2 LINEAR AND PROJECTILES MOTION, SPEED, VELOCITY, ACCELERATION AND PROJECTILE MOTION

LINEAR MOTION

OBJECTIVES

At the end of the topic, students should be able to:

- use the concept of velocity as a vector to find the resultant velocity of objects with one or two changes in direction;
- analyse motion of objects with constant velocity, uniform acceleration and variable (changing) acceleration using velocity – time graph;
- use the velocity-time graph to find distance travelled between two time intervals.
- use the velocity-time graph for a body moving at constant acceleration from an initial velocity to obtain three equations of linear motions; and
- explain the variables or terms of the equations of linear motion and identify the correct equation to use for each practical problem.

Velocity–time graph for variable acceleration

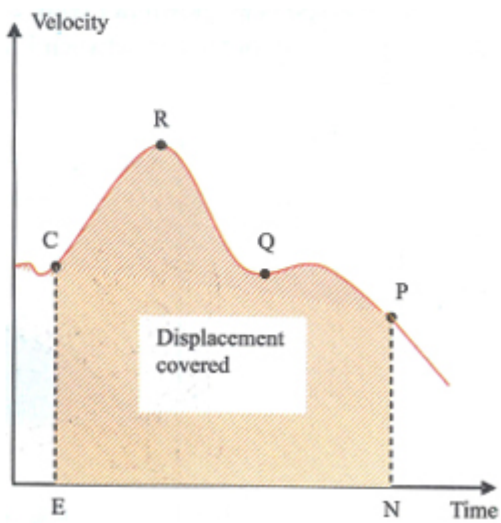


Figure 2.1: V – t graph for a variable acceleration

It is possible for the acceleration of a body to change between two points. The increase or decrease in velocity varies between the points for the same time interval. The velocity-time graph is a curve or a combination of a curve and a straight line. This is illustrated in Figure 2.1.

The velocity varies along the curve and the acceleration at different points C, R, Q and P on the curve varies. The acceleration at each point C, R, Q and P can be obtained by evaluating the slope at the respective points. The distance or displacement covered in the time interval EN is the area under the curve between C and P.

Equations of linear motion with constant acceleration

One of the four equations of motion listed below describes the motion of a body moving with a constant or uniform acceleration.

$$s = \tfrac{1}{2}(u+v)t............i$$

$$v = u + at................ii$$

$$s = ut + \tfrac{1}{2}at^2...........iii$$

$$v^2 = u^2 + 2as..........iv$$

These equations involve only five variables as follows: **s, u, v, a, t.**

***s* = distance or displacement covered**
***u* = initial velocity (velocity at t = 0)**
***v* = final velocity (velocity at time t)**
***a* = constant or uniform acceleration**
***t* = time to change velocity from u to v**

How to derive equations of linear motion

A car passes two towns A and B travelling at constant acceleration (a), in a time (t) seconds, the car changes its initial velocity (u) at E to a final velocity (v) at C. If the distance between the towns is (s), then the velocity – time graph is as shown in Figure 2.2.

From Figure 2.2 below, the average velocity of the car between E and C is given by:

$$\text{Average velocity} = \frac{u+v}{2} = \frac{1}{2}(u+v)$$

$$\text{Average velocity} = \frac{\text{total distance covered}}{\text{time}}$$

$$\text{Average velocity} = \frac{s}{t}$$

$$\therefore \frac{s}{t} = \frac{1}{2}(u+v)$$

$$s = \frac{1}{2}(u+v)t.......i$$

The total distance travelled (s) is the area of the trapezium OECN.

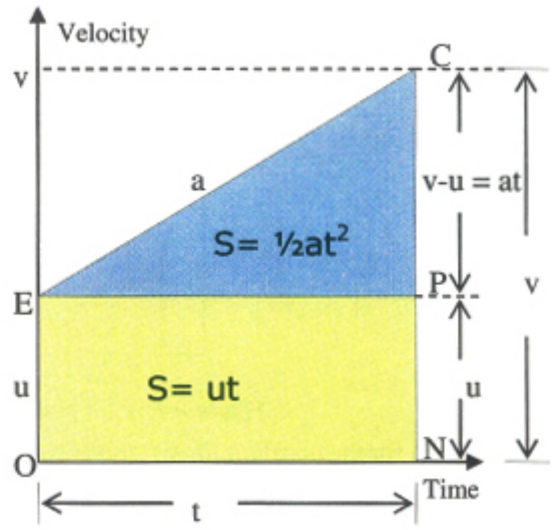


Figure 2.2: v-t graph of a body moving with constant acceleration

The car increases its velocity from (u) to (v) in t seconds; its acceleration during this time is given by;

$$\text{Acceleration} = \frac{\text{increase in velocity}}{\text{time}}$$

$$a = \frac{v-u}{t} \quad \Rightarrow v - u = at$$

$$v = u + at \text{.........}ii$$

The total distance travelled by the car from E to C is the area of the trapezium OECN. This area is divided into two parts; the area of triangle ECP and the area of rectangle OEPN.

Total distance travelled = Area of rectangle OEPN + Area of triangle ECP

$$s = ut + \frac{1}{2}at^2 \text{ ....}iii$$

Eliminating (t) in equations (i) and (ii), that is $t = \dfrac{v-u}{a}$ *and* $s = \frac{1}{2}(u+v)t$

$$\therefore s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right) \Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$v^2 = u^2 + 2as \text{ ...}iv$$

The equations (i – iv) contain five variables **s, u, v, a, t.**

Each of these equations has four of these variables; the one chosen to solve any problem on motion depends on the variables given and the quantity to be calculated. Table 2.1 is a summary to help you choose the correct equation based on the quantities given and the quantity to be calculated. A tick (✓) indicates quantity given in the problem while (?) stands for the quantity to be calculated.

Equation	Quantities				
	s	u	v	a	T
$s = \frac{1}{2}(v+u)t$	?	✓	✓	-	✓
$v = u + at$	✓	✓	?	✓	✓
$s = ut + \frac{1}{2}at^2$	?	✓	-	✓	✓
$v^2 = u^2 + 2as$	✓	✓	?	✓	-

Table 2.1: Equations and their quantities

Guide to solving problems on linear motions

â€¢ Write the **s, u, v, a, t** as shown in Table 2.1

â€¢ Identify the quantities given in the problem and select the correct equation that could solve the problem.

