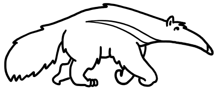
Cloud-Assisted On-Sensor Observation Classification in Latency-Impeded IoT Systems

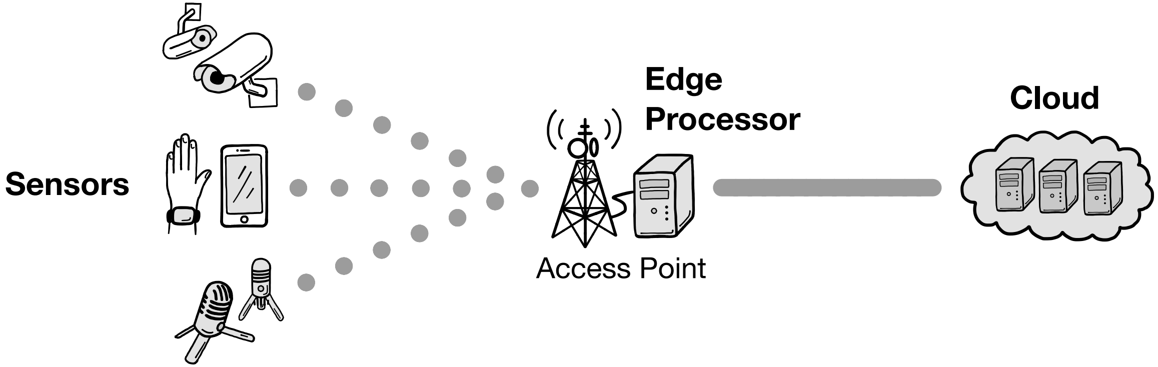
Igor Burago   Marco Levorato

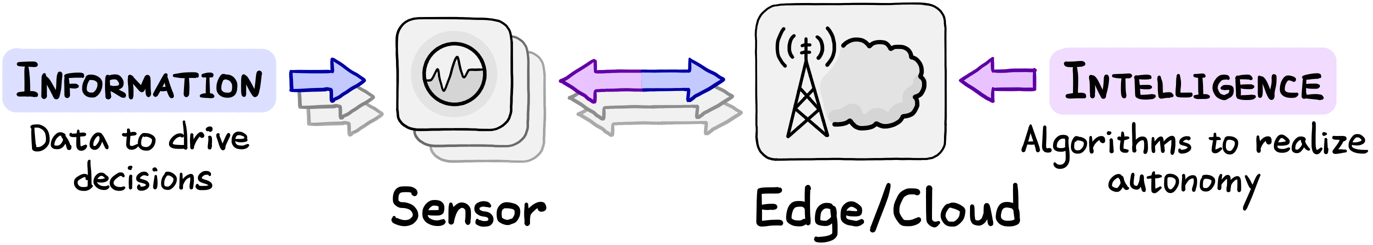
Department of Computer Science
University of California, Irvine



2019

Intelligent IoT in Principle



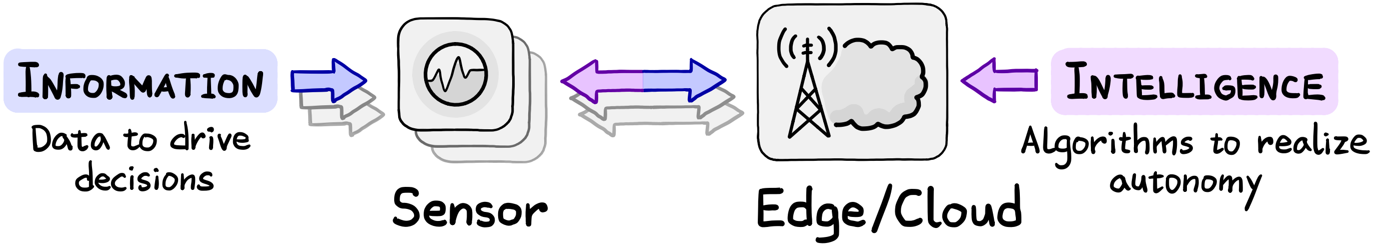


- Observes the environment
- Selectively informs the edge
- Adjusts its own operation

- Accumulates relevant observations
- Exercises high-level control
- Assists on-sensor decision-making

Goal: Realize the alliance between information completeness and best intelligence

