$$se(3) \ni \hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 000 & 0 \end{bmatrix} \qquad \xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 0 & -\omega_3 & \omega_2 & v_1 \\ \omega_3 & 0 & -\omega_1 & v_2 \\ -\omega_2 & \omega_1 & 0 & v_3 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

$$e^{\hat{\xi}\theta} = \left[\begin{array}{c|c} R & p \\ \hline 000 & 1 \end{array}\right] \in SE(3)$$

Proof Case(1) $\omega = 0$ $\quad \hat{\xi} = \begin{bmatrix} 0 & v \\ 000 & 0 \end{bmatrix}$

$$\hat{\xi}^2 = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix} = 0 \qquad \hat{\xi}^3 = 0 = \hat{\xi}^4 = \dots$$

$$e^{\hat{\xi}\theta} = I + \hat{\xi}\theta + \frac{\hat{\xi}^2\theta^2}{2!} + \frac{\hat{\xi}^3\theta^3}{3!} + \dots$$

Pure Translation

$$= I_4 + \hat{\xi}\theta$$

$$= \begin{bmatrix} I & v\theta \\ 000 & 1 \end{bmatrix} \in SE(3)$$

Case(2) $\omega \neq 0$

$$e^{\hat{\xi}\theta} = e^{\underbrace{g_0 g_0^{-1}}\,\hat{\xi}\,\underbrace{g_0 g_0^{-1}}\theta} \qquad g_0 \in SE(3)$$

$$= g_0\, e^{\underbrace{g_0^{-1}\hat{\xi} g_0}\theta} g_0^{-1}$$

$$g_0 = \begin{bmatrix} I & \omega \times v \\ 000 & 1 \end{bmatrix} \qquad g_0^{-1} = \begin{bmatrix} I & -\omega \times v \\ 000 & 1 \end{bmatrix}$$

W.L.O.G.

$$g_0^{-1}\hat{\xi} g_0 = \begin{bmatrix} I & -\omega \times v \\ 000 & 1 \end{bmatrix}\begin{bmatrix} \hat{\omega} & v \\ 000 & 0 \end{bmatrix}\begin{bmatrix} I & \omega \times v \\ 000 & 1 \end{bmatrix}$$

$\|\omega\|=1$

$$= \begin{bmatrix} I & -\omega\times v \\ 000 & 1 \end{bmatrix} \begin{bmatrix} \hat{\omega} & \hat{\omega}^2 v + v \\ 000 & 0 \end{bmatrix} \qquad \hat{\omega}^2 = \omega\omega^T - I$$

$$= \begin{bmatrix} I & -\omega\times v \\ 000 & 1 \end{bmatrix} \begin{bmatrix} \hat{\omega} & \omega\omega^T v \cancel{- v} \cancel{+ v} \\ 000 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & -\omega\times v \\ 000 & 1 \end{bmatrix} \begin{bmatrix} \hat{\omega} & \omega\omega^T v \\ 000 & 0 \end{bmatrix}$$

$$\hat{\xi}' = g_0^{-1}\hat{\xi} g_0 = \begin{bmatrix} \hat{\omega} & \omega\omega^T v \\ 000 & 0 \end{bmatrix}$$

$$e^{\hat{\xi}\theta} = g_0 e^{\hat{\xi}'\theta} g_0^{-1}$$

$$e^{\hat{\xi}'\theta} = I + \hat{\xi}'\theta + \frac{\hat{\xi}'^2\theta^2}{2!} + \frac{\hat{\xi}'^3\theta^3}{3!} + \dots$$

$$\longrightarrow \quad \hat{\xi}' = \begin{bmatrix} \hat{\omega} & \omega\omega^T v \\ 000 & 0 \end{bmatrix} \longleftarrow$$

$$\hat{\xi}'^2 = \begin{bmatrix} \hat{\omega} & \omega\omega^T v \\ 000 & 0 \end{bmatrix} \begin{bmatrix} \hat{\omega} & \omega\omega^T v \\ 000 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\omega}^2 & \cancel{\hat{\omega}(\omega\omega^T v)}\ 0 \\ 000 & 0 \end{bmatrix}$$

$$\Big[\ \hat{\omega}^2 \quad 0\ \Big]$$

$$= \begin{bmatrix} \hat\omega & \\ 0\,0\,0 & 0 \end{bmatrix}$$

$$\hat\xi^{13} = \begin{bmatrix} \hat\omega^3 & 0 \\ 0\,0\,0 & 0 \end{bmatrix}$$

$$e^{\hat\xi^1\theta} = I + \hat\xi^1\theta + \frac{\hat\xi^2\theta^2}{2!}$$

$$= \left[\begin{array}{c|c} I + \hat\omega\theta + \frac{\hat\omega^2\theta^2}{2!} + \cdots & \omega\omega^T v\theta \\ \hline 0\;0\;0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} e^{\hat\omega\theta} & \omega\omega^T v\theta \\ 0\,0\,0 & 1 \end{bmatrix}$$

$$e^{\hat\xi\theta} = g_0 \boxed{e^{\hat\xi^1\theta}} g_0^{-1}$$

$$= \begin{bmatrix} I & \omega\times v \\ 0\,0\,0 & 1 \end{bmatrix} \begin{bmatrix} e^{\hat\omega\theta} & \omega\omega^T v\theta \\ 0\,0\,0 & 1 \end{bmatrix} \begin{bmatrix} I & -\omega\times v \\ 0\,0\,0 & 1 \end{bmatrix}$$

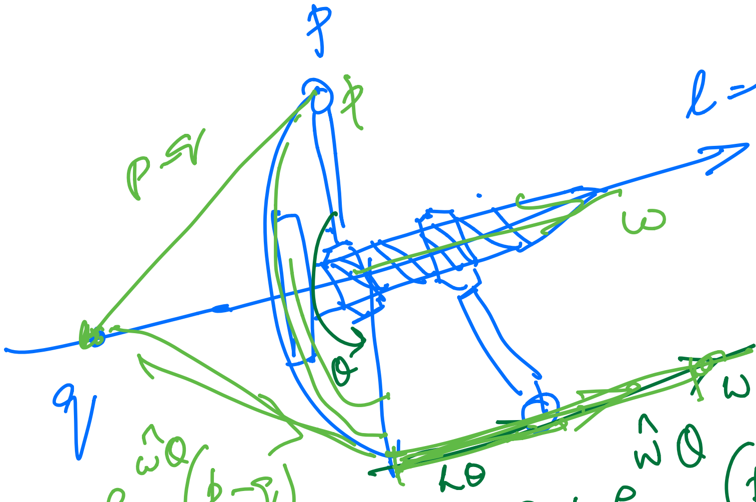
$$= \begin{bmatrix} I & \omega\times v \\ 0\,0\,0 & 1 \end{bmatrix} \begin{bmatrix} e^{\hat\omega\theta} & e^{\hat\omega\theta}(-\omega\times v) + \omega\omega^T v\theta \\ 0\,0\,0 & 1 \end{bmatrix}$$

$$SO(3) \quad = \begin{bmatrix} \boxed{e^{\hat\omega\theta}} & \boxed{(I - e^{\hat\omega\theta})\,\omega\times v + \omega\omega^T v\theta} \\ 0\,0\,0 & \boxed{1} \end{bmatrix} \quad p \in \mathbb{R}^3$$

$$e^{\hat\omega\theta} = I + \hat\omega\sin\theta + \hat\omega^2(1-\cos\theta)$$

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 000 & 0 \end{bmatrix} \longleftrightarrow e^{\hat{\xi}\theta} = \begin{bmatrix} R & p \\ 000 & 1 \end{bmatrix} \in SE(3)$$

$$l = \{ q + \lambda\omega : \lambda \in \mathbb{R} \}$$



$$gp = q + e^{\hat{\omega}\theta}(p-q) + h\theta \cdot \omega$$

$$\begin{bmatrix} gp \\ 1 \end{bmatrix} = \begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} e^{\hat{\omega}\theta} & \boxed{(I - e^{\hat{\omega}\theta})q + h\theta\omega} \\ 000 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix}$$

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 000 & 0 \end{bmatrix} \qquad \|\omega\| = 1$$

h.

$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})\omega \times v + \omega \omega^T v \theta \\ 000 & 1 \end{bmatrix}$$

$h = \omega^T v$ PITCH

Ans ω is axis of twist
θ is mag. of screw = angle of twist

Claim $\boxed{v = -\omega \times q + h\omega}$

Proof of Claim

$$(I - e^{\hat{\omega}\theta})\,\omega \times v + \omega h \theta$$

$$= (I - e^{\hat{\omega}\theta})\,\hat{\omega}(-\hat{\omega}q + h\omega) + \omega h \theta$$

$$= (I - e^{\hat{\omega}\theta})(-\hat{\omega}^2 q + \cancel{\hat{\omega} h \omega}) + \omega h \theta$$

$$= (I - e^{\hat{\omega}\theta})(-\omega\omega^T + I)q + \omega h \theta$$

$$(I - e^{\hat{\omega}\theta})(\omega\omega^T) = 0$$

$$\left(\cancel{I} - \left(\cancel{I} + \cancel{\hat{\omega}\theta} - \cancel{\frac{\hat{\omega}^2\theta^2}{2!}} + \cdots\right)\right)(\cancel{\omega}\omega^T)$$

$$= (I - e^{\hat{\omega}\theta})q + \omega h \theta \qquad \square$$