Chapter 5

Chapter 5

Application of Linear and Quadratic Equations to Capital Markets

OBJECTIVES

At the end of this chapter, students should be able to:

1. solve simultaneous linear and quadratic equations.
2. solve word problems on linear, quadratic and simultaneous linear and quadratic equations.
3. solve problems on linear equations involving capital markets.

I. Simultaneous Linear and Quadratic Equations (One Linear and One Quadratic) (Revision)

Two simultaneous equations in which one is linear and the other quadratic are referred to as simultaneous linear and quadratic equations, respectively. They usually have two pairs of solutions. They can be solved either by substitution or by graphical method.

Worked Example 1

Solve the equations $y - x = 2$; $2x^2 - xy = 3$.

SOLUTION

$$y - x = 2 \quad \text{..........} (1)$$

$$2x^2 - xy = 3 \quad \text{..........} (2)$$

From (1)

$$y = 2 + x \quad \text{..........} (3)$$

Substituting (3) into (2)

$$2x^2 - x(2 + x) = 3$$

$$2x^2 - 2x - x^2 = 3$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$\therefore x = 3 \text{ or } x = -1$$

Substituting each of the value of x into (3)

When $x = 3$, $y = 2 + 3 = 5$

When $x = -1$, $y = 2 + 1 = 3$

$\therefore$ The solution is either

$$x = 3, y = 5 \text{ or } x = -1, y = 3$$

Hence, two pairs of solutions are (3, 5) and (–1, 3).

Worked Example 2

Solve the equations $3x + y = 13$, $xy = 14$.

SOLUTION

$$3x + y = 13 \quad \text{..........} (1)$$

$$xy = 14 \quad \text{..........} (2)$$

From (1) $\Rightarrow y = 13 - 3x$ .... (3)

Substituting (3) into (2)

$$x(13 - 3x) = 14$$

$$\Rightarrow \quad 13x - 3x^2 = 14$$

$$\Rightarrow \quad -3x^2 + 13x - 14 = 0$$

$$\Rightarrow \quad -3x^2 + 6x + 7x - 14 = 0$$

$$\Rightarrow \quad (-3x^2 + 6x) + (7x - 14) = 0$$

$$\Rightarrow \quad 3x(-x + 2) + 7(x - 2) = 0$$

$$\Rightarrow \quad -3x(x - 2) + 7(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(7 - 3x) = 0$$

Either $x = 2$ or $x = \frac{-7}{3}$

Substituting each of the value of x into (1)

When $x = 2$

$$3 \times 2 + y = 13$$

$$6 + y = 13$$

$$y = 13 - 6$$

$$y = 7$$

When $x = \frac{-7}{3}$

$$3\left(\frac{-7}{3}\right) + y = 13$$

$$-7 + y = 13$$

$$y = 13 + 7$$

$$y = 20$$

$\therefore$ The solution is either

$$x = 2, y = 7 \text{ or } x = \frac{-7}{3}, y = 20.$$

Thus, the two pairs of solutions are (2, 7) and $\left(\frac{-7}{3}, 20\right)$

Worked Example 3

Solve the equations $x^2 - y^2 = 9$, $x + y = 9$.

SOLUTION

$$(x^2 - y^2) = (x+y)(x-y) = 9$$
$$\Rightarrow \quad (x+y)(x-y) = 9 \text{ .... } (1)$$
$$x + y = 9 \text{ .... } (2)$$

From (2), $x + y = 9$

Substituting into (1)

$$\Rightarrow \quad 9(x-y) = 9$$
$$\Rightarrow \quad x - y = 1 \text{ .... } (3)$$
$$\therefore \quad (2)+(3) \Rightarrow 2x - 8$$
$$x = 4$$

When $x = 4$

$$4 + y = 9$$
$$y = 9 - 4$$
$$y = 5$$

Exercise 1

Solve the following equations:

1. $2x + y = 11, x - 4y^2 = -97$
2. $8x + 9y^2 = 33, 3x + 2y - 7 = 0$
3. $3x + 3y = 2, xy = \frac{1}{12}$
4. $2x - 5y = 0, xy = 10$
5. $3x - 4y = 13$, $xy = 15$
6. $3x + 4y = 20, 2xy = -16$
7. $x^2 + y^2 = 34, x + y = 2$
8. $4x^2 - 9y^2 = 0, 2x + 3y + 12 = 0$
9. $4x^2 - y^2 = 30, 5 + y - 2x = 0$
10. $2x + 3y = 5, 4x^2 - 9y^2 = 15$

II. Revision on Graphical Solution of Simultaneous Quadratic and Linear Equations

Worked Example 4

Use a graphical method to solve the equations $y = 2x^2 + x - 2$, $2y + x = 6$ simultaneously using the scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis for the values of x from -3 to $+2$.

