EECS/BioE C106A/206A Introduction to Robotics

Lost Section 1

Sep 18 Fri 7 – 9 PM

For any issue in this material, please email to donggun_lee@berkeley.edu

Contents

- Theory
  - Rigid body transformation
  - Twist, exponential coordinates
  - Screw

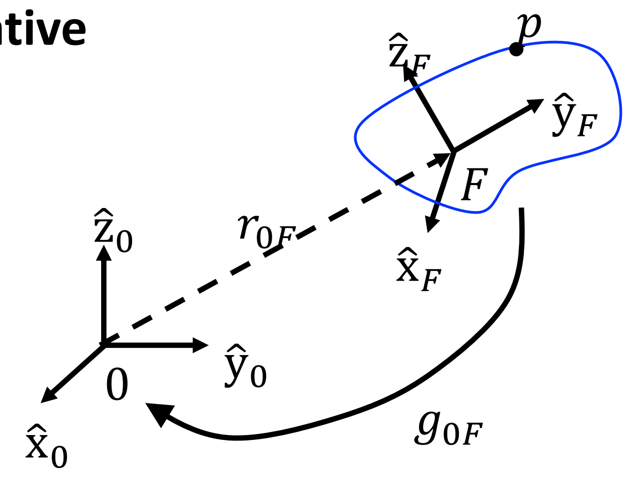
- Example
  - The satellites system (Homework1 Problem 4)

One message in this section:

Check a frame in which a coordinate is defined.

Rigid body transformation (relative+absolute)

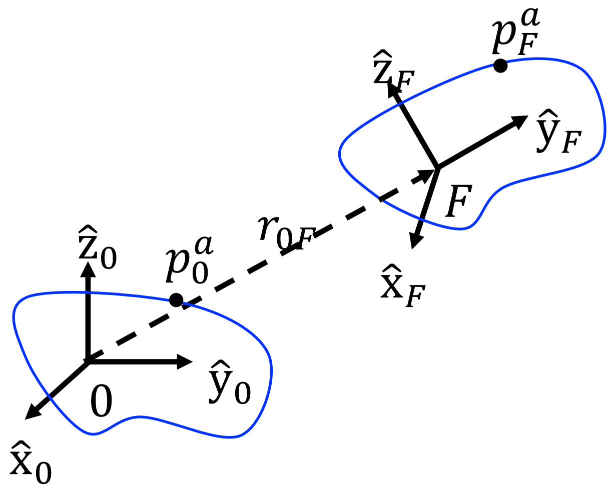
relative



ω: the coordinate for the rotation axis of frame 0
r_{0F}: the coordinate for the translation of frame 0

p: a point
p_F: the coordinate for q of frame F
p_0: the coordinate for q of frame 0

absolute

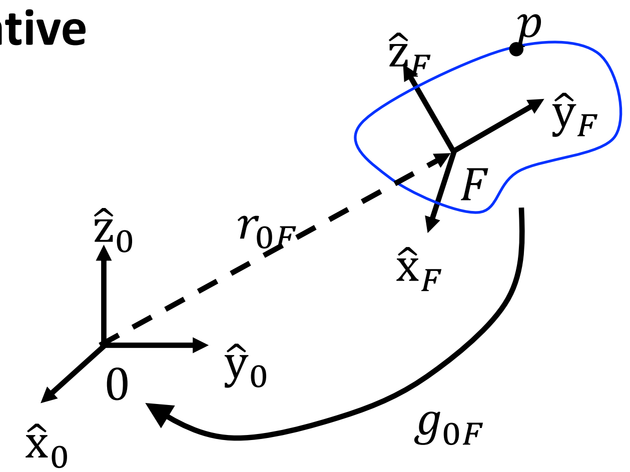


ω: frame 0
r_{0F}: frame 0

p_0^a: the coordinate of frame 0, for the fixed point on frame 0
p_F: the coordinate of frame 0, for the fixed point on frame F

