

CHAPTER 1: Logarithms

OBJECTIVES

At the end of the chapter, students should be able to:

1. Express numbers in standard form.
2. Use logarithm tables to perform calculations involving numbers greater than one (1).
3. Compare characteristics of logarithms with standard form of numbers.
4. Use logarithm tables to perform calculations involving multiplication, division, powers and roots of numbers less than one (1).
5. Solve simple logarithmic equations.

I. Numbers in Standard Form (Revision)

Numbers such as 1, 10, 100, 1000, 10 000, 1 000 000, etc. can be expressed as positive powers of 10, while decimal

fractions such as one tenth $\left(\frac{1}{10} = 0.1\right)$, one hundredth $\left(\frac{1}{100} = 0.01\right)$, one thousandth

$\left(\frac{1}{1000} = 0.0001\right)$, etc. can be expressed as negative powers of 10 (see Table 1.1).

Table 1.1

Number	Power of 10
1 000 000	10^6
100 000	10^5
10 000	10^4
1 000	10^3
100	10^2
10	10^1
1	10^0
0.1	10^{-1}
0.01	10^{-2}
0.001	10^{-3}
0.0001	10^{-4}
0.00001	10^{-5}
0.000001	10^{-6}

A number expressed in the form $P \times 10^m$, where P is a number between 1 and 10 ($1 \leq P < 10$) and m is an integer ($m \in \mathbb{Z}$), is said to be in *standard form*. The numbers 7.095×10^3 , 4.011×10^2 and 5.113×10^{-5} are in standard form.

Worked Example 1

Write the following numbers in standard form:

- 45.005
- 4 500.26
- 712.32
- 115
- 0.00521
- 0.0004567

SOLUTION

(a) $45.005 = 4.5005 \times 10^1$

(b) $4\,500.26 = 4.50026 \times 10^3$

(c) $712.32 = 7.1232 \times 10^2$

(d) $115 = 1.15 \times 10^2$

(e) $0.00521 = \frac{521}{100\,000} = \frac{5.21 \times 10^2}{10^5} = 5.21 \times 10^{2-5} = 5.21 \times 10^{-3}$
 $\hat{a} \hat{'} 0.00521 = 5.21 \times 10^{-3}$

(f) $0.0004567 = \frac{4\,567}{10\,000\,000} = \frac{4.567 \times 10^3}{10^7} = 4.567 \times 10^{3-7}$

$$\hat{\wedge}^{\wedge} 0.0004567 = 4.567 \times 10^{\hat{\wedge}^4}$$

Worked Example 2

The approximate population of three schools in a city is given as 3.7×10^3 , 2.85×10^5 and 5.82×10^7 . What is the total population of the schools?

SOLUTION

$$\begin{aligned} &\text{The total population of the schools} \\ &= 3.7 \times 10^3 + 2.85 \times 10^5 + 5.82 \times 10^7 \\ &= 10^3(3.7 + 2.85 \times 10^2 + 5.82 \times 10^4) \\ &= 10^3(3.7 + 285 + 58\,200) \\ &= 10^3(58\,488.7) \\ &= 58\,488.7 \times 10^3 \\ &= 5.84887 \times 10^4 \times 10^3 \\ &= 5.84887 \times 10^7 \end{aligned}$$

Exercise 1

Write the following numbers in standard form:

1. 465.05
2. 1 161.45
3. 146
4. 100.154
5. 0.0046
6. 16.78
7. 0.00812
8. 0.0068905
9. 468.25
10. 52.48
11. 3 005.85
12. 116.72
13. 0.0568
14. 0.00068
15. 14.7806
16. 0.000789
17. 0.1689
18. 3.00278
19. 14.6891
20. 13.38
21. 116.75
22. 114.689
23. 169.78
24. 11 687.75
25. 74.68

26. Evaluate the following, leaving your answers in standard form:

- (a) $1.5 \times 10^3 + 5.2 \times 10^3$
- (b) $4.58 \times 10^4 + 1.24 \times 10^5 \hat{\wedge}^{\wedge} 1.5 \times 10^3$

27. Write the reciprocal of 1.5×10^7 in standard form.

28. The approximate population of three schools in an urban centre is given as 4.6×10^2 , 3.14×10^8 and 1.09×10^3 . What is the total population of the schools?

