Lecture 14: Grasp Planning

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14.1 Planning Optimal Grasps

Some of the the lecture's material is from Canny and Ferrari's paper on optimal grasp planning, found here.

14.2 Grasp quality measurement

Given an object characterized with external forces $\mathbf{w} = [F, \tau]$ and contact points C_i, and the generalized force vector $g = [f_1^{\perp} \dots f_n^{\perp}]$, the local grasp quality measure is defined as,

$$LQ = \max \frac{||\mathbf{w}||}{||\mathbf{g}||}$$

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$$B_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \tau \end{bmatrix}$$

s.t. $f_1^2 + f_2^2 \leq \mu_1^2 f_3^2$, $f_3 > 0$, $\tau \leq \mu_2 f_3$

14.4 Grasp Solutions

Given grasp map G and wrench $\mathbf{w} = [F_{ext}, \tau_{ext}]^T$, the solution to $\mathbf{w} = G\mathbf{x}$, where $x = [x_1, x_2, \dots, x_k]^T$, $x_i \in \mathbb{R}^{n_i}$ ($n_i \in \{1, 3, 4\}$ corresponds to a frictionless finger, finger with friction, and soft finger, respectively), are the grasp forces that resist the external forces on the object. No solution exists when G is not surjective and x_i does not satisfy $f_1^2 + f_2^2 \leq \mu_1^2 f_3^2$ and $f_3 > 0$. The existence of a solution indicates force-closure.

Lemma. If $x_0 \in \mathcal{N}(G \in \text{FC})$ and G is onto, then given any $\mathbf{w} \in \mathbb{R}^6, \exists \mathbf{x} \in \text{FC}$ that solves $\mathbf{w} = G\mathbf{x}$.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} + \lambda \mathbf{x_0}$$

In other words, this means that increasing the strength of the normal force (x_0 is in the null space of G and does not apply torque), which is in the friction cone, can result in the resulting sum being pulled into the friction cone. Intuitively, this is like grasping a bar of soap harder so it doesn't slip out.

14.4.1 Optimal Grasping

From the equation $\mathbf{w} = G\mathbf{x}$, it follows that,

$$LQ = \max \frac{||\mathbf{w}||}{||\mathbf{g}||} = (\sigma_{min}(G))^{-1}$$

where σ_{min} is the smallest singular value of G. This is a measure of grasp stability. Another measure mentioned is $(\sigma_{max}(G))/(\sigma_{min}(G))$. Grasp planning, in essence, is optimizing $\max \frac{||\mathbf{w}||}{||\mathbf{g}||}$.

Steinitz Theorem. If $S \subset \mathbb{R}^p$ and $q \in \text{int}(co(s))$, then there exists $X = (v_1, \ldots, v_k) \subset S$ such that $q \in \text{int}(co(X))$ and $k < 2p$.

This gives us an upper bound on the number of minimum fingers needed for grasping.

14.5 Planar Antipodal Grasp

A planar grasp with two point contacts with friction is force-closure iff the line connecting the contact point lies inside both friction cones.

For example, a ball held by two parallel surfaces.