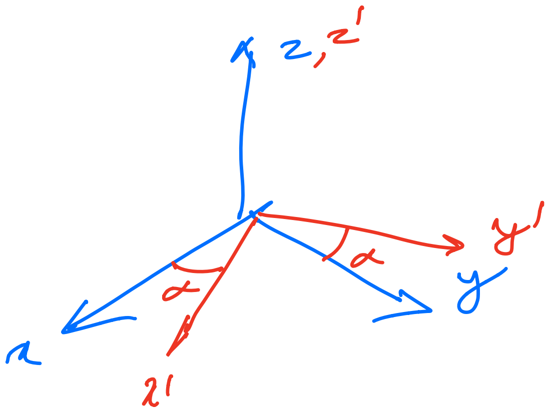
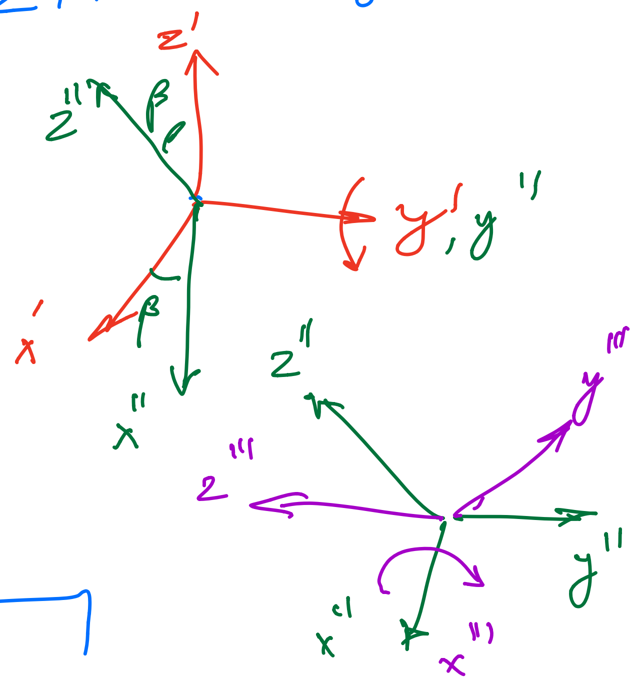


ZYX Euler Angles



$$R_{x''}(\gamma)\, R_{y'}(\beta)\, R_z(\alpha)$$

$$\hat{y}' = R_z(\alpha)\, y = R_z(\alpha)\, \hat{y}\, R_z^T(\alpha)$$

$$R_{y'}(\beta) = e^{\hat{y}'\beta}$$

$$= R_z(\alpha)\, e^{\hat{y}\beta}\, R_z^T(\alpha) \qquad I$$

$$R_{y'}(\beta)\, R_z(\alpha) = R_z(\alpha)\, e^{\hat{y}\beta}\, \cancel{R_z^T(\alpha) R_z(\alpha)}$$

$$= R_z(\alpha)\, R_y(\beta)$$

$$\hat{x}'' = R_z(\alpha) R_y(\beta)\, \hat{x}\, R_y^T(\beta)\, R_z^T(\alpha)$$

$$e^{\hat{x}''\gamma} = R_z(\alpha)\, R_y(\beta)\, e^{\hat{x}\gamma}\, R_y^T(\beta)\, R_z^T(\alpha) \qquad I$$

