

CHAPTER 3

INDICES

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Objectives

At the end of the chapter, students should be able to:

1. Solve problems on standard form.
2. Represent indices and give examples using standard notation.
3. Identify indices as a shorthand notation of the standard form.
4. State the laws of indices.
5. Solve problems on indicial equations.

I. Revision of Standard Form

A number is in standard form when it is expressed in the form $p \times 10^n$, where p is a number between 1 and 10 and n is an integer. That is: $1 \leq p < 10$ and n is either a positive or negative integer. A number can be expressed in standard form from the expanded form.



Worked Example 1

(a) $50 = 5 \times 10 = 5.0 \times 10^1$

(b) $8\,752 = 8\,752.0 = 8.752 \times 1\,000$
 $= 8.752 \times 10^3$

(c) $584.22 = 5.8422 \times 100 = 5.8422 \times 10^2$

Note: Decimal numbers can also be expressed in standard form. See the examples below.



Worked Example 2



Express the following in standard form:

- (a) 0.468

- (b) 3.489
(c) 0.0049

Solution



$$\begin{aligned}\text{(a)} \quad 0.468 &= \frac{468}{1\,000} = \frac{4.68 \times 10^2}{10^3} \\ &= 4.6 \times 10^{-1}\end{aligned}$$

$$\text{(b)} \quad 3.489 = 3.489 \times 10^0$$

$$\begin{aligned}\text{(c)} \quad 0.0049 &= \frac{49}{10\,000} = \frac{4.9 \times 10^1}{10^4} \\ &= 4.9 \times 10^{-3}\end{aligned}$$

Note: Any number in standard form can also be written in expanded form. For example,

$$\text{(a)} \quad 4.68 \times 10^2 = 4.68 \times 100 = 468$$

$$\begin{aligned}\text{(b)} \quad 3.691 \times 10^{-2} &= 3.691 \times \frac{1}{100} \\ &= 0.03691\end{aligned}$$

Numbers in standard form can be added together or deducted from one another. This is done by changing the numbers to ordinary form or factoring out the common factor.



Worked Example 3

Evaluate the following leaving your answers in standard form:

- (a) $7.36 \times 10^3 + 2.045 \times 10^4$
(b) $9.025 \times 10^8 - 3.75 \times 10^2$



Solution

$$(a) 7.36 \times 10^3 + 2.045 \times 10^4$$

1st Method: Change the numbers to ordinary form.

$$\begin{aligned}7.36 \times 10^3 + 2.045 \times 10^4 \\&= 7\ 360 + 20\ 450 \\&= 27\ 810 \\&= 2.781 \times 10^4\end{aligned}$$

2nd Method: Factor out the common factor.

$$\begin{aligned}7.36 \times 10^3 + 2.045 \times 10^4 \\&= 10^3 (7.36 + 2.045 \times 10) \\&= 10^3 (7.36 + 20.45) \\&= 27.81 \times 10^3 \\&= 2.781 \times 10 \times 10^3 \\&= 2.781 \times 10^4\end{aligned}$$

$$(b) 9.025 \times 10^8 - 3.75 \times 10^2$$

1st Method: Change the numbers to ordinary form.

$$\begin{aligned}9.025 \times 10^8 - 3.75 \times 10^2 \\&= 902\ 500\ 000 - 375 \\&= 902\ 499\ 625 \\&= 9.02499625 \times 10^8\end{aligned}$$

Note: Numbers in standard form can also be multiplied or divided.