Rigid body transformation (relative+absolute)

relative



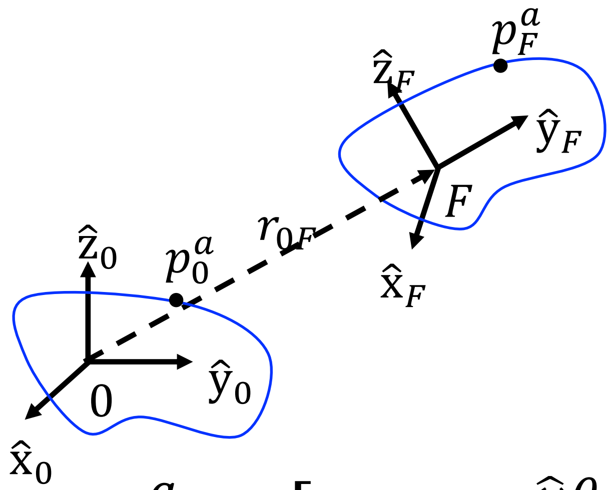
$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_{0F} = e^{\widehat{\omega}\theta} & r_{0F} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_F \\ 1 \end{bmatrix}$$

ω: the coordinate for the rotation axis of frame 0
r_{0F}: the coordinate for the translation of frame 0

p: a point
p_F: the coordinate for q of frame F
p_0: the coordinate for q of frame 0

There is a single p, but multiple coordinates p_F, p_0.
$(R, \omega, r_{0F}, p_F, p_0)$ are defined in the relative frame.

absolute



$$\begin{bmatrix} p_F^a \\ 1 \end{bmatrix} = \begin{bmatrix} R_{0F} = e^{\widehat{\omega}\theta} & r_{0F} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_0^a \\ 1 \end{bmatrix}$$

ω: frame 0
r_{0F}: frame 0

p_0^a: the coordinate of frame 0, for the fixed point on frame 0
p_F^a: the coordinate of frame 0, for the fixed point on frame F

There are multiple p_0^a, p_F^a.
$(R, \omega, r_{0F}, p_0^a, p_F^a)$ are defined in the frame 0.

Exponential Coordinates, Twist + Screw **(absolute)**

$(v, \widehat{\omega}) \in se(3)$: twist

$\xi := (v, \omega) \in \mathbb{R}^6$: the twist coordinates of $\hat{\xi} = \begin{bmatrix} \widehat{\omega} & v \\ 0 & 0 \end{bmatrix}$

(ξ, θ): exponential coordinate

$$\frac{d}{dt}\begin{bmatrix} p(t) \\ 1 \end{bmatrix} = \begin{bmatrix} \dot{p}(t) \\ 0 \end{bmatrix} = \hat{\xi}\begin{bmatrix} p(t) \\ 1 \end{bmatrix} = \begin{bmatrix} \widehat{\omega} & v \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p(t) \\ 1 \end{bmatrix}$$

$p(t)$: frame 0

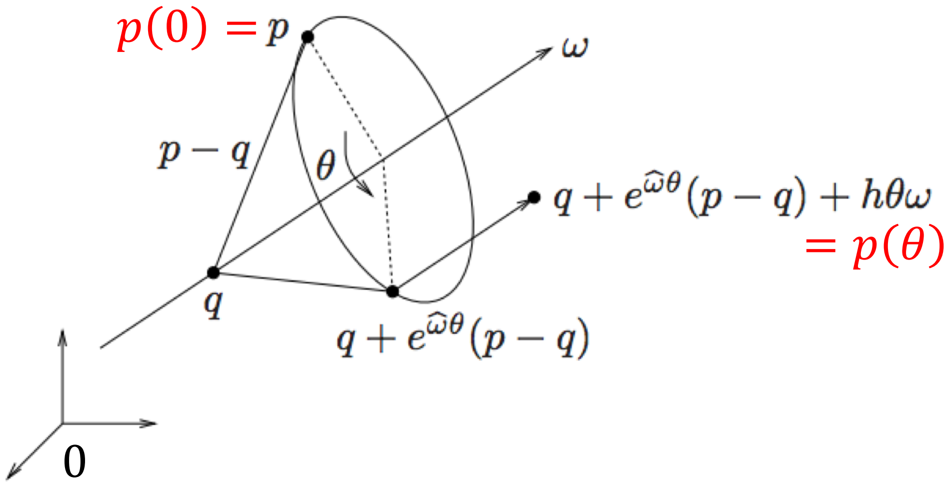
ω: the coordinate for the rotation axis of frame 0

