EECS/BioE C106A/206A Introduction to Robotics

Lost Section 5

Oct 30 Fri 7 – 9 PM

For any issue in this material, please email to donggun_lee@berkeley.edu

Velocities and Jacobian, and Singularities

- The spatial velocity and Jacobian: $V^s = J^s\dot{\theta}$
- The body velocity and Jacobian: $V^b = J^b\dot{\theta}$
- Singularities

$$\xi = (v, \omega)^T \in \mathbb{R}^6$$

$$\hat{\xi} = \begin{bmatrix} \widehat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times4}$$

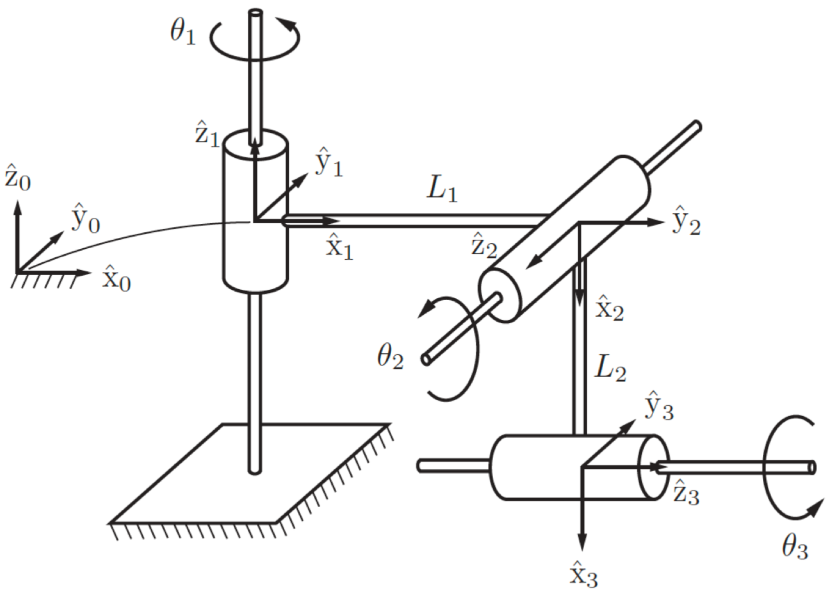
Spatial Jacobian: $V^s = J^s\dot{\theta}$

$\xi = (v, \omega)^T \in \mathbb{R}^6$

$\hat{\xi} = \begin{bmatrix} \widehat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times4}$

Body Jacobian: $V^b = J^b\dot{\theta}$

Example



Spatial Jacobian at $\theta = 0$

$$q_1' = [0,0,0]^T, \omega_1' = [0,0,1]^T$$
$$q_2' = [L_1, 0,0]^T, \omega_2' = [0,-1,0]^T$$
$$q_3' = [0,0,-L_2]^T, \omega_3' = [1,0,0]^T$$

$$J^s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -L_2 \\ 0 & -L_1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{\theta}$$

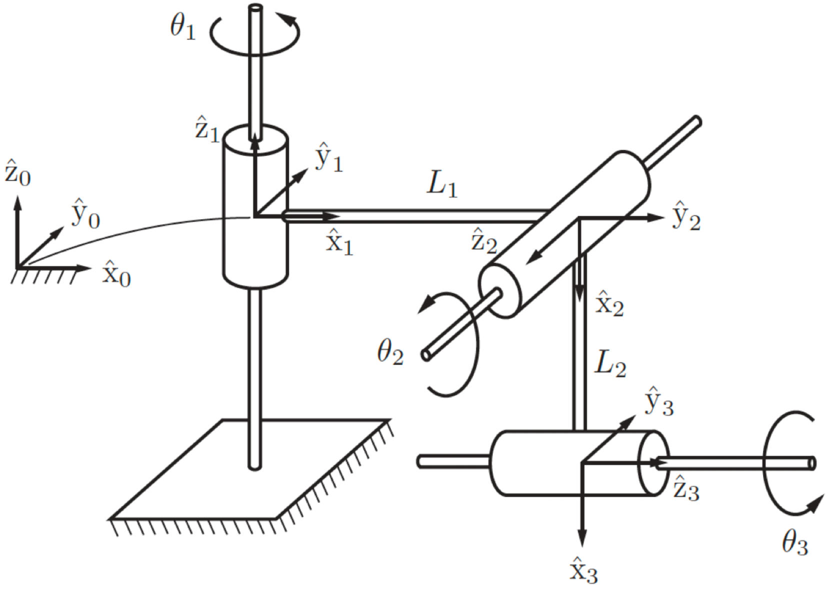
Body Jacobian at $\theta = 0$

$$q_1^\dagger = [0,0,-L_1]^T, \omega_1^\dagger = [-1,0,0]^T$$
$$q_2^\dagger = [-L_2, 0,0]^T, \omega_2^\dagger = [0,-1,0]^T$$
$$q_3^\dagger = [0,0,0]^T, \omega_3^\dagger = [0,0,1]^T$$

$$J^b = \begin{bmatrix} 0 & 0 & 0 \\ L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\theta}$$

You can choose other points for $q_1', q_2', q_3', q_1^\dagger, q_2^\dagger, q_3^\dagger$.

