EECS/BioE C106A/206A Introduction to Robotics

Lost Section 2

Sep 25 Fri 7 – 9 PM

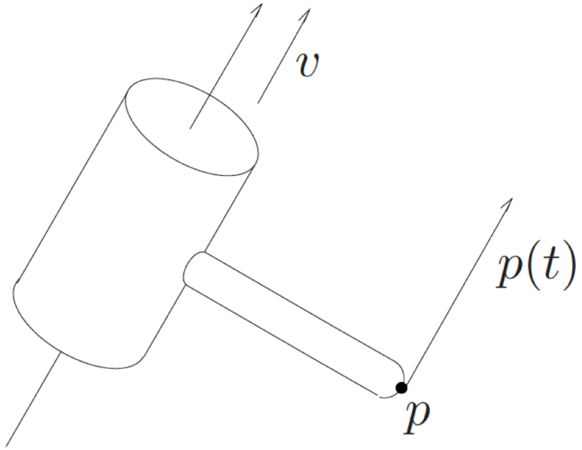
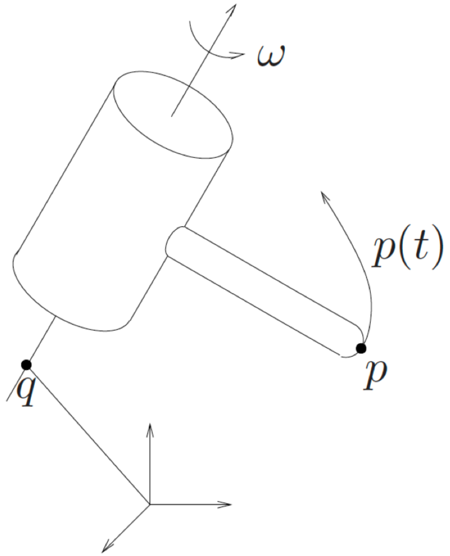
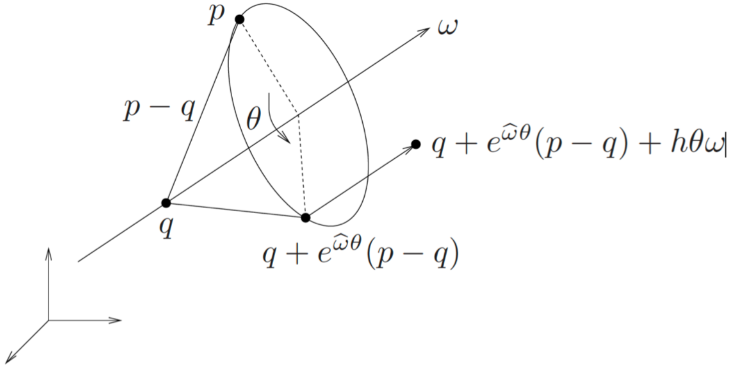
For any issue in this material, please email to donggun_lee@berkeley.edu

