTION

$$\log_3 125 \times \log_5 81$$

$$\log_3 5^3 \times \log_5 3^4$$

$$= 3\log_3 5 \times 4\log_5 3$$

$$= 3 \times 4 \times \log_3 5 \times \log_5 3$$

$$= 12 \times \log_3 5 \times \frac{1}{\log_3 5}$$

$$\left(\text{recall } \log_B A = \frac{1}{\log_A B}\right)$$

$$= 12 \times \frac{\log_3 5}{\log_3 15}$$

$$= 12 \times 1$$

$$= 12$$

Worked Example 13

Calculate the values of x given that $\log_9 (8x - 15) = 4 \log_{81} x$.

SOLUTION

$$\Rightarrow \quad \frac{\log_3 (8x - 15)}{\log_3 9} = \frac{4\log_3 x}{\log_3 81}$$

$$\Rightarrow \quad \frac{\log_3 (8x - 15)}{\log_3 3^2} = \frac{4\log_3 x}{\log_3 3^4}$$

$$\Rightarrow \quad \frac{\log_3 (8x - 15)}{2\log_3 3} = \frac{4\log_3 x}{4\log_3 3}$$

$$\Rightarrow \quad \frac{\log_3 (8x - 15)}{2} = \log_3 x$$

$$\Rightarrow \quad \log_3 (8x - 15) = \log_3 x^2$$

$$\Rightarrow \quad 8x - 15 = x^2$$

$$\Rightarrow \quad x^2 - 8x + 15 = 0$$

$$\Rightarrow \quad (x - 3)(x - 5) = 0$$

$$\Rightarrow \quad x = 3 \text{ or } 5$$

Note

1. $\log_C AB \neq (\log_C A)(\log_C B)$; the log of a product is not the product of the log.
2. $\log_C (A + B) \neq \log_C A + \log_C B$; the log of a sum is not the sum of the log.
3. $\log_C \left(\frac{A}{B}\right) \neq \frac{\log_C A}{\log_C B}$; the log of a quotient is not the quotient of the log.
4. $(\log_C A)^x \neq x \log_C A$; the power of the log is not the exponent times the log.

Exercise 5

1. If $A = \log_y z$, $B = \log_z x$, $C = \log_x y$, show that $A \times B \times C = 1$.
2. Prove that $\log_B A = \frac{1}{\log_A B}$.
3. Solve the following equation leaving the answers in index form:

$$5\log_x 7 = \log_7 x$$

4. Solve for x in the following equation

$$\log_8 y + \log_2 x = 4$$

5. Solve $(\log_{27} x)^2 - \log_3 x + 2 = 0$

VI. Use of Logarithm Tables

Worked Example 14

Evaluate the following using log tables:

(a) $(45.34)^2$ (b) $(0.3054)^3$

(c) $\sqrt{0.8144}$ (d) $0.00376^{\frac{2}{3}}$

SOLUTION

(a) $\log (45.34)^2$

$$= 2\log(45.34)$$
$$= 2 \times 1.6565$$
$$= 3.313$$

$$\therefore (45.34)^2 = \text{antilog of } 3.313$$
$$= 2.056 \times 10^3$$
$$= 2\,056$$

Aliter

$\log (45.34)^2$

Number	log
$(45.34)^2$	1.6565
	$\times$ 2
	3.3130
2 056.0	2056
	+ 0
	2056

(b) $\log (0.3054)^3$

$$= 3\log (0.3054)$$

$$= 3 \times \bar{1}.4849 \text{ (from tables)}$$

$$= \bar{2}.4544$$

$$\therefore (0.3054)^3 = \text{antilog of } \bar{2}.4544$$

$$= 2.847 \times 10^{-2}$$

$$= 0.02847$$

Allter

Number	log
(0.3054)	$\bar{1}.4849$
	$\times\ 3$
	$\bar{2}.4447$
	2780
0.02784	+ 4
	2784

(c) $\log\sqrt{0.8144} = \log (0.8144)^{\frac{1}{2}}$

$$= \frac{1}{2}\log 0.8144$$

$$= \frac{1}{2} \times \bar{1}.9108 \text{ (from tables)}$$

$$= \bar{2} + 1.9108$$

$$= \bar{1}.9554$$

$$\therefore (0.8114)^{\frac{1}{2}} = \text{antilog of } \bar{1}.9554$$

$$= 9.024 \times 10^{-1}$$

$$= 0.9024$$

Allter

Number	log
$(0.8144)^{\frac{1}{2}}$	$\bar{1}.9108 \div 2$
	$\frac{-2 + 1.9108}{2}$
	$\bar{1}.9554$
	9016
0.9024	+ 8
	9024

