EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2020**

Lecture 16: Overview of Soft Robotics

Scribes: Fayyaz Ahamed

1 Lecture Overview

This lecture serves as a general introduction to the field of soft robotics, building upon the prior lecture's discussion of the concepts of stress and strain from statics. Much of the material from this lecture was based upon material from K. L. Johnson's book, ***Contact Mechanics***. We start by considering various examples of sensing and actuation within soft robotic systems, integrated soft robotic systems, and the advantages and disadvantages of using soft robotics systems as opposed to rigid robots. We then introduce the field of contact mechanics by discussing different types of elasticity, Hertz's Elastic Theory of Contact, sphere inflation, and various constitutive laws.

2 Examples of Current Soft Robotics Systems

2.1 Sensing

A functional robot, whether soft or rigid, is able to sense its environment, utilize its onboard processing and produce appropriate motion via its actuators to respond to said environment. Some of the discussed methods of soft sensing included geometry-enabled sensors, fluidic sensors, and bio-hybrid sensors through the use of ultrathin materials, microfluidic electronics, ionic hydrogels, and more. Several types of these sensors measure internal properties of the robot and others sense properties of the environment.

2.2 Actuation

Soft actuation is somewhat more complicated than soft sensing, but ultimately requires that some form of energy be converted to mechanical energy that allows the robot to perform the desired motion. Some common types of soft actuation discussed included pneumatic actuators, dielectric elastomer actuators, and resistive heating actuators. Some of these methods of actuation were compared to animal muscles, which contract and expand to produce desired motion.

2.3 Integrated Soft Robotic Systems

To discuss current soft robotic systems that utilize integration between sensing and actuation, we examined the concept of an ideal soft robot based upon current state-of-the-art octopus-inspired robots with artificial sensors, artifical actuation, and onboard processing. In addition, the importance of having an efficient onboard power source was discussed. Some examples of autonomous soft robots discussed included a jumping robot powered by combustion, a swimming robot powered by hydraulics, and a walking robot powered by pneumatics.

3 Advantages and Disadvantages of Soft Robotics

Next, we discussed several advantages and disadvantages of using soft robotic systems compared to rigid ones. Before discussing the individual advantages, a distinction between continuous soft robots, articulated soft robots, and rigid robots must be made. Continuous soft robots are entirely composed of materials that experience significant strain from forces applied to them. Articulated soft robots generally have rigid components that resist strain as well as soft components that are much more pliable, and they generally fall into two major categories. The first category is elastically actuated robots, which use actuator mechanisms discussed in the previous section to produce motion. Baxter falls into this category since it uses serial elastic actuators. The human hand also is also of this category since it contains rigid bone with soft muscle and tendons. The second category is robots with elastic joints; their extra ability to deform at joints allows for a larger range of motion and factors into some of next advantages and disadvantages.

Advantages of Soft Robots:

- A major advantage of soft robots is that their softness allows for safe contact with humans. Since soft robots experience significant strain upon contact with other objects, if the other object in question is a human or animal, the soft robot will not apply dangerous amounts of force to the organism at risk. Instead, once the robot starts exerting excess force, it will deform under that stress and allow the organism to pull away before further damage is done. Purely rigid robots do not have this safety feature and can easily apply dangerous amounts of forces to humans, making them a hazard. Because soft robots have this feature of deforming with excess force, it also makes it significantly easier to facilitate human-robot interaction.
- Because of soft robots' deformation ability, they can achieve much better grasps than rigid robots. Since a soft robot can make its hand or fingers conform to the object it's grasping, it can ensure a grasp with many points of contact that allow it to exert a more stable, continuous set of forces to keep the object within force closure.

Disadvantages of Soft Robots:

- The dynamics of rigid robots are better understood than soft robot dynamics, so it is possible to make more precise dynamic motions with rigid robots than soft robots. $M(q)\ddot{q}+C(q,\dot{q})+G(q)=\tau$ represents the dynamics of most rigid robots, and it has been very well understood, whereas research into the dynamics of soft robots is still currently underway.