Contents

- Theory
  - Forward kinematics

- Example

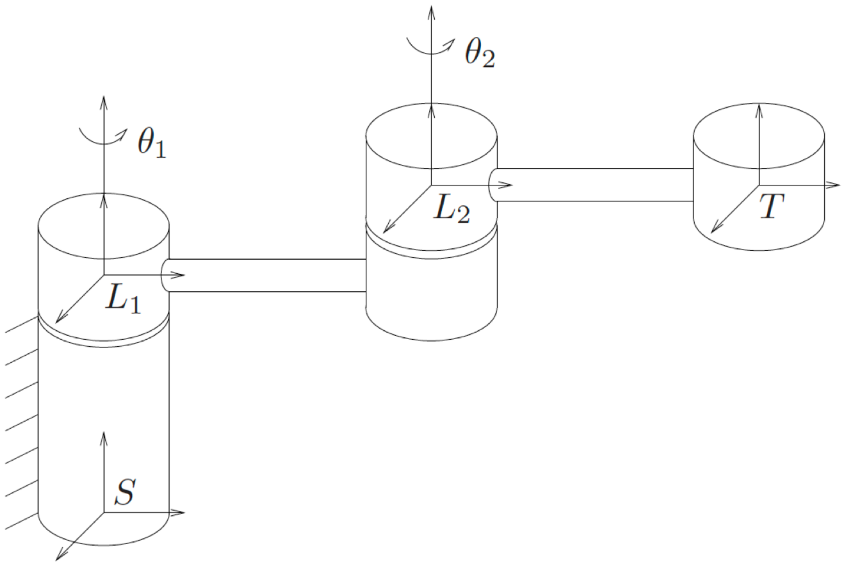
Types of Joints and Twists

<table>
<tr><th>Prismatic joint</th><th>Revolute joint</th><th>Screw joint</th></tr>
<tr>
<td>
</td>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v \\ 0 \end{bmatrix}$</td>
<td>$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$</td>
<td>$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -\omega \times q + h\omega \\ \omega \end{bmatrix}$</td>
</tr>
</table>

ω: rotation axis, v: translational direction/velocity, q: a point on the rotation axis (center of the rotation), h: pitch

Product of exponentials

A two degree of freedom manipulator



$$g_{ST}(\theta_1, \theta_2) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} g_{ST}(0)$$

$\xi_1 = (v_1, \omega_1)$ and $\xi_2 = (v_2, \omega_2)$ are coordinates with respect to frame S.

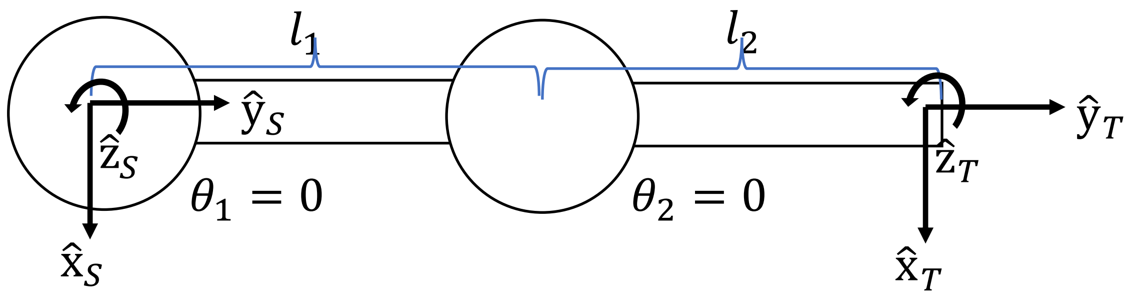
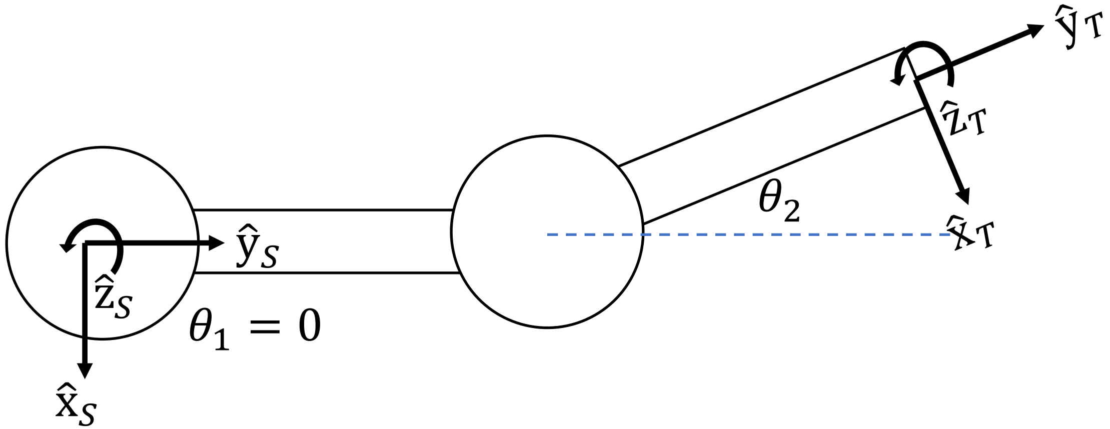
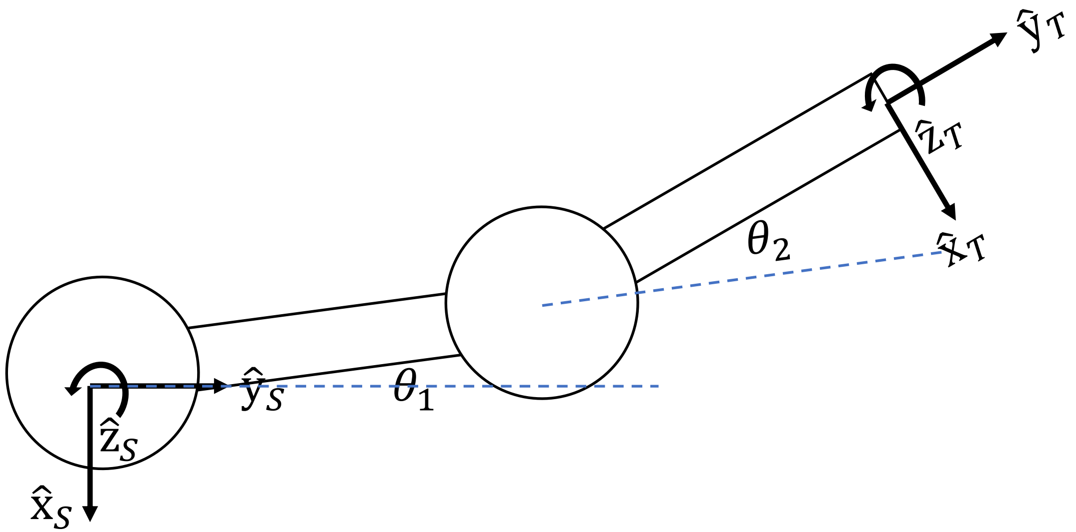
Step1. Fix $\theta_1 = 0$ and move θ_2

