EECS106B 2/9 Lecture; Spring 2021

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Contents

1 Announcement: E-robot competition

Department of Energy is sponsoring an E-robot prize competition with $5M prize. See this link: https://americanmadechallenges.org/EROBOT/ Prof. Zakhor is considering assembling a robotics team to meet this challenge. We are particularly interested in students who have actually assembled actual robots and associated sensors in hardware, are familiar with ROS, and have some experience with SLAM algorithms. Expertise in manipulation and grasping would be a bonus. If interested, please contact avz@berkeley.edu.

- The deadline for Phase 1 (planning) is May 12, deadline for phase ends May of 2022
- You will have to assist in identifying building retrofitting, through concept and design, and eventually actuation.
- Because many of the buildings in the US are old and not up to date in energy efficiency standards, this project seeks to automate this updating process in some capacity
- There are 3 parts, and your team can pick any combination of these 3 assignments to work on:
  1. Sensing and inspection tools: Collecting real time information
  2. Retrofitting tools: semi/fully autonomous robots that automate some aspects of building retrofitting
  3. Mapping tools: Mapping the building envelope geometry and other defects

2 Lie bracket properties review

- For a review of Lie brackets needed for these notes, review the scribe notes from 2/4/2021

3 The Nonholonomic Integrator

Consider the so called nonholonomic integrator, on our state space q:

$$\begin{aligned} \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \\ \dot{q}_3 &= q_1 u_2 - q_2 u_1 \end{aligned} \tag{1}$$

The system has:

$$g_1 = \begin{bmatrix} 1 \\ 0 \\ -q_2 \end{bmatrix} \quad g_2 = \begin{bmatrix} 0 \\ 1 \\ q_1 \end{bmatrix} \quad [g_1, g_2] = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \tag{2}$$

3.1 Optimal Control

The optimal input minimizing our chosen cost function:

$$\int_0^1 \|u(t)\|^2 dt \tag{3}$$

From an initial point q_0 to a final q_1 was shown by Roger W. Brockett to be sinusoidal. The frequency λ of the optimal input occurs when $q_1(0) = q_1(1)$ and $q_2(0) = q_2(1)$ to be integer multiples of $2n\pi$ ...

The generalization of this system to m>2 inputs is stated as a control system on a space $q \in \Re^m \times Y \in so(m)$ such that

$$\begin{aligned} \dot{q} &= u \\ \dot{Y} &= qu^T - uq^T \end{aligned} \tag{4}$$

If $q_1(0) = q_1(1)$ and $Y(1) \in so(m)$ is given then it can be shown that the optimal input is multiples of 2π that is to say:

$$\begin{array}{ll} 2\pi.2, \ldots, \frac{m}{2} & \text{m even} \\ 2\pi, 2\pi.3, \ldots, \frac{m-1}{2} & \text{m odd} \end{array}$$

4 Can this system be steered?

4.1 Hopper

We have the system

$$q = \begin{bmatrix} \psi \\ l \\ \theta \end{bmatrix} \tag{5}$$

$$\dot{q} = \begin{bmatrix} 1 \\ 0 \\ -\frac{m(l+d)^2}{l+m(l+d)^2} \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2 \tag{6}$$

For equation 5.3, define the first matrix as g_1 and the second as g_2. Using Frobenius's Theorem, we want to know if we can get $\Delta(q) = span\{g_1(q), g_2(q), ...g_m(q)\}$ to span R^m by taking successive lie brackets because that would mean the system is completely nonholonomic.

We have two vectors g_1 and g_2 in R^3 so far, so let's take another lie bracket:

$$[g_1, g_2] = D_{g_2}g_1 - D_{g_1}g_2 = ... = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{2lm(l+d)}{l+m(l+d)^2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{2lm(l+d)}{l+m(l+d)^2} \end{bmatrix} := g_3$$

where $D_{g_i} = \frac{dg_i}{dq}$. The matrix $[g_1, g_2, g_3]$ is full rank, so $\bar{\Delta} = R^3$. Thus, the hopper can be steered.

