Problem Set 5: Dynamics and Control
EECS C 106A/C206A, Fall 2019

Due: Monday November 11th, 2019 at 11:59 PM on Gradescope
Only one slip day may be used on this assignment

1. Dynamics of a Mass Spring

   Use Lagrangian Dynamics to find the equations of motion for the system in Figure 1.

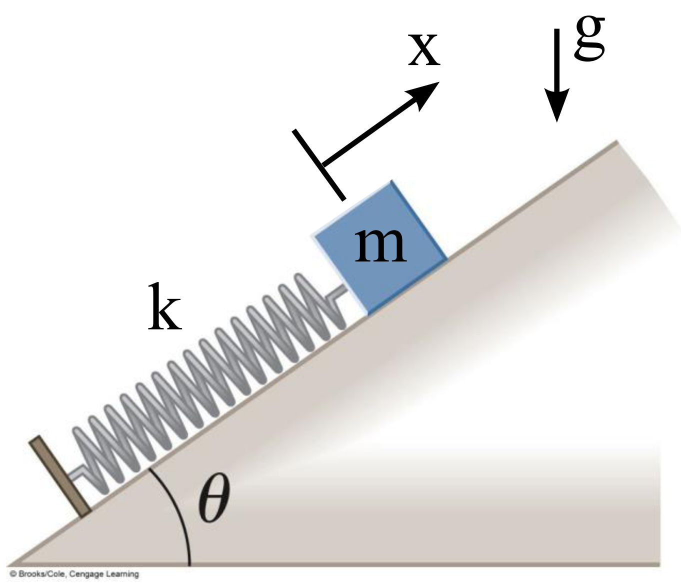

Figure 1: Mass Spring on an Inclined Plane

   State the generalized coordinates you're using (you should only need one). What are the Inertia matrix, Coriolis matrix, and gravity vectors for this system (since this is a one dimensional system these will all just be scalars)? What does the generalized force vector Υ represent?

2. Dynamics of a Double Pendulum

   Use Lagrangian Dynamics to find the equations of motion for the system in Figure 2.

   State the generalized coordinates you're using (you should need two). What are the Inertia matrix, Coriolis matrix, and gravity vectors for this system? What does the generalized force vector Υ represent?

3. Forward and Inverse Dynamics

   When we worked with kinematics, we learned about both forward and inverse kinematics. Forward kinematics allows us to compute the pose of some object (end effector) given parameters (joint angles). Inverse kinematics allows us to compute the parameters given the object pose. Similarly, we can define forward and inverse dynamics problems.

   (a) Forward Dynamics

   Forward dynamics allows you to compute the derivative of your state given your current state and your inputs to the system. Our state will be $\{\theta, \dot{\theta}\}$, and our input vector vector will be τ, the joint torques of our system. Write down the forward dynamics for a general open-chain robot manipulator in the following form

$$\ddot{\theta} = f(\theta, \dot{\theta}) + g(\theta, \dot{\theta})\tau$$

(b) Inverse Dynamics

Inverse dynamics allows you to compute the inputs to your system given your state and a desired state derivative. Write down the inverse dynamics for a general open-chain robot manipulator in the following form

$$\tau = a(\theta, \dot{\theta}) + b(\theta, \dot{\theta})\ddot{\theta}_d$$

(c) Joint Torque Control

Say you want your robot to follow the following PD control law:

$$\ddot{\theta}_d = \ddot{\theta}_{ff} + K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$$

What torques would you need to apply to your system to have your system follow this control law? Show that if you put this torque through your system forward dynamics you'll get

$$\ddot{\theta} = \ddot{\theta}_{ff} + K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$$

(d) Feedback Linearization

Here we've taken a nonlinear system and designed a controller that makes it act like a linear system. This is an example of *feedback linearization* which you may encounter in your controls classes. What is a potential pitfall of this control method?

4. Trajectory Generation

In this problem, we will find a constant twist trajectory from any initial pose $g^i_{st} \in SE(3)$ to any final pose $g^f_{st} \in SE(3)$.

Let the position of the origin of tool frame at the initial location be $p^i \in \mathbb{R}^3$ and the orientation at the initial location $R^i \in SO(3)$. Similarly at the final location let $p^f \in \mathbb{R}^3, R^f \in SO(3)$.

Now define $\hat{\omega}T = \log\{R^f(R^i)^{-1}\}$, with $||\omega|| = 1$. Here log is the matrix logarithm, which is the inverse of the matrix exponential. T is the time in which we want our trajectory to execute.

(a) Find a formula for ω. (Hint: note that $\{R^f(R^i)^{-1}\} = \exp^{(\hat{\omega}T)}$ and use the results of Section 2.2 of MLS.)

