# CHAPTER 13


At the end of the chapter, students should be able to:
1. Find the circular length measurements of circles.
2. Find the circular area measurements of circle.
3. Calculate the surface area, volume and capacity of cubes, cuboids, cylinders, cones, prisms and pyramids.
4. Find the surface area and volume of a frustum of a cone and pyramid.
5. Find the surface area and volume of compound shapes.

## I. Circular Length Measurement

### Objectives

At the end of this sub-chapter, students should be able to:
1. Find the length of arcs practically and by using formula.
2. Determine the perimeter of sectors of circles.
3. Calculate the length of chords of circles.
4. Find the perimeter of segments of circles.

## A. Components of a circle (Revision)

A circle is an enclosed plane figure in which the set of all points in the plane are of equal distance from a fixed point called the *centre*, while the distance from the point to any point on the circumference is the *radius*.
The components of a circle can be classified into three broad headings:
(a) Point
(b) Length and
(c) Region
**(a) Point Centre:** It is a fixed point within the circle that is of equal distance from all points on the circle (see Fig. 13.1).

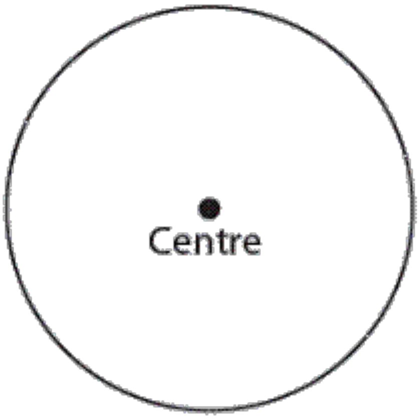



Fig. 13.1

**(b) Length**
**(i) Circumference:** It is the length around the circle (see Fig. 13.2a).
**(ii) Arc:** It is a part of the circumference of the circle. The shorter arc is referred to as the minor arc while the longer one is called the major arc. However, in solving problems, the word arc is used to represent the minor arc (see Fig. 13.2a).
**(iii) Radius:** It is a straight line that joins the centre and a point on the circumference of the circle (see Fig. 13.2a).
**(iv) Diameter:** It is a straight line that joins two points on the circumference passing through the centre of the circle. It is two times the radius.

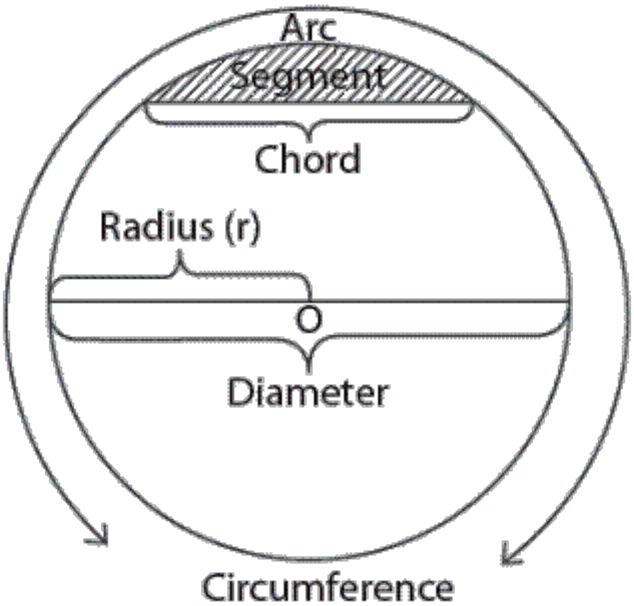



Fig. 13.2a

**(v) Chord:** It is a straight line that joins any two points on the circumference with or without passing through the centre of the circle. This is why a diameter is referred to as a special chord (see Fig. 13.2a).
**(c) Region**
**(i) Sector:** It is a portion of the circle enclosed by an arc and two radii. While the smaller sector is referred to as the minor, the bigger sector is called the major sector (see Fig. 13.2b).
**(ii) Segment:** It is a portion of the circle enclosed by an arc and a chord. The smaller segment is called the minor segment, while the bigger one is called the major segment (see Fig. 13.2a).

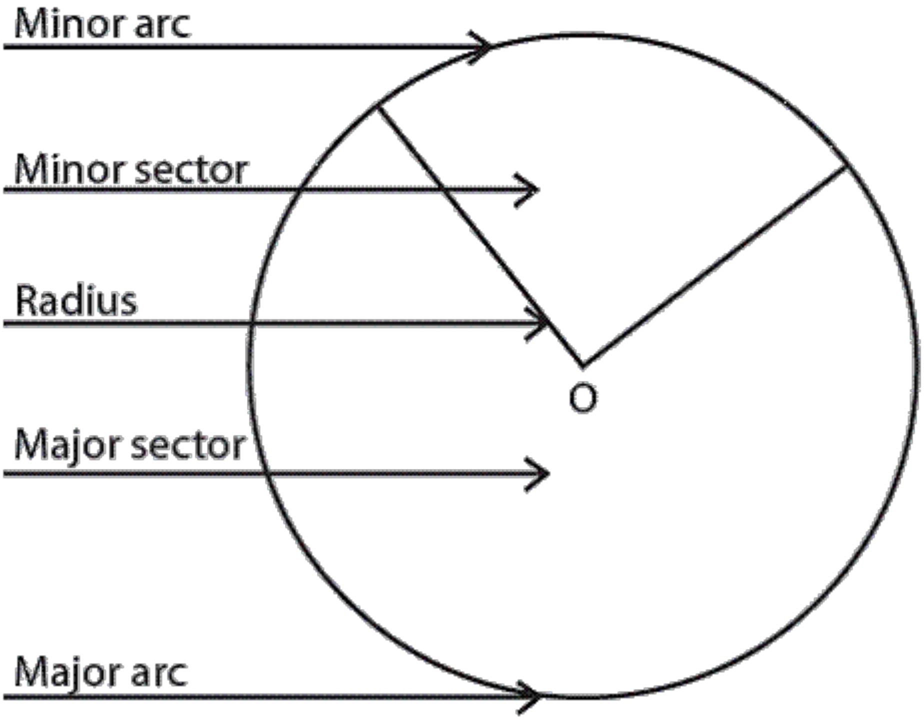



Fig. 13.2b

**(iii) Semi-circle:** It is half of any given circle. The diameter divides the circle into two equal parts. In Fig. 13.3a, line AB is the diameter of the circle ABCD while
ABC and ABD are semi-circles.

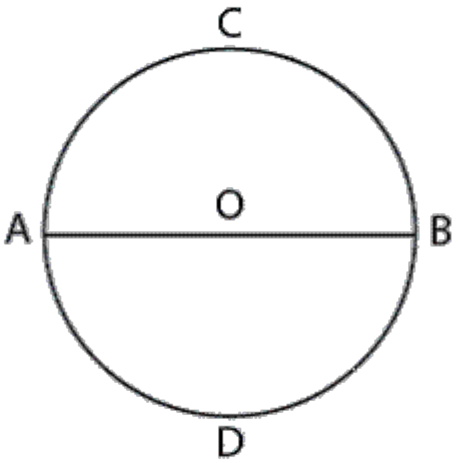



Fig. 13.3a

**(iv) Quadrant:** It is a quarter of the circle as shown in Fig. 13.3b.

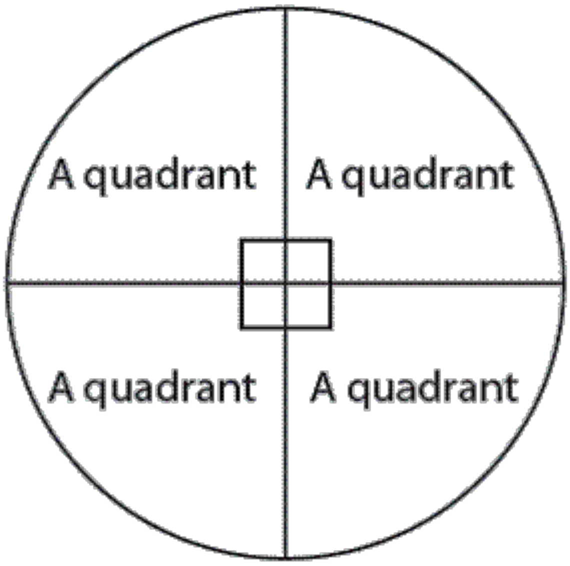



**Fig. 13.3b**

## C. Length of an arc of a circle

**Activity 13.1**

**Step 1:** *Make the following items available: strings, protractor, pair of ompasses, divider, cardboard paper and ruler.*
**Step 2:** *Using a pair of compasses and a ruler, construct five different circles with radius 3.5, 7, 10, 12 and 14 cm respectively on the cardboard paper.*
*Mark out two points "A" and "B" on the circumference of each of the five circles and join each of these two points to the centre "O" of the circle.*
**Step 3:** *Now using a protractor and a ruler or a divider or a string, measure the following on each of the five circles:*
*(i) Circumference*
*(ii) Length of arc AB*
*(iii) Angle AOB (angle subtended at the centre by arc AB) and use the result to copy and complete Table 13.1.*

**Table 13.1**

| S/N | Radius ($r$ cm) | Circumference (Cir cm) | Length of arc ($L$ cm) | Angle of the circle | Angle at centre ($\theta°$) | $\frac{L \text{ cm}}{\text{Cir}}$ | $\frac{\theta°}{360°}$ |
|---|---|---|---|---|---|---|---|
| 1 | 3.5 | | | 360° | | | |
| 2 | 7 | | | 360° | | | |
| 3 | 10 | | | 360° | | | |
| 4 | 12 | | | 360° | | | |
| 5 | 14 | | | 360° | | | |

From the results in Table 13.1, we will observe that the ratios

$$\frac{\text{Length of arc } (L)}{\text{Circumference } (2\pi r)} = \frac{\text{Angle subtended at the centre by arc } (\theta°)}{360°}$$

$$\Rightarrow \qquad \frac{L}{2\pi r} = \frac{\theta}{360°}$$

$$\text{Therefore, } L = \frac{\theta}{360°} \times 2\pi r$$

*We can now conclude that the length of an arc of a circle is proportional to the angle it subtends at the centre.*

## Worked Example 1

Find the length of an arc of a circle whose radius is 3.5 cm and subtends an angle of 30° at the centre of the circle given that $\pi = \frac{22}{7}$.

### Solution

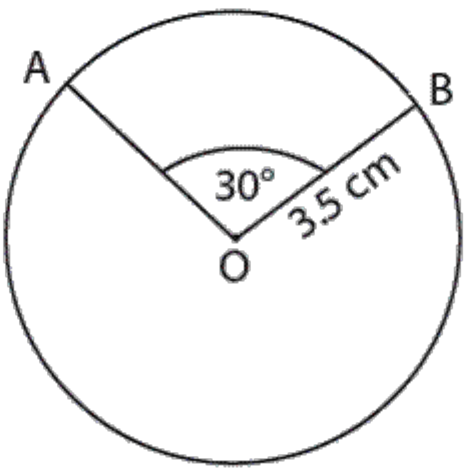



**Fig. 13.4**

$$\text{Arc AB} = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{30°}{360°} \times 2 \times \frac{22}{7} \times \frac{3.5}{1}$$

$$= \frac{1}{12} \times 22$$

$$= 1.83 \text{ cm (2 d.p)}$$

## Worked Example 2

An arc of a circle 176 cm subtends an angle of 120° at the centre. Calculate the
radius of the circle. Use $\pi = \frac{22}{7}$.

### Solution

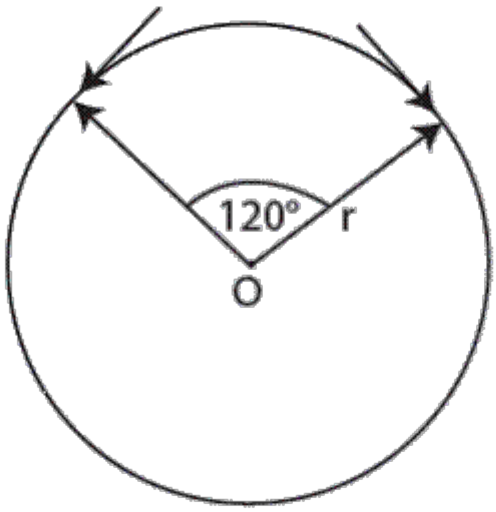



Fig. 13.5

$$\frac{\theta}{360°} \times 2\pi r = \text{Arc PQ}$$

$$\Rightarrow \frac{120°}{360°} \times 2 \times \frac{22}{7} \times r = 176 \text{ cm}$$

$$\Rightarrow r = \frac{176 \times 360 \times 7}{120° \times 2 \times 22}$$

$$\Rightarrow r = \frac{443\,520}{5\,280}$$

$$\therefore r = 84 \text{ cm}$$

## Wo.rked Example 3

Calculate the angle of an arc of length 19.8 cm subtended at the centre, if the radius is 10.5 cm. Use $\pi = \frac{22}{7}$.

## Solution

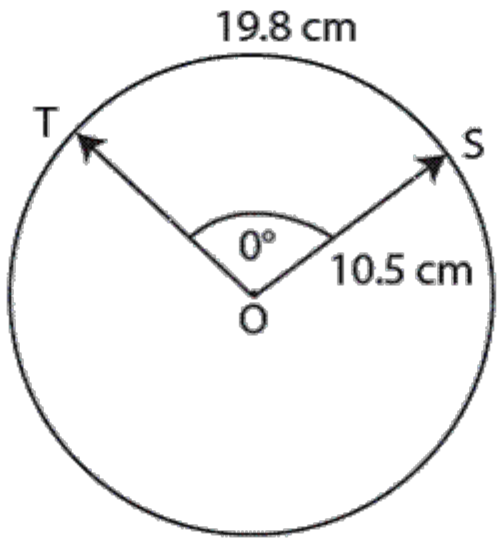



Fig. 13.6

$$\frac{\theta}{360°} \times 2\pi r = \text{arc TS}$$

$$\Rightarrow \quad \frac{\theta}{360°} \times 2 \times \frac{22}{7} \times 10.5 = 19.8$$

$$\Rightarrow \quad \theta = \frac{19.8 \times 360 \times 7}{2 \times 22 \times 10.5}$$

$$\Rightarrow \quad \theta = \frac{49\,896}{462}$$

Therefore, $\theta = 108°$

## D. Perimeter of a sector

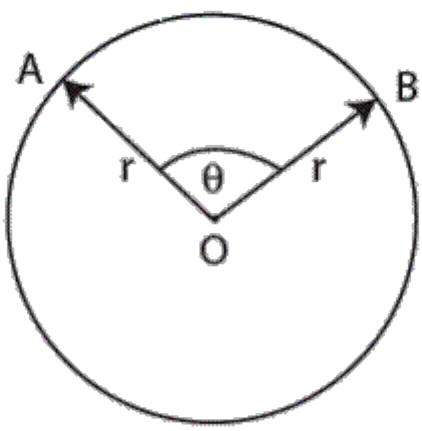



Fig. 13.7

Using Fig. 13.7, the perimeter of sector AOB = arc AB + two radii

$$= \left(\frac{\theta}{360^\circ} \times 2\pi r\right) + (2r)$$

## Worked Example 4

Find the perimeter of a sector AOB that subtends an angle of 80° at the centre

and radius 7 cm. Given that $\pi = \frac{22}{7}$.

### Solution

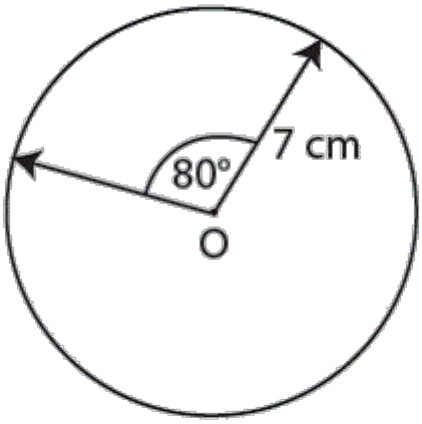



**Fig. 13.8**

Perimeter of the sector AOB = arc AB + 2 radii

$$= \left(\frac{\theta}{360^\circ} \times 2\pi r\right) + (2r)$$

$$= \left(\frac{80^\circ}{360^\circ} \times 2 \times 7 \times \frac{22}{7}\right) + (2 \times 7)$$

$$= 9.7778 + 14$$

$$= 23.78 \text{ cm (4 s.f.)}$$

## Worked Example 5

Calculate the radius of a sector that subtends an angle of 135° at the centre whose perimeter is 30.5 cm. Given that $\pi = \frac{22}{7}$.

### Solution

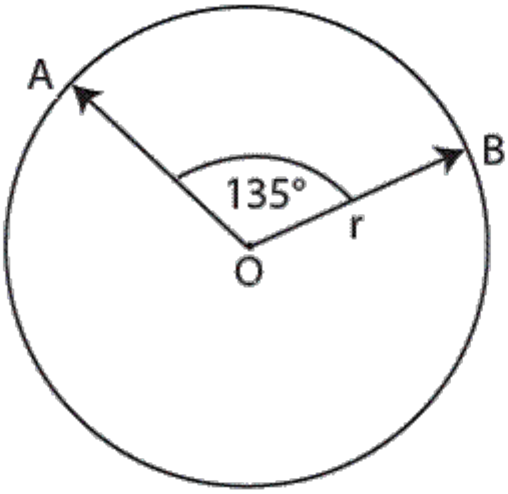



**Fig. 13.9**

$$\left(\frac{\theta}{360°} \times 2\pi r\right) + (2r) = \text{Perimeter of the sector AOB}$$

$$\left(\frac{135°}{360°} \times 2 \times \frac{22}{7} \times r\right) + (2r) = 30.5$$

$$2.35r + 2r = 30.5$$

$$r(2.35 + 2) = 30.5$$

$$r(4.35) = 30.5$$

$$r = \frac{30.5}{4.35} = 7.01$$

Therefore, $r = 7$ cm (to the nearest whole number)

## E. Length of a chord

Consider the chord AB as shown in Fig. 13.10. Arc *AB* subtends an angle of $\theta$ at the centre of the circle with "$r$" as the radius and *OC i*s the perpendicular bisector of the chord *AB*.

Using trigonometric ratio from ΔAOC

$$\sin\frac{\theta}{2} = \frac{|AC|}{|AO|}$$

$$= \frac{|AC|}{r}$$

$$\Rightarrow \quad |AC| = r\sin\frac{\theta}{2}$$

$$\Rightarrow \quad 2|AC| = 2r\sin\frac{\theta}{2}$$

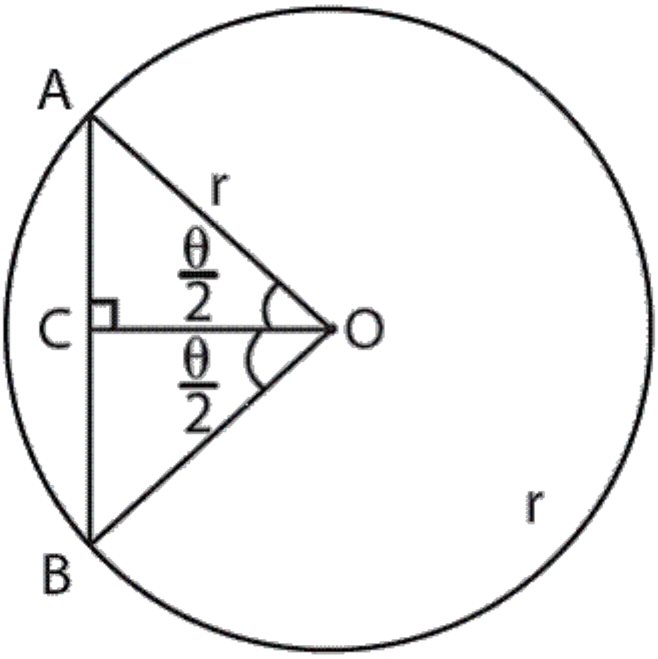



**Fig. 13.10**

Therefore, $|AB| = 2r\sin\frac{\theta}{2}$ $(2|AC| = |AB|)$
Generally, the length of the chord of any given circle is $2r\sin\frac{\theta}{2}$.

**Note:** The perpendicular bisector of a chord passes through the centre of the circle.

## Worked Example 6

Find the length of a chord AB that subtends an angle of 105° at the centre of a circle of radius 10 cm.

### Solution

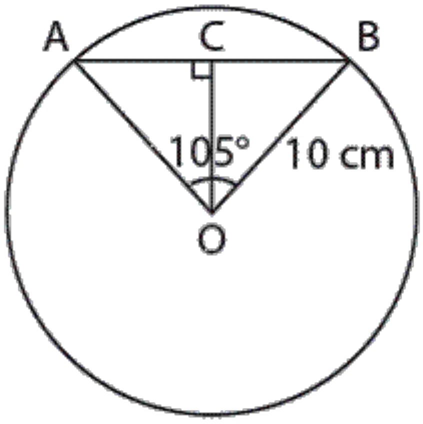



**Fig. 13.11**

$$\text{Chord AB} = 2r\sin\frac{\theta}{2}$$
$$= 2 \times 10 \times \sin\frac{105°}{2}$$
$$= 20 \times \sin 52.5°$$
$$= 20 \times 0.7934$$
$$= 15.86\text{cm}$$

## Worked Example 7

A chord of length 16 cm subtends an angle of 60° at the centre of the circle. Find the radius of the circle.

### Solution

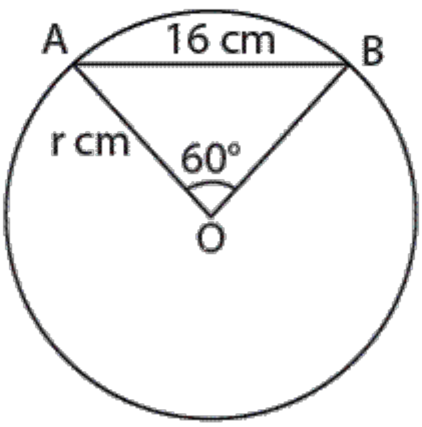



**Fig. 13.12**

$$2r\sin\frac{\theta}{2} = |AB|$$

$$\Rightarrow \quad 2r \times \sin\frac{60^\circ}{2} = 16$$

$$\Rightarrow \quad 2r \times \sin 30^\circ = 16$$

$$\Rightarrow \quad 2r \times 0.5 = 16$$

$$\Rightarrow \quad r = 16\text{ cm}$$

## Worked Example 8

A chord PQ of length 8.19 cm is in a circle of radius 5 cm. Find the angle subtended by the chord at the centre of the circle. Express the answer to the nearest degree.

