EECS/BioE C106A/206A
Introduction to Robotics

Lost Section 3

Sep 28 Mon 8 – 9 PM

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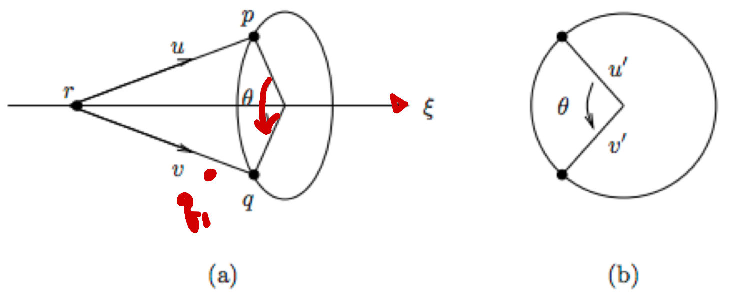
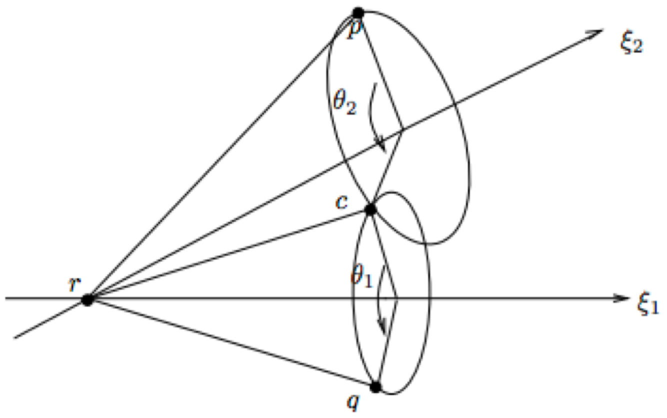
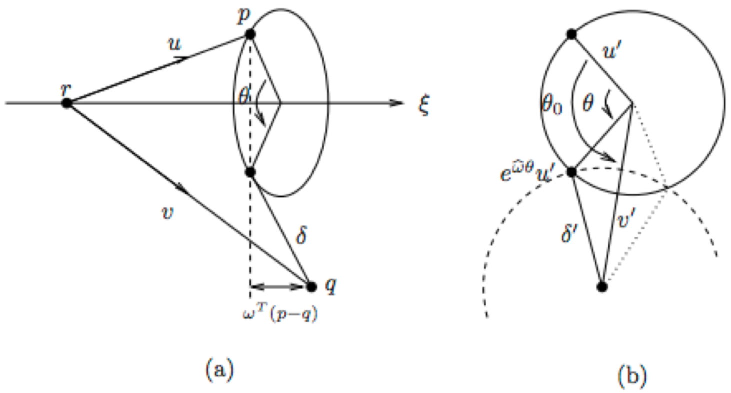
Inverse Kinematics

Given g_d, find $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ such that

$$e^{\hat{\xi}_1 \theta_1} \ldots e^{\hat{\xi}_n \theta_n} g_{st}(0) = g_d.$$

Padan-Kahan (PK) subproblems for revolute joints

subproblem 1	subproblem 2	subproblem 3
		
Find θ such that $e^{\hat{\xi}\theta}p = q$	Find θ_1, θ_2 such that $e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}p = q$	Find θ such that $\parallel q - e^{\hat{\xi}\theta}p \parallel = \delta$
The maximal number of solutions is 1.	The maximal number of solutions is 2.	The maximal number of solutions is 2.

Subproblems

subproblem 1: $e^{\hat{\xi}\theta} p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$

subproblem 3: $\| e^{\hat{\xi}\theta} p - q \| = \delta$

One approach: which angle we get first

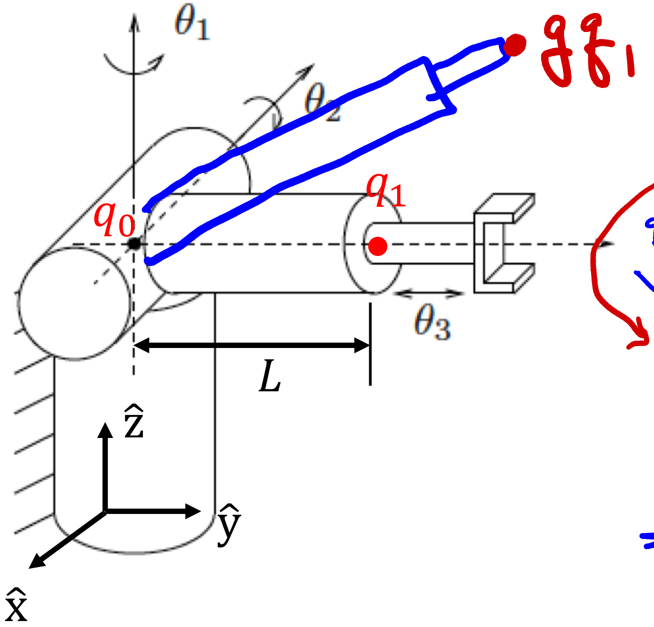
Step 1. find θs that decide the length between the end position and the origin

