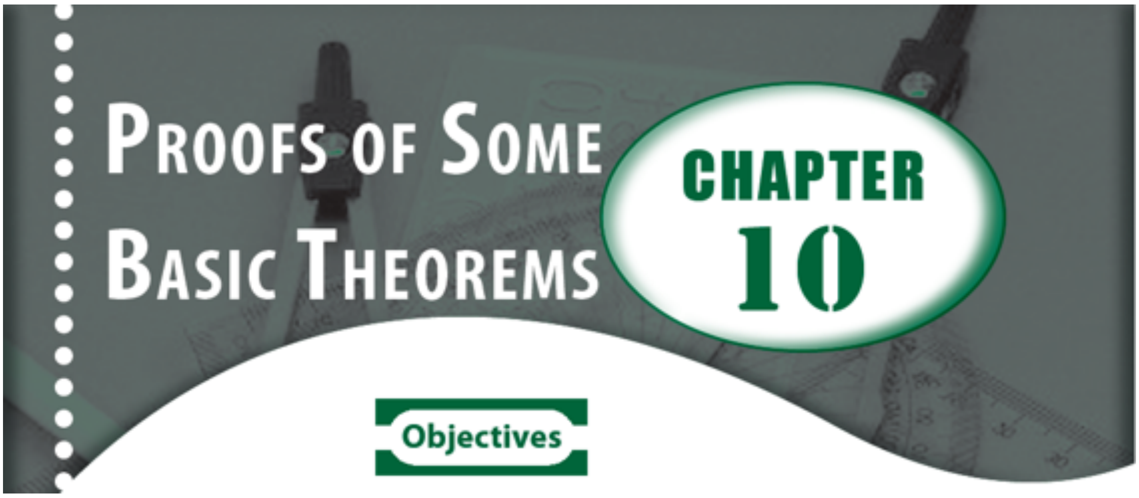
CHAPTER 10



At the end of the chapter, students should be able to:
1. Write out formal proofs of some basic theorems in Euclidean geometry.
2. Apply the proofs in solving practical problems in Euclidean geometry.
3. Apply the skills of deduction in proving riders on:
    - i. Angles on parallel lines.
    - ii. Angles in a polygon.
    - iii. Conditions of congruence of triangles.
    - iv. Properties of a parallelogram.
    - v. Intercept theorem.
4. Solve problems on riders in Euclidean geometry.

I. Introduction

In deductive geometry, we do not accept any other geometrical statement as being true unless it can be proved (or deduced) from the axioms. An axiom is a statement that is simply accepted as being true. So a statement that is proved by a sequence of logical steps is called a theorem.
To prove a theorem, we start by using one or more axioms in a particular situation to get some true statements. We then have to apply logical reasoning to these statements to produce new statements that are true. The proof ends when we arrive at the statement of the theorem.

II. Interior and Exterior Angles of a Triangle

Angles of a Triangle

Theorem 1

Sum of angles of a triangle is 180°.

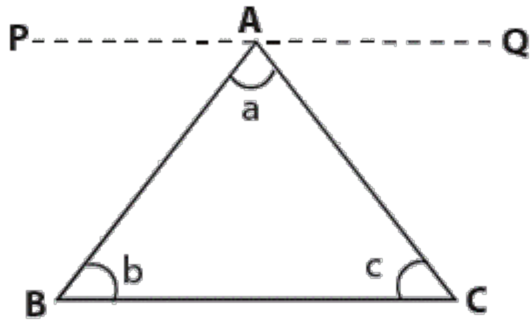


Fig. 10.1

Given: Any triangle ABC.

To prove $\hat{A} + \hat{B} + \hat{C} = 180°$.

Construction: Draw a line through A parallel to $\overline{BC}$.

Proof: $\angle PAB = b$ (alternate angles)
$\angle QAC = c$ (alternate angles)

Now, $\angle PAB + \angle BAC + \angle QAC = 180°$ (angles on a straight line)

$\therefore b + a + c = 180°$

$\therefore a + b + c = 180°$

$\therefore \hat{A} + \hat{B} + \hat{C} = 180°$

Hence, the sum of angles of a triangle is 180°.

Worked Example 1

Find the value of the lettered angle in Fig. 10.2.

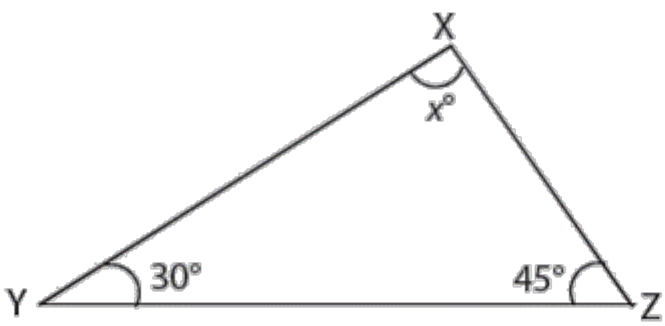


Fig. 10.2

Solution

$x + 30° + 45° = 180°$ (sum of angles of a triangle is 180°)

$x + 75° = 180°$

$\therefore x = 180° - 75°$

$x = 105°$

Worked Example 2

Find the value of the lettered angles in Fig. 10.3.

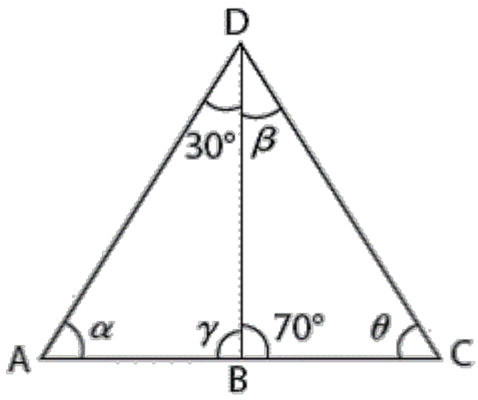


Fig. 10.3

ΔBCD is isosceles

$\beta = \theta$ (base angles of isosceles)

$70° + \beta + \theta = 180°$ (sum of angles of a triangle)

$$70° + \beta + \beta = 180°$$

$$70° + 2\beta = 180°$$

$$\therefore 2\beta = 180° - 70°$$

$$= 110°$$

$$\therefore \beta = \frac{110°}{2}$$

$$= 55°$$

since $\beta = \theta$

$$\therefore \theta = 55°$$

From ΔABD

$\alpha + \gamma + 30° = 180°$ (sum of angles in a triangle)

$\gamma + 70° = 180°$ (angles on a straight line)

$$\therefore \gamma = 180° - 70°$$

$$\gamma = 110°$$

$$\therefore \alpha + 110° + 30° = 180°$$

$$\alpha + 140° = 180°$$

$$\therefore \alpha = 180° - 140°$$

$$\alpha = 40°$$

Note: Theorem 1 can be used to prove theorem 2.

Theorem 2

The exterior angle of a triangle is equal to the sum of two interior opposite angles.

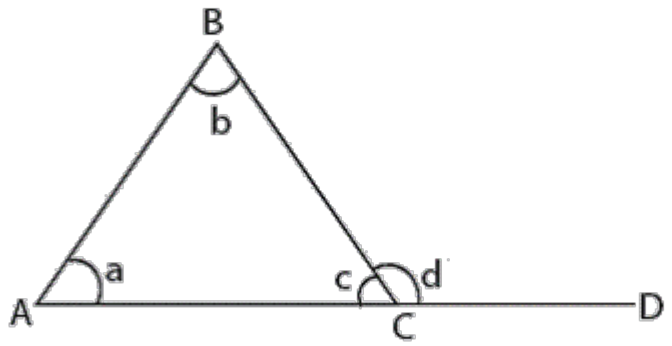


Fig. 10.4

Given: ΔABC

To prove: $B\hat{C}D = B\hat{A}C + A\hat{B}C$

Construction: Produce $\overline{AC}$ to point D.

Proof: With the lettering in Fig. 10.4,

$b + a + c = 180°$ (angle sum of a triangle)

$c + d = 180°$ (angles on a straight line)

$\therefore c + d = b + a + c$

$d = b + a$ (subtract c from both sides)

$d = a + b$

$\therefore B\hat{C}D = B\hat{A}C + A\hat{B}C$

Hence, the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Note: The theorem that has already been proved can be applied to prove other theorems.

Worked Example 3

Find the values of the lettered angles in Fig. 10.5.

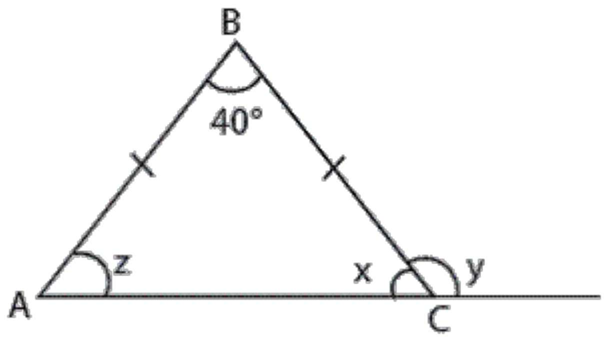


Fig. 10.5

Solution

In ΔABC,

$z = x$ (base angles of an isosceles triangle)

Also, $z + 40° + x = 180°$ (sum)

$z + 40° + z = 180°$ ($z = x$)

$\therefore 2z + 40° = 180°$

$2z = 180° - 40°$

$2z = 140°$

$\therefore z = 140° \div 2$

$z = 70°$

Since $z = x$

$x = 70°$

Also, $x + y = 180°$ (angles on a straight line)

$70° + y = 180°$

$\therefore y = 180° - 70°$

$y = 110°$

Worked Example 4

What is the value of the lettered angles in Fig. 10.6?