29. A barrel of oil costs N7 000. What is the cost of 5.68×10^3 barrels of oil?

30. The moon is 186 000 km from the earth. Express this figure in standard form.

II. Logarithm of Numbers Greater Than 1 (Revision)

In Functional Mathematics Book 1, we learnt that the logarithm of a number to a given base is the power (index) to which the base must be raised to give that number. That is, $a^x = M$ implies $\log_a M = x$.

For example, $\log_3 9 = 2$ implies $3^2 = 9$, $\log_2 64 = 6$ implies $2^6 = 64$ and $\log_{10} 100 = 2$ implies that $10^2 = 100$. If the base of the logarithm is 10, we call the logarithm in base 10 a common logarithm. Hence, the base 10 logarithm of a number is the power to which 10 is raised to give that number. For example,

$$\begin{aligned} 5\,681 &= 5.681 \times 10^3 \text{ (in standard form)} \\ &= 10^{0.7544} \times 10^3 \text{ (using table)} \\ &= 10^{3.7544} \end{aligned}$$

$$\hat{\wedge}^{\wedge} \log 5\,681 = 3.7544$$

From the example above, we can see that the logarithm of any number consists of two parts—the integer part (whole part) called the characteristics and the decimal part (fractional part) called the mantissa.

For example,

$$\log 5681 = 3 \quad \cdot \quad 7544$$

integer whole (characteristics) decimal or fractional part (mantissa)

To obtain the characteristics of a number, we first need to express the number in the standard form and obtain the mantissa from a table called the four-figure table.

Exercise 2

Read the logarithms of the following from the table:

1. 152.001
2. 15.2001
3. 45.32
4. 153
5. 361.46
6. 16.81
7. 196.14
8. 168.75
9. 362.81
10. 182
11. 196
12. 8678.98
13. 5
14. 900
15. 86500
16. 7859
17. 9.44
18. 19005
19. 1168
20. 12.32

III. Logarithm of Numbers Less Than 1 (Revision)

From Section II, we recall that the logarithm of numbers between 1 and 10 is a number between 0 and 1 (see Table 1.2).

Table 1.2

Number	1	2	3	4	5	6	7	8	9	10
Log	0	0.3010	0.4771	0.6021	0.6990	0.7782	0.8451	0.9031	0.9542	1

We can also find the logarithm of numbers less than 1. Here, we use negative powers of 10. For example, $0.5 = 5 \times 10^{-1}$, $0.05 = 5 \times 10^{-2}$, $0.005 = 5 \times 10^{-3}$, etc. To find the logarithm of a number less than 1, we follow the same procedure as for numbers greater than 1. We first express the number in standard form and later read the mantissa from the table. The negative power of 10 here is the characteristics of the number.

For example,

$$0.00456 = 4.56 \times 10^{-3} \text{ (in standard form)}$$

$$= 10^{0.6590} \times 10^{-3} \text{ (from the table)}$$

$$= 10^{0.6590 + (-3)} = 10^{-3 + 0.6590}$$

$$\therefore \log 0.00456 = -3 + 0.6590$$

In practice, $-3 + 0.6590$ is written as $\bar{3}.6590$ which is read as 'bar 3 point 6590'.

We should note that $\bar{3}.6590$ simply means $-3 + 0.6590$ and $\bar{3}65.90$ means $-3 + (0.6590)$. Hence, we can see that the logarithm of a number less than 1 has a negative integer as its characteristics and a positive fraction.

$$\log 0.00456 = \bar{3} \cdot 6590$$

negative integer (characteristics) decimal fraction (mantissa)

The mantissa 6590 is read from the table by reading the logarithm of 45 under 6 from the four-figure table.

