CHAPTER 11 Probability

Probability is a mathematical tool that helps topredict the likelihood of an event, and probabilities are expressed (quantified) as fractions of a collection of all possible outcomes. Hence, since the probabilities of possibilities are fractions, the sum total of all the possibilities related to an event can be called a whole (that is one). If I go to a restaurant that prepares chicken, beef and snail; given that I have equal likelihood of taking any one of the three varieties of meat, the probability that I will take chicken will be $\frac{1}{3}$, the probability that I will take beef will be $\frac{1}{3}$, and the probability that I will take snail is $\frac{1}{3}$.

As there are **only** three possibilities that relate to the event of 'eating meat at a restaurant,' then, the summation (addition) of the probabilities of the three possibilities **MUST** be equal to 1.

That is $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$, which is indeed equal to 1. In addition, the possibilities of my experience at the restaurant can be listed as the elements in sample space, *S*:

S = {*chicken, beef, snail*}

The sample space is a set of **all** the likely outcomes in a particular event, while the event space is a subset of the sample space and it contains element that satisfies the conditions of event *E*. Should we be asked to find the probability that I will take a chicken in the restaurant, then, the event space, *E* will be the event that 'I will take chicken at the restaurant.' Thus, *E* = {*chicken*}. The probability that an event *E* will happen at the expense of other possibilities in the sample space

is $\dfrac{\text{Number of elements in event space, } E}{\text{Number of elements in sample space, } S}$.

Therefore, the probability that I will eat chicken in the restaurant will be $\frac{1}{3}$.

The probabilities of taking beef or snail were also gotten by the same method.

Complimentary Events

Considering the restaurant example again, permit *P* to be the event that 'I will eat chicken at the restaurant,' and event *Q*, the event that 'I will NOT eat chicken at the restaurant.' If 'I will **NOT** eat chicken at the restaurant,' then I will eat beef **OR** snail (as these are the other alternatives).

Therefore, the elements of event *Q* are as follows: *Q* = {*beef, snail*}.

A careful look at events *P* and *Q* suggests that there are **NO** element common to them; thus,

$P \cap Q = \{\}$. For this reason, events *P* and *Q* are called complimentary events.

Recall that the probability that an event, E will occur

is $\frac{\textit{Number of elements in event space, E}}{\textit{Number of elements in sample space, S}}$.

Hence, $\Pr(\textit{that event P will occur}) = \frac{1}{3}$, while
$\Pr(\textit{that event Q will occur}) = \frac{2}{3}$.

As a convention in probability, the probability that 'an event will occur' is referred to as the probability of success, and it is represented as p, while the probability that the event will **not** occur' is named the probability of failure and is represented as q; and, as both events are complimentary events, $p + q = 1$. For this reason, in the case of our example about the restaurant,

$$p + q = \frac{1}{3} + \frac{2}{3} = 1.$$

Independent Events

Two events A and B are said to be independent of each other if the occurrence of event A **does not in any way** affect the likelihood of event B occurring. And so, if events A and B are independent events, the probability that events A **and** B will occur is expressed as Pr(that event A will occur) × Pr(that event B will occur).

Mutually Exclusive Events

Two events are mutually exclusive of each other if both events **CANNOT** happen together at a time. For instance, in a single throw of a fair die (the ludo game die), 'the event of getting an even number' and 'the event of getting a prime number' are **NOT** mutually exclusive events. This is because, should 2 show up, 2 is an even number and also a prime number, hence, the two events can occur at a time, therefore 'the event of getting an even number' and 'the event of getting a prime number' are **NOT** mutually exclusive.

Binomial Probability

The Binomial probability distribution presents a short method for dealing with potentially cumbersome probability problems. There is no big deal in Binomial probability. Let us examine the fundamental principle of the Binomial probability distribution, and we will do these with the help of the illustration below.

A survey carried out shows that 1 out of every 10 students in Ogbomoso Grammar School is left handed. If 5 students are chosen from the school at random, find the probability that 2 out of the 5 students are left handed.

We will first use the conventional probability method (you may find this tiring; wait and see!), later, we will make use of the shorter Binomial probability method to solve the same problem.

The conventional method

As the survey revealed that 1 out of every 10 students are left handed, the probability that a student from the school is left handed will be

$$\frac{\textit{number of left handed students surveyed}}{\textit{Total number of students surveyed}} = \frac{1}{10}.$$

The event that 'a student chosen is left handed' and the event that 'a student chosen is **NOT** left handed' are complimentary events, therefore,

$$\Pr\begin{pmatrix} \text{that a student} \\ \text{chosen is} \\ \text{left handed} \end{pmatrix} + \Pr\begin{pmatrix} \text{that a student} \\ \text{chosen is NOT} \\ \text{left handed} \end{pmatrix} = 1;$$

$$\Pr\begin{pmatrix} \text{that a student} \\ \text{chosen is NOT} \\ \text{left handed} \end{pmatrix} = 1 - \Pr\begin{pmatrix} \text{that a student} \\ \text{chosen is} \\ \text{left handed} \end{pmatrix}$$

$$= 1 - \tfrac{1}{10} = \frac{9}{10}.$$

If a student is not left handed however, the only alternative is that he will be right handed; for this reason, the event that 'a student chosen is **NOT** left handed' is equivalent to the event that 'a student is right handed.'

$$\text{Therefore, } \Pr\begin{pmatrix} \text{that a student} \\ \text{chosen is NOT} \\ \text{left handed} \end{pmatrix} = \Pr\begin{pmatrix} \text{that a student} \\ \text{chosen is} \\ \text{right handed} \end{pmatrix}.$$

The 5 students in question are represented below as *A, B, C, D* and *E*; the left handed students are identified with letter *L*, while the right handed students are identified with letter *R*. The following is a list of **all the possible outcomes** of the event of finding 2 left handed students out of 5 students sampled:

H: A_L *and* B_L *and* C_R *and* D_R *and* E_R *or*

J: A_L *and* B_R *and* C_L *and* D_R *and* E_R *or*

K: A_L *and* B_R *and* C_R *and* D_L *and* E_R *or*

M: A_L *and* B_R *and* C_R *and* D_R *and* E_L *or*

N: A_R *and* B_L *and* C_L *and* D_R *and* E_R *or*

S: A_R *and* B_L *and* C_R *and* D_L *and* E_R *or*

T: A_R *and* B_L *and* C_R *and* D_R *and* E_L *or*

U: A_R *and* B_R *and* C_L *and* D_L *and* E_R *or*

V: A_R *and* B_R *and* C_L *and* D_R *and* E_L *or*

W: A_R *and* B_R *and* C_R *and* D_L *and* E_L *or*

The expression *H*: *AL and BL and CR and DR and ER* means that event *H* is the outcome that student *A* is left handed, student *B* is also left handed, student *C* is right handed, student *D* is right handed and student *E* is also right handed.

