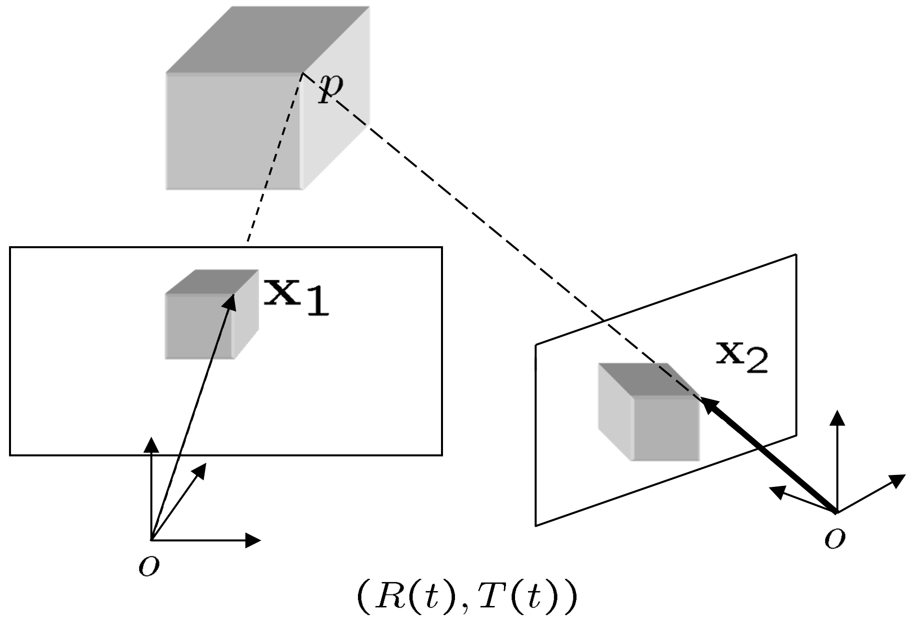
Lecture 3

Image Primitives and Correspondence

Image Primitives and Correspondence

Given an image point in left image, what is the (corresponding) point in the right image, which is the projection of the same 3-D point

Matching - Correspondence



Lambertian assumption

$$I_1(\mathbf{x}_1) = \mathcal{R}(p) = I_2(\mathbf{x}_2)$$

Rigid body motion

$$\mathbf{x}_2 = h(\mathbf{x}_1) = \frac{1}{\lambda_2(\mathbf{X})}(R\lambda_1(\mathbf{X})\mathbf{x}_1 + T)$$

Correspondence

$$I_1(\mathbf{x}_1) = I_2(h(\mathbf{x}_1))$$

Local Deformation Models

- Translational model

$$h(\mathbf{x}) = \mathbf{x} + d \qquad\qquad I_1(\mathbf{x}_1) = I_2(h(\mathbf{x}_1))$$

- Affine model

$$h(\mathbf{x}) = A\mathbf{x} + d \qquad\qquad I_1(\mathbf{x}_1) = I_2(h(\mathbf{x}_1))$$

- Transformation of the intensity values and occlusions

$$I_1(\mathbf{x}_1) = f_o(\mathbf{X}, g) I_2(h(\mathbf{x}_1) + n(h(\mathbf{x}_1))$$

Feature Tracking and Optical Flow

- Translational model

$$I_1(\mathbf{x}_1) = I_2(\mathbf{x}_1 + \Delta \mathbf{x})$$

- Small baseline

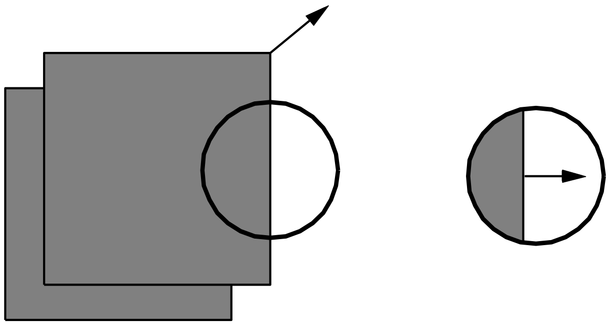
$$I(\mathbf{x}(t), t) = I(\mathbf{x}(t) + \mathbf{u}dt, t + dt)$$

- RHS approx. by first two terms of Taylor series

$$\boxed{\nabla I(\mathbf{x}(t), t)^T \mathbf{u} + I_t(\mathbf{x}(t), t) = 0}$$

- Brightness constancy constraint

Aperture Problem



- Normal flow

$$\mathbf{u}_n \doteq \frac{\nabla I^T \mathbf{u}}{\|\nabla I\|} \cdot \frac{\nabla I}{\|\nabla I\|} = -\frac{I_t}{\|\nabla I\|} \cdot \frac{\nabla I}{\|\nabla I\|}$$

Optical Flow

- Integrate around over image patch

$$E_b(\mathbf{u}) = \sum_{W(x,y)} [\nabla I^T(x,y,t)\mathbf{u}(x,y) + I_t(x,y,t)]^2$$

- Solve

$$\begin{aligned} \nabla E_b(\mathbf{u}) &= 2 \sum_{W(x,y)} \nabla I(\nabla I^T \mathbf{u} + I_t) \\ &= 2 \sum_{W(x,y)} \left(\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \mathbf{u} + \begin{bmatrix} I_x I_t \\ I_y I_t \end{bmatrix} \right) \end{aligned}$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \mathbf{u} + \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} = 0$$

$$G\mathbf{u} + \mathbf{b} = 0$$

Optical Flow, Feature Tracking

$$\boxed{\mathbf{u} = -G^{-1}\mathbf{b}}$$

$$G = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

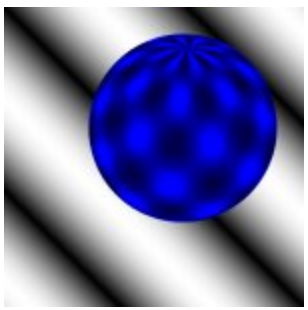
Conceptually:

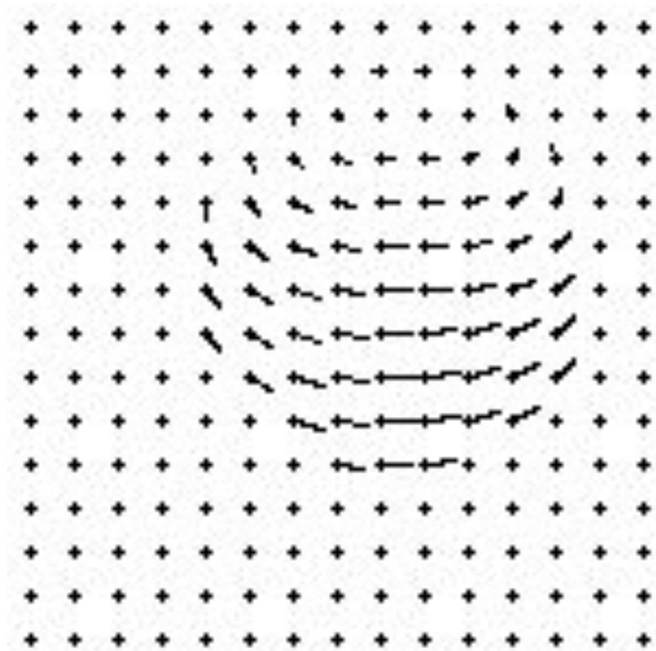
rank(G) = 0 blank wall problem
rank(G) = 1 aperture problem
rank(G) = 2 enough texture – good feature candidates

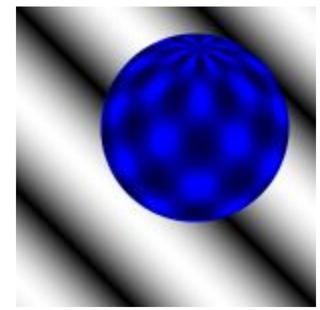
In reality: choice of threshold is involved

