Problem Set 3: Inverse Kinematics
EECS C 106A/C206A, Fall 2019

Due: Monday, Sept 30th 2019 at 11:59 PM on Gradescope

1 Inverse Kinematics of a 3 DOF Manipulator

a) Solve the inverse kinematics problem on the manipulater below using Paden-Kahan subproblems. By which we mean do the following:

   i Break down the problem into the subproblems you use (in the order you use them) as the book does on page 104-105.

   ii For each subproblem, state which axes, points, and lengths you'd use. However, there's no need to actually solve the subproblems.

b) Give the count of the number of inverse kinematic solutions.

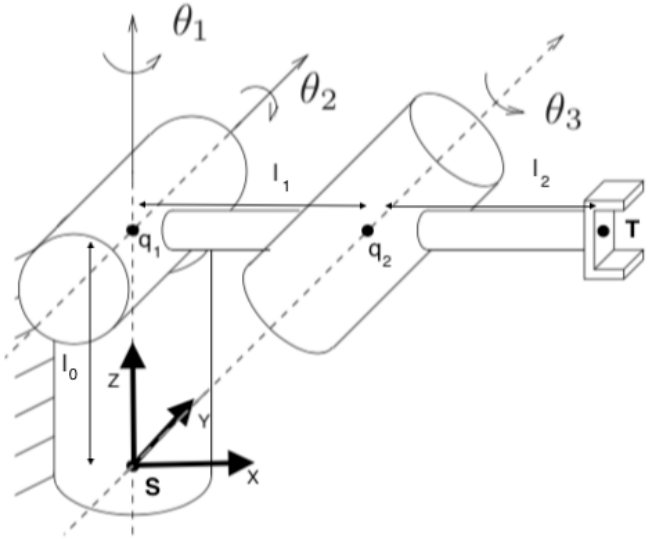

Figure 1: Three-dof manipulator

2 More Inverse Kinematics of the Inverse Elbow Manipulator

a) Solve the inverse kinematics problem for the manipulator below using Paden Kahan subproblems. By which we mean do the following:

   i Break down the problem into the subproblems you use (in the order you use them) as the book does on page 104-105.

   ii For each subproblem, state which axes, points, and lengths you'd use. However, there's no need to actually solve the subproblems.

b) Give the count of the number of inverse kinematic solutions.

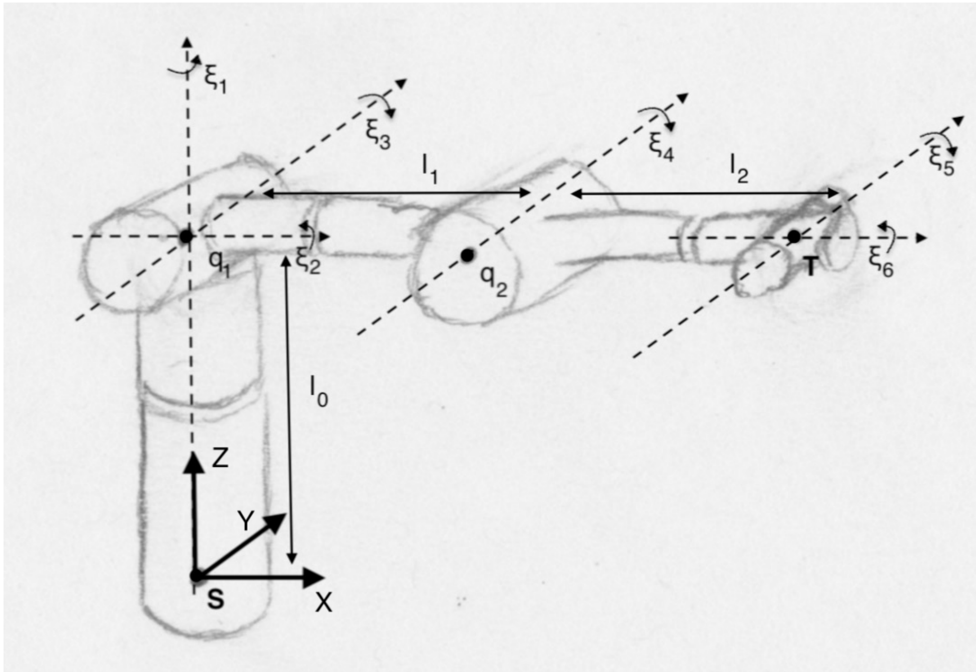

Figure 2: Inverse elbow manipulator

3 Subproblem 2' adapted from Problem 5 on page 148

Solve Subproblem 2 when the two twist axes ξ_1, ξ_2 do not intersect but are not parallel. That is, given ξ_1, ξ_2 zero pitch unit magnitude twists (not intersecting but not parallel) and points $p, q \in \mathbb{R}^3$, find θ_1, θ_2 such that

$$e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}p = q$$

Hint: Follow the derivation as done in the chapter, but modify Figure 3.9 to have two different points r_1, r_2 on the twist axes ξ_1, ξ_2 respectively that rotate p, q respectively onto a common point c. Define $z_1 = c - r_1$, $z_2 = c - r_2$, where c is the point of final intersection. Write $z_2 = \alpha\omega_1 + \beta\omega_2 + \gamma(\omega_1 \times \omega_2)$. Using $u = p - r_2$ and $v = q - r_1$ check that

$$\omega_2^T z_2 = \omega_2^T u = \alpha\omega_1^T\omega_2 + \beta \qquad \omega_1^T z_2 = \omega_1^T v + \omega_1^T(r_1 - r_2) = \alpha + \beta\omega_1^T\omega_2$$

Solve for α, β as in the book using these two equations. Then, as in the Subproblem 2 proof in the book, by checking that $||z_2||^2 = ||u||^2$ find the formula for γ to get z_2. Now $c = r_2 + z_2$ and $z_1 = c - r_1$. From

$$e^{\hat{\omega}_2\theta_2}u = z_2 \qquad e^{-\hat{\omega}_1\theta_1}v = z_1$$

complete the derivation. When do we have $0, 1, 2$ solutions?