Homework1 Report

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1 Question 1

1.1 1.a

Given an arbitrary 3D rotation matrix R,

$$\begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \tag{1}$$

Assume there is a 3D vector A =

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

The magnitude of A is 1. If A is rotated through the arbitrary rotation matrix, its magnitude would not change. Therefore, we have

$$A' = A{\cdot}R = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_4 \\ r_7 \end{bmatrix} \tag{3}$$

Since the magnitude A′ is equal to A, therefore

$$||A^{'}|| = \sqrt{r_1^2 + r_4^2 + r_7^2} = 1 \tag{4}$$

Equation 4 proves that $r_1 \leq 1$, $r_4 \leq 1$, $r_7 \leq 1$, otherwise their sum will be larger than 1. Similarly, If we use a 3D vector B along y-axis and C along z-axis, it can be proved that $r_2 \leq 1$, $r_5 \leq 1$, $r_8 \leq 1$ and $r_3 \leq 1$, $r_6 \leq 1$, $r_9 \leq 1$ respectively. Hence, for any arbitrary 3D rotation matrix, $||r_i|| \leq 1$, where i = 1,2,...,9.

1.2 1.b

From axis-angle to rotation matrix, we have

$$R = \begin{bmatrix} \cos\theta + u_x^2(1-\cos\theta) & u_x u_y(1-\cos\theta) - u_z\sin\theta & u_x u_z(1-\cos\theta) + u_y\sin\theta \\ u_y u_x(1-\cos\theta) + u_z\sin\theta & \cos\theta + u_y^2(1-\cos\theta) & u_y u_z(1-\cos\theta) + u_x\sin\theta \\ u_z u_x(1-\cos\theta) - u_y\sin\theta & u_z u_y(1-\cos\theta) + u_x\sin\theta & \cos\theta + u_y^2(1-\cos\theta) \end{bmatrix}$$

Since

$$\cos\theta + u_x^2(1-\cos\theta) = \cos(-\theta) + (-u)_x^2(1-\cos(-\theta)) \tag{5}$$

$$u_x u_y(1-\cos\theta) - u_z\sin\theta) = (-u_x)(-u_y)(1-\cos(-\theta)) - (-u_z\sin(-\theta)) \tag{6}$$

$$u_x u_z(1-\cos\theta) + u_y\sin\theta = (-u_x)(-u_z)(1-\cos(-\theta)) + u(-_y)\sin(-\theta) \tag{7}$$

$$u_y u_x(1-\cos\theta) + u_z\sin\theta = (-u_y)(-u_x)(1-\cos(-\theta)) + (-u_z)\sin(-\theta) \tag{8}$$

$$\cos\theta + u_y^2(1-\cos\theta) = \cos(-\theta) + (-u_y)^2(1-\cos(-\theta)) \tag{9}$$

$$u_y u_z(1-\cos\theta) + u_x\sin\theta = (-u_y)(-u_z)(1-\cos(-\theta)) + (-u_x)\sin(-\theta) \tag{10}$$

$$u_z u_x(1-\cos\theta) - u_y\sin\theta = (-u_z)(-u_x)(1-\cos(-\theta)) - (-u_y)\sin(-\theta) \tag{11}$$

$$u_z u_y(1-\cos\theta) + u_x\sin\theta = (-u_z)(-u_x)(1-\cos(-\theta)) - (-u_y)\sin(-\theta) \tag{12}$$

$$\cos\theta + u_y^2(1-\cos\theta) = \cos(-\theta) + (-u_y)^2(1-\cos(-\theta)) \tag{13}$$

Therefore, for any rotation matrix R, we have $R_{k,\theta} = R_{-k,-\theta}$

1.3 1.c

Given two arbitrary Cartesian coordinate frames a and b, each row in a rotation matrix ${}^aR_b$ represents the projection of each b coordinate axis to each of the a coordinate axes.

For example, ${}^aR_b = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$, then r_{11}, r_{12}, r_{13} are the projection of X_b on X_a, Y_a, Z_a. Similarly, r_{21}, r_{22}, r_{23} are the projection of Y_b on X_a, Y_a, Z_a, and r_{31}, r_{32}, r_{33} are the projection of Z_b on X_a, Y_a, Z_a.