Example



Spatial Jacobian at $\theta = 0$

$$J^s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -L_2 \\ 0 & -L_1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{\theta}$$

Body Jacobian at $\theta = 0$

$$J^b = \begin{bmatrix} 0 & -L_2 & 0 \\ 0 & 0 & 0 \\ -L_1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\theta}$$

Using the rigid body transformation $g(0) = \begin{bmatrix} 0 & 0 & 1 & L_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

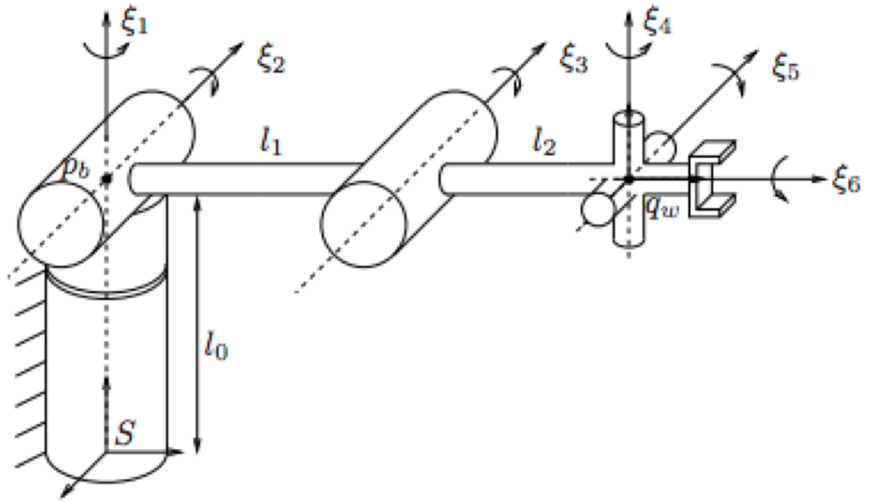
we can check $J^b = \mathrm{Ad}_{g^{-1}} J^s$.

Note that $\mathrm{Ad}_{g(0)} = \begin{bmatrix} 0 & 0 & 1 & 0 & L_2 & 0 \\ 0 & 1 & 0 & L_1 & 0 & -L_2 \\ -1 & 0 & 0 & 0 & L_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$, $\mathrm{Ad}_{g^{-1}(0)} = \begin{bmatrix} 0 & 0 & -1 & 0 & L_1 & 0 \\ 0 & 1 & 0 & L_2 & 0 & L_1 \\ 1 & 0 & 0 & 0 & -L_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

Singularity

The Jacobian J^s (or J^b) is singular if J^s is not full rank.

- check the linear dependency of the columns in J^s with a particular θ
- *find a particular θ such that the robot cannot achieve a certain direction of translational or angular velocity*
- find a particular θ such that two axes are on the same line



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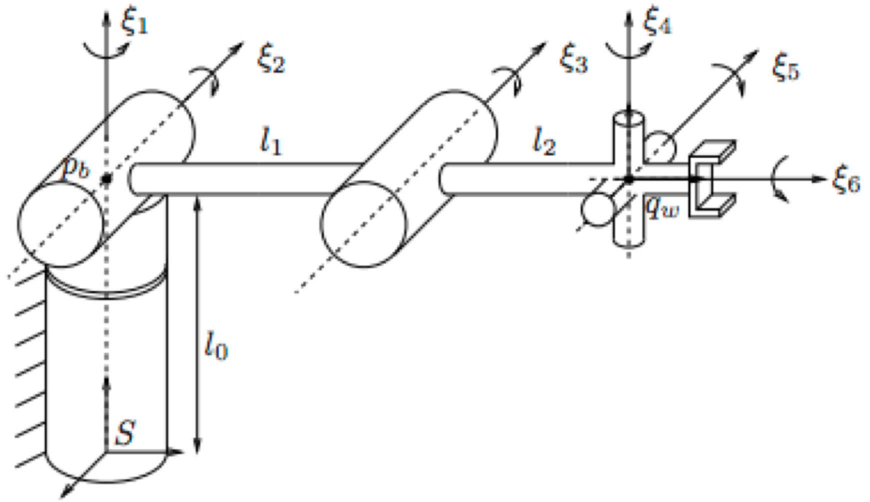

- The *y*-translational velocity cannot be achieved at the zero configuration.
- Six joints must cannot achieve the six dimensional velocities (translation + rotation)
- Thus, we have singularity at the zero configuration.

