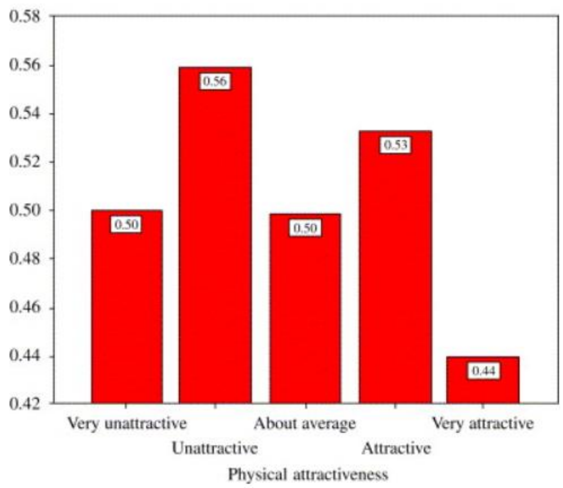
So we are looking at the sex ratios example. Remember that in the original data from the Kanazawa paper, parents were classed into five categories based on attractiveness. The very attractive parents had the most daughters:



Dichotomizing the data into very attractive vs all other patents:

	Daughters	Total Children	Proportion Daughters
Very Attractive Parents	170	300	0.567
Other Parents	1310	2700	0.485

The question at hand is whether very attractive parents have a different newborn sex ratio than other parents. Last session, Yuan presented two different models.

Model 1 – Uniform Prior for Daughter Probability

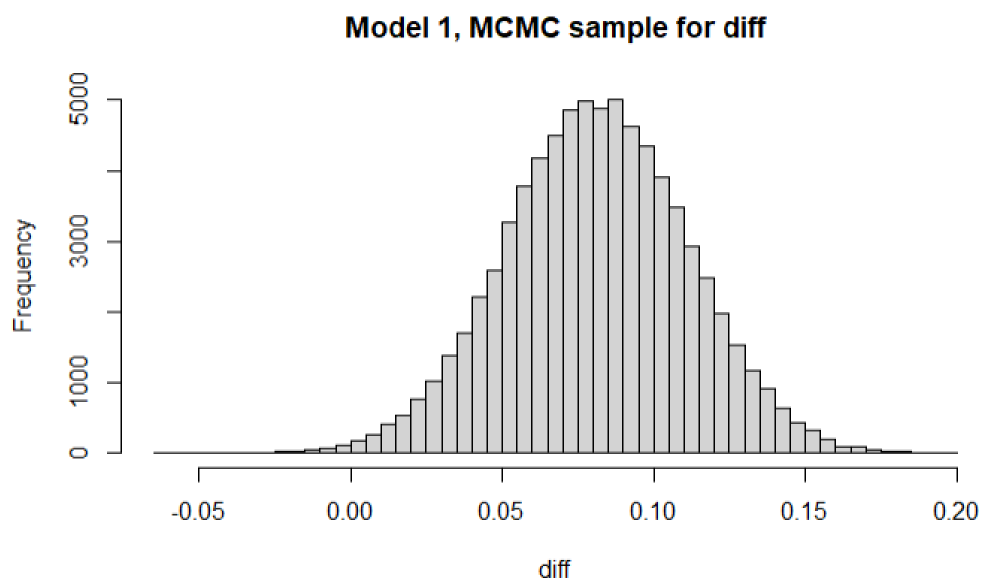
$$d_{att} \sim Binom(n_{att}, p_{att})$$

$$d_{oth} \sim Binom(n_{oth}, p_{oth})$$

$$p_{att}, p_{oth} \sim Uniform(0,1)$$

To examine whether the two groups had a different sex ratio, Yuan focused on the difference between these two probabilities: $diff = p_{att} - p_{oth}$.

The posterior distribution of this parameter HAS a 95% CI of (0.030, 0.140). This excludes 0. 99.6% of the posterior samples for difference were above 0.