Optical Flow

- Previous method - assumption locally constant flow



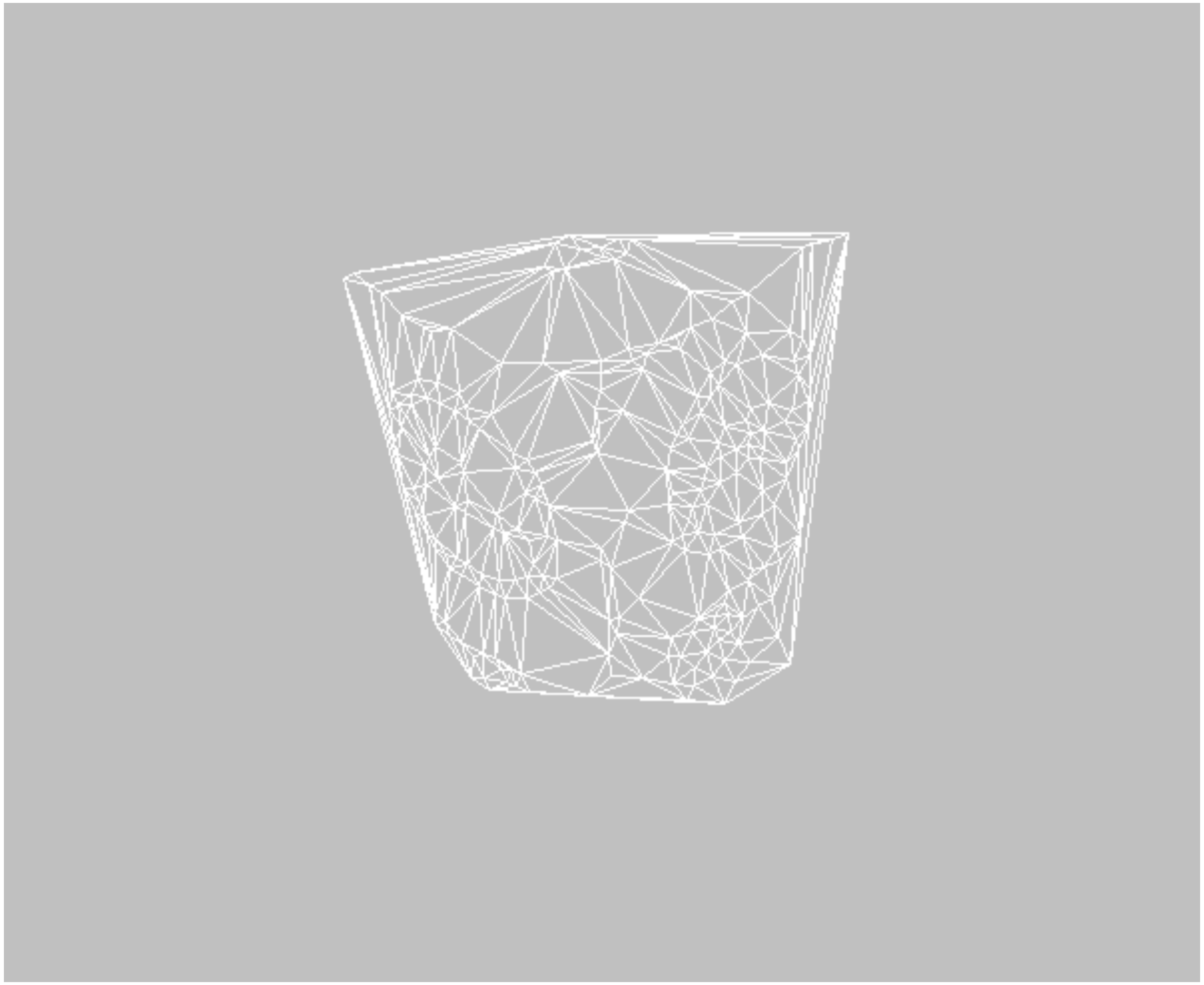




- Alternative regularization techniques (locally smooth flow fields, integration along contours)
- Qualitative properties of the motion fields

