Chapter 17: Cumulative Frequency Graph

OBJECTIVES
At the end of the chapter, students should be able to:
1. Compute the cumulative frequency of grouped data.
2. Draw a cumulative frequency curve (ogive).
3. Estimate median, quartiles, percentiles, etc. from a cumulative frequency curve.
4. Apply cumulative frequency curves to real life situations.

I. Computation of Cumulative Frequency of Grouped Data

(i) Meaning

Cumulative frequency is the sum of the frequencies within a particular class interval added to those of the class intervals above it. In other words, cumulative frequency is the addition of the frequencies of the initial class intervals to the frequency of a specified class interval.
From the frequency Table 17.1, the frequency of the initial class interval is 5 and that of the specified class interval is 2. So the cumulative frequency of the specified class is the addition of all frequencies from that of the initial class up to that of the specified class.
Hence, the cumulative frequency of the specified class 31–35 is 5 + 9 + 2 = 16.

Table 17.1

Class interval	Frequency
21–25	5
26–30	9
31–35	2
36–40	8
41–45	1
46–50	7

Worked Example 1

The table below shows the frequency distribution of the marks of 800 candidates in an examination:

Marks (%) Frequency

Marks (%)	Frequency
1–10	10
11–20	40
21–30	80
31–40	140
41–50	170
51–60	130
61–70	100
71–80	70
81–90	40
91–100	20

Construct a cumulative frequency table for the frequency distribution.

SOLUTION

Table 17.2: Cumulative frequency table showing the distribution of 800 candidates in an examination

Marks (%)	Frequency	Class boundaries	Cumulative frequency
1–10	10	0.5–10.5	10
11–20	40	10.5–20.5	10 + 40 = 50
21–30	80	20.5–30.5	50 + 80 = 130
31–40	140	30.5–40.5	130 + 140 = 270
41–50	170	40.5–50.5	270 + 170 = 440
51–60	130	50.5–60.5	440 + 130 = 570
61–70	100	60.5–70.5	570 + 100 = 670
71–80	70	70.5–80.5	670 + 70 = 740
81–90	40	80.5–90.5	740 + 40 = 780
91–100	20	90.5–100.5	780 + 20 = 800

Worked Example 2

The data below shows the lengths in centimetres of 30 pieces of roofing sheet.

64 63 71 59 64 53 57 74 55 57
50 68 72 68 80 67 70 62 79 67
45 55 65 60 61 68 59 54 64 76

(a) Using class intervals of 45–49, 50–54, 55–59, etc., construct a frequency table for the data.
(b) Construct a cumulative frequency table.

SOLUTION

(a)

Class interval	Tally	Frequency
45–49	/	1
50–54	///	3
55–59	𝍸 /	6
60–64	𝍸 //	7
65–69	𝍸 /	6
70–74	////	4
75–79	//	2
80–84	/	1

(b) Cumulative frequency table

Class interval	Tally	Frequency	Cumulative frequency
45–49	/	1	1
50–54	///	3	1 + 3 = 4
55–59	~~////~~ /	6	4 + 6 = 10
60–64	~~////~~ //	7	10 + 7 = 17
65–69	~~////~~ /	6	17 + 6 = 23
70–74	////	4	23 + 4 = 27
75–79	//	2	27 + 2 = 29
80–84	/	1	29 + 1 = 30

Exercise 1

Compute the cumulative frequency tables for the following frequency distributions:

1.

Weight (kg)	Number of iron bars
100–104	4
105–109	5
110–114	9
115–119	14
120–124	13
125–129	9
130–134	4
135–139	2

2.

Kegs of palm-wine	Number of tappers
1–10	0
11–20	9
21–30	32
31–40	42
41–50	23
51–60	10
61–70	3
71–80	7

3.

Weekly profit (₦)	1–10	11–20	21–30	31–40	41–50	51–60
Frequency	5	10	11	12	6	6

4.

Number of oranges	Number of trees
11–20	68
21–30	98
31–40	164
41–50	184
51–60	294
61–70	310
71–80	402
81–90	408

5.