SOLUTION

$$y = 2x^2 + x - 2$$

Table 5.1(a)

x	−3	−2	−1	0	1	2
y	13	4	−1	−2	1	8

$$2y + x = 6$$

Table 5.1(b)

x	−3	−1	2
y	4.5	3.5	2

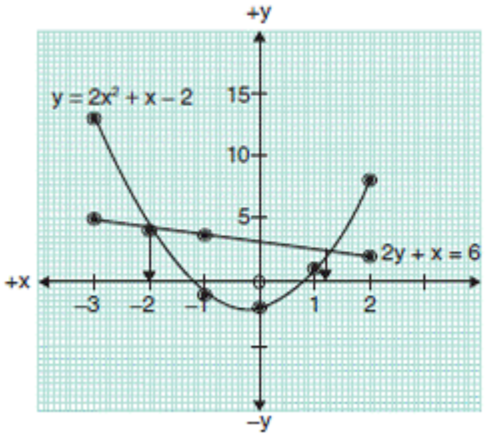


Figure 5.1

Figure 5.1 shows the graphs of the relations. The points of intersection of the line and the curve are $(-2, 4)$ and $(1.2, 2.5)$. These give the values of x and y which satisfy both the equations simultaneously.
The solutions of the simultaneous equations are

$$x = -2, \quad y = 4$$

$$x = 1.2, \quad y = 2.5$$

Worked Example 5

(a) Copy and complete the table of values for the relation

$y = -x^2 + x + 2$ for $-3 \leq x \leq 3$

x	−3	−2	−1	0	1	2	3
y		−4		2			−4

(b) Using scales of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the

y-axis, draw a graph of the relation $y = -x^2 + x + 2$.

(c) From the graph, find the following:

(i) Minimum value of y

(ii) Roots of equation $x^2 - x - 2 = 0$

(iii) Gradient of the curve at $x = 0.5$

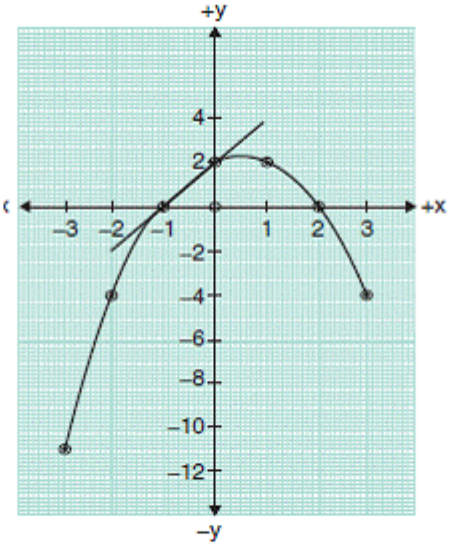
SOLUTION

x	–3	–2	–1	0	1	2	3
y	–11	–4	0	2	2	0	–4

(c) (i) Minimum value of $y = -11$

(ii) $x = -1$ or $x = 2$

(iii) Gradient at $0.5 = 1$



Note

1. Linear equations have only one solution.
2. Simultaneous linear equations have two solutions.
3. Simultaneous linear/quadratic equations have four solutions or two pairs of solutions.

Exercise 2

1. (a) Copy and complete the following table of values for $y = 6 + x - 2x^2$.

Table 5.2

x	–3	–2	–1	0	1	2	3
y	–15						

(b) Using a scale of 2 cm to 1 unit on the x-axis and a scale of 2 cm to 2 units on the y-axis, draw the graph of the relation $y = 6 + x - 2x^2$ for $-3 \leq x \leq 3$.

(c) Use your graph to

(i) Find the greatest value of y;

(ii) Solve the equation $2x^2 - x - 11 = 0$.

