EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2020**

Lecture 12: (Introduction to Grasping)

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12.1 Types of Grippers

Grippers differ by their functionality and ability to grasp different objects. For some objects, a "pinch grasp" executed with a parallel jaw is sufficient, while other objects can be grasped more easily with a suction cup, for example.
The human hand can be viewed as an extremely advanced gripper, able to grasp arbitrary objects, perform fine manipulation and provide an interface with the external world through an abundance of tactile sensors located under the skin. It has 27 degrees of freedom.

12.2 History

The history of artificial hands goes back to the middle ages. In 1509 the knight Goetz Von Berlichingen received a metal prosthetic hand that was able to mechanically grasp objects. The modern grippers, however, were developed in the late twentieth century:

1. 1982 - The Salisbury hand, with its 3 fingers, was the first robotic hand designed for dexterous manipulation.
2. 1987 - The Utah-MIT hand had 25 degrees of freedom with 4 fingers and was intended for research purpose.
3. 1993 - The HKUST hand, developed in Japan, showed a different paradigm for dexterous manipulation. The system had 3 one-finger arms operating and coordinating as a single system.

Why aren't we using these hands nowadays in the field of robotics? The more fingers and degrees of freedom a gripper has, the more challenging it is to control. It turns out that many commercial objectives can be achieved using the simple parallel-jaw grippers, or suction cups.

12.3 Grasp Planning

Our assumptions for this lecture are:

1. The grasped objects are rigid bodies
2. We have accurate models of the gripper and the objects

From static mechanics we know that the condition for equilibrium requires the sum of forces applied on an object to be zero. In the following case this translates to the application of forces from 3 directions such that they intersect in the middle of the triangle:

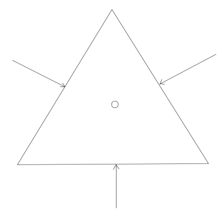

Figure 12.1: The statics condition

However, in robotics we are interested in motion, so our initial goal is to exert external forces to resist gravity and hold an object still. From that point on, additional forces can be applied to execute the required manipulation.
At the beginning, the field of grasping was geometry-based. A 3D CAD model or cameras were used to measure the 3D geometry of the objects and plan accordingly. In addition, the dynamics of the system were required for grasp planning. To plan a grasp (i.e., determine a set of contact locations for the object and the gripper fingers) and estimate whether or not a grasp will work, several physical properties of the grasped object need to be taken into consideration - the shape of the object, its density and the materials from which it is made.

12.4 Contact Models

A *contact* between a finger and an object can be described as a mapping between forces exerted by the finger at the point of contact and the resultant wrenches at some reference point on the object. we choose the center of mass of the object to be the object's reference point, and assume that the location of the contact point on the object is fixed. The forces at the contacts and on the object can be represented in terms of a set of coordinate frames attached at each contact location and the object reference point.

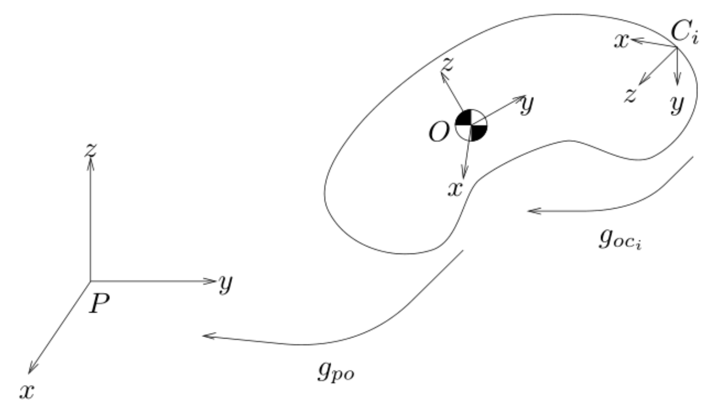

Figure 12.2: Coordinate frames for contact and object forces

The z-axis of the contact coordinate frame, C_i, is always chosen to be normal to the surface at the contact point and pointing inwards (as shown in figure 12.2). We use g_{oc_i} to denote the configuration from the contact point C_i to the object reference frame O, g_{po} to denote the configuration from the object to the fixed palm frame, where $g_{oc_i}, g_{po} \in SE(3)$. The force applied by a contact is modeled as a wrench F_{c_i} applied at the origin of the contact frame, C_i.