Worked Example 4



Evaluate the following leaving your answers in standard form:

- (a) $3.81 \times 10^2 \times 4.025 \times 10^5$
- (b) $15.32 \times 10^3 \div 8.251 \times 10^3$

Solution



$$\begin{aligned}(a) \quad & 3.81 \times 10^2 \times 4.025 \times 10^5 \\&= 3.81 \times 4.025 \times 10^2 \times 10^5 \\&= 15.33525 \times 10^7 \\&= 1.533525 \times 10^8\end{aligned}$$

$$\begin{aligned}(b) \quad & 15.32 \times 10^5 \div 8.251 \times 10^3 \\&= \frac{15.32 \times 10^5}{8.251 \times 10^3} = \frac{15.32 \times 10^2}{8.251} \\&= 1.856\,744\,637 \times 10^2\end{aligned}$$



Exercise 1

Express the following in standard form:

1. 4 689
2. 505
3. 816
4. 46 000
5. 1 684 321
6. 49 689
7. 896 407
8. 592.325
9. 1 689
10. 405

Express the following in standard form:

11. 0.075
12. 0.00086
13. 0.000069
14. 0.0007856
15. 0.001

Write the following in ordinary form:

16. 4.78×10^3
17. 1.68×10^{-3}
18. 9.68×10^5
19. 1.69×10^{-5}
20. 3.69×10^7

$$21. 5.505 \times 10^{-8}$$

$$22. 6.89 \times 10^{-3}$$

$$23. 5.695 \times 10^5$$

$$24. 4.19 \times 10^{13}$$

$$25. 6.895 \times 10^5$$

Evaluate the following leaving your answers in standard form:

$$26. 1.2 \times 10^7 + 3.06 \times 10^2$$

$$27. 8.69 \times 10^{-2} - 1.89 \times 10^5$$

$$28. \frac{6.82 \times 10^6 - 4.12 \times 10^2}{3.12 \times 10^7}$$

29. If a barrel of oil costs ₦350, what is the cost of 8.78×10^7 barrels of oil?

30. Calculate the area of a rectangular field measuring 4.68×10^6 cm by 8.28×10^3 cm.

II. Concept of Indices

(i) Introduction of indices

Consider the following statements:

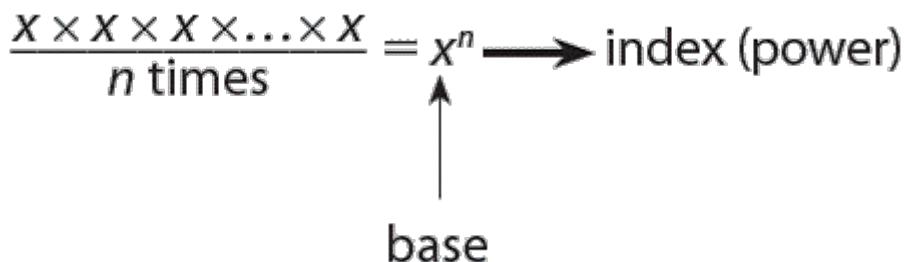
$$(a) 2 \times 2 \times 2 \times 2 = 2^4$$

$$(b) 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$$

$$(c) a \times a \times a = a^3 (a \neq 0)$$

In each case, each number is multiplied by itself repeatedly. For example, in (a), the multiplication of number 2 is repeated 4 times. Also, in (c), the multiplication of 'a' is repeated 3 times.

The number that we use to multiply repeatedly is called the base, while the number of times the number is repeated is called the index (power). Hence, in (b), '5' is the base, while '6' is the index (power). The plural of index is indices and the numbers expressed as in the examples above are said to be in index notation. Hence, in general



Express the following in index notation:

- (a) $21 \times 21 \times 21 \times 21 \times 21 \times 21 \times 21$
- (b) $y \times y \times y \times y$
- (c) $3 \times 3 \times 3 \times 3 \times 3$

Solution

- (a) $21 \times 21 \times 21 \times 21 \times 21 \times 21 \times 21 = 21^7$
- (b) $y \times y \times y \times y = y^4$
- (c) $3 \times 3 \times 3 \times 3 \times 3 = 3^5$



Worked Example 5

Write down the value of

- (a) $2_3 \times 2_6$ (b) $3_4 \times 3_7$
- (c) $y_6 \div y_2$ in index notation