Model 2 – Weakly Informative Prior

$$d_{att} \sim Binom(n_{att}, p_{oth} + diff)$$

$$d_{oth} \sim Binom(n_{oth}, p_{oth})$$

$$p_{oth} \sim Uniform(0,1)$$

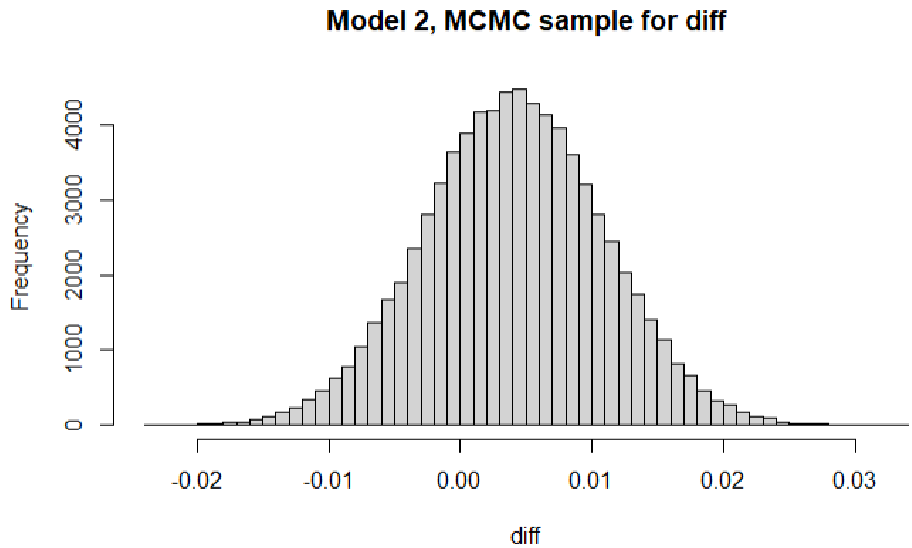
$$diff \sim Normal(0,0.007)$$

$$p_{att} = p_{oth} + diff$$

$$p_{att} \in [0,1]$$

Applying Model 2 to our data

When we apply the model to our data, the 95% CI is (-0.009, 0.018). 72.4% of the MCMC sample were above 0.



Examining Model 2's prior

This was selected based on the prior assumption that $diff < 1.5\%$ with 95% probability

Before going any further, let's plot the prior distribution of p_{att}. The joint distribution of $(p_{oth}, diff)$ is:

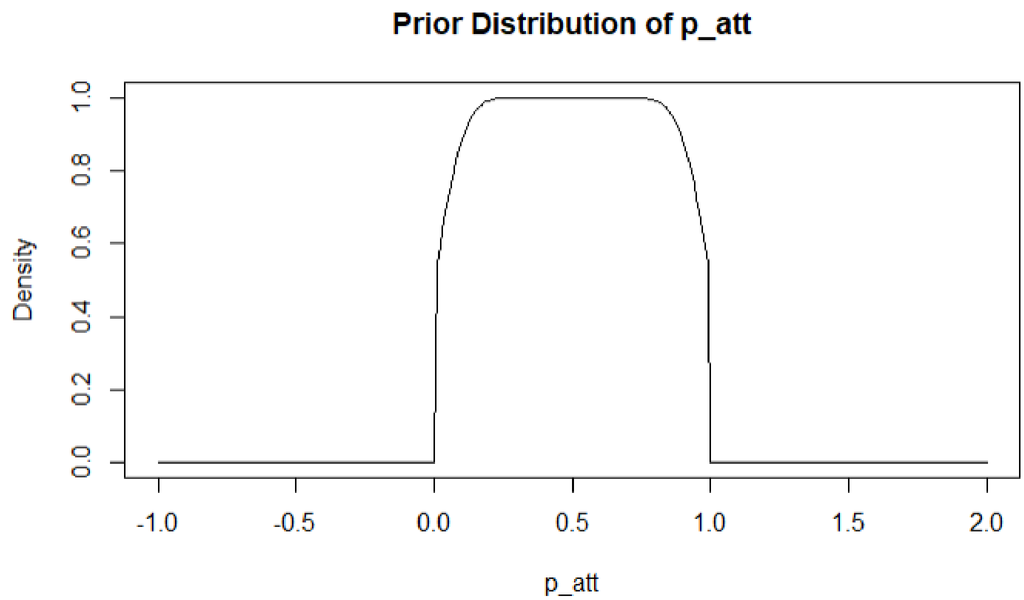
$$f_{p_{oth},diff}(p_{oth}, diff) = \frac{1}{\sqrt{2\pi * 0.007}} \exp\left[\frac{-diff^2}{2 * 0.007}\right] I_{[0,1]}(p_{oth})$$
$$= \frac{1}{\sqrt{2\pi * 0.007}} \exp\left[\frac{-(p_{att} - p_{oth})^2}{2 * 0.007}\right] I_{[0,1]}(p_{oth})$$

