Multiple-View Reconstruction from Scene Knowledge

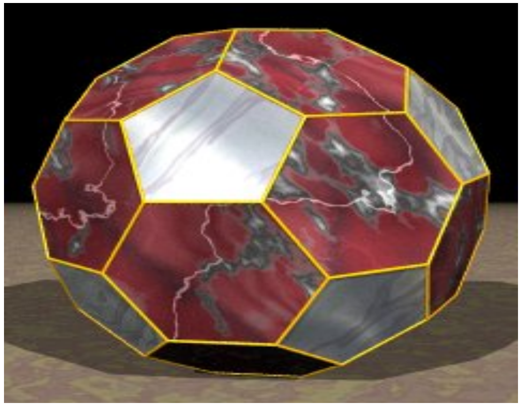
Yi Ma
EECS, UC Berkeley

"Mathematics is the art of giving the same name to different things."

– Henri Poincare

INTRODUCTION: Scene knowledge and symmetry






Symmetry is ubiquitous in man-made or natural environments

INTRODUCTION: Scene knowledge and symmetry

Parallelism (vanishing point)

Orthogonality

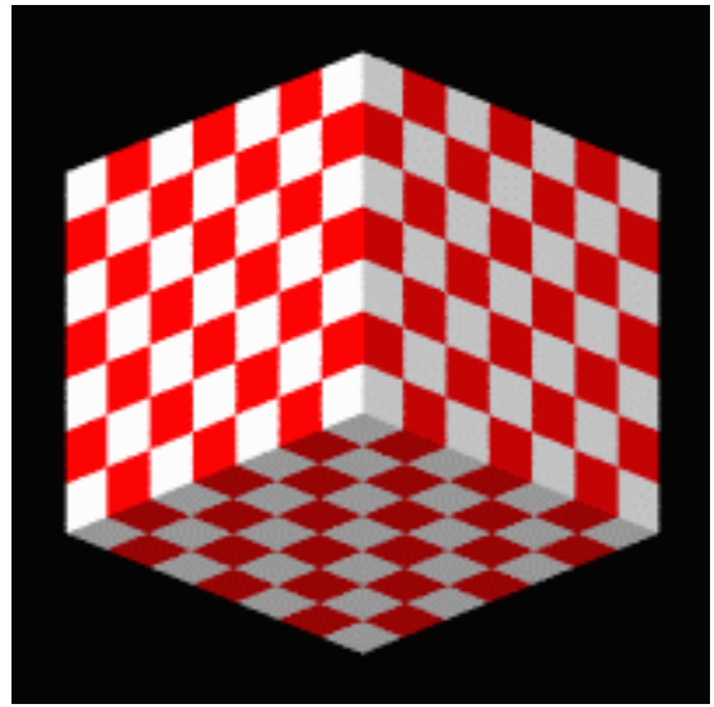
Congruence

Self-similarity

INTRODUCTION: Wrong assumptions

Ames room illusion

Necker’s cube illusion



INTRODUCTION: Related literature

Mathematics: Hilbert 18th problem: Fedorov 1891, Hilbert 1901, Bieberbach 1910, George Polya 1924, Weyl 1952

Cognitive Science: Figure "goodness": Gestalt theorists (1950s), Garner'74, Chipman'77, Marr'82, Palmer'91'99…

Computer Vision (isotropic & homogeneous textures): Gibson'50, Witkin'81, Garding'92'93, Malik&Rosenholtz'97, Leung&Malik'97…

Detection & Recognition (2D & 3D): Morola'89, Forsyth'91, Vetter'94, Mukherjee'95, Zabrodsky'95, Basri & Moses'96, Kanatani'97, Sun'97, Yang, Hong, Ma'02

Reconstruction (from single view): Kanade'81, Fawcett'93, Rothwell'93, Zabrodsky'95'97, van Gool et.al.'96, Carlsson'98, Svedberg and Carlsson'99, Francois and Medioni'02, Huang, Yang, Hong, Ma'02,03

Multiple-View Reconstruction from Scene Knowledge

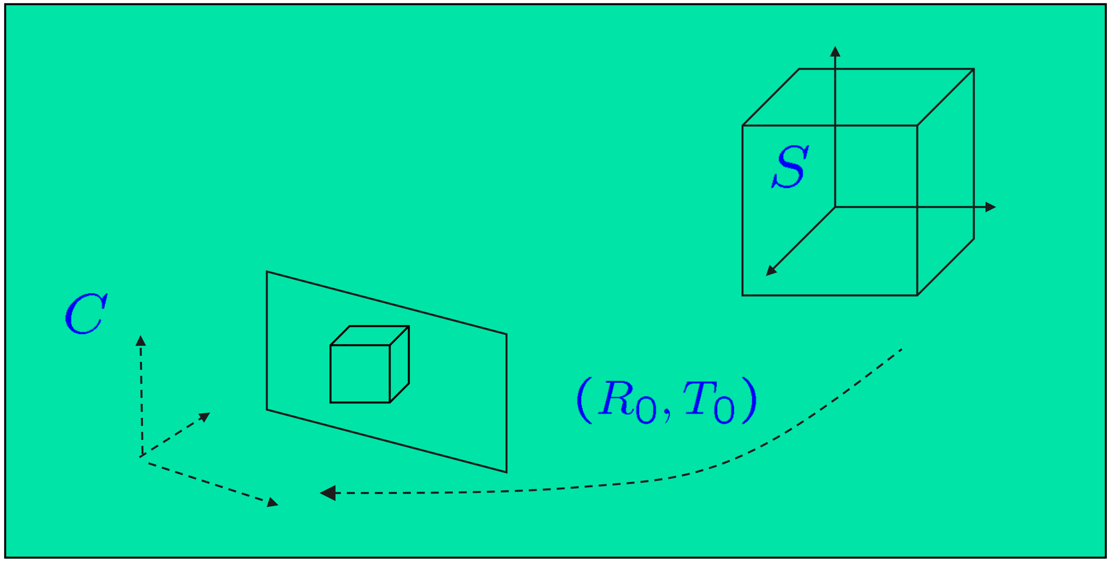
SYMMETRY & MULTIPLE-VIEW GEOMETRY
- Fundamental types of symmetry
- Equivalent views
- Symmetry based reconstruction

MUTIPLE-VIEW MULTIPLE-OBJECT ALIGNMENT
- Scale alignment: adjacent objects in a single view
- Scale alignment: same object in multiple views

ALGORITHMS & EXAMPLES
- Building 3-D geometric models with symmetry
- Symmetry extraction, detection, and matching
- Camera calibration

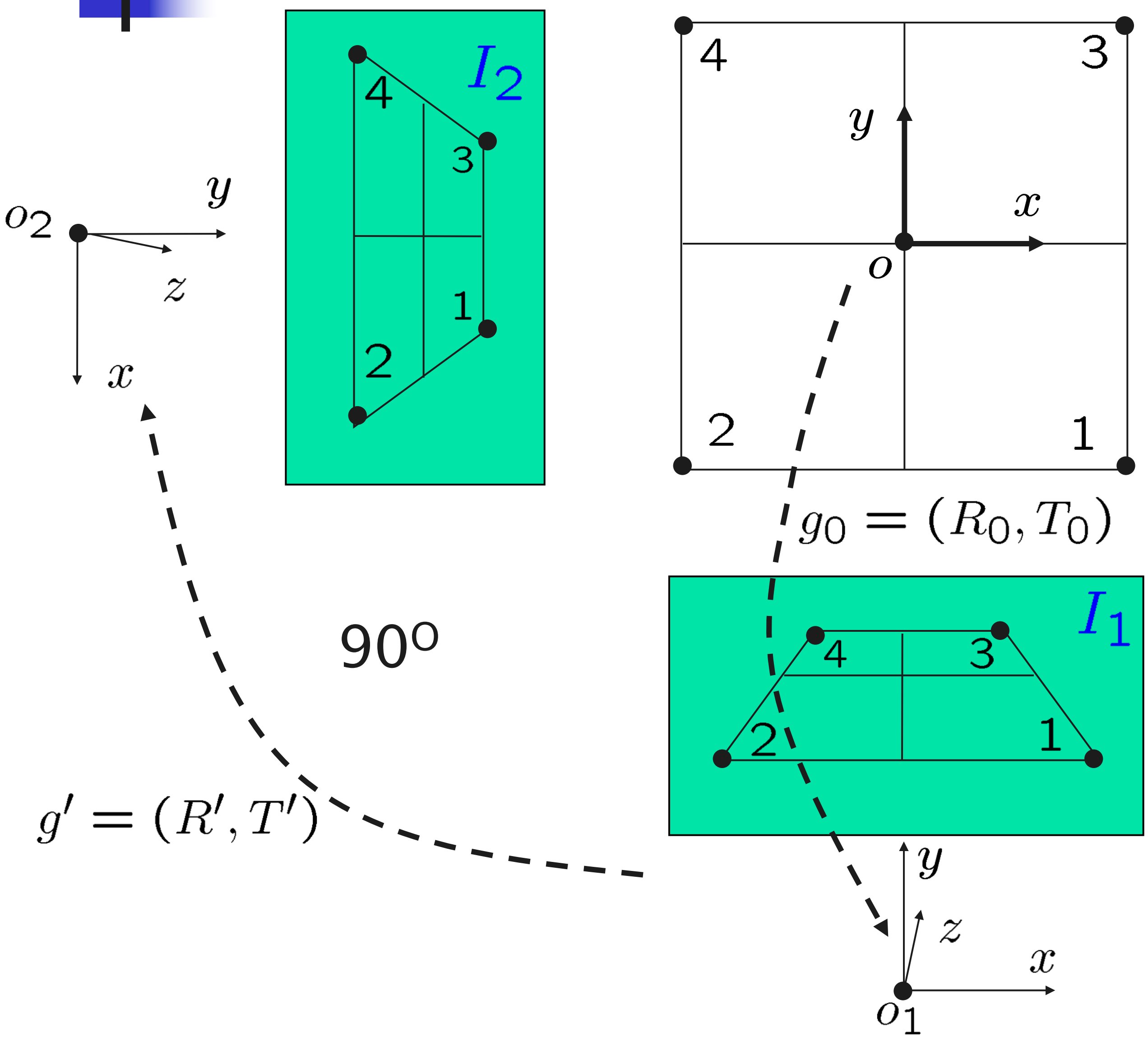
SUMMARY: Problems and future work

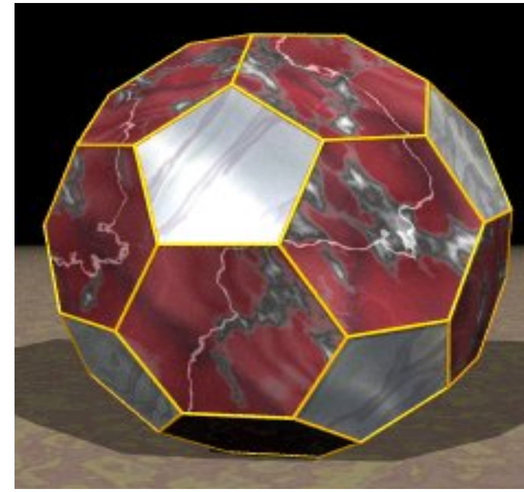
SYMMETRY & MUTIPLE-VIEW GEOMETRY



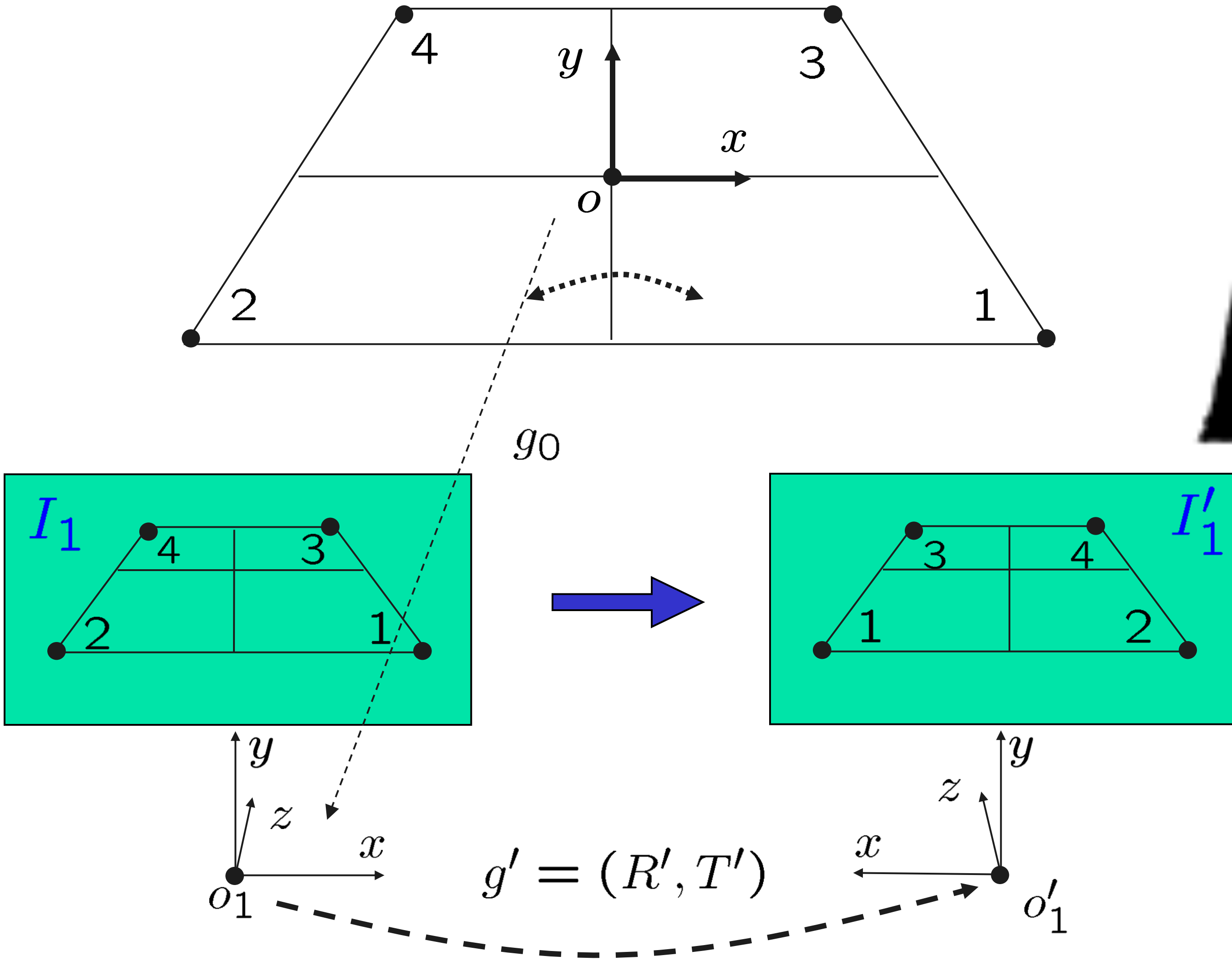
- Why does an image of a symmetric object give away its structure?
- Why does an image of a symmetric object give away its pose?
- What else can we get from an image of a symmetric object?