This same explanation holds for each of the subscript of each student in the other sets of events listed.

All the 5 sub events under event *H* are independent of each other. For instance, student *A* being left handed does **NOT** make it certain that another student chosen will be left handed, or will **NOT**. Thus, the occurrence of any one of the five events does **NOT IN ANY WAY** affect the outcome of other 4 sub events, so the 5 events are independent of each other. Therefore,

$\Pr(H) = \Pr(A_L) \times (B_L) \times (C_R) \times (D_R) \times (E_R)$

All the 5 sub events under event H are independent of each other. For instance, student A being left handed does **NOT** make it certain that another student chosen will be left handed, or will **NOT**. Thus, the occurrence of any one of the five events does **NOT IN ANY WAY** affect the outcome of other 4 sub events, so the 5 events are independent of each other. Therefore,

$\Pr(H) = \Pr(A_L) \times (B_L) \times (C_R) \times (D_R) \times (E_R)$

$$= \frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

$$= \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3$$

$$= \frac{81}{100{,}000}.$$

Recall that the probability of choosing a left handed student is $\frac{1}{10}$, while the probability of choosing a right handed student is $\frac{9}{10}$.

By the same explanation, the solutions for the probabilities of events J, K, M, N, S, T, U, V and W will be as follows:

$$\Pr(J) = \frac{1}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(K) = \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(M) = \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(N) = \frac{9}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(S) = \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(T) = \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(U) = \frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(V) = \frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{1}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3;$$

$$\Pr(W) = \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{1}{10} = \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3.$$

You may go through events H to W to see that all the ten events are subsets of event E, which is the event that 2 out of 5 students sampled are left handed. Thus, $E = H \cup J \cup K \cup M \cup N \cup S \cup T \cup U \cup V \cup W$. You will also observe that it is **NOT** possible for two (or more) of these events to happen at a time; it is only one of the ten events that can happen at a time. Hence, the ten events are mutually exclusive of each other. So, $\Pr(E) = \Pr(H \cup J \cup K \cup M \cup N \cup S \cup T \cup U \cup V \cup W)$

$= \Pr(H) + \Pr(J) + \Pr(K) + \Pr(M) + \Pr(N) + \Pr(S)$

$+ \Pr(T) + \Pr(U) + \Pr(V) + \Pr(W) = 10 \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3$. Take a careful look at $10 \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3$

to notice that rather than go through this cumbersome method, one can get the number '10' by finding the number of ways that 2 students can be picked from 5 different students, and this is given by ${}^5C_2 = \frac{5!}{(5-2)2!} = 10.$

So, this is how the ten subset of event E came about. Furthermore, one observes that the probabilities of each of the possible outcomes of set E has the value $\left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^3$, which is the product of each of the independent outcomes of each sub sets (or sub event) of E. Since the problem demanded for the probability of getting 2 left handed students , the probability, p of success will be the probability of getting left handed students, while probability of failure, q, for this problem, will be the probability of **NOT** getting a left handed student; **for this question**, the probability of **NOT** getting a left handed student is equivalent to the probability of getting right handed students. Hence, instead of the long and cumbersome method, one can identify p (in this case it is _ 11_0_) and multiply it by another p (since we are looking at getting 2 left handed students).

Moreover, we can identify q (in this case _ 19_0_) and multiply it by two other qs as $q \times q \times q$, and in all this gives ($p \times p \times q \times q \times q$). Adding this in the 10 places expresses will give $10 \times (p \times p \times q \times q \times q)$, and inserting the values of p and q into this will give the answer. All we have explained can be summarized as follows:

$$10 \times (p \times p \times q \times q \times q) = 10(p)^2(q)^3 = 5C^2(p)^2(q)^3$$
$$= 5C^2(p)^2(q)^{5-2}.$$

This means that Probability of getting 2 success in 5 trial is given by the binomial probability distribution, $5C^2(p)^2(q)^{5-2}$. This is the Binomial probability in a nut shell. We can then make a general statement that Probability of getting r success in n trial is given by the binomial probability distribution, ${}^nC_2(p)r(q)^{n-r}$.

Kindly note that if the illustration had demanded that we find the probability that out of 5 students surveyed 3 are right handed, then the event of success will be the event of choosing a right handed student. And so, the probability p, of success will be the probability of choosing a right handed student, which is $\frac{9}{10}$.

In case you do not fully understand the concept of the Binomial probability as explained, please read through the illustration once more, then, come along to the workshop.

Probability

1. Two identical bags X and Y contain 10 and 6 identical balls respectively. X contains 4 black, 4 green and 2 red balls while Y contains 2 black, 3 green and 1 red balls. If a bag is selected at random, followed by a selection of a ball from it, what is the probability of drawing

(a) a green ball from X,

(b) a red ball,

(c) either a green or a black ball. **(WAEC)**

Workshop

(a) Before a green ball can be drawn from bag X, bag X must have been selected from the two available bags, X and Y. So, two events are involved in drawing a green ball from X. The first is an event that 'bag X is selected from the two available bags'; the second event is that 'a green ball was drawn from bag X'.

Looking at these two events carefully, you will see that selecting bag X from the two available bags does not in any way, affect the event of drawing a green ball from X. For example, selecting bag X from the two available bags does not make it certain that a green ball will be selected, or will **not** be selected from bag X, as any one of the black or green or red balls in X, can be selected. For this reason, these two events are independent of each other. Moreover, if events A and B are independent of each other, then

$$\begin{pmatrix} \textit{Probaibility that events} \\ A \text{ and } B \textit{ will occur} \end{pmatrix} = \Pr\begin{pmatrix} \textit{that event } A \\ \textit{will occur} \end{pmatrix} \times \Pr\begin{pmatrix} \textit{that event } B \\ \textit{will occur} \end{pmatrix}.$$