â€¢ Substitute into the correct equation and simplify to get the correct answer.

â€¢ Write your calculated answer to required significant figure with the correct unit.

Worked examples

1. A truck starts from rest and moves with a constant acceleration of 5 ms^{-1} for 20 seconds. Calculate;
   (a) Final velocity of the truck;
   (b) The distance it travels after 20 seconds.

Solution

Quantity	s	u	v	a	t
$v = u + at$	-	0	?	5	20

(a) $v = u + at$ = 0 + 5 Ã— 20 = 100 ms^{-1}

(b)

Quantity	s	u	v	a	t
$s = ut + \frac{1}{2}at^2$	?	0	100	5	20

$s = ut + \frac{1}{2}at^2 = 0\times20 + ½ \times 5 \times 20^2 = 1000$ m

Alternative method;

Quantity	s	u	v	a	t
$s = \frac{1}{2}(v+u)t$	?	0	100	-	20

$s = \frac{1}{2}(v+u)t = ½\,(100+0)20 = 1000$ m.

2. A car travelling on an expressway decreases its speed to 12 ms^{-1}in 3 seconds. If the car covers a total distance 60 m during this time, calculate:
   (a) the initial velocity of the car;

(b) the carâ€™s average retardation.

Solution

(a)

Quantity	s	u	v	a	t
$s = \frac{1}{2}(v+u)t$	60	?	12	-	3

$s = \frac{1}{2}(v+u)t$

$60 = ½\,(12 + u)\,3$

$12 + u = \frac{60 \times 2}{3} = 40$

$u = 40 - 12 = 28\ ms^{-1}$

(b)

Quantity	s	u	v	a	t
$v = u + at$	60	28	12	?	3

$v = u + at$

$12 = 28 + 3a \quad \Rightarrow a = -5\ m\ s^{-1}$

Note: Any of the equations of linear motion containing acceleration could be used to solve the problem.

Solution

Graphical method:

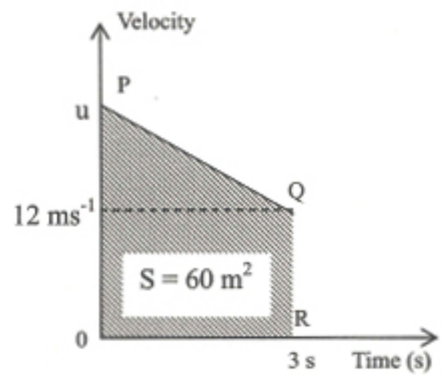
Distance covered = Area of trapezium OPQR.

s = Â½(OP+RQ)OR

60 = Â½2(U + 12) 3

u+12 = 40

u = 40 – 12 = 28 ms^{-1}



Retardation = slope (s) of line PQ

$= \frac{12-28}{3} = \frac{-16}{3} = 5\frac{1}{3}\,ms^{-1}$ or 5.3 m/s

3. A train started from rest and travelled with a constant acceleration of 2 ms^{-2} for 6 seconds. It maintained the velocity reached for 30 seconds and was brought to rest after 5 seconds when the brake was

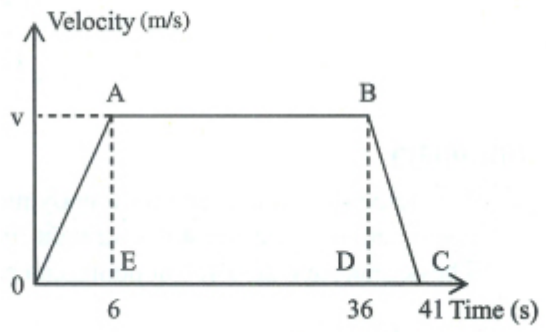
applied. Find for the motion:
(a) the maximum velocity reached;
(b) the total distance travelled;
(c) the average speed for the whole journey
(d) the average retardation.

Graphical solution

(a) Acceleration = slope (s) of line OA

$$Acceleration = \frac{AE}{EO}$$

$$2 = \frac{v}{6} \quad \Rightarrow v = 2 \times 6 = 12\ m\ s^{-1}$$



(b) Total distance travelled = Area of trapezium.

Total distance travelled = ½ (30+41) 12 = 426 m.

(c) average speed $= \frac{\text{Total distance covered}}{\text{time}} = \frac{426}{41} = 10.39 ms^{-1}$

(d) Retardation = slope (s) of line BC

$$= \frac{12}{5} = 2.4\ ms^{-2}$$

Using equations of linear motion

(a)

Quantity	s	u	v	a	t
$v = u + at$	-	0	?	2	6

$v = u + at = 0 + 2\times6 = 12\ ms^{-1}$

(b) Distance travelled during retardation.

Quantity	s	u	v	a	t
$s = \frac{1}{2}(v+u)t$	?	12	0	-	5

$s = \frac{1}{2}(v+u)t$ = ½ (0+12) 5 = 30 m

Quantity	s	u	v	a	t
$s=\frac{1}{2}(v+u)t$	?	0	12	2	6

$s=\frac{1}{2}(v+u)t$ = ½ (12+0) 6 = 36 m

Distance travelled when the train was moving with constant velocity.

S = vÃ—t = 12Ã—30 =360 m

Total distance travelled = 30+36+360 = 426 m.

(c) Average speed= $\frac{\text{Total distance covered}}{\text{time}}$

Average speed = $\frac{426}{41}=10.39\ ms^{-1}$

(d)

	s	u	v	a	t
$v=u+at$	30	12	0	?	5

$$v=u+at$$
$$0=12+5a$$

$$a=-\frac{12}{5}=-2.4ms^{-2}$$

Summary

â€¢ The velocity – time graph for a body moving with a variable acceleration is a curve or a combination of a curve and a straight line depending on the nature of the motion.

â€¢ The acceleration at different points can be obtained by evaluating the slope at these points.

