

Problem Set 3: Inverse Kinematics

EECS C 106A/C206A, Fall 2019

Due: Monday, Sept 30th 2019 at 11:59 PM on Gradescope

1 Inverse Kinematics of a 3 DOF Manipulator

- Solve the inverse kinematics problem on the manipulator below using Paden-Kahan subproblems.
- Give the count of the number of inverse kinematic solutions.

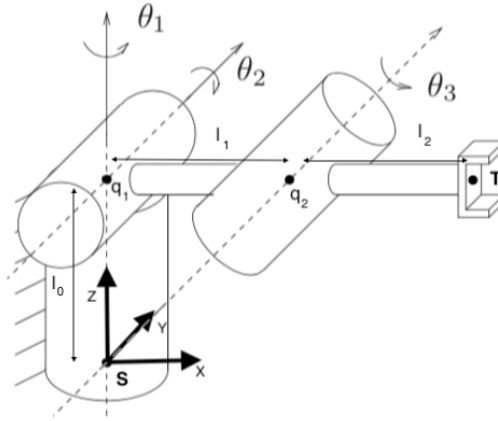


Figure 1: Three-dof manipulator

2 More Inverse Kinematics of the Inverse Elbow Manipulator

- Solve the inverse kinematics problem for the manipulator below using Paden Kahan subproblems.
- Give the count of the number of inverse kinematic solutions.

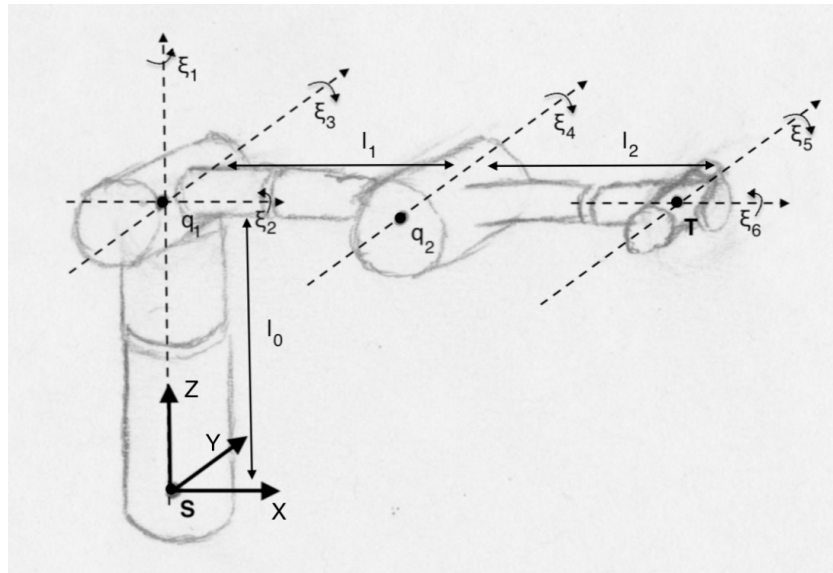


Figure 2: Inverse elbow manipulator

3 Subproblem 2' adapted from Problem 5 on page 148

Solve Subproblem 2 when the two twist axes ξ_1, ξ_2 do not intersect but are not parallel. That is, given ξ_1, ξ_2 zero pitch unit magnitude twists (not intersecting but not parallel) and points $p, q \in \mathbb{R}^3$, find θ_1, θ_2 such that

$$e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} p = q$$

Hint: Follow the derivation as done in the chapter, but modify Figure 3.9 to have two different points r_1, r_2 on the twist axes ξ_1, ξ_2 respectively that rotate p, q respectively onto a common point c . Define $z_1 = c - r_1$, $z_2 = c - r_2$, where c is the point of final intersection. Write $z_2 = \alpha \omega_1 + \beta \omega_2 + \gamma(\omega_1 \times \omega_2)$. Using $u = p - r_2$ and $v = q - r_1$ check that

$$\omega_2^T z_2 = \omega_2^T u = \alpha \omega_1^T \omega_2 + \beta \quad \omega_1^T z_2 = \omega_1^T v + \omega_1^T (r_1 - r_2) = \alpha + \beta \omega_1^T \omega_2$$

Solve for α, β as in the book using these two equations. Then, as in the Subproblem 2 proof in the book, by checking that $\|z_2\|^2 = \|u\|^2$ find the formula for γ to get z_2 . Now $c = r_2 + z_2$ and $z_1 = c - r_1$. From

$$e^{\hat{\omega}_2 \theta_2} u = z_2 \quad e^{-\hat{\omega}_1 \theta_1} v = z_1$$

complete the derivation. When do we have 0, 1, 2 solutions?