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H. W. 1

The 3 Demand equations are:

$$P_a = 56 - 4Q_a$$
$$P_s = 32 - 4Q_s$$
$$P_{sn} = 16 - 4Q_{sn}$$

Cost funct^n

$T(Q) = 100 + 12Q$

Marginal cost:

$MC = T'(Q) = \boxed{12}$

Further calculations assume a monopolistic market:

1 a) If no price-discrimination:

$$Q_a = 14 - \frac{P_a}{4} \; ; \quad P_a \leq 56$$
$$Q_s = 8 - \frac{P_s}{4} \; ; \quad P_s \leq 32$$
$$Q_{sn} = 4 - \frac{P_{sn}}{4} \; ; \quad P_{sn} \leq 16$$

Aggregating demand,

$$Q \begin{cases} = 14 - \frac{P}{4} \; ; & 32 \leq P \leq 56 \\ = 22 - P/2 \; ; & 16 \leq P < 32 \\ = 26 - 3P/4 \; ; & P < 16 \end{cases}$$

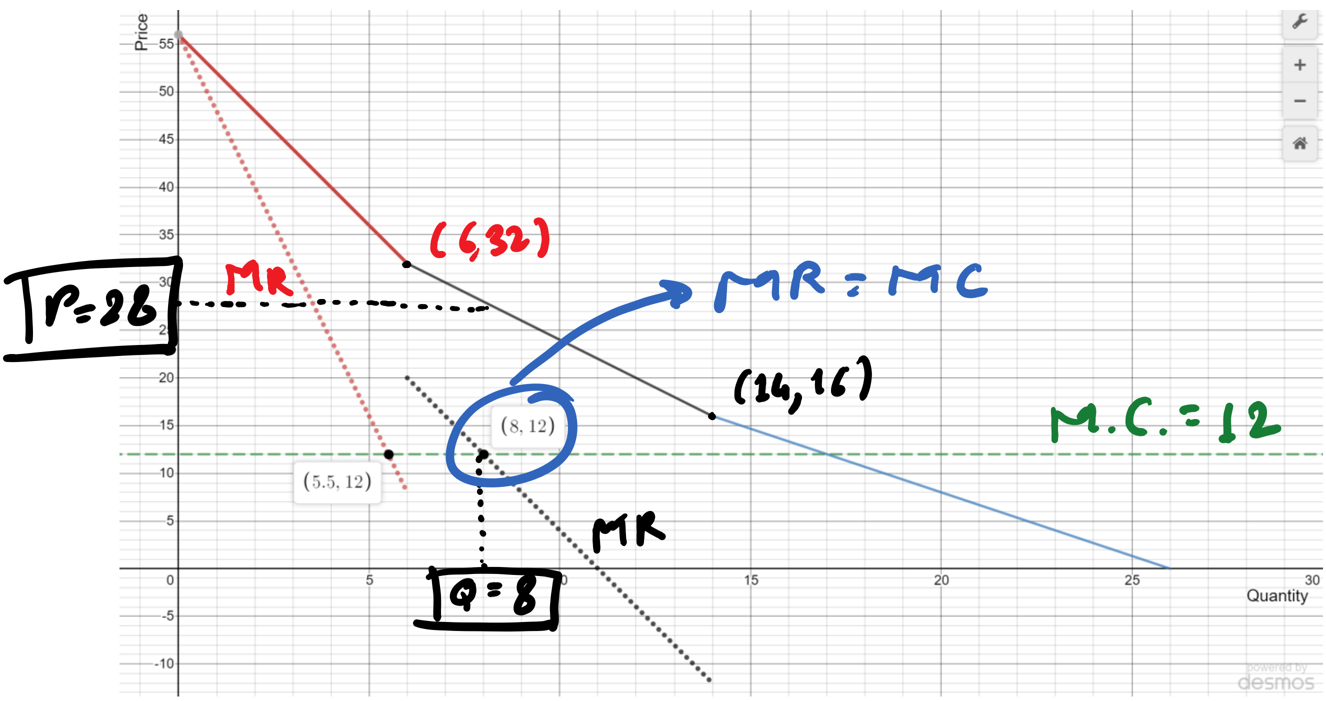
Inverting these functions;

$$P \begin{cases} = 56 - 4Q & ; \; Q \le 6 \\ = 44 - 2Q & ; \; 6 < Q \le 14 \\ = \dfrac{104 - 4Q}{3} & ; \; Q > 14 \end{cases}$$

Calculating M.R. curves (monopoly market)

$$MR \begin{cases} 56 - 8Q & ; \; Q \le 6 \\ 44 - 4Q & ; \; 8 < Q \le 14 \\ \dfrac{104 - 8Q}{3} & ; \; Q > 14 \end{cases}$$

Plotting the functions above:



Graph plotted on: desmos.com

We observe that there are 2 intersection points for MC = MR

Naturally, to generate higher profits, we take the point where MR is higher.

Hence, total quantity sold = 8 units

$\Downarrow$

Corresponding price = 28 ; Profit = 244

$\therefore Q_A = 14 - \left(\frac{28}{4}\right) = \boxed{7}$

$Q_S = 8 - \left(\frac{28}{4}\right) = \boxed{1}$

Add upto 8. No tickets are sold to seniors.

1 b) Assuming 3rd degree P.D.

For adults,

$P_A = 56 - 4Q_A$

$MR = 56 - 8Q_A = MC$

$\Rightarrow 56 - 8Q_A = 12$

$\Rightarrow \boxed{Q_A = 5.5}$

$\therefore Price_A = 56 - 4(5.5) = \boxed{34}$

For students,

$P_S = 32 - 4Q_S$

$MR = 32 - 8Q_S = 12$

$\Rightarrow \boxed{Q_S = 2.5}$

$\boxed{\therefore P_S = 22}$

For seniors,

$P_{sn} = 16 - 4Q_{sn}$

$MR = 16 - 8Q_{sn} = 12$

$\Rightarrow \boxed{Q_{sn} = 0.5}$

$\boxed{P_{sn} = 14}$

Total Profit = 187 + 55 + 7 = 249 (+2.05%)