Formulating Laplacian Matrix Generation as a N-D Stencil Problem

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As per Professor James Demmel's lecture note[1], we can generate the heat equation matrix as $T = I - z * L$ with $z = \frac{C\Delta t}{h^2}$. C is the heat diffusivity constant and h as the position step size, or distance between two sample points. L, the Laplacian matrix, assumes that the mesh in question is fully embedded within a larger superstructure, so that there are no true boundary positions.

The Laplacian matrix is defined as follows for a n-D space S where p_i is the coordinate of point i of degree n [2]

$$S_{p_i,p_j} = \begin{cases} 2^n & \text{if } p_i = p_j \\ -1 & \text{if } ||p_i p_j||_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

We can thus express S as a matrix of dimension $2n$, the first n being p_i and the second being p_j. To obtain our final 2D matrix, we can simply reshape by linearizing the first n dimensions and the last n.

[1] http://www.cs.berkeley.edu/~demmel/cs267_Spr14/Lectures/lecture21_structured_jwd14_4pp.pdf

[2] http://en.wikipedia.org/wiki/Laplacian_matrix