EECS 106/206A HW5: Jacobians and Wrenches

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Homework due: 10/18/2018 at 11:59 PM,

Problem-sets are due on Gradescope. Feel free to use a computer to help you with this problem set. If you do write any code, to help you solve the problem, attach the code at the end of your problem set. If you use any pre-made code (such as MATLAB's pseudo-inverse function pinv()), state that you use it as a step in your solution.

Question 1: Jacobians, Singularities and Wrenches . . . . . 20 points

For this question, consider the sagittal view of Alyx shown in Figure 1.

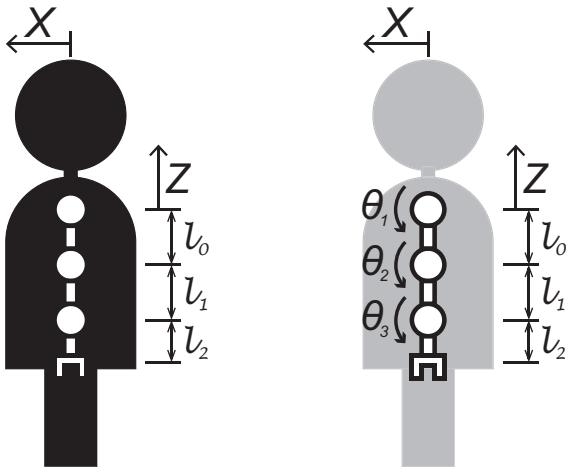

Figure 1: Alyx's arm in the Saggital plane in the initial configuration. The origin is defined at the shoulder. The motion of the hand relative to the arm is simplified to be three revolute joints, rotating about the y axis.

(a) Draw a sketch of Alyx's arm in the configuration $\{\theta_1, \theta_2, \theta_3\} = \{0, -\frac{\pi}{2}, 0\}$. (1)

(b) Using this sketch (not through direct computation), show that the spatial and body Jacobians of the system in this configuration are: (2)

$$\boldsymbol{J}^s = \begin{bmatrix} 0 & l_0 & l_0 \\ 0 & 0 & 0 \\ 0 & 0 & l_1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{J}^b = \begin{bmatrix} -l_1 - l_2 & -l_1 - l_2 & -l_2 \\ 0 & 0 & 0 \\ l_0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) Is the configuration $\{\theta_1, \theta_2, \theta_3\} = \left\{0, -\frac{\pi}{2}, 0\right\}$ singular? What end effector velocities and forces are instantaneously achievable? (1)

(d) Draw a sketch of Alyx's arm in the configuration $\{\theta_1, \theta_2, \theta_3\} = \{-\pi, 0, 0\}$. (1)

(e) Using this sketch (not through direct computation), show that the spatial and body Jacobians of the system in this configuration are: (2)

$$\boldsymbol{J}^s = \begin{bmatrix} 0 & -l_0 & -l_0 - l_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{J}^b = \begin{bmatrix} -l_0 - l_1 - l_2 & -l_1 - l_2 & -l_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(f) Is the configuration $\{\theta_1, \theta_2, \theta_3\} = \{-\pi, 0, 0\}$ singular? What end effector velocities and forces are instantaneously achievable? (1)

(g) Show that the spatial and body Jacobians for this system can be written symbolically as: (4)

$$\boldsymbol{J}^s = \begin{bmatrix} 0 & l_0 c_1 & l_0 c_1 + l_1 c_{1+2} \\ 0 & 0 & 0 \\ 0 & -l_0 s_1 & -l_0 s_1 - l_1 s_{1+2} \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{J}^b = \begin{bmatrix} -l_0 c_{2+3} - l_1 c_3 - l_2 & -l_1 c_3 - l_2 & -l_2 \\ 0 & 0 & 0 \\ -l_0 s_{2+3} - l_1 s_3 & -l_1 s_3 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(h) Using configurations $\{\theta_1, \theta_2, \theta_3\} = \left\{0, -\frac{\pi}{2}, 0\right\}$ and $\{\theta_1, \theta_2, \theta_3\} = \{-\pi, 0, 0\}$ as references, explain in words/illustrations how ξ' and $\xi^\dagger$ relate to ξ. (2)

(i) Using these symbolic expressions, when will Alyx's arm be in a singular configuration? (2)

(j) Compute Alyx's joint torques when they hold a mass m in their hand, in both the $\{\theta_1, \theta_2, \theta_3\} = \left\{0, -\frac{\pi}{2}, 0\right\}$ and $\{\theta_1, \theta_2, \theta_3\} = \{-\pi, 0, 0\}$ configurations. (2)

(k) Show that these joint torques are the same as when a Spatial Wrenches $\boldsymbol{F}_1^s$, and $\boldsymbol{F}_2^s$ are applied to the system. Explain in words/illustrations how a spatial wrench is defined for a given configuration. (2)

$$\boldsymbol{F}_1^s = \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0 \\ (l_1 + l_2)\, mg \\ 0 \end{bmatrix} \qquad \boldsymbol{F}_2^s = \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0 \\ 0 \\ 0 \end{bmatrix}$$