Solution

$$(a) 2^3 \times 2^6 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \\ \times 2 \times 2 \times 2)$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ \times 2 \times 2$$

$$= 2^9$$

$$(b) 3^4 \times 3^7 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \\ \times 3 \times 3 \times 3 \times 3 \times 3)$$

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ \times 3 \times 3 \times 3$$

$$= 3^{11}$$

$$\begin{aligned}
 (c) \quad & y^6 \div y^2 \\
 &= (y \times y \times y \times y \times y \times y) \div (y \times y) \\
 &= \frac{y \times y \times y \times y \times y \times y}{y \times y} \\
 &= y^4
 \end{aligned}$$

Note: In the example above, (a) and (b) show that by multiplying powers of the same base, we add the indices. Also, (c) shows that to divide powers of the same base, we subtract the indices.

(ii) Laws of indices

1. Consider $x_3 \times x_2 = (x \times x \times x) \times (x \times x)$

$$= x \times x \times x \times x \times x$$

$$= x_5$$

Here, we can see that the index $5 = 3 + 2$. Thus, we say that if numbers of equal base are to be multiplied together, we can equally add their powers (indices) together to get the same result. Hence, if m and n are integers then:

$$a^m \times a^n = a^{m+n}, a \neq 0$$



Worked Example 6



Evaluate the following:

(a) $p^2 \times p^5$

(b) $10^3 \times 10^5$

(c) $y^7 \times y^6$

(d) $(3 \times 2 \times b)^2$



Solution



(a) $p^2 \times p^5 = p^{2+5} = p^7$

(b) $10^3 \times 10^5 = 10^{3+5} = 10^8$

(c) $y^7 \times y^6 = y^{7+6} = y^{13}$

(d) $(3 \times 2 \times b)^2 = 3^2 \times 2^2 \times b^2 = 36b^2$

Note:

$$(a \times b \times c)^n = a^n \times b^n \times c^n$$

2. Consider (i) $x^4 \div x^2 = \frac{x \times x \times x \times x}{x \times x}$
 $= x \times x = x^2$

Similarly (ii) $k^6 \div k^2$

$$= \frac{k \times k \times k \times k \times k \times k}{k \times k}$$
 $= k^4$

In each of the two cases above, we can see that when numbers of the same or equal bases are dividing each other, we subtract their index since in (i) the index $2 = 4 - 2$ and in (ii) the index $4 = 6 - 2$. In general, if m and n are integers,

$$a^m \div a^n = a^{m-n}, a \neq 0$$

Note:

$$a^m \div a^n = a^{m-n}$$

$m - n$ = positive, if $m > n$

$m - n = 0$, if $m = n$

$m - n$ = negative, if $m < n$



Worked Example 7

Write down the values of each of the following in index notation.

(a) $4^5 \div 4^2$ (b) $w^8 \div w^4$ (c) $\left(\frac{a}{2b}\right)^4$

Solution

- (a) $4^5 \div 4^2 = 4^{5-2} = 4^3$
- (b) $w^8 \div w^4 = w^{8-4} = w^4$
- (c) $\left(\frac{a}{2b}\right)^4 = \frac{a^4}{(2b)^4} = \frac{a^4}{16b^4}$

Note:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

- (a) $4^5 \div 4^2 = 4^{5-2} = 4^3$
- (b) $w^8 \div w^4 = w^{8-4} = w^4$
- (c) $\left(\frac{a}{2b}\right)^4 = \frac{a^4}{(2b)^4} = \frac{a^4}{16b^4}$

Note:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

3. Consider (i) $(5^2)^3 = 5^2 \times 5^2 \times 5^2$

$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$$

$$\begin{aligned} \text{(ii)} \quad (4^3)^4 &= 4^3 \times 4^3 \times 4^3 \times 4^3 \\ &= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \\ &\quad \times 4 \times 4 \times 4 \times 4 \times 4 \\ &= 4^{12} \end{aligned}$$

We can see that from (i), the index 6 is obtained by multiplying 2 by 3, that is, $6 = 2 \times 3$, and also from (ii) the index 12 is obtained by multiplying 3 by 4, that is, $12 = 3 \times 4$. In general, if m and n are integers $(a_m)_n = a_{mn}$, $a \neq 0$.