Then, $\begin{bmatrix} p(\theta) \\ 1 \end{bmatrix} = e^{\hat{\xi}\theta}\begin{bmatrix} p(0) \\ 1 \end{bmatrix}$

Here, we only have a single frame: frame 0.

$$g = e^{\hat{\xi}\theta} = \begin{cases} \begin{bmatrix} e^{\widehat{\omega}\theta} & v\theta \\ 0 & 1 \end{bmatrix} & \omega = 0 \\ \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} & \omega \neq 0, \| \omega \| = 1 \end{cases}$$

Exponential Coordinates, Twist + **Screw (absolute)**



Case 1. $\omega \neq 0$

$$\begin{bmatrix} p(\theta) \\ 1 \end{bmatrix} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p(0) \\ 1 \end{bmatrix}$$

Case 2. $\omega = 0$: pure translation $h = \infty$

$$\begin{bmatrix} p(\theta) \\ 1 \end{bmatrix} = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p(0) \\ 1 \end{bmatrix}$$

, where v is a velocity vector and a unit vector.

p: the coordinate of the point of frame 0
ω: the coordinate for the rotation axis of frame 0
q: the coordinate for the center of the rotation of frame 0

h: pitch
θ: rotation angle

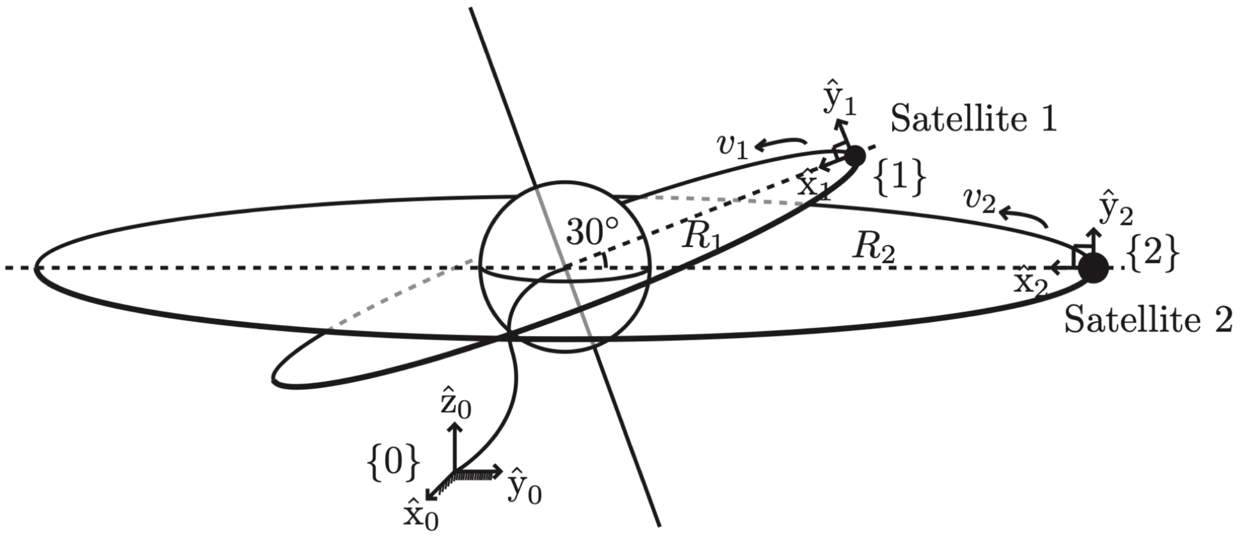
Here, we only have a single frame: frame 0.