(b) Now consider the differential equation for the tool frame given by

$$\dot{g} = \hat{\xi}g \quad g(0) = g^i_{st}$$

with the constant twist ξ having the rotational component ω and the translational component v. Using Proposition 2.9 in Chapter 2 of MLS find the value of v required to give $g_{st}(T) = g^f_{st}$. The resulting $g_{st}(t) \;\; t \in [0, T]$ is the generated trajectory of the manipulator tool frame from the initial to the final location.

(c) If the tool frame was at the end of a 6 degree of freedom manipulator, show how you can use (the inverse of?) the manipulator spatial Jacobian to find the joint angle trajectory, that is $\theta(t) \;\; t \in [0, T]$, corresponding to this trajectory.

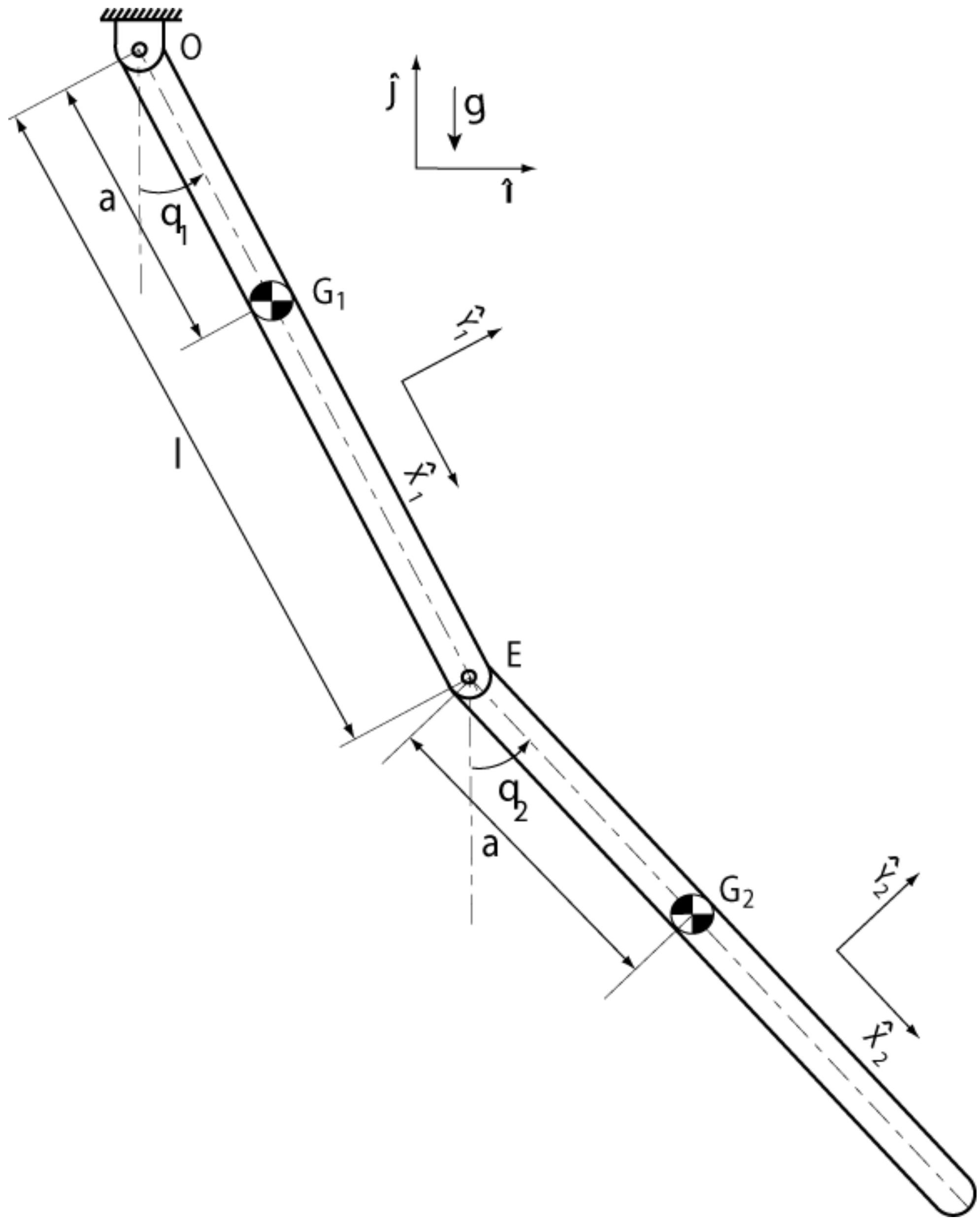

Figure 2: Double Pendulum