Worked Example 8



Simplify the following:

- (a) $(5_7)_2$
 (b) $(2_3)_{10}$
 (c) $(p_6)_7$

Solution

(a) $(5^7)^2 = 5^{7 \times 2} = 5^{14}$

(b) $(2^3)^{10} = 2^{3 \times 10} = 2^{30}$

(c) $(p^6)^7 = p^{6 \times 7} = p^{42}$

4. Consider (i) $2^4 \div 2^4 = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 1$

(ii) $x^3 \div x^3 = \frac{x^3}{x^3} = \frac{x \times x \times x}{x \times x \times x} = 1$

In (i), $2^4 \div 2^4 = 2^{4-4} = 2^0 = 1$ and in

(ii), $x^3 \div x^3 = x^{3-3} = x^0 = 1$

Hence, in general, $a^0 = 1, a \neq 0$.



Worked Example 9



What is the value of the following:

- (a) 3^0 (b) 9^0 (c) 100^0



Solution

- (a) $3^0 = 1$ (b) $9^0 = 1$ (c) $100^0 = 1$

Note: Any number (except zero) raised to the power zero (0) is one (1).

5. Recall that $a^m \times a^{-m} = a^{m-m} = a^0 = 1$

$$\therefore a^m \times a^{-m} = 1$$

$$\text{Hence, } a^{-m} = \frac{1}{a^m}$$

Note: Any number raised to a negative index denotes reciprocal.



Worked Example 10

Evaluate the following:

(a) 2^{-4}

(b) 3^{-5}



Solution

$$(a) 2^{-4} = \frac{1}{2^4} = \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{16}$$

$$(b) 3^{-5} = \frac{1}{3^5} = \frac{1}{3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{243}$$

6. Consider $(a^{\frac{1}{3}})^3 = a^{\frac{1}{3} \times 3} = a$

Similarly, $(a^{\frac{1}{3}})^3 = (3\sqrt{a})^3$

Hence, $a^{\frac{1}{n}} = n\sqrt{a}$ and $a^{\frac{m}{n}} = (n\sqrt{a})^m$

$$= n\sqrt{a^m}, a \neq 0$$



Worked Example 11

Evaluate the following:

(a) $9^{\frac{1}{2}}$

(b) $27^{\frac{2}{3}}$

(c) $4^{-\frac{3}{2}}$

Solution



(a) $9^{\frac{1}{2}} = \sqrt{9} = 3$

(b) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

(c) $4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$

7. Recall that $(a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

For example:

(i) $(27^2)^{\frac{1}{3}} = 27^{\frac{2}{3}} = (\sqrt[3]{27})^2$
 $= 3^2 = 9$

(ii) $(m^4)^{\frac{1}{2}} = m^{\frac{4}{2}} = m^2$

Also that $a^{\frac{-m}{n}} = \frac{1}{a^{\frac{m}{n}}}$



Exercise 2

Evaluate the following and write your answers in index form:

1. $3^2 \times 3^4$
2. $2k^2 \times 4k^5$
3. $q^3 \times q^4$
4. $xy \times 4xy^2$
5. $5p^2 \times 25pq$
6. $9x^2 \times 2x^3$
7. $5u^2 \times 2u^3$
8. $pq^2 \times 3pq^4$
9. $10^2 \times 10^7$
10. $abc \times a^2b^3c^4$

Write down the values of these in index notation.

