EECS 106A/206A
Dynamics

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Announcements

- Final project proposals due Sunday
- Homework 4 due Monday
- Shankar back next week
- Isabella gone next week(s) :(

Lecture outline

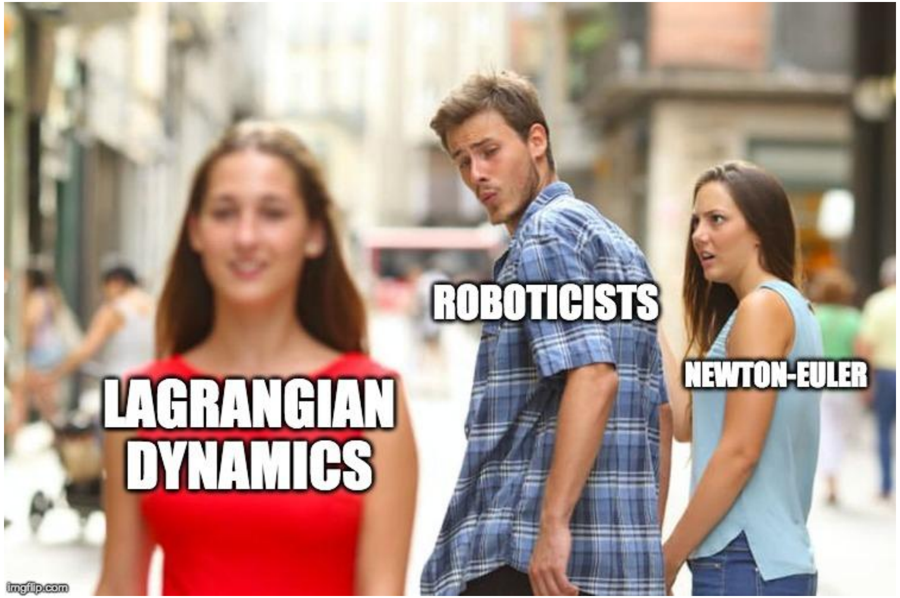
- Lagrangian Dynamics
- Lagrangian Dynamics of an Open Chain Manipulator

Lagrangian Dynamics: A Better Way

Rather than basing dynamics on conservation of momentum, we instead use conservation of energy

Since constraint forces do no work, they don't contribute to the total system energy (and we can ignore them)

Thus, we can write our dynamics in terms of *generalized coordinates* (joint angles) instead of rigid body transforms



The Lagrangian

We define the lagrangian

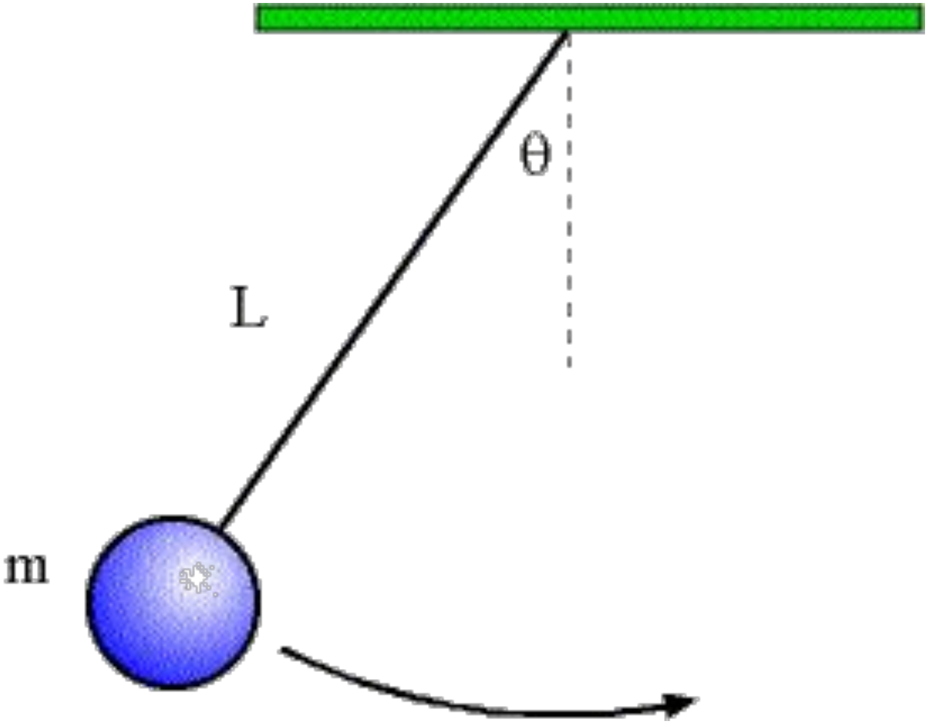
$$L(q, \dot{q}) = T(q, \dot{q}) - V(q),$$

