EECS/BioE C106A/206A Introduction to Robotics

Lost Section 4

Oct 23 Fri 7 – 9 PM

For any issue in this material, please email to donggun_lee@berkeley.edu

Velocities and Twists

- The spatial velocity: $\widehat{V}_{ab}^{s}$
- The body velocity: $\widehat{V}_{ab}^{b}$
- Adjoint matrix: Ad_g such that

$$V_{ab}^{s} = \mathrm{Ad}_g V_{ab}^{b}$$

Linear Algebra and Adjoint Matrix

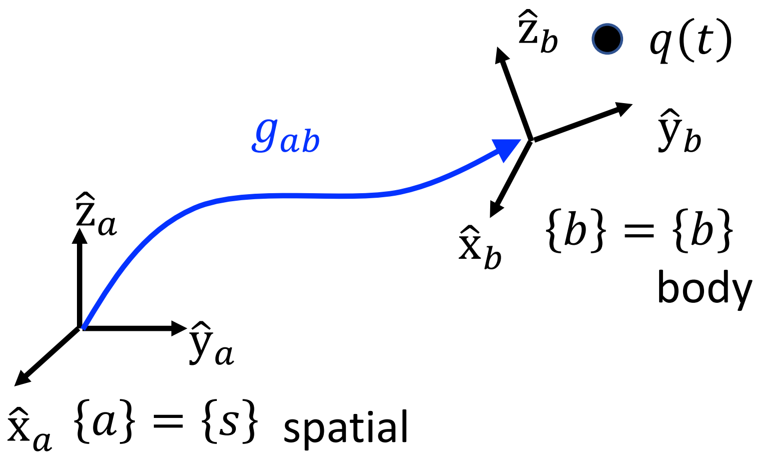
$$R(\omega_1 \times \omega_2) = (R\omega_1) \times (R\omega_2)$$

$$R\widehat{\omega}R^T = (R\omega)^\wedge$$

$$g\hat{\xi}g^{-1} = \left(\mathrm{Ad}_g \xi\right)^\wedge$$

$$\mathrm{Ad}_g = \begin{bmatrix} R & \hat{p}R \\ 0 & R \end{bmatrix}$$

Velocities and Twist Motion



$q(t)$: moving point

$q_a(t)$: coordinate of the point w.r.t. $\{a\}$
q_b: coordinate of the point w.r.t. $\{b\}$

$v_{q_a}(t)$: coordinate of the velocity of the point w.r.t. $\{a\}$
$v_{q_b}(t)$: coordinate of the velocity of the point w.r.t. $\{b\}$

Definition of the spatial velocity:

$$v_{q_a}(t) = \hat{V}^s_{ab} q_a(t)$$

In the book, we have $\hat{V}^s_{ab} = \dot{g}_{ab} g^{-1}_{ab}$.

Definition of the body velocity:

$$v_{q_b}(t) = \hat{V}^b_{ab} q_b$$

In the book, we have $\hat{V}^b_{ab} = g^{-1}_{ab} \dot{g}_{ab}$.

$$V^s_{ab} = \mathrm{Ad}_{g_{ab}} V^b_{ab}$$

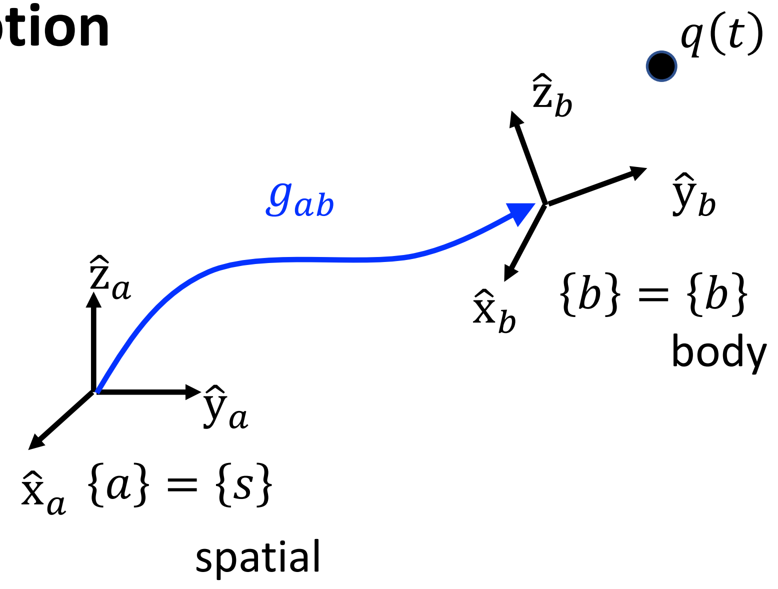
Velocities and Twist Motion

Definition of the spatial velocity:

$$v_{q_a}(t) = \hat{V}_{ab}^{s} q_a(t)$$

EX. Constant twist motion between the two frames: $g_{ab} = e^{\hat{\xi}\theta(t)}$

$$q_a(t) = e^{\hat{\xi}\theta(t)} q_b$$



Then, the velocity of q_a becomes

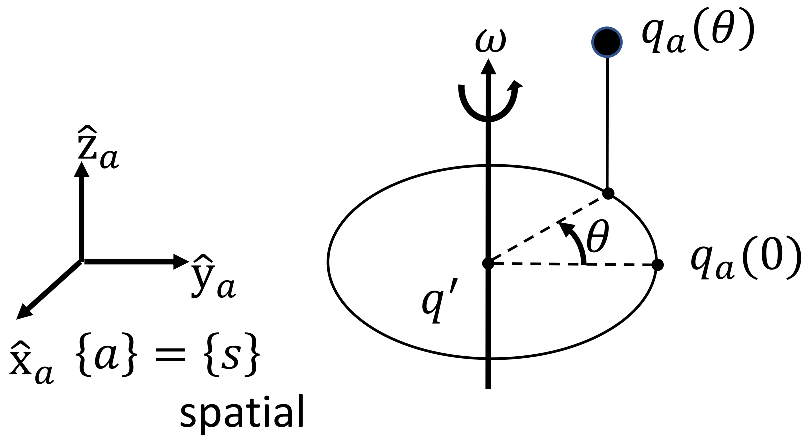
$$v_{q_a}(t) = \hat{\xi}\dot{\theta} e^{\hat{\xi}\theta(t)} q_b = \hat{\xi}\dot{\theta} q_a(t)$$

Here, we observe that $\hat{\xi}\dot{\theta} = \hat{V}_{ab}^{s} \in \mathbb{R}^{4\times 4}$ and $\xi\dot{\theta} = V_{ab}^{s} = \begin{bmatrix} v_{ab}^{s} \\ \omega_{ab}^{s} \end{bmatrix} \in \mathbb{R}^{6}$.

The spatial velocity is the multiplication of the twist and the angular speed.

$$V_{ab}^{s} = \xi\dot{\theta}$$

Short Review: Screw Motion and Twist



q': the center of the rotation, or a point on the rotation axis.
ω: the direction of the rotation axis.

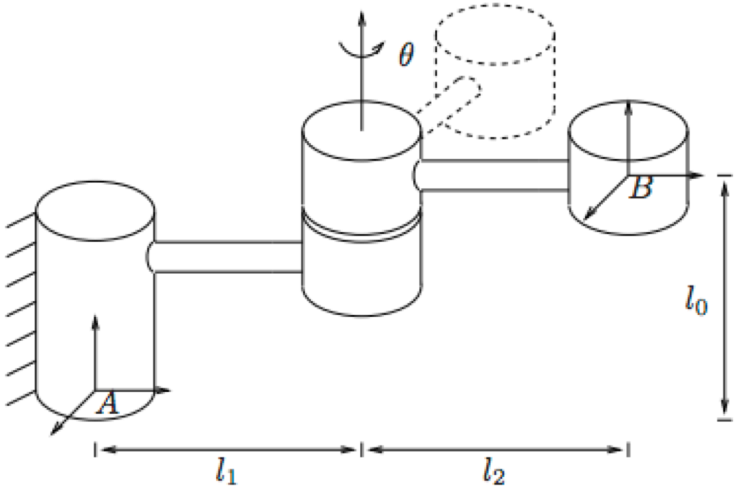
We can find the twist $\xi = [v, \omega]^T$ *using the given* q' *and* ω.

$$v = -\omega \times q' + h\omega$$

By the screw motion, we can find the spatial velocity V_{ab}^s using q' and ω.

$$V_{ab}^s = \xi\dot{\theta} = \begin{bmatrix} -\omega \times q' + h\omega \\ \omega \end{bmatrix} \dot{\theta}$$

Example: MLS example 2.5 in pg 56 - 57



Spatial velocity: find V_{ab}^s

$$
\begin{aligned}
q' &= [0, l_1, 0]^T \\
\omega' &= [0,0,1]^T \\
v' &= -\omega' \times q' = [l_1, 0,0]^T
\end{aligned}
$$

Then, $V_{ab}^s = \begin{bmatrix} l_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}$

Body velocity: find V_{ab}^b

$$
\begin{aligned}
q^\dagger &= [0, -l_2, 0]^T \\
\omega^\dagger &= [0,0,1]^T \\
v^\dagger &= -\omega^\dagger \times q^\dagger = [-l_2, 0,0]^T
\end{aligned}
$$

Then, $V_{ab}^b = \begin{bmatrix} -l_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}$

Find $q^\dagger$ and $\omega^\dagger$ w.r.t. the frame B.