Homework 1 Solutions

1 System Interconnect

You have been tasked with building a system-interconnect for a system-on-chip with 256 processing cores. You are considering several options for the on-chip system interconnect network: (1) a 16-ary 2-mesh, (2) a 16-ary 2-cube, (3) an 8-ary 2-Cmesh4, and (4) 16-ary 3-fly Clos. The application requires a packet size of 1024 bits.

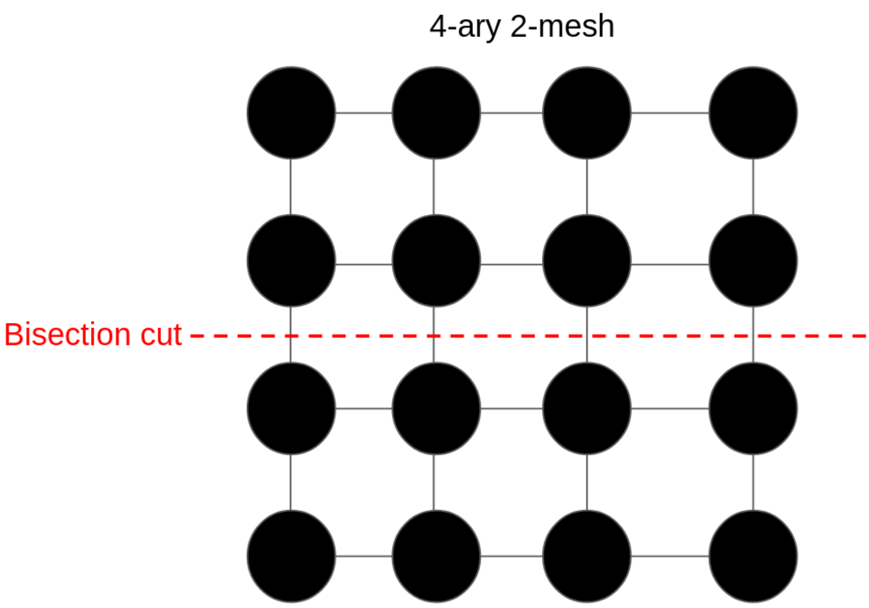
1. a) Calculate the required number of bits per unidirectional channel, for each network option, such that each of the networks can support the ideal throughput of 64 bits/cycle/core under uniform random traffic.

Question 1. a) - 16-ary 2-mesh

First, let's calculate the total throughput (Θ_{total}) of the network.

$$\Theta_{total} = N * \Theta_{core} = 256 \text{ cores} * 64 \text{ bits/core/cycle} = 16,384 \text{ bits/cycle} \tag{1}$$

Now we need to determine the bisection bandwidth (B_B) and the number of unidirectional bisection channels (B_C). A 4-ary 2-mesh network is shown below. Let's solve for its B_B and B_C and then generalize the results.



The above figure's bisection cut goes through 4 bidirectional channels. Thus we get $B_C = 4 * 2 = 8$ unidirectional channels. We can also see that under uniform random traffic this topology should see a bisection bandwidth that is exactly half of the total throughput. Generalizing our results gives us:

$$B_C = \sqrt{N} * 2 \tag{2}$$

$$B_B = \frac{\Theta_{total}}{2} \tag{3}$$

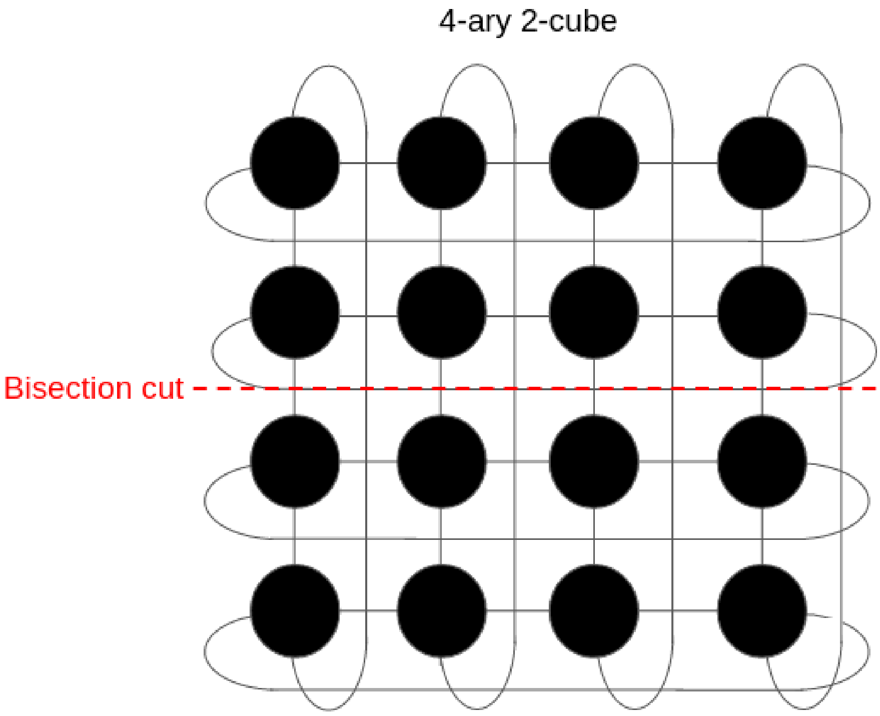
Now we can solve for the unidirectional bandwidth (b_C) by dividing the bisection bandwidth (B_B) by the number of bisection channels (B_C). Plugging in the numbers for a 16-ary 2-mesh gives us:

$$b_C = \frac{B_B}{B_C} = \frac{\frac{16{,}384}{2}}{16 * 2} = 256 \text{ bits/channel} \tag{4}$$

Question 1. a) - 16-ary 2-cube

A similar analysis to "Question 1. a) - 16-ary 2-mesh" is performed for this topology.

A smaller network, 4-ary 2-cube, is shown below



The above figure's bisection cut goes through 8 bidirectional channels. Thus we get $B_C = 8 * 2 = 16$ unidirectional channels. We can also see that under uniform random traffic this topology should see a bisection bandwidth that is exactly half of the total throughput. Generalizing our results gives us:

$$B_C = \sqrt{N} * 4 \tag{5}$$

$$B_B = \frac{\Theta_{total}}{2} \tag{6}$$

Now we can solve for the unidirectional bandwidth (b_C) by dividing the bisection bandwidth (B_B)

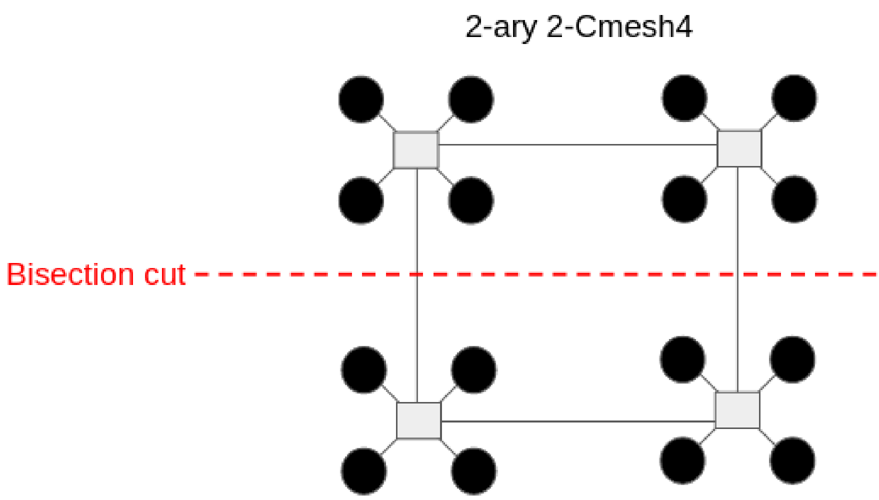
by the number of bisection channels (B_C). Plugging in the numbers for a 16-ary 2-mesh gives us:

$$b_C = \frac{B_B}{B_C} = \frac{\frac{16{,}384}{2}}{16 * 4} = 128 \text{ bits/channel} \tag{7}$$

