**Chapter 8**

Chapter 8

# Surface Area and Volume of Solids

**OBJECTIVES**

At the end of the chapter, students should be able to calculate the surface area and volume of the following:

1. Cuboid
2. Cube
3. Cylinder
4. Cone
5. Prism
6. Pyramid
7. Sphere

## 1. Cuboid

### <u>(ii)Total Surface Area</u>

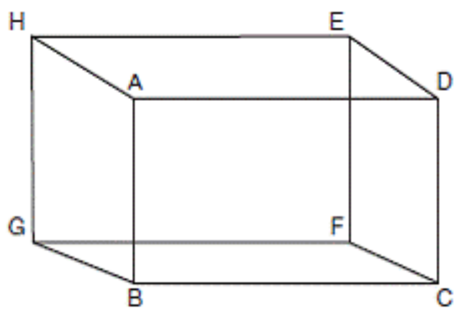



Figure 8.1

Total surface area (TSA) of the cuboid

= Area of shape (ABCD + EFGH + ADEH + BCFG + ABGH + CDEF)

$= lb + lb + bh + bh + lh + lh$

$= 2lb + 2bh + 2lh$

$= 2(lb + bh + lh)$

**Worked Example 1**

What is the total surface area of a cuboid whose dimensions are 12 cm × 8 cm ×3 cm?

SOLUTION

**Given:** $l = 12$ cm, $b = 8$ cm, $h = 3$ cm

**Formula:** TSA of the cuboid = $2[(12 \times 8) + (8 \times 3) + (12 \times 3)]$ cm$^2$

$2(96 + 24 + 36)$ cm$^2$

$2 \times 156$ cm$^2$

$312$ cm$^2$

## Worked Example 2

An open rectangular box externally measures 4 m × 3 m × 4 m . Find the total cost of painting the box externally if it costs N2.00 to paint 1 m$^2$.

SOLUTION

Surface area of the side faces

$2bh + 2lh$ $[2(3 \times 4) + 2(4 \times 4)]$ m$^2$

$= [(2 \times 12) + (2 \times 16)]$ m$^2$

$= (24 + 32)$ m$^2$

$= 56$ m$^2$

Cost of painting 1 m$^2$ = N2.00

$\therefore$Cost of painting 56 m$^2$ = 56 × N2.00

= N112.00

## Worked Example 3

The base of an open tank is a square of sides $x$ cm and its volume is 200 m$^2$. Show that the total surface area, $y$ m$^2$ of the base and sides is given by the relation $y = x^2 + \frac{800}{x}$.

SOLUTION

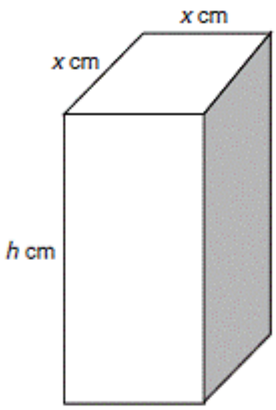



Figure 8.2

Volume of a cuboid (prism)

$$= \text{Area of base} \times \text{perpendicular height}$$

$$200 = (x \times x) \times h$$

$$= \frac{200}{x^2} h$$

Area of the base $= (x \times x) \times \text{m}^2 = x^2 \text{ m}^2$

Area of the four sides $= 4 \times (h \times x) \text{ m}^2$

$$= 4h\,x \text{ m}^2$$

Surface area of the open tank $= (x^2 + 4hx)\text{m}^2$

$$y = x^2 + (4 \times \frac{200}{x^2} \times x)$$

$$y = x^2 + \frac{800}{x}$$

## Worked Example 4

How deep is a rectangular water tank whose length = 20 m, breadth =15 m and total surface area = 1 300 m$^2$?

SOLUTION

**Given:**

TSA of the rectangular tank $= 1\,300\ \text{m}^2$

Length $= 20$ m

Breadth $= 15$ m

Depth $= ?$

**Formula:** $\text{TSA} = 2(lb + bd + ld)1\,300$

$$= 2[(20 \times 15) + (15 \times d) + (20 \times d)]$$

$$\frac{1\,300}{2} = 300 + 15d + 20d$$

$$650 - 300 = 35d$$

$$350 = 35d$$

$$d = \frac{350}{35}$$

$$d = 10\ \text{m}$$

*Depth is* 10 m

**(ii)Volume of cuboid**

The unit of volume is cubic centimetre, cubic metre or cubic kilometre.

The volume of a cuboid is calculated as the product of the length, the breadth and the height.

**Worked Example 5**

The length, breadth and height of a cuboid are 8 cm, $7\frac{1}{2}$ cm and $6\frac{1}{2}$ cm, respectively. What is the volume in $\text{m}^3$? (Leave your answer in standard form).

**SOLUTION**

$$1\ \text{cm} = \frac{1}{100}\ \text{m}$$

$$\text{Length} = 8\ \text{cm} = 8 \times \frac{1}{100}\ \text{m} = 0.08\ \text{m}$$

$$\text{Breath} = 7\frac{1}{2}\ \text{cm} = 7\frac{1}{2} \times \frac{1}{100}\ \text{m} = 0.075\ \text{m}$$

$$\text{Height} = 6\frac{1}{2}\ \text{cm} = 6\frac{1}{2} \times \frac{1}{100}\ \text{m} = 0.065\ \text{m}$$

$$\begin{aligned}\text{Volume} &= (0.08 \times 0.075 \times 0.065)\ m^3\\ &= (8 \times 10^{-2} \times 7.5 \times 10^{-2} \times 6.5 \times 10^{-2})\ m^3\\ &= (390 \times 10^{-2+-2+-2}\ m^3\\ &= (3.9 \times 10^2 \times 10^{-6})\ m^3\\ &= (3.9 \times 10^{2+(-6)})\ m^3\\ &= (3.9 \times 10^{2-6})\ m^3\\ &= (3.9 \times 10^{-4})\ m^3\end{aligned}$$

## Worked Example 6

A cuboid of base 12.5 cm by 20 cm holds exactly 1 L of water. What is the height of the cuboid? (1 L = 1 000 $cm^3$)

