Chapter 19: Probability

OBJECTIVES
At the end of the chapter, students should be able to:
1. Define and explain some important terms used in probability.
2. Demonstrate practical examples of each term.
3. List the chance instruments used in probability.
4. Explain the frequentist approach to theoretical probabilities.
5. Describe with examples, the equiprobable sample space.
6. Solve examples on equiprobable sample space.
7. Solve simple problems on probability.
8. Identify and solve various practical problems on probability involving health, finance, population, etc.

I. Meaning of Some Important Terms Used in Probability

(i) Experiment
This is any process that generates raw data. For example, throwing of a die.
(ii) Sample space
This is the set whose elements represent all possible outcomes of an experiment. It is represented by the symbol S. For example, when a die is thrown, the sample space:
S = {1, 2, 3, 4, 5, 6}
(iii) Sample point
An element of a sample space is called a sample point. Example: given that a sample space S = {1, 2, 3, 4}, each member of the sample space is a sample point.
(iv) Event
An event is a subset of the sample space S governed by a given rule. Example: when a fair die is tossed, the sample space S = {1, 2, 3, 4, 5, 6}. If the rule is that a prime number appears, the event E = {2, 3, 5}.
(v) Number of elements
If S represents a sample space, then n(S) is defined as the number of elements in the sample space S.
(vi) Probability of an event (experimental probability)
(a) In general sense, the chance of an event happening when expressed quantitatively is called probability.
(b) Let S be the sample space and E an event in S. If n(S) is the number of points in S and
n(E) the number of points in E, then the probability of event E is defined as:

$$P(E) = \frac{\text{Number of points in } E}{\text{Number of points in } S}$$

$$= \frac{n(E)}{n(S)}$$

(vii) Sure event
This is an event whose probability is one.
Example: Given that S = {2, 3, 5, 7, 11}
âˆ´ n(S) = 5. Let B represent the event that a prime number occurs. âˆ´ B = {2, 3, 5, 7, 11} and n(B) = 5.
Hence, P(B) = n(B) = 5 = 1.
(viii) Impossible event
This is an event whose probability is zero. Example: In tossing a die, the sample space S = {1, 2, 3, 4, 5, 6}. If A is the event that a number greater than 6 appears, P(A) = 0. Hence, A is an impossible event.
(ix) Equiprobable sample space
This is the sample space in which we assign each of the sample elements equal probability. Example: In throwing a fair die once, the sample space S = {1, 2, 3, 4, 5, 6}. The probability of 1 appearing in a throw is the same as the probability of any of the other numbers appearing. Each having 1/6 as the probability occurrence.
(x) Properties of probability
Let A be an event and S the sample space.
(a) 0 â‰¤ P(A) â‰¤ 1.
(b) Î£P(A) = 1.
(c) If A and A_c are complementary events, then P(A) + P(A_c) = 1 or P(A) = 1 âˆ’ P(A_c).
(xi) Probability
Probability is a numerical measure of the likelihood of the occurrence of an event. Probability of some events

could be by experiment or theory as the case may be.

(xii) Chance instruments

Chance instruments are used in probability experiments such as throwing of die, tossing of coin and playing of cards. The chance instruments include the following:

(a) The dice: It has six faces numbered 1, 2, 3, 4, 5 and 6.

(b) The coin: It has two faces. These are head (H) and tail (T).

(c) Park of playing cards: There are 52 in number. It consists of 13 spades, 13 hearts, 13 clubs and 13 diamonds. The diamond further consists of an ace, a king, a queen, a jack and 9 other ordinary cards.

(xiii) Unbiasedness

An experiment (probability) is said to be unbiased, if there exist an outcome of a coin. If there is an appearance of head or tail, we say the experiment is unbiased, i.e. without favour. Otherwise, it is biased, if there is no appearance of either head or tail.

Worked Example 1

A fair coin is tossed twice. Find the probability that at least one head appears.

SOLUTION

The sample space S = {HH, HT, TH, TT}

âˆ´ n(S) = 4

Let D represent the event that at least one head appears.

D= {HH, HT, TH}

âˆ´ n(E) = 3

Probability that at least one head occurs

$$=\frac{n(E)}{n(S)}=\frac{3}{4}$$

Worked Example 2

A fair die is thrown. Find the probability of obtaining a prime number.

SOLUTION

When a fair die is thrown, the sample space S = {1, 2, 3, 4, 5, 6}, n(S) = 6.

Let A represent the event that a prime number appears.

A = {2, 3, 5}, n(A) = 3

$$\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2}$$

Worked Example 3

A number is chosen at random from the set S = {x: 5 â‰¤ x < 15} where x is a natural number. Find the probability that the number is an even number.

SOLUTION

5 â‰¤ x < 15 implies that 5 is included in the set but 15 is not included.

âˆ´S **= {**x**: 5 â‰¤** x **< 15}**

= {5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

n(S) **= 10**

Let E **represent the event that the number chosen is an even number.**

âˆ´E **= {6, 8, 10, 12, 14},** n(E) **= 5**

$$P(E)=\frac{n(E)}{n(S)}=\frac{5}{10}=\frac{1}{2}$$

Worked Example 4

Table 19.1 shows the marks obtained by 15 students in a Mathematics test in which the total mark is 10.

Table 19.1

Marks	2	3	4	5	6	7	8
Numbers of students	2	1	2	3	5	1	1

Use Table 19.1 to fi nd the following:

(a) The probability that a student scored exactly 6 marks.

(b) (i) If the pass mark is greater than 4, find the probability that a student failed the test.

(ii) Hence, fi nd the probability that a student passed the test.

SOLUTION

The total number of students **n**(**S**) = 15.
(a) If **R** is the event that a student scored exactly 6 marks, then from the table **n**(**R**) = 5. Hence, the probability that a student scored exactly 6 marks is

$$P(R)=\frac{n(R)}{n(S)}=\frac{5}{15}=\frac{1}{3}$$

(b) (i) Let E represent the event that a student failed the test. If the pass mark is greater than 4, then from the table, number of students who failed the test n(E) = 2 + 1 + 2 = 5.
Hence, the probability that a student failed the test

$$\frac{n(E)}{n(S)}=\frac{5}{15}=\frac{1}{3}$$

(ii) The probability that a student passed the test is P(E') = 1 âˆ’ P(E), where E is the event that a student failed the test.

= 1 âˆ’ $\frac{1}{3}$

= $\frac{2}{3}$

Exercise 1

1. A number is chosen at random from the set X = {p: 3 â‰¤ p â‰¤ 12}. Find the probability that the number chosen is a prime.
2. The data below shows the number of lecturers employed in various departments in a particular University in Nigeria

Mathematics	20
Physics	15
Chemistry	9
Biology	8
Geography	25
Music	3

If a member of these departments is promoted, what is the probability that he is from either Mathematics or Biology?
3. Table 19.2 shows the distribution of the amount of money shared by 9 students in an essay competition.