Age (years)	Number of people
1–10	5
11–20	6
21–30	12
31–40	20
41–50	18
51–60	15
61–70	15
71–80	9

6.

Marks	Number of students
1–10	90
11–20	50
21–30	20
31–40	10
41–50	10
51–60	40
61–70	70
71–80	110

7.

Score	Number of pupils
1–20	10
21–40	21
41–60	35
61–80	12
81–100	2

8.

Weight (kg)	Frequency
20–29	12
30–39	16
40–49	24
50–59	32
60–69	10
70–79	6

9.

Marks (%)	Frequency
0–9	100
10–19	70
20–29	40
30–39	20
40–49	130
50–59	170
60–69	140
70–79	10
80–89	40
90–99	80

0.

Age (years)	Frequency
10–14	6
15–19	12
20–24	18
25–29	26
30–34	16
35–39	8
40–44	3

II. Drawing of Cumulative Frequency Curve (Ogive)

Worked Example 3

The table below shows the distribution of weights in kilogrammes of 50 employees in a company:

Weight (kg)	Frequency
20–29	6
30–39	8
40–49	12
50–59	16
60–69	5
70–79	3

(a) Construct a cumulative frequency table for the distribution.
(b) Draw the cumulative frequency curve or ogive of the distribution.

SOLUTION

Table 17.3: Cumulative frequency table showing the distribution of weights in kilogrammes of 50 employees in a company.

Weight (kg)	Frequency	Class boundaries	Cumulative frequency
20–29	6	19.5–29.5	6
30–39	8	29.5–39.5	6 + 8 = 14
40–49	12	39.5–49.5	14 + 12 = 26
50–59	16	49.5–59.5	26 + 16 = 42
60–69	5	59.5–69.5	42 + 5 = 47
70–79	3	69.5–79.5	47 + 3 = 50

III. Use of Cumulative Frequency Curve to Estimate Median, Quartile and Percentile

Estimation of quartiles, percentiles, deciles, etc. can be done on the cumulative frequency curve or ogive. The quartiles are the lower quartile ($\mathbf{Q}_1$), the median ($\mathbf{Q}_2$) and the upper quartile ($\mathbf{Q}_3$). The percentiles can be 25th percentile ($\mathbf{P}_{25}$), 30th percentile ($\mathbf{P}_{30}$), 60th percentile ($\mathbf{P}_{60}$), 75th percentile ($\mathbf{P}_{75}$), etc. The deciles can be 1st decile ($\mathbf{D}_1$), 5th decile ($\mathbf{D}_5$), 6th decile ($\mathbf{D}_6$), etc. It should be noted that calculation of quartiles involves division of the total frequency by 4. Hence, lower quartile ($\mathbf{Q}_1$), is calculated as 1/4 **x Î£ f** and upper quartile ($\mathbf{Q}_3$) as 3/4 **x Î£ f.** Percentiles are calculated by dividing the total frequency by 100. Hence, 60th percentile $\mathbf{P}_{60}$ is calculated as 60/1000 **x Î£ f.** Deciles are calculated by dividing the total frequency by 10. Hence, 4th decile $\mathbf{D}_4$ is calculated as 4/10 **x Î£ f.**

However, estimation of the quartiles, percentiles, deciles, etc. are done on the cumulative frequency curve as follows:

Steps

1. Locate any of the calculated values i.e. quartiles, percentiles or deciles on the cumulative frequency axis.
2. Trace the cumulative frequency curve from the located point using a horizontal line.
3. Draw a perpendicular line from the point of contact of the horizontal line and the ogive to a point on the **x**-axis.
4. Read the value at the point of contact of the perpendicular line and the **x**-axis as the actual value of the estimation.

Worked Example 4

The table below shows the weekly profit of a mini market in Naira:

Weekly profit ₦	Frequency
1–10	6
11–20	6
21–30	12
31–40	11
41–50	10
51–60	5

(a) Draw the cumulative frequency graph of the data.
(b) From your graph, estimate the value of the
(i) Median.
(ii) 80th percentile.