Thus,

$$\Pr\begin{pmatrix} \textit{of drawing} \\ \textit{a green} \\ \textit{ball} \\ \textit{from } X \end{pmatrix} = \Pr\begin{pmatrix} \textit{of selecting} \\ \textit{bag } X \end{pmatrix} \times \Pr\begin{pmatrix} \textit{of drawing} \\ \textit{a green} \\ \textit{ball from} \\ \textit{bag } X \textit{ after} \\ \textit{selecting} \\ \textit{bag } X \end{pmatrix}$$

$$\Pr\begin{pmatrix} \textit{of selecting} \\ \textit{bag } X \text{ (1 bag)} \\ \textit{out of 2 bags} \\ X \textit{ and } Y \end{pmatrix} = \frac{\begin{matrix}\textit{number of elements} \\ \textit{in event space}\end{matrix}}{\begin{matrix}\textit{number of elements} \\ \textit{in sample space}\end{matrix}} = \frac{1}{2}.$$

$$\Pr\begin{pmatrix} \textit{of drawing} \\ \textit{a green ball} \\ \textit{from bag } X, \\ \textit{after selecting} \\ \textit{bag } X \textit{ from} \\ \textit{the two bags} \end{pmatrix} = \frac{4}{4+4+2} = \frac{4}{10} = \frac{2}{5}$$

This is because there are 4 green balls in bag X, which contains a total of 10 balls.

$$\Pr(\textit{of drawing a green ball from } X) = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}.$$

(b) Let E be an event that a red ball was drawn. The possible subsets of event E are as listed below: A, bag X was selected **and** a red ball was drawn from bag X, B, bag Y was selected **and** a red ball was drawn from bag Y. A and B are the **only** two subsets of E; therefore, $E = A \cup B$. Events A and B are mutually exclusive since they cannot occur together at the same time. Thus, $\Pr(E) = \Pr(A \cup B) = \Pr(A) + \Pr(B)$. By the same explanation for question (a) above, before a red ball is drawn from bag X, bag X must first be selected, **and** later, a red ball will be drawn from it.

So, $\Pr(A) = \Pr\begin{pmatrix} \textit{that bag X was selected,} \\ \textbf{and } \textit{a red ball was} \\ \textit{selected from bag X,} \\ \textit{after selecting bag X} \end{pmatrix}$.

These two events are independent of each other, therefore,

$$\Pr(A) = \Pr\begin{pmatrix} \textit{that bag} \\ \textit{X was} \\ \textit{selected} \end{pmatrix} \times \Pr\begin{pmatrix} \textit{that a red ball} \\ \textit{was selected} \\ \textit{from bag X} \\ \textit{after selecting} \\ \textit{bag X} \end{pmatrix}$$

$$= \frac{1}{2} \times \frac{2}{4+4+2} = \frac{1}{2} \times \frac{2}{10} = \frac{2}{10} = \frac{1}{10}.$$

$$\Pr(B) = \Pr\begin{pmatrix} \textit{that bag Y was selected,} \\ \textbf{and } \textit{a red ball was} \\ \textit{selected from bag Y,} \\ \textit{after selecting bag Y} \end{pmatrix}.$$

These two events are also independent, and so,

$$\Pr(B) = \Pr\begin{pmatrix} \textit{that bag Y} \\ \textit{was selected} \end{pmatrix} \times \Pr\begin{pmatrix} \textit{that a red} \\ \textit{ball was} \\ \textit{later selected} \\ \textit{from bag Y} \\ \textit{after select-} \\ \textit{ing bag Y} \end{pmatrix}$$

$$= \frac{1}{2} \times \frac{2}{2+3+1} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}.$$

Hence, Pr(*of drawing a red ball*) = Pr(E) = Pr(A) + Pr(B).

$\Pr(E) = \frac{1}{10} + \frac{1}{12} = \frac{11}{60}$. Therefore, probability of drawing a red ball is $\frac{11}{60}$.

*While attempting probability questions, pay CAREFUL attention to the words, '**or**' and '**and**', especially the ones linking events, in the question. Often times in probability, '**or**' implies **addition** of the events joined by '**or**', while '**and**' implies **multiplication** of the events joined by '**and**'. However, one should not write this in an exam. This is just a trick to let you know when events are mutually exclusive, or independent of each other. At times, these two words does not show up in probability question, but one or both of them can be inferred by careful analysis of the problem: a very good example is the way we inferred 'or' in solution to 1(b) and (c), leading to the addition sign (+) that we used.*

(c) Let F be an event that either a green or a black ball was drawn; the following will be the possible subsets of F:

G, bag X was selected and a green ball was drawn from bag X,

H, bag Y was selected and a green ball was drawn from bag Y,

J, bag X was selected and a black ball was drawn from bag X,

K, bag Y was selected and a black ball was drawn from bag Y.

G, H, J and K are the subsets of F, thus,

$F = G \cup H \cup J \cup K$

The events G, H, J and K are mutually exclusive, since they cannot occur together at the same time, hence,

$\Pr(F) = \Pr(G \cup H \cup J \cup K)$

$= \Pr(G) + \Pr(H) + \Pr(J) + \Pr(K)$

$\Pr(G)$ is the probability that bag X was selected, **and** a green ball was later drawn from it. There are 2 sub events in event G, which are the event that **'bag X was selected'**, and the event that **'a green ball was selected from bag X'.** These two events are independent of each other, since the occurrence of one does not, in any way, affect the occurrence of the other, so that:

$$\Pr(G) = \Pr\begin{pmatrix}\text{that bag } X \\ \text{was selected}\end{pmatrix} \times \Pr\begin{pmatrix}\text{that a green ball} \\ \text{was later selected} \\ \text{from bag } X\text{, after} \\ \text{selecting bag } X\end{pmatrix}$$

$$\Pr(G) = \frac{1}{2} \times \frac{2}{4+4+2} = \frac{1}{2} \times \frac{4}{10} = \frac{4}{10} = \frac{1}{5}.$$

$\Pr(H)$ is the probability that bag Y was selected, **and** a green ball was later drawn from it. These two events are also independent of each other, therefore,

$$\Pr(H) = \Pr\begin{pmatrix}\text{that bag } Y \\ \text{was selected}\end{pmatrix} \times \Pr\begin{pmatrix}\text{that a green ball} \\ \text{was later selected} \\ \text{from bag } Y\text{, after} \\ \text{selecting bag } Y\end{pmatrix}$$

$\Pr(H) = \frac{1}{2} \times \frac{3}{2+3+1} = \frac{1}{2} \times \frac{3}{6} = \frac{3}{12} = \frac{1}{4}$.

$\Pr(J)$ is the probability that bag X was selected, and a black ball was later drawn from it. Also, these two events are independent of each other, hence,

$$\Pr(J) = \Pr\begin{pmatrix}\textit{that bag } X \\ \textit{was selected}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{that a black ball} \\ \textit{was later selected} \\ \textit{from bag } X\textit{, after} \\ \textit{selecting bag } X\end{pmatrix}$$

$\Pr(J) = \frac{1}{2} \times \frac{2}{4+4+2} = \frac{1}{2} \times \frac{4}{10} = \frac{4}{10} = \frac{1}{5}$.

