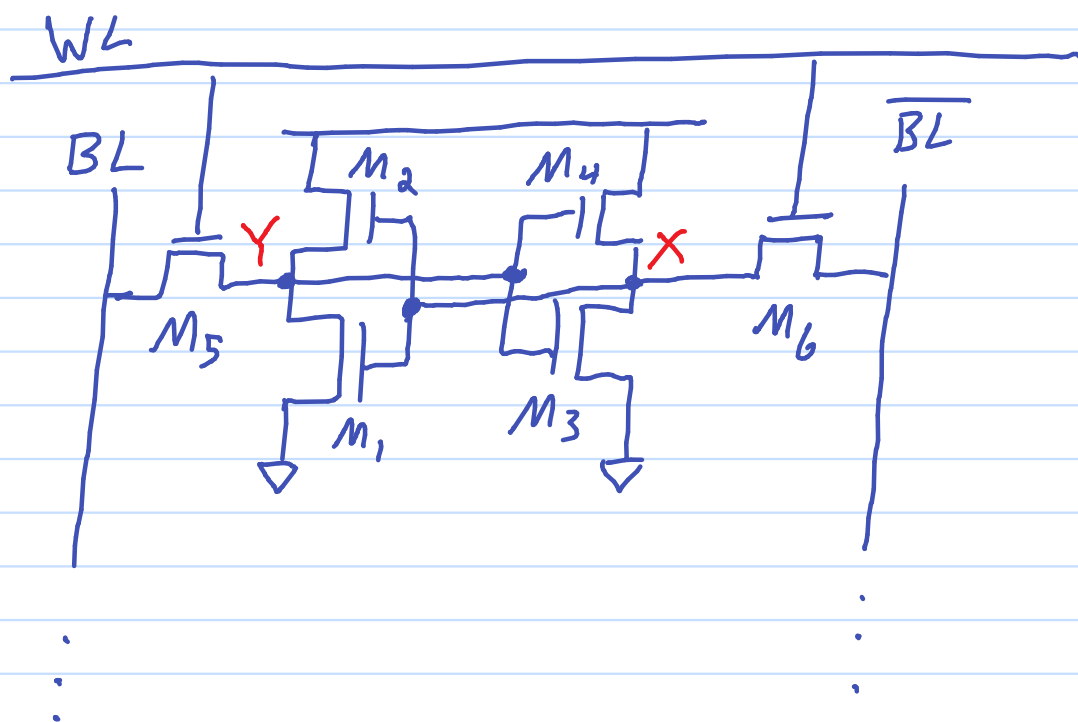# EECS 251B HW3 Solutions

(1)

a) $V_{WL} \downarrow \Rightarrow M_5$ & M_6 effective resistance increases relative to $M1$ & $M3$. This __increases read stability__ by lowering the voltage seen across nodes X/Y during a read.

b) $V_{WL} \downarrow \Rightarrow$ Assuming M_1 and M_3 are not limiting the current from the bitline, then decreasing the word line voltage will __increase the read access time__

c) $V_{WL} \downarrow \Rightarrow M_5$ & M_6 R_{eff} increases relative to M_2 and M_4. This __decreases write stability__ by increasing the voltage at nodes X/Y during a write.

d)
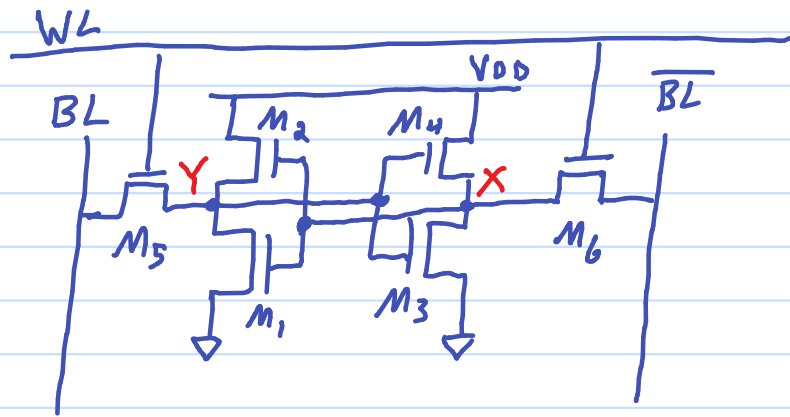
$\uparrow V_{DD} \Rightarrow$ Increased cell supply voltage will decrease R_{eff} of M_2 & M_4 relative to the access transistors M_5 & M_6. This decreases write stability as the voltage drop will be larger across the access transistors M_5 & M_6 during a write.

