

Network Science or Applied Graph Theory? Examining the network of indirect collaboration in a Department of Mathematics

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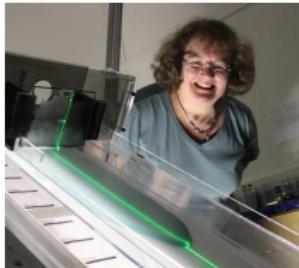
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I. Networks in/and Mathematics

Examples of Mathematics Departments offering Networks Courses

- ▶ UCLA Mathematics (**Andrea Bertozzi, Mason Porter**)
- ▶ University of North Carolina Mathematics (**Peter Mucha**)
- ▶ University of Vermont Mathematics & Statistics (**Peter Dodds**)
- ▶ Queen Mary University of London Mathematical Sciences & Statistics (**Ginestra Bianconi**)
- ▶ Oxford Mathematical Institute (UK) (**Sam Howison**)
- ▶ University of Namur (Belgium) Mathematics (**Renaud Lambiotte**)
- ▶ University of Limerick (Ireland) Mathematics & Statistics (**James Gleeson**)
- ▶ Stockholm University (Sweden) Statistics (**Ove Frank**)
- ▶ Chalmers Gothenburg (Sweden) Mathematical Sciences
- ▶ Caltech Computing & Mathematical Sciences
- ▶ US Military Academy West Point
- ▶ University of Chicago Mathematics
- ▶ Dartmouth Mathematics
- ▶ University of Texas Austin Simons Center on Network Mathematics
- ▶ Bar-Ilan University (Israel) Mathematics



Andrea Bertozzi (UCLA)



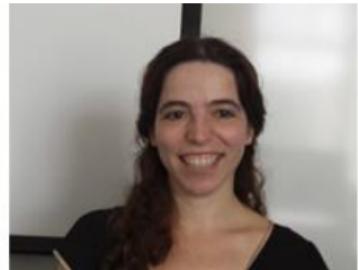
Mason Porter (UCLA)



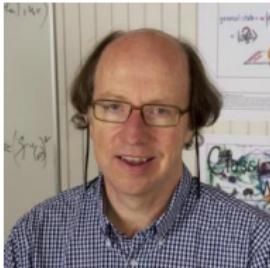
Peter Mucha (UNC)



Peter Dodds (UVM)



Ginestra Bianconi (QMU)



Sam Howison (Oxford)



Renaud Lambiotte (Namur)



James Gleeson (Limerick)

Networks in Mathematics Subject Classification (MSC2010)

05-xx Combinatorics

05Cxx Graph theory

05C82 Small world graphs, complex networks

35-xx Partial differential equations

35Rxx Miscellaneous topics

35R02 Partial differential equations on graphs and networks (ramified or polygonal spaces)

68-xx Computer science

68Mxx Computer system organization

68M10 Network design and communication

90-xx Operations research, mathematical programming

90Bxx Operations research and management science

90B10, 15 Network models, deterministic, stochastic

90Cxx Mathematical programming

90C35 Programming involving graphs or networks

91-xx Game theory, economics, social and behavioral sciences

91Dxx Mathematical sociology (including anthropology)

91D30 Social networks

94-xx Information and communication, circuits

94Cxx Circuits, networks

94C15 Applications of graph theory

From Numerical Mathematics

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GRAPH THEORY AND FLUID DYNAMICS*

KARL GUSTAFSON† AND ROBERT HARTMAN‡

Abstract. We describe recent applications of network-theoretic graph theory to the analysis of certain discretizations of fluid flow. Also given are a natural extension to elliptic equations of divergence form and a computation of a sparseness matrix previously inaccessible due to lack of a general basis method.

AMS(MOS) subject classifications. 65N30, 68E10, 35Q10, 76D05

1. Introduction. Consider a viscous incompressible liquid in a vessel Ω in Euclidean space. Let the velocity and pressure be given by $\mathbf{u}(x, t)$ and $p(x, t)$, where a scale has been chosen so that the density equals one. Then the *Navier-Stokes equations* describing the motion of the liquid are (in three dimensions):

$$(1.1) \quad \mathbf{u}_t - \gamma \Delta \mathbf{u} + \sum_{k=1}^3 u_k \mathbf{u}_{x_k} = -\operatorname{grad} p + \mathbf{f}, \quad x \in \Omega, \quad t > 0,$$

$$(1.2) \quad \operatorname{div} \mathbf{u} = 0, \quad x \in \Omega, \quad t > 0,$$

$$(1.3) \quad \mathbf{u}(x, t) = 0, \quad x \in \partial\Omega, \quad t > 0,$$

$$(1.4) \quad \mathbf{u}(x, 0) = \mathbf{a}(x) \quad x \in \Omega, \quad t = 0.$$

Equations (1.1) and (1.2) are coupled partial differential equations called the momentum equation and continuity equation. The boundary condition (1.3) is called the noslip condition, (1.4) is the known initial condition, \mathbf{f} represents known external forces, $\gamma = 1/R$ is the viscosity, R is the Reynolds number.

It is generally accepted that for smooth known \mathbf{f} and \mathbf{a} the problem (1.1)–(1.4) is *well-posed* for all $t \geq 0$. This means that the equations possess a unique solution stable under small changes of data and regular for all time. This has been shown for two dimensions and has been a longstanding open problem, which we will not discuss, in three dimensions. We shall instead look at the construction of approximate solutions following [1], [2], [3], [4].

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II. Domination in Wikipedia Graphs

Wikipedia graphs

Definition

A **Wikipedia graph** is a directed graph $G = (V, E)$ such that

- ▶ nodes $v \in V$ represent **webpages** from the **Wikipedia** (Internet) domain, from now on, called **Wikipedia pages** or just **pages**, and
- ▶ edges $e = (u, v) \in E$ represent **hyperlinks** directed (or pointing) from page u to page v .

Moreover, for two pages u, v in the Wikipedia graph $G = (V, E)$ (i.e., for $u, v \in V$),

- ▶ if page v is hyperlinked (pointed) by page u (i.e., if $(u, v) \in E$), we say that page u **dominates** page v (or page v is **dominated by** page u or page v is a **dominant** of page u), and
- ▶ if page u both dominates and is dominated by page v (i.e., if (u, v) and $(v, u) \in E$), we say that pages u, v are **equidominant**.

Wikipedia graphs stemming from fixed pages

Definition

Let $W = \{w_1, \dots, w_k\}$ be a finite collection of k (distinct) Wikipedia pages. A Wikipedia graph $G = (V, E)$ is said to be a **graph of Wikipedia pages stemming from W** , whenever

- ▶ $V = W \cup Z$, where $W \cap Z = \emptyset$ and $|Z| > 0$,
- ▶ every page $z \in Z$ is dominated by (at least) one page $w \in W$, there is no page in W without any dominant pages in Z , and
- ▶ if there exists a hyperlink $(z, y) \in E$ such that $z \in Z$ and $y \in V$, then either $y \in W$, in which case pages z, y are equi-dominant, or $y \in Z$, in which case page z dominates or is dominated by page y .

Using the terminology of **ego-centric networks** (or **ego-nets**), pages in W are called **egos** and pages in Z are called **alters**. Moreover, a hyperlink $(u, v) \in E$ is called **reciprocating** if $(v, u) \in E$ (i.e., if u, v are equi-dominant).