Feature Tracking

3D Reconstruction - Preview



Point Feature Extraction

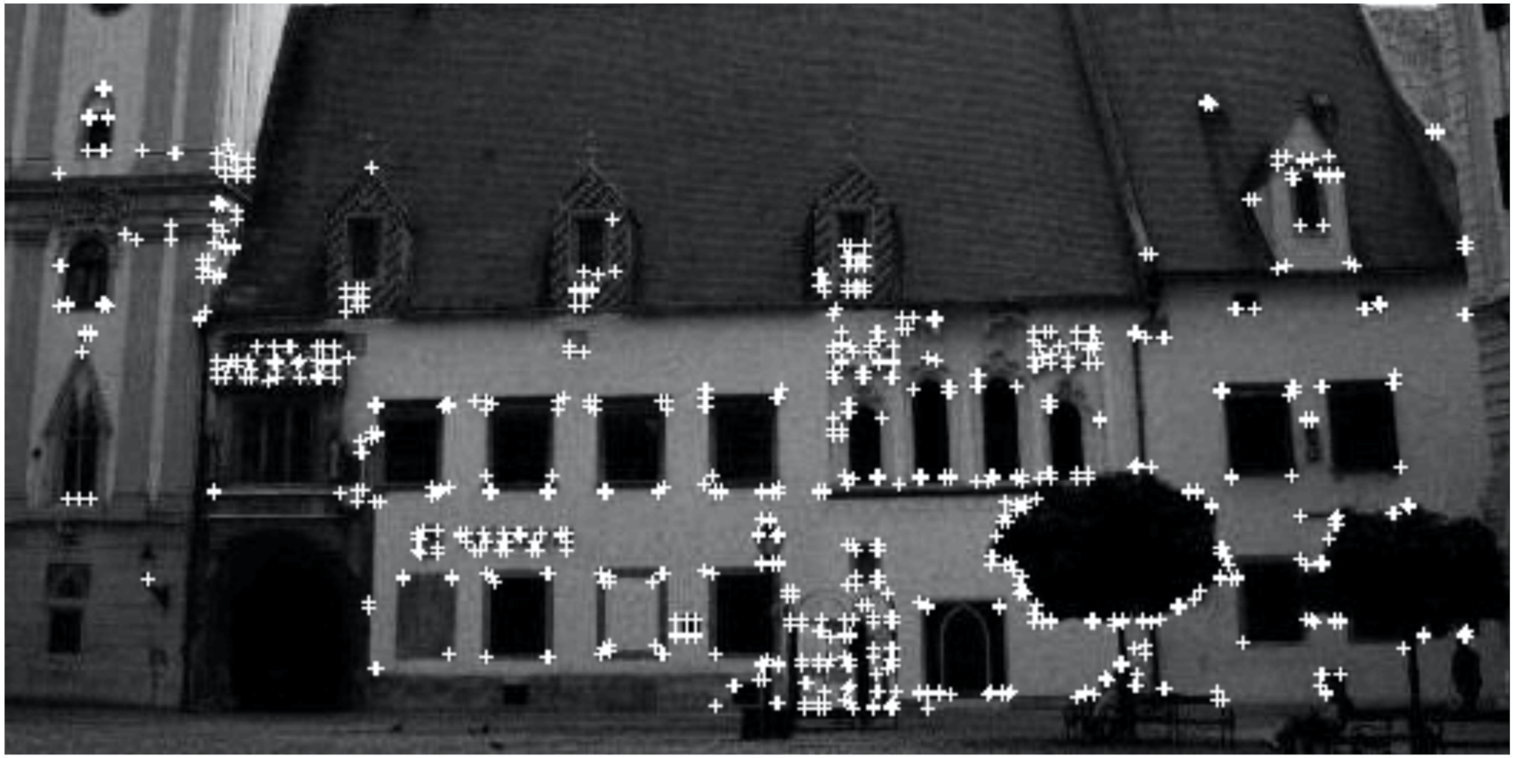
$$G = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

- Compute eigenvalues of G
- If smalest eigenvalue σ of G is bigger than τ - mark pixel as candidate feature point

- Alternatively feature quality function (Harris Corner Detector)

$$C(G) = \det(G) + k \cdot \mathrm{trace}^2(G)$$

Harris Corner Detector - Example



Wide Baseline Matching

Region based Similarity Metric

- Sum of squared differences

$$SSD(h) = \sum_{\tilde{\mathbf{x}} \in W(\mathbf{x})} \|I_1(\tilde{\mathbf{x}}) - I_2(h(\tilde{\mathbf{x}})\|^2$$