4 Contact Mechanics

Here, we introduce the field of contact mechanics, in contrast to classical mechanics, which deals solely with bulk material properties. Contact mechanics, on the other hand, deals with bulk properties that consider surface and geometrical constraints. It is defined as the study of the deformation of solids that touch each other at one or more points. Contact mechanics considers both the friction and frictionless cases, and differentiates between normal and tangential stresses. Normal stresses are caused by applied forces and adhesion interactions between surfaces, which we will discuss further with Hertzian contact.

4.1 Hertzian Contact

Hertzian contact refers to the stresses that build up as two curved surfaces come into contact and deform slightly as a result of applied forces. It is used as the basis for understanding the load bearing capabilities and fatigue life of bearings, gears, and other configurations with two bodies in contact. The experienced deformation depends on the modulus of elasticity of the materials in Hertzian contact. This modulus of elasticity, E, is very similar to the spring constant k, in the force-extension curve of a spring, $F = kx$, and it is defined as:

$$\sigma_{ij} = \sum_{kl} E_{ijkl} u_{kl}$$

where u is strain. Hertzian contact gives the contact stress from the known quantities of the normal contact force, the radii of curvatures of both objects, and the moduli of elasticities of both objects. In this next example, we will discuss Hertz's Elastic Theory of Contact.

4.2 Hertz's Elastic Theory of Contact

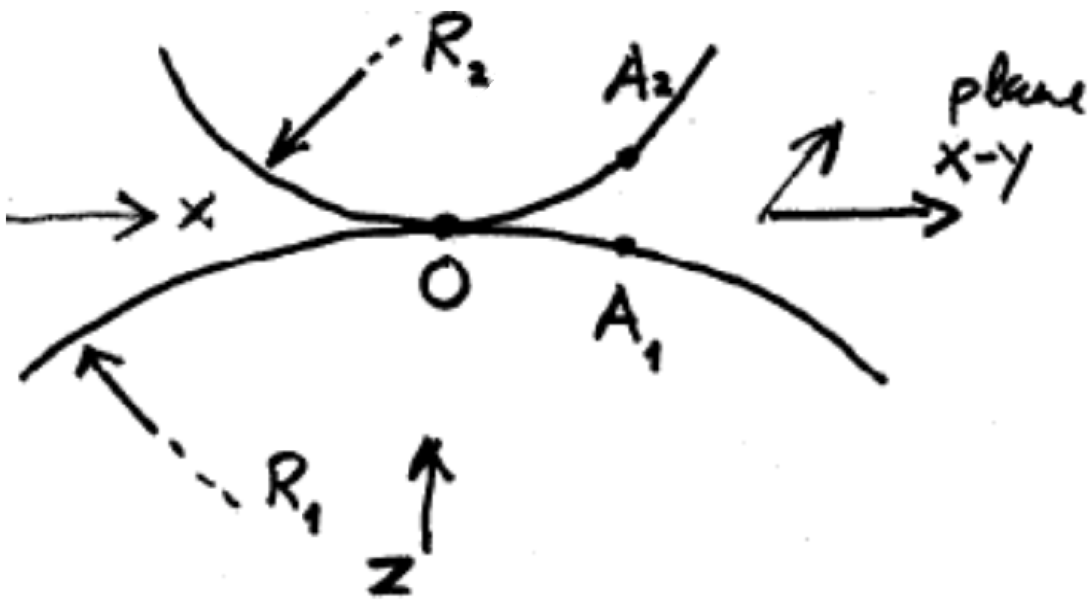

Figure 1: Spheres Prior To Deformation

Here we consider two spheres of deformable materials in contact, shown in Figure 1 prior to deformation; this mathematical formulation also extends to the example in Figure 3 with a sphere and elastic half-space in contact. In Figure 1, the bottom sphere has radius R_1, the top sphere has radius $R2$, and the points of interest on each respectively are A_1 and A_2. The objects meet at point O and force is only applied in the z direction.