**SOLUTION**

Capacity = 1 L
∴ Volume = 1 000 $cm^3$
Base area = 12.5 cm × 20 cm
Volume = Base area × height
1000 = 12.5 × 20 × height

$$\begin{aligned}\text{Height} &= \frac{1\,000}{12.5 \times 20}\ \text{cm}\\ &= \frac{1\,000}{250}\ \text{cm}\\ &= 4\ \text{cm.}\end{aligned}$$

## Worked Example 7

A cylindrical tank of radius 35 cm contains water 20 cm high. The water is poured into a barrel of rectangular base 42 cm by 30 cm. Find, correct to two significant figures, the height of the water level in the barrel. (Take $\pi = \frac{22}{7}$)

**SOLUTION**

Volume of the cylinder = Volume of the cuboid

$$\pi r^2 h = l \times b \times h$$

$$\frac{22}{7} \times \frac{35}{1} \times \frac{35}{1} \times \frac{20}{1} = 42 \times 30 \times h$$

$$h = \frac{22}{7} \times \frac{35}{1} \times \frac{35}{1} \times \frac{20}{1} \times \frac{1}{42} \times \frac{1}{30}\ \text{cm}$$

$$h = \frac{550}{9}\ \text{cm}$$

$$h = 61\frac{1}{9}\ \text{cm}$$

**Exercise 1**

1. A closed rectangular tank externally measures 4 m × 3 m × 4 m. Find the total cost of painting the box externally if it costs ₦10 to paint 1 $m^2$.
2. An open rectangular tank is made of a steel plate of area 1 440 $m^2$. Its length is twice its width. If the depth of the tank is 4 m less than its width, find its length. (WAEC)
3. A box contains 40.1 $m^3$ of air. If the length and breadth of the box are 5.23 m and 2.34 m, respectively, calculate its height.
4. The length, breadth and height of a cuboid are 8 cm, $7\frac{1}{2}$ cm and $6\frac{1}{2}$ cm, respectively. What is its volume in $m^3$? (Leave your answer in standard form). (NECO)
5. A water tank of height $\frac{1}{2}$ m is filled with water from a water tanker holding 1500 L. How many litres of water are left in the water tanker?
   (1 000 L = 1 $m^3$) (WAEC)

Calculate the total surface area of the cuboids whose dimensions are as follows:

6. Length = 12 cm,
   breadth = 8 cm,
   height = 6 cm
7. Length = 9 cm,
   breadth = 6 cm,
   height = 10 cm
8. Length = 15 cm,
   breadth = 11 cm,
   height = 7 cm
9. Length = 11 cm,
   breadth = 6 cm,
   height = 6 cm
10. Length = 18 cm,
    breadth = 12 cm,
    height = 10 cm

## II. Cube

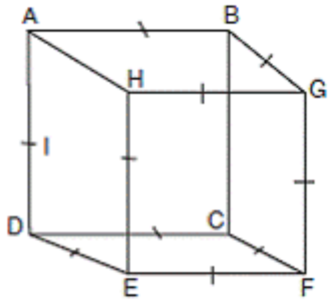



Figure 8.3

### (i) Total surface area of a cube

TSA of the cube

$= \textit{Area of shape } (ABCD + EFGH + ADEH + BCFG + ABGH + CDEF)$

$= (l \times l) + (l \times l) + (l \times l) + (l \times l) + (l \times l) + (l \times l)$

$= l^2 + l^2 + l^2 + l^2 + l^2 + l^2$

$= 6l^2$

**Worked Example 8**

Calculate the total surface area of a cube whose length of side is √3 cm.

**SOLUTION**

Given: Length of side of cube $\sqrt{3}$ cm.

TSA of a cube $= 6l^2$

$= [(6 \times \sqrt{3})^2]\ \text{cm}^2$

$= (6 \times 3)\ \text{cm}^2$

$= 18\ \text{cm}^2$

**Worked Example 9**

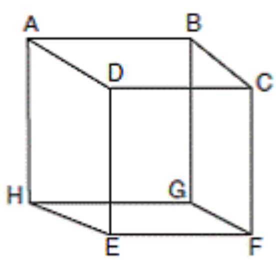



Figure 8.4

Calculate the total surface area of the cube in Figure 8.4.

**SOLUTION**

$$(\sqrt{2})^2 = /DE/^2 + /HE/^2$$

$$2 = 2/DE/^2$$

$$/DE/^2 = \frac{2}{2} = 1$$

$$/DE/ = \sqrt{1} = 1$$

Total surface area of the cube $= 6l^2$

$$= 6 \times 1^2$$

$$= (6 \times 1 \times 1)\ \text{cm}^2$$

$$= 6\ \text{cm}^2$$

### (ii) Volume of cube

The formula for calculating the volume of a cube is the cube of the length of side.

## Worked Example 10

A cube and a cuboid have the same volume. The length of cuboid is 3 cm greater, the width 2 cm greater and the height 3 cm smaller than the edge of the cube. Find the length of the edge of the cube.

### SOLUTION

**Given:** Volume of the cube = $x$ cm

Length of the cube = $x$ cm

Breadth of the cube = $x$ cm

Height of the cube = $x$ cm

Length of the cuboid = $(x + 3)$ cm

Breadth of the cuboid = $(x + 2)$ cm

Height of the cuboid = $(x - 3)$ cm

Volume of the cube

$= (x \times x \times x)\ \text{cm}^3 = x^3\ \text{cm}^3$

So, $x^3 = (x + 3)\ (x + 2)\ (x - 2)$

$x^3 = (x^2 + 9)\ (x + 2)$

$x^3 = x^3 + 2\ x^2 - 9x - 18$

$x^3 + 2\ x^2 - 9 - 18 - x^3 = 0$

$2\ x^2 - 12x + 3x - 18 = 0$

$(2\ x^2 - 12x) + (3x - 18) = 0$

$2x\ (x - 6) + 3(x - 6) = 0$

$(x - 6)(2x + 3) = 0$

*either* $x - 6 = 0$ *or* $2x + 3 = 0$

$x = 6$ *or* $2x = -3$

$x = 6$ *or* $x = -\frac{3}{2}$

$x = 6, x \neq -\frac{3}{2}$

*length of the edge of the cube is* 6 cm.