- Normalize cross-correlation

$$NCC(h) = \frac{\sum_{W(\mathbf{x})}(I_1(\tilde{\mathbf{x}}) - \bar{I}_1)(I_2(h(\tilde{\mathbf{x}})) - \bar{I}_2))}{\sqrt{\sum_{W(\mathbf{x})}(I_1(\tilde{\mathbf{x}}) - \bar{I}_1)^2 \sum_{W(\mathbf{x})}(I_2(h(\tilde{\mathbf{x}})) - \bar{I}_2)^2)}}$$

- Sum of absolute differences

$$SAD(h) = \sum_{\tilde{\mathbf{x}} \in W(\mathbf{x})} |I_1(\tilde{\mathbf{x}}) - I_2(h(\tilde{\mathbf{x}}))|$$

Edge Detection

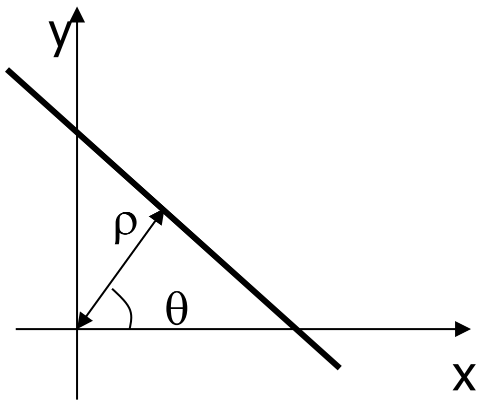
original image

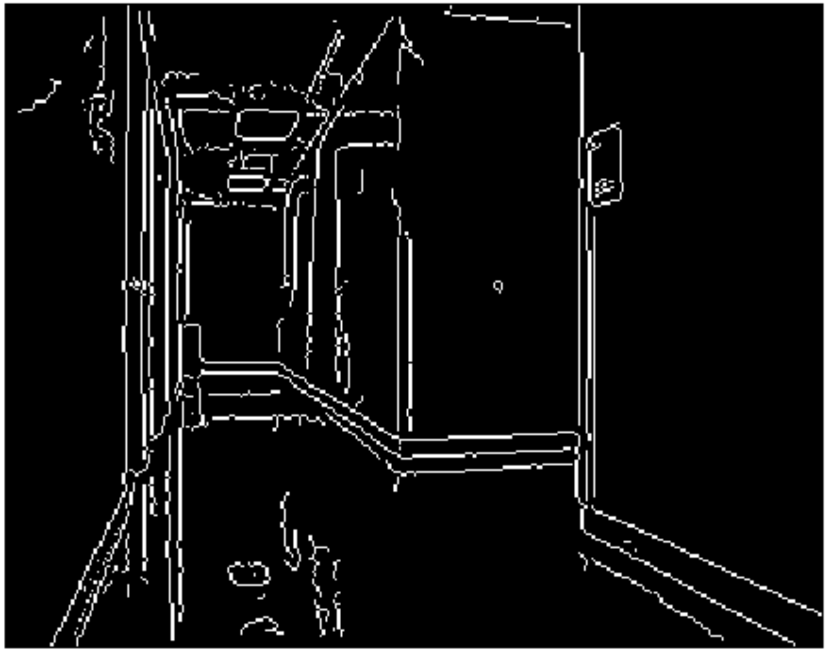
gradient magnitude

Canny edge detector

- Compute image derivatives
- if gradient magnitude > τ and the value is a local maximum along gradient direction – pixel is an edge candidate

Line fitting





Non-max suppressed gradient magnitude

- Edge detection, non-maximum suppression
  (traditionally Hough Transform – issues of resolution, threshold selection and search for peaks in Hough space)
- Connected components on edge pixels with similar orientation
  - group pixels with common orientation

Line Fitting

$$A = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i y_i & \sum y_i^2 \end{bmatrix}$$

second moment matrix
associated with each
connected component

- Line fitting Lines determined from eigenvalues and eigenvectors of A
- Candidate line segments - associated line quality