

Discussion #2

*Author: Amay Saxena***Problem 1 - Velocities are Twists, Twists are Velocities**

Consider a rigid body (body frame B , spatial frame A) performing a uniform twist motion $\xi = (v, \omega)$, so that its configuration at time t is given by:

$$g_{ab}(t) = e^{\hat{\xi}t} g_{ab}(0) \quad (0.1)$$

1. Let p_a be the spatial coordinates for a point on the rigid body. While the body is performing this motion, write down an expression for \dot{p} in terms of p and the coordinates of the twist ξ .
2. Compute the spatial rigid body velocity \hat{V}_{ab}^s of the frame $g_{ab}(t)$ as a function of time (Recall the derivative of a matrix exponential: $d(e^{At})/dt = Ae^{At}$).
3. Compute the body velocity \hat{V}_{ab}^b of the frame $g_{ab}(t)$ as a function of time.
4. Interpret screw motions as simply the analog of moving with a constant velocity, and hence interpret twists as velocities.

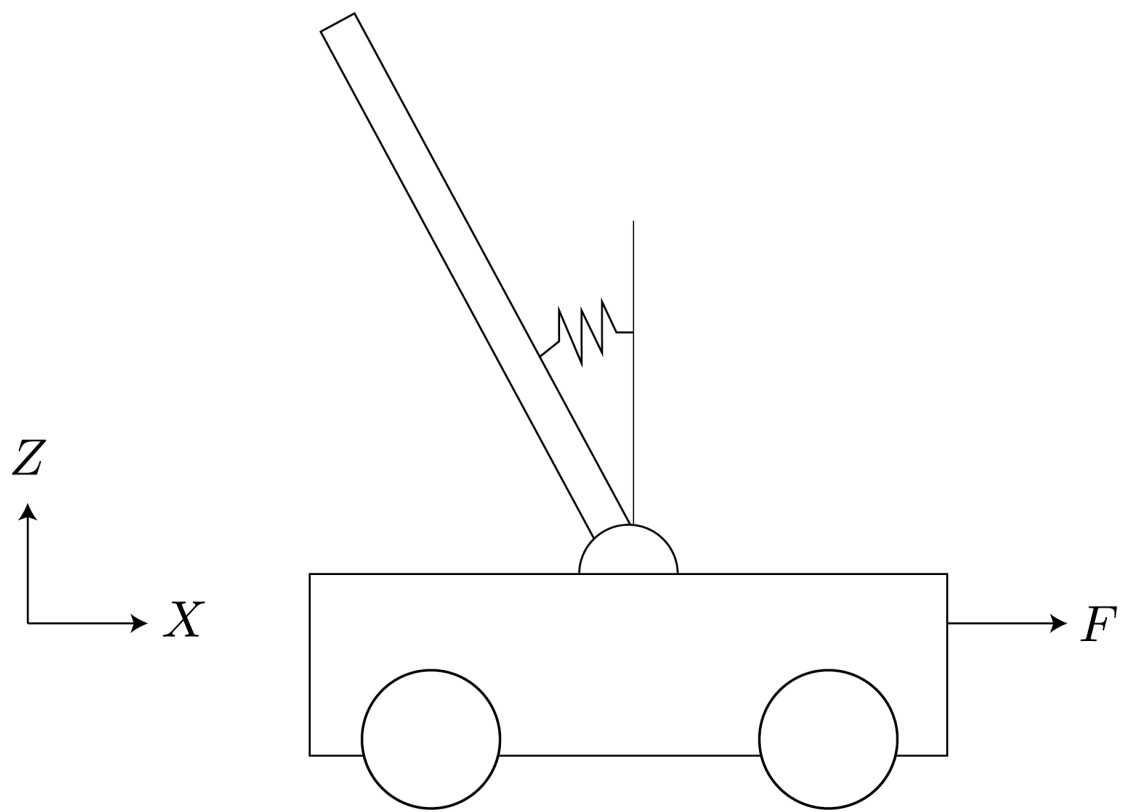
Problem 2 - Workspace Tracking with Jacobians

We would like the end effector of our robot arm to perform some trajectory in its workspace. This trajectory is given to us as a trajectory of rigid transforms, $g(t)$. Assume we have access to both $g(t)$ and $\dot{g}(t)$ for $t \in [0, T]$, and that the trajectory starts from the current position of the robot, $g(0)$. Recall that we can only give the robot jointspace commands, so we want to convert this workspace trajectory into a jointspace trajectory $\theta(t)$ such that $g_{ST}(\theta(t)) = g(t)$ for $t \in [0, T]$ where $g_{ST}(\theta)$ is the forward kinematics map.

1. How would you solve this problem using an inverse kinematics solver? Why might this be undesirable?
2. Write down an expression for the desired spatial workspace velocity \hat{V} that we want the end effector to perform at time t , in order to perform the trajectory g .
3. Assume we also have efficient access to the spatial jacobian J . Write down an expression for $\dot{\theta}(t)$ in terms of the velocity you computed in the previous part (recall the Moore-Penrose pseudoinverse that we spoke about in lecture).
4. Write down an expression for $\theta(t)$, using your answers to the previous two parts.
5. Can we use the previous part to do inverse kinematics?

Problem 3 - Lagrangian Dynamics

A cart with mass M is free to move in the x-direction without resistance. A uniform rod of length L with mass m and moment of inertia I (about its center of mass) is mounted on a frictionless pivot on top of the cart. An external force F is applied as an input. The pivot also has mounted in it a torsional spring with spring constant κ , as shown. Pick a suitable set of generalized coordinates, and find the equations of motion of the system in terms of those coordinates using Lagrangian dynamics.



(0.2)