EECS 106A/206A

Dynamics

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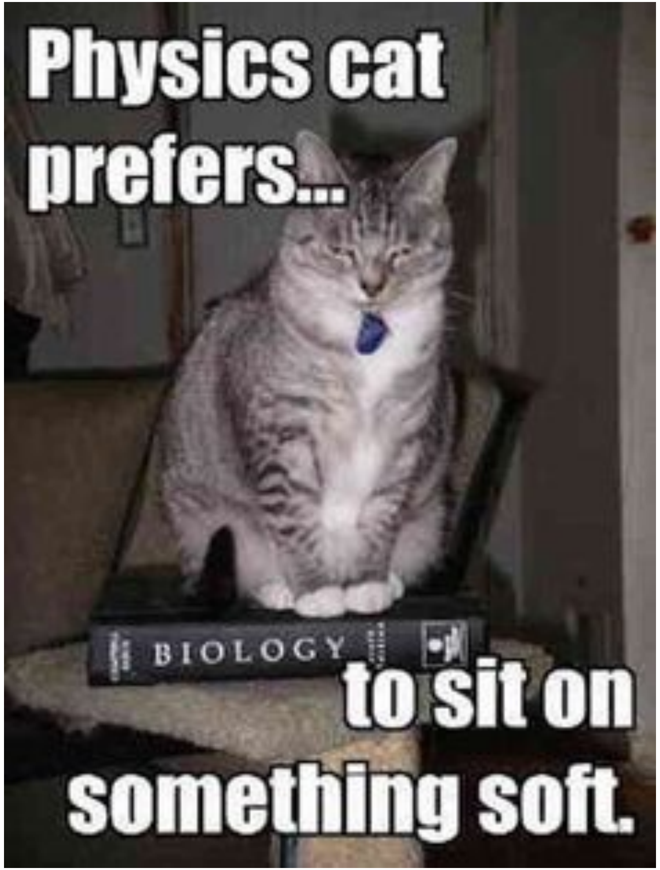
Announcements

- Final project proposals due Sunday
  - I'll upload stuff after lecture
- Lab cleanliness

Lecture outline

- A review of physics
- Wrenches and the jacobian transpose
- Newton-Euler dynamics
- Lagrangian Dynamics



What is dynamics?

Kinematics: motion

Dynamics: Forces

Basic Physics

- Position, m
- Velocity, m/s
  - Angular Velocity, rad/s
- Acceleration, m / s^2
  - Angular acceleration, rad / s^2
- Mass, kg
  - Moment of inertia kg m^2
- Momentum, kg m / s
  - Angular momentum, kg m^2 / s
- Force, N, kg m / s^2
  - Torque, m•N, kg m^2 / s^2
- Energy/Work, Nm, kg m^2 / s^2
- Power, Nm/s, kg m^2 / s^3

Wrenches: The duals of twists

A wrench represents an instantaneous force

$$F = \begin{bmatrix} f \\ \tau \end{bmatrix} \begin{array}{l} \text{linear component } f \in \mathbb{R}^3 \\ \text{angular component } \tau \in \mathbb{R}^3 \end{array}$$

The work caused by a wrench and a velocity is $W = \int V^T F dt$

Two wrenches are equivalent if they generate the same work for all possible V

To change coordinate frames, we use the transpose adjoint: $F_b = Ad_{g_{ab}}^T F_a$