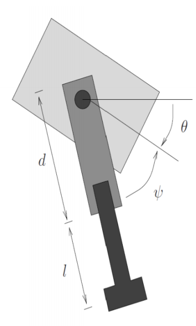

Figure 1: Hopper system

4.2 Car

$$\dot{q} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{l}\tan\phi \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2 := g_1 u_1 + g_2 u_2$$

We perform the same procedure of taking the lie brackets of the g vectors to get new directions:

$$g_3 := [g_1, g_2] = ... = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{l\cos^2\phi} \\ 0 \end{bmatrix}$$

$$g_4 := [g_1, g_3] = ... = \begin{bmatrix} -\frac{\sin\theta}{l\cos^2\phi} \\ \frac{\cos\theta}{\cos^2\phi} \\ 0 \\ 0 \end{bmatrix}$$

Since $[g_1, g_2, g_3, g_4]$ is full rank when ϕ is not π or $-\pi$, the front wheel drive car is completely non-holonomic with degree of nonholonomy 3.

We can interpret g_1 as drive, g_2 as steer, g_3 as wiggle, and g_4 as slide.

The new vectors we got from taking lie brackets can be interpreted as new directions.

5 1-chain form

This can also be called chained form or a Goursat normal form system.

5.1 Definition

A system in 1-chain form is written as:

$$\dot{q_1} = u_1$$
$$\dot{q_2} = u_2$$
$$\dot{q_3} = q_2 u_1$$
$$\dot{q_4} = q_3 u_1$$
$$...$$
$$\dot{q_n} = q_{n-1} u_1$$

Where u_i is an input and q_i is a state. This extends a non-holonomic integrator to dimension n.

This form implies that

$$g_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ ... \end{bmatrix}$$

and

$$g_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ ... \end{bmatrix}$$

Also, through brute force calculation, we can see that

$$g_3 = [g_1, g_2] = ... = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ ... \end{bmatrix}$$

In fact,

$$[g_1, [..., [g_1, \text{k times}, g_2]...]] = \begin{bmatrix} 0 \\ ...0 \\ (-1)^k \\ 0 \\ ... \\ 0 \end{bmatrix}$$

This form makes it easy to see if the system can be steered because the kth column has a single element on the kth row.

Also, for ease of notation, let's define "little ad" which is very different from "big ad" (Ad) we saw in 106A:

$$ad_{g_1} g_2 = [g_1, g_2]$$
$$ad_{g_1}^{k+1} = [g_1, ad_{g_1}^k g_2]$$

This means

$$\text{span of the system} = \{g_1, g_2 ad_{g_1}^k g_2, k = 1, ..., n-2\} = R^n$$

and a chained form system can always be steered.

Algorithm 3. Steering chained form systems

1. Steer q_1 and q_2 to their desired values.
2. For each q_{k+2}, $k \geq 1$, steer q_k to its final value using $u_1 = a\sin 2\pi t$, $u_2 = b\cos 2\pi kt$, where a and b satisfy

$$q_{k+2}(1) - q_{k+2}(0) = \left(\frac{a}{4\pi}\right)^k \frac{b}{k!}.$$

Figure 2: Algorithm for steering chained form systems

5.2 Steering chained form systems

We can use the algorithm in Figure 2 to steer a chained form system. q_1 and q_2 can be steered to their desired positions by simply setting u_1 and u_2 by definition. To get the other q_i to their desired positions, we use sinusoidal signals $u_1(t) = sin(2\pi t)$ and $u_2 = cos(2\pi kt)$ to steer the next state q_k that is wrong, while also keeping the previously corrected states the same (after a 1 second movement period).

The integral calculations for making these corrections are below:

$$\begin{aligned}
q_1 &= \frac{a}{2\pi}(1 - \cos 2\pi t) \\
q_2 &= \frac{b}{2\pi k}\sin 2\pi kt \\
q_3 &= \int \frac{ab}{2\pi k}\sin 2\pi kt \sin 2\pi t dt \\
&= \frac{1}{2}\frac{ab}{2\pi k}\left(\frac{\sin 2\pi(k-1)t}{2\pi(k-1)} - \frac{\sin 2\pi(k+1)t}{2\pi(k+1)}\right) \\
q_4 &= \frac{1}{2}\frac{a^2 b}{2\pi k \cdot 2\pi(k-1)}\int \sin 2\pi(k-1)t \cdot \sin 2\pi t dt + \cdots \\
&= \frac{1}{2^2}\frac{a^2 b}{2\pi k \cdot 2\pi(k-1) \cdot 2\pi(k-2)}\sin 2\pi(k-2)t + \cdots \\
&\vdots \\
q_{k+2} &= \int \frac{1}{2^{k-1}}\frac{a^k b}{2\pi k \cdot 2\pi(k-1)\cdots 2\pi}\sin^2 2\pi t dt + \cdots \\
&= \frac{1}{2^{k-1}}\frac{a^k b}{(2\pi)^k k!}\frac{t}{2} + \cdots
\end{aligned}$$