1.4 1.d

The eigenvalues of an orthogonal rotation matrix are 1, $e^{i\theta}$ and $e^{-i\theta}$, where θ is the angle of rotation. The eigenvector of the orthogonal rotation matrix is the axis of rotation.

2 Question 2

2.1 2.a

$\mathrm{R}_z(\alpha)R_y(\beta)R_z(\gamma)$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \cdot \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma & \cos\alpha\cos\beta(-\sin\gamma) - \sin\alpha\cos\alpha & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\gamma & \sin\alpha\cos\beta(-\sin\gamma) + \cos\alpha\cos\gamma & \sin\alpha\sin\beta \\ -\sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{3}}{4} - \frac{\sqrt{6}}{8} & -\frac{\sqrt{2}}{4} & -\frac{3}{4} + \frac{\sqrt{2}}{8} \\ \frac{1}{4} - \frac{3\sqrt{2}}{8} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{3}}{4} + \frac{\sqrt{6}}{8} \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

Therefore, β=$\cos^{-1}(\frac{\sqrt{2}}{4})$= 1.2094 rad or -1.2094 rad,

For the first case, if β=1.2094, then $\alpha = \tan^{-1}(\frac{\sin\alpha\sin\beta}{\cos\alpha\sin\beta})$= 2.2304 rad, $\gamma = \tan^{-1}(\frac{\sin\beta\sin\gamma}{-\sin\beta\cos\gamma})$= 0.8571 rad. $[\alpha, \beta, \gamma]$ = [2.2304, 1.2094, 0.8571]

For the second case, if β=-1.2094 rad, $\alpha = \tan^{-1}(\frac{\sin\alpha\sin\beta}{\cos\alpha\sin\beta})$= -0.9112 rad, $\gamma = \tan^{-1}(\frac{\sin\beta\sin\gamma}{-\sin\beta\cos\gamma})$= -2.2845 rad. $[\alpha, \beta, \gamma]$ = [-0.9112, -1.2094, -2.2845]

2.2 2.b

The rotation matrix represents a final state orientation, while the Z-Y-X Euler angles represents the rotation process. Since the rotation direction 'clockwise' or 'anti-clockwise' and the order of axis are not specified in the Z-Y-X Euler angles representation, there are various ways to reach the final orientation state.

2.3 2.c

The limitation of using Euler angle representation for rotation matrix is the Gimbal lock problem.

Gimbal lock is the loss of one degree of freedom in a three-dimensional space when two axes of euler angle representation are parallel. For example, given two poses of two different frames, there are twelve ways to rotate from one to another.

For example, when doing the Gyroscope measurement, one degree of freedom might lost. The roll and yaw probably cannot be distinguished. Another example is the robotic arm control, the arm might not move in the expected direction due to Gimbal lock.

Using quaternion representation, instead of euler angle representation, can avoid gimbal lock problem. Quaternion is a 4D representation which represents 3D rotation, that's why it is sufficient to avoid any ambiguities.

3 Question 3

3.1 3.a

The quaternion to rotation matrix is

$$\mathrm{R} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} 1-2q_y^2-2q_z^2 & 2q_xq_y-2q_zq_w & 2q_xq_z+2q_yq_w \\ 2q_xq_y+2q_zq_w & 1-2q_x^2-2q_z^2 & 2q_yq_z-2q_xq_w \\ 2q_xq_z-2q_yq_w & 2q_yq_z+2q_xq_w & 1-2q_x^2-2q_y^2 \end{bmatrix}$$

