EECS C106B / 206B Robotic Manipulation and Interaction — Spring 2021

Lecture 20: (Grasp Stability, Manipulability, Grasp Planning)

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20.1 Grasp Statics

20.1.1 Properties

Property (1):

1.1. FC_i is a closed subset of $\mathbb{R}^{m_i}$, with nonempty interior.

1.2. $\forall x_1, x_2 \in FC_i, \alpha x_1 + \beta x_2 \in FC_i, \forall \alpha, \beta > 0$

Property (2):

2.1. $P_i = P_i^T, P = P^T$

2.2. $x_i \in FC_i$ (or x $\in$ FC) $\iff$ $\mathrm{P_i(x_i)} \geq 0$ (or $\mathrm{P(x)} \geq 0$)

2.3. $x_i \in FC_i \iff P_i(x_i) = S_i^0 + \sum_{j=1}^{m_i} S_i^j x_{i,j} \geq 0, {S_i^j}^T = S_i^j = S_i^j, j = 1, ..., m_i$.

$x \in FC \iff P(x) = S_0 + \sum_{i=1}^{m} S_i x_i \geq 0, S_i^T = S_i, i = 1, ..., m$

2.4. Let $Q(x) = S_0 + \sum_{i=1}^{mx} {}_iS_i, S_i^T = S_i$, then $A_Q = x \in \mathbb{R}^m \| Q(x) \geq 0 and B_Q = x \in \mathbb{R}^m \| Q(x) > 0$ are bot convex.

Property (3): (G, FC) is force closure iff $G(FC) = \mathbb{R}^p$ and $\exists x_N \in kerG$ s.t. $x_N \in int(FC)$.

Property (4): Let $G = \{G_1, ..., G_k\}$, the following are equivalent:

4.1. (G, FC) is force-closure.

4.2. The columns of G positively span $\mathbb{R}^p, p = 3, 6$.

4.3. The convex hull of G_i contains a neighborhood of the origin.

4.4. There does not exist a vector $v \in \mathbb{R}^p, v \neq 0$ s.t. $\forall i = 1, ..., k, v \cdot G_i \geq 0$.

20.1.2 The Grasp Map

The Grasp Map is comprised of 2 components: (1) Single Contact and (2) Multifingered Grasp. They are defined as follows:

Single Contact: $F_o = Ad^T_{g^{-1}_{oc_i}} F_i$

Multifingered Grasp: $F_o = \sum_{i=1}^{k} G_i x_i = [Ad^T_{g_{oc_1}^{-1}} B_1, ..., Ad^T_{g_{oc_k}^{-1}} B_k] \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_k \end{bmatrix} \triangleq G \cdot x$

In the case of the Single Contact, F_i is the generalized force which is a wrench composed of a force and torque piece. It is transformed by the adjoint transpose of $g_{oc_i^{-1}}$, where c_i is the contact points, o is the central mass of the object.

P_{oc_i}: position of contact point in the body's coordinate frame.

R_{oc_i}: rotation of contact rotation of the contact coordinate frame.

page 10 figure, o is object coordinate frame, c_i is the contact coordinate frame, P represents the "palm" as the base frame.

The force component is transformed by the first 3 rows of G_i while the torque component is transformed by the second 3 rows of G_i.

The forces can only be applied in directions corresponding to the basis of B_i. B_I is determined by the kinds of contacts that we have. The two that were mentioned in a previous lecture were (1) Frictionless Point Contact (FPC), (2) Point Contact with Friction (PCWF), and (3) Soft Finger Contact (SFC). Please refer to the previous lecture for notes on these contact models.

page 12 single contact: FC_i is the friction column... G_i is the grasp matrix of the i^{th} finger.

Multifingered Grasp extends on the math and reasoning from Single Contact, Adding up all forces from all fingers from 1 to k. The $x_1, ..., x_k$ matrix represents the force of the first finger contact force to the k^{th} finger contact force. Adding up the resulting transformations between finger contact forces and their respective adjoints results in $G \cdot x$, which is known as the Grasp Map.

20.1.3 Friction Cone Representation

The friction cones are written as semi-definite matrices. We represent our contact models FPC, PCWF, and SFC from last lecture as the following matrices.

