Lecture EECS 106A/206A

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

$\theta \in \mathbb{R}^n \qquad \tau \in \mathbb{R}^n.$

1. Computed Torque

$\theta_d(t) \qquad t \in [0, T]$

$$\tau = M(\theta)\left(\ddot{\theta}_d - K_v(\dot{\theta} - \dot{\theta}_d) - K_p(\theta - \theta_d)\right) + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta})$$

$K_v, K_p \succ 0 \in \mathbb{R}^{n \times n}$ positive definite

$$\theta - \theta_d = e_\theta$$

$$\underline{M}(\theta)\left(\ddot{e}_\theta + K_v \dot{e}_\theta + K_p e_\theta\right) = 0$$

Since $M^{-1}(\theta)$ exists

$$\ddot{e}_\theta + K_v \dot{e}_\theta + K_p e_\theta = 0 \qquad \in \mathbb{R}^n \qquad (1)$$

$K_v, K_p > 0 \Rightarrow$ e.v. of [illegible] are $\in \mathbb{C}_-$

(open left half plane)

$[0, T]$

Speed of dynamics $\lambda_1, \lambda_2 \in \mathring{\mathbb{C}}_-$

$e^{\lambda_i t} \to 0$ as $t \to \infty$

$$e^{(\mathrm{Re}\,\lambda_i)t} = e^{-3} = .05$$

Settling time $t_s = \dfrac{3}{|(\mathrm{Re}\,\lambda_i)|} \ll T$

$|\mathrm{Re}\,\lambda_i|$ large

too large {control signals are high in magnitude}

→ COMPUTED TORQUE ←

(K_v, K_p).

GAIN SCHEDULING

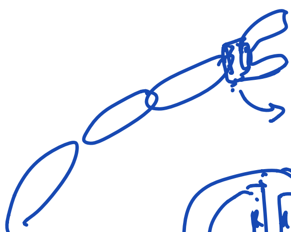
i = payload
1. light
⋮
10 heavy

$M^i(\theta)$ → GEN MOM. OF INERTIA

$C^i(\theta, \dot\theta)$ $N^i(\theta, \dot\theta)$

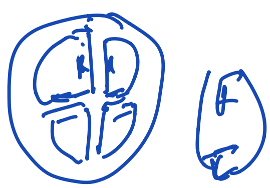
$$\tau^i_{CT}(\cdot) = M^i(\theta)(\ddot\theta_d + ---) + C^i(\theta,\dot\theta)\dot\theta + N^i(\theta, \dot\theta)$$

$i = 1, \ldots, 10$



WRIST FORCE/TORQUE SENSOR

LORD'S (CARY, NC)



MASS BEING HELD

↓ i

Blend

$i=1$ .. $i=2$

MANUFACTURER   Specifications

$\theta_d(t)$ in 0-10 Hz

Max overshoot of ( )...

Payload - [0, 10] lbs.

ADEPT ROBOTS   GOOGOL

Modified P. D.

$$\tau = M(\theta)\ddot{\theta}_d + C(\theta,\dot{\theta})\dot{\theta}_d + N(\theta,\dot{\theta}) - K_v(\dot{\theta}-\dot{\theta}_d) - K_p(\theta-\theta_d)$$

$\theta_d \equiv \bar{\theta}_d \quad \ddot{\theta}_d = 0 \quad \dot{\theta}_d = 0$

$$\tau = -K_v \dot{e}_\theta - K_p e_\theta + N(\theta,\dot{\theta})$$

P.D. are a good model for control of limbs, muscles

{ Golgi tendon organs e
{ Muscle spindle $\dot{e}$ }

P. I. D.
integral term

$K_I > 0$

$$\tau = M\left(\ddot{\theta} - K_v\dot{e} - K_p e - K_I\int_0^t e(\tau)\,d\tau\right) + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta})$$