② 2 MB
128 × 128 bit arrays
4 columns/sense amp
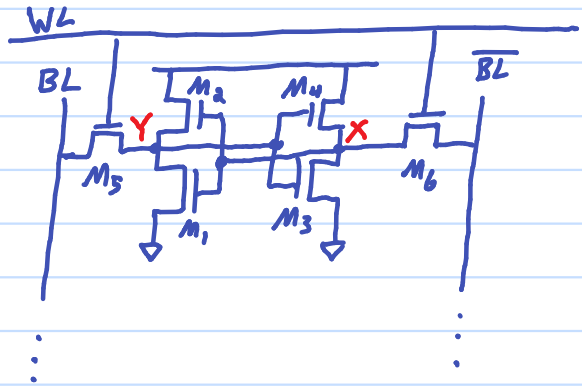$C_{BL} = 1 pF$
$V_{DD} = 1 V$
$I_{ON} = 100 mA$
$I_{OFF} = 400 nA$
$t_{read} = 1 ns$
$\left.\begin{array}{l} M_{offset} = 0 V \\ \\ \sigma_{offset} = 15 mV \end{array}\right\}$ ← sense amp offset
normally distributed

The I_{ON} and I_{OFF} currents are the currents
from the bitline through the access transistor and
through either M_1 or M_3 during a read (depends on
whether node X or Y is at 0V). We want to
know the minimum ΔV_{BL} and make sure it is
greater than the sense amp's offset voltage. This
is what determines whether or not the sense amp
"yields".

(2.) cont...

First step is to calculate the minimum (i.e. worst case) ΔV_{BL}. This is seen when all cells in the column store 0's while the cell being read stores a 1 (or vice versa).

The ΔV_{BL} in this case becomes

All cells in column except 1

change in BL   change in $\overline{BL}$

$$\Delta V_{BL} = \Delta V_{BL-1} - \Delta V_{BL-2} = \frac{I_{on} \cdot t_{read}}{C_{BL}} - \frac{127 \cdot I_{off} \cdot t_{read}}{C_{BL}}$$

Plugging in we get

$$\Delta V_{BL} = \frac{(100mA)(1ns)}{1pF} - \frac{(127)(400nA)(1ns)}{1pF}$$

$$\Delta V_{BL} = 100mV - 51mV = 49mV$$

$$\boxed{\Delta V_{BL} = 49mV}$$

Now calculate the number of columns in the SRAM array.

$MiB = 2^{20}$ bytes

$$\#_{cols} = \frac{\text{Size of Mem}}{\text{Column Length}} = \frac{2MiB}{128b} = 131,072$$

Now get the # of sense amps

$$\#_{SA} = \frac{\#_{cols}}{4} = \frac{131,072}{4} = 32,768$$

(2) cont...

Now let's get the probability that a single
Sense amp will yield. This is done by first
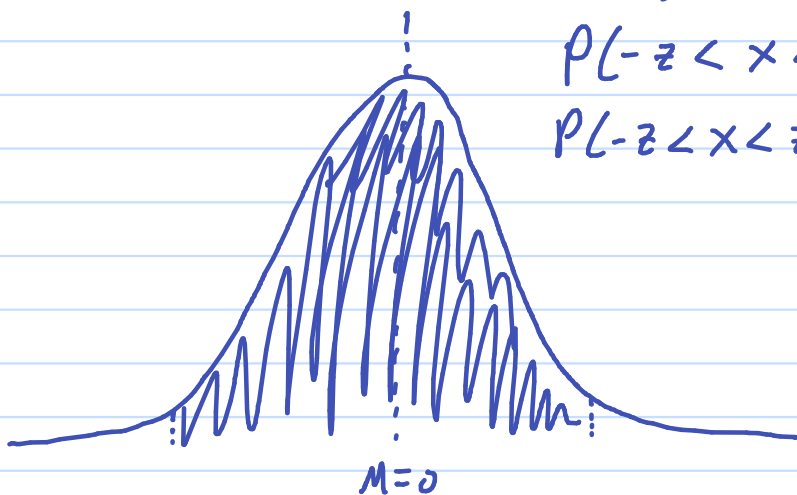computing the z-score of our minimum ΔV_{BL}