Since

$$q_x^2+q_y^2+q_z^2+q_w^2 = 1. \tag{14}$$

We can have

$$m_{11}+m_{22}+m_{33}+1 = 4-4*q_x^2-4*q_y^2-4*q_z^2 = 4*q_w^2 \tag{15}$$

$$m_{32}-m_{23} = 4q_xq_w \tag{16}$$

$$m_{13}-m_{31} = 4q_yq_w \tag{17}$$

$$m_{21}-m_{12} = 4q_zq_w \tag{18}$$

Therefore,

$$q_w = \frac{\sqrt{m_{11}+m_{22}+m_{33}+1}}{4} = 0.02226 \tag{19}$$

$$q_x = \frac{m_{32}-m_{23}}{4q_w} = -0.36042 \tag{20}$$

$$q_y = \frac{m_{13}-m_{31}}{4q_w} = 0.43968 \tag{21}$$

$$q_z = \frac{m_{21}-m_{12}}{4q_w} = 0.82236 \tag{22}$$

$$q = (q_x, q_y, q_z, q_w) = (-0.36042, 0.43968, 0.82236, 0.02226) \tag{23}$$

3.2 3.b

R = $\begin{bmatrix} 1-2q_y^2-2q_z^2 & 2q_xq_y-2q_zq_w & 2q_xq_z+2q_yq_w \\ 2q_xq_y+2q_zq_w & 1-2q_x^2-2q_z^2 & 2q_yq_z-2q_xq_w \\ 2q_xq_z-2q_yq_w & 2q_yq_z+2q_xq_w & 1-2q_x^2-2q_y^2 \end{bmatrix}$

If q = -q, then $q_x = -q_x$, q_y=$-q_y$, q_z=$-q_z$, q_w=$-q_w$, then

$$R' = \begin{bmatrix} 1-2(-q_y)^2-2(-q_z)^2 & 2(-q_x)(-q_y)-2(-q_z)(-q_w) & 2(-q_x)(-q_z)+2(-q_y)(-q_w) \\ 2(-q_x)(-q_y)+2(-q_z)(-q_w) & 1-2(-q_x)^2-2(-q_z)^2 & 2(-q_y)(-q_z)-2(-q_x)(-q_w) \\ 2(-q_x)(-q_z)-2(-q_y)(-q_w) & 2(-q_y)(-q_z)+2(-q_x)(-q_w) & 1-2(-q_x)^2-2(-q_y)^2 \end{bmatrix}$$

$= R$

Therefore, q and -q represent a same rotation matrix, meaning q is equal to -q.

3.3 3.c

Assume $AB = BA$ and $Av = \lambda v$, v is an eigenvector of A. Then applying B to both sides, we obtain $BA\lambda = B(\lambda v)$ $\Rightarrow$ABv=B(λv), which is to say that Bv is an eigenvector of A. Since A is a rotation matrix, meaning it has distinct real eigenvalues, eacho of it eigenspaces is one dimensional. Since Bv and v live in the same one dimensional vector space, thus B has the same real eigenvectors as A.

Suppose v_1, v_2, v_3 are the eigenvectors of A. Since B also has the same eigenvectors, then $T_{-1}AT$ and $T_{-1}BT$ are both diagonal matrices, with their eigenvalues on the diagonal. Then we can have

$$T^{-1}ABT = (T_{-1}AT)(T_{-1}BT) = (T_{-1}BT)(T_{-1}AT) = T_{-1}BAT \tag{24}$$

If we add T on the left side and T_{-1} on the right side on the equation, we can obtain $TT^{-1}ABTT^{-1} = TT^{-1}BATT^{-1} \Rightarrow AB = BA$.

Hence, if rotation matrix R_a and rotation matrix R_b has the same eigenvectors, then they are commutative.

4 Question 4

5 Question 5

5.1 Q5.a

The frames coordinates are shown in figure 1, the standard Denavit-Hartenberg convention is shown in table 1.