Integral Controller $K_I \int_0^t e(\tau) d\tau$

K_I small hedge against modeling mismatches

large could cause instability

$\int_0^{t} e(\tau) d\tau$ $t \uparrow$ need to forget old $e(t)$

Reset Integral Wind up

$$\int_0^t e^{-\alpha(t-\tau)} e(\tau) d\tau$$

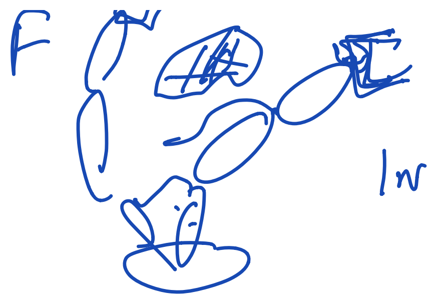
Forgetting Factor

EECS 222 / ME 237

PD & PID → Lab 5

Task Space Control

F I $\theta_{d1}(\cdot)\ \theta_{d2}\ \theta_{d3}$



Inv. Kin

$\downarrow$

θ CT

CAN ONE WORK DIRECTLY IN TASK COORDINATES

$g \in SE(3) \sim x \in \mathbb{R}^6$

$\theta \xrightarrow{f} x$ f FORWARD KIN.

θ ~~to~~ a parameter. of $g \in SE(3)$

$\theta \underset{f^{-1}}{\overset{f}{\rightleftarrows}} x$ is bijective

one to one

onto

$\theta \in \mathbb{R}^6 \xrightarrow{f} x \in \mathbb{R}^6$ | $\theta \in \mathbb{R}^4 \rightarrow x \in \mathbb{R}^4 \subset SE(3)$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

$x = f(\theta)$

SEC. 5.4

$$\dot{x} = Df(\theta)\,\dot{\theta} \in \mathbb{R}^{6\times 6}$$

OF CHAP 4 (MLS)

$$= \left[\frac{\partial f_i}{\partial \theta_j}\right]_{ik} \quad jk$$

$$\dot{x} = \underline{J(\theta)}\,\dot{\theta}$$

J of map from config space to work space.

$$\dot{\theta} = J^{-1}(\theta)\,\dot{x}$$

$$\ddot{\theta} = J^{-1}(\theta)\,\ddot{x} + \frac{d}{dt}\left(J^{-1}(\theta)\right)\dot{x}$$

$$J^{-T}(\theta)\times\Big[M(\theta)J^{-1}(\theta)\ddot{x} + M(\theta)\frac{d}{dt}\left(J^{-1}(\theta)\right)\dot{x} + C(\theta,\dot{\theta})J^{-1}(\theta)\dot{x} + N(\theta,\dot{\theta})\Big] = J^{-T}(\theta)\,\tau$$

$$\underbrace{J^{-T}(\theta)M(\theta)J^{-1}(\theta)}\ddot{x} + \Big[J^{-T}(\theta)M(\theta)\frac{d}{dt}\left(J^{-1}(\theta)\right) + J^{-T}(\theta)C(\theta,\dot{\theta})J^{-1}(\theta)\Big]\dot{x} + J^{-T}(\theta)N(\theta,\dot{\theta}) = J^{-T}(\theta)\,\tau$$

$$\theta = f^{-1}(x)$$

$$\tilde{M}(x)\ddot{x} + \tilde{C}(x,\dot{x})\dot{x} + \tilde{N}(x,\dot{x}) = F = J^{-T}(f^{-1}(x))\tau$$

$$\tilde{M}(x)\ddot{x} + \tilde{C}(x,\dot{x}) + \tilde{N}(x,\dot{x}) = F$$

$$\tilde{M} = \underbrace{J^{-T}(f^{-1}(x))\, M(f^{-1}(x))\, J^{-1}(f^{-1}(x))}_{\text{SYMMETRIC}}$$

$\dot{M} - 2C$ skew symmetric

$\dot{\tilde{M}} - 2\tilde{C}$ "

$$\underline{\tilde{M}(x)\ddot{x} + \tilde{C}(x,\dot{x})}\dot{x} + \tilde{N}(x,\dot{x}) = F$$