Equivalent views from rotational symmetry

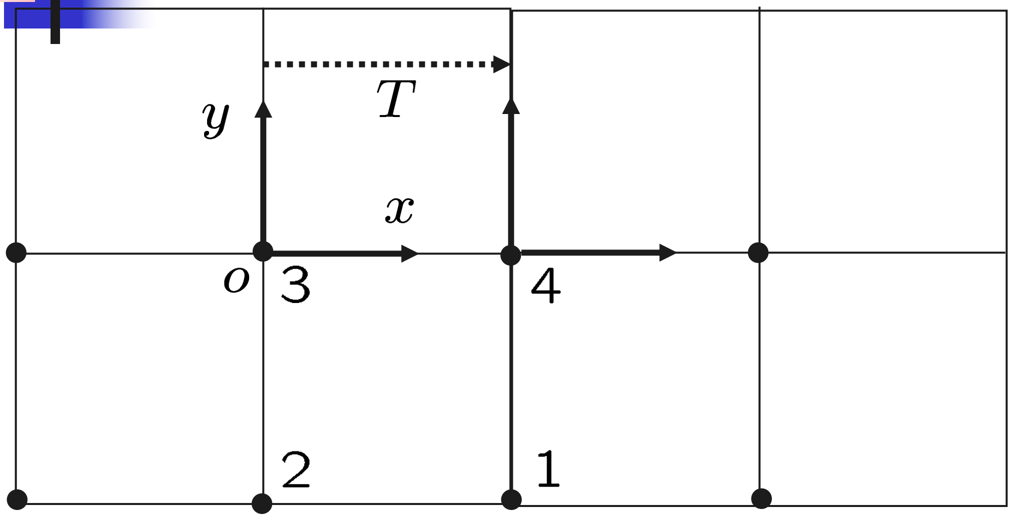


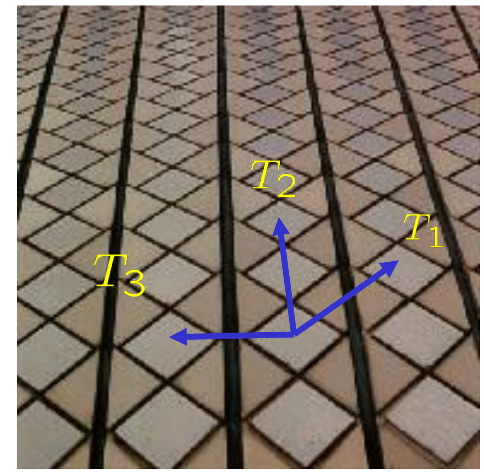


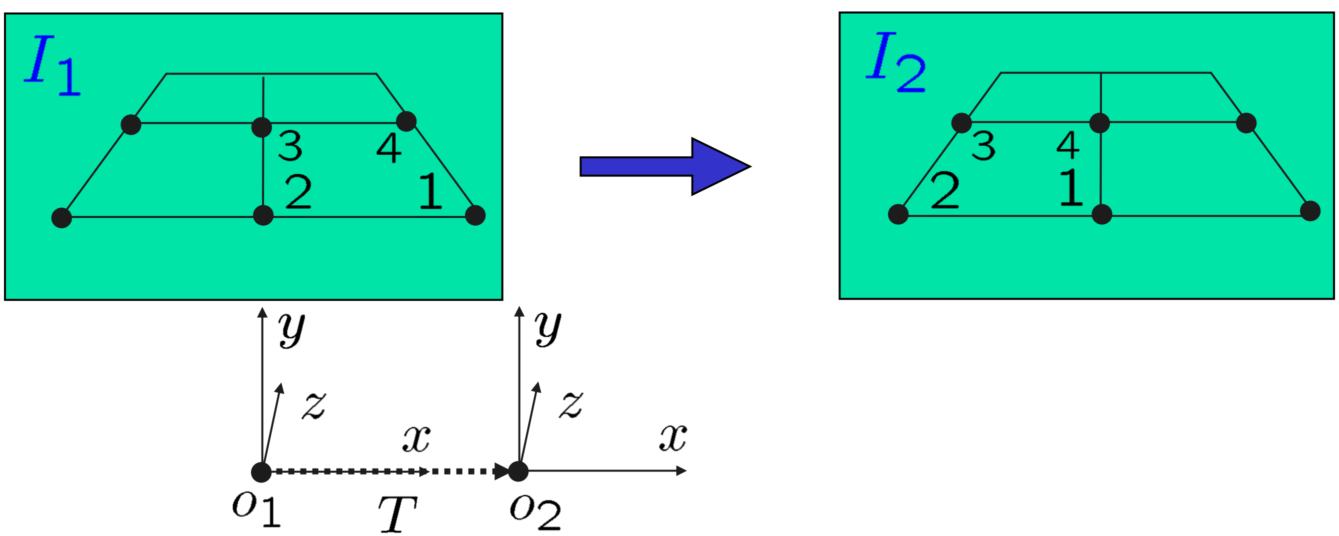
Equivalent views from reflectional symmetry



Equivalent views from translational symmetry







Definition. A set of 3-D features S is called a symmetric structure if there exists a nontrivial subgroup G of E(3) that acts on it such that for every g in G, the map

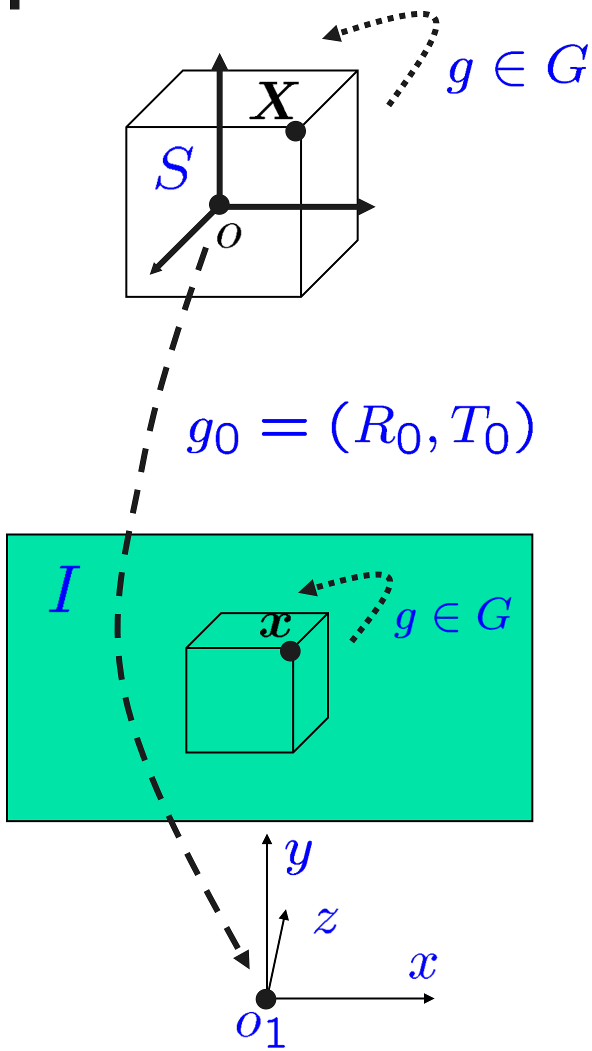
$$\boxed{g \in G : \; S \to S}$$

is an (isometric) automorphism of S. We say the structure S has a group symmetry G.

$$\boldsymbol{X} = [X, Y, Z, 1]^T \in \mathbb{R}^4, \quad \boldsymbol{x} = [x, y, z]^T \in \mathbb{R}^3$$
$$g_0 = \begin{bmatrix} R_0 & T_0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}, \quad \Pi_0 = [I, 0] \in \mathbb{R}^{3\times 4}$$

$$\boxed{\boldsymbol{x} \sim \Pi_0 g_0 \boldsymbol{X}} \implies \boxed{g(\boldsymbol{x}) \sim \Pi_0 g_0 g \boldsymbol{X}}$$

GEOMETRY FOR SINGLE IMAGES – **Multiple "Equivalent" Views**



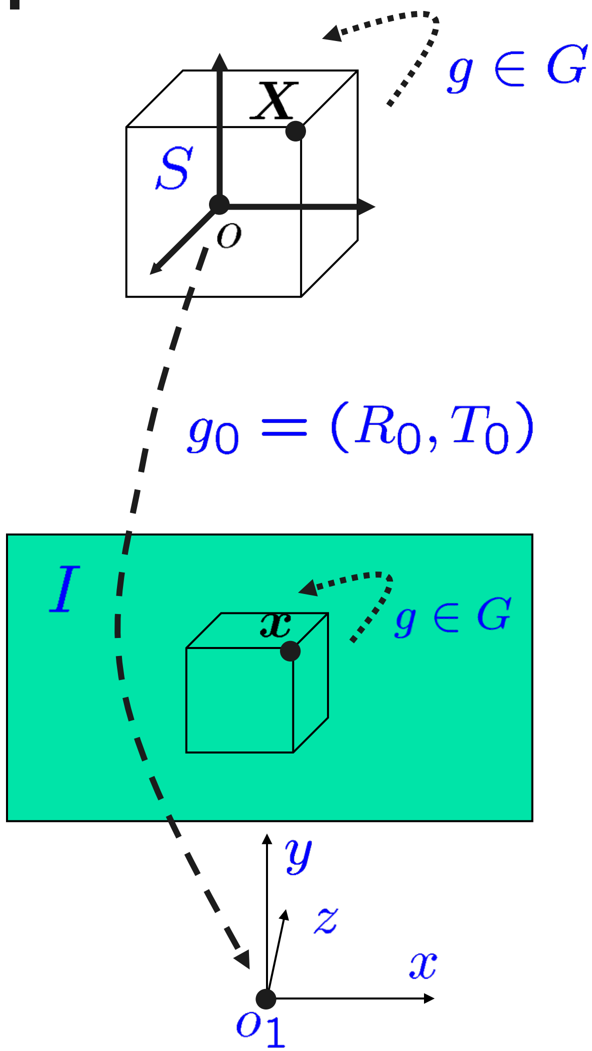
$$g(\boldsymbol{x}) \sim \Pi_0 g_0 g \boldsymbol{X} = \Pi_0 g_0 g g_0^{-1} g_0 \boldsymbol{X}$$

$$\begin{aligned} g_1(\boldsymbol{x}) &\sim \Pi_0 g_0 g_1 g_0^{-1}(g_0 \boldsymbol{X}), \\ g_2(\boldsymbol{x}) &\sim \Pi_0 g_0 g_2 g_0^{-1}(g_0 \boldsymbol{X}), \\ &\vdots \\ g_m(\boldsymbol{x}) &\sim \Pi_0 g_0 g_m g_0^{-1}(g_0 \boldsymbol{X}). \end{aligned}$$

$$g = (R, T), \; g' = g_0 g g_0^{-1}$$

$$g' : \begin{cases} R' \doteq R_0 R R_0^T \in O(3), \\ T' \doteq (I - R')T_0 + R_0 T \in \mathbb{R}^3. \end{cases}$$

GEOMETRY FOR SINGLE IMAGES – **Symmetric Rank Condition**



$$M \doteq \begin{bmatrix} \widehat{g_1(\boldsymbol{x})} R_1' \boldsymbol{x} & \widehat{g_1(\boldsymbol{x})} T_1' \\ \widehat{g_2(\boldsymbol{x})} R_2' \boldsymbol{x} & \widehat{g_2(\boldsymbol{x})} T_2' \\ \vdots & \vdots \\ \widehat{g_m(\boldsymbol{x})} R_m' \boldsymbol{x} & \widehat{g_m(\boldsymbol{x})} T_m' \end{bmatrix}$$

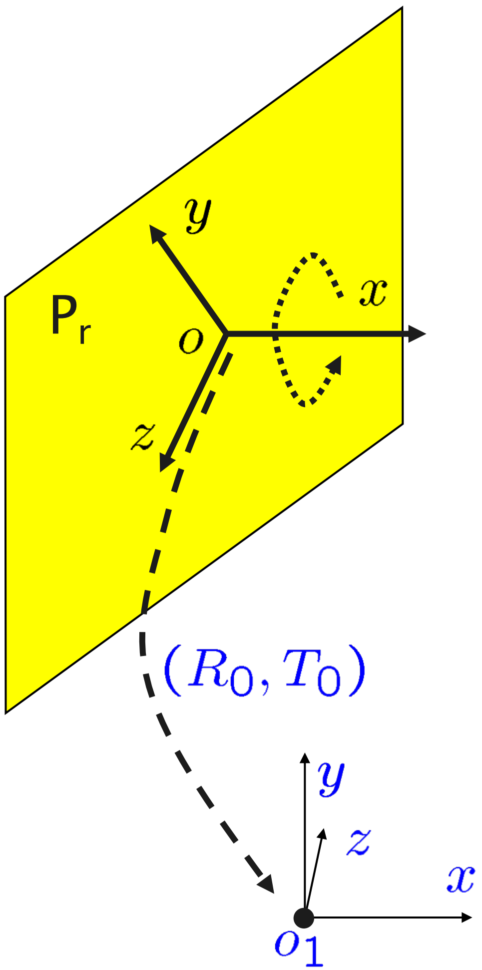
$$\boxed{\mathrm{rank}(M) = 1}$$

Solving g_0 from Lyapunov equations:

$$\boxed{g_i' g_0 - g_0 g_i = 0,\ i = 1, \ldots, m}$$

with g'_i and g_i known.