### Solution

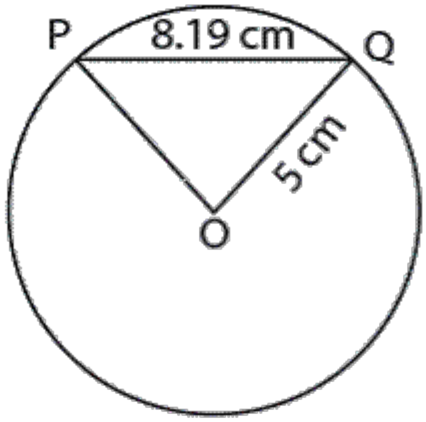



**Fig. 13.13**

$$2r\sin\frac{\theta}{2} = \text{chord } PQ$$

$$\Rightarrow \quad 2 \times 5 \times \sin\frac{\theta}{2} = 8.19$$

$$\Rightarrow \quad 10 \times \sin\frac{\theta}{2} = 8.19$$

$$\Rightarrow \quad \sin\frac{\theta}{2} = \frac{8.19}{10}$$

$$\Rightarrow \quad \sin\frac{\theta}{2} = 0.819$$

$$\Rightarrow \quad \frac{\theta}{2} = \sin^{-1} 0.819$$

$$\Rightarrow \quad \frac{\theta}{2} = 55°$$

$$\Rightarrow \quad \theta = 2 \times 55°$$

$$\text{Therefore,} \quad \theta = 110°$$

## Worked Example 9

A chord of a circle is 24 cm long. If the midpoint of the chord is 5 cm from the centre of the circle, find the radius of the circle.

### Solution

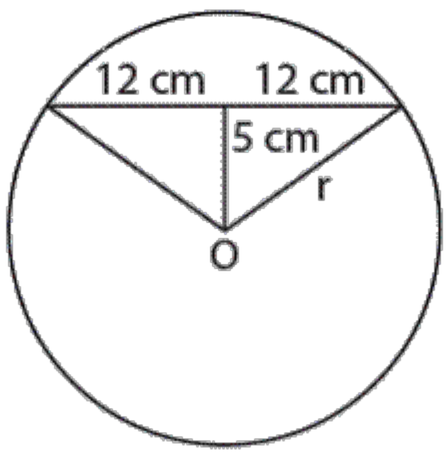



**Fig. 13.14**

$r^2 = 12^2 + 5^2$

$r^2 = 144 + 25$

$r^2 = 169$

$r = 13$ cm

## Worked Example 10

The radius of a circle is 17 cm. The centre of the circle is 8 cm from the midpoint of the chord. Find the length of the chord.

### Solution

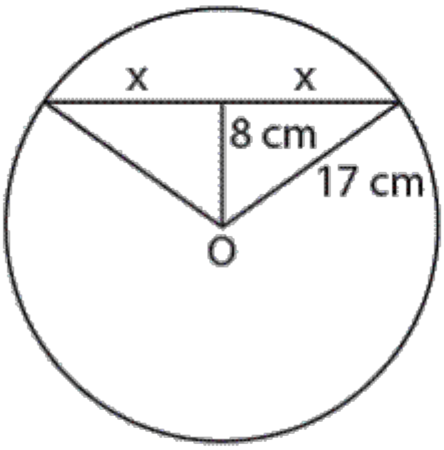



Fig. 13.15

$$17^2 = 8^2 + x^2$$

$$x^2 = 17^2 - 8^2$$

$$x^2 = 289 - 64 = 225$$

$$x = 15 \text{ cm}$$

Length of the chord $= 15 \times 2 = 30$ cm

## F. Perimeter of the minor segment.

Consider the minor segment or the shaded portion in Fig. 13.16 of a circle of radius $r$ in which arc PQ subtends an angle $\theta$ at the centre "O" of the circle.

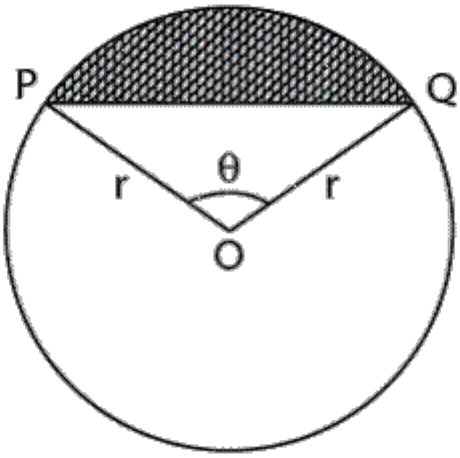



**Fig. 13.16**

Hence, the perimeter of the minor segment (the shaded portion) will be

$$= \text{arc } PQ + \text{chord } PQ$$

$$= PQ + |PQ| \text{ (symbolically)}$$

$$= \left(\frac{\theta}{360^\circ} \times 2\pi r\right) + \left(2r\sin\frac{\theta}{2}\right)$$

## Worked Example 11

Figure 13.17 shows a chord AB of a circle that subtends an angle of 120° at the
Centre and radius 8 cm. Find the perimeter of the minor segment. Use $\pi = \frac{22}{7}$

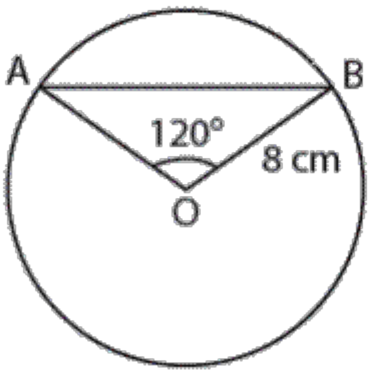



**Fig. 13.17**

## Solution

Perimeter of the minor segment

$= AB + |AB|$

$= \left(\frac{\theta}{360°} \times 2\pi r\right) + \left(2r \sin\frac{\theta}{2}\right)$

$= \left(\frac{120°}{360°} \times 2 \times \frac{22}{7} \times 8\right)$

$+ \left(2 \times 8 \times \sin\frac{120°}{2}\right)$

$= \left(\frac{1}{3} \times 2 \times \frac{22}{7} \times 8\right) + (2 \times 8 \times \sin 60°)$

$= 16.76 + 13.856$

$= 30.616$

$= 30.62$ cm (2 d.p.)

## Worked Example 12

The perimeter of a minor segment of a circle that subtends an angle 84° at the

centre is 29.45 m. Calculate the value of the radius of the circle. (Use $\pi = \frac{22}{7}$ )

### Solution

$= \left(\frac{\theta}{360} \times 2\pi r\right) + \left(2r \sin\frac{\theta}{2}\right)$

$=$ Perimeter of the minor segment

$\Rightarrow \quad \left(\frac{84°}{360°} \times 2 \times \frac{22}{7} \times r\right)$

$+ \left(2r \sin\frac{84}{2}\right) = 29.45$

$$\Rightarrow \quad \frac{22}{15}r + 2r\sin 42° = 29.45 \text{ m}$$

$$\Rightarrow \quad \frac{22}{15}r + 2r \times 0.6691 = 29.45$$

$$\Rightarrow \quad 1.46r + 1.33r = 29.45$$

$$\Rightarrow \quad r(2.79) = 29.45$$

$$\Rightarrow \quad r = \frac{29.45}{2.79}$$

$$\Rightarrow \quad r = 10.55$$

Therefore, $r = 10.6$ m (3 s.f.)

## Exercise 1

1. Find the lengths of arc AB in Fig. 13.18. Express the answers to the nearest whole number. Use $\pi = \frac{22}{7}$.

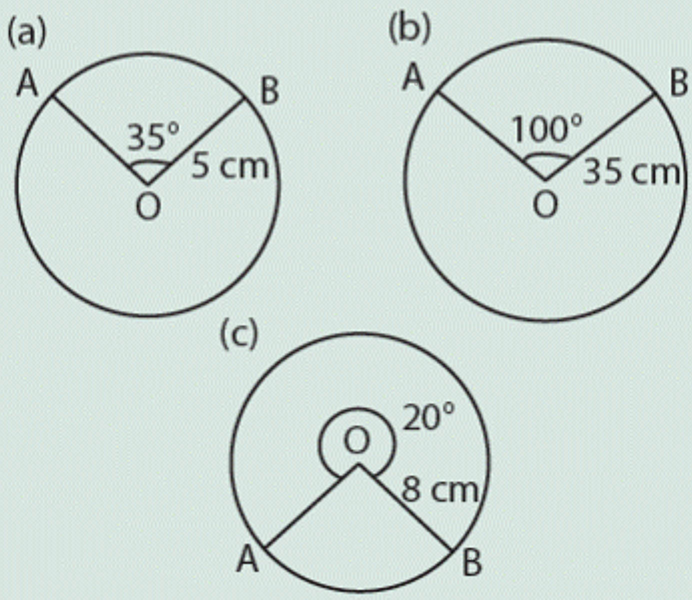



Fig. 13.18

2. Calculate the arc lengths of the circles that subtend the given angle $\theta$ at the centre with their specified radius in questions a–c.

   Take $\pi = \frac{22}{7}$.

   (a) $r = 8$ cm, $\theta = 60°$

   (b) $r = 10.3$ cm, $\theta = 75°$

   (c) $r = 15$ cm, $\theta = 100°$

3. Calculate and correct to 3 significant figures, the length of an arc of a circle of radius 16.5 cm that subtends an angle 90° at the centre. Use $\pi = 3.142$.

4. Find the length of an arc of a circle which subtends an angle of 70° with radius 10.5 cm.

5. Given an arc of a circle of radius 10 cm that subtends an angle of 190° at the centre.

   (a) Is the given arc a minor or a major arc?

   (b) Find the length of the major arc. $\left(\text{Use } \pi = \frac{22}{7}\right)$

6. Calculate the radius of the circles in Fig. 13.19. Express all answers to 2 s.f.

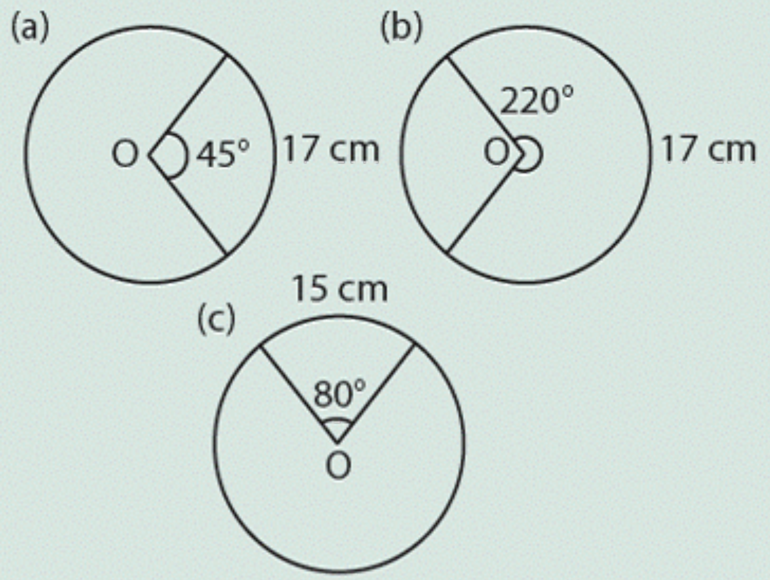



**Fig. 13.19**

7. Calculate correct to 2 decimal places, the radius of circles whose arcs AB subtend the given angles $\theta$ as specified in questions a–c. Take $\pi = \frac{22}{7}$.
   (a) AB = 9.15 cm, $\theta = 35°$
   (b) AB = 10.25 cm, $\theta = 60°$
   (c) AB = 15.55 cm, $\theta = 80°$

8. Find to the nearest whole number, the radius of a circle whose arc is 330 cm and subtends an angle of 150° at the centre.
9. Calculate, correct to 3 significant figures, the radius of a circle whose arc is 24 m, which subtends an angle of 12° at the centre.
10. An arc of length 15 cm subtends 45° at the centre of the circle. What is the radius of the circle? Use $\pi = \frac{22}{7}$.
11. Find to the nearest degree, angles $\theta$ subtended at the centre of the circles in Fig. 13.20 when both the arcs and the radii are given.

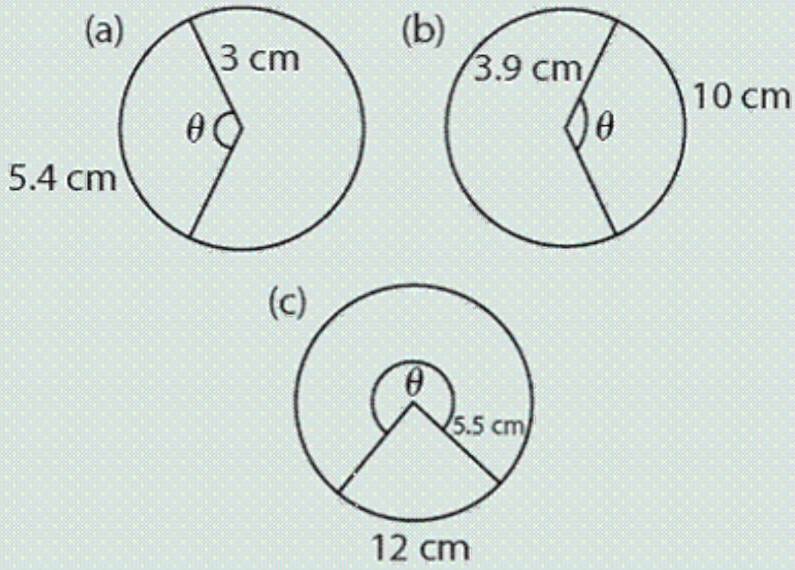



**Fig. 13.20**

12. Calculate, correct to the nearest degree, the angles $\theta$ subtended at the centre of the circles when the radius and the arcs are given as specified in questions a–c.

    (a) $r = 3.7$ cm, arc $= 5$ cm

    (b) $r = 5.5$ cm, arc $= 15$ cm

    (c) $r = 10.2$ cm, arc $= 17$ cm

13. An arc of length 13 cm of a circle has a radius of 7 cm. What angle has the arc subtended at the centre?

14. An arc of a circle whose length is 23 cm has a radius of 10 cm. Find the angle the arc subtends at the centre.
15. What angle does an arc of length 8.5 cm subtend at the centre of a circle of radius 2.5 cm?
16. Find the perimeter of a sector AOB whose angle at the centre $\theta$ and radius $r$ are specified in questions a–c.

    Express all the answers to the nearest whole number. Take $\pi = \frac{22}{7}$.

    (a) $\theta = 30°,\ r = 7$ cm

    (b) $\theta = 55°,\ r = 10$ cm

    (c) $\theta = 13°,\ r = 20$ cm.

17. Calculate the radius(r) of a sector that subtends an angle $\theta$ and whose perimeter ($p$) are given as stated in questions a–c. Express all the answers to 2 d.p. Use $\pi = \frac{22}{7}$.
(a) $\theta = 30°$, $p = 13$ cm
(b) $\theta = 45°$, $p = 25$ cm
(c) $\theta = 140°$, $p = 55$ cm

18. Calculate the lengths of the chords of the circles in Fig. 13.21. Express all answers to 2 decimal places.

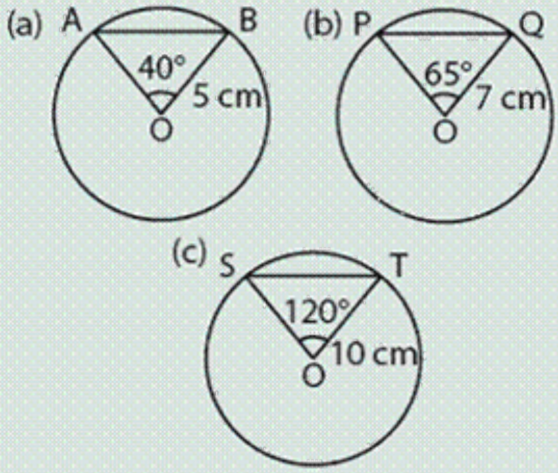



**Fig. 13.21**

19. Find the lengths of the chords of a circle whose angles at the

centre $\theta$ and radii $r$ are specified in questions a–c correct to 2 d.p. Use $\pi = \frac{22}{7}$.

(a) $\theta = 25°$, $r = 10$ cm

(b) $\theta = 35°$, $r = 15$ cm

(c) $\theta = 50°$, $r = 18$ cm

20. Calculate the length of a chord PQ that subtends an angle of 139° at the centre of a circle with radius 15 cm.

# II. Circular Area Measurement

## Objectives

At the end of this sub-chapter, the students should be able to:

1. Find the area of a sector of a circle.
2. Determine the area of a segment of a circle.
3. Determine the relationship between the area of a sector of a circle and the surface area of a cone.

## A. Area of a sector of a circle

Recall that the area of a plane is the amount of surface covered by the plane. It is solved by finding the number of square units in a given plane. The formula $\pi r^2$ is used to find the area of a circle.

### (i) Area of a sector when its radius and the angle it subtends at the centre are given

#### Activity 13.2

**Step 1:** *Draw a circle of radius 7 cm and divide the circle into sectors in the ratio 1:2:3 such that the sectors subtend angles of 60°, 120° and 180° respectively, as shown in Fig. 13.22.*

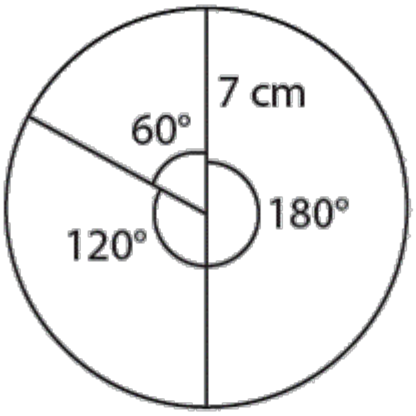



**Fig. 13.22**

**Step 2:** *Use the given figures and facts to copy and complete Table 13.2.*

**Table 13.2**

| | Radius ($r$) | Angle of sector ($\theta$) | Angle of circle ($360°$) | Area of the circle ($\pi r^2$) | Area of the sector (A of S) | Area of the sector (A of S) / Area of the circle ($\pi r^2$) | Angle of the sector ($\theta$) / Angle of the circle ($360°$) |
|---|---|---|---|---|---|---|---|
| a. | 7 cm | 60° | 360° | $\frac{22}{7}\times\frac{7}{1}\times\frac{7}{1}$ $= 154\ cm^2$ | $\frac{1}{6}\times\frac{154}{1}$ $= 25.66\ cm^2$ | $\frac{25.67}{154} = 0.16$ | $\frac{60°}{360°} = 0.16$ |
| b. | 7 cm | 120° | 360° | $\frac{22}{7}\times\frac{7}{1}\times\frac{7}{1}$ $= 154\ cm^2$ | $\frac{2}{6}\times\frac{154}{1}$ $= 51.33\ cm^2$ | | |
| c. | 7 cm | 180° | 360° | | | | |

**Step 3:** *Observation*

*Comparing the values of the ratios in columns 7 and 8, you will observe that in each row:*

$$\frac{\text{Area of the sector (A of S)}}{\text{Area of the circle } (\pi r^2)} = \frac{\text{Angle of the sector } (\theta)}{\text{Angle of the circle } (360°)}$$

$$\Rightarrow \frac{\text{A of S}}{\pi r^2} = \frac{(\theta)}{360°}$$

$$\Rightarrow \text{Area of the sector} = \frac{(\theta)}{360°} \times \pi r^2$$

*That is, the area of the sector of a circle with radius r and angle $\theta$ at the centre.*

***Note:*** *We have established the fact that the area of a sector of a circle with a given radius (r) is proportional to the angle which the sector makes at the centre of the circle.*

## (ii) Area of a sector when only its radius and arc length are given

Given a sector AOB of a circle with centre O, arc length (AB), radius ($r$) that subtends an angle $\theta$ at the centre as shown in Fig. 13.23.

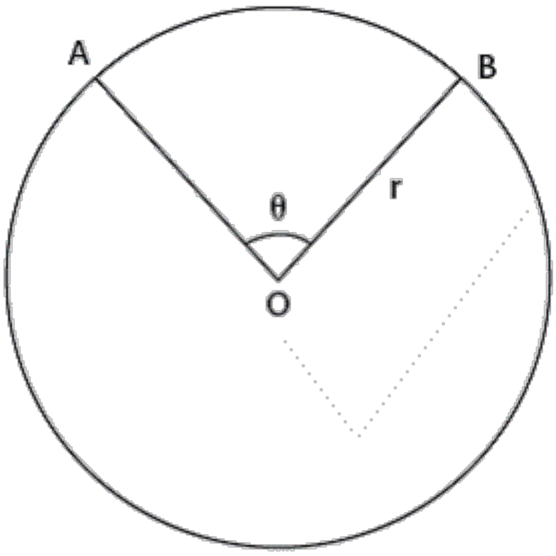



**Fig. 13.23**

We have previously established the fact that:

$$\frac{AB}{2\pi r} = \frac{\theta}{360^\circ}$$

$$\Rightarrow \theta = \frac{AB \times 360^\circ}{2\pi r} \qquad \text{(i)}$$

And that the area of the sector AOB

$$= \frac{\theta}{360^\circ} \times \pi r^2 \qquad \text{(ii)}$$

Then substitute the value of $\theta$ in (i) and (ii). Therefore, area of the sector

$$= \frac{AB \times 360^\circ}{2\pi r} \div \frac{360}{1} \times \pi r^2$$

$$= \frac{AB \times 360 \times \pi r^2}{2\pi r \times 360^\circ}$$

$$= \frac{AB \times r}{2}$$

## Worked Example 13

Find the area of a sector of a circle of radius 10 cm that subtends an angle of 30° at the centre.

### Solution

$$\text{Area} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{30°}{360°} \times \frac{22}{7} \times \frac{10 \times 10}{1}$$

$$= 26.19 \text{ cm}^2$$

## Worked Example 14

Calculate, correct to 1 decimal place, the radius of a circle, if the area of its sector that subtends an angle of 100° is 10.69 $cm^2$.

### Solution

$$\frac{\theta}{360°} \times \pi r^2 = \text{Area}$$

$$= \frac{100°}{360°} \times \frac{22}{7} \times r^2 = 10.69$$

$$r^2 = \frac{10.69 \times 360 \times 7}{100 \times 22}$$

$$r^2 = 12.24$$

$$r = \sqrt{12.24}$$

$$r = 3.4986$$

$$r = 3.5 \text{ cm (1 d.p.)}$$

## Worked Example 15

Calculate the angle to the nearest degree, subtended by a sector at the centre of the circle, if the radius is 12 cm and the area of the sector is 100.57 $cm^2$.

### Solution

$$\frac{\theta}{360^\circ} \times \pi r^2 = \text{Area}$$

$$= \frac{\theta}{360^\circ} \times \frac{22}{7} \times 12 \times 12 = 100.57 \text{ cm}^2$$

$$\theta = \frac{100.57 \times 360 \times 7}{22 \times 12 \times 12}$$

$$\theta = 79.99^\circ$$

$$\theta = 80^\circ \text{ (to the nearest degree)}$$

## Worked Example 16

Find the area of a sector bounded by an arc of length 15 cm and radius 6.5 cm.

$$\text{Area} = \frac{\text{Arc length} \times \text{radius}}{2}$$

$$= \frac{15 \times 6.5}{2}$$

$$= 48.75 \text{ cm}^2$$

### (iii) Area of a segment of a circle

Given a sector POQR as shown in Fig. 13.24.

