Practice Midterm II
EECS C 106A/C206A, Fall 2019

Two 8.5×11 crib sheets allowed, double sided.

Remember if something is true you have to prove it. If on the other hand something is false, you need to give a counterexample.
Some Notes:

1. **This midterm is approximately 1.5x as long as the actual midterm. I gave you extra questions on the short problems/ROS portions.**
2. **This midterm is not guaranteed to be 100% comprehensive. That said, I think it reasonably spans the actual midterm material.**
3. **Remember that *any* material from the class is fair game, be it from lecture, homework, discussion, lab, or the sections of the book that we've covered.**

Name:

SID:

Problem	Score / Max.
Stuff	/ 30
Total	/ 30

Question 1: Jacobian Multiple Choice .... 13 points

Let $\boldsymbol{J}^s(\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \boldsymbol{\theta_3})$ be the manipulator Jacobian of a robot parameterized by the joint position vector $\boldsymbol{\theta}$. In the current configuration, all joint positions are 0. The Jacobian, joint velocities, and joint torques/forces are:

$$\boldsymbol{J}^s(0,0,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

All of the robot's joints are either revolute or prismatic.

(a) How many joints does this robot have? (1)

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8

(b) How many revolute joints? (1)

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8

(c) This robot is in a singular configuration. (2)

○ True ○ False ○ Not enough info

(d) In this configuration, in which directions is it possible to induce a nonzero velocity? (2)

☐ v_x ☐ v_y ☐ v_z ☐ ω_x ☐ ω_y ☐ ω_z

(e) In this configuration, in which directions is it possible to induce a nonzero force/torque? (2)

☐ f_x ☐ f_y ☐ f_z ☐ τ_x ☐ τ_y ☐ τ_z

(f) Which column(s) of the spatial Jacobian are constant for all values of $\boldsymbol{\theta}_1$? (1)

☐ First column ☐ Second column ☐ Third column

(g) Which column(s) of the spatial Jacobian are constant for all values of $\boldsymbol{\theta}_2$? (1)

☐ First column ☐ Second column ☐ Third column

(h) Which column(s) of the spatial Jacobian are constant for all values of $\boldsymbol{\theta}_3$? (1)

☐ First column ☐ Second column ☐ Third column

(i) How many singular configurations does this robot have? (2)

○ 0 ○ 1 ○ 3 ○ 6 ○ 18 ○ 36 ○ ∞ ○ Not enough info

Question 2: Dynamics .... 8 points

Consider the mass-pulley system shown in Figure 2 in the Appendix. This system is classically referred to as Atwood's Machine. The pulley is mounted on a frictionless bearing with a moment of inertia I and a radius r. Two masses, m_1 and m_2, are suspended across the pulley by a rope of fixed length l. The position, x, is defined as the vertical distance of the mass m_1 from the center of the pulley. You may assume that the rope does not slip with respect to the pulley.

(a) Determine the position of the second mass m_2 as a function of the position of the first mass m_1 with the given parameters. The following equation for the circumference of a circle might be useful: (2)

$$C = 2\pi r$$

(b) Derive an equation for the acceleration of the mass m_1 using Lagrangian mechanics. (6)

Question 3: Path Planning .... 14 points

You have a simple 2-dof manipulator shown below in Figure 1

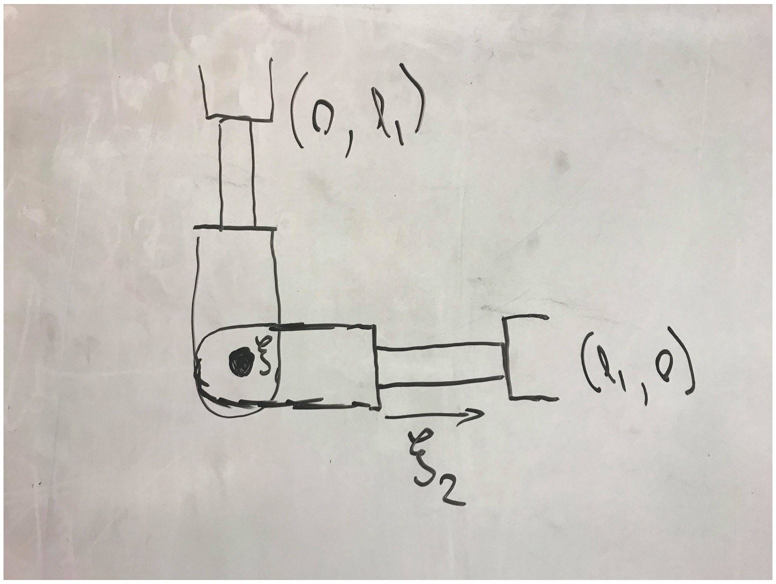

Figure 1: 2-dof manipulator

This manipulator has one revolute joint ξ_1 and one prismatic joint ξ_2. It starts in the horizontal configuration g^i ($\theta = [0, l_1]$) and ends in the vertical configuration g^f ($\theta = [\frac{\pi}{2}, l_1]$).

