EECS C106B - Robotic Manipulation and Interaction **(Week 6)**

Discussion #6

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1 Camera Intrinsic Matrix

The camera intrinsic parameter matrix K can be represented

$$\begin{bmatrix} fs_x & s_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1}$$

What do each of these terms represent?

2 Projections Matrix

A basic model of a camera is the following:

$$\lambda \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Given some point $p \in \mathbb{R}^3$ in the camera frame, we can apply the camera transformation to get the image of that point $q \in \mathbb{R}^2$. Show that given any point $r \in \mathbb{R}^3$ that lies on the line between o (the origin of the camera frame) and p, the image of r is q.

3 Vanishing Points

A straight line in the 3D world becomes a straight line in the image. However, two parallel lines in the 3D world will often intersect in the image. The point of intersection is called the *vanishing point*.

1. Given two parallel lines, how do you compute the vanishing point?
2. When does the vanishing point not exist (the two lines do not intersect)?
3. Show that the vanishing points of lines on a plane lie on the vanishing line of the plane.

4 Homography Matrix Transform

When the image features lie in a plane in the real world, the two-image correspondence problem is called planar homography. The real world coordinates of a point X are X_1 in the frame of camera 1, and X_2 in the frame of camera 2. We assume that X lies on a plane with normal vector N (defined with respect to camera 1). We know that the transform between the cameras is of the form $X_2 = RX_1 + T$, an affine transformation, but by assuming that X lies on the plane we can represent this transformation with a *homography matrix* H where

$$X_2 = HX_1 \tag{4.1}$$

where $H = R + \frac{1}{d}TN^T$, where d is the shortest distance between the plane and camera 1 ($d = N^T X_1$).
If we switch the roles of the first and second cameras, we should still be able to define a homography matrix $\tilde{H}$ such that $X_1 = \tilde{H}X_2$. Assume that $d_1 = 1$, so $H = R + TN^T$. Show that the new homography matrix $\tilde{H}$ is defined

$$\tilde{H} = \left(R^T + \frac{-R^T T}{1 + N^T R^T T} N^T R^T \right) \tag{4.2}$$