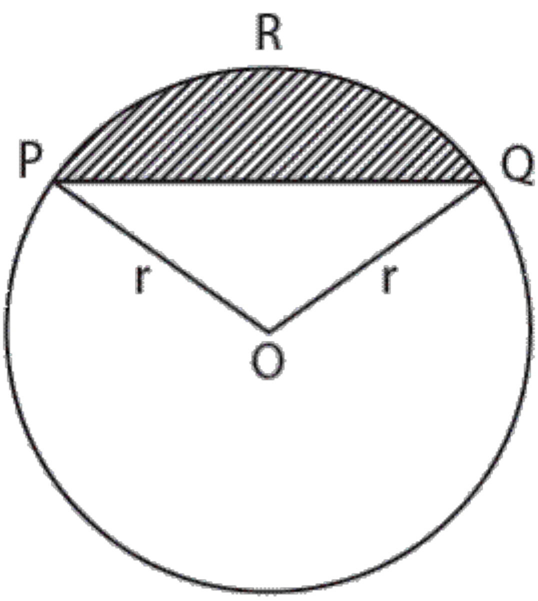



**Fig. 13.24**

Sector POQR = Segment PQR + Triangle POQ

$\Rightarrow$ Segment PQR = Sector POQR − Triangle POQ

For more details, see illustrations in Fig. 13.29.

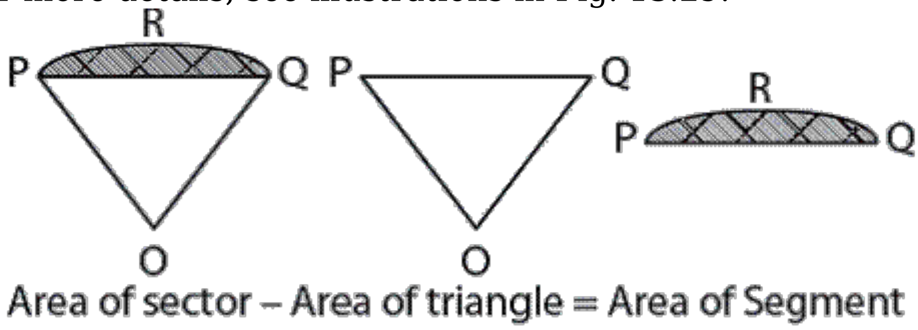



**Fig. 13.25**

Recall that the area of sector with radius ($r$) and $\theta$ as the angle at the centre is

$$\frac{\theta}{360^\circ} \times \pi r^2$$

And the area of a triangle when an angle $\theta$ and the adjacent sides $a$ and $b$ are given as shown in Fig. 13.26 is

$$\frac{a \times b \times \sin\theta}{2}$$

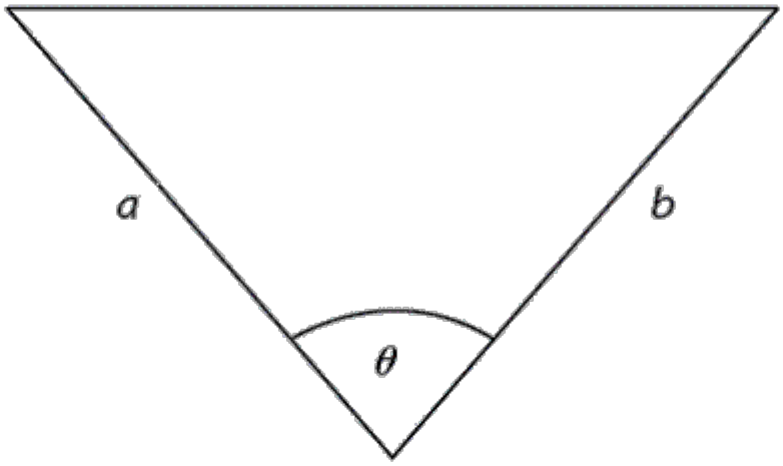



**Fig. 13.26**

Therefore, using Fig. 13.26, the area of the segment PQR (the shaded portion) is equal to the area of the sector POQR minus the area of the triangle POQ; i.e.,

Area of segment PQR =

$$\left(\frac{\theta}{360^\circ} \times \pi r^2\right) - \left(\frac{r^2 \sin\theta}{2}\right)$$

## Worked Example 17

Calculate the area of a segment of a circle of radius 10 cm which subtends an angle of 95° at the centre. (Use $\pi = \frac{22}{7}$)

## Solution

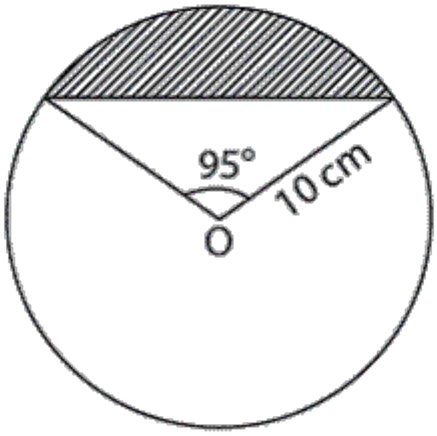



**Fig. 13.27**

Area of segment = Area of sector − Area of triangle

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{r^2 \sin \theta}{2}$$

$$= \left(\frac{95}{360} \times \frac{22}{7} \times 10 \times 10\right)$$

$$- \left(\frac{10 \times 10 \times \sin 95°}{2}\right)$$

$$= 82.9365 - 49.8097$$

$$= 33.1268$$

$$= 33.13 \text{ cm}^2 \text{ (2 d.p.)}$$

## Worked Example 18

Find the area, correct to 2 decimal places, of the major segment of a circle of radius 5.5 cm, if the minor segment subtends an angle of 100° at the centre.

## Solution

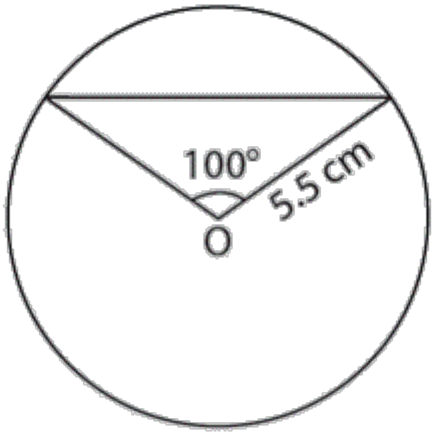



Fig. 13.28

Area of major segment = Area of circle − Area of minor segment

$$= \pi r^2 - \frac{\theta}{360°} \times \pi r^2 - \frac{r^2 \sin\theta}{2}$$

$$= \pi r^2 - \left(\frac{\theta}{360°} \times \pi r^2 - \frac{r^2 \sin\theta}{2}\right)$$

Therefore, area of major segment

$$= \left(\frac{22}{7} \times 5.5 \times 5.5\right)$$

$$- \left(\frac{100}{360} \times \frac{22}{7} \times 5.5 \times 5.5\right)$$

$$+ \left(\frac{5.5 \times 5.5 \times \sin 100°}{2}\right)$$

$$= 95.0714 - 26.4087 + 14.8952$$

$$= 83.5579$$

$$= 83.56 \text{ cm}^2 \text{ (2 d.p)}$$

## Worked Example 19

12.98 $cm^2$ is the area of a segment of a circle that subtends an angle of 150° at the centre. Find its radius, correct to 1 decimal place.

### Solution

Area of sector − Area of Δ = Area of segment

$$\frac{\theta}{360^\circ} \times \pi r^2 - \frac{r^2 \sin\theta}{2} = \text{Area of segment}$$

$$\frac{150^\circ}{360} \times \frac{22}{7} \times r^2 - \frac{r^2 \sin 150^\circ}{2} = 12.98 \text{ cm}^2$$

$$\Rightarrow r^2\left(\frac{150^\circ}{360^\circ} \times \frac{22}{7} - \frac{\sin 150^\circ}{2}\right) = 12.98$$

$$\Rightarrow r^2\,(1.3095 - 0.25) = 12.98$$

$$\Rightarrow r^2\,(1.0595) = 12.98$$

$$\Rightarrow r^2 = \frac{12.98}{1.0595}$$

$$r^2 = 12.2511$$

$$r = \sqrt{12.2511}$$

$$r = 3.5002$$

$$r = 3.5 \text{ (1 d.p.)}$$

**(iv) The relationship between the area of a sector of a circle and the surface area of a cone**

**Activity 13.3**

*Draw a fairly large circle on a cardboard paper. Cut off a sector from the drawn circle. Fold up the sector until the two radii AO and BO meet. A right circular cone is formed. See Fig. 13.29.*

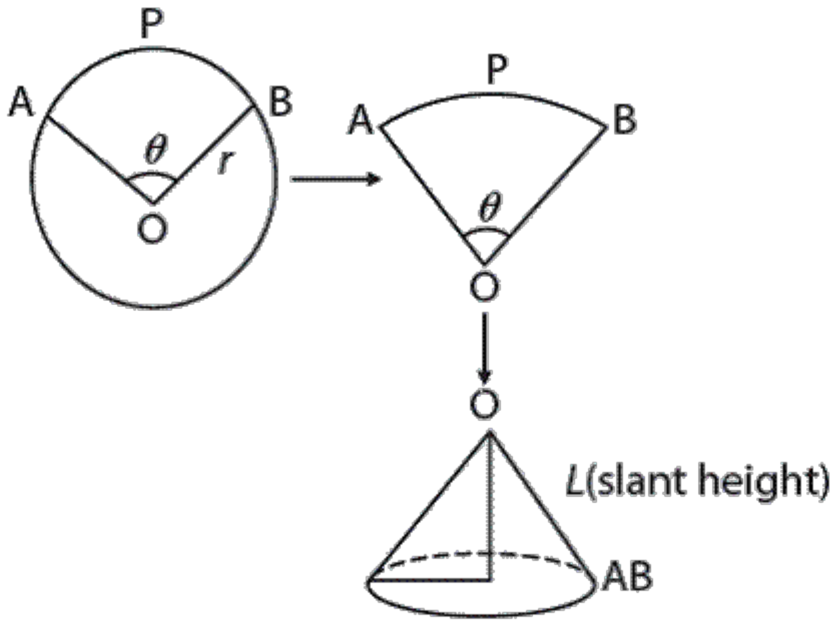



**Fig. 13.29**

From the illustration in Fig. 13.29, you will observe that:

i. The length of the radius of the sector AO becomes the slant height of the cone.

ii. The length of the arc APB becomes the circumference of the base of the cone.

$$\text{i.e. } \frac{\theta}{360°} \times 2\pi \times AO = 2\pi r$$

$$\frac{\theta}{360°} \times 2\pi \times \mathrm{L} = 2\pi r$$

$$\Rightarrow r = \frac{\theta}{360°} \times L \text{ (slant height)}$$

Therefore, the circular base radius ($r$)

$$= \frac{\theta}{360°} \times L \text{ (slant height)}$$

iii. The area of the sector of the circle is equal to the curved surface area of the cone.

$$\text{Area of the sector (A)} = \frac{\theta}{360°}\pi r^2$$

$$= \frac{\theta}{360°} \times \pi \times |AO|^2$$

$$= \frac{\theta}{360°} \times \pi \times |AO| \times |AO|$$

$$= \frac{\theta}{360°} \times L \times \pi L \text{ (since } |AO| = L\text{)}$$

$$= r \times \pi L \text{ (since in statement (ii),}$$

$$r = \frac{\theta}{360°} \times L\text{)}$$

$$= \pi L r$$

This formula is now used to find the curved surface area of a cone whose radius is "*r*" and slant height is "*L*".

## Worked Example 20

Find the curved surface area of a cone whose radius is 10 cm and the slant height is 14 cm.

### Solution

$$\text{Curved surface area (CSA)} = \pi L r$$

$$= \frac{22}{7} \times 10 \times 14$$

$$= 440 \text{ cm}^2$$

## Worked Example 21

Calculate the circular base radius of a cone formed by folding a sector whose radius is 12 cm and angle at the centre is 110°.

### Solution

$$R = \frac{\theta}{360°} \times L$$
$$= \frac{110}{360} \times 12$$
$$= 3.6667$$
$$= 3.67 \text{ (2 d.p.)}$$

## Worked Example 22

A sector of a circle of radius 8 cm and angle at the centre 120° is folded to form a cone. Calculate the
(a) radius of the cone and
(b) vertical angle of the cone

## Solution

(a) $r = \frac{\theta}{360°} \times L$

$$= \frac{120}{360} \times 8$$
$$= 2.667$$
$$= 2.67 \text{ (2 d.p)}$$

(b) $\sin\theta = \frac{2.667}{8}$

$$= 0.3334$$
$$\theta = \sin^{-1} 0.3334$$
$$\theta = 19.4692$$
$$2\theta = 2 \times 19.47$$
$$= 38.94°$$
$$= 39° \text{ (to the nearest degree)}$$

## Worked Example 23

A sector whose angle at the centre is 130° and radius 15 cm is used to produce a cone. Find the perpendicular height of the cone.

Solution

$$\text{Base radius of the cone } (r) = \frac{\theta}{360^\circ} \times L$$

$$= \frac{130}{360} \times 15$$

$$= 5.4167$$

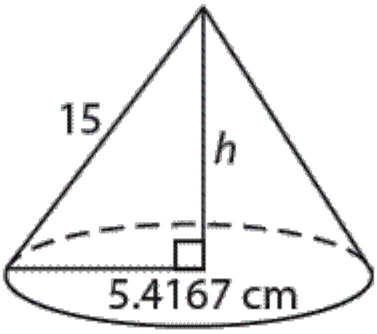



**Fig. 13.30**

$$h^2 = (15)^2 - (5.4167)^2 \text{ (Pythagoras theorem)}$$

$$= 225 - 29.3406$$

$$= 195.6594$$

$$h = 13.9878$$

$$h = 14 \text{ cm (to the nearest whole number)}$$

## Worked .Example 24

If a cone of radius 5 cm and perpendicular height 12 cm is opened up and turned into a sector of a circle, calculate the angle of the sector.

Solution

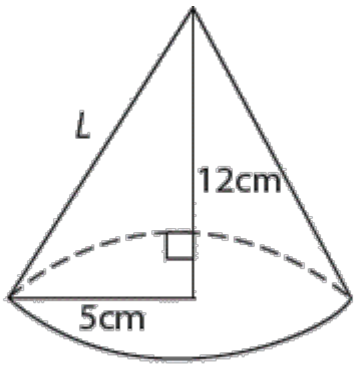



**Fig. 13.31**

$$L^2 = 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169$$
$$L = \sqrt{169} = 13 \text{ cm}$$

But, $\frac{\theta}{360°} \times L = r$

$$= \frac{\theta}{360} \times 13 = 5$$

$$\theta = \frac{5 \times 360}{13}$$

$$\theta = 138.46$$

## Exercise 2

1. Find the area of the sectors in Fig. 13.32. Express your answers to the nearest whole number. Use $\pi = \frac{22}{7}$.

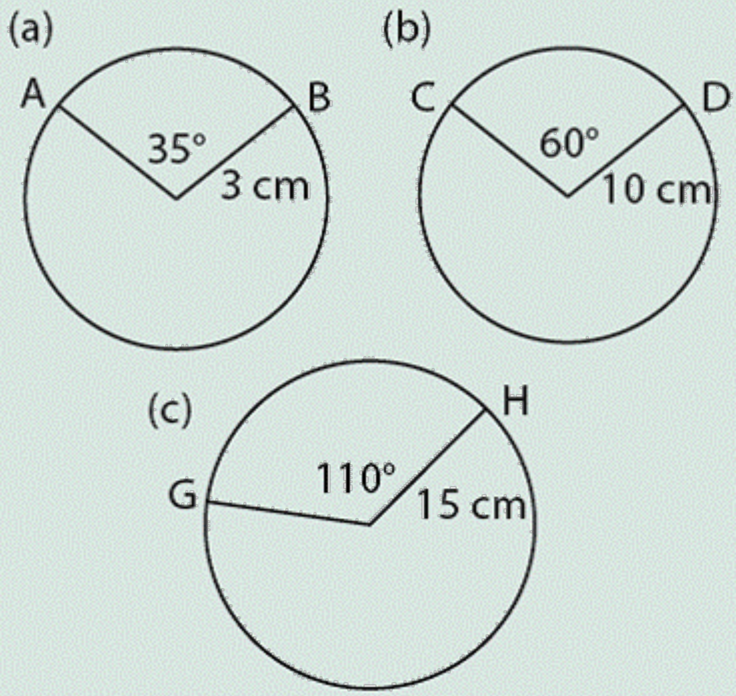



**Fig. 13.32**

2. Calculate the area of the sectors that subtend the given angles $\theta$ at their centre with their specified radius ($r$) in questions a–c.

(a) $\theta = 35°$, $r = 25$ cm

(b) $\theta = 120°$, $r = 10$ cm

(c) $\theta = 155°$, $r = 5$ cm

3. Find the area of the major sector of a circle of radius 8 cm, if the angle subtended by the minor sector at the centre is 95°. Correct your answer to 2 decimal places.

4. 65° sector plot of land with 55 m radius is meant for clearance. Find the total cost of clearing the plot at the rate of ₦ 45.00 per square metre.

5. Figure 13.33 shows a composite plane made up of a semi-circle and an equilateral triangle PQR. If the radius is 3.5 cm, find the total surface area of the composite plane.

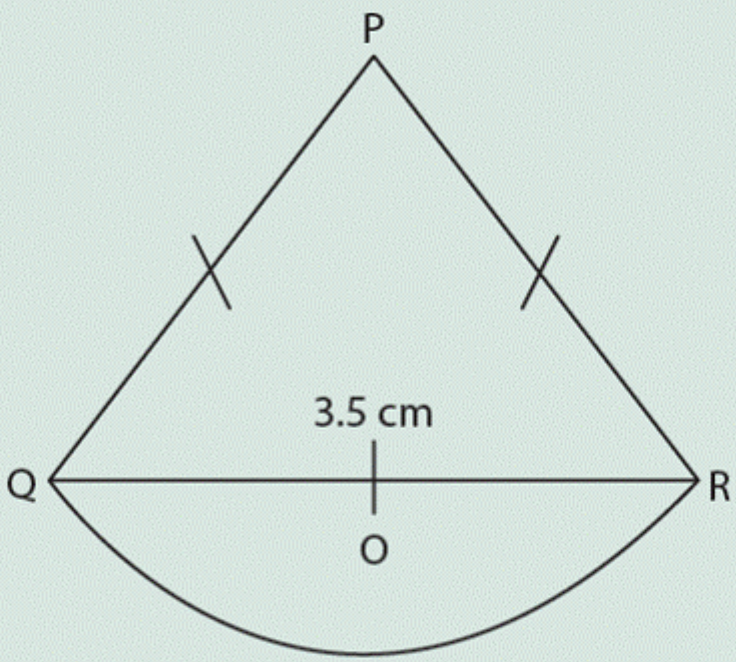



**Fig. 13.33**

6. Calculate, correct to 1 decimal place, the radius of a circle, if the area of its sector and the angle the sector subtends at the centre are given as specified in Fig. 13.34.

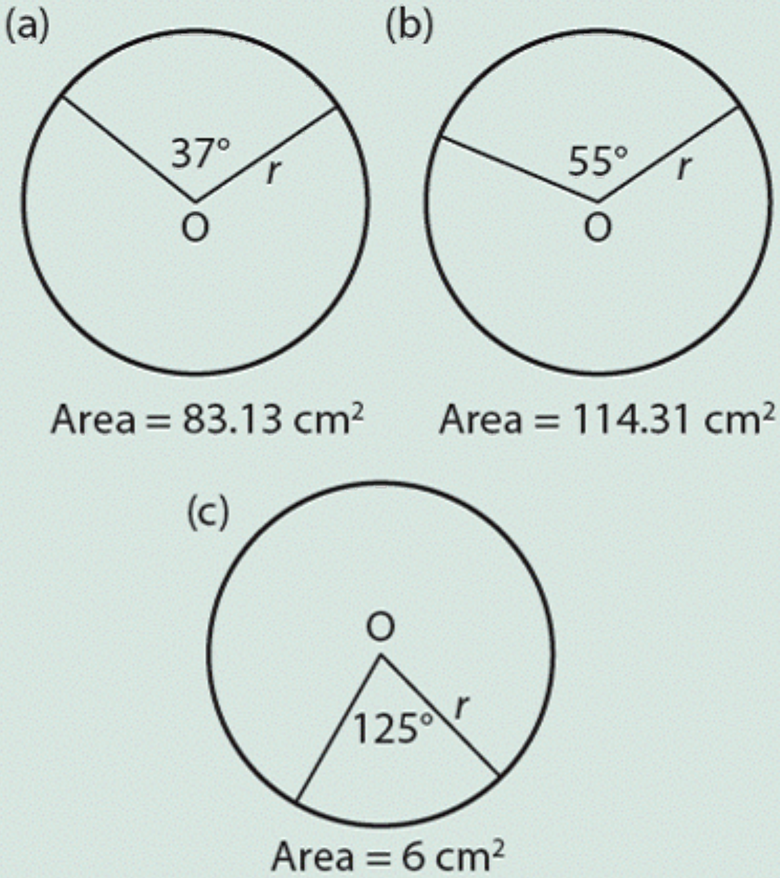



**Fig. 13.34**

7. Find the radii of the sector of a circle which subtends the following angles at the centre and with the following areas.
   (a) $75°$ and area $= 145 \text{ cm}^2$
   (b) $100°$ and area $= 140 \text{ cm}^2$
   (c) $135°$ and area $= 120 \text{ cm}^2$

8. Calculate, correct to 2 decimal places, the radius of a circle, if the area of its sector that subtends an angle of 83° is 15.35 $cm^2$.

9. Find the radius of a sector whose area is 84.35 $cm^2$ and which enclosed an angle of 95°.

10. A pendulum swept through 25.0° on each side of the vertical. If the area swept through by the pendulum is 43.65 $cm^2$, find the length of the pendulum.

11. Find to the nearest degree the angle a sector subtends at the

centre, if the area of the sector and its radius are given as shown in Fig. 13.35.

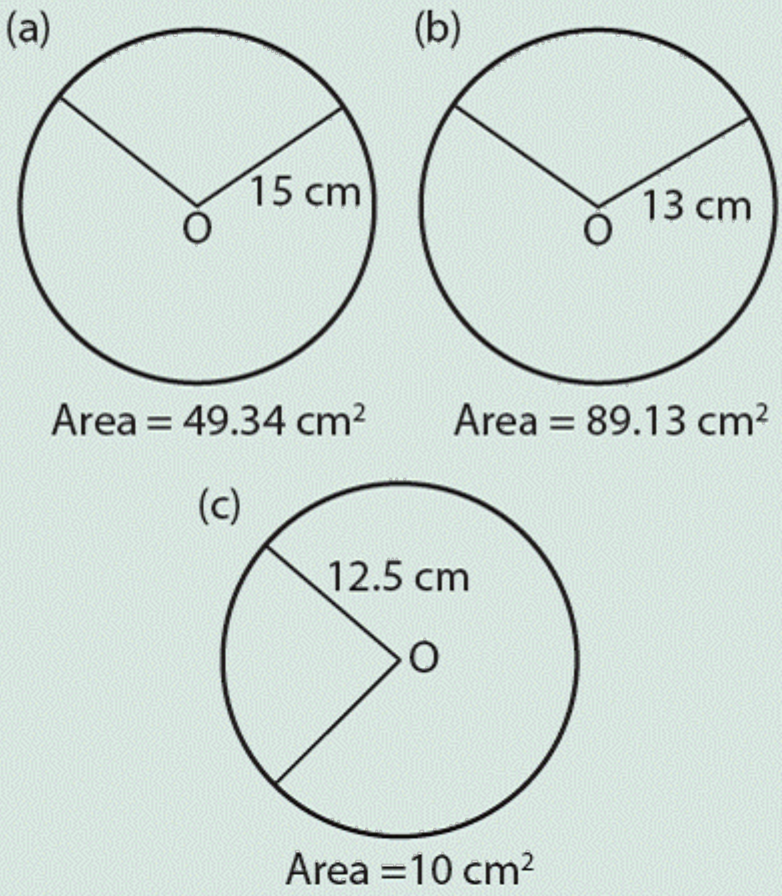



**Fig. 13.35**

12. Calculate the angle to the nearest degree subtended by a sector at the centre of a circle while its radius and area are given as specified in questions a–c.

    (a) radius = 5.5 cm
        area = 46.1 $cm^2$

    (b) radius = 7.3 cm
        area = 110.3 $cm^2$

    (c) radius = 9.2 cm
        area = 141.5 $cm^2$

13. A sector of area 15 $cm^2$ is cut off from a circle of radius 3.5 cm. Find the angle of the sector.

14. Given a sector AOB of a circle whose area is 55.69 $cm^2$ with a radius 13.5 cm; find the angle the sector subtends at the centre.

15. A sector of area 143 $cm^2$ is cut off from a circle of radius 11 cm. Calculate the sectorial angle to the nearest degree.

16. Calculate the area of the sectors in Fig. 13.36. Express the answers to 2 decimal places.

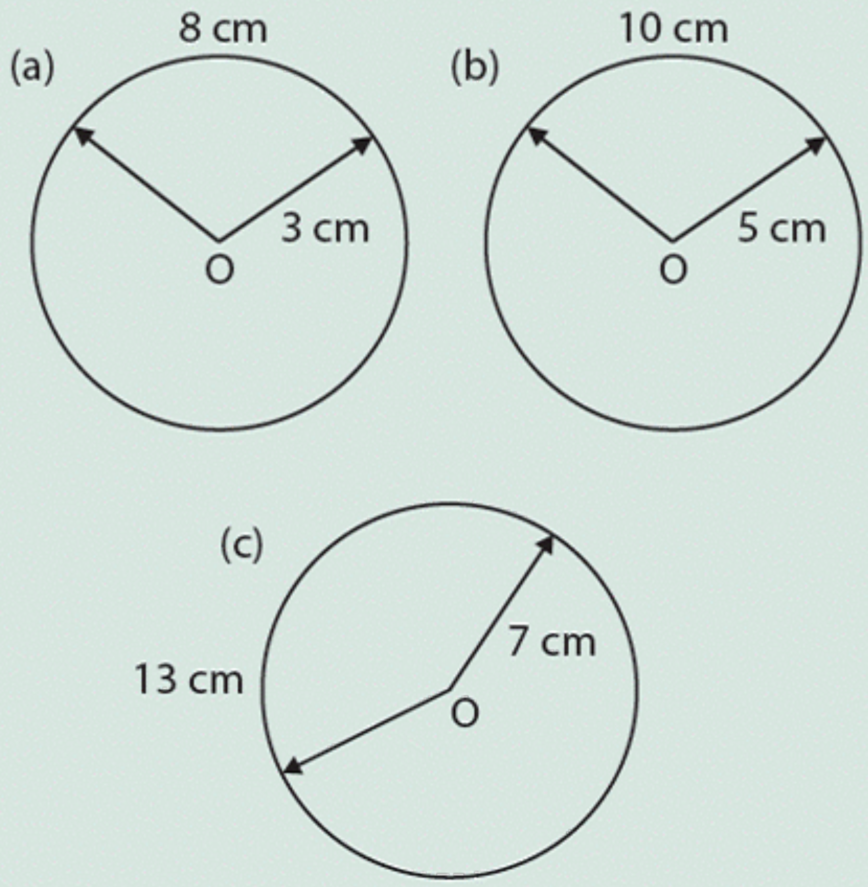



**Fig. 13.36**

17. Find the area of the sectors whose arc length and radius are given in questions a–c. Correct all answers to 3 significant figures.
    (a) Arc length = 3 cm
    Radius = 4.0 cm
    (b) Arc length = 8 cm
    Radius = 4.5 cm
    (c) Arc length = 10 cm
    Radius = 5.5 cm
18. Find the area of a sector bounded by an arc of length 10 cm and radius 5.5 cm.
19. Calculate the arc length, if the area is 135 $cm^2$ and radius 8 cm.

20. Show that the area "$A$" of a sector whose arc length "$L$" and radius "$r$" is $\frac{Lr}{2}$.

21. Find the area of the shaded segments of the circles in Fig. 13.37.

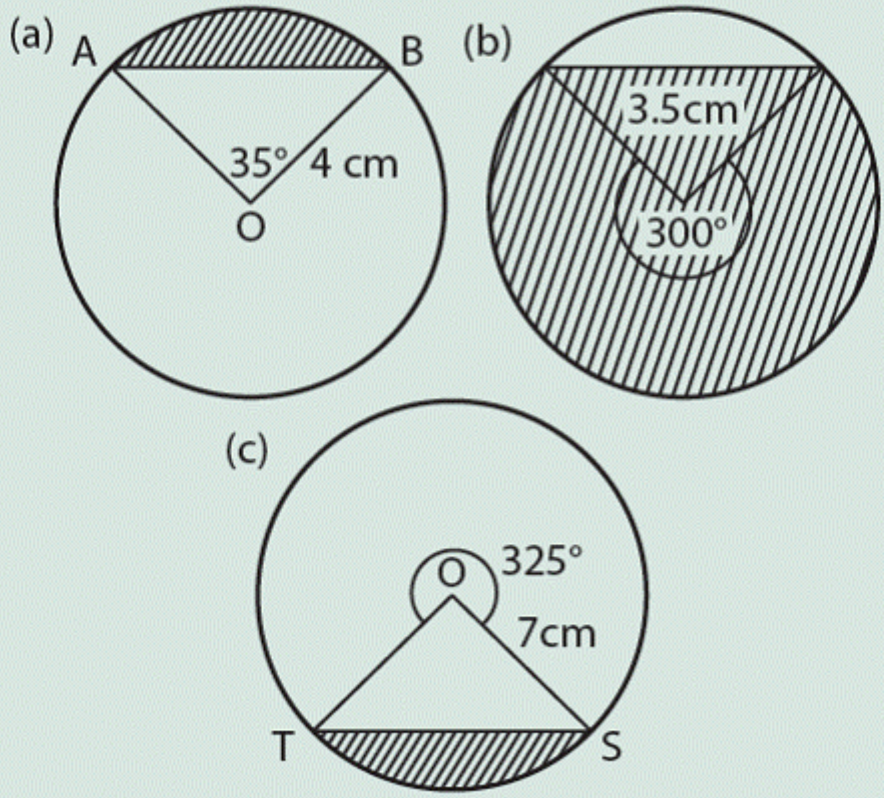



**Fig. 13.37**

22. Calculate the area of the minor segment of a circle of radius 8 cm which subtends an angle of 100° at the centre.

23. Find the area of a segment, if its chord subtends an angle of 110° at the centre of a circle of radius 15 cm.

24. Calculate, correct to 2 decimal places, the area of the major segment of a circle of radius 6.5 cm, if the minor segment subtends an angle of 75° at the centre.

25. Find the radii, correct to 2 decimal places, of the segments, if the area of the segments and the angles their chord subtends at the centre are given as shown in Fig. 13.38.

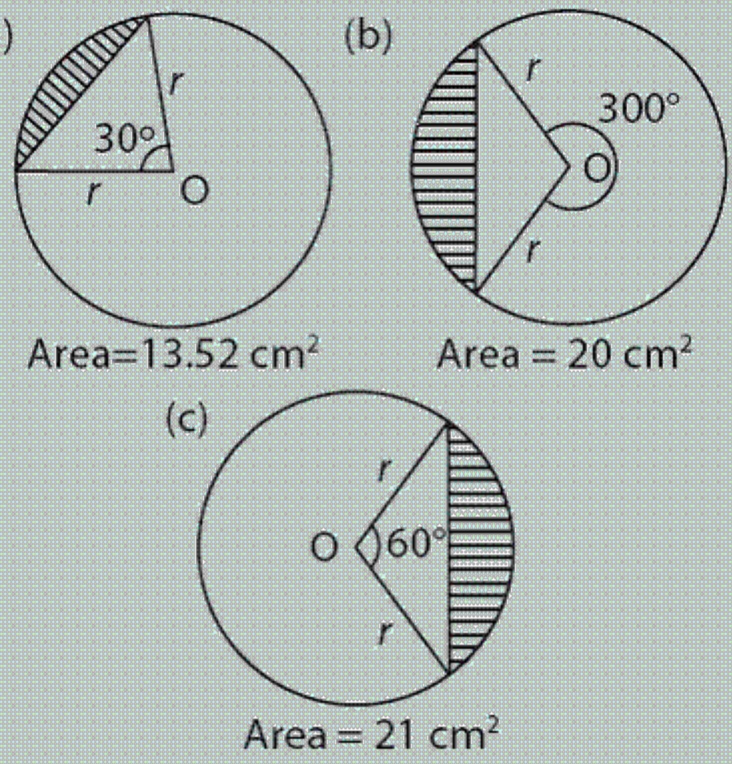



Fig. 13.38

$\theta = 138°$ (to the nearest degree)

# III. Surface Area, Volume and Capacity of Cubes, Cuboids, Cylinders, Cones, Prisms, Pyramids

## (i) Cube

Briefly recall the properties of a cube:

➤ 12 equal edges

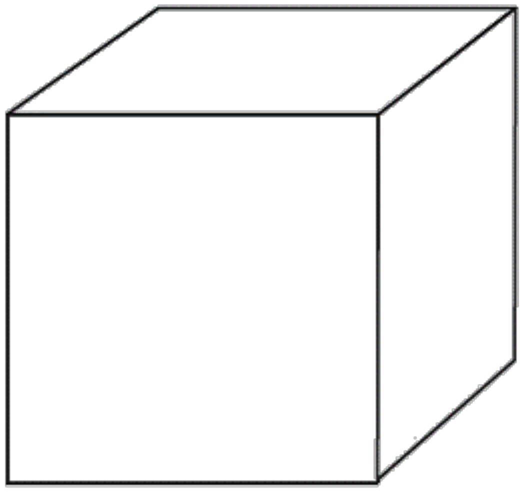
➤ 8 vertices

➤ 6 equal square faces

Examples of cubes are sugar cube, magi cube, etc.

## (ii) Area of a cube

Recall that area is the measurement of the amount of surface covered by a plane shape. It is measured in square units.



Cube

**Fig. 13.39**

$$\begin{aligned}\text{Total surface area} &= (a \times a) + (a \times a) \\ &\quad + (a \times a) + (a \times a) \\ &\quad + (a \times a) + (a \times a) \\ &= a^2 + a^2 + a^2 + a^2 \\ &\quad + a^2 + a^2 \\ &= 6\, a^2 \text{ square units}\end{aligned}$$

**Note:** The total surface area of any cube is 6 times the area of a square face.

## (iii) Volume of a cube

Recall that volume measures how much space a given solid occupies. The unit of measurement is CUBIC UNITS. Using the cube in Fig. 13.39 whose single side is "*a*", Volume ($V$) = length × width × height

$= a \times a \times a$

$= a^2 \times a$ (area × height)

$= a^3$ cubic units

## (iv) Capacity of a cube

Recall that capacity is the measurement of the amount of liquid or gas a given solid can contain. The unit of measurement is litres. One litre is equal to the capacity of a cubical container with side 10 cm.

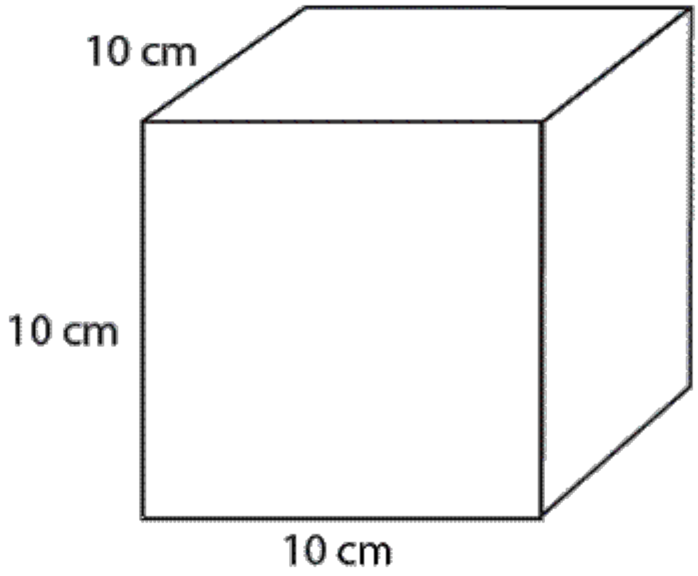



**Fig. 13.40**

Volume = 10 × 10 × 10
= 100 × 10
= 1 000 $cm^3$
= 1 litre

## Worked Example 25

Calculate the
(a) Total surface areas (b) Volumes (c) Capacities of cubes of length
(1) 15 cm (2) 12.5 cm (3) 8.5 cm (4) 3 m (5) 2 m

## Solution

1. (a) T.S.A. $= 6a^2 = 6 \times 15^2 = 1\,350\ \text{cm}^2$

(b) Volume $= a^3 = 15 \times 15 \times 15$

$= 3\,375\ \text{cm}^3$

(c) Capacity $= \dfrac{\text{Volume}}{1\,000} = \dfrac{a^3}{1\,000}$

$= \dfrac{15 \times 15 \times 15}{1\,000} = \dfrac{3\,375}{1\,000}$

$= 3.375$

$= 3.38$ litres (2 d.p)

2. (a) T.S.A. $= 6a^2 = 6 \times (12.5)^2$

$= 937.5\ \text{cm}^2$

(b) Volume $= a^3 = (12.5)^3$

$= 1\,953.125\ \text{cm}^3$

(c) Capacity $= \dfrac{\text{Volume}}{1\,000} = \dfrac{a^3}{1\,000}$

$= \dfrac{(12.5)^3}{1\,000} = \dfrac{1\,953.125}{1\,000}$

$= 1.9533125$

$= 1.95$ litres

3. (a) T.S.A. $= 6a^2 = 6 \times (8.5)^2 = 433.5 \text{ cm}^2$

(b) Volume $= a^3 = (8.5)^3 = 614.125 \text{ cm}^3$

(c) Capacity $= \dfrac{\text{Volume}}{1\,000} = \dfrac{a^3}{1\,000}$

$$= \frac{(8.5)^3}{1\,000} = \frac{614.125}{1\,000}$$

$$= 0.614125$$

$$= 0.614 \text{ litres}$$

4. (a) T.S.A. $= 6a^2 = 6 \times 3^2 = 54 \text{ m}^2$

(b) Volume $= a^3 = (3)^3 = 27 \text{ m}^3$

(c) Capacity $=$ Volume $(\text{m}^3) \times 1\,000$

$$= a^3 \times 1\,000 = 3^3 \times 1\,000$$

$$= 27\,000 \text{ litres}$$

5. (a) T.S.A. $= 6a^2$

$$= 6 \times 2^2$$

$$= 6 \times 4$$

$$= 24 \text{ cm}^2$$

(b) Volume $= a^3 = 2^3 = 8 \text{ cm}^3$

(c) Capacity $=$ Volume $(\text{m}^3) \times 1\,000$

$$= a^3 \times 1\,000 = 2^3 \times 1\,000$$

$$= 8 \times 1\,000$$

$$= 8\,000 \text{ litres}$$

## Worked Example 26

Find the length of a cube whose total surface area is $150 \text{ cm}^2$.

**Solution**

$$\text{T.S.A} = 6a^2 \text{ (formula)}$$

$$\text{T.S.A} = 150 \text{ cm}^2 \text{ (given)}$$

$$6a^2 = 150$$

$$a^2 = \frac{150}{6}$$

$$a^2 = 25$$

$$a = \sqrt{25} = 5 \text{ cm}$$

**Worked Example 27**

Find the length of a cube whose volume is 343 $cm^3$.

**Solution**

$$\text{Volume} = a^3 \text{ (formula)}$$

$$\text{Volume} = 343 \text{ cm}^3 \text{ (given)}$$

$$a^3 = 343$$

$$a = \sqrt[3]{343}$$

$$a = 7 \text{ cm}$$

## (i) Cuboid

Briefly recall the properties of a cuboid:

- 8 vertices
- 12 edges
- 3 pairs of different plane faces Examples of cuboids are chalkboard, textbook, etc.

## (ii) Area of a cuboid

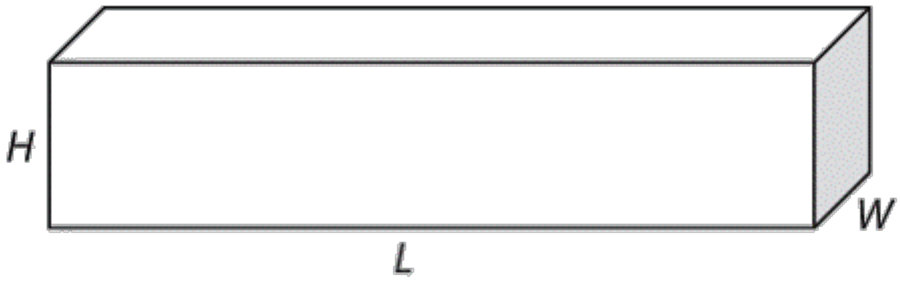



Fig. 13.41

Total surface area = $HL + HL + HW + HW$
$+ LW + LW$
$= 2\,HL + 2\,HW + 2\,LW$
$= 2(HL + HW + LW)$
unit square

## (iii) Volume of a cuboid

Volume (V) = length × width × height (cubic unit)

### Worked Example 28

Find the (a) Total surface area (b) Volume (c) Capacity of a cuboid whose
1. Length = 5 cm, width = 8 cm and height = 10 cm.
2. Length = 20 cm, width = 13 cm and height = 5 cm.
3. Length = 3 m , width = 2 m and height = 2.5 cm.

### Solution

1. (a) Total surface area
$= 2(hl + hw + lw)$
$= 2\,((10 \times 5) + (10 \times 8) + (5 \times 8))$
$= 2\,(50 + 80 + 40)$
$= 2\,(170)$
$= 340 \text{ cm}^2$

(b) Volume $= 5 \times 8 \times 10$
$= 400 \text{ cm}^3$

(c) Capacity $= \dfrac{\text{Volume}}{1\,000}$
$= \dfrac{400}{1\,000}$
$= 0.4$ litres

2. (a) Total surface area

$= 2(hl + hw + lw)$

$= 2(5 \times 20) + (5 \times 13) + (20 \times 13)$

$= 2\ (100 + 65 + 260)$

$= 2\ (425)$

$= 850\ \text{cm}^2$

(b) Volume $= l \times w \times h$

$= 20 \times 13 \times 5$

$= 1\ 300\ \text{cm}^3$

(c) Capacity $= \dfrac{\text{Volume}}{1\ 000}$

$= \dfrac{1\ 300}{1\ 000}$

$= 1.3$ litres

3. (a) Total surface area $= 2(hl + hw + lw)$

$= 2\ ((2.5 \times 3) + (2.5 \times 2) + (3 \times 2))$

$= 2\ ((7.5) + (5.0) + (6))$

$= 2\ (18.5)$

$= 37\ \text{cm}^2$

(b) Volume $= l \times w \times h$

$= 3 \times 2 \times 2.5$

$= 15\ \text{m}^3$

(c) Capacity $=$ Volume $\times\ 1\ 000$

$= 15 \times 1\ 000$

$= 15\ 000$ litres

## Worked Example 29

Calculate the height of a cuboid whose total surface area is 92 $cm^2$, width = 2 cm
and length = 8 cm.

### Solution

$$T.S.A = 2\,(hl + hw + lw) = \text{area}$$

$$92 = 2\,((h \times 8) + (h \times 2) + (8 \times 2))$$

$$92 = 2\,(8h + 2h + 16)$$

$$92 = 2\,(10h + 16)$$

$$92 = 20h + 32$$

$$92 - 32 = 20h$$

$$60 = 20h$$

$$60 \div 20 = h$$

$$3 = h$$

$$h = 3 \text{ cm}$$

Therefore, $h = 3$ cm

## Worked Example 30

Find the length of a cuboid whose volume is 400 $cm^3$, width = 5 cm and height
= 8 cm.

### Solution

$$l \times w \times h = \text{volume}$$

$$l \times 5 \times 8 = 400 \text{ cm}^3$$

$$40\,l = 400$$

$$l = \frac{400}{40}$$

Therefore, $l = 10$ cm

## (i) Cylinder

Briefly recall the properties of a cylinder:

- 1 curved face and 2 flat circular faces
- 2 curved edges
- no vertex

Examples of cylinders are tin milk, pipe, rod, log of wood, etc.

## (ii) Area of a cylinder

**Activity 13.4**

*Open up a cylindrical object to produce a rectangular shape and two circular ends. The height of the cylinder becomes the length, while the circumference of the cross-section becomes the width of the rectangle as shown in Fig. 13.42.*

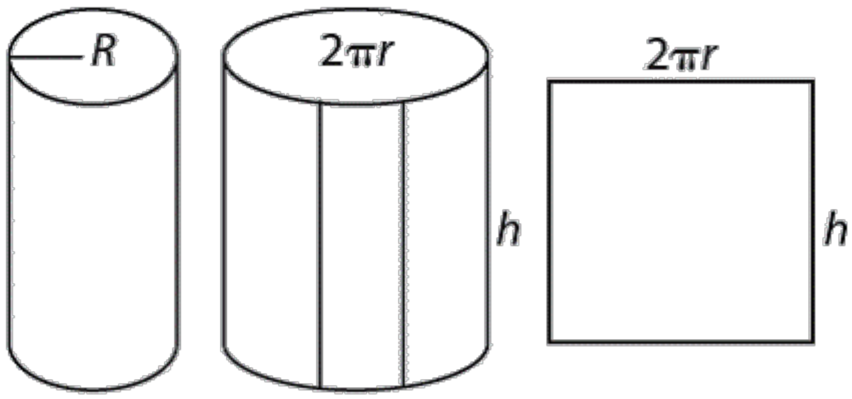



**Fig. 13.42**

Curved surface area $= 2\pi r$ square unit

Total surface area (when only one end is closed)

$= $ Curved surface area $+$ area of the closed end

$= 2\pi r h + \pi r^2$

$= \pi r (2h + r)$ square unit

Total surface area (when both ends are closed)

$= $ Curved surface area $+$ area of both ends

$= 2\pi r h + 2\pi r^2$

$= 2\pi r(h + r)$ square unit

### (iii) Volume of a cylinder

Volume (V) = base area × perpendicular height

$= \pi r^2 h$ cubic unit

## Worked Example 31

Given a cylinder whose radius is 10 cm and perpendicular height is 25 cm; calculate its:

(a) Curved surface area.
(b) Total surface area when one side is open.
(c) Total surface area when both sides are closed.
(d) Volume.
(e) Capacity in litres.

## Solution

(a) Curved surface area $= 2\pi r h$

$$= 2 \times \frac{22}{7} \times 10 \times 25$$

$$= \frac{11\,000}{7}$$

$$= 1\,571.4286$$

$$= 1\,571.43 \text{ cm}^2 \text{ (2 d.p.)}$$

(b) Total surface area when one side is open $= 2\pi r h + \pi r^2$

$$= \left(2 \times \frac{22}{7} \times 10 \times 25\right) + \left(\frac{22}{7} \times 10 \times 10\right)$$

$$= \frac{11\,000}{7} + \frac{2\,200}{7}$$

$$= \frac{13\,200}{7}$$

$$= 1\,885.7143$$

$$= 1\,885.71 \text{ cm}^2 \text{ (2 d.p.)}$$

(c) Total surface area when both sides are closed $= 2\pi r h + 2\pi r^2$

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 10\ (25 + 10)$$

$$= \frac{440}{7} \times 35$$

$$= \frac{15\,400}{7}$$

$$= 2\,200 \text{ cm}^2$$

(d) Volume $= \pi r^2 h$

$$= \frac{22}{7} \times 10 \times 10 \times 25$$

$$= \frac{55\,000}{7}$$

$$= 7\,857.1429$$

$$= 7\,857.14 \text{ cm}^2$$

(e) Capacity (litre) $= \frac{\text{Volume}}{1\,000}$

$$= \frac{7\,857.14 \text{ cm}^3}{1\,000}$$

$$= 7.8571$$

$$= 7.86 \text{ litres (2 d.p.)}$$

## Worked Example 32

Find the radius of a cylindrical object whose total surface area is 297 cm$^2$ and height = 10 cm.

## Solution

$$2\pi r(h + r) = \text{T.S.A.}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r(10 + r) = 297$$

$$\Rightarrow \quad \frac{44r}{7}(10 + r) = 297$$

$$\Rightarrow \quad 10r + r^2 = \frac{297 \times 7}{44}$$

$$\Rightarrow \quad r^2 + 10r = \frac{189}{4}$$

$$\Rightarrow \quad 4r^2 + 40r - 189 = 0$$

$$4r^2 \times -189 = -756r^2$$

Sum of $40r$ Factor of $-756r^2$

$54r - 14r = 40r$ $54r \times -14r = -756r^2$

$$4r^2 + 54r - 14r - 189 = 0$$

$$(4r^2 + 54r) - (14r + 189) = 0$$

$$2r(2r + 27) - 7(2r + 27) = 0$$

$$(2r + 27)(2r - 7) = 0$$

either $2r + 27 = 0$

$$2r = -27$$

$$r = \frac{-27}{2}$$

$$r = -13.5$$

or $2r - 7 = 0$

$$2r = 7$$

$$r = \frac{7}{2}$$

$$r = 3.5$$

The radius ($r$) must be positive therefore, radius ($r$) = 3.5 cm.

## Worked Example 33

Find, correct to the nearest whole number, the radius of a cylinder whose volume is 942.86 $cm^3$ and height = 12cm.

### Solution

$$\pi r^2 h = \text{volume}$$

$$\frac{22}{7} \times r^2 \times 12 = 942.86$$

$$r^2 = \frac{942.86 \times 7}{22 \times 12}$$

$$r^2 = \frac{6600.02}{264}$$

$$r^2 = 25.0000$$

$$r^2 = \sqrt{25} \text{ therefore, } r = 5 \text{ cm.}$$

## (i) Cone

Briefly recall the properties of a cone:

➤ 1 vertex

➤ 1 curved edge

➤ 2 faces: 1 curved surface area and 1 circular base area

➤ the line segment VO joining the vertex (V) to the centre O of the circular base is called the PERPENDICULAR HEIGHT (h) of the cone. See Fig. 13.43.

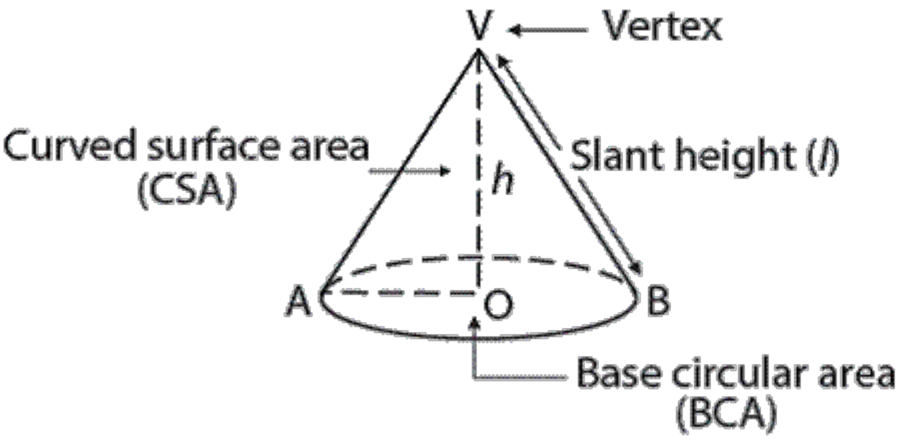



**Fig. 13.43**

➤ angle AVB of the cone is called the *vertical angle* of the cone, while angle AVO and angle BVO are called the *semi-vertical angles* of the cone (see Fig.

13.43).

### (ii) Area of a cone

We have previously established in this chapter, the fact that the area of the curved surface of a cone of slant height ($l$) is equal to the area of a sector of a circle that subtends angle $\theta$ at the centre with radius ($r$) and arc length $2\pi r$.

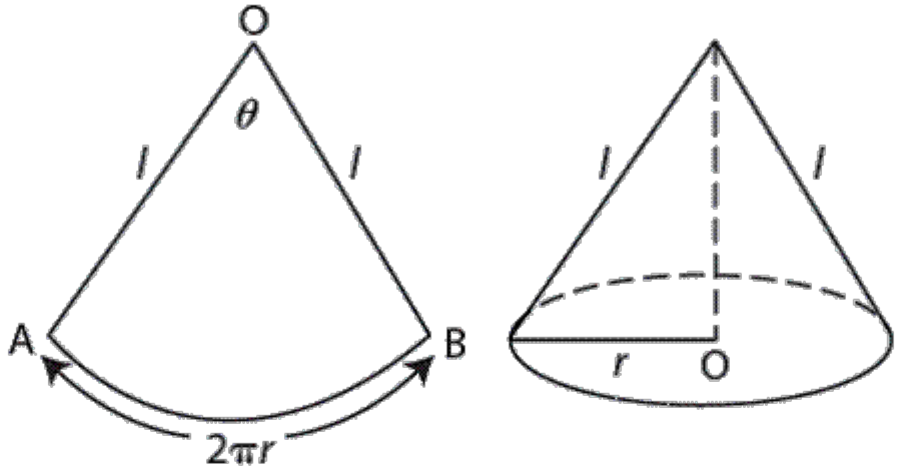



Fig. 13.44

Using Fig. 13.44, recall that the area of sector

$$AOB = \frac{\theta}{360} \times \pi l^2 \text{ (when } \theta \text{ is given)}$$

$$= \frac{1}{2} \times \text{arc length} \times l \text{ (when the arc length is given)}$$

$$= \frac{1}{2} \times 2\pi r \times l \text{ (Remember } 2\pi r$$

$$= \text{arc length of the sector)}$$

$$= \text{circumference of the circular base of the cone)} = \pi r l$$

Therefore, the *curved surface area of the cone* = $\pi r l$ square unit (where $r$ is the radius of the circular base and $l$ is the slant height).

**Total surface area** = Curved surface area + Base circular area

$$= \pi r l + \pi r^2$$

$$= \pi r(l + r) \text{ square unit}$$

### (iii) Volume of a cone

Given a cone of the same perpendicular height ($h$) and radius ($r$) with a cylinder, it will take the filled up cone three times (3x) to fill the cylinder to the brim when water or sand from the cone is poured into the cylinder as shown in Fig. 13.45.

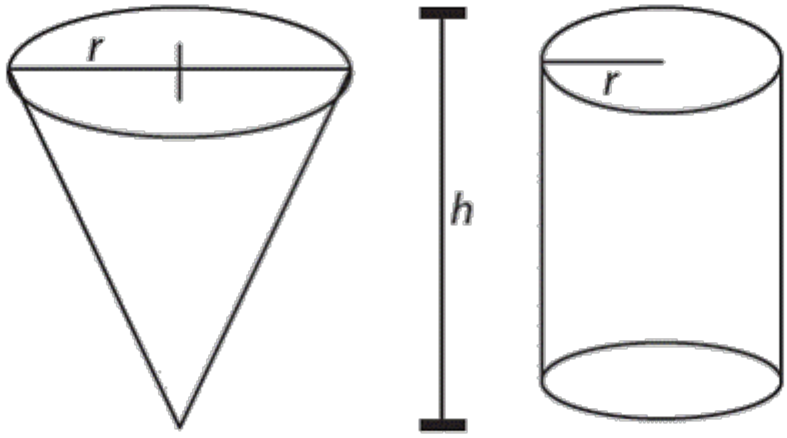



**Fig. 13.45**

With this practically done, we conclude that the volume ($V$) of a cone is one-third
of the volume of a cylinder whose height and radius is the same.
Now, since the volume of a cylinder is $\pi r^2 h$. Then the volume of a cone ($v$) = 1/3
($\pi r^2 h$).

## Worked Example 34

A sector of a circle of radius 3.5 cm that subtends an angle of 120° at the centre is folded to form a cone. Using $\pi = \frac{22}{7}$
Calculate the:
(a) Radius of the base of the cone.
(b) Circumference of the base of the cone.
(c) Perpendicular height of the cone.
(d) Vertical angle of the cone.
(e) Semi-vertical angle of the cone.
(f) Curved surface area of the cone if closed.
(g) Total surface area of the cone.
(h) Volume of the cone.
(i) Capacity of the cone in litres.

## Solution

(a) Radius of the base of the cone ($r$)

$$= \frac{\theta}{360} \times l \text{ (slant height)}$$

$$r = \frac{120}{360} \times 3.5$$

$$r = 1.1667$$

$$r = 1.17 \text{ cm (2 d.p.)}$$

(b) Circumference of the base of the cone

$$\text{Cir} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 1.1667$$

$$= 7.3335$$

$$= 7.33 \text{ cm (2 d.p.)}$$

(c) Perpendicular height of the cone

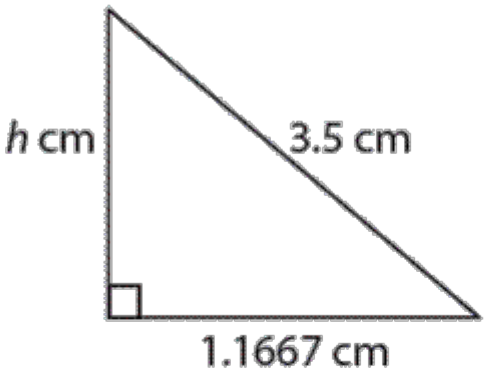



**Fig. 13.46**

$$h^2 = (3.5)^2 - (1.1667)^2$$

$$h^2 = 12.25 - 1.3612$$

$$h^2 = 10.8888$$

$$h = \sqrt{10.8888}$$

$$h = 3.2998$$

$$h = 3.30 \text{ (2 d.p.)}$$

(d) Vertical angle of the cone

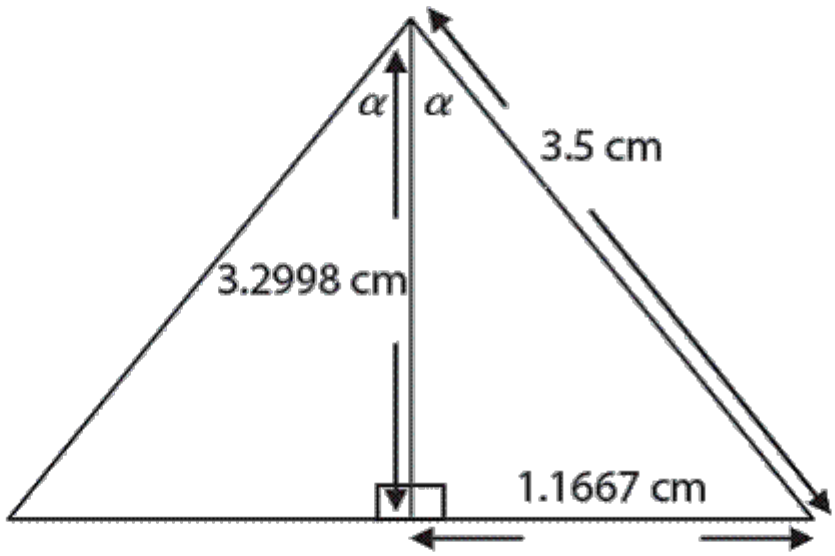



**Fig. 13.47**

$$\sin \alpha = \frac{1.1667}{3.5}$$

$$= 0.3333$$

$$\alpha = \sin^{-1} 0.3333$$

$$\alpha = 19.4692°$$

Therefore, $2\alpha = 2 \times 19.4692°$

$$2\alpha = 38.9384$$

$$= 38.94°$$

(e) Semi-vertical angle of the cone

$$\frac{38.94}{2} = 19.4692$$
$$= 19.47° \text{ (2 d.p.)}$$

(f) Curved surface area of the cone

$$\text{C.S.A} = \pi r l$$
$$= \frac{22}{7} \times 1.1667 \times 3.5$$
$$= 12.8337$$
$$= 12.83 \text{ cm}^2 \text{ (2 d.p.)}$$

(g) Total surface area of the cone

$$\text{T.S.A} = \pi r l \times \pi r^2$$
$$= \left(\frac{22}{7} \times 1.1667 \times 3.5\right)$$
$$+ \frac{22}{7} \times (1.1667)^2$$
$$= 12.8337 + 4.2780$$
$$= 17.1117$$
$$= 17.11 \text{ cm}^2 \text{ (2 d.p.)}$$

(h) Volume of the cone

$$V = \pi r^2 h$$
$$V = \frac{22}{7} \times 1.1667 \times 1.1667 \times 3.2998$$
$$V = 14.1166$$
$$V = 14.12 \text{ cm}^3$$

(i) Capacity of the cone of Litre

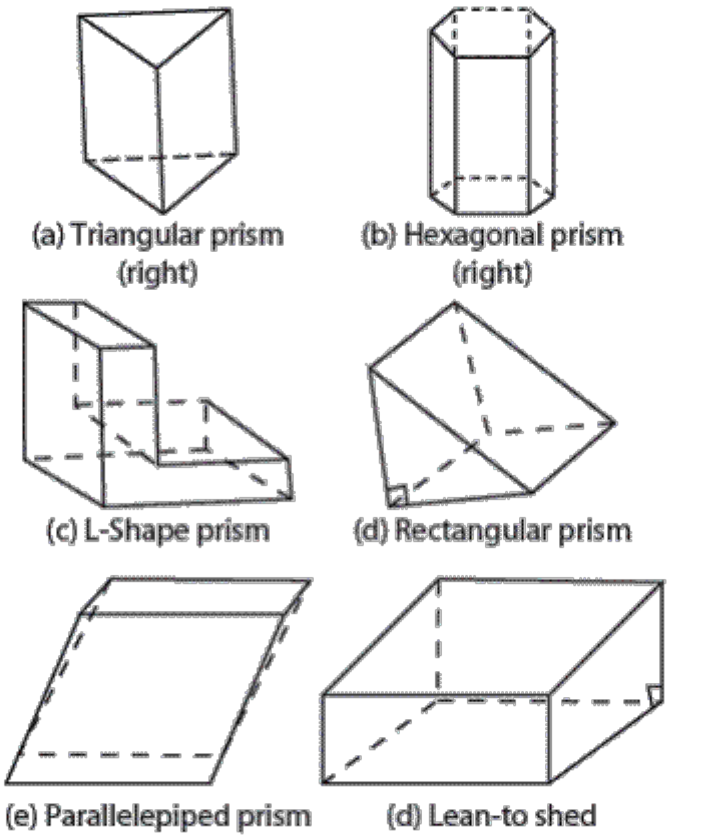
$$\text{Capacity (litre)} = \frac{Vcm^3}{1\,000}$$

$$= \frac{14.12}{1\,000}$$

$$= 0.014 \text{ litres}$$

**Prism**

A uniform cross-sectional solid that is either a triangle, quadrilateral, or any polygon is called a *prism*. A right prism has all its vertical edges perpendicular to the plane of its base (cross section). The name given to a prism is derived from the shape of the base and the top face which is always the same. Some examples of prisms are rectangular prisms (cubes and cuboids), cylinders which have already been treated, while others yet to be treated are triangular prisms, hexagonal, septahedron (5 sides), E or H or L shaped prisms, parallelepiped, etc. See Fig. 13.48.



(a) Triangular prism (right)

(b) Hexagonal prism (right)

(c) L-Shape prism

(d) Rectangular prism

(e) Parallelepiped prism (not right)

(d) Lean-to shed (It has a trapezium cross section)

**Fig. 13.48**

**Surface area of prisms**

The total surface area (sum of the area of the surfaces) of a prism has no given rule. In other words, no singular formula exists for solving its total surface area.

**Volume of prisms**

Volume of a right prism is the product of the base area (cross section) and its height (depth).

## Worked Example 35

Find the (a) total surface area of a scalene triangular prism of height 10 cm and

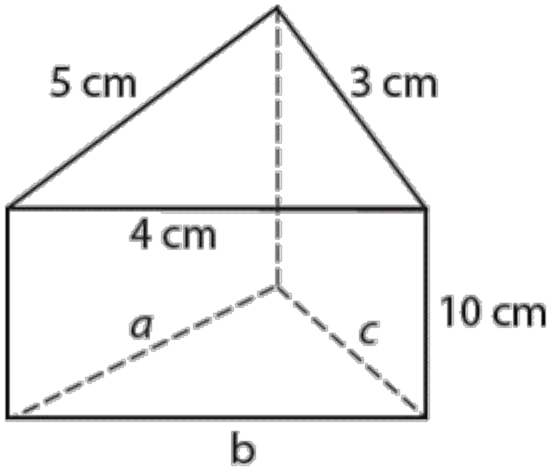



**Fig. 13.49**

base 3 cm, 4 cm and 5 cm respectively (b) volume.

**Solution**

(a) The height of the prism ($h$) = 10 cm.
Therefore, the area of the three rectangular faces will be
$(3 \times 10) + (4 \times 10) + (5 \times 10)$
$= 30 + 40 + 50$
$= 120 \text{ cm}^2$

$$\text{The area of a base} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{3+4+5}{2} = \frac{12}{2} = 6 \text{ cm}$$

Therefore, the area of the two bases

$$= 2\sqrt{6(6-3)(6-4)(6-5)}$$

$$= 2\sqrt{6 \times 3 \times 2 \times 1}$$

$$= 2\sqrt{36}$$

$$= 2 \times 6$$

$$= 12 \text{ cm}^2$$

Therefore, the total surface area of the prism = the sum of the area of the three rectangular faces and the area of the two bases
$= (120 + 12) \text{ cm}^2$
$= 132 \text{ cm}^2$
(b) Volume = Product of the base (cross section) area and height
$= 6 \times 10$
$= 60 \text{ cm}^3$

## Worked Example 36

A right prism of height 10 cm has a regular 5 sided base and the length of each edge of the base is 2.5 cm. calculate its
(a) Total surface area
(b) Volume

## Solution

(a)

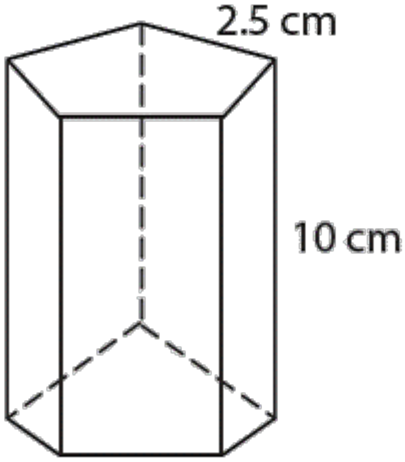



Fig. 13.50

The area of the 5 rectangular faces
= 5 (10 cm × 2.5 cm)
= 5 × 25 cm
= 125 cm2

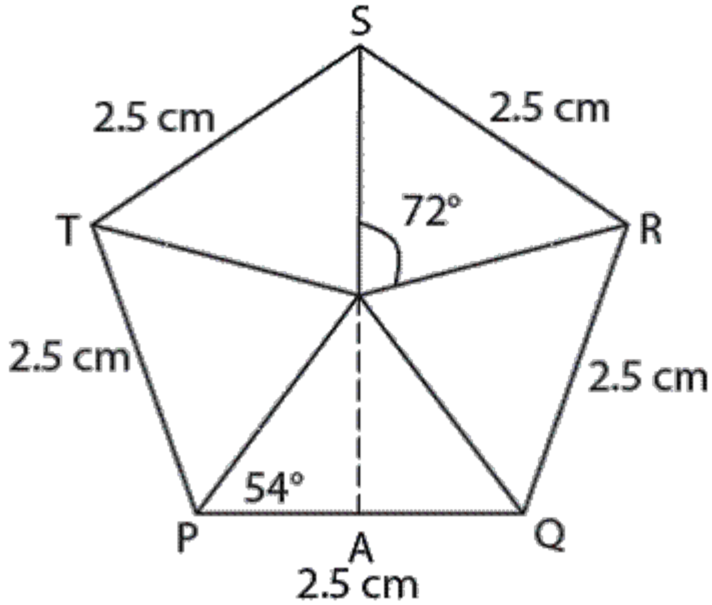



Fig. 13.51

Using the cross section in Fig. 13.51 to calculate the area of the five faces

$$P\hat{O}Q = Q\hat{O}R = R\hat{O}S = S\hat{O}T = T\hat{O}P$$

$$Q<OR = R<SO = S<TO = T<OP$$

$$= \frac{360°}{5} = 72°$$

Using any of the isosceles $\Delta$, say POQ

OA is the perpendicular bisector of PQ

$$= |PA| = \frac{1}{2}(PQ) = \frac{1}{2} \times 2.5 = 1.25 \text{ cm}$$

While OPA = 54°

$$\tan 54° = \frac{|OA|}{|PA|} = \frac{|OA|}{1.25}$$

$$|OA| = 1.25 \times \tan 54°$$

$$= 1.25 \times 1.3764$$

$$= 1.7205$$

$$\text{Area of } \Delta \text{ POQ} = \frac{1}{2}|PQ| \times |OA|$$

$$= \frac{1}{2} \times 2.5 \times 1.7205$$

$$= 2.1506$$

$$\text{Area of cross-section} = 5 \times 2.1506$$

$$= 10.753$$

Total surface area of the five-sided prism
$= 2(10.753) \text{ cm}^2 + 125 \text{ cm}^2$

$$= 21.506 + 125$$

$$= 146.506 = 146.51 \text{ cm}^2 \text{ (2dp)}$$

**Aliter:**

Given an oblique triangle as shown in Fig. 13.52

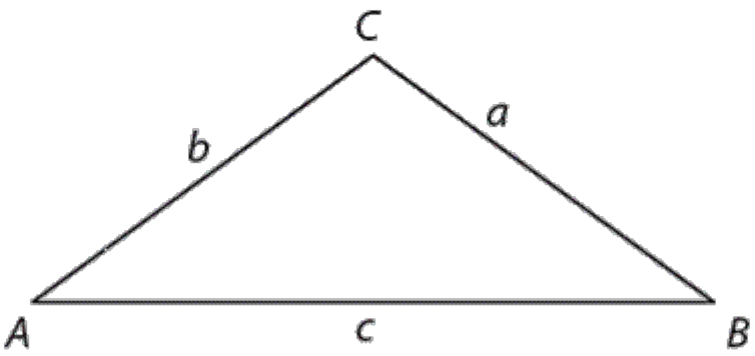



**Fig. 13.52**

The area would be

$$\frac{a^2 \sin B \sin C}{2 \sin A} = \frac{b^2 \sin A \sin C}{2 \sin B} = \frac{c^2 \sin A \sin B}{2 \sin C}$$

Therefore, the area of the Δ POQ

$$= \frac{2.5^2 \sin 54^0 \sin 54^0}{2 \times \sin 72^0}$$

$$= \frac{6.25 \times 0.8090 \times 0.8090}{2 \times 0.9511}$$

$$= \frac{4.0905}{1.9022}$$

$$= 2.1504 \text{ cm}^2$$

Area of cross-section = 5 × 2.1504 $cm^2$
= 10.752
Area of the 5 rectangular faces
= 5(10 × 2.5)
= 5 × 25
= 125 cm
Therefore, total surface area of the five sided prism
= 2 × 10.752 + 125
= 21.504 + 125
= 146.504
= 146.50 (2 d.p.)
(b) Volume = product of the area and the height
= 21.504 $cm^2$ × 10 cm
= 215.04
= 215 $cm_3$ (3 s.f )

**Worked Example 37**

Calculate the (a) total surface area (b) volume of the prism shown in Fig. 13.53.

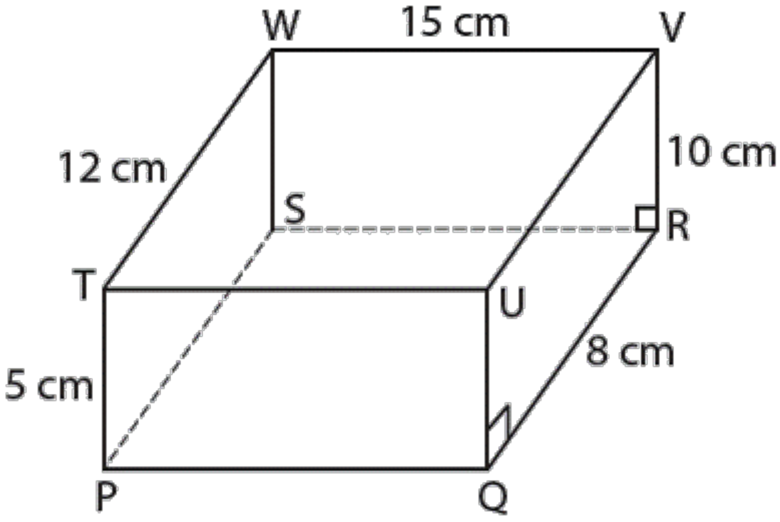



**Fig. 13.53**

(a). Area of rectangle PQUT = $5 \times 15$

$= 75 \text{ cm}^2$

Area of rectangle PQRS = $15 \times 8$

$= 120 \text{ cm}^2$

Area of rectangle SRVW = $15 \times 10$

$= 150 \text{ cm}^2$

Area of rectangle QRVU

$= 10 \times 8 = 80 \text{ cm}^2$

Area of rectangle TPSW

$= 12 \times 5 = 60 \text{ cm}^2$

Therefore, total surface area of the prism

$= (75 + 120 + 150 + 80 + 60) \text{ cm}^2$

$= 485 \text{ cm}^2$

(b) Volume = Product of the base (cross section) area and the height = $60 \times 15 =$

$900 \text{ cm}^3$

**Pyramid**

It is a polyhedron with one base and the same number of triangular faces as there are sides of the base. Precisely, the shape of its base determines the number of its triangular faces. The names of pyramids come from the shape of their base faces. Some of them are:

(i) Triangular-based pyramid

(ii) Square-based pyramid

(iii) Rectangular-based pyramid

(iv) Hexagonal-based pyramid

The figure below is a rectangular-based pyramid

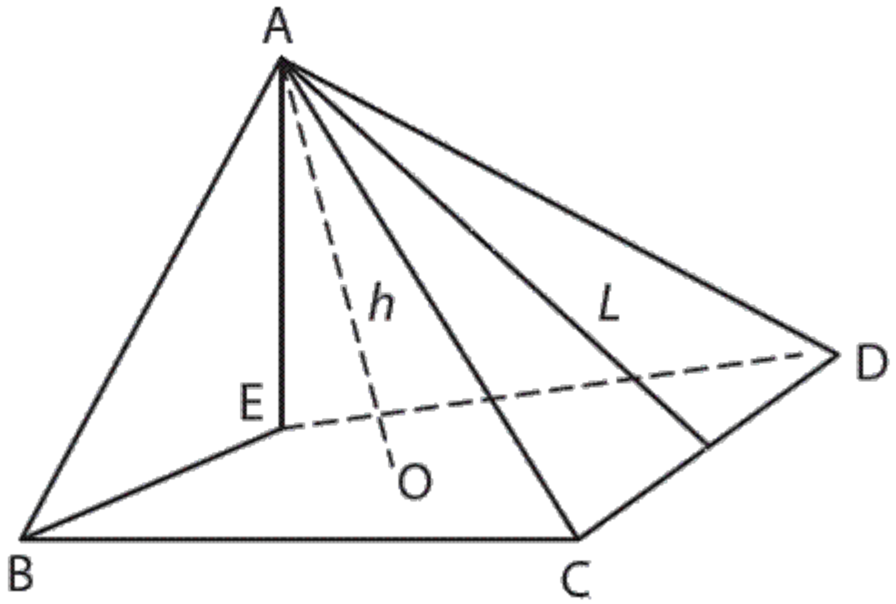



**Fig. 13.54**

A skeletal view of a rectangular-based pyramid is illustrated in Fig. 13.54. $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, and $\overline{AE}$ are the slanting edges because these are the edges that join the adjacent triangular faces. The distance from the vertex "A" to the centre "O" of the base is called the perpendicular height ($h$). See Fig. 13.54. Every slanting triangular face of the pyramid has a slanting height ($l$) which is used to calculate its area. See Fig. 13.54.

**Note:** A pyramid that has a base which is bounded by a regular polygon and slanting edges of equal length is called a *regular pyramid*. Not all pyramids have
regular bases and equal slanting edges. If the slanting edges of a pyramid are equal in length, it is called a *right pyramid*. In other words, a right pyramid is a
pyramid that has its vertex directly over the centroid of its base. See Fig. 13.54.

**Total surface area of a pyramid** = sum of the area of the base of the pyramid and the area of its triangular faces.

**Volume of a pyramid** $= \frac{1}{3} \times$ base area $\times$ perpendicular height

## Worked Example 38

Given a right pyramid whose perpendicular height is 15cm and a rectangular base of 8 cm by 12 cm. Calculate its.

(a) Total surface area

(b) Volume

## Solution

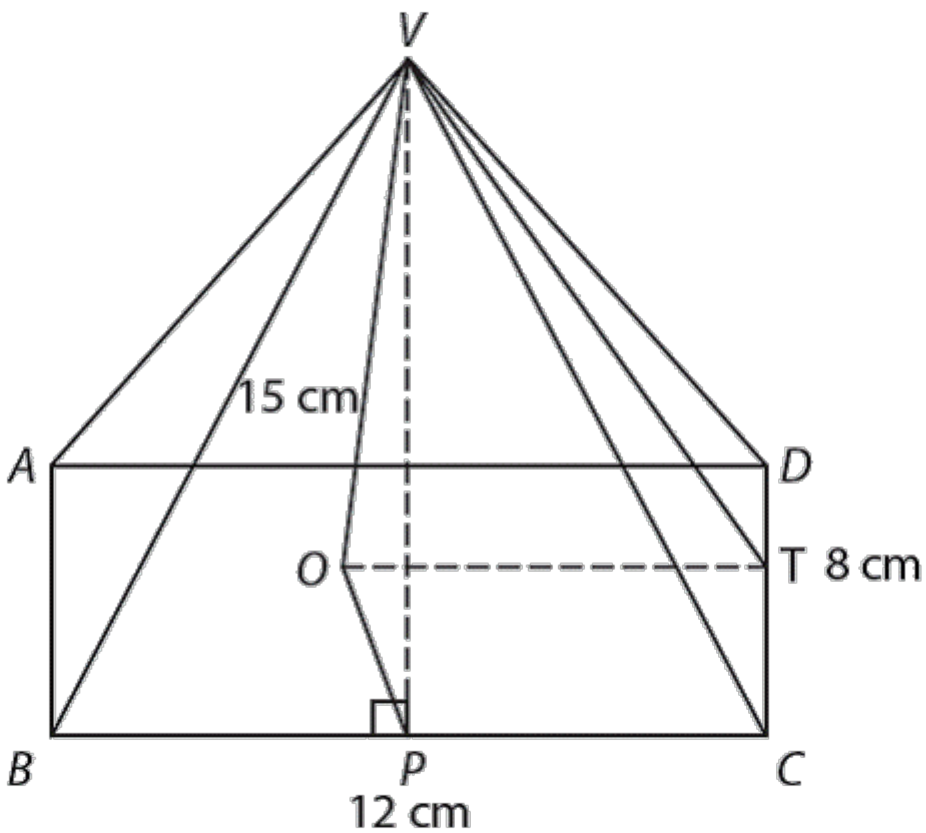



**Fig. 13.55**

(a) $|VP|$ is the slanting height of $\Delta$ VBC

$|PO| = 8 \text{ cm} \div 2 = 4 \text{ cm}$

$$\begin{aligned} \text{Therefore, } |VP|^2 &= |VO|^2 + |PO|^2 \\ &= 15^2 + 4^2 \\ &= 225 + 16 \\ &= 241 \end{aligned}$$

$$\begin{aligned} |VP| &= \sqrt{241} \\ &= 15.52 \text{ cm} \end{aligned}$$

Area of triangular face

$$VBC = \frac{12 \times 15.52}{2} = 93.12 \text{ cm}^2$$

Sum of the areas of $\Delta VBC$ and $\Delta VAD$

$$= 2 \times 93.12$$
$$= 186.24 \text{ cm}^2$$

$|VT|$ is the slanting height of $\Delta VCD$

Therefore,

$$\begin{aligned} |VT|^2 &= |VO|^2 + |TO|^2 \\ &= 15^2 + 6^2 \\ &= 225 + 36 \\ &= 261 \\ |VT| &= \sqrt{261} \\ &= 16.16 \text{ cm} \end{aligned}$$

Area of triangular face

$$VCD = \frac{8 \times 16.16}{2} = 64.64 \text{ cm}$$

Sum of the areas of $\Delta VCD$ and $\Delta VAB$

$$= 2 \times 64.64$$
$$= 129.28 \text{ cm}^2$$

Area of the base $= 12 \times 8 = 96 \text{ cm}^2$

Total surface area of the pyramid

$$= (186.24 + 129.28 + 96) \text{ cm}^2$$
$$= 411.52 \text{ cm}^2$$

(b) Volume $= \frac{1}{3} \times$ area of the base $\times$ perpendicular height

$$= \frac{1}{3} \times (12 \times 8) \times 15$$

$$= 480 \text{ cm}^3$$

## Worked Example 39

If a right pyramid with vertex V has a rectangular base of 4.3 cm by 6.5 cm and the length of a slanting edge is 4.5 cm, calculate the:

(a) perpendicular height of the pyramid

(b) volume and

(c) total surface area

### Solution

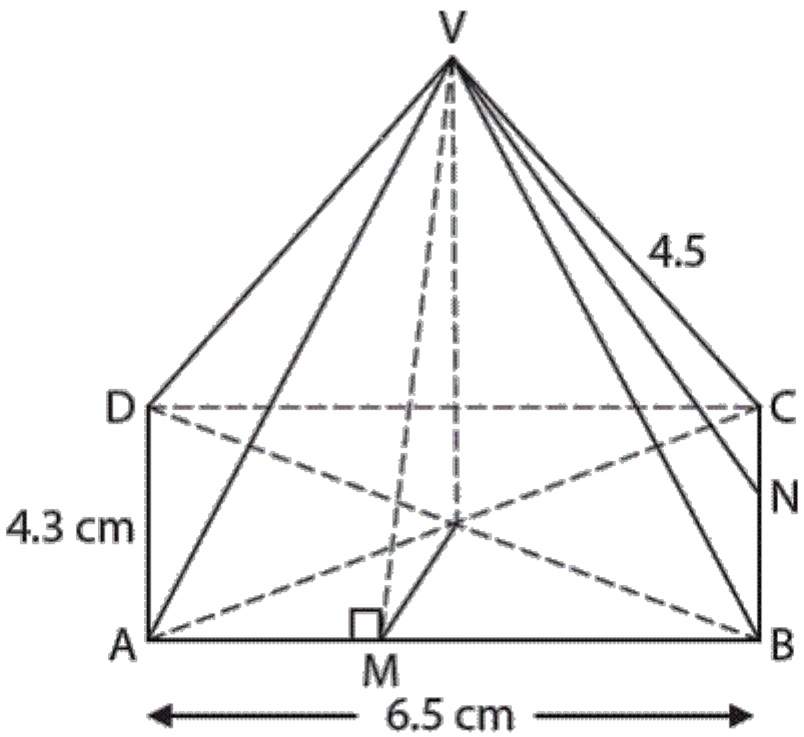



Fig. 13.56

(a) $|AC|^2 = |AB|^2 + |BC|^2$

$$= 6.5^2 + 4.3^2$$

$$= 42.25 + 18.49$$

$$= 60.74$$

$$|AC| = \sqrt{60.74}$$

$$|AC| = 7.79$$

$$|OC| = \frac{|AC|}{2}$$

$$= \frac{7.79}{2}$$

$$|OC| = 3.896$$

Hence, $|VO|^2 = |VC|^2 - |CO|^2$

$$= 4.5^2 - 3.9^2$$

$$= 20.25 - 15.21$$

$$= 5.04$$

$$|VO| = 2.2450$$

$$= 2.25 \text{ cm (2 d.p.)}$$

(b) Volume $= \frac{1}{3} \times$ base area $\times$ perpedicular height

$$= \frac{1}{3} \times 6.5 \times 4.3 \times 2.2450$$

$$= 20.9159$$

$$= 20.92 \text{ cm}^3$$

(iii) Since N is the midpoint of |BC|, <VNC = 90° and |VB| = |VC|. Therefore, $|VN|^2 = |VC|^2 - |NC|^2$

$$= 4.5^2 - \left(\frac{4.3}{2}\right)^2$$

$$= 4.5^2 - 2.15^2$$

$$= 20.25 - 4.62$$

$$= 15.63$$

$$|VN| = \sqrt{15.63}$$

$$= 3.95$$

Therefore, area of $\Delta VBC = \frac{1}{2} \times 4.3 \times 3.95$

$$= 8.4925 \text{ cm}^2$$

Since M is the midpoint of |AB|, <VMB = 90° and |VA| = |VB| Therfore, $|VM|^2 = |MO|^2 + |VO|^2$

$$= \left(\frac{4.3}{2}\right)^2 + 2.2450^2$$

$$= 2.15^2 + 2.2450^2$$

$$= 4.6225 + 5.040$$

$$= 9.6625$$

$$|VM| = \sqrt{9.6625}$$

$$= 3.11 \text{ cm}$$

Therefore, area of $\Delta VAB = \frac{1}{2} \times 6.5 \times 3.11$

$$= 10.11 \text{ cm}^2$$

Therefore, area of the four faces of the pyramid

$$= 2(8.4925 + 10.11)$$

$$= 37.205 \text{ cm}^2$$

Therefore, area of the base ABCD

$$= 6.5 \times 4.3$$

$$= 27.95 \text{ cm}^2$$

Therefore, total surface area

$$= 37.205 + 27.95$$

$$= 65.155$$

$$= 65.16 \text{ cm}^2 \text{ (2 d.p.)}$$

## Exercise 3

1. Copy and complete the table below of a given right pyramid.

| Sl.no | Height | Base | Volume |
|---|---|---|---|
| (a) | 12 cm | Rectangular base 5 cm by 7 cm | ? |
| (b) | 8 cm | Square base of side ? | 24 $cm^3$ |
| (c) | ? | Triangular base 13 cm by 12 cm by 5 cm | 80 $cm^3$ |

2. Given a right pyramid with a slant edge 24 cm and a rectangular base 10 cm by 6 cm. Find the (a) total surface area (b) volume

3. A cone and a right pyramid have equal heights and volumes. If the area of the base of the pyramid is 154 $cm^2$, find the base radius of the cone.
   $\left(\text{Take } \pi = \frac{22}{7}\right)$ **(WASSCE)**

4. A pyramid has a rectangular base with dimension 13 cm by 8 cm and a perpendicular height of 10 cm. Find
   (a) its volume and
   (b) total surface area.
5. OPQRS is a pyramid on a square base PQRS of side 10 cm, If $|OP| = |OQ| = |QR| = |SO| = 16$ cm, find the
   (a) volume.
   (b) total surface area.

6. The base ABCD of a right-pyramid of vertex V is a rectangle 8.4 cm by 5.5 cm and the length of a slant edge is 6.9 cm, find
   (a) the volume and
   (b) the total surface area of the pyramid.
7. Find
   (a) the total surface area.
   (b) the volume of a right pyramid of slant height 10 cm whose rectangular base is 3 cm by 5 cm.

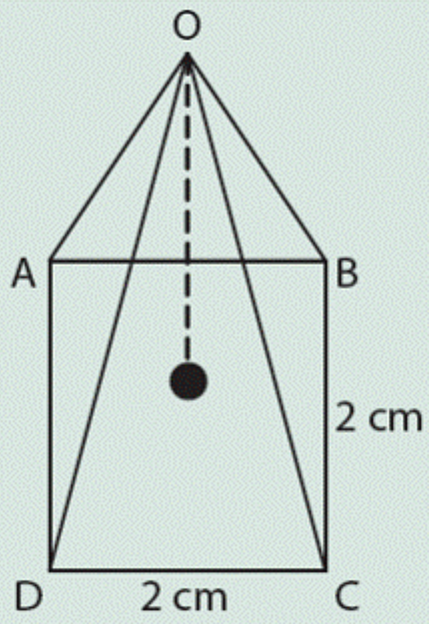


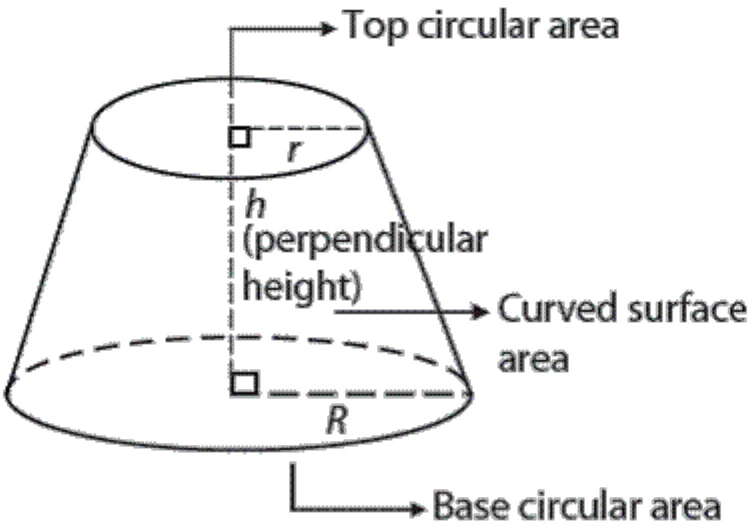

**Fig. 13.57**

8. In the diagram, OABCD is a pyramid with a square base of 2 cm and a slant height of 4 cm. Calculate, correct to three significant figures,
   (a) the vertical height of the pyramid.
   (b) the volume of the pyramid.
9. A pyramid stands on a rectangular base with sides of length 15 cm and 7 cm. If the length of each slant edge is 12 cm, find the
   (a) volume.
   (b) total surface area.

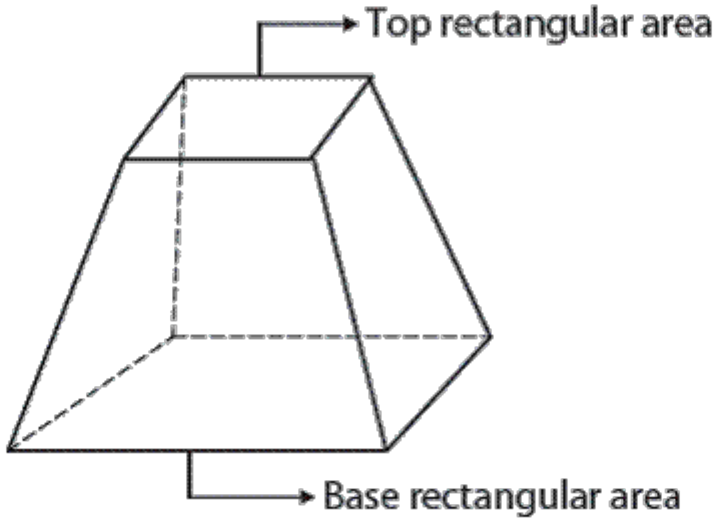
10. Given a solid square-based pyramid of side 10 m and slant edge length of 13 m. Calculate the
    (a) volume.
    (b) density, if the weight is 10 kg.
    (c) total surface area.
    (d) cost of filling the pyramid, if a square metre costs ₦15.00.