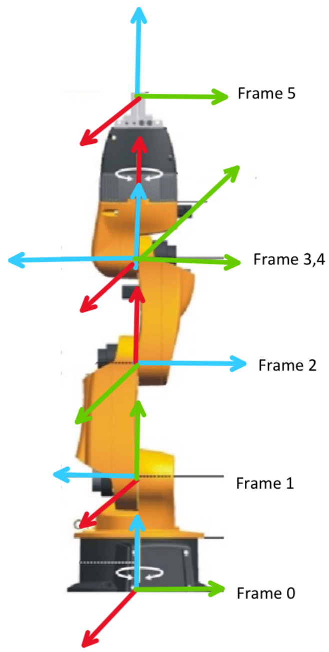

Figure 1: Question5

i	θ_i	d_i	a_i	α_i
1	$0+\theta_1$	0.147	0	$\pi/2$
2	$\pi/2+\theta_2$	0	0.155	π
3	$0+\theta_3$	0	0.135	π
4	$3\pi/2+\theta_4$	0	0	$3\pi/2$
5	$0+\theta_5$	0.218	0	0

Table1: standard Denavit-Hartenberg

The transformation matrix of standard DH is:

$$\begin{bmatrix} cos(\theta_i) & -sin(\theta_i)cos(\alpha_i) & sin(\theta_i)sin(\alpha_i) & a_i cos(\theta_i) \\ sin(\theta_i) & cos(\theta_i)cos(\alpha_i) & -cos(\theta_i)sin(\alpha_i) & a_i sin(\theta_i) \\ 0 & sin(\alpha_i) & cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the end-effector pose with respect to the origin can be calculated by combining all the transformations:

$$ {}^0T_5 = {}^0T_1{}^1T_2{}^2T_3{}^3T_4{}^4T_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.655 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25} $$

5.2 Q5.b

The modified Denavit-Hartenberg convention is shown in table 2:

i	a_i	α_i	θ_i	d_i
1	0	0	$0+\theta_1$	0.147
2	0	$\pi/2$	$\pi/2+\theta_2$	0
3	0.155	π	$0+\theta_3$	0
4	0.135	π	$3\pi/2+\theta_4$	0
5	0	$3\pi/2$	$0+\theta_5$	0.218
6	0	0	$0+\theta_6$	0

Table2: modified Denavit-Hartenberg

The transformation matrix of standard DH is:

$$ \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) & 0 & a_{i-1} \\ sin(\theta_i)cos(\alpha_{i-1}) & cos(\theta_i)cos(\alpha_{i-1}) & -sin(\alpha_{i-1}) & -d_i sin(\alpha_{i-1}) \\ sin(\theta_i)sin(\alpha_{i-1}) & cos(\theta_i)sin(\alpha_{i-1}) & cos(\alpha_{i-1}) & d_i cos(\alpha_{i-1}) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Therefore, the end-effector pose with respect to the origin can be calculated by combining all the transformations:

$$ {}^0T_6 = {}^0T_1{}^1T_2{}^2T_3{}^3T_4{}^4T_5{}^5T_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.655 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26} $$

5.3 Q5.c

The differences between standard Denavit-Hartenberg and modified Denavit-Hartenberg are:
1. In standard DH, Joint axis z_{i-1} is for joint i, while in modified DH, z_i is for joint i.
2. In standard DH, the order of transformation is translation first, and then rotation. In modified DH, the order of transformation is rotation first, and then translation.

6 Question 6

6.1 Q6.a

Let $\mathrm{T}_{gt} = \begin{bmatrix} R_{gt} & t_{gt} \\ 0_{1\times 0} & 1 \end{bmatrix}, T_{est} = \begin{bmatrix} R_{est} & t_{est} \\ 0_{1\times 0} & 1 \end{bmatrix}$

So $\mathrm{T}_{gt}^{-1} = \begin{bmatrix} R_{gt} & t_{gt} \\ 0_{1\times 0} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R_{gt}^T & -R_{gt}^T t_{gt} \\ 0_{1\times 0} & 1 \end{bmatrix}, T_{est}^{-1} = \begin{bmatrix} R_{est} & t_{est} \\ 0_{1\times 0} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R_{est}^T & -R_{est}^T t_{est} \\ 0_{1\times 0} & 1 \end{bmatrix}$

Therefore,

$T_{gt}T_{est}^{-1} = \begin{bmatrix} R_{gt}R_{est}^T & -R_{gt}R_{est}^T t_{est} + t_{gt} \\ 0_{1\times 0} & 1 \end{bmatrix}, T_{gt}^{-1}T_{est} = \begin{bmatrix} R_{gt}^T R_{est} & R_{gt}t_{est} - R_{gt}^T t_{gt} \\ 0_{1\times 0} & 1 \end{bmatrix}$