We should check all translational and angular velocities.

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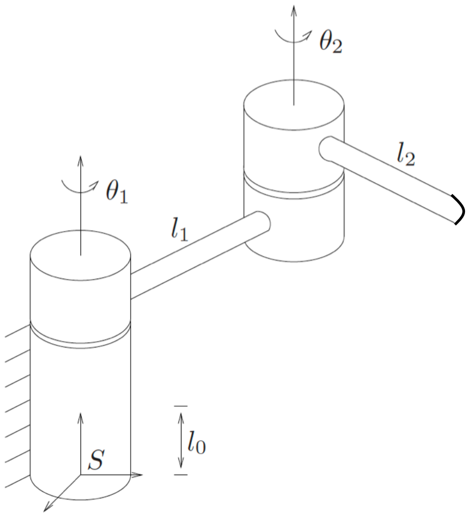

$$J^s = \begin{bmatrix} 0 & l_1 \cos\theta_1 \\ 0 & l_1 \sin\theta_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

- In this problem, for any θ, the robot is not singular.
- Note: at $\theta = 0$, y-directional velocity cannot be achieved. However, the robot is not singular. We should also consider the angular velocity of the robot, which always span 1-dimensional space.

Warning: We should check all translational and angular velocities.

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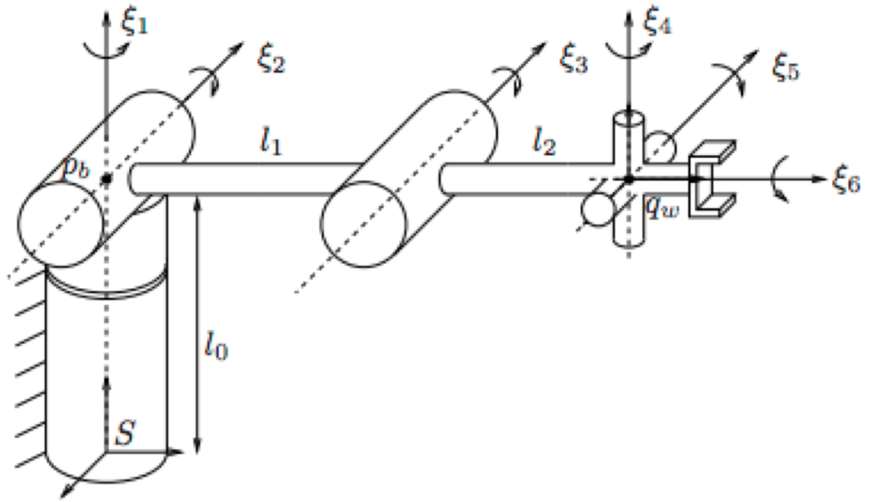


Goal: find θ_2 and j such that $\xi_1 = \pm\xi_j$

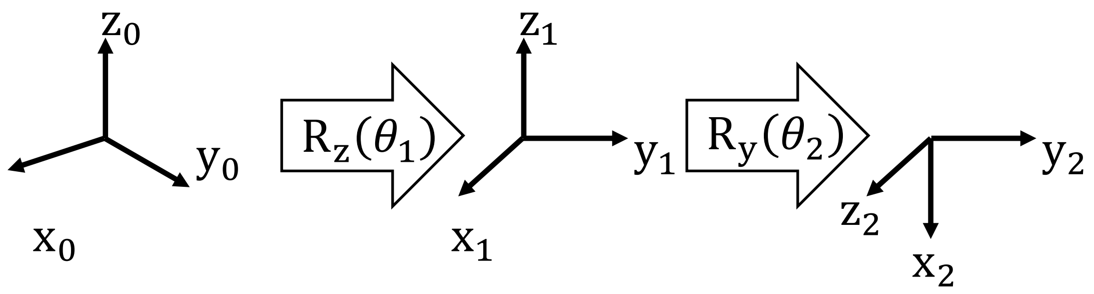
Answer:

Singularity

The Jacobian J^s (or J^b) is singular if J^s is not full rank.

- check the linear dependency of the columns in J^s with a particular θ
- find a particular θ such that the robot cannot achieve a certain direction of translational or angular velocity
- *find a particular θ such that two axes are on the same line*

Euler ZYX



Goal: find θ_2 such that $z_0 = \pm x_2$.

Answer: