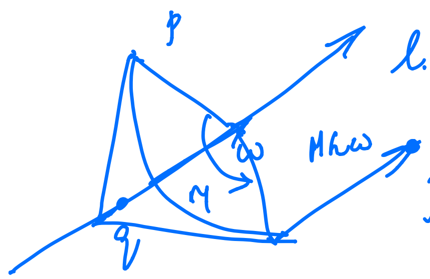
EECS 106A LECTURE NOTES 09/10/20

REVIEW OF SCREWS & TWISTS



$\|\omega\| = 1 \quad \omega \in \mathbb{R}^3$

$l = \{ q + \lambda \omega : \lambda \in \mathbb{R} \}$

M = magnitude

h = pitch of the screw

$$\begin{bmatrix} p' \\ 1 \end{bmatrix} = \begin{bmatrix} e^{M\hat{\omega}} & (I - e^{M\hat{\omega}})q + hM\omega \\ 000 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix}$$

$$(p' - q) = \underbrace{e^{M\hat{\omega}}(p - q)}_{\text{Rotation}} + \underbrace{hM\omega}_{\text{Translation}}$$

SCREW MOTION

$$g = \begin{bmatrix} e^{M\hat{\omega}} & (I - e^{M\hat{\omega}})q + hM\omega \\ 000 & 1 \end{bmatrix}$$

$$\overset{\|}{e^{\hat{\xi}\theta}}$$

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 000 & 0 \end{bmatrix}$$

ω screw axis $\|\omega\| = 1$

$\theta = M$

Claim: If $v = -\omega \times q + h\omega$

Then $e^{\hat{\xi}\theta} = g$

$\square$

Proof $$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & \boxed{(I - e^{\hat{\omega}\theta})\omega \times v + \omega\omega^T v}\,\theta \\ 0 & 1 \end{bmatrix}$$ ②

$\theta = M.$

$$v = -\omega \times q + h\omega$$

$$(I - e^{\hat{\omega}\theta})(\omega \times (-\omega \times q + \cancel{h\omega})) + \omega\omega^T(-\omega \times q + h\omega)\theta$$

$$= (I - e^{\hat{\omega}\theta})\,-\hat{\omega}^2 q + \omega\omega^T h\omega\theta$$

$$= (I - e^{\hat{\omega}\theta})(-\omega\omega^T + I)q + h\theta\omega\,\cancel{\omega^T\omega}^{\,1}$$

$$= -(I - e^{\hat{\omega}\theta})(\omega\omega^T)q + (I - e^{\hat{\omega}\theta})q + h\theta\omega$$

$$= -(\cancel{I} - \cancel{I} + \hat{\omega}\theta + \frac{\hat{\omega}^2\theta^2}{2!} + \cdots)\omega\omega^T q + (I - e^{\hat{\omega}\theta})q + h\theta\omega$$

$$= 0 + (I - e^{\hat{\omega}\theta})q + h\theta\omega$$

SCREW MOTION $$g = e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0\,0\,0 & 1 \end{bmatrix}$$

$\boxtimes$

$$\xi = \begin{bmatrix} \hat{\omega} & v \\ 0\,0\,0 & 1 \end{bmatrix}$$

$\|\omega\| = 1$

$\omega \neq 0$

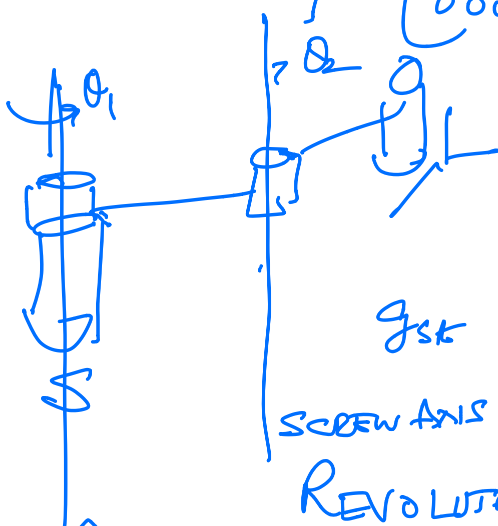
Define $q = \omega \times v$ $\qquad M = \theta$

