Spiral Down Effect, Elasticity

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Static Airline model

At price p_1

At price p_2

- C seats in total
- 2 classes - Business (D_1, p_1), Leisure (D_2, p_2)

$p_1 < p_2$

- Assumption: leisure passengers arrive before business passengers
- If you know D_1 and D_2 exactly, what would you do?

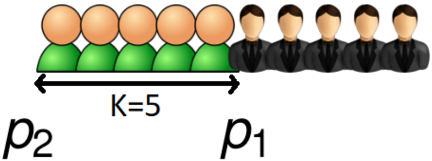
Reserve D_1 seats first, then sell D_2

Question

- In reality, D_1 and D_2 are random
- How many Business class seats to reserve?
- Equivalently, how many seats to sell at price p_2?

Model

- decide how many seats K to sell at price p_2, given capacity C and price p_1, p_2
- Pricing policy: Set price to p_2 at beginning, change to p_1 after selling K seats
- What is the trade-off?



Solution

Expectation to deal with randomness

Expected revenue

$$r(K) = p_2 E[\min(D_2, K)] + p_1 E[\min(D_1, \{C - \min(D_2, K)\})]$$

Take min - because if $D_2 < K \Rightarrow D_2$; $D_2 > K \Rightarrow K$

- select K to maximize $r(K)$
- $K = C - F_1^{-1}\left(1 - \dfrac{p_2}{p_1}\right)$
- $F_1^{-1}\left(1 - \dfrac{p_2}{p_1}\right) = C - K =$ number of seats to protect

  seats reserved

- Littlewood's model

cdf of business customers

1st order differential

Assumptions

- Low WTP passengers arrive before high WTP passengers
- Independent demands
- No group bookings

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All models are wrong, but some are useful - George Box

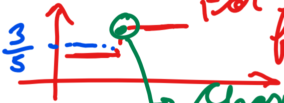
Spiral down effect

- Repeated flight at times $k = 1, 2, 3, \ldots$
- Capacity $C = 10$, with two fare classes $p_1 = 500$, $p_2 = 200$
- 8 customers in each period
- Demand function: buy the cheapest fare available
- Demand update: $F_1^k(x) = \frac{1}{k}\sum_{j=1}^{k} I_{\{\text{demand of fare 1 at period } j \le x\}}$
- Recall: $F_1^{-1}\left(1 - \frac{p_2}{p_1}\right) = F_1^{-1}\left(\frac{3}{5}\right) = \min\{x | F(x) \ge \frac{3}{5}\}$

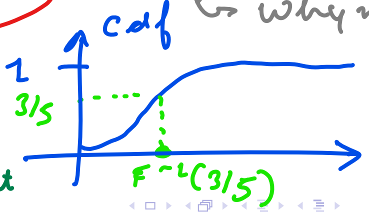
why min?

- Intial protection level: $L^0 = 10$

For discrete funct^n



Choosing this point implies min.



Spiral down effect

k	Protection level L^{k-1}	Class 1 sales	Class 2 sales	Revenue
1	10	8	0	4000

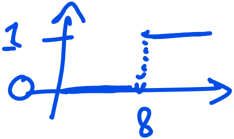
Indicator func' (0 or 1)

$$F_1^1(x) = \frac{1}{1}\sum_{j=1}^{1} I_{\{\text{demand of fare 1 at period } j \leq x\}}$$

$$= I_{\{\text{demand of fare 1 at period } j \leq x\}}$$

$$= \begin{cases} 0 & \text{if } x < 8 \\ 1 & \text{if } 8 \leq x \end{cases}$$

$\Rightarrow$ cdf



k	Protection level L^{k-1}	Class 1 sales	Class 2 sales	Revenue
2	8	6	2	3400

Spiral down effect

k	Protection level L^{k-1}	Class 1 sales	Class 2 sales	Revenue
1	10	8	0	4000
2	8	6	2	3400
3	8	6	2	3400

$$F_1^3(x) = \frac{1}{3}\sum_{j=1}^{3} I_{\{\text{demand of fare 1 at period } j \le x\}}$$

$$= \begin{cases} 0 & \text{if } x < 6 \\ \frac{1}{3}(0+1+1) = \frac{2}{3} & \text{if } 6 \le x < 8 \to \text{eg. for } F(7) \\ \frac{1}{3}(1+1+1) = 1 & \text{if } 8 \le x \end{cases}$$

k	Protection level L^{k-1}	Class 1 sales	Class 2 sales	Revenue
4	6	4	4	2800

Spiral down effect

Table 1. Spiral down with $c = 10$, $d = 8$, $f_1 = 500$, $f_2 = 200$, and $L^0 = 10$.