## Worked Example 11

In Figure 8.5, GHIJKLMN is a cube whose volume is $a^3$ cm$^3$. Find the length of HN.

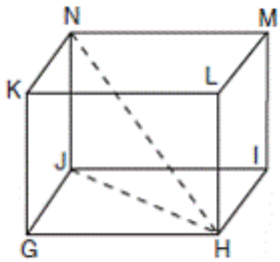



Figure 8.5

### SOLUTION

**Given:** Volume of the cube $= a^3$ cm.

**Formula:**

Volume of cube = (length of side)$^3$

$a^3 =$ (length of side)$^3$

$\therefore$ length of side $= a$

But $JH^2 = JG^2 + GH^2$

$JH^2 = a^2 + a^2 = 2a^2$

$JH = \sqrt{2a^2} = \sqrt{a^2} \times \sqrt{2} = a\sqrt{2}$

$JN = a$

$HN^2 = JN^2 + JH^2$

$= a^2 = 2a^2 = 3a^2$

$HN = \sqrt{3a^2} = \sqrt{a^2} \times \sqrt{3} = a\sqrt{3}$ cm

Length of HN is $a\sqrt{3}$.

### Exercise 2

1. If the volume of a cube is $8 \times 10^3$ cm$^3$, what is the surface area of the cube? (NECO)
2. What is the volume of a cube if the diagonal of one of the sides is $\sqrt{50}$ cm? (NECO)

3. If the volume of a cube is the same as the volume of a cuboid of length 12 cm, breadth 6 cm and height 3 cm, calculate the total surface area of the cube.
4. Calculate the diagonal of a side face of a cube whose volume is 125 $cm^3$. Leave your answer in surd form.
5. Find the total surface area of a cube, which has the diagonal of one of its side faces as $3\sqrt{2}$.

Calculate the total surface area and the volume of a cube with the following as length of side:

6. 12 cm
7. 9 cm
8. 10 cm
9. 3.5 cm
10. $6\frac{1}{4}$ cm

## III. Cylinder

A cylinder is a solid shape with two circular faces and a curved face.

(i) Total Surface area of a Cylinder

The total surface area of a cylinder is the sum of the areas of the two circular faces and that of the remaining curved face.

Area of the curved face = $2\pi rh$

Area of the two circular faces = $2\pi r^2$

$\therefore$ Total surface area of a cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r + h)$

But the surface area of a cylinder closed at one end

$= \pi r^2 + 2\pi rh$ (*are of one circle plus the curved surface area*).

**Worked Example 12**

Calculate the surface area of a hollow cylinder which is closed at one end with base radius 3.5 cm and height 8 cm. (Take $\pi = \frac{22}{7}$ )

SOLUTION

**Given :** $r = 3.5$ cm

$h = 8$ cm

$\pi = \frac{22}{7}$

Surface area of the hollow cylinder closed at one end $= \pi r^2 + 2\pi rh$

$$= \pi r(r + 2h)$$

$$= \left\{\left(\frac{22}{7} \times \frac{3.5}{1}\right)[3.5 + 2(8)]\right\} \text{ cm}^2$$

$$= \{(22 \times 0.5)(3.5 + 16)\} \text{ cm}^2$$

$$= (11 \times 19.5) \text{ cm}^2$$

$$= 214.5 \text{ cm}^2$$

## Worked Example 13

The curved surface area of a cylindrical tin is 704 $cm^2$. Calculate the height when the radius is 8 cm. (Take $\pi = \frac{22}{7}$)

**SOLUTION**

Curved surface area of cylinder = 704 $cm^2$
Radius = 8 cm
Height = ?

**Formula:** Curved surface area $= 2\pi rh$

$$704 = 2 \times \frac{22}{7} \times \frac{8}{1} \times h$$

$$\frac{704 \times 7}{2 \times 22 \times 8} = h$$

$$h = \frac{88 \times 7}{44} \text{ cm}$$

$$h = 14 \text{ cm}$$

Therefore, height is 14 cm.

## Worked Example 14

A cylindrical container, closed at both ends, has a radius of 7 cm and height 5 cm. Find the total surface area of the container. **(WAEC)**

**SOLUTION**

**Given:** Radius = 7 cm

Height = 5 cm

TSA = ?

**Formula:**

TSA of a closed cylinder $= 2\pi r(r + h)$

$$= \left\{\left(\frac{2}{1} \times \frac{22}{7} \times \frac{7}{1}\right)(7 + 5)\right\} \text{ cm}^2$$

$$= (44 \times 12) \text{ cm}^2$$

$$= 528 \text{ cm}^2$$

**Worked Example 15**

Calculate the surface area of a cylinder that is open at one end if its curved surface area and height are 88 cm2 and 8 cm, respectively.
(Use $\pi = \frac{22}{7}$)

**SOLUTION**

**Given:** $CSA = 88 \text{ cm}^2$

Height = 8 cm and $\pi = \frac{22}{7}$

**Formula:** $CSA = 2\pi rh$

$$88 = 2 \times \frac{22}{7} \times r \times 8$$

$$\frac{88 \times 7}{44 \times 8} = r$$

But, surface area of a cylinder open at one end $= \pi r(r + 2h)$

$$= \left\{\left(\frac{22}{7} \times \frac{7}{4}\right)\left(\frac{7}{4} + (2 \times 8)\right)\right\} \text{ cm}^2$$

$$= \left(\frac{22}{7} \times \frac{7}{4}\right)\left(\frac{7}{4} + \frac{16}{1}\right) \text{ cm}^2$$

$$= \left(\frac{11}{2} \times \frac{7 + 64}{4}\right) \text{ cm}^2$$

$$= \frac{11}{2} \times \frac{71}{4} \text{ cm}^2$$

$$= \frac{781}{8} \text{ cm}^2$$

$$= 97\frac{5}{8} \text{ cm}^2$$

### (ii) Volume of a cylinder

The volume of a cylinder is calculated as the area of the circular base ($\pi r^2$) multiplied by the perpendicular height ($h$).
This is expressed as $V = \pi r^2 h$.

**Worked Example 16**

A cylindrical well of radius 1 m is dug out to a depth of 8 m

(a) Calculate, in m3, the volume of soil dug out.

(b) If the soil is used to raise the level of the rectangular floor of a room 4 m by 12 m, calculate, correct to the nearest cm, the thickness of the new layer of solid.

(Take $\pi = \frac{22}{7}$) (WAEC)

SOLUTION

(a) Volume of a cylinder $= \pi r^2 h$.

$$= \frac{22}{7} \times \frac{1}{1} \times \frac{1}{1} \times \frac{8}{1} \text{ m}^3$$

$$= \frac{176}{7} \text{ m}^3$$

$$= 25.14 \text{ m}^3 \text{ (2 d.p.)}$$

(b) Volume of the concrete of floor of room = Volume of cylinder

$4 \text{ m} \times 12 \text{ m} \times \text{thickness} =$

$$\frac{22}{7} \times 1 \text{ m} \times 1 \text{ m} \times 8 \text{ m}$$

$$\text{Thickness} = \frac{22}{7} \times \frac{100}{1} \times \frac{100}{1} \times \frac{800}{1} \times \frac{1}{400} \times \frac{1}{1\,200}$$

$$\text{Thickness} = \frac{1\,100}{21} \text{ cm}$$

Thickness = 52.38 cm = 52 cm (nearest cm)

**Worked Example 17**

The curved surface area of a cylindrical tin is 704 $\text{cm}^2$. Calculate the volume of the tin when the radius is 8 cm.

(Take $\pi = \frac{22}{7}$)

SOLUTION

**Given:** CSA of a cylinder $= 704\ cm^3$

$$r = 8\ cm$$

$$h = ?$$

**Formula:** CSA of a cylinder $= 2\pi rh$

$$704 = 2 \times \frac{22}{7} \times \frac{8}{1} \times h$$

$$\frac{704 \times 7}{2 \times 22 \times 8} = h$$

$$h = \frac{88 \times 7}{44}\ cm$$

$$h = 14\ cm$$

Volume of a cylindrical tin

$$= 2 \times \frac{22}{7} \times \frac{8}{1} \times \frac{8}{1} \times \frac{14}{1}\ cm^3$$

$$= \frac{22 \times 8 \times 8 \times 14}{7}\ cm^3$$

$$= (44 \times 64)\ cm^3$$

$$= 2\,816\ cm^3.$$

**Exercise 3**

1. Given that a container closed at both ends has a radius of 7 cm and height 5 cm. Find the total surface area of the container. (WAEC)
2. A cylindrical drum of diameter 56 cm contains 123.2 L of oil when full. Find the height of the drum in centimetres. ($\pi = \frac{22}{7}$) (UME)
3. The volume of a cylinder of radius 14 cm is 210 $cm^3$. What is the curved surface area of the cylinder? (WAEC)

4. The volume of a cylinder of height 40 m is 260 $m^2$. Find the radius of the cylinder.
5. Find the volume of a cylindrical tin of height 16 cm with curved surface area 348 $cm^2$.
6. A cylindrical tank contains 50 L of water. The diameter of the tank is 42 cm. Find the height of the water in the tank. (Take $\pi = \frac{22}{7}$, 1 litre = 1000 $cm^3$) (WAEC)
7. A tin has radius 3 cm and height 6 cm. Find
   (a) The total surface area of the tin.
   (b) The volume, in litres, of the liquid that will fill the tin to capacity, correct to two decimal places. (Take $\pi = \frac{22}{7}$) (WAEC)
8. A cylindrical pipe is 28 m long, its internal radius is 3.5 cm and external radius is 5 cm. Calculate
   (a) The volume, in $cm^3$, of metal used in making the pipe.
   (b) The volume of water that the pipe can hold when full, in litres and correct to one decimal place. (Take $\pi = \frac{22}{7}$) (WAEC)
9. A solid cylinder of radius 3 cm has a total surface area of $36\pi cm^2$. Find its height. (UME)
10. Find the area of the curved surface of a cylinder whose base radius is 6 cm and whose height is 8 cm. (Take $\pi = \frac{22}{7}$)

## IV. Cone

A cone is a solid with a circular base and a curved face.

### (i) Total surface area of a cone

The total surface area of a cone is the sum of the curved surface area of the cone and its circular base area. The curved surface area of the cone is $\pi rl$ while the area of the circular base is $\pi r^2$. Hence, the TSA of a cone is $\pi r^2 + \pi rl$, where $r$ is the radius, $\pi$ is $\frac{22}{7}$ and $l$ is the slant height.

**Worked Example 18**

Find the total surface area of a solidcone of radius 2 √3 cm and slanting side 4 √3 cm.

**SOLUTION**

**Given:** $r = 4\sqrt{3}$ cm
$l = 2\sqrt{3}$ cm
$\pi = \frac{22}{7}$

**Formula:** TSA = $\pi r^2 + \pi rl = \pi r(r + l)$

$$= \left\{\left(\frac{22}{7} \times \frac{2\sqrt{3}}{1}\right)(2\sqrt{3} + 4\sqrt{3})\right\} \text{cm}^2$$

$$= \frac{44\sqrt{3}}{7} \times \frac{6\sqrt{3}}{1} \text{cm}^2$$

$$= \frac{264 \times 3}{7} \text{cm}^2 = \frac{792}{7} \text{cm}^2$$

$$= 113\frac{1}{7} \text{cm}^2$$

## Worked Example 19

Calculate the total surface area of a cone of height 12 cm and base radius 5 cm.

(Take $\pi = \frac{22}{7}$) **(WAEC)**

**SOLUTION**

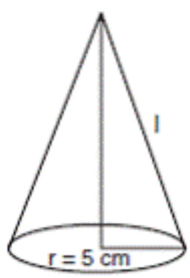



Figure 8.6

**Given:** $h = 12$ cm, $r = 5$ cm
$l = ?$ π = _ 27_2_
$l^2 = h^2 + r^2$
$= 12^2 + 5^2$
$= 144 + 25$
$= 169$
$= \sqrt{169}$ $l = 13$

$$\begin{aligned} \text{TSA} &= \pi r(r + l) \\ &= \left\{\left(\frac{22}{7} \times \frac{5}{1}\right)(5 + 13)\right\} \text{cm}^2 \\ &= \frac{110}{7} \times \frac{18}{1} \text{cm}^2 \\ &= \frac{1\,980}{7} \text{cm}^2 \\ &= 282\frac{6}{7} \text{cm}^2. \end{aligned}$$

## Worked Example 20

Find the curved surface area of a cone of radius 3 cm and slant height 7 cm. (Take $\pi = \frac{22}{7}$.)

### SOLUTION

**Given:** $r = 3$ cm, $l = 7$ cm and $\pi = \frac{22}{7}$.

**Formula:** $CSA = \pi r l$

$$\begin{aligned} &= \frac{22}{7} \times \frac{3}{1} \times \frac{7}{1} \text{cm}^2 \\ &= 66 \text{ cm}^2 \end{aligned}$$

## Worked Example 21

A closed cone of radius $r$ cm and slant height $2r$ cm has a total surface area of 462 $\text{cm}^2$. Find its radius. (Take $\pi = \frac{22}{7}$.) **(WAEC)**

### SOLUTION

**Given:** $r = ?$

$l = 2r$

$\text{TSA} = 462 \text{ cm}^2$

**Formula:** $\text{TSA} = \pi r(r + l)$

$$\begin{aligned} 462 &= \left\{\left(\frac{22}{7} \times r\right)(r + 2r)\right\} \text{cm}^2 \\ 462 &= \left(\frac{22}{7} r \times 3r\right) \text{cm}^2 \\ 462 &= \frac{66r^2}{7} \\ \frac{462}{1} \times \frac{7}{66} &= r^2 \\ r &= \sqrt{49} \\ r &= 7 \text{ cm} \end{aligned}$$

The radius is 7 cm

## (ii) Volume of a cone

The formula for the calculation of the volume of a cone is one-third the formula for calculating the volume of a cylinder.

This is expressed as $V = \frac{1}{3}\pi r^2 h$, where $V$ is the volume of a cone, $\pi = \frac{22}{7}$, $r$ is the radius and $h$ the perpendicular height.

### Worked Example 22

Find the volume of a closed cone of radius 3 cm and slant height 5 cm.

**SOLUTION**

**Given:** Volume = $V$

$r = 3$ cm

$l = 5$ cm

$5^2 = VM + 3^2$

$VM^2 = 25 - 9 = 16$

$VM = \sqrt{16} = 4$ cm

Volume of a cone $= \frac{1}{3}r^2h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{3}{1} \times \frac{3}{1} \times \frac{4}{1}\ \text{cm}^3$

$= \frac{264}{7}\ \text{cm}^3$

$= 37\frac{5}{7}\ \text{cm}^3$

### Worked Example 23

The volume of a cone of height 9 cm is 1 848 $\text{cm}^3$. Find the slant height of the cone. (Take $\pi = \frac{22}{7}$)

**SOLUTION**

**Given:** Volume of cone = 1 848$\text{cm}^3$

$h = 9$ cm

$r = ?$

**Formula:** Volume of cone $= \frac{1}{3}r^2h$

$$1\,848 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 9$$

$$\frac{1\,848 \times 3 \times 7}{22 \times 9} = r^2$$

$$196 = r$$

$$r = \sqrt{196}$$

$$r = 14 \text{ cm}$$

$$l^2 = 9^2 + 14^2$$

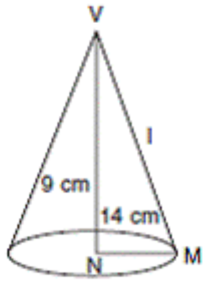



Figure 8.7

$$= 81 + 196$$

$$= 277$$

$$l = \sqrt{277}$$

$$l = 16.64 \text{ cm}$$

Slant height is 16.6 cm (1 d.p.)

## Exercise 4

1. Find the curved surface area of a cone with circular base diameter of 10 cm and height 12 cm.
2. Find the volume of a cone of radius 3.5 cm and vertical height 13 cm. (Take $\pi = \frac{22}{7}$) (WAEC)
3. A cone is 14 cm deep and the base radius is $4\frac{1}{2}$ cm. Calculate the volume of water that is exactly half the volume of the cone. (Take $\pi = \frac{22}{7}$) (WAEC)
4. Find the total surface area and the volume of a solid right cone of diameter 7 cm and height 12 cm. (Take $\pi = \frac{22}{7}$) (WAEC)
5. Find the total surface area of a solid circular cone with base radius 3 cm and slant height 4 cm. (Take $\pi = \frac{22}{7}$) (WAEC)

6. Calculate the total surface area of solid cone of slant height 15 cm and base radius 8 cm in terms of $\pi$. (WAEC)
7. Find the curved surface area of a cone of radius 3 cm and perpendicular height 4 cm. (Take $\pi = \frac{22}{7}$) (WAEC)
8. Find the volume of a right solid cone of base radius 4 cm and perpendicular height 6 cm. (Take $\pi = \frac{22}{7}$) (WAEC)
9. Find the area of the curved surface of a cone whose base radius is 6 cm and whose height is 8 cm. (Take $\pi = \frac{22}{7}$)
10. Calculate the height of a cone of surface area $172\frac{1}{8}$ cm² and base diameter 8 cm, to the nearest cm. (Take $\pi = \frac{22}{7}$)

## V. Pyramid

A pyramid is a solid shape with a plane base and triangular side faces.

### (i) Total surface area of a pyramid

The total surface area of a pyramid is the sum of the area of its base and the surface area of its side faces. In a square-based pyramid, the total surface area is the sum of the area of the square base and the surface area of the four triangular faces.