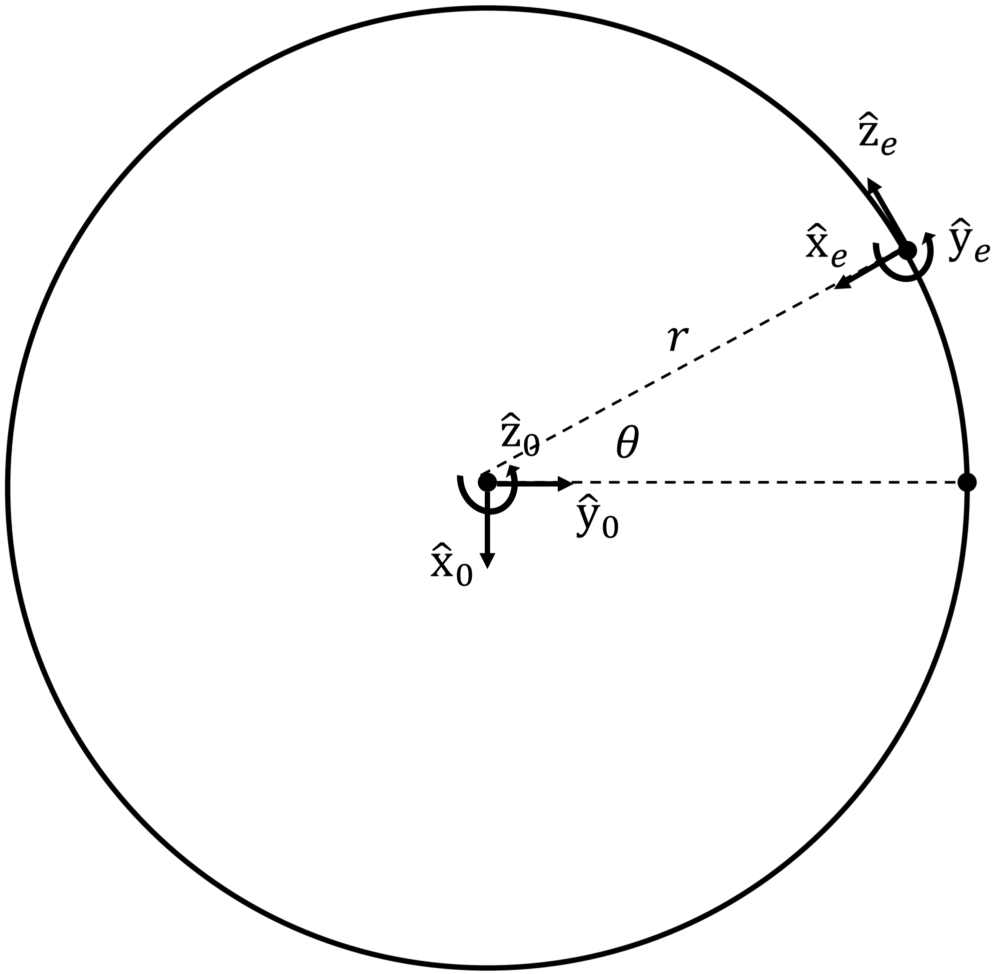
Exponential Coordinates, Twist + Screw (absolute)

	Exponential Coordinates, Twist	Screw
All v, ω, q are defined in the single frame: frame 0.	(v, ω, θ): exponential coordinate	ω, q, h, θ
$\omega \neq 0$	$\begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix}$
$\omega = 0$	$\begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$

If $v = -\omega \times q + h\omega$,
the two transformations are the same.

HW1 Problem 4. Satellite System





Find g_{0e}?

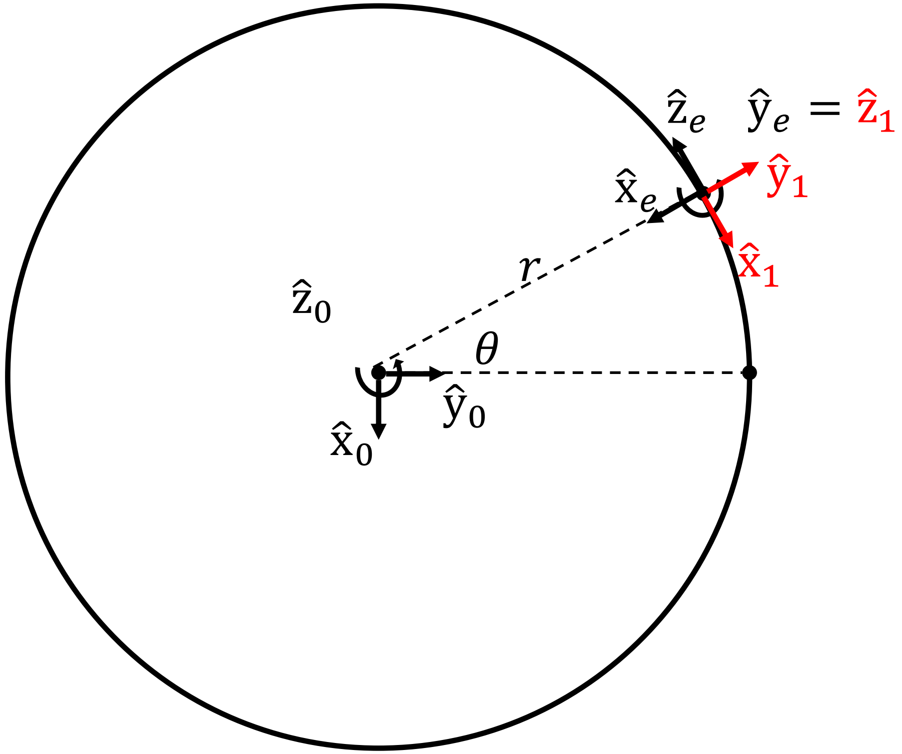
Relative transformation: method 1

rotation θ and translation R (g_{01}) -> coordinate change (g_{1e})

$$g_{0e} =$$

$$g_{1e} =$$

$$g_{01} =$$



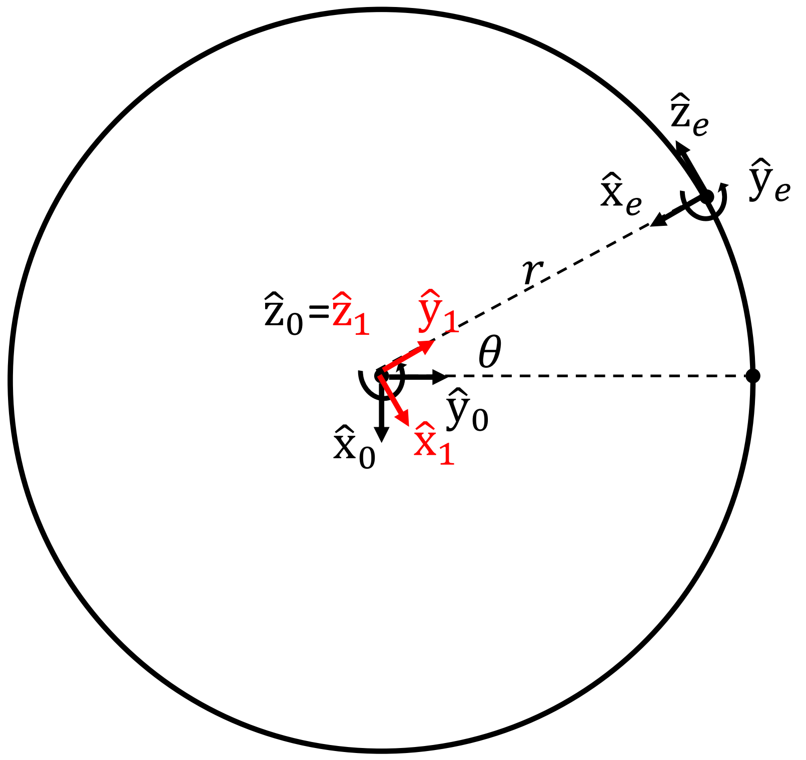
Relative transformation: method 2

rotation θ (g_{01}) -> coordinate change and translation R (g_{1e})

$$g_{0e} =$$

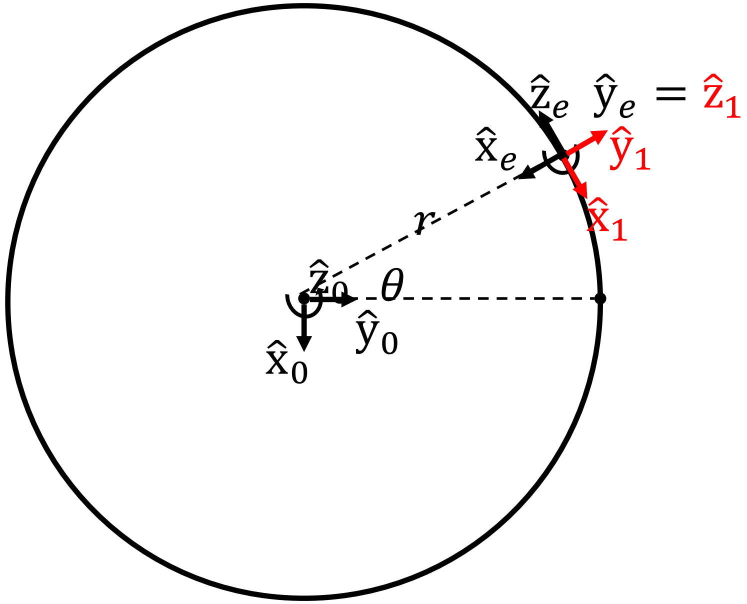
$$g_{1e} =$$

$$g_{01} =$$

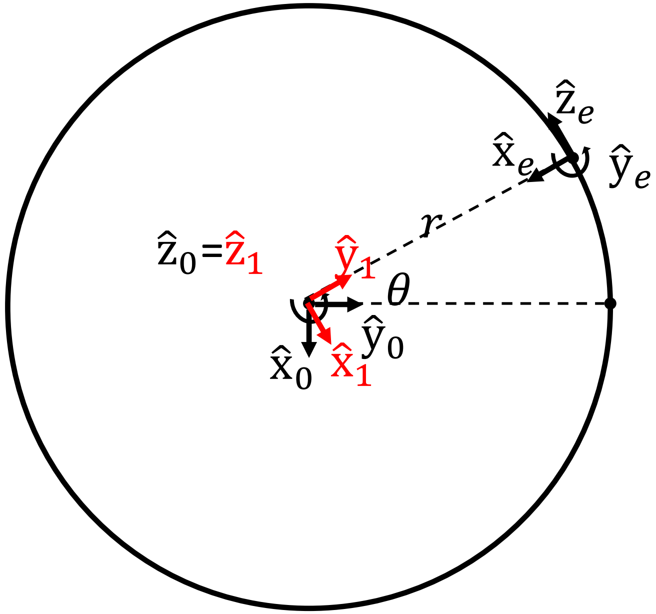


Relative transformation: other methods

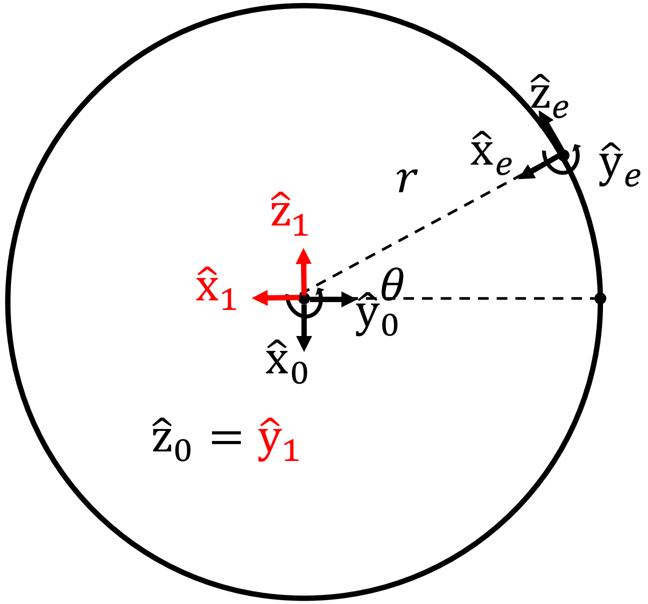
method 1



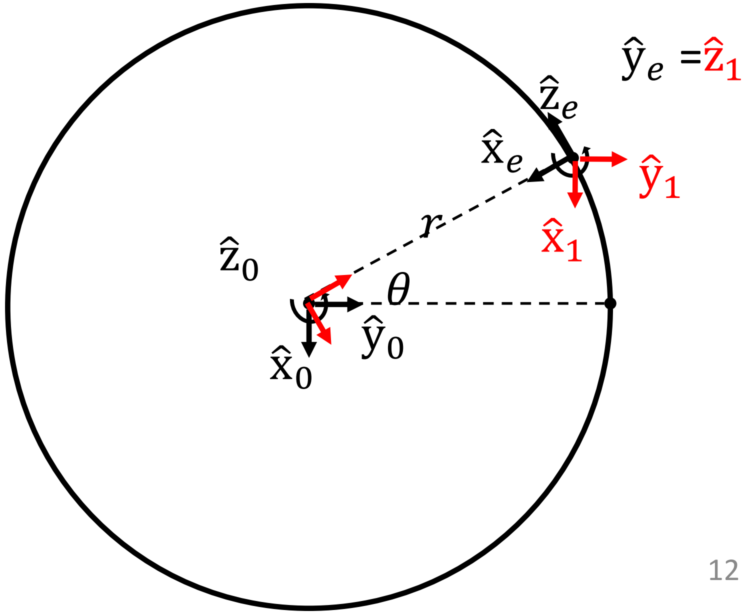
method 2



other method 1



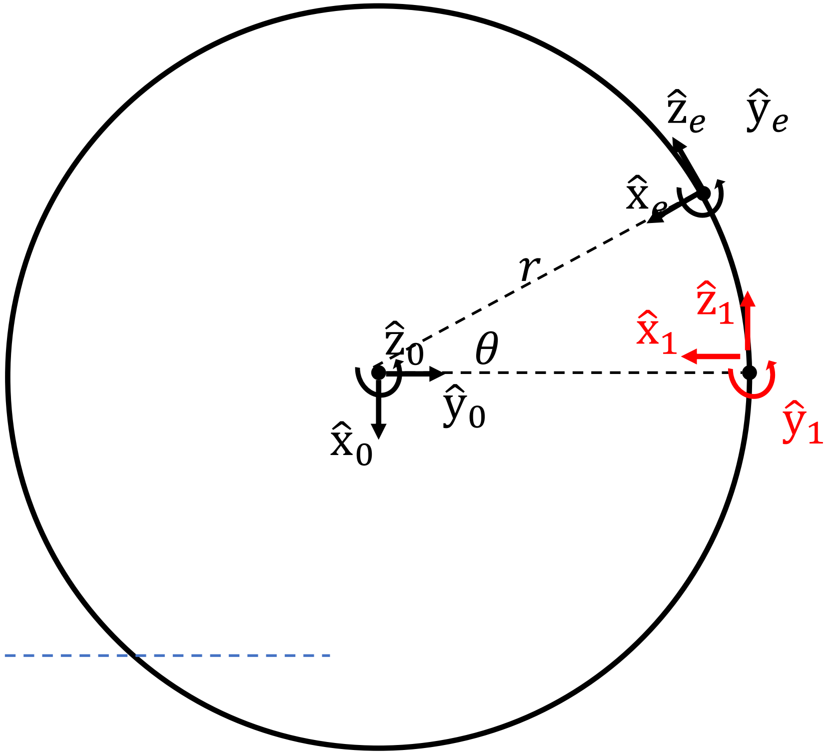
other method 2



Absolute transformation: method 3

translation R and coordinate change (g_1) -> rotation θ (g_2)

$$g_{0e} =$$



$$g_1 =$$

screw

$$g_2 =$$

Absolute transformation: method 4

translation R (g_1) -> coordinate change (g_2) -> rotation θ (g_3)

$g_{0e} =$

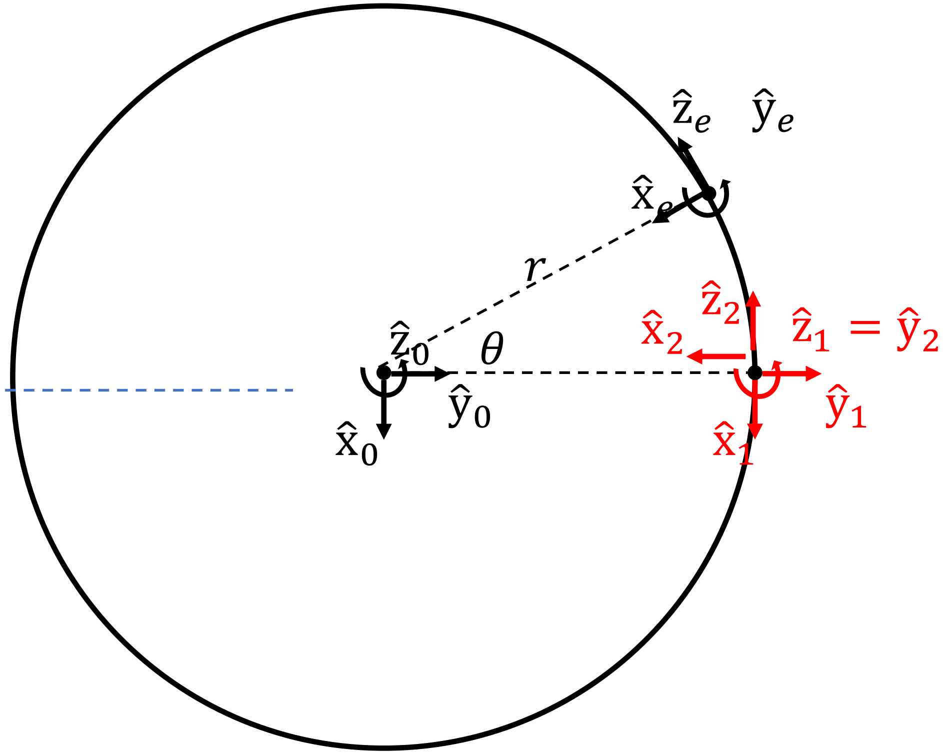
$g_1 =$

screw

$g_2 =$

screw

$g_3 =$



Absolute transformation: method 5

translation R (g_1) -> rotation θ (g_2) -> coordinate change (g_3)

$$g_{0e} =$$

$$g_1 =$$

screw

$$g_2 =$$

screw

$$g_3 =$$