(a) We can define a "straight line path" or constaint velocity path of a coordinate q as a path that goes from q_i to q_f at constant velocity. What's a straight line path in joint space? W and a straight line path in workspace? Draw them. (4)

(b) Assume that you want the robot to complete this trajectory in T seconds. What will your trajectories $\theta(t)$ be for both the joint space and work space trajectories? Note: You shouldn't need to do any nontrivial integrals. (6)

(c) What is the jacobian of this manipulator at the initial configuration. Assume that the z axis extends out of the page. (2)

(d) How many singularities will this manipulator have? Where will they occur? (2)

Question 4: Physics Review .... 10 points

An object of mass m is launched at a velocity of v m/s at some angle θ from horizontal. The initial position of the object is $x = 0$, $y = 0$. The ground is at $y = 0$ for all x. (See figure 3)

(a) What are the equations of motion for the mass? (use cartesian coordinates) (2)

(b) What is the maximum height reached by the object? (2)

(c) What is the total flight time? (2)

(d) How far does the object travel? (2)

(e) At what velocity does the mass strike the ground? (2)

Question 5: Singular Configuration .... 12 points

Three revolute joint axes with twists $\xi_i = [q_i \times \omega_i, \omega_i]^T, i = 1, 2, 3$ are said to be parallel if

$$\omega_i = \pm\omega_j,\ \ i, j = 1, 2, 3$$

A prismatic joint with twist $\xi_4 = [v, 0]^T$ is said to be perpendiuclar to these revolute joints if

$$v^T\omega_i = 0,\ \ i = 1, 2, 3$$

Show that a six degree of freedom manipulator with three parallel revolute axes and a prismatic axis perpendicular to all three is at a singular configuration, that is, that $J(\theta)$ is singular.

1 Appendix: Figures and Useful Formulae

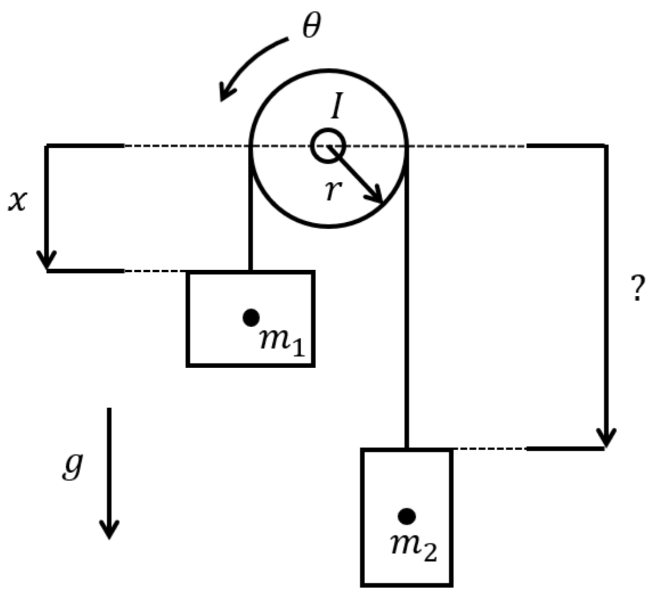

Figure 2: Atwood's Machine

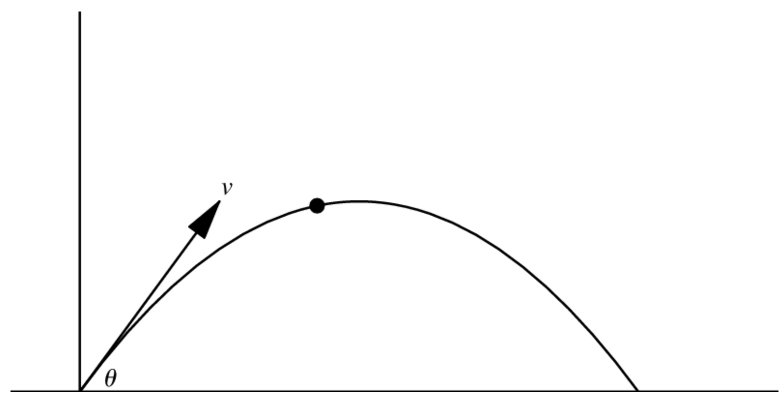

Figure 3: Projectile motion