Practice Midterm
EECS C 106A/C206A, Fall 2019

One 8.5×11 **crib sheet allowed, double sided.**

Remember if something is true you have to prove it. If on the other hand something is false, you need to give a counterexample.
Some Notes:

1. **This midterm is approximately 1.5x as long as the actual midterm. I gave you extra questions on the short problems/ROS portions.**
2. **This midterm is not guaranteed to be 100% comprehensive. That said, I think it reasonably spans the actual midterm material.**
3. **Remember that *any* material from the class is fair game, be it from lecture, homework, discussion, lab, or the sections of the book that we've covered.**

Name:

SID:

Problem	Score / Max.
Stuff	/ 30
Total	/ 30

1. **Short Answer Questions**

   (a) We have a rotation matrix $R = e^{\hat{\omega}\theta}$ where $\hat{\omega} \in so(3), ||\omega|| = 1$. Show that ω is an eigenvector of R. What's the associated eigenvalue? Why is this the case?

   (b) Prove that the matrix exponential of a skew symmetric matrix $\hat{\omega} \in so(n)$ is a rotation matrix ($e^{\hat{\omega}\theta} \in SO(n)$).

   (c) What is the minimum number of variables needed to express a rotation $R \in SO(3)$? What is this representation? What are some pro's and cons of using it?

(d) A quaternion Q can be generated by $Q = (\cos(\theta/2), \omega \sin(\theta/2))$. Prove that a unit ω will produce a unit quaternion.

(e) Prove that $SE(n)$ is a group under matrix multiplication.

2. **Order of Operations**

(a) Select all operations that are always commutative:

- ☐ Multiple rotation matrices, about orthogonal axes
- ☐ Multiple rotation matrices, about parallel axes
- ☐ Multiple homogenous transforms, where all $R = I$
- ☐ Multiple homogenous transforms, where all $R = R_X(\frac{\pi}{4})$
- ☐ Multiple exponential mappings, with parallel revolute joints.
- ☐ Multiple exponential mappings, with parallel prismatic joints.

(b) Select all options that are always associative:

- ☐ Multiple rotation matrices, about orthogonal axes
- ☐ Multiple rotation matrices, about parallel axes
- ☐ Multiple homogenous transforms, where all $R = I$
- ☐ Multiple homogenous transforms, where all $R = R_X(\frac{\pi}{4})$
- ☐ Multiple exponential mappings, with parallel revolute joints.
- ☐ Multiple exponential mappings, with parallel prismatic joints.

3. **Programming with ROS**

(a) List the file structure of a ros workspace in the root directory of your computer. There should be a workspace called `my_workspace`, and package called `my_package`, containing a python script `my_code.py`, a launch file `my_launch.launch`, and a service `my_service.srv`.

(b) Describe at least one function of the `package.xml`

(c) Name two types of communication that you can use to connect your ROS nodes. When should we use each?

(d) `rostopic info /mobilebase/commands/velocity` says that the topic type is `geometry_msgs/Twist`. What is `geometry_msgs`? What is `Twist`?

(e) What is an AR tag? Why might one use one?

4. **Forward Kinematics**

   Please write down the forward kinematics of the arm with the axes labelled as shown in Figure 3 below. You do not need to multiply out the full product of exponentials formula, but please indicate the twist axes and conveniently chosen points on the axes.

5. **Inverse Kinematics**

Use the Paden Kahan sub problem to do the inverse kinematics of the arm with the axes as labeled in the previous problem. You do not need to do the details of the inverse kinematics, but indicate how you would break down the inverse kinematics to get the angles. Please indicate the number of possible solutions. Describe the reachable workspace of the manipulator, that is the subset of $\mathbb{R}^3$ that the origin of the tool frame can reach.

1 Appendix: Figures and Useful Formulae

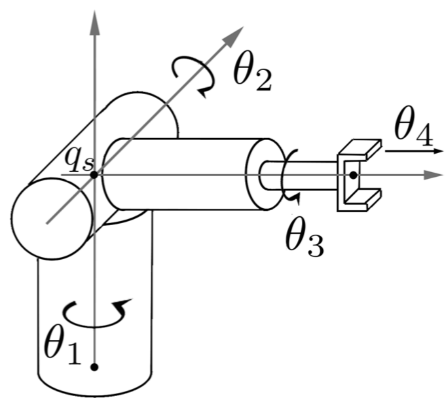

Figure 1: 4DOF Arm. Joints 1,2,3 are revolute. Joint 4 is prismatic

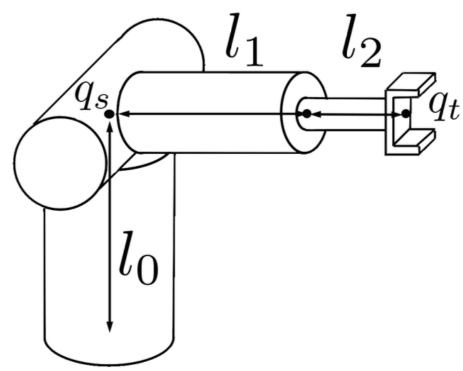

Figure 2: Manipulator lengths in zero configuration

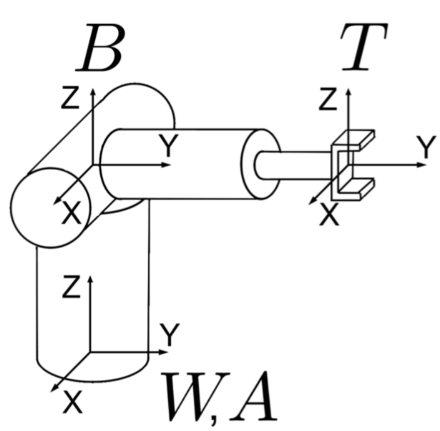

Figure 3: Coordinate Axes at zero configuration