EECS/BioE C106A/206A Introduction to Robotics

Lost Section 3

Sep 28 Mon 8 – 9 PM

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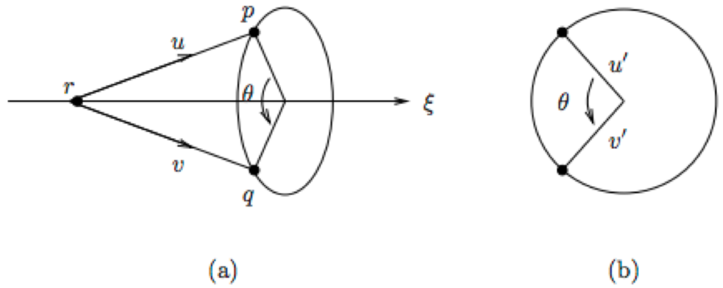
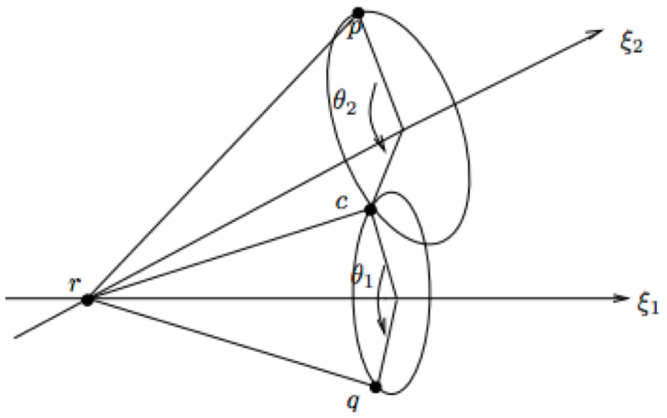
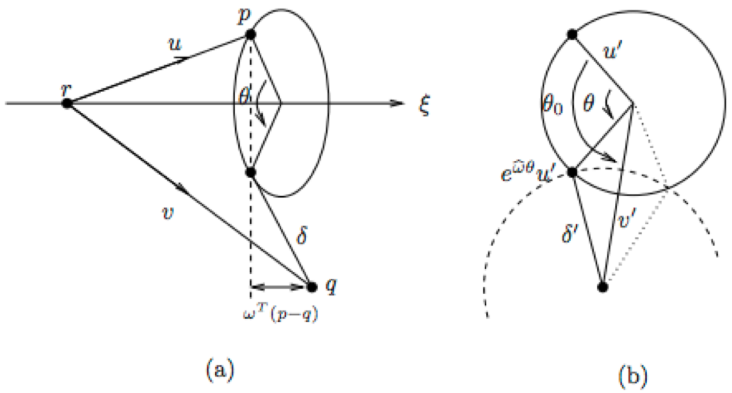
Inverse Kinematics

Given g_d, find $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ such that

$$e^{\hat{\xi}_1 \theta_1} \ldots e^{\hat{\xi}_n \theta_n} g_{st}(0) = g_d.$$

Padan-Kahan (PK) subproblems for revolute joints

subproblem 1	subproblem 2	subproblem 3
 (a) (b)		 (a) (b)
Find θ such that $$e^{\hat{\xi}\theta}p = q$$	Find θ_1, θ_2 such that $$e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}p = q$$	Find θ such that $$\parallel q - e^{\hat{\xi}\theta}p \parallel = \delta$$
The maximal number of solutions is 1.	The maximal number of solutions is 2.	The maximal number of solutions is 2.

Subproblems

subproblem 1: $e^{\hat{\xi}\theta}p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}p = q$

subproblem 3: $\| e^{\hat{\xi}\theta}p - q \| = \delta$

One approach: which angle we get first

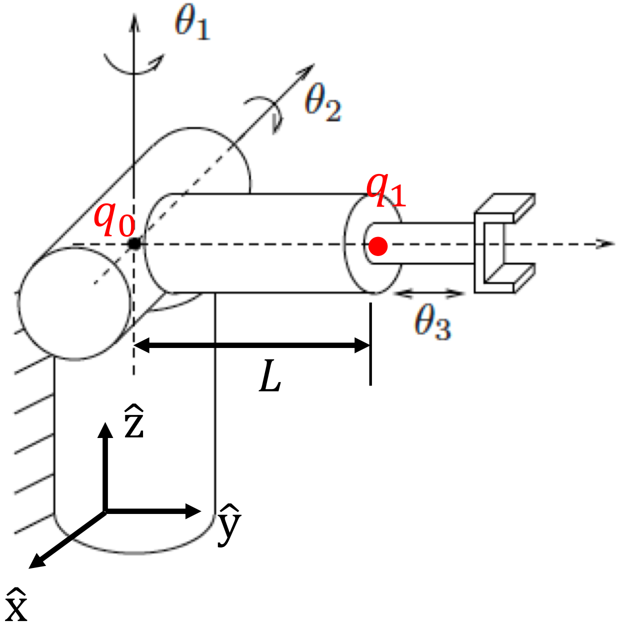
Step 1. find θs that decide the length between the end position and the origin

- θs of the prismatic joints (not subproblems)
- θs of the revolute joints (subproblem 3)

Step 2. find the other θs (all revolute joints): subproblem 1 and 2

- find the first part of θs by freezing the effect of the transformation of the latter part of θs (by choosing q on the rotation axis)

$g = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}$ is given.



