**Chapter 6**

# Chapter 6

# Trigonometry

**OBJECTIVES**

At the end of this chapter, students should be able to:

1. draw graphs of sine and cosine for angles $0^\circ \le \theta \le 360^\circ$.
2. interpret/read graphs of trigonometric ratios.
3. solve graphical equations of simultaneous linear equations and trigonometric equations.

## I. Trigonometric Ratios of 0°, 30°, 45°, 60° and 90°

### (i) Length measurement of right-angled triangle

In any given right-angled triangle, the measurements of lengths of its sides are related to each other by Pythagoras rule as shown in Figure 6.1.

Given a right-angled triangle ABC, we have

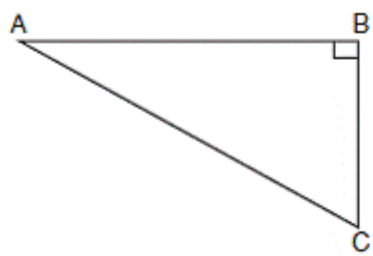



Figure 6.1

$$|AB|^2 + |BC|^2 = |AC|^2$$

$$\Rightarrow \quad |AB|^2 = |AC|^2 - |BC|^2$$

Also,

$$|BC|^2 = |AC|^2 - |AB|^2$$

## Worked Example 1

A cuboid has a diagonal whose length is 13 m, and base dimension of 2.5 m by 6 m. Calculate the height of the cuboid, correct to three significant figures.

### SOLUTION

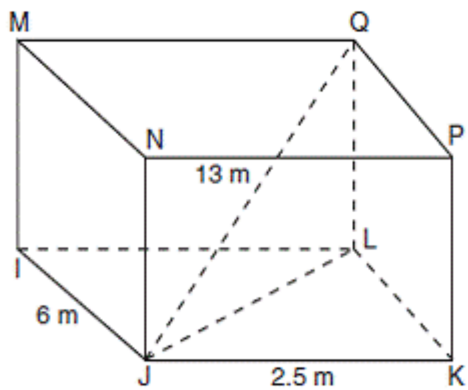



Figure 6.2

Consider the right-angled triangle JKL.

By Pythagoras rule,

$$|JL|^2 = |JK|^2 + |KL|^2$$

$$\Rightarrow \quad |JL|^2 = 2.5^2 + 6^2$$

$$= 6.25 + 36$$

$$= 42.25$$

$$|JL| = \sqrt{42.25}$$

$$= 6.5 \text{ m}$$

In $\Delta JLQ$,

$$|LQ|^2 = |JQ|^2 - |JL|^2$$
$$= 13^2 - 6.5^2$$
$$= (169 - 42.25)$$
$$= 126.75 \text{ m}$$
$$|LQ| = 11.258 \text{ m}$$
$$= 11.3 \text{ m (3 s.f.)}$$

## (II) Angular measurements of a right-angled triangle (Reviision)

There exists a relationship between the angles of a right-angled triangle and the length of its sides.

The relationship is always established through the use of trigonometric ratios.

Given a right-angled triangle as shown in Figure 6.3

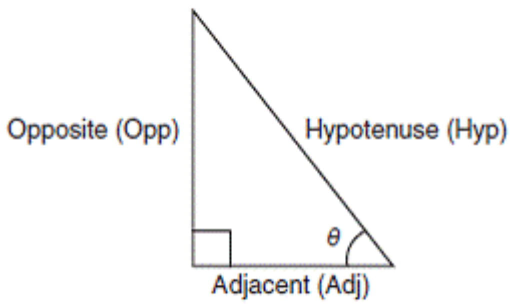



Figure 6.3

$$\text{sine } \theta = \frac{\text{Opp}}{\text{Hyp}} \quad \text{cosecant } \theta = \frac{\text{Hyp}}{\text{Opp}} = \frac{1}{\sin \theta}$$

$$\text{cosine } \theta = \frac{\text{Adj}}{\text{Hyp}} \quad \text{secant } \theta = \frac{\text{Hyp}}{\text{Adj}} = \frac{1}{\cos \theta}$$

$$\text{tangent } \theta = \frac{\text{Opp}}{\text{Adj}} \quad \text{cotangent } \theta = \frac{\text{Adj}}{\text{Opp}}$$
$$= \frac{1}{\tan \theta}$$

### (III) Special angles

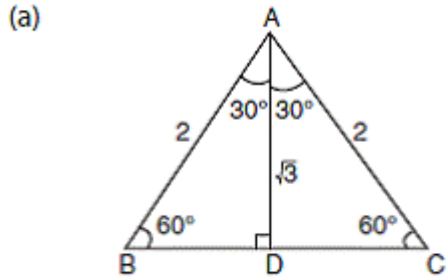
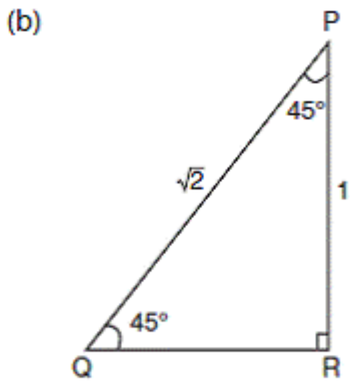



