Precautionary Labor Supply: Calibrated Income Shocks and Age Heterogeneity

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Introduction

Facts:

1. People don't seem to save enough:
   "The thriftlessness of early times was in great measure due to the want of security that those who made provision for the future would enjoy it"(Marshall, Alfred (1920))
2. Old people save more than young people:
   A potential way for young people to self-insure - increasing labor supply in early period.

Research Question:
Do people substitute potential labor supply for savings as insurance against future income shocks?

Hypothesis:
When young, people use latent potential labor as "precautionary savings" and need less actual savings.

Literature Review

Literature related with Endogenous Labor Supply:

Theoretical Analysis:

1. Eaton and Rosen (1980): Future labour supply can increase in response to increased wage uncertainty if risk aversion is sufficiently high.
2. Bodie et al. (1992): Analyze how labour-supply flexibility influences investors' portfolio decisions and find that labor supply flexibility raises precautionary motives when wages are stochastic.
3. Floden (2006): Labour-supply flexibility tends to raise saving when future wages are uncertain and that future wage uncertainty tends to raise current labour supply and future leisure.

Literature Review

Empirical Studies:

1. Low (2004) uses a calibrated model finds that young workers with much unresolved wage uncertainty work longer hours than old workers with little remaining wage uncertainty.
2. The empirical relationship between risk and hours of work has been documented to be positive for self-employed men in the USA (Parker et al., 2005), male employees in the USA (Kuhn and Lozano, 2008), and German and US workers (including self-employed) of both sexes (Bell and Freeman, 2001).
3. Jessen et al. (2018) conducted an empirical study to quantify the importance of precautionary labour supply and they find that individuals choose an additional 2.8% of their hours of work (i.e. about one week per year) to shield against wage shocks.

General Model

A model with s-period-lived agents, endogenous labor supply and stochastic ability.

Stochastic Ability

Initially, each individual is randomly assigned one of J discrete ability types $e_{j,t} \in \epsilon = \{e_1, e_2, ..., e_J\}$.

Define a Markov transition matrix $\Pi(e_{k,t+1}|e_{j,t})$ that gives the probability of being ability type $e_{k,t+1}$ next period given that you are type $e_{j,t}$ today with $k, j \in 1, 2, ...J$.

$$\Pi(e_{k,t+1}|e_{j,t}) = \begin{bmatrix} \pi_{1,1} & \pi_{1,2} & \dots & \pi_{1,J} \\ \pi_{2,1} & \pi_{2,2} & \dots & \pi_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{J,1} & \pi_{J,2} & \dots & \pi_{J,J} \end{bmatrix}$$

General Model

Disutility of Labor

Model the disutility of labor using a constant Frisch elasticity (CFE) functional form.

$$u(c_t, n_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \chi^n \frac{\gamma n_t^{1+\frac{1}{\gamma}}}{1+\gamma}$$

where $\sigma \geq 1$ is the coefficient of relative risk aversion on consumption and $\gamma \geq 0$ is the Frisch elasticity of labor supply.

Household Problem Budget Constraint

$$c_{s,t} + a_{s+1,t+1} = R_t a_{s,t} + w_t e_{j,t} n_{s,t} \quad \forall j, s, t$$
$$\text{where} \quad a_{1,t}, a_{S+1,t} = 0 \quad \forall t$$

General Model

The household will choose a sequence of consumption and labor supply to maximize lifetime utility:

$$\max_{\{c_t, n_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t u(c_t, n_t)$$

The Bellman equation for this problem is:

$$V_t(e_t, a_t) = \max_{\{c_t, n_t\}} u(c_t, n_t) + \beta E_t V_{t+1}(e_{t+1}, a_{t+1})$$

The FOCs (after applying the envelope conditions) are:

$$u_1(c_t, n_t) = \beta E_t R_{t+1} u_1(c_{t+1}, n_{t+1}) \ \forall t$$

and

$$e_t w_t u_1(c_t, n_t) = -u_2(c_t, n_t) \ \forall t$$

Computational Methodology

Generalized Endogenous Grid Method (Barillas and Fernandez-Villaverde (2007))

1. Create a grid for a_T and e_T
2. Use the FOC for the choice of labor supply to find $n_T = n(c_T)$.
3. Plug $n(c_T)$ into the budget constraint. Use a root finder to solve for c_T for every point on the (a_T, e_T) grid.
4. Interpolate $c_T(a_T, e_T)$ so that have a function for c_T for all points.
5. Use the FOC for the choice of a_T to find c_{T-1}.
6. Use $n(c_{T-1})$ to analytically solve for n_{T-1} for all points on the (a_T, e_{T-1}) grid.
7. Use the budget constraint to solve for a grid of a_{T-1} that is endogenous to the choice of a_T.
8. Interpolate $c_{T-1}(a_{T-1}, e_{T-1})$ so that have a function for c_T for all points.

Solutions

Analytical Solution:

To simplify, we solve a two-period models with four different settings:
(1) deterministic income & exogenous labor supply
(2) deterministic income & endogenous labor supply
(3) stochastic income & exogenous labor supply
(4) stochastic income & endogenous labor supply

Numerical Solution:

Parameters	**Description**	**Value**
S	Life Periods	80
β	Discount Factor	0.96
σ	CRRA Coefficient	2.2
γ	Frisch Elasticity	0.9
χ^n	CFE constant	10

Table: Model Calibration

Key Results and Conclusions

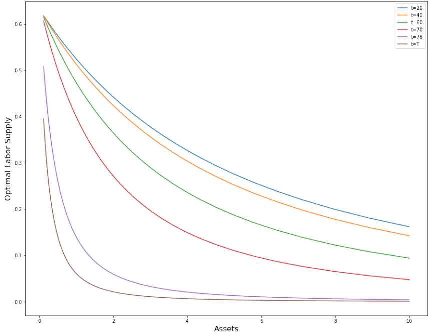

Figure: Different Period's Labor Supply

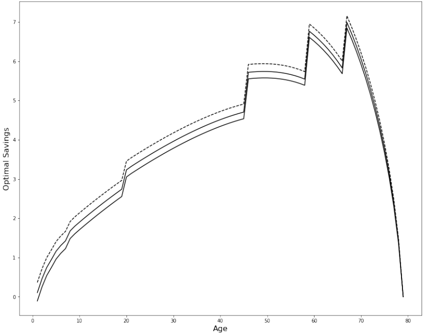

Figure: Optimal Savings over Ages

Conclusions

Young generations have less actual savings when labor supply are flexible, and they provide more labor supply compared with old generations.