Table 19.2

Amount (₦) shared	30	50	100	200
Number of students	3	3	2	1

From Table 19.2, find the probability of the students who collected:
(a) above 50 naira.
(b) below 100 naira.
(c) between 30 and 200 naira.
4. A bag contains eight red, four white and six blue balls; (all of the same material and size). If a ball is picked at random, what is the probability of obtaining
(a) a red ball?
(b) a white ball?
(c) a blue ball?
(d) a ball which is not white?
5. What is the chance of choosing at random a three digit number which is divisible by 77?
6. A fair die is tossed once, what is the probability that 4 shows up?
7. In a deck of 52 cards, there are 13 cards in each of the four suits: spades, hearts, clubs and diamonds. Each suit has an ace, a king, a queen and a jack. Use the diagram below to:
(a) draw an ace.
(b) draw a diamond.
(c) draw the king of spades.

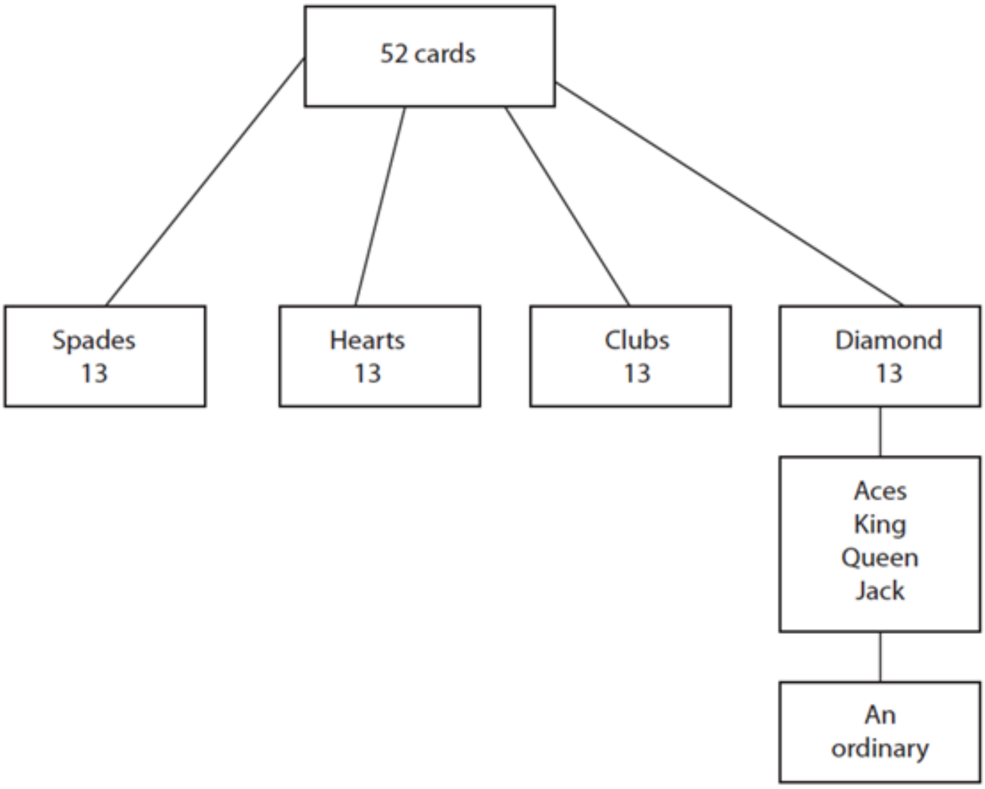
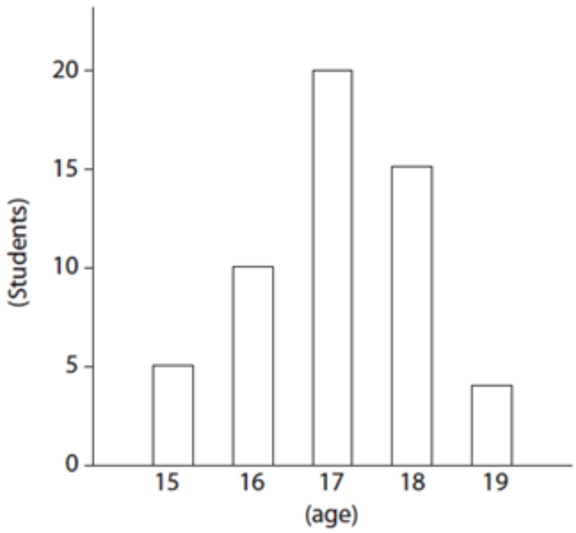


Figure 19.1

8. In a well-shuffl ed deck of 52 cards, what is the probability of picking a queen in the first draw?
9. In question 8, what is the probability of not picking a queen?
10. What is the probability of obtaining a number greater than 2 in a throw of a single fair die? (WAEC)
11. Two fair dice are tossed together. Find the probability that the total score is at most 4. (WAEC)
12. The bar graph shows the ages of students in a senior class of a school.



(a) How many students are there in the class?
(b) If a student is chosen at random from the class for the post of a prefect, what is the probability of choosing:
(i) A 16 year old or an 18 year old?
(ii) A student in the modal age?
13. A fair die has its faces marked 3, 3, 3, 5, 5 and 4. What is the probability that it shows a 3 when thrown?
14. A letter is chosen at random from the alphabets. What is the probability that it is (a) M? (b) E?
15. Table 19.3 shows the distribution of relief materials for 450 refugees in Somalia

Table 19.3

Rice	50 tonnes
Blanket	80 tonnes
Flour	75 tonnes
Wheat	35 tonnes
Grains	60 tonnes

If a person among the refugees is selected at random, what is the probability that (a) wheat (b) grains or rice was given to him/her?

16. In a class of 56, 32 are girls. What is the probability that a boy is picked randomly to represent the class in a quiz competition?

17. If a die is thrown and later a coin is tossed, the possible outcomes are:

(1, H) (2, H) (3, H) (4, H) (5, H) (6, H)

(1, T) (2, T) (3, T) (4, T) (5, T) (6, T)

Repeat the experiment 100 times and find the experimental probability of obtaining (a) a 5 and a head (b) a 1 and a tail (c) a 4 and a tail.

18. Table 19.4 shows the number of cars parked at a particular car park.

Table 19.4

Car	Mazda	Camry	Honda	Kia
Number	42	18	12	6

Find the probability that any car leaving the park is

(a) Camry

(b) Honda

(c) Kia.

19. Table 19.5 shows the numbers of students in each age group in a class.

Table 19.5

Age (years)	12	13	14	15	16
Number of pupils	4	8	6	10	12

How many pupils are in the class? What is the probability that a student chosen at random from the class will be

(a) 14 years old?

(b) over 12 years old?

(c) 15 years old or less?

20. A die is rolled 250 times. The outcomes are shown in Table 19.6.

Table 19.6

Number	1	2	3	4	5	6
Number of times	50	35	55	38	40	32

Find the experimental probability of obtaining a

(a) 4

(b) 6

(c) 3.