$\Pr(K)$ is the probability that bag Y was selected, and a black ball was later drawn from it. You can also see that the occurrence of the event that bag Y was selected does not, in any way, affect the event that a blackball was drawn from bag Y. Therefore, the two events are independent of each other, so that

$$\Pr(K) = \Pr\begin{pmatrix}\textit{that bag } Y \\ \textit{was selected}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{that a black ball} \\ \textit{was later selected} \\ \textit{from bag } Y\textit{, after} \\ \textit{selecting bag } Y\end{pmatrix}$$

$\Pr(K) = \frac{1}{2} \times \frac{2}{2+3+1} = \frac{1}{2} \times \frac{2}{6} = \frac{2}{12} = \frac{1}{6}$.

Hence, $\Pr(F) = \Pr(G) + \Pr(H) + \Pr(J) +$

$\Pr(K) = \frac{1}{5} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{49}{60}$.

Therefore, the probability of drawing either a green or black ball is $= \frac{49}{60}$.

2. A bag **L** contains 5 white and 3 black balls. Another bag **M** contains 4 white and 3 black balls. A ball is picked at random from **L** and put into **M**. A ball is then drawn at random from **M**. Find the probability that the ball drawn from **M** is:

(a) of the same colour as that drawn from **L**;

(b) of a different colour from that drawn from **L**. **(WAEC)**

Workshop

$$\Pr\begin{pmatrix}\textit{that an event } E\\ \textit{will occur}\end{pmatrix} = \frac{\begin{array}{c}\textit{number of elements}\\ \textit{in event space}\end{array}}{\begin{array}{c}\textit{number of elements}\\ \textit{in sample space}\end{array}}.$$

$$\text{Therefore, } \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a white ball}\\ \textit{from } \mathbf{L}\end{pmatrix} = \frac{\begin{array}{c}\textit{number of white}\\ \textit{balls in bag } \mathbf{L}\end{array}}{\begin{array}{c}\textit{total number of}\\ \textit{balls in bag } \mathbf{L}\end{array}}$$

$$= \frac{5}{5+3} = \frac{5}{8}.$$

$$\Pr\begin{pmatrix}\textit{of drawing a}\\ \textit{white ball from}\\ \mathbf{M}\textit{, after putting}\\ \textit{a white ball from}\\ \textit{bag } \mathbf{L} \textit{ into } \mathbf{M}\end{pmatrix} = \frac{4+1}{7+1} = \frac{5}{8},$$

since by adding a white ball to the balls in **M**, the number of white balls in **M** will increase by one, and in turn, the total number of balls will increase by 1.

$$\Pr\begin{pmatrix}\textit{of drawing a black}\\ \textit{ball from } \mathbf{L}\end{pmatrix} = \frac{3}{5+3} = \frac{3}{8}.$$

$$\Pr\begin{pmatrix}\textit{of drawing a black}\\ \textit{ball from bag } \mathbf{M}\\ \textit{after putting a}\\ \textit{black ball from}\\ \textit{bag } \mathbf{L} \textit{ into } \mathbf{M}\end{pmatrix} = \frac{3+1}{7+1} = \frac{4}{8} = \frac{1}{2}.$$

$$\Pr\begin{pmatrix}\textit{of drawing a white ball from}\\ \textit{bag } \mathbf{M} \textit{ after putting a black}\\ \textit{ball from bag } \mathbf{L} \textit{ into } \mathbf{M}\end{pmatrix} = \frac{4}{7+1} = \frac{4}{8} = \frac{1}{2},$$

as a black ball was added, the number of white balls is the same; i.e 4, while the total number of balls had increased by one (1).

$$\Pr\begin{pmatrix}\textit{of drawing a black ball}\\ \textit{from } \mathbf{M} \textit{ after putting a}\\ \textit{white ball from } \mathbf{L} \textit{ into } \mathbf{M}\end{pmatrix} = \frac{3}{7+1} = \frac{3}{8}.$$

(a) Let E be an event that the ball drawn from **M**, is of **the same colour** as that, drawn from **L.** The possible subsets of event E are as follows:

A, an event that a white ball was drawn from **L, and** a white ball was later drawn from **M,** after putting the white ball from bag **L** into bag **M,**

B, an event that a black ball was drawn from **L, and** a black ball was later drawn from bag **M** after putting the black ball drawn from bag **L,** into bag **M.**

From the explanation above, $E = A \cup B$. Events A and B are mutually exclusive, since the two cannot occur at the same time. So, $\Pr(E) = \Pr(A \cup B) = \Pr(A) + \Pr(B)$ (*mutually exclusive events*).

$$\Pr(A) = \Pr\begin{pmatrix}\textit{that a white ball is drawn}\\ \textit{from } \mathbf{M}\textit{, given that a}\\ \textit{white ball has been drawn}\\ \textit{from } \mathbf{L}\textit{, and put into } \mathbf{M}\end{pmatrix}$$

The Probability that an event X will occur, given that event Y had occurred is calculated as, $\Pr(X) \times \Pr(Y)$.

$$\text{Therefore, } \Pr(A) = \Pr\begin{pmatrix}\textit{that a white ball}\\ \textit{is drawn from}\\ \mathbf{M}\textit{, given that a}\\ \textit{white ball has}\\ \textit{been drawn}\\ \textit{from } \mathbf{L}\textit{, and put}\\ \textit{into } \mathbf{M}\end{pmatrix}$$

$$= \Pr\begin{pmatrix}\textit{that a}\\ \textit{white ball}\\ \textit{was drawn}\\ \textit{from } \mathbf{L}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{that a white ball}\\ \textit{was drawn from}\\ \mathbf{M}\textit{, after putting}\\ \textit{the white ball}\\ \textit{from } \mathbf{L}\textit{, into } \mathbf{M}\end{pmatrix}$$

$\Pr(A) = \frac{5}{8} \times \left[\frac{4+1}{7+1}\right] = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$. By the same argument as above

$$\Pr(B) = \Pr\begin{pmatrix}\textit{that a black ball is drawn}\\ \textit{from } \mathbf{M}\textit{, given that a black}\\ \textit{ball has been drawn from}\\ \mathbf{L}\textit{, and put into } \mathbf{M}\end{pmatrix}$$

$$= \Pr\begin{pmatrix}\textit{that a}\\ \textit{black ball}\\ \textit{was drawn}\\ \textit{from } \mathbf{L}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{that a black}\\ \textit{ball was also}\\ \textit{drawn from } \mathbf{M}\textit{,}\\ \textit{after putting the}\\ \textit{black ball from}\\ \mathbf{L}\textit{, into } \mathbf{M}\end{pmatrix}$$

$\Pr(B) = \frac{3}{8} \times \left(\frac{3+1}{7+1}\right)$ *(as 1 black ball had been added to* **M***.)*

$\Pr(B) = \frac{3}{8} \times \frac{4}{8} = \frac{3}{16}$. Thus,

$\Pr(E) = \Pr(A) + \Pr(B) = \frac{25}{64} + \frac{3}{16} = \frac{37}{64}$.

