EECS C106B / 206B Robotic Manipulation and Interaction **Spring 2021**

Lecture 19: (Types of Contact, Modeling Grasp by Multi-Fingered Hands)

Scribes: William Loo, Adrian Liu

19.1 Introduction

19.1.1 Hand Function:

The hand is the interface with the external world. Some hand functions include:

- Grasping
- Dextrous manipulation
- Fine manipulation
- Exploration
- Pen Twirling

Fingers may be modeled as robots in their own right.

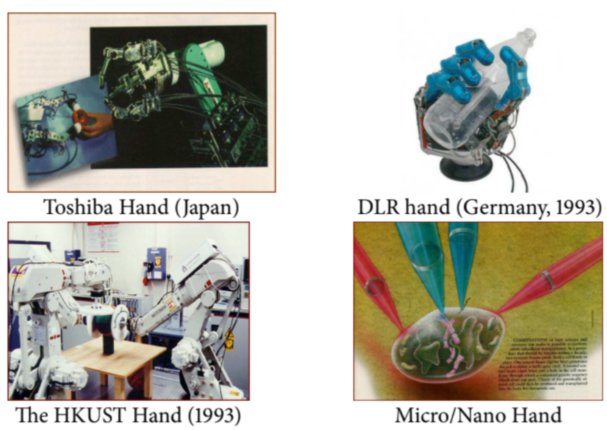


Figure 19.1: clockwise from top left: Toshiba Hand (Japan), DLR hand (1993), The HKUST Hand (1993), Micro/Nano Hand

19.1.2 Lessons from Biological Systems

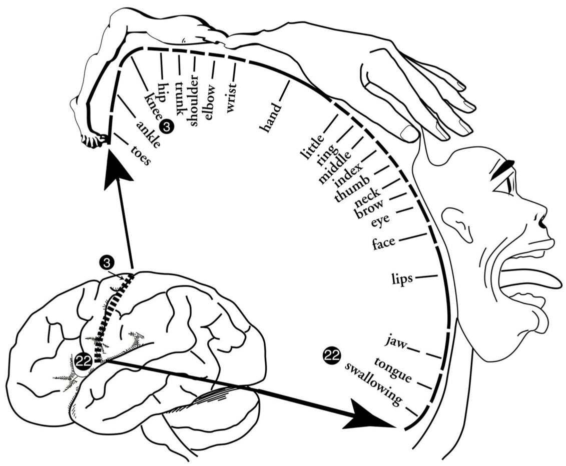


Figure 19.2: The Homunculus

In Latin, Homonculus means little man. It is a lateral view of the motor cortex. Different parts of the motor cortex maps onto different parts of the body. For humans, 30 to 40 percent of the motor cortex corresponds to hand control. The mouth and lips also take up large portions for eating and vocalizing.

percent of motor cortex for hand control:

- Human: 30% to 40%
- Monkey: 20% to 30%
- Dog: less than 10%

Goal: A manipulation theory for robotic hands based on physical laws and rigorous mathematical models!

19.2 Grasp Statics

19.2.1 Contact Models

The are many ways to model the wrench produced at a contact point by a given force. In all these models, F_i represents the wrench associated with the contact and is expressed in the contact frame. That is, a frame centered at the point of contact with the z axis pointing into the object.

19.2.1.1 Frictionless Point Contact Model

The most simple model is the Frictionless Point Contact (FPC) Model. In this model there is no friction between the fingertip and the object and forces can only be applied in the direction normal to the surface of the object. This force needs to be non-negative because you cannot "pull" on a contact point.

$$F_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_i,\ x_i \geq 0$$

Figure 19.3: Frictionless Point Contact (FPC)

19.2.1.2 Point Contact with Friction model

When friction exists between the fingertip and the object, we can use the Point Contact with Friction (PCWF) model.

$$F_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x_i,\ x_i \in FC_i$$

Figure 19.4: Point Contact with Friction (PCWF)

FC_i is the friction cone and represents the set of possible forces that can be exerted at the contact point. It takes the shape of a cone because the magnitude of friction in any direction is less than or equal to $\mu_i x_{i,3}$, where μ_i is the Coulomb coefficient of friction.

$$FC_i = \{x_i \in \mathbb{R}^3 : \sqrt{x_{i,1}^2 + x_{i,2}^2} < \mu_i x_{i,3}, x_{i,3} \geq 0\} \tag{19.1}$$

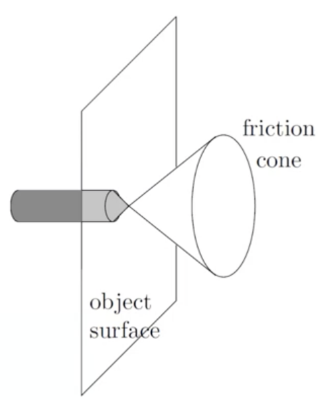


Figure 19.5: Interaction between the finger and object surface.