We define h to be the separation of points A_1 and A_2 prior to deformation:

$$h = |A_1 - A_2| = \frac{1}{2R_1}x^2 + \frac{1}{2R_2}y^2$$

A detailed diagram of the contact location after deformation is shown in Figure 2, where δ_1 is the displacement of the distant point of the bottom sphere, δ_2 is the displacement of the distant point of the top sphere, A_1

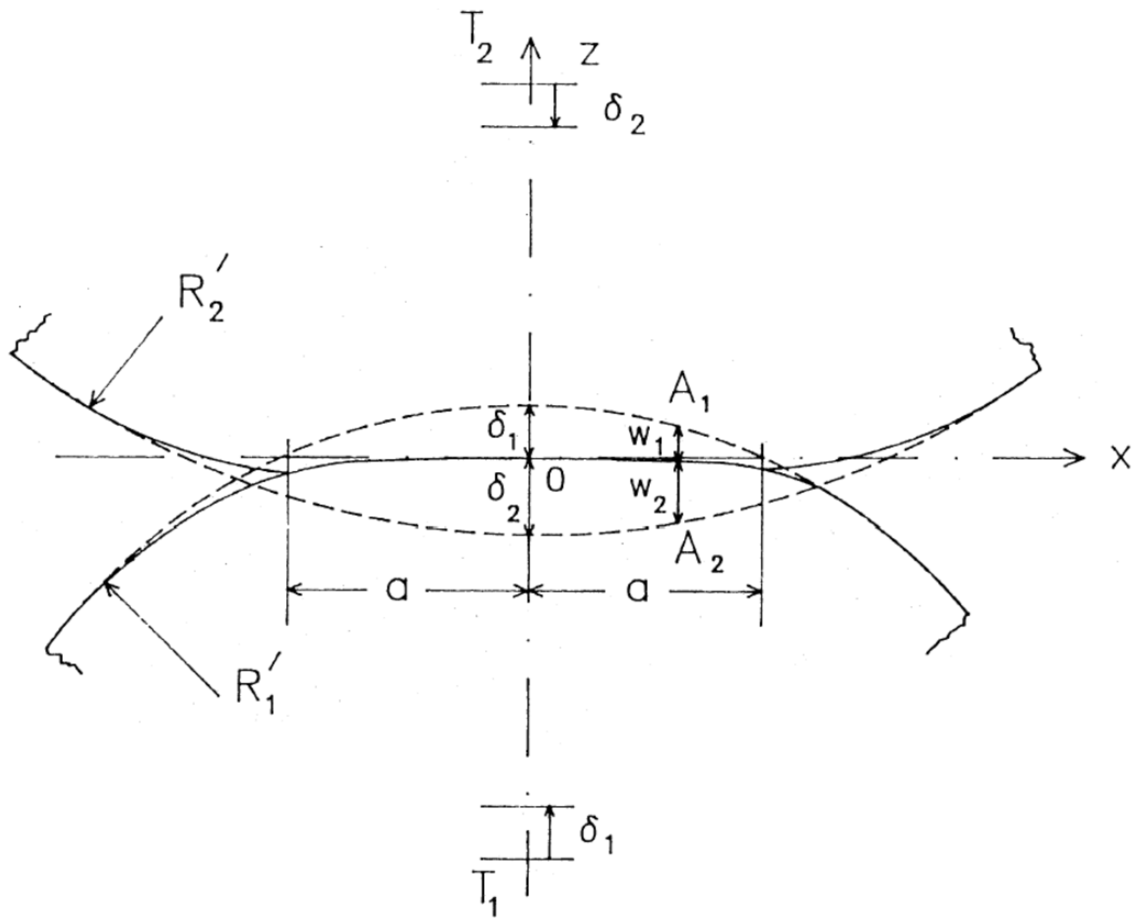

Figure 2: Spheres After Deformation

and A_2 are the points of interest on the bottom and top spheres respectively, R_1 and R_2 are the sphere's radii of curvature, and w_1 and w_2 are the vertical distances from A_1 to A_2 to their sphere edges. The deformation extends in the $x - y$-plane forming a circle of radius a.

We now define h' to be the separation of A_1 and A_2 after deformation:

$$h' = h - (\delta_1 + \delta_2) + (w_1 + w_2)$$

We then consider two different cases: having A_1 and A_2 within the contact area, and having them outside the contact area.

