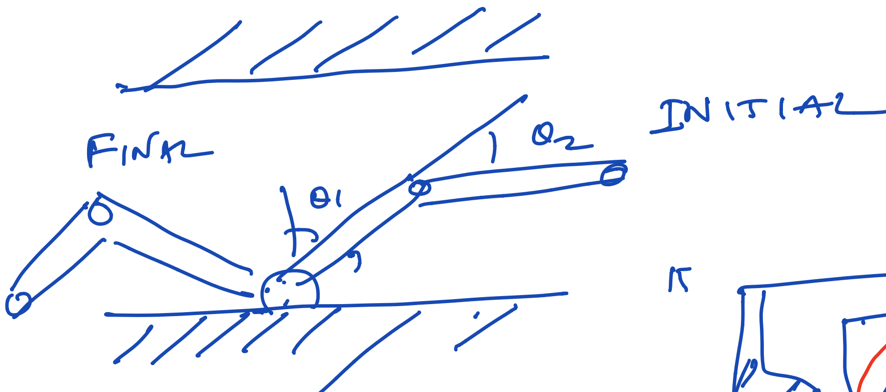
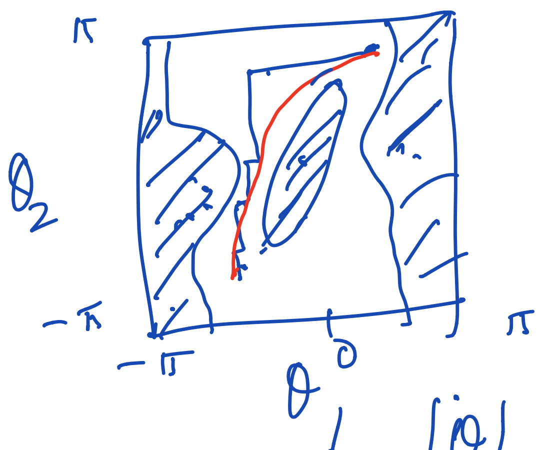
EECS 106A / 206A 11/05/20

MANIPULATOR CONTROL





CONFIGURATION SPACE

$|\dot{\theta}|_{max}$

ACCEL $|\ddot{\theta}|_{max}$

JERK $|\dddot{\theta}|_{max}$

CONTROLLER BOARDS

$\theta_d(t) \quad t \in [0, T]$

PATH PLANNER GIVES YOU

d desired $\theta_d(t)$ $t \in [0, T]$

CONTROL

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau$$

$n \times n$ · $n \times 1$ ($\ddot{\theta}$) + $n \times n$ · $n \times 1$ + $n \times 1$ = $n \times 1$

GEN. MOM. OF INERTIA

CORIOLIS

GRAVITY + FRICTION

θ angle for revolute joints

θ dist for prismatic joints

MOTOR TORQUES, FORCES

ENCODER ← θ

VELOCITY ← $\dot{\theta}$

$$\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^{2n}$$

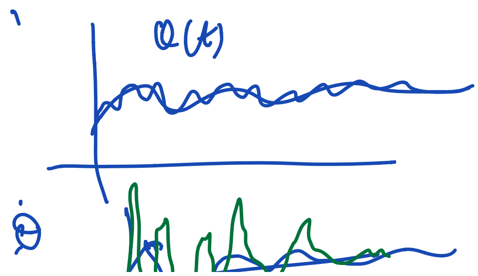
$\boxed{\text{FULL STATE MEASURABLE}}$ ✓ ✓

only measure θ

NOT ALLOWED TO DIFFERENTIATE $\left(\frac{d}{dt}\right)$

SENSOR

SIGNALS



$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = \tau$$

$\theta(t) \rightarrow \theta_d(t)$ TRY

FEED FORWARD CONTROL

$$\tau_I = M(\theta_d)\ddot{\theta}_d + C(\theta_d,\dot{\theta}_d)\dot{\theta}_d + N(\theta_d,\dot{\theta}_d)$$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta}) = M(\theta_d)\ddot{\theta}_d + C(\theta_d,\dot{\theta}_d)\dot{\theta}_d + N(\theta_d,\dot{\theta}_d)$$

If $\theta(t) \equiv \theta_d(t)$ equation would be satisfied.

$$\boxed{\begin{aligned} \theta(0) &= \theta_d(0) \\ \dot{\theta}(0) &= \dot{\theta}_d(0) \end{aligned}}$$

ROBUST M, C, N unknown; partially known

OPEN LOOP!

FEEDBACK

$$\theta(t) - \theta_d(t) = e_\theta(t)$$
$$\dot{\theta}(t) - \dot{\theta}_d(t) = \dot{e}_\theta(t)$$

e, $\dot{e}$ to make changes in the control.

$$\tau_2 = M(\theta)\ddot{\theta}_d + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta})$$

$$M(\theta)\ddot{\theta} + \cancel{C(\theta,\dot{\theta})\dot{\theta}} + \cancel{N(\theta,\dot{\theta})} = M(\theta)\ddot{\theta}_d + \cancel{C(\theta,\dot{\theta})\dot{\theta}} + \cancel{N(\theta,\dot{\theta})}$$

$$M(\theta)(\ddot{\theta} - \ddot{\theta}_d) = 0$$

$$M^{-1}(\theta)M(\theta)(\ddot{\theta} - \ddot{\theta}_d) = M^{-1}(\theta)\cdot 0 = 0$$

$$\ddot{\theta} - \ddot{\theta}_d = 0 \Rightarrow \boxed{\ddot{e} = 0}$$

$\ddot{e} = 0 \not\Rightarrow$ $e(t) \to 0$ as $t \to \infty$, $\dot{e}(t) \to 0$

$\underline{e(0) \neq 0 \quad \dot{e}(0) \neq 0}$

$$\tau_3 = M(\theta)\left[\ddot{\theta}_d - K_v(\dot{\theta} - \dot{\theta}_d) - K_p(\theta - \theta_d)\right] + C(\theta,\dot{\theta}) + N(\theta,\dot{\theta})$$

$K_v, K_p \in \mathbb{R}^{n\times n}$ $\quad K_v \succ 0$ pos.def $\quad K_p \succ 0$

$K_v = K_v^T$ $\quad K_p = K_p^T$

$x^T K_v x \geq 0$ $\quad x^T K_p x \geq 0$

only if $x = 0$ $= 0$ $\quad$ with equality $= 0$ only if $x = 0$

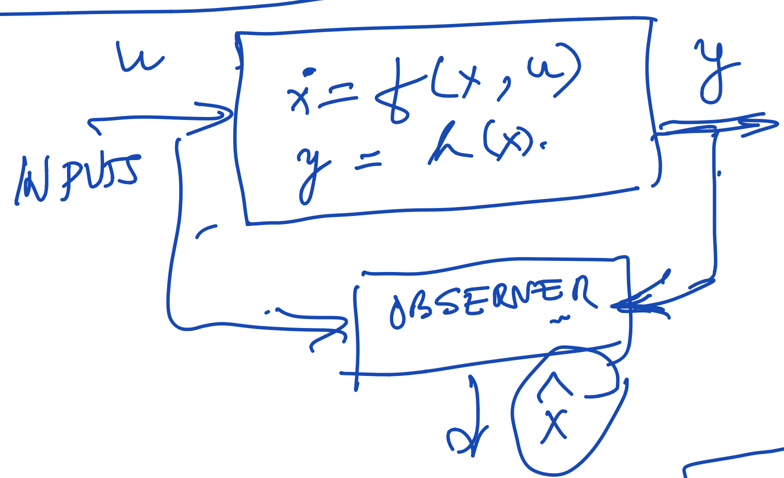
<u>Claim</u>: Any $K_p \succ 0$, $K_v \succ 0$ results in $e(t), \dot{e}(t) \to 0$ as $t \to \infty$ regardless of

$$\tau_3 = \underbrace{M(\theta)\ddot{\theta}_d + C(\theta,\dot{\theta})\dot{\theta} + N(\theta,\dot{\theta})}_{\tau_{ff}\ \text{FEED FORWARD}} + \underbrace{M(\theta)\left(-K_v\dot{e} - K_p e\right)}_{\tau_{fb}\ \text{FEED BACK}}$$

$$\boxed{\begin{matrix}\theta(0)\\ \dot{\theta}(0)\end{matrix}}$$

[COMPUTED TORQUE CONTROLLER]

"OBSERVABLE" $x \in \mathbb{R}^n$ ← "MEASURABLE" (sensors) MEASUREMENTS



If $y = x$ no observer

$$x \begin{pmatrix}\theta\\ \dot{\theta}\end{pmatrix} \qquad y = x \qquad y \begin{pmatrix}\theta\\ \dot{\theta}\end{pmatrix}$$

$$M(\theta)\ddot{\theta} + \cancel{C(\theta,\dot{\theta})\dot{\theta}} + \cancel{g(\theta)} \ldots - K_p(\theta-\theta_d)$$

$$= M(\theta)\left[\ddot{\theta}_d - K_v(\dot{\theta}-\dot{\theta}_d) - K_p(\theta - \theta_d)\right] + \cancel{C(\theta,\dot{\theta})\dot{\theta}} + \cancel{g(\theta,\dot{\theta})}$$

$$M(\theta)\left[(\ddot{\theta} - \ddot{\theta}_d) + K_v(\dot{\theta}-\dot{\theta}_d) + K_p(\theta-\theta_d)\right] = 0$$

$M^{-1}(\theta)$ exists $\Longrightarrow$

$$\ddot{\theta} - \ddot{\theta}_d + K_v(\dot{\theta}-\dot{\theta}_d) + K_p(\theta-\theta_d) = 0$$

$$\boxed{\ddot{e} + K_v\dot{e} + K_p e = 0}$$ LINEAR

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \dot{e} \\ -K_v\dot{e} - K_p e \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}}_{\in \mathbb{R}^{2n\times 2n}} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

Claim All eigenvalues of $\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}$ lie in open left half complex plane. $\mathbb{C}$

$\mathbb{C}_-$

Let $\lambda \in \mathbb{C}$ be an eigenvalue of

$$\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \lambda \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$v_1 \in \mathbb{C}^n$, $v_2 \in \mathbb{C}^n$

$\lambda \neq 0$. Indeed if $\lambda = 0$

$$(*) \quad \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_2 = 0$

$-K_p v_1 \cancel{- K_v v_2} = 0 \Rightarrow K_p v_1 = 0$

$\Rightarrow v_1 = 0$

If $\lambda = 0 \Rightarrow v_1, v_2 = 0$.

Since Eigenvectors have to be non zero $\Rightarrow$

$\lambda = 0$ cannot be

[illegible] (*)

an eigenvalue of ...

$$\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \lambda \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$v_2 = \lambda v_1$

$-K_p v_1 - K_v v_2 = \lambda v_2$

If $v_2 = 0 \implies K_p v_1 = 0 \implies v_1 = 0$

Thus $v_2 \neq 0$ then because $\lambda \neq 0$

$v_1 = \frac{1}{\lambda} v_2 \neq 0.$

$\lambda \neq 0 \quad v_1 \neq 0 \quad v_2 \neq 0$

Let us choose v_1 to be $v_1^* v_1 = 1$

$$\lambda^2 = \lambda^2 v_1^* v_1$$
$$= \lambda \cdot v_1^* v_2$$
$$= v_1^* (\lambda v_2)$$
$$= v_1^* (-K_p v_1 - K_v v_2)$$

$$= -v_1^* K_p v_1 - \lambda v_1^* K_v v_1$$

$$\lambda^2 + \lambda \underbrace{v_1^* K_v v_1}_{\alpha > 0} + \underbrace{v_1^* K_p v_1}_{\beta > 0} = 0$$

(because $K_v \succ 0$) (because $K_p \succ 0$)

$$\lambda^2 + \alpha \lambda + \beta = 0 \qquad \boxed{\alpha, \beta > 0}$$

If λ were not real.

$\lambda_1 = \lambda_2^*$ $\qquad \lambda_1 + \lambda_2 = -\alpha$

$(\lambda - \lambda_1)(\lambda - \lambda_2) = 0 \qquad 2\operatorname{Re}\lambda_1 = -\alpha = 2\operatorname{Re}\lambda_2$

$\operatorname{Re}\lambda_1 = -\alpha/2 < 0$

If λ were real λ_1, λ_2

$\lambda_1 + \lambda_2 = -\alpha$

$\lambda_1 \cdot \lambda_2 = \beta$

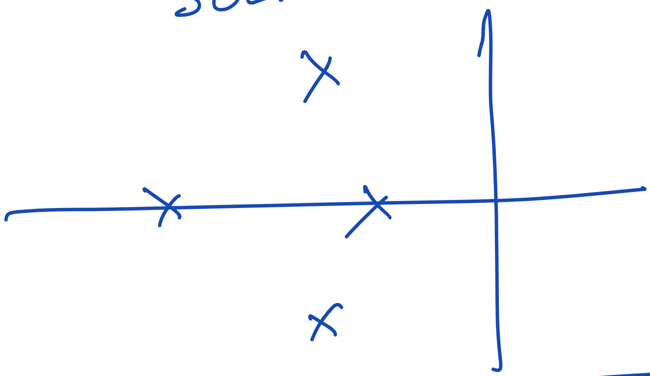
λ_1, λ_2 are both + or both −

Impossible because $\lambda_1 + \lambda_2 = -\alpha$

$\lambda_1, \lambda_2 < 0$

$\lambda_1^2 + \alpha \lambda_1 + \beta = 0$ $\quad \mathbb{C}_-^o$

SOLNS LIE IN



$K_v \succ 0 \qquad K_p \succ 0$

$$\begin{bmatrix} k_{v_1} & & 0 \\ & \ddots & \\ 0 & & k_{v_n} \end{bmatrix} \quad \begin{bmatrix} k_{p_1} & & 0 \\ & \ddots & \\ 0 & & k_{p_n} \end{bmatrix}$$

diagonal matrices

$$\ddot{e} + \begin{bmatrix} k_{v_1} & & 0 \\ & \ddots & \\ 0 & & k_{v_n} \end{bmatrix} \dot{e} + \begin{bmatrix} k_{p_1} & & 0 \\ & \ddots & \\ 0 & & k_{p_n} \end{bmatrix} e = 0$$

$$\ddot{e}_1 + k_{v_1}\dot{e}_1 + k_{p_1}e_1 = 0$$

$$\ddot{e}_2 + k_{v_2}\dot{e}_2 + k_{p_2}e_2 = 0$$

$$\vdots$$

$$\ddot{e}_n + k_{v_n}\dot{e}_n + k_{p_n}e_n = 0$$

decoupled

COMPUTED TORQUE → FEEDBACK LINEARIZATION

GM – FANUC PAINTING,

P. D. CONTROLLER

CONTROL OF LIMBS (BIOLOGICAL MOTOR CONTROL)

N. BERNSTEIN Computes? CONTROL LAWS

MUSCLES

P.D.

Let us say $\theta_d(t) \equiv \theta^f$.

$\dot{\theta}_d \equiv 0$

PD

$$\tau = -K_v \dot{e} - K_p e.$$

$$= \underbrace{-K_v(\dot{\theta} - \cancel{\dot{\theta}_d})}_{-K_v \dot{\theta}} \quad \checkmark D \quad -K_p(\theta - \theta_d) \quad \checkmark P$$

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = -K_v \dot{e} - K_p e.$$

$K_v > 0 \quad K_p > 0$

$\theta \to \theta_d$ as $t \to \infty$ $\quad e \to 0$

$\dot{e} \to 0 \quad \ddot{e} \Rightarrow 0$

$\theta_d(t)$

$$\tau = \boxed{M(\theta)}\ddot{\theta}_d + \boxed{C(\theta, \dot{\theta})}\dot{\theta}_d + \boxed{N(\theta, \dot{\theta})} - K_v \dot{e} - K_p e$$

BIOLOGICAL MOTOR CONTROL

LEARNING