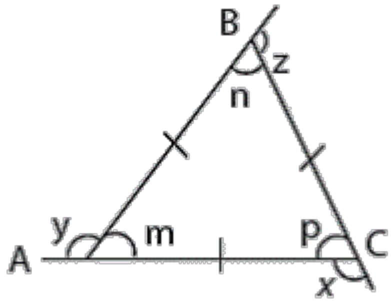


Fig. 10.6

Solution

In ΔABC,

$m = n = p = 60°$ (angles of an equilateral triangle)

$y = n + p$ (exterior angle of a triangle)

$= 60° + 60°$

$= 120°$

$z = m + p$ (exterior angle of a triangle)

$z = 60° + 60°$

$z = 120°$

$x = m + n$ (exterior angle of a triangle)

$x = 60° + 60°$

$x = 120°$

Worked Example 5

The sides $\overline{PQ}$ and $\overline{PR}$ of ΔPQR are produced to T and S respectively, such that $\angle TQR = 131°$ and $\angle QRS = 98°$. Find $\angle QPR$.

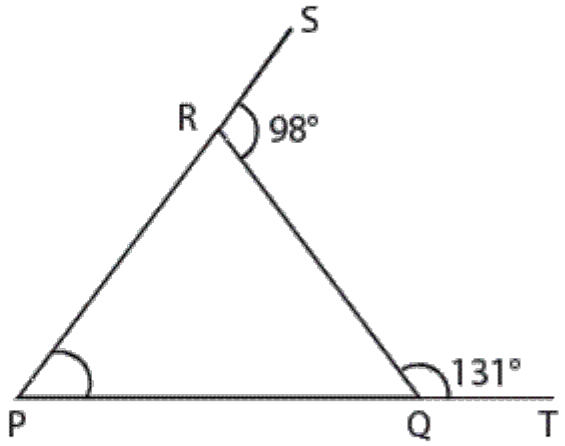


Fig. 10.7

Solution

$131° + \angle PQR = 180°$ (angles on a straight line)

$\therefore \angle PQR = 180° - 131°$

$= 49°$

$98° = \angle QPR + \angle PQR$ (exterior angle of a triangle)

$98° = \angle QPR + 49°$

$\therefore \angle QPR = 98° - 49°$

$= 49°$

Hence, $\angle QPR = 49°$.

Worked Example 6

In Fig. 10.8, side $\overline{BC}$ of ΔABC is produced to D. The bisector of ACD meets BA produced at E. Prove that $E = \frac{1}{2}(B\hat{A}C - B)$.

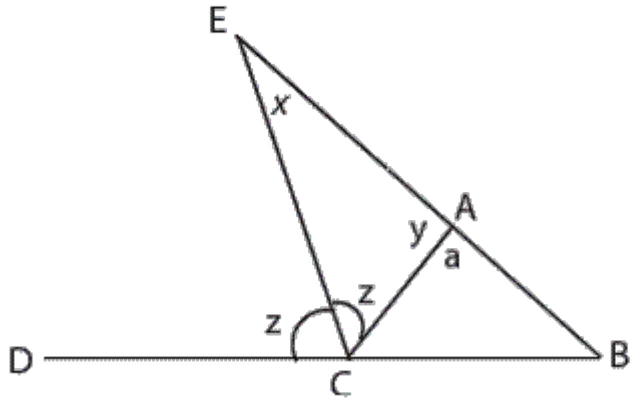


Fig. 10.8

Solution

Since $\overline{CE}$ is the bisector of ACD, ACE = DCE = 2 ACD

In ΔEAC,

$x + y + z = 180°$ (sum of angles in a triangle)

$$\therefore x = 180° - y - z \qquad \text{(i)}$$

At point A, $a + y = 180°$ (angles on a straight line)

$$\therefore y = 180° - a$$

In Δ ABC,

$2z = a + b$ (exterior angle of a triangle)

$\therefore z = \frac{1}{2}a + \frac{1}{2}b$

substituting y and z in (i)

$x = 180° - (180° - a) - (\frac{1}{2}a + \frac{1}{2}\text{b})$

$= 180° - 180° + a - \frac{1}{2}a - \frac{1}{2}b$

$= \frac{1}{2}a - \frac{1}{2}b$

$= \frac{1}{2}(a - b)$

$\therefore \text{E} = \frac{1}{2}(\text{B}\hat{\text{A}}\text{C} - \text{B})$

Exercise 1

1. Find the values of the lettered angles in Fig. 10.9.

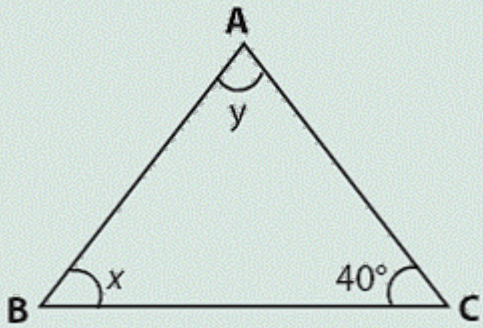


Fig. 10.9

2. In Fig. 10.10, $\overline{PQ} = \overline{PR} = \overline{RS}$ and $\angle RPS = 32°$. Find the value of $\angle QPR$.

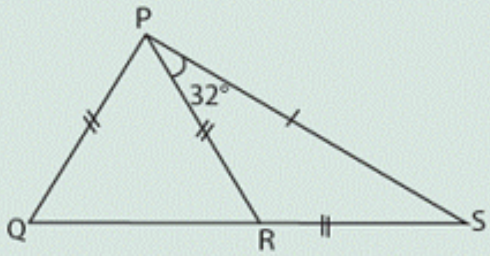


Fig. 10.10 (WAEC)

3. Find the size of the lettered angles in Fig. 10.11.

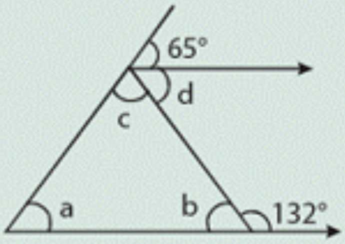


Fig. 10.11

4. What are the values of the lettered angles in Fig. 10.12?

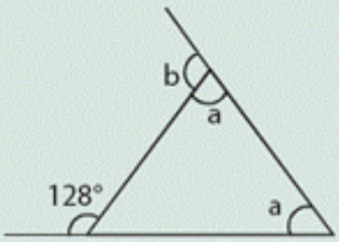


Fig. 10.12

5. In Fig. 10.13, triangle ABE is isosceles, $\overline{AB} = \overline{BE}$, $\angle DCB = 35°$, $\angle \overline{EAB} = 20°$ and ABC is a straight line. Calculate $\angle EDC$.

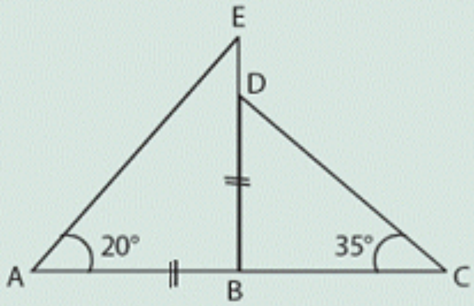


Fig. 10.13

6. Two exterior angles of a triangle are 120° and 130°. Find the third exterior angle.

7. In ΔABC, the bisector of B and C meet at D. Prove that $\angle BDC = 90° + \frac{1}{2}\hat{A}$.

8. In ΔABC, D is a point on BC such that $C\hat{A}D = \angle B$. Prove that $\angle ADC = \angle BAC$.

9. The angles of a triangle are x, $2x$ and $5x$. Find the value of x in degrees.

10. Prove that the sum of a triangle is two right angles. **(WAEC)**

11. In a triangle LMN, the side $\overline{NM}$ is produced to P and the bisector of $\angle LNP$ meets $\overline{ML}$ produced at Q. If $\angle LMN = 46°$ and $\angle MLN = 80°$, calculate $\angle LQN$. Give a reason for your answer. **(WAEC)**

12. The side $\overline{AB}$ of a triangle ABC is produced to a point D. The bisector of $\angle ACB$ cuts $\overline{AB}$ at E. Prove that $\angle CAE + \angle CND = 2\angle CEB$.

13. An isosceles triangle is such that each of the base angles is twice

the vertical angle. Find the angles of the triangle.

14. In Fig. 10.14, $\angle PTQ = \angle URP = 25°$ and $\angle XPU = 4\angle URP$. Calculate $\angle USQ$.

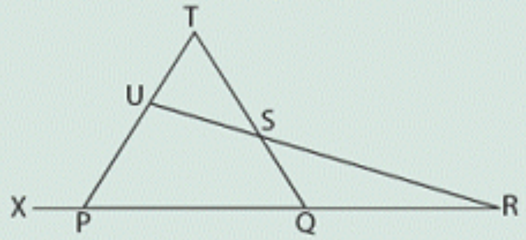


Fig. 10.14 (WAEC)

15. In ΔABC, the bisector of angle $B\hat{A}C$ meets $\overline{BC}$ at D. The line $\overline{AB}$ is produced to F and the bisector of $\angle FBD$ meets $\overline{AD}$ produced at E. If $\angle BAC = 46°$ and $\angle BDE = 75°$, calculate $\angle ABC$ and $\angle BED$.

16. In a right-angled triangle, one of the acute angles is 20° greater than the other. Find the angles of the triangle.

17. In ΔABC, the side $\overline{BC}$ is produced to D. If the bisector of $\angle ACD$ is parallel to $\overline{AB}$, prove that two angles of ΔABC are equal.