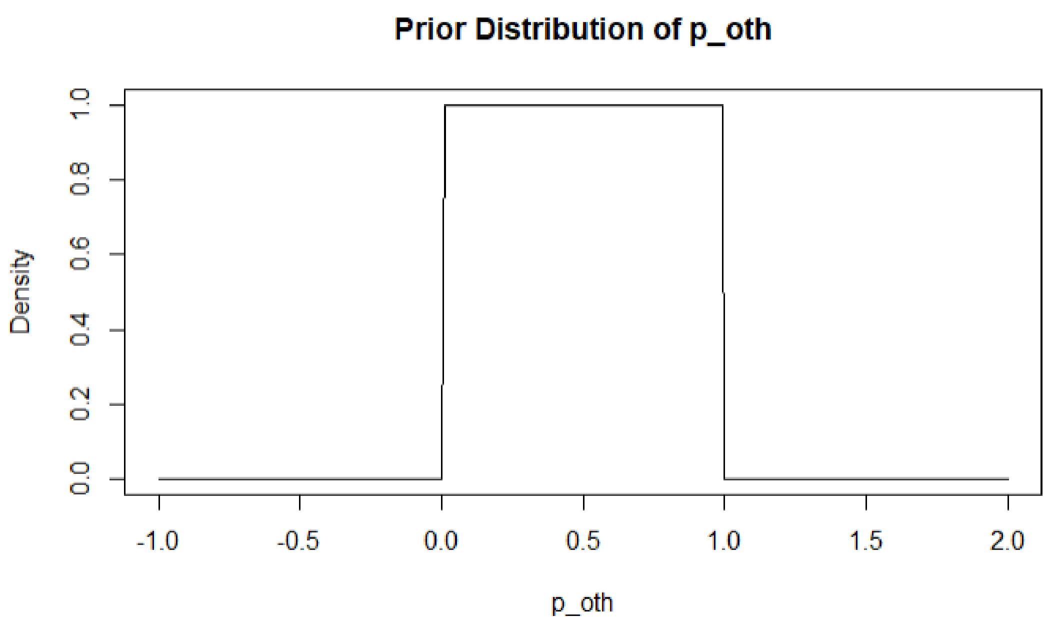
The Jacobian is 1 for the conversion from $(p_{oth}, diff)$ to (p_{oth}, p_{att}), so this is also the joint density for (p_{oth}, p_{att}) as written. Integrating out p_{oth}, we obtain the prior distribution of p_{att}:

$$\int_0^1 f_{p_{oth},diff}(p_{oth}, diff) dp_{oth} = \int_0^1 \frac{1}{\sqrt{2\pi * 0.07}} \exp\left[\frac{-(p_{att} - p_{oth})^2}{2 * 0.007}\right] I_{[0,1]}(p_{oth}) dp_{oth}$$