- θs of the prismatic joints (not subproblems)
- θs of the revolute joints (subproblem 3)

Step 2. find the other θs (all revolute joints): subproblem 1 and 2

- find the first part of θs by freezing the effect of the transformation of the latter part of θs (by choosing q on the rotation axis)

$g = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3}$ is given.



Step 1. find θ_3

$\| g q_1 - q_0 \| = \text{func}(\theta_3)$

$= \| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} q_1 - q_0 \|$

$q_0 = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} q_0$

$= \| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} q_1 - q_0) \| = \| e^{\hat{\xi}_3\theta_3} q_1 - q_0 \|$

$= L + \theta_3$

$\rightarrow \theta_3 = -L + \| g q_1 - q_0 \|$

Step 2. find θ_1, θ_2

Let $g' := g e^{-\hat{\xi}_3\theta_3} = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2}$

$\underbrace{g' q_1}_{\text{known}} = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} q_1$: subprob 2.

Subproblems

subproblem 1: $e^{\hat{\xi}\theta} p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$

subproblem 3: $\| e^{\hat{\xi}\theta} p - q \| = \delta$

One approach: which angle we get first

Step 1. find θs that decide the length between the end position and the origin

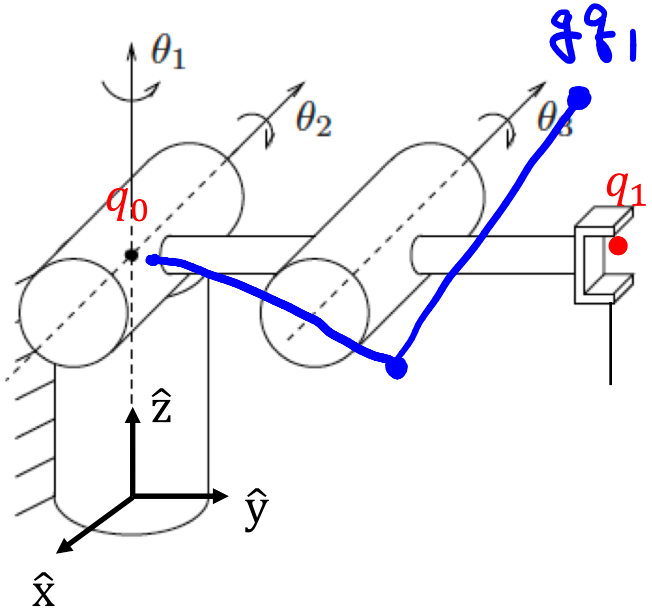
- θs of the prismatic joints (not subproblems)
- θs of the revolute joints (subproblem 3)

Step 2. find the other θs (all revolute joints): subproblem 1 and 2

- find the first part of θs by freezing the effect of the transformation of the latter part of θs (by choosing q on the rotation axis)

$e^{\hat{\xi}_1\theta_1}\left(e^{\hat{\xi}_2\theta_2} q_0\right) = q_0$

$g = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3}$ is given.



Step 1. find θ_3

$\delta = \underbrace{\| g q_1 - q_0 \|}_{} = \text{func}(\theta_3)$

$\| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} q_1 - q_0 \|$

$q_0 = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} q_0$

$\| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} q_1 - q_0) \|$

$\| e^{\hat{\xi}_3\theta_3} \overset{p}{q_1} - \overset{q}{q_0} \|$

Step 2. find θ_1, θ_2

Let $\boxed{g'} := \underline{g e^{-\hat{\xi}_3\theta_3}} = \underline{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2}}$

$\underbrace{g' q_1}_{q} = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \underbrace{q_1}_{p}$: subprob 2.