II. Theoretical Probability

Majorly, experimental probability uses numerical records of past events to predict the future. Hence, its predictions are not always accurate. However, the probability that a die shows a 6 is equal to the probability that it shows a 5 or 4,

i.e. **P**(6) = **p**(5) = **p**(4) = **p**(3) = **p**(2) = **p**(1) = 1/6.

Each face of a fair die can show up if tossed because of the equally likely nature of the faces of the die. This is an example of theoretical probability. Theoretical probability is accurate. Exact values can be calculated by considering the physical feature of the given events/ situations.

Worked Example 5

The variables (**a**, **b**) can take values from the set **S** = {0, 1, 2, 3, 4, 4}. Find the probability that (a) **a** = **b** (b) **a** + **b** > 4.

SOLUTION

Table 19.7

		0	1	2	3	4
	0	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)
	1	(1, 0)	(1, 1)	(1, 2)	(1, 3)	(1, 4)
	2	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(2, 4)
B	3	(3, 0)	(3, 1)	(3, 2)	(3, 3)	(3, 4)
	4	(4, 0)	(4, 1)	(4, 2)	(4, 3)	(4, 4)

The sample space S = {(0, 0), (0, 1), â€¦, (4, 3), (4, 4)}, n(s) = 25.
Let E represent the event that a = b.
E = {(0,0),(1, 1),(2,2),(3, 3)(4, 4)}

$$P(E)=\frac{n(E)}{n(S)}=\frac{5}{25}=\frac{1}{5}$$

Let B represent the event that a + b > 4
B = {(1, 4), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4,1), (4, 2), (4, 3), (4, 4)}
n(B) = 10

$$P(B)=\frac{n(B)}{n(S)}$$

$$\frac{10}{25}=\frac{2}{5}$$

Worked Example 6

Three coins are tossed once. What is the probability of obtaining
(a) 3 tails? (b) 2 tails and a head? (c) no tail?

SOLUTION

The following are the eight possible outcomes: HHH, HHT, HTH, THH, TTH, THT, HTT and TTT.

(a) $P(\text{3 tails})=\frac{1}{8}$

(b) $P(\text{2 tails and one head})=\frac{3}{8}$

(c) $P(\text{no tails})=P(\text{all heads})=\frac{1}{8}$

Worked Example 7

A pack of 52 playing cards is shuffled and a card is drawn at random. Calculate the probability that it is either a five or a red nine.
Hint: There are 4 fives and 2 red nines in a pack of 52 cards.

SOLUTION

Let **E**1 be the event drawing a fi ve
âˆ´n(E1) = 4
There are 52 cards **âˆ´ n(S)** = 52

$$\therefore P(E_1)=\frac{n(E_1)}{n(S)}=\frac{4}{52}=\frac{1}{13}$$

Let E_2 be the event drawing a red nine

$$n(E_2)=2$$

$$\therefore P(E_2)=\frac{n(E_2)}{n(S)}=\frac{2}{52}=\frac{1}{26}$$

Since E_1 and E_2 are disjoint

$$\therefore P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

$$\frac{1}{13} + \frac{1}{26} = \frac{3}{26}$$

or

Number of fives and red nines

$$= 4 + 2 = 6$$

$$P(\text{a five or a red nine}) = \frac{6}{52} = \frac{3}{26}$$

Exercise 2

1. If a die is tossed twice, what is the probability of showing:
(a) a sum of 9 points?
(b) a sum of 12 points?
2. Abu and Uche have their birthdays in the same week. Calculate the probability that their birthdays falls on the same day.
3. A die is loaded in such a way that an odd number is twice as likely to appear as an even number. If E is the event that a number less than 5 appears in a single toss of the die, find the probability of E.
4. A coin and a die are tossed simultaneously. Find the probability that
(a) a head and an even number appears
(b) a tail and an odd number appears.
5. Ade throws a fair six-sided die. What is the probability that he throws
(a) a 5? (b) a 4? (c) a number greater than 3? (d) an odd number?
6. A letter is chosen at random from the alphabets. Find the probability that it is
(a) P (b) T or Y
(c) one of the letters of the word EXPECTANCY
(d) not one of the letters of the word CHAIR.
7. A sack contains two white balls, three blue balls and four purple balls. A ball is picked from the bag at random. What is the probability that it is
(a) white? (b) blue? (c) purple? (d) green? (e) not white? (f) either white or purple?
8. Table 19.8 shows the number of pupils in different age groups in a school.

Table 19.8

Age (years)	6	7	8	9	10	11	12
Number	96	35	135	250	160	180	280

Find the probability that a pupil chosen at random is
(a) 8 (b) 8 or less.
9. There are 450 men and 325 women in a political rally. If a person is chosen at random, what is the probability that a man is chosen?
10. A die is tossed. What is the probability that the outcome is divisible by 3?
11. A card is picked at random from a perfectly shuffled deck of 52 cards. What is the probability that it is 6 of clubs?
12. Table 19.9 shows the age distribution of some pupils in a certain school.

Table 19.9

Age (years)	7	8	9	10	11
Number of pupils	10	12	6	4	3

(a) How many pupils were their ages given in Table 19.9?
(b) If a pupil is selected at random, what is the probability that he is 9 years old?
13. Table 19.10 shows the types of food stuffs in Bodija market and their sellers.

Table 19.10

Type of food stuff	Garri	Bean	Rice	Yam
Number of sellers	25	16	34	50

Find the probability that
(a) a garri seller
(b) a yam seller
(c) a bean seller
(d) a rice seller
is chosen to clean the market on each environmental day.
14. A number is chosen at random from integers 1 to 15. Find the probability that the number is
(a) a prime (b) a multiple of 3 (c) either a prime or a multiple of 3.
15. A fair octahedral (eight sided) die has the following numbers on its faces: 2, 3, 1, 2, 1, 3, 1, 1. Each time the die is thrown, the score is the number on the top face. If the die is thrown
(a) once, what is the probability that the score is not 3?
(b) twice, find the probability of scoring a total of 6.
(c) twice, find the probability of scoring a total of 5. (WAEC)
16. From a deck of 52 well-shuffled cards, a card is picked at random: what is the probability that it is
(a) a king of diamonds?
(b) an ace of clubs?
(c) a jack of spades?

III. Addition and Multiplication of Probabilities

(i) Mutually exclusive events
Two events A and B are said to be mutually exclusive, if they cannot occur simultaneously. In other words, two events A and B are mutually exclusive, if A â^© B = f or P(A â^© B) = 0.
Under this condition, for two mutually exclusive events A and B, the addition law of probability becomes P(A â^ª B) = P(A) + P(B).

