CHAPTER 6

At the end of the chapter, students should be able to:
1. Change the subject of a given equation.
2. Solve problems involving direct, inverse, joint and partial variations.
3. Apply variations to physical laws and real life situations.

I. Solving Linear Equations

A linear equation is an equation of the form $ax + b = c$, where a, b and c are constants. Examples are $2x + 4 = 0$, $4y + 3 = 0$ etc.

Worked Example 1

Solve $3x - 1 = 5 - 6x$.

Solution

$3x - 1 = 5 - 6x$
Add $6x$ to both sides
$3x - 1 + 6x = 5 - 6x + 6x$
$9x - 1 = 5$
Add 1 to both sides
$9x - 1 + 1 = 5 + 1$
$9x = 6$
Divide both sides by 9

$$\frac{9 \times x}{9} = \frac{6}{9}$$

$$x = \frac{2}{3}$$

Worked Example 2

$5(7 - n) = 3\ (2n + 1)$
Open the bracket on both sides
$35 - 5n = 6n + 3$

Add $5n$ to both sides

$35 - 5n + 5n = 6n + 3 + 5n$

$35 = 11n + 3$

Subtract 3 from both sides

$35 - 3 = 11n + 3 - 3$

$32 = 11n$

Divide both sides by 11

$$\frac{32}{11} = \frac{\cancel{11} \times n}{\cancel{11}}$$

$$n = 2\frac{10}{11}$$

Worked Example 3

Solve $2 - (5 - k) + 6 = (3 - 6k) - (3 + k)$.

Solution

$2 - (5 - k) + 6 = (3 - 6k) - (3 + k)$

Open brackets on both sides

$2 - 5 + k + 6 = 3 - 6k - 3 - k$

$3 + k = -7k$

Add $7k$ to both sides

$3 + k + 7k = -7k + 7k$

$3 + 8k = 0$

Subtract 3 from both sides

$3 + 8k - 3 = 0 - 3$

$8k = -3$

Divide both sides by 8

$$\frac{8 \times k}{8} = \frac{-3}{8}$$

$$k = \frac{-3}{8}$$

$$\frac{4x}{5} - 2 = \frac{5x}{8} + 5$$

Subtract $\frac{5x}{8}$ from both sides

$$\frac{4x}{5} - 2 - \frac{5x}{8} = \frac{5x}{8} + 5 - \frac{5x}{8}$$

$$\frac{4x}{5} - \frac{5x}{8} - 2 = 5$$

Add 2 to both sides

$$\frac{32x-25x}{40}-2+2=5+2$$

$$\frac{7x}{40}=7$$

Multiply both sides by $\frac{40}{7}$

$$\frac{\cancel{7}}{\cancel{40}}\times x\times\frac{\cancel{40}}{\cancel{7}}=\frac{7}{1}\times\frac{40}{7}$$

$$x=40$$

Worked Example 5

Solve $\frac{4}{5}(4p-1)=\frac{2}{3}(3p-4)-\frac{1}{5}$.

Solution

$$\frac{4}{5}(4p-1)=\frac{2}{3}(3p-4)-\frac{1}{5}$$

Open brackets on both sides

$$\frac{4}{5}\times\frac{4p}{1}-\frac{4}{5}\times\frac{1}{1}=\frac{2}{\cancel{3}}\times\frac{\cancel{3}p}{1}-\frac{2}{3}\times\frac{4}{1}-\frac{1}{5}$$

Multiply through by 15

$$\frac{16p}{\cancel{5}}\times\frac{\overset{3}{\cancel{15}}}{1}-\frac{4}{\cancel{5}}\times\frac{\overset{3}{\cancel{15}}}{1}=\frac{2p}{1}\times\frac{15}{1}$$

$$-\frac{8}{\cancel{3}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{1}}-\frac{\cancel{1}}{\cancel{5}}\times\frac{\overset{3}{\cancel{15}}}{\cancel{1}}$$

$$48p-12=30p-40-3$$

$$48p-12=30p-43$$

Subtract $30p$ from both sides

$48p - 30p - 12 = 30p - 30p - 43$

$18p - 12 = -43$

Add 12 to both sides

$18p - 12 + 12 = -43 + 12$

$18p = -31$

Divide both sides by 18

$$\frac{18 \times p}{18} = -\frac{31}{18}$$

$$P = -1\frac{13}{18}$$

Exercise 1

Solve the following equations:

1. $a = 2a + 3$
2. $b = 1 - b$
3. $5c + 3 = 3c + 5$
4. $d + 2d + 3d = 1$
5. $e - 1 - 2e = 2 + 5e$
6. $7f - 18 = 4 + 2f$
7. $40 = 8 + 24g$
8. $11 - 3h - 2 = 8 - 4h$
9. $2k - 15\frac{1}{2} = 2\frac{1}{2} + 11k$
10. $5\frac{1}{2}m = 44$

11. $5n - 100 = 0$

12. $\frac{1}{4}p = \frac{3}{4}p - 1$

13. $8q + 7q = 15$

14. $4r = 9 + r$

15. $2a + b = b$

16. $[4(1 + a)] - 5a = 0$

17. $4b - 2(b + 6) = 2$

18. $3(7c + 5) - 15(3c - 4) = 3$

19. $0 = 8 - (3x - 7)$

20. $4m = 18 - (6m - 12)$

21. $10 - (5 - 8n) = 11 - 4n$

22. $3v - 4(1 - v) = 1 - (v + 2)$

23. $8k - [4(1 + k) - 3k] = 6$

24. $9u - (10u + 15) = 7 + u$

25. $5(y + 2) - 2(3y - 5) = 4$

26. $\frac{3a}{2} + \frac{5a}{2} + \frac{5a}{4} = 7$

27. $\frac{4p - 2}{5} + \frac{3}{4} = \frac{8p}{15}$

28. $\frac{5k}{6} - \frac{4}{15} = \frac{8k - 1}{9}$

29. $\frac{x}{4} - \frac{5x - 1}{2} = 1 - \frac{3x}{8}$

30. $\frac{6x + 5}{7} - \frac{x - 4}{14} = \frac{x}{21}$

31. $6c - 4\frac{7}{12} = \frac{5c}{6}$

32. $\frac{n}{8} - 1\frac{1}{4} + \frac{9}{12} = \frac{8n + 3}{4}$

33. $\frac{1}{7}y - \frac{8}{21} = \frac{5}{14} - \frac{16y}{21}$

34. $\frac{d - 5}{6} - \frac{3 - 2d}{3} = 4$

35. $\frac{10e - 1}{6} - \frac{5e}{12} = \frac{e}{2}$

II. Change of Subject of a Formula

Formula is a rule expressed in the form of an equation. Examples are:

(i) $V = lbh$, where V stands for the volume of a cuboid, l the length, b the breadth and h the height.