18. Find the size of the lettered angles in Fig. 10.15.

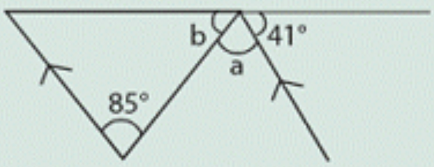


Fig. 10.15

19. Find the values of x and y in Fig. 10.16.

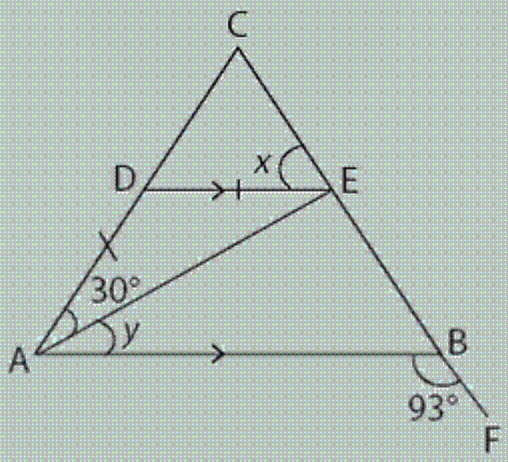


Fig. 10.16

20. Find the values of the lettered angles in Fig. 10.17.

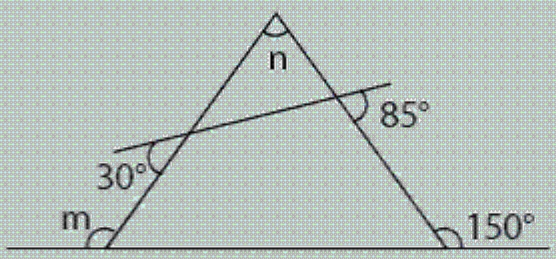


Fig. 10.17

III. Riders Including

(i) Angles of parallel lines

Parallel lines are lines that are side by side, neither converging nor diverging. They are lines that are always the same distance apart as shown in Fig. 10.18.

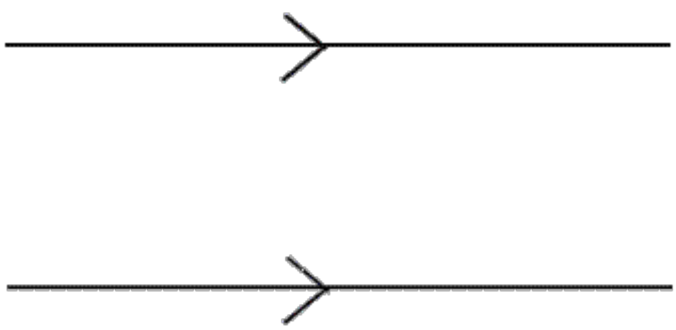


Fig. 10.18

The intersection of two parallel lines with a third line (called a transversal) defines several angles as shown in Fig. 10.19.

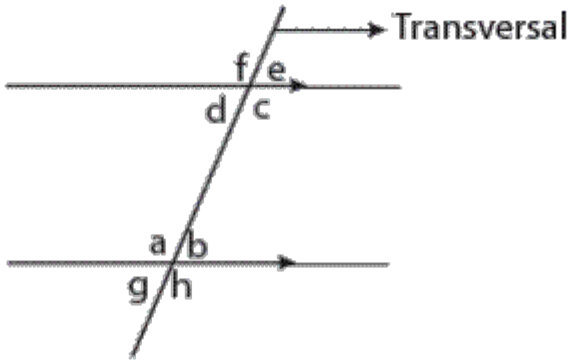


Fig. 10.19

Alternate angles: In Fig. 10.19, the following pairs of angles which are formed by the transversal are called alternate angles: a and c, b and d

Corresponding angles: In Fig. 10.19, the following pairs of angles which are formed by the transversal are called corresponding angles: a and f, b and e, d and g, and c and h

Interior opposite angles: In Fig. 10.19, the following pairs of angles which are formed by the transversal are called interior opposite angles or interior angles on the same side of the transversal. a and d, b and c.

Vertically opposite angles: In Fig 10.19, the following pairs of angles which are formed by the transversal are called vertically opposite angles: d and e, c and f, a and h, b and g

Theorem 3

If two parallel lines are intersected by a transversal:

(i) The alternate angles are equal.

For example, in Fig. 10.20 $a = c$ and $b = d$

(ii) The corresponding angles are equal. For example, in Fig. 10.20 $a = f$, $b = e$, $d = g$ and $c = h$

(iii) The interior angles on the same side of the transversal are supplementary.

For example, in Fig. 10.19

$a + d = 180°$ and $b + c = 180°$.

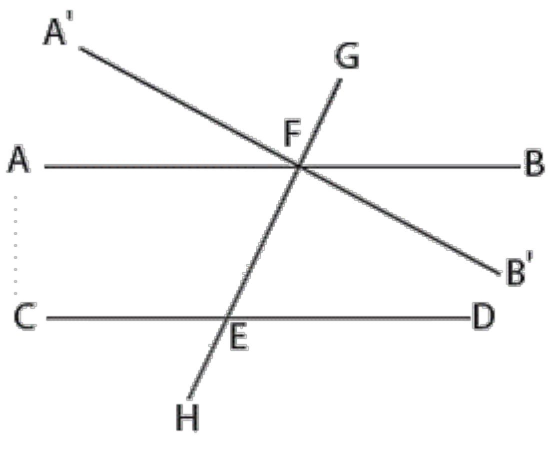


Fig. 10.20

Given: $\overline{AB}$ is parallel to $\overline{CD}$. $\overline{GH}$ intersects $\overline{AB}$ and $\overline{CD}$ at F and E respectively.

To prove: (i) $\angle AFE = \angle FED$

(ii) $\angle GFB = \angle FED$

(iii) $\angle BFE + \angle FED = 180°$

Construction: Through F draw $\overline{AB}$ such that $\angle AFE = \angle FED$.

Using indirect proof, by assuming that AFE $\neq$ FED.

Proof:

(i) If $\angle AFE \neq \angle FED$.
Let $\angle A'FE = \angle FED$ and alternate to it (by construction).
Then $\overline{A'B'}$ and CD are parallel (definition of $\parallel$ lines).
But we are given that $\overline{AB} \parallel \overline{CD}$. This means that through F, there are two lines $\overline{AB}$ and $\overline{A'B'}$ both parallel to $\overline{CD}$

Hence, the statement $\angle AFE \neq \angle FED$ is false.

$$\therefore \angle AFE = \angle FED$$

Hence, alternate angles are equal.

(ii) Again, because $\angle GFB = \angle AFE$ (vertically opposite angles) and $\angle AFE = \angle FED$ (alternate angles), $\angle GFB = \angle AFE$.

Hence, corresponding angles are equal.

(iii) $\angle GFB = \angle FED$ (corresponding angles).

To each angle add $\angle BFE$ then,

$\angle GFB + \angle BFE = \angle FED + \angle BFE$.

But $\angle GFB + \angle BFE = 180°$ (angles on a straight line)

$\therefore \angle FED + \angle BFE = 180°$

$\therefore \angle BFE + \angle FED = 180°$

Hence, the interior angles of the same side of the transversal are supplementary.

Worked Example 7

In Fig. 10.21, $\overline{AB} \parallel \overline{CD}$. Find the size of the angles a, b and c.

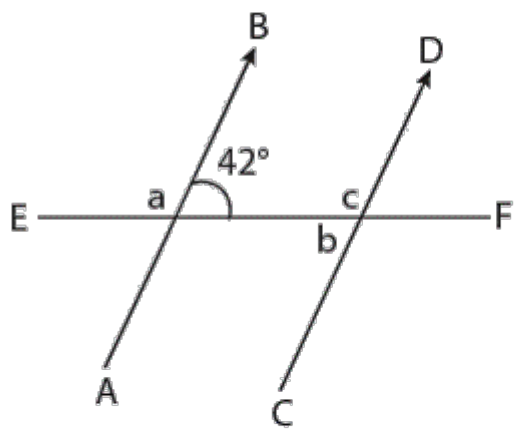


Fig. 10.21

Solution

$a + 42° = 180°$ (angles on a straight line)

$\therefore a = 180° - 42°$

$= 138°$

$b = 42°$ (alternate angles)

$c = a = 138°$ (corresponding angles)

Worked Example 8

In Fig. 10.22, find the value of y if $\overline{AB} \parallel \overline{DE}$, $\angle ABC = 80°$ and $\angle CDE = 120°$.

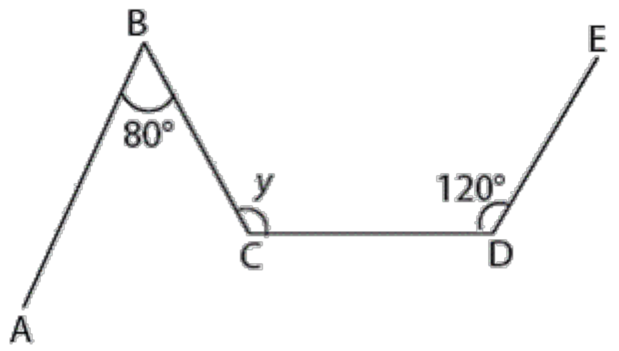


Fig. 10.22

Solution

Produce line $\overline{CD}$ to meet line $\overline{AB}$ at F, then line $\overline{DF}$ is transversal to the parallel lines $\overline{AB}$ and $\overline{DE}$ as shown in Fig. 10.23.

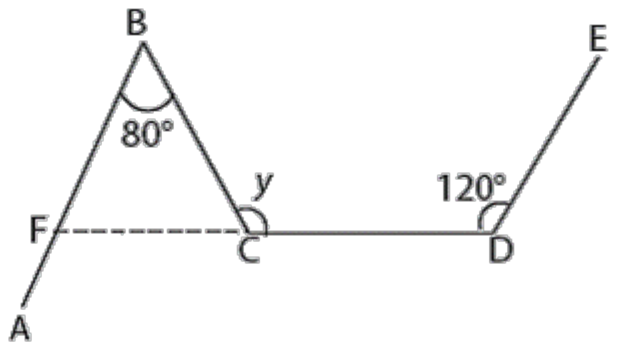


Fig. 10.23

$\angle BFC + \angle EDC = 180°$ (opposite interior angles)

$\therefore \angle BFC = 180° - \angle EDC$
$= 180° - 120°$
$= 60°$

Now, $\angle BCD$ is an exterior angle to ΔBCF.

$\angle BCF = \angle CBF + \angle BFC$
$= 80° + 60°$
$= 140°$

(ii) Angles in a polygon

A polygon is a closed plane completely bounded by n straight lines, where $n \geq 3$.

Examples of polygons are shown in Table 10.1.

Table 10.1

Number of sides	Names of polygon	Number of angles
3	Triangle	3
4	Quadrilateral	4
5	Pentagon	5
6	Hexagon	6
7	Heptagon	7
8	Octagon	8
9	Nonagon	9
10	Decagon	10
.	.	.
.	.	.
n	n-sided polygon	n

Theorem 4

The sum of the interior angles of any n-sided convex polygon is $(2n - 4)$ right angles $= (2n - 4)90°$.

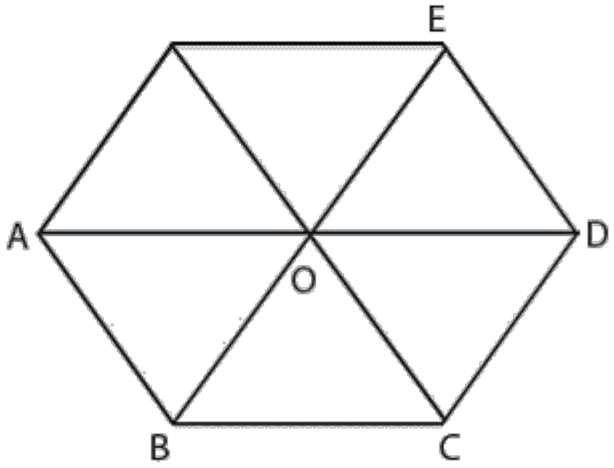


Fig. 10.24

Given: Any convex polygon ABCDE… with n sides.

To prove: $\angle A + \angle B + \angle C + \ldots = (2n - 4)$ right angles.

Construction: Join the vertices $A, B, C, \ldots$ to any point O inside the polygon.

Proof: By construction, there are n triangles (polygon ABCDE… has n sides).

Sum of angle of 1Δ = 180° or 2 right angles (sum of angles in a triangle).

$\therefore$ Sum of angles of $n\Delta$s = $2n$ right angles.

Sum of angles at O = 360° or 4 right angles (angles at a point).

Sum of angles of polygon ABCDE…

= Sum of angles of $n\Delta$s − sum of angles at centre O.

$\therefore \angle A + \angle B + \angle C + \ldots = (2n - 4)$ right angles.

Note: A *convex polygon* is a polygon in which all the interior angles are less than 180° (does not contain reflex angles). A polygon which contains reflex angles is called a *re-entrant polygon* (Fig. 10.25).

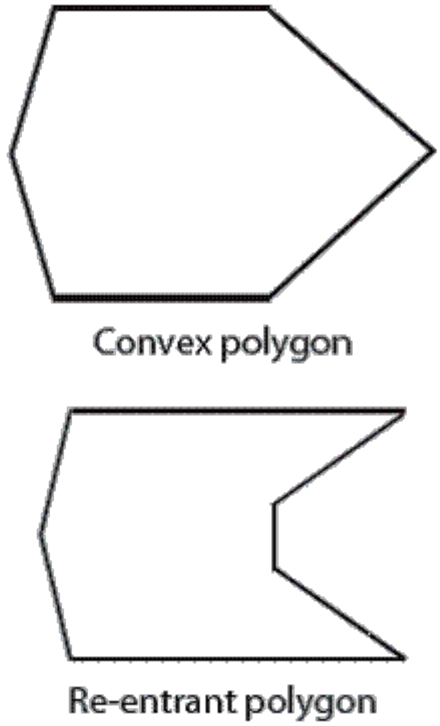


Fig. 10.25

Theorem 5
The sum of the exterior angles of any convex polygon is 4 right angles.

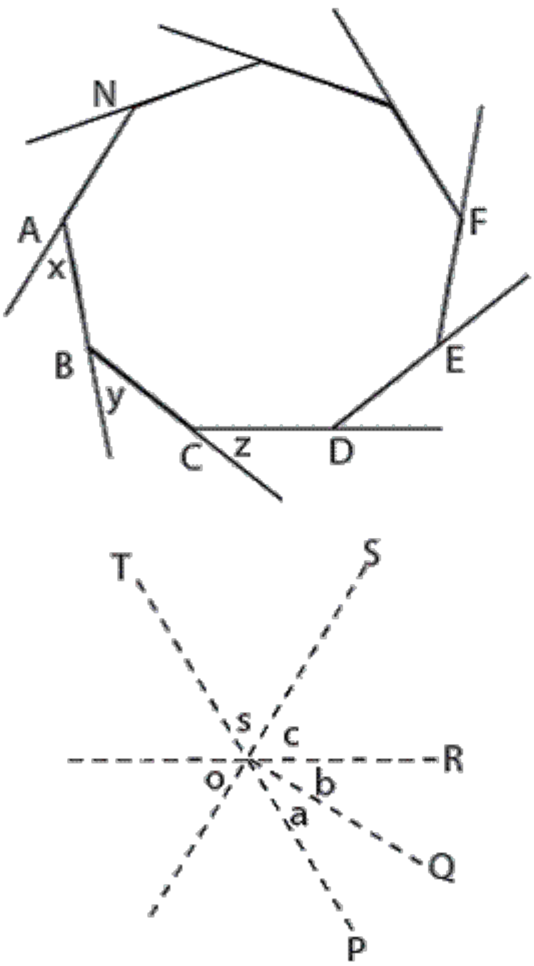


Fig. 10.26

Given: Any convex polygon ABCDE … N with n sides. Each side produced to give exterior angles $x, y, z, \ldots$

To prove: $x + y + z + \ldots = 4$ right angles

Construction: From any point O, draw line $\overline{OP}$, $\overline{OQ}$, $\overline{OR}$, $\overline{OS}$, … parallel to the sides of ABCD … N in turn.

Proof: With the lettering of Fig. 10.26,

$a = x$ ($\overline{OP} \parallel \overline{AB}$ and $\overline{OQ} \parallel \overline{BC}$)

Similarly, $b = y$, $c = z$

But $a + b + c + \ldots = 4$ right angles (angles at a point)

$\therefore x + y + z + \ldots = 4$ right angles

Hence, the sum of the exterior angles of any convex polygon is 4 right angles.

Worked Example 9

Find angle x in Fig. 10.27.

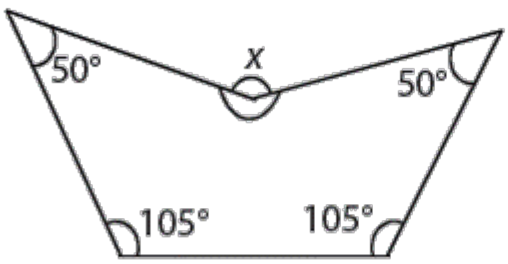


Fig. 10.27

Solution

Sum of interior angles $= (2n - 4) \times 90°$

$y + 50° + 105° + 105° + 50° = (2 \times 5 - 4) \times 90°$

$y + 310° = 540°$

$\therefore y = 540 - 310°$

$= 230°$

$\therefore x = 360° - y$ (angles at a point)

$= 360° - 230°$

$= 130°$

(iii) Congruent triangles

Two figures are congruent, if they have exactly the same shape and size as shown in Fig. 10.28.

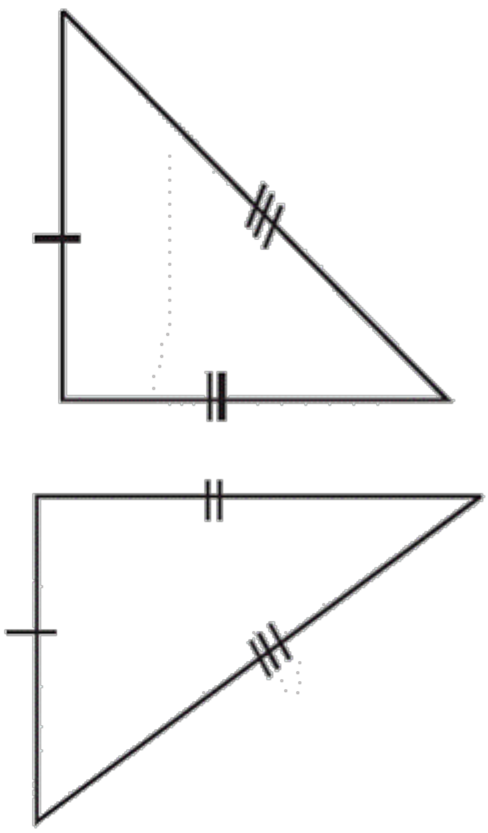


Fig. 10.28

Note: Two congruent triangles are equal in all respect, including sides, angles and areas. Two triangles are congruent, if they satisfy any of the following conditions:

1. Two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle (abbreviated as SAS). For example, in Fig. 10.29

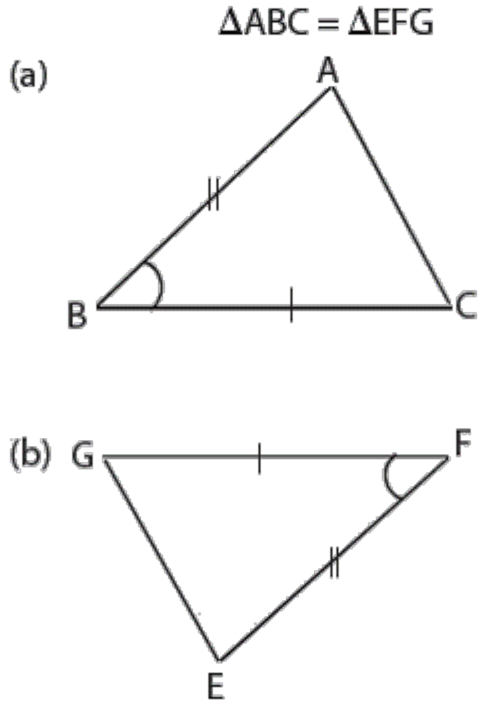


Fig. 10.29

In Fig. 10.29(a), ABC is called the included angle because it lies between, or is included between, the given sides $\overline{AB}$ and $\overline{BC}$.

2. Two angles and one side of one triangle are equal to the corresponding two angles and one side of the other triangle (abbreviated as ASA). For example, in Fig. 10.30

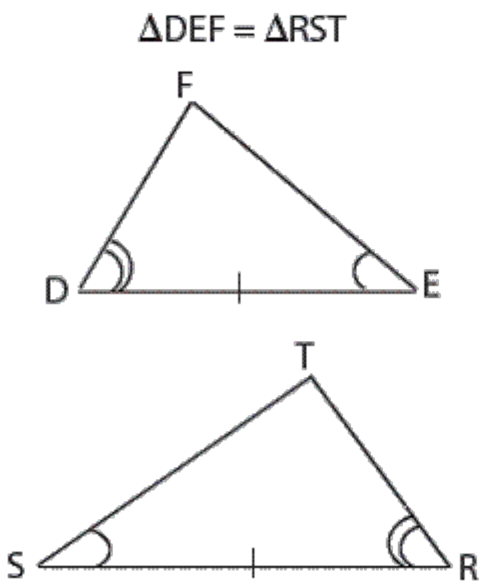


Fig. 10.30

In Fig. 10.30, the corresponding sides are between the pairs of given angles.

However, it is not necessary that the sides should be between the angles. If two angles of one triangle are equal to the two angles of another triangle, then from the sum of the angles of a triangle, the third angle in the first triangle will be equal to the third angle in the second. Hence, it does not matter which two angles are given, as long as the sides correspond. For example, in Fig. 10.31, ΔDEF is congruent to ΔRST.

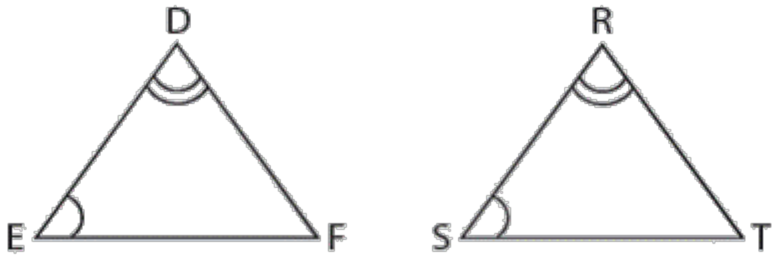


Fig. 10.31

3. All the corresponding sides of the two triangles are equal (abbreviated as SSS). For example, in Fig. 10.32

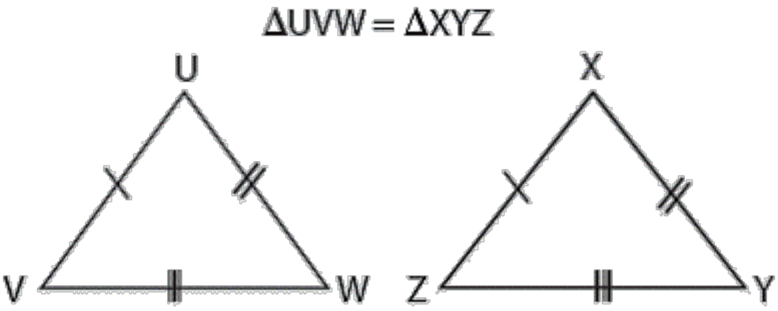


Fig. 10.32

4. The two triangles are right-angled triangles such that the hypotenuse and one other side of the triangle are equal to the hypotenuse and corresponding side of the other triangle (abbreviated as RHS). For example, in Fig. 10.33

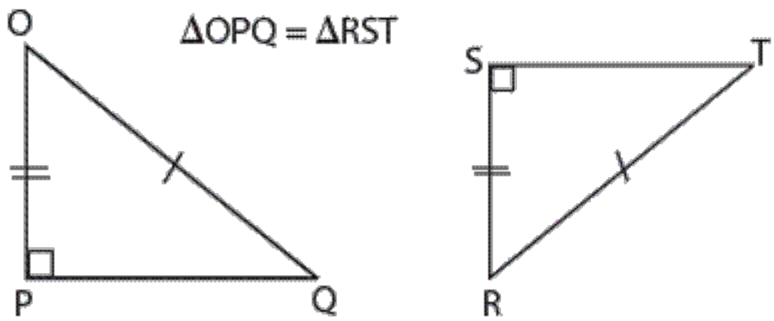


Fig. 10.33

IV. Naming Congruent Triangles

One way of naming congruent triangles is to give the letters in the correct order so that it is easy to relate all corresponding sides and angles to each other as shown in Figs. 10.29 –10.33.

Worked Example 10

ABCD is a quadrilateral in which $\overline{AD} = \overline{BC}$ and $\overline{AD} \parallel \overline{BC}$. Prove that triangle ABC is congruent to triangle CDA.

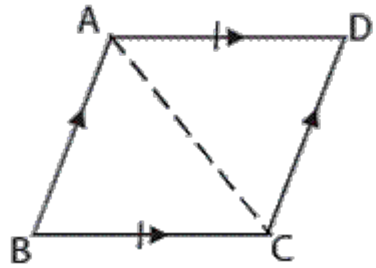


Fig. 10.34

Given: ABCD is a quadrilateral, $\overline{AD} = \overline{BC}$ and $\overline{AD} \parallel \overline{BC}$.

To prove: $\Delta ABC = \Delta CDA$

Construction: Join $\overline{AC}$

Proof: In ΔABC and ΔCDA,

$\overline{AD} = \overline{BC}$ (given)

$\overline{AC}$ is common

$\angle ACB = \angle CAD$ (alternate angles)

$\therefore \Delta ABC \equiv \Delta CDA$ (SAS)

Note: In the above example, we write $\Delta ABC \equiv \Delta CDA$ to mean triangle ABC is congruent to triangle CDA.

(iv) Properties of a parallelogram

A parallelogram is a quadrilateral (foursided figure) in which the opposite sides are equal. There are properties of a parallelogram arising from the parallel nature of its opposite sides such as:

1. The opposite sides are parallel.
2. The opposite sides are equal.
3. The opposite angles are equal.
4. The diagonals bisect one another.
5. The diagonals make pairs of alternate angles which are equal.
6. The allied angles forming adjacent vertices of a parallelogram are supplementary.

There are theorems which clearly state some of the properties of a parallelogram.

Theorem 6

In a parallelogram (i) The opposite sides are equal. (ii) The opposite angles are equal.

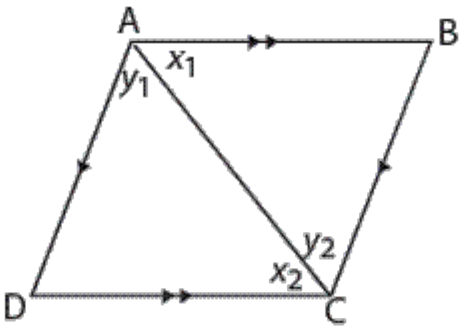


Fig. 10.35

Given: Parallelogram ABCD

To Prove: (i) $\overline{AB} = \overline{CD}, \overline{BC} = \overline{AD}$

(ii) $\angle B = \angle D, \angle A = \angle C$

Construction: Draw the diagonal $\overline{AC}$

Proof: In Δs ABC and CDA,

$x_1 = x_2$ (alternate angles, $\overline{AB} \parallel \overline{DC}$)

$y_1 = y_2$ (alternate angles, $\overline{AD} \parallel \overline{BC}$)

$\overline{AC}$ is common

$\therefore \Delta ABC = \Delta CDA$ (ASA)

$\therefore$ (i) $\overline{AB} = \overline{CD}, \overline{BC} = \overline{AD}$ (correspon ing sides)

(ii) $\angle B = \angle D$ (corresponding angl

$\angle A = x_1 + y_1$

$\angle C = x_2 + y_2$

$\angle A = \angle C$

Hence, in a parallelogram,
(i) The opposite sides are equal.
(ii) The opposite angles are equal.

Note: Most proofs are based on comparison of triangles because of the useful properties of congruency of triangles. In the above proofs, ΔABC ≡ ΔCDA. A diagonal bisects a parallelogram into two triangles of equal area.

Theorem 7

The diagonals of a parallelogram bisect one another.

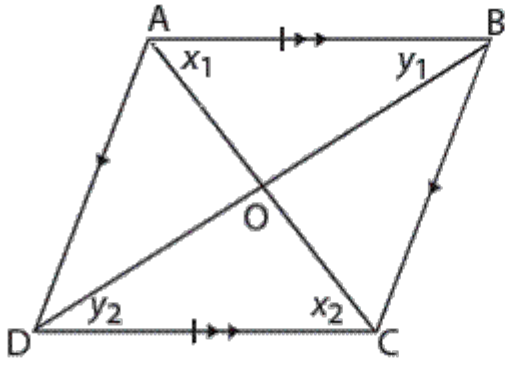


Fig. 10.36

Given: Parallelogram ABCD with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at O

To prove: $\overline{AO} = \overline{OC}$, $\overline{BO} = \overline{OD}$

Proof: In Δs AOB and COD,

$x_1 = x_2$ (alternate angles, $\overline{AB} \parallel \overline{CD}$)

$y_1 = y_2$ (alternate angles, $\overline{AB} \parallel \overline{CD}$)

$\overline{AB} = \overline{CD}$ (opposite sides of a parallelogram)

$\therefore \Delta AOB = \Delta COD$ (ASA)

$\therefore \overline{AO} = \overline{CO}$ (corresponding sides)

and $\overline{BO} = \overline{DO}$ (corresponding sides)

Worked Example 11

One angle of a parallelogram is 55°. Find the other angles.

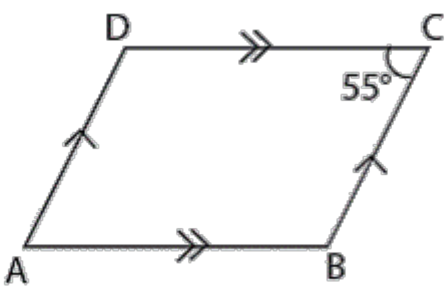


Fig. 10.37

$$\angle A = \angle C = 55^\circ \text{ (opposite angles of a parallelogram)}$$

$$\angle A + \angle D = 180^\circ \text{ (sum of interior angles, } AB \parallel DC\text{)}$$

$$\therefore \angle D = 180^\circ - \angle A$$

$$= 180^\circ - 55^\circ$$

$$= 125^\circ$$

$$\angle B = \angle D = 125^\circ \text{ (opposite angles of a parallelogram)}$$

(v) Intercept theorem

In Fig. 10.38, the line $\overline{AB}$ and $\overline{CD}$ cut the transversal $\overline{EF}$ into three parts. The part of the transversal cut off between the lines

is called an *intercept*. In Fig. 10.38, the line segment $\overline{GH}$ is the intercept.

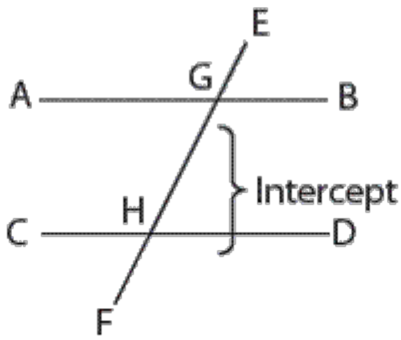


Fig. 10.38

Theorem 8

If there are three or more parallel straight lines and the intercepts made by them on any transversal are equal, then the corresponding intercepts on any other transversal are also equal.

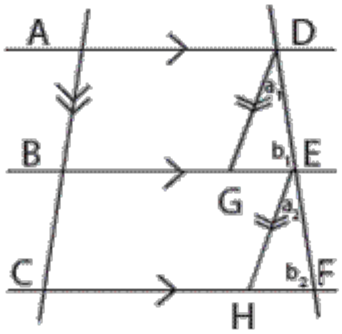


Fig. 10.39

Given: Three parallel lines cutting a fourth line at ABC, so that $\overline{AB} = \overline{BC}$ and cutting another line at D, E and F, respectively

To prove: $\overline{DE} = \overline{EF}$

Construction: Through D and E, draw $\overline{DG}$ and $\overline{EH}$ parallel to $\overline{AB}$.

Proof: ADGB is a parallelogram (opposite sides are parallel).

$\therefore \overline{AB} = \overline{DG}$ (opposite sides are equal)

Similarly, $\overline{EH} = \overline{BC}$ (in parallelogram BECH)

$\therefore \overline{DG} = \overline{EH}$ (given $\overline{AB} = \overline{BC}$)

In ΔDGE and ΔEHF,

$\overline{DG} = \overline{EH}$ (proved)

$a_1 = a_2$ (corresponding angles)

$b_1 = b_2$ (corresponding angles)

$\therefore \Delta DGE = \Delta EHF$ (AAS)

$\therefore \overline{DE} = \overline{EF}$

Worked Example 12

Find K in the figure below.

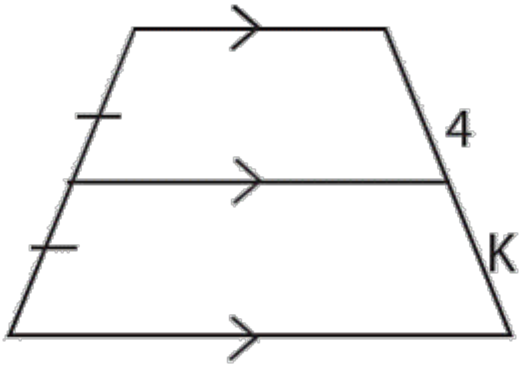


Fig. 10.40

Solution

$K = 4$ (intercept theorem)

Exercise 2

For questions 1 and 2, find the value of the marked angles.

1.

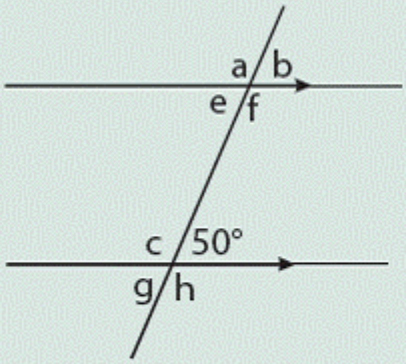


Fig. 10.41

2.

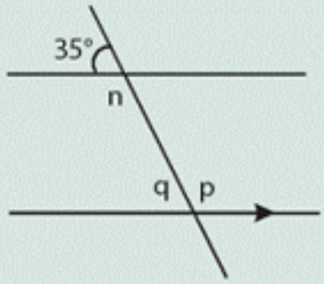


Fig. 10.41 Continued

3. In Fig. 10.42, $\overline{LK} \parallel \overline{PQ}$, reflex angle KLM = 241° and ∠QPM = 89°. What is the value of ∠LMP?

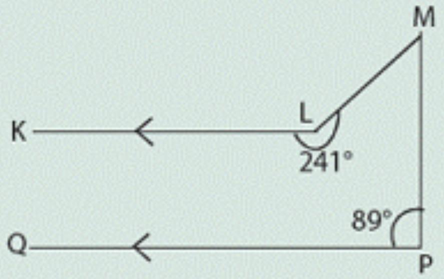


Fig. 10.42

4. In Fig. 10.43, $\overline{PQ} \parallel \overline{RS}$ and the angles are as shown. Find x. **(WAEC)**

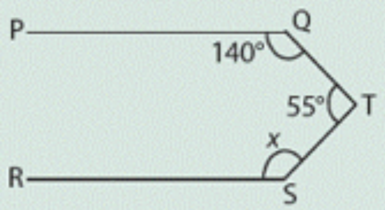


Fig. 10.43

5. In Fig. 10.44, $\overline{AB} \parallel \overline{CD}$. Find the values of n, p, q. **(WAEC)**

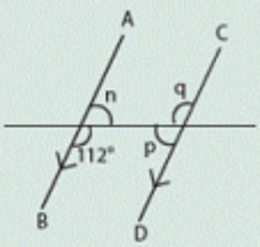


Fig. 10.44

6. In Fig. 10.45, $\overline{ML} \parallel \overline{PQ}$ and $\overline{NP} \parallel \overline{QR}$. If $\angle LMN = 40°$ and $\angle MNP = 55°$, find $\angle PQR$. **(WAEC)**

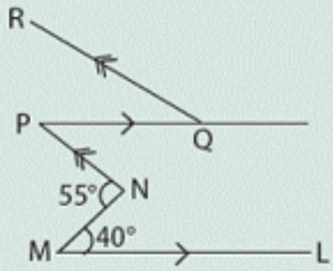


Fig. 10.45

7. In Fig. 10.46, QRS is a straight line. $\overline{QP} \parallel \overline{RT}$, $\angle PRQ = 56°$, $\angle QPR = 84°$ and $\angle TRS = x$, find x. **(WAEC)**

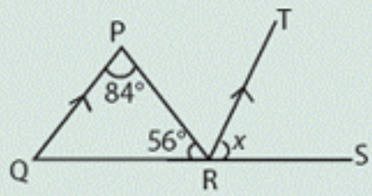


Fig. 10.46

8. In Fig. 10.47, NP = NO and MC ∥ OP. If $\overline{MNP}$ is a straight line, calculate angle x. **(WAEC)**

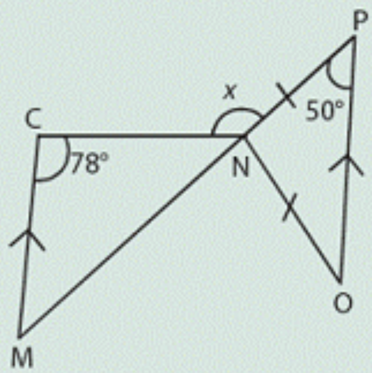


Fig. 10.47

9. The sum of the interior angles of a polygon with n sides is $900°$. Find the value of n. **(WAEC)**

10. The sum of the angles of a regular polygon is $2\,520°$. How many sides does the polygon have? **(WAEC)**

11. The angles of a pentagon are $x°$, $2x°$, $(x+60)°$, $(x+10)°$ and $(x-10)°$. Find the value of x.

12. ABCDE is a regular pentagon and rectangle AXYE is drawn on the side AE such that the vertices X and Y lie on the sides $\overline{BC}$ and $\overline{CD}$ respectively. Calculate the size of:
    (a) An interior angle of the pentagon.
    (b) $\angle BXA$. **(WAEC)**

13. If the angles of a pentagon are $52°$, $100°$, $110°$, $(z - 30)°$ and $42°$, what is the value of z? **(WAEC)**

14. If the exterior angles of a quadrilateral are $y°$, $(y + 5)°$, $(y + 15)°$ and $(3y + 10)°$, find $y°$. **(WAEC)**

15. In Fig. 10.48, $\angle BAE = \angle CDE = 90°$ and $\angle BCD = 80°$. Find x.

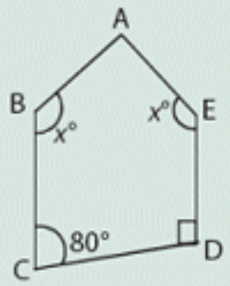


Fig. 10.48

16. In Fig 10.49, the angles marked are measured in degrees. Find x.

(WAEC)

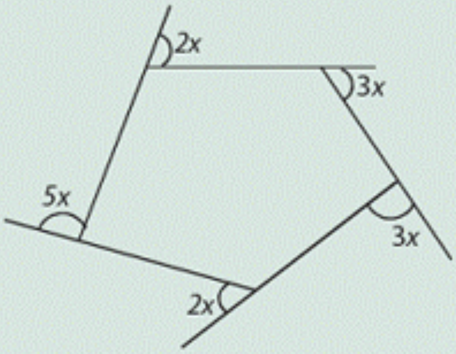


Fig. 10.49

17. Calculate the value of d in Fig. 10.50.

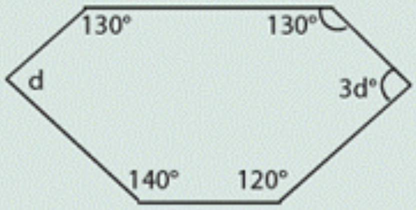


Fig. 10.50

18. In each problem in Fig. 10.51, state whether the pairs of triangles are congruent. Give a reason for your answer.

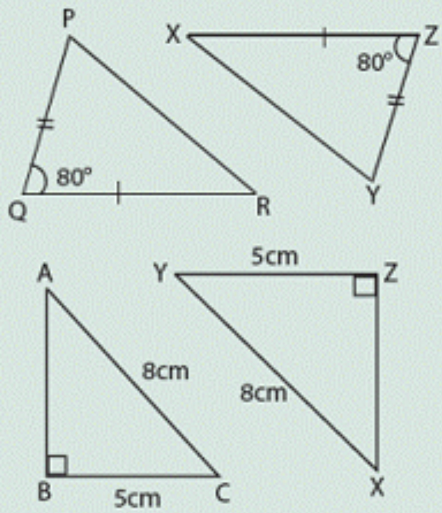


Fig. 10.51

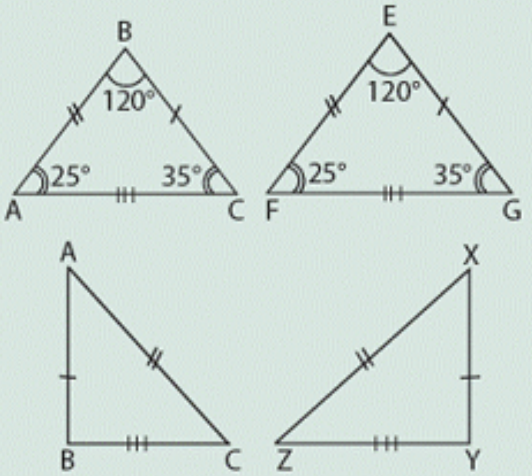


Fig. 10.51 Continued

19. For each part of Fig. 10.52, name the triangle which is congruent to ΔXYZ giving the letters in the correct order. In each case, state the condition for congruency.

(a)

(b)

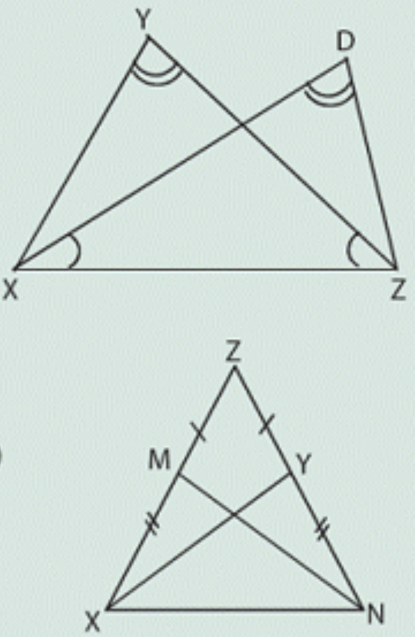


Fig. 10.52

20. ABCD is a quadrilateral in which $\overline{AB} = \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$. Prove that ABCE is a parallelogram.

21. PQRS is a quadrilateral in which the diagonals bisect each other at K. Prove that PQRS is a parallelogram.

22. In the parallelogram ABCD, a line is drawn through R parallel to the bisector of angle B to cut $\overline{AB}$ produced at X and $\overline{AD}$ produced at Y. Prove that ΔAXY is isosceles.

23. Find the unknown angles marked in Fig. 10.53.

Fig. 10.53

24. Find the lettered lengths in (a) and (b) in Fig. 10.54. All dimensions are in cm. **(WAEC)**

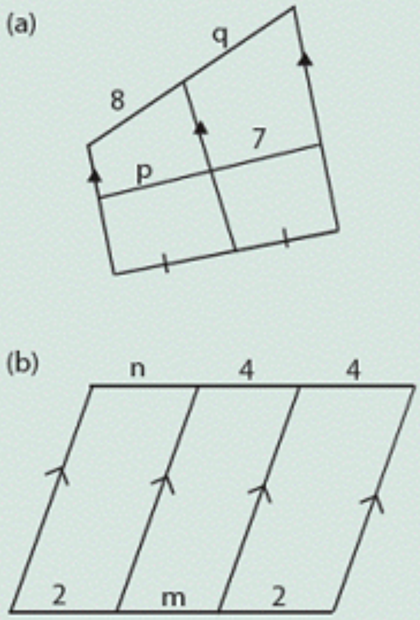


Fig. 10.54

SUMMARY

In this chapter, we have learnt the following:

- The four main steps used when writing out the proof of a statement are: Given, To Prove, Construction and Proof.
- The sum of angles in a triangle is 180°.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.
- If two parallel lines are cut by a transversal:
  - **(i)** The alternate angles are equal.
  - **(ii)** The corresponding angles are equal.
  - **(iii)** The interior angles on the same side of the transversal are supplementary.
- The sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles.
- The sum of the exterior angles of a polygon is 4 right angles.
- Two figures are congruent, if they have exactly the same shape and size.
- Two triangles are congruent, if
  - **(i)** Two sides and the included angle (SAS) or
  - **(ii)** Two angles and a corresponding side (ASA) or
  - **(iii)** All three sides (SSS) or
  - **(iv)** A right angle, hypotenuse and side (RHS) are equal.
- The properties of a parallelogram are listed on p. 165.
- If there are three or more parallel straight lines, and the intercepts made by them on any transversal are equal, then the corresponding intercepts on any other transversal are also equal. This is called Intercept Theorem.

GRADUATED EXERCISE

1. Find the size of each lettered angles in Fig. 10.55.

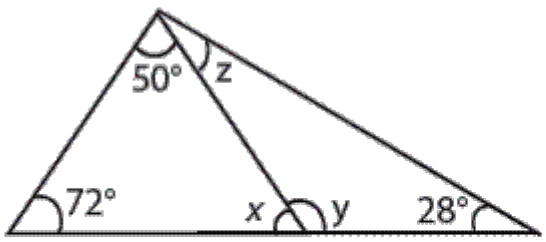


Fig. 10.55

2.

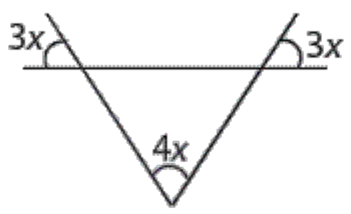


Fig. 10.56

Determine the value of x in Fig. 10.56.

(WAEC)

3. In Fig. 10.57, $z = 2x$.

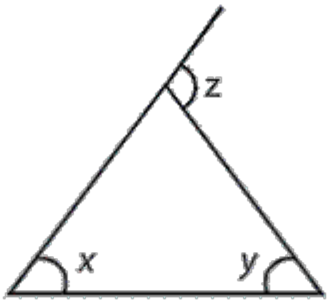


Fig. 10.57

Prove that: (a) $x = y$ and (b) $y = \frac{1}{2}z$.

4. In Fig. 10.58, what is the value of angle m?

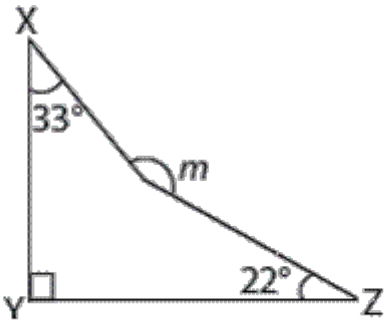


Fig. 10.58 (WAEC)

5. In Fig. 10.59, $\overline{PQT}$ is an isosceles triangle, $\overline{PQ} = \overline{QT}$, $\angle SRQ = 75°$, $\angle QPT = 25°$ and PQR is a straight line, find $\angle RST$.