Wikipedia graphs stemming from the pages of Graph theory, Network science and Complex network

Let $G = (V, E)$ be the Wikipedia graph stemming from

$W = \{\text{Graph theory, Network science, Complex network}\}$,

- ▶ G is composed of 733 pages and 31,435 hyperlinks,
- ▶ the density of G is 0.059,
- ▶ the density of reciprocating hyperlinks is 0.391,
- ▶ G is weakly connected,
- ▶ G is not strongly connected and it has 11 strongly connected components,
- ▶ denoting by G_{LSCC} the largest strongly connected component of G , then:
 - ▶ G_{LSCC} is composed of 722 pages and 31,277 hyperlinks,
 - ▶ the density of G_{LSCC} is 0.06,
 - ▶ the density of reciprocating hyperlinks is 0.393,
 - ▶ the diameter of G_{LSCC} is 8.

Domination number

Definition

Let S be a set of dominating pages in the Wikipedia graph G . S is called a **minimum dominating set** if the cardinality of S is minimum among the cardinalities of any other dominating set on G . The cardinality of a minimum dominating set is called the **domination number** of graph G and is denoted by $\gamma(G)$.

Apparently, the domination number of the Wikipedia graph stemming from W is

$$\gamma(G) = |W| = 3$$

. However, if the relationship of domination was defined conversely (i.e., for a hyperlink $(u, v) \in E$, v would be called **conversely dominating** u), then the **converse domination number** of the Wikipedia graph stemming from W would have found to be

$$\gamma_{\text{converse}}(G) = 61.$$

A categorical attribute on pages

For every page $v \in V \setminus W$, let

$$A(v) = \{w \in W : w \text{ dominates } v\} \neq \emptyset.$$

If $v \in W$, by reflexivity, take $A(v) = \{v\}$. So, this association defines a set-valued mapping $A : V \rightarrow 2^W$. Now, instead of taking all possible (nonempty) subsets of W , we consider the partition of 2^W

$$2^W = \mathcal{W}_1 \cup \mathcal{W}_2 \cup \mathcal{W}_3 \cup \mathcal{W}_{123}$$

into the following four disjoint subsets:

$$\mathcal{W}_1 = \{\text{Graph theory}\},$$

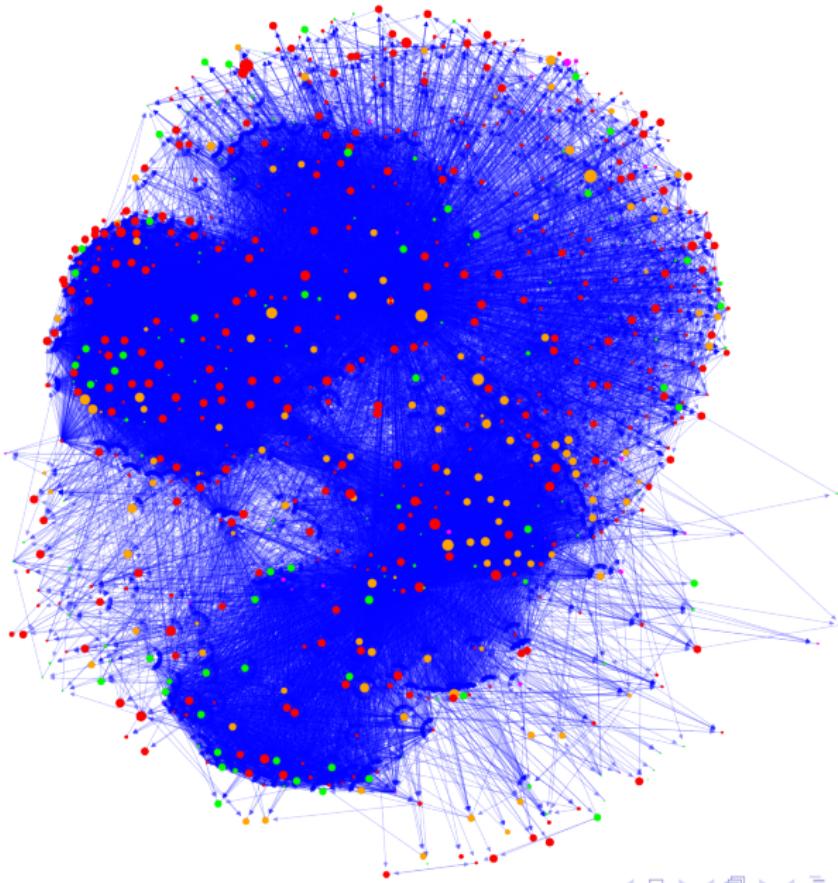
$$\mathcal{W}_2 = \{\text{Network science}\},$$

$$\mathcal{W}_3 = \{\text{Complex network}\},$$

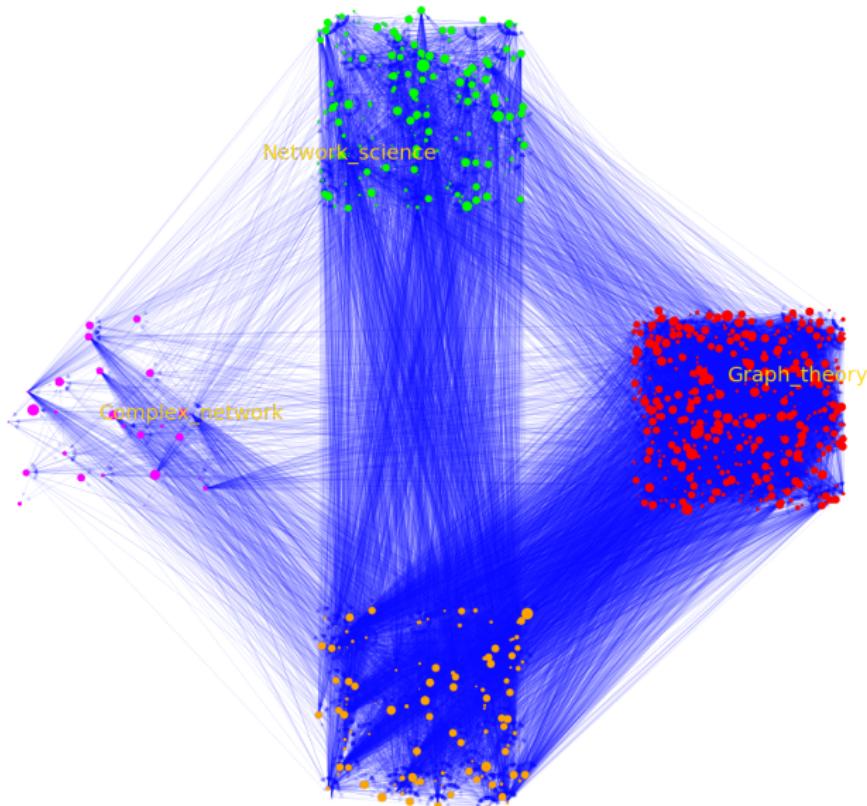
$$\mathcal{W}_{123} = \{A \in 2^W : |A| \geq 2\}.$$

In this way, setting $\mathcal{W} = \{\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_{123}\}$, a (single-valued) attribute $\alpha : V \rightarrow \mathcal{W}$ is defined on all pages stemming from W .