$h = \omega^T v \leftarrow$

SCREW $\{ q + \lambda \omega \}$ $\quad M$, pitch $h = \omega^T v$

associated with

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0\,0\,0 & 0 \end{bmatrix}$$



g_{sk} $\qquad e^{\hat{\xi}_2 \theta_2}$

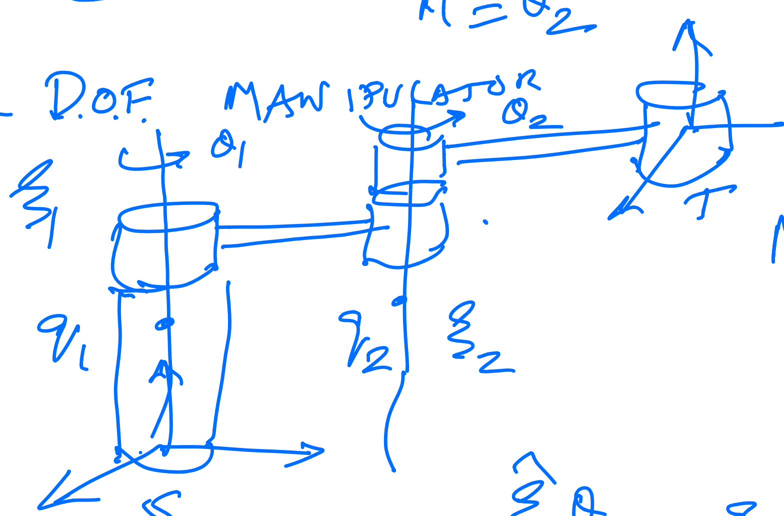
$e^{\hat{\xi}_1 \theta_1}$

REVOLUTE JOINT

SCREW WITH $h = 0$

$M = \theta_2$

2 D.O.F. MANIPULATOR



MOVE JOINT2

$e^{\hat{\xi}_2 \theta_2}$ $\qquad g_{ST}(0)$

$g_{ST}(\theta_1, \theta_2) =$

INITIAL POSN & ORIENTATION

$$e^{\hat{\xi}_2 \theta_2} = \begin{bmatrix} e^{\hat{\omega}_2 \theta_2} & (I - e^{\hat{\omega}_2 \theta_2}) q_2 + h_2 \theta_2 \omega_2 \\ 0\;0\;0 & 1 \end{bmatrix}$$

h_2 = PITCH OF SCREW 2 = 0

BECAUSE JOINT 2 IS ROTARY

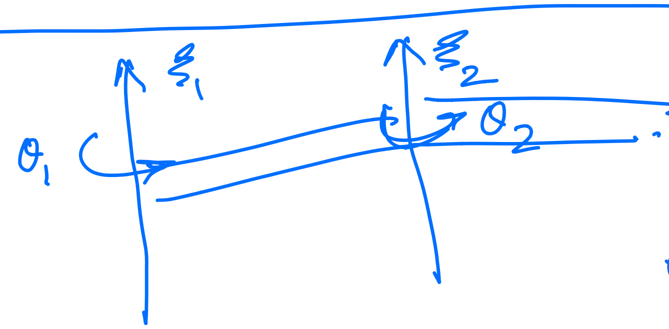
$$e^{\hat{\xi}_2 \theta_2} = \begin{bmatrix} e^{\hat{\omega}_2 \theta_2} & (I - e^{\hat{\omega}_2 \theta_2}) q_2 \\ 0\;0\;0 & 1 \end{bmatrix}$$

Now move joint 1

$$e^{\hat{\xi}_1 \theta_1} = \begin{bmatrix} e^{\hat{\omega}_1 \theta_1} & (I - e^{\hat{\omega}_1 \theta_1}) q_1 \\ 0\;0\;0 & 1 \end{bmatrix}$$

$$g_{ST}(\theta) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} g_{ST}(0)$$

$$g_{ST}(\theta) = \begin{bmatrix} e^{\hat{\omega}_1 \theta_1} & (I - e^{\hat{\omega}_1 \theta_1}) q_1 \\ 0\;0\;0 & 1 \end{bmatrix} \begin{bmatrix} e^{\hat{\omega}_2 \theta_2} & (I - e^{\hat{\omega}_2 \theta_2}) q_2 \\ 0\;0\;0 & 1 \end{bmatrix} *$$



KINEMATICS

$$e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2}$$

IF YOU STARTED rotating about $\xi_2 \rightsquigarrow$ ξ_1 is left unchanged

$$e^{\hat{\xi}_1\theta_1} \; e^{\hat{\xi}_2\theta_2}$$

IF YOU START by rotating about ξ_1 first

$$e^{\hat{\xi}_1\theta_1} \qquad \xi_2 \rightsquigarrow \xi_2'$$

$$\xi_2 \xrightarrow{g = e^{\hat{\xi}_1\theta_1}} \xi_2'$$

$$\hat{\xi}_2' = \underbrace{g\,\hat{\xi}_2\,g^{-1}}$$

$$\hat{\xi}_2' = e^{\hat{\xi}_1\theta_1}\,\hat{\xi}_2\,e^{-\hat{\xi}_1\theta_1}$$