1. A_1 and A_2 are within the contact area:

- If points A_1 and A_2 are within the contact area of radius a, then we find that $h' = 0$, such that

$$w_1 + w_2 = (\delta_1 + \delta_2) - h$$

  where $w_1 + w_2$ is the elastic displacement, $\delta_1 + \delta_2$ is the displacement of the bodies, and h is the separation of A_1 and A_2 for a single point contact.

Thus we can simplify the expression by defining $\delta \equiv \delta_1 + \delta_2 = w_1(0) + w_2(0)$:

$$w_1 + w_2 = \delta - \frac{1}{2R_1}x^2 - \frac{1}{2R_2}y^2$$

2. A_1 and A_2 are outside the contact area:

- If A_1 and A_2 are outside the contact area, then we know that $h' > 0$ because A_1 and A_2 are not in contact and we find the inequality:

$$w_1 + w_2 > \delta - \frac{1}{2R_1}x^2 - \frac{1}{2R_2}y^2$$

Lastly we seek to calculate the resultant forces after this deformation. We split the forces into a normal load L and tangential force components Q_x and Q_y which are caused by friction. We define these forces as follows:

$$L = \int_S p dS$$

$$Q_x = \int_S q_x dS$$

$$Q_y = \int_S q_y dS$$

where p is pressure, and q_x and q_y are lateral traction.

4.3 Assumptions of Hertz's Theory

Hertz's theory largely built upon Contact Stress Theory and assumed that an elastic half-space was loaded with a circular/elliptical object. This required that two conditions were satisfied:

- The contact area must be small relative to the dimension of each body: $a << R$.
- The relative radii of curvature must be small, so that the stress field calculation of the solid everywhere away from O is not influenced by the distance away from O, the highly stressed region. (Metals meet this condition, but rubber doesn't.)

Another assumption is that the surfaces are frictionless, such that $q_x = q_y = 0$; a minor assumption that the theory also requires is that the strains are small enough that $a << R$. It is important to note that the linear theory of elasticity doesn't account for changes in boundary forces arising from deformation, but the problem statement of Hertz's theory, whereby mutual pressure $p(x, y)$ acts over an area S on the surface of two elastic half surfaces and produces normal displacements of the surfaces by δ_1 and δ_2, does take these boundary forces into account.

4.4 Summary of Hertz's Equations for Elastic Contact

Here we state Hertz's equations for elastic contact from the aforementioned example. First we calculate load L:

$$L = \int_S p dS = \int_0^a p_0 \sqrt{1 - r^2/a^2} 2\pi r dr = \frac{2}{3} p_0 \pi a^2$$

This allows us to calculate Hertz's three equations for a, the radius of contact, δ, the mutual approach of distant points, both of which are defined identically to before, and p_0, the maximum pressure experienced by the interaction of these spheres:

$$a = (\frac{3LR}{4E_*})^{1/3}$$

$$\delta = \frac{a^2}{R} = (\frac{9L^2}{16RE_*})^{1/3}$$

$$p_0 = \frac{3L}{2\pi a^2} = \frac{3}{2} p_m = (\frac{6LE_*^2}{\pi^3 R^2})^{1/3}$$

where p_m is the mean pressure and E_* is defined by $\frac{1}{E_*} = \frac{1}{2}(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2})$.

5 Examples

5.1 Sphere on Elastic Half-Space

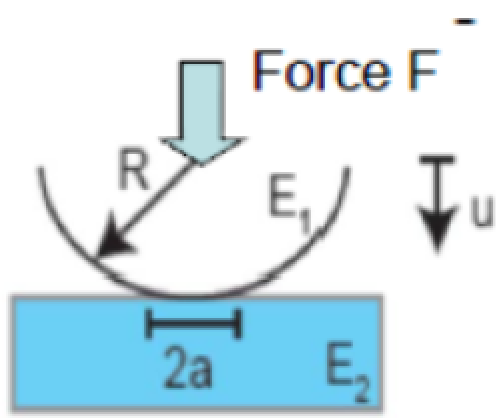

Figure 3: Elastic Sphere Indents Elastic Half-Space

In this example shown in Figure 3, an elastic sphere of radius R indents an elastic half-space to depth u, and creates a contact area of radius:

$$a \approx (\frac{3RF}{2E_*})^{1/3}$$

where E_* is defined by the equation:

$$\frac{1}{E_*} = \frac{1}{2}(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2})$$

where E_1 and E_2 are the elastic moduli and v_1 and v_2 are the Poisson's ratios associated with each body.