• $s=\frac{1}{2}(u+v)t$.......i

$v=u+at$.........ii

$s=ut+\frac{1}{2}at^2$ iii

$v^2=u^2+2as$ iv

These equations involve only five variables as follows: **s, u, v, a, t.**

s = distance or displacement covered

u = initial velocity (velocity at t = 0)

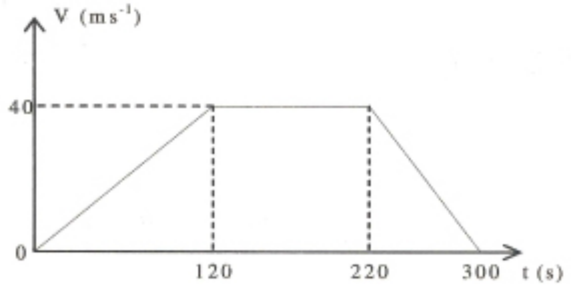
v = final velocity (velocity at time t)

a = constant or uniform acceleration

t = time to change velocity from v to u

Practice question 2a

1. The diagram below represents the velocity – time graph of a bus beginning its motion from rest. Use the graph to find:



(a) the distance covered by the bus when travelling at a constant velocity.

(b) the acceleration and retardation of the bus.

(c) the total distance travelled.

(d) the average speed of the bus during the entire journey.

2. Deji Aliyu won the 100m race in a record time of 9.95 seconds at the 2003 all African games in Abuja.

   (a) Calculate his average velocity for the race.

   (b) If he takes off from rest and he reaches a constant speed of v ms^{-1} in 6.5 seconds and braced the tape in 9.95 seconds. Find:

   i) his maximum speed;

   ii) his acceleration before reaching the maximum speed.

3. An athlete took off with a speed of 8 ms^{-1} in a 200 m race and attained a maximum speed of 12 ms^{-1} in 8 seconds. If he continued with the speed till he won the race, calculate the time he braced the tape.

4. A cyclist passed two police posts 1200 m apart. If his speed at the first police post is 20 ms^{-1} and the speed at the second police post is 35 ms^{-1} calculate:

   (a) the time it takes the cyclist to pass the two police posts.

   (b) his acceleration between the two police posts.

5. A particle, moving in a straight line with a uniform acceleration, has velocities 20 ms^{-1},40 ms^{-1} and 80 ms^{-1} points A, B and C respectively. If the distance between B and C is 100m find:

   (a) the acceleration of the particle;

   (b) the distance covered between A and B;

   (c) the time taken to cover the distance of A from C. **F-math WASSCE.**

6. A train initially travelling at a speed of 10ms^{-1}, increases its speed to 25 ms^{-1} in 20 seconds. Calculate the acceleration and the distance covered during the same time.

7. (a) Explain the terms:

   i) Uniform velocity and uniform acceleration;

   ii) Average speed.

   (b) Use a velocity – time graph to show how to get uniform acceleration and the total distance travelled.

(c) A car slows down from an initial velocity of 50 ms^{-1} to rest in 2 minutes.

i) Sketch the velocity – time graph of the car as it slows down.

ii) Find the deceleration of the car.

iii) What is the distance travelled by the car before it comes to rest?

MOTION UNDER GRAVITY

OBJECTIVES

At the end of the topic, students should be able to:

- identify projectile motion and give examples of projectiles;
- explain range, maximum height, time of flight and derive their formulae;
- state the applications of projectiles in warfare and sports;
- solve simple problems on projectiles.

All objects, if allowed to fall under the influence of gravity alone, fall with a constant or uniform acceleration called **gravity or acceleration due to gravity.** The magnitude of acceleration due to gravity is about 9.8 ms^{-2} or 10 ms^{-2}. If an object falls towards the earth, its acceleration of free fall is taken as positive because it is in the same direction as gravity.

Vertical and horizontal motion of objects under gravity

All objects released inside the earthâ€™s gravity fall to the earth with a constant acceleration.

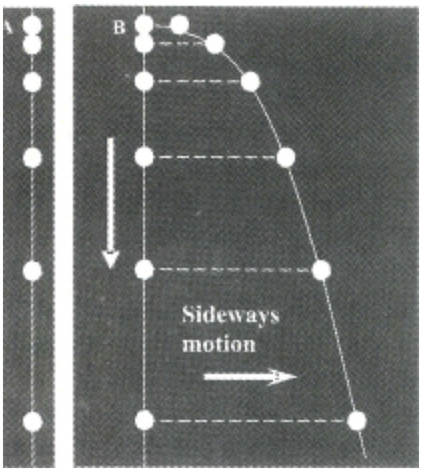


Figure 2.3: Vertical and horizontal motion of a ball given a horizontal velocity (u)

If the object is released from rest, it falls vertically downwards and if it is given a horizontal push with a velocity (u), it falls vertically downwards and at the same time moves sideways as shown in Figure 2.3.

A close inspection of the motion shows that:

- â€¢ the balls A and B fall at constant rate vertically towards the earth and reach the ground at the same time.
- â€¢ the vertical or downward motions of the balls depend on gravity (g).
- â€¢ the ball B falls vertically and moves horizontally. The vertical motion depends on gravity alone while the horizontal motion depends on the initial velocity alone. The horizontal velocity is constant as long as the ball is still moving.

Therefore, if a body is given a horizontal push with an initial velocity (u), it falls vertically and moves sideways. The two motions are treated separately.

Projectiles

A projectile is an object whose curved motion in space is affected or influenced by gravity alone.

An object thrown vertically or obliquely (at an angle) into space such that the effect of air resistance on it is negligible is a projectile. A feather is not a projectile because gravity and air resistance influence its motion in space.

Examples of projectiles

i) A missiles launched from a rocket;

ii) A bomb dropped from a moving war plane

iii) Arrow or bullet shot into space;

iv) A stone shot from a catapult;

v) A basket or football kicked into space.

Projectile motion

Projectile motion is a plane-curved motion of an object thrown vertically or obliquely into space and moving with constant acceleration

Vertical projection

An object thrown up vertically, has maximum initial velocity at the time of projection. As the projectile gains height, gravity opposes its motion until the velocity is zero (0) at the maximum height. The projectile stops for a short time, changes direction and falls back to the earth. Figure 2.4 shows the displacement – time and velocity – time graphs.

