Endogenous Income Lottery, Risk Sharing and Negative Assortative Matching

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Motivation

1. The analysis of matching pattern started with Beck (1973) and Shubik (1971). In general, those works rely on transferable utility (TU) assumption.
2. Mazzocco (2004) analyzed a condition if ISHARA holds, then we can treat the matching utility as TU holds.

Motivation

1. The analysis of matching pattern started with Beck (1973) and Shubik (1971). In general, those works rely on transferable utility (TU) assumption.
2. Mazzocco (2004) analyzed a condition if ISHARA holds, then we can treat the matching utility as TU holds.
3. Legros and Newman (2007) consider a risk sharing problem in which individual has different risk aversion (ordered by Arrow-Pratt measure) in which each couple's joint income can be either high or low.
4. Chiappori and Reny (2016) give a general treatment for income risk involving an arbitrary number of states of the world and for general risk averse preferences.
5. My paper extend Chiappori and Reny (2016) in a sense in their paper, the income risk is ex-ante identical for any men and women, my paper consider a situation where the income lottery is endogenous choosen.

Model

1. Agent: a group of men $w \in \{1, ..., n\}$ and a group of women $m \in \{1, ..., n\}$.
2. Income lottery: w_i, which will generate a pair of income lottery with a summary statistics $(r(w_i), V[w_i])$
3. Variance and mean trade-off frontier of income lottery:

$$r(w_i) = f(V[w_i]) \tag{A1}$$

such that the function f satisfies $f'(x) > 0, f''(x) < 0$.

4. Agent has a quadratic utility function

$$u(w) = (w - b)^2 \tag{A2}$$

Model 2

1. Now Consider a two stage game, in first stage the men m and women w choose their desired income lottery w_m, w_w
2. In the second stage, men and women matched together.
3. Is the NAM stable matching results still holds?

Theorem

Unique NAM Stable Matching

For general stable matching mean variance lottery choose such that satisfies (A1), and a quadratic preference (A2). If we order the men and women by their risk preference (measured by Arrow - Pratt sense), There is a unique negative assortative stable matching.

Exmaple 1

1. Suppose that There are n men and women, each have a quadratic utility is given by
$$u(w_i) = aE[w_i] - b_i V[w_i] \tag{A1'}$$
2. in the first stage the agent is free to choose a variance of mean-variance lottery where the mean of the lottery is given by
$$r(w_i) = r_f + \beta V[w_i], \forall V[w_i] \in [0,1] \tag{A2'}$$

in this case we have

$$\begin{aligned} u(w_i) =& aE[w_i] - b_i V[w_i] \\ =& aE[r_f + \beta V[w_i] - b_i V[w_i] \\ =& ar_f + (a\beta - b_i) V[w_i] \end{aligned}$$

Example 2

From the above function we can see that the optimization is

$$U_{ij}(v_j) = \max_{r_i', V_i'} u(r_i', V_i') = ar_f + (a\beta - b_i)V_i'$$

$$\begin{aligned} \text{such that } u(r_j, V_j) &\geq v_j \\ r_i &= r_f + \beta V_i \\ r_j &= r_f + \beta v_j \\ r_i + r_j &= r_i' + r_j' \\ V_i + V_j &= V_i' + V_j' \end{aligned}$$

Example 3

so this optimization is optimized if $max\{a\beta - b_i, a\beta - b_i\} > 0$, then $V_i = V_j = 1$, and $max\{a\beta - b_i, a\beta - b_i\} < 0$, then $V_i = V_j = 0$. Now, we shall consider the following three cases

1. $a\beta - b_i > a\beta - b_j > 0$, the optimal is achieved when $V_j = V_j = V_i' = V_j' = 1$
2. $a\beta - b_i > 0 > a\beta - b_j$, the optimal is achieved when $V_j = V_j = 1, V_i' = 2, V_j' = 0$
3. $0 > a\beta - b_i > a\beta - b_j$, the optimal is achieved when $V_j = V_j = V_i' = V_j' = 0$

Example 4

From above derivation we can see that

$$U_{11}(v_1) > U_{12}(v_2) \Rightarrow U_{21}(v_1) > U_{22}(v_2)$$

then by theorem 1 of Chaippori and Reny (2016), NAM results holds. Intuitively, it's just the most risk averse men will be matched with the least risk averse women, because the least risk averse women are willing to pay risk at a higher price while the most risk averse men are willing to pay more.