



Regime switching fuzzy AHP model for choice-varying priorities problem and expert consistency prioritization: A cubic fuzzy-priority matrix design

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ABSTRACT

The aim of this paper is to develop a regime switching design of the fuzzy analytic hierarchy process (FAHP) and to improve its functionality under the choice-varying priority (CVP) problem. In the conventional AHP decision process, priority matrices are identical and their values are invariant for a specific objective. However, in many Multi-Criteria Decision Making (MCDM) problems, the relative importance of criteria may differ according to the choices. A regime switching process is proposed for improving the CVP problem. Under the fuzzy-AHP (FAHP) framework, choice-varying priorities are presented in a cubic matrix form. Another novel contribution is suggested in the prioritization of the level of expert consistency. During the decision-making practice, experts may have different attitudes and their individual matrix consistencies might be superior or inferior in their overall practices. Individual consistency is one of the objective indicators of the quality of judgment. An expert consistency prioritization approach is proposed to deal with the classification of response stability. For the financial risk assessment part of the study, the loss probability of the intended projects is calculated by the fuzzy Monte-Carlo simulation framework.

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1. Introduction

Multi-Criteria Decision Making (MCDM) problems are investigated and several methods are suggested in the existing literature. The Analytic Hierarchy Process (AHP) and its fuzzy extended models are frequently used in many practical applications. The AHP method has many advantages such as linguistic analysis, subjective reasoning, top-down analysis etc. On the other hand, incompatibility with Rational Choice theory is criticized because of a limited scale of judgment, lack of transitivity and the rank reversal phenomenon. Although several alternative scales are recommended, none of them completely ensures the mentioned characteristics. Therefore, Saaty (1977)'s fundamental scale is still the most used in the literature and the most convenient way for the prioritization procedure.

The AHP method is usually applied in a single process and experts are asked for completing pairwise comparison matrix once. The conventional AHP method assumes that the relative importance of criteria remains identical for every alternative of the objective. However, in business practice, a specific criterion may have particular importance for a specific choice and vice versa. Therefore, the priorities are choice-varying in some cases and priority matrices should be defined separately for every distinct

choice of the intended objective. To overcome such incompetency, the regime switching FAHP (here after RS-FAHP) method is proposed. In the RS-FAHP process, every choice has its own particular regime which is associated with a two-dimensional priority matrix (e.g. criteria vs. criteria). The cumulative priority matrix will be in a cubic matrix form including every regime and their priority matrices. In the conventional AHP or FAHP, the estimation is designed in a top-down process and criterion vs. choice matrices are weighted by the result of a single identical priority matrix. In the RS-FAHP process, every particular choice is weighted by its own regime matrix. If the relative importance of criteria is the same for all choices, then the result will be a traditional AHP outcome. Otherwise, a specific choice has its particular criteria priorities and these priorities differ than other choices. In the calculation process, regimes alternate due to the corresponding choice.

Finally, this paper considers the importance of expert consistency in the FAHP applications. The consistency degree of experts differs and this may cause a structural change on the hierarchical design. For classification of the degree of the consistency, the expert prioritization algorithm is proposed by relative consistency of experts (a normalized priority coefficient). The prioritization of the level of experience (i.e. experience in years) is a solution for expert prioritization. However, classification of the experience is still a questionable point. In the case of year experience prioritization, the facilitator assumes that greater experience has particular superiority on accurate decisions. In practice, however, while

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experience has specific importance, it is not a robust indicator of accurate decisions at all. The degree of consistency may derive such particulars of the expert and the quality of the pairwise analysis. Therefore, the degree of the consistency is proposed to classify priority levels of the experts. Rather than the classical consistency index, Crawford and Williams (1985) suggested the geometric consistency index (GCI) by simplifying optimization problem in the row geometric mean method (RGMM) to ensure individual consistency measurement. By using the GCI, the aggregated individual priorities are taken into consideration of consistency assessment which satisfies the Pareto principle of social choice theory. A fuzzy extended GCI is calculated by using the centre of gravity in fuzzy sets which is named the centric consistency index (CCI) (Bulut et al., 2012). Therefore, the proposed RS-FAHP method is performed under the CCI framework and expert priorities are based on the inverse normalization of the individual degree of the CCI indicator. The remainder of this paper is organized as follows. Section 2 overviews the shipping asset management and particulars of the intended investment problem. In Section 3, FAHP, the Monte-Carlo simulation (MCS) and regime switching algorithm are explained for the choice varying priority problem. Section 4 presents application and results for the assessment of shipping assets. Section 5 concludes the paper.

2. Shipping asset management and particulars of the ship investment

Transportation is of prime importance in world trading activities and maritime transportation has a special niche in the transport industry. In recent figures, maritime transport is around 90% of import–export transportation. The cost of maritime transport-named freight rate is included in the price of the products and the shipping price fluctuations directly affect retail price levels. In the period 2003–2008, the enormous upturn of freight rates was caused by the limited shipping fleet under rapid development of in the financial climate. The price of a single route shipment increased ten times over prices of the year of 2003. In conventional economic analysis, shipping price plays a critical role and is used as a leading indicator (“Hurting the real economy”, *The Economist*, October 15th, 2008; Israely, 2009).

Particularly in the rapid growth of the world economy, shipping capacity is more important and a certain size of ship has advantages on employment and profit margins. An investor is basically asked to define characteristics of his shipping asset. The ship size is one of the critical dimensions of ship investment. Investors have three optional entry strategies: building a new shipping asset, purchasing a second hand hull or hiring an existing asset for a period of time. In practice in the shipping business, charterers of ships are large-scale industry players or their intermediaries. A shipping investor usually tends to purchase a new construction or an existing shipping asset.

This paper investigates such a ship investment problem for the dry bulk transport industry. Most of the raw materials of the fundamental industries (ore, coal, grains, fertilizers, etc.) are carried by ships and their cargo size is generally over 30,000 metric tons. Handymax (around 30,000–60,000 deadweight tons) and Panamax (around 60,000–80,000 deadweight tons) size ships have an important role in raw material shipments. In the present paper, these two tonnages are selected for the empirical work of the intended shipping asset problem.

Under the FAHP framework, several criteria and alternatives are defined by an expert consultation and the industrial survey. Table 1 shows the major criteria which have selective capabilities. These criteria can be classified into two groups: financial features and technical features. An extended discussion of the ship investments and the criteria for the asset selection are performed in Bulut, Duru,

Table 1

The criteria for the shipping asset selection and their symbols.

Criterion of the shipping asset selection	The symbols of each criterion
Return on equity	RE
Loss probability (FMC)	LP
Fuel consumption	FC
Loaded draught	LD
Ship's speed	SS
Cargo crane existence	CE

and Yoshida (2010a), Bulut, Yoshida, and Duru (2010b). In the financial assessment of an investment project, one of the most used indicators is the return rate of the invested equity. Every investment project has its own profit expectations and a marginal return on equity is required over the risk free alternatives of the capital markets. Return on equity (ROE) is the ratio of net profit/deficit over the invested equity except financial aids, loans, etc. Another financial indicator is the loss probability of the project under the Monte Carlo Bayesian simulation model. Duru, Bulut, and Yoshida (2010) first developed the fuzzy-Monte Carlo simulation method for financial assessment and applied it to the assessment of shipping assets and their financial particulars. One of the outcomes of that study is the loss probability value. The loss probability is the ratio of deficits in the simulation and completely differs from the terminology of that BASEL accords in banking and finance (see Section 3.2 for a numerical description). Fuel consumption is a financial concern, but it is also partly a technical feature. Since the majority of the operational cost of a merchant ship is based on its fuel consumption, it is usually included in the subjective assessment of the asset.