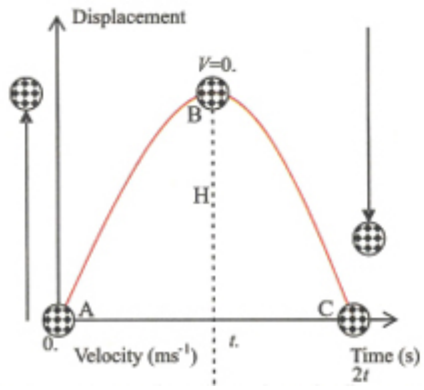


Figure 2.4a: Ball projected vertically upward

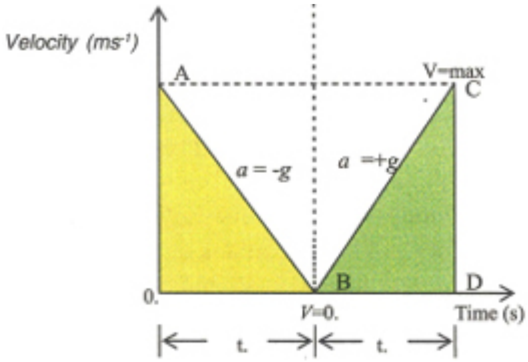


Figure 2.4b: Displacement – time and velocity – time graphs of an object projected vertically upwards

Displacement is a vector, its direction is as important as its magnitude. When the ball is moving upward its direction is positive and when it is moving downward, the direction is negative. *The total displacement is zero when the ball returns to the same level where it was projected but the total distance travelled is twice the maximum height.*

The ball has a maximum velocity at A when it was projected and at C when it returns to the same level or plane of projection. The velocity is zero at B the maximum height (the greatest height the ball can reach).

Maximum height

Maximum height (H) is the greatest vertical displacement of the projectile from the plane of projection.

â€¢ The maximum height (H) is the area of triangle OAB or area of triangle BCD.

â€¢ The total distance covered by the projectile (ball) during its motion is the sum of the areas of the triangles OAB and BCD.

â€¢ The acceleration due to gravity is the slope of the velocity – time graph. The slope AB of triangle OAB is negative (– g) because the ball is moving upward or against gravity. The slope BC of triangle BCD is positive (+ g) since the ball is moving downward in the same direction as gravity.

Object dropped from a height: An object dropped from a height (H) above the ground, has a zero initial velocity. As it falls with constant acceleration, its velocity increases from zero to maximum just before it hits the ground.

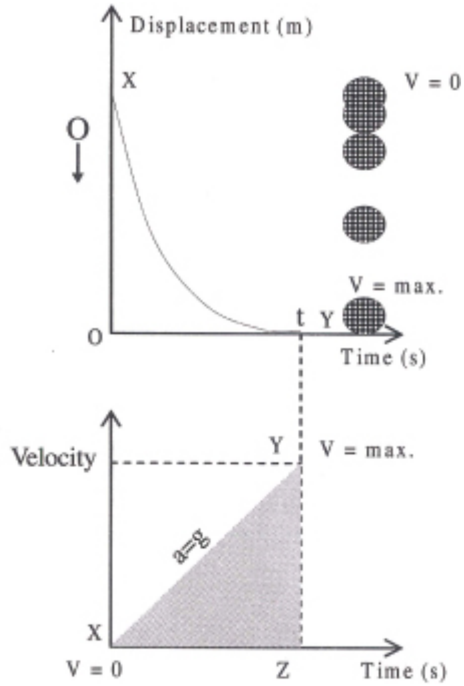


Figure 2.5: Displacement – time and Velocity – time graphs of an object dropped from a height

Maximum height for vertical projection

s *=H =maximum height*

u *= velocity of projection*

v *= velocity at maximum height*

u *= -g = acceleration*

t *= time of flight (T)*

$v^2 = u^2 + 2as$

$0 = u^2 - 2gH$

$$H = \frac{u^2}{2g}$$

Time of flight (T): The total time the projectile remains in flight is the time of flight (T).

Time of flight is the time it takes the projectile to move from the point of projection to the point where it hits the ground

When the projectile hits the same plane from which it was projected, the time to reach the maximum height (H) is equal to the time (t) to return to the same plane of projection.

At the maximum height, s=H, u=u, v=0, a= – g and t (time to the maximum height) is given by

$v = u + at$

$0 = u - gt$

$$t = \frac{u}{g}$$

Time of flight (T) =2t

$$T = \frac{2u}{g}$$

Note: These equations are not applicable if the projectile lands somewhere above or below the plane of projection.

Horizontal projection

Figure 2.6 shows a ball projected horizontally with an initial velocity (u). The ball falls vertically and sideways. The vertical distance (H) is covered as the ball falls under the influence of gravity alone while the horizontal distance or the range (R) depends on the initial velocity (u).

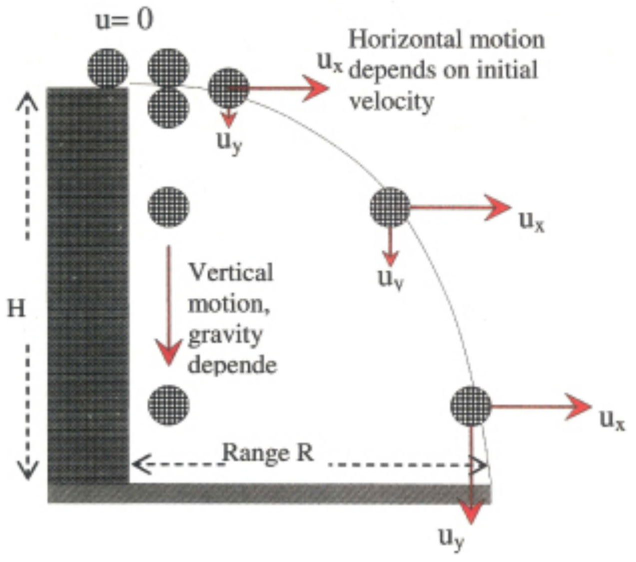


Figure 2.6

The Range (R) is the horizontal distance covered by the projectile from the point of projection to the point of landing.

Range is the horizontal displacement of the projectile from the point where it is projected to the point where it returns to the plane of projection.