The graph of Wikipedia pages stemming from "Graph Theory, Network Science and Complex Network"
(red nodes are pages hyperlinked only from "Graph theory",
lime nodes are pages only hyperlinked from "Network science",
magenta nodes are pages only hyperlinked from "Complex network",
and orange nodes are pages hyperlinked by at least two of the starting pages)



The graph of Wikipedia pages stemming from "Graph Theory, Network Science and Complex Network"
(red nodes are pages hyperlinked only from "Graph theory",
lime nodes are pages only hyperlinked from "Network science",
magenta nodes are pages only hyperlinked from "Complex network",
and orange nodes are pages hyperlinked by at least two of the starting pages)



Attribute assortativity coefficient

Following Mark Newman (2003), the (normalized) **enumerative attribute assortativity** (or **discrete assortativity**) **coefficient** of attribute α is defined as:

$$r_\alpha = \frac{\text{tr } \mathbf{M}_\alpha - \|\mathbf{M}_\alpha^2\|}{1 - \|\mathbf{M}_\alpha^2\|},$$

where \mathbf{M}_α is the $|\mathcal{W}| \times |\mathcal{W}|$ (normalized) **mixing matrix** of the values (or categories) of attribute α on edges of G . Equivalently:

$$r_\alpha = \frac{\sum_{i,j \in V} (A_{ij} - \frac{k_i k_j}{2m}) \delta(\alpha(i), \alpha(j))}{2m - \sum_{i,j \in V} (\frac{k_i k_j}{2m}) \delta(\alpha(i), \alpha(j))},$$

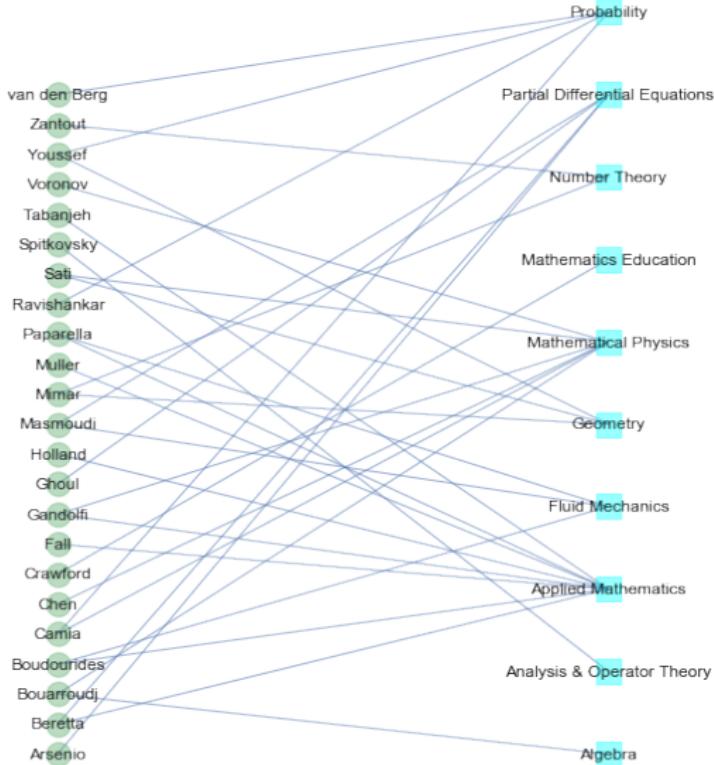
where A_{ij} is the adjacency matrix of graph G , m is the total number of edges of G , k_i is the degree of vertex i and $\delta(x, y)$ is the Kronecker delta. Here, we find that

$$r_\alpha = 0.473$$

implying that the Wikipedia graph stemming from W is moderately assortative for the previously defined attribute α .

III. Math Faculty and Research Areas

The bipartite graph of faculty members and research areas
in the NYUAD Math Faculty Network



The basics of Formal Concept Analysis (FCA)

Definition

- ▶ A **formal context** is a triple (G, M, I) , where G is a set of **objects**, M is a set of **attributes**, and I is a **relation** between G and M , in which we read $(g, m) \in I$ as "object g has attribute m ."
- ▶ For $A \subseteq G$, the **prime derivation operator on sets of objects** is defined as $A' = \{m \in M : \forall g \in A, (g, m) \in I\}$.
- ▶ For $B \subseteq M$, the **prime derivation operator on sets of attributes** is defined as $B' = \{g \in G : \forall m \in B, (g, m) \in I\}$.

Properties:

For $A, A_1, A_2 \subseteq G$,

- ▶ $A_1 \subseteq A_2 \Rightarrow A'_2 \subseteq A'_1$,
- ▶ $A \subseteq A''$ and $A' \subseteq A'''$.

For $B, B_1, B_2 \subseteq M$,

- ▶ $B_1 \subseteq B_2 \Rightarrow B'_2 \subseteq B'_1$,
- ▶ $B \subseteq B''$ and $B' \subseteq B'''$.

Definition

A **formal concept** is a pair (A, B) , where $A \subseteq G$ and $B \subseteq M$, such that $A' = B$ and $B' = A$. A is called the **extent** and B the **intent** of the concept (A, B) .

Lemma

For $A \subseteq G$ and $B \subseteq M$, (A, B) is a formal concept if and only if A and B are both maximal with respect to $A \times B \subseteq I$, meaning that every concept corresponds to a maximal rectangle in the relation I .

An ordering on concepts:

$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 (\Leftrightarrow B_1 \subseteq B_2).$$

Definition

The set of all concepts $\mathfrak{B}(G, M, I)$ together with the partial order \leq is the **concept lattice** of the context (G, M, I) and it is denoted as $\underline{\mathfrak{B}}(G, M, I)$.

Definition

An **implication** $X \rightarrow Y$ is said to hold in a context, if every object that has all attributes from X also has all attributes from Y .

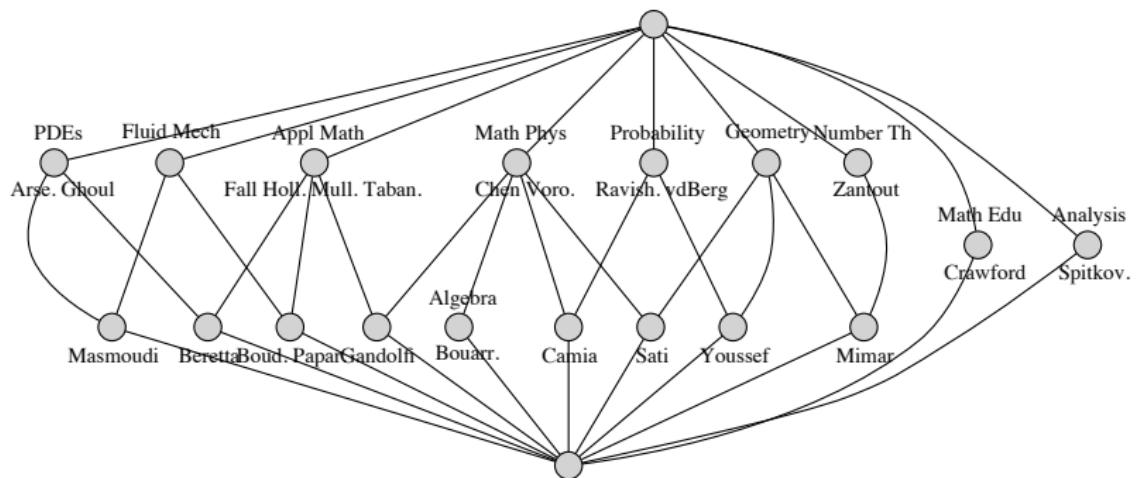
Definition

Let (G, M, I) be a context. Then (M, G, I^{-1}) is the **dual context** of (G, M, I) , with the understanding that
 $(m, g) \in I^{-1} \iff (g, m) \in I$.