THREE TYPES OF SYMMETRY – **Reflective Symmetry**



$$\begin{cases} R' \doteq R_0 R R_0^T \in O(3), \\ T' \doteq (I - R')T_0 \in \mathbb{R}^3. \end{cases}$$

$$R' R_0 - R_0 R = 0.$$

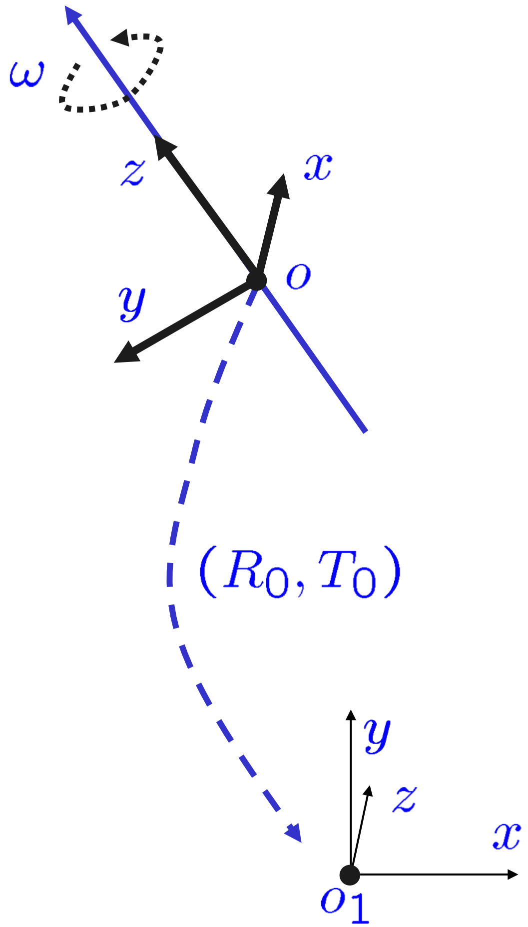
$$\dim(Ker(L)) = 5.$$

$$R_0 \in Ker(L) \cap SO(3) = SO(2).$$

$$T_0 \in (I - R')^\dagger T' + Null(I - R').$$

$$\dim(Null(I - R')) = 2.$$

THREE TYPES OF SYMMETRY – **Rotational Symmetry**



$$\begin{cases} R' \doteq R_0 R R_0^T \in O(3), \\ T' \doteq (I - R')T_0 \in \mathbb{R}^3. \end{cases}$$

$$R' R_0 - R_0 R = 0.$$

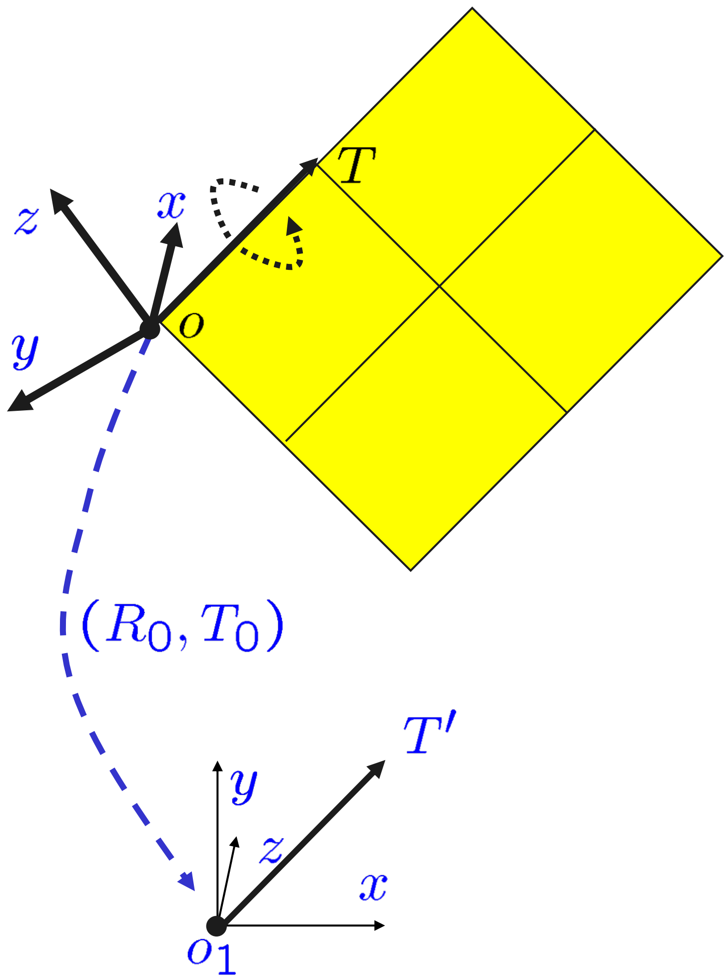
$$\dim(Ker(L)) = 3.$$

$$R_0 \in Ker(L) \cap SO(3) = SO(2).$$

$$T_0 \in (I - R')^{\dagger} T' + Null(I - R').$$

$$\dim(Null(I - R')) = 1.$$

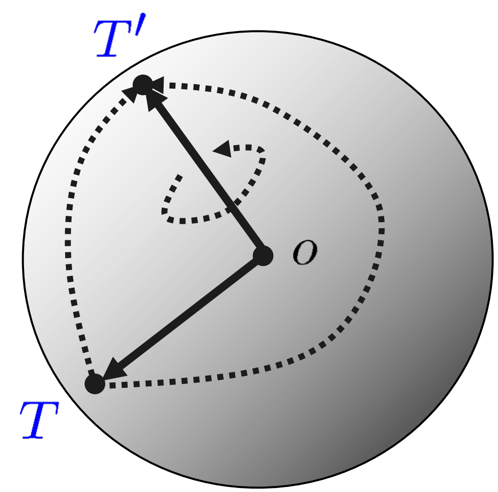
THREE TYPES OF SYMMETRY – **Translatory Symmetry**



$$\begin{cases} R = R' \doteq I, \\ T' \doteq R_0 T. \end{cases}$$

$$R_0 T = T'.$$

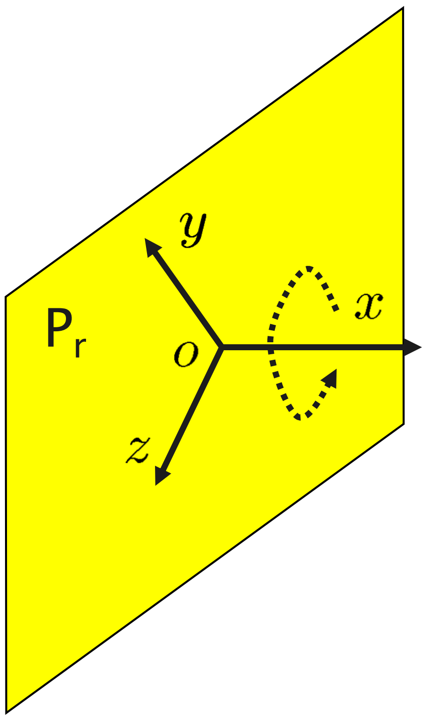
$$R_0 \in SO(2).$$
$$T_0 \in \mathbb{R}^3.$$

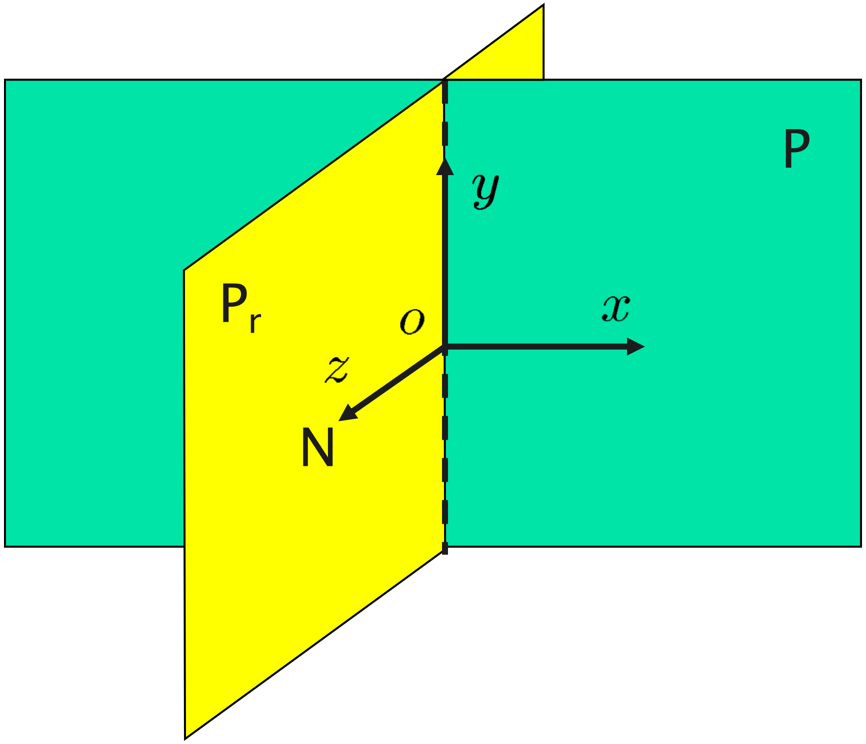


SINGLE-VIEW GEOMETRY WITH SYMMETRY – **Ambiguities**

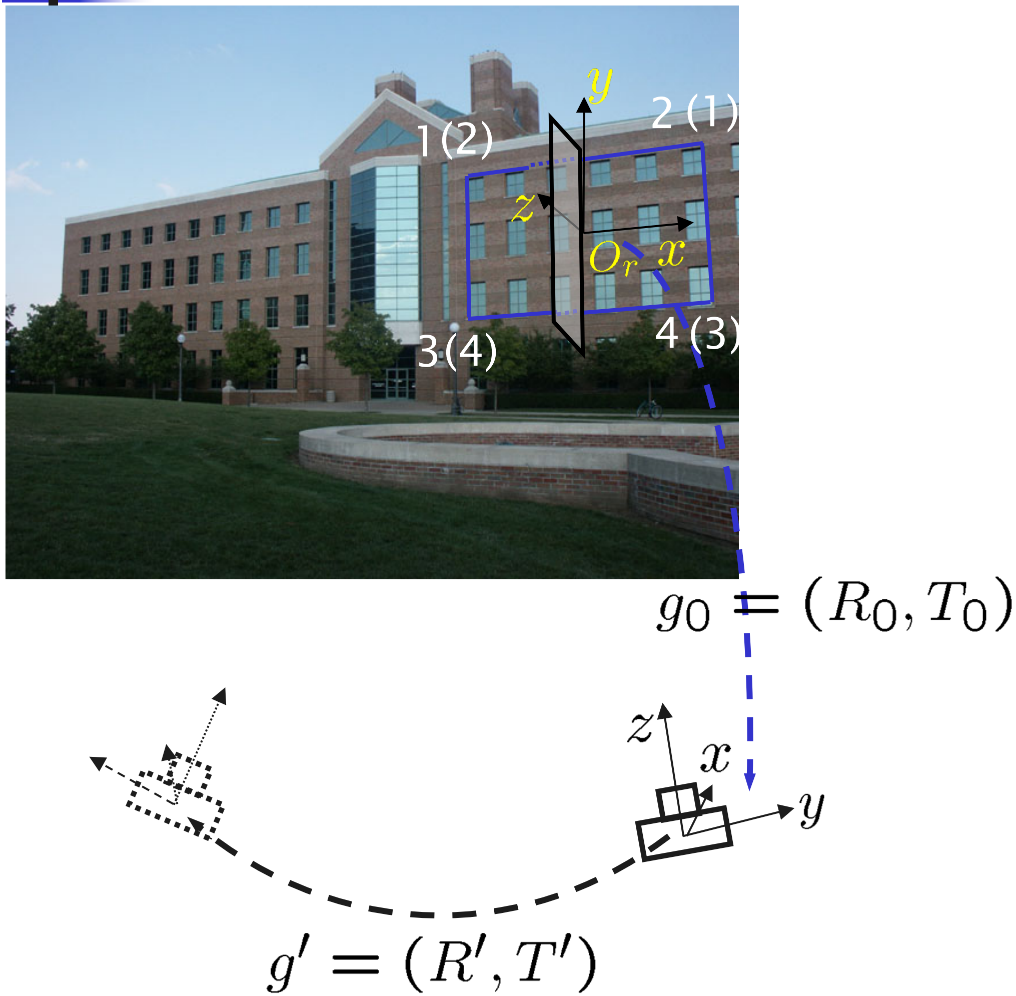
Ambiguity	dim(Ker(L))	g_0 (general scene)	g_0 (planar scene)
Reflective	5-dimensional	(1+2)-parameter*	(0+1)-parameter
Rotational	3-dimensional	(1+1)-parameter	(1+0)-parameter
Translational	9-dimensional	(1+3)-parameter	(0+2)-parameter

"(a+b)-parameter" means there are an a-parameter family of ambiguity in R_0 of g_0 and a b-parameter family of ambiguity in T_0 of g_0.





Symmetry-based reconstruction (reflection)



Reflectional symmetry

$$g = (R, 0)$$

$$R = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

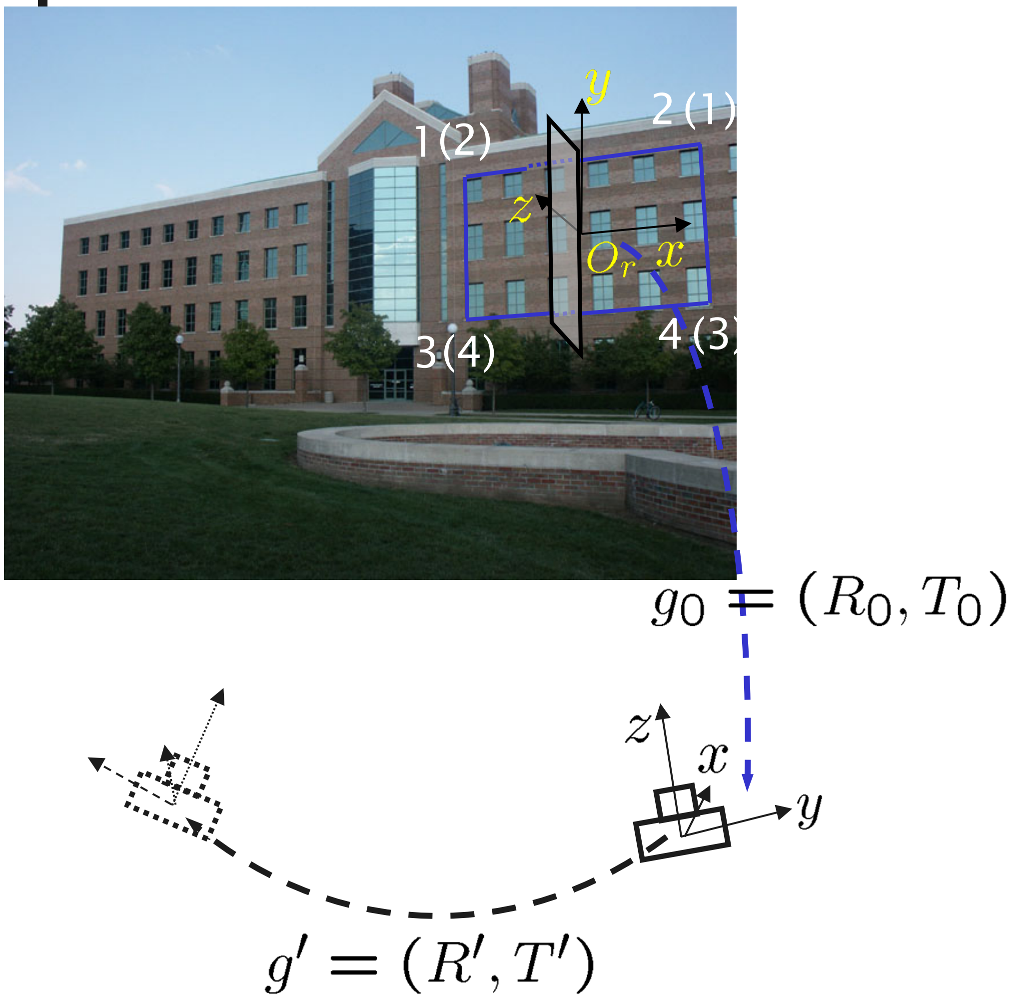
Virtual camera-camera

$$g' = (R', T')$$

$$R' = R_0 R R_0'$$

$$T' = (I - R')T_0$$

Symmetry-based reconstruction



Epipolar constraint

$$g(\mathbf{x})^T E \mathbf{x} = 0$$

$$E = \widehat{T'} R'$$

Homography

$$\widehat{g(\mathbf{x})} H \mathbf{x} = 0$$

$$H = R' + \frac{1}{d} T' N^T$$

Symmetry-based reconstruction (algorithm)

