EECS C106B - Robotic Manipulation and Interaction **(Week 1)**

Discussion #1

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Problem 1 - Matrix Exponential and Linear ODEs

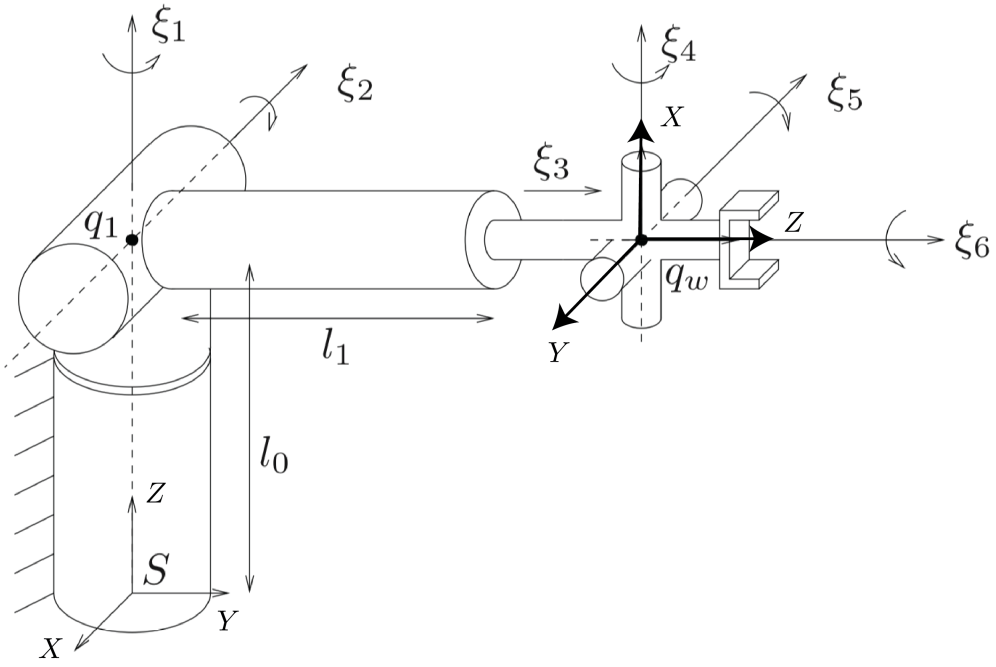
Recall that the exponential of a square matrix $A \in \mathbb{R}^{n \times n}$ is defined by the following infinite series:

$$e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!} \tag{0.1}$$

1. Let $Y(t) = e^{At}$. By differentiating the series representation, show that $\dot{Y}(t) = Ae^{At}$.
2. Show that $\left(e^A\right)^{-1} = e^{-A}$.
3. Show that $x(t) = e^{At}x_0$ is the *unique* solution to the differential equation $\dot{x} = Ax$ with initial condition $x(0) = x_0$, for $x(t) \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$. (Do this by first considering the function $y(t) = e^{-At}x(t)$. What is the time derivative of $y(t)$?)

Problem 2 - Forward Kinematics

Write down the twists for each of the joints in the following 6DOF manipulator. Use the product of exponentials formula to find the forward kinematics map $g_{st}(\theta) \in \mathbb{R}^{4 \times 4}$.



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