If $R_{gt} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}$, $R_{est} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{14} & r_{15} & r_{16} \\ r_{17} & r_{18} & r_{19} \end{bmatrix}$

Then $R_{gt}^T R_{est} =$

$$\begin{bmatrix} r_1 & r_4 & r_7 \\ r_2 & r_5 & r_8 \\ r_3 & r_6 & r_9 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{14} & r_{15} & r_{16} \\ r_{17} & r_{18} & r_{19} \end{bmatrix} = \begin{bmatrix} r_1r_{11} + r_4r_{14} + r_7r_{17} & r_1r_{12} + r_4r_{15} + r_7r_{18} & r_1r_{13} + r_4r_{16} + r_7r_{19} \\ r_2r_{11} + r_5r_{14} + r_8r_{17} & r_2r_{12} + r_5r_{15} + r_8r_{18} & r_2r_{13} + r_5r_{16} + r_8r_{19} \\ r_3r_{11} + r_6r_{14} + r_9r_{17} & r_3r_{12} + r_6r_{15} + r_9r_{18} & r_3r_{13} + r_6r_{16} + r_9r_{19} \end{bmatrix}$$

Then $R_{gt}R_{est}^T =$

$$\begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \begin{bmatrix} r_{11} & r_{14} & r_{17} \\ r_{12} & r_{15} & r_{18} \\ r_{13} & r_{16} & r_{19} \end{bmatrix} = \begin{bmatrix} r_1r_{11} + r_2r_{12} + r_3r_{13} & r_1r_{14} + r_2r_{15} + r_3r_{16} & r_1r_{17} + r_2r_{18} + r_3r_{19} \\ r_4r_{11} + r_5r_{12} + r_6r_{13} & r_4r_{14} + r_5r_{15} + r_6r_{16} & r_4r_{17} + r_5r_{18} + r_6r_{19} \\ r_7r_{11} + r_8r_{12} + r_9r_{13} & r_7r_{14} + r_8r_{15} + r_9r_{16} & r_7r_{17} + r_8r_{18} + r_9r_{19} \end{bmatrix}$$

The Rodrigues' formula are,

$$\theta = arcos(\frac{R_{1,1} + R_{2,2} + R_{3,3} - 1}{2}) \tag{27}$$

$$u = \frac{1}{2\sin\theta} \begin{bmatrix} R_{3,2} - R_{2,3} \\ R_{1,3} - R_{3,1} \\ R_{2,1} - R_{1,2} \end{bmatrix} \tag{28}$$

Since $R_{1,1}+R_{2,2}+R_{3,3}$ are the same for both $R_{gt}^T R_{est}$ and $R_{gt}R_{est}^T$. Therefore, the rotation is the same for both $T_{gt}T_{est}^{-1}$ and $T_{gt}^{-1}T_{est}$.

However, for $T_{gt}^{-1}T_{est}$, $R_{3,2}$ - $R_{2,3}$ = $r_3r_{12} + r_6r_{15} + r_9r_{18}$ - $r_2r_{13} + r_5r_{16} + r_8r_{19}$, for $T_{gt}T_{est}^{-1}$, $R_{3,2}$ - $R_{2,3}$ = $r_7r_{14} + r_8r_{15} + r_9r_{16}$ - $r_4r_{17} + r_5r_{18} + r_6r_{19}$. $R_{3,2}$ - $R_{2,3}$ is not the same in this case unless T_{gt} and T_{est} are the same symmetrical matrix, which is not likely to happen. Similarly, $R_{1,3}$ - $R_{3,1}$ and $R_{2,1}$ - $R_{1,2}$ are not the same unless T_{gt} and T_{est} are the same symmetrical matrix.

Since the rotation axis is not likely to be the same, the rotation of $T_{gt}T_{est}^{-1}$ and $T_{gt}^{-1}T_{est}$ are basely not equal to each other.