11. $t^3 \div t^2$
12. $p^4 \div p^{-4}$

$$13. 8x^2 \div 2x^4$$

$$14. w^7 \div w^2$$

$$15. 25k^3 \div 125k^5$$

$$16. 92y^3 \div 70y^0$$

$$17. 3a^3 \div a$$

$$18. 9 \times \frac{9^{\frac{-1}{2}}}{27^{\frac{2}{3}}}$$

$$19. \frac{2(3^5 - 1)}{1}$$

$$20. \frac{4^{-\frac{1}{2}} \times 16^{\frac{1}{2}}}{4^{\frac{3}{2}}}$$

$$21. 10a^2 \div 2a$$

$$22. (p^3)^{\frac{1}{2}}$$

$$23. (4^{\frac{1}{2}})^3$$

$$24. (-p^2)^4$$

$$25. (3u^2pq)^5$$

Simplify the following:

$$26. 0.125^{\frac{-1}{3}}$$

$$27. 2^{-3} \times 3^{-2}$$

$$28. 3^{-2} \div 3^{-3}$$

$$29. (0.01)^3$$

$$30. (64^{\frac{1}{2}})^5$$

$$31. 32^{\frac{-1}{5}}$$

$$32. 27^4 \div 81^3$$

$$33. \left(\frac{64}{81}\right)^{\frac{-1}{4}}$$

$$34. \left(\frac{27}{125}\right)^{\frac{-1}{3}} \times 3 \div \left(\frac{9}{25}\right)^{\frac{-1}{2}}$$

$$35. \left(\frac{25}{144}\right)^{\frac{1}{2}}$$

III. Application of Indices: Simple Indicial Equations

A simple equation that involves indices is called an *indicial equation*. These equations are otherwise called *exponential equations*.



Worked Example 12



Find x , if $81^{-x} = 3 \times 9^{1+x}$



Solution



$$81^{-x} = 3 \times 9^{1+x}$$

$$(3^4)^{-x} = 3 \times (3^2)^{1+x}$$

$$3^{-4x} = 3 \times 3^{2+2x}$$

$$3^{-4x} = 3^{3+2x}$$

Now, equating the powers, we have:

$$-4x = 3 + 2x$$

$$-3 = 2x + 4x$$

$$\therefore 6x = -3$$

$$x = -\frac{3}{6} = -\frac{1}{2} \quad \therefore x = -\frac{1}{2}$$



Worked Example 13



Solve the following:

(a) $2(2^x) = 64$

(b) $7^x = 49^{-2}$

(c) $(0.25)^{x+3} = 16$



Solution

$$(a) \ 2(2^x) = 64$$

$$2^{1+x} = 2^6$$

Equating the powers, we have:

$$1 + x = 6$$

$$x = 6 - 1 = 5$$

$$\therefore x = 5$$

$$(b) \ 7^x = 49^{-2}$$

$$7^x = (7^2)^{-2} = 7^{-4}$$

$$\therefore x = -4$$

$$(c) \ (0.25)^{x+3} = 16$$

$$\left[\frac{25}{100}\right]^{x+3} = 16$$

$$\left(\frac{1}{4}\right)^{x+3} = 4^2$$

$$(2^{-2})^{x+3} = (2^2)^2$$

$$2^{-2(x+3)} = 2^4$$

Equating the powers:

$$-2(x + 3) = 4$$

$$x + 3 = -2$$

$$x = -2 - 3$$

$$\therefore x = -5$$

Some exponential (indicial) equations can be reduced to quadratic forms (see the examples below).