The range (R) is given by:

R = uT

$T = \frac{u}{g}$ = time of flight.

$$R = UT = U\frac{U}{g} = \frac{U^2}{g} \quad \therefore R = \frac{u^2}{g}$$

Worked examples

1. An object is thrown vertically upwards with a velocity of 20 ms^{-1}. Calculate:

(i) the maximum height reached;
(ii) the time to reach the maximum height;
(iii) the time of flight. ($g = 10\ ms^{-2}$)

Solution

(i) $v^2 = u^2 - 2gs$

$0 = 20^2 - 2\times10\times H$

$$H = \frac{20^2}{20} = 20\ m$$

or

$$H = \frac{u^2}{2g} = \frac{20^2}{20} = 20\ m$$

(ii) $v = u - gt \Rightarrow 0 = 20 - 10t$

$$t = \frac{20}{10} = 2\ s \text{ or } t = \frac{u}{g} = \frac{20}{10} = 2\ s$$

(iii) Time of flight T = 2t

$T = 2\times2 = 4\ s$

2. A ball is projected vertically with an initial velocity u and returns to the same level 5 seconds later. Find the;
(i) initial velocity of the ball;
(ii) maximum height reached by the ball.
($g = 10ms^{-1}$)

Solution

(a)
$$v = u - at$$
$$0 = u - 10\times2.5$$
$$u = 25\ m\ s^{-1}$$

or

$$T = \frac{2u}{g} \Rightarrow 5 = \frac{2u}{10}$$
$$\therefore u = 25\ m\ s^{-1}$$

(b)
$$v^2 = u^2 - 2gH$$
$$0 = 25 - 2\times10\times H$$
$$\therefore H = \frac{25^2}{20} = 31.25\ m$$

or

$$H = \frac{u^2}{2g} = \frac{25^2}{20} = 31.25\ m$$

3. A stone is dropped from a height 80 m above the ground. Calculate;
(i) the time it takes the stone to hit the ground;
(ii) the velocity of the stone as it hits the ground.($g = 10ms^{-1}$)

Solution

(i) $t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times80}{10}} = \sqrt{16} = 4\ s$

(ii) $v = \sqrt{2gh} = \sqrt{2\times10\times80} = \sqrt{1600}$
$= 40\ m\ s^{-1}$

4. A ball is projected horizontally from the top of a hill with an initial velocity of 20 ms^{-1}. If the height of the hill is 80 m calculate;

(a) the time it takes the ball to hit the ground;

(b) the horizontal distance from the foot of the hill where the ball hits the ground.(g =10ms^{-2})

Solution

(a) $s = \frac{1}{2}gt^2 \Rightarrow 80 = \frac{1}{2}\times 10\times t^2$

$$t^2 = \frac{80\times 2}{10} = 16 \Rightarrow t = \sqrt{16} = 4\ s$$

or

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times 80}{10}} = \sqrt{16} = 4\ s$$

(b) Range $(R) = uT = 20\times 4 = 80\ m$

Projectiles in x - y plane

An object thrown obliquely (at an angle) into space performs two motions which are independent of each other. Suppose a football is kicked at angle (Î¸) on a level field with an initial velocity (u). The path traced by the ball as it moves through the space is as shown in Figure 2.7.

Vertical motion of projectiles

The motion of the projectile depends on the acceleration due to gravity (g). The vertical component of the velocity of ball decreases as it goes higher. The magnitude of the vertical component of the velocity v_y at any time during the motion of the ball is given by:

$$v_y = u_y - gt$$
$$v_y = u\sin - gt$$

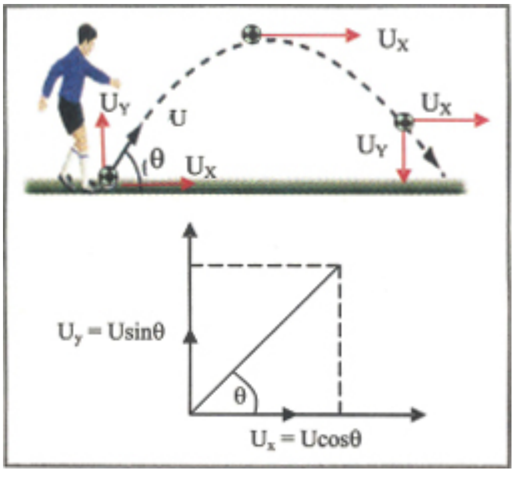


Figure 2.7: Projectile in Ã— – y plane

$s = H$= maximum height attained

$u_y = usin\ Î¸$ = component of u alone the x – axis

$v = v_y$= vertical component of velocity at a time (t)

a = *g* = acceleration due to gravity
t = time
Î¸ = angle of projection

Deriving the formula for projectiles in the x – y plane

Time of flight: The vertical component of the velocity is zero (0) at the maximum height.

$$t = \frac{u\sin}{g} = \frac{u}{g}\sin$$

The time of flight or the time for the projectile to return to the level or plane of projection is twice the time to reach the maximum height.

$$T = 2t \qquad \Rightarrow \qquad T = \frac{2u}{g}\sin$$

The maximum height: vertical component of the velocity is zero at the maximum height. The equation of linear motion connecting v, a, u and s is used to find the formula for the maximum height.

$$v_y^2 = u_y^2 - 2gH = 0$$
$$0 = u^2\sin^2 \quad - 2gH$$
$$H = \frac{u^2}{2g}\sin^2$$

The horizontal motion of projectiles: The horizontal motion of a projectile is constant as long as the projectile remains in space. The horizontal or sideway motion of the projectile depends on the component of the initial velocity along the x – axis. The distance covered by the projectile along the horizontal axis is called the *range.* The range is given by: s = u_xT

R = ucos Î¸Ã—T

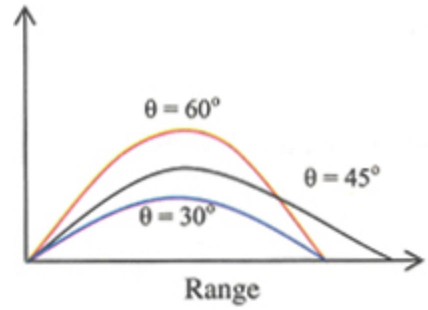
$$But\ T = \frac{2u}{g}\sin$$

$$\therefore R = u\cos \quad \times \frac{2u}{g}\sin$$

$$R = \frac{u^2}{g}\sin 2$$

Note 2sin cos = sin 2

The maximum range (R_{max}): The horizontal displacement or range of a projectile depends on the *initial velocity and the angle (Î¸) through which the projectile is thrown up.* If the initial velocity is kept constant, the range varies according to angle of projection *(Î¸). Maximum range is obtained when the angle of projection (Î¸) is 45Â°.*



$$R = \frac{u^2}{g}\sin 2$$

Maximum range occurs when sin 2Î¸ = 1
Sin 29 = sin 90Â° (1= sin 90Â°)
2Î¸ = 90Â° ⇒ Î¸ = 45Â°

$$R_{max} = \frac{u^2}{g}$$

Worked examples

1. A particle is projected with a velocity of 100 ms^{-1} from the ground level at an angle (Î¸) to the horizontal. If the time of flight of the projectile is 10 seconds, calculate the angle of projection. [g = 10 ms^{-2}]