Question 1. a) - 8-ary 2-Cmesh4

A similar analysis to "Question 1. a) - 16-ary 2-mesh" is performed for this topology.

A smaller network, 2-ary 2-Cmesh4, is shown below



The above figure's bisection cut goes through 2 bidirectional channels. Thus we get $B_C = 2 * 2 = 4$ unidirectional channels. We can also see that under uniform random traffic this topology should see a bisection bandwidth that is exactly half of the total throughput. Generalizing our results gives us:

$$B_C = \sqrt{\frac{N}{4}} * 2 \tag{8}$$

$$B_B = \frac{\Theta_{total}}{2} \tag{9}$$

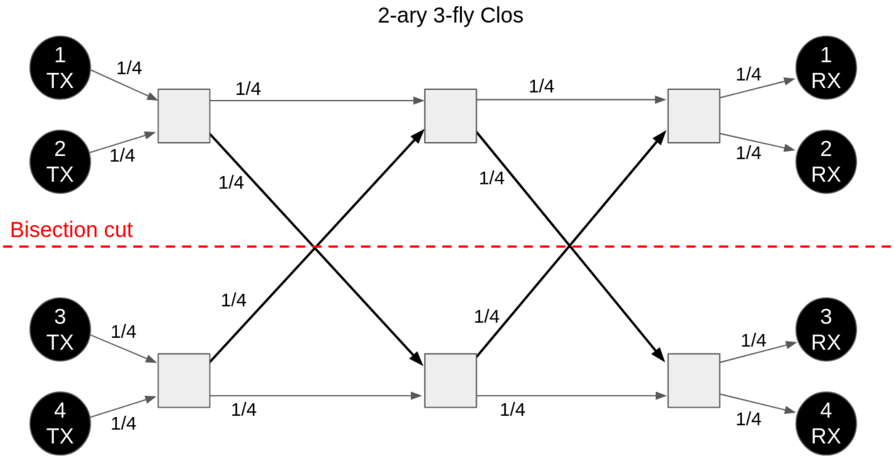
Now we can solve for the unidirectional bandwidth (b_C) by dividing the bisection bandwidth (B_B) by the number of bisection channels (B_C). Plugging in the numbers for a 8-ary 2-Cmesh4 gives us:

$$b_C = \frac{B_B}{B_C} = \frac{\frac{16{,}384}{2}}{\sqrt{64} * 2} = 512 \text{ bits/channel} \tag{10}$$

Question 1. a) - 16-ary 3-fly

A similar analysis to "Question 1. a) - 16-ary 2-mesh" is performed for this topology.

A smaller network, 2-ary 3-fly Clos, is shown below



The above figure's bisection cut goes through 4 bidirectional channels. Thus we get $B_C = 2 * 2 = 8$ unidirectional channels. We can also see that under uniform random traffic this topology should see a bisection bandwidth that is exactly equal to the total throughput. This is different than the previous topologies.

In the above image, the labeled "1/4" on all the channels represents the portion of the total network throughput that any given channel is seeing under uniform random traffic. Each core inputs 1/4 of the total throughput into the first stage (because we have 4 cores total in this example). The first stage switches will split the 1/4 input throughput from one node to two 1/8 throughput streams onto the outgoing channels. This will give us 1/4 of the total network throughput on both outgoing channels (1/8 from one input and 1/8 from the other). This is repeated until we output 1/4 of the total network throughput to each of the core's receivers. Now we can draw our bisection line, shown in red, and calculate the ratio of total throughput in our bisection cut. It should be very clear that the ratio is 1:1 because we have 4 channels crossing the bisection each carrying 1/4 of the total data throughput.