2 pairs of symmetric image points

↓

Recover essential matrix or homography

$$E = \widehat{T'}R' \qquad H = R' + \frac{1}{d}T'N^T$$

↓

Decompose E or H to obtain $\{R', T', N\}$

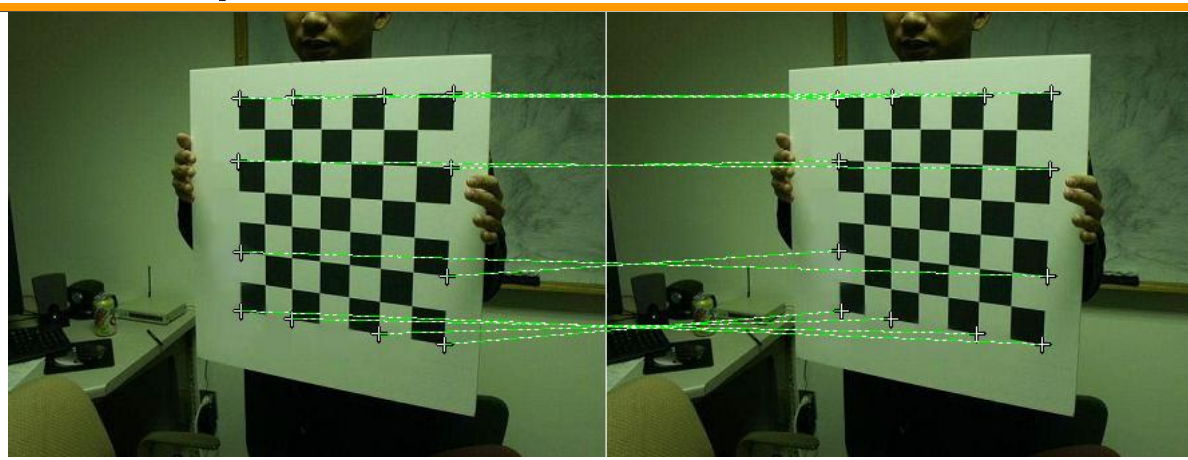
↓

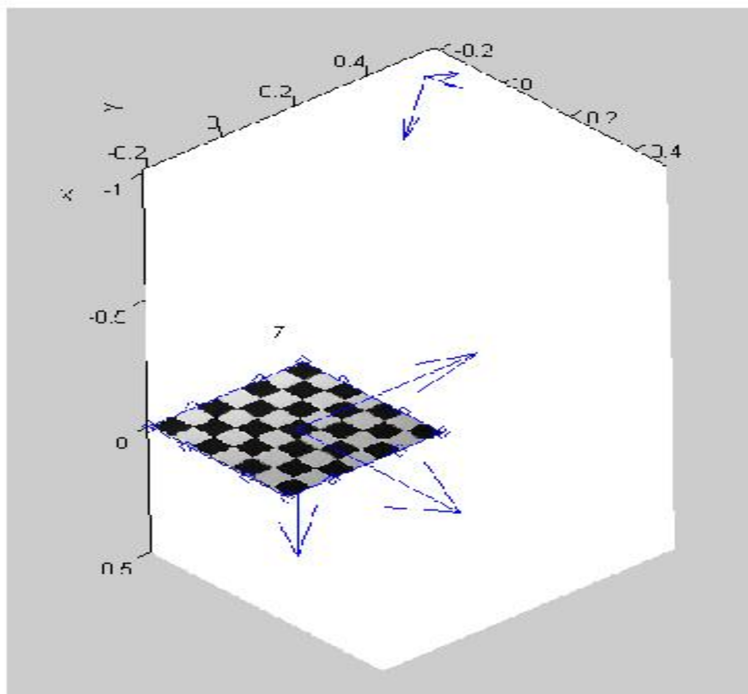
Solve Lyapunov equation

$$R'R_0 - R_0R = 0$$

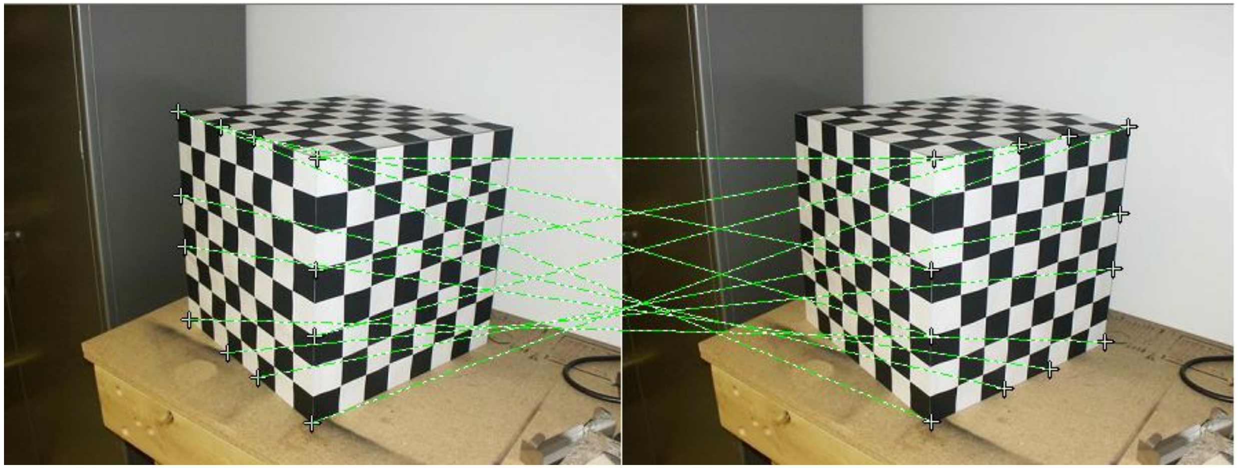
to obtain R_0 and then T_0.

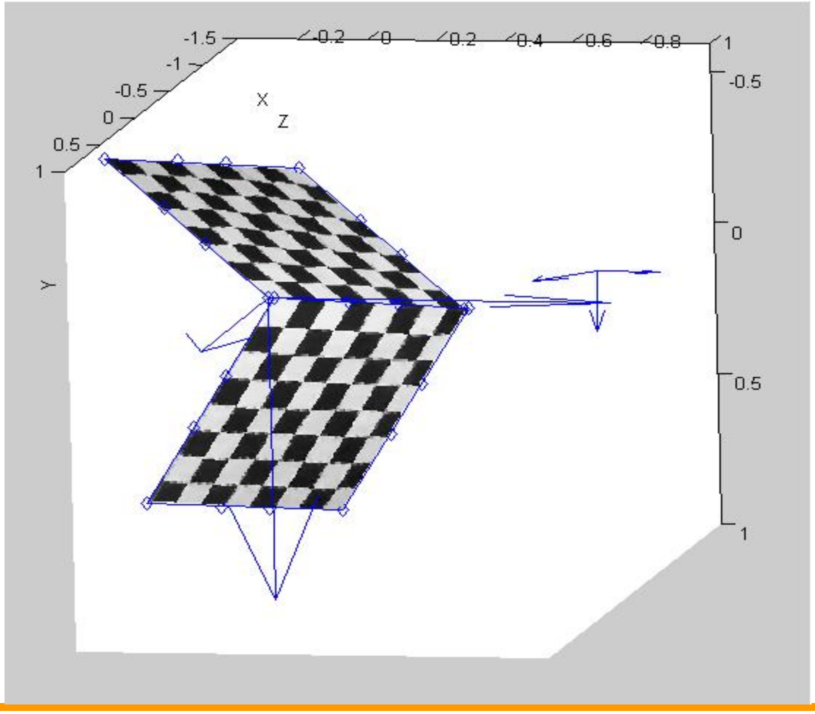
Symmetry-based reconstruction (reflection)



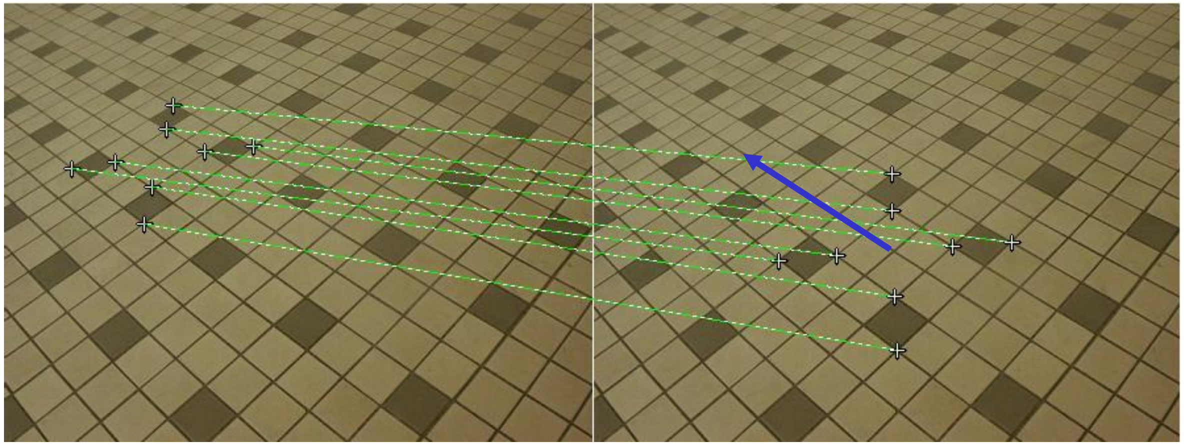


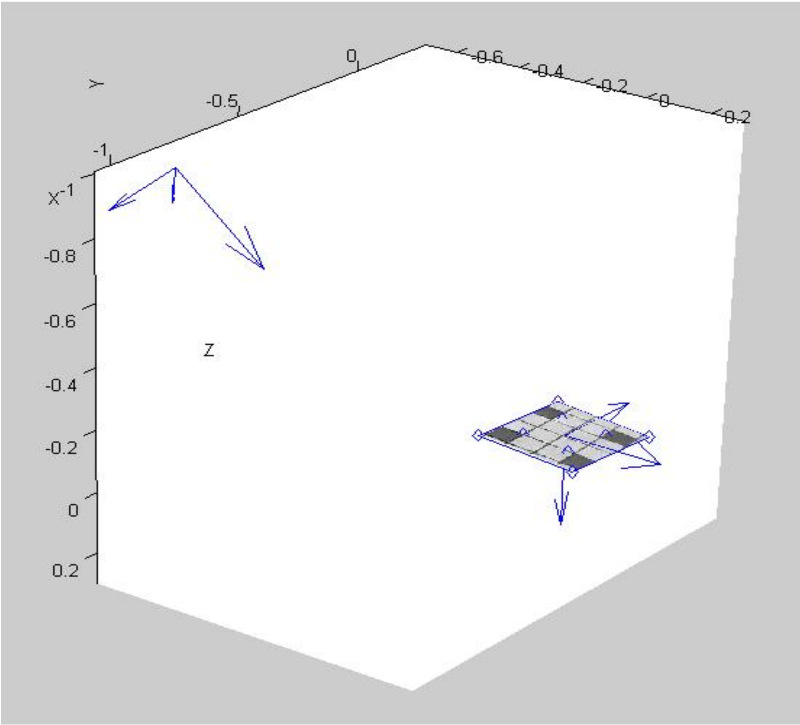
Symmetry-based reconstruction (rotation)



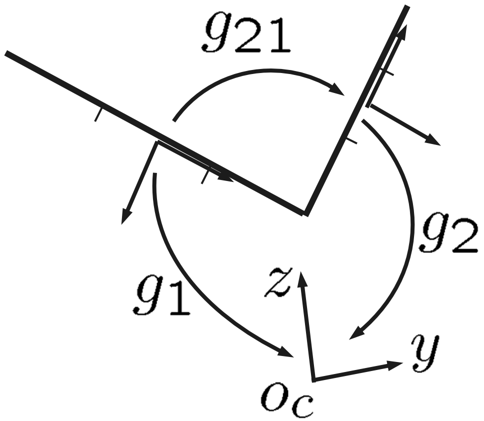


Symmetry-based reconstruction (translation)