Worked Example 3

Read the logarithms of the following:

- (a) 0.041
- (b) 0.00325
- (c) 0.000031

SOLUTION

$$(a) \log (0.041) = \log (4.1 \times 10^{-2}) = \bar{2}.6128$$

$$(b) \log (0.00325) = \log (3.25 \times 10^{-3}) = \overline{3}.5079$$

$$(c) \log (0.000031) = \log (3.1 \times 10^{-5}) = \overline{5}.4914$$

Exercise 3

Write down the logarithms of the following:

1. 0.046
2. 0.00314
3. 0.0104
4. 0.0156
5. 0.145
6. 0.114
7. 0.1505
8. 0.00000034
9. 0.000000004
10. 0.000001213
11. 0.00468
12. 0.000685312
13. 0.000146
14. 0.00156
15. 0.7685

Use antilogarithm table to find the number whose logarithm is:

16. $\overline{2}.4190$
17. $\overline{3}.1468$
18. $\overline{4}.3105$
19. $\overline{1}.1900$
20. $\overline{5}.3169$
21. $\overline{4}.1490$
22. $\overline{3}.0128$
23. $\overline{4}.1395$
24. $\overline{1}.0596$
25. $\overline{7}.3194$

IV. Revision on Roots and Powers of Numbers Greater Than 1

In Functional Mathematics Book 1, we learnt that $(a^m)^n = a^{mn}$ and $\log M^n = n \log M$.

Also, $\log M^{\frac{1}{n}} = \frac{1}{n} \log M$ which means that the logarithm of a number to a given power is the product of the power and the logarithm of the number. If the power is a fraction whose numerator equals 1 (say $\frac{1}{n}$), then the logarithm is the logarithm of the number divided by the denominator of the fraction.

Worked Example 4

Evaluate using logarithm.

- (a) $(15.72)^3$
- (b) $\sqrt[3]{352.13}$
- (c) $(13.62)^2$
- (d) $\sqrt[4]{68.32}$

SOLUTION

$$(a) \log (15.72)^3 = 3 \log (15.72)$$

Number	log	log
(15.72)	1.1965	
$(15.72)^3$	1.1965×3	3.5895
3 885		

Procedures

Step 1: Read log of 15.72 from the table, which gives 1.1965.

Step 2: Multiply log number by 3, which gives 3.5893.

Step 3: Read the antilogarithm of 0.5893, which gives 3 885.

Step 4: Add one to characteristics 3 to have 4, which shows that the result is a four-digit number, that is, $(15.72)^3 = 3\,885$.

$$(b) \sqrt{(352.13)} = (352.13)^{\frac{1}{2}}$$

$$\therefore \log \sqrt{(352.13)} = \log 352.13^{\frac{1}{2}}$$

$$= \frac{1}{2} \log 352.13$$

Number	log	log
$(352.13)^{\frac{1}{2}}$	$2.5466 \times \frac{1}{2}$	
$352.13^{\frac{1}{2}}$	$2.5466 \div 2$	1.2733
18.76		

Procedures

Step 1: Write $\sqrt{352.13}$ to have fractional index, that is, $\sqrt{352.13} = (352.13)^{\frac{1}{2}}$.

Step 2: Read log of 352.13 from the table, which gives 2.5466.

Step 3: Divide the log number by 2, which gives 1.2733.

Step 4: Read the antilogarithm of 0.2733, which gives 1 876.

Step 5: Add one to characteristics 1 in 1.2733 to have 2, which shows that the result has two digits before the decimal point, that is, $\sqrt{352.13} = 18.76$.

$$(c) \log (13.62)^2 = 2 \log 13.62$$

Number	log	log
13.62	1.1341	
13.62^2	1.1341×2	2.2682
185.5		

$$\hat{\wedge}^{\wedge} (13.62)^2 = 185.5$$

$$(d) \sqrt[4]{68.32} = (68.32)^{\frac{1}{4}}$$

$$\hat{\wedge}^{\wedge} \log 68.32^{\frac{1}{4}} = \frac{1}{4} \log 68.32$$

Number	log	log
$(68.32)^{\frac{1}{4}}$	$1.8345 \times \frac{1}{4}$	
$68.32^{\frac{1}{4}}$	$1.8345 \div 4$	0.4586
2.875		

$$\hat{\wedge}^{\wedge} \sqrt[4]{68.32} = 2.875$$

V. Roots and Powers of Numbers Less Than 1

To find the logarithm of roots and powers of numbers less than 1, we follow the same procedure that we used for finding the roots and powers of numbers greater than 1. See the following example.