k	Protection level L^{k-1}	Obs. qty. X^k	Class-1 sales	Class-2 sales	Revenue ($)
1	10	8	8	0	4,000
2	8	6	6	2	3,400
3	8	6	6	2	3,400
4	6	4	4	4	2,800
⋮	⋮	⋮	⋮	⋮	⋮
8	6	4	4	4	2,800
9	4	2	2	6	2,200
⋮	⋮	⋮	⋮	⋮	⋮
20	4	2	2	6	2,200
21	2	0	0	8	1,600
⋮	⋮	⋮	⋮	⋮	⋮
50	2	0	0	8	1,600
51	0	0	0	8	1,600

Result

- Over the long run the estimate will be that there is no class 1 passenger
- revenue manager implements protection level 0
- will never see a class 1 passenger again
- What is the remedy?

The problem is endogeneity

reducing protection reduces demand.

Go for IV and exogenous variables

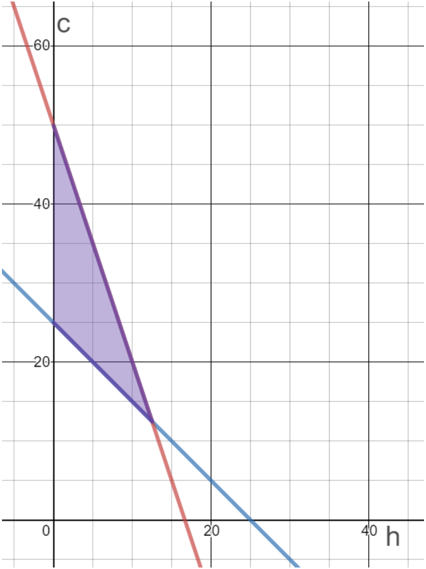
Lunch

- Budget \$50 for lunch per week
- Consume at least 25 units of protein
- 2 types of food: chicken breast and hamburger
- 1 protein/serving
- Chicken breast: \$1/serving, Hamburger: \$3/serving
- 1 happiness/serving of chicken, 4 happiness/serving of hamburger

How to maximize happiness given the budget and protein constraints?

Model

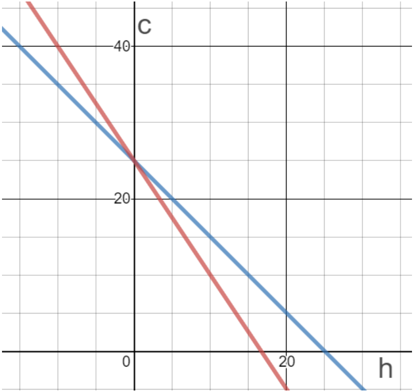
$$\begin{aligned} \max\ & 4h + c \\ \text{s.t. } & c + h \geq 25 \\ & c + 3h \leq 50 \\ & c \geq 0, h \geq 0 \\ OPT = & (12.5, 12.5) \end{aligned}$$



Price increase of chicken

Chicken is more expensive: \$2

$$
\begin{aligned}
& \max 4h + c \\
\text{s.t. } & c + h \geq 25 \\
& 2c + 3h \leq 50 \\
& c \geq 0, h \geq 0 \\
OPT = & (0, 25)
\end{aligned}
$$



Price Elasticity of Demand (PED)

$$\epsilon_D = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\frac{25-12.15}{12.5}}{\frac{2-1}{1}} = 1$$

- PED ≥ 0
- Effect is coming from the constraints
- Income effect bigger than substitution effect

Increasing price of chicken actually increased the demand!! eg If NUS increases its food price, people have lower income to spend outside, so they eat more in NUS!!