(iii) Find the range of values of x for which $6 + x - 2x^2 = 4$. (WAEC)

2. (a) Copy and complete the table of values for the relation

$y = -x^2 + x + 2$ for $-3 \le x \le 3$

Table 5.3

x	−3	−2	−1	0	1	2	3
y		−4		2			−4

(b) Using scales of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw a graph of the relation $y = -x^2 + x + 2$.

(c) From the graph, find the following:

(i) Minimum value of y.

(ii) Roots of the equation $x^2 - x - 2 = 0$.

(iii) Gradient of the curve at $x = -0.5$. (WAEC)

3. (a) Copy and complete the following table of values for the relation

$y = x^2 - 2x - 1$ for $-2 \le x \le 4$.

Table 5.4

x	−2	−1	0	1	2	3	4
y			−1			2	7

(b) Draw the graph of the relation using a scale of 2 cm to 1 unit on both axes.

(c) Using your graph, find the following:

(i) Roots of the equation $x^2 - 2x - 1 = 0$.

(ii) Minimum value of y.

(d) (i) Using the same axes, draw the graph of $y = 2x - 3$.

(ii) From your graphs determine the roots of the equation $x^2 - 2x - 1 = 2x - 3$.

4. (a) Copy and complete table of values of the relation $y = 5 - 7x - 6x^2$ for $-3 \leq x \leq 2$.

Table 5.5

x	-3	-2	-1	-0.5	0	1	2
y	-18		6		5		

(b) Using scales of 2 cm to 1 unit on the x-axis and 2 cm to 4 units on the y-axis, draw the graph of $y = 5 - 7x - 6x^2$.

(c) Use the graph to find the following:

(i) The root of the equation $5 - 7x - 6x^2 = 0$

(ii) The solution of the equation $5 - 7x - 6x^2 = x - y - 5$. (WAEC)

5. (a) (i) Copy and complete the following table of values for the relation $y = 2 + x - x^2$.

Table 5.6

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3
y			0	1.25	2		2	1.25			−4

(ii) Draw the graph of the relation using a scale of 2 cm to 1 unit on each x-axis.

(iii) From your graph, find the greatest value of y, and the value of x at which this occurs.

(b) Using the same axes, draw the graph of $y = 1 - x$.

(c) From your graphs determine the roots of the equation $1 + 2x - x^2 = 0$.

III. Application of Linear and Quadratic Equations to Capital Markets

To solve linear and quadratic equations involving capital markets, which always appear in words, the following steps should be adopted:

1. Read the word problems several times, be sure the meaning is clear.
2. Identify all the unknown concepts or quantities involved in the problem.
3. When forming equations from a given information so as to establish a relationship, clearly state, at the beginning, the concepts or quantity being represented by x or y (say) and ensure that the representation is consistent.
4. Use the appropriate system (linear or quadratic or both) to solve the problem.
5. Finally, check to ensure that the solution has actually solved the problems in question.

Read and carefully work through the following examples, so as to see how each word problem is translated into an equation and eventually solved.

Worked Example 6

The cost of seven shares from a publishing company and eight shares from a bank is N1 750.00. Eight shares from the same publishing company and seven shares from the same bank cost N1 700.00.
Calculate the price of each share from the publishing company and bank.

SOLUTION

Let P = Shares from the publishing company

B = Shares from the bank

$$\therefore \quad 7P + 8B = 1\,750 \text{ .... } (1)$$

$$8P + 7B = 1\,700 \text{ .... } (2)$$

$$(1) \times 8 \Rightarrow 56P + 64B = 14\,000 \text{ .... } (3)$$

$$(2) \times 7 \Rightarrow 56P + 49B = 11\,900 \text{ .... } (4)$$

$$(3) - (4) \Rightarrow \quad 15B = 2\,100$$

$$B = \frac{2\,100}{5}$$

$$B = \text{₦}140.00$$

To solve for P, substitute the numerical value of B into (1)

$$\Rightarrow 7P + 8 \times 140 = 1\,750$$

$$7P + 1\,120 = 1\,750$$

$$7P = 1\,750 - 1\,120$$

$$7P = 630$$

$$\therefore P = \frac{630}{7}$$

$$P = \text{₦}90.00$$

Each publishing company share would cost ₦90.00.

Each bank share would cost ₦140.00.

Worked Example 7

The price of an ordinary share is N4.00 more than the price of a debenture share, if the product of the prices of the two shares is N45.00. Determine the prices of the ordinary and the debenture shares.

SOLUTION

Let the price of debenture share be ₦x.

$\therefore$ The price of the ordinary share will be ₦$(x + 4)$

Then,

$$x(x+4) = 45$$
$$\Rightarrow \quad x^2 + 4x = 45$$
$$\Rightarrow \quad x^2 + 4x - 45 = 0$$

By factorisation

$$x^2 + 4x - 45 = (x+9)(x-5) = 0$$
$$\Rightarrow \quad \text{Either } x = -9 \text{ or } x = 5$$

Since we are dealing with the prices, we take the positive value of *x.* Hence, if debenture share costs N5, ordinary share will cost N9 and vice versa.

Worked Example 8

Half of a man's shares plus one-fifth of his wife's shares in a year is N25 800.00.

Two-thirds of the husband's shares plus two-fifths of wife's shares is N34 933.33.

Then how much is the

(a) Husband's share?

(b) Wife's share?

SOLUTION

Let H = Husband

W = Wife

$$\left(\frac{1}{2}\right)H + \left(\frac{1}{5}\right)W = ₦25\,800.00$$

$\therefore$ Multiplying throughout by 10

$$5H + 2W = ₦258\,000.00 \text{ .... } (1)$$

$$\left(\frac{2}{3}\right)H + \left(\frac{2}{5}\right)W = ₦34\,933.33$$

$\therefore$ Multiplying throughout by 15

$$10H + 6W = ₦525\,000.00 \text{ .... } (2)$$

$$(1) \times 2 \Rightarrow 10H + 4W = ₦516\,000.00$$

$$(2) - (3) \Rightarrow 2W = 8\,000.00$$

$$\Rightarrow \quad W = \frac{8\,000.00}{2}$$

$$\therefore W = ₦4\,000.00$$

To solve for H, substitute the numerical value of W into (1)

$$\Rightarrow 5H + 2 \times ₦4\,000.00 = ₦258\,000.00$$

$$5H + ₦80\,000 = ₦258\,000.00$$

$$5H = ₦258\,000.00 - ₦8\,000.00$$

$$= ₦250\,000$$

$$H = ₦\frac{250\,000}{5}$$

$$\therefore H = ₦50\,000.00$$

$\therefore$ The husband's share is ₦50 000.00 while the wife's share is ₦4 000.00.

Exercise 3

1. A man bought two shares from company A and two shares from company B. If the sum of the amount of the four shares is ₦2 400 and product of their prices is ₦3 200, find the price of each of the shares.
2. Three times the number of shares from a manufacturing company plus four times the number of shares from a bank is 11, and their product is 2. Find the number of shares from the
   (a) Manufacturing company
   (b) Bank.
3. The price of an ordinary share and a preference share of a stock exchange differ by ₦12.00 and their product is ₦325.00. Calculate the prices of the ordinary and preference shares.

SUMMARY

In this chapter, we have learnt the following:

❖ Revised simultaneous linear and quadratic equations.

❖ Steps to be adopted while using simultaneous linear and quadratic equations to solve problems in capitalmarkets.

❖ Revised the use of graphs to solve simultaneous linear and quadratic equations.

GRADUATED EXERCISES

Solve the following equations:

1. $3x + \frac{1}{2}y = 8$

   $\frac{1}{2}x + 2y = 9$ (WAEC)

2. $x + y = 8$

   $2x^2 + y^2 = 43$

3. $3x = 2y$

   $x^2 + xy + y^2 = 19$

4. (a) Draw the table of values for the relation $y = x^2$ for the interval $-3 \le x \le 4$.

   (b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of

   (i) $y = x^2$

   (ii) $y = 2x + 3$ for $-3 \le x \le 4$.

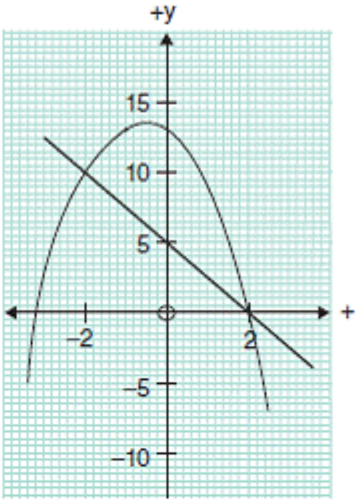
   (c) Use your graph to find the following:

   (i) The roots of the equation $x^2 = 2x + 3$.

   (ii) The gradient of $y = x^2$ at $x = -2$. (WAEC)

5. Find the solution of simultaneous linear and quadratic equations in Figure 5.2.

(a)



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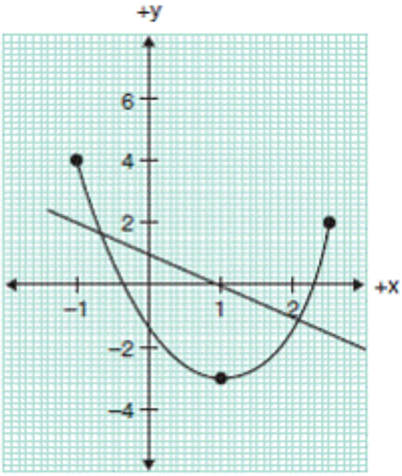

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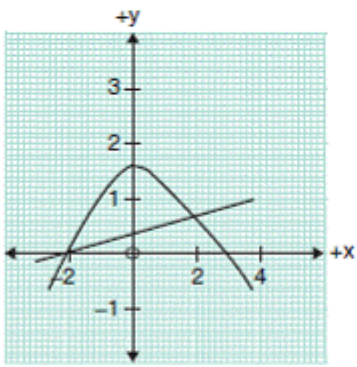

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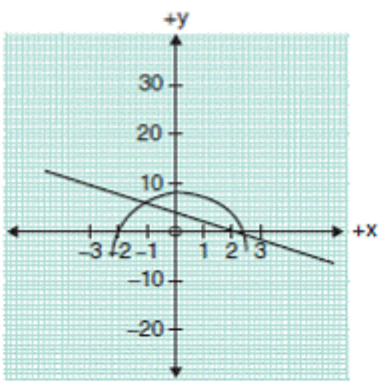

Figure 5.2

6. The difference between the dividend from a preference share and an ordinary share is N20.00. If twice one of the shares is three times the square of the other, find the amount of each share.

7. (a) Draw the graph of $y = 3x^2 - 5x - 2$ for $-1 \leq x \leq 5$, using a scale of 2 cm to represent 1 unit on x-axis and 2 cm to represent 4 units on y-axis.
   (b) From the graph, find the following:
      (i) Gradient at the point where x has the value 1.5.
      (ii) Roots of the equation $3x^2 - 5x + 1 = 0$. **(NECO)**

8. (a) Draw the table of values for the relation $y = x^2$ for the internal $-3 \leq x \leq 4$.
   (b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the

y-axis, draw the graph of

(i) $y = x^2$

(ii) $y = 2x + 3$ for $-3 \le x \le 4$.

(c) Use your graph to find the following:

(i) The roots of the equation $x^2 = 2x + 3$.

(ii) The gradient of $y = x^2$ at $x = -2$.

9. (a) Copy and complete the table for the relation $y = x^2 - 3x - 4$ for $-1.5 \le x \le 4.5$:

x	-1.5	-1	0	1	2	3	4	4.5
y				-0.6			0	2.8

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of $y = x^2 - 3x - 4$ for $-1.5 \le x \le 4.5$.

(c) From your graph, obtain the following, correct to one decimal place:

(i) Roots of the equation $x^2 - 3x - 4 = 0$

(ii) Roots of the equation $x^2 - 3x = 5$

(iii) Minimum value of $y = x^3 - 3x - 4$. **(WAEC)**

10. (a) Prepare a table of specification of values for the relation $y = 2x^2 - 3$ from $x = -4$ to $x = 4$ at interval of limit.

(b) Use your table to draw the graph of the relation.

(c) From your graph, find the following:

I. The roots of $2x^2 - 3 = 0$, correct to one decimal place.

II. The least value of y and the corresponding value of x.

III. The gradient of the tangent to the graph at $x = 1$.