19.2.1.3 Soft Finger Model

This logical model attempts to approximate how a soft finger makes contact with an object.

$$F_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_i,\, x_i \in FC_i$$

Figure 19.6: Soft Finger Contact(SFC)

There can be frictional twist in the z axis. The magnitude of this friction twist is bounded by the magnitude of the normal force times a coefficient, just like friction forces. This coefficient is known as the Torsional coefficient of friction (μ_{it}), which is in addition to the Coulomb coefficient of friction (μ_i).
Here are two models of contact that describe the friction cone:

The Elliptic Model:

$$FC_i = \{x_i \in \mathbb{R}^4 | x_{i,3} \geq 0, \sqrt{\frac{1}{\mu_i^2}(x_{i,1}^2 + x_{i,2}^2) + \frac{1}{\mu_{it}^2} x_{i,4}^2} \leq x_{i,3}\} \tag{19.2}$$

The Linear Model:

$$FC_i = \{x_i \in \mathbb{R}^4 | x_{i,3} \geq 0, \frac{1}{\mu_i}\sqrt{x_{i,1}^2 + x_{i,2}^2} + \frac{1}{\mu_{it}}|x_{i,4}| \leq x_{i,3}\} \tag{19.3}$$

19.2.1.4 Wrench Basis

The matrices in the models above are called wrench basis because they map contact forces into contact wrenched

$$F_i = B_i \cdot x_i, x_i \in FC_i, B_i \in \mathbb{R}^{6xm_i} \tag{19.4}$$

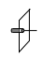
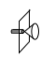
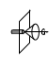
F_i is force exerted by finger acting in the finger tip coordinate frame which has the z axis pointing to the object is B_i times x_i where x_i belongs to friction cone. B_i has always has 6 rows, because the wrenches consist of 3 forces, and 3 torques. The number of columns depends on the contact model used. m_i is one for frictionless contact, and three for frictional contact, and four for soft finger contact.

19.2.1.5 Property 1

- FC_i is a closed subset of R^{m_i}, with nonempty interior.
- $\forall x_1, x_2 \in FC_i, \alpha x_1 + \beta x_2 \in FC_i, \forall \alpha, \beta > 0$

19.2.1.6 Summary table from MLS

Table 5.2: Common contact types.

Contact type	Picture	Wrench basis	FC
Frictionless point contact		$\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}$	$f_1 \geq 0$
Point contact with friction		$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\sqrt{f_1^2+f_2^2} \leq \mu f_3$ $f_3 \geq 0$
Soft-finger		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$	$\sqrt{f_1^2+f_2^2} \leq \mu f_3$ $f_3 \geq 0$ $\lvert f_4\rvert \leq \gamma f_3$

Note the textbook uses a different model for the Friction Cone of the Soft Finger Model.

19.2.2 The Grasp Map:

19.2.2.1 Single contact

The grasp map transforms the force (exerted at a single contact point) from the contact frame to the object frame. Assuming the configuration of the ith contact frame relative to the object frame to be (p_{oci}, R_{oci}), we have

$$F_o = Ad^T_{g^{-1}_{oc_i}} F_i = \begin{bmatrix} R_{oc_i} & 0 \\ \hat{p}_{oc_i} R_{oc_i} & R_{oc_i} \end{bmatrix} B_i x_i \tag{19.5}$$

The Contact Map transforms contact forces (in the contact frame) to wrenches in the object frame, and is defined as

$$G_i = Ad^T_{g^{-1}_{oc_i}} B_i \tag{19.6}$$

19.2.2.2 Multifingered grasp

If we have k fingers contacting an object, the total wrench on the object is the sum of the object wrenches due to each finger. The map between the contact forces and the total object force is called the grasp map, $G : \mathbb{R}^m \Rightarrow \mathbb{R}^p, m = m_1 + \cdots + m^k$. Since each contact map is linear and wrenches can be superposed (as long as they are all written in the same coordinate frame), the net object wrench is

$$F_0 = G_i x_1 + \cdots + G_k x_k = \begin{bmatrix} G1 & \dots & G_k \end{bmatrix} \begin{bmatrix} x_i \\ \vdots \\ x_k \end{bmatrix} \tag{19.7}$$

we define the grasp maps to be

$$G = \begin{bmatrix} Ad^T_{g^{-1}_{oc_1}} B_1 & \dots & Ad^T_{g^{-1}_{oc_k}} B_k \end{bmatrix} \tag{19.8}$$

Finally, the net wrench can be expressed as

$$F_0 = Gx, x \in FC, FC = FC_i \times \cdots \times FC_k, FC \subset \mathbb{R}^m \tag{19.9}$$

19.2.3 Grasp Defintion

Definition: Grasp
(G,FC) is called a grasp.