Three attributes are involved in the assessment of technical features: loaded draught, navigating speed of the ship and the availability of a cargo transfer crane. Draught is the vertical height of the under water part of the ship which differs according to whether loaded or cargo-free. The maximum level of the draught exists in the loaded condition and it defines whether the ship can be accepted for a port or if it is limited for specific depth waterways. Speed of the ship defines its service time and cargo crane existence defines whether the ship can operate cargo transfers using its own facilities.

Table 2

The alternatives for the shipping asset selection and their symbols.

Alternatives for the shipping asset selection	The symbols of each alternatives
Panamax new building	PNB
Handymax new building	HNB
Panamax second hand	PSH
Handymax second hand	HSH

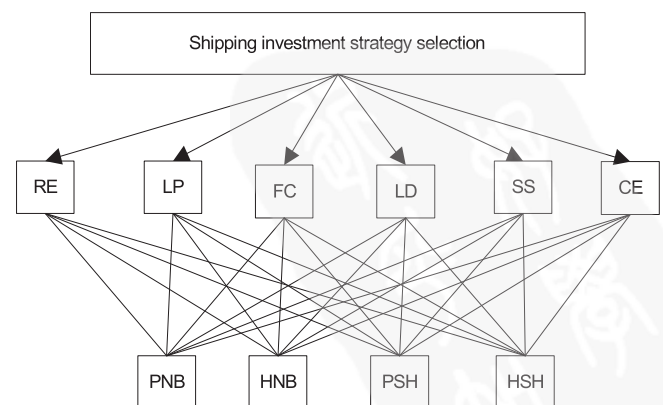


Fig. 1. Decision hierarchy of shipping asset selection problem.

Shipping assets are identified in four ways: a new building and a second hand of Panamax size dry bulk carrier, a new building and a second hand of Handymax size dry bulk carrier. Table 2 indicates alternative ship projects and their corresponding symbols.

The final structure of the AHP hierarchy is designed as in Fig. 1.

3. Methodology

3.1. Fuzzy sets and triangular fuzzy numbers (TFNs)

The definitions below briefly present particulars of fuzzy sets and their arithmetic rules (Buckley, 1985; Kaufmann & Gupta, 1985; Van Laarhoven & Pedrycz, 1983; Zadeh, 1965; Zimmermann, 1991). These basic definitions will be used throughout the paper unless otherwise stated.

Definition 1. Let X be universe of discourse, \tilde{A} is a fuzzy subset of X such that for all $x \in X$, $\mu_{\tilde{A}}(x) \in [0, 1]$ which is assigned to stand for the membership of x to \tilde{A} , and $\mu_{\tilde{A}}(x)$ is called the membership function of fuzzy set \tilde{A} .

Definition 2. A fuzzy number \tilde{A} is a convex and normalized fuzzy set of $X \subseteq \mathbb{R}$.

Definition 3. A triangular fuzzy number (TFN) is defined by its basic particulars which is:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ (x - a)/(b - a), & a \leq x < b, \\ 1, & x = b, \\ (c - x)/(c - b), & b < x \leq c, \\ 0, & c < x. \end{cases} \quad (1)$$

where a and c are the lower and upper bounds of the fuzzy number \tilde{A} , respectively, and b is the midpoint (Fig. 2). The triangular fuzzy numbers (TFNs) are indicated as $\tilde{A} = (a, b, c)$. Consider two TFNs $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$. Their operational law is as follows (Kaufmann and Gupta, 1985):

Fuzzy number addition \oplus :

$$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2). \quad (2)$$

Fuzzy number subtraction \ominus :

$$\tilde{A}_1 \ominus \tilde{A}_2 = (a_1, b_1, c_1) \ominus (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2). \quad (3)$$

Fuzzy number multiplication \otimes :

$$\begin{aligned} \tilde{A}_1 \otimes \tilde{A}_2 &= (a_1, b_1, c_1) \otimes (a_2, b_2, c_2) \\ &= (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2) \text{ for the } a_i > 0, b_i > 0, c_i > 0. \end{aligned} \quad (4)$$

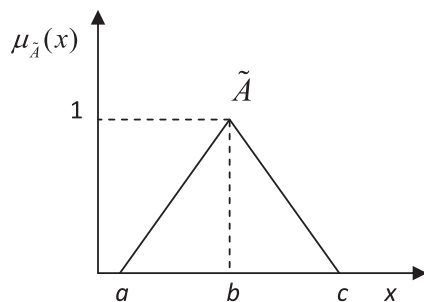


Fig. 2. A triangular fuzzy number \tilde{A} .

Fuzzy number division \oslash :

$$\begin{aligned} \tilde{A}_1 \oslash \tilde{A}_2 &= (a_1, b_1, c_1) \oslash (a_2, b_2, c_2) \\ &= (a_1/c_2, b_1/b_2, c_1/a_2) \text{ for the } a_i > 0, b_i > 0, c_i > 0. \end{aligned} \quad (5)$$

3.2. Fuzzy Monte Carlo simulation

The fuzzy Monte-Carlo (FMC) simulation method is previously used for fault tree analysis (Zonouz & Miremadi, 2006) and health risk assessment (Kentel and Aral, 2005). Originally this paper improves FMC for financial assessment purposes. The FMC is developed for random variable selection and Bayesian process design. Rather than the classical Monte-Carlo method, fuzzy extension provides data clusters and reduces data noise. These clusters are based on fuzzy intervals and their corresponding discrete probabilities are used for random selection of income-cost inputs. Fuzzy Monte-Carlo simulation is designed in six steps (Fig. 3). The process is simply commencing with fuzzification of all simulation inputs, and then simulations are carried out. Finally, the net results of financial period are calculated over fuzzy inputs and a fuzzy output will be produced for all iterations. The fuzzy output is transformed to a crisp result by calculating centre of gravity.

Step 1 & 2. Inputs of the simulation are fuzzified according to the defined fuzzy intervals. The number of clusters and their intervals is generally based on judgmental decisions which should be predetermined by the practitioner. In the example of ship investment, four sample projects are used in the empirical works. Samples are selected for a Panamax and a Handymax bulk carrier with different characteristics. Table 3 shows particulars of the projects which have different purchasing prospects, but the same risk premiums. For the cost

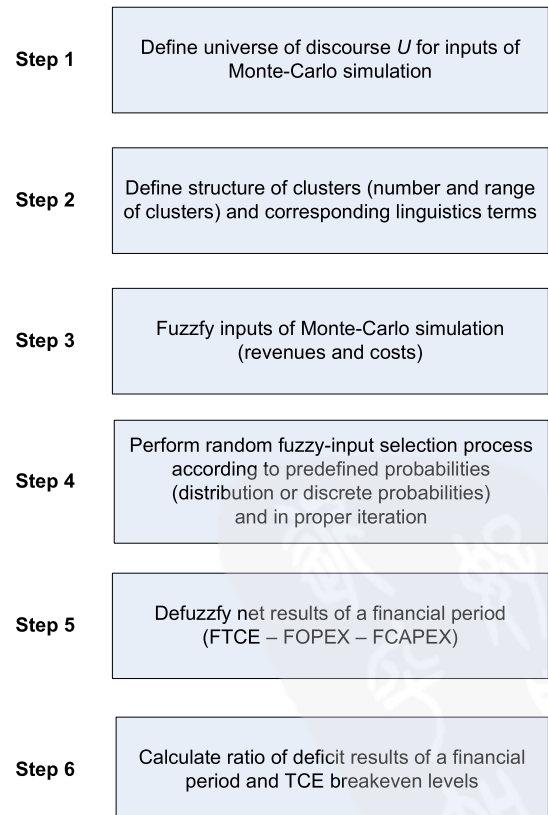


Fig. 3. The process of FMC simulation.

Table 3
Particulars of empirical ship investment projects.