Worked Example 14

Solve the following equations:

$$(a) 2^{2x} - 8(2^x) + 16 = 0$$

$$(b) 5^{2y+1} - 26(5^y) + 5 = 0$$

$$(c) (4^x) - 12(2^x) + 32 = 0$$

Solution

$$(a) 2(2^x) = 64$$

$$2^{1+x} = 2^6$$

Equating the powers, we have:

$$1 + x = 6$$

$$x = 6 - 1 = 5$$

$$\therefore x = 5$$

$$(b) 7^x = 49^{-2}$$

$$7^x = (7^2)^{-2} = 7^{-4}$$

$$\therefore x = -4$$

$$(c) (0.25)^{x+3} = 16$$

$$\left[\frac{25}{100}\right]^{x+3} = 16$$

$$\left(\frac{1}{4}\right)^{x+3} = 4^2$$

$$(2^{-2})^{x+3} = (2^2)^2$$

$$2^{-2(x+3)} = 2^4$$

Equating the powers:

$$-2(x + 3) = 4$$

$$x + 3 = -2$$

$$x = -2 - 3$$

$$\therefore x = -5$$

Some exponential (indicial) equations can be reduced to quadratic forms (see the examples below).



Worked Example 14

Solve the following equations:

(a) $2^{2x} - 8(2^x) + 16 = 0$

(b) $5^{2y} + 1 - 26(5^y) + 5 = 0$

(c) $(4^x) - 12(2^x) + 32 = 0$

Solution

(a) $2^{2x} - 8(2^x) + 16 = 0$

$$2^2(2^x)^2 - 8(2^x) + 16 = 0$$

Now substitute $z = 2^x$

$$z^2 - 8z + 16 = 0$$

$$(z - 4)^2 = 0 \text{ (factorising)}$$

$z = 4$ (twice).

But $z = 2^x$, $2^x = 4 = 2^2$

Hence, $x = 2$

$$(b) \quad 5^{2y+1} - 26(5^y) + 5 = 0$$

$$5 \cdot 5^{2y} - 26(5^y) + 5 = 0$$

$$5 \cdot (5^y)^2 - 26(5^y) + 5 = 0$$

Now substitute $x = 5^y$

$$\therefore 5x^2 - 26x + 5 = 0$$

$$(x - 5)(5x - 1) = 0 \text{ (factorising)}$$

$$\therefore x - 5 = 0 \text{ or } 5x - 1 = 0$$

$$x = 5 \text{ or } x = \frac{1}{5}$$

Now, if $x = 5$, it follows that $5^y = 5^1$.

Hence, $y = 1$. Similarly, if $x = \frac{1}{5}$,

it follows that $5^y = \frac{1}{5} = 5^{-1}$

$$\therefore y = -1$$

Finally, $y = 1$ or -1

$$(c) \quad (4^x) - 12(2^x) + 32 = 0$$

$$(2^2)^x - 12(2^x) + 32 = 0$$

$$(2^x)^2 - 12(2^x) + 32 = 0$$

Now substitute $k = 2^x$

$$\therefore k^2 - 12k + 32 = 0$$

$$(k-4)(k-8) = 0$$

$$\therefore k = 4 \text{ or } 8$$

When $k = 4$, we have $2^x = 4$ (since $k = 2^x$)

$$\therefore 2^x = 2^2$$

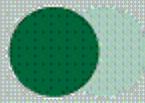
$$x = 2$$

When $k = 8$, $2^x = 8$

$$2^x = 2^3$$

$$\therefore x = 3$$

Finally, $x = 2$ or 3



Exercise 3

Solve the following equations:

$$1. \left(\frac{1}{3}\right)^x = 9$$

$$2. 2(2^x) = 8$$

$$3. 7x = \frac{1}{49}$$

$$4. 10^x = 0.0001$$

$$5. 2^{2x} + 2(2^x) - 8 = 0$$

$$6. 3^{2y} + 2(3^y) - 3 = 0$$

$$7. 3^y = 243$$

$$8. 9^{2x+1} = \frac{81^{x-2}}{3^x} \quad (\text{NECO})$$

$$9. 4^x = 2^{\frac{1}{2}} \times 8 \quad (\text{WAEC})$$

$$10. \frac{1}{3^{5n}} \times 9^{n-1} \times 27^{n+1} \quad (\text{WAEC})$$

$$11. \frac{6^{2n+1} \times 9^n \times 4^{2n}}{18^n \times 2^n \times 12^{2n}} \quad (\text{UME})$$