$$g_{ST}(0, \theta_2) = e^{\hat{\xi}_2 \theta_2} g_{ST}(0)$$

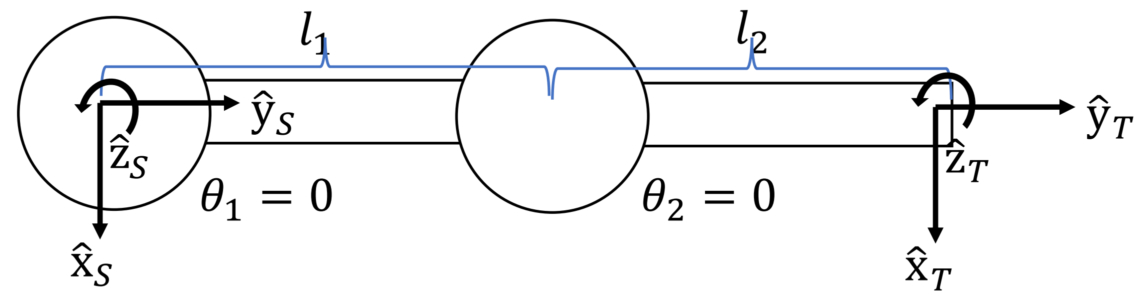
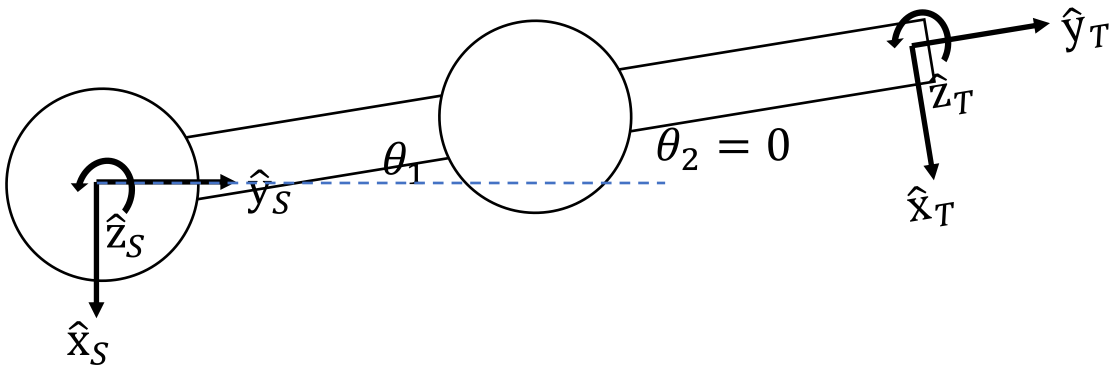
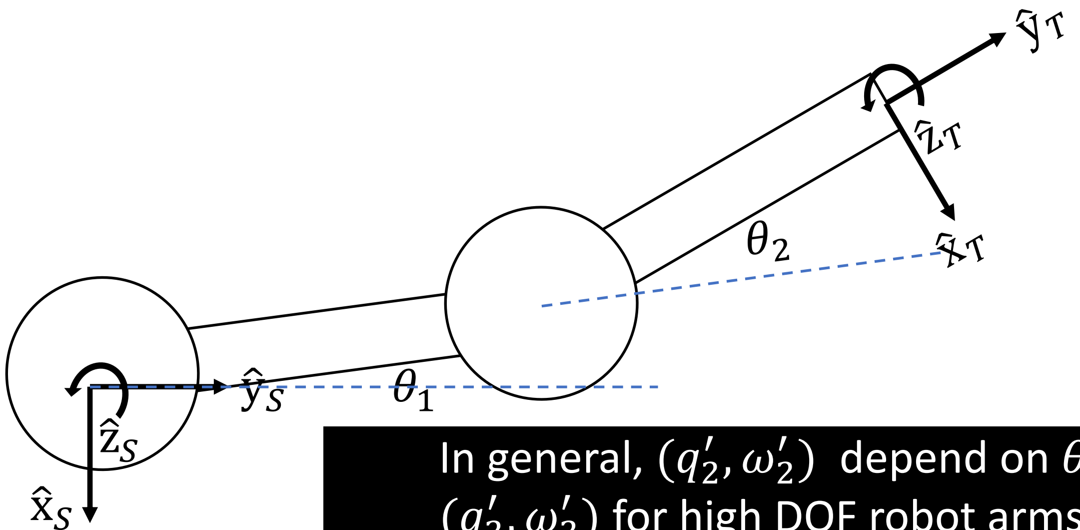
Step2. Move θ_1