(b) Let F be an event that the ball drawn from bag **M** is of a different colour from that drawn from bag **L**, then the possible subsets of this event are as follows:

C, a white ball was drawn from **L**, and a black ball was later drawn from **M**, after putting the white ball drawn from **L** into **M**,

D, a black ball was drawn from **L** and a white ball was later drawn from **M** after putting the black ball drawn from **L**, into **M**.

From the explanations above, events C and D are the only subsets of F, hence, $F = C \cup D$. Events C and D are mutually exclusive, and for this reason, $\Pr(F) = \Pr(C \cup D) = \Pr(C) + \Pr(D)$. Going by the same argument, as explained in question 13a,

$$\Pr(C) = \Pr\begin{pmatrix} \text{that a black ball is drawn} \\ \text{from } \mathbf{M}\text{, given that a white} \\ \text{ball has been drawn from } \mathbf{L}\text{,} \\ \text{and put into } \mathbf{M} \end{pmatrix}$$

$$= \Pr\begin{pmatrix} \text{of drawing} \\ \text{a white ball} \\ \text{from } \mathbf{L} \end{pmatrix} \times \Pr\begin{pmatrix} \text{of drawing a} \\ \text{black ball from} \\ \mathbf{M}\text{, after putting} \\ \text{the white ball} \\ \text{drawn from } \mathbf{L}\text{,} \\ \text{into } \mathbf{M} \end{pmatrix}$$

$\Pr(C) = \frac{5}{8} \times \frac{3}{7+1} = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$. By the same argument, a s in question 13a,

$$\Pr(D) = \Pr\begin{pmatrix} \text{that a white ball is drawn} \\ \text{from } \mathbf{M}\text{, given that a black} \\ \text{ball has been drawn from} \\ \mathbf{L}\text{, and put into } \mathbf{M} \end{pmatrix};$$

$$\Pr(D) = \Pr\begin{pmatrix} \text{of drawing} \\ \text{a black ball} \\ \text{from bag } \mathbf{L} \end{pmatrix} \times \Pr\begin{pmatrix} \text{of drawing a} \\ \text{white ball from} \\ \mathbf{M}\text{, after putting} \\ \text{a black ball from} \\ \mathbf{L}\text{, into } \mathbf{M} \end{pmatrix}.$$

So, $\Pr(D) = \frac{3}{8} \times \frac{4}{7+1} = \frac{3}{8} \times \frac{4}{8} = \frac{12}{64} = \frac{3}{16}$.

$\Pr(F) = \Pr(C) + \Pr(D) = \frac{15}{64} + \frac{3}{16} = \frac{27}{64}$.

Therefore, the probability that the ball drawn from M is of a different colour from that, drawn from L, is $\frac{27}{64}$.

3. Two fair dice with faces numbered 1–6 and two fair coins are thrown and tossed respectively, at the same time. Find the probability that the sum of the numbers on the faces of the dice that show up is greater than seven and only one coin shows a head.

Workshop

All the **possible** outcomes of the **sum** of the two numbers that can appear on two fair dice, in a single throw of the dice, is as shown in the table below.

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Note that the numbers 1 to 6 written in bold along the first row and the first column on the table above are the possible outcomes that could show on the dice while the possible sums of these possible outcome are on the other rows and columns in the table.

$$\begin{pmatrix} \textit{Probability} \\ \textit{that an event} \\ \textit{E, will occur} \end{pmatrix} = \frac{\begin{matrix}\textit{number of elements} \\ \textit{in event space}\end{matrix}}{\begin{matrix}\textit{number of elements} \\ \textit{in sample space}\end{matrix}} = \frac{n(E)}{n(S)}.$$

$$\Pr\begin{pmatrix} \textit{that the sum of the} \\ \textit{numbers that show} \\ \textit{up on the two dies} \\ \textit{is greater than 7} \end{pmatrix} =$$

$$\frac{\begin{pmatrix}\textit{number of elements (sum of the two} \\ \textit{numbers in the two dice) greater than 7}\end{pmatrix}}{\begin{pmatrix}\textit{number of possible sums that} \\ \textit{could be gotten from the two dice}\end{pmatrix}}$$

Carefully counting from the table above, the number of elements greater than 7 is 15, while the total number of elements is 36. And so,

$$\Pr\begin{pmatrix} \textit{that the sum of the num-} \\ \textit{bers that show up on the} \\ \textit{two dies is greater than 7} \end{pmatrix} =$$

$$\frac{\textit{number of elements in event space}}{\textit{number of elements in sample space}} = \frac{15}{36} = \frac{5}{12}.$$

In a single toss of **two** fair coins, the possible outcomes are $\{HH, HT, TH, TT\}$, where H stands for head and T for tail. For example, on Nigeria's One Naira coin, we can call the side where Herbert Macaulay's portrait is engraved the head, and the side where the coat of arms is found the tail. Therefore, the number of element in sample space is 4. In this case, the event space is that only one coin shows a head. The number of elements that showed only one head is 2 which are *HT and TH.*

$$\Pr\begin{pmatrix} \textit{that only one} \\ \textit{coin shows} \\ \textit{a head} \end{pmatrix} = \frac{\begin{matrix}\textit{number of elements} \\ \textit{in event space}\end{matrix}}{\begin{matrix}\textit{number of elements} \\ \textit{in sample space}\end{matrix}} = \frac{2}{4} = \frac{1}{2}.$$