In a triangular-based pyramid, the total surface area is the sum of the area of the triangular base and surface area of the three side faces.

In a nutshell, the total surface area of a pyramid is calculated considering the nature of the pyramid involved.

**Worked Example 24**

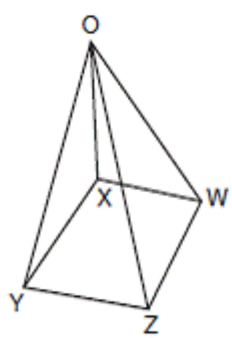



Figure 8.8

OXYZW is a pyramid with a square base such that OX = OY = OZ = OW = 5

cm and
XY = XW = YZ = WZ = 6 cm. Find the area of the pyramid.

**SOLUTION**

Area of WXYZ (square base) = (6 × 6) $cm^2$

From $\Delta OXY$, $S = \frac{5+5+6}{2}$ cm

$= \frac{16}{2}$ cm = 8 cm

Area of $\Delta OXY = \sqrt{8(8-5)(8-5)(8-6)}\ cm^2$

$= \sqrt{8 \times 3 \times 3 \times 2}\ cm^2$

$= \sqrt{144}\ cm^2$

$= 12\ cm^2$

Surface area of side faces (OXY, OXW, OWZ, OYZ)

= 4 × 12 $cm^2$

= 48 $cm^2$

Total surface area of the pyramid = Area of base + Surface area of side faces

= 36 $cm^2$ + 48 $cm^2$

## Worked Example 25

The base of right pyramid, vertex *V*, is a rectangle PQRS. |PQ| = 10 cm and |QR| =
16 cm. If the face VPQ of the pyramid makes an angle of
60° with the base, calculate, correct to three significant figures, the total surface area of the pyramid.

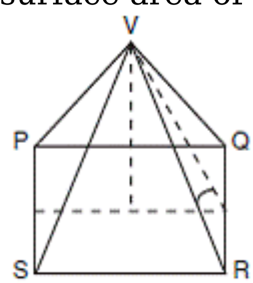



Figure 8.9

**SOLUTION**

Area of PQRS (rectangular base)

= |PQ| × |QR|

= 10 cm × 16 cm

= 160 $cm^2$

$$\cos 60^\circ = \frac{MN}{VN} = \frac{8\text{ cm}}{VN}$$

$$\frac{1}{2} = \frac{8\text{ cm}}{VN}$$

$$VN = 2 \times 8\text{ cm} = 16\text{ cm}$$

$$\text{Area of } \Delta VPQ = \frac{1}{2} \times /PQ/ \times /VN/$$

$$= \frac{1}{2} \times 10\text{ cm} \times 16\text{ cm}$$

$$= 80\text{ cm}^2$$

Also, area of $\Delta VRS = 80\text{ cm}^2$

### (ii) Volume of a pyramid

The formula for calculating the volume of a pyramid is one-third the area of the base multiplied by the perpendicular height.
This is the volume of the pyramid, *a* is the base area of the pyramid and *h* is the height.

**Worked Example 26**

A square-based pyramid has a perpendicular height 6 cm. Calculate the volume of the pyramid of length 9 cm.

**SOLUTION**

$$\text{Volume of pyramid } (V) = \frac{1}{3}Ah$$

$$\text{Area of square base } (A) = l^2 = (9\text{ cm})^2$$

$$\text{Volume} = \left(\frac{1}{3} \times 81 \times 6\right)\text{cm}^3$$

$$= 162\text{ cm}^3$$

**Worked Example 27**

Calculate the height of a pyramid whose base area is 303 $m^2$ and volume 2272.5 $m^2$.

**SOLUTION**

**Given:** Base area = 303 $m_2$
Height = *h*
Volume = 2 272.5 $m_3$
**Formula:** Volume = Base area × height
2 272.5 = 303 × *h*
$h = \frac{2\,272.5}{303}$ m
$h = 7.5$ m

## Exercise 5

1. The base of a solid pyramid is a square of side 6 cm. If the height of the pyramid is 7 cm, calculate the volume of the pyramid. (WAEC)

2. A right pyramid is on a square base of side 4 cm. The slanting side of the pyramid is $2\sqrt{3}$ cm. Calculate the volume of the pyramid. (WAEC)

3. The diagram shows the net of a pyramid consisting of a square PQRS of side 14 cm and four congruent isosceles triangles of altitude 25 cm.

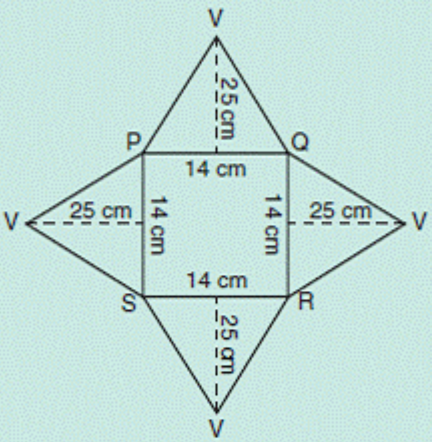



Figure 8.10

   (a) Sketch the pyramid.
   (b) Calculate:
      (i) The total surface area
      (ii) The height
      (iii) The volume. (WAEC)

4. A pyramid with vertex O stands on a square base ABCD and $|OA| = |OB| = |OC| = |OD| = 4$ cm, when $|AD| = |AB| = 5$ cm. Calculate
   (a) The height of the pyramid.
   (b) Volume of the pyramid.
   (c) The total surface area of the triangular faces.

5. If a pyramid ABCDV with a square base ABCD of side 10 cm has triangle faces with altitudes of 12 cm, calculate, giving your answer to three significant figures, the
   (a) Total surface area.
   (b) Volume of the pyramid.
   (NECO)

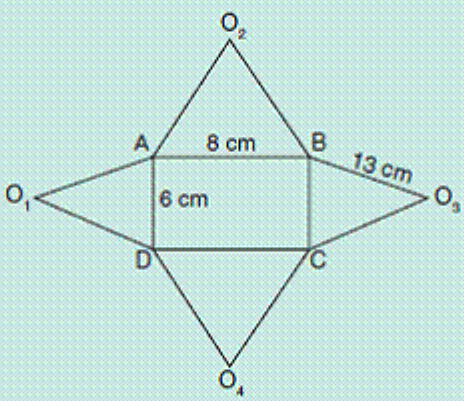



Figure 8.11

The diagram above shows the net of a right rectangular pyramid in which ABCD is the base and $O_1$, $O_2$, $O_3$, $O_4$ are brought together to form the vertex O. When the solid is formed, ABCD is the base /AB/ = 8 cm, /AD/ = 6 cm and $BO_3$ = 13 cm.

