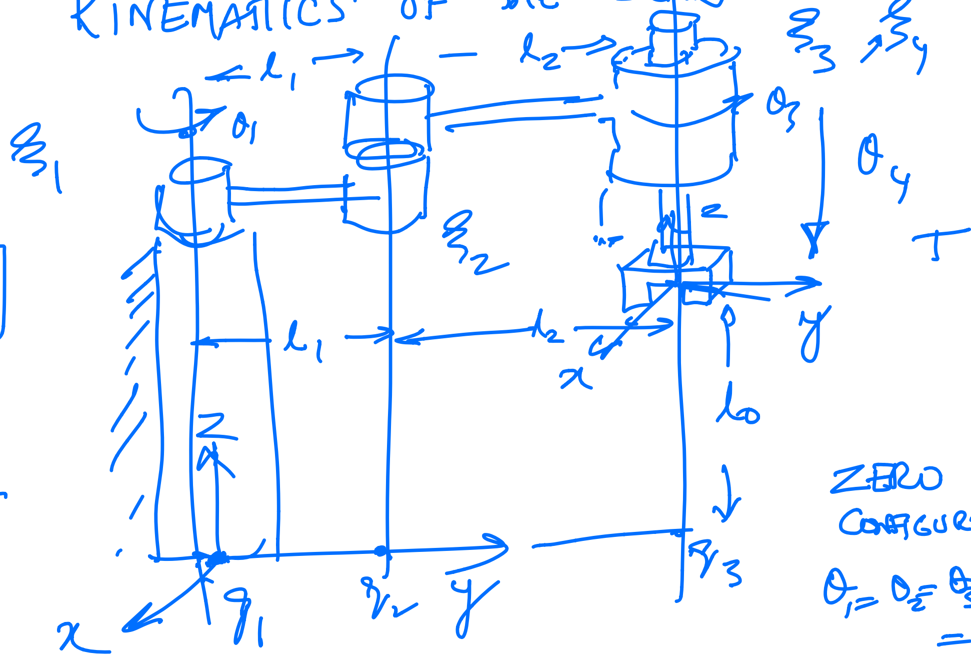
Lecture Sept. 15th

Kinematics of the SCARA Robot

$\boxed{RRRP}$



ZERO CONFIGURATION

$\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$

ξ_1, ξ_2, ξ_3 are all zero pitch screws

ξ_4 is infinite pitch.

$$\omega_1 = \omega_2 = \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$g_{st}(0) = \left[\begin{array}{c|c} I & \begin{matrix} 0 \\ l_1 + l_2 \\ l_0 \end{matrix} \\ \hline 0 \; 0 \; 0 & 1 \end{array}\right]$$

$$g_{st}(\theta) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} e^{\hat{\xi}_3 \theta_3} e^{\hat{\xi}_4 \theta_4} g_{st}(0)$$

$$q_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad q_2 = \begin{pmatrix} 0 \\ l_1 \\ 0 \end{pmatrix} \qquad q_3 = \begin{pmatrix} 0 \\ l_1 + l_2 \\ 0 \end{pmatrix}$$

$$e^{\hat{\xi}_1\theta_1} = \begin{bmatrix} e^{\hat{\omega}_1\theta_1} & (I-e^{\hat{\omega}_1\theta})q_1 \\ 0\ 0\ 0 & 1 \end{bmatrix}, \qquad \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$e^{\hat{\xi}_2\theta_2} = \begin{bmatrix} e^{\hat{\omega}_2\theta_2} & (I-e^{\hat{\omega}_2\theta_2})q_2 \\ 0\ 0\ 0 & 1 \end{bmatrix} \qquad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$c_2 = \cos\theta_2$

$s_2 = \sin\theta_2$

$$= \begin{bmatrix} c_2 & -s_2 & 0 & \begin{bmatrix} 1-c_2 & s_2 & 0 \\ -s_2 & 1-c_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix} \\ s_2 & c_2 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} c_2 & -s_2 & 0 & l_1 s_2 \\ s_2 & c_2 & 0 & l_1(1-c_2) \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$(I-e^{\hat{\omega}_3\theta_3})q_3$$

$$e^{\hat{\xi}_3\theta_3} = \left[\begin{array}{ccc|c} c_3 & -s_3 & 0 & (l_1+l_2)s_3 \\ s_3 & c_3 & 0 & (l_1+l_2)(1-c_3) \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \hat{\omega}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1-c_3 & s_3 & 0 \\ -s_3 & 1-c_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ l_1+l_2 \\ 0 \end{bmatrix} \qquad q_3 = \begin{bmatrix} 0 \\ l_1+l_2 \\ 0 \end{bmatrix}$$

$$e^{\hat{\xi}_4 \theta_4} = \left[\begin{array}{c|c} I & \begin{matrix} 0 \\ 0 \\ \theta_4 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$\omega_4 = 0 \qquad v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$g_{st}(\theta) = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & l_1 s_2 \\ s_2 & c_2 & 0 & l_1(1-c_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} c_3 & -s_3 & 0 & (l_1+l_2)s_3 \\ s_3 & c_3 & 0 & (l_1+l_2)(1-c_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c|c} I & \begin{matrix} 0 \\ 0 \\ \theta_4 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$\times \left[\begin{array}{c|c} I & \begin{matrix} 0 \\ l_1+l_2 \\ l_0 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$\cos(\theta_1+\theta_2+\theta_3)$

$$= \left[\begin{array}{ccc|c} c_{123} & -s_{123} & 0 & (l_1+l_2)s_{123} - l_1 s_1 - l_2 s_{12} \\ s_{123} & c_{123} & 0 & -(l_1+l_2)c_{123} + l_1 c_1 + l_2 c_{12} \\ 0 & 0 & 1 & \theta_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & \begin{matrix} 0 \\ l_1+l_2 \\ l_0 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$s_{12} = \sin(\theta_1+\theta_2)$

$$g_{st}(\theta) = \left[\begin{array}{ccc|c} c_{123} & -s_{123} & 0 & -l_1 s_1 - l_2 s_{12} \\ s_{123} & c_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ 0 & 0 & 1 & l_0 + \theta_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R(\theta) = \begin{bmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(\theta_1 + \theta_2 + \theta_3)$

$$p(\theta) = \begin{bmatrix} -l_1 s_1 & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} \\ l_0 + \theta_4 \end{bmatrix}$$

no dependance on θ_3

ELBOW MANIPULATOR