Worked Example 8

Find the probability that a number chosen at random from a set of integers between 2 and 13 inclusive is either a prime or a multiple of 4.
SOLUTION
The total outcome consists of the set:
E = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}
A = {2, 3, 5, 7, 11, 13}, where A is the set of primes numbers.
B = {4, 8, 12}, where B is the set of multiples of 4.
Thus, n(A) = 6, n(B) = 3 and n(E) = 12.
The event favourable to either a prime or a multiple of 4 is A â^ª B.
But, n(A â^ª B) = n(A) + n(B) since
A â^© B = 0.
n(A â^© B) = 0
â^´ P(A â^ª B) = P(A) + P(B)

$= \frac{6}{12} + \frac{3}{12} = \frac{9}{12}$

Worked Example 9

If A is the event that an even number appears and B is the event that an odd number greater than one occurs in a single throw of a die.
(a) Show that A and B are mutually exclusive.
(b) Find the probability of obtaining either an even number or an odd number greater than one.
SOLUTION
(a) The sample space S = {1, 2, 3, 4, 5, 6}
â^´n(S) = 6.
A = {2, 4, 6}; n(A) = 3

â^´ $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$
B = {3, 5}; n(B) = 2

âˆ´ $P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$
Since A = {2, 4, 6} and B = {3, 5}
âˆ´A âˆ© B = f.
Hence, A and B are mutually exclusive.

(b) P(A âˆª B) = P(A) + P(B) = $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

Exercise 3

1. If A and B are mutually exclusive events such that P(A_c) = 0.6 and P(B) = 0.5, find P(A âˆª B).
2. If A and B are mutually exclusive events such that P(B_c) = 0.7 and P(A_c) = 0.8. Find P(A âˆª B).
3. Find the probability that a three or an even number appears when a fair die is tossed once.
4. Find the probability that a letter chosen at random from the alphabet is either a vowel or one of the letters P, Q and R.
5. A number is chosen at random from the set {2, 4, 6, 8, 10, 12, 14, 16, 16, 18, 20). Find the probability that it is either a factor of 18 or a multiple of 6.
6. A letter is chosen at random from the word MATHEMATICS. What is the probability that it is
(a) either in the word MAT or in the word TICS?
(b) not in the word MET?
7. A card is chosen at random from a pack of playing cards. What is the probability that it is either a heart or the queen of spades?
8. Given A = {5, 7, 12} and B = {8, 16, 32, 64}.
(a) If one element is selected at random from A, what is the probability that it is even?
(b) If one element is selected at random from B, what is the probability that it is a multiple of 4?
(c) If one element is selected at random from A âˆª B, what is the probability that it is either a prime factor of 35 or a multiple of 16?
9. The probability of picking a white ball from a bag is 1/3 and that of picking a red ball is 5/6. What is the probability of picking a white or a red ball?
10. There are 10 N10 notes in Ade's pocket and 6 N50 notes. What is the probability of picking a N10 note or a N50 note?

Worked Example 10

If A and B are two events such that $P(A \cup B) = \frac{5}{8}$, $P(A^c) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{8}$, find (a) $P(A)$ and (b) $P(B)$.

Since A and A^c are complementary events

$$P(A) + P(A^c) = 1$$
$$\therefore P(A) = 1 - P(A^c)$$
$$= 1 - \frac{3}{4}$$
$$= \frac{1}{4}$$
$$\therefore P(A) = \frac{1}{4}$$

By the addition law of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$
$$\therefore \frac{5}{8} = \frac{1}{4} + P(B) - \frac{1}{8}$$
$$P(B) = \frac{5}{8} + \frac{1}{8} - \frac{1}{4}$$
$$= \frac{4}{8} = \frac{1}{2}$$
$$P(B) = \frac{1}{2}$$

Worked Example 11

Chukwu sat for Common Entrance Examination in English language and Mathematics. The probability that he passes English language is **3/4** and the probability that he passes Mathematics is **4/**9. If the probability of passing at least one subject is **2/3**, what is the probability that he will pass both subjects?

SOLUTION

Let E and M represent the events that Chukwu passes English language and Mathematics, respectively.

$$\therefore P(E) = \frac{3}{4} \text{ and } P(M) = \frac{4}{9}$$

Probability of passing at least one subject $P(M \cup E) = \frac{2}{3}$. To find the probability that he will pass both subjects

$$P(M \cap E) = P(M) + P(E) - P(M \cup E)$$

$$= \frac{4}{9} + \frac{3}{4} - \frac{2}{3}$$

$$= \frac{19}{36}$$

Exercise 4

1. Given that P and Q are two events such that P(P âˆª Q) = 1/8 , P(P_c) = 5/8 and P(P âˆ© Q) = 1/4, find P(Q).
2. The probability of an event P = 1/4_ . The probability of Q is 3/4_ . The probability of both P and Q is 4/7_ . What is the probability of the event P or Q or both.
3. A number is chosen at random from the set of integers 1 to 10. Find the probability that the number is (a) prime (b) a multiple of 2 (c) either a prime or a multiple of 2.
4. Mrs. Jones is expecting a baby. The probability that it will be a boy is 1/2 and the probability that the baby will have blue eyes is 1/4. What is the probability that she will have a blueeyed baby boy? (WAEC)
5. A group of eleven people can speak either English language or French or both. Seven can speak English language and six can speak French. What is the probability that a person chosen at random can speak both English language and French? (WAEC)
6. The probabilities of the events X and Y are 1/4 and 3/7, respectively. If the probability of P and Q is 3/28, what is the probability of P or Q or both?
7. Given that A = {1, 2, 3, 4, 5, 6} and B = {1, 5, 7, 9, 11} where A and B are subsets of the set of integers, find the probability that any number picked at random is from A or B.
8. The probability of the event A is 4/9 and the probability of the event B is 1/ 4 . If the probability of the events
A or B or both is 1/3, what is the probability of the outcomes A and B?
9. The probability of an event P is 2/7, the probability of Q is 2/5. The probability of both P and Q occurring is 4/35. What is the probability of P and Q or both occurring?

(ii) Independent events

Two or more events are said to be independent, if the probability of the occurrence of any of them is not influenced by the occurrence of the other. In other words, for two events A and B to be independent, P(A/B) = P(A).

Recall that $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ i.e.

$$P(A \cap B) = P(A/B) \cdot P(B) \text{ .... (i)}$$

If A and B are independent, P(A/B) = P(A). Using this condition, equation (i) becomes
P(A âˆ© B) = P(A) Â· P(B) â€¦â€¦â€¦. (ii)
Equation (ii) gives the multiplication law for independent events.

Worked Example 12

A bag contains 6 oranges, 4 bananas and 2 mangoes. If three fruits are picked from the bag, fi nd the probability that they are picked in the order orange, banana and mango, if the selection is done with replacement.

SOLUTION

Let R represent the event that the fruit picked is an orange, B the event that the fruit picked is a banana and M the event that the fruit picked is a mango.