6. Draw a sketch of the pyramid in Figure 8.11 and calculate its height.
7. Calculate the volume of the pyramid in Figure 8.11.
8. Calculate the height of a rectangular-based pyramid whose volume is 140 $cm^3$ and base area is 56 $cm^2$.

Find the volumes of the pyramids with the following data:

9. Base area = 35 $cm^2$, height = 7 cm

10. Base area = 18.5 $cm^2$, height = 6.5 cm.

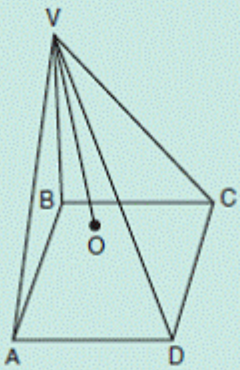
11. 


Figure 8.12

The diagram above shows a pyramid VABCD standing on horizontal ground. The base ABCD is a square of side 15 cm. The diagonals of ABCD intersect at O and V is vertically above O. Given that $V$ = 20 cm, calculate:

(a) The height, in cm to one decimal place, of the pyramid.

(b) The volume, in $cm^3$ to three significant figures, of the pyramid.

## VI. Sphere

A sphere is a solid shape that has the shape of a football or table tennis egg.

### (i) Surface area of a sphere

The surface area of a sphere is $4\pi r^2$.

**Worked Example 28**

Calculate the surface area of a sphere of radius 7 cm.

(Take $\pi = \frac{22}{7}$) (WAEC)

SOLUTION

**Given:** Radius $r = 7$ cm, $\pi = \frac{22}{7}$

Surface area = ?

**Formula:** Surface area = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

**Worked Example 29**

A sphere has a surface area of 30,184 $cm^2$. Calculate the radius of the sphere. (Take $\pi = \frac{22}{7}$).

**SOLUTION**

**Given:** Surface area of sphere = 30,184 $cm^2$

$$\text{Radius} = r$$

$$\text{Surface area} = 4\pi r^2$$

$$30\,184 = 4 \times \frac{22}{7} \times r^2$$

$$\frac{30\,184 \times 7}{4 \times 22} = r^2$$

$$r^2 = \frac{7\,546 \times 7}{4 \times 22}$$

$$= \frac{7\,546 \times 7}{22}$$

$$r^2 = 2\,401$$

$$r = \sqrt{2\,401}$$

$$r = 49 \text{ cm}$$

### (ii) Volume of a sphere

The formula for calculating the volume ofa sphere is $\frac{4}{3}\pi r^3$, where $r$ is the radius and $\pi = \frac{22}{7}$ or 3.142 or $3\frac{1}{7}$.

**Worked Example 30**

A sphere of radius 2 cm is of mass 11.2 g.
Find:
(a) The volume of the sphere.
(b) The density of the sphere.
(c) The mass of the sphere of the same material but with radius 3 cm.
(Take $\pi = \frac{22}{7}$).

**SOLUTION**

(a) **Given:** $r = 2$ cm

Mass $= 11.2$ g

Volume $= V$

**Formula:** $V = \frac{4}{3}\pi r^3$

$$V = \frac{4}{3} \times \frac{22}{7} \times 2^3$$

$$= \frac{704}{21}\ \text{cm}^3$$

$$= 33.52\ \text{cm}^3$$

(b) Density $= \frac{mass}{vol}$

$= 11.2 \div 33.52\ \text{g/cm}^3$

$= 0.334\ \text{g/cm}^3$

(c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{27}{1}$

$= 113.14\ \text{cm}^3$

So, Mass $=$ density $\times$ volume

$= 0.334 \times 113.14$

$= 37.789$ g

$= 37.8$ g

## Worked Example 31

A hollow sphere has a volume of $K\ \text{cm}^3$ and a surface area of $K\ \text{cm}^2$. Calculate the diameter of the sphere. **(WAEC)**

### SOLUTION

**Given:** Volume $= K\ \text{cm}^3$

Surface area $= K\ \text{cm}^2$

Diameter $= ?$

**Formula:** Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Hence, $K = \frac{4}{3}\pi r^3$

and $K = 4\pi r^2$

$$\therefore\ \frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\frac{r^3}{r^2} = 4 \times \pi \times \frac{3}{4} \times \frac{1}{\pi}$$

$$r = 3\ \text{cm}$$

So, $d = 2 \times 3\ \text{cm} = 6\ \text{cm}$

The diameter of the sphere is 6 cm.

## Exercise 6

1. The surface area of a sphere is 616 $m^2$. What is the volume of the sphere, correct to two significant figures? (Take $\pi = 3.142$) (NECO)
2. The radius of a sphere is 21 cm. If $s$ and $v$ represent the curved surface area and volume respectively, what is $s{:}v$? (Take $\pi = 3.142$) (NECO)
3. If the surface area of a sphere is 616 $cm^2$, calculate its volume. (NECO)
4. Find the radius of a sphere whose surface area is 154 $cm^2$. (Take $\pi = \frac{22}{7}$) (UME)
5. Find the surface area of a sphere whose radius is 3.5 cm. correct your answer to one decimal place. (Take $\pi = \frac{22}{7}$) (NECO)
6. Calculate the volume of a sphere whose surface area is 314 $cm^2$.
7. Calculate the surface area of a sphere whose diameter is 42 cm.
8. The area of a circular plate is one-sixteenth the surface area of a ball. If the area of the plate is 154 $cm^2$, find the radius of the ball.
9. A solid sphere has radius 3 cm, a solid right cone has radius 3 cm and height 12 cm and a solid right circular cylinder has radius 3 cm and height 4 cm. Which of the three solids has the greatest volume?