The depth of indentation, u, is calculated as:

$$u \approx (\frac{2F^2}{E_*^2 R})^{1/3}$$

This allows for computation of the stiffness constant, k, maximum stress, $(\sigma_c)_{max}$, and maximum torque τ_{max}:

$$k = \frac{dF}{du} \approx (E_*^2 RF)^{1/3}$$

$$(\sigma_c)_{max} \approx \frac{3F}{2\pi a^2} = 0.4(\frac{E_*^2 F}{R^2})^{1/2} = 0.4\frac{k}{R}$$

$$\tau_{max} \approx (\sigma_c)_{max}/3$$

5.2 Contact Between Two Cylinders with Parallel Axes

In this scenario shown in Figure 4, two cylinders of differing radii are compressed against one another. To calculate the equivalent of a, the contact area, we define a new variable b, that is half the width of the rectangular contact area between the two cylinders (the long end being the length L). The force applied is linearly proportional to the indentation depth. We find that

$$b = \sqrt{\frac{4F(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2})}{\pi L(\frac{1}{R_1} + \frac{1}{R_2})}}$$

where E_1 and E_2 are the moduli of elasticity for cylinders 1 and 2, and v_1 and v_2 are the Poisson's ratios respectively.

We find that the maximum contact pressure, located at the longer center line inside the rectangular contact area, is:

$$P_{max} = \frac{2F}{\pi bL}$$

5.3 Sphere Inflation

In this example, we examine a sphere being expanded at constant pressure p, demonstrated in Figure 5. We define several strain terms as follows:

$$\lambda_1 = \lambda_2 = \lambda \equiv \frac{R}{R_0}$$

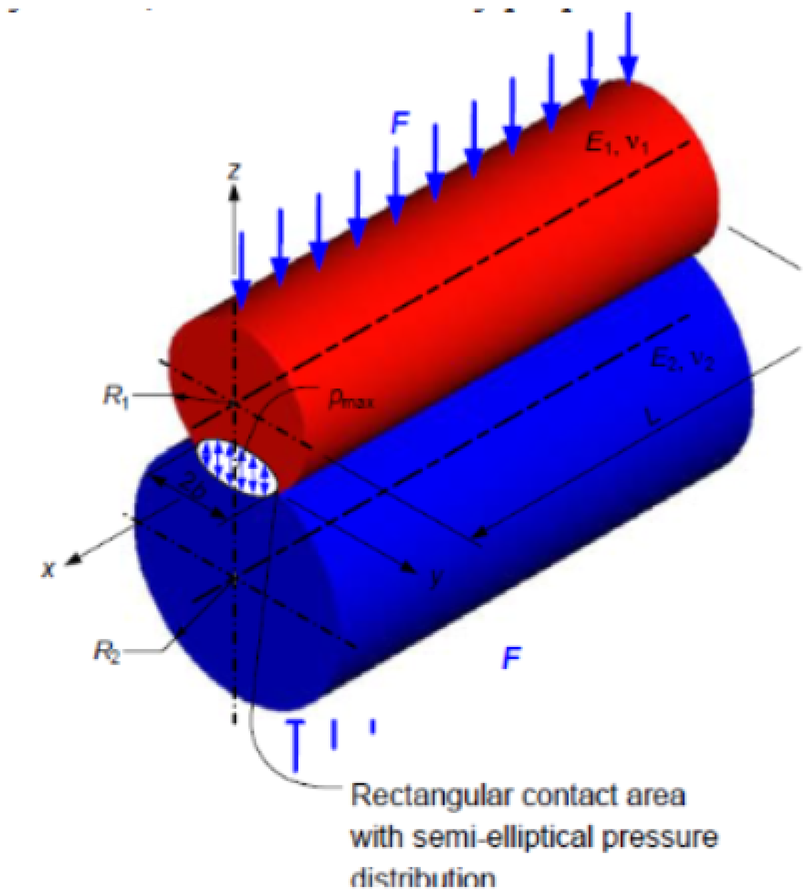

Figure 4: Contact Between Two Cylinders

$$\lambda_3 = \frac{1}{\lambda_1 \lambda_2} = \frac{1}{\lambda^2}$$

We next define W to be the radial stress as a function of the aforementioned 3 strains

$$W = W(\lambda_1 . \lambda_2, \lambda_3)$$

We define Π as a force-balance describing the system:

$$\Pi = \Pi(R) = 4\pi R_0^2 t_0 W - \frac{4}{3}\pi R^3 p$$

We know that the force across the sphere is uniform, so:

$$\frac{d\Pi}{dR} = 0$$

which allows us to solve for p:

$$p = \frac{t_0 R_0^2}{R^2} \frac{dW}{dR}$$

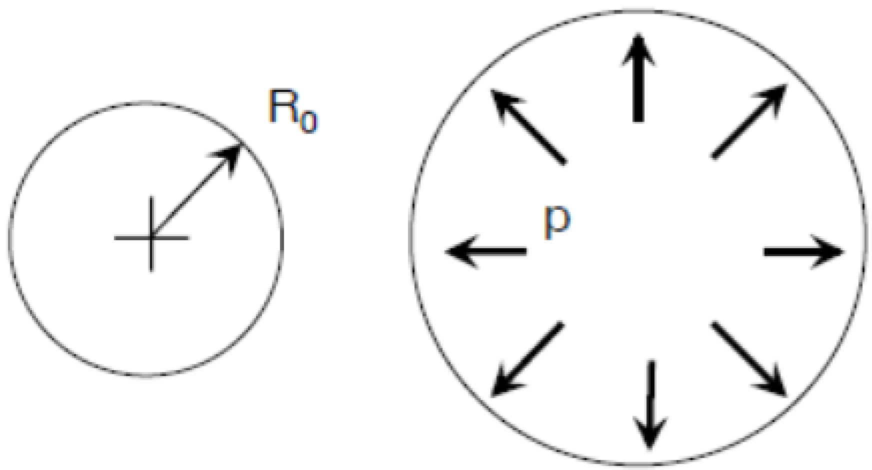

Figure 5: Sphere Inflation

5.4 Constitutive Laws

Here we examine various constitutive laws that characterize the relationship between stress, W, as a function of various elastic moduli, E, and various strains, λ:

NeoHookean

$$W = \frac{E}{6}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)$$

"1-param" Ogden

$$W = \frac{E}{24}(\lambda_1^4 + \lambda_2^4 + \lambda_3^4 - 3)$$

"Gent" Solid

$$W = \frac{-EJ_m}{6} \ln{(1 - \frac{J_1}{J_m})}$$

where we define J_m as the maximum possible value of J_1 due to strain hardening and J_1 is defined as:

$$J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3$$

After defining these constitutive laws, we examine their stress-strain relationships in Figure 6, and depending upon our application, we can choose the constitutive law that best matches our model.

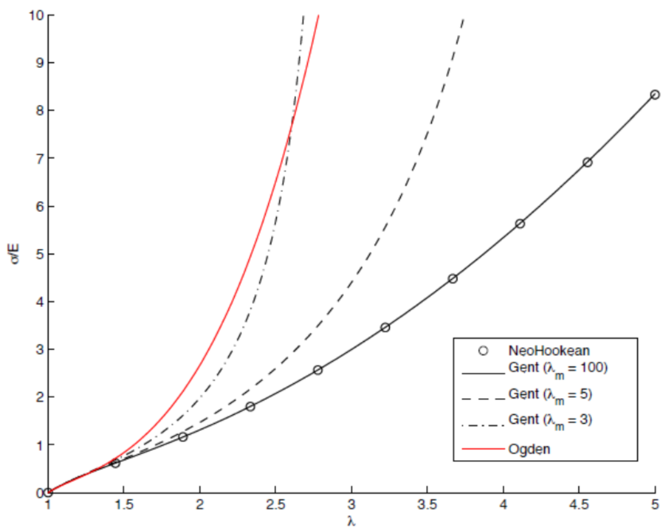

Figure 6: Constitutive Law Plots of Stress Normalized by Elastic Modulus vs. Strain