Total number of fruits $= 6 + 4 + 2 = 12$

Number of oranges $n(R) = 6$

$$\therefore P(R) = \frac{6}{12} = \frac{1}{2}$$

Number of bananas $n(B) = 4$

$$\therefore P(B) = \frac{4}{12} = \frac{1}{3}$$

Number of mangoes $n(M) = 2$

$$\therefore P(M) = \frac{2}{12} = \frac{1}{6}$$

$$\therefore P(R \cap B \cap M) = P(R)P(B)P(M)$$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{36}$$

Worked Example 13

If the probability that Obinna will pass Mathematics examination is _23_ and the probability that Ade will pass the same examination is _14_ , what is the probability
that they will both pass the examination?

SOLUTION

Since the events of passing the examination by Obinna and Ade are both independent, then **P**(Obinna and Ade) passing the examination will be **P**(Obinna passed). **P**(Ade passed)

$= \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}.$

Worked Example 14

A box contains 6 green, 7 orange and 4 red square shapes (all of identical sizes). A square shape is selected at random from the box and replaced. A second shape is then selected. Find the probability of obtaining:

(a) two red square shapes.

(b) two green square shapes or two orange square shapes.

(c) one orange square shape and one red square shape in any order.

SOLUTION

Since the shapes are drawn and replaced, the events are independent.

(a) P (two red shapes)

= P (1st red shape) and P (2^{nd} red shape)

= P (R_1) Ã…~ P (R_2) (multiplication rule)

$$= \frac{4}{17} \times \frac{4}{17} = \frac{16}{289} \text{ or } 0.055$$

(b) P (two green or two orange shapes)

$= P$ (1st green and 2nd green or 1st orange and 2nd orange)

$= P(G_1) \times P(G_2) + P(O_1) \times P(O_2)$

(multiplication and addition rule)

$$= \frac{6}{17} \times \frac{6}{17} + \frac{7}{17} \times \frac{7}{17}$$

$$= \frac{36}{289} + \frac{49}{289} = \frac{85}{289} \text{ or } 0.294$$

(c) P (1st orange and 2nd red or 1st red and 2nd orange)

$$= P(\text{O}) \times P(\text{R}) + P(\text{R}) \times P(\text{O})$$

$$= \frac{7}{17} \times \frac{4}{17} + \frac{4}{17} \times \frac{7}{17}$$

$$= \frac{28}{289} + \frac{28}{289} = \frac{56}{289} \text{ or } 0.194$$

Worked Example 15

If the probability of Asiru solving a problem is **1**/6 and Mba solving it is 3/5, what is the probability that
(a) at least one of them will solve it?
(b) one will and one will not?

SOLUTION

(a) The probability of Asiru solving the problem is 1/6 . Probability that Asiru does not solve the problem is 1 **â^'**1/6 = 5/6 .
The probability of Mba solving it is 3/5.
Probability that Mba does not solve the problem is 1 **â^'** 3/5 = 2/5 . The probability that at least one of them will solve it is the same as:
The probability that either Asiru solves the problem and Mba does not or Mba solves the problem and Asiru does not or both of them solves the problem.
Hence, the probability that at least one of them will solves the problem =

$$\left(\frac{1}{6} \times \frac{2}{5}\right) + \left(\frac{3}{5} \times \frac{5}{6}\right) + \left(\frac{1}{6} \times \frac{3}{5}\right) = \frac{2}{3}$$

(b) The probability that one will and one will not is the same as either Asiru solves the problem and Mba does not or Mba solves the problem and Asiru does not.
Hence, the probability that one will and one will not

$$= \left(\frac{1}{6} \times \frac{2}{5}\right) + \left(\frac{3}{5} \times \frac{5}{6}\right)$$

$$= \frac{2}{30} + \frac{15}{30} = \frac{17}{30}$$

Exercise 5

1. In a game, a fair die is rolled and two unbiased coins are tossed once. What is the probability of obtaining 3 and a tail? (WAEC)
2. Two dice are thrown together. What is the probability of obtaining:
(a) a total score of at least 6?
(b) a double (i.e. same number in each die)?
(c) a total score greater than 7?
(d) a double or a total score greater than 8?
3. The probabilities that two hunters P and Q hit their targets are 2/3 and 3/4 respectively. The two hunters hit the target together.

(a) What is the probability that they both miss the target?
(b) If the target is hit, what is the probability that
(i) only hunter P hits it?
(ii) only one of them hits it?
(iii) both hunters hit the target? (WAEC)
4. A box contains four green balls, five white balls and three blue balls, all of the same size. A ball is selected at random from the box and replaced.
A second ball is then selected. Find the probability of obtaining:
(a) two blue balls.
(b) two green balls or two white balls.
(c) one white ball and one blue ball in any order.
5. The probability that a male is born is equal to the probability that a female is born in any given family. Each probability is equal to 1/2. What is the probability of having at least one male in a family of 3?
6. What is the probability of having a male and two females in question 5?
7. Three coins are thrown once. What is the probability of
(a) obtaining three heads?
(b) obtaining at least one head?
8. What is the probability of obtaining at least 9 in two tosses of a die?
9. A bag contains green, white and purple balls. One ball is selected at random. If the probability that it is green is 3/4, and the probability that is white is 2/3, what is the probability that it is purple?
10. A box contains four blue, five white, eight green and three red balls, all of the same size. What is the probability of selecting at random:
(a) a green or a white ball?
(b) a ball that is not red?
(iii) Dependent events
Let A and B be two events that are dependent. The probability that an event A occurs given that B has occurred is the conditional probability of A given B. It is defined as:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \text{ with } P(B) > 0 \,\ldots\ldots\ldots\, \text{(i)}$$

From (i)
P (A â^© B) = P(B) Â· P(A / B) â€¦â€¦â€¦â€¦â€¦â€¦ (ii)
Equation (ii) gives the multiplication law for conditional probability.

Worked Example 16

A box contains a bunch of 15 keys. Seven of them are union while the remaining are orion type. If two keys are taken one after the other from the box without replacement, find:
(a) the probability that the first key taken is union and the second is orion.
(b) the two keys are of the same type.
SOLUTION
(a) Total number of keys = 15
Number of union keys = 7
Number of orion keys = 8
Let A represent the event that the first key taken is union and B the event that the second key taken is orion.

