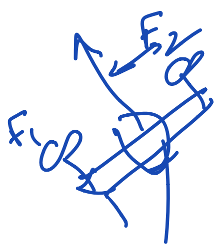
$u_1 = F_1 + F_2$

$u_2 = (F_2 - F_1)\ell$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\\ \dot x_5\\ \dot x_6\end{bmatrix} = \underbrace{\begin{bmatrix}x_4\\ x_5\\ x_6\\ 0\\ -g\\ 0\end{bmatrix}}_{f(x)} + \begin{bmatrix}\underbrace{\begin{matrix}0\\ 0\\ 0\\ -\frac{1}{m}\sin x_3\\ \frac{1}{m}\cos x_3\\ 0\end{matrix}}_{g_1(x)} & \underbrace{\begin{matrix}0\\ 0\\ 0\\ 0\\ 0\\ \frac{1}{I}\end{matrix}}_{g_2(x)}\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$



$\bar x_e, \bar u_e$

$$\dot x = f(x) + g_1(x)u_1 + g_2(x)u_2$$

$$0 = \dot x_1 = \bar x_4$$
$$0 = \dot x_2 = \bar x_5$$
$$0 = \dot x_3 = \bar x_6$$
$$0 = \dot x_4 = -\frac{1}{m}\sin \bar x_3\, \bar u_1$$
$$0 = \dot x_5 = -g + \frac{1}{m}\cos \bar x_3\, \bar u_1$$
$$0 = \dot x_6 = \frac{1}{I}\bar u_2$$

$\bar u_2 = 0 \quad \bar x_4 = 0 = \bar x_5 = \bar x_6 \qquad \bar x_1, \bar x_2$ arbitrary

$$\left\{\begin{aligned} 0 &= -\frac{1}{m}\sin\bar{x}_3\,\bar{u}_1 \\ g &= \frac{1}{m}\cos\bar{x}_3\,\bar{u}_1 \end{aligned}\right\} \Rightarrow \bar{u}_1 = g$$

$$\sin\bar{x}_3 = 0 \qquad \cos\bar{x}_3 = 1$$

$$\bar{x}_3 = 0 \qquad \cancel{\bar{x}_3 = \pi}$$

NOT PHYS. REALIZABLE

$$\bar{u}_1 = mg$$

$$\bar{u}_2 = 0$$

$\bar{x}_1$

$\bar{x}_2$

$\bar{x}_3 = 0$

$\bar{x}_4 = 0$

$\bar{x}_5 = 0$

$\bar{x}_6 = 0$

$$\dot{x} = \underbrace{f(x) + g_1(x)u_1 + g_2(x)u_2}_{f(x,u)}$$

$$D_1 f(x,u) = \frac{\partial}{\partial x}\left[f(x) + g_1(x)u_1 + g_2(x)u_2\right]\Big|_{\bar{x},\bar{u}}$$

$$= \left(\begin{bmatrix} \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} + \begin{bmatrix} 0_3 & 0_3 \\ \begin{matrix} 0 & 0 & -\frac{1}{m}\cos\bar{x}_3 \\ 0 & 0 & -\frac{1}{m}\sin\bar{x}_3 \\ 0 & 0 & 0 \end{matrix} & 0_3 \end{bmatrix} \bar{u}_1 \right.$$

$$\left. + \; 0\,\bar{u}_2 \right)$$

$$A = \begin{bmatrix} 0_3 & I \\ \cdots & \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 \;\; 0 \;\; -g & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -\frac{1}{m}u_1 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \end{bmatrix}$$

$$B = D_2 f(x, u)$$

$$= \frac{\partial}{\partial u}\left(f(x) + g_1(x)u_1 + g_2(x)u_2\right)$$

$$= \begin{bmatrix} \bar{g}_1(\bar{x}) & g_2(\bar{x}) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{m}\sin\bar{x}_3 & 0 \\ \frac{1}{m}\cos\bar{x}_3 & 0 \\ 0 & \frac{1}{I} \end{bmatrix} \qquad \bar{x}_3 = 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & \frac{1}{I} \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$

$$= \left[\begin{array}{c|c} 0 & I \\ \hline \begin{matrix} 0 & 0 & -g \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & 0 \end{array}\right] x + \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 1/m & 0 \\ 0 & 1/I \end{array}\right] u$$