Biostat 216 Homework 2

Due Oct 11 @ 11:59pm

Submit a PDF (scanned/photographed from handwritten solutions, or converted from RMarkdown or Jupyter Notebook or Quarto) to Gradescope in BruinLearn.

1 Q1. Sub-multiplicity of Frobenius norm

Show the matrix norm property

$$\|\mathbf{AB}\|_{\mathrm{F}} \le \|\mathbf{A}\|_{\mathrm{F}}\|\mathbf{B}\|_{\mathrm{F}}$$

for the Frobenius norm. Hint: Cauchy-Schwartz inequality.

BV exercise 6.14 is a special case of this result.

2 Q2. Induced matrix norm

For any vector norm $\|\mathbf{x}\|$ on $\mathbb{R}^m$ and $\mathbb{R}^n$, there is an induced matrix norm $\|\mathbf{A}\|$ on $m \times n$ matrices defined by

$$\|\mathbf{A}\| = \sup_{\mathbf{x} \ne \mathbf{0}} \frac{\|\mathbf{Ax}\|}{\|\mathbf{x}\|} = \sup_{\|\mathbf{x}\|=1} \|\mathbf{Ax}\|.$$

1. Show the second equality in the above equation.

2. Show the four properties (positive definiteness, homogeneity, triangle inequality, sub-multiplicity) for the induced matrix norm.

3. Show that the Frobenius norm is different from the induced matrix-2 norm.

3 BV exercises

5.1, 5.2, 5.4, 5.6, 5.8, 5.9, 6.3, 6.11, 6.17, 6.21, 6.22