T is kinetic energy, V potential energy

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \Upsilon$$

Υ is a vector of external forces

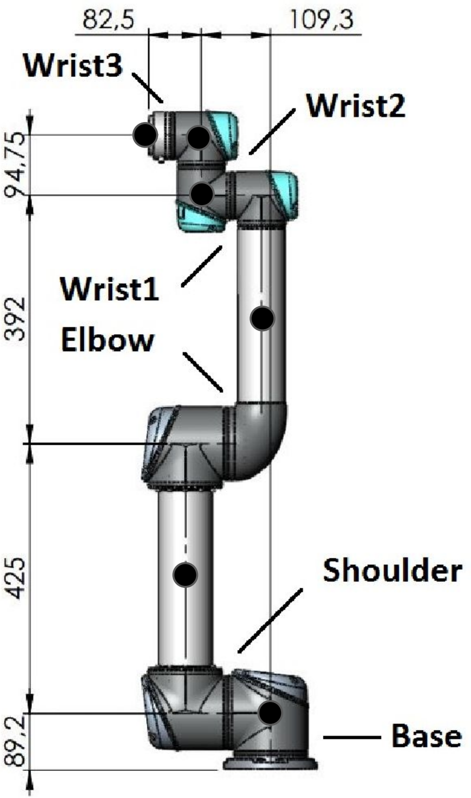
Example: Simple Pendulum



Lagrangian of an Open Chain Manipulator

We define a coordinate frame for each link

$$g_{sl_i}(\theta) = e^{\widehat{\xi}_1\theta_1} \cdots e^{\widehat{\xi}_i\theta_i} g_{sl_i}(0)$$



Define the Lagrangian

We work in the body frame of each link!

$$M(\theta) = \sum_{i=1}^{n} J_i^T(\theta)\mathcal{M}_i J_i(\theta). \quad \mathcal{M}_i = \left[\begin{array}{ccc|ccc} m_i & & 0 & & & \\ & m_i & & & 0 & \\ 0 & & m_i & & & \\ \hline & & & I_{xi} & & 0 \\ & 0 & & & I_{yi} & \\ & & & 0 & & I_{zi} \end{array}\right]$$

$$T(\theta,\dot{\theta}) = \frac{1}{2}\dot{\theta}' M(\theta)\dot{\theta}$$

$$V(\theta) = \sum_{i=1}^{n} V_i(\theta) = \sum_{i=1}^{n} m_i g h_i(\theta).$$

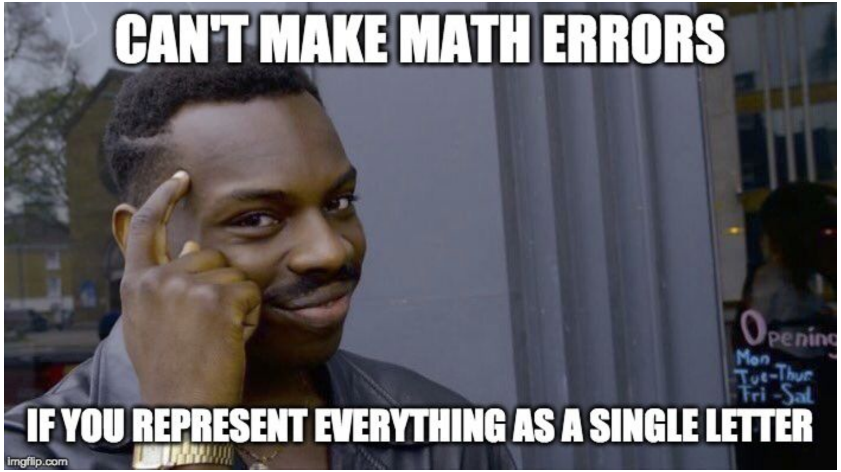
Take Derivatives

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i} = \Upsilon_i,$$

Coriolis Forces

Robot Dynamics

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \Upsilon$$



APPENDIX