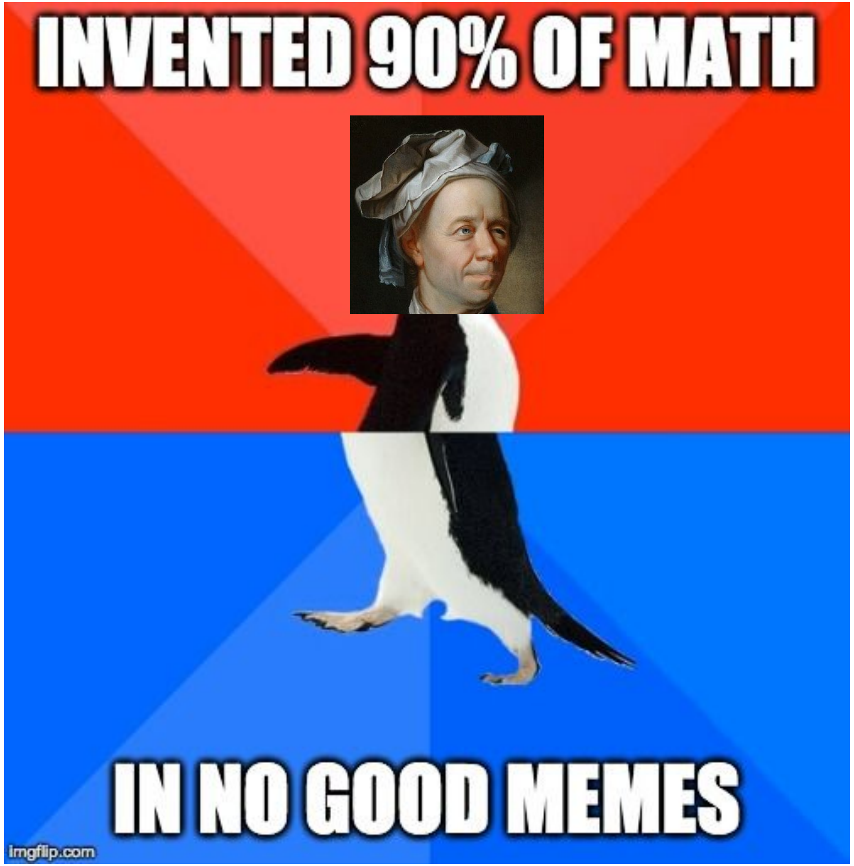
To get joint torques from a wrench, we use J^T: $\tau = J^T(\theta) F$

Newton’s Laws of Motion

1. An object will stay at rest or in motion unless acted upon by an external force
2. F = ma (or $F = \frac{dp}{dt}$)
3. Every action has an equal and opposite reaction

Euler's Laws of Motion

1. Linear momentum: $p = mv_{cm}$
2. Moments: $M = \frac{dL}{dt}$



Newton-Euler equations (inertial coordinates)

$$\begin{bmatrix} mI_3 & 0 \\ 0 & \mathcal{I}' \end{bmatrix} \begin{bmatrix} \ddot{p} \\ \dot{\omega}^s \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_s \times \mathcal{I}'\omega_s \end{bmatrix} = \begin{bmatrix} f \\ \tau \end{bmatrix} = F$$

Moment of inertia in spatial frame
$\mathcal{I}' = R\mathcal{I}R^T$

Linear and angular accelerations

This term (w x L) occurs when the angular velocity vector and angular momentum vectors don’t line up. It causes precession in tops and other “fun” behavior

The first line is Newton’s second Law

The second line is conservation of angular momentum (Euler’s equation)

Newton-Euler equations (body coordinates)

Now the linear velocity depends on the orientation

$$\begin{bmatrix} mI & 0 \\ 0 & \mathcal{I} \end{bmatrix} \begin{bmatrix} \dot{v}^b \\ \dot{\omega}^b \end{bmatrix} + \begin{bmatrix} \omega^b \times mv^b \\ \omega^b \times \mathcal{I}\omega^b \end{bmatrix} = F^b$$

Usually this is a diagonal matrix (if you define your body frame nicely)

You've still got precession, but at least the moment of inertia's a constant

Inertia Tensor

The inertia tensor is *symmetric positive definite*

$$\mathcal{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} = -\int_V \rho(r)\widehat{r}^2\, dV$$

Thus there will always be a basis that makes $\mathcal{I}$ diagonal.

This is the intelligent choice for the body frame.

$$I_{xx} = \int_V \rho(r)(y^2 + z^2)\, dx\, dy\, dz$$

$$I_{xy} = -\int_V \rho(r)(xy)\, dx\, dy\, dz,$$

Solving multi-body dynamics with Newton-Euler

Define body dynamics for each body (including any external body forces)