Solve the following equations:

$$12. a^{\frac{1}{2}} = 3$$

$$13. m^{-1} = 5$$

$$14. 9^y = 81$$

$$15. 9^{2-1} = 27$$

$$16. 2x^{-3} = -16$$

$$17. 10y = \frac{80}{y^{\frac{1}{2}}}$$

$$18. (7^y)^2 + 1 = 2 \times 7^y$$

$$19. (2^w)^2 + 8 - 6(2^w) = 0$$

$$20. 5^{2k} - 125 = -4 \times 5^{k+1}$$

SUMMARY

In this chapter, we have learnt the following:

◆ A number is in standard form when it is expressed in the form $P \times 10_n$, where P is a whole number between 1 and 10 and n is an integer.

◆ **Laws of Indices**

$$(a) a^m \times a^n = a^{m+n}$$

$$(b) a^m \div a^n = a^{m-n}, m - n$$

= positive, if $m > n$

= 0 if $m = n$

= negative, if $m < n$

$$(c) (a^m)^n = a^{mn}$$

$$(d) a^0 = 1, a \neq 0$$

$$(e) \ a^{-n} = \frac{1}{a^n}$$

$$(f) \ a^{\frac{m}{n}} = (\sqrt[n]{a})^m = n \sqrt{a^m}$$

$$(g) \left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}}$$

$$(h) \ a^{\frac{-m}{n}} = \frac{1}{a^{\frac{m}{n}}}$$

GRADUATED EXERCISES

Write the following in standard form:

1. 468.17
2. 598 056
3. 0.0001
4. 8 675.97
5. 5 689
6. 0.000898

Write the following in ordinary form:

7. 4.76×10^3
8. 1.69×10^{-4}
9. 7.86×10^6
10. 9.86×10^{-5}
11. 9.69×10^8

Write down the values of the following in index notation:

$$12. a^2 \times a^{-2}$$

$$13. 4xy \times 2x^2y^3$$

$$14. 5p \times 4p^3$$

$$15. 3^4 \div 9^2$$

$$16. 2^{-2} \times 2^3$$

$$17. 15^{\frac{1}{3}} \times 45^{\frac{2}{3}} \times 3^{\frac{1}{3}}$$

Simplify the following:

$$18. (8x^{-1})^{\frac{2}{3}}$$

$$19. 9^{\frac{1}{2}}$$

$$20. (125x^6)^{\frac{1}{3}}$$

$$21. \frac{3^n - 3^{n-1}}{3^3 \times 3^n - 27 \times 3^n - 1}$$

$$22. \frac{9^{\frac{1}{2}} \times 27^{\frac{1}{3}}}{3 - \frac{1}{8} \times 3 - \frac{1}{2}}$$

$$23. (2^0 + 4^{\frac{1}{2}})^2$$

Solve the equation:

$$24. 81^x = 3 \times 9^{1+x}$$

$$25. 4^{\frac{x}{2}} = 16$$

$$26. 2^{2x} - 6(2^x) + 8 = 0$$

$$27. 4x = \frac{1}{4}$$

$$28. \left(\frac{1}{10}\right)^x = 100$$

$$29. 5^{2-x} = 125$$

$$30. 10^{2x} = 1000$$

$$31. 5^x = 1 - 3^x = -27$$

$$32. 10^{x-1} = 0.01$$

$$33. \left(\frac{1}{2}\right)^x = 8$$

Let $f(x) = 2^x$, $g(x) = \left(\frac{1}{3}\right)^x$, $h(x) = 10^x$, $m(x) = e^x$.

Find the values of x in each of the following equations:

$$34. f(x) = 32$$

$$35. g(x) = 27$$

$$36. g(x) = \frac{1}{9}$$

$$37. h(x) = 1000$$

$$38. m(x) = e^3$$

$$39. m(x) = \frac{1}{e}$$

$$40. h(x) = 0.1$$