These events that 'the sum of the numbers on the faces of the dice that show up is greater than 7' **and** 'only one coin shows a head' are two independent events because the outcome of one does not in any way affect the outcome of the other. For example the event that a double six appeared on the two dice

will not make it **certain** that one of the coins tossed will show a head and the other coin a tail. If events A and B are independent of each other,

$$\Pr\begin{pmatrix}\textit{that events}\\ \text{A and B}\\ \textit{will occur}\end{pmatrix} = \Pr\begin{pmatrix}\textit{that event}\\ \text{A } \textit{will}\\ \textit{occur}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{that events}\\ \text{B } \textit{will}\\ \textit{occur}\end{pmatrix};$$

for this reason,

$$\Pr\begin{pmatrix}\textit{that the sum of the}\\ \textit{numbers on the faces of}\\ \textit{the die that show up is}\\ \textit{greater than 7 } \textbf{and} \textit{ only}\\ \textit{one coin showa a head}\end{pmatrix} =$$

$$\Pr\begin{pmatrix}\textit{that the sum of the}\\ \textit{numbers that show}\\ \textit{up on the two dies is}\\ \textit{greater than 7}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{that only one}\\ \textit{coin shows a}\\ \textit{head}\end{pmatrix}$$

$$= \frac{5}{12} \times \frac{1}{2} = \frac{5}{24}.$$

Therefore, probability that the sum of numbers on the faces of the die that show up is greater than 7 **and** only one coin shows a head is $\frac{5}{24}$.

It is good to note that in probability, the probability that event A ***and*** *B will occur* = Pr(*that A will occur*) × Pr(*that B will occur*), *provided A and B are independent of each other, that is, the occurrence of one event does not affect the occurrence of the other event. In addition, the probability that event A* ***or*** *B will occur is equal to* Pr(*that A will occur*) + Pr(*that B will occur*)*, provided A and B are mutually exclusive (that is, they cannot both occur together at the same time. Lastly you need to keep your eyes open while reading the question since the "****and****" and "****or****" will not be written in capital letters or in bold letters, in the question. Therefore, the question should be well understood, before an attempt is made at answering it.*

4. A bag contains 6 red, 5 blue and 4 green identical balls. Three balls are drawn from the bag at random without replacement.

Calculate, correct to 3 decimal places, the probability that:

(a) the balls are of the same colour;

(b) a ball from each of the three colours is drawn. **(WAEC)**

Workshop

The bag contains 6 red, 5 blue and 4 green balls. So, the total number of balls in the bag is 15.

$$\Pr\begin{pmatrix}\textit{of drawing a red ball}\\ \textit{at the first pick}\end{pmatrix} = \frac{6}{15} = \frac{2}{5},$$

$$\Pr\begin{pmatrix}\textit{of drawing another red}\\ \textit{ball } \textbf{without} \textit{ replacing}\\ \textit{the first ball picked}\end{pmatrix} = \frac{6-1}{15-1} = \frac{5}{14}.$$

Remember a red ball had been drawn first without replacement so that the total number of red balls left is 6 − 1 = 5 and the total number of balls left is 15 − 1 = 14.

$$\Pr\begin{pmatrix}\textit{of drawing another red}\\ \textit{ball } \textbf{without} \textit{ replacing}\\ \textit{the first 2 red ball picked}\end{pmatrix} = \frac{6-2}{15-2} = \frac{4}{13},$$

$$\Pr\begin{pmatrix}\textit{of drawing a blue ball}\\ \textit{at the first pick}\end{pmatrix} = \frac{5}{15} = \frac{1}{3},$$

$$\Pr\begin{pmatrix}\textit{of drawing another blue}\\ \textit{ball } \textbf{without} \textit{ replacing}\\ \textit{the first ball picked}\end{pmatrix} = \frac{5-1}{15-1} = \frac{4}{14}; = \frac{2}{7};$$

$$\Pr\begin{pmatrix}\textit{of drawing the third blue}\\ \textit{ball } \textbf{without} \textit{ replacing the}\\ \textit{first 2 blue balls drawn}\end{pmatrix} = \frac{5-2}{15-2} = \frac{3}{13};$$

$$\Pr\begin{pmatrix}\textit{of drawing a green}\\ \textit{ball at the first pick}\end{pmatrix} = \frac{4}{15},$$

$$\Pr\begin{pmatrix}\textit{of drawing another green}\\ \textit{ball } \textbf{without} \textit{ replacing}\\ \textit{the first ball picked}\end{pmatrix} = \frac{4-1}{15-1} = \frac{3}{14},$$

$$\Pr\begin{pmatrix}\textit{of drawing the third green}\\ \textit{ball } \textbf{without} \textit{ replacing the}\\ \textit{first 2 green balls drawn}\end{pmatrix} = \frac{4-2}{15-2} = \frac{2}{13}.$$

(a) Let E be the event that the three balls drawn one after the other from the bag are of the same colour. The following are the **possible** sub events (subsets) of E:

A, all the three balls drawn are red;

B, all the three balls drawn are blue;

C, all the three balls drawn are green.

Thus, $E = A \cup B \cup C$; events A, B and C are mutually exclusive since only one of the three events can occur at a time; therefore,

$\Pr(E) = \Pr(A \cup B \cup C) = \Pr(A) + \Pr(B) + \Pr(C)$.

The probability that an event, ***G***, will occur, given that event F **had** occurred is expressed as $\Pr(F) \times \Pr(G)$

$$\text{Therefore, Pr}\begin{pmatrix}\textit{of drawing a red ball at the}\\ \textit{second pick, given that a red}\\ \textit{ball was drawn at the first pick}\\ \textit{without replacement}\end{pmatrix}$$

$$= \Pr\begin{pmatrix}\textit{of drawing a red}\\ \textit{ball at the first pick}\\ \textit{without replacement}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{of drawing a}\\ \textit{red ball at the}\\ \textit{second pick}\end{pmatrix}.$$

Thus, the probability that an event H will occur, given that events F and G had occurred will be $[\Pr(F) \times \Pr(G)] \times \Pr(H) = \Pr(F) \times \Pr(G) \times \Pr(H)$.