We use *Coulomb friction model* to model friction. The model asserts that the allowed tangential force is proportional to the applied normal force, and the proportionality constant is a function of the materials which are in contact. If we denote tangential force as $f^t \in \mathbb{R}$ and normal force as $f^n \in \mathbb{R}$, Coulomb's law states that slipping begins when $|f^t| > \mu f^n$, where $\mu > 0$ is the coefficient of friction. This implies that the range of tangential forces can be applied at a contact is given by

$$|f^t| \leq \mu f^n \tag{12.1}$$

Geometrically, the set of forces can be applied at a contact must lie in a cone, as shown in figure 12.3.

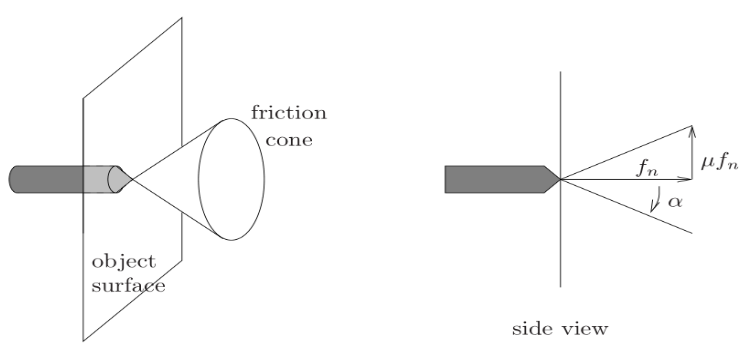

Figure 12.3: Geometric interpretation of the Coulomb friction model

12.4.1 Approximating the Friction Cone

We can approximate the friction cone by a set of facet vectors. Assumimg that we approximate the cone to n facet vectors, then we can use the following formula to represent the ith facet vector, as shown in 12.4.

$$f_i = \begin{bmatrix} \mu\cos\frac{2\pi i}{n} \\ \mu\sin\frac{2\pi i}{n} \\ 1 \end{bmatrix} \tag{12.2}$$

It is obtained by partition the cone to n parts and parameterize the equation $\sqrt{f_1^2 + f_2^2} = \mu f_3,\ f_3 = 1$. Doing so, we can simplify the constraints to a positive linear combination of facet vectors.

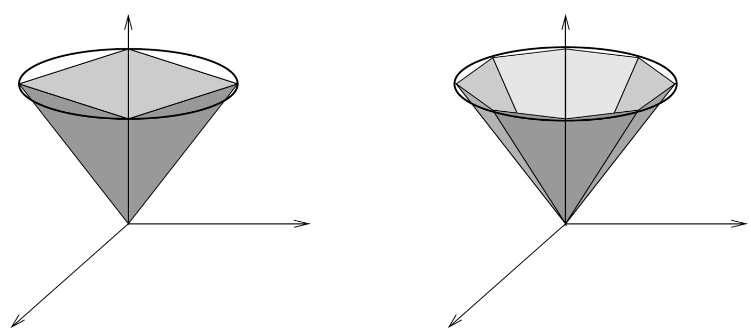

Figure 12.4: Approximations of a spatial friction cone

For example, to approximate the friction cone using 4 vectors, as in figure 12.4, we obtain

$$F = \begin{bmatrix} \mu & 0 & -\mu & 0 \\ 0 & \mu & 0 & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

In general, we model a contact using a wrench basis, $B_{c_i} \in \mathbb{R}^{p\times m_i}$, and a friction cone, FC_{c_i}. We require that FC_{c_i} satisfy the following properties:

1. FC_{c_i} is a closed subset of $\mathbb{R}^{m_i}$ with non-empty interior.
2. $f_1, f_2 \in FC_{c_i} \implies \alpha f_1 + \beta f_2 \in FC_{c_i}$, for $\alpha, \beta > 0$.