Intelligent IoT Challenge

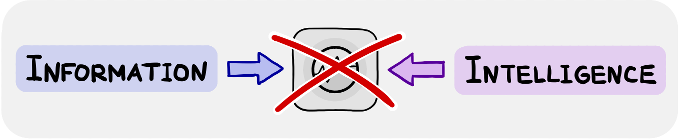


System is **constrained** by:

AND

→ **Sensor**

High-quality decision cannot be made locally

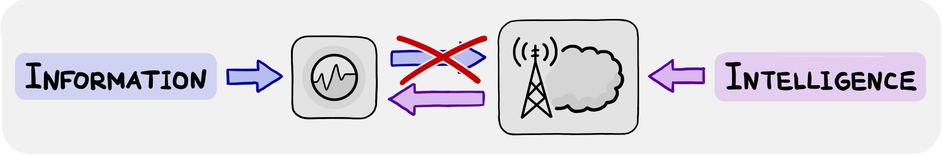


→ **Channel** with bottleneck in:

OR

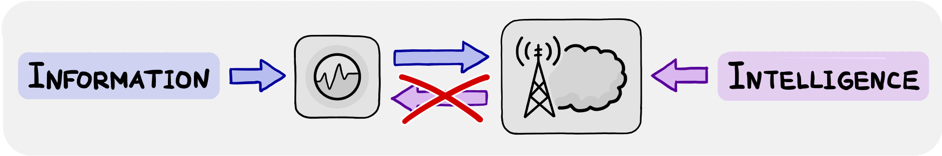
→ **Bandwidth**

Edge/Cloud cannot see all observations in full



→ **Latency**

Decision turnaround is too slow for reactive sensor adjustments



Approaches to Computation-Communication Tradeoff

Strategies vary in the amount of:

Decision
Quality

Approaches to Computation-Communication Tradeoff

Strategies vary in the amount of:

Decision Quality	On-Sensor Computation

Approaches to Computation-Communication Tradeoff

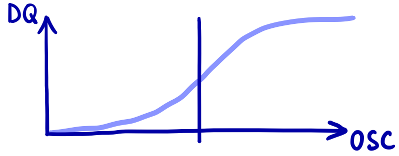
Strategies vary in the amount of:

Decision Quality	On-Sensor Computation	Sensor-Cloud Communication

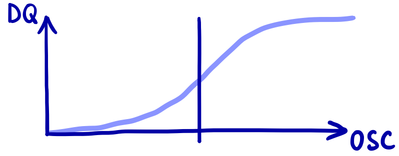
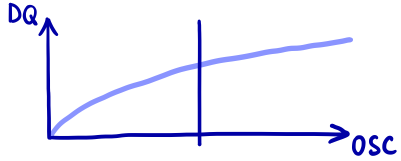
Approaches to Computation-Communication Tradeoff

	Decision Quality	On-Sensor Computation	Sensor-Cloud Communication
• Offload all decisions to cloud	Best	None	High

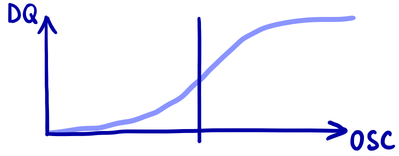
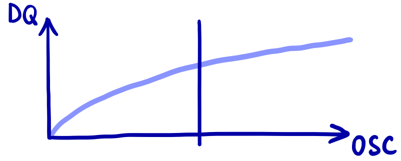
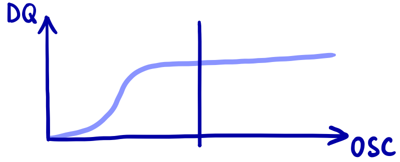
Approaches to Computation-Communication Tradeoff

	Decision Quality / On-Sensor Computation		Sensor-Cloud Communication
• Offload all decisions to cloud	Best	None	High
• Make all decisions on sensor			Low

Approaches to Computation-Communication Tradeoff

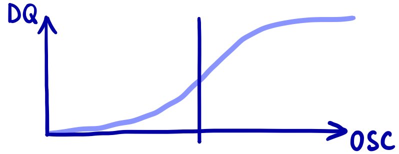
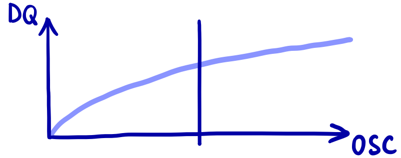
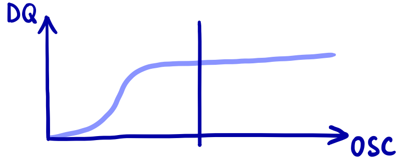
	Decision Quality	On-Sensor Computation	Sensor-Cloud Communication
• Offload all decisions to cloud	Best	None	High
• Make all decisions on sensor			Low
• Compress observations and offload all decisions to cloud			Medium

Approaches to Computation-Communication Tradeoff

	Decision Quality	On-Sensor Computation	Sensor-Cloud Communication
• Offload all decisions to cloud	BEST	NONE	HIGH
• Make all decisions on sensor			LOW
• Compress observations and offload all decisions to cloud			MEDIUM
• Selectively transmit observations to cloud for decisions*			LOW

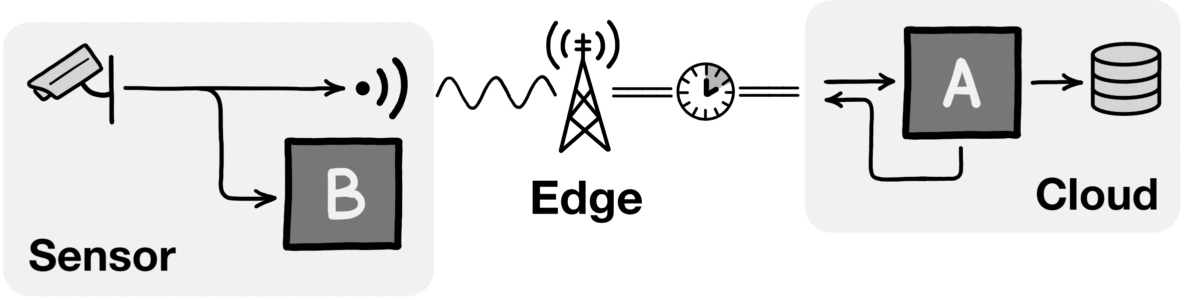
* I. Burago and M. Levorato, "Randomized Edge-Assisted On-Sensor Information Selection for Bandwidth-Constrained Systems", Asilomar Conference on Signals, Systems, and Computers, 2018.

Approaches to Computation-Communication Tradeoff

	Decision Quality	On-Sensor Computation	Sensor-Cloud Communication	Decision Turnaround
• Offload all decisions to cloud	BEST	NONE	HIGH	DELAYED
• Make all decisions on sensor			LOW	NONE
• Compress observations and offload all decisions to cloud			MEDIUM	DELAYED
• Selectively transmit observations to cloud for decisions*			LOW	DELAYED

* I. Burago and M. Levorato, "Randomized Edge-Assisted On-Sensor Information Selection for Bandwidth-Constrained Systems", Asilomar Conference on Signals, Systems, and Computers, 2018.

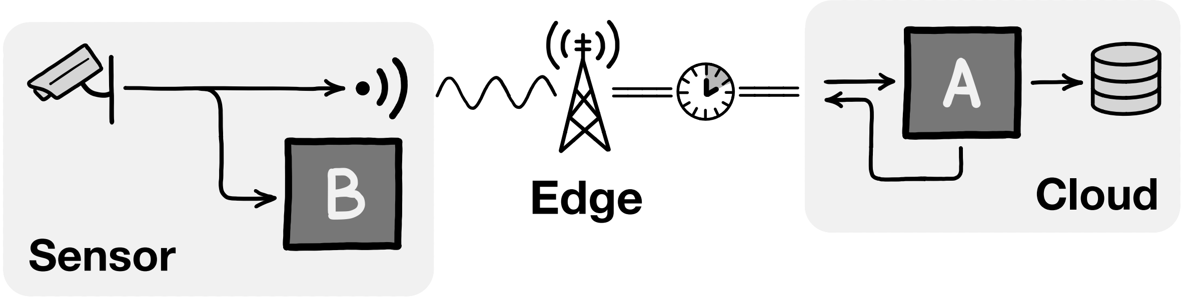
Sensor-Cloud Cooperation Model



Model assumptions:

- Observations fall into two classes:
  - of interest to the sensor (positives)
  - and not (negatives)
- Sensor adjusts to its environment by reacting to positives
- Cloud can run the optimal classifier δ; sensor can not
- Sensor-cloud link has sufficient bandwidth but unacceptable latency for sensor adaptivity
- B: Sensor runs a low-complexity classifier, supplied by cloud, to make local decisions
- A: Cloud, via supervision of its classifier, trains the sensor's

Sensor-Cloud Cooperation Model

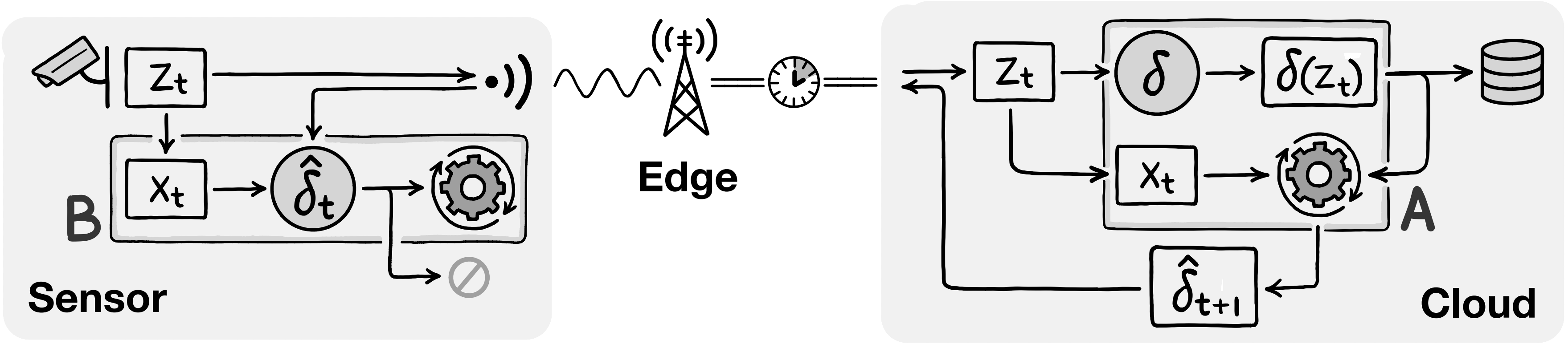


Factors at play

- Delayed channel
- No tolerance for missing decisions
- Auxiliary decision $\hat{\delta}$ to learn

- CPU, battery, etc.
- Computation constraints
- Complexity of $\hat{\delta}$

- Need to react to environment
- Cost of error in feature space
- Expected risk V to minimize

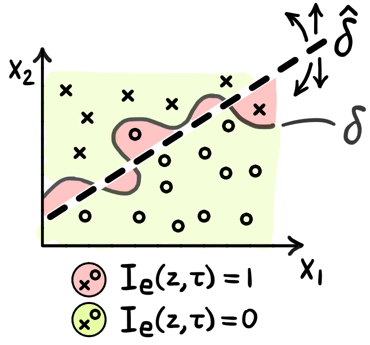


Sensor-Cloud Cooperation Model

Problem Formulation

For the on-sensor decision rule $\hat{\delta}: f(x,\theta) \gtrless \mu$

$$\underbrace{V(\tau) \triangleq \mathbb{E}\left[I_E(z,\tau)\, |r(u_\theta - \mu)| \right]}_{\text{Expected penalty for errors}} \longrightarrow \min_{\tau} \quad \text{s.t.} \quad \varphi(\tau) \leq \varphi_*$$



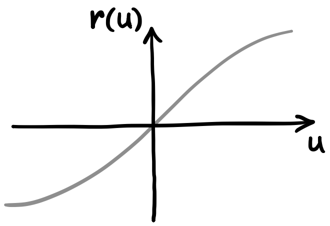
$\underbrace{\tau = \mathrm{vec}[\theta,\mu]}_{\text{Parameters}}$ $\underbrace{u_\theta \triangleq f(x,\theta)}_{\text{Decision function}}$ $\underbrace{\mu}_{\text{Threshold}}$ $\underbrace{\varphi(\tau) \triangleq \mathbb{E}[\hat{I}^{(1)}_{\mu}(u_\theta)]}_{\text{Probability of positives}}$ $\underbrace{\varphi_* > \pi_1 \triangleq \mathbb{E}[I_1(z)]}_{\text{Potential positive classifiability}}$

$\hat{I}^{(1)}_{\mu}$, I_1 $\in \{0,1\}$ indicate whether an observation z with features x is classified as a positive and is a true positive, respectively.

$I_E(z,\tau) \triangleq (1 - I_1(z))\hat{I}^{(1)}_{\mu}(u_\theta) + I_1(z)(1 - \hat{I}^{(1)}_{\mu}(u_\theta))$ is the error indicator.

r is a loss function based on the "distance" $u_\theta - \mu$.

In the simplest case, $r(u_\theta - \mu) = u_\theta - \mu$.



Choice of Criterion

Error probability ($r(\cdot) \triangleq 1$):

$$\mathbb{E}[\boxed{I_E(z,\tau)}] \longrightarrow \min_\tau$$

Challenging to build a gradient estimator.

For instance, let $F(\tau) \triangleq \int_{\{x: f(x,\tau)>0\}} p(x)\,dx$.

Then $\nabla F(\tau) = -\int_{\{x:\, f(x,\tau)>0\}} \mathrm{div}_x [\Lambda(x,\tau)p(x)]\,dx,$ (A)

$$\Lambda(x,\tau) \triangleq \frac{\nabla_\tau f(x,\tau)(\nabla_x f(x,\tau))^T}{\|\nabla_x f(x,\tau)\|}$$

or $\nabla F(\tau) = \int_{\{x:\, f(x,\tau)=0\}} \frac{\nabla_\tau f(x,\tau)}{\|\nabla_x f(x,\tau)\|} p(x)\,d\sigma.$ (B)

(A) requires estimating $p(x)$ for $\nabla p(x)$. ← Practically impossible

(B) requires estimating a surface integral in X. ← Slow

Choice of Criterion

Error probability ($r(\cdot) \triangleq 1$):

$$\mathbb{E}\left[I_E(z,\tau) \right] \longrightarrow \min_\tau$$

Challenging to build a gradient estimator.

For instance, let $F(\tau) \triangleq \int_{\{x : f(x,\tau) > 0\}} p(x)\,dx$.

Then $\nabla F(\tau) = -\int_{\{x: f(x,\tau)>0\}} \mathrm{div}_x \left[\Lambda(x,\tau) p(x) \right] dx,$ (A)

$$\Lambda(x,\tau) \triangleq \frac{\nabla_\tau f(x,\tau) \left(\nabla_x f(x,\tau)\right)^T}{\|\nabla_x f(x,\tau)\|}$$

or $\nabla F(\tau) = \int_{\{x: f(x,\tau)=0\}} \frac{\nabla_\tau f(x,\tau)}{\|\nabla_x f(x,\tau)\|} p(x)\, d\sigma.$ (B)

(A) requires estimating $p(x)$ for $\nabla p(x)$. ← Practically impossible

(B) requires estimating a surface integral in X. ← Slow

Expected risk (nontrivial $r(\cdot)$):

$$V(\tau) \triangleq \mathbb{E}\left[I_E(z,\tau) \, |r(u_\theta - \mu)| \right] \longrightarrow \min_\tau$$

Practical stochastic gradient is possible when

$I_E(z,\tau)$ and $r(\cdot)$ concord to allow $I_E(z,\tau) \cdot |r(\cdot)|$ to be differentiable

In $V(\tau)$: $r(u_\theta - \mu) > 0$ when $I_E(z,\tau) = 1$

$r(u_\theta - \mu) \leq 0$ when $I_E(z,\tau) = 0$

Decision Rule Update

Algorithm

$$\tau_{t+1} = \mathcal{H}(\tau_t - \gamma_t \widehat{G}(z_{t+1}, \tau_t)),$$

where $\widehat{G}(z,\tau) \triangleq \mathrm{vec}[J(z,\tau)\nabla_\theta f(x,\theta), -J(z,\tau)]$ is a stochastic quasi-gradient of $V(\tau)$;

$$J(z,\tau) \triangleq \begin{cases} +1, & \text{for false positive;} \\ 0, & \text{for correct;} \\ -1, & \text{for false negative;} \end{cases}$$

is the signed error indicator;

γ_t is learning rate; and

$\mathcal{H}(\tau)$ is a transformation encapsulating the a priori knowledge on parameter localization (or $\mathcal{H}(\tau) \equiv \tau$).

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^{\kappa}\right] \leq C_{f,\kappa}, \qquad \kappa > 1,$$
$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leq C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leq C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t.
the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies
$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leq C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$
4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leq V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall\tau;$
5) a sequence of learning rates $\gamma_t > 0$, s.t.
$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty;$
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:
$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \mathrm{vec}\Big[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\ -\mathbb{E}[J(z,\tau)]\Big].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \le C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^{\kappa}\right] \le C_{f,\kappa}, \qquad \kappa > 1,$$
$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \le C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \le C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t. the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies
$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \le C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$
4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \le V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau$;
5) a sequence of learning rates $\gamma_t > 0$, s.t.
$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty$;
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:
$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \mathrm{vec}\Big[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\ -\mathbb{E}[J(z,\tau)]\Big].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^\kappa\right] \leq C_{f,\kappa}, \qquad \kappa > 1,$$
$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leq C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leq C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t. the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies
$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leq C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$
4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leq V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau$;
5) a sequence of learning rates $\gamma_t > 0$, s.t.
$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty$;
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:
$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \operatorname{vec}\Big[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\ -\mathbb{E}[J(z,\tau)]\Big].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leqslant C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^\kappa\right] \leqslant C_{f,\kappa}, \qquad \kappa > 1,$$
$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leqslant C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leqslant C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t. the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies

$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leqslant C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$

4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leqslant V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau$;
5) a sequence of learning rates $\gamma_t > 0$, s.t.

$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty$;
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:

$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \mathrm{vec}\Big[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\, -\mathbb{E}[J(z,\tau)]\Big].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^\kappa\right] \leq C_{f,\kappa}, \qquad \kappa > 1,$$

$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leq C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leq C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t.
the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies

$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leq C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$

4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leq V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau$;
5) a sequence of learning rates $\gamma_t > 0$, s.t.

$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty$;
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:

$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \mathrm{vec}\Big[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\ -\mathbb{E}[J(z,\tau)]\Big].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^{\kappa}\right] \leq C_{f,\kappa}, \quad \kappa > 1,$$
$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leq C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leq C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t.
the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies

$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leq C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$

4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leq V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau;$
5) a sequence of learning rates $\gamma_t > 0$, s.t.

$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty;$
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:

$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \mathrm{vec}\Big[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\ -\mathbb{E}[J(z,\tau)]\Big].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^{\kappa}\right] \leq C_{f,\kappa}, \qquad \kappa > 1,$$

$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leq C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leq C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t. the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies

$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leq C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$

4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leq V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau$;
5) a sequence of learning rates $\gamma_t > 0$, s.t.

$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty;$
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:

$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \operatorname{vec}\left[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\, -\mathbb{E}[J(z,\tau)]\right].$$

Decision Rule Update Guarantees

Theorem 1 (Algorithm convergence) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) a continuously differentiable decision function $f(x,\theta)$, s.t. $\forall x, \theta', \theta''$

$$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[(C_f(x))^{\kappa}\right] \leq C_{f,\kappa}, \qquad \kappa > 1,$$

$$|\nabla_\theta f(x,\theta'') - \nabla_\theta f(x,\theta')| \leq C_{\nabla f}(x)\,\|\theta''-\theta'\|, \qquad \mathbb{E}\left[C_{\nabla f}(x)\right] \leq C_{\nabla f};$$

3) a feature-extraction function $\chi(z)$, s.t.
the c.d.f. F_{u_θ} of the random variable $u_\theta \triangleq f(\chi(z),\theta)$ satisfies

$$|F_{u_\theta}(u'') - F_{u_\theta}(u')| \leq C_F\,|u''-u'|, \qquad \forall \theta, u', u'';$$

4) a mapping $\mathcal{H}(\tau)$, s.t. $V(\mathcal{H}(\tau)) \leq V(\tau) + C_{\mathcal{H}}\|\mathcal{H}(\tau)-\tau\|^2 \quad \forall \tau$;
5) a sequence of learning rates $\gamma_t > 0$, s.t.

$$\sum_{t=1}^{\infty} \gamma_t = \infty, \qquad \sum_{t=1}^{\infty} \gamma_t^{2\lambda} < \infty, \qquad \lambda \triangleq \frac{\kappa}{\kappa+1};$$

the algorithm:

- converges in criterion: $\lim\limits_{t\to\infty} V(\tau_t) \overset{a.s.}{=} V_*, \quad \mathbb{E}V_* < \infty$;
- achieves the necessary condition of extremum for $V(\tau)$ on a subsequence:

$$\liminf_{t\to\infty} \|G(\tau_t)\| \overset{a.s.}{=} 0, \qquad G(\tau) \triangleq \mathrm{vec}\left[\mathbb{E}[J(z,\tau)\nabla_\theta f(x,\theta)],\, -\mathbb{E}[J(z,\tau)]\right].$$

Decision Rule Update Guarantees

Theorem 2 (Constraint satisfaction)

Under the assumptions of Theorem 1, if

1) condition 2) of Theorem 1 holds for $\kappa = \infty$, i.e.,

   $|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta'' - \theta'\|$, where $\mathbb{E}\left[(C_f(x))^{\kappa}\right] \leq C_{f,\kappa} \quad \forall \kappa$;

2) p.d.f. $p(x)$ of observations in the feature space that is continuous and has compact support;

3) $\mathcal{H}(\tau + \gamma\Delta) - \tau = \gamma \Psi(\tau)\Delta + O(\gamma^2)$ for some bounded matrix, $\|\Psi(\tau)\| \leq C_\Psi \quad \forall \tau$;

then, on the trajectory of the algorithm,
$\varphi(\tau_t)$ converges to $\pi_1 < \varphi_*$ in mean square and w.p. 1.

Decision Rule Update Guarantees

Theorem 2 (Constraint satisfaction)

Under the assumptions of Theorem 1, if

1) condition 2) of Theorem 1 holds for $\kappa = \infty$, i.e.,
   $|f(x,\theta'') - f(x,\theta')| \leqslant C_f(x)\,\|\theta'' - \theta'\|$, where $\mathbb{E}\left[(C_f(x))^\kappa\right] \leq C_{f,\kappa} \quad \forall \kappa$;
2) p.d.f. $p(x)$ of observations in the feature space that is continuous and has compact support;
3) $\mathcal{H}(\tau + \gamma\Delta) - \tau = \gamma\Psi(\tau)\Delta + O(\gamma^2)$ for some bounded matrix, $\|\Psi(\tau)\| \leqslant C_\Psi \ \forall \tau$;

then, on the trajectory of the algorithm,
$\varphi(\tau_t)$ converges to $\pi_1 < \varphi_*$ in mean square and w.p. 1.

Decision Rule Update Guarantees

Theorem 2 (Constraint satisfaction)

Under the assumptions of Theorem 1, if

1) condition 2) of Theorem 1 holds for $\kappa = \infty$, i.e.,
$|f(x,\theta'') - f(x,\theta')| \leq C_f(x) \|\theta'' - \theta'\|$, where $\mathbb{E}\left[(C_f(x))^\kappa\right] \leq C_{f,\kappa} \quad \forall \kappa$;
2) p.d.f. $p(x)$ of observations in the feature space that is continuous and has compact support;
3) $\mathcal{H}(\tau + \gamma\Delta) - \tau = \gamma \Psi(\tau)\Delta + O(\gamma^2)$ for some bounded matrix, $\|\Psi(\tau)\| \leq C_\Psi \quad \forall \tau$;

then, on the trajectory of the algorithm,
$\varphi(\tau_t)$ converges to $\pi_1 < \varphi_*$ in mean square and w.p. 1.

Decision Rule Update Guarantees

Theorem 2 (Constraint satisfaction)

Under the assumptions of Theorem 1, if

1) condition 2) of Theorem 1 holds for $\kappa=\infty$, i.e.,
$|f(x,\theta'') - f(x,\theta')| \leq C_f(x)\,\|\theta''-\theta'\|$, where $\mathbb{E}\left[(C_f(x))^\kappa\right] \leq C_{f,\kappa} \quad \forall\kappa;$

2) p.d.f. $p(x)$ of observations in the feature space that is continuous and has compact support;

3) $\mathcal{H}(\tau+\gamma\Delta) - \tau = \gamma\Psi(\tau)\Delta + O(\gamma^2)$ for some bounded matrix, $\|\Psi(\tau)\| \leq C_\Psi \;\forall\tau;$

then, on the trajectory of the algorithm,
$\varphi(\tau_t)$ converges to $\pi_1 < \varphi_*$ in mean square and w.p. 1.

Decision Rule Update Guarantees

Theorem 2 (Constraint satisfaction)

Under the assumptions of Theorem 1, if

1) condition 2) of Theorem 1 holds for $\kappa=\infty$, i.e.,
   $|f(x,\theta'')-f(x,\theta')| \leqslant C_f(x)\,\|\theta''-\theta'\|$, where $\mathbb{E}\left[(C_f(x))^\kappa\right] \leqslant C_{f,\kappa} \quad \forall\kappa$;
2) p.d.f. $p(x)$ of observations in the feature space that is continuous and has compact support;
3) $\mathcal{H}(\tau+\gamma\Delta)-\tau=\gamma\Psi(\tau)\Delta+O(\gamma^2)$ for some bounded matrix, $\|\Psi(\tau)\|\leqslant C_\Psi \quad \forall\tau$;

then, on the trajectory of the algorithm,
$\varphi(\tau_t)$ converges to $\pi_1<\varphi_*$ in mean square and w.p. 1.

Decision Rule Update Guarantees

Algorithm (Hyperplane separator):

$$\hat{\theta}_{t+1} = \theta_t - \gamma_t J(z_{t+1}, \tau_t)\left(\mathbf{I} - \theta_{t+1}\theta_{t+1}^T\right) x_{t+1},$$

$$\mu_{t+1} = \mu_t + \gamma_t J(z_{t+1}, \tau_t)$$

$$\tau_{t+1} = \mathrm{vec}\left[\hat{\theta}_{t+1} / \|\hat{\theta}_{t+1}\|, \; \mu_{t+1}\right]$$

In other words,

$$f(x, \theta) \triangleq \frac{\langle x, \theta \rangle}{\|\theta\|},$$

$$\mathcal{H}(\tau) \triangleq \mathrm{vec}\left[\theta / \|\theta\|, \mu\right].$$

Decision Rule Update Guarantees

Algorithm (Hyperplane separator):

$$\hat{\theta}_{t+1} = \theta_t - \gamma_t J(z_{t+1}, \tau_t)\left(\mathbf{I} - \theta_{t+1}\theta_{t+1}^T\right) x_{t+1},$$
$$\mu_{t+1} = \mu_t + \gamma_t J(z_{t+1}, \tau_t)$$
$$\tau_{t+1} = \mathrm{vec}\left[\hat{\theta}_{t+1}/\|\hat{\theta}_{t+1}\|,\ \mu_{t+1}\right]$$

In other words,

$$f(x,\theta) \triangleq \frac{\langle x, \theta\rangle}{\|\theta\|},$$
$$\mathcal{H}(\tau) \triangleq \mathrm{vec}[\theta/\|\theta\|, \mu].$$

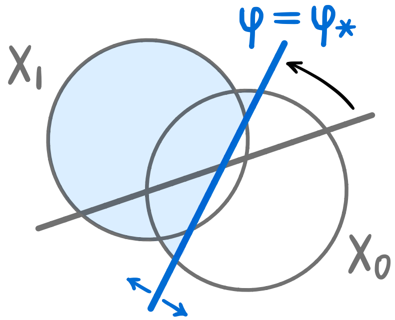
Theorem 3 (Hyperplane separator) Given

1) a sample of i.i.d. observations $\{z_t\}$;
2) p.d.f. $p(x)$ of observations in the feature space that is continuous and have compact support;
3) $\sum_{t=1}^{\infty} \gamma_t = \infty, \quad \sum_{t=1}^{\infty} \gamma_t^2 < \infty$;

the algorithm converges in criterion and achieves

$$\lim_{t\to\infty} \left\| \mathbb{E}_z\left[J(z_{t+1}, \tau_t)\left(\mathbf{I} - \theta_{t+1}\theta_{t+1}^T\right) x_{t+1}\right]\right\| \overset{a.s.}{=} 0,$$
$$\lim_{t\to\infty} \mathbb{E}_z\left[J(z_{t+1}, \tau_t)\right] \overset{a.s.}{=} 0,$$

i.e., $\varphi(\tau_t)$ converges to $\pi_1 < \varphi_*$ w.p. 1.



Cloud-Assisted On-Sensor Observation Classification in Latency-Impeded IoT Systems

Igor Burago
iburago@uci.edu

Marco Levorato
levorato@uci.edu

Intelligent and Autonomous Systems Lab
iasl.ics.uci.edu

Papers

- ib.fyi/isit19
- ib.fyi/acscc18