## SUMMARY

**In this chapter, we have learnt the following:**

v The unit of measure for area is $cm^2$, $m^2$ or $km^2$, while the unit of measure for volume is $cm^3$, $m^3$ or $km^3$.

❖ (i) The total surface area of a cuboid is calculated as $A = 2(lb + bh + lh)$, where $l$, $b$ and $h$ are respectively the length, the breadth and the height.

(ii) The volume of a cuboid is calculated as $V = lbh$, where $l$ is the length, $b$ is the breadth and $h$ isthe height.

❖ The total surface area of a cube is calculated as $A = 4l^2$, where $l$ is the length of side.

❖ (i) The total surface area of a cylinder is calculated as $A = 2\pi r(r + h)$, where $r$ is the radius, $h$ is the height and $\pi = \frac{22}{7}$ or 3.142.

(ii) Volume of a cylinder is calculated as $V = \pi r^2 h$, where $r$ is the radius, $h$ is the height and $\pi = \frac{22}{7}$ or 3.142.

❖ (i) The total surface area of a cone is calculated as $A = \pi r(r + l)$, where $r$ is the radius, $l$ is the slant height and $\pi = \frac{22}{7}$ or 3.142 and;

(ii) The volume of a cone is calculated as $V = \frac{1}{3}\pi r^2 h$, where $r$ is the radius, $h$ is the height and $\pi = \frac{22}{7}$

❖ (i) The surface area of a pyramid is the addition of the area of the base and that of the side faces.

(ii) Volume of the pyramid is calculated as $V = \frac{1}{3}Ah$, where $A$ is the area of the base and $h$ is the perpendicular height.

❖ (i) The surface area of a sphere is calculated as $A = 4\pi r^2$, where $r$ is the radius and $\pi = \frac{22}{7}$ or 3.142

(ii) Volume of a sphere is calculated as $V = \frac{4}{3}\pi r^3$, where $r$ is the radius and $\pi = \frac{22}{7}$ or 3.142.

## GRADUATED EXERCISES

1. A cuboid has a diagonal of length 9 cm and a square base of side 4 cm. What is its volume?
2. Calculate the total surface area of a cube of length √2 cm.
3. Find the total surface area of the solid below.

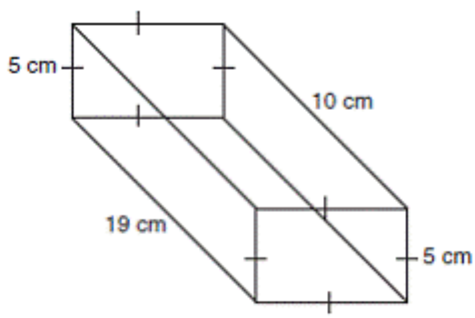



Figure 8.13

4. A cylindrical container closed at both ends has a radius of 7 cm and height of 5 cm. Find the following:
   (a) Total surface area of the container.
   (b) Volume of the container. **(WAEC)**
5. A cylindrical pipe is 28 m long. Its internal radius is 3.5 cm and external radius 5 cm. Calculate:
   (a) The volume, in $cm^3$, of metal used in making the pipe.
   (b) The volume of water, in litres that the pipe can hold when full, correct to one decimal place.
   (Take $\pi = \frac{22}{7}$)
6. A cylinder with radius 3.5 cm hasits two ends closed. If the total

surface area is 209 $cm^2$, calculate the height of the cylinder. (Take $\pi = \frac{22}{7}$) **(WAEC)**

7.

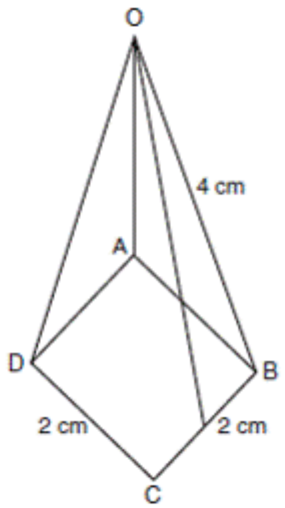



Figure 8.14

In the diagram, OABCD is a pyramid with a square base of side 2 cm and a slant height of 4 cm. Calculate the following, correct to three significant figures:

(a) The vertical height of the pyramid.

(b) The volume of the pyramid. **(WAEC)**

8. The total surface area of the walls of a room, 7 m long, 5 m wide and *x* m high is 96 $cm^2$. Find the value of *x*. **(WAEC)**

9. A pyramid of volume 120 $cm^3$ has a rectangular base which measures 5 cm by 6 cm. Calculate the height of the pyramid.

10.

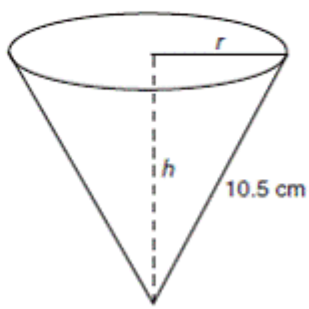



Figure 8.15

The diagram shows a cone with a slant height 10.5 cm. If the curved surface area of the cone is 115.5 $cm^2$, calculate the following, correct to three significant figures: (a) Base radius, *r*

(b) Height, *h;*

(c) Volume of the cone (Take $\pi = \frac{22}{7}$) **(WAEC)**

11. Calculate the total surface area of a solid cone of slant height 15 cm and base radius 8 cm in terms of π.

12. The base of a pyramid is a square of side 8 cm. if its vertex is directly above the centre, fi nd the height, given that the edge is 3 cm. **(UME)**

13. A cylindrical tank has a capacity of 3080 $m^3$. What is the depth

of the tank if the diameter of its base is 14m? **(UME)**

14. Calculate (a) The curved surface area and (b) The volume of a cone of height 16cm and base diameter 24cm. express the answer in terms of $\pi$.

15. Find the volume of a sphere whose radius is 1.4 m. (Take $\pi = \frac{22}{7}$ )