Worked Example 5

Evaluate the following using logarithm:

$$(a) (0.0056)^3 \quad (b) \sqrt[3]{0.045} \quad (c) (0.00589)^4$$

SOLUTION

$$(a) \log (0.0056)^3 = 3 \log (0.0056)$$

Number	log	log
0.0056	$\bar{3}.7482$	
0.0056^3	$\bar{3}.7482 \times 3$	$\bar{7}.2446$

$$\hat{\wedge}^{\wedge} (0.0056)^3 = 0.000001756$$

$$(b) \sqrt[3]{0.045} = (0.045)^{\frac{1}{3}}$$

$$\hat{\wedge}^{\wedge} \log 0.045^{\frac{1}{3}} = \frac{1}{3} \log 0.045$$

Number	log	log
$(0.045)^{\frac{1}{3}}$	$\bar{2}.6532 \times \frac{1}{3}$	
$\sqrt[3]{0.045}$	$\bar{2}.6532 \div 3$	$\bar{1}.3276$
0.2126		

$$\hat{a}^{\wedge'} \hat{a} \approx 0.045 = 0.2121$$

$$(c) \log (0.00589)^4 = 4 \log 0.00589$$

Number	log	log
0.00589	$\bar{3}.7701$	
0.00589^4	$\bar{3}.7701 \times 4$	9.0804
0.000000001203		

$$\hat{a}^{\wedge'} (0.00589)^4 = 0.000000001203$$

Note: When adding or subtracting numbers in logarithm, note the following:

(a) The fractional parts of logarithms called the mantissa are positive. These are added or subtracted in the usual way.

(b) The integral parts of logarithms called the characteristics may be positive or negative. These are to be added or subtracted as directed numbers.

For example,

$$(a) \bar{2}.3149 + \bar{5}.3495$$

$$\begin{array}{r} \bar{2} + 0.3149 \\ + \bar{5} + 0.3495 \\ \hline \bar{7} + 0.6644 \\ \therefore \bar{2}.3149 + \bar{5}.3495 = 7.6644 \end{array}$$

$$(b) \bar{5}.4192 - \bar{2}.3209$$

$$\begin{array}{r} \bar{5} + 0.4192 \\ - \bar{2} + 0.3209 \\ \hline -5 - (-2) + 0.983 = \bar{3} + 0.0983 \\ = 3.0983 \\ \therefore \bar{5}.4192 - \bar{2}.3209 = \bar{3}.0983 \end{array}$$

Exercise 4

Evaluate the following using logarithm table:

$$1. (45.09)^{\frac{1}{2}}$$

$$2. (105.46)^5$$

$$3. \hat{a}^{\wedge} \hat{s} 465.36$$

$$4. \hat{s} \hat{a}^{\wedge} \hat{s} 6.215$$

$$5. (3.2103)^{15}$$

$$6. \hat{a}^{\wedge} \hat{s} 6.321$$

$$7. (3.8)^5$$

$$8. 56.89^{\frac{1}{3}}$$

$$9. 15.69^2$$

$$10. (14.68)^4$$

$$11. (23.46)^8$$

$$12. (17.82)^{\frac{1}{3}}$$

$$13. 242.86^3$$

$$14. (14.69)^{\frac{1}{3}}$$

$$15. (135.009)^4$$

$$16. (0.0456)^4$$

$$17. (0.008905)^{\frac{1}{2}}$$

$$18. (0.0046809)^3$$

$$19. \hat{a}^{\wedge} \hat{s} (0.25)$$

$$20. (0.098)^{\frac{2}{3}}$$

$$21. (0.045)^{\frac{3}{7}}$$

$$22. \hat{s} \hat{a}^{\wedge} \hat{s} 0.009$$

$$23. \hat{a}^{\wedge} \hat{s} 45.39$$

$$24. \hat{a} \hat{a}^{\wedge} \hat{s} 0.00568$$

$$25. \hat{a}^{\wedge} \hat{s} 0.042$$

$$26. (0.64)^{\frac{1}{2}}$$

$$27. (0.025)^3$$

$$28. (0.1846)^{\frac{3}{5}}$$

$$29. (0.006)^{\frac{1}{2}}$$

30. $(0.0014)^4$

VI. Multiplication and Division of Numbers Greater Than 1

In Functional Mathematics for Senior Secondary Schools 1, we learnt that when numbers are multiplied, we add their logarithms and when numbers are divided, we subtract their logarithms.

That is,

(i) $\log MN = \log M + \log N$

(ii) $\log \frac{M}{N} = \log M - \log N$

Worked Example 6

Evaluate using logarithm

(a) 97.5×310.35 (b) $81.32 \div 45.005$

SOLUTION

(a) $\log (97.5 \times 310.35) = \log 97.5 + \log 310.35$

Number	log
97.5	1.9890
310.35	+2.4922
30 280	<u>4.4812</u>

$\therefore 97.5 \times 310.35 = 30\,280$

(b) $\log (81.32 \div 45.005) = \log 81.32 - \log 45.005$

Number	log
81.32	1.9102
45.005	<u>-1.6533</u>
1.807	<u>0.2569</u>

Worked Example 7

Evaluate the following using logarithm:

(a) $\frac{45.05 \times 3.419}{81.002}$ (b) $\frac{(3168)^2 \times (451.02)^3}{\sqrt{431.09}}$

SOLUTION

(a) $\frac{45.05 \times 3.419}{81.002}$

Number	log	
45.05	1.6537	
3.419	<u>0.5339</u>	
	2.1876	} Add
81.002	<u>1.9085</u>	
1.906	<u>0.2801</u>	} Subtract
$\therefore \frac{45.05 \times 3.419}{81.002} = 1.906$		

(b) $\frac{(3\,168)^2 \times (451.02)^3}{\sqrt{431.09}}$

Number	log	log	
$(3\,168)^2$	3.5008×2	7.0016	
$(451.02)^3$	2.6542×3	<u>7.9626</u>	
		14.9642	} Add
$\sqrt{431.09}$	$2.6346 \div 2$	<u>1.3173</u>	
		13.6469	} Subtract
4435000000000			
$\therefore \frac{(3\,168)^2 \times (451.02)^3}{\sqrt{431.09}} = 4435000000000$			

Exercise 5

Use tables to evaluate the following:

1. 25.68×46.07
 $\frac{81.09 \times 15.69}{21.09}$
2. $48.5 \div 62.15$
 $\frac{81.43^2 \times 8.96^3}{45.09^3}$
3. $\frac{(14.6 \times 8.4)^3}{\sqrt{46.89}}$
4. $\left(\frac{189.3}{175}\right)^3$
5. $\sqrt[3]{\frac{405.5 \times 31}{48.95}}$
6. $\frac{\sqrt{404.9}}{\sqrt{78.93}}$
7. $\left(\frac{296.45 \times 48.9}{35.42}\right)^2$
8. $\frac{1.978 \times 48.9}{68.5}$
9. $\sqrt[3]{39.96^2 \times 46.09^3}$
10. $\sqrt[5]{\frac{5698}{49.05}}$
11. $\frac{49.32 \times 41.32}{6.92 \times 3.14}$
12. $\frac{25.42 \times 15.68}{3.142}$
13. $\frac{49.05 \times 41.32^3}{6.95}$
14. $\frac{49.05 \times 41.32^3}{6.95}$

VII. Multiplication and Division of Numbers Less Than 1

To multiply or divide numbers less than 1, we follow the same rules that we follow in multiplication and division of numbers greater than 1. Now study the following examples:

Worked Example 8

Use table to evaluate the following:

- (a) 0.02×0.14
 $\frac{0.045^2 \times 0.15}{\sqrt{0.09}}$
- (b)

SOLUTION

- (a) $\log (0.02 \times 0.14) = \log 0.02 + \log 0.14$

Number	log	
0.02	$\bar{2}.3010$	} Add
0.14	$\bar{1}.1461$	
0.002800	$\bar{3}.4471$	

$$\therefore 0.02 \times 0.14 = 0.0028$$

- (b) $\frac{0.045^2 \times 0.15}{\sqrt{0.09}}$

Number	log	log	
0.045^2	$\bar{2}.6532 \times 2$	$\bar{3}.3064$	} Add
0.15	$\bar{1}.1761 \times 1$	$\bar{1}.1761$	
		4.4825	} Subtract
$0.09^{\frac{1}{2}}$	$\bar{2}.9542 \div 2$	$\bar{1}.4771$	
0.001013		$\bar{3}.0054$	

$$\therefore \frac{0.045^2 \times 0.15}{\sqrt{0.09}} = 0.001013$$

Exercise 6

Use logarithm tables to evaluate the following:

1. 0.03×0.15

2. $0.4505^2 \div 0.012$

$$\frac{0.016 \times 0.1896}{0.1568}$$

3. $0.478 \div 196^2$

4. 0.3461×0.0056^2

$$\frac{0.046^2 \times 0.048}{\sqrt[3]{0.09}}$$

5. $\frac{0.496 \times 0.073^3}{0.15}$

6. $\frac{0.00346 \times 0.157}{\sqrt[4]{0.14}}$

7. $0.23_2 \div 0.54^2$

$$\frac{0.0468 \times \sqrt{0.09}}{0.05^2}$$

8. $\sqrt{\frac{0.068^2}{0.168^3 \times 0.165}}$

9. $\frac{0.94 \times 0.568}{0.0017}$

10. $\left(\frac{0.15 \times 0.1568}{\sqrt{0.027}} \right)$

11. $\frac{0.6504^3 \times 0.2275}{0.16^2}$

12. $\left(\frac{0.162 \times 0.0068}{0.35^2} \right)$

13. $\left(\frac{0.162 \times 0.0068}{0.35^2} \right)$

14. $\left(\frac{0.162 \times 0.0068}{0.35^2} \right)$

VIII. Solutions of Simple Logarithmic Equations

Simple logarithmic equations are equations that involve logarithms. These equations are solved similar to ordinary equations. For example, if $x^2 = \log_{10} 10\,000$, we can find the value of x as follows:

$$x^2 = \log_{10} 10\,000$$

$$\Rightarrow x^2 = \log_{10} 10^4$$

$$x^2 = 4 \log_{10} 10 = 4 \times 1 \quad (\log_{10} 10 = 1)$$

$$\therefore x^2 = 4 \text{ and}$$

$$x = \pm \sqrt{4} = \pm 2$$

Hence, $x = 2$ or -2

Worked Example 9

Solve the equation $\log x^2 = \log 2(x + 4)$

SOLUTION

$$\log x^2 = \log 2(x + 4)$$

$$\Rightarrow x^2 = 2(x + 4)$$

$$\Rightarrow x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x + 2)(x - 4) = 0$$

$$x + 2 = 0 \text{ or } x - 4 = 0$$

$$x = -2 \text{ or } 4$$

That is, $x = 4$ or $x = -2$

Worked Example 10

Solve the equation $3^{3x+1} = 5^x + 1$.

SOLUTION

$$3^{3x+1} = 5^x + 1$$

Now take logarithm on both sides to base 10.

$$\begin{aligned}
 (3x+1)\log_{10} 3 &= (x+1)\log_{10} 5 \\
 3x\log_{10} 3 + \log_{10} 3 &= x\log_{10} 5 + \log_{10} 5 \\
 x\log_{10} 3^3 + \log_{10} 3 &= x\log_{10} 5 + \log_{10} 5 \\
 x(\log_{10} 27 - \log_{10} 5) &= \log_{10} 5 - \log_{10} 3 \\
 \therefore x &= \frac{\log_{10} 5 - \log_{10} 3}{\log_{10} 27 - \log_{10} 5} \\
 x &= \frac{\log_{10} \left(\frac{5}{3}\right)}{\log_{10} \left(\frac{27}{5}\right)} = \frac{\log_{10} 1.67}{\log_{10} 5.4}
 \end{aligned}$$