(ii) $A = 2\pi r(r + h)$, where A stands for the total surface area of a cylinder, r the radius, h the height and π equals $\frac{22}{7}$ or 3.142.

(iii) $P = 2(l + b)$, where P stands for the perimeter of a rectangle, l the length and b the breadth.

In each of the three formulae above, there are two sides to the equation, namely the left-hand side (LHS) and the right-hand side (RHS).
The subject of the formula:

(i) $V = lbh$ is V

(ii) $A = 2\pi r(r + h)$ is A

(iii) $P = 2(l + b)$ is P

At times, there may be a need to make any of the variables the subject of the formula. In $V = lbh$, l, b or h can be made the subject as follows:

$$l = \frac{V}{bh},\ b = \frac{V}{lh} \text{ and } h = \frac{V}{lb}$$

Worked Example 6

If $\frac{x}{a+1} + \frac{y}{b} = 1$, make y the subject of the relation.

$$\frac{x}{a+1} + \frac{y}{b} = 1$$

Subtract $\frac{x}{a+1}$ from both sides

$$\left(\frac{x}{a+1}\right) + \left(\frac{y}{b}\right) - \left(\frac{x}{a+1}\right) = 1 - \left(\frac{x}{a+1}\right)$$

$$\left(\frac{y}{b}\right) = 1 - \left(\frac{x}{a+1}\right)$$

Multiply both sides by b

$$\frac{y}{\cancel{b}} \times \frac{\cancel{b}}{1} = \left(\frac{1(a+1) - x}{a+1}\right) \times \frac{b}{1}$$

$$y = \frac{b(a + 1 - x)}{a+1}$$

$$y = \frac{b(a - x + 1)}{a+1}$$

Worked Example 7

Make h the subject of the formula in: $S = 2\pi rh + 2\pi r^2$.

Solution

$S = 2\pi rh + 2\pi r^2$

Subtract $2\pi r^2$ from both sides

$S - 2\pi r^2 = 2\pi rh + 2\pi r^2 - 2\pi r^2$

$S - 2\pi r^2 = 2\pi rh$

Divide both sides by $2\pi r$

$$\frac{S - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$

$$\frac{S - 2\pi r^2}{2\pi r} = h$$

$$h = \frac{S - 2\pi r^2}{2\pi r}$$

Worked Example 8

Make R the subject of the relation $n = \frac{2R}{R - r}$.

Solution

$$n = \frac{2R}{R - r}$$

Multiply both sides by $(R - r)$

$$n(R - r) = \frac{2R}{(R - r)} \times \frac{(R - r)}{1}$$

Open the brackets on the LHS

$nR - nr = 2R$

Add nr to both sides

$nR - nr + nr = 2R + nr$

$nR = 2R + nr$

Subtract $2R$ from both sides

$nR - 2R = nr$

Factorize the LHS

$R(n-2) = nr$

Divide $\dfrac{R \times \cancel{(n-2)}}{\cancel{(n-2)}} = \dfrac{nr}{n-2}$

$R = \dfrac{nr}{n-2}$

Worked Example 9

Make w the subject of the relation

$\left(\dfrac{a+bc}{wd+f}\right) = g.$

Solution

$\left(\dfrac{a+bc}{wd+f}\right) = g.$

Multiply both sides by $(wd+f)$

$\dfrac{a+bc}{\cancel{(wd+f)}} \times \dfrac{\cancel{(wd+f)}}{1} = g \times (wd+f)$

Open the brackets on the RHS

$a + bc = gwd + gf$

Subtract gf from both sides

$a + bc - gf = gwd + gf - gf$

$a + bc - gf = gwd$

Divide both sides by gd

$\dfrac{a+bc}{gd} - gf = \dfrac{\cancel{g}w\cancel{d}}{\cancel{gd}}$

$\dfrac{a+bc}{gd} - gf = w$

$w = \dfrac{a+bc}{gd} - gf$

Change of subject of a formula involving brackets, roots and powers

When the formula involves a bracket, open the bracket before carrying out other arithmetic operations.

Worked Example 10

If $h(m + n) = m(h + r)$, find h in terms of m, n and r. (WAEC)

Solution

$h(m + n) = m(h + r)$

Open the brackets on both sides

$hm + hn = hm + mr$

Subtract hm from both sides

$hm + hn - hm = hm + mr - hm$

$hn = mr$

Divide both sides by n

$$\frac{hn}{n} = \frac{mr}{n}$$

$$h = \frac{mr}{n}$$

Worked Example 11

If $y = \frac{(b + 2x)}{4(b + x)}h$, make b the subject of the formula.

Solution

$$y = \frac{(b + 2x)}{4(b + x)}h$$

Open the brackets on the RHS

$$y = \frac{bh + 2xh}{4b + 4x}$$

Multiply both sides by $(4b + 4x)$

$$y(4b + 4x) = \frac{(bh + 2hx)}{\cancel{(4b + 4x)}} \times \cancel{(4b + 4x)}$$

Open the brackets on both sides

$4by + 4xy = bh + 2hx$

Subtract $4xy$ from both sides

$4by + 4xy - 4xy = bh + 2hx - 4xy$

$4by = bh + 2hx - 4xy$

Divide both sides by $4y$

$$\frac{\cancel{4}b\cancel{y}}{\cancel{4}\cancel{y}} = \frac{bh + 2hx - 4xy}{4y}$$

$$b = \frac{bh + 2hx - 4xy}{4y}$$

Worked Example 12

Make m the subject of the relation

$$y = \sqrt{\frac{m + k}{k(1 + km)}}.$$

Solution

$$y = \sqrt{\frac{m + k}{k(1 + km)}}$$

Square both sides

$$y^2 = \left(\sqrt{\frac{m + k}{k(1 + km)}}\right)^2$$

$$y^2 = \frac{m + k}{k(1 + km)}$$

Multiply both sides by $k(1 + km)$

$$y^2 \times k(1 + km) = \frac{m + k}{\cancel{k(1 + km)}} \times \frac{\cancel{k(1 + km)}}{1}$$

$$ky^2(1 + km) = m + k$$

Open the bracket on the LHS

$$ky^2 + k^2my^2 = m + k$$

Subtract m from both sides

$$ky^2 + k^2my^2 - m = m + k - m$$

$$ky^2 + k^2my^2 - m = k$$

Subtract ky^2 from both sides

$$ky^2 + k^2my^2 - m - ky^2 = k - ky^2$$

$$k^2my^2 - m = k - ky^2$$

Factorise both sides

$$m(k^2y^2 - 1) = k(1 - y^2)$$

Divide both sides by $(k^2y^2 - 1)$

$$\frac{m\cancel{(k^2y^2 - 1)}}{\cancel{(k^2y^2 - 1)}} = \frac{k(1 - y^2)}{(k^2y^2 - 1)}$$

$$m = \frac{k(1 - y^2)}{(k^2y^2 - 1)}$$

Worked Example 13

Express y in terms of a, b and x, if $x = \sqrt{\frac{a}{y} + \frac{b}{y}}$.