Solution

$$T = \frac{2u}{g}\sin \quad \Rightarrow \sin \quad = \frac{Tg}{2u}$$

$$\sin \quad = \frac{10\times10}{2\times100} = 0.5000$$

$$= \sin^{-1} \quad = 30^o$$

2. A particle is projected with a velocity of 20 ms^{-1} from the ground level at an angle 50Â°, find the;
   (a) maximum height reached by the particle;
   (b) magnitude and direction of the velocity 0.5 seconds later. = (g=10ms^{-2})

Solution

$$\text{(a)} \quad H = \frac{u^2}{2g}\sin^2 \quad = \frac{20^2}{2\times10}\sin^2 50^o$$

$$H = 20\times0.7660\times0.7660 = 11.74\, m$$

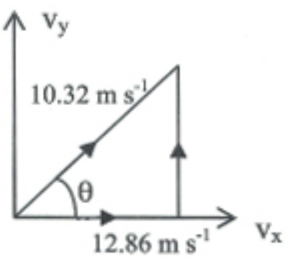
(b) $V_x = u_x = u\cos \quad = 20\cos 50^o = 12.86 ms^{-1}$

$$v_y = u\sin \quad - gt = 20\sin 50^o - 10\times 0.5$$

$$v_y = 15.32 - 5 = 10.32\ ms^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10.32^2 + 12.86^2}$$

$$v = 16.45\ ms^{-1}$$



$$\tan \quad = \frac{v_y}{v_x} = \frac{10.32}{12.86}$$

$$\tan \quad = 0.8025$$

$$= \tan^{-1} 0.8025 = 38.7^o$$

3. An arrow is shot into space with a speed of 125 ms^{-1} at an angle of 15Â° to the level ground. Calculate the:

(a) time of flight

(b) range of the arrow.

Solution

(a) $T = \frac{2u}{g}\sin \quad = \frac{2\times 125}{10}\sin 15^o = 25\sin 15^o$

$$T = 6.47\ seconds$$

(b) $R = \frac{u^2}{g} sin\,2 \quad = \frac{125^2}{10} sin\,(2\times 15)$

$$R = 1562.5 sin\, 30^o = 781.25\ m$$

4. A hunter sitting on a tree branch, 120m above the ground, shot at a bird on another tree and narrowly missed the bird. If the angle of elevation of his gun is 30Â° and the bullet leaves the gun with an initial speed of 50 ms^{-1}, calculate the:

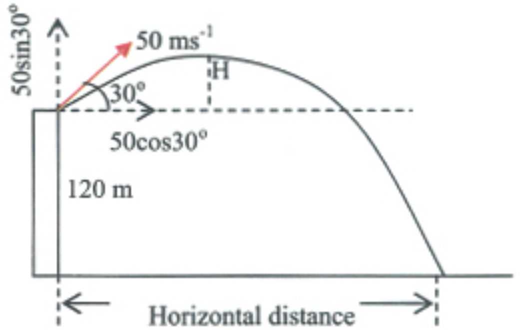
(a) time it takes the bullet to hit the ground;

(b) the distance from the foot of the tree to where the bullet hits the ground;

(c) the greatest vertical distance reached by the bullet. (g = 10 ms^{-2})

Solution

(a) The hunterâ€™s height above the ground is negative because it is above level ground where the bullet hits the ground.



$\therefore -s = u\sin\ \ t - \frac{1}{2}gt^2$

$-120 = 50\sin 30^o t - \frac{1}{2}\times 10t^2$

$-120 = 25t - 5t^2$

$t^2 - 5t - 24 = 0 \Rightarrow (t-8)(t+3) = 0$

t = 8 seconds since time cannot be negative.

(b) Horizontal distance = u cos Î¸t

= 50cos 300 x8 = 346.4 m

(c) $H = \frac{u^2}{2g}\sin\ \ = \frac{50^2}{2\times 10}\sin 30^o = 31.25\ m$

Highest height reached = 120 m + 31.25 m

= 151.25 m.

Summary

- â€¢ A projectile is any object whose motion in space is affected or influenced by gravity alone.
- â€¢ Projectile motion is a plane curved motion of an object thrown vertically or obliquely into space and moving with a constant acceleration.
- â€¢ The vertical and horizontal motions of a projectile are independent of each other. The vertical motion depends on gravity while the horizontal or sideways motion depends on the initial velocity.
- â€¢ The maximum height (H) is the greatest vertical displacement of the projectile from the plane of projection.
- â€¢ The maximum height (H) is the area of the velocity – time graph when the projectile moving upwards or when it is moving downwards.
- â€¢ The acceleration due to gravity is the slope of the velocity – time graph.
- â€¢ Time of flight is the time it takes the projectile to move from the point of projection to the point where it hits the ground.
- â€¢ Range is the horizontal displacement of the projectile from the point where it is projected to the point where it returns to the plane of projection.

â€¢ Maximum range is obtained when the angle of projection (Î¸) is 45Â°.

Practice questions 2b

1. (a) What is a projectile?

(b) The vertical and horizontal components of a projectile are $6ms^{-1}$ and $8ms^{-1}$ respectively find;

(i) the initial velocity of the projectile;

(ii) the angle of projection;

(iii) the greatest height reached by the projectile;

(iv) the time of flight;

(v) the range of the projectile. ($g=10\ ms^{-2}$)

2. (a) Define the time of flight.

(b) A hawk flying at a height of 250 m above the ground, dropped a chick of mass 300 g. Calculate:

(i) the time it takes the chick to strike the ground;

(ii) the velocity with which the chick hits the ground. {$g = 10\ ms^{-2}$}

3. (a) What is the range of a projectile?