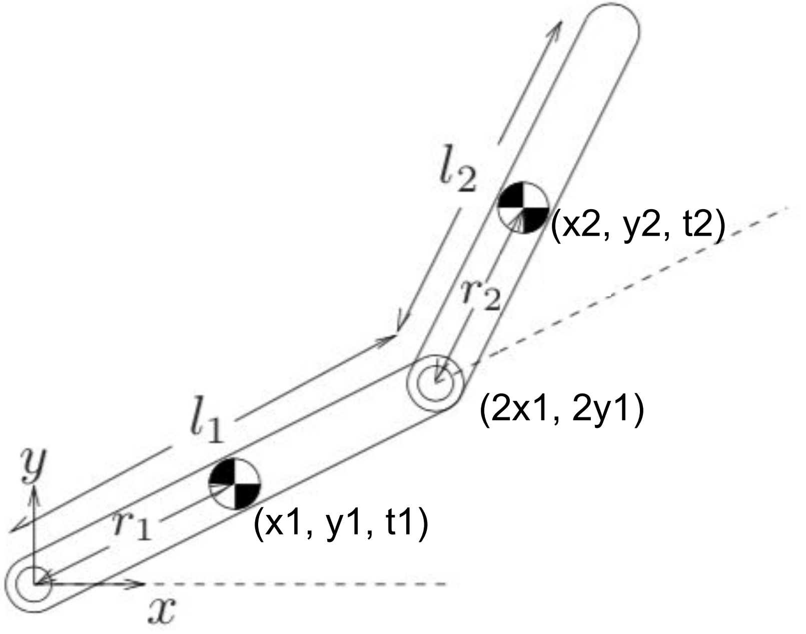
Add external forces and joint torques

Define constraints between rigid bodies

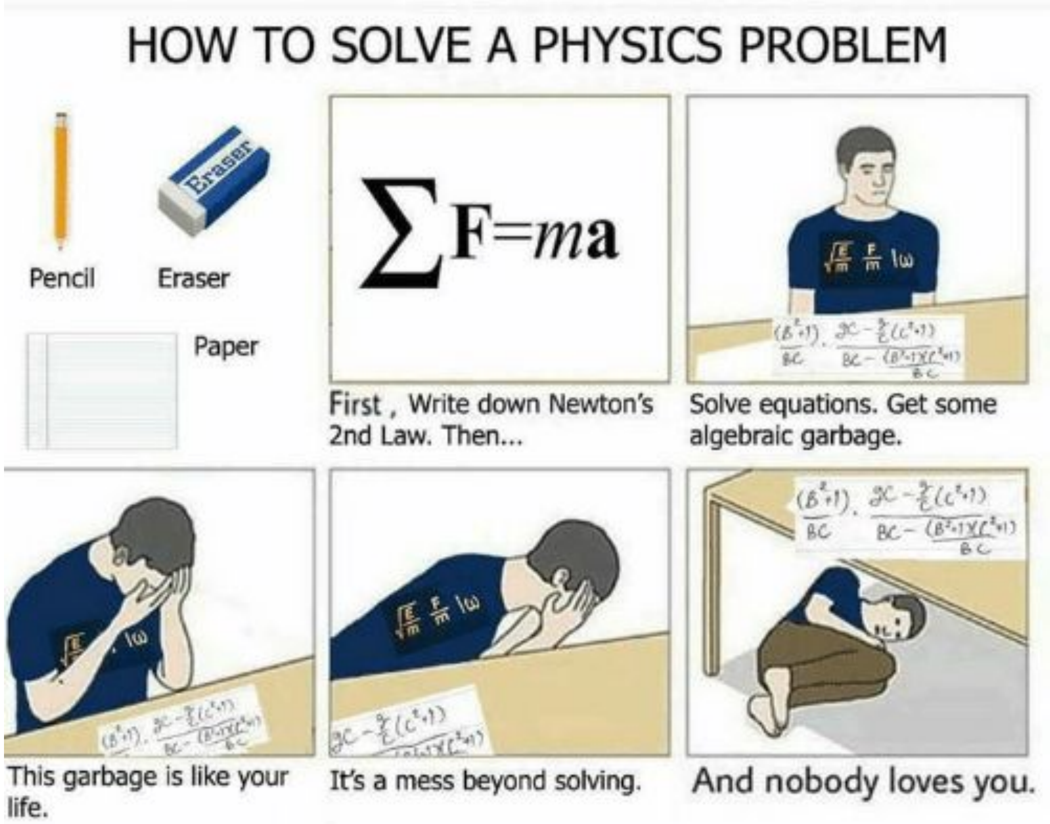
$$x_1^2 + y_1^2 = r_1^2$$
$$(x_2 - 2x_1)^2 + (y_2 - 2y_1)^2 = r_2^2$$

Differentiate them to get the constraint forces

Do a ton of algebra to cancel out constraint forces and arrive at a closed form solution