These friction cone matrix representations can be written as equivalent quadratic constraints defined in Property 2 from earlier. The explanation and proof for Property 2 were skipped over in lecture. Both have been included in this set of notes for the reader's reference anyway.

20.1.4 Force Closure

Definition: A grasp (G, FC) is force closure if $\forall F_o \in \mathbb{R}^p (p = 3 \text{ or } 6), \exists \text{x} \in \text{FC s.t. } Gx = F_o$.

Given an external force, F_o, that belongs to $\mathbb{R}^p$, there exists x such that it resists the force F_o.

3 problems exist the realm of Force Closure:

(1) Force-closure Problem: Determine if a grasp (G, FC) is force-closure or not.

(2) Force Feasibility Problem: Given $F_o \in \mathbb{R}^p, p = 3$ or 6, determine if there exists $x \in FC$ s.t. $Gx = F_o$

(3) Force Optimization Problem: Given $F_o \in \mathbb{R}^p, p = 3$ or 6, find $x \in FC$ s.t. $Gx = F_o$ and x minimizes $\phi(x)$. This is essential problem two with an added optimization problem to minimize $\phi(x)$. There are many cost functions that can be used to achieve this, such as placing constraints on the norm of x or the strength of the fingers, as discussed briefly in lecture.

Definition: Internal Force: $x_N \in FC$ is an internal force if $Gx_n = 0$ or $x_n \in (kerG \cap FC)$

20.1.5 Constructive Force-closure for PCWF

Definition:

a. $v_1, ..., v_k, v_i \in \mathbb{R}^p$ is positively dependent if $\exists \alpha_i > 0$ such that $\sum \alpha_i v_i = 0$

b. $v_1, ..., v_k, v_i \in \mathbb{R}^p$ positively span $\mathbb{R}^n$ if $\forall x \in \mathbb{R}^n, \exists \alpha_i > 0$, such that $\sum \alpha_i v_i = x$.

Definition:

a. A set K is convex if $\forall x, y \in K, \lambda x + (1-\lambda)y \in K, \lambda \in [0,1]$.

b. Given $S = v_1, ..., v_k, v_i \in \mathbb{R}^p$, the convex hull of $S : co(S) = \{v = \sum \alpha_i v_i, \sum \alpha_i = 1, \alpha_i \geq 0\}$

20.2 Kinematics of Contact

20.2.1 Surface Model

$c : U \subset \mathbb{R}^2 \to \mathbb{R}^3, c(U) \subset S$.

Differential Geometry of Curves and Surfaces. We parameterize the surface using coordinate charts by flattening the curved surface. The corresponding x coordinate is u and the corresponding y coordinate is v. c is a mapping that maps a point (u, v) on the 2D coordinate chart to the 3D surface of the original manipulated object. c has 3 coordinates, which we can generate:

$c_u = \frac{\partial c}{\partial u} \in \mathbb{R}^3$

$c_v = \frac{\partial c}{\partial v} \in \mathbb{R}^3$

To determine the relationship between the curvature of the object and the fingers, we must define the First Fundamental Form: $I_p = \begin{bmatrix} c_u^T c_u & c_u^T c_v \\ c_v^T c_u & c_v^T c_v \end{bmatrix}$ which is a $2x2$ symmetrical matrix.

Orthogonal Coordinates Chart: $c_u^T c_v = 0$ under the assumption that we have found an Orthogonal Coordinate Chart. As such, $I_p = \begin{bmatrix} \|c_u\|^2 & 0 \\ 0 & \|c_v\|^2 \end{bmatrix} = M_p \cdot M_p$

Removing the squares of the previous, First Fundamental Form, we get the Metric Tensor: $M_p = \begin{bmatrix} c_u^T c_u & c_u^T c_v \\ c_v^T c_u & c_v^T c_v \end{bmatrix}$
This is used for measuring distances on the curved surface of the object.

The normal of the surface, defined as the Gauss Map: $N : S \to s^2 : N(u, v) = \frac{c_u \times c_v}{\|c_u \times c_v\|} := n$. n is the normal vector of unit length. n_u shows the changes of the normal vector in the u direction and n_v shows the changes of the normal vector in the v direction. Taking the inner products of these values multiplied by c_u and c_v gives the 2^{nd} Fundamental Form.