Code of project	Ship type & size	Loan amount	Term structure & interest
PNB	Panamax (75,000 DWT) new building (2 years) (post-delivery finance)	26,950,000 USD	LIBOR base 1% spread for risk premium paid in 8 years, semi-annually
HNB	Handymax (45,000 DWT) new building (2 years) (post-delivery finance)	23,450,000 USD	LIBOR base 1% spread for risk premium paid in 8 years, semi-annually
PSH	Panamax (75,000 DWT) second hand	23,100,000 USD	LIBOR base 1% spread for risk premium paid in 10 years, semi-annually
HSH	Handymax (75,000 DWT) second hand	18,900,000 USD	LIBOR base 1% spread for risk premium paid in 10 years, semi-annually

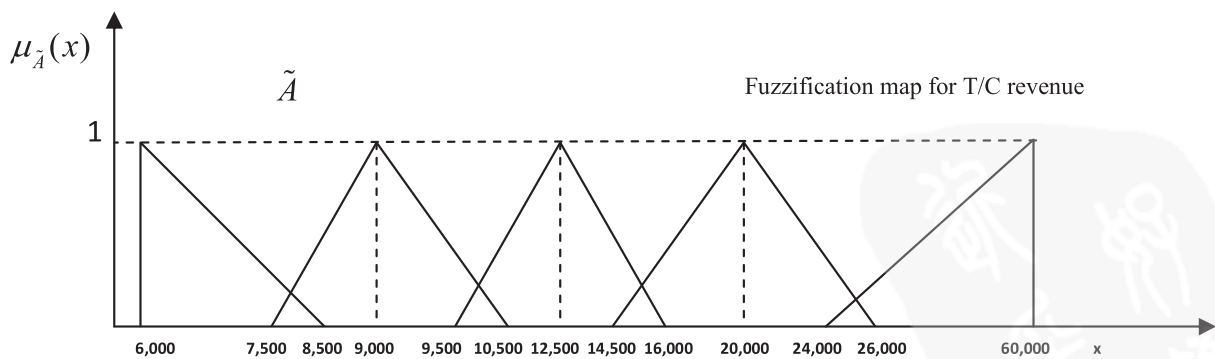
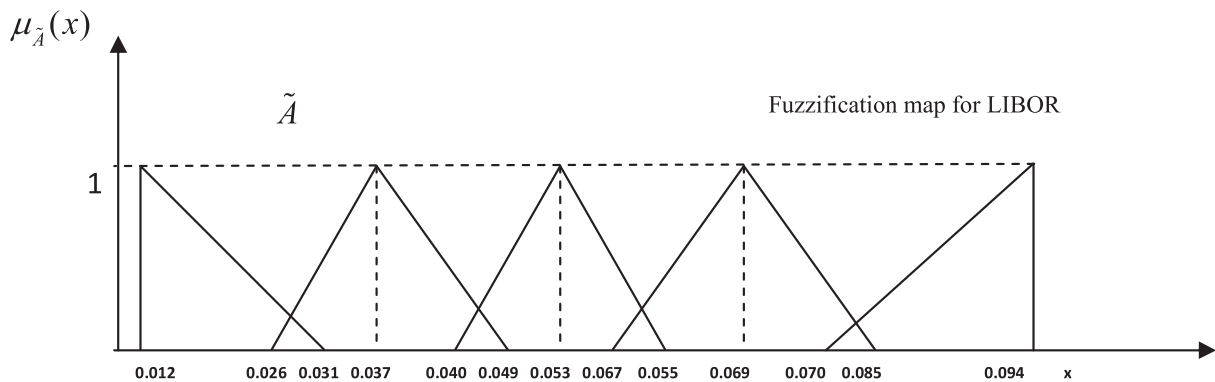
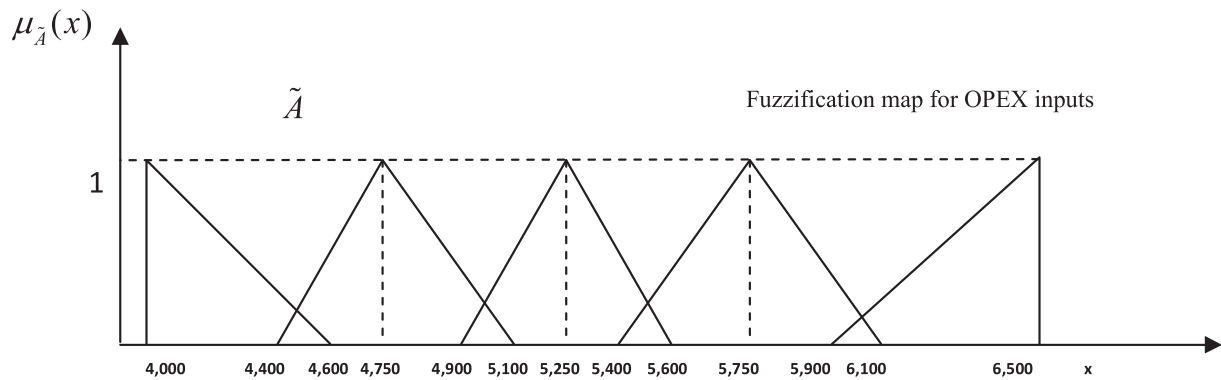


Fig. 4. Fuzzification maps of the FMC simulation inputs.

inputs and revenue inputs (TCE), fuzzification maps are defined as in Fig. 4. Datasets are divided into five fuzzy intervals which have corresponding linguistic terms

such as very low, low, moderate, high and very high (u_1, u_2, u_3, u_4 and u_5 , respectively). Their corresponding fuzzy sets are A_1, A_2, A_3, A_4 and A_5 , respectively.

Step 3. The operating cost, LIBOR (London Interbank Offer Rate) and TCE base income are transformed into fuzzy sets. The operating cost (OPEX-Operating Expense) means the fixed cost of a ship which does not include the cost of the voyage (bunkers, port dues, commissions, etc.). Since the income is based on TCE base, voyage costs are out of scope for the current study. Another important fixed cost is the capital cost which is arises from project financing. One of the critical and volatile criterias of financial deal is the rate of interest (conventionally LIBOR rate plus a risk premium based on risk perception by the lender). According to the LIBOR rate, capital cost of a financial period differs. As a cost input, LIBOR rate is also fuzzified and a pre-determined spread of risk premium is applied over the random selection of fuzzy-LIBOR rate.

Step 4. Inputs of FMC simulation are selected randomly in a thousand iteration according to the scenario characteristics. Since the intended simulation is not performed for an optimization, the higher number of iteration is just for smoothing. Scenarios are based on historical densities, Gaussian densities (normally distributed), lower case and higher case. Table 4 shows the probability character of inputs based on historical record. OPEX data is collected from Drewry Shipping Cost Annual reports for 2000–2009 term on annual average base. The LIBOR rate series include monthly averages between 1990 and 2009. T/C rates of Panamax and Handymax bulkers are supplied by Clarkson Shipping Co. for 1987–2009 period. Probabilities that are presented on Table 4 are calculated from the historical probability distribution.

For every scenario, a thousand iteration of input is performed and the result of output fuzzy sets is transformed to crisp numbers by the calculating centre of gravity of triangle fuzzy numbers.

Step 5. The net result of a financial period (semi-annual in the present empirical work) is calculated by subtracting Fuzzy-OPEX and Fuzzy-CAPEX (Capital Expense) from Fuzzy-TCE as follows:

$$\text{Profit/Loss} = \text{FTCE} - \text{FOPEX} - \text{FCAPEX}. \quad (6)$$

CAPEX is based on straight line principals rather than straight line payments. Therefore, term payments decrease by the declining interest payment. Subtracting process is a fuzzy arithmetic operation which is proposed by Zadeh (1965). The final crisp result is defined by the calculating centre of gravity point on X-axis.

Step 6. The ratio of deficit results and deficits less than 5% of equity invested are recorded for every candidate project. Fuzzy interval of loss probability is bounded by these indications and average value is the mid-point of the fuzzy set. Table 5 presents the calculated sets of loss probabilities. The loss probability sets are direct numerical inputs (DNIs) which are used for calculating cross fractions to define comparative priorities in the FAHP process.

Table 5
Loss probability sets.

Code of project	Lower-bound	Mid-point	Upper-bound
PNB	0.379	0.462	0.545
HNB	0.539	0.583	0.626
PSH	0.154	0.267	0.379
HSH	0.316	0.400	0.484

3.3. Fuzzy-analytic hierarchy process (FAHP)

Beside the conventional AHP method, the FAHP is designed to improve decision support for uncertain valuations and priorities. In the process of FAHP, both priorities and data are evaluated under fuzzy set environment. In the existing literature, many studies applied the FAHP method to analyze the selection problems (Buckley, 1985; Chang, 1996; Cheng, 1996; Leung & Cao, 2000; Mikhailov & Tsvetnikov, 2004; Van Laarhoven & Pedrycz, 1983). Van Laarhoven and Pedrycz (1983) first extended the AHP method to the FAHP by using triangular fuzzy numbers in the pairwise comparison matrix of the AHP. Buckley (1985) used trapezoidal fuzzy numbers to state the decision maker's evaluation of alternatives based on each criterion.

In this paper, the extent FAHP is utilized (Chang, 1996) under the regime switching framework, which is a frequently used algorithm for handling FAHP. The unique contribution of the extent FAHP is based on the use of the extent analysis method for the synthetic extent values of pairwise comparisons. However, many FAHP studies ignore the matrix consistency problem. Even the judgments are inconsistent; many FAHP studies present empirical works completing whole process. This paper proposes the centric consistency index (CCI) which is an extended version of the geometry consistency index (GCI) to investigate consistency for each pairwise matrix (Crawford & Williams, 1985).

Chang (1996) introduces the extent synthesis method as follows:

Let $X = \{x_1, x_2, \dots, x_n\}$ be an object set and $U = \{u_1, u_2, \dots, u_m\}$ be a goal set. According to the method of extent analysis, each object is taken and extent analysis for each goal is performed, respectively (Chang, 1996). Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M_{g_i}^1, M_{g_i}^2, \dots, M_{g_i}^m, \quad i = 1, 2, \dots, n, \quad (7)$$

where all the $M_{g_i}^j (j = 1, 2, \dots, m)$ are TFNs.

The steps of Chang's extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the i th object is defined as:

$$S_i = \sum_{j=1}^m M_{g_i}^j \otimes \left[\sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j \right]^{-1}. \quad (8)$$

To obtain $\sum_{j=1}^m M_{g_i}^j$, the fuzzy addition operation of m extent analysis values for a particular matrix is performed such as:

$$\sum_{j=1}^m M_{g_i}^j = \left(\sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j \right), \quad (9)$$

and to obtain $\left[\sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j \right]^{-1}$, the fuzzy addition operation of $M_{g_i}^j (j = 1, 2, \dots, m)$ values is performed such as:

Table 4
Historical probabilities of the FMC simulation inputs.

	OPEX	LIBOR	T/C rate
u_1	0.10	0.21	0.18
u_2	0.35	0.20	0.18
u_3	0.35	0.39	0.32
u_4	0.15	0.15	0.14
u_5	0.05	0.05	0.18

$$\sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j = \left(\sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j \right) \quad (10)$$

and then the inverse of the vector in Eq. (10) is computed, such as:

$$\left[\sum_{i=1}^n \sum_{j=1}^m M_{g_i}^j \right]^{-1} = \left(\frac{1}{\sum_{i=1}^n u_i}, \frac{1}{\sum_{i=1}^n m_i}, \frac{1}{\sum_{i=1}^n l_i} \right). \quad (11)$$

Step 2: The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as:

$$V(M_2 \geq M_1) = \sup_{y \geq x} [\min(\mu_{M_1}(x), \mu_{M_2}(y))] \quad (12)$$

and can be expressed as follows:

$$V(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 \geq u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise.} \end{cases} \quad (13)$$

Fig. 5 illustrates Eq. (13) where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2} . To compare M_1 and M_2 , we need both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 3: The degree possibility for a convex fuzzy number to be greater than k convex fuzzy $M_i (i = 1, 2, \dots, k)$ numbers can be defined by

$$V(M \geq M_1, M_2, \dots, M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \dots \text{ and } (M \geq M_k)] = \min V(M \geq M_i), \quad i = 1, 2, 3, \dots, k. \quad (14)$$

Assume that $d'(A_i) = \min V(S_i \geq S_k)$ for $k = 1, 2, \dots, n; k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_n))^T, \quad (15)$$

where $A_i (i = 1, 2, \dots, n)$ are n elements.

Step 4: Via normalization, the normalized weight vectors are:

$$W = (d(A_1), d(A_2), \dots, d(A_n))^T, \quad (16)$$

where W is a non-fuzzy number.

Table 6 presents the evaluation scale for the linguistic comparison terms and Fig. 6 indicates their equivalent fuzzy numbers in this paper (Wang & Chen, 2008; Cebeci, 2009; Gumus, 2009; Hsu, Lee, & Kreng, 2010).

3.4. Centric consistency index (CCI)

Bulut et al. (2012) proposed the centric consistency index (CCI) which is based on the geometric consistency index (GCI) framework (Crawford & Williams, 1985) and centric consistency index

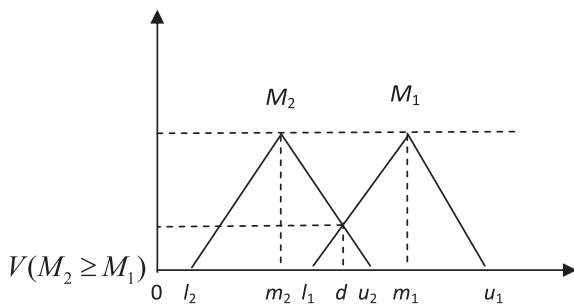


Fig. 5. The intersection between M_1 and M_2 .

Table 6
Membership function for the TFNs.

Fuzzy number	Linguistic scales	Membership function	Inverse
\tilde{A}_1	Equally important	(1, 1, 1)	(1, 1, 1)
\tilde{A}_2	Moderately important	(1, 3, 5)	(1/5, 1/3, 1)
\tilde{A}_3	More important	(3, 5, 7)	(1/7, 1/5, 1/3)
\tilde{A}_4	Strongly important	(5, 7, 9)	(1/9, 1/7, 1/5)
\tilde{A}_5	Extremely important	(7, 9, 9)	(1/9, 1/9, 1/7)

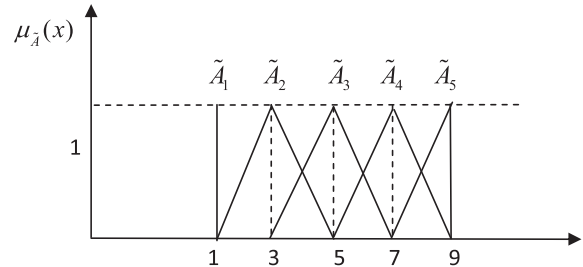


Fig. 6. Fuzzy number of linguistic variable set.

(CCI) is fuzzy extended version of the GCI. The procedure of the CCI is as follows:

Let $A = (a_{Lij}, a_{Mij}, a_{Uij})_{n \times n}$ be a fuzzy judgment matrix, and let $w = [(w_{L1}, w_{M1}, w_{U1}), (w_{L2}, w_{M2}, w_{U2}), \dots, (w_{Ln}, w_{Mn}, w_{Un})]^T$ be the priority vector derived from Ausing the RGMM. The centric consistency index (CCI) is computed by

$$CCI(A) = \frac{2}{(n-1)(n-2)} \sum_{i < j} \left(\log \left(\frac{a_{Lij} + a_{Mij} + a_{Uij}}{3} \right) - \log \left(\frac{w_{Li} + w_{Mi} + w_{Ui}}{3} \right) + \log \left(\frac{w_{Lj} + w_{Mj} + w_{Uj}}{3} \right) \right)^2, \quad (17)$$

when $CCI(A) = 0$, we consider A fully consistent. Aguarón and Moreno-Jiménez (2003) also provide the thresholds (\overline{CCI}) as $\overline{CCI} = 0.31$ for $n = 3$; $\overline{CCI} = 0.35$ for $n = 4$ and $\overline{CCI} = 0.37$ for $n > 4$. When $CCI(A) < \overline{CCI}$, it is considered that the matrix A is sufficiently consistent. Since the CCI is a fuzzy extended version of the GCI, thresholds remain identical.

3.5. Design of the proposed method (RS-FAHP)

Let $A = (a_{ij})_{n \times n}$, where $a_{ij} > 0$ and $a_{ij} \times a_{ji} = 1$, be a judgment matrix. The prioritization method refers to the process of deriving a priority vector of criteria $w = (w_1, w_2, \dots, w_n)^T$, where $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$, from the judgment matrix A .

Let $D = \{d_1, d_2, \dots, d_m\}$ be the set of decision makers, and $\lambda_k = \{\lambda_{1k}, \lambda_{2k}, \dots, \lambda_{mk}\}$ be the priority vector of decision makers. The priority vector of decision makers (λ_k) is the normalized I_k for the group of experts which is calculated as follows:

$$I_k = \frac{1}{CCI_k}, \quad (18)$$

where I_k is the inverse of the CCI normalization,

$$\lambda_k = \frac{I_k}{\sum_{k=1}^m I_k}, \quad (19)$$

where $\lambda_k > 0$, $k = 1, 2, \dots, m$, and $\sum_{k=1}^m \lambda_k = 1$.

Let $A^{(k)} = (a_{ij}^{(k)})_{n \times n}$ be the judgment matrix provided by the decision maker d_k .

$w_i^{(k)}$ is the priority vector of criteria for each decision maker calculated by

$$w_i^{(k)} = \frac{\left(\prod_{j=1}^n a_{ij}\right)^{1/n}}{\sum_{i=1}^n \left(\prod_{j=1}^n a_{ij}\right)^{1/n}}. \quad (20)$$

The aggregation of individual priorities is defined by

$$w_i^{(w)} = \frac{\prod_{k=1}^m \left(w_i^{(k)}\right)^{\lambda_k}}{\sum_{i=1}^n \prod_{k=1}^m \left(w_i^{(k)}\right)^{\lambda_k}}, \quad (21)$$

where $w_i^{(w)}$ is the aggregated weight vector. After the aggregation process, the extent synthesis methodology of Chang (1996) is applied for subsequent choice selection.

In the case of non-judgmental criteria, numerical inputs are capable of indicating relative importance among the intended alternatives. The direct numerical inputs (DNIs) method is designed to express priorities without time-consuming expert consultation. DNI is based on ranking-and-assignment scale and converts numerical inputs to priority fuzzy sets. Table 7 presents sample scales of five alternative and four alternative cases for the DNI purpose. A pairwise comparison between alternatives i and j on criterion, C , is defined by

$$a_{ij}^C = \frac{A_r^i}{A_r^j}, \quad (21)$$

where A_r^i is the rank valuation set of alternative i .

For the ship investment problem, DNIs are converted to fuzzy priority sets according to their numerical superiority (Table 8).

The comparative priority is defined by cross fractions. For example, if the rank valuation of alternative i is A_3^i and alternative j is A_1^j , then the pairwise comparison value is $(3/7, 5/9, 7/9)$. Since the posterior process is based on a normalized matrix, use of a specific scale matrix does not deteriorate the entire procedure. For the loss probability sets, pairwise comparisons are similarly based on the cross fractions between the intended alternatives without any transformations to the standard scale.

The regime switching algorithm of the FAHP method is defined by alternating priority matrices of each regime. The final priority matrix is defined by

Table 7
The scale of the DNI.

Rank	Linguistic variable	TFNs	Rank	Linguistic variable	TFNs
Scale for five alternatives			Scale for four alternatives		
A_r			A_r		
A_1	Very high	(7,9,9)	A_1	Very high	(7,9,9)
A_2	High	(5,7,9)	A_2	High	(5,7,9)
A_3	Medium	(3,5,7)	A_3	Medium	(3,5,7)
A_4	Low	(1,3,5)	A_4	Low	(1,3,5)
A_5	Very low	(1,1,3)			

Table 8
Direct numerical input for the result of ROE.

Project	Result of ROE (%)	Linguistic variable	TFNs
PNB	−12.5	Low	(1,3,5)
HNB	28.0	Medium	(3,5,7)
PSH	183.1	High	(5,7,9)
HSH	355.4	Very high	(7,9,9)

$$\rho_j = w_c^R \times w_j^R, \quad (22)$$

where ρ_j is the final priority value of alternative j and c is the criteria. $R \rightarrow (P, H)$ is the regime of alternative and criteria (P – Panamax, H – Handymax).

The rule-based process is based on IF-THEN framework as follows:

Program

1. IF the alternative j is a Panamax tonnage project, THEN the final priority of alternative j is $\rho_j = w_c^P \times w_j^P$,
2. IF the alternative j is a Handymax tonnage project, THEN the final priority of alternative j is $\rho_j = w_c^H \times w_j^H$.

4. Application and results

The fuzzy pairwise comparison matrix relevant to the goal is presented in Table 9. Lambda (λ) coefficients correspond to the expert priority ratio.

Tables 10 and 11 present the result of the individual fuzzy priority vectors and the aggregated weight vector of Handymax and Panamax regimes respectively. As it is seen in Table 10, the coefficient of return on equity has the considerable contribution on the final outcome with its 0.41 value and the lost probability has the second major contribution on the final outcome. Fuel consumption, crane existence, ship speed and loaded draught have the remaining contributions of 0.17, 0.07, 0.05, and 0.03 respectively.

For the Panamax ship, the aggregated weight of each criterion is different than the Handymax contribution of criteria (Table 11). Return on equity and lost probability have the major contribution of the aggregated weight vector for both Panamax and Handymax regimes. However, the ranking of the remaining criteria in Panamax regime is quite different than the aggregated weight matrix of the Handymax regime. The contributions of the fuel consumption, loaded draught, ship speed and crane existence in Panamax regime are 0.16, 0.09, 0.06 and 0.03 respectively. Fig. 7 illustrates the cubic priority matrix for the intended problem including Panamax (PRW) and Handymax regimes (HRW).

The aggregated fuzzy judgment matrix (AFJM) of the Handymax regime (see Table 12) is formed from Tables 9 and 10. For Panamax regime (see Table 13), it is based on the individual fuzzy judgment matrix of the Panamax ship and Table 11. In this study, there are two different mean aggregated weights (MAW) for Handymax and Panamax regimes which are comprised from Tables 10 and 11 respectively.

AFJM is found consistent since the CCI is less than the threshold of 0.37. By the extent analysis method for Handymax, the computation is performed for the shipping asset selection problem as follows:

$$S_{RE} = (19.18, 28.17, 34.57) \otimes (1/105.23, 1/79.34, 1/51.74) \\ = (0.18, 0.36, 0.67),$$

$$S_{LP} = (13.74, 21.37, 28.42) \otimes (1/105.23, 1/79.34, 1/51.74) \\ = (0.13, 0.27, 0.55),$$

$$S_{FC} = (10.42, 16.77, 23.35) \otimes (1/105.23, 1/79.34, 1/51.74) \\ = (0.10, 0.21, 0.45),$$

$$S_{LD} = (1.89, 2.23, 3.46) \otimes (1/105.23, 1/79.34, 1/51.74) \\ = (0.02, 0.03, 0.07),$$

Table 9

The individual fuzzy judgment matrix for criteria of Handymax ship investment strategy.

DM ₁	$\lambda = 0.13$	RE	LP	FC	LD	SS	CE
	RE	(1, 1, 1)	(1, 1, 1)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	LP	(1, 1, 1)	(1, 1, 1)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)
	FC	(1/7, 1/5, 1/3)	(1/9, 1/7, 1/5)	(1, 1, 1)	(3, 5, 7)	(3, 5, 7)	(1, 3, 5)
	LD	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
	SS	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
	CE	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CCI = 0.05						
DM ₂	$\lambda = 0.17$						
	RE	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)
	LP	(1/5, 1/3, 1)	(1, 1, 1)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	FC	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 1, 1)	(5, 7, 9)	(5, 7, 9)	(1, 3, 5)
	LD	(1/9, 1/7, 1/5)	(1/9, 1/7, 1/5)	(1/9, 1/7, 1/5)	(1, 1, 1)	(1/5, 1/3, 1)	(1/5, 1/3, 1)
	SS	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CE	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(3, 5, 7)	(1, 3, 5)	(1, 1, 1)
	CCI = 0.04						
DM ₃	$\lambda = 0.11$						
	RE	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(7, 9, 9)	(7, 9, 9)	(3, 5, 7)
	LP	(1/5, 1/3, 1)	(1, 1, 1)	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	FC	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 1, 1)	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)
	LD	(1/9, 1/9, 1/7)	(1/9, 1/7, 1/5)	(1/9, 1/7, 1/5)	(1, 1, 1)	(1/5, 1/3, 1)	(1/5, 1/3, 1)
	SS	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CE	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CCI = 0.06						
DM ₄	$\lambda = 0.24$						
	RE	(1, 1, 1)	(1, 1, 1)	(3, 5, 7)	(7, 9, 9)	(7, 9, 9)	(5, 7, 9)
	LP	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(7, 9, 9)	(5, 7, 9)	(3, 5, 7)
	FC	(1/7, 1/5, 1/3)	(1, 1, 1)	(1, 1, 1)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	LD	(1/9, 1/9, 1/7)	(1/9, 1/9, 1/7)	(1/9, 1/7, 1/5)	(1, 1, 1)	(1/5, 1/3, 1)	(1/5, 1/3, 1)
	SS	(1/9, 1/9, 1/7)	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CE	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CCI = 0.03						
DM ₅	$\lambda = 0.14$						
	RE	(1, 1, 1)	(3, 5, 7)	(5, 7, 9)	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)
	LP	(1/7, 1/5, 1/3)	(1, 1, 1)	(1, 3, 5)	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)
	FC	(1/9, 1/9, 1/7)	(1/5, 1/3, 1)	(1, 1, 1)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	LD	(1/9, 1/9, 1/7)	(1/9, 1/7, 1/5)	(1/9, 1/7, 1/5)	(1, 1, 1)	(1/5, 1/3, 1)	(1/5, 1/3, 1)
	SS	(1/9, 1/9, 1/7)	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CE	(1/9, 1/9, 1/7)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 1, 1)	(1, 1, 1)
	CCI = 0.05						
DM ₆	$\lambda = 0.20$						
	RE	(1, 1, 1)	(1, 3, 5)	(1, 3, 5)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)
	LP	(1/5, 1/3, 1)	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)
	FC	(1/5, 1/3, 1)	(1/5, 1/3, 1)	(1, 1, 1)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)
	LD	(1/9, 1/7, 1/5)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 1, 1)	(1, 1, 1)	(1/5, 1/3, 1)
	SS	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 1, 1)	(1, 1, 1)	(1/5, 1/3, 1)
	CE	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1/7, 1/5, 1/3)	(1, 3, 5)	(1, 3, 5)	(1, 1, 1)
	CCI = 0.04						

Table 10

The individual fuzzy priority vector of decision-makers and aggregated weight vector for criteria of shipping investment – Handymax regime.

	RE	LP	FC	LD	SS	CE
DM ₁	(0.34, 0.37, 0.39)	(0.30, 0.31, 0.32)	(0.13, 0.14, 0.15)	(0.04, 0.05, 0.05)	(0.05, 0.05, 0.06)	(0.07, 0.08, 0.10)
DM ₂	(0.31, 0.35, 0.36)	(0.31, 0.34, 0.34)	(0.13, 0.15, 0.17)	(0.03, 0.03, 0.03)	(0.04, 0.04, 0.05)	(0.08, 0.09, 0.12)
DM ₃	(0.40, 0.42, 0.44)	(0.27, 0.28, 0.29)	(0.15, 0.15, 0.16)	(0.03, 0.03, 0.03)	(0.04, 0.05, 0.05)	(0.06, 0.07, 0.08)
DM ₄	(0.38, 0.40, 0.40)	(0.26, 0.28, 0.29)	(0.18, 0.19, 0.21)	(0.03, 0.03, 0.04)	(0.05, 0.05, 0.05)	(0.05, 0.05, 0.06)
DM ₅	(0.46, 0.52, 0.69)	(0.22, 0.24, 0.25)	(0.14, 0.16, 0.17)	(0.02, 0.03, 0.03)	(0.04, 0.05, 0.06)	(0.05, 0.05, 0.06)
DM ₆	(0.36, 0.37, 0.40)	(0.26, 0.26, 0.27)	(0.18, 0.20, 0.20)	(0.04, 0.04, 0.05)	(0.04, 0.04, 0.06)	(0.07, 0.08, 0.08)
Aggregated weight	(0.37, 0.41, 0.41)	(0.28, 0.28, 0.28)	(0.16, 0.16, 0.18)	(0.03, 0.04, 0.04)	(0.05, 0.05, 0.05)	(0.06, 0.07, 0.08)

Table 11

The individual fuzzy priority vector of decision-makers and aggregated weight vector for criteria of shipping investment (Panamax).

	RE	LP	FC	LD	SS	CE
DM ₁	(0.39, 0.44, 0.44)	(0.24, 0.25, 0.27)	(0.15, 0.15, 0.16)	(0.09, 0.09, 0.10)	(0.05, 0.06, 0.06)	(0.02, 0.02, 0.03)
DM ₂	(0.46, 0.51, 0.55)	(0.21, 0.23, 0.24)	(0.12, 0.13, 0.15)	(0.06, 0.06, 0.07)	(0.05, 0.05, 0.06)	(0.02, 0.02, 0.02)
DM ₃	(0.31, 0.32, 0.33)	(0.32, 0.33, 0.34)	(0.15, 0.15, 0.18)	(0.07, 0.07, 0.07)	(0.09, 0.09, 0.10)	(0.02, 0.03, 0.03)
DM ₄	(0.34, 0.37, 0.38)	(0.33, 0.33, 0.34)	(0.15, 0.15, 0.17)	(0.08, 0.08, 0.09)	(0.04, 0.04, 0.05)	(0.02, 0.02, 0.02)
DM ₅	(0.33, 0.38, 0.60)	(0.32, 0.32, 0.44)	(0.14, 0.15, 0.18)	(0.09, 0.12, 0.13)	(0.03, 0.04, 0.06)	(0.03, 0.03, 0.05)
DM ₆	(0.36, 0.39, 0.41)	(0.22, 0.23, 0.25)	(0.20, 0.20, 0.21)	(0.10, 0.10, 0.11)	(0.05, 0.05, 0.05)	(0.02, 0.02, 0.03)
Aggregated weight	(0.37, 0.40, 0.40)	(0.28, 0.29, 0.29)	(0.15, 0.15, 0.17)	(0.08, 0.08, 0.09)	(0.05, 0.06, 0.06)	(0.02, 0.02, 0.03)

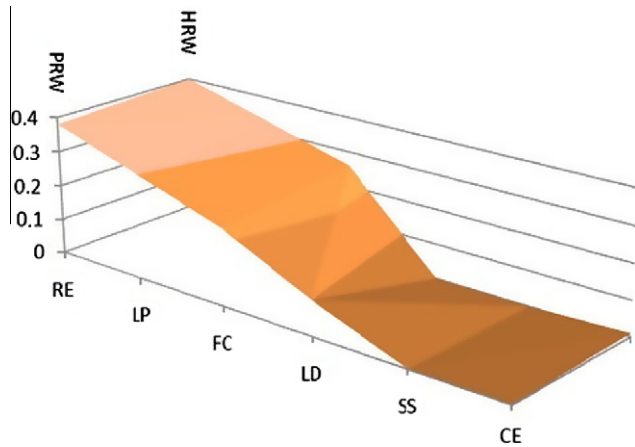


Fig. 7. The cubic priority matrix design for the ship investment problem.

$$S_{SS} = (2.84, 4.18, 5.72) \otimes (1/105.23, 1/79.34, 1/51.74) \\ = (0.03, 0.05, 0.11),$$

$$S_{CE} = (3.66, 6.63, 9.69) \otimes (1/105.23, 1/79.34, 1/51.74) \\ = (0.03, 0.08, 0.19),$$

$$\begin{aligned} V(S_{RE} \geq S_{LP}) &= 1 \\ V(S_{RE} \geq S_{FC}) &= 1 \\ V(S_{RE} \geq S_{LD}) &= 1 \\ V(S_{RE} \geq S_{SS}) &= 1 \\ V(S_{RE} \geq S_{CE}) &= 1 \\ V(S_{LP} \geq S_{RE}) &= (0.18 - 0.55) / \\ &[(0.27 - 0.55) - (0.36 - 0.18)] = 0.81 \\ V(S_{LP} \geq S_{FC}) &= 1 \\ V(S_{LP} \geq S_{LD}) &= 1 \\ V(S_{LP} \geq S_{SS}) &= 1 \\ V(S_{LP} \geq S_{CE}) &= 1 \\ V(S_{FC} \geq S_{RE}) &= 0.65 \\ V(S_{FC} \geq S_{LP}) &= 0.85 \\ V(S_{FC} \geq S_{LD}) &= 1 \\ V(S_{FC} \geq S_{SS}) &= 1 \end{aligned}$$

Table 12

The aggregated fuzzy judgment matrix for criteria of ship investment-Handymax regime.

	RE	LP	FC	LD	SS	CE	MAW
RE	(1, 1, 1)	(1.17, 1.78, 2.20)	(2.13, 4.33, 6.39)	(6.26, 8.28, 9.00)	(4.80, 6.89, 8.28)	(3.82, 5.89, 7.70)	(0.40)
LP	(0.46, 0.56, 0.86)	(1, 1, 1)	(1.58, 2.86, 3.92)	(4.57, 6.65, 8.28)	(3.98, 6.02, 8.05)	(2.15, 4.28, 6.32)	(0.28)
FC	(0.16, 0.23, 0.47)	(0.25, 0.35, 0.63)	(1, 1, 1)	(3.86, 5.91, 7.93)	(3.00, 5.00, 7.00)	(2.15, 4.28, 6.32)	(0.17)
LD	(0.11, 0.12, 0.16)	(0.12, 0.15, 0.22)	(0.13, 0.17, 0.26)	(1, 1, 1)	(0.34, 0.48, 1.00)	(0.19, 0.31, 0.83)	(0.03)
SS	(0.12, 0.15, 0.21)	(0.12, 0.17, 0.25)	(0.14, 0.20, 0.33)	(1.00, 2.08, 2.93)	(1, 1, 1)	(0.45, 0.58, 1.00)	(0.05)
CE	(0.13, 0.17, 0.26)	(0.16, 0.23, 0.46)	(0.16, 0.23, 0.46)	(1.21, 3.28, 5.30)	(1.00, 1.71, 2.20)	(1, 1, 1)	(0.07)
CCI = 0.27							

Table 13

The aggregated fuzzy judgment matrix for criteria of shipping investment-Panamax regime.

	RE	LP	FC	LD	SS	CE	MAW
RE	(1, 1, 1)	(1.14, 1.75, 2.16)	(2.00, 4.19, 6.25)	(3.32, 5.37, 7.22)	(4.84, 6.91, 8.44)	(6.42, 8.44, 9.00)	(0.39)
LP	(0.46, 0.57, 0.88)	(1, 1, 1)	(1.38, 2.89, 4.20)	(2.42, 4.53, 6.56)	(3.52, 5.56, 7.58)	(5.96, 7.98, 9.00)	(0.29)
FC	(0.16, 0.24, 0.50)	(0.24, 0.35, 0.73)	(1, 1, 1)	(1.77, 3.95, 6.01)	(1.98, 3.11, 4.09)	(5.00, 7.00, 9.00)	(0.16)
LD	(0.14, 0.19, 0.30)	(0.15, 0.22, 0.41)	(0.17, 0.25, 0.57)	(1, 1, 1)	(1.34, 2.59, 3.62)	(4.13, 6.17, 8.19)	(0.09)
SS	(0.12, 0.14, 0.21)	(0.13, 0.18, 0.28)	(0.24, 0.32, 0.50)	(0.28, 0.39, 0.75)	(1, 1, 1)	(2.42, 3.65, 4.79)	(0.06)
CE	(0.11, 0.12, 0.16)	(0.11, 0.13, 0.17)	(0.11, 0.14, 0.20)	(0.12, 0.16, 0.24)	(0.21, 0.27, 0.41)	(1, 1, 1)	(0.03)
CCI = 0.29							

$$\begin{aligned} V(S_{FC} \geq S_{CE}) &= 1 \\ V(S_{LD} \geq S_{RE}) &= 0 \\ V(S_{LD} \geq S_{LP}) &= 0 \\ V(S_{LD} \geq S_{FC}) &= 0 \\ V(S_{LD} \geq S_{SS}) &= 0.62 \\ V(S_{LD} \geq S_{CE}) &= 0.37 \\ V(S_{SS} \geq S_{RE}) &= 0 \\ V(S_{SS} \geq S_{LP}) &= 0 \\ V(S_{SS} \geq S_{FC}) &= 0.07 \\ V(S_{SS} \geq S_{LD}) &= 1 \\ V(S_{SS} \geq S_{CE}) &= 0.71 \\ V(S_{CE} \geq S_{RE}) &= 0.02 \\ V(S_{CE} \geq S_{LP}) &= 0.23 \\ V(S_{CE} \geq S_{FC}) &= 0.41 \\ V(S_{CE} \geq S_{LD}) &= 1 \\ V(S_{CE} \geq S_{SS}) &= 1 \end{aligned}$$

The priority weights for criteria are calculated by using Eq. (12):

$$\begin{aligned} d'(RE) &= \min(1, 1, 1, 1, 1) = 1 \\ d'(LP) &= \min(0.81, 1, 1, 1, 1) = 0.81 \\ d'(FC) &= \min(0.65, 0.85, 1, 1, 1) = 0.65 \\ d'(LD) &= \min(0, 0, 0, 0.62, 0.37) = 0 \\ d'(SS) &= \min(0, 0, 0.07, 1, 0.71) = 0 \\ d'(CE) &= \min(0.02, 0.23, 0.41, 1, 1) = 0.02 \end{aligned}$$

Via normalization, the priority weights of the main attributes for the Handymax (H) and Panamax (P) are calculated as follows:

$$\begin{aligned} d(H) &= (0.40, 0.33, 0.26, 0, 0, 0.01) \\ d(P) &= (0.37, 0.31, 0.22, 0.10, 0, 0) \end{aligned}$$

After the calculation of weight for the Handymax and Panamax, the aggregated fuzzy judgment matrix for the alternatives under each criterion is computed from the individual fuzzy judgment matrix of decision makers (Table 14).

The results of the priority weights of the alternatives are as follows:

$$\begin{aligned} d(RE) &= (0.00, 0.20, 0.36, 0.44) \\ d(LP) &= (0.31, 0.50, 0.00, 0.19) \\ d(FC) &= (0.32, 0.53, 0.00, 0.16) \end{aligned}$$

Table 14

The aggregated fuzzy judgment matrix for alternatives of shipping investment under each criterion.

Criteria		PNB	HNB	PSH	HSH	MAW
RE	PNB	(1, 1, 1)	(0.33, 0.60, 0.71)	(0.20, 0.43, 0.56)	(0.14, 0.33, 0.33)	0.11
	HNB	(1.40, 1.67, 3.00)	(1, 1, 1)	(0.60, 0.71, 0.78)	(0.43, 0.56, 0.78)	0.21
	PSH	(1.80, 2.33, 5.00)	(1.29, 1.40, 1.67)	(1, 1, 1)	(0.43, 0.78, 1.00)	0.29
	HSH	(3.00, 3.00, 7.00)	(1.29, 1.80, 2.33)	(1.00, 1.29, 2.33)	(1, 1, 1)	0.40
CCI = 0.00						
LP	PNB	(1, 1, 1)	(0.70, 0.79, 0.87)	(1.44, 1.73, 2.46)	(1.13, 1.16, 1.20)	0.27
	HNB	(1.15, 1.26, 1.42)	(1, 1, 1)	(1.65, 2.19, 3.50)	(1.29, 1.46, 1.71)	0.35
	PSH	(0.41, 0.58, 0.70)	(0.29, 0.46, 0.61)	(1, 1, 1)	(0.49, 0.67, 0.78)	0.15
	HSH	(0.83, 0.87, 0.89)	(0.59, 0.69, 0.77)	(1.28, 1.50, 2.05)	(1, 1, 1)	0.23
CCI = 0.00						
FC	PNB	(1, 1, 1)	(0.26, 0.34, 0.57)	(1.63, 3.63, 5.49)	(0.94, 1.57, 2.05)	0.23
	HNB	(1.77, 2.90, 3.85)	(1, 1, 1)	(3.84, 5.63, 7.48)	(1.00, 2.88, 4.72)	0.48
	PSH	(0.18, 0.28, 0.61)	(0.13, 0.18, 0.26)	(1, 1, 1)	(0.37, 0.51, 1.00)	0.08
	HSH	(0.49, 0.64, 1.06)	(0.21, 0.35, 1.00)	(1.00, 1.95, 2.67)	(1, 1, 1)	0.17
CCI = 0.01						
LD	PNB	(1, 1, 1)	(0.14, 0.20, 0.33)	(1, 1, 1)	(0.14, 0.20, 0.33)	0.23
	HNB	(3.00, 5.00, 7.00)	(1, 1, 1)	(3.00, 5.00, 7.00)	(1.00, 1.00, 1.00)	0.48
	PSH	(1.00, 1.00, 1.00)	(0.14, 0.20, 0.33)	(1, 1, 1)	(0.14, 0.20, 0.33)	0.08
	HSH	(3.00, 5.00, 7.00)	(1.00, 1.00, 1.00)	(3.00, 5.00, 7.00)	(1, 1, 1)	0.17
CCI = 0.01						
SS	PNB	(1, 1, 1)	(0.51, 0.63, 1.00)	(3.47, 5.62, 7.68)	(1.59, 3.40, 5.06)	0.34
	HNB	(1.00, 1.59, 1.97)	(1, 1, 1)	(4.74, 6.77, 8.65)	(3.21, 5.34, 7.34)	0.49
	PSH	(0.13, 0.20, 0.26)	(0.12, 0.15, 0.21)	(1, 1, 1)	(0.29, 0.43, 0.96)	0.06
	HSH	(0.20, 0.29, 0.63)	(0.14, 0.19, 0.31)	(1.04, 2.34, 3.43)	(1, 1, 1)	0.11
CCI = 0.01						
CE	PNB	(1, 1, 1)	(0.16, 0.22, 0.45)	(1.00, 1.26, 1.41)	(0.18, 0.28, 0.71)	0.12
	HNB	(2.25, 4.45, 6.40)	(1, 1, 1)	(2.90, 4.69, 6.51)	(1.00, 1.30, 1.46)	0.45
	PSH	(0.71, 0.79, 1.00)	(0.15, 0.21, 0.34)	(1, 1, 1)	(0.22, 0.37, 1.08)	0.11
	HSH	(1.42, 3.56, 5.61)	(0.68, 0.77, 1.00)	(0.93, 2.71, 4.52)	(1, 1, 1)	0.32
CCI = 0.02						

Table 15

Final assessment of alternatives of shipping investment.

Panamax-regime weight	RE	LP	FC	LD	SS	CE	Alternative priority weight
	0.37	0.31	0.22	0.10	0.00	0.00	
Handymax-regime weight	0.40	0.33	0.26	0.00	0.00	0.01	
PNB	0.00	0.31	0.32	0.00	0.43	0.03	0.17
HNB	0.20	0.50	0.53	0.50	0.54	0.53	0.39
PSH	0.36	0.00	0.00	0.00	0.00	0.02	0.13
HSH	0.44	0.19	0.16	0.50	0.03	0.43	0.29

$$d(LD) = (0.00, 0.50, 0.00, 0.50)$$

$$d(SS) = (0.43, 0.54, 0.00, 0.03)$$

$$d(CE) = (0.03, 0.53, 0.02, 0.43)$$

The results by regime switching analytic hierarchy process (RS-FAHP) for the shipping investment selection problem are summarized in Table 15. The priority weights of alternatives, PNB, HNB, PSH and HSH, are 0.17, 0.39, 0.13 and 0.29, respectively. As it is seen in Table 15, the priority weight of the Handymax new building is superior to other alternatives.

5. Conclusion

The proposed methodology, RS-FAHP, improves functionality of the traditional AHP and FAHP methods by using the regime switching framework. In business practice, expert valuations may differ

according to the concerning alternatives. The investigative space of the alternatives is not constant in many practical problems. By utilizing the RS-FAHP approach, different candidates are able to be analyzed in different evaluation spaces and particulars.

The Monte-Carlo simulation method is frequently used in business practice for assessment of investments. This paper utilizes the fuzzy Monte-Carlo method by embedding into the RS-FAHP decision process a financial risk criterion. Experts indicated its relative importance as a secondary indicator behind the return on equity.

Another novel contribution is the recommendation for the consistency prioritization. In many practical examples, it is very difficult to define a robust and efficient way for expert prioritization. Although this approach was suggested previously, several drawbacks exist in the case of pure judgmental prioritization or use of experience in the intended business field. Level of consistency is

one of unique indicators of the decision quality and robustness. Therefore, the former expert prioritization algorithm is redesigned by embedding consistency level superiority among the other participating experts.

The proposed method is illustrated by an empirical work about the ship investment selection problem. The Handymax size new building bulk carrier project is found superior.

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