My method for solving dynamics with Newton-Euler



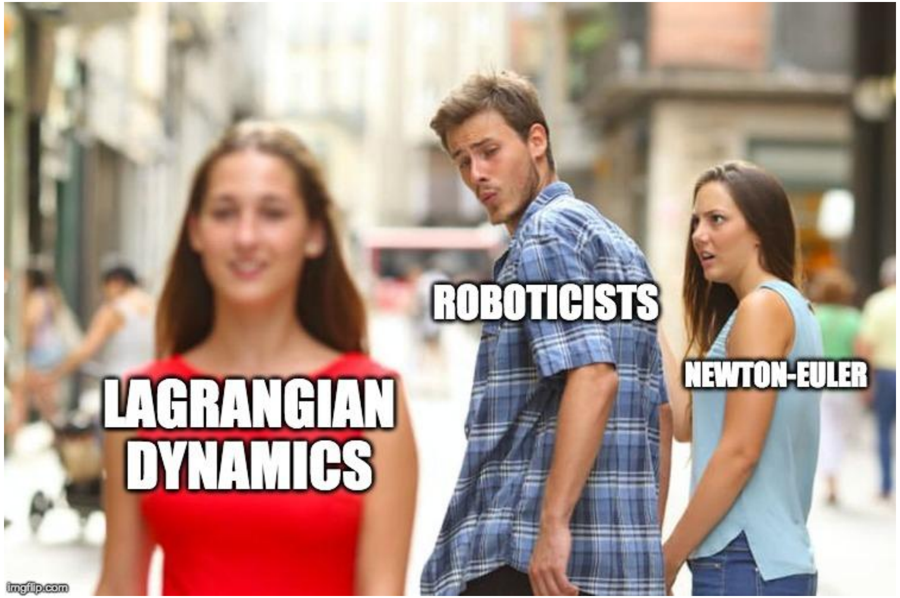
Credit: pairode jaroensri

Lagrangian Dynamics: A Better Way

Rather than basing dynamics on conservation of momentum, we instead use conservation of energy

Since constraint forces do no work, they don't contribute to the total system energy (and we can ignore them)

Thus, we can write our dynamics in terms of *generalized coordinates* (joint angles) instead of rigid body transforms



The Lagrangian

We define the lagrangian

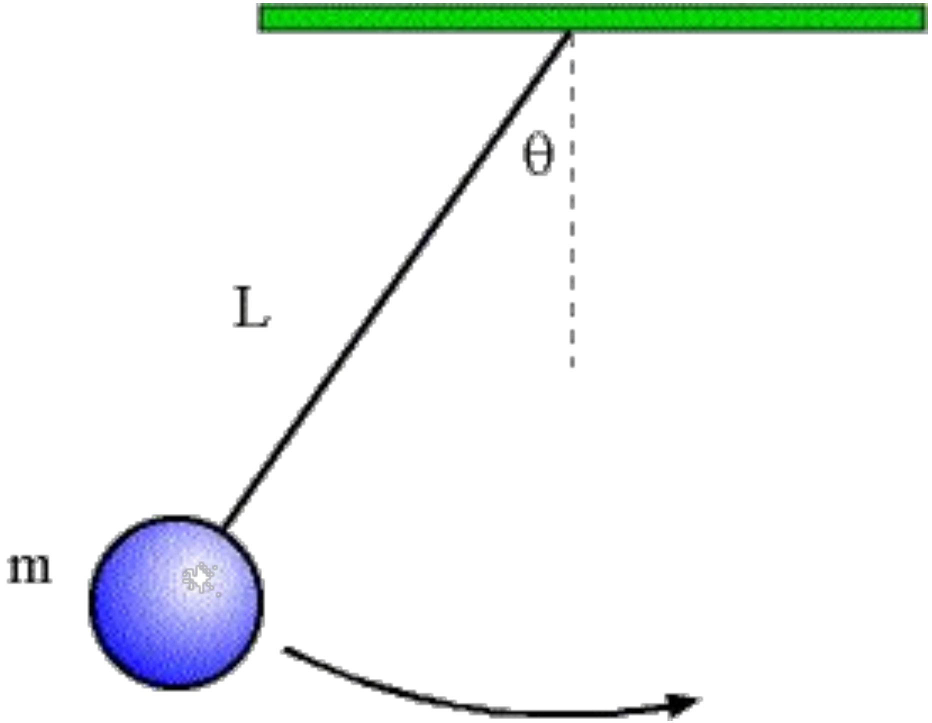
$$L(q, \dot{q}) = T(q, \dot{q}) - V(q),$$

T is kinetic energy, V potential energy

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \Upsilon$$

Υ is a vector of external forces

Example: Simple Pendulum



When is Newton-Euler Better?

Lagrangian Dynamics implicitly requires state to be a vector

SE(3) (and SO(3)) cannot be represented as vectors without singularities

Thus, for single rigid bodies in 3D space we often use Newton-Euler

- Drones
- Tops
- Projectiles
- Satellites

APPENDIX