Robotic Systems Engineering
Coursework 1: Rigid Transformations and Forward Kinematics

Prof. Danail Stoyanov
Department of Computer Science
University College London
Gower Street, London, WC1E 6BT

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To get full credit for an answer, you are *required* to provide a full working solution. For some questions, you will be asked to write code; for these you are expected to include a *full print out* of any requested results or graphs in the report. Furthermore, you will be required to *upload* your code to Moodle along with your submitted coursework manuscript in .zip extension. The necessary packages are available on https://github.com/surgical-vision/comp0127_lab

Rigid Transformations

1. a. Given an arbitrary 3D rotation matrix,

$$\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}$$

Prove that $||r_i|| \leq 1$ where $i = 1, 2, ..., 9$. [3 marks]

   b. For any rotation matrix $\mathbf{R}$, prove that $\mathbf{R}_{k,\theta} = \mathbf{R}_{-k,-\theta}$, where k is the unit vector defined axis of rotation and θ is the angle of rotation. [3 marks]

   c. Given two arbitrary Cartesian coordinate frames a and b, what does each row in a rotation matrix ${}^a\mathbf{R}_b$ represent? [3 marks]

   d. Identify the relationship between axis/angle of rotation and the eigenvector/eigenvalue of a rotation matrix. [3 marks]

2. a. Convert this rotation matrix into Z-Y-Z Euler angle representation, i.e. $\mathbf{R}_z\mathbf{R}_y\mathbf{R}_z$.

$$\mathbf{R} = \begin{bmatrix} -\frac{\sqrt{3}}{4} - \frac{\sqrt{6}}{8} & -\frac{\sqrt{2}}{4} & -\frac{3}{4} + \frac{\sqrt{2}}{8} \\ \frac{1}{4} - \frac{3\sqrt{2}}{8} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{3}}{4} + \frac{\sqrt{6}}{8} \\ -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

[5 marks]

b. Why can a rotation matrix have more than one Z-Y-X Euler angle representation? [3 marks]

c. What is the limitation of using Euler angle representation for rotation matrix? How could one avoid this condition in practice? [3 marks]

3. a. Convert the rotation matrix in Question 2.a into a quaternion. [5 marks]

   b. Prove that a rotation quaternion $\mathbf{q}$ and $-\mathbf{q}$ are equivalent. [3 marks]

   c. When do two arbitrary rotation matrices $\mathbf{R}_a$ and $\mathbf{R}_b$ become commutative? [5 marks]

4. Complete the following tasks by filling the code in the packages ”cw1_stack/cw1q4_srv” and ”cw1_stack/cw1q4”. Please note that you can either use the provided cpp template ”cw1q4_node.cpp” or ”cw1q4_node.py” to complete the tasks. **The report is not required for this question.**

   a. Fill in the appropriate input and output for each service. [3 marks]
      - The quaternion message is ”geometry_msgs/Quaternion”, namely ”q”.
      - The rodrigues representation message is three ”std_msgs/Float64”, namely ”x” ,”y” and ”z”.
      - The Z-Y-X euler angle representation message is three ”std_msgs/Float64”, namely ”z” ,”y” and ”x”.
      - The rotation matrix is three ”std_msgs/Float64MultiArray” representing three rows vectors in the matrix, namely ”r1” ,”r2” and ”r3”.

   b. Write a ROS node for converting a quaternion representation to an euler angle representation $\mathbf{R}_z\mathbf{R}_y\mathbf{R}_x$. [3 marks]

   c. Write a ROS node for converting a quaternion representation to rodrigues representation. [3 marks]

   d. Write a ROS node for converting a rotation matrix to a quaternion representation. [3 marks]

Robot Kinematic

5. Apply forward kinematics on the KUKA YouBot manipulator to identify the end-effector pose with respect to the origin using the simplified dimensions of the Youbot shown in Figure 1.

   a. Use standard Denavit-Hartenberg convention. [5 marks]

   b. Use modified Denavit-Hartenberg convention. [4 marks]

   c. What is the difference between these two conventions? [1 mark]

d. Complete this task by filling in the code in the package "cw1_stack/cw1q5d". **The report is not required for this subquestion.** Write a ROS node for computing the transformation using forward kinematics. Your code should be able to

- Take DH variables from the rostopic that publishes joint data.
- Publish a set of transformations relating the frame "base_link" and each frame on the arm "arm5d_link_i" where i is the frame, using tf messages.

[12 marks]

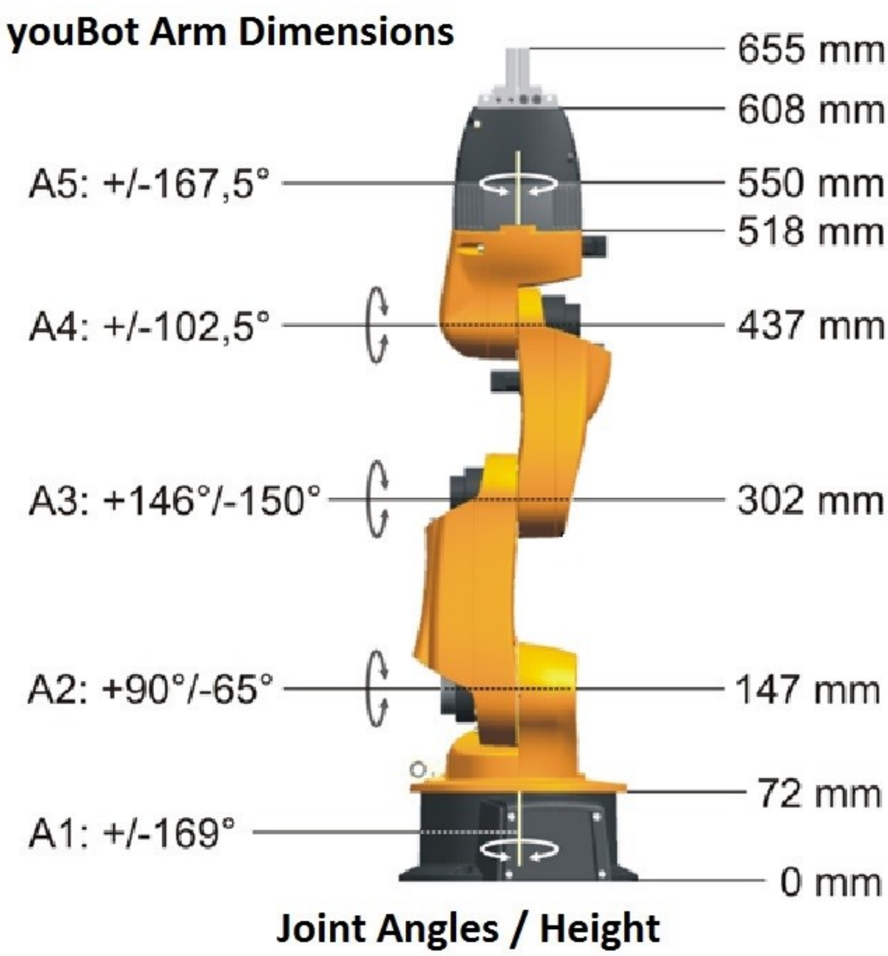

Figure 1: Kuka youBot Manipulator's simplified dimensions

6. In robotic applications, the error in rigid transformation is represented as $\mathbf{T}_{\text{error}} = \mathbf{T}_{\text{gt}}^{-1}\mathbf{T}_{\text{est}}$, where $\mathbf{T}_{\text{gt}}$ is the ground truth of the transformation and $\mathbf{T}_{\text{est}}$ is the estimation of the transformation. The residue in rotation is represented by the rotation part of $\mathbf{T}_{\text{error}}$ (also depending on the interested representation) whereas the error in translation is represented by the norm of the translation component of $\mathbf{T}_{\text{error}}$. Use **THIS FORMULA** and **SIMPLIFIED MODEL** of Youbot (from question 5) to answer the following questions.

   NOTE: For the subquestions 6b-6d, The full calculation of $\mathbf{T}_{\text{gt}}$ and $\mathbf{T}_{\text{est}}$ is not required.

a. If we use $\mathbf{T}_{\text{gt}}\mathbf{T}_{\text{est}}^{-1}$ to calculate the error instead of using $\mathbf{T}_{\text{gt}}^{-1}\mathbf{T}_{\text{est}}$, will the error in the rotation (in rodrigues representation) and the translation be the same? [3 marks]

b. If we use $\mathbf{T}_{\text{est}}^{-1}\mathbf{T}_{\text{gt}}$ to calculate the error instead of using $\mathbf{T}_{\text{gt}}^{-1}\mathbf{T}_{\text{est}}$, will the error in the translation be the same? [1 marks]

c. Suppose the Youbot is in the zero position, i.e. $\theta_i = 0$ for $i = 1, 2, ..., 5$, but there is an error in joint position reading which produces $\theta_1 = 0.5°$. Compute the positioning error in the translation component. [1 mark]

d. Suppose the Youbot is in the zero position, i.e. $\theta_i = 0$ for $i = 1, 2, ..., 5$, but there is an error in joint position reading which produces $\theta_4 = 0.5°$. Compute the positioning error in the translation component. [1 mark]

e. Suppose the Youbot is in the zero position, i.e. $\theta_i = 0$ for $i = 1, 2, ..., 5$, but there is an error in joint position reading which produces $\theta_2 = 0.5°$. Compute the positioning error in the translation component. [1 mark]

f. Explain why the error in position reading in each joint produce different errors. What is the implication of this situation in a bigger robot? [6 marks]

7. Apply forward kinematics on the KUKA YouBot manipulator to identify the end-effector pose with respect to the origin using the complete dimensions of the Youbot described in the xacro file ”youbot_stack/youbot_description/urdf/youbot_arm/arm.urdf.xacro”.

   a. Use **STANDARD** Denavit-Hartenberg convention to determine the location of each frame. [7 marks]

   b. Complete this task by filling in the code in the package ”cw1_stack/cw1q7b”. **The report is not required for this subquestion.** Write a ROS node for computing the transformation using forward kinematics. Your code should be able to

      - Take DH variables from the rostopic that publishes joint data.
      - Publish a set of transformations relating the frame ”base_link” and each frame on the arm ”arm7b_link_i” where i is the frame, using tf messages.

[10 marks]

END OF COURSEWORK