



Today's agenda

- ↳ Searching basics
- ↳ why mid at half
- ↳ search in sorted array
- ↳ floor in a sorted array
- ↳ Every element occurs twice except for 1.



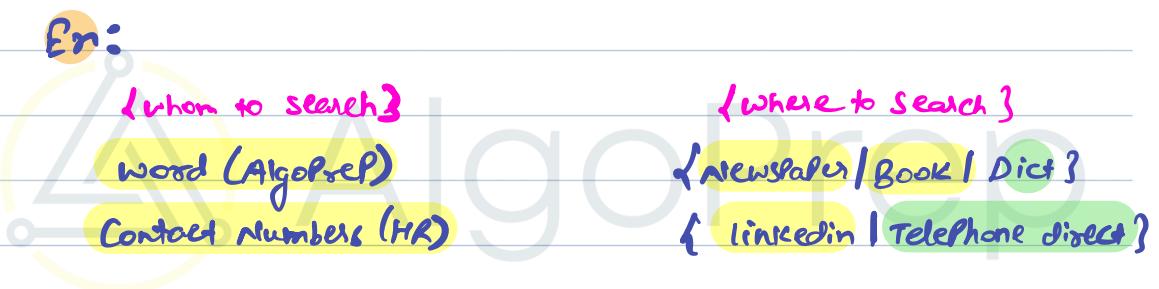
AlgoPrep



Story:



Ex:



↳ Search space is ordered, searching becomes easier.

→ linear search: Searching one by one.

→ binary search: Divide your search space in 2 parts, if we can discard one half, we can apply BS.

↳ Array should not necessarily be sorted.



Q) Given a sorted arr[N] search if K is present or not?

arr[10]: { 0 2 2 3 4 5 6 7 9 3 } $K=13$
↓
true

Idea 1

↳ Linear Search

T.C: $O(n)$

S.C: $O(1)$

Idea 2

↳ binary search

Search space: given array

target: K

Case 1:



$$mid = \frac{i_0 + hi}{2}$$

if ($arr[mid] == k$) {return true; }

Case 2:



if ($arr[mid] < k$) {

 discard left side / search on right.

}



Case 3:



if ($\text{arr}[\text{mid}] > \text{k}$) {

discard right side / search on left.

3

$\text{arr}[10] = \{ 4, 7, 10, 13, 15, 20, 21, 24, 26, 28 \}$ $\text{k} = 13$

lo	hi	$m = \frac{lo+hi}{2}$	
0	9	4	if ($\text{arr}[m] > \text{k}$): search on left: $hi=m-1$
0	3	1	if ($\text{arr}[m] < \text{k}$): search on right: $lo=m+1$
2	3	2	if ($\text{arr}[m] < \text{k}$): search on right: $lo=m+1$
3	3	3	if ($\text{arr}[m] == \text{k}$): return true



II) Pseudo code

```
boolean search (int arr[], int k){
```

```
    int lo = 0;
```

```
    int hi = N-1;
```

```
    while (lo <= hi) {
```

```
        int m = (lo+hi)/2;
```

\sim

$\sim/2$

```
        if (arr[m] == k) {
```

```
            return true;
```

$\sim/4$

```
        else if (arr[m] < k) {
```

```
            lo = m + 1;
```

$\sim/4$

```
        } else {
```

```
            hi = m - 1;
```

\sim

```
    }
```

3

3

```
return false;
```

T.C: $O(\log n)$

S.C: $O(1)$



int lo = 0;
int hi = n-1;

arr[lo] = {~~0 1 2 3 4 5 6 7 8 9~~ 10} K=13

```
while ( ) {
    int m = (lo+hi)/2;
    if (arr[m] == k) {
        return true;
    }
    else if (arr[m] < k) {
        lo = m + 1;
    }
    else {
        hi = m - 1;
    }
}
```

lo

hi

$m = \frac{lo+hi}{2}$

go to

0 3 2 3 3 2 3

9 1 3 3 3 3 2

if (arr[m] > k): left
if (arr[m] < k): right
if (arr[m] < k): right
if (arr[m] > k): left

→ exit



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Q) Given a sorted arr[n], find floor of given num k.

b) just smaller {greatest no. $\leq k$ in arr[]}
or equal

Ex: arr[9] = { -4 3 4 7 10 11 12 15 19 }

K = 5 : 4

K = 7 : 7

K = 13 : 12

1/idea1

↳ linear search

T.C: $O(n)$

S.C: $O(1)$

1/idea2

↳ binary search

Case 1:



if (arr[mid] == k) ↳ return k; 3



Case 2:



$\text{if } (\text{arr}[mid] < k) \leftarrow$

$\text{if } (\text{arr}[mid] > \text{ans}) \{ \text{ans} = \text{arr}[mid]; \}$

discard left / go to right

}

Case 3:



$\text{if } (\text{arr}[mid] > k) \leftarrow$

discard right / go to left

}



// Pseudo Code

```
int floor ( int arr[n], int k ) {  
    int lo = 0;  
    int hi = N-1;  
    int ans = -∞;
```

```
    while ( lo <= hi ) {  
        int m = (lo + hi) / 2;
```

T.C: $O(\log n)$

S.C: $O(1)$

```
        if (arr[m] == k) {  
            return k;  
        }
```

```
        else if (arr[m] < k) {  
            if (arr[mid] > ans) { ans = arr[mid]; }  
            lo = m+1;
```

```
        }  
    }
```

```
    else {  
        hi = m-1;  
    }
```

return ans;

ans = $\lceil \frac{3}{4} \rceil$



```
int floor ( int arr[], int k ) {
    int lo = 0;
    int hi = N-1;
    int ans = -∞;
```

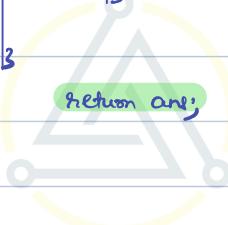
$arr[0] = \{ \dots, 4, 3, \cancel{2}, \cancel{1}, \cancel{0}, 1, 2, 3, 4, 5, 6, 7, 8 \}$ $k=6$

```
while ( lo <= hi ) {
    int m = (lo + hi) / 2;
    if (arr[m] == k) {
        return k;
    }
    else if (arr[m] < k) {
        ans = arr[m];
        lo = m+1;
    }
    else {
        hi = m-1;
    }
}
```

lo	hi	m	
0	8	4	if ($arr[m] > k$): go to left
0	3	1	if ($arr[m] < k$): go to right
2	3	2	if ($arr[m] < k$): go to right
3	3	3	if ($arr[m] > k$): go to left
3	2	→ exit	

↳ ans = 4

3



return ans;

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Break till 9:47 PM



Q) Every element occurs twice except for 1, find unique element.

Note: duplicates are adjacent to each other

Ex: arr[15]: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
arr[15]: 4 4 1 1 9 9 11 11 20 7 7 3 3 5 5

II idea 1

↳ Take XOR of all elements.

T.C: $O(N)$

S.C: $O(1)$

II idea 2

Binary Search

arr[15]: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
arr[15]: 4 4 1 1 9 9 11 11 20 7 7 3 3 5 5

Pre single no. occ: Numbers are starting at even idm

Post single no. occ: Numbers are starting at odd idm

arr[15]: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
arr[15]: 4 4 1 1 9 9 11 11 7 7 3 3 5 5 20

arr[15]: 20 4 4 1 1 9 9 11 11 7 7 3 3 5 5



Case 1:



if (arr[mid] != arr[mid-1] && arr[mid] != arr[mid+1]) {
 return arr[mid];

}

→ my mid is at 1st occ. of any number.

↳ if (arr[m] == arr[m+1])

Case 2:



if (mid % 2 == 0) {

discard left / go to right

}

↖ m

arr[15]: [4 4 1 1 9 9 11 11 20 7 7 3 3 5 5]

Case 3: → my mid is at 1st occ. of any number.

if (mid % 2 == 1) {

discard right / go to left.

}

arr[15]: [4 4 1 1 9 9 11 11 20 7 7 3 3 5 5]



→ my mid is at 2nd occ. of any number.

do mid-- and then solve above cases.

↳ if ($\text{arr}[m] == \text{arr}[m-1]$)

//Pseudo Code

```
int uniqueElement (int arr[n]) {
    //0th index
    if (arr[0] != arr[i]) { return arr[0]; }
    //N-1th index
    if (arr[N-1] != arr[N-2]) { return arr[N-1]; }
```

int lo = 2;

int hi = N-3;

T.C: $O(\log n)$

S.C: $O(1)$

while (lo <= hi) {

int mid = (lo + hi) / 2;

if ($\text{arr}[mid] != \text{arr}[mid-1]$ && $\text{arr}[mid] != \text{arr}[mid+1]$) {

return arr[mid];

} else if ($\text{arr}[mid] == \text{arr}[mid+1]$) { mid--; }

if (mid % 2 == 0) { lo = mid + 2; }

else { hi = mid - 1; }

}

3



$\text{arr}[15]:$ 4 4 1 1 9 9 11 11 [8] 20 7 7 3 3 5 5

10th index if ($\text{arr}[10] \neq \text{arr}[1]$) { return arr[0]; }
11N-1th index if ($\text{arr}[11N-1] \neq \text{arr}[11N-2]$) { return arr[11N-1]; }

```

int lo = 2;
int hi = N-3;
while (lo <= hi) {
    int mid = (lo + hi) / 2;
    if (arr[mid] != arr[mid+1]) {
        arr[mid] = arr[mid+1];
        return arr[mid];
    }
    if (arr[mid] == arr[mid+1]) {
        mid--;
    }
    if (mid % 2 == 0) {
        lo = mid + 2;
    } else {
        hi = mid - 1;
    }
}

```

lo	hi	mid	*
2	12	7	$m = 6$ $m \times 2 = 12$, go to right
8	12	10	$m = 9$ $m \times 2 = 18$, go to left
8	8	8	

3