$$z = \frac{\Delta V_{BL} - M}{\sigma} = \frac{49 nV}{15 nV} = 3.27$$

Now we need to get the two sided cumulative
probability when using a z-score table because
an offset of $> 49 nV$ or $< -49 nV$ will render
the sense amp useless in this worst-case scenario.

$$z = 3.27$$
$$P(-z < x < z) = 1 - 2(0.0005)$$
$$P(-z < x < z) = 0.999$$

$M = 0$

Thus, $P_{SA} = 0.999$

Without redundancy our array would have a yield of

$$yield = (P_{SA})^{\# SAs} = (0.999)^{32,768} = 5.8e-15$$

This is crazy low... Now we'll use redundancy!

② cont...

Redundant columns can only be used to replace defective columns in their own sub-array. Let's calculate the yield needed for each sub-array to get $\geq 99\%$ yield in the entire SRAM

$$\#_{\text{sub-arrays}} = \frac{\text{Size of Mem}}{\text{Size of sub}} = \frac{2\text{MiB}}{128 \times 128} = 1024$$

$$P_{\text{sub}} = (0.99)^{\frac{1}{1024}} = 0.99999$$

Now we need to figure out how many additional sense amps we need to get our sub array to have this yield.

$$\#_{\text{SAs}/\text{Sub}} = \frac{\#\text{cols}}{4} = \frac{128}{4} = 32 \text{ SAs/sub-array}$$

0 additional

Too low

$$P_{\text{sub-work}} = P_{\text{SA}}^{32} = 0.968$$

All 33 work

Only 1 SA is broken

1 additional

$$P_{\text{sub-work}} = P_{33\text{-work}} + P_{32\text{-work}} = P_{\text{SA}}^{33} + \binom{33}{32} P_{\text{SA}}^{32}(1 - P_{\text{SA}})$$

33    $\binom{33}{32} P_{\text{SA}}^{32}(1 - P_{\text{SA}})$
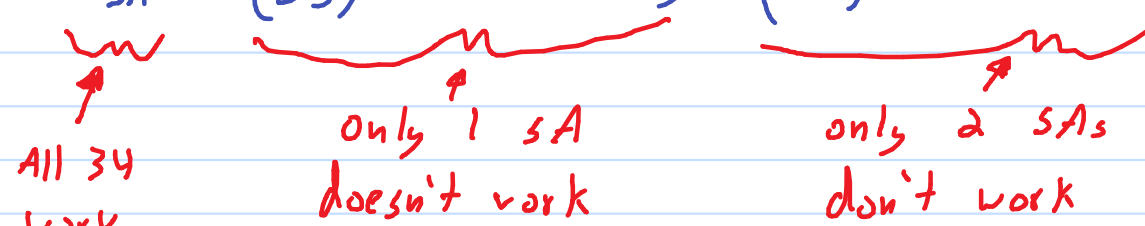
$$P_{\text{sub-work}} = 0.9995 \quad \longleftarrow \quad \text{still too low}$$

② cont...

2 additional sense amps

$$P_{sub-work} = P_{34-work} + P_{33-work} + P_{32-work}$$

$$P_{sub-work} = P_{SA}^{34} + \binom{34}{33} P_{SA}^{33}(1-P_{SA}) + \binom{34}{32} P_{SA}^{32}(1-P_{SA})^2$$

All 34 work

only 1 SA doesn't work

only 2 SAs don't work

$$P_{sub-work} = 0.999994 \quad \text{This is } \underline{is} \text{ large enough!}$$

We have found that adding 2 additional sense amps gives us a sub-array yield greater than what is needed to get the entire array's yield to 99%.

2 sense amps means we need 2×4 = 8 additional columns per sub-array

8 redundant columns per sub-array