

Problem Set 4: Manipulator Jacobians

EECS C 106A/C206A, Fall 2019

Due: Monday October 28th, 2019 at 11:59 PM on Gradescope

1. Singularities of Euler Angles

- (a) What's the adjoint of a rotation about the origin?
- (b) Using this, demonstrate that the singularity of ZYX (extrinsic) Euler angles occurs when $\theta_2 = \frac{\pi}{2}$. Intuitively, why does this occur?
- (c) Prove that any rotation represented by three rotations about arbitrary axes ($R = R_1 R_2 R_3$) will have a singularity.
Hint 1: First prove the case where $a\omega_1 + b\omega_2 = \omega_3$ (such as with ZYZ Euler angles). Then prove the harder case where $\omega_1, \omega_2, \omega_3$ span \mathbb{R}^3
Hint 2: Doing the second part by brute force is very difficult

2. Jacobian Calculation with the Circle Method

Use the Circle Method to calculate the Jacobian of the manipulator below at the zero (current) configuration with respect to (a) the S frame and (b) the T frame. In the book these are referred to as the spatial and body jacobians, respectively. You will need to draw additional pictures.

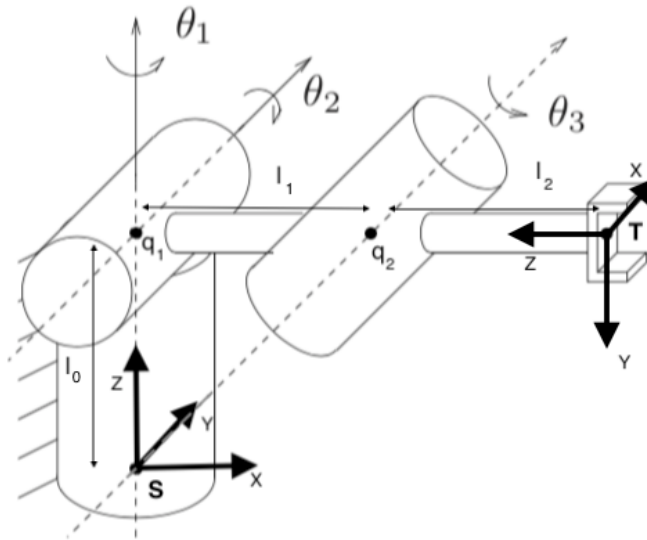


Figure 1: Three-dof manipulator

3. Kinematic Singularity: four coplanar revolute joints

Four revolute joint axes with twists $\xi_i = (q_i \times \omega_i, \omega_i)$, $i = 1 \dots 4$ are said to be coplanar if there exists a plane with unit normal n such that:

- Each axis direction is orthogonal to n : $n^T \omega_i = 0$.
- The vector from q_i to q_j is orthogonal to n : $n^T (q_i - q_j) = 0$.

Show that when four of its revolute joint axes are coplanar, any six degree of freedom manipulator is at a singular configuration. Give an example of a manipulator exhibiting such a singularity.

4. Kinematic Singularity: prismatic joint perpendicular to two parallel coplanar revolute joints

A prismatic joint with twist $\xi_3 = (v_3, 0)$ is normal to a plane containing two parallel revolute axes $\xi_i = (q_i \times \omega_i, \omega_i)$, $i = 1, 2$ if

- $v_3^T \omega_i = 0$
- $v_3^T (q_1 - q_2) = 0$
- $\omega_1 = \pm \omega_2$

Show that when this occurs, any six degree of freedom manipulator is at a singular configuration. Give an example of a manipulator exhibiting such a singularity.