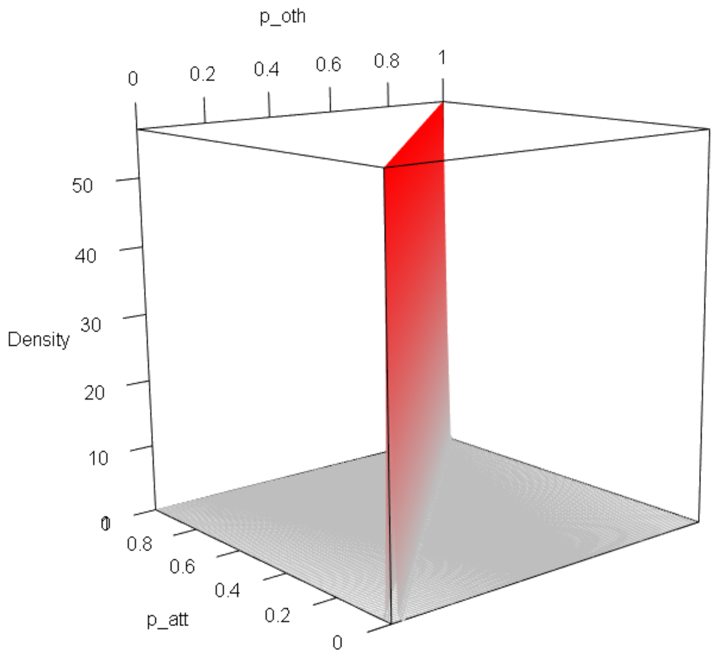
$$= \int_0^1 \frac{1}{\sqrt{2\pi * 0.007}} \exp\left[\frac{-(p_{att} - p_{oth})^2}{2 * 0.007}\right] dp_{oth} = \Phi\left(\frac{1 - p_{att}}{\sqrt{0.007}}\right) - \Phi\left(-\frac{p_{att}}{\sqrt{0.007}}\right)$$

We plot the prior distribution of p_{oth} and p_{att}:





In 3 dimensions:



In this model, how is the choice of variance for $diff$ justified? We used $diff \sim N(0,0.007)$. Why 0.007? Yuan said that Dr Wang had challenged this, thinking it was too restrictive. It is a subjective choice and should be made by consulting with domain experts and looking at the amount of past evidence available. But another way to view this choice is to step outside of the Bayesian framework and examine its frequentist operating characteristics. We can simulate the Type 1 error and the power under different choices of variance.

Simulating Frequentist Operating Characteristics

To simulate the Type 1 error, assume that 49% of babies are female regardless of parent attractiveness, and then simulate random draws from $Binom(2700, 0.49)$ and $Binom(300, 0.49)$ distributions. For each draw, run an MCMC simulation for the posterior distribution and determine whether the 95% CI for difference covers 0. The percent of intervals that exclude 0 is the frequentist Type 1 error for this Bayesian design.

For power, let's consider the power for detecting a difference of 10%. We can simulate random draws from $Binom(2700, 0.49)$ and $Binom(300, 0.59)$ distributions. For each draw, run an MCMC simulation for the posterior distribution and determine whether the 95% CI for difference covers 0. The percent of intervals that exclude 0 is the frequentist power error for this Bayesian design.

We can then plot this frequentist power for different values of the prior variance for $diff$.

Note that making these assumptions used in the simulations is a switch from a Bayesian to a frequentist mindset, because the Bayesian framework does not admit "true values" of the parameter. Alternatively, this can be seen as a special case of a Bayesian sample size determination technique that is called "design priors" vs "analysis priors". In this approach, the experimental designer sets an optimistic prior as the “design prior” and then calculates the predictive probability of success under this design prior. This is similar to the frequentist power, and one would want a large enough sample size so that it is reasonably high to give the trial a reasonable chance of success. Then the experimenter uses a neutral “analysis prior” in the actual analysis. You can view the assumptions we are making to compute frequentist power as a point mass version of a design prior.

200 simulations run for the following four conditions:

Prior Variance	**Type 1 Error**	**Power**
0.007	0%	0%
100	3%	91%