## IV. Surface Area and Volume of a Fraction (Frustum) of a Cone and Pyramid

A frustum of a cone or pyramid is a cone or pyramid whose fraction top part has been cut off by a plane which is parallel to the base as shown in Fig. 13.58.




Frustum of a cone




Frustum of a pyramid

**Fig. 13.58**

## Worked Example 40

A frustum of a cone has top and bottom radii of 16 cm and 24 cm respectively, and a perpendicular height of 26 cm. Calculate its

(a) Curved surface area (C.S.A)
(b) Total surface area (T.S.A)
(c) Volume
(d) Capacity (litres)

### Solution

Figure 13.59a is the frustum while Fig. 13.59b is the frustum reproduced and completed into a full cone labeled as follows:

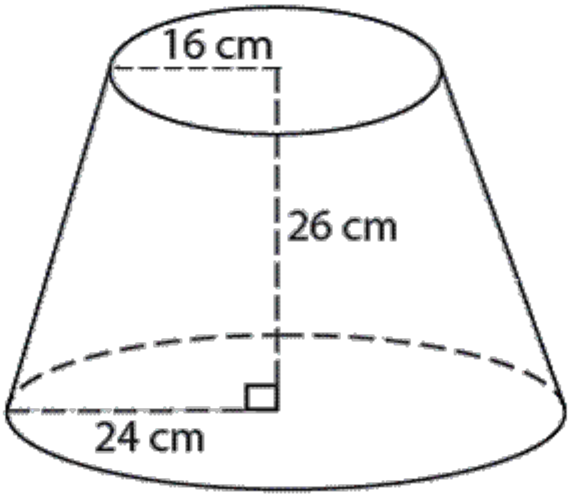



**Fig. 13.59a**

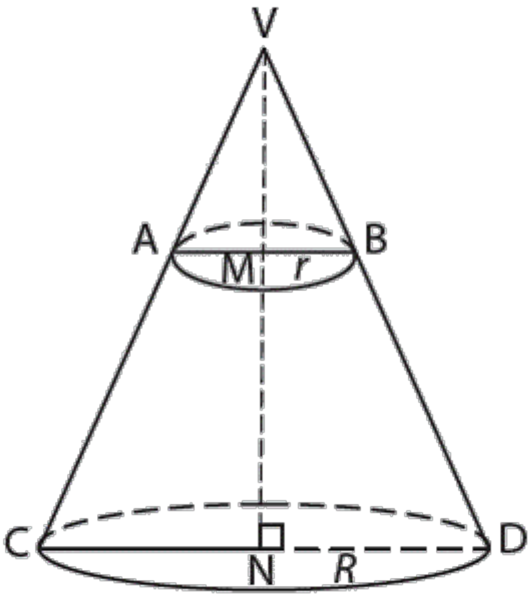



**Fig. 13.59b**

Using similar triangle rule:

$$\frac{VM}{MB} = \frac{VM + MN}{ND}$$

$$\Rightarrow \quad \frac{VM}{16} = \frac{VM + 26}{24}$$

$$\Rightarrow \quad 24\,VM = 16(VM + 26)$$

$$\Rightarrow \quad 24\,VM = 16VM + 416$$

$$\Rightarrow \quad 24VM - 16VM = 416$$

$$\Rightarrow \quad 8VM = 416$$

$$\Rightarrow \quad VM = \frac{416}{8}$$

Therefore, VM = 52 cm

Perpendicular height of the cone VAB = 52 cm

Perpendicular height of the bigger cone VCD = 52 + 26 = 78 cm

using Pythagoras theorem:

$$
\begin{aligned}
VB^2 &= VM^2 + MB^2 \\
&= 52^2 + 16^2 \\
&= 2\,704 + 256 \\
&= 2\,960 \\
VB &= \sqrt{2\,960} \\
&= 54.41
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
VD^2 &= VN^2 + ND^2 \\
&= 78^2 + 24^2 \\
&= 6\,084 + 576 \\
&= 6\,660 \\
VD &= \sqrt{6\,660} \\
&= 81.61
\end{aligned}
$$

(a) Curved surface area of the frustum

$= \text{C.S.A of cone VCD} - \text{C.S.A of cone VAB}$

$= \left(\frac{22}{7} \times 24 \times 81.61\right)$

$- \left(\frac{22}{7} \times 16 \times 51.41\right)$

(a) Curved surface area of the frustum

$= \text{C.S.A of cone VCD} - \text{C.S.A of cone VAB}$

$$= \left(\frac{22}{7} \times 24 \times 81.61\right) - \left(\frac{22}{7} \times 16 \times 51.41\right)$$

**Aliter**

C.S.A of the frustum $= \pi l (r + R)$

$l$ = slant height of the frustum

$r$ = radius of the top circular area

$R$ = radius of the base circular area

$$\text{C.S.A (frustum)} = \frac{22}{7} \times 27.2\,(16 + 24)$$

$$= \frac{22 \times 27.2 \times 40}{7}$$

$$= \frac{23\,936.0}{7}$$

$$= 3\,419.43 \text{ cm}^2 \text{ (2 d.p.)}$$

(b) But the top circular area of the frustum

$$\pi r^2 = \frac{22}{7} \times 16 \times 16$$

$$= 804.57 \text{ cm}^2$$

and the base circular area of the frustum

$$\pi R^2 = \frac{22}{7} \times 24 \times 24$$

$$= 1\,810.29 \text{ cm}^2$$

Therefore T.S.A (Frustum) = (Top circular + base circular + curved surface area)

$$= 804.57 + 1\,810.29 + 3\,419.43$$
$$= 6\,034.29 \text{ cm}^2$$

(c) Volume (frustum)

$$= \text{volume (cone VCD)} - \text{volume (cone VAB)}$$
$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi (R^2 H - r^2 h)$$
$$= \frac{1}{3} \times \frac{22}{7} (24^2 \times (52 + 26) - 16^2 \times 52)$$
$$= \frac{22}{21} [(24^2 \times 78) - (16^2 \times 52)]$$
$$= \frac{22}{21} (44\,928 - 13\,312)$$
$$= \frac{22}{21} \times 31\,616$$
$$= \frac{695\,552}{21}$$
$$= 33\,121.5238$$
$$= 33\,121.52 \text{ cm}^2 \text{ (2 d.p.)}$$

**Aliter**

$$\text{Volume} = \frac{1}{3}\pi h\,(R^2 + Rr + r^2)$$

$h$ = height of the frustum

$R$ = radius of the base area of the frustum

$r$ = radius of the top area of the frustum

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 26\,(24^2 + (24 \times 16) + 16^2)$$

$$= \frac{22 \times 26}{21}\,(576 + 384 + 256)$$

$$= \frac{572 \times 1\,216}{21}$$

$$= 33\,121.5238$$

$$= 33\,121.52\ \text{cm}^2\ (2\ \text{d.p.})$$

(d) $\text{Capacity (litre)} = \dfrac{Vcm^3}{1000}$

$$= 33\,121.5238$$

$$= 33.1215$$

$$= 33.12\ \text{litres}\ (2\ \text{d.p.})$$

## Worked Example 41

A boundary pillar moulded in the form of a frustum of a pyramid 25 cm high has
a square top and base of side 15 cm and 30 cm respectively. Calculate its
(a) Volume
(b) Total surface area and (c) Density, if the weight of the pillar is 52 564 grams.

### Solution

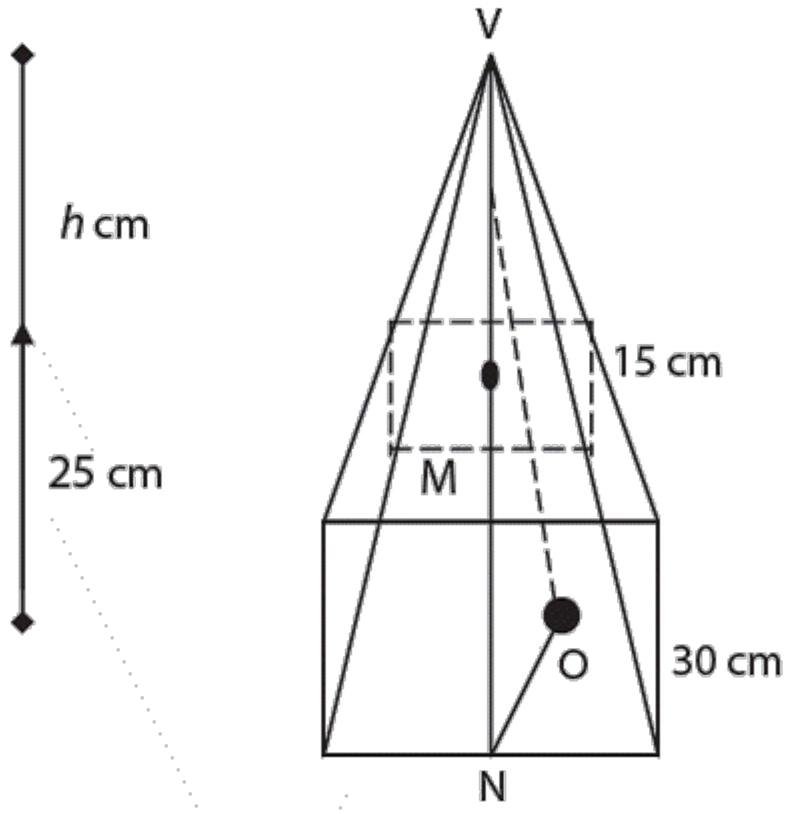



**Fig. 13.60**

(a) Let $h$ be the height of the removed pyramid.

Therefore, by similar triangles:

$$\frac{h}{15} = \frac{h+25}{30}$$

$$30h = 15(h+25)$$

$$30h = 15h + 375$$

$$30h - 15h = 375$$

$$15h = 375$$

$$h = \frac{375}{15}$$

$$h = 25 \text{ cm}$$

Hence, the height of the original pyramid is $h + 25 = 25 + 25 = 50$ cm.

But the volume of the frustum = volume of the original pyramid − volume of the removed pyramid.

$$= \left(\frac{1}{3} \times 30 \times 30 \times 50\right)$$

$$- \left(\frac{1}{3} \times 15 \times 15 \times 25\right)$$

$$= \frac{1}{3}((30 \times 30 \times 50) - (15 \times 15 \times 25))$$

$$= \frac{1}{3}(45\,000 - 5625)$$

$$= \frac{39375}{3}$$

$$= 13\,125 \text{ cm}^3$$

(b) Using the diagram in Fig. 13.60

$$VN^2 = VO^2 + ON^2$$

$$VN^2 = 50^2 + 15^2$$

$$= 2\,500 + 225$$

$$= 2\,725$$

$$VN = \sqrt{2\,725}$$

$$VN = 52.2015 \text{ cm}$$

And by similar triangle

$$\frac{VN}{30} = \frac{VM}{15}$$

$$= \frac{52.2015}{30} = \frac{VM}{15}$$

$$VM = \frac{15 \times 52.2015}{30}$$

$$VM = 26.1008 \text{ cm}$$

Therefore, MN = VN − VM
= 52.2015 − 26.1008
= 26.1007 cm

The areas of the four slanting faces of the frustum

$$= 4\left(\frac{1}{2} \times 30 \times 52.2015 - \frac{1}{2} \times 15 \times 26.1008\right)$$

$$= \frac{4}{2}((30 \times 52.2015) - (15 \times 26.1008))$$

$$= 2\ (1\ 566.045 - 391.512)$$

$$= 2 \times 1\ 174.533$$

$$= 2\ 349.066\ \text{cm}^2$$

Top square area $= 15 \times 15 = 225\ \text{cm}^2$

Base square area $= 30 \times 30 = 900\ \text{cm}^2$

Therefore, total surface area

$$= (225 + 2\ 349.066 + 900)\ \text{cm}^2$$

$$= 3\ 474.066$$

$$= 3\ 474.06\ \text{cm}^2\ (2\ \text{d.p.})$$

## Aliter

Using the formula for the area of a trapezium: $\frac{1}{2}h(a + b)$.

The area of the four slanting faces of the frustum

$$= 4\left(\frac{1}{2} \times MN\right) \times (15 + 30)$$

$$= 4\left(\frac{1}{2} \times 26.1008\right) \times (15 + 30)$$

$$= \frac{4}{2}(26.1008) \times (45)$$

$$= 2\ (1\ 174.536)$$

$$= 2\ 349.072\ \text{cm}^2$$

$$= 2349.07\ \text{cm}^2\ (2\ \text{d.p.})$$

(c) Density = g/cm$^3$

$$= \frac{52\ 564}{14\ 125}$$

$$= 3.7213$$

$$= 3.7\ \text{g/cm}^3$$

## Exercise 4

1. A frustum of a cone has its upper and lower radii as 8 cm and 15 cm, respectively. If the perpendicular height of the frustum is 10 cm, find the
   (a) volume.
   (b) total surface area (take $\pi = \frac{22}{7}$).
2. Given a frustum of a cone whose slant height is 10 cm top and bottom radii are 4 cm and 9 cm respectively. Calculate its,
   (a) volume
   (b) total surface area (take $\pi = \frac{22}{7}$).
3. A cone of height 24 cm and radius of base 16 cm has its top cut off by a plane parallel to its base and 8 cm from its vertex. Calculate the
   (a) volume and
   (b) the total surface area of the frustum.

4. A dustbin is in the form of a frustum of a cone and has a height of 46 cm, top and lower diameter of 30 cm and 12 cm respectively. Find the
   (a) volume.
   (b) capacity in litres.
5. A solid cone of height 10 cm and radius 4 cm has its top cut off by a plane parallel to its base and 5 cm from it. Calculate the
   (a) volume.
   (b) total surface area of the frustum.

6. A top open-ended dustbin in the form of a frustum of a pyramid 40 cm high has a square top and base of 20 cm and 35 cm respectively. Calculate its
   (a) volume and
   (b) surface area.
7. Both open-ended frustum of a pyramid with a rectangular base has a top dimension of 5 m by 3 m and base dimension of 10 m by 7 m. If the height of the frustum is 12 m, calculate to 2 decimal places the
   (a) volume and
   (b) surface area of the frustum.
8. A pyramid on a rectangular base of dimension 15 cm by 12 cm is 25 cm high. If the top height 14 cm of the pyramid is cut off, find
   (a) the volume of the removed pyramid.
   (b) the volume of its frustum.

9. Figure 13.61 shows a lamp shade with top width and base of sides 4 cm and 8 cm respectively. If the perpendicular height is 5 cm, calculate the volume of the lamp shade.

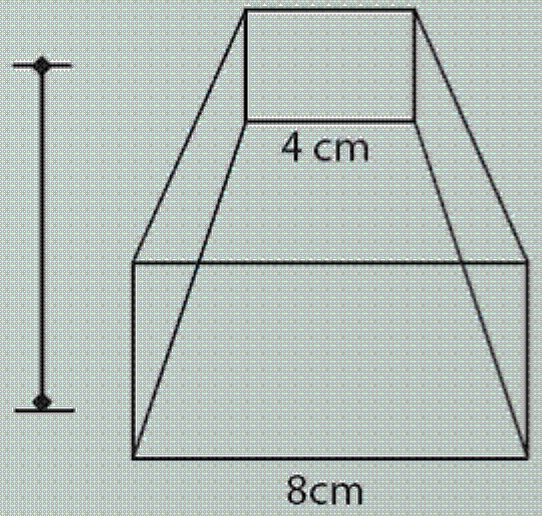



**Fig. 13.61**

10. Given a solid rectangular base frustum of a pyramid that has a top dimension of 3 cm by 5 cm, base dimension of 8 cm by 10 cm and a vertical height of 5 cm. Find its volume.

## V. Surface Area and Volume of Compound Shapes (Composite)

A compound shape is one that is made up of two or more solid shapes. Its volume is calculated by adding the volumes of the solid shapes it is made up of, while the total surface area is found by adding up the surface areas that are visible.

**Note:** Some solid shapes are obtained by removing a part of the given solid. Such solids are called pipes.

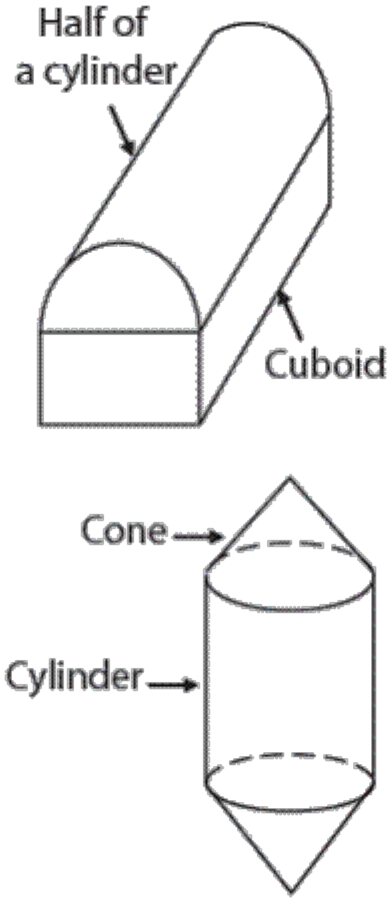
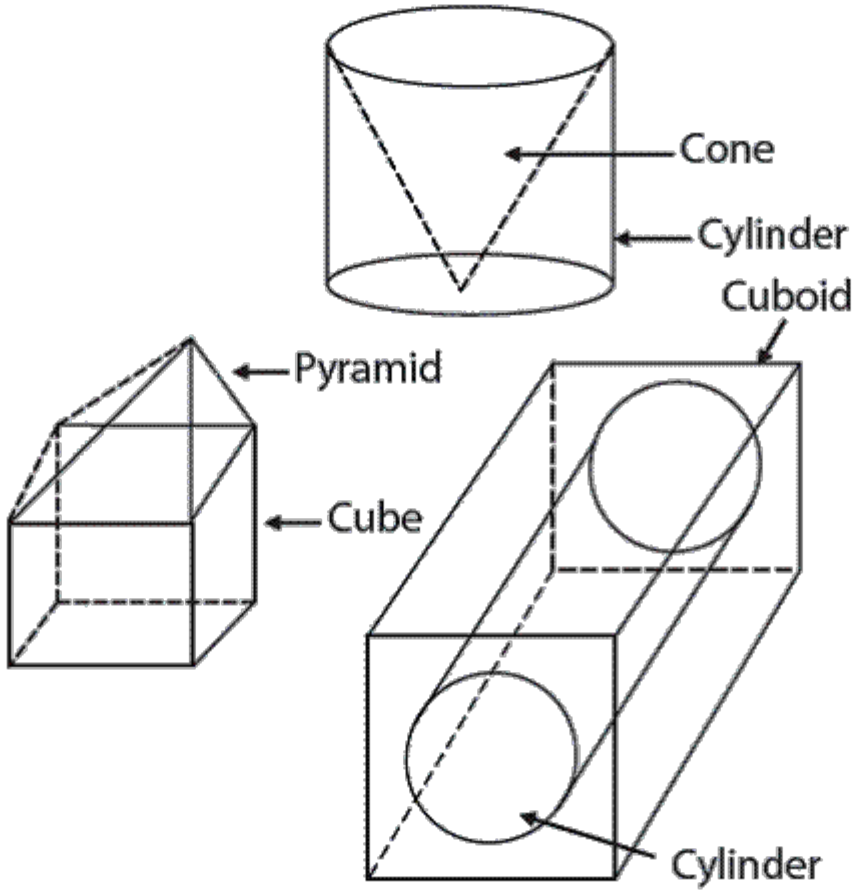



**Fig. 13.62**




Compound shapes (composite)

**Fig. 13.62** Continued

## Worked Example 42

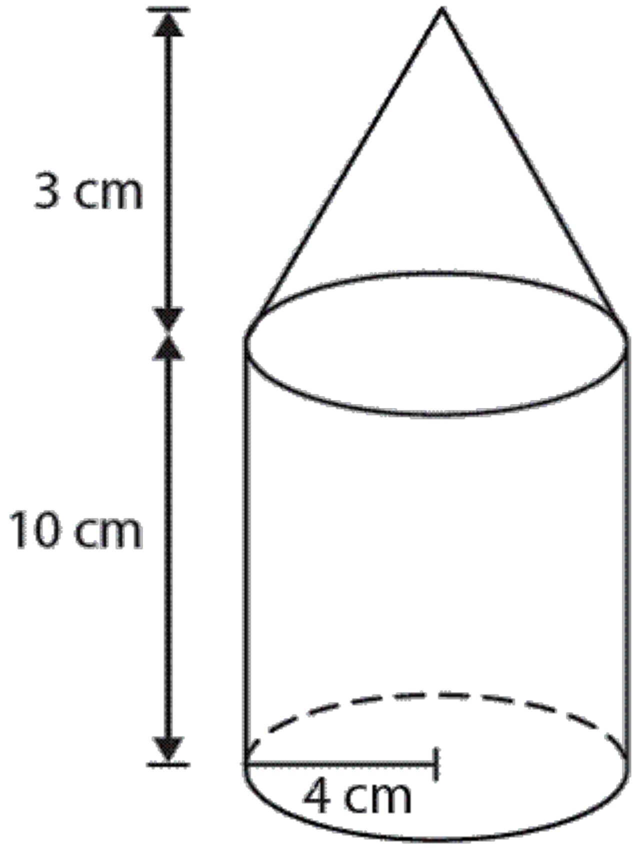



**Fig. 13.63**

Figure 13.63 shows a composite made up of a cone and a cylinder with its discasion shown. Find the:

(a) Volume and

(b) Total surface area (take $\pi = \frac{22}{7}$).

**Solution**

(a) Let $h_1$ = height of the cone $h_2$ = height of the cylinder

Volume = volume (cone) + volume (cylinder)

$$= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3\right)$$

$$+ \left(\frac{22}{7} \times 4 \times 4 \times 10\right)$$

$$= \frac{22}{7} \times 4 \times 4 \left(\frac{1}{3} \times \frac{3}{1}\right) + \left(\frac{10}{1}\right)$$

$$= \frac{22 \times 16}{7} (1 + 10)$$

$$= \frac{352}{7} \times 11$$

$$= 553.1429$$

$$= 553.14 \text{ cm}^3 \text{ (2 d.p.)}$$

(b) Total surface area = curved surface area of the cone + curved surface area of the cylinder + base circular area of the cylinder

$$= \pi r l + 2\pi r h_2 + \pi r^2$$

$$= \pi r(l + 2h_2 + r)$$

$$= \frac{22}{7} \times 4 (5 + (2 \times 10) + 4)$$

$$= \frac{88}{7} (5 + 20 + 4)$$

$$= \frac{88}{7} \times \frac{29}{1}$$

$$= 364.5714$$

$$= 364.57 \text{ cm}^2 \text{ (2 d.p.)}$$

Since $l^2 = 4^2 + 3^2$

$$= 16 + 9$$

$$= 25$$

$$l = \sqrt{25} = 5$$

## Worked Example 43

Figure 13.64 shows a composite made up of a rectangular-based right pyramid of height 2 m and a cuboid of length 5 m, width 3 m and height 4 m. Find the

(a) Volume.
(b) Total surface area of the solid.
(c) Cost of painting the solid, if every square metre costs N15.00.