5.3 Converting a hopper system to 1-chained form

We use the same hopper system as before:

$$\dot{q} = \begin{bmatrix} 1 \\ 0 \\ -\frac{m(l+d)^2}{l+m(l+d)^2} \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2$$

The optimal solution is uses the Fourier series and sinusoids due to the fact about optimal control stated above. Let's choose

$$u_1 = a_1 \sin(2\pi t)$$
$$u_2 = a_2 \cos(2\pi t)$$

We can break the original equation down with the Fourier series

$$f(l) = f\left(\frac{a_2}{2\pi} \sin 2\pi t\right) = \beta_1 \sin 2\pi t + \beta_2 \sin 4\pi t + \cdots$$

By suitably choosing a_1 and a_2, the first two elements ψ and l are back to their initial values in 1 second, and θ can be anything we want.

5.4 Converting a car system to 1-chained form

The control system for the car from before is

$$\begin{aligned} \dot{x} &= \cos\theta u_1 \\ \dot{y} &= \sin\theta u_1 \\ \dot{\theta} &= \tfrac{1}{l} \tan\phi u_1 \\ \dot{\phi} &= u_2 \end{aligned}$$

To change this into the 1-chain form, let's define states $z_1 = x, z_2 = \phi, z_3 = \sin\theta, z_4 = y$ and inputs $v_1 = \cos\theta u_1, v_2 = u_2$. This gives us the 1-chain form:

$$\begin{aligned} \dot{z}_1 = &\quad v_1 \\ \dot{z}_2 = &\quad v_2 \\ \dot{z}_3 = &\quad \tfrac{1}{l} \tan z_2 v_1 \\ \dot{z}_4 = &\quad \tfrac{z_2}{\sqrt{1-z_3^2}} v_2 \end{aligned}$$

We can then steer this system with the Fourier series method and double frequency sinusoids from before.

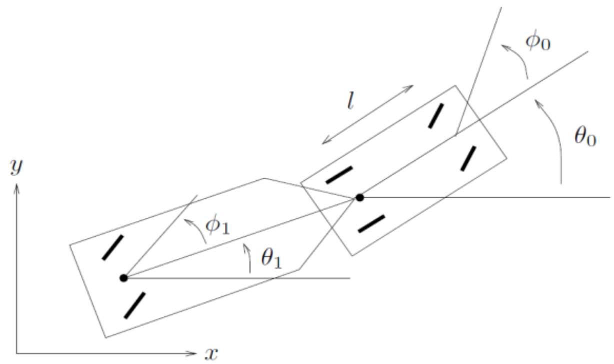

Figure 3: Layout of the firetruck model

5.5 Multi Chained Forms

Given a control system of the form in $\Re^n$ with two inputs:

$$\dot{q} = g_1(q)u_1 + g_2(q)u_2$$

If and only if the following three distributions are regular (constant dimension) and involutive:

$$\begin{aligned}
\Delta_0 &= \operatorname{span}\left\{g_1, g_2, ad_{g_1}g_2, \ldots, ad_{g_1}^{n-2}g_2\right\} = \Re^n \\
\Delta_1 &= \operatorname{span}\left\{g_2, ad_{g_1}g_2, \ldots, ad_{g_1}^{n-2}g_2\right\} \\
\Delta_2 &= \operatorname{span}\left\{g_2, ad_{g_1}g_2, \ldots, ad_{g_1}^{n-3}g_2\right\}
\end{aligned}$$

then there exists a choice of coordinates $z_1(q), \ldots, z_n(q)$ and inputs $v = \alpha(q) + \beta(q)u$ such that:

$$\begin{aligned}
\dot{z}_1 &= v_1 \\
\dot{z}_2 &= v_2 \\
\dot{z}_3 &= z_2 u_1 \\
&\vdots \\
\dot{z}_n &= z_{n-1} v_1
\end{aligned}$$

When there are m $\geq$ 3 inputs for

$$\dot{q} = g_1(q)u_1 + g_2(q)u_2 + \cdots + g_m(q)u_m$$

there are also analogous necessary and sufficient conditions for the conversion into m - 1 chains. For example, the below firetruck model. The driver in front has two inputs: drive and steer and the one at the back of the ladder can steer. This can be converted into a two chain system.