(b) A bomber moving with a horizontal velocity of 200 ms^{-1} sighted an enemyâ€™s armoured tank 1.6 km ahead and dropped a bomb. If the bomb struck the target, find:

(i) the time it took the bomb to reach the ground;

(ii) the height at which the bomber was flying before he dropped the bomb. ($g = 10\ ms^{-2}$)

4. A man standing on top of a hill 60 m high fires a bullet from a rifle when the nozzle is inclined at 30Â° to the horizontal. If the bullet leaves the rifle with a velocity of 80 ms^{-1}, find the:

(i) maximum height reached from the ground;

(ii) time it takes the bullet to strike the ground;

(iii) displacement from the foot of the hill where the projectile strikes the ground. ($g = 10\ ms^{-2}$)

5. (a) Define the terms: maximum height, range and time of flight.

(b) A free kick specialist kicks a ball giving it a velocity of 20 ms^{-1}. If the ball leaves the spot at an angle of 30Â° to the level field find:

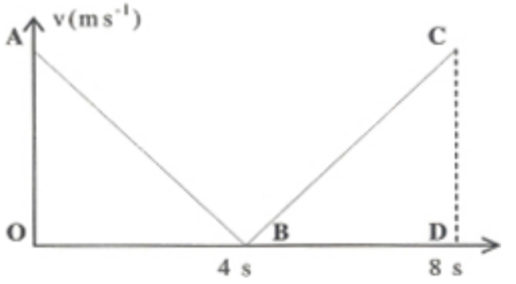
(i) the greatest height reached by the ball;

(ii) the time for the ball to strike the field again;

(iii) the velocity of the ball at the point where it strikes the ground. {$g = 10\ ms^{-2}$}

6. (a) Explain how to find the acceleration due to gravity and the maximum height from a velocity – time graph.

(b) A stone is projected from the ground and reached a height of 80 m before returning to the ground again. The velocity – time graph is shown below;



Use the graph above to find;

(i) the acceleration due gravity

(ii) the velocity with which the stone was projected;

(iii) the total distance covered by the stone.

7. A stone was vertically projected with a velocity of 20 ms^{-1} and 2 seconds later, another stone is projected vertically with a velocity of 20 ms^{-1}. Find the time and distance from the ground when they meet. (g=10ms^{-2})

Past questions

1. An object moving due east with an initial velocity of 10 ms^{-1} undergoes uniform acceleration of 2 ms^{-2} until its velocity reaches 40 ms^{-1}. Calculate the distance covered by the object.

   A. 25 m

   B. 80 m

   C. 375 m

   D. 750 m

   WASSCE

2. A car took off from rest and covered a distance of 80m on a straight road in 10 s. Calculate the magnitude of its acceleration.

   A. 1.25 ms^{-2}

   B. 1.60 ms^{-2}

   C. 4.00 ms^{-2}

   D. 8.00 ms^{-2}

   WASSCE

3. An object is released from rest at a height of 25 m. Calculate the time it takes to fall to the ground. [g =10 ms^{-2}]

   A. 25.00 s

   B. 10.00 s

   C. 2.50 s

   D. 2.24 s

   WASSCE

4. A body, which is uniformly retarded, comes to rest in 5s after travelling a distance of 10m. What is the initial velocity?

A. 0.25ms^{-1}
B. 0.50ms^{-1}
C. 2.00ms^{-1}
D. 4.00ms^{-1}

NECO

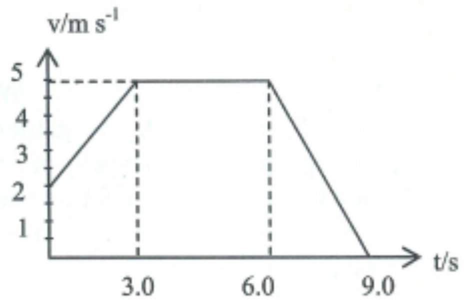
5. An object is projected with a velocity of 50ms^{-1} from the ground level at an angle Î¸ to the vertical. If the total time of flight of the projectile is 5s, what is the value of Î¸? (g = 10 ms^{-2})
A. 0^{o}
B. 30Â°
C. 45Â°
D. 60Â°
E. 90Â°

NECO

6. Which of the following equations represents the distance x travelled by a body moving on a straight road with a constant speed?
A. $x = ut + \frac{1}{2}at^2$
B. $x = ut$
C. $x = \frac{v^2 - u^2}{2a}$
D. $x = \frac{1}{2}at^2$

WASSCE

7. From the velocity – time graph shown below, the magnitude of the acceleration of the moving body is



A. 0.50 ms^{-2}
B. 0.56 ms^{-2}
C. 0.83 ms^{-2}
D. 1.00 ms^{-2}
E. 1.67 ms^{-2}

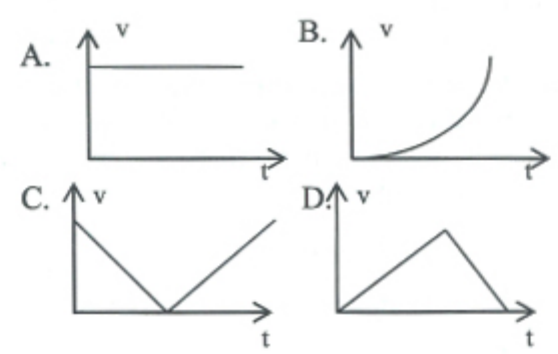
NECO

8. A ball is thrown up from the ground vertically upwards. If the maximum height reached by the ball is 80 m, calculate the speed of projection. [g = 10 ms^{-2}]
A. 10ms^{-1}
B. 20ms^{-1}
C. 40ms^{-1}

WASSCE

D. 50ms-

9. Which of the following sketches represents the velocity – time graph of the motion of a stone projected vertically upwards and allowed to return to the point of projection?