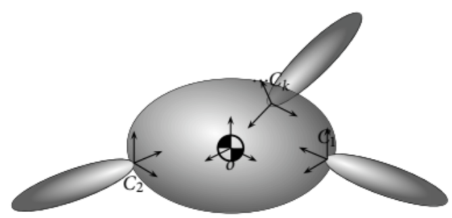


Figure 19.7: Example interaction between three fingers and an object.

Again, the z axis of the contact point is just the norm into the surface at that point. The X_is have to be positive. When a finger is contacting an object, it is pushing on the object, not pulling back (this happens if the force is negative).

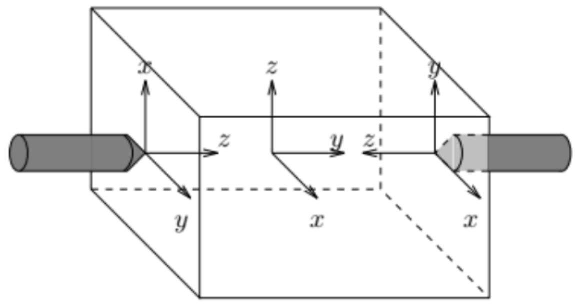
19.2.4 Example: Grasp map of frictionless point contact

$$F_0 = \begin{bmatrix} R_{ci} & 0 \\ \hat{p_{ci}} R_{ci} & R_{ci} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_i = \begin{bmatrix} n_{ci} \\ p_{ci} \times n_{ci} \end{bmatrix} x_i, x_i \geq 0$$

$$\Rightarrow F_0 = \begin{bmatrix} n_{ci} & \dots & n_{ck} \\ p_{ci} \times n_{ci} & \dots & p_{ck} n_{ck} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} = Gx$$

$$F_0 \in \mathbb{R}^6, x \geq 0.$$

19.2.5 Example: Soft finger grasp of a box



$$R_{c1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \; p_{c1} = \begin{bmatrix} 0 \\ -r \\ 0 \end{bmatrix}, \; R_{c2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \; p_{c2} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}$$

$$G_i = \begin{bmatrix} R_{0ci} & 0 \\ \hat{p_{0ci}} R_{0ci} & R_{0ci} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

An Introduction: What we are controlling here are the normal forces of the fingers (left and right). We counter gravitational force by applying more normal force which in turn creates more frictional force to balance the gravity. When coefficient of friction is low, grab on harder to keep holding the "slippery object". Of course, to keep the object steady, the two forces from both sides will need to cancel out as well.

19.2.5.1 Force Closure:

Definition: A grasp is force closure if it can resist any wrench on the object. In the example above, if someone twisted the object about the z-axis, generate a counter-torque through a differential between f1 and f2. A grasp (G, FC) is force closure if $\forall F_0 \in \mathbb{R}^p$ (p = 3 in the planar case and p = 6 one in the three-dimensional case) $\exists x \in$ FC, s.t. Gx = F_0. A grasp (G, FC) is force closure if $\forall F_0 \in \mathbb{R}^p$, $x_0 = Gx$. G having rank 6 is necessary for force closure. However, it is not sufficient since for any solution of x, x_{i3} needs to be non-negative for all i.

- **Problem 1: Force-closure Problem:** Determine if a grasp (G, FC) is force-closure or not.
- **Problem 2: Force Feasibility Problem:** Given $F_0 \in \mathbb{R}^p$, p = 3 or 6, determine if there exists $x \in FC s.t. Gx = F_0$
- **Problem 3: Force Optimization Problem:** Given $F_0 \in \mathbb{R}^p$, p = 3 or 6, find $x \in FC s.t. Gx = F_0$ and x minimizes $\phi(x)$

Definition: internal force $X_N \in FC$ is an internal force if $Gx_N = 0 \in (ker G \cap FC)$
An internal force is when each finger has forces which lies within the friction cone but g of x_N is equal to zero. When two fingers are pushing on an object with the same force (like in the figure above), then it's an

internal force; the forces will cancel out.

Property 3: (G,FC) is force closure iff G(FC) = $\mathbb{R}^p$ and $\exists x_N \in$ ker G s.t. $x_N \in int(FC)$

Proof of Property 3:

Sufficiency:

For $F_o \in \mathbb{R}^p$, let x' be s.t. $F_o = Gx'$. Since $\lim_{\alpha\to\infty} \frac{x'+\alpha x_N}{\alpha} = x_N \in$ int(FC), there $\exists\alpha'$, sufficiently large, s.t.

$$\frac{x' + \alpha' x_N}{\alpha'} \in \text{int}(FC) \subset FC$$

$\Rightarrow x = x' + \alpha' x_N \in \text{int}(FC)$
$\Rightarrow Gx = Gx' = F_o$

Necessity:

Choose $x_1 \in \text{int}(FC)$ s.t. $F_o = Gx_1 \neq 0$, and choose $x_2 \in FC$ s.t. $Gx_2 = -F_o$. Define $x_N = x_1 + x_2$, $Gx_N = 0 \Rightarrow x_N \in \text{int}(FC)$ $\square$