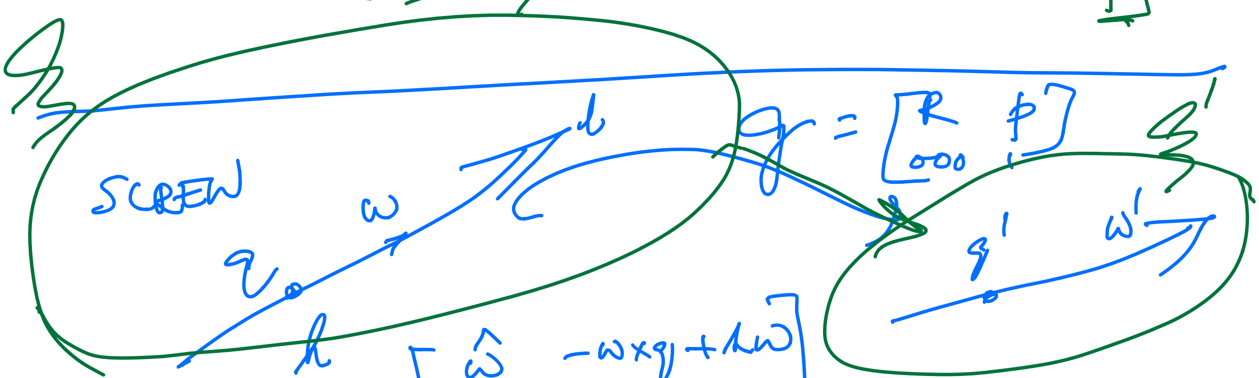
$R_z(\alpha)$

$R_y(\beta)$

$$R = \underbrace{R_z(\alpha)\, R_y(\beta)\, R_x(\gamma)}$$

→ ZYX Euler

→ XYZ Euler



$$g = \begin{bmatrix} R & p \\ 000 & 1 \end{bmatrix}$$

TWIST $\hat{\xi} = \begin{bmatrix} \hat{\omega} & -\omega \times q + h\omega \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} q' \\ 1 \end{bmatrix} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ 1 \end{bmatrix}$$

$q' = Rq + p$

$\hat{\omega}' = R\hat{\omega}R^T$

$\omega' = R\omega$

$$\hat{\xi}' = \begin{bmatrix} \hat{\omega}' & -\omega' \times q' + h\omega' \\ 0 & 0 \end{bmatrix}$$

$\hat{\omega}' = R\hat{\omega}R^T$

RES 1

Claim: $\boxed{\hat{\xi}' = g\,\hat{\xi}\,g^{-1}}$

Proof

$$RHS = \begin{bmatrix} R & p \\ 000 & 1 \end{bmatrix} \begin{bmatrix} \hat{\omega} & -\omega \times q + h\omega \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R\hat{\omega} & -R\omega \times q + hR\omega \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R^T & -R^T p \\ 00 & 1 \end{bmatrix}$$

$v = -\omega \times q + h\omega$

$v' = -\omega' \times q' + h\omega'$

$= -R\omega \times (Rq + p)$

$+ hR\omega$

$$= \begin{bmatrix} R\hat{\omega}R & -R\omega R p - R\omega \times q + hR\omega \\ 0 & 0 \end{bmatrix}$$

$$LHS = \begin{bmatrix} \hat{\omega}' & -\hat{\omega}' \times q' + h\omega' \\ 0\,0\,0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} R\hat{\omega}R^T & -R\omega \times (Rq + p) + hR\omega \\ 0 & 0 \end{bmatrix}$$

$$\mathbb{R}^{4\times 4} \quad \hat{\xi}' = g\hat{\xi}g^{-1}$$

$$\xi' = \begin{pmatrix} v' \\ \omega' \end{pmatrix} \in \mathbb{R}^6 \qquad \xi = \begin{pmatrix} v \\ \omega \end{pmatrix} \in \mathbb{R}^6$$

$$\omega' = R\omega$$

$$\boxed{v' = -R\hat{\omega}R^T p + Rv}$$

$$= -(R\omega) \times p + Rv$$

$$= p \times R\omega + Rv$$

$$= \hat{p}R\omega + Rv$$

$$\hat{p} = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$$

RES 2

$$\mathbb{R}^6 \quad \underbrace{\begin{pmatrix} v' \\ \omega' \end{pmatrix}}_{\xi'} = \underbrace{\begin{pmatrix} R & \hat{p}R \\ 0 & R \end{pmatrix}}_{6} \underbrace{\begin{pmatrix} v \\ \omega \end{pmatrix}}_{\xi} \quad \mathbb{R}^6$$

S

6

Ad_g adjoint

$$\left\{\begin{aligned} \omega' &= R\omega \\ v' &= Rv + \hat{p}R\omega \end{aligned}\right\}$$

$$\hat{\xi}' = g\hat{\xi}g^{-1}$$

$$v = -\omega \times q + h\omega$$

$$\begin{aligned}
v' &= -\omega' \times q' + h\omega' \\
&= -R\omega \times (Rq + p) + hR\omega \\
&= -\widehat{R\omega}Rq - \widehat{R\omega}p + hR\omega \\
&= -R(\omega \times q) + hR\omega - R\hat{\omega}p \\
&= Rv - R\hat{\omega}p \\
&= Rv - (R\omega) \times p \\
&= Rv + p \times R\omega \\
v' &= Rv + \hat{p}R\omega
\end{aligned}$$