The set of allowable contact forces applied by a given contact is:

$$F_{c_i} = B_{c_i} f_{c_i} \quad f_{c_i} \in FC_{c_i} \tag{12.3}$$

The following sections show three examples of different B_{c_i}'s.

12.4.2 Frictionless Point Contact

A *frictionless point contact* is obtained when there is no friction between the fingertip and the object. In this case, forces can only be applied in the z direction of the contact coordinate frame and hence the applied wrench can be represented as

$$F_{c_i} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_{c_i}, \quad f_{c_i} \geq 0 \tag{12.4}$$

where $f_{c_i} \in \mathbb{R}$ is the magnitude of the force applied by the finger in the normal direction. The requirement that f_{c_i} be positive models the fact that a contact of this type can push on an object, but it cannot pull on the object.

12.4.3 Point Contact with Friction

In reality, we do want to make use of friction. A *point contact with friction* model is used when friction exists between the fingertip and the object. The applied wrench can be represented as

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f_{c_i}, \quad f_{c_i} \in FC_{c_i} \tag{12.5}$$

where FC_{c_i} is the friction cone, $FC_{c_i} = \{f \in \mathbb{R}^3 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3,\ f_3 \geq 0\}$. f_3 is the normal force, and f_1, f_2 are the tangential forces.

12.4.4 Soft-finger

A more realistic contact model is the *soft-finger* contact. Different to *point contact with friction* model, the soft-finger contact allows torques about the surface normal. For simplicity, torques are limited by a torsional friction coefficient, γ. The applied wrench can be represented as

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} f_{c_i}, \quad f_{c_i} \in FC_{c_i} \tag{12.6}$$

where the fiction cones is $FC_{c_i} = \{f \in \mathbb{R}^4 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3,\ f_3 \geq 0,\ |f_4| \leq \gamma f_3\}$.

12.4.5 The Grasp Map

To determine the effect of the contact forces, we need to transform the forces to the object reference frame. A force exerted by a single contact can be written in the object coordinates as

$$F_o = \mathrm{Ad}^T_{g^{-1}_{oc_i}} F_{c_i} = \begin{bmatrix} R_{oc_i} & 0 \\ \hat{p}_{oc_i} R_{oc_i} & R_{oc_i} \end{bmatrix} B_{c_i} f_{c_i}, \quad f_{c_i} \in FC_{c_i} \tag{12.7}$$

The matrix $\mathrm{Ad}^T_{g^{-1}_{oc_i}}$ is the wrench transformation matrix which maps contact wrenches to object wrenches. The *contact map* is defined to be the linear map between contact forces and object wrench: $G_i := \mathrm{Ad}^T_{g^{-1}_{oc_i}} B_{c_i}$. If there are K fingers contacting an object, the net force on the object is:

$$F_o = G_1 f_{c_1} + \ldots + G_k f_{c_k} = \begin{bmatrix} G_1 & \ldots & G_k \end{bmatrix} \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix} = G f_c \tag{12.8}$$

and the grasp map is

$$G = \begin{bmatrix} \mathrm{Ad}^T_{g^{-1}_{oc_1}} B_{c_1} & \ldots & \mathrm{Ad}^T_{g^{-1}_{oc_k}} B_{c_k} \end{bmatrix} \tag{12.9}$$

Thus, a grasp is completely described by the grasp map G and the friction cone FC.

12.5 Force Closure

A grasp is considered a "force-closure" grasp if for any external wrench $F_e \in \mathbb{R}^P$ applied to the object, we can find contact forces $f_c \in FC$ to resist it:

$$G f_c = -F_e \tag{12.10}$$

12.5.1 Necessity of Internal Forces

A key feature of force-closure grasp is the existence of *internal forces.* An internal force is a set of contact forces f_N which result in no net force on the object ($G f_N = 0$), i.e., an internal force lies in the null space of grasp map G. If f_N is in the interior of the friction cone, then it is called a *strictly internal force.* The internal forces can be used to insure that contact forces satisfy friction cone constraints.