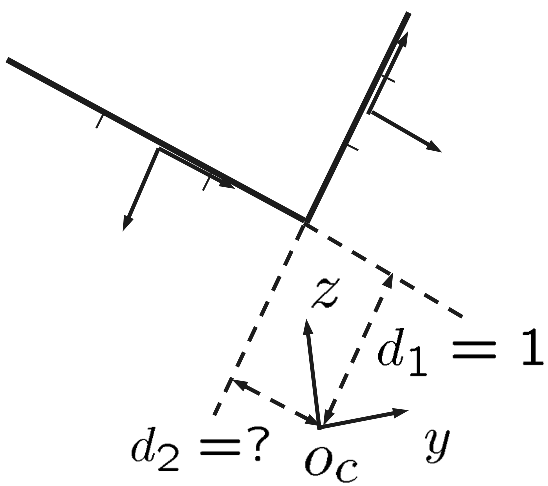


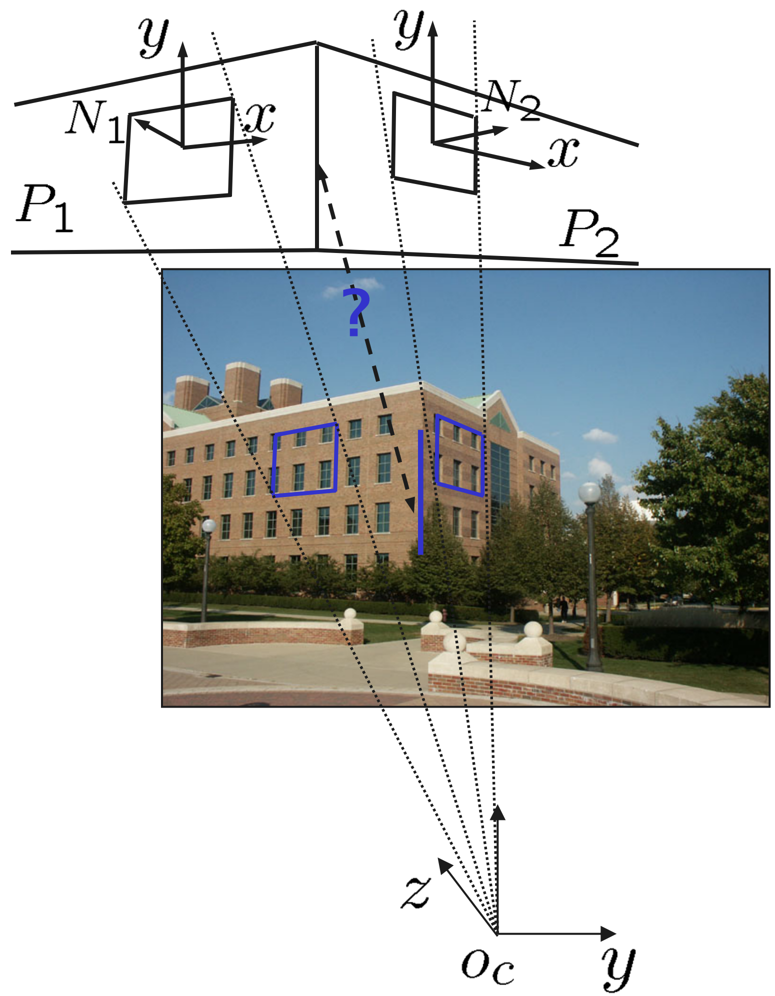


ALIGNMENT OF MULTIPLE SYMMETRIC OBJECTS



$$g_{21} = g_2 g_1^{-1}$$





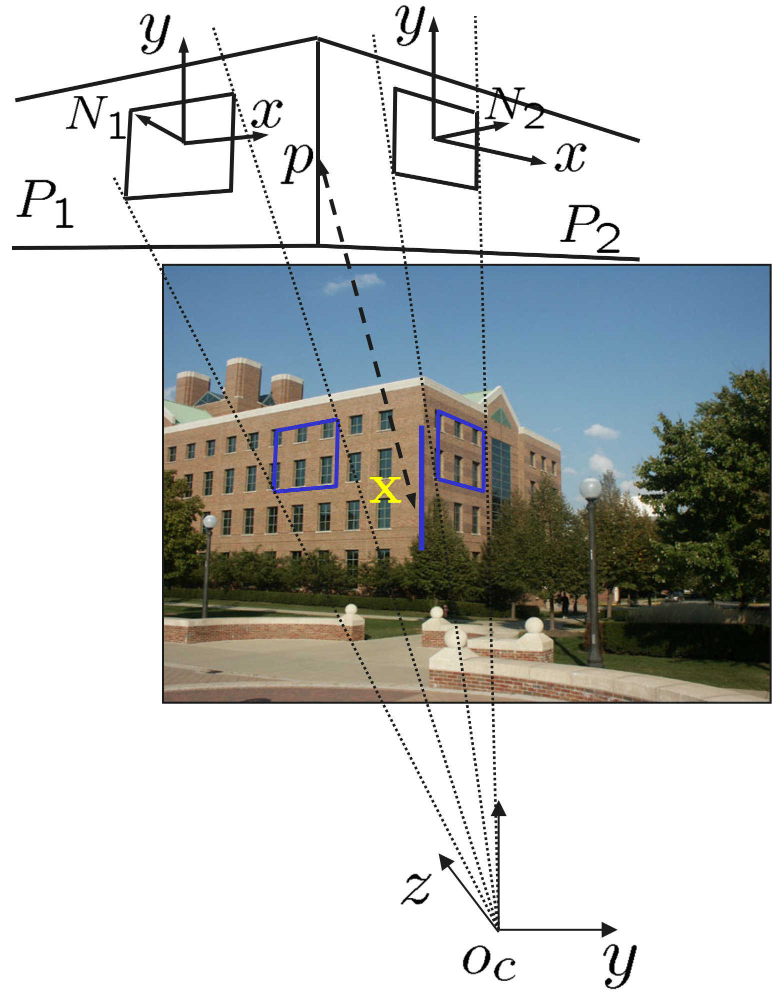
Correct scales within a single image

Pick the image $\mathbf{x}$ of a point p on the intersection line

$$\lambda_1 = \frac{d_1}{N_1^T \mathbf{x}} = \lambda_2 = \frac{d_2}{N_2^T \mathbf{x}}$$

$$\alpha = \frac{d_2}{d_1} = \frac{N_1^T \mathbf{x}}{N_2^T \mathbf{x}}$$

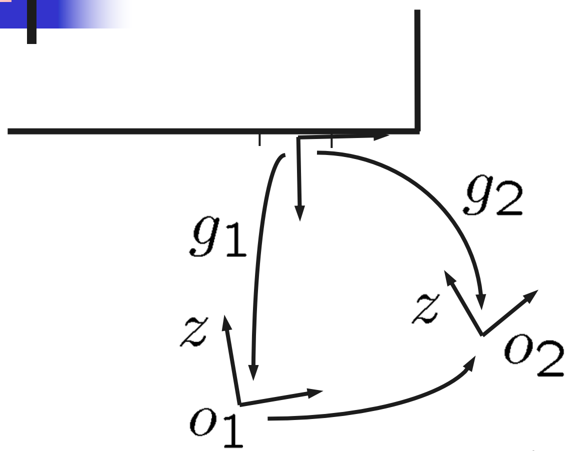
$$g_2 \leftarrow (R_2, \alpha T_2)$$
$$g_{21} = g_2 g_1^{-1}$$



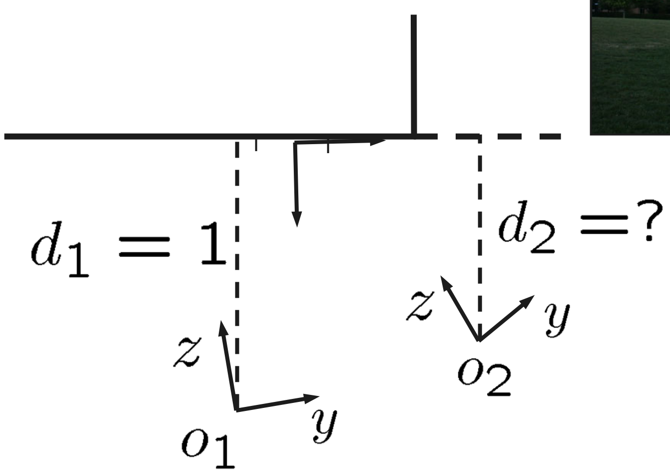
Correct scale within a single image

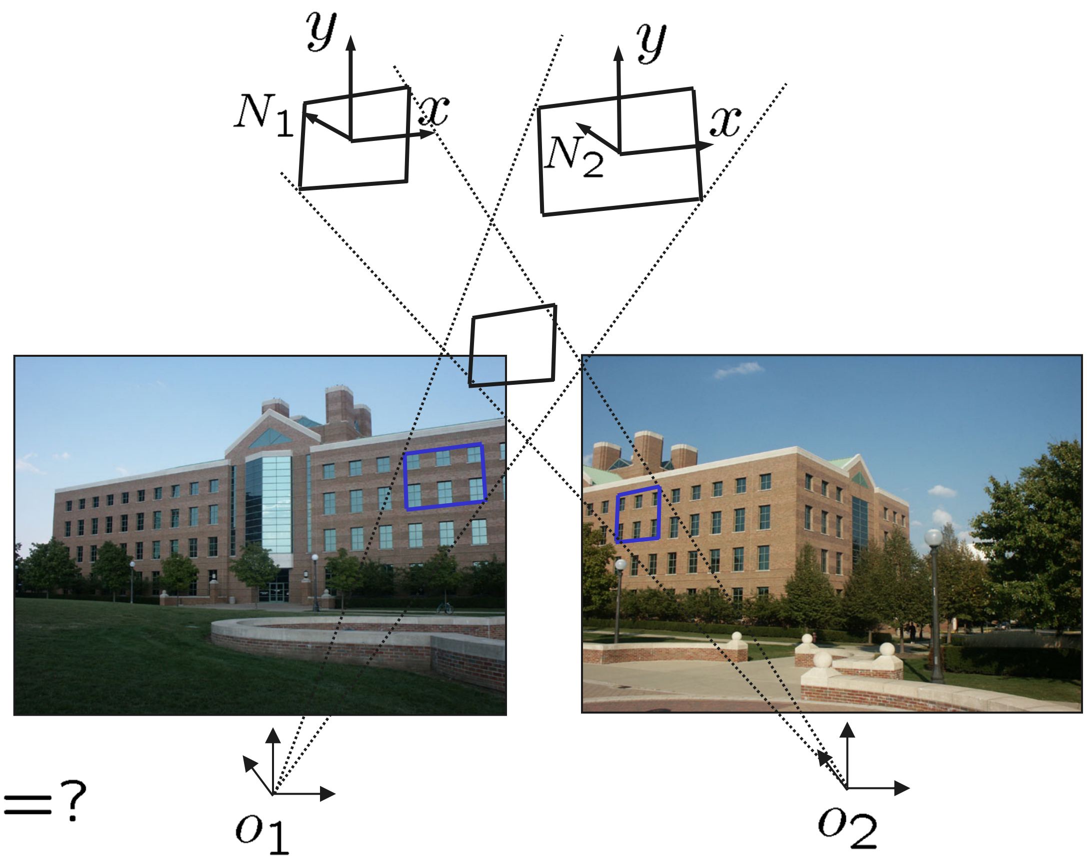
$$\alpha = 0.7322 \quad \theta(N_1, N_2) = 90.36^o$$

Correct scales across multiple images



$$g_{21} = g_2 g_1^{-1}$$





Correct scales across multiple images

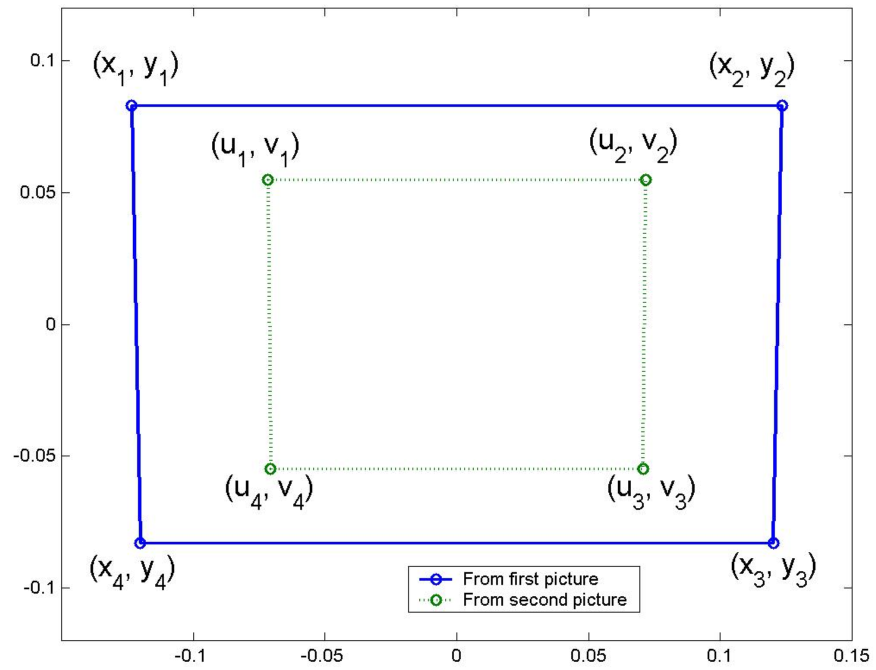
$$\begin{bmatrix} x_i - \bar{x} \\ y_i - \bar{y} \end{bmatrix} = \alpha \begin{bmatrix} u_i - \bar{u} \\ v_i - \bar{v} \end{bmatrix}$$

$$i = 1, 2, 3, 4$$

$$d_2 = \alpha$$

$$g_2 \leftarrow (R_2, \alpha T_2)$$

$$g_{21} = g_2 g_1^{-1}$$



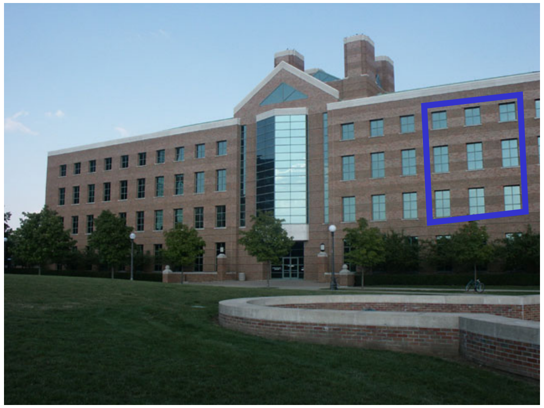
Correct scales across multiple images

$$\alpha = 0.7433$$

Image alignment after scales corrected

ALGORITHM: Building 3-D geometric models

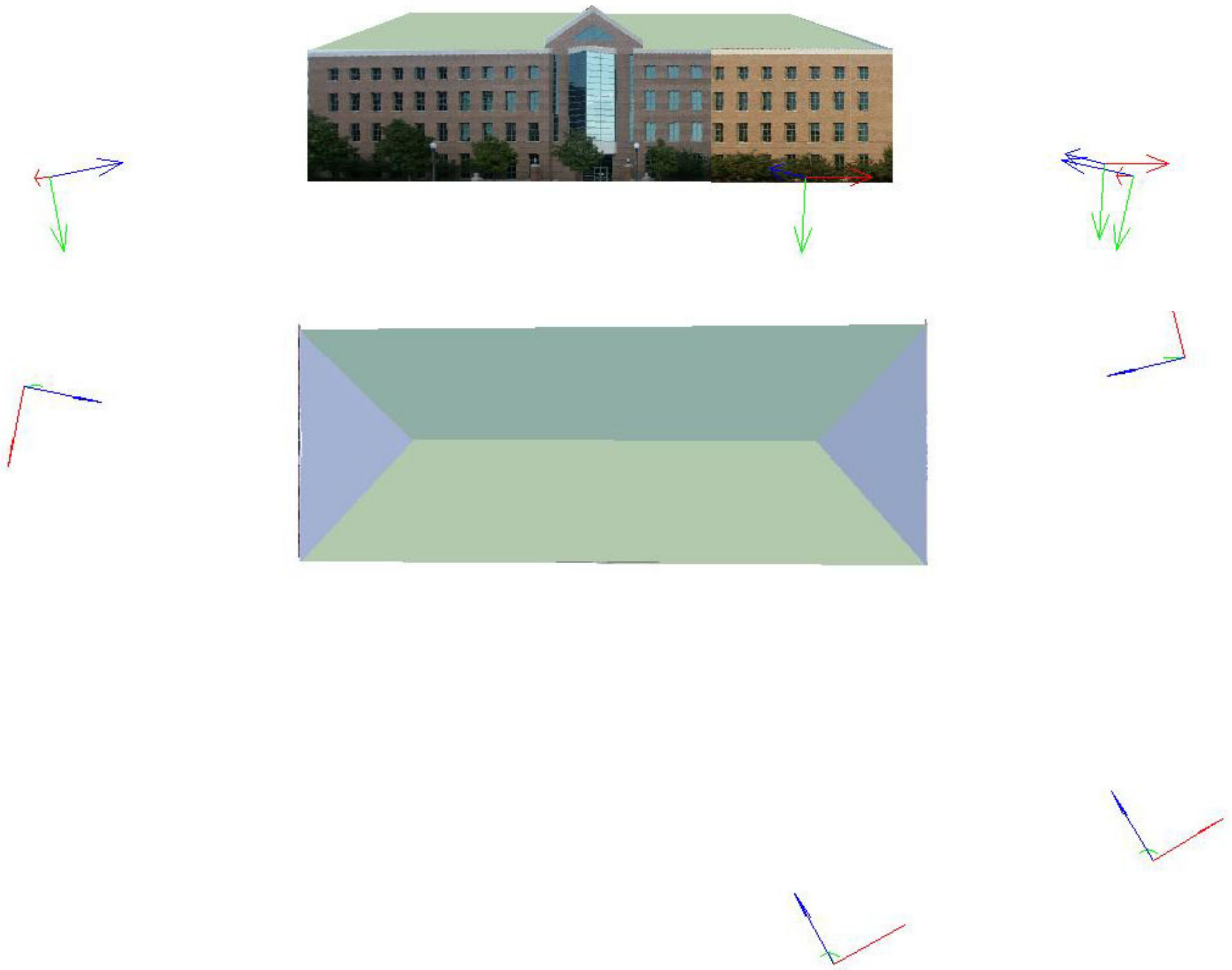
1. Specify symmetric objects and correspondence





Building 3-D geometric models

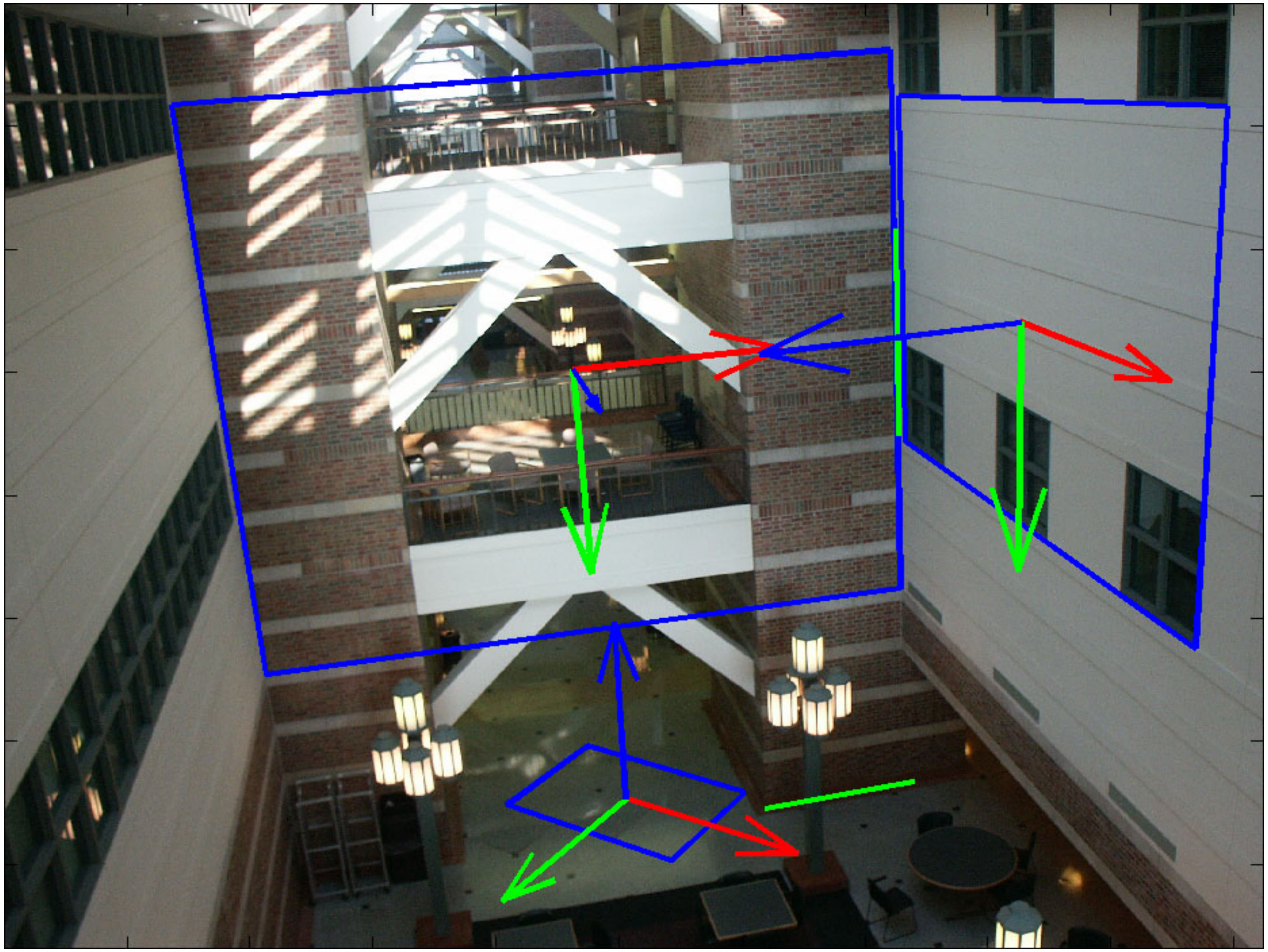
2. Recover camera poses and scene structure



3. Obtain 3-D model and rendering with images

APPLICATIONS – Photo Editing (Beckman Institute, UIUC)

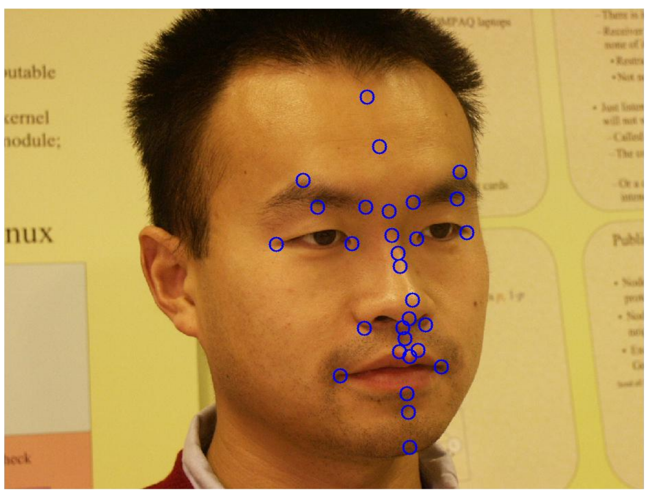
APPLICATIONS – Photo Editing (Beckman Institute, UIUC)

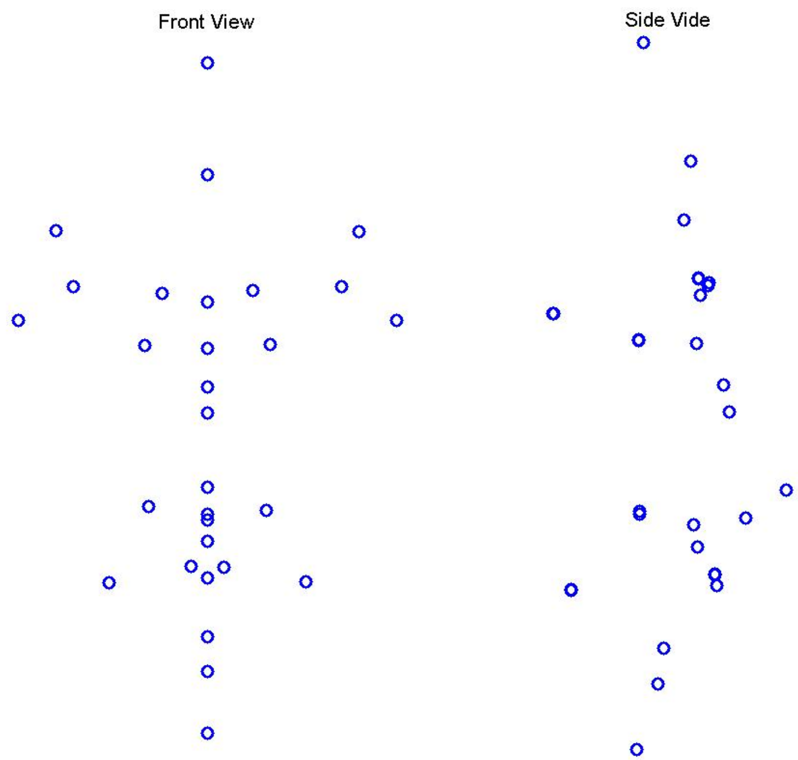


APPLICATIONS – Photo Editing (Beckman Institute, UIUC)

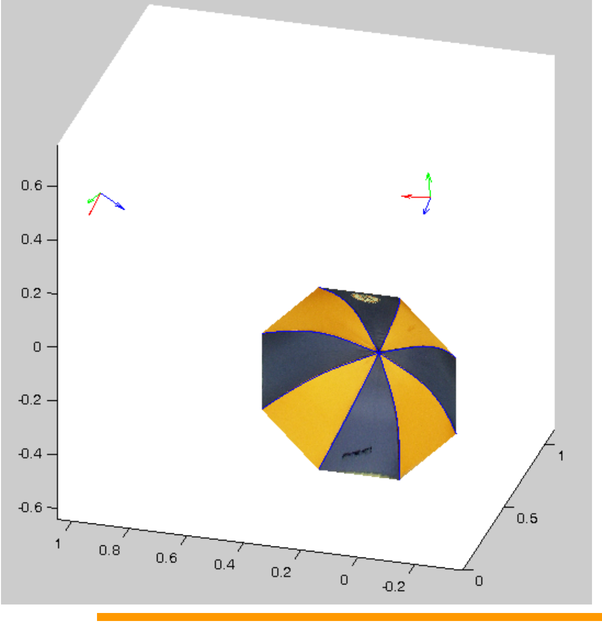
APPLICATIONS – Human Face Features from a Single View

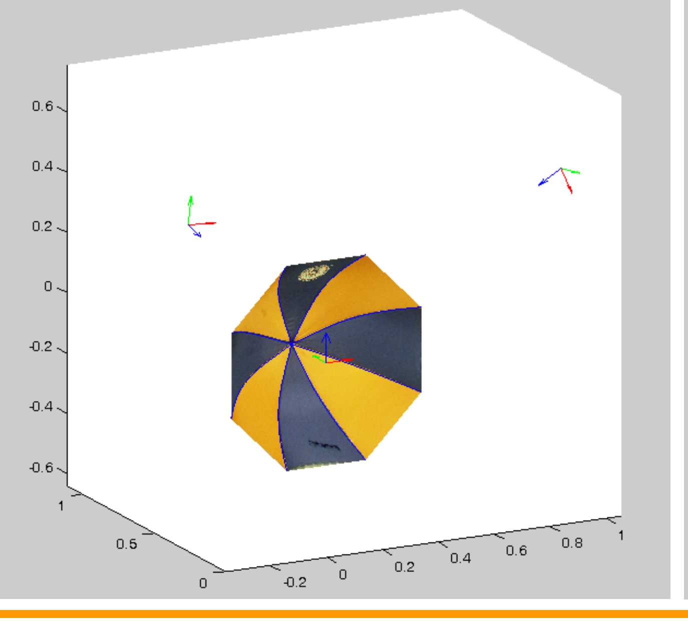
INS canonical views (3/4 view)

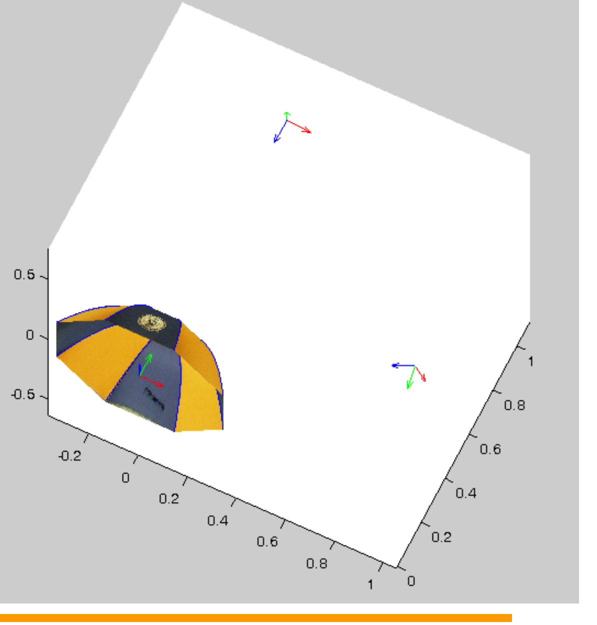




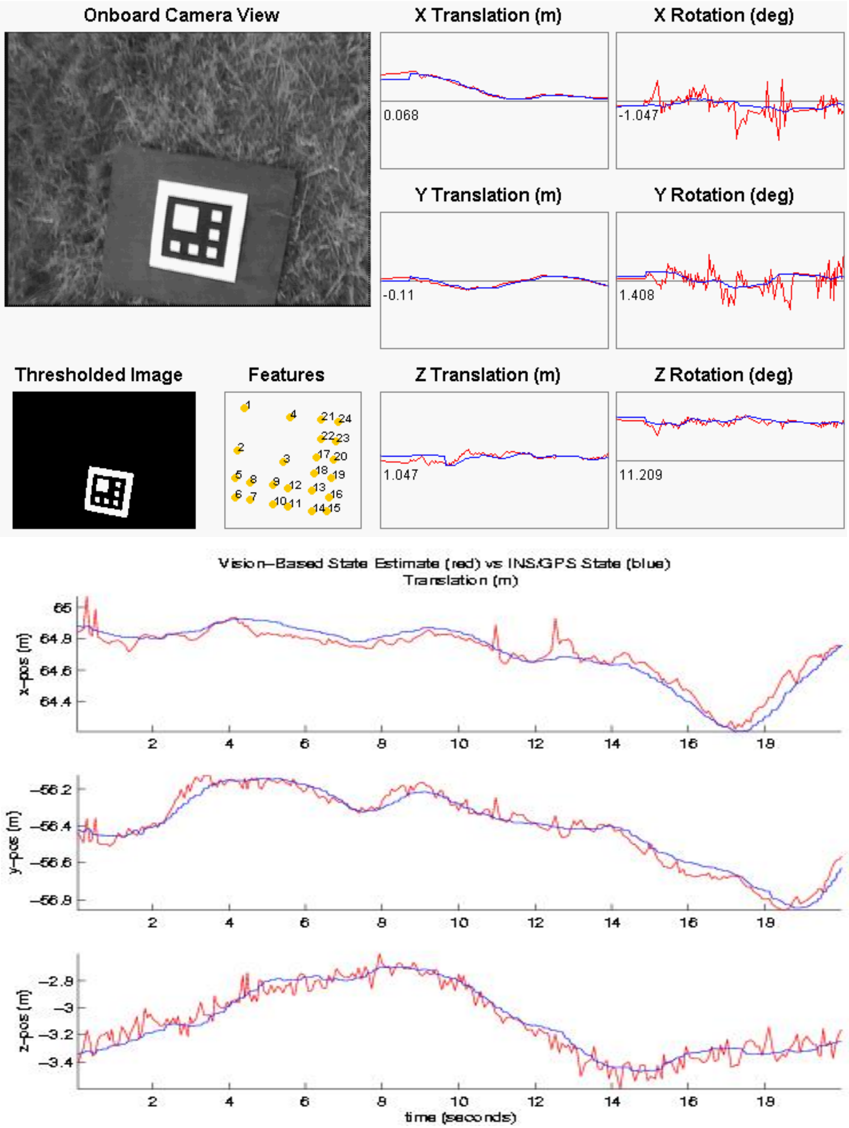
APPLICATIONS – Symmetric Curves and Surfaces







APPLICATIONS – Unmanned Aerial Vehicles (UAVs)



Rate: 10Hz
Accuracy: 5cm, 4º

Berkeley Aerial Robot (BEAR) Project

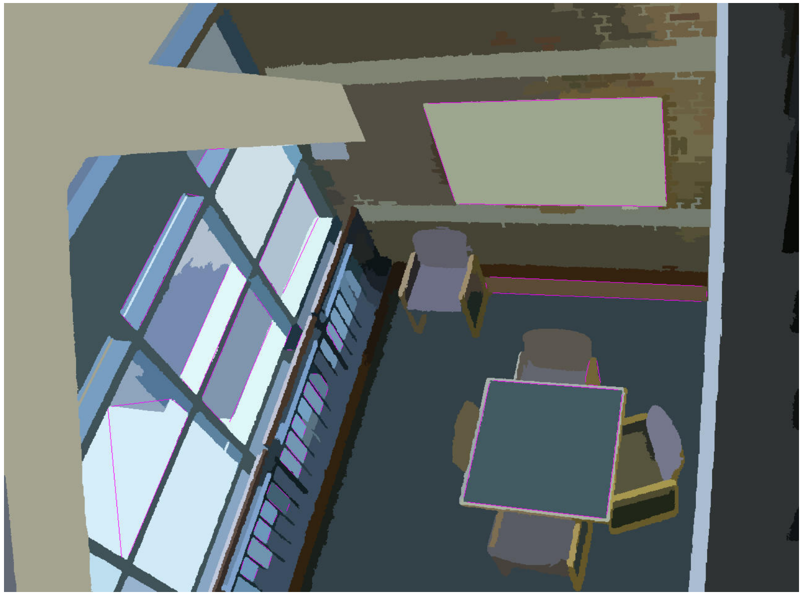
ALGORITHM: Symmetry detection and matching

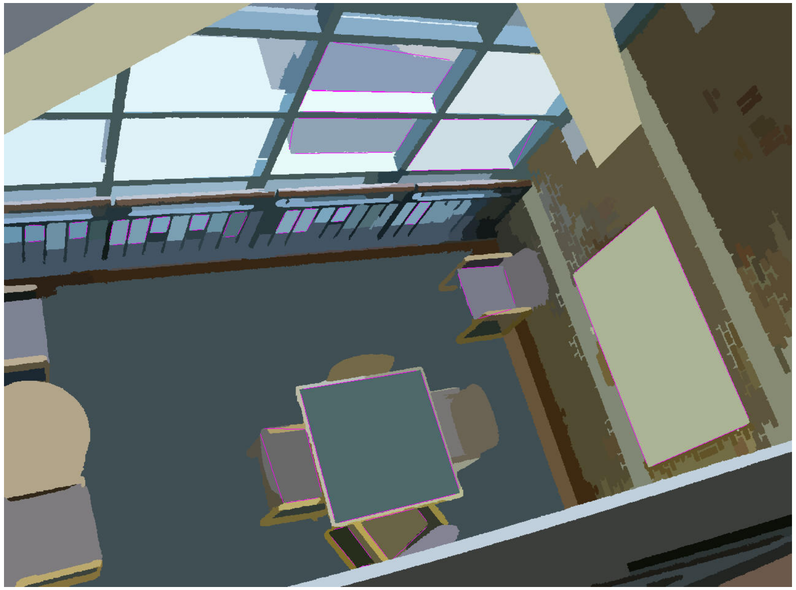
Extract, detect, match symmetric objects across images, and recover the camera poses.

Segmentation & polygon fitting

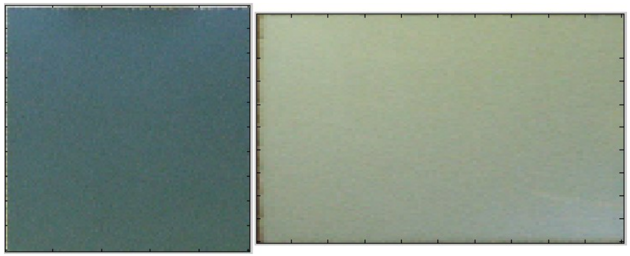
1. Color-based segmentation (mean shift)

2. Polygon fitting

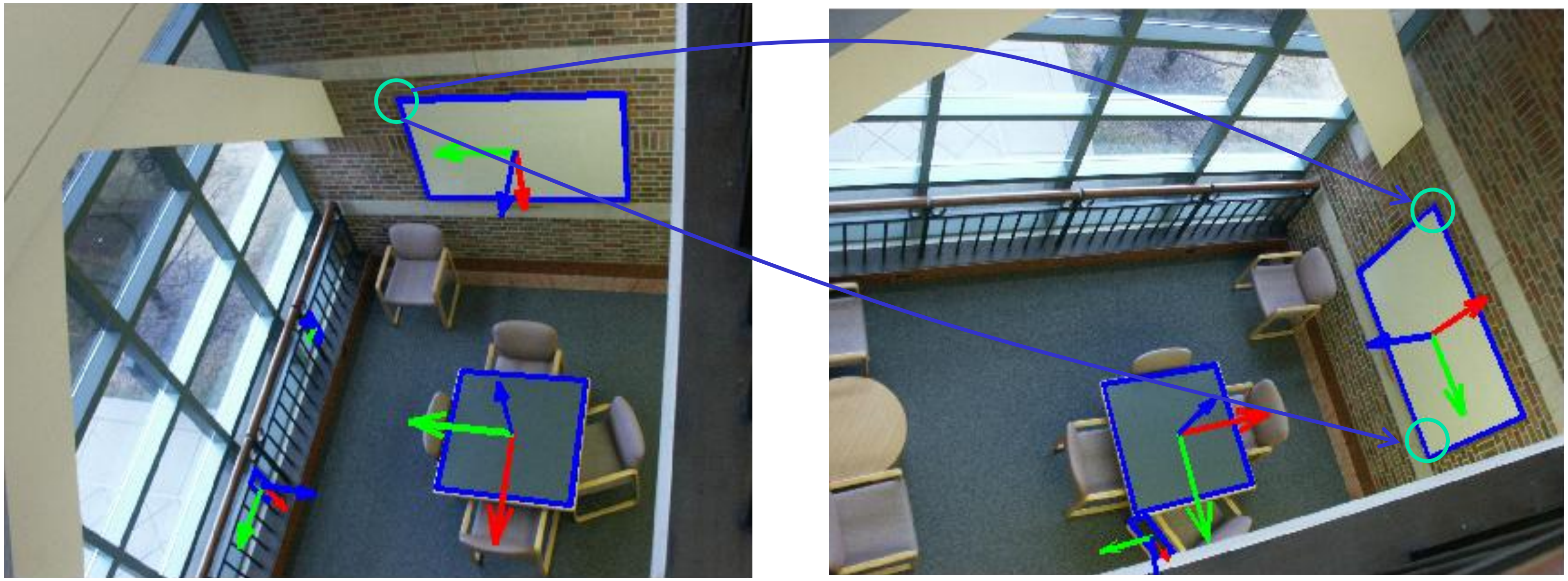




3. Symmetry verification (rectangles,…)

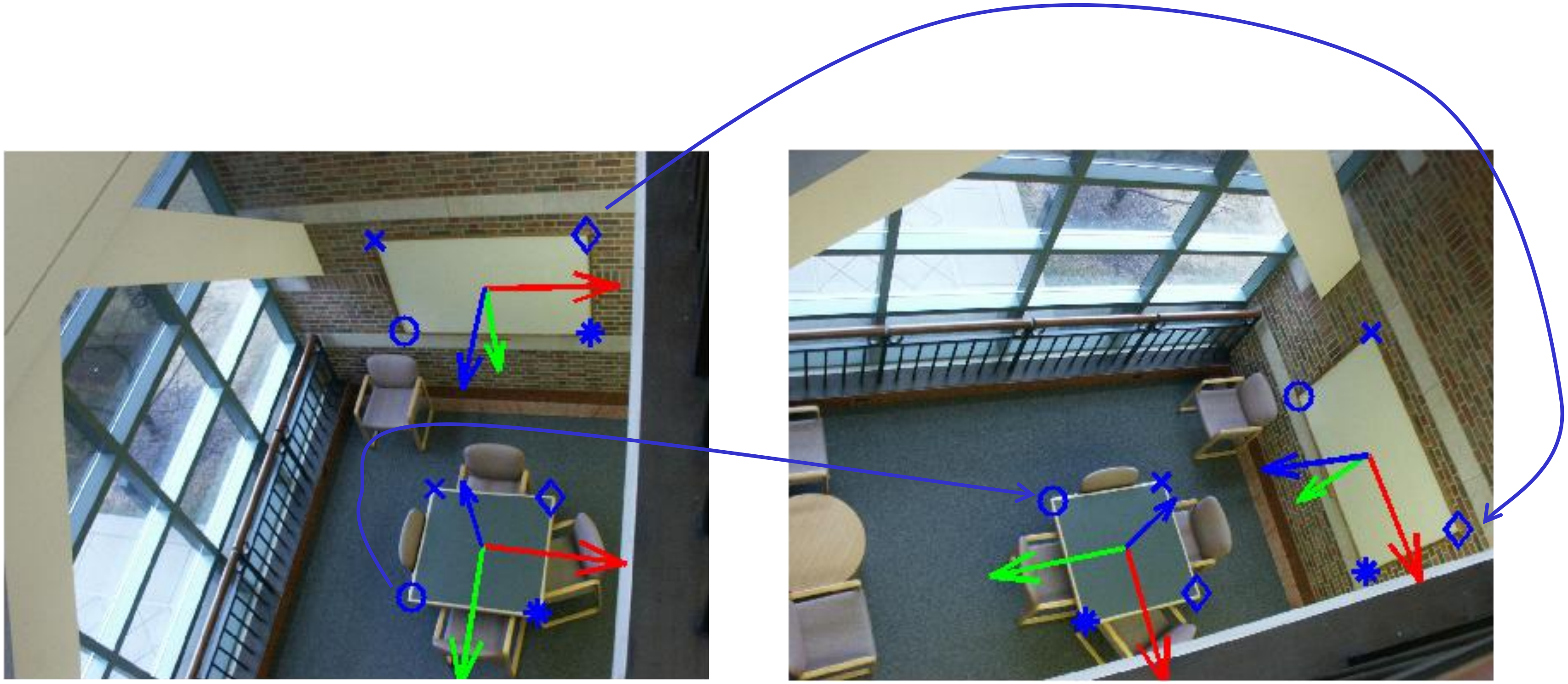


4. Single-view recovery

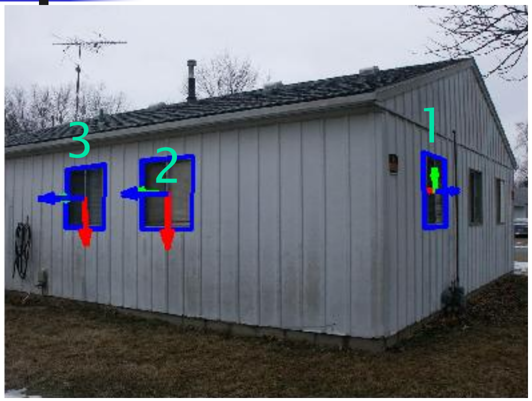


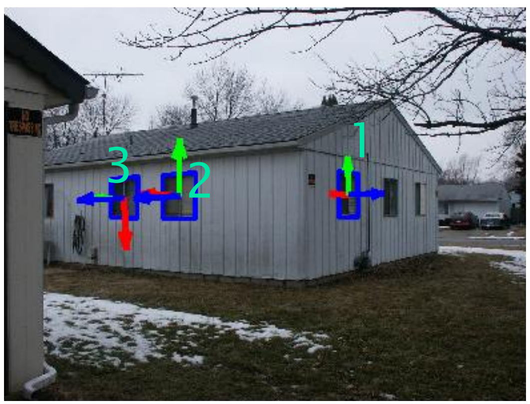
Symmetry-based matching

5. Find the only one set of camera poses that are consistent with all symmetry objects



MATCHING OF SYMMETRY CELLS – **Graph Representation**

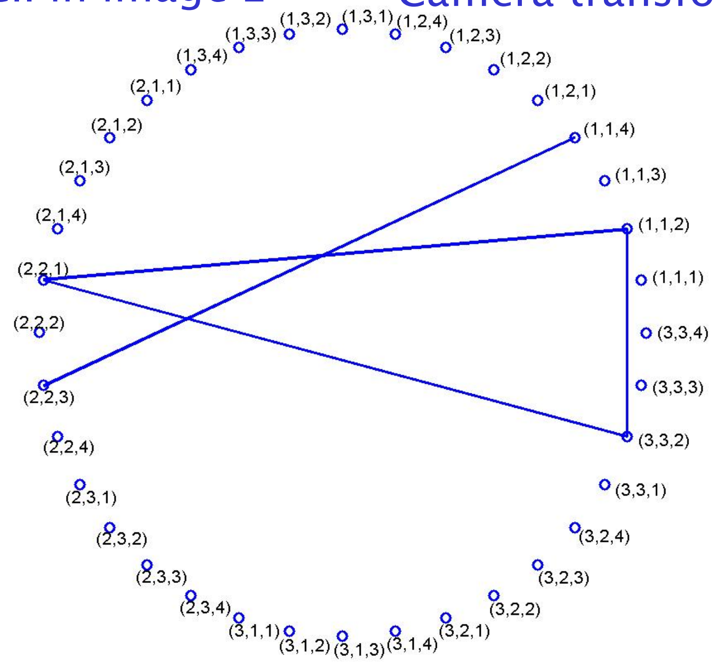




Cell in image 1   # of possible matching

(1,2,1, g)

Cell in image 2   Camera transformation



36 possible matchings

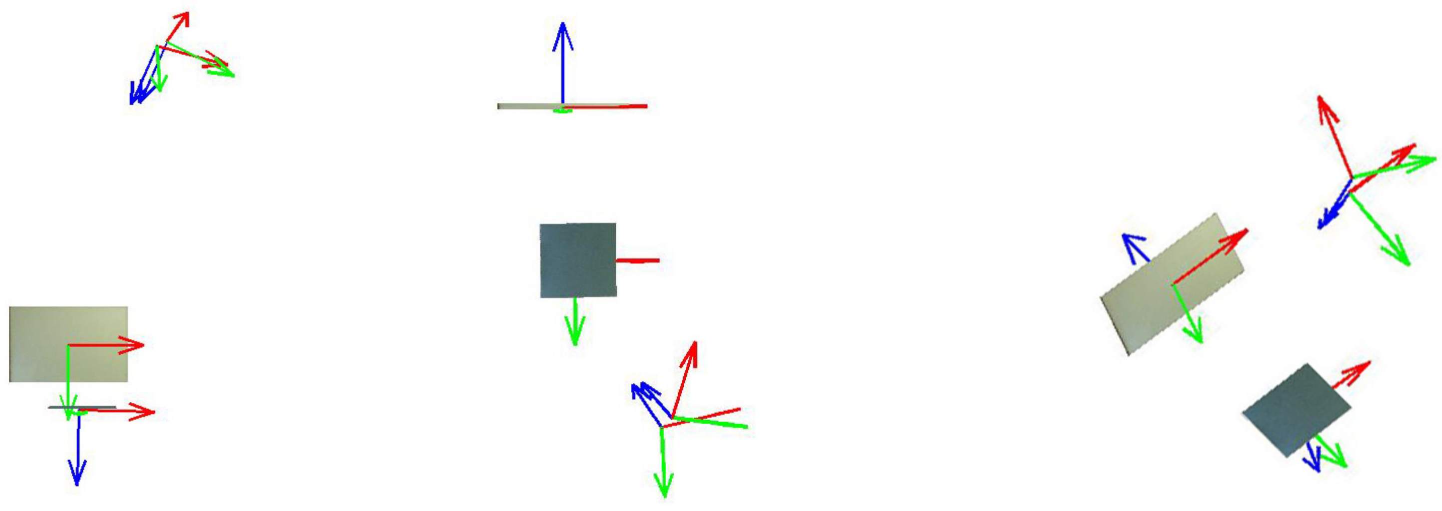
The problem of finding the largest set of matching cells is equivalent to the problem of finding the *maximal complete subgraphs (cliques)* in the matching graph.

Camera poses and 3-D recovery

Side view

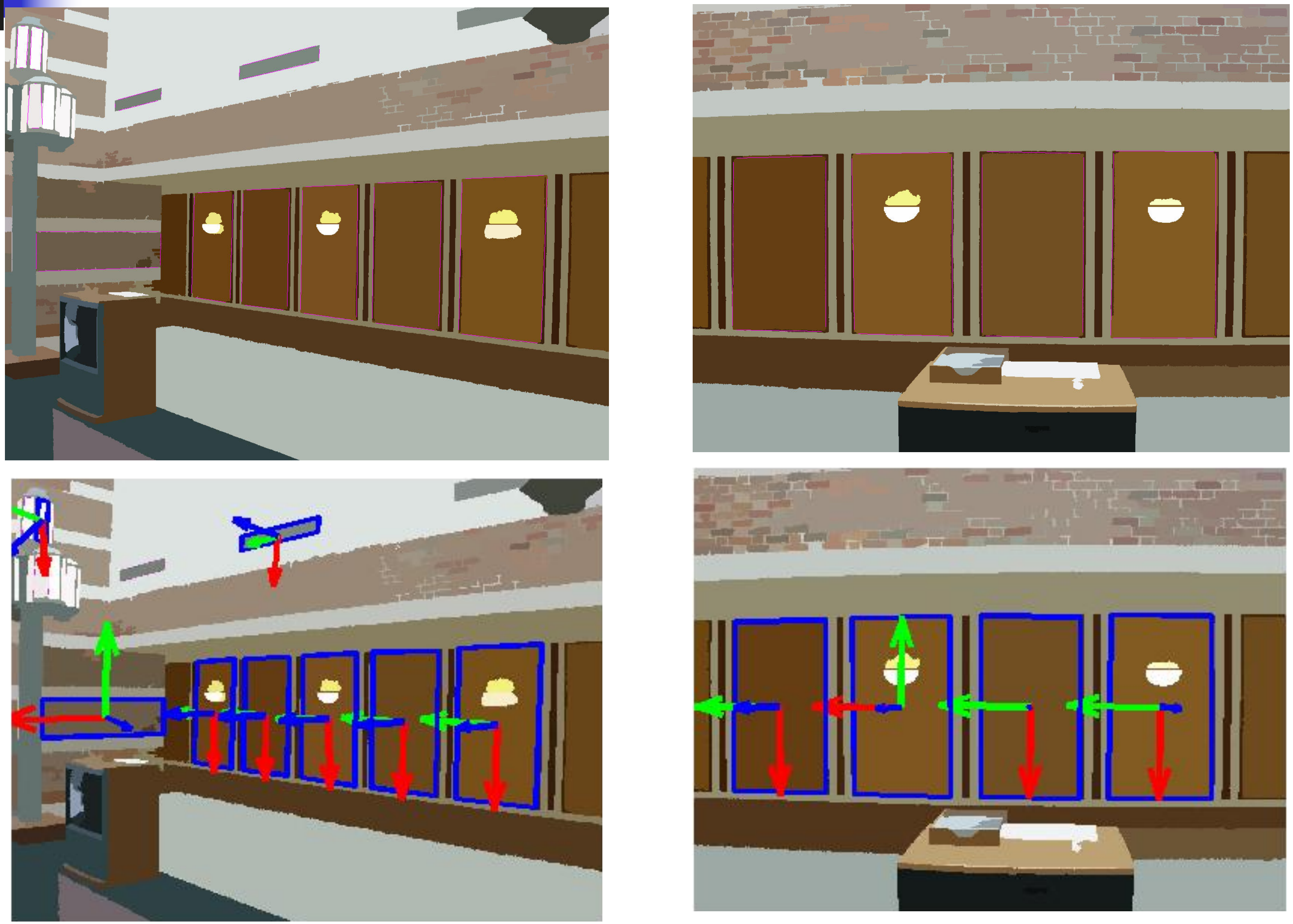
Top view

Generic view

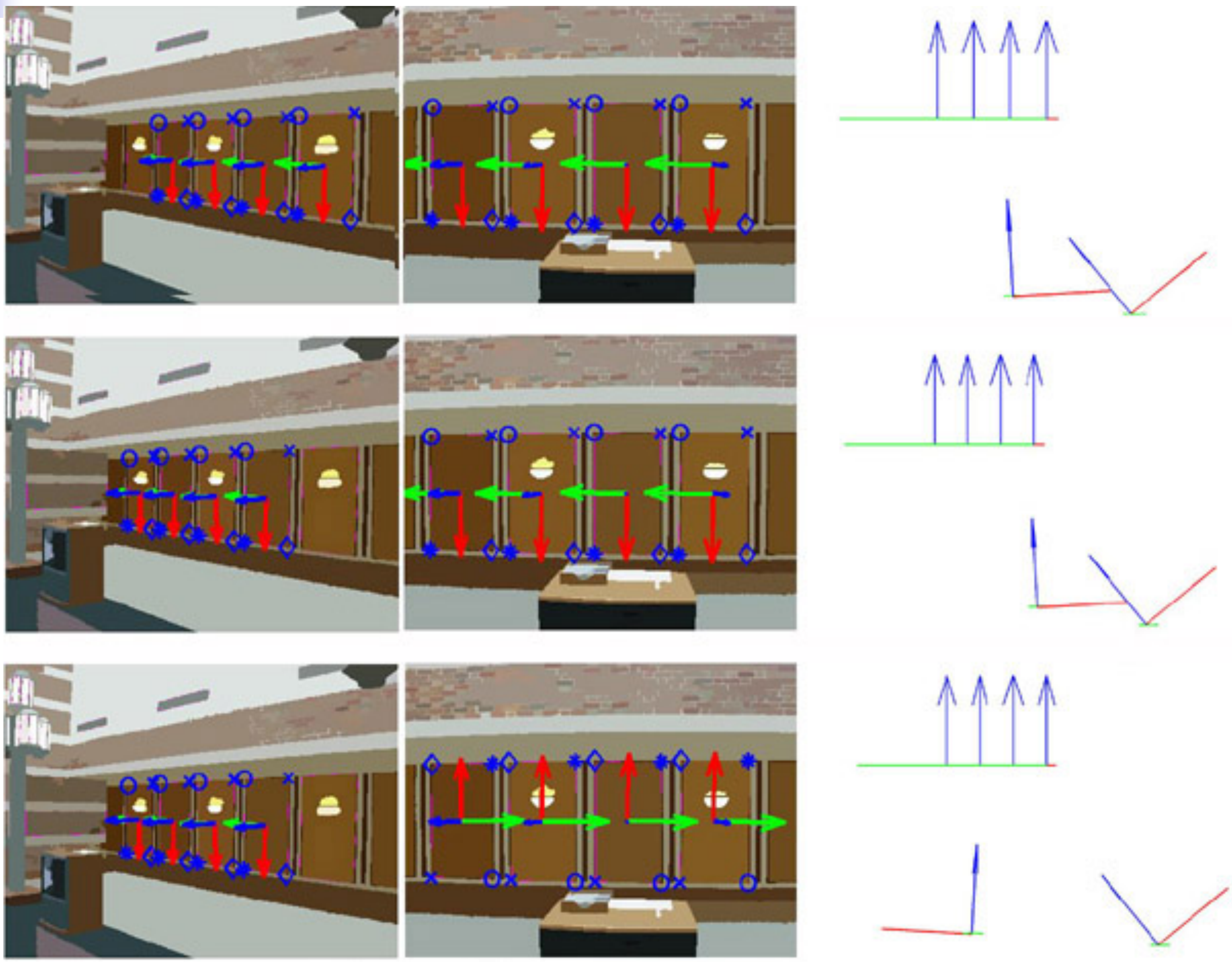


Length ratio	Reconstruction	Ground truth
Whiteboard	1.506	1.51
Table	1.003	1.00

Multiple-view matching and recovery (Ambiguities)



Multiple-view matching and recovery (Ambiguities)



ALGORITHM: Calibration from symmetry

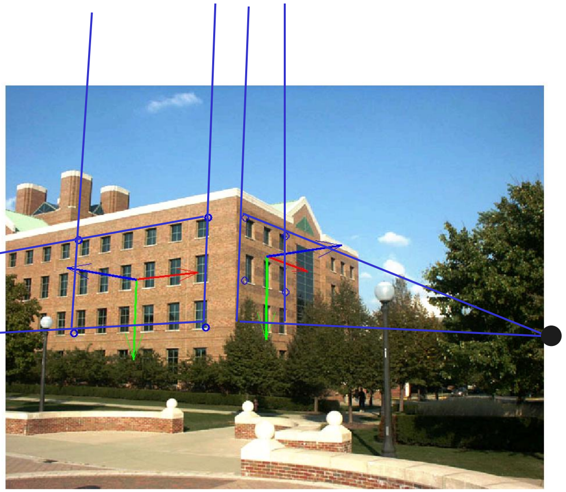
Calibrated homography $H = R' + \frac{1}{d} T' N^T$

Uncalibrated homography

$$\tilde{H} = K(R' + \frac{1}{d} T' N^T) K^{-1}$$

$$\tilde{H}(KT') = KT'$$

(vanishing point)

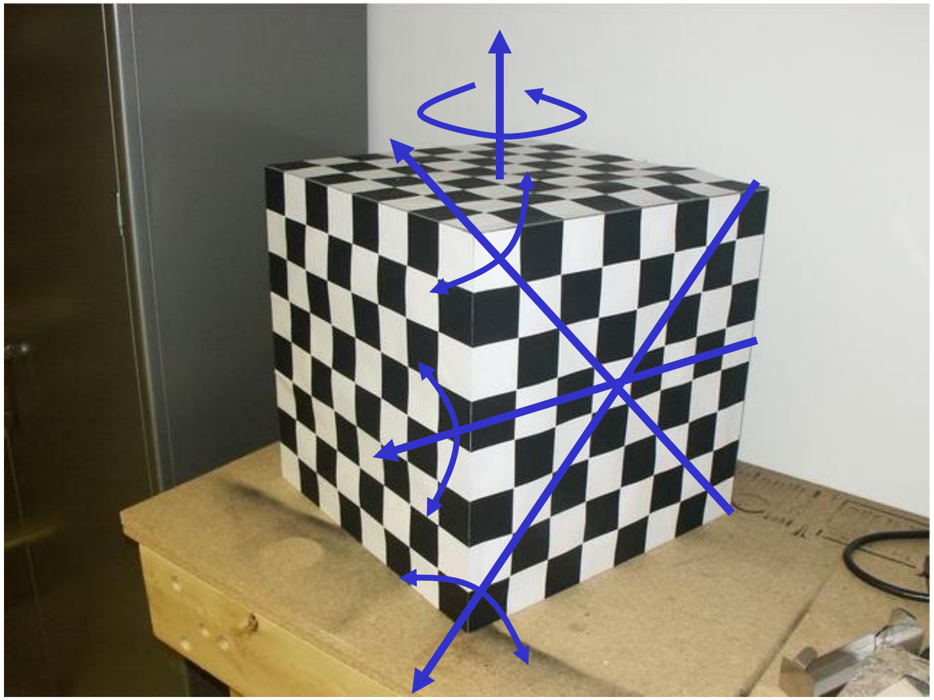


$$S = K^{-1} K^{-T}$$

$$(KT'_{1x})^T S (KT'_{1y}) = 0,$$
$$(KT'_{2x})^T S (KT'_{2y}) = 0,$$
$$(KT'_{1y})^T S (KT'_{2y}) = 0.$$

ALGORITHM: Calibration from symmetry

Calibration with a rig is also simplified: we only need to know that there are sufficient symmetries, **not** necessarily the 3-D coordinates of points on the rig.



SUMMARY: Multiple-View Geometry + Symmetry

Multiple (perspective) images	=	multiple-view rank condition
Single image + symmetry	=	"multiple-view" rank condition
Multiple images + symmetry	=	rank condition + scale correction
Matching + symmetry	=	rank condition + scale correction + clique identification

SUMMARY

Multiple-view 3-D reconstruction in presence of symmetry

- Symmetry based algorithms are accurate, robust, and simple.
- Methods are baseline independent and object centered.
- Alignment and matching can and should take place in 3-D space.
- Camera self-calibration and calibration are simplified and linear.

Related applications

- Using symmetry to overcome occlusion.
- Reconstruction and rendering with non-symmetric area.
- Large scale 3-D map building of man-made environments.