Step 1. find θ_3

Step 2. find θ_1, θ_2

Let $g' := ge^{-\hat{\xi}_3\theta_3} = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}$

Subproblems

subproblem 1: $e^{\hat{\xi}\theta} p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$

subproblem 3: $\| e^{\hat{\xi}\theta} p - q \| = \delta$

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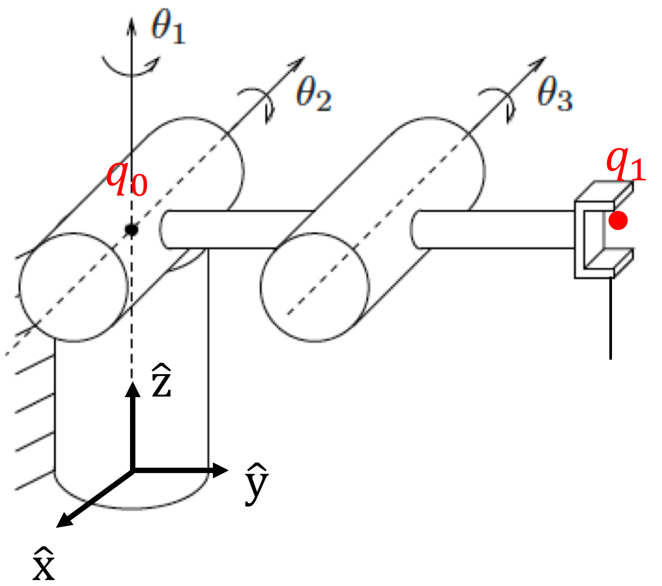

Step 1. find θ_3

Step 2. find θ_1, θ_2

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Subproblems

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subproblem 2: $e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}p = q$

subproblem 3: $\| e^{\hat{\xi}\theta}p - q \| = \delta$

One approach: which angle we get first

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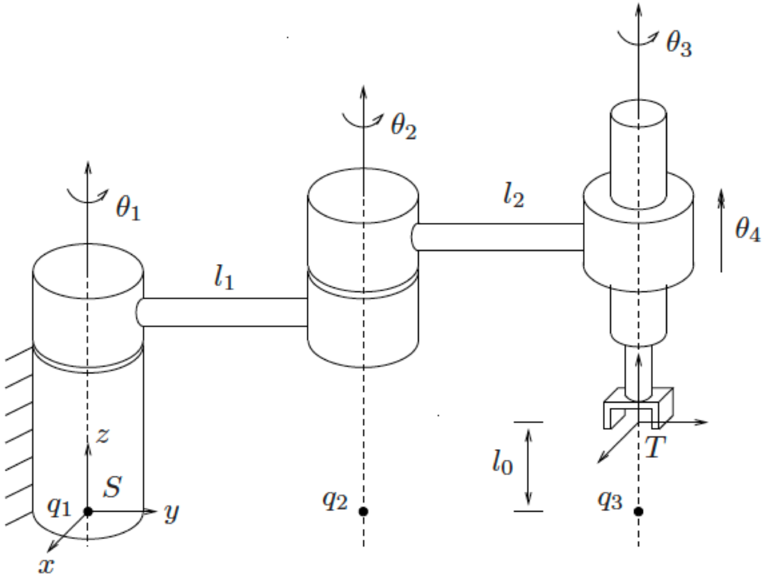
$g = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}e^{\hat{\xi}_4\theta_4}$ is given.



Step 1. find θ_2, θ_4

Step 2. find θ_1 and then θ_3

Subproblems

subproblem 1: $e^{\hat{\xi}\theta}p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}p = q$

subproblem 3: $\| e^{\hat{\xi}\theta}p - q \| = \delta$

One approach: which angle we get first

Step 1. find θs that decide the length between the end position and the origin

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Step 2. find the other θs (all revolute joints): subproblem 1 and 2

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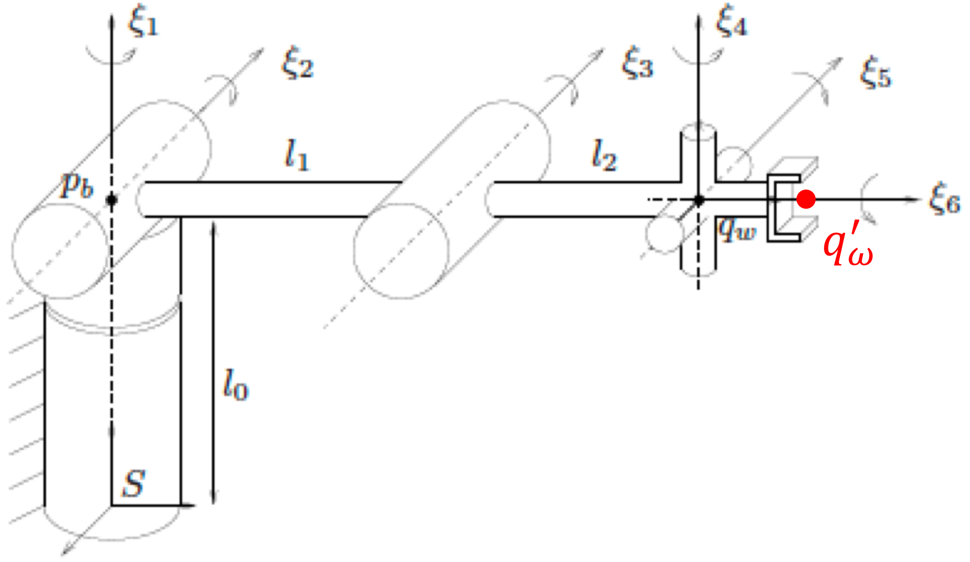
$g = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}$ is given.



Step 1. find θ_3

Step 2.1 find θ_1, θ_2 by choosing q_ω

Step 2.2 find $\theta_4, \theta_5, \theta_6$

- find θ_4, θ_5 by choosing q'_ω
- find θ_6