EECS C106B - Robotic Manipulation and Interaction **(Week 11)**

Discussion #10

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Problem 1 - A return to Velocities

In 106A, you learned two ways to represent the velocity of a rigid body transform $g_{AB} \in SE(3)$. You learned about spatial velocities $\hat{V}^s_{AB} \in \mathfrak{se}(3)_A$ and body velocities $\hat{V}^b_{AB} \in \mathfrak{se}(3)_B$. How do you represent the velocity of g_{AB} in terms of a third coordinate frame C?

Problem 2 - Projections

A basic model of a camera is the following:

$$\lambda \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Given some point $p \in \mathbb{R}^3$ in the camera frame, we can apply the camera transformation to get the image of that point $q \in \mathbb{R}^2$. Show that given any point $r \in \mathbb{R}^3$ that lies on the line between o (the origin of the camera frame) and p, the image of r is q.

Problem 3 - Vanishing Points

A straight line in the 3D world becomes a straight line in the image. However, two parallel lines in the 3D world will often intersect in the image. The point of intersection is called the *vanishing point*.

1. Given two parallel lines, how do you compute the vanishing point?
2. When does the vanishing point not exist (the two lines do not intersect)?