So,

$$\Pr(A) = \Pr\begin{pmatrix}\textit{of drawing a red}\\ \textit{ball at the third}\\ \textit{pick, given that}\\ \textit{a red ball was}\\ \textit{drawn at the}\\ \textit{first and second}\\ \textit{picks, all without}\\ \textit{replacement}\end{pmatrix} = \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a red ball at}\\ \textit{the first pick}\\ \textbf{without}\\ \textit{replacement}\end{pmatrix}$$

$$\times \Pr\begin{pmatrix}\textit{of drawing a red ball}\\ \textit{at the second pick}\\ \textbf{without}\ \textit{replacement}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a red ball at}\\ \textit{the third pick}\end{pmatrix}$$

$$= \frac{2}{5} \times \frac{5}{14} \times \frac{4}{13} = \frac{40}{910} = \frac{4}{91}.$$

By the same explanation,

$$\Pr(B) = \Pr\begin{pmatrix}\textit{of drawing a blue}\\ \textit{ball at the first pick}\\ \textbf{without}\ \textit{replacement}\end{pmatrix}$$

$$\times \Pr\begin{pmatrix}\textit{of drawing a blue ball}\\ \textit{at the second pick}\\ \textbf{without}\ \textit{replacement}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a blue ball at}\\ \textit{the third pick}\end{pmatrix}$$

$$= \frac{1}{3} \times \frac{2}{7} \times \frac{3}{13} = \frac{6}{273} = \frac{2}{91}.$$

$$\Pr(C) = \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a green ball}\\ \textit{at the first}\\ \textit{pick}\ \textbf{without}\\ \textit{replacement}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a green ball}\\ \textit{at the second}\\ \textit{pick}\ \textbf{without}\\ \textit{replacement}\end{pmatrix} \times$$

$$\Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a green}\\ \textit{ball at the}\\ \textit{third pick}\end{pmatrix}$$

$$= \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \frac{24}{2{,}730} = \frac{4}{455}.$$

$$\Pr(E) = \Pr(A) + \Pr(B) + \Pr(C)$$

$$= \frac{4}{91} + \frac{2}{91} + \frac{4}{455} = \frac{20 + 10 + 4}{455}$$

$$= \frac{34}{455} = 0.075.$$

Therefore, the probability that the 3 balls drawn one after the other without replacement are of the same colour is 0.075 to 3 decimal places.

(b) If a ball from each of the three colours is drawn, it means that a blue ball (B) was drawn, a green ball (G) was drawn, a red ball (R) was drawn in any possible order. The number of ways 3 balls of 3 different colours can be picked from a bag is the possible number of ways 3 different colours of balls can be arranged picking 3 at a time, which is expressed as

$$^3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3 \times 2 \times 1 = 6 \text{ ways.}$$

*Note: permutation was used to know the number of possible arrangements because we are interested in the order in which the balls are drawn. For example, since the selection is without replacement, the event of picking the different balls in the order R B G is **different** from the event of picking the different balls in the order B R G because the*

$$\Pr\begin{pmatrix}\textit{of drawing a red ball} \\ \textit{(R) at the first pick}\end{pmatrix} = \frac{6}{15}$$

while the $\Pr\begin{pmatrix}\textit{of drawing a red ball} \\ \textit{(R) at the second pick} \\ \textit{without replacement}\end{pmatrix}$

$$= \frac{6-1}{15-1} = \frac{5}{14}.$$

So, events R B G is different from event B R G. Hence, the order in which the different balls are picked is very important. So, permutation was used instead of combination. That is BGR is different from GBR, GBR is also different from BRG, and so on.

The 6 possible ways of selecting a blue, a green, a red ball from a bag without replacement is as below:

R **and** B **and** G or R **and** G **and** B or

B **and** G **and** R or B **and** R **and** G or

G **and** R **and** B or G **and** B **and** R

The 6 underlined events above are mutually exclusive events, since only one out of the 6 events could occur **at a time**. For example, in a single outcome, one cannot pick a 'red, blue, green' combination and a 'red, green, blue' combination. One can only have at a time, either 'red, blue, green' combination **or** 'red, green, blue' combination. Therefore, these two events are mutually exclusive. Applying this to the other four possible outcomes, you will notice that all the six events are mutually exclusive.

Also, in a single throw of a fair die (ludo game die), one **cannot** get a '3' and a '6' in **a single throw** of the fair die. So, the event of 'getting a 3' and the event of 'getting a 6' in **a single throw** of a fair die are mutually exclusive events, as the two events cannot occur together. On the other hand, the event of 'getting a prime number' and

the event of 'getting an even number' in a single throw of a fair die are **not** mutually exclusive. This is because in a single throw of a fair die, the number 2 may be obtained, which is a prime number and an even number.

Thus,

$$\Pr\begin{pmatrix}\textit{of drawing R and B and G } \textbf{or} \textit{ R and G and} \\ \textit{B } \textbf{or} \textit{ B and R and G } \textit{or} \textit{ B and G and R } \textbf{or} \\ \textit{G and R and B } \textbf{or} \textit{ G and B and R balls}\end{pmatrix}$$

$$= \Pr\begin{pmatrix}R \text{ and } B \\ \text{and } G\end{pmatrix} + \Pr\begin{pmatrix}R \text{ and } G \\ \text{and } B\end{pmatrix} + \Pr\begin{pmatrix}B \text{ and } R \\ \text{and } G\end{pmatrix} +$$

$$\Pr\begin{pmatrix}B \text{ and } G \\ \text{and } R\end{pmatrix} + \Pr\begin{pmatrix}G \text{ and } R \\ \text{and } B\end{pmatrix} + \Pr\begin{pmatrix}G \text{ and } B \\ \text{and } R\end{pmatrix}$$

As we learnt earlier that probability that an event H will occur, given that events F and G had occurred will be $\Pr(F) \times \Pr(G) \times \Pr(H)$

$$\Pr\begin{pmatrix}\textit{of drawing a green ball at the third pick,} \\ \textit{given that a red ball was drawn at the} \\ \textit{first pick and a blue ball was drawn at the} \\ \textit{second picks, all without replacement}\end{pmatrix}$$

$$= \Pr\begin{pmatrix}\textit{of drawing a} \\ \textit{red ball at the} \\ \textit{first pick } \textbf{without} \\ \textit{replacement}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{of drawing a blue} \\ \textit{ball at the second} \\ \textit{pick } \textbf{without} \\ \textit{replacement}\end{pmatrix}$$

$$\times \Pr\begin{pmatrix}\textit{of drawing} \\ \textit{a green ball at} \\ \textit{the third pick}\end{pmatrix}.$$

$$\Pr(\textit{of drawing a red ball at the first pick}) = \frac{6}{15}.$$

$$\Pr\begin{pmatrix}\textit{drawing a blue } \textbf{without} \\ \textit{replacing the red ball} \\ \textit{first drawn}\end{pmatrix} = \frac{5}{15-1} = \frac{5}{14}.$$

*Note that a red ball had been drawn first without replacement, then, the total number of balls in the bag will reduce by 1 but the number of blue balls is still the same because a red ball was first picked and **not** a blue ball.*

$$\Pr\begin{pmatrix}\textit{of drawing a green ball} \\ \textbf{without} \textit{ replacing the red and} \\ \textit{blue balls already drawn}\end{pmatrix} = \frac{4}{15-2} = \frac{4}{13}.$$