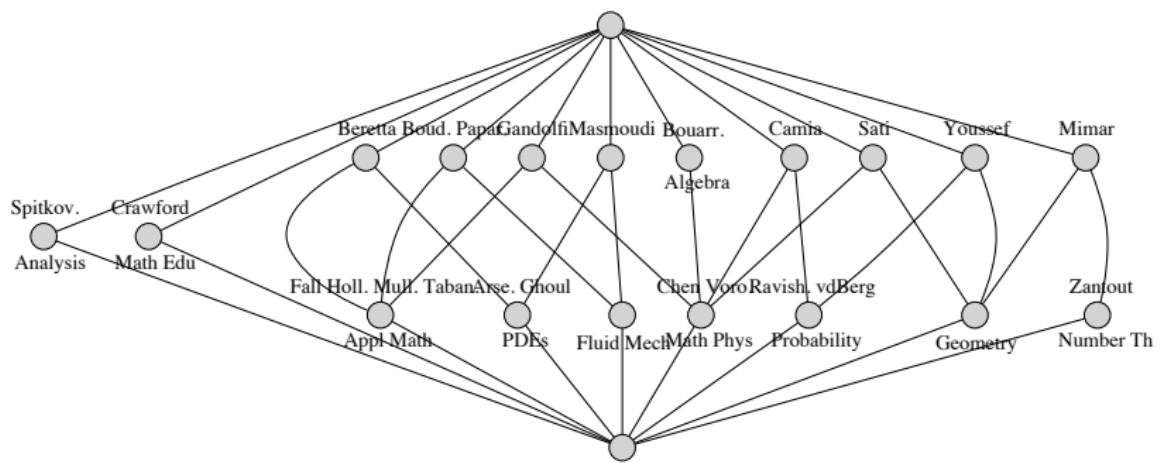
Theorem

The concept lattice of the dual context is isomorphic to the concept lattice of a context.

The concept lattice of the context of Math Faculty and Research Areas



The dual concept lattice of the context of Math Faculty and Research Areas



IV. Math Faculty Collaboration Network

The AMS MathSciNet tool to find collaboration distances among any two authors of Math publications



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MR Collaboration Distance = 3

Federico Camia	coauthored with	Sandro Graffi	MR1627552
Sandro Graffi	coauthored with	Barry Simon	MR0332068
Barry Simon	coauthored with	Ilya Matvey Spitkovsky	MR3735507

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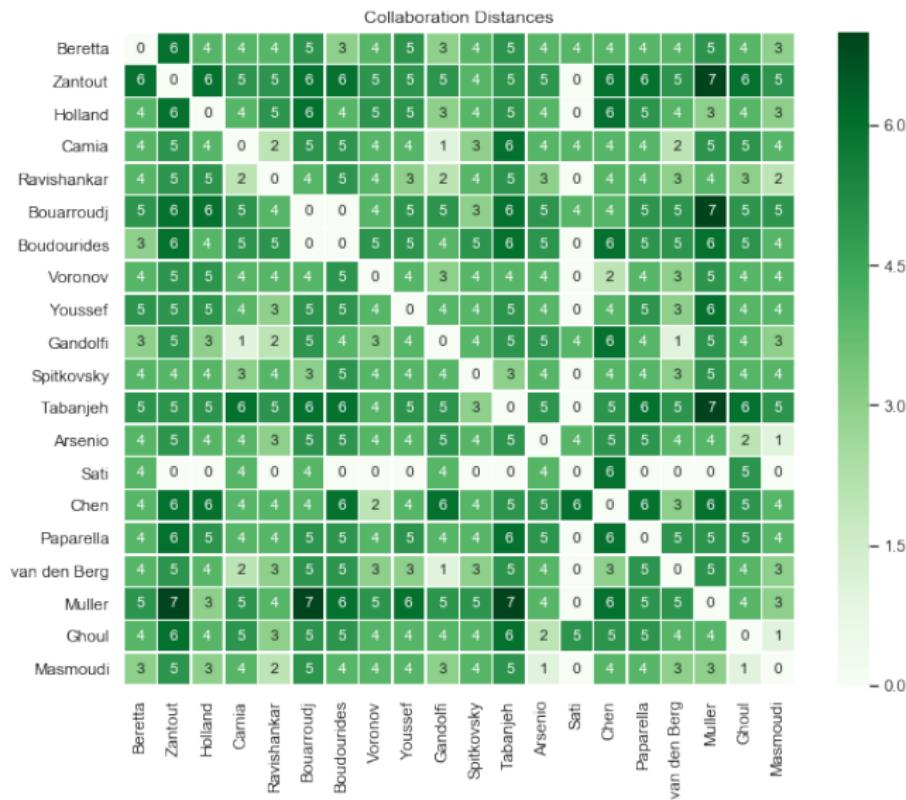
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Collaboration distances among any two members of the NYUAD Math Faculty

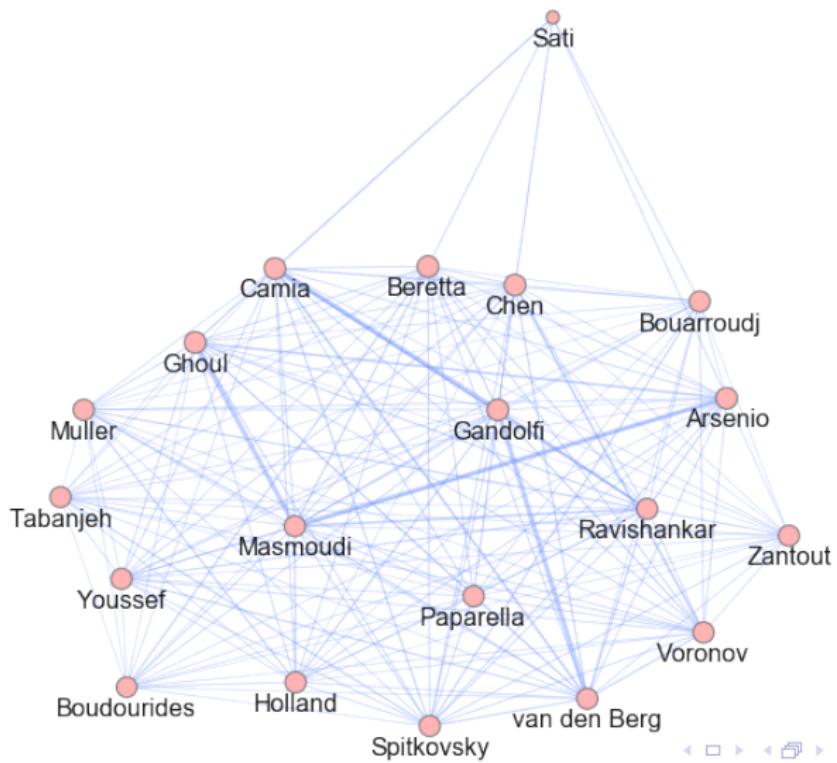


The mean value of all collaboration distances is 3.98

	mean
Masmoudi	3.10
Ravishankar	3.30
van den Berg	3.35
Spitkovsky	3.50
Gandolfi	3.55
Voronov	3.60
Camia	3.75
Arsenio	3.85
Youssef	3.95
Beretta	3.95
Ghoul	4.00
Holland	4.00
Boudourides	4.20
Paparella	4.30
Bouarroudj	4.45
Chen	4.50
Muller	4.60
Tabanjeh	4.70
Zantout	4.90

The network of (indirect) collaboration among Math Faculty

The NYUAD Math Faculty Network of (indirect) collaborations
with link weights equal to the inverses of collaboration distances

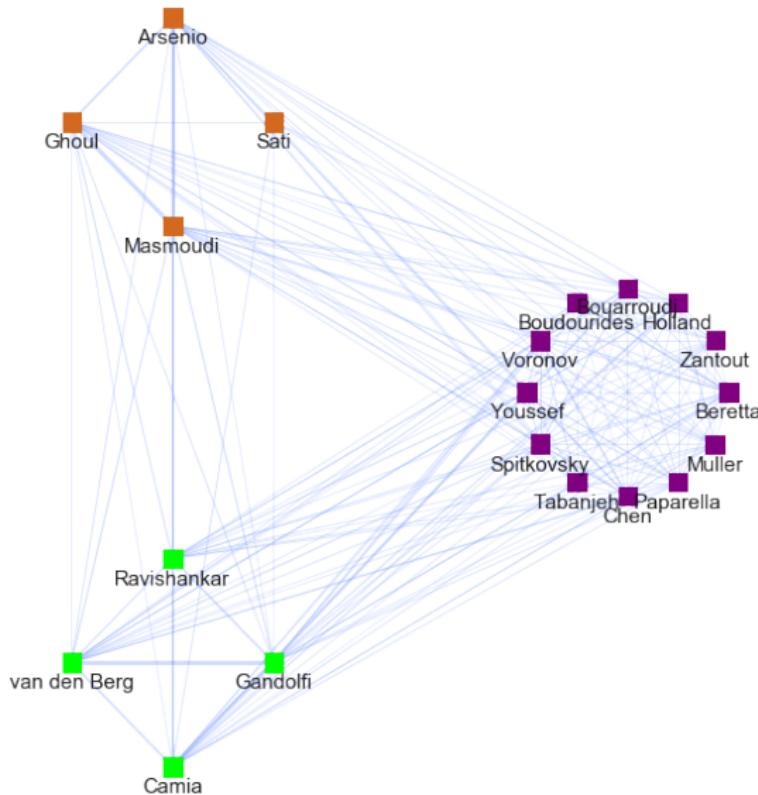


Various centrality indices (sorted by PageRank)

	page	degree	closeness	betweenness	eigenvector	HITS	Katz	PageRank	load	communicability	current flow
6	Gandolfi	1.000000	1.000000	0.010508	0.232639	0.071479	0.232361	0.068262	0.010508	0.626138	0.493149
9	Masmoudi	0.947368	0.950000	0.000344	0.227933	0.072013	0.227461	0.067978	0.000344	0.605377	0.478942
4	Camia	1.000000	1.000000	0.010508	0.232639	0.063661	0.232361	0.061309	0.010508	0.626138	0.493149
19	van den Berg	0.947368	0.950000	0.000344	0.227933	0.064667	0.227461	0.060270	0.000344	0.605377	0.478942
0	Arsenio	1.000000	1.000000	0.010508	0.232639	0.060384	0.232361	0.058861	0.010508	0.626138	0.493149
7	Ghoul	1.000000	1.000000	0.010508	0.232639	0.059814	0.232361	0.057614	0.010508	0.626138	0.493149
12	Ravishankar	0.947368	0.950000	0.000344	0.227933	0.060391	0.227461	0.057418	0.000344	0.605377	0.478942
1	Beretta	1.000000	1.000000	0.010508	0.232639	0.049723	0.232361	0.052006	0.010508	0.626138	0.493149
14	Spitkovsky	0.947368	0.950000	0.000344	0.227933	0.050632	0.227461	0.051684	0.000344	0.605377	0.478942
16	Voronov	0.947368	0.950000	0.000344	0.227933	0.050922	0.227461	0.051460	0.000344	0.605377	0.478942
5	Chen	1.000000	1.000000	0.010508	0.232639	0.046118	0.232361	0.048374	0.010508	0.626138	0.493149
8	Holland	0.947368	0.950000	0.000344	0.227933	0.046733	0.227461	0.047013	0.000344	0.605377	0.478942
17	Youssef	0.947368	0.950000	0.000344	0.227933	0.046543	0.227461	0.046839	0.000344	0.605377	0.478942
11	Paparella	0.947368	0.950000	0.000344	0.227933	0.042163	0.227461	0.043292	0.000344	0.605377	0.478942
2	Bouarroudj	0.947368	0.950000	0.009190	0.221280	0.039808	0.222033	0.043165	0.009190	0.586469	0.479956
10	Muller	0.947368	0.950000	0.000344	0.227933	0.041567	0.227461	0.042240	0.000344	0.605377	0.478942
15	Tabanjeh	0.947368	0.950000	0.000344	0.227933	0.038576	0.227461	0.040619	0.000344	0.605377	0.478942
3	Boudourides	0.894737	0.904762	0.000000	0.216313	0.039364	0.216888	0.040422	0.000000	0.563833	0.464990
18	Zantout	0.947368	0.950000	0.000344	0.227933	0.036717	0.227461	0.038723	0.000344	0.605377	0.478942
13	Sati	0.368421	0.612903	0.000000	0.089623	0.018724	0.102886	0.022451	0.000000	0.127093	0.267151

Community partitioning

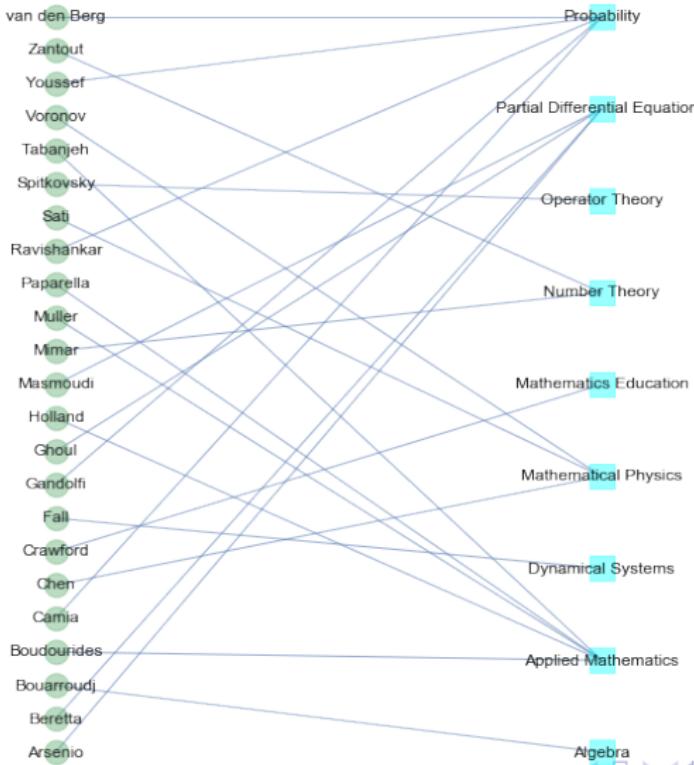
Communities of the NYUAD Math Faculty Network of (indirect) collaborations



V. Assortativity of Research Areas in the Collaboration Network

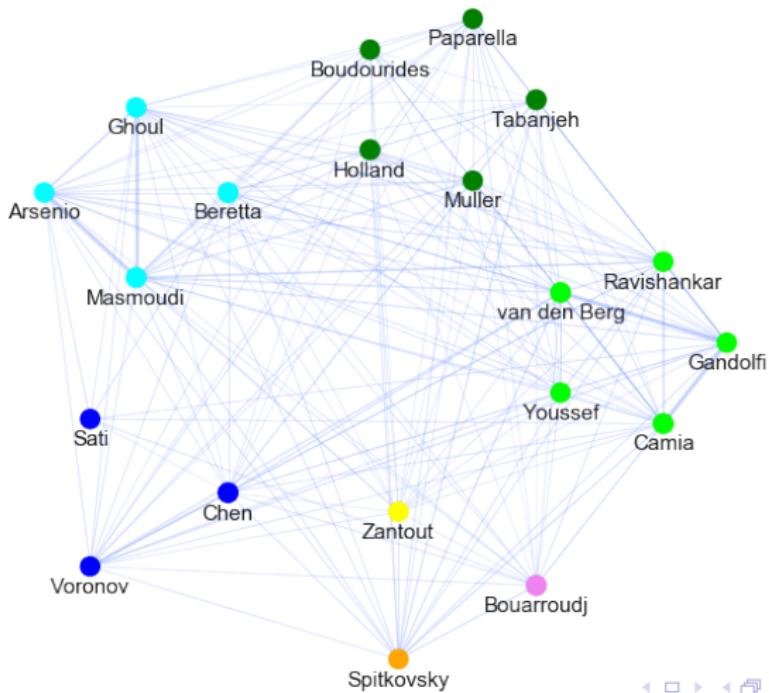
Assigning unique Research Area to each Math Faculty

The bipartite graph of NYUAD Math Faculty and unique Research Areas



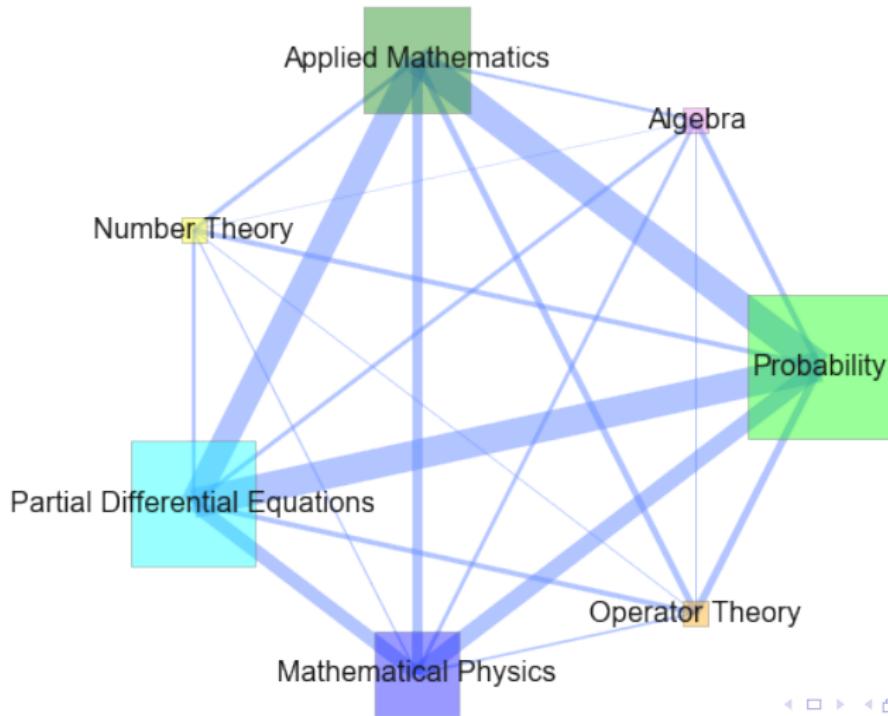
The network of Math Faculty collaboration assortet in Research Areas

The network of the NYUAD Math Faculty
with link weights equal to the inverses of collaboration distances
(green = Applied Mathematics, blue = Mathematical Physics,
lime = Probability, cyan = Partial Differential Equations,
yellow = Number Theory, orange = Operator Theory, violet = Algebra)
Research Area assortativity coefficient = -0.05



The Network of Research Areas of the Math Faculty

The network of Research Areas
with link weights proportional to the sum of weights among
NYUAD Math Faculty members working on these areas
and size of nodes proportional to the sum of weights of collaborations inside areas



VI. A Model of a Social Influence Process

The Friedkin–Johnsen Model of Social Influence on Graph

- ▶ Let $G = (V, E)$ be a (weakly) connected **graph** with n **nodes** representing **persons**. Given two persons (nodes) $i, j \in V$, one denotes by $i \sim j$ that i, j are **adjacent** (i.e., i, j are **neighbors** to each other). Let **A** denote the **adjacency matrix** of G .
- ▶ For each person (node) $i \in V$ and each time step $k = 0, 1, 2, \dots$, the **opinion** of i (with regards to a certain issue) at time k is denoted by $x_i^{(k)} \in \mathbb{R}$.
- ▶ The opinion of i at time k is updated at the subsequent time step $k + 1$ according to the following iterative scheme of the **Friedkin–Johnsen Social Influence Model**:

$$x_i^{(k+1)} = s_i N x_i^{(k)} + (1 - s_i) x_i^{(0)},$$

- ▶ where $N x_i^{(k)}$ is the *average* opinion held by i 's neighbors at time step k , $x_i^{(0)}$ is the initial opinion of i and
- ▶ the scalar parameter $s_i \in [0, 1]$ is called **susceptibility coefficient** of person (node) i .

Solving the Friedkin–Johnsen system of equations

- ▶ Disregarding the time exponent (k), $N x_i$ is defined as:

$$N x_i = \frac{1}{k_i} \sum_{j \sim i} x_j,$$

where k_i denotes the degree of i . In other words, \mathbf{N} is a **random walk matrix** on G :

$$\mathbf{N} = \mathbf{D}^{-1} \mathbf{A},$$

where \mathbf{D} denotes the diagonal matrix of degrees of G .

- ▶ As \mathbf{N} is a *row-stochastic* matrix, $\mathbf{I} - \mathbf{S} \mathbf{N}$ becomes *diagonally dominant*, where \mathbf{S} denotes the the diagonal matrix of susceptibility coefficients of persons (nodes).
- ▶ Therefore, the solution to this problem converges (relatively fast, depending on \mathbf{S}) to the vector $\bar{\mathbf{x}} = \bar{\mathbf{x}}(\mathbf{S}; \mathbf{x}^{(0)})$ of equilibrium (steady state) opinions of persons (nodes) of G :

$$\bar{\mathbf{x}} = (\mathbf{I} - \mathbf{S} \mathbf{N})^{-1} (\mathbf{I} - \mathbf{S}) \mathbf{x}^{(0)}.$$

Remarks on influence susceptibility coefficients

- ▶ When $s_i = 0$, i 's opinion does not change ($x_i^{(k)} = x_i^{(0)}$, for every time iteration $k = 1, 2, \dots$). Such a person (node) is called **persistent** or **stubborn** in holding the same opinion without being influenced by anybody else.
- ▶ When $s_i = 1$, i is always adopting the average neighbors' opinion $N x_i^{(k)}$ without taking into account the initial opinion $x_i^{(0)}$. Such a person (node) is called **malleable** or fully **conforming** to the neighbors' influences.
- ▶ When $0 < s_i < 1$, i 's opinion is interpolated in between the average neighbors' opinion $N x_i^{(k)}$ and the initial opinion $x_i^{(0)}$, in such a way that the exact position of the resulting i 's opinion is weighted as a convex combination through s_i .

Boundary conditions

We partition the set of nodes (persons) of the Friedkin–Johnsen social influence system as follows:

$$V = \Omega \cup \partial\Omega \quad (\Omega \cap \partial\Omega = \emptyset),$$

where:

- ▶ $\Omega = \{i \in V : s_i > 0\}$ is the domain of the Friedkin–Johnsen social influence system composed *exclusively* of nonpersistent persons (nodes).
- ▶ $\partial\Omega = \{i \in V : s_i = 0\}$ is the **boundary** of the domain of the social influence system composed *exclusively* of persistent persons (nodes).

Furthermore, we assume that:

- ▶ $|\partial\Omega| < n$.
- ▶ $\forall j \in \partial\Omega, \exists i \in \Omega, i \sim j$.

The Initial Boundary Value Friedkin–Johnsen Problem

- ▶ When $\partial\Omega \neq \emptyset$, any person (node) on the social influence boundary $\partial\Omega$ is called **source of a boundary stimulation** or just a **persistent source**. Moreover, we use the notation $\partial\Omega = \{j_a\}_{a=1,\dots,|\partial\Omega|}$.
- ▶ Thus, the **Initial Boundary Value Problem (IBVP)** of the Friedkin–Johnsen social influence system is the following:

$$\begin{aligned}x_i^{(k+1)} &= s_i N x_i^{(k)} + (1 - s_i) x_i^{(0)}, \text{ for all } i \in \Omega, k = 0, 1, \dots, \\x_{j_a}^{(k)} &= 1, \text{ for all } j_a \in \partial\Omega, k = 0, 1, \dots, \\x_i^{(0)} &= \phi_i, \text{ for all } i \in \Omega,\end{aligned}$$

where ϕ_i is the initial opinion of person/node $i \in \Omega$.

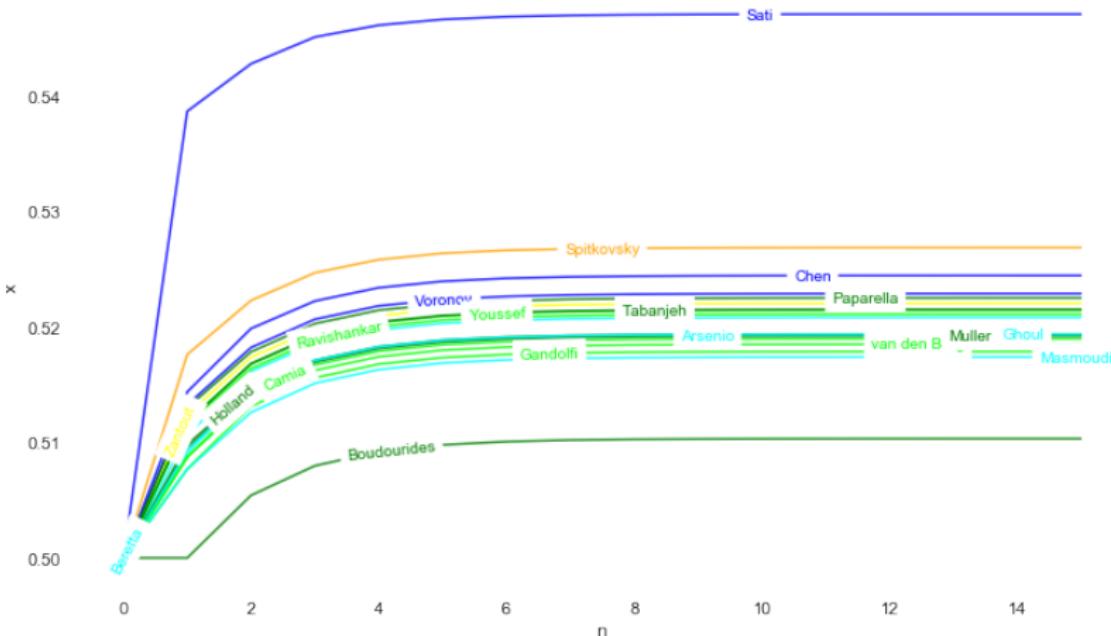
Solving the Initial Boundary Value Friedkin–Johnsen Problem on a Math Faculty collaboration network with Research Areas as sources of boundary simulation

Now, let G be the network of (indirect) collaboration among Math Faculty and let \mathcal{R} denote the 7 Research Areas to a (unique) of which each Faculty member is involved:

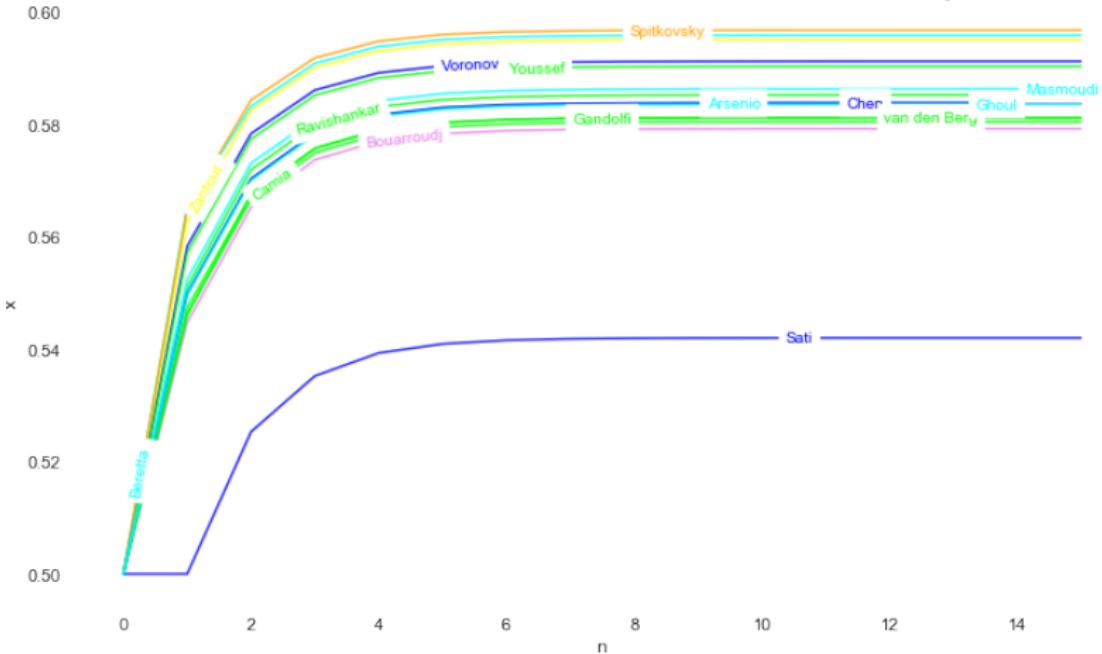
$$\mathcal{R} = \{\text{Algebra, Applied Mathematics, Mathematical Physics, Number Theory, Operator Theory, Partial Differential Equations, Probability}\}.$$

In what follows, for each Research Area in \mathcal{R} , the Math Faculty working on this Research Area will be taken as a source of boundary simulation to the remaining Math Faculty involved in all other Research Areas and the influence exerted by the former to the latter will be plotted time (iteration step).

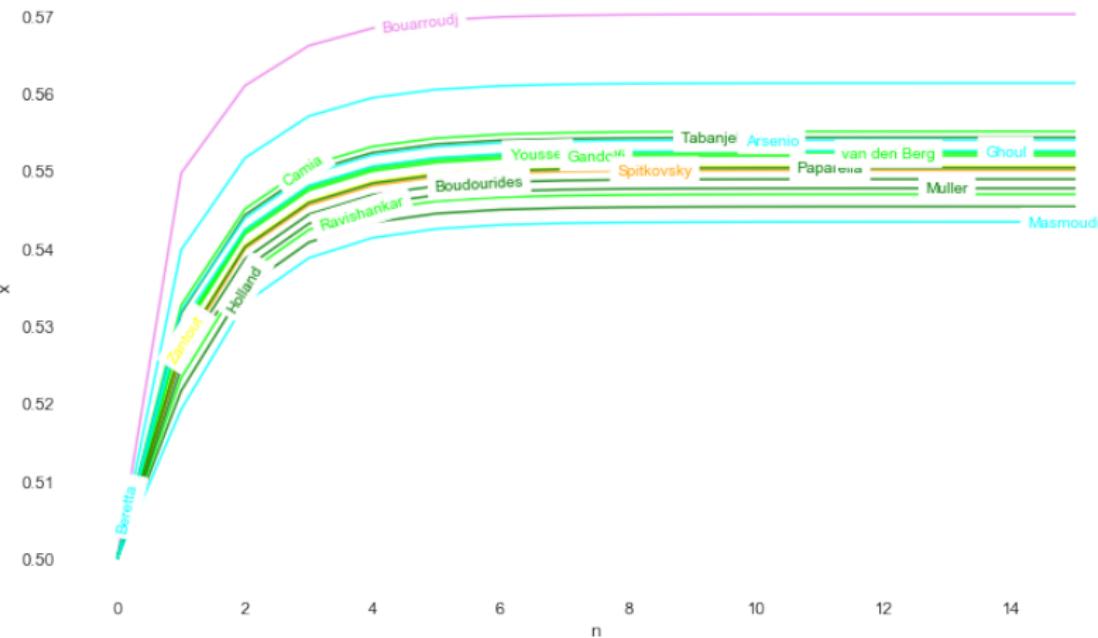
Influence of Algebra on the Network of Research Fields of NYUAD Math Faculty



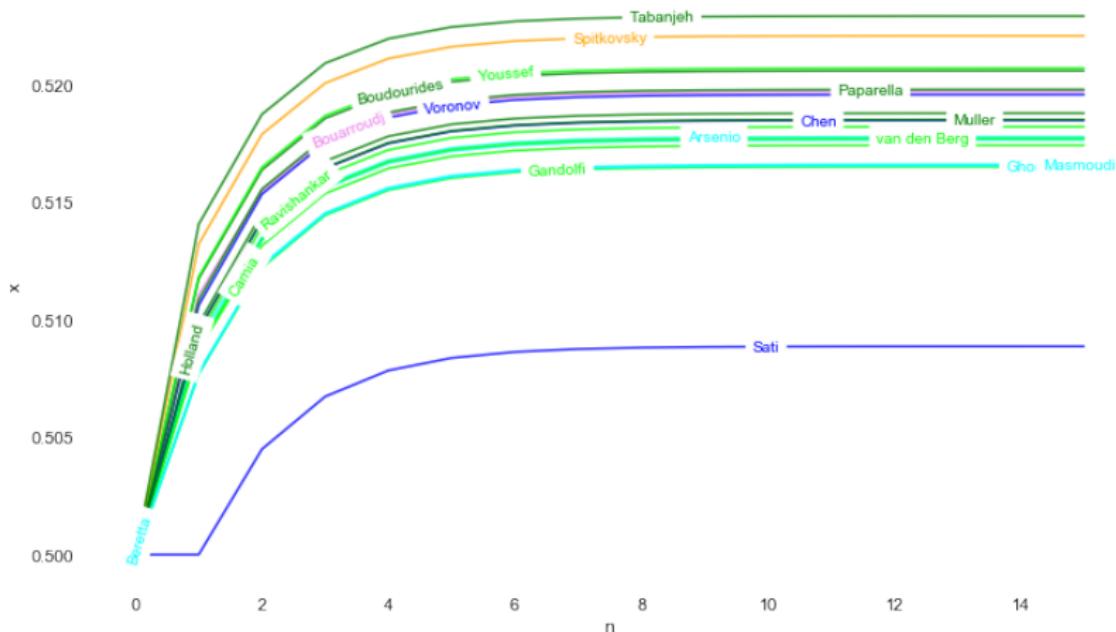
Influence of Applied Mathematics on the Network of Research Fields of NYUAD Math Faculty



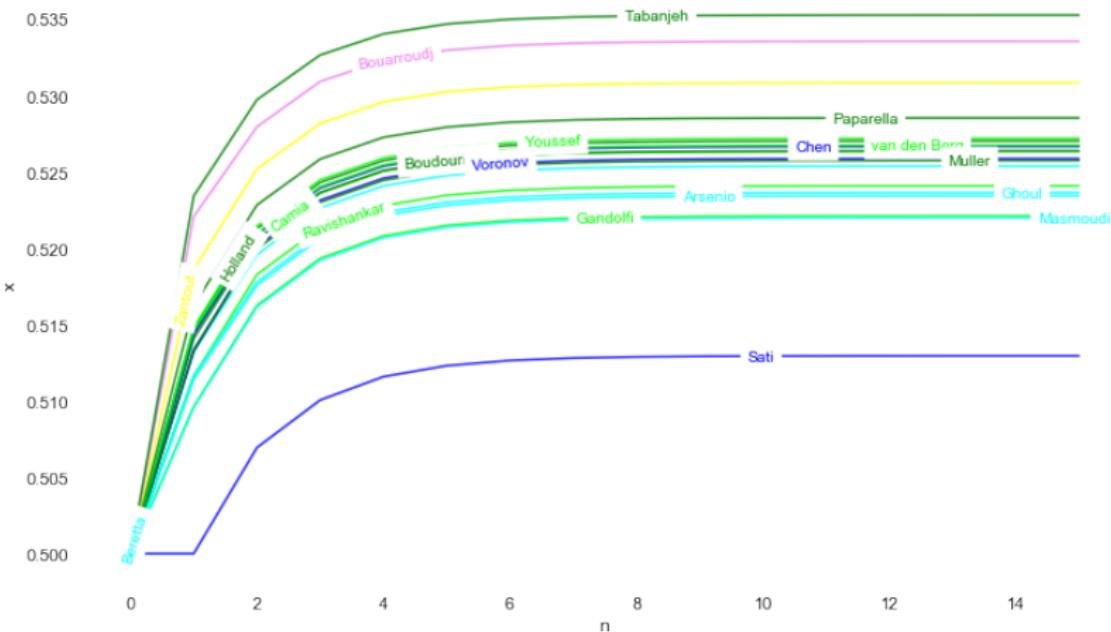
Influence of Mathematical Physics on the Network of Research Fields of NYUAD Math Faculty



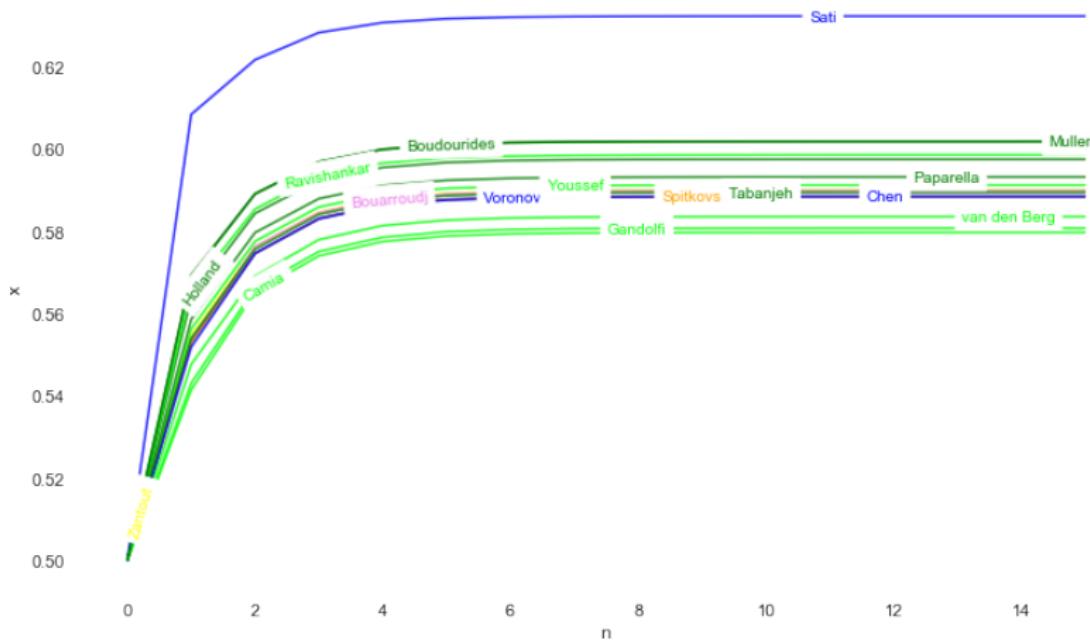
Influence of Number Theory on the Network of Research Fields of NYUAD Math Faculty



Influence of Operator Theory on the Network of Research Fields of NYUAD Math Faculty



Influence of Partial Differential Equations on the Network of Research Fields of NYUAD Math Faculty



Influence of Probability on the Network of Research Fields of NYUAD Math Faculty