Generalizing our results gives us:

$$B_C = N \tag{11}$$

$$B_B = \Theta_{total} \tag{12}$$

Now we can solve for the unidirectional bandwidth (b_C) by dividing the bisection bandwidth (B_B) by the number of bisection channels (B_C). Plugging in the numbers for a 16-ary 3-fly gives us:

$$b_C = \frac{B_B}{B_C} = \frac{16,384}{256} = 64 \text{ bits/channel} \tag{13}$$

Question 1. b) - 16-ary 2-mesh

Assuming cut-through flow control, we can calculate the zero-load latency as:

$$T_0 = H * (t_r + \frac{D}{v}) + \frac{L}{b} \tag{14}$$

Where H is the hop count, t_r is the router delay, D is the physical distance the signal must traverse, v is the speed of the signal, L is the length of the signal in bits, and b is the channel bandwidth.

We have been given a channel latency of 1 cycle. This latency covers both the routing delay (t_r) and the time of flight delay ($\frac{D}{v}$) for a single hop. We also know that our serialization delay is the number of cycles required to serialize the packet across a channel of width b_C (this is the number we calculated in the previous question). Thus, our latency equation can be rewritten in terms of "cycles":

$$T_0 = H + \frac{\text{flits per packet}}{\text{cycles per flit}} \tag{15}$$

The flit size is set to a static 64 bits regardless of the channel width b_C. With a packet size of 1024 bits, this gives us $\frac{\text{1024 bits per packet}}{\text{64 bits per flit}} = 16$ flits per packet.

"cycles per flit" is greater than 1 only when the channel width (b_C) is less than the flit size. The routers will allocate buffer space at the flit level and thus channels w/ widths greater than the flit size will be underutilized (i.e. flits are sent sequentially across channels even for channels with widths greater than the flit size. It is important to note that networks will generally never be designed such that the flit size is less than the channel width, but this question is an exception).

We want the worst-case zero-load latency and thus we have to find the diameter (H_{max}) of the network to plug into our latency equation. H_{max} is the longest minimum-hop path between two nodes. For this topology, H_{max} is found from one corner node to the opposite corner node. We can now solve for the latency of the 16-ary 2-mesh:

$$H_{max} = (\sqrt{N} * 2) - 1 = 31 \tag{16}$$

$$T_0 = H_{max} + \frac{\text{flits per packet}}{\text{cycles per flit}} + \frac{16}{1} = 47 \text{ cycles} \tag{17}$$

Question 1. b) - 16-ary 2-cube

H_{max} for this topology is found by going from one of the middle nodes to one of the corner nodes on the opposite side of the network. Thus,

$$H_{max} = \sqrt{N} = 16 \tag{18}$$

$$T_0 = H_{max} + \frac{\text{flits per packet}}{\text{cycles per flit}} = 16 + \frac{16}{1} = 32 \text{ cycles} \tag{19}$$

Question 1. b) - 8-ary 2-Cmesh4

H_{max} for this topology is found by going from one of the corner node clusters to a node in the opposite corner node cluster.

$$H_{max} = (\sqrt{N/4} * 2) - 1 = 15 \tag{20}$$

$$T_0 = H_{max} + \frac{\text{flits per packet}}{\text{cycles per flit}} = 15 + \frac{16}{1} = 31 \text{ cycles} \tag{21}$$

Question 1. b) - 16-ary 3-fly

H_{max} for this topology is found by going from any node to any other node.

$$H_{max} = 3 \tag{22}$$

$$T_0 = H_{max} + \frac{\text{flits per packet}}{\text{cycles per flit}} = 3 + \frac{16}{1} = 19 \text{ cycles} \tag{23}$$