$$\therefore P(A) = \frac{7}{15} \text{ and } P(B/A) = \frac{8}{14}$$

$$\therefore P(A \cap B) = P(A) \cdot P(B/A)$$

$$= \frac{7}{15} \times \frac{8}{14} = \frac{4}{15}$$

(b) There are two possibilities that (a) the first key taken is union and the second
one is also union or (b) the first key taken is orion and the second key taken is also orion.
Let $\mathbf{A}_1$ represent the event that the first key taken is union and $\mathbf{A}_2$ the event that the second key taken is union; $\mathbf{B}_1$ the event that the first key taken is orion and $\mathbf{B}_2$ the event that the second key taken is orion.
â^´P ($\mathbf{A}_1$ **â^©** $\mathbf{A}_2$**) = P (**$\mathbf{A}_1$**)** Â· **P (**$\mathbf{A}_1$**/**$\mathbf{A}_2$**)**

$$= \frac{7}{15} \times \frac{6}{14} = \frac{1}{5}$$

$$\therefore P(B_1 \cap B_2) = P(B_1) \cdot P(B_1/B_2)$$

$$= \frac{8}{15} \times \frac{7}{14} = \frac{4}{15}$$

Hence, the probability that the two keys are of the same type

$$= P(A_1 \cap A_2) \cup P(B_1 \cap B_2)$$

$$= P(A_1 \cap A_2) + P(B_1 \cap B_2)$$

$$= \frac{1}{5} + \frac{4}{15} = \frac{7}{15}$$

(iv) Bayes rule

Let $\mathbf{E}_1$, $\mathbf{E}_2$, â€¦, $\mathbf{E}_k$ be a set of mutually non-overlapping exclusive events with probability **P(X)** > 0, then the conditional probability of event **E** given that **X** has occurred is:

$$P(E/X) = \frac{P(E_i \cap X)}{P(X)} = \frac{P(E_i)\, P(X/E_i)}{\Sigma\, P(E_j)\, P(X/E_j)}$$

Worked Example 17

There are three local hotels, $\mathbf{E}_1$, $\mathbf{E}_2$ and $\mathbf{E}_3$ where the invitees can lodge during the period of an accreditation exercise. 20% of them were assigned to hotel $\mathbf{E}_1$, 45% to hotel $\mathbf{E}_2$ and the remaining 35% to hotel E_3. Given that the probability of faulty plumbing in hotels E_1, E_2 and E_3 are 0.01, 0.05 and 0.03, respectively, find the probability that a person in a room having faulty plumbing was assigned to hotel E_1.

SOLUTION

E_1, E_2 and E_3 are the three local hotels.

$$P(E_1) = \frac{20}{100} = 0.20$$

$$P(E_2) = \frac{45}{100} = 0.45$$

$$P(E_3) = \frac{35}{100} = 0.35$$

Let X represent a room with faulty plumbing.

$$P(X/E_1) = 0.01$$

$$P(X/E_2) = 0.05$$

$$P(X/E_3) = 0.03$$

By Bayes law,

$$P(E_1/X) = \frac{P(E_1) \times P(X/E_1)}{P(E_1) \times P(X/E_1) + P(E_2) \times P(X/E_2) + P(E_3) \times P(X/E_3)}$$

$$= \frac{0.20 \times 0.01}{(0.20 \times 0.01) + (0.45 \times 0.05) + (0.35 \times 0.03)}$$

$$= 0.0571$$

(v) Tree diagram

Tree diagram method can also be used to solve problems on probability involving independent and dependent events. Each of the branches of the tree diagram represents different possibilities in which probability can be written or formed. The following examples use tree diagram method to solve probability problems.

Worked Example 18

A bag contains four blue and three yellow balls.

(a) A ball is taken from the bag and then replaced. A second ball is chosen. What is the probability that

(i) they are both blue?

(ii) one is blue and one is yellow?

(iii) at least one is blue?

(iv) at most one is blue?

(b) Find out how these possibilities are affected, if two balls are chosen without replacement.

SOLUTION

Using the tree diagram method as in Figure 19.2,

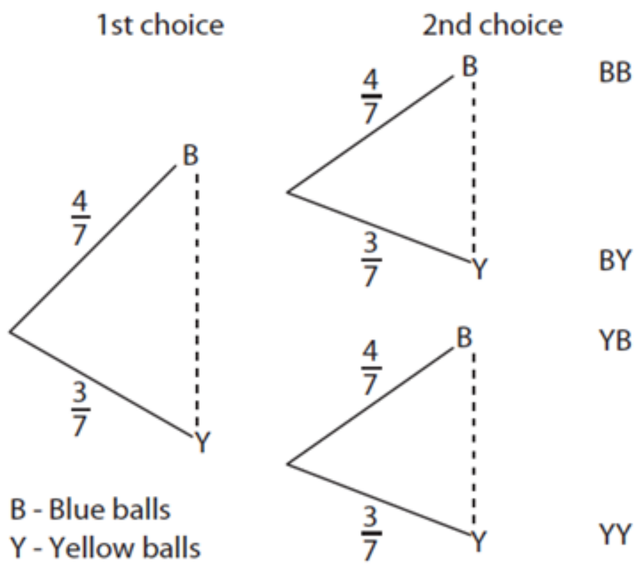


Figure 19.2

Using the branches of the tree diagram,

(a) (i) The probability that the 1st two balls are blue is

$$P(\text{BB}) = \frac{4}{7} \times \frac{4}{7} = \frac{16}{49}.$$

(ii) The probability that the first is blue and the second is yellow is

$$P(\text{BY}) = \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$$

The probability that the first is yellow and the second is blue is

$$P(\text{YB}) = \frac{3}{7} \times \frac{4}{7} = \frac{12}{49}$$

Note: $P(\text{BY})$ and $P(\text{YB})$ are probabilities of mutually exclusive events. Hence, the probability of obtaining a blue ball and a yellow ball when the order does not matter is

$$P(\text{BY}) + P(\text{YB}) = \frac{12}{49} + \frac{12}{49} = \frac{24}{49}$$

(iii) The probability that at least one is blue is equal to the probability that one is blue plus the probability that two are blue.

Note: At least one is blue means either one is blue or two are blue.

$$\begin{aligned} P(\text{one blue}) &= P(\text{BY}) + P(\text{YB}) \\ &= \frac{12}{49} + \frac{12}{49} \\ &= \frac{24}{49} \end{aligned}$$

(calculated in (ii))

$$\begin{aligned} P(\text{two blue}) &= P(\text{BB}) \\ &= \frac{4}{7} \times \frac{4}{7} = \frac{16}{49} \end{aligned}$$

(calculated in (i))

$$P(\text{at least one blue}) = \frac{24}{49} + \frac{16}{49}$$
$$= \frac{40}{49}$$

(iv) At most one is blue means either one is blue or none is blue, i.e. one is blue or two are yellow.

$$P(\text{two yellow}) = P(YY)$$
$$= \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$$
$$P\text{ (one blue)} = \frac{24}{49}$$

(calculated in (ii))

$$P(\text{at most one is blue}) = \frac{9}{49} + \frac{24}{49}$$
$$= \frac{33}{49}$$

(b) Without replacement means there are only six balls left after the first is picked. See Figure 19.3.