$x_d(t)$ — motion of robot in task coordinates

$$F = \tilde{M}(x)(\ddot{x}_d - K_v(\dot{x} - \dot{x}_d) - K_p(x - x_d)) + \tilde{C}(x,\dot{x})\dot{x} + \tilde{N}(x,\dot{x})$$

COMPUTED TORQUE CONTROLLER IN TASK COORDINATES

F?

$$F = J^{-T}(f^{-1}(x))\,\tau$$

TORQUES

$$\tau = J^{T}(f^{-1}(x))\,F$$

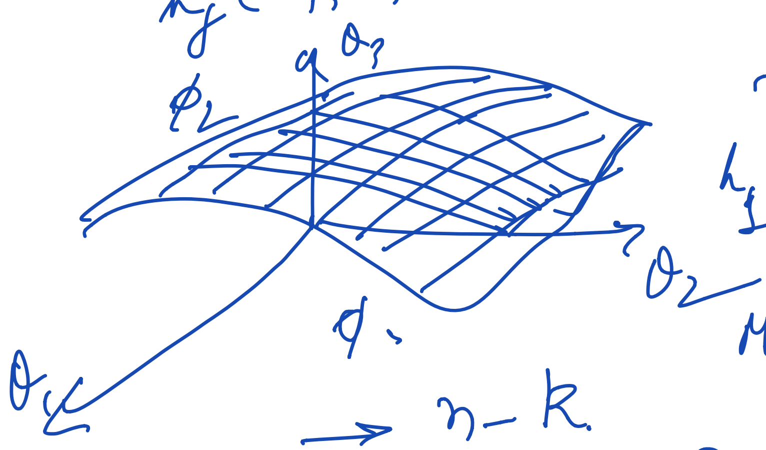
$$= \underline{J^{T}(\theta)}\,F$$

CONTROL OF CONSTRAINED MANIPULATION

Model Constraint

$\theta_1 \ldots \theta_n = q$ CONFIGURATION SPACE

$$h_j(\theta_1, \ldots, \theta_n) = 0 \quad j = 1, \ldots, K$$



$n = 3$

$h_1(\theta_1, \theta_2, \theta_3) = 0$

Manifold

$\rightarrow n - K$

$3 - 1 = 2$ dimension

$\phi_1, \phi_2 \ldots \phi_{n-k}$ parameterize constraints.

SURFACES OF CONST.

$$h_j(f_1(\phi) \ldots f_n(\phi)) = 0 \quad j = 1 \ldots k$$

$$\begin{aligned} \theta_1 &= f_1(\phi) \\ \theta_2 &= f_2(\phi) \\ &\vdots \\ \theta_n &= f_n(\phi) \end{aligned}$$

$f : \phi \to \theta$

$\mathbb{R}^{n-k} \to \mathbb{R}^n$

WANT ROBOT TO LIVE ON

$$h_j(\theta_1 \ldots \theta_n) = 0 \quad j = 1 \ldots k$$

$$\theta = f(\phi)$$

$$\dot{\theta} = \frac{df}{d\phi} \dot{\phi}$$

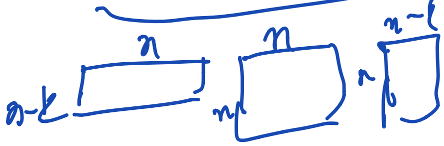
$$\dot{\theta} = \underbrace{J(\phi)}_{n \times (n-k)} \dot{\phi}$$

$$\ddot{\theta} = J(\phi) \ddot{\phi} + \frac{d}{dt} J(\phi) \dot{\phi}$$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