The translation of $T_{gt}T_{est}^{-1}$ and $T_{gt}^{-1}T_{est}$ are $-R_{gt}R_{est}^{T}t_{est}+t_{gt}$ and $R_{gt}t_{est}-R_{gt}^{T}t_{gt}$ respectively, both of them look completely different. Hence, the translation of $T_{gt}T_{est}^{-1}$ and $T_{gt}^{-1}T_{est}$ is not likely to be the same.

6.2 Q6.b

Since the rotation transformation matrices are full rank, so they are invertible. Assume

$$(T_{gt}^{-1}T_{est})^{-1} = \begin{bmatrix} R & t \\ 0_{1\times 0} & 1 \end{bmatrix}^{-1} \tag{29}$$

Therefore

$$T_{est}^{-1}T_{gt} = \begin{bmatrix} R^T & -R^T t \\ 0_{1\times 0} & 1 \end{bmatrix} \tag{30}$$

Equation 30 shows that the translation between $T_{gt}^{-1}T_{est}$ and $T_{est}^{-1}T_{gt}$ does not change, their translation is the same, although their rotation is different.

6.3 Q6.c

Suppose the Youbot is in the zero position, there is an error in joint position reading which produces $\theta_1 = 0.5°$. The positioning error in the translation component is 0.

6.4 Q6.d

Suppose the Youbot is in the zero position, there is an error in joint position reading which produces $\theta_4 = 0.5°$. The positioning error in the translation component is 0.001902m.

6.5 Q6.e

Suppose the Youbot is in the zero position, there is an error in joint position reading which produces $\theta_2 = 0.5°$. The positioning error in the translation component is 0.004433m.

6.6 Q6.f

The error in position reading in each joint cause different error because the errors accumulate during the transformation from base link to the end-effector. Basically, if the error occurs in the joint that is closer to the base link, the position reading error detected in the end-effector will be larger. However, joint 1 of KUKA robot rotates horizontally, so the reading position error of the end-effector will be zero when the entire robotic arm erects vertically.

When it comes to a bigger robot, the error in reading position would be even lager, especially if the errors occur in the joints that are near the base link.

7 Question 7

7.1 Q7.a

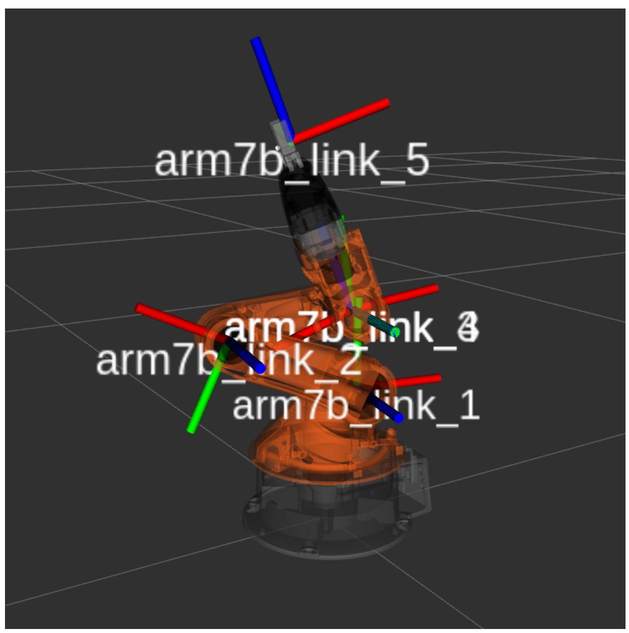

Figure 2: Question7

i	θ_i	d_i	a_i	α_i
1	$170\pi/180 + \theta_1$	0.145	0.033	$\pi/2$
2	$155\pi/180 + \theta_2$	0	0.155	0
3	-$146\pi/180 + \theta_3$	0	0.135	0
4	-$167.5\pi/180 + \theta_4$	0	0	$\pi/2$
5	$167.5\pi/180 + \theta_5$	0.185	0.002	0

Table3: standard Denavit-Hartenberg

Figure 2 shows the coordinates of different frames. Table 3 shows the stand Denavit-Hartenberg of the complete dimension of the Youbot.