1 Camera Intrinsic Matrix

The camera intrinsic parameter matrix K can be represented

$$\begin{bmatrix} fs_x & s_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1}$$

→ normally

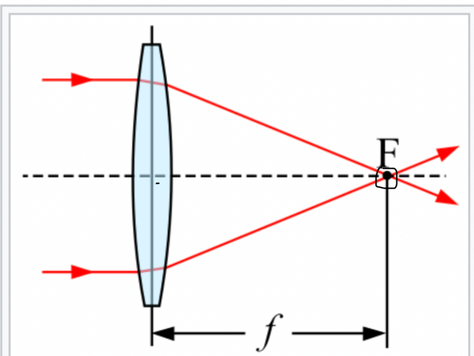
Ox = Image_width/2

Oy = Image_height/2

focal length

What do each of these terms represent?

http://ksimek.github.io/2013/08/13/intrinsic/



In practice, f_x and f_y can differ for a number of reasons:

- Flaws in the digital camera sensor.
- The image has been non-uniformly scaled in post-processing.
- The camera's lens introduces unintentional distortion.
- The camera uses an anamorphic format, where the lens compresses a widescreen scene into a standard-sized sensor.
- Errors in camera calibration.

2 Projections Matrix

A basic model of a camera is the following:

$$\lambda \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Given some point $p \in \mathbb{R}^3$ in the camera frame, we can apply the camera transformation to get the image of that point $q \in \mathbb{R}^2$. Show that given any point $r \in \mathbb{R}^3$ that lies on the line between o (the origin of the camera frame) and p, the image of $\cancel{r}$ is q.

The matrix $\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ is called projection matrix.

See recording for detail.

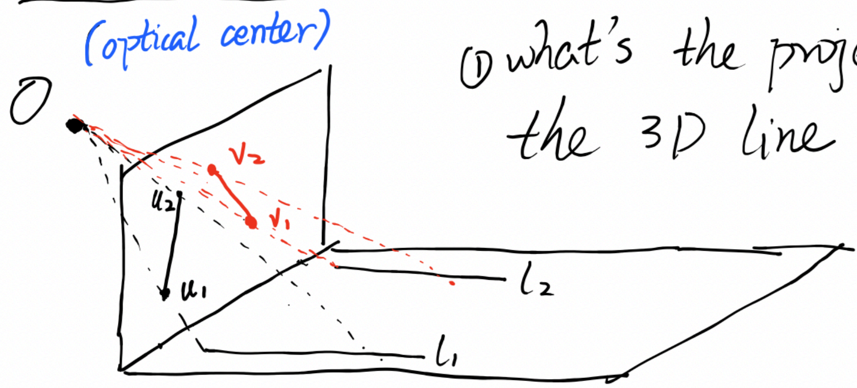
3 Vanishing Points

A straight line in the 3D world becomes a straight line in the image. However, two parallel lines in the 3D world will often intersect in the image. The point of intersection is called the *vanishing point*.

1. Given two parallel lines, how do you compute the vanishing point?
2. When does the vanishing point not exist (the two lines do not intersect)?
3. Show that the vanishing points of lines on a plane lie on the vanishing line of the plane.

1) Given two lines in an image, and you know they are parellel in 3D.

Then what is the direction of this 3D lines?



① what's the projection of the 3D line on image plane?

$\Rightarrow (U_1, U_2)$

$\Rightarrow (V_1, V_2)$

$U_1, U_2, V_1, V_2 \in R^2$

This part is missing in the live discussion:

we assume the image plane at depth = 1

(why? → we have the intrinsic matrix)

Consider the normal vector of $\triangle OU_1U_2$, $\triangle OV_1V_2$

$$\vec{n}_{\triangle OU_1U_2} = \overrightarrow{OU_1} \times \overrightarrow{OU_2}$$
$$\vec{n}_{\triangle OV_1V_2} = \overrightarrow{OV_1} \times \overrightarrow{OV_2}$$

Consider the normal vector of $\triangle OU_1U_2$, $\triangle OV_1V_2$

$$\vec{n}_{\triangle OU_1U_2} = \vec{OU_1} \times \vec{OU_2}$$
$$\vec{n}_{\triangle OV_1V_2} = \vec{OV_1} \times \vec{OV_2}$$

$$\begin{cases} \vec{d} \perp \vec{n}_{\triangle OU_1U_2} \\ \vec{d} \perp \vec{n}_{\triangle OV_1V_2} \end{cases} \quad \boxed{\vec{d} = \vec{n}_{\triangle OU_1U_2} \times \vec{n}_{\triangle OV_1V_2}}$$

direction of l_1, l_2

2) when $\vec{d} \parallel$ image plane

3) ① what is vanishing line of a plane?