From the tree diagram in Figure 19.3,

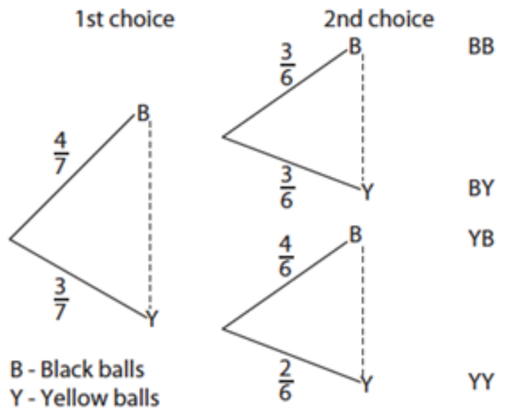


Figure 19.3

(i) $P(BB) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42} = \frac{2}{7}$

(ii) $P(BY) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42} = \frac{2}{7}$

$$\therefore P(\text{one is blue and one is yellow}) = \frac{2}{7} + \frac{2}{7}$$
$$= \frac{4}{7}$$

(iii) $P(BB) + P(BY) + P(YB) = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}$

$$\text{or } 1 - \frac{1}{7} = \frac{6}{7}$$

i.e. $1 - P(YY)$.

(iv) $P(YY) = \frac{3}{7} \times \frac{2}{6} = \frac{6}{42} = \frac{1}{7}$

$$P(BY) + P(YB) + P(YY) = \frac{2}{7} + \frac{2}{7} + \frac{1}{7}$$
$$= \frac{5}{7}$$
$$\text{or } 1 - P(BB) = 1 - \frac{2}{7} = \frac{5}{7}$$

Worked Example 19

If three cards are chosen from a pack without replacement, what is the probability of obtaining:

(a) at least two spades?

(b) at most two spades?

SOLUTION

Figure 19.4 illustrates different possibilities of choosing the three cards.

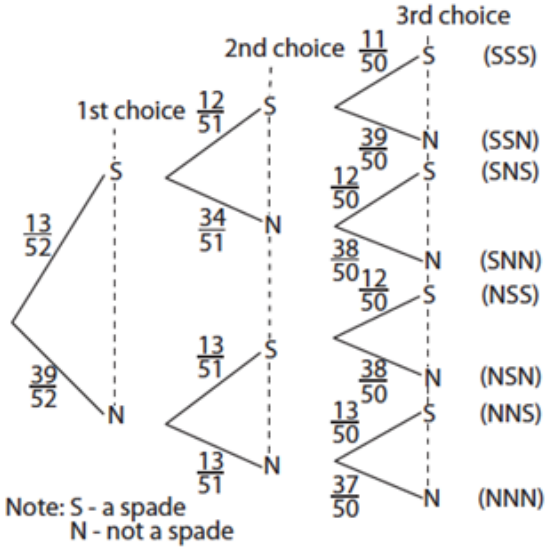


Figure 19.4

(a) $P(3 \text{ spades}) = \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{11}{850}$

$$P(2 \text{ spades}) = \frac{13}{52} \times \frac{12}{51} \times \frac{39}{50} + \frac{13}{52} \times \frac{39}{51} \times \frac{12}{50} + \frac{39}{52} \times \frac{13}{51} \times \frac{12}{50}$$

$$= \frac{117}{850}$$

(i.e. two spades or three spades)

(b) P (at most 2 spades) (i.e. 0, 1 or 2 spades) is P(0 spade)

$$= \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} = \frac{703}{1\,700}$$

$$P(1 \text{ spade}) = \left(\frac{39}{52} \times \frac{38}{51} \times \frac{13}{50}\right) + \left(\frac{39}{52} \times \frac{13}{51} \times \frac{38}{50}\right) + \left(\frac{13}{52} \times \frac{39}{51} \times \frac{38}{50}\right)$$

$$= \frac{347}{1\,700} \times 3 = \frac{741}{1\,700}$$

$$P(2 \text{ spades}) = \frac{117}{850}$$

(calculated in (a) above)

$$P(\text{at most 2 spades}) = \frac{703}{1\,700} + \frac{741}{1\,700} + \frac{117}{850}$$

$$= \frac{839}{850}$$

Aliter

P (at least 2 spades) = 1 – P (3 spades)

Note: Sum of all probabilities add up to 1.

$$P(3 \text{ spades}) = \frac{11}{850}$$

(calculated in (a) above)

$$P(\text{at least 2 spades}) = 1 - \frac{11}{850}$$

$$= \frac{839}{850}$$

Exercise 6

1. A bag contains four red and three blue identical balls. Two balls are drawn at random one after the other without replacement. What is the probability that both balls are of the same colour?
2. The fi rst 5 students to arrive in a school on a Monday morning were two boys and three girls. Two students

were chosen at random for an assignment. Find the probability that:
(a) both were boys.
(b) both were of diff erent sexes. (WAEC)

3. Two balls are drawn successively without replacement from a bag containing 8 blue and 14 white balls. Find the probability that they are both blue. (WAEC)

4. A box contains seven red, eight white and three blue balls. Three balls are drawn at random from the box without replacement. Calculate the probabilities, if:
(a) two are red and one is blue.
(b) at least two are white and one is blue. (WAEC)

5. A box contains identical balls of which 12 are red, 16 white and 8 blue. Three balls are drawn from the box after the other without replacement. Find the probability that:
(a) three are red.
(b) the fi rst is blue while the other two are red.
(c) two are white and one is blue. (WAEC)

6. A box contains six red and five green identical balls. Two balls are drawn at random one after the other without replacement. What is the probability that both balls are of the same colour? (WAEC)

7. If three cards are chosen from a pack without replacement, what is the probability of obtaining:
(a) at least two spades?
(b) at most two spades?

8. If two cards are drawn from a pack without replacement, what is the probability of obtaining:
(a) an ace and a king?
(b) two aces?

9. A bag contains three black balls, four white balls and fi ve red balls. Three balls are removed without replacement. What is the probability of obtaining:
(a) one of each colour?
(b) at least two red balls?

10. Three machines X, Y and Z produce 55%, 35% and 10% respectively of the total number of items in a factory. The percentages of defective items produced by the machines are 3%, 2% and 1%, respectively. An item is selected at random and found defective. Find the probability that the item was produced by machine Y

IV. Practical Application of Probability
Probability concepts can be used to solve various practical problems involving health, fi nance, population issues, etc..

Worked Example 20
The probability that Dr. Tella will receive high dividends from his shares is _ 25_ . What is the probability that he will not receive high dividends?
SOLUTION
Recall that
P(**A**) + **P**(**A**c) = 1
where
A = receiving high dividends
Ac = not receiving high dividends.

$$\therefore P(A^c) = 1 - P(A)$$

$$= \frac{1}{1} - \frac{2}{5}$$

$$= \frac{5-2}{5} = \frac{3}{5}$$

âˆ´ Probability that Dr. Tella will not receive high dividends is 3/5.

Worked Example 21
Four female and 6 male teachers are involved in a collective investment scheme called â€˜AJOâ€™. What is the probability that a female teacher will collect the â€˜AJOâ€™ fi rst?
SOLUTION
Total = 4 + 6 = 10 teachers.
P(female teacher collecting first)

$$= \frac{4}{10} = \frac{2}{5}$$

Exercise 7
1. The first five students to arrive in a school on Tuesday morning were 2 boys and 3 girls. Of these 2 were chosen at random for an experiment. Find the probability that
(a) both were boys (b) the two are of different sexes.
2. A number is chosen at random from the integers 1 to 10. Find the probability that the number is (a) (i) prime
(ii) multiple of 2 (iii) either prime or a multiple of 2 (b) the probability that a candidate passes an examination is 1/5. Find the probability that
(i) if 2 candidates are selected at random, only one passes the examination
(ii) if 3 candidates are selected at random, all pass the examination.
3. The probability that a boy does the addition correctly is 3/4 and the probability that he does subtraction

correctly is 2/5. Find the probability
that he evaluates 170 + 80 – 50.
4. The probability of three independent
events A, B and C occuring are
2/5, 1/3 and 3/5 respectively. Determine
the probability that only 2 of the
events occur.
5. The probability of an event A is _1/3 .
The probability of event B is 1/2 . The
probability of both A and B is 1/12.
What is the probability of either
event A or B or both?
6. The chances of 2 independent
events X and Y occuring are 1/4 and 2/3
respectively. What are the chances
of only one of them occuring.
7. A bag contains 6 blue and 10 red
identical balls. Two balls are drawn
at random without replacement.
Calculate the probability that the
balls are of (a) the same colour (b)
different colours.
8. The probability that any of the fi ve
telephone lines in a company is
engaged at instant is 1/4. What is the probability that at least one of the
lines is engaged.
9. Two perfect dice are thrown
together. Calculate the probability
that (a) they show different numbers
(b) there is one and only one 5.
(c) the sum is 9 or 10. (d) the sum is
at most 5.
10. Three unbiased dice are thrown.
Find the probability of obtaining (a)
3 fives (b) 2 fives and one six.

SUMMARY

In this chapter, we have learnt the following:

➢– Probability of an event A =

$$= \frac{\text{Number of equiprobable favourable outcomes}}{\text{Total number of equiprobable favourable outcomes}}$$

➢– Probability of an event lies between 1 and 0. If an event is certain to happen, its probability is 1, if an event is certain not to happen, its probability is 0.

➢– Two events A and B are mutually exclusive, if they cannot happen together, i.e.
P(A âˆª B) = P(A) + P(B) since A âˆ© B = âˆ….

➢– If two events A and B are not mutually exclusive then P(A âˆª B) = P(A) + P(B) âˆ’ P(A âˆ© B).

➢– If two events A and B are said to be independent then P(A âˆ© B) = P(A) Ã…~ P(B).

➢– If two events A and B are said to be dependent then P(A âˆ© B) = P(A) Ã…~ P(B / A), where P(B / A) means the probability of B given that A has occurred.

GRADUATED EXERCISES

1. A fair die is thrown. Find the probability of obtaining a number greater than 6.
2. A pair of unbiased dice each numbered 1 to 4 is thrown. If R is the event that the same number is obtained in a single toss of the dice, fi nd the probability of R.
3. A die is loaded in such a way that an even number is thrice as likely to occur as an odd number. What is the probability of obtaining a number greater than 2?
4. The number of children in a particular school is 300. If 120 of them are girls, what is the probability that a number selected at random is a boy?

5. Table 19.14 shows the mark distribution of 30 students in a Mathematics test.

Table 19.14

Marks (%)	25	55	60	70	80	90
Number of students	3	10	7	5	2	3

Find the probability that a student randomly selected scores:
(a) at most 60%.
(b) between 55% and 70% inclusive.
6. A box contains five green balls labelled 1 to 5 and another box contains five white balls labelled A to E. If two balls are selected at the same time one from each box
(with replacement), find the probability that (a) one of the balls has a number less
than four (b) one of the balls has either the alphabet B or E on it.
7. Two events B and C are such that P(B âˆª C) = 3/4, P(B âˆ© C) =1/8 and P(B_c) = 1/2. Find
(a) P(B) and (b) P(C).
8. Given a set S = {x: 3 â‰¤ x < 12} where x is a natural number. If a number is chosen at
random, fi nd the probability that it is either a prime or a factor of 8.
9. A and B are two events such that P(A) = 1/2 , P(B) = 2/3 and P(A / B) = 1/2 . Find
(a) P (A âˆ© B) and (b) P(B âˆª C).
10. A basket contains 6 mangoes and 4 oranges. If two fruits are to be picked one after
the other from the basket without replacement, fi nd the probability that (a) the
fi rst fruit picked is mango and the second is orange (b) the two fruits picked are
of the same type.
11. A bag contains four green balls, fi ve white balls and three blue balls. If three balls
are picked from the bag, fi nd the probability that they are picked in the order
green, white and blue balls. (The selection is done with replacement.)

12. The probability of a seed germinating is $\frac{1}{3}$. If three seeds are planted, what is the probability that (a) at least one will germinate? (b) exactly one will germinate?

13. In a particular city in Nigeria, a brand of "grape" drink comes from plants X, Y and Z. Plant X produces 25% of the whole consignment, plant Y produces 40% and plant Z produces 35%. If 3% of the bottles of "grape" drink produced in plant X is contaminated, 1% in plant Y is contaminated and 4% in plant Z is contaminated, given a bottle of "grape" drink that is contaminated, find the probability that the bottle comes from plant Y.

14. A pure water seller realises after a day sales that her sales include ₦5 notes worth ₦ 55 and ₦10 notes worth ₦80. If she pulls the money out of her bag at random, what is the probability that the money pulled out is in (i) ₦5 notes? (ii) ₦10 notes?

15. Three balls are drawn without replacement from a bag containing 8 blue, 12 red and 10 yellow balls. Find the probability that:
   (a) the three balls are of the same colour.
   (b) two of the balls are red and one is blue.
   (c) the three balls are of different colours. (WAEC)

16. A bag contains four red, three white and five green balls.
   (a) If one ball is picked at random, what is the probability that it is not green?
   (b) If two balls are picked at random without replacement, what is the probability that one is red and the other white? (WAEC)

17. A man has 12 identical marbles in his pocket consisting of 5 green, 2 white, 3 blue and 2 red. He draws out a marble at random:
   (a) What is the probability that it is either a green or a blue marble?
   (b) If the man makes two draws one after the other from his pocket without replacement, what is the probability that:
      (i) the first draw will be a green marble and the second a blue marble?
      (ii) neither draw is a red marble? (WAEC)

18. A man has identical balls in a bag. Out of these, three are black, two are blue and the remaining are red.
   (a) If a ball is drawn at random, what is the probability that it is (i) not blue? (ii) not red?
   (b) If two balls are drawn at random, one after the other, what is the probability that both of them will be
      (i) black, if there is no replacement?
      (ii) blue, if there is a replacement? (WAEC)

19. A number is selected at random from each of the set {2, 3, 4} and {1, 3, 5}. What is the probability that the sum of the two numbers will be less than 7 but greater than 3? (WAEC)

20. Two fair dice are tossed. Find the probability:
(a) of getting a total of 7 or 8.
(b) of not getting a total of 9.
(c) that the two dice show the same number. (WAEC)