Homework 4

EECS/BioE C106B/206B
Robotic Manipulation and Interaction

Due: March 16, 2021

Problem 1. A rank condition for the epipolar constraint

Exercise 5.4 from An Invitation to 3D Vision (Ma, Soatto, Kosecka, Sastry).

Show that $x_2^T \widehat{T} R x_1 = 0$ if and only if

$$\operatorname{rank}\left[\widehat{x_2} R x_1, \widehat{x_2} T\right] \leq 1$$

Problem 2. Two physically possible solutions for the homography decomposition

Exercise 5.19 from An Invitation to 3D Vision (Ma, Soatto, Kosecka, Sastry).

Let us study in the nature of the two physically possible solutions for the homography decomposition. Without loss of generality, suppose that the true homography matrix is $H = I + ab^T$ with $\|a\| = 1$

(a) Show that $R' = -I + 2aa^T$ is a rotation matrix.

(b) Show that $H' = R' + (-a)(b + 2a)^T$ is equal to $-H$.

(c) Since $\left(H'\right)^T H' = H^T H$, conclude that both $\{I, a, b\}$ and $\{R', -a, (b + 2a)\}$ are solutions from the homography decomposition of H.

(d) Argue that, under certain conditions on the relationship between a and b, the second solution is also physically possible.

(e) What is the geometric relationship between these two solutions? Draw a figure to illustrate your answer.

Problem 3. Two-view geometry from W_p

Exercise 8.5 from An Invitation to 3D Vision (Ma, Soatto, Kosecka, Sastry).

Given two images x_1, x_2 with respect to two camera frames specified by Π_1, Π_2, then:

(a) Derive the epipolar constraint from the rank condition

$$\operatorname{rank}(W_p) = \operatorname{rank}\begin{bmatrix} \widehat{\boldsymbol{x}_1}\Pi_1 \\ \widehat{\boldsymbol{x}_2}\Pi_2 \end{bmatrix} \leq 3$$

(b) Assuming that the translation between the first and second camera frame is nonzero, show that

$$\operatorname{rank}(W_p) = 3$$

except for a special case. What is this special case? This property means that the (homogeneous) coordinates of the 3 -D point are uniquely determined by the null space of W_p when translation is present.

(Hint: to simplify the proof, one can consider the first camera frame to be the reference frame.)

This can be considered as a triangulation procedure for lines.

Problem 4. Rectilinear motion and points

Exercise 8.10 from An Invitation to 3D Vision (Ma, Soatto, Kosecka, Sastry).

Use the rank condition of the M_p matrix to show that if the camera is calibrated and only translating on a straight line, then the relative translational scale between frames cannot be recovered from bilinear constraints, but it can be recovered from trilinear constraints.
(Hint: use $R_i = I$ to simplify the constraints.)

Problem 5. Research Comprehension

Read the paper *On Symmetry and Multiple-View Geometry: Structure, Pose, and Calibration from a Single Image* (link) and answer the following questions. This paper is fairly long, so remember the paper reading techniques you've learned.

(a) Mathematically, how do the authors characterize the types of symmetries being considered in this paper?

(b) The authors claim that *"One image of a symmetric object is equivalent to multiple images"*. In your own words, explain what they mean by this. You do not need to go too deep into the math, but feel free to use equations to supplement your response.

(c) Explain, in your own words, two applications of the techniques developed in this paper.