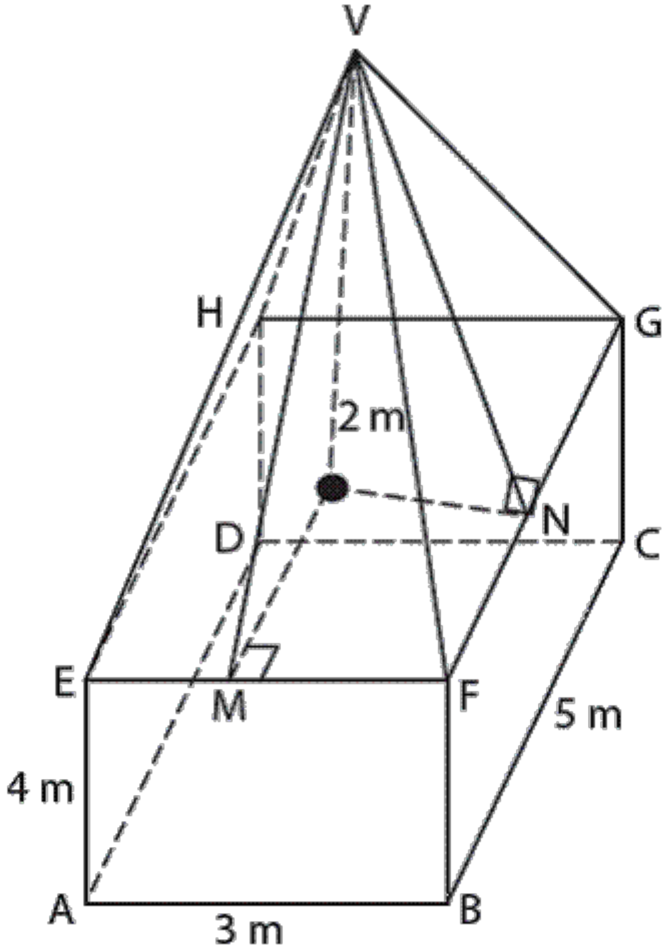



**Fig. 13.64**

### Solution

(a) $\text{Volume} = \text{volume (pyramid)} + \text{volume (cuboid)}$

$$= \left(\frac{1}{3}|EF| \times |FG| \times |VO|\right) + (|AB| \times |BC| \times |AE|)$$

$$= \left(\frac{1}{3} \times 3 \times 5 \times 2\right) + (3 \times 5 \times 4)$$

$$= 10 + 60$$

$$= 70 \text{ m}^3$$

(b) Total surface area = surface area of the pyramid + surface area of the cuboid. But M is the midpoint of |EF|, <EMV = 90°

$$|VM|^2 = |OM|^2 + |VO|^2$$

$$= (2.5)^2 + 2^2$$

$$= 6.25 + 4$$

$$= 10.25 \text{ m}$$

$$|VM| = \sqrt{10.25}$$

$$= 3.2016 \text{ m}$$

Therefore, area of $\Delta VEF = \frac{1}{2} \times 3 \times 3.2016$

$$= 4.8024 \text{ m}^2$$

N is the midpoint of |FG|

$\angle VNG = 90°$ then $|FN| = |NG|$

$$|VN|^2 = |VO|^2 + |ON|^2$$

$$= 2^2 + 1.5^2$$

$$= 4 + 2.25$$

$$= 6.25$$

$$|VN| = \sqrt{6.25}$$

$$= 2.5 \text{ m}$$

$$\text{Therefore, area of } \Delta VFG = \frac{1}{2} \times 5 \times 2.5$$
$$= 6.25 \text{ m}^2$$

Therefore, area of the four faces of the pyramid = 2(4.8024 + 6.25)

= 2 × 11.0524

= 22.1048 $m^2$

Surface area of the cuboid = 2(|AB| × |AE|) + 2(|BC| × |BF|) + (|AB| × |BC|)

= 2 (3 × 4) + 2(5 × 4) + (3 × 5)

= 24 + 40 + 15

= 79 $m^2$

Therefore, total surface area of the composite

= 22.1048 + 79

= 101.1048

= 101.10 $m_2$ (2 d.p.)

(c) N15.00 × 101.1048

= N1 516.572

= N1 516.57

## Exercise 5

Find the volume and total surface area of the composites in questions 1 to 5. Express your answer to 3 significant figures.

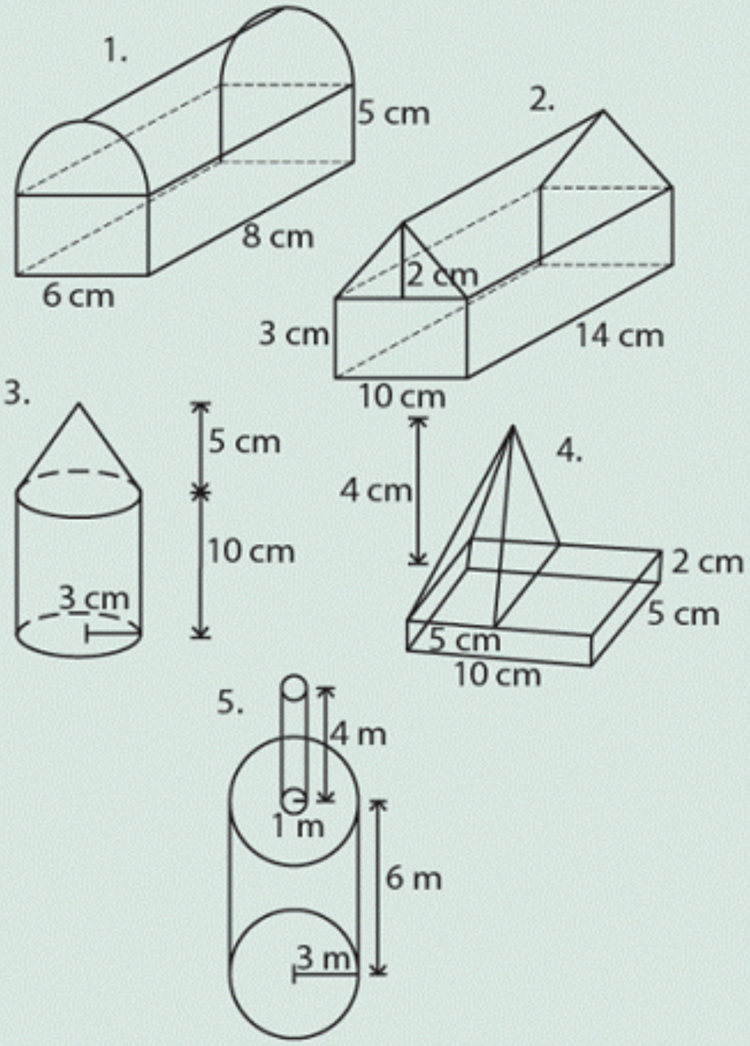



**Fig. 13.65**

6. Given a cylindrical concrete block mounted by a right concrete

circular cone whose common radius is 8 cm while the slant height of the cone is 10 cm and the height of the cylinder is 25 cm. Find the

(a) Total surface area of the compound shape.

(b) Volume of the shapes.

7. A wooden right circular cone whose height is 15 cm and radius 2 cm is surmounted on a wooden cuboid with dimension 5 cm by 10 cm by 3 cm. Find the

   (a) Volume and

   (b) Total surface area of the composite.

8. Figure 13.66 shows a half of a cylinder surmounted on a cuboid of dimension 3 cm by 10 cm by 12 cm.

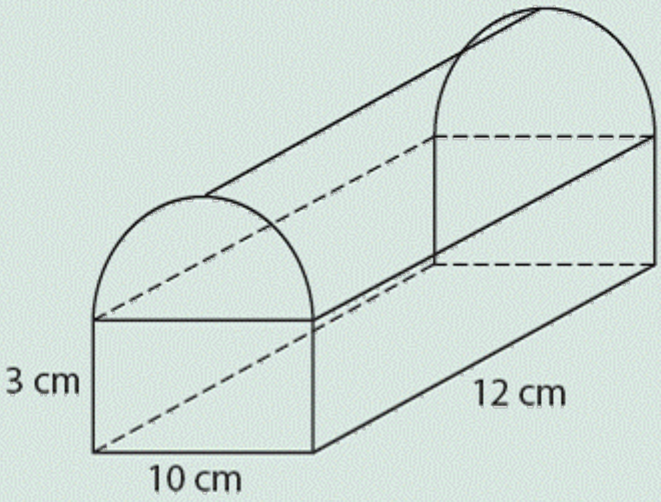



**Fig. 13.66**

Calculate the

(a) Volume and

(b) Total surface area of the compound shape.

9. Figure 13.67 shows a frustum of a cone mounted on a cuboid. Find the

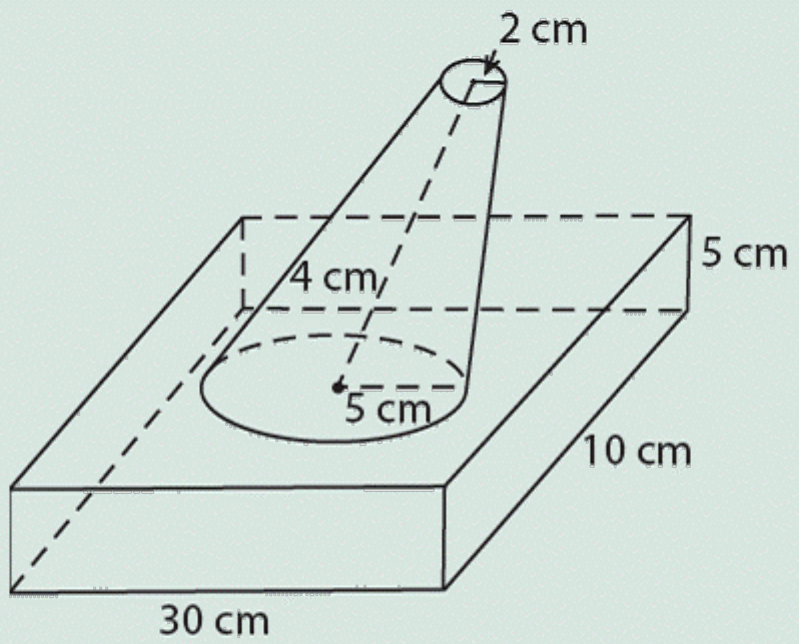



**Fig. 13.67**

(a) Volume and

(b) Total surface area of the compound shape.

10. A concrete right squared-based pyramid is placed on top of a cylindrical iron as shown in Fig. 13.68.

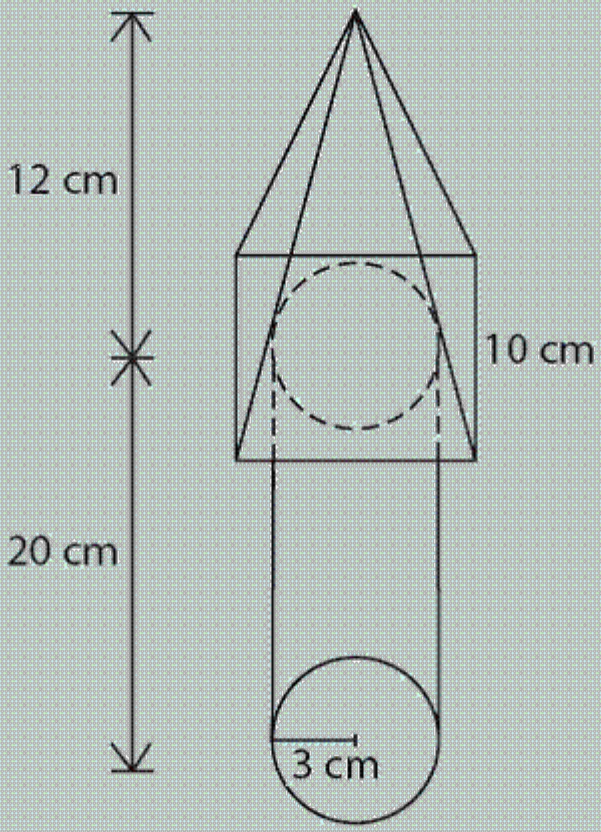



**Fig. 13.68**

Calculate the

(a) Volume of the compound shape and

(b) Total surface area.

## SUMMARY

In this chapter, we have learnt the following:

- (a) Circumference of a circle is $2\pi r$ while '$r$' is the radius of the circle.

  (b) Length of an arc of a circle is proportional to the angle it subtends at the centre and therefore an arc $= \frac{\theta}{360} \times 2\pi r$.

  (c) The perimeter of a sector that subtends an angle $\theta$ at the centre is the sum of the arc and the two radii, that is, $\frac{\theta}{360} \times 2\pi r + (2r)$.

  (d) The length of a chord that subtends an angle $\theta$ at the centre is $2r \sin\left(\frac{\theta}{2}\right)$.

  (e) The area of a segment of a circle of radius '$r$' which subtends an angle $\theta$ at the centre is $\left(\frac{\theta}{360} \times \pi r^2\right) - \left(\frac{r^2 \sin\theta}{2}\right)$

- (a) The curved surface area of the cone whose circular base radius is "$r$" and slant height is "$l$" is $\pi r(l + r)$ square unit.

  (b) Volume is $\frac{1}{3}(\pi r^2 h)$.

  (c) Volume of the frustum of a cone is $\frac{1}{3}\pi h(R^2 + Rr + r^2)$.

- Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ perpendicular height.

- (a) Compound shape (composite) is one that is made up of two or more solid shapes.

  (b) Its volume is solved by adding the volumes of the solid shapes it is made up of, while its total surface area is found by adding up the areas that are visible.

## GRADUATED EXERCISE

1. If a pyramid *ABCDV* with a square base *ABCD* of side 10 cm has triangular faces with altitudes of 12 cm. Calculate giving your answer to 3 significant figures the (a) total surface area. (b) volume of the pyramid and (c) angle between the face VCB and the base. (NECO)

2. The diagram below shows a piece of cardboard in the form of a sector of a circle. The radii OP and OQ are each equal to 12 cm. A and B are the mid points of OP and OQ respectively, and <POQ = 120o. Calculate, correct to 1 decimal place, the

(a) area of ABQP.

(b) perimeter of ABQP (Take $\pi = \frac{22}{7}$).

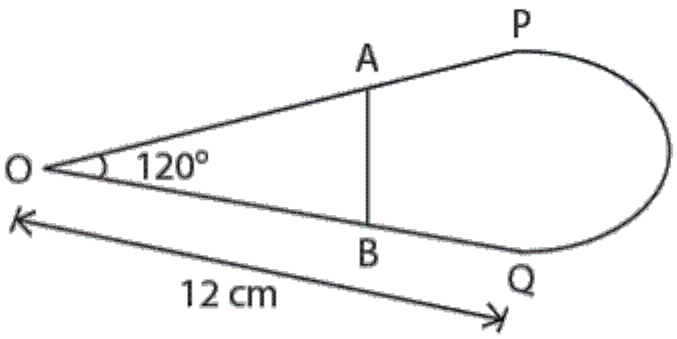



**Fig. 13.69**

3. (a) In the diagram below, ABCDEF represents a triangular prism in which |CE| = 8 cm, |DE| = 15 cm, |BC| = 10 cm and <BFA = <CED = 90o. Calculate

(i) |BD|.

(ii) the angle which $\overline{BD}$ makes with $\overline{BE}$ and

(iii) the volume of the prism.

(b) If the prism is made up of a material which has mass 2.5 g for every 1 cm3, then calculate the total mass of the prism in kilogramme. (WASSCE)

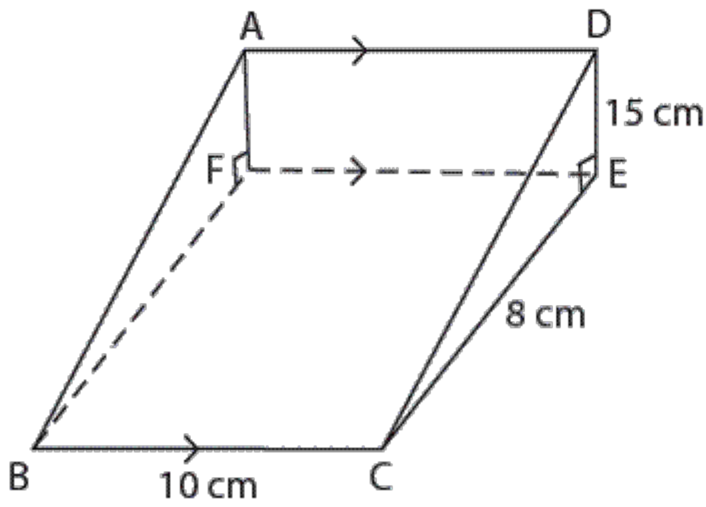



**Fig. 13.70**

4. The diagram below shows the net of a right rectangular pyramid in which ABCD is base and $O_1$, $O_2$, $O_3$ and $O_4$ are brought together to form the vertex O. When the solid is formed, ABCD is the base with |AB| = 8 cm, |AD| = 6 cm and |BO| = 13 cm.

(a) Draw a sketch of the pyramid.
(b) Calculate the
(i) height of the pyramid.
(ii) volume of the pyramid.
(iii) area of triangle AOB and
(iv) inclination of triangle BOC to the base ABCD.

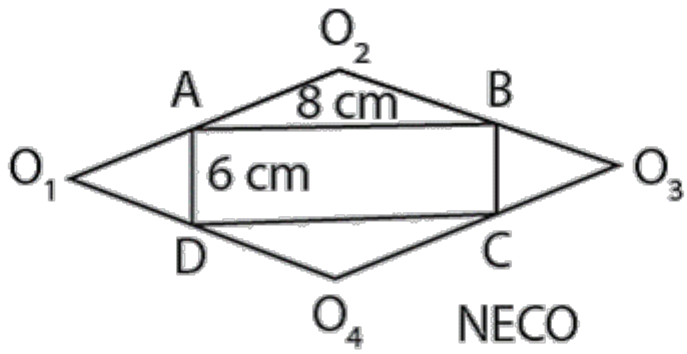



**Fig. 13.71**

5. In the diagram, PQRO is one quarter of a circle with centre R; PRO is a semicircle with centre O. |RQ| = |PR| = 7 cm.
Calculate, correct to 2 decimal places, the area of the shaded portion.

(Take $\pi = \frac{22}{7}$ )

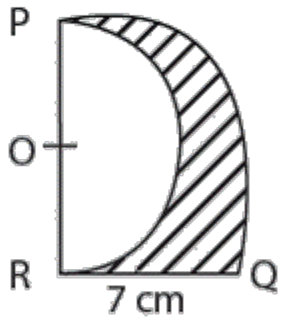



Fig. 13.72

6. A bucket is 12 cm in diameter at the bottom, 20 cm in diameter at the open end and 16 cm deep. If the bucket is filled with water and emptied into a cylindrical tin of diameter 14 cm, calculate the depth of water in the tin. (Take $\pi = \frac{22}{7}$ )

7. The base of a pyramid is a 4.5 m by 2.5 m rectangle. The height of the pyramid is 4 m. Calculate its volume. (WASSCE)

8. (a) The diagram below shows a circle ABCD with centre O and radius 7 cm. The reflex angle AOC = 190° and angle DAO = 35°. Find

(i) <ABC

(ii) <ADC

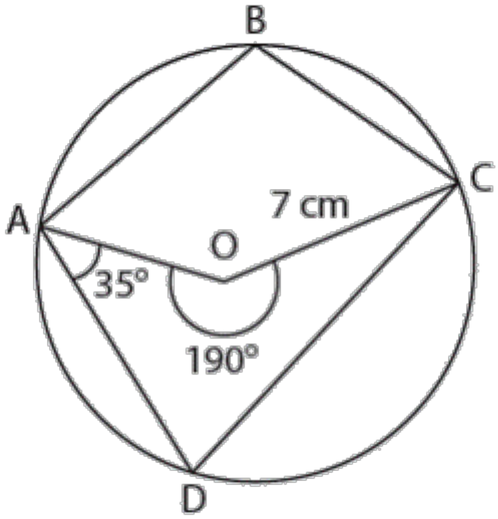



Fig. 13.73

(b) Using the diagram above, calculate, correct to 3 significant figures, the length of

(i) arc ABC

(ii) the chord AD (Take $\pi = 3.14$ ) (WASSCE)

9. A solid cone is cut into two as shown in Fig. 13.74. The base radii of cone "A" and frustum "B" are 7 cm and 10 cm respectively. If the height of the frustum is 8 cm and that of "A" is 12 cm. Calculate correct to 3 significant figures

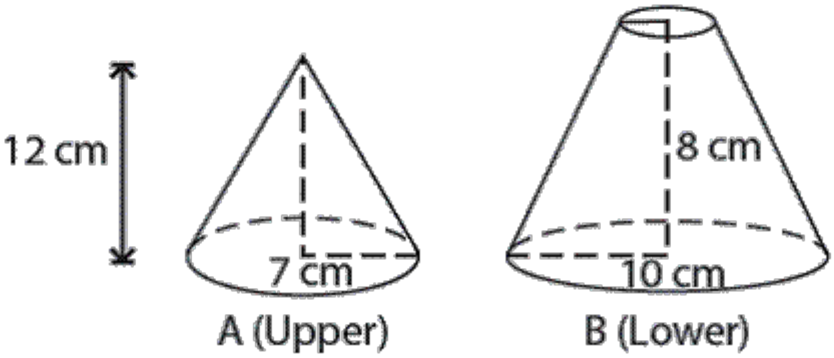



**Fig. 13.74**

(a) the total surface area of cone "A".

(b) the total surface of cone A and B joined together as one cone.

(c) the total surface area of frustum B. (NECO)

10. Calculate, correct to 3 significant figures

(a) the volume.

(b) the total surface area of the compound shape in Fig. 13.75 below.

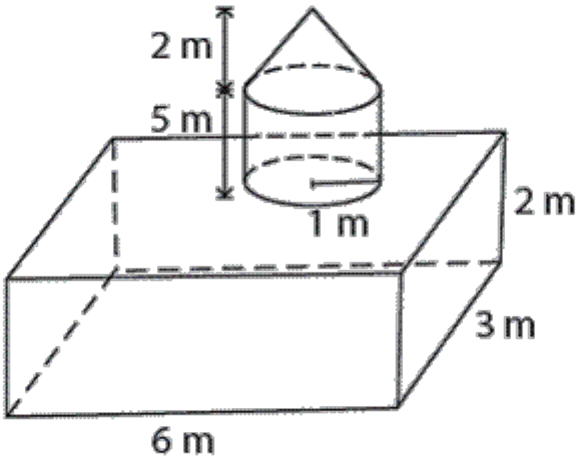



**Fig. 13.75**