$$e^{\hat{\xi}_2'\theta_2} = e^{\left(\underbrace{e^{\hat{\xi}_1\theta_1}}\,\hat{\xi}_2\,\underbrace{e^{-\hat{\xi}_1\theta_1}}\right)\theta_2}$$

$$= e^{\hat{\xi}_1\theta_1}\,e^{\hat{\xi}_2\theta_2}\,e^{-\hat{\xi}_1\theta_1}$$

$$e^{\hat{\xi}_2'\theta_2}\,e^{\hat{\xi}_1\theta_1} \cdot g_{st}(0)$$

$$\underbrace{e^{\hat{\xi}_1\theta_1}\,e^{\hat{\xi}_2\theta_2}}\,\underbrace{\cancel{e^{-\hat{\xi}_1\theta_1}\,e^{\hat{\xi}_1\theta_1}}}_{I}\,g_{st}(0)$$

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} \cdot e^{\hat{\xi}_2\theta_2} \cdot e^{\hat{\xi}_3\theta_3} \cdots e^{\hat{\xi}_n\theta_n}\,g_{st}(0)$$

NUMBERING FROM BASE S to TOOL T

$\mathbb{R}^6 \ni \xi_1 = \begin{bmatrix} v_1 \\ w_1 \end{bmatrix} \quad \xi_2 = \begin{bmatrix} v_2 \\ w_2 \end{bmatrix} \quad \cdots$

$\mathbb{R}^{4\times4} \quad \hat{\xi}_1 = \begin{bmatrix} \hat{\omega}_1 & v_1 \\ 000 & 0 \end{bmatrix} \quad \hat{\xi}_2 = \begin{bmatrix} \hat{\omega}_2 & v_2 \\ 000 & 0 \end{bmatrix}$

SCREW q_i on axis ξ_i & h_i pitch of i-th joint

geometric

$v_i = -\omega_i \times q_i + h_i \omega_i$

$h_i = 0$ REV.

$h_i = \infty$ PRISMATIC

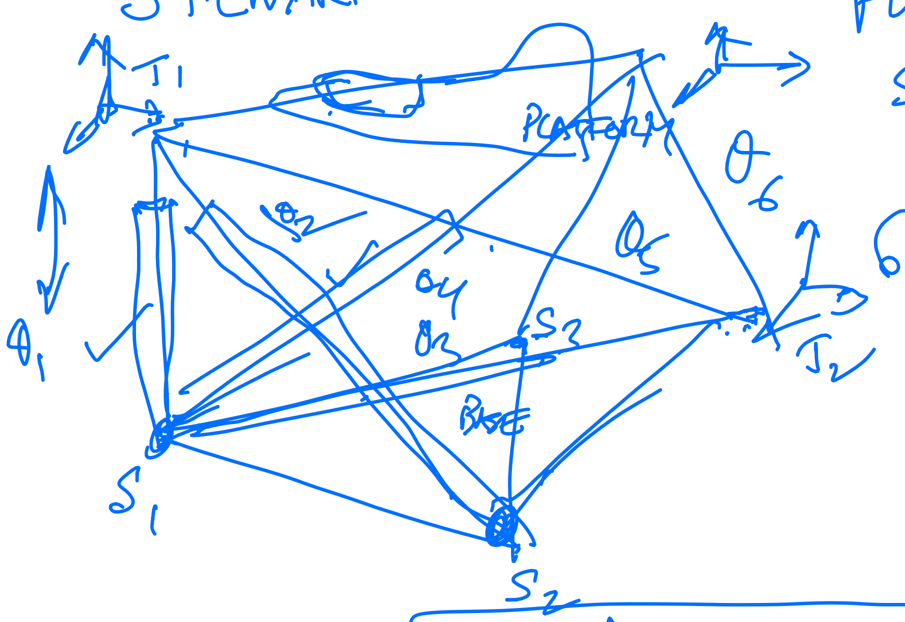
q_i

$\mathbb{R}^3 \ni \omega \qquad \hat{\omega} \in so(3) \quad \mathbb{R}^{3\times3}$

$$\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

STEWART PLATFORM

FLIGHT SIMULATORS



$$
\begin{aligned}
g_{s_1 T_1} &= e^{\xi_1 \theta_1} e^{\xi_2 \theta_2} g_{s_1 T_1}(0) \\
g_{s_2 T_2} &= \cdot \\
g_{s_3 T_3} &=
\end{aligned}
$$

Additional relationships between

$T_1, T_2, T_3 \longrightarrow$ See 5.4 of Chap 3 of MLS