Since a red and blue ball had been drawn without replacement, the total number of balls in the bag will reduce by 2 while the total number of green ball is still 4.

$$\Pr\left(\begin{array}{l}\textit{of drawing a green ball at the third pick,}\\ \textit{given that a red ball was drawn at the}\\ \textit{first pick and a blue ball was drawn at the}\\ \textit{second picks, all without replacement}\end{array}\right)$$

$$= \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} = \frac{120}{2{,}730}$$

$$\Pr\left(\begin{array}{l}\textit{of drawing a blue ball at the third pick,}\\ \textit{given that a red ball was drawn at the}\\ \textit{first pick and a green ball was drawn at}\\ \textit{the second picks, all without replacement}\end{array}\right)$$

$$\Pr\left(\begin{array}{l}\textit{of draw-}\\ \textit{ing a red}\\ \textit{ball at the}\\ \textit{first pick}\\ \textbf{without}\\ \textit{replacement}\end{array}\right) \Pr\left(\begin{array}{l}\textit{of draw-}\\ \textit{ing a green}\\ \textit{ball at the}\\ \textit{second pick}\\ \textbf{without}\\ \textit{replacement}\end{array}\right) \times$$

$$\Pr\left(\begin{array}{l}\textit{of draw-}\\ \textit{ing a blue}\\ \textit{ball at the}\\ \textit{third pick}\end{array}\right).$$

$$\Pr\,(\textit{of drawing a red ball at the first pick}) = \frac{6}{15}.$$

$$\Pr\left(\begin{array}{l}\textit{of drawing a green ball}\\ \textbf{without}\ \textit{replacing}\\ \textit{the red ball first drawn}\end{array}\right) = \frac{4}{15-1} = \frac{4}{14}.$$

$$\Pr\left(\begin{array}{l}\textit{of drawing a blue ball}\\ \textbf{without}\ \textit{replacing}\\ \textit{the red and green balls}\\ \textit{drawn}\end{array}\right) = \frac{5}{15-2} = \frac{4}{13}.$$

$$\Pr\left(\begin{array}{l}\textit{of drawing a blue ball at the third pick,}\\ \textit{given that a red ball was drawn at the}\\ \textit{first pick and a green ball was drawn at the}\\ \textit{second picks, all without replacement}\end{array}\right)$$

$$= \frac{6}{15} \times \frac{4}{14} \times \frac{5}{13} = \frac{120}{2{,}730}$$

$$\Pr\left(\begin{array}{l}\textit{of drawing a green}\\ \textit{ball at the third pick,}\\ \textit{given that a blue ball}\\ \textit{was drawn at the}\\ \textit{first pick and a red}\\ \textit{ball was drawn at}\\ \textit{the second picks, all}\\ \textit{without replacement}\end{array}\right) = \Pr\left(\begin{array}{l}\textit{of drawing}\\ \textit{a blue}\\ \textit{ball at the}\\ \textit{first pick}\\ \textbf{without}\\ \textit{replacement}\end{array}\right) \times$$

$$\Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a red ball at}\\ \textit{the second}\\ \textit{pick } \textbf{without}\\ \textit{replacement}\end{pmatrix} \times \Pr\begin{pmatrix}\textit{of drawing}\\ \textit{a green ball}\\ \textit{at the third}\\ \textit{pick}\end{pmatrix}.$$

$\Pr(\textit{of drawing a blue ball at the first pick}) = \frac{5}{15}$.

$$\Pr\begin{pmatrix}\textit{drawing a red ball}\\ \textbf{without } \textit{replacing}\\ \textit{the blue ball first}\\ \textit{drawn}\end{pmatrix} = \frac{6}{15-1} = \frac{6}{14}.$$

$$\Pr\begin{pmatrix}\textit{of drawing a green}\\ \textit{ball } \textbf{without } \textit{replac-}\\ \textit{ing the blue and red}\\ \textit{balls drawn}\end{pmatrix} = \frac{4}{15-2} = \frac{4}{13}.$$

Therefore,

$$\Pr\begin{pmatrix}\textit{of drawing a green ball at the third pick,}\\ \textit{given that a blue ball was drawn at the}\\ \textit{first pick and a red ball was drawn at the}\\ \textit{second picks, all without replacement}\end{pmatrix}$$

$$= \frac{5}{15} \times \frac{6}{14} \times \frac{4}{13} = \frac{120}{2{,}730}.$$

By this same explanation for the other three events, we will get $\frac{120}{2{,}730}$ for the probability that each of the other 3 events will occur. You can try your hands at the other calculations.

So,

$$\Pr\begin{pmatrix}\textit{of drawing R and B and G } \textbf{or } \textit{R and G and}\\ \textit{B } \textbf{or } \textit{B and R and G or B and G and R } \textbf{or}\\ \textit{G and R and B } \textbf{or } \textit{G and B and R balls}\end{pmatrix}$$

$$= \Pr\begin{pmatrix}R \text{ and } B\\ \text{and } G\end{pmatrix} + \Pr\begin{pmatrix}R \text{ and } G\\ \text{and } B\end{pmatrix} + \Pr\begin{pmatrix}B \text{ and } R\\ \text{and } G\end{pmatrix} +$$

$$\Pr\begin{pmatrix}B \text{ and } G\\ \text{and } R\end{pmatrix} + \Pr\begin{pmatrix}G \text{ and } R\\ \text{and } B\end{pmatrix} + \Pr\begin{pmatrix}G \text{ and } B\\ \text{and } R\end{pmatrix}$$

$$= \frac{120}{2{,}730} + \frac{120}{2{,}730} + \frac{120}{2{,}730} + \frac{120}{2{,}730} + \frac{120}{2{,}730} + \frac{120}{2{,}730}$$

$$= 6\left(\frac{120}{2{,}730}\right) = \frac{720}{2{,}730} = 0.264.$$

Therefore, the probability that a ball from each of the 3 colours was drawn without replacement is 0.264 to 3 decimal places.

5. A man P has 5 red, 3 blue and 2 white buses. Another man Q has 3 red, 2 blue and 4 white buses. A bus owned by P is involved in an accident with a bus belonging to Q. Calculate the probability that the two buses are **not** of the same colour. **(WAEC)**

Workshop

(a) P has 5 red buses, 3 blue buses and 2 white buses; Q has 3 red buses, 2 blue buses and 4 white buses. The probability