(d) $\log (0.0376)^{\frac{2}{3}}$

$$= \frac{2}{3} \log (0.0376)$$

$$= \frac{2}{3} \times \bar{2}.5752$$

Next, we divide by 3 and multiply the result by 2.

$$\bar{2}.5752 \div 3$$

$$\bar{3} + 1.5752 \div 3$$

$$= \bar{1}.5251$$

$$= \bar{1}.5251 \times 2$$

$$= \bar{1}.0502$$

$\therefore (0.0376)^{\frac{2}{3}} = \text{antilog of } \bar{1}.0502$

$$= 1.122 \times 10^{-1}$$

$$= 0.1122$$

Aliter

Number	log
$(0.0376)^{\frac{2}{3}}$	$\bar{2}.5752 \times \frac{2}{3}$
	$\frac{\bar{3}.1504}{3}$
	$\bar{1}.0501$
	1122
0.1122	+ 0
	1122

Worked Example 15

Use log tables to evaluate, correct to 2 decimal places

$$\left(\sqrt[5]{\frac{0.364 \times 0.0489}{3.96}}\right)^3$$

SOLUTION

Number	log
0.364	$\bar{1}.5611$
0.0489	$\bar{2}.6893$ +
0.364×0.0489	$\bar{2}.2504$
3.96	0.5977 −
	$\bar{3}.6527$
$\sqrt{\dfrac{0.364 \times 0.0489}{3.96}}$	$\dfrac{\bar{3}.6527}{5}$
	$= \dfrac{\bar{5} + 2.6527}{5}$
	$= \bar{1}.5305$

$\left(\sqrt{\dfrac{0.364 \times 0.489}{3.96}}\right)^3$	$\bar{1}.5305$
	$\times \quad 3$
	$\bar{2}.5915$
$\left(\sqrt{\dfrac{0.364 \times 0.489}{3.96}}\right)^3$	$=$ antilog $\bar{2}.5915$
	$= 3.904 \times 10^{-2}$
	$= 0.03904$
	$= 0.04$ (2 d.p.)

Exercise 6

Use logarithm tables to evaluate the following, correct to 3 significant figures:

1. $(143.1)^2$
2. $(0.0764)^{\frac{1}{3}}$
3. $(0.00467)^{\frac{2}{3}}$
4. $\dfrac{12.15^3 \times \sqrt{0.3413}}{161.5}$ (WAEC)
5. $\dfrac{846.2^2 \times \sqrt{0.05436}}{462.4^{\frac{1}{3}}}$ (NECO)
6. $\left(\dfrac{(14.34)^2 \times \sqrt{0.44}}{0.143}\right)^3$ (NECO)
7. $(3.14)2 \times (0.145)^{\frac{2}{3}} \times (0.0069)^{\frac{1}{4}}$
8. $\sqrt{\dfrac{13.69 \times 164}{(145)^{\frac{1}{3}}}}$
9. $\left(\dfrac{36.14 \times 0.691}{14.64 \times 14.34}\right)^{\frac{2}{3}}$
10. $\left(\dfrac{0.336 \times 14.34}{14479}\right)^{\frac{-2}{5}}$
11. $\dfrac{\sqrt{1.34} \times (36.3)^3}{\sqrt{0.145}}$
12. $\left(\dfrac{69.4 \times 6.39 \times 0.67}{16.34 \times 3.11}\right)^{\frac{-2}{3}}$
13. $\sqrt{\left(\dfrac{16.34 \times 17.36}{0.0075}\right)^3}$
14. $\left(\dfrac{1.34}{3.64}\right)^2 \times \left(\dfrac{6.345}{8.162}\right)^3$
15. $\sqrt{\dfrac{(0.345)^3 \times (1.369)^2}{(1.314)^5}}$

SUMMARY

In this chapter, we learnt the following:

- Laws of positive indices

  (a) $a^x + a^y = a^{x+y}$

  (b) $a^x \div a^y = a^{x-y}$

  (c) $(a^x)^y = a^{xy}$

  (d) $a^x + b^x \times c^x = (abc)^x$

  (e) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

  (f) If $a^x = a^y$ then $x = y$

- Laws of negative indices

  (a) $a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x$

  (b) $ab^{-x} = \frac{a}{b^x}$

  (c) $(ab)^{-x} = \frac{1}{(ab)^x} = \left(\frac{1}{ab}\right)^x$

  (d) $\frac{1}{a^{-x}} = \left(\frac{a}{1}\right)^x$

  (e) $\frac{a}{b^{-x}} = a(b)^x$

- Fractional indices

  (a) $a^{\frac{1}{x}} = (\sqrt[x]{a})^1$

  (b) $a^{\frac{x}{y}} = (\sqrt[y]{a})^x$

  (c) $a = \left(\frac{1}{a}\right)^{\frac{x}{y}} = \left(\sqrt[y]{\frac{1}{a}}\right)^x$

- Logarithm

  Given that A and B are positive integers, where $C > 0$ and $C \neq 1$, then

  (a) $\log_c A + \log_c B = \log_c AB$

  (b) $\log_c \left(\frac{A}{B}\right) = \log_c A - \log_c B$

  (c) $\log_c A^x = x \log_c A$, where x is any real number.

  (d) $\log_c A = \log_c B$, if and only if $A = B$.

  (e) $\log_c C = 1$

  (f) $\log_c 1 = 0$

  (g) $\log_B A = \frac{\log_c A}{\log_c B}$ (where C is any positive real number $\neq 0$)

  (h) $\log_B A = \frac{1}{\log_A B}$

GRADUATED EXERCISES

1. Simplify $\sqrt[3]{(3^9 \times 5^3)}$.

2. $32^{\frac{-3}{5}} \div 64^{\frac{-7}{3}}$

3. (a) Solve the equation $3^{x+1} \div 9^{-1} = 1$.

   (b) Solve the equation $\frac{9^{2x-3}}{3^{x+3}} = 1$.

4. If $8^{x+1} = \frac{1}{4}$, find x.

5. Solve the simultaneous equations $5^{6x+y} = 1$ and $2^{2y-3x} = \frac{1}{32}$.

6. Evaluate $\log_{10}6 + \log_{10}45 - \log_{10}27$ without using logarithm tables.

7. Solve $3\log a + 5\log a - 6\log a = \log 64$.

8. Find x, given that $6\log(x+4) = \log 64$.

9. Simplify $\dfrac{\log\sqrt{8}}{\log 8}$.

10. Using logarithm tables, evaluate

    $\dfrac{\sqrt[3]{1.376}}{\sqrt[5]{0.007}}$, correct to 3 significant figures.

11. If $\log_{10}(2x+1)-\log_{10}(3x-2)=1$, find x.

12. Simplify the following without using logarithm tables:

    $\log_{10}\left(\frac{30}{16}\right)-2\log_{10}\left(\frac{5}{9}\right)+\log_{10}\left(\frac{400}{243}\right)$

13. Evaluate

    $\log_{10}\sqrt{35}+\log_{10}\sqrt{2}-\log_{10}\sqrt{7}$.

14. (a) Given that $\log_{10}2 = 0.3010$, $\log_{10}7 = 0.8451$ and $\log_{10}5 = 0.6990$. Evaluate the following without using logarithm tables:

    (i) $\log_{10}35$ (ii) $\log_{10}2.8$

    (b) Given that $N^{0.8942}=2.8$, use your result in (a.ii) to find the value of N.

15. Evaluate

    $\log_5\left(\frac{3}{5}\right)+3\log_5\left(\frac{15}{2}\right)-\log_5\left(\frac{81}{8}\right)$.

16. Evaluate

    $\frac{1}{2}\log_{10}\left(\frac{25}{4}\right)-2\log_{10}\left(\frac{4}{5}\right)+\log_{10}\frac{320}{125}$.

17. Simplify

    $\log_{10}\sqrt{25}-\log_{10}\sqrt{4}+\log_{10}\sqrt{16}$.

18. Evaluate, without log tables,

$$\frac{\log 8}{\log 12 - \log 3}.$$ (WAEC)

19. If $\log_{10}(3x - 1) - \log 2 = 3$, find the value of x. (WAEC)

20. Using tables evaluate $\log_5 6$.

21. Evaluate

$$\frac{1}{2}\log_{10}25 - \frac{1}{3}\log_{10}64 + \frac{2}{3}\log_{10}8.$$

22. Evaluate

$$\log_5 81 \times \log_9 25.$$

23. Simplify

$$\frac{2\log 8 + \log 4 - \log 16}{\log 32}.$$

(WAEC)

24. Simplify

$$\frac{1}{2}\log_{10}25 - 2\log_{10}3 + \log_{10}18.$$

(WAEC)