Assume $N = (N_x, N_y, N_z)$ is the norm of plane.
$P = (X, Y, Z)$ is a point in the plane.

$\Rightarrow N^T P = d$ (plane equation, d is a constant)

Consider P's projection on image plane

(recall the pinhole camera model)
$x = f\frac{X}{Z}$, $y = f\frac{Y}{Z}$

$\Rightarrow x N_x + y N_y + f N_z = \frac{fd}{Z}$ (x, y is Projection of P)

Let $Z \to \infty$, we get a line in image plane

$\vec{l} =$ $x N_x + y N_y = -f N_z$

② For any line in the plane, its vanishing point lies in $\vec{l}$

For any line in the plane

$$\vec{a} + \lambda \vec{d} \qquad \begin{cases} \vec{a} = (a_x, a_y, a_z) \\ \vec{d} = (d_x, d_y, d_z) \\ N^T d = 0 \end{cases}$$

The vanishing point is :

$$\left(\frac{f(a_x+\lambda d_x)}{a_z+\lambda d_z}, \frac{f(a_y+\lambda d_y)}{a_z+\lambda d_z}\right)$$

$$\overset{\lambda\to\infty}{=} \underbrace{\left(f\frac{d_x}{d_z}, f\frac{d_y}{d_z}\right)}$$

substitude in the plane equation

$$N_x d_x + N_y d_y + N_z d_z = 0$$

$$\Downarrow$$

$$\underline{f\frac{d_x}{d_z} N_x + f\frac{d_y}{d_z} N_y = -f N_z}$$

4 Homography Matrix Transform

When the image features lie in a plane in the real world, the two-image correspondence problem is called planar homography. The real world coordinates of a point X are X_1 in the frame of camera 1, and X_2 in the frame of camera 2. We assume that X lies on a plane with normal vector N (defined with respect to camera 1). We know that the transform between the cameras is of the form $X_2 = RX_1 + T$, an affine transformation, but by assuming that X lies on the plane we can represent this transformation with a *homography matrix* H where

$$X_2 = HX_1 \tag{4.1}$$

where $H = R + \frac{1}{d}TN^T$, where d is the shortest distance between the plane and camera 1 ($d = N^T X_1$).
If we switch the roles of the first and second cameras, we should still be able to define a homography matrix $\tilde{H}$ such that $X_1 = \tilde{H}X_2$. Assume that $d_1 = 1$, so $H = R + TN^T$. Show that the new homography matrix $\tilde{H}$ is defined

$$\tilde{H} = \left(R^T + \frac{-R^T T}{1 + N^T R^T T} N^T R^T\right) \tag{4.2}$$

① why $X_2 = HX_1$ where $H = R + \frac{1}{d}TN^T$

$$\Leftarrow X_2 = RX_1 + \frac{1}{d}TN^T X_1$$

$$\xLeftarrow{X_2 = RX_1 + T} T = \frac{1}{d}TN^T X_1$$

$$\Leftarrow d = N^T X_1$$

(distance between a point and a plane)

② why $X_1 = \tilde{H}X_2$ where $d=1$, $\tilde{H} = (R^T + \cdots\cdots)$

$$X_1 = \tilde{H} X_2$$

$$\Leftarrow X_1 = R^T X_2 + \frac{-R^T T}{1+N^T R^T T} N^T R^T X_2$$

$X_1 = R^T (X_2 - T)$

$$\Leftarrow -R^T T = \frac{-R^T T}{1+N^T R^T T} N^T R^T X_2$$

$$\Leftarrow 1 + N^T R^T T = N^T R^T X_2$$

And because

$$N^T X_1 = 1 \quad , \quad X_1 = R^T (X_2 - T)$$

$$\Rightarrow N^T X_1 = N^T R^T (X_2 - T)$$

$$\Rightarrow 1 + N^T R^T T = N^T R^T X_2$$