Subproblems

subproblem 1: $e^{\hat{\xi}\theta} p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$

subproblem 3: $\| e^{\hat{\xi}\theta} p - q \| = \delta$

One approach: which angle we get first

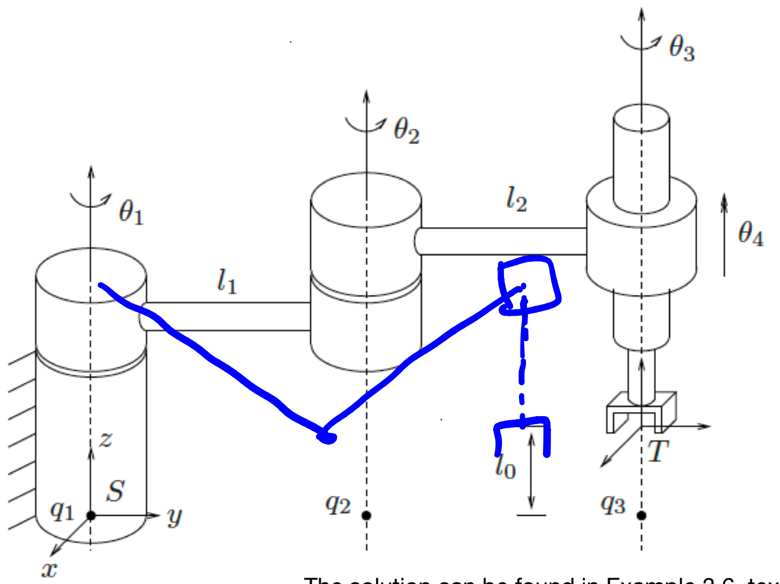
Step 1. find θs that decide the length between the end position and the origin

- θs of the prismatic joints (not subproblems)
- θs of the revolute joints (subproblem 3)

Step 2. find the other θs (all revolute joints): subproblem 1 and 2

- find the first part of θs by freezing the effect of the transformation of the latter part of θs (by choosing q on the rotation axis)

$g = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4}$ is given.



The solution can be found in Example 3.6, textbook pg 106 - 107.

Step 1. find θ_2, θ_4

Step 2. find θ_1 and then θ_3

Subproblems

subproblem 1: $e^{\hat{\xi}\theta} p = q$

subproblem 2: $e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = q$

subproblem 3: $\| e^{\hat{\xi}\theta} p - q \| = \delta$

One approach: which angle we get first

Step 1. find θs that decide the length between the end position and the origin

- θs of the prismatic joints (not subproblems)
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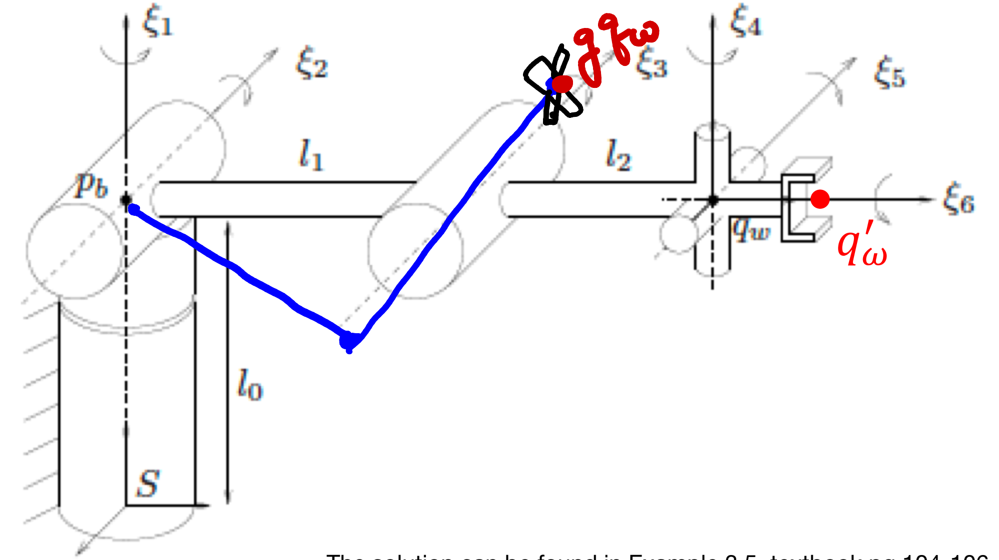
Step 2. find the other θs (all revolute joints): subproblem 1 and 2

- find the first part of θs by freezing the effect of the transformation of the latter part of θs (by choosing q on the rotation axis)

$g = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6}$ is given.



The solution can be found in Example 3.5, textbook pg 104-106.

$g q_w = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} q_w$, $p_b = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p_b$

Step 1. find θ_3

$\underbrace{\| g q_w - p_b \|}_{\delta} = \text{func}(\theta_3) = \| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} q_w - p_b \| = \| e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} (e^{\hat{\xi}_3\theta_3} q_w - p_b) \|$

$= \| e^{\hat{\xi}_3\theta_3} \underbrace{q_w}_{p} - \underbrace{p_b}_{q} \|$

Step 2.1 find θ_1, θ_2 by choosing q_ω

Step 2.2 find $\theta_4, \theta_5, \theta_6$

- find θ_4, θ_5 by choosing q'_ω

- find θ_6