DSC5101 In-class Project

No collaboration is allowed and online access is only to IVLE, no other websites. You are allowed to have one A4 size help sheet, written on both sides. The duration of the in-class project is two hours. Submit your work to IVLE. Please remember to include your name and student ID on the cover page.

1. Bespresso is a coffee machine company which sells coffee machines and coffee pods for its machines. In order to use the coffee pods, the coffee machine must be purchased. The firm knows it serves two types of customers, A and B. Furthermore, the inverse demand for each person in group A is given by $P = 50 - Q_A$, where Q_A is the quantity of coffee pods demanded by each person in A. The inverse demand for each person in Group B is given by $P = 40 - 2Q_B$, where Q_B is the quantity of coffee pods demanded by each person in B. The firm faces a constant marginal cost of \$2 for the coffee pods, with no fixed cost.

   (a) The firm decides to sell two types of machines, BespressoA at \$1152 and BespressoB at \$361, and its pods at \$2 each. The firm displays both machines at its stores and allows both to be sold. Customers enter the store and chooses which machine they want to buy. If there are 20 type A customers and 30 type B customers, what is the firm's profit? Assume for simplicity that the machines have zero cost.

   > **Solution:** Consumer surplus for type A = 1152. Consumer surplus for Group B = 361. Type A will buy BespressoB at \$361, and 49 coffee pods for \$49. This leaves type A with a consumer surplus of $1152 - 361 = 791$, compared with 0 consumer surplus if it chooses BespressoA.
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   > Total profit $= 361 * 50 = 18050$

   (b) Can the firm improve its profit by changing its pricing policy? If so, give a pricing policy and the new profit. If not, give a short explanation.

   > **Solution:** The firm can perform second degree price discrimination. It should sell BespressoB together with 19 pods for \$399. Doing so will give type A customers a surplus of $19(29 + 10)/2 = 370.5$ if they buy this package. It should sell another package consisting of BespressoA and 48 pods for 877.5. The new profit will be $361 * 30 + 877.5 * 20 = \$28380$.
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   > Alternatively, if the firm can identify the customer types, then they can perform first degree price discrimination. Then the profit will be \$33870. Any scheme which improves upon part-a is acceptable here.

2. A company is trying to choose between four banner ads – A, B, C, and D. Each banner ad has a certain probability of being clicked by a user. These probabilities are given by 0.1, 0.3, 0.5, and 0.95 for the ads A, B, C, and D respectively. So banner ad D is optimal but the company does not know this upfront. When modelled as a Multi-Armed Bandit problem, each banner ad is thought of as an arm. The rewards given by each arm can be found in the file 'rewardDF.csv'. There are $10,000$ time periods corresponding to 10,000 rows. In each row, the reward obtained when pulling arm A, B, C, or D is represented by the respective column. For example, pulling arm A in period 1 gives a reward of 0, while pulling arm D in period 2 gives a reward of 1. In each period, the company can only use one banner ad and thus has to decide which banner ad to use.

   (a) Implement the UCB1 algorithm to identify the best arm. A starter code (in python and R) is provided to help you. You may choose to use that or implement in any other language of your choice (including Excel). Before submission of your code, ensure that you comment important sections.

   > **Solution:** See the implementation files.

   (b) How many times in total was each arm pulled in the 10,000 time periods?

   > **Solution:** 19 47 65 9869

(c) Assume that an algorithm experiences zero regret if the optimal arm is pulled. Otherwise it experiences a regret of one. Given this, what is the regret seen in this run?

Solution: Here, the regret is the number of times arms other than D is pulled. This turns out to be 131.