Number	log
1.67	0.2227
5.4	0.7324
0.3092	̄1.4903

$$\therefore x = 0.31 \text{ (2 decimal places)}$$

Exercise 7

Solve the following logarithmic equations:

- $\log_{10} (2y^2 + 5y + 97)$
- $\log (3z^2 + 8) = 1 + \log_{10} \left(\frac{z}{2} + 1\right)$
- $\log_{10} (2w^2 + 5w + 2) = 1$
- $\log_2 (3y^2 + 8y + 1) = 1$
- $\log_3 (t^2 + t + 2) = 2\log_3 (t + 1)$
- $\log_{10} (u + 9) = 1 + \log_{10} (u + 1) + \log_{10} (u + 2)$
- Express x in terms of y , if $\log_2 (x + 2) = 2\log_2 y$.
- Solve for x , if $\log_2 (x^2 + 5x + 6) = 3$.
- Find the value of x for $\log_{10} (x^2 + x + 7) = 0.1$.
- Find the value of $\log_7 12$.

SUMMARY

In this chapter, we have learnt the following:

- A number in the form $P \times 10^n$, where P is a number between 1 and 10 and n is an integer (positive or negative whole number), is said to be in standard form.
- When numbers are multiplied, we add their logarithms and when numbers are divided, we subtract their logarithms.
- The logarithm of a number less than 1 always has negative characteristics. For example, $\log 0.02 = \bar{2}.3010$, which is read as 2 point three zero, one, zero.
- $\bar{2}.3010$ means $-2 + (0.3010)$ and not -2.3010 .
- The logarithm of a number to a given power is the product of the logarithm of the number and the power.
- The fractional parts of logarithms called mantissa are always positive and these are added or subtracted in the usual way.
- The integral parts of logarithms called characteristics may be positive or negative.
- These are added or subtracted as directed numbers.

GRADUATED EXERCISES

1. Write down the logarithms of the following:

- 46.07
- 180.46
- 19.05
- $(45.009)^2$
- $(168.48)^{\frac{1}{2}}$
- $\sqrt[3]{48.09}$
- $(81.74)^4$
- 14.46^2
- 6.231
- $(9.25)^{\frac{1}{3}}$

2. Write down the logarithms of the following:

(a) 0.048

(b) 0.00879

(c) 0.14079

(d) $(0.25)^{\frac{1}{2}}$

(e) $(0.64)^{\frac{1}{2}}$

(f) $(0.3589)^{\frac{1}{3}}$

(g) $(0.319)^4$

(h) 0.49723^3

(i) 0.0084^3

(j) ≈ 0.418

3. Evaluate the following:

(a) $\log 3 + \log \left(1 + \frac{x}{100}\right)$ (take $x = 5$)

(b) $\frac{7\,144 \times 982.41}{0.002^3}$

(c) $\left(\frac{95.86 \times 9.81}{215}\right)^2$

4. Given that $ry^n = 15800$, find the value of n when $r = 9.82$ and $y = 85.4$.

5. A gas is expands according to the law $PV^n = C$. Find

(a) V , when $C = 415$, $P = 98$ and $n = 14$.

(b) n , when $C = 340$, $P = 91$ and $V = 2.75$.

6. Use mathematical tables to evaluate:

$$\sqrt[4]{\frac{4.097}{0.0468 \times 0.346}}$$

(WAEC)

7. Evaluate

$$\frac{16.475 \times 13.096}{495^{\frac{1}{2}}}$$

(WAEC)

8. Evaluate

$$\frac{821.5 \times 7.143}{0.0014}$$

to 3 s.f. (WAEC)

9. Given that

$$V = \frac{1}{6} \pi h (3r^2 - h^2)^2$$

, make r the subject of the formula. Hence, calculate r (to 3 s.f.) when $\pi = 3.142$, $V = 134.3 \text{ cm}^3$ and $h = 4.3$. (WAEC)

$$\sqrt{\frac{82.84 \times 76.43}{8.949}}$$

10. Use mathematical tables to evaluate: