

(c) Earthquakes-1 Exercise 1.22 :

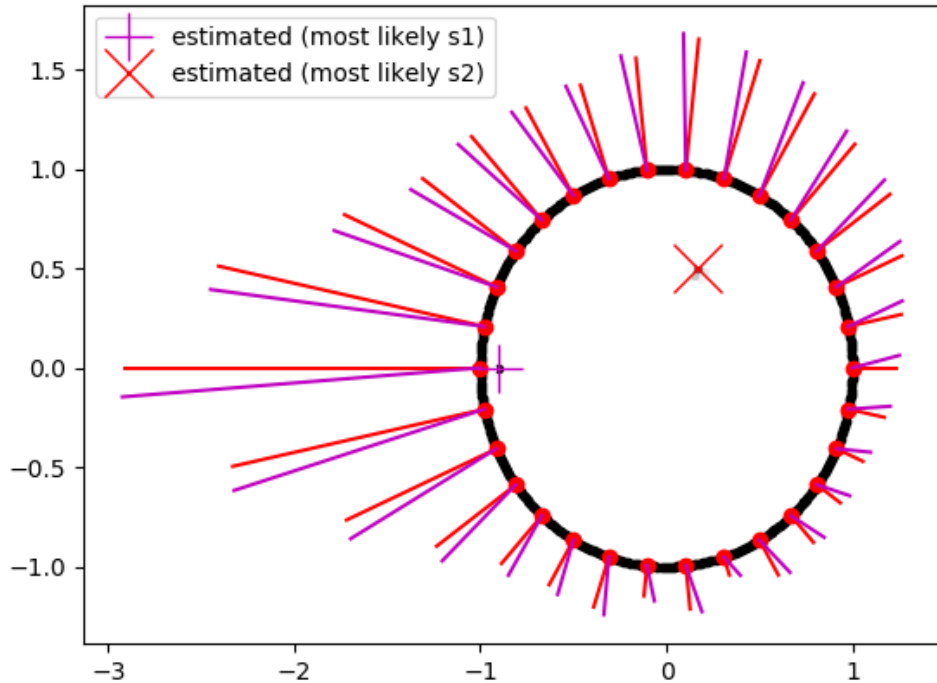
$$1. \quad p(s1|v) = \sum_{s2} p(s1, s2|v) = \sum_{s2} \frac{p(v|s1, s2) * p(s1, s2)}{p(v)}$$

$$= \sum_{s2} \frac{p(s1, s2)}{p(v)} * \prod_{i=1}^N p(v_i|s1, s2) \quad \left(\frac{p(s1, s2)}{p(v)} \text{ is constant} \right)$$

The posterior is calculated by Julia code (earthquakeExerciseSetup.jl), which can be found in appendix:

$$p(s1|v) = 0.49899688076583376$$

The visualization of the posterior is showed below:



2. To calculate $\log p(v|H2)$:

$$\log p(v_i|H2) = \sum_{s1, s2} \log p(v_i, s1, s2|H2) = \sum_{s1, s2} \log p(v_i|s1, s2, H2) * p(s1, s2)$$

since the 30 sensors are independent, we get: $\log p(v|H2) = \sum_v \log p(v_i|H2)$

To calculate $\log p(v|H1)$:

$$\log p(v_i|H1) = \sum_s \log p(v_i, s|H1) = \sum_s \log p(v_i|s, H1) * p(s)$$

since the 30 sensors are independent, we get: $\log p(v|H1) = \sum_v \log p(v_i|H1)$

the value of $\log p(v|H2) - \log p(v|H1)$ is calculated by Julia code (oneOrTwo_122.jl), which can be found in appendix:

$$\log p(v|H2) - \log p(v|H1) = 261.870365746503$$

3. Since $\log p(v|H2) - \log p(v|H1)$ is positive, we may argue that $\log p(v|H2) > \log p(v|H1)$. The log function is monotonically increasing, therefore, $p(v|H2) > p(v|H1)$. The probability of explosion will increase as the observed sensor increase. Therefore, the probability of two explosions is bigger than that of one.

4. $Computational\ complexity = N * S^k * (4k - 1) + N * S^k * 6 + N * S^k * 2 + 30$

$$= N * S^k * (4k - 1 + 6 + 2) + 30 = O(k * S^k)$$

3. (d) Earthquakes-1 Exercise 1.23 :

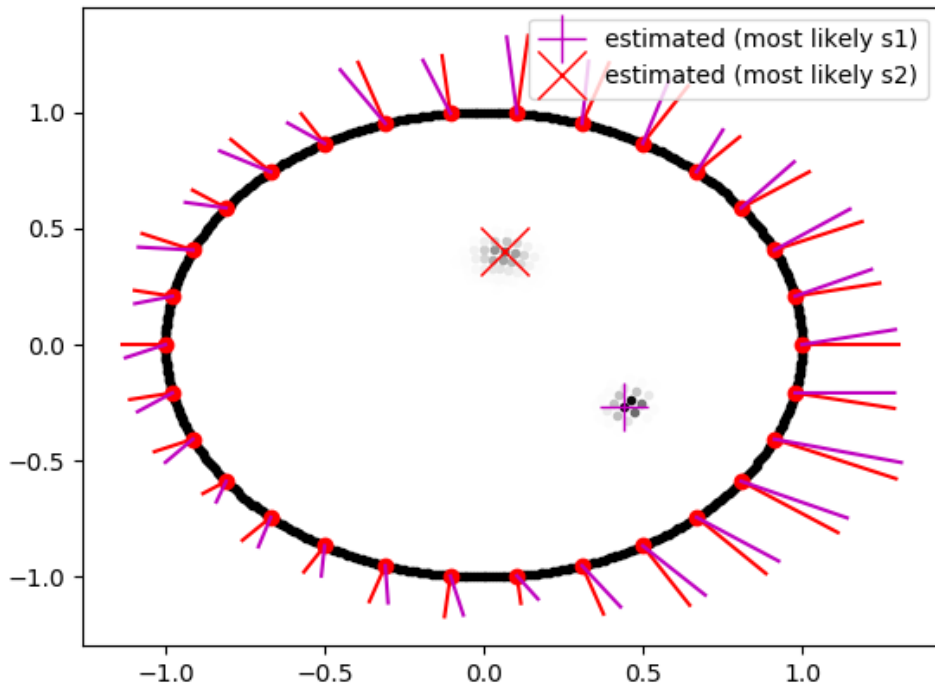
1. $p(s1|v) = \sum_{s2} p(s1, s2|v) = \sum_{s2} \frac{p(v|s1, s2) * p(s1, s2)}{p(v)}$

$$= \sum_{s2} \frac{p(s1, s2)}{p(v)} * \prod_{i=1}^N p(v_i|s1, s2) \quad \left(\frac{p(s1, s2)}{p(v)} \text{ is constant} \right)$$

The posterior is calculated by Julia code (earthquake_123.jl), which can be found in appendix:

$$p(s1|v) = 0.1103265594730385$$

The visualization of the posterior is showed below:



2. To calculate $\log p(v|H2)$:

$$\log p(v_i|H2) = \sum_{s1,s2} \log p(v_i, s1, s2|H2) = \sum_{s1,s2} \log p(v_i|s1, s2, H2) * p(s1, s2)$$

since the 30 sensors are independent, we get: $\log p(v|H2) = \sum_v \log p(v_i|H2)$

To calculate $\log p(v|H1)$:

$$\log p(v_i|H1) = \sum_s \log p(v_i, s|H1) = \sum_s \log p(v_i|s, H1) * p(s)$$

since the 30 sensors are independent, we get: $\log p(v|H1) = \sum_v \log p(v_i|H1)$

the value of $\log p(v|H2) - \log p(v|H1)$ is calculated by Julia code (oneOrTwo_122.jl), which can be found in appendix:

$$\log p(v|H2) - \log p(v|H1) = 294.8538271503182$$

3. This might be because many different combinations of explosion points can result to a same mean value of incoming explosions. Therefore, for such combinations, a sensor, which just measures the mean of incoming explosion, can only records one value, while the sensor in exercise 1.22 may record many different observed values for these case. There, it can be more difficult for the sensors in exercise 1.23 to accurately estimate the true explosion location.