Labor Supply

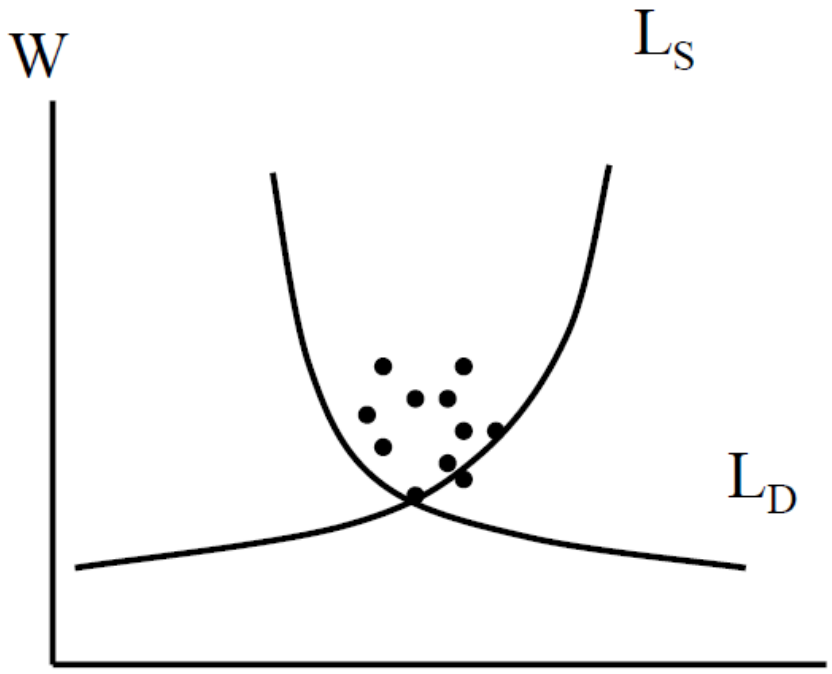
Contents

- Simultaneous equation models

- Stating the problem
- How to estimate given simultaneity
- Instrumental variables and two stage least squares (TSLS)

Example

- Example1.xls : examining the relationship between price and quantity
- Two variables: hours and wages
- Can construct a graph of labor demand and supply



We want to identify the relationship between hours and wages for workers

Structural model

- In terms of labor demand and labor supply:

$$(D): H_i = a_1 + b_1 W_i + u_i$$
$$(S): H_i = a_2 + b_2 W_i + c_2 X_i + d_2 NK_i + v_i$$

We can only estimate (D); because IV's are present for it only

where X_i = age and NK_i = number of kids ← IV's for (D)

- We estimate the labor supply curve for married women in California in 1980

Structural model (2)

- We assume labor market equilibrium, or $H_i^D = H_i^S$
- We also assume that hours and wages are endogenous
  - Both are determined at the same time: either could be placed as the dependent variable for the model
  - Since hours and wages are determined simultaneously, there is a relationship between wages and the error term u_i

Simultaneous Equation

Reduced forms

- Assuming labor market equilibrium

$$a_1 + b_1 W_i + u_i = a_2 + b_2 W_i + c_2 X_i + d_2 NK_i + v_i$$

- Working with the equation, we have:

$$W_i = \frac{(a_2 - a_1)}{(b_1 - b_2)} + \frac{c_2}{(b_1 - b_2)} X_i + \frac{d_2}{(b_1 - b_2)} NK_i + \frac{(v_i - u_i)}{(b_1 - b_2)}$$

  - This gives us an expression for wages in terms of the exogenous variables
  - Here, our exogenous variables are: X_i (age) and NK_i (number of kids)

Reduced forms (2)

- We can rewrite the expression for wages in a simpler form:

$$W_i = \pi_{10} + \pi_{11} X_i + \pi_{12} NK_i + \varepsilon_{1i}$$

  - This is the reduced form for wages

- We can also get a reduced form for hours:
  - Original expression: $H_i = a_1 + b_1 W_i + u_i$
  - By rewriting it and substituting into the labor supply equation, we arrive at:

$$H_i = \pi_{20} + \pi_{21} X_i + \pi_{22} NK_i + \varepsilon_{2i}$$

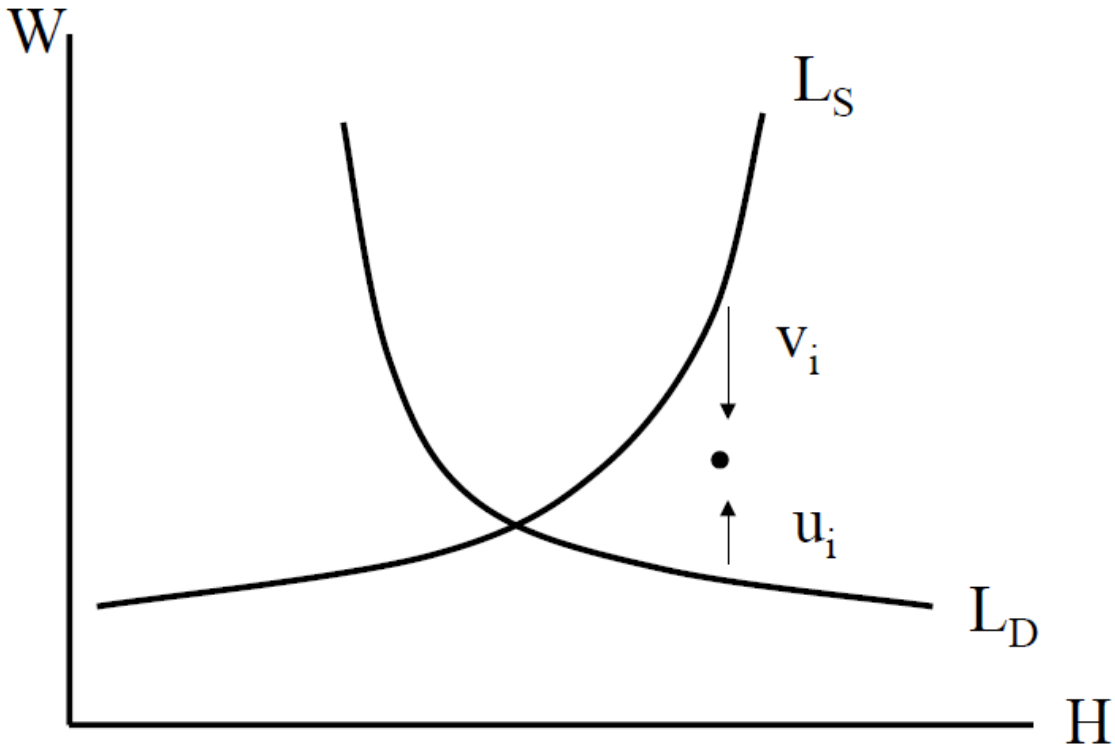
Reduced forms (3)

- The reduced forms give the relationship between each of the endogenous variables and the exogenous variables
  - also show how the errors are related

- Looking back at the derivation of the reduced form equations
  - see that each ε is composed of the error terms from both the labor demand and labor supply equation u_i and v_i

Reduced forms (4)

- Errors from both the labor demand and labor supply curves show up in both of the reduced form equations:



Reduced forms (5)

- If faced with the problem of simultaneity
  - model can be restated in terms of reduced forms
  - endogenous variables can be expressed solely in terms of exogenous variables
  - can estimate reduced forms using OLS if we have strictly exogenous variables on the right-hand-side
  - cannot recover structural parameters from the reduced form estimation
  - reduced forms only give an indication of the correlation between the endogenous and exogenous variables

Reduced forms (6)

- Our example: can only get estimates of the effects of age and the number of kids on hours of work and wages separately

- Three problems with reduced form estimation:
  - X_i and NK_i are assumed to be exogenous - might not be true
  - Structural model isn't estimated
  - Need structural model to examine more complex matters (example: effect of a tax cut on hours worked)

Instrumental variables and TSLS

↳ 2 stage Least Squares

- Since we think wages and hours are simultaneously determined, we expect:

$$E(W_i, u_i) \neq 0, L^D$$

$$E(W_i, v_i) \neq 0, L^S$$

- To estimate the model we will use instrumental variables, or two stage least squares (TSLS)
- Our model is: $H_i = a_1 + b_1 W_i + u_i$
  - we need a proxy for W_i in order to get an unbiased estimate b_1

Instrumental variables and TSLS (2)

- Why we need a proxy
  - Assuming wages and hours are correlated, the errors on H_i will be correlated with W_i
  - if we can find a proxy for W_i that is correlated with W_i but not with the errors on H_i, we can get a consistent estimate of b_1

- Our proxy will be an instrumental variable

Instrumental variables and TSLS (3)

- We already know from the reduced form that wages are partially dependent on age and number of kids:

$$W_i = \pi_{10} + \pi_{11} X_i + \pi_{12} NK_i + \varepsilon_{1i}$$

- What is a good proxy that's independent of the endogenous variable?
  - Predicted wages because by construction, it has the errors taken out:

$$\hat{W}_i = W_i - \varepsilon_{1i}$$

Instrumental variables and TSLS (4)

- We can substitute in the predicted wage term into the expression for hours:

$$H_i = a_1 + b_1 \hat{W}_i + u_i$$

  - the predicted wage term is related to W_i and independent from the u_i by construction
  - This is called two stage least squares

Two stage least squares

- First stage: use OLS to estimate reduced form for the right-hand side endogenous variable W_i:

$$W_i = \pi_{10} + \pi_{11} X_i + \pi_{12} NK_i + \varepsilon_{1i}$$

- Second stage: use predictions from the first stage for $\hat{W}_i$:

$$H_i = a_1 + b_1 \hat{W}_i + u_i$$

  - Then we estimate this model using the proxy for wages

Two stage least squares (2)

- Why should TSLS work?
  - We have a set of exogenous, predetermined variables
  - The more exogenous variables we have, the better we can determine the endogenous variable of interest
  - Can test for over-identification (how many ext ra variables you have) using the Hausman test or Sargan test

Good empirical practice

1) Look at the reduced forms

2) Report the OLS results for the structural model
   ($H_i = a_1 + b_1 W_i + u_i$)
   - Since $E(W_i, u_i) \neq 0$, the OLS estimate for b_1 will be biased !!

3) Report the two stage least squares results
   - consistent estimate with instrumental variables
   - inefficient because standard errors on coefficient of interest are inflated

Example

- Example1.xls : sample of married women in CA in 1980
  - information on hours of work, age, wages, and number of kids
- What do we notice?
  - The b_1 is much larger & standard error on b_1 is larger
  - Using instrumental variables: consistent but inefficient estimate
  - Consistent: no correlation between proxy variable and error
  - Still no clear relationship between predicted wages and hours of work!
  - Variance has increased