(iii) 6th decile.

SOLUTION

Weekly profit	Frequency	Class boundaries	Cumulative frequency
1–10	6	0.5–10.5	6
11–20	6	10.5–20.5	6 + 6 = 12
21–30	12	20.5–30.5	12 + 12 = 24
31–40	11	30.5–40.5	24 + 11 = 35
41–50	10	40.5–50.5	35 + 10 = 45
51–60	5	50.5–60.5	45 + 5 = 50

b (i) $\text{Median} = Q_2 = \frac{2}{4} \times \sum f$

$$= \frac{\cancel{2}^{1}}{\cancel{4}^{2}} \times \frac{\cancel{50}^{25}}{1} = 25$$

Median is the 25th frequency value. From the ogive, the corresponding value of the 25th frequency on the **x**-axis is 31.5. So, median is N31.50.

(ii) $\text{80th percentile} = P_{80} = \frac{80}{100} \times \sum f$

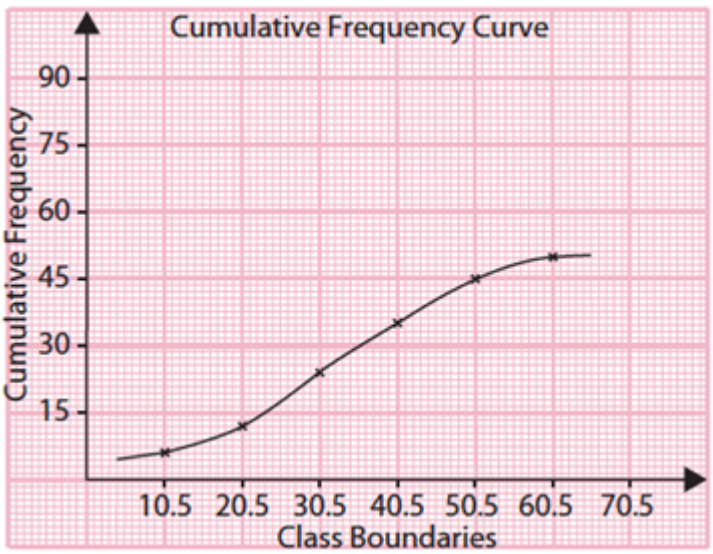
$$= \frac{4}{\cancel{5}_{1}} \times \frac{\cancel{50}^{10}}{1} = 40$$

80th percentile is the 40th frequency value. From the ogive, the corresponding value of the 40th frequency on the **x**-axis is 44.5. So, the 80th percentile is N44.50

(iii) $\text{6th decile} = D_6 = \frac{6}{10} \times \sum f$

$$= \frac{3}{\cancel{5}} \times \frac{\cancel{50}^{10}}{1} = 30$$

6th decile is the 30th frequency value. From the ogive, the corresponding value of the 30th frequency on the x-axis is 35.5. So, the 6th decile is N35.50.



Exercise 2

1. The table below shows the examination marks of 120 students:

Marks (%)	Frequency
30–39	11
40–49	25
50–59	45
60–69	15
70–79	10
80–89	14

(a) (i) Draw the cumulative frequency table for the distribution.
(ii) Draw the cumulative frequency curve.
(b) Use the cumulative frequency curve to estimate the:
(i) median.
(ii) Upper quartile. (WAEC)

2. The frequency distribution below shows the marks of 100 students in a Mathematics test:

Marks	Frequency
1–10	2
11–20	4
21–30	9
31–40	13
41–50	18
51–60	32
61–70	13
71–80	5
81–90	3
91–100	1

(a) Draw a cumulative frequency curve for the distribution.
(b) Use your curve to estimate the
(i) Median.
(ii) Lower quartile.
(iii) 60th percentile. (WAEC)

3. The table below shows the distribution of the heights of trees in a farm:

Height of trees (cm)	Number of trees
151–175	1
176–200	4
201–225	17
226–250	15
251–275	11
276–300	2

(a) Construct the cumulative frequency table for the distribution.
(b) Draw the cumulative frequency curve.

(c) Estimate the
(i) Median.
(ii) Interquartile range.
(WAEC)

4. The table below shows the frequency distribution of the scores obtained by 100 students in an examination:

Marks	Frequency
30–39	9
40–49	14
50–59	32
60–69	20
70–79	15
80–89	7
90–99	3

(a) Draw a cumulative frequency curve for the distribution.
(b) Use your curve to determine the
(i) Median.
(ii) Lower quartile.
(iii) Lowest mark of distinction, if 5% of the students passed with distinction. (WAEC)

5. The table below shows the marks scored by 400 candidates in a certain examination

Marks	Frequency
1–10	12
11–20	18
21–30	24
31–40	38
41–50	56
51–60	74
61–70	82
71–80	56
81–90	26
91–100	14

(a) Construct the cumulative frequency table.
(b) Using a scale of 2 cm to 10 marks on the mark axis and 2 cm to 40 units on the frequency axis, draw the cumulative frequency graph.
(c) From your graph, estimate the
(i) Pass mark, if 80% of the candidates passed the examination.
(ii) Minimum mark scored by the top 25% of the candidates.
(iii) Number of candidates who scored less than 25 marks. (WAEC)

6. The table below shows the masses of various quantities of maize sold by a farmer during the year 1980:

Mass (kg)	Frequency
40–43	7
44–47	18
48–51	31
52–55	48
56–59	41
60–63	28
64–67	17
68–71	10

(a) Draw the cumulative frequency table.
(b) Using a scale of 2 cm to 4 kg on the mass axis and 2 cm to 20 units on the frequency axis, draw the cumulative frequency curve.
(c) Use your curve to estimate the
(i) Median mass.
(ii) Interquartile range of the distribution. (WAEC)

7. The table below shows the frequency distribution of the marks of 800 candidates in an examination:

Marks (%)	Frequency
0–9	10
10–19	40
20–29	80
30–39	140
40–49	170
50–59	130
60–69	100
70–79	70
80–89	40
90–99	20

(a) (i) Construct a cumulative frequency table.
(ii) Draw the ogive.
(iii) Use your ogive to determine the 50th percentile.
(b) The candidates that scored less than 25% are to be withdrawn from the institution while those that scored more than 75% are to be awarded scholarship.
Estimate the number of candidates that will be retained but will not enjoy the award. (WAEC)

8. The following is the record of marks of 40 candidates in an examination:

65 84 91 58 43 86 73 33 76 80
57 33 53 29 40 27 72 19 51 67
37 14 18 92 13 45 61 39 23 22
22 41 27 51 63 47 19 35 39 76

Using class intervals 11–20, 21–30, etc., prepare a
(a) Frequency table.
(b) Cumulative frequency table for the distribution. (WAEC)

9. The following data represents the scores of students in a Mathematics mode examination:

90 40 60 70 46 53 40 60 91 70
55 35 24 63 66 20 55 25 45 78
66 25 38 68 78 30 38 82 63 55
78 82 51 59 80 45 51 66 60 50

(a) Prepare a grouped cumulative frequency table with class intervals 20–29, 30–39, 40–49, etc.
(b) Draw a cumulative frequency curve for the distribution.
(c) Use your curve to estimate the
(i) Failure mark, if 35% of the students failed the examination:
(ii) Interquartile range. (NECO)

10. The number of eggs collected daily by a poultry farmer for 50 days is as follows:

75 45 84 69 28 49 61 42 50 76
69 53 75 64 63 48 60 35 25 78
45 68 35 66 62 54 79 43 57 80
67 72 52 78 62 56 74 57 50 65
30 50 51 40 73 63 82 58 69 66

Using the above data:
(a) Construct a grouped cumulative frequency table using intervals of 20–29, 30–39, etc.
(b) Draw a cumulative frequency curve.
(c) Estimate the median. (NECO)