$$g_{ST}(\theta_1, \theta_2) = e^{\hat{\xi}_1 \theta_1} g_{ST}(0, \theta_2)$$

Why $g_{ST}(\theta_1, \theta_2) = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}g_{ST}(0)$?

step 0.		Initially, we have $g_{ST}(0)$.
step 1.		Fix $\theta_1 = 0$ and move θ_2 $q_2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}, \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, then $v_2 = -\omega_2 \times q_2$.
step 2.		Move θ_1 $q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, then $v_1 = -\omega_1 \times q_1$.

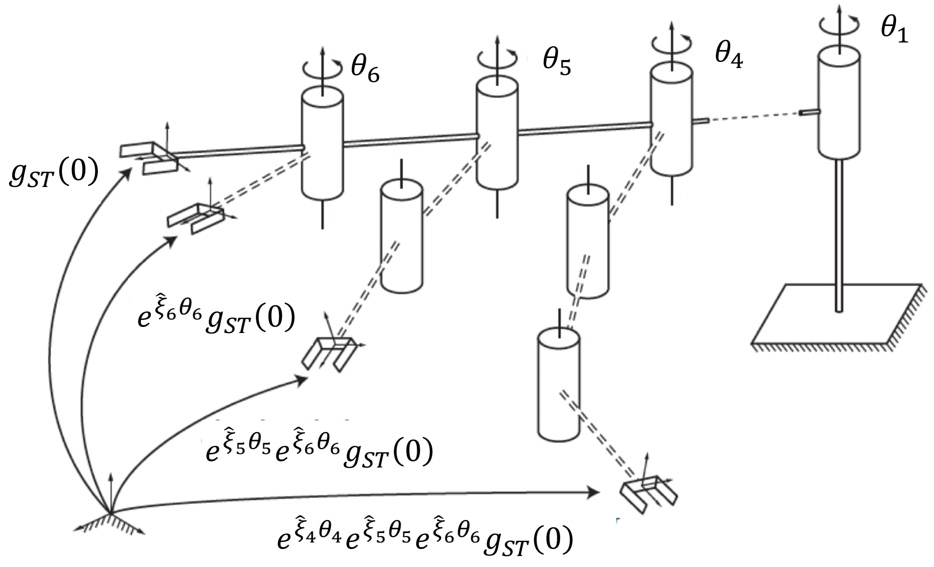
Why not $g_{ST}(\theta_1, \theta_2) = e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_1\theta_1}g_{ST}(0)$? Instead, $g_{ST}(\theta_1, \theta_2) = e^{\hat{\xi}_2'\theta_2}e^{\hat{\xi}_1\theta_1}g_{ST}(0)$

step 0.		Initially, we have $g_{ST}(0)$.
step 1.		Fix $\theta_2 = 0$ and move θ_1 $q_1 = \begin{bmatrix}0\\0\\0\end{bmatrix}, \omega_1 = \begin{bmatrix}0\\0\\1\end{bmatrix}$, then $v_1 = -\omega_1 \times q_1$.
step 2.		Move θ_2 $q_2 = \begin{bmatrix}0\\l_1\\0\end{bmatrix}, \omega_2 = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $q_2' = l_1\begin{bmatrix}-\sin\theta_1\\\cos\theta_1\\0\end{bmatrix}, \omega_{2'} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

In general, (q_2', ω_2') depend on θ_1, and it could be difficult to find (q_2', ω_2') for high DOF robot arms.

Summary

Consider a higher DOF robot arm.



Forward kinematics:

$$g_{ST}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) \\ = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} g_{ST}(0)$$

Step 0. Fix $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = 0$ and find $g_{ST}(0)$.

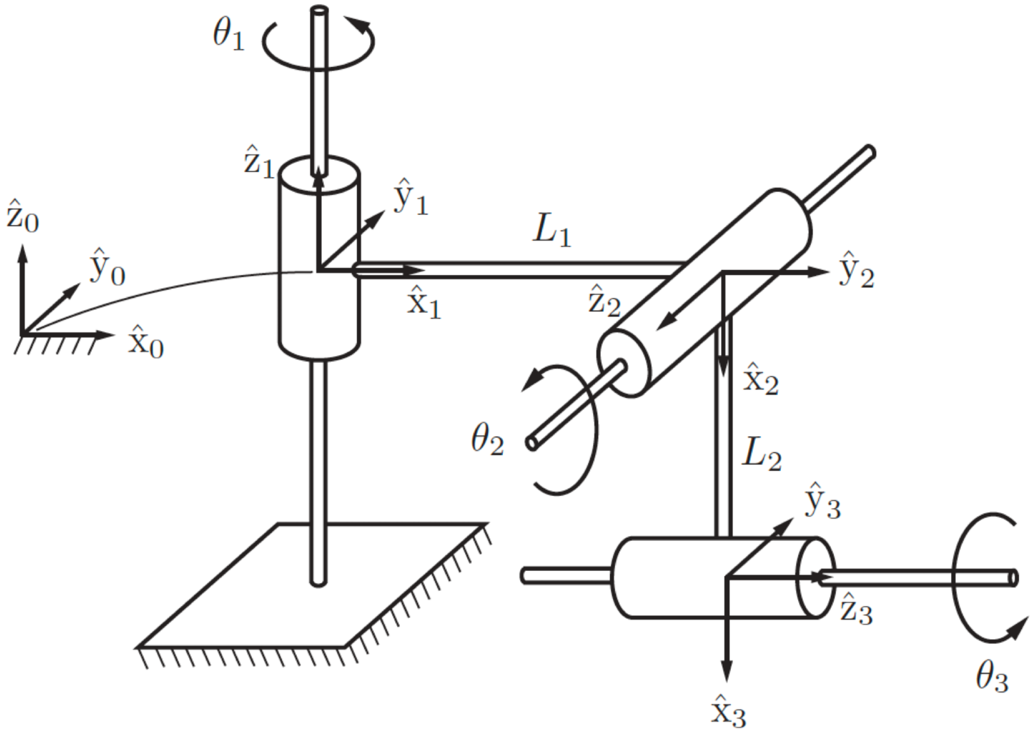
Step 1. Fix $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = 0$ and move θ_6
(find $\xi_6 = (v_6, \omega_6)$ with respect to frame S)

Step 2. Fix $(\theta_1, \theta_2, \theta_3, \theta_4) = 0$ and move θ_5
(find $\xi_5 = (v_5, \omega_5)$ with respect to frame S)

⋮

Step 6. Move θ_1
(find $\xi_1 = (v_1, \omega_1)$ with respect to frame S)

Example



Forward kinematics:

$$g_{03}(\theta_1, \theta_2, \theta_3) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} g_{03}(0)$$

Find $g_{03}(0), \xi_1, \xi_2, \xi_3$.