Figure 6.4



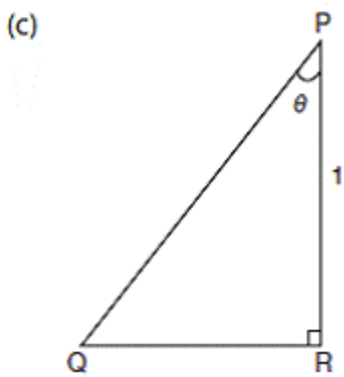




Figure 6.4 Continued

Using Figure 6.4,

Table 6.1

| | (a) | | (b) | (c) | |
|---|---|---|---|---|---|
| **Trig. ratio** | **30°** | **60°** | **45°** | **0°** | **90°** |
| **Sine** | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{0}{1}=0$ | $\frac{1}{1}=1$ |
| **Cosine** | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{1}=1$ | $\frac{0}{1}=0$ |
| **Tangent** | $\frac{1}{\sqrt{3}}$ | $\frac{\sqrt{3}}{1}$ | $\frac{1}{1}=1$ | $\frac{0}{1}=0$ | $\frac{1}{0}=0$ |
| **Cosecant** | $\frac{2}{1}$ | $\frac{2}{\sqrt{3}}$ | $\frac{\sqrt{2}}{1}$ | | |
| **Secant** | $\frac{2}{\sqrt{3}}$ | 2 | $\frac{\sqrt{2}}{1}$ | | |
| **Cotangent** | $\sqrt{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{1}=1$ | | |

## Worked Example 2

Simplify without using tables:

(a) $\dfrac{\sin 30^\circ - \cos 45^\circ}{\tan 60^\circ}$

(b) $\dfrac{\sin 60^\circ + \tan 45^\circ}{\sin 30^\circ}$

### SOLUTION

(a) $\dfrac{\sin 30^\circ - \cos 45^\circ}{\tan 60^\circ} = \dfrac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\sqrt{3}}$

Numerator: $\dfrac{1}{2} - \dfrac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{2} - 2}{2\sqrt{2}}$

Denominator: $\sqrt{3}$

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\sqrt{2} - 2}{2\sqrt{2} \times \sqrt{3}} = \frac{\sqrt{2} - 2}{2\sqrt{6}}$$

$$= \frac{(\sqrt{2} - 2)\sqrt{6}}{(2\sqrt{6})(\sqrt{6})} = \frac{\sqrt{12} - 2\sqrt{6}}{2 \times 6}$$

$$= \frac{2\sqrt{3} - 2\sqrt{6}}{12} = \frac{2(\sqrt{3} - \sqrt{6})}{12}$$

$$= \left(\frac{\sqrt{3} + \sqrt{6}}{6}\right)$$

$$\therefore \frac{\sin 30^\circ - \cos 45^\circ}{\tan 60^\circ} = \frac{\sqrt{3} - \sqrt{6}}{6}$$

(b) $\dfrac{\sin 60^\circ + \tan 45^\circ}{\sin 30^\circ} = \dfrac{\frac{\sqrt{3}}{2} + \frac{1}{1}}{\frac{1}{2}}$

Numerator: $\dfrac{\sqrt{3}}{2} + \dfrac{1}{1}$

$= \dfrac{\sqrt{3}+2}{2}$

Denominator: $\dfrac{1}{2}$

$\dfrac{\text{Numerator}}{\text{Denominator}} = \dfrac{\frac{\sqrt{3}+2}{2}}{\frac{1}{2}}$

$= 2(\sqrt{3}+2)$

$\therefore \dfrac{\sin 60^\circ + \tan 45^\circ}{\sin 30^\circ} = 2(\sqrt{3}+2)$

## Worked Example 3

Given that $\sin \frac{1}{2}$ and $\theta$ is less than 90°, evaluate $\dfrac{\cos\theta + \tan\theta}{\operatorname{cosec}\theta}$, without using tables.

**SOLUTION**

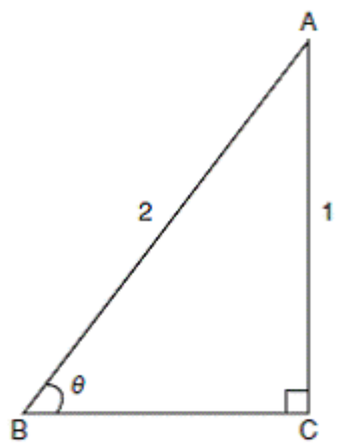



Figure 6.5

From Figure 6.5, $\sin\theta = \frac{1}{2}$

Using Pythagoras rule,

$$|BC|^2 = |AB|^2 - |AC|^2$$

$$= 2^2 - 1^2$$

$$= 4 - 1$$

$$= 3$$

$$|BC| = \sqrt{3}$$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad$ $\tan\theta = \frac{1}{\sqrt{3}}$ $\quad$ $\operatorname{cosec}\theta = 2$

$$\therefore \frac{\cos\theta + \tan\theta}{\operatorname{cosec}\theta} = \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$$

Numerator: $\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$

$$= \frac{3+2}{2\sqrt{3}}$$

$$= \frac{5}{2\sqrt{3}}$$

Denominator: $\quad 2$

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{5}{2\sqrt{3}\times 2}$$

$$= \frac{5}{4\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{4\sqrt{3}\times\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{4\times 3}$$

$$= \frac{5\sqrt{3}}{12}$$

$$\therefore \quad \frac{\cos\theta + \tan\theta}{\operatorname{cosec}\theta} = \frac{5\sqrt{3}}{12}$$

### (iv) Trigonometry identities

Consider Figure 6.6.

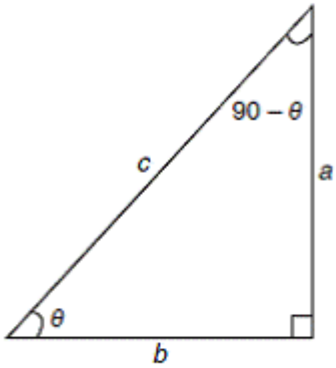



Figure 6.6

1. $$\left.\begin{array}{l} \sin\theta = \frac{a}{c} \\ \cos(90^\circ - \theta) = \frac{a}{c} \end{array}\right\} \sin\theta = \cos(90^\circ - \theta)$$

2. $$\left.\begin{array}{l} \cos\theta = \frac{b}{c} \\ \sin(90^\circ - \theta) = \frac{b}{c} \end{array}\right\} \cos\theta = \sin(90^\circ - \theta)$$

3. $$\left.\begin{array}{l} \tan\theta = \frac{a}{b} \\ \frac{\sin\theta}{\cos\theta} = \frac{a}{b} \div \frac{b}{c} = \frac{a}{c} \times \frac{c}{b} = \frac{a}{b} \end{array}\right\} \tan\theta = \frac{\sin\theta}{\cos\theta}$$

4. $$\sin^2\theta + \cos^2\theta = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$$

$$= \frac{a^2 + b^2}{c^2}$$

But as per Pythagoras rule, $c^2 = a^2 + b^2$

$$\therefore \quad \frac{a^2 + b^2}{c^2} = \frac{a^2 + b^2}{a^2 + b^2} = 1$$

$$\therefore \quad \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \sin^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$

5. $\left.\begin{array}{l} \operatorname{cosec}\theta = \dfrac{c}{a} \\ \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{a}{c}} = \dfrac{c}{a} \end{array}\right\} \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

6. $\left.\begin{array}{l} \sec\theta = \dfrac{c}{b} \\ \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{b}{c}} = \dfrac{c}{b} \end{array}\right\} \sec\theta = \dfrac{1}{\cos\theta}$

7. $\left.\begin{array}{l} \cot\theta = \dfrac{b}{a} \\ \dfrac{1}{\tan\theta} = \dfrac{1}{\frac{a}{b}} = \dfrac{b}{a} \end{array}\right\} \cot\theta = \dfrac{1}{\tan\theta}$

8. $\sin^2\theta + \cos^2\theta = 1 \ldots(*)$

   Dividing throughout by $\sin^2\theta$

   $$\Rightarrow \quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

   $$\cot^2\theta = \operatorname{cosec}^2\theta - 1$$

   Dividing throughout by $\cos^2\theta$

   $$\Rightarrow \quad \tan^2\theta + 1 = \sec^2\theta$$

   $$\tan^2\theta = \sec^2\theta - 1$$

## Worked Example 4

Show that $\tan^2\theta + 1 = \sec^2\theta$.

**SOLUTION**

$$\tan^2\theta + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1$$

$$= \frac{\sin^2 + \cos^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} \text{ (since } \sin^2\theta + \cos^2\theta = 1)$$

$$= \sec^2\theta$$

$\therefore \tan^2\theta + 1 = \sec^2\theta.$

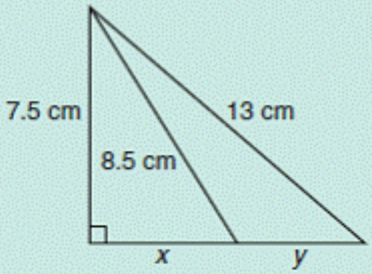
### Exercise 1

1. If $\cos\theta = \frac{1}{2}$, find the value of

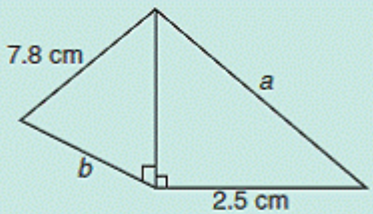
$$\frac{\sin\theta - 2\cos^2\theta}{1+\cos\theta}.$$

2. Calculate the values of the unknown lengths of the following figures:

(a)




(b)




(c)

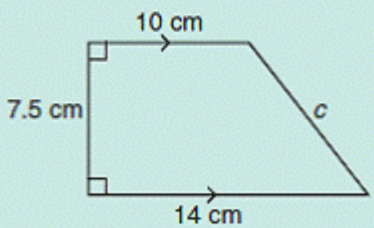



Figure 6.7

3. Three observation posts P, Q and R are such that Q is due east of P and R is due north of Q. If |PQ| = 5 km and |PR| = 10 km, find |QR|. (WAEC)

4. In Figure 6.8, ABCD is a kite. Calculate the length of the diagonals.

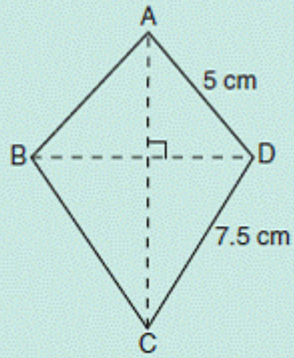



Figure 6.8

## II. The General Angle

The Cartesian plane is divided into four parts, each referred to as a quadrant.

1st Quadrant ($0^\circ < \theta < 90^\circ$)

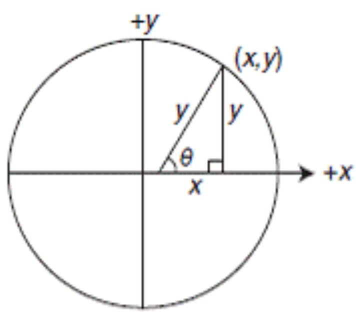



Figure 6.9

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

For acute angles, all trigonometric ratios are positive.

2nd Quadrant $90° < \theta < 180°$

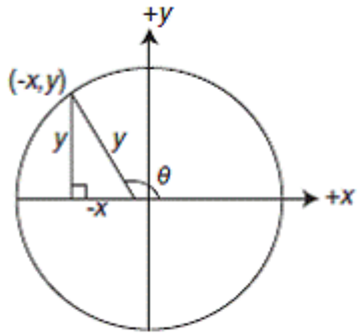



Figure 6.10

$$\sin\theta = \frac{y}{r} = \sin(180° - \theta)$$

$$\cos\theta = \frac{-x}{r} = -\cos(180° - \theta)$$

$$\tan\theta = \frac{y}{-x} = -\tan(180° - \theta)$$

For $90° < \theta < 180°$, both cos and tan are negative but sin is positive.

3rd Quadrant $180° < \theta < 270°$

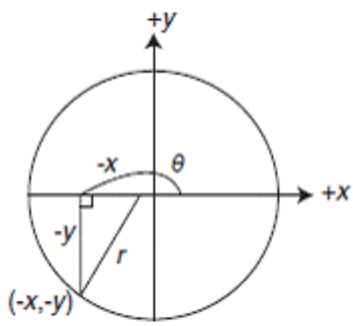



Figure 6.11

$$\sin\theta = \frac{-y}{r} = \sin(\theta - 180°)$$

$$\cos\theta = \frac{-x}{r} = -\cos(\theta - 180°)$$

$$\tan\theta = \frac{-y}{-x} = \frac{y}{x} = \tan(\theta - 180°)$$

For $180^0 < \theta < 270°$, sin and cos are negative, but tan is positive.

4th Quadrant $270° < \theta < 360°$

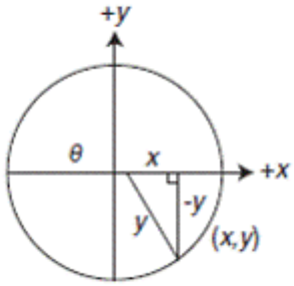



Figure 6.12

$$\sin\theta = \frac{-y}{r} = -\sin(360° - \theta)$$

$$\cos\theta = \frac{x}{r} = \cos(360° - \theta)$$

$$\tan\theta = \frac{-y}{x} = -\tan(360° - \theta)$$

For $270° < \theta < 360°$, sin and tan are negative but cos is positive.

**Note:** When $\theta = 360°$, OX = 1, OY = 0

$$\therefore \sin 360° = 0$$

$$\cos 360° = 1$$

$$\tan 360° = 0$$

Figure 6.11 shows the summary of the sign of the trigonometric ratios in the four quadrants.

| | |
|---|---|
| S; sin is +ve<br>cos is –ve<br>tan is –ve | A; all are positive<br>sin is +ve<br>cos is +ve<br>tan is +ve |
| T; tan is +ve<br>sin is –ve<br>cos is –ve | C; cos is +ve<br>sin is –ve<br>tan is –ve |

Figure 6.13 CAST rule

## Worked Example 5

Evaluate the following:

(a) sin 205° (b) sin 330°

(c) cos 300° (d) $\tan(-137^0)$

(e) $\cos(-195^0)$

### SOLUTION

(a) 205° is in the third quadrant where sine is negative.

$$\therefore \sin 205° = -\sin (205° - 180°)$$

$$= -\sin 25°$$

But from the tables, $\sin 25° = 0.4226$

$$\therefore \sin 205° = -0.4226$$

(b) 330° is in the fourth quadrant where sine is negative.

$$\therefore \sin 330° = -\sin (360° - 330°)$$

$$= -\sin 30°$$

From the tables, $\sin 30° = 0.5$

$$\therefore \sin 330° = -0.5 \text{ or } -\frac{1}{2}$$

(c) 300° is in the fourth quadrant where cosine is positive.

$$\therefore \cos 300° = \cos (360° - 300°)$$

$$= \cos 60°$$

From the tables, $\cos 60° = 0.5$

$$\therefore \cos 300° = 0.5 \text{ or } \frac{1}{2}$$

(d) $\tan (-137°) = -\tan 137°$

$$\therefore \tan (-\theta) = -\tan \theta$$

But 137° is in the second quadrant where tangent is negative.

Then,

$$-\tan 137° = -[-\tan (180° - 137°)]$$

$$= -(-\tan 43°)$$

$$= \tan 43°$$

From the tables, $\tan 43° = 0.9325$

$$\therefore \tan (-137) = 0.9325$$

(e) $\cos(-195°) = -\cos 195°$

$(\therefore \cos(-\theta) = -\cos\theta)$

But 195° is in the third quadrant where cosine is negative.

That is,

$$-\cos 195° = -[-\cos(195° - 180°)]$$

$$= -(-\cos 15°)$$

$$= \cos 15°$$

From the tables, $\cos 15° = 0.9659$

$$\therefore \cos(-195°) = 0.9659$$

## Worked Example 6

From the tables, determine the angle $\theta$ such that $\theta$ lies between 0° and 360°, if

(a) $\sin\theta = 0.8660$ (b) $\cos\theta = -0.7986$
(c) $\tan\theta = 11.43$ (d) $\cos\theta = 0.9744$
(e) $\tan\theta = -1.732$

**SOLUTION**

(a) $\sin\theta = 0.8660$ Since $\theta$ is positive, it must be either in the first or in the second quadrant.

$$\therefore \theta = 60°$$

$$\text{or } \theta = 180° - 60° = 120°$$

$$\therefore \theta = 60° \text{ or } 120°$$

(b) $\cos\theta = -0.7986$

$\Rightarrow \theta = \cos^{-1}(-0.7986) = -37°$

Since $\theta$ is negative, it must be either in the second or in the third quadrant.

$\therefore \theta = 180° - 37° = 143°$

or $\theta = 180° + 37° = 217°$

(c) $\tan\theta = 11.43$

Since $\theta$ is positive, it will be either in the first or in the third quadrant.

$\therefore \theta = 85°$

or $\theta = 180° + 85°$

(d) $\cos\theta = 0.9744$

Since $\theta$ is positive, it will be either in the first or in the third quadrant.

$\therefore \theta = 13°$

or $\theta = 360° - 13° = 247°$

(e) $\tan\theta = -1.732$

Since $\theta$ is negative, it will be either in the second or in the fourth quadrant.

Either $\theta = 60°$

or $\theta = 360° - 60° = 300°$

**Exercise 2**

**Evaluate the following:**

1. $\cos 80°$
2. $\sin(-125°)$
3. $\sin 205°$
4. $\cos 120°$

5. $\tan 320°$
6. $\tan(-60°)$
7. $\sin 244°$
8. $\sin(-266°)$
9. $\tan(-359°)$
10. $\sin 140°$
11. $\sin(-330°)$
12. $\cos(-298°)$
13. $\tan(-115°)$
14. $\sin 320°$
15. $\cos(-299°)$
16. $\tan(-45°)$
17. $\sin 135°$
18. $\cos(-120°)$
19. $\sin 240°$
20. $\cos(-122°)$

From the table, determine angle $\theta$ such that $\theta$ lies between 0° and 360°.

21. $\sin\theta = 0.9285$
22. $\sin\theta = 0.2410$
23. $\cos\theta = 0.7109$
24. $\tan\theta = 0.6741$
25. $\tan\theta = -2.145$
26. $\cos\theta = -0.8273$
27. $\tan\theta = -1.743$
28. $\cos\theta = -0.9778$
29. $\sin\theta = -0.9778$

## III. Sine, Cosine and Tangent Graphs Between 0° and 360°

### (I) Sine and cosine graphs

We shall use the CAST rule to find the trigonometric ratios of sine and cosine of
angles 0°to 360°.
While plotting the graphs of sine, cosine and tangent, we must first prepare its table of values and then use the table to plot the graphs.

**Worked Example 7**

Draw the graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$, using scale of 2 cm to 30° on the $\theta$-axis and 2 cm to 1 unit on the $y$-axis.
Find
(a) The maximum value of $y$ and the corresponding value of $\theta$.
(b) The minimum value of $y$ and the corresponding value of $\theta$.
(c) The value of $y$ for which $\theta = 135°$.
(d) The value of $\theta$ for which $y = 0.5$.

## SOLUTION

First: Prepare the table of values from $0° \leq \theta \leq 360°$ in the interval of 30°.

$$y = \sin \theta$$

Table 6.2

| $\theta$ | $y = \sin \theta$ |
|---|---|
| 0° | 0 |
| 30° | 0.5 |
| 60° | 0.9 |
| 90° | 1 |
| 120° | 0.9 |
| 150° | 0.5 |
| 180° | 0 |
| 210° | –0.5 |
| 240° | –0.9 |
| 270° | –1 |
| 300° | –0.9 |
| 330° | –0.5 |
| 360° | 0 |

Second: Plot the graph. Use a convenient scale when scales are not given. But when scales are given, ensure you abide by the scales strictly.

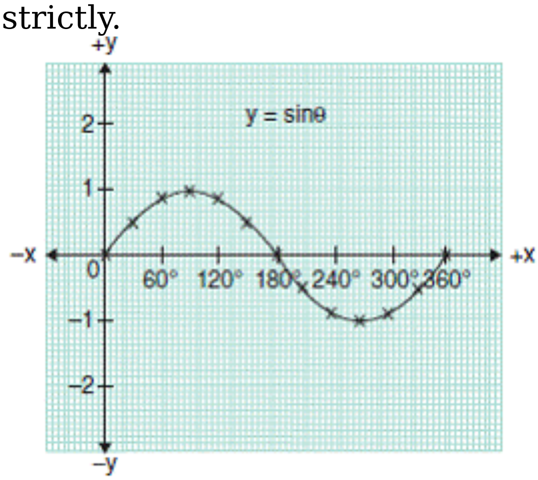



Figure 6.14

(a) $y = 1$ and $\theta = 90°$

(b) $y = -1$ and $\theta = 270°$

(c) $y = 0.7$

(d) $\theta = 30°$ or $150°$

## Worked Example 8

Draw the graph of $y = \cos\theta$ from $0° \leq \theta \leq 360°$, using the scale of 2 cm to 30° on the $\theta$-axis and 2 cm to 1 unit on the $y$-axis. Find

a) The root of the equation $\cos\theta = 0$

b) The minimum value of $y$ and the corresponding value of $\theta$

c) The value of $y$ for which $\theta = 75°$

**SOLUTION**

Table 6.3

| $\theta$ | $y = \cos\theta$ |
|---|---|
| 0° | 0 |
| 30° | 0.9 |
| 60° | 0.5 |
| 90° | 1 |
| 120° | 0.5 |
| 150° | 0.9 |
| 180° | 0 |
| 210° | –0.9 |
| 240° | –0.5 |
| 270° | –1 |
| 300° | –0.5 |
| 330° | –0.9 |
| 360° | 0 |

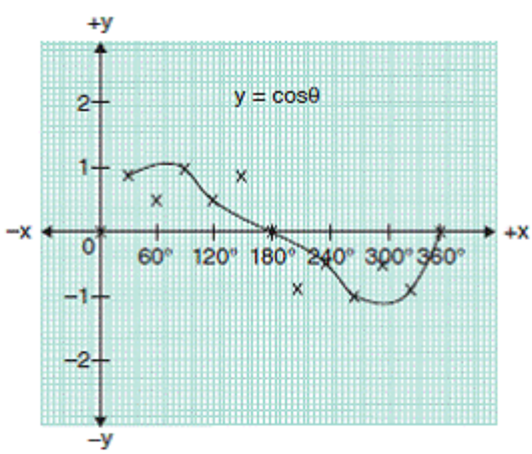



Figure 6.15

(a) either $\theta = 90°$ or $270°$

(b) $y = -1$ and $\theta = 180°$

(c) $y = 0.3$

### (ii) Tangent graph

The graph of a tangent is entirely different from that of sine and cosine. Unlike sine and cosine graphs, tangent graph tends to infinity when $\theta$ is an odd multiple of 90°. That is, tan $\theta$ is not defined when $\theta = 90°, 270°$, etc. This is because tan $\theta$ = $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cos\theta = 0$ when $\theta$ is an odd multiple of 90°.

## Worked Example 9

Draw the graph of tan $\theta$ from $\theta = 0°$ to $360°$.

Table 6.4

| $\theta$ | tan $\theta$ |
|---|---|
| 0° | 0 |
| 30° | 0.6 |
| 60° | 1.7 |
| 90° | 0 |
| 120° | –1.7 |
| 150° | –0.6 |
| 180° | 0 |
| 210° | 0.6 |
| 240° | 1.7 |
| 270° | 0 |
| 300° | –1.7 |
| 330° | –0.6 |
| 360° | 0 |

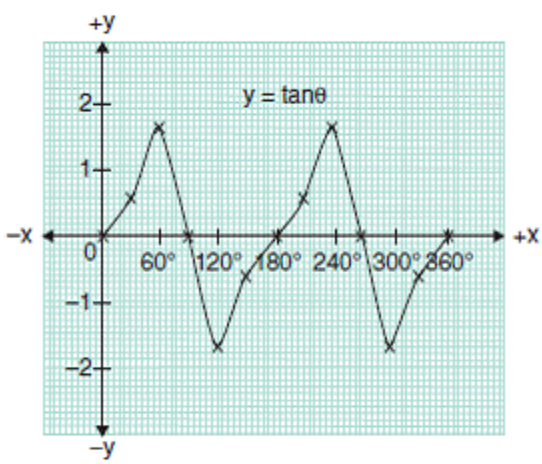



Figure 6.16

## Worked Example 10

(a) Copy and complete the table for the relation $y = 2 \sin x - \cos 2x$.

Table 6.5(a)

| $x$ | $y$ |
|---|---|
| 0 | |
| 30 | 0.5 |
| 60 | |
| 90 | |
| 120 | |
| 150 | |
| 180° | –1.0 |

Using a scale of 2 cm to 30° on the $x$-axis and 2 cm to 0.5 unit on the $y$-axis, draw the graph of $y = 2 \sin x - \cos 2x$ for $0° \leq \theta \leq 180°$.

(b) Using the same axes, draw the graph of $y = 1.25$.

(c) Use your graphs to find the following:

(i) Values of $x$ for which $2 \sin x - \cos 2x = 0$.

(ii) Roots of the equation $2 \sin x - \cos 2x = 1.25$.

(d) The equation of the line of symmetry.

## SOLUTION

Table 6.5(b)

| $x$ | $y$ |
|---|---|
| 0 | –1.0 |
| 30 | 0.5 |
| 60 | 2.2 |
| 90 | 3.0 |
| 120 | 2.2 |
| 150 | 0.5 |
| 180 | –1.0 |

When $x = 0°$

$$y = 2\sin 0° - \cos(2 \times 0°)$$
$$= 2 \times 0 - 1$$
$$= 0 - 1$$
$$= -1$$

When $x = 60°$

$$y = 2 \sin 60° - \cos(2 \times 60°)$$
$$= 2 \sin 60° - \cos 120°$$
$$= 2 \times 0.8660 - (-0.5)$$
$$= 1.732 + 0.5$$
$$= 2.232$$
$$= 2.2$$

When $x = 90°$

$$
\begin{aligned}
y &= 2\sin 90° - \cos(2 \times 90°) \\
&= 2\sin 90° - \cos 180° \\
&= 2 \times 1 - (-1) \\
&= 2 + 1 \\
&= 3
\end{aligned}
$$

When $x = 120°$

$$
\begin{aligned}
y &= 2\sin 120° - \cos(2 \times 120°) \\
&= 2\sin 120° - \cos 240° \\
&= 2 \times 0.8660 - (-0.5) \\
&= 1.732 + 0.5 \\
&= 2.232 \\
&= 2.2
\end{aligned}
$$

When $x = 150°$

$$
\begin{aligned}
y &= 2\sin 150° - \cos(2 \times 150°) \\
&= 2\sin 150° - \cos 300° \\
&= 2 \times 0.5 - 0.5 \\
&= 1 - 0.5 \\
&= 0.5
\end{aligned}
$$

## Exercise 3

1. Draw the graph of the following trigonometric functions in the interval of 30° within the range of 0° to 360° and in each equation, identify the roots:
   (a) $y = \sin 2x$
   (b) $y = \cos 5x$
   (c) $y = \cos 2x - 2\sin x$

(d) $y = \frac{1}{2}\cos 2x - \sin 3x$

(e) $y = \sin(x + 30°)$

(f) $y = 1 + \sin x$

(g) $y = 2\sin x$

(h) $y = 3\cos 3x - 2\sin x$

(i) $y = 3\sin x$

(j) $y = \frac{1}{2}\sin x$

2. Draw the graph of $2\sin\theta + 3\cos\theta$ between $\theta = 0°$ and $\theta = 90°$. Find the following from your graph:

   (a) The maximum value of $2\sin\theta + 3\cos\theta$.

   (b) The value of $\theta$ when $2\sin\theta + 3\cos\theta = 3.2$.

   (c) The value of $\theta$ when $2\sin\theta + 3\cos\theta = \frac{0}{10}$.

3. Solve the following equations graphically:

   (a) $2\sin x + \cos x = -1.5$, $0 \le \theta \le 180°$

   (b) $\sin 3x - 3\sin x = 2.5$, $0 \le \theta \le 180°$

   (c) $2\sin x + 3\cos x = 2.5$, $0 \le \theta \le 360°$

   (d) $3\cos 2x = 1, 0 \le \theta \le 180°$

   (e) $3\cos x + 2 = 0, 0 \le \theta \le 180°$

   (f) $4\cos 2x = 3, 0 \le \theta \le 180°$

   (g) $2\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) = 0$, $0 \le \theta \le 180°$

   (h) $2\cos 3x - 3\sin 3x = 2$, $0 \le \theta \le 180°$

   (i) $4\sin\theta + 2\cos\theta = 1$, $0 \le \theta \le 180°$

## SUMMARY

**In this chapter, we have learnt the following:**

❖ Given a right-angled triangle ABC where the right angle is at B,

$$|AB|^2 + |BC|^2 = |AC|^2$$

❖ There exists a relationship between the angle of a right-angled triangle and the length of it sides. Given an acute angle $\theta$ (say)

$$\text{sine } \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{cosine } \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{tangent } \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\text{cosecant } \theta = \frac{\text{Hypotenuse}}{\text{Opposite}}$$

$$\text{secant } \theta = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

$$\text{cotangent } \theta = \frac{\text{Adjacent}}{\text{Opposite}}$$

❖ (a) $\sin\theta = \cos(90° - \theta)$

(b) $\cos\theta = \sin(90° - \theta)$

(c) $\tan\theta = \frac{\sin\theta}{\sin\theta}$

(d) $\sin^2\theta + \cos^2\theta = 1$

(e) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(f) $\sec\theta = \frac{1}{\cos\theta}$

(g) $\cot\theta = \frac{1}{\tan\theta}$

(h) $\tan^2\theta + 1 = \sec^2\theta$

❖ Special angles of the trigonometric ratios

Table 6.6

| $\theta$ | 30° | 60° | 45° |
|---|---|---|---|
| sin | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ |
| cos | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ |
| tan | $\frac{1}{\sqrt{3}}$ | $\frac{\sqrt{3}}{1}$ | 1 |

❖ We can use the CAST rule to determine the trigonometric ratios of sine, cosine and tangent of angles from 0°to 360°.

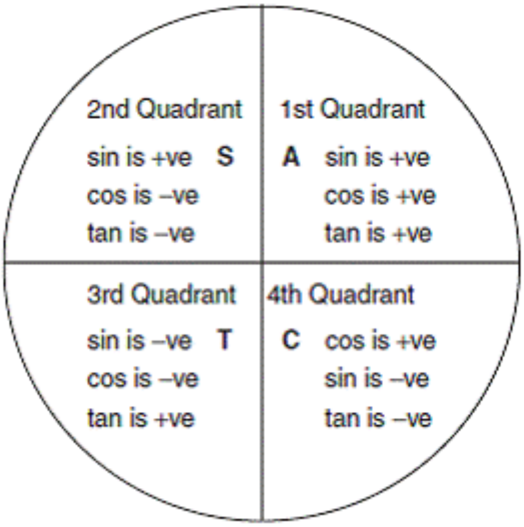



Figure 6.17

Both the sign (positive and negative) and the magnitude (size) of the trigonometric ratios of angles are shown below:

1st Quadrant: $0° < \theta < 90°$

$\sin\theta = +\sin\theta$

$\cos\theta = +\cos\theta$

$\tan\theta = +\tan\theta$

2nd Quadrant: $90° < \theta < 180°$

$\sin\theta = +\sin(180° - \theta)$

$\cos\theta = -\cos(180° - \theta)$

$\tan\theta = -\tan(180° - \theta)$

3rd Quadrant: $180° < \theta < 270°$

$\sin\theta = -\sin(\theta - 180°)$

$\cos\theta = -\cos(\theta - 180°)$

$\tan\theta = +\tan(\theta - 180°)$

4th Quadrant: $270° < \theta < 360°$

$\sin\theta = -\sin(360° - \theta)$

$\cos\theta = +\cos(360° - \theta)$

$\tan\theta = -\tan(360° - \theta)$

## GRADUATED EXERCISES

1. (a) Determine the value of sin 0° cos 45° − cosec 60° cos 30° without using tables.
   (b) If $\sin(x + 30°) = \cos 40°$, find the value of $x$.

2. If $\sin x = -\frac{1}{2}$, find all the values of $x$ between 0°and 360°.
3. In Figure 6.18, PR is the perpendicular from P to QS, PQ = 2 cm, QR = 1 cm and PR = RS. Find angle QPS.

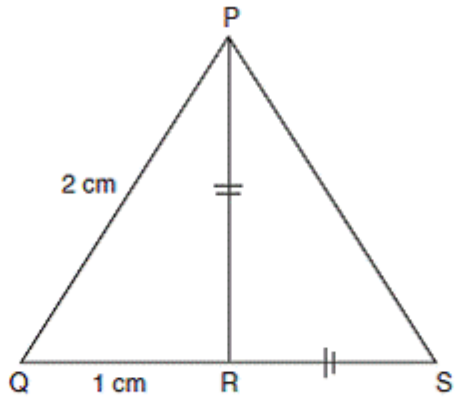



Figure 6.18

(WAEC)

4. Triangle PQR is right-angled at Q, PQ = 3$a$ cm and QR = 4$a$ cm. Determine PR in terms of $a$. **(WAEC)**
5. In the following fi gure, ABCD is a trapezium in which AD||BC and ∠ABC is a right angle. If |AD| = 15 cm, |BD| = 17 cm and |BC| = 9 cm, calculate the following:
   (a) |AB|
   (b) The area of the triangle BCD.
   (c) |CD|
   (d) The perimeter of the trapezium. **(WAEC)**
6. From the top of a cliff , the angle of depression of a boat on the sea is 60°. If the top of the cliff is 35 m above the sea level, calculate the horizontal distance from the bottom of the cliff to the boat. Solve without using tables.
7. If $\tan x = \frac{1}{\sqrt{3}}$, find $\cos x - \sin x$, such that $0° \leq \theta \leq 90°$.
8. (a) Copy and complete the following values for $y = 3\sin 2\theta - \cos \theta$.
   (b) Using a scale of 2 cm to 30° on the $\theta$-axis and 2 cm to 1 unit on the $y$-axis, draw a graph of $y = 3\sin 2\theta - \cos\theta$ for $0° \leq \theta \leq 180°$.
   (c) Use your graph to find:
      (i) The solution of the equation $3\sin 2\theta - \cos \theta = 0$, correct to the nearest degree.
      (ii) Maximum value of $y$, correct to one decimal place.

Table 6.7

| $\theta$ | 0° | 30° | 60° | 90° | 120° | 150° | 180° |
|---|---|---|---|---|---|---|---|
| $y$ | –1.0 | | | 0 | | | 1.0 |

9. (a) Copy and complete the table for the relation $y = 2\cos 2x - 1$.

Table 6.8

| $x$ | 0° | 30° | 60° | 90° | 120° | 150° | 180° |
|---|---|---|---|---|---|---|---|
| $y = 2\cos 2x - 1$ | 1.0 | | | | | | 1.0 |

(b) Using a scale of 2 cm = 30° on the $x$-axis and 2 cm = 1 unit on the $y$-axis, draw the graph of $y = 2\cos 2x - 1$ for $0° \leq x \leq 180°$.

(c) On the same axes, draw the graph of $y = \sin x$.

(d) Use your graphs to fi nd the

I. values of $x$ for which $2\cos 2x + 2 = 0$.

II. roots of the equation $2\cos 2x - 1 = \sin x$.

10. (a) Copy and complete the following table of values for $y = 3\cos x + 2\sin x$ to one decimal place.

(b) Using a scale of 2 cm to 30° on the $x$-axis and 2 cm to 1 unit on the $y$-axis, draw the graph of $y = 3\cos x + 2\sin x$ for $0° \leq x \leq 210°$.

(c) Use your graph to solve the following equations:

(i) $3\cos x + 2\sin x = 0$

(ii) $3\cos x + 2\sin x = 1.5$

(d) Find the maximum value of $y$ and the corresponding value of $x$.