2^{nd} Fundamental Form: $II_p = \begin{bmatrix} c_u^T n_u & c_u^T n_v \\ c_v^T n_u & c_v^T n_v \end{bmatrix}, n_u = \frac{\partial n}{\partial u}, n_v = \frac{\partial n}{\partial v}$

Normalizing the 2^{nd} Fundamental Form with the curvature arrives us at the Curvature Tensor:

$$K_p = M_p^{-T} II_p^{-1} = \begin{bmatrix} \frac{c_u^T n_u}{\|c_u\|^2} & \frac{c_u^T n_v}{\|c_u\|\|c_v\|} \\ \frac{c_v^T n_u}{\|c_u\|\|c_v\|} & \frac{c_v^T n_v}{\|c_v\|^2} \end{bmatrix}$$

The $SO(3)$ Gauss Frame: $[x, y, z] = [\frac{c_u}{\|c_u\|} \frac{c_v}{\|c_v\|} n], K_p = \begin{bmatrix} x^T \\ y^T \end{bmatrix} [\frac{n_u}{\|c_u\|} \frac{n_u}{\|c_v\|}]$ A Gauss Frame is associated with a finger tip touching and object and another Gauss Frame is associated with the object at the point of contact. We observe how the Gauss Frames evolve with respect to each other.

Torsion tells you how the curvature changes we move the point along the surface. We arrive at the Torsion Form: $T_p = y^T [\frac{x_u}{\|c_u\|} \frac{x_v}{\|c_v\|}]$, where y^T is a 3×1 matrix and T_p is 1×2.

Our previous three, intrinsic quantities of the surface together is: (M_p, K_p, T_p): Geometric parameter of the surface.

20.2.2 Gauss Frame

Keep track of a finger on an object. Both the finger and the object have a Gauss Frame associated with them.

$p_{oc}(t) = p(t) = c(\alpha(t)), R_{oc}(t) = [x(t), y(t), z(t)] = [\frac{c_u}{\|c_u\|} \frac{c_u}{\|c_u\|} \frac{c_u \times c_u}{\|c_u \times c_v\|}]$

p_{oc} is the contact point and R_{oc} is the Gauss Frame associated.

$v_{oc} = R_{oc}^T \dot{p}_{oc} = \begin{bmatrix} M\dot{\alpha} \\ 0 \end{bmatrix}$

v_{oc} is the slippage of the contact point.

$\hat{\omega}_{oc} = R_{oc}^T \dot{R}_{oc} = \begin{bmatrix} x^T \\ y^T \\ z^T \end{bmatrix} [\dot{x} \ \dot{y} \ \dot{z}]$

ω_{oc} is the roll of the contact point.

20.2.3 Contact Kinematics

$p_t \in S_0 \to p_f(t) \in S_f$

Local coordinate:

$c_o = U_0 \subset \mathbb{R}^2 \to S_0$

$c_f = U_f \subset \mathbb{R}^2 \to S_f$

Coordinate chart on the object: $\alpha_0(t) = c_0^{-1}(p_o(t))$

Coordinate chart on the finger: $\alpha_f(t) = c_f^{-1}(p_f(t))$

Angle of contact: ϕ, which is the angle between the two x axis.

Contact coordinates: $\eta = (\alpha_f, \alpha_0, \phi)$

Montana Equations of Contact:

$\dot{\alpha}_f = M_f^{-1}(K_f + \tilde{K}_0)^{-1}(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - \tilde{K}_0 \begin{bmatrix} v_x \\ v_y \end{bmatrix})$

$\dot{\alpha}_0 = M_0^{-1} R(K_f + \tilde{K}_0)^{-1}(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + \tilde{K}_f \begin{bmatrix} v_x \\ v_y \end{bmatrix})_\Psi$

$\dot{\Psi} = \omega_z + T_f M_f \dot{\alpha_f} + T_0 M_0 \dot{\alpha 0}$

$v_z = 0$

We can arrive at the set of equations:

$$\begin{bmatrix} \dot{u_f} \\ \dot{v_f} \\ \dot{u_0} \\ \dot{v_0} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ sec(u_f) \\ -psin(\psi) \\ -pcos(\psi) \\ -tan(u_f) \end{bmatrix} \omega_x + \begin{bmatrix} -1 \\ 0 \\ -pcos(\psi) \\ psin(\psi) \\ 0 \end{bmatrix} \omega_y$$