WASSCE

10. An object is projected with a velocity of 100 ms^{-1} at an angle of 60Â° to the vertical. Calculate the time taken by the object to reach the highest point. (g = 10 ms^{-2})

A. 5.0 s

B. 8.7 s

C. 10.0 s

D. 17.3 s

E. 20.0 s

WAEC

11. A stone of mass 0.7 kg is projected vertically upwards with a velocity of 5 ms^{-1}. Calculate the maximum height reached. (g = 10 ms^{-2})

A. 1.00 m

B. 1.25 m

C. 1.50 m

D. 3.75 m

WASSCE 1999J

12. A body accelerates from rest at 0.2 ms^{-2} for 20 seconds. Calculate the distance covered by the body.

A. 2m

B. 4m

C. 20 m

D. 40m

E. 80 m

WAEC

13. A stone is projected horizontal from the top of a tower with a speed of 10 ms^{-1}. It lands at a horizontal distance of 20m from the foot of the tower. Calculate the height if the tower. [g=10 ms^{-2}]

A. 4.9 m

B. 9.8 m

C. 10.0 m

D. 19.6 m

WAEC

E. 20.0 m

14. An object falls freely from a height of 25 m onto the roof of a building 5 m high. Calculate the velocity with which the object strikes the roof. [g = 10 ms^{-2}]

A. 17.3 ms^{-1}

B. 20.0 ms^{-1}

C. 24.5ms^{-1}

D. 125.0 ms^{-1}

E. 200.0ms^{-1}

WAEC

15. A mango fruit drops from a branch 10 m above the ground. Just before hitting the ground its velocity is {g= 10 ms^{-2}}

A. $10\sqrt{2}\ m\,s^{-1}$

B. $\frac{10}{\sqrt{2}}\ m\,s^{-1}$

C. $100\ m\,s^{-1}$

D. $5\sqrt{2}\ m\,s^{-1}$

E. $200\ \mathrm{m\,s^{-1}}$

JAMB

16. The distance travelled by a particle starting from rest is plotted against the square of the time elapsed from the commencement of the motion. The resulting graph is linear. The slope of this graph is a measure of

A. initial displacement

B. initial velocity

C. acceleration

D. half the acceleration

E. half the velocity.

JAMB

17. (a) Explain the terms: *uniform acceleration* and *average speed.*

(b) A body at rest is given an initial uniform acceleration of 8.0 ms^{-2} for 30 seconds after which the acceleration is reduced to 5.0 ms^{-2} for the next 20 seconds. The body maintains the speed for 60 seconds after which it is brought to rest in 20 seconds. Draw the velocity – time graph of the motion using the information given above.

(c) Using the graph, calculate the:

(i) maximum speed attained during the motion;

(ii) average retardation as the body is being brought to rest;

(iii) total distance travelled during the first 50s;

(iv) average speed during the same interval as in (iii).

WAEC

18. (a) Using a suitable diagram, explain how the following can be determined from a velocity – time graph; (i) acceleration (ii) retardation (iii) total distance covered.

(b) Show that the displacement *s* of a body moving with a uniform acceleration *a* is given by $s = ut + \frac{1}{2}at^2$. Where *u* is the initial velocity of the body at time t =0.

(c) A particle moving in a straight line with uniform deceleration has a velocity of 40 ms^{-1}at a point P, 20 ms^{-1} at a point Q and comes to rest at a point R where QR = 50 m. Calculate the:

(i) distance PQ

(ii) time to cover PQ

(iii) time to cover PR.

WAEC

19. (a) Define *speed, velocity* and *acceleration.*

(b) A stone is released at the top of a tall tower. Draw a distance – time graph of its free fall under gravity during the first 6s. Show your table of values.

(c) A bullet, fired vertically upwards from a gun held 2m above the ground, reaches its maximum height in 4s. Calculate:

(i) the initial velocity of the bullet;

(ii) the total distance the bullet travelled by the time it hits the ground. [g = 10 ms^{-2}]

WAEC

20. (a) Explain the term *uniform acceleration;*

(b) (i) Sketch and describe the velocity – time graph for the motion of a ball from the time it is projected vertically upwards until it returns to the point of projection.

(ii) Neglecting air resistance and using your sketch, explain how the acceleration of free fall due to gravity *g* and the maximum height attained when the ball is projected vertically upwards can be determined.

(c) A stone is projected vertically upwards with a velocity of 20 ms^{-1}. Two seconds later, a second stone is similarly projected with same velocity. When the two stones meet, the second one is rising at a velocity of 10 ms^{-1}. Neglecting air resistance, calculate the:

(i) the length of time the stone is motion before they meet;

(ii) velocity of the first stone when they meet. [g =10 ms^{-2}]

WAEC

21. (a) State two objects, one in each case, which may be considered as projectiles in sports and warfare;

(b) An object is projected with a speed of 30 ms^{-1} in a direction that makes an angle of 30Â° with the ground level. Calculate the length of time the object takes to reach the highest point of flight. [$g=10\ ms^{-2}$]

WASSCE

22. (a) What is meant by the *range* of a projectile?

(b) An object is projected into the air with a speed of 50 ms^{-1} an angle of 30Â° above the ground level. Calculate the maximum height attained by the object. [$g = 10\ ms^{-1}$]

WASSCE

23. (a) The horizontal range R, of a projectile is given by the expression $R = \frac{u^2}{g} sin$;

State what the symbols *u,* Î¸, and *g* represent

(b) A projectile has a range of 100 m when the angle of projection is 20Â°. Calculate the initial velocity of the projectile. [$g = 10\ ms^{-1}$]

WASSCE

24. In his first attempt, a long jumper took off from the spring board with a speed of 8 ms^{-1} at 30Â° to the horizontal. He made a second attempt with the same speed at an angle of 45Â° to the horizontal. Given that the expression for the horizontal range of a projectile is

$$R = \frac{v^2}{g} sin2$$

Where all the symbols have their usual meanings, show that he gained a distance of 0.8576 m in his second attempt. [$g = 10\ ms^{-2}$]

WASSCE

25. (a) What is a *trajectory?*

(b) A ball is kicked with a velocity of 8ms^{-1} at an angle of 30Â° to the horizontal. Calculate the time of flight of the ball. [$g = 10\ ms^{-2}$]

NECO

26. A body is projected from the ground at an angle of Î¸ to the horizontal with a velocity of 30 ms^{-1}. It reached a maximum height of 11.25 m. Calculate:

(i) the value of Î¸;

(ii) the time taken to strike the ground;

(iii) the range;

(iv) its velocity 2s after projection;

[Neglect air resistance and take *g* as 10 ms^{-2}]


stop please! I must not miss this flight.
Hey, what do think you are doing? You want to stop plane by running after it?