Surjective Function: a function $f(x)$ from a set X to a set Y is surjective, if for every element $y \in Y$ of f, there is at least one element $x \in X$ of f such that $f(x) = y$.

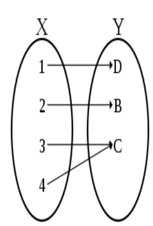

Figure 12.5: A surjective function from X to Y

Proposition 5.2. Necessity of Internal Forces: A grasp is "force-closure" if and only if the contact map G is surjective and there exists a vector of contact forces $f_N \in Null\{G\}$ such that $f_N \in int(FC)$, i.e., the contact forces lie within the interior of the friction cone.

The intuition for this proposition - the null space of G means that we apply forces that result with zero motion. We need to check if there is a member of the null space that lies inside the friction cone. When we try to grasp a wet bottle of shampoo in the shower and it slips from our hand, we try to grasp harder, which is analogous to adding more forces in the null space of our grasp.

12.5.2 Convexity Conditions for Force-closure Grasps

A set of vectors $\{v_1, \ldots, v_k\}$ *positively spans* $\mathbb{R}^n$ if for every $x \in \mathbb{R}^n$ there exists $\alpha_i > 0, \forall i = 1, \ldots, k$ such that

$$\sum_{i=1}^{k} \alpha_i v_i = x.$$

A set K is said to be *convex* if for every $x, y \in K,\ \lambda x + (1-\lambda)y \in K, \lambda \in [0,1]$. Geometrically, any line connecting two points in a convex set lies in that set, as shown in figure 12.6.

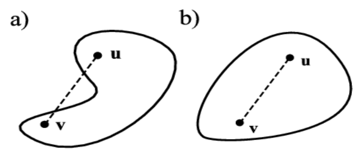

Figure 12.6: a) non-convex set, b) convex set

Given a set $S = \{v_1, \ldots, v_k\}$, the *convex hull* of S is $\text{co}(S) = \{v = \sum \alpha_i v_i : \sum \alpha_i = 1, \alpha_i \geq 0\}$, as shown in figure 12.7.

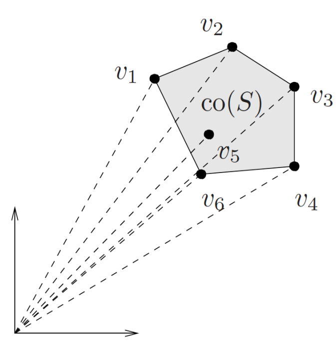

Figure 12.7: Convex hull

Proposition 5.3. Convexity conditions for force-closure grasps: Consider a fixed contact grasp which contains only frictionless point contacts. Let $G \in \mathbb{R}^{p \times m}$ be the associated grasp matrix and let $\{G_i\}$ denote the columns of G. The following statements are equivalent:

1. The grasp is force-closure.
2. The columns of G positively span $\mathbb{R}^p$.
3. the convex hull of $\{G_i\}$ contains a neighborhood of the origin.
4. There does not exist a vector $v \in \mathbb{R}^p, v \neq 0$, such that for $i = 1, \ldots, m, v \cdot G_i \geq 0$.

Since the fraction cone cannot be represented as a sum of finite set of vectors, we cannot use the convex hull of a finite basis to determine if a grasp is force closure. In practice, we use formula 12.2 to represent the friction cone by n vectors. The force closure condition can then be checked by evaluating the convex hull of the approximation.

12.5.3 Limitations of Force Closure

There are some limitation of force closure because of our assumptions. We assume that the fingers can move in any direction, and also assume that the applied forces can be as much as infinite, none of which are possible in real world. Sometimes, it's better to execute suboptimal grasps when optimal grasp might require a force that is too large to break the hardware.