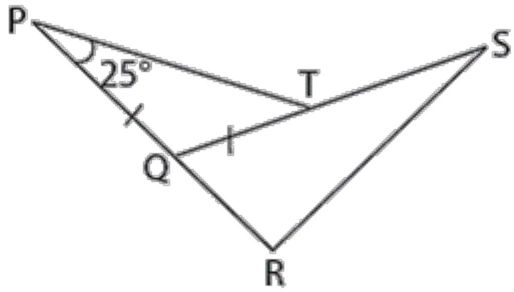


Fig. 10.59 (WAEC)

6. In Fig. 10.60, $\overline{PQ} = \overline{QR}$, $\overline{PS} = \overline{RS}$, $\angle PSR = 30°$ and $\angle PQR = 80°$. find $\angle SPQ$.

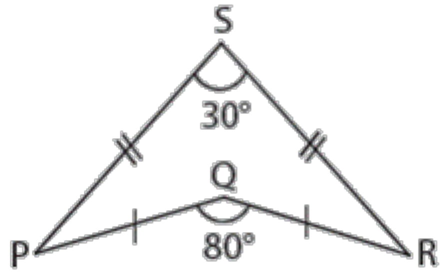


Fig. 10.60 (WAEC)

7. In Fig. 10.61, find the value of x.

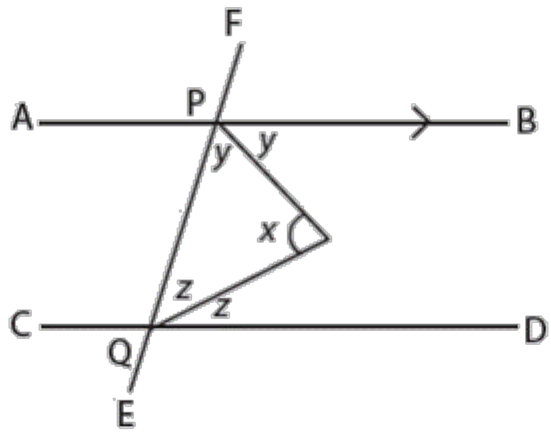


Fig. 10.61 (JAMB)

8. In Fig. 10.62, $\overline{AD} \parallel \overline{EG} \parallel \overline{BC}$ and $|AE| = |EB|$. Prove that $|EG| = \frac{1}{2}(|BC| + |AD|)$.

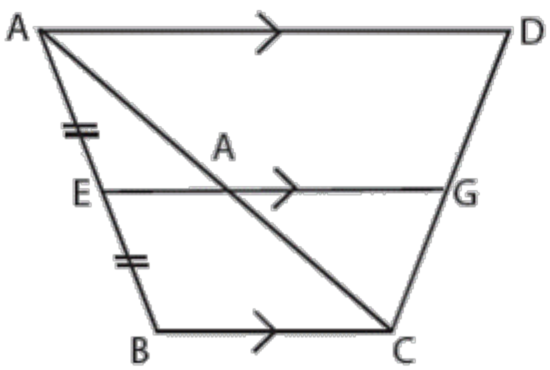


Fig. 10.62 (WAEC)

9. In Fig. 10.63, P, Q, R are parallel lines. What is the value of angle x?

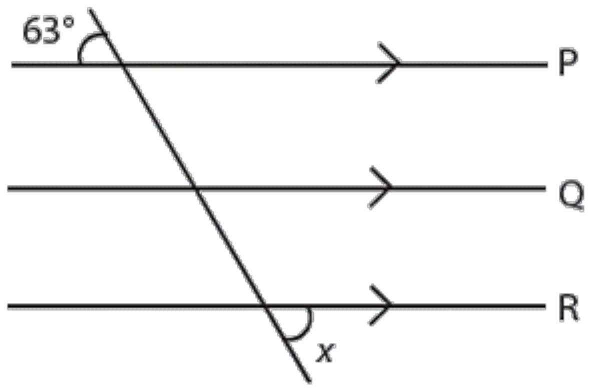


Fig. 10.63

10. In Fig. 10.64, $\overline{WU} \parallel \overline{YZ}$, $\overline{WY} \parallel \overline{VU}$, $|UZ| =$ 12 cm, $|VZ|$ = 6 cm and $|XU|$=8 cm. Determine the length of WU.

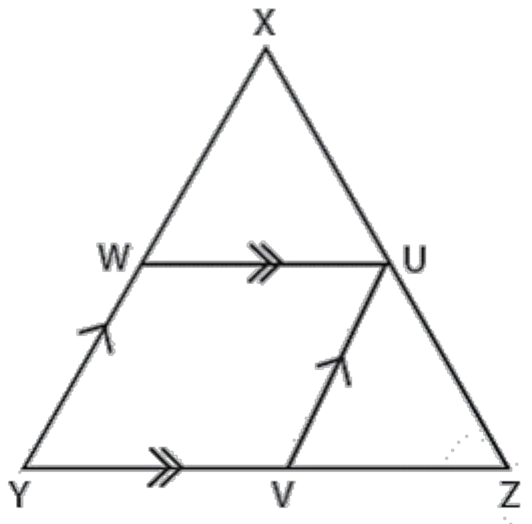


Fig. 10.64 (JAMB)

11. In Fig. 10.65, ΔPQT isosceles, $|PQ| = |QT|$, $\angle SRQ = 35°$, $\angle TPQ = 20°$ and $\overline{PQR}$ is a straight line. Calculate $\angle TSR$.

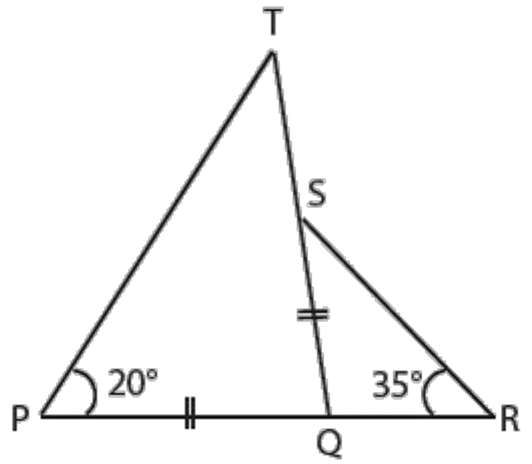


Fig. 10.65 (JAMB)

12. In Fig.10.66, find angle x.

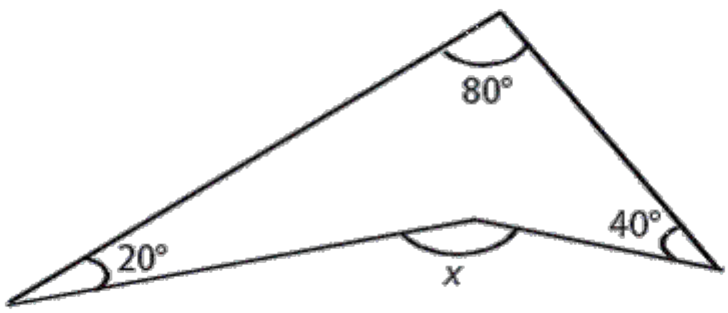


Fig. 10.66 (JAMB)

13. In Fig.10.67, FGHK is a rhombus. What is the value of angle x?

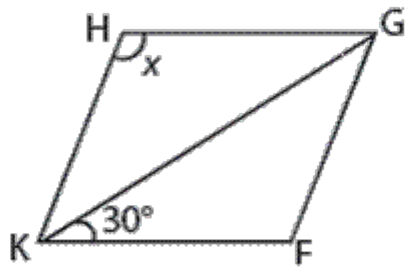


Fig. 10.67

14. In Fig. 10.68, the parallelogram WXYZ is such that |WZ| = 3 cm, |WX| = 8 cm and the perpendicular from Z to |WX| = 1.5 cm. What is the length of the perpendicular from Z to $\overline{XY}$?

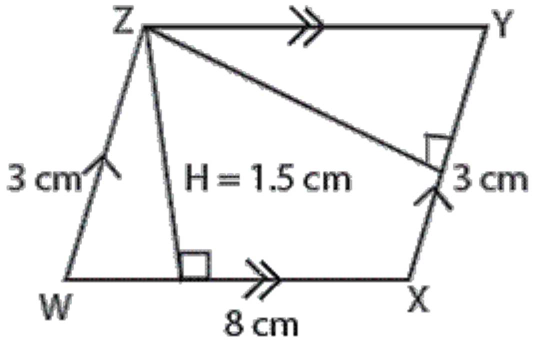


Fig. 10.68

15. In Fig. 10.69, |PS| = 7 cm and |RY| = 9 cm. The area of parallelogram PQRS is 56 cm². Find the area of trapezium PQTS.

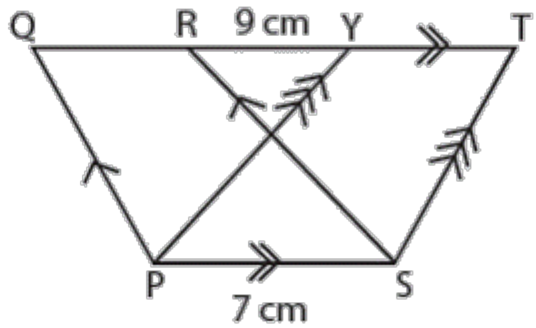


Fig. 10.69