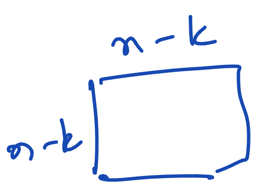
$$J^T(\phi)\Big[\underline{M(\theta)\,J(\phi)}\,\ddot{\phi} + \boxed{M(\theta)\,\frac{d}{dt}J(\phi)\,\dot{\phi} + C(\theta,\dot{\theta})\,J(\phi)\,\dot{\phi}} + \underline{N(f(\phi), J\dot{\phi})} = \tau\Big]$$

$$\underbrace{J^T(\phi)\,M(f(\phi))\,J(\phi)}\,\ddot{\phi} + J^T(\phi)\left[M(f(\phi))\,\dot{J}(\phi) + C(f(\phi), J\dot{\phi})\right]\dot{\phi} + J^T(\phi)\,N(f(\phi), J\dot{\phi})$$



$$= J^T(\phi)\,\tau$$

$$\tilde{M}(\phi)\,\ddot{\phi} + \tilde{C}(\phi,\dot{\phi})\,\dot{\phi} + \tilde{N}(\phi,\dot{\phi}) = F = J^T(\phi)\,\tau$$



$\phi_d(t)$ trajectory on constant surface

$$F = \tilde{M}(\phi)\left(\ddot{\phi}_d - K_v(\dot{\phi} - \dot{\phi}_d) - K_p(\phi - \phi_d)\right) + \tilde{C}(\phi, \dot{\phi})\dot{\phi} + \tilde{N}(\phi, \dot{\phi})$$

$$\phi^{(\cdot)} \longrightarrow \phi_d^{(\cdot)}$$

What about τ ?

$$F = J^T \tau.$$

$$F = {}_{k}\underbrace{\overset{n}{\boxed{\qquad\qquad}}}\ \tau$$

Give F, how do you solve for τ,

$$F = J^T \tau$$

J^T has rank $(n-k)$

$\mathcal{N}(J^T)$

$$h(f_1(\phi) \ldots f_n(\phi)) = 0$$

$$h = \begin{bmatrix} h_1 \\ \vdots \\ h_k \end{bmatrix}$$

$$Dh \cdot J = 0$$

$${}_{k}\overset{n}{\boxed{\qquad}}\ {}_{n}\overset{n-k}{\boxed{\quad}}$$

$$J^T Dh^T = 0 \qquad \mathcal{N}(J^T)$$

$x^T x \ldots$

$$\frac{\nabla h_j}{\partial \theta} = \left(\frac{\partial h_j}{\partial \theta}\right)^T \qquad = \mathrm{Span}\left\{\frac{\partial h_1}{\partial \theta}, \frac{\partial h_2}{\partial \theta}, \dots \frac{\partial h_k}{\partial \theta}\right\}$$

$$n(J^T) = \sum_{i=1}^{k} \lambda_i \frac{\partial h_i}{\partial \theta}$$

$$F = J^T \tau$$

$$\tau_{LS} = J\mu \quad \in R(J)$$

$$F = J^T J \mu \Rightarrow \mu = (J^T J)^{-1} F$$

$$\tau_{LS} = J (J^T J)^{-1} F$$

$$\tau = \tau_{LS} + \sum_{i=1}^{k} \lambda_i \frac{\partial h_i}{\partial \theta}^T$$

$$\tau = J(J^T J)^{-1} F + \underbrace{\sum_{i=1}^{k} \lambda_i \frac{\partial h_i}{\partial \theta}}_{\text{Can be chosen}}$$

$J^{\dagger T}$

GEN FORCES

$\perp$ to surface

$$J^T \tau = (J^T J)(J^T J)^{-1} F + J^T(\cancel{\ \ }) \quad 0$$

Contact Forces

$$J^T \underbrace{\left[J(J^TJ)^{-1}\right]}_{(J^T)^{+}} = I$$

$$F = J^T \tau$$

$$\tau = (J^T)^{+} F$$

$$+ \tau_N$$

$$= (J^T)^{+} F + \boxed{\sum_{i=1}^{k} \lambda_i \left(\frac{\partial h_i}{\partial \theta}\right)^T}$$

F_N^d FORCE SPECIFIED

$$F_N^d = \sum_{i=1}^{k} \lambda_i \left(\frac{\partial h_i}{\partial \theta}\right)$$

$$\boxed{\lambda_i \geq 0}$$

SEC 6.1 OF Chapter 4