Solution

$$x = \sqrt{\frac{a}{y} + \frac{b}{y}}$$

Square both sides

$$x^2 = \left(\sqrt{\frac{a}{y} + \frac{b}{y}}\right)^2$$

$$x^2 = \frac{a}{y} + \frac{b}{y}$$

Simplify the RHS as a single fraction

$$x^2 = \frac{a + b}{y}$$

Multiply both sides by y

$$x^2 \times y = \frac{a+b}{\cancel{y}} \times \cancel{y}$$

$$x^2 \times y = a + b$$

Divide both sides by x^2

$$\frac{\cancel{x^2} \times y}{\cancel{x^2}} = \frac{a+b}{x^2}$$

$$y = \frac{a+b}{x^2}$$

Worked Example 14

If $WP^2 x = \frac{Rml}{50} + P^2$,

make P the subject of the formula.

Solution

$$WP^2 x = \frac{Rml}{50} + P^2$$

Subtract P^2 from both sides

$$WP^2 x - P^2 = \frac{Rml}{50} + P^2 - P^2$$

$$WP^2 x - P^2 = \frac{Rml}{50}$$

Factorise the LHS

$$P^2 (Wx - 1) = \frac{Rml}{50}$$

Divide both sides by $Wx - 1$

$$\frac{P^2 \cancel{(Wx-1)}}{\cancel{(Wx-1)}} = \frac{Rml}{50} \times \frac{1}{(Wx-1)}$$

$$P^2 = \frac{Rml}{50(Wx-1)}$$

Square root both sides

$$\sqrt{P^2} = \sqrt{\frac{Rml}{50(Wx-1)}}$$

$$P = \sqrt{\frac{Rml}{50(Wx-1)}}$$

Worked Example 15

Make K the subject of the relation

$$V = \frac{mg\cos\alpha}{K^3}.$$

Solution

$$V = \frac{mg\cos\alpha}{K^3}$$

Multiplying both sides by K^3

$$V \times K^3 = \frac{mg\cos\alpha}{\cancel{K^3}} \times \frac{\cancel{K^3}}{1}$$

$$K^3V = mg\cos\alpha$$

Divide both sides by V

$$\frac{K^3\cancel{V}}{\cancel{V}} = \frac{mg\cos\alpha}{V}$$

$$K^3 = \frac{mg\cos\alpha}{V}$$

Cube root both sides

$$\sqrt[3]{K^3} = \sqrt[3]{\frac{mg\cos\alpha}{V}}$$

$$K = \sqrt[3]{\frac{mg\cos\alpha}{V}}$$

Exercise 2

1. Make 'a' the subject of the formula, if $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (NECO)
2. If $\frac{1}{v} + \frac{1}{u} = \frac{2}{f}$, find u in terms of v and f. (NECO)
3. Make h the subject of the equation $A = \pi r\sqrt{h^2 - r^2}$. (NECO)
4. Make r the subject of the relation $A = \pi[(r + l)^2 - r^2]$. (WAEC)
5. If $\sqrt{x^2 + 1} = \frac{5}{4}$, find the positive value of x. (WAEC)
6. Make A the subject of the equation $g = \sqrt{\frac{h}{A - 3}}$. (NECO)
7. If $z = \frac{x}{m} - \frac{h}{y}$, make y the subject of the formula. (NECO)
8. If $s = \sqrt{\frac{a^2}{x^2} - \frac{b^2}{y^2}}$, express y in terms of a, b and x. (NECO)
9. Make f the subject of the formula $s = ut + \frac{1}{2}ft^2$. (WAEC)
10. Make x the subject of the formula $v = w\sqrt{a^2 + x^2}$.
11. Make r the subject of the formula $\frac{R - r}{R + r} = a$. (WAEC)
12. Given that $V = \pi h^2 r - \frac{4}{5}h$, express r in terms of V, h and π. (WAEC)
13. Make r the subject of the expression $\frac{1}{t} = \frac{1 - r}{100}$. (WAEC)

14. Make s the subject of the formula $V = \frac{K}{\sqrt{T-s}}$. (WAEC)

15. Given the equation $m = pq + rq^2$, express p in terms of m, q and r. (WAEC)

16. If $y = \sqrt{ax - b}$, express x in terms of y, a and b. (WAEC)

17. Make t the subject of the formula in $K = m\sqrt{\frac{t-p}{r}}$. (WAEC)

18. If $x = \frac{a-rt}{1-r}$, express r in terms of a, s and t. (WAEC)

19. Make m the subject of the relation $g = \frac{m+bc}{md+p}$.

20. If $P = \left(\frac{Q(R-T)}{15}\right)^{\frac{1}{3}}$, make T the subject of the relation. (WAEC)

III. Subject of Formula and Substitution

In any given formula, the value of the subject can be obtained by substituting the various given values for other variables in the formula.

Worked Example 16

Given that $D = L\left(\frac{M}{K} + \frac{U^2}{25}\right)$

(a) Make U the subject of the formula.
(b) Find the value of U, when $D = 6$, $L = 8$, $M = 1$ and $K = 18$.

Solution

(a) $D = L\left(\frac{M}{K} + \frac{U^2}{25}\right)$

Divide both sides by L

$$\frac{D}{L} = \frac{L}{L}\left(\frac{M}{K} + \frac{U^2}{25}\right)$$

$$\frac{D}{L} = \left(\frac{M}{K} + \frac{U^2}{25}\right)$$

Subtract $\frac{M}{K}$ from both sides

$$\frac{D}{L} - \frac{M}{K} = \frac{M}{K} + \frac{U^2}{25} - \frac{M}{K}$$

$$\frac{D}{L} - \frac{M}{K} = \frac{U^2}{25}$$

Multiply both sides by 25

$$25 \times \left(\frac{D}{L} - \frac{M}{K}\right) = \frac{U^2}{\cancel{25}} \times \frac{\cancel{25}}{1}$$

$$25 \times \left(\frac{D}{L} - \frac{M}{K}\right) = U^2$$

Find the square root of both sides

$$\sqrt{25} \times \sqrt{\left(\frac{D}{L} - \frac{M}{K}\right)} = \sqrt{U^2}$$

$$\pm 5 \times \sqrt{\left(\frac{D}{L} - \frac{M}{K}\right)} = U$$

$$U = \pm 5 \times \sqrt{\left(\frac{D}{L} - \frac{M}{K}\right)} = 5\sqrt{\left(\frac{D}{L} - \frac{M}{K}\right)}$$

(b) Given that $D = 6$, $L = 8$, $M = 1$ and $K = 18$.

$$U = 5 \times \sqrt{\left(\frac{6}{8} - \frac{1}{18}\right)}$$

$$U = 5 \times \sqrt{\left(\frac{3}{4} - \frac{1}{18}\right)}$$

$$U = 5 \times \sqrt{\frac{54 - 4}{72}}$$

$$= 5 \times \sqrt{\frac{50}{72}}$$

$$= 5 \times \sqrt{\frac{25}{36}}$$

$$= 5 \times \frac{5}{6} = \frac{25}{6} = 4\frac{1}{6}$$

Worked Example 17

Make t the subject of the relation

$K = M\sqrt{\frac{t-3}{t-9}}$.

Find the value of t, if $K = 2$ and $M = 1$.

Solution

$$K = M\sqrt{\frac{t-3}{t-9}}$$

Divide both sides by M

$$\frac{K}{M} = \frac{\cancel{M}}{\cancel{M}}\sqrt{\frac{t-3}{t-9}}$$

$$\frac{K}{M} = \sqrt{\frac{t-3}{t-9}}$$

Square both sides

$$\left(\frac{K}{M}\right)^2 = \left(\sqrt{\frac{t-3}{t-9}}\right)^2$$

$$\frac{K^2}{M^2} = \frac{t-3}{t-9}$$

Multiply both sides by $t-9$

$$\frac{K^2}{M^2}(t-9) = \frac{t-3}{\cancel{(t-9)}} \times \cancel{(t-9)}$$

$$\frac{K^2}{M^2}(t-9) = t-3$$

Multiply both sides by M^2

$$\frac{K^2(t-9)}{\cancel{M^2}} \times \frac{\cancel{M^2}}{1} = (t-3) \times M^2$$

Open the brackets on both sides

$$K^2t - 9K^2 = M^2t - 3M^2$$

Add $3M^2$ to both sides

$K^2t - 9K^2 + 3M^2 = M^2t - 3M^2 + 3M^2$

$K^2t - 9K^2 + 3M^2 = M^2t$

Subtract K^2t from both sides

$\cancel{K^2t} - 9K^2 + 3M^2 - \cancel{K^2t} = M^2t - K^2t$

$3M^2 - 9K^2 = M^2t - K^2t$

Factorise both sides

$3(M^2 - 3K^2) = t(M^2 - K^2)$

Divide both sides by $M^2 - K^2$

$$\frac{3(M^2 - 3K^2)}{M^2 - K^2} = \frac{t\cancel{(M^2 - K^2)}}{\cancel{(M^2 - K^2)}}$$

$$t = \frac{3(M^2 - 3K^2)}{(M^2 - K^2)}$$

Given that $K = 2$ and $M = 1$

$$t = \frac{3[(1)^2 - 3\,(2)^2]}{(1)^2 - (2)^2}$$

$$= \frac{3[1 - 3 \times 4]}{1^2 - 4}$$

$$= \frac{3[1 - 12]}{-3}$$

$$= \frac{3 \times -11}{-3}$$

$$= \frac{\cancel{-33}}{\cancel{-3}} = \frac{11}{1}$$

$= 11$

Worked Example 18

Given that $E = \dfrac{W(R - r)}{2RP}$,

(a) Make R the subject of the formula.

(b) Find the value of R, correct to three significant figures, when $r = 12$, $P = 60$, $W = 1\,024$ and $E = \frac{5}{6}$.

Solution

$$E = \frac{W(R-r)}{2RP}$$

Multiply both sides by $2RP$

$$E \times 2RP = \frac{W(R-r)}{\cancel{2RP}} \times \frac{\cancel{2RP}}{1}$$

$$E \times 2RP = W(R-r)$$

Open the bracket on the RHS

$$2RPE = WR - Wr$$

Add Wr to both sides

$$2RPE + Wr = WR - Wr + Wr$$

$$2RPE + Wr = WR$$

Subtract $2RPE$ from both sides

$$\cancel{2RPE} + Wr - \cancel{2RPE} = WR - 2RPE$$

$$Wr = WR - 2RPE$$

Factorise the RHS

$$Wr = R[W - 2PE]$$

Divide both sides by $W - 2PE$

Divide both sides by $W - 2PE$

$$\frac{Wr}{W - 2\,PE} = \frac{R[W - 2\,PE]}{[W - 2\,PE]}$$

$$R = \frac{Wr}{W - 2PE}$$

Given that $r = 12$, $P = 60$, $W = 1\,024$ and $E = \frac{5}{6}$.

$$R = \frac{1\,024 \times 12}{1\,024 - \frac{2}{1} \times \frac{60}{1} \times \frac{5}{6}}$$

$$R = \frac{12\,288}{1\,024 - 100}$$

$$R = \frac{12\,288}{924}$$

$$R = 13.29$$

$$R = 13.3 \text{ (3 s.f.)}$$

Exercise 3

1. If $a^2 - m^2 = \sqrt{\frac{a}{n} - \frac{m}{a}}$, find n when $a = 3$ and $m = 2$. (WAEC)

2. Use the relation $C = \frac{5}{9}(F - 32)$ to find F, when $C = 40$. (WAEC)

3. If $E = \frac{1}{2}m(v^2 - u^2)$, find m when $E = 270$, $v = 10$ and $u = 8$. (WAEC)

4. If $Q = P\left(\frac{1 + r^2}{100}\right)$, find, correct to 3 significant figures, the value of r when $Q = 625$ and $P = 225$.

5. If $P = \frac{m}{2} - \frac{n^2}{5m}$, find, correct to 3 significant figures, the value of n when $P = 14$ and $m = -8$.

6. Make w the subject of the formula in $R - d = \sqrt{R^2 - w^2}$. Given that $R = 125$ and $d = 0.25$, calculate w. (WAEC)

7. The volume V of a segment of a sphere is given by the formula $V = \frac{1}{6}\pi h\,(3r^2 + h^2)$, where h is the height of the segment and r is the radius of the circular face.

   (a) Make r the subject of the formula.

   (b) If $V = 124.3$ cm^3 and $h = 4.20$ cm, calculate r, correct to 3 significant figures. Take $\pi = 3.142$. (WAEC)

8. Given that $\frac{x}{a} + \frac{y^2}{b} = r^2$
   (a) Make y the subject of the expression.
   (b) Find y, correct to the nearest whole number, when $x = 6$, $a = 4$, $b = 2$ and $r = 3$.
   (WAEC)

9. Make x the subject of the relation $\frac{1 + ax}{1 - ax} = \frac{p}{q}$ and then find the value of x if $a = -\frac{1}{5}$.

10. If $t = \frac{1}{t} - \frac{100}{r}$,
   (a) Make r the subject of the formula.
   (b) Find r if $t = \frac{1}{2}$.

IV. Variation

Mathematicians are very keen about studying the relationship that exists between quantities whereby a change in a quantity necessitates corresponding change in others. A study of this nature is referred to as *variation*. Any of the quantities whose value remains the same in the course of the relationship is regarded as a constant. The relationship $p \propto q$ implies, p is proportional to q, that is $p = kq$, where k is a constant.

In another form, $p \propto \frac{1}{q}$ implies p is inversely proportional to q, that is $p = \frac{k}{q}$, where k is a constant.

Types of variation

There are four types of variation. These are direct, inverse, joint and partial variation.

Direct variation m varies directly with n, when the ratio of m to n is always constant. Hence, the equation connecting quantities m and n is $m \propto k$ or $m = kn$. This kind of a relationship can as well be expressed as $m \propto n$, which implies that 'm is proportional to n'. When the graph of m is plotted against n, the result is a straight line graph through the origin. Whenever n is tripled, m is also tripled and when n is halved, m is also halved. Example of direct variation is the circumference of a circle that is directly proportional to the radius.

Worked Example 19

ked Example 19

If A varies directly as the square root of B, and A = 35 when B = 25, find B when
A = 14.

Solution

$$A \propto \sqrt{B}$$

$$A = k\sqrt{B} \qquad \text{(i)}$$

where k is a constant, hence

$$k = \frac{A}{\sqrt{B}}$$

But $A = 35$ when $B = 25$

Hence, $k = \frac{35}{\sqrt{25}} = \frac{35}{5} = 7$

Substitute 7 for k in (i)

$$A = 7\sqrt{B} \qquad \text{(ii)}$$

When $A = 14$, $B = ?$

Substitute 14 for A in (ii)

$$14 = 7\sqrt{B}$$

Divide both sides by 7

$$\frac{14}{7} = \frac{7\sqrt{B}}{7}$$

$$2 = \sqrt{B}$$

Square both sides

$$2^2 = (\sqrt{B})^2$$

$$4 = B$$

$$\therefore B = 4$$

Worked Example 20

If $P \propto Q$ and $P = 10$ when $Q = 40$,
(a) Find the relationship between P and Q.
(b) Calculate the value of Q when $P = 20$.

Solution

(a) $P \propto Q$

$\Rightarrow P = kQ$ (i)

where k is a constant.

When $P = 10; Q = 40$

$\Rightarrow 10 = 40 \times k$

Divide both sides by 40

$$\frac{10}{40} = \frac{\cancel{40} \times k}{\cancel{40}}$$

$$\frac{1}{4} = k$$

$$k = \frac{1}{4}$$

Substitute $\frac{1}{4}$ for k in (i)

$P = \frac{1}{4}Q$ (ii)

(b) When $P = 20, Q = ?$

Substitute 20 for P in (ii)

$$20 = \frac{1}{4}Q$$

Multiply both sides by 4

$$20 \times 4 = \frac{1}{4} \times Q \times \frac{4}{1}$$

$$Q = 80$$

Worked Example 21

The area of a circle varies directly as the square of the diameter. If a circle of diameter 14 cm has an area of 154 cm^2, find the diameter of a circle whose area is 66 cm^2. (WAEC)

Solution

Let A represent the area of the circle and d the diameter

Divide both sides by $(14)^2$

Hence, $A \propto d^2$

$A = kd^2$ (i)

where k is a constant.

But $d = 14$ when $A = 154$

$\Rightarrow 154 = k \times (14)^2$

Divide both sides by $(14)^2$

$$\frac{154}{(14)^2} = k \times \frac{\cancel{(14)^2}^{1}}{\cancel{(14)^2}_{1}}$$

$$\frac{\cancel{154}^{11}}{\cancel{14} \times 14} = k$$

$$k = \frac{11}{14}$$

Substitute $\frac{11}{14}$ for k in (i)

$A = \frac{11d^2}{14}$ (ii)

When $A = 66$, $d = ?$,

Substitute 66 for A in (i)

$$66 = \frac{11d^2}{14}$$

Multiply both sides by $\frac{14}{11}$

$$\frac{\cancel{66}^{6}}{1} \times \frac{14}{\cancel{11}_{1}} = \frac{\cancel{11}}{\cancel{14}} \times d^2 \times \frac{\cancel{14}}{\cancel{11}}$$

$84 = d^2$

Take square root of both sides

$\sqrt{84} = \sqrt{d^2}$

$d = 9.165$

The diameter of the circle is 9.165 cm $\simeq$ 9.17 cm (2 d .p.)

Exercise 4

1. The time of oscillation of a pendulum varies as the square root of its length. If the length of the pendulum which oscillates for 35 seconds is 49 cm, find the time of oscillation of a pendulum with length 121 cm. **(NECO)**
2. $W \propto L^2$ and $W = 6$ when $L = 4$, if $L = \sqrt{17}$, find W. **(UME)**
3. What is the relationship between x and y if $x \propto \sqrt{y}$ and $x = \frac{9}{2}$ when $y = 9$? **(PCE)**
4. If y varies directly as the square of x and $y = 98$, when $x = 7$, calculate y when $x = 5$.
5. If P varies directly as r^2 and $P = 3.2$ when $r = 4$, find the value of P when $r = 6.5$. **(WAEC)**

Inverse variation

Quantity m is said to vary inversely as quantity n, if $m \propto \frac{1}{n}$.

Hence, the equation that connects n to m is $m = k\frac{1}{n}$ or $mn = k$.

Whenever m is plotted against n, the graph is always a straight line through the origin. So, m varies inversely as n and is written as m varies directly as $\frac{1}{n}$.

Examples of inverse variation are

(i) The time for a piece of job is inversely proportional to the number of men employed.

(ii) The greater the average speed of a vehicle, the less the time taken to complete the journey.

(iii) The greater the number of sectors of a circle, the smaller the angle of each sector.

Worked Example 22

If x varies inversely as y and $x = \frac{2}{3}$ when $y = 9$, find the value of y when $x = \frac{3}{4}$.

Solution

$x \propto \frac{1}{y}$

$x = \frac{k}{y}$ (i)

where k is a constant.

But $x = \frac{2}{3}$ when $y = 9$

$\frac{2}{3} = \frac{k}{9}$ [from (i)]

Multiply both sides by 9

$\frac{2}{3} \times \frac{9}{1} = \frac{k}{\cancel{9}} \times \frac{\cancel{9}}{1}$

$k = 6$

Substitute 6 for k in (i)

$x = \frac{6}{y}$ (ii)

When $x = \frac{3}{4}$, $y = ?$

Substitute $\frac{3}{4}$ for x in (ii)

$\frac{3}{4} = \frac{6}{y}$

Multiply both sides by 4

$3y = 6 \times 4$

Divide both sides by 3

$\frac{\cancel{3}y}{\cancel{3}} = \frac{6 \times 4}{3}$

$y = 8$

Worked Example 23

y varies inversely as the cube root of x. If $y = 4$ when $x = 0.125$, find y when $x = 8$.

Solution

$y \propto \frac{1}{\sqrt[3]{x}}$

Implies: $y = k\frac{1}{\sqrt[3]{x}}$ (i)

where k is a constant.

Given: $y = 4$ when $x = 0.125$

$4 = k\frac{1}{\sqrt[3]{0.125}}$

Multiply both sides by $\sqrt[3]{0.125}$

$k = 4 \times \sqrt[3]{\frac{125}{1000}}$

$k = 4 \times \sqrt[3]{\left[\frac{5}{10}\right]^3}$

$k = \frac{20}{10} = 2$

Substitute $k = 2$ in (i)

$y = 2\frac{1}{\sqrt[3]{x}}$ (ii)

When $x = 8, y = ?$

Substitute 8 for x in (ii)

$y = \frac{2}{\sqrt[3]{8}} = \frac{2}{\sqrt[3]{2^3}}$

$y = \frac{2}{2}$

$y = 1$

Worked Example 24

The time taken to do a piece of work is inversely proportional to the number of men employed. If it takes 25 men 6 days to do the work, how long will it take 12 men working at the same rate?

Solution

Let t represent time and n represent number of men employed.

$$t \propto \frac{1}{n}$$

$$t = \frac{k}{n} \qquad \text{(i)}$$

where k is a constant.

But $n = 25$, when $t = 6$

$$6 = \frac{k}{25}$$

$$k = 6 \times 25 = 150$$

Substitute 150 for k in (i)

$$t = \frac{150}{n} \qquad \text{(ii)}$$

$n = 12, t = ?$

Substitute 12 for n in (ii)

$$t = \frac{\cancel{150}^{25}}{\cancel{12}_{2}}$$

$$t = 12\frac{1}{2}$$

Time is $12\frac{1}{2}$ hours.

Exercise 5

1. If $y \propto \frac{1}{\sqrt{x}}$ and $y = 1$ when $x = 36$, find y when $x = 2\frac{1}{4}$.
2. x varies inversely as the square of y. When $x = 3, y = 16$, find the value of y, when $x = 48$. (NECO)

QUADRATIC EQUATION

CHAPTER 7

Objectives

(i) The relation between R and d.

(ii) The value of d when R is 18 ohms. **(WAEC)**

11. If $y \propto \frac{1}{\sqrt{x}}$ and $y = 1$ when $x = 36$, find y when $x = 2\frac{1}{4}$. **(WAEC)**

12. If $y \propto \frac{1}{\sqrt{x}}$ and $x = 16$ when $y = 2$, find x when $y = 24$.

13. If x varies inversely as the square root of y and $x = 2$ when $y = 9$, find y when $x = 18$. **(WAEC)**

14. If $y \propto \frac{1}{\sqrt{x}}$ and $y = 1\frac{1}{4}$ when $x = 4$, find the value of y when $x = \frac{1}{2}$. **(WAEC)**

15. At constant temperature, the volume V of a fixed mass of gas is inversely proportional to the applied pressure P. If $V = 9$ m^3 when $p = 150$ Nm^{-2}, calculate the pressure at which the volume will be reduced to a third. **(WAEC)**

Joint variation

The formula for the volume of a cuboid is $V = lbh$. This means that the volume is a function of three variables l, b and h. However, the volume varies directly as the length, the breadth and the height. Simply put, V varies jointly as the length, breadth and height. Symbolically, $V \propto lbh$ while the equation is $V = klbh$ where k is a constant. The curved surface area of a cone also varies jointly as the product of its radius and its slant height. This is written as $A \propto rl$ where $A = \pi rl$ and π is a constant.

Worked Example 25

If M varies directly as the square of m and inversely as n and if $M = 36$ when

Solution

$$M \propto \frac{m^2}{n}$$

$$\Rightarrow M = k\frac{m^2}{n} \qquad \text{(i)}$$

$m = 6$ and $n = 8$, find M when $m = 10$ and $n = 4$.

where k is a constant.

But $M = 36$, when $m = 6$ and $n = 8$.

Implies: $36 = \frac{k \times 6^2}{8}$

Multiply both sides by $\frac{8}{6^2}$

$$36 \times \frac{8}{6^2} = k \times \frac{\cancel{6^2}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{6^2}}$$

$$36 \times \frac{8}{36} = k$$

$$k = 8$$

Substitute 8 for k in (i)

$$M = \frac{8m^2}{n}$$

When $m = 10, n = 4, M = ?$

Substitute 10 for m and 4 for n in (ii)

$$M = 8 \times \frac{10^2}{4}$$

$$= \frac{\overset{2}{\cancel{8}} \times 100}{\cancel{4}}$$

$$= 200$$

Worked Example 26

The volume V of a cuboid is directly proportional to the length l, the breadth b, and the height h. When $l = 10$ cm, $b = 4$ cm, $h = 5$ cm and $V = 100$ cm^3, find h when $l = 5$ cm, $b = 2$ cm and $V = 50$ cm^3.

Solution

$V \propto lbh$

$\Rightarrow V = k\,lbh$ (i)

where k is a constant.

But: $l = 10$ cm, $b = 4$ cm, $h = 5$ cm, when $V = 100$ cm^3.

$100 = k \times 10 \times 4 \times 5$

$100 = k \times 200$

Divide both sides by 200

$\frac{100}{200} = \frac{k \times 200}{200}$

$k = \frac{1}{2}$

Substitute $\frac{1}{2}$ for k in (i)

$V = \frac{1}{2} lbh$

Given: $V = 50$ cm^3, $l = 5$ cm, $b = 2$ cm, $h = ?$

$50 = \frac{1}{\cancel{2}} \times 5 \times \cancel{2} \times h$ [from (ii)]

$50 = 5 \times h$

Divide both sides by 5

$\frac{50}{5} = \frac{\cancel{5} \times h}{\cancel{5}}$

$10 = h$

$h = 10$

The height is 10cm.

Exercise 6

1. r varies directly as the cube of p and inversely as the square of q. If k is a constant, r is the first prime number, p is the first 2-digit number and q is the first 2-digit multiple of 4, find the equation connecting p, q and r.

2. P varies directly as Q and inversely as the square of R. When $Q = 5$, $R = 3$ and $P = 20$. Find P when $Q = 6$ and $R = 4$. **(NECO)**

3. x, y and z are related such that x varies directly as the cube of y and inversely as the square of z. If $x = 108$ when $y = 3$ and $z = 4$, find z when $x = 4\,000$ and $y = 10$. **(WAEC)**

4. The electrical resistance R ohms, of a copper wire varies directly as its length, l m and inversely as the square of its radius, r cm. A copper wire of length 625 m and radius 0.25 cm has a resistance of 24 ohms.

   (a) Find an equation connecting R, l and r.

   (b) Calculate the radius of the same type of copper wire of length 500 m and resistance 30 ohms. **(WAEC)**

5. The time t taken to buy fuel at a petrol station varies directly as the number of vehicles V on queue and jointly varies inversely as the number of pumps P available

in the station. In a station with 5 pumps, it took 10 minutes to fuel 20 vehicles. Find:

(a) The relationship between t, P and V.

(b) The time it will take to fuel 50 vehicles in the station with 2 pumps.

(c) The number of pumps required to fuel 40 vehicles in 20 minutes. (WAEC)

6. A number p varies directly as q and partly inversely as q^2. Given that $p = 11$ when $q = 2$ and $p = 25.16$ when $q = 5$, calculate the value of p when $q = 7$.(WAEC)

7. p, x and n are related in such a way that p varies directly as the cube of x and inversely as the square root of n. If $p = 108$ when $x = 3$ and $n = 4$, find n when $p = 4\,000$ and $x = 10$. (WAEC)

8. The energy (E) varies directly as the resistance (R) and inversely as the square of the distance (D). If $E = \frac{32}{25}$ when $R = 16$ and $D = 10$, calculate the value of R when $E = 32$ and $D = 7$. (NECO)

9. Three quantities P, Q and R are connected so that P varies directly as R and inversely as the square root of Q. If $P = 6$ when $R = 12$ and $Q = 25$, find

(a) The expression for P in terms of Q and R.

(b) The value of Q when $P = 30$ and $R = 9$. (WAEC)

Partial variation

The cost of running an evening coaching programme is a function of the cost of renting a suitable accommodation that is always constant and the amount of time schedule for the level of education of the participants. Time schedule for Junior Secondary School classes is expected to differ from General Certificate in Education (GCE) preparatory classes. One hour time schedule for each subject could be enough for a Junior Secondary School class but a preparatory class for General Certificate in Education (GCE) may need one-and-half hours or more. This simply means that amount of time schedule is not constant.

Hence, the cost of running a successful evening coaching programme is

partly constant and partly varies with the amount of time. Symbolically, $C = c + Kt$ where c and K are constants and t is the amount of time taken. Examples of partial variation are as follows:
1. $V = 2\pi r_2 + 2\pi rh$ meaning ‘V’ varies partly as the square of the radius ‘r’ and jointly as the radius ‘r’ and the height ‘h’.2. $P = 2l + 2b$ meaning the perimeter ‘P’ of a rectangle varies partly as the length ‘l’ and the breadth ‘b’.

Worked Example 27

Worked Example 27

x varies partly as y and partly as the square of y. When $y = 2$, $x = 5$ and when $y = 5$, $x = 57.5$. Find x when $y = 4$. (NECO)

Solution

$x = ay + by^2$ (i)

where a and b are constants.

Given: $y = 2$ when $x = 5$

$\Rightarrow 5 = 2a + 2^2b$ [from (i)]

$2a + 4b = 5$ (ii)

Given: $y = 5$ when $x = 57.5$

$\Rightarrow 57.5 = 5a + 5^2b$ [from (i)]

$5a + 25b = 57.5$ (iii)

Multiply (ii) by 5

$10a + 20b = 25$ (iv)

Multiply (iii) by 2

$10a + 50b = 115$ (v)

Subtract (iv) from (v) $\Rightarrow 30b = 90$

Divide both sides by 30

$$\frac{\cancel{30} \times b}{\cancel{30}} = \frac{\cancel{90}^{3}}{\cancel{30}}$$

$b = 3$ (vi)

Substitute 3 for b in (ii)

$2a + 4(3) = 5$

$2a + 12 = 5$

Subtract 12 from both sides

$2a + 12 - 12 = 5 - 12$

$2a = -7$

Divide both sides by 2

$$\frac{\cancel{2} \times a}{\cancel{2}} = -\frac{7}{2}$$

$$a = -\frac{7}{2}$$

Substitute $-\frac{7}{2}$ for a and 3 for b in (i)

$$x = -\frac{7}{2}y + 3y^2$$

When $y = 4$, $x = ?$

Substitute 4 for y in (vii)

$$x = -\frac{7}{\cancel{2}} \times \cancel{4}^{2} + 3 \times 4^2$$

$= -14 + (3 \times 16)$

$= -14 + 48$

$= 34$

Worked Example 28

The quantity y is partly constant and partly varies inversely as the cube of x.
(i) Given that $x = 1$ when $y = 5$ and $x = 2$ when $y = -1$, find the relationship between x and y.
(ii) Hence, find the value of y when $x = 3$.

Solution

$y = c + \frac{k}{x^3}$ (i)

where c and k are constants

Given: $x = 1$ when $y = 5$

$\Rightarrow 5 = c + \frac{k}{1^3}$ [from (i)]

$5 = c + \frac{k}{1}$

$c + k = 5$ (ii)

Given: $x = 2$ when $y = -1$

$-1 = c + \frac{k}{2^3}$

$-1 = c + \frac{k}{8}$

Multiply through by 8

$-1 \times 8 = (c \times 8) + \left(\frac{k}{8} \times \frac{8}{1}\right)$

$-8 = 8c + k$

$8c + k = -8$ (iii)

Subtract (ii) from (iii)

$7c = -13$

Divide both sides by 7

$\frac{7 \times c}{7} = \frac{-13}{7}$

$c = \frac{-13}{7}$

Substitute $\frac{-13}{7}$ for c in (ii)

$$\frac{-13}{7} + k = 5$$

Add $\frac{13}{7}$ to both sides

$$\frac{-13}{7} + \frac{13}{7} + k = 5 + \frac{13}{7}$$

$$k = \frac{35 + 13}{7}$$

$$k = \frac{48}{7}$$

Substitute $\frac{-13}{7}$ for c and $\frac{48}{7}$ for k in (i)

$$y = \frac{-13}{7} + \frac{48}{7x^3}$$

To find y when $x = 3$

$$\Rightarrow y = \frac{-13}{7} + \frac{48}{7 \times 3^3}$$

$$y = \frac{-13}{7} + \frac{\overset{16}{\cancel{48}}}{7 \times 3 \times \cancel{3} \times 3}$$

$$y = \frac{-13}{7} + \frac{16}{63}$$

$$y = \frac{-117 + 16}{63}$$

$$y = \frac{-101}{63}$$

$$y = -1\frac{38}{63}$$

Exercise 7

1. The quantity w varies partly directly as q and partly inversely as the square of q. When $q = 2$, $w = 8$ and when $q = 3$, $w = 12$. Find w when $q = 6$. (PCE)
2. The quantity y is partly constant and partly varies inversely as the square of x:
   (a) Write down the relationship between x and y.
   (b) When $x = 1, y = 11$ and when $x = 2, y = 5$. Find the value of y when $x = 4$. (WAEC)
3. The cost of maintaining a school is partly constant and partly varies as the number of pupils. With 50 pupils, the cost is \$15705.00 and with 40 pupils, it is \$13305.00.
   (a) Find the cost when there are 44 pupils.
   (b) If the fee per pupil is \$360.00, what is the least number of pupils for which the school can be run without loss? (WAEC)
4. y varies partly as d and partly as the cube root of d. When $y = 6$, $d = 27$ and when $y = 11$, $d = \frac{1}{8}$. Find y when $d = 3$ (correct to two decimal places). (NECO)
5. If A is partly constant and partly varies as P. When $A = 15, P = 21$ and when $A = 21, P = 24$. Find

(a) The law of variation between A and P.

(b) A when $P = 33$.

(NECO)

6. The quantity y is partly constant and partly varies inversely as the cube of x.

   (a) Given that $x = 1$ when $y = 5$ and $x = 8$ when $y = -1$, find the relationship between x and y.

   (b) Hence, find the value of y when $x = 64$.

7. P is partly constant and partly varies as the cube of y. When $p = 5.4$, $y = 2$ and when $p = 8.5$, $y = 1$.
   Find the value of p, correct to 1 decimal place, when $y = 7$.
   (NECO)

8. The cost of making a door frame is the sum of two parts. One is proportional to the area and the other to the square of the length. If the cost of a door frame 4 m by 6 m is Le100 and the cost of a door frame 3 m by 8 m is Le128, find the cost of a door frame 5 m^2.

9. L is partly constant and partly varies with M. When $L = 215$, $M = 500$ and when $L = 135$, $M = 900$. Find

   (a) The formula which connects L and M.

   (b) L when $M = 650$.

SUMMARY

In this chapter, we have learnt the following:

- A linear equation is an equation of the form $ax + b = c$ where a, b and c are constants. For example $2x + 4 = 0$, $4y + 3 = 0$ etc.
- A formula is a rule expressed in the form of an equation. For example $V = lbh$, V is the volume of a cuboid, l is the length, b is the breadth and h is the height.
- A change in the subject of a formula means replacing the subject of the formula by any of the other variables in such formula.
- The value of the subject of a formula can be obtained by substituting given values for other variables in the formula.
- The four types of variation are direct, inverse, joint and partial variation.
- m varies inversely as n, if $m \propto \frac{1}{n}$ and the equation connecting m and n is $m = \frac{k}{n}$ or $mn = k$.
- The volume V of a cuboid varies jointly as the length 'l', the breadth 'b' and the height 'h', that is $V \propto lbh$ and the equation connecting the volume, length, breadth and height is $V = lbh$.
- $V = 2\pi r^2 + 2\pi rh$ means 'V' varies partly as the square of the radius 'r' and jointly as the radius 'r' and the height 'h'.

GRADUATED EXERCISE

1. Solve the following equations:

   (i) $5m = 40 - 3m$

   (ii) $4y + z = 2y + z + 5$

   (iii) $3 + 3v - 14 = 24 - 2v$

2. Solve the following equations:

   (i) $2(6b - 10) = 5(b + 4) - 5$

   (ii) $5 - 6u = 18 - (3u - 5)$

   (iii) $6d - 8(2 - d) = 2 - 2(d + 1)$

3. Solve

   (i) $\frac{4p}{5} - \frac{8}{15} = \frac{4p-2}{10}$

   (ii) $\frac{q}{2} - \frac{5q-4}{8} = 1 - \frac{3q}{16}$

   (iii) $\frac{3r}{8} - 2\frac{1}{2}r + \frac{3}{4} = \frac{5r-6}{16}$

4. Make f the subject of the formula $\frac{d}{e-f} = c$.

5. Make k the subject of the relation

$y = \sqrt{\frac{k^2 + x^2}{t}}$

6. Find T in terms of K, Q and S, if

$s = 2r\sqrt{\pi(QT + K)}$. (UME)

7. Make r the subject of the formula in

$\frac{L}{E} = \frac{2a}{R-r}$. (UME)

8. Make f the subject of the formula

$t = \sqrt{\frac{v}{\frac{1}{f} + \frac{1}{g}}}$. (UME)

9. If $P = \frac{rd^3}{t}$, express r in terms of p, d and t. (UME)

10. The quantity y is partly constant and partly varies inversely as the square of x. With P and Q as constants, find the relationship between x and y.

11. Make p the subject of the formula

$$y = \frac{a + p}{a - p}$$

12. Given that $S = K\sqrt{m^2 + n^2}$

(i) Make m the subject of the relation.

(ii) If $S = 12.2$, $K = 0.02$ and $n = 1.1$, find, correct to the nearest whole number, the positive value of m.

13. If $4a^3 = c - b^2$, find the value of b when $a = -3$ and $c = 24$ (correct to 2 d.p.). (UME)

14. $W \propto L^2$ and $W=6$ when $L=4$. If $L=\sqrt{17}$, find W. (UME)

15. What is the relationship between x and y, if $x \propto \sqrt{y}$ and $x=\frac{9}{2}$ when $y=9$? (UME)

16. The quantity w varies partly directly as q and partly inversely as the square of q. When $q=2$, $w=8$ and when $q=3$, $w=12$, find w when $q=6$. (PCE)

17. The weight W kg of a metal bar varies jointly as its length L m and the square of its diameter d m. If $W=140$, when $d=\frac{42}{3}$ and $L=54$, find d in terms of W and L. (UME)

18. The time taken to do a piece of work is inversely proportional to the number of men employed. If it takes 30 men to do the piece of work in 6 days, how many men are required to do the work in 4 days?

19. If y varies directly as the square of x and $y=98$ when $x=7$, calculate y when $x=5$. (PCE)

20. W is directly proportional to u. If $W=5$ when $u=3$, find u when $W=\frac{2}{7}$. (UME)

21. Make c the subject of the equation

$$a(b+c)+\frac{5}{d}-2=0.$$

22. y varies partly as the square of x and partly as the inverse of the square root of x. Write down the expression for y, if $y=2$ when $x=1$ and $y=6$ when $x=4$. (UME)

23. The quantity y is partly constant and partly varies inversely as the square of x.

(a) Write down the relationship between x and y.

(b) When $x = 1$, $y = 11$ and when $x = 2$, $y = 5$. Find the value of y when $x = 4$.

(WAEC)

24. M varies directly as n and inversely as the square of p. If $M = 3$ when $n = 2$ and $p = 1$, find M in terms of n and p.

25. (a) Make "A" the subject of the equation $y = 1 - \frac{x}{3}\left(B + \frac{5}{7}A\right)$.

(b) Hence, find the value of A, if $y = -3, x = 4$ and $B = -2$.