Biostat 216 Homework 1

Due Oct 4 @ 11:59pm

Submit a PDF (scanned/photographed from handwritten solutions, or converted from RMarkdown or Jupyter Notebook or Quarto) to Gradescope in BruinLearn.

1 Q1. Average and norm

Use the Cauchy-Schwarz inequality to prove that

$$-\frac{1}{\sqrt{n}}\|\mathbf{x}\| \le \frac{1}{n}\sum_{i=1}^n x_i \le \frac{1}{\sqrt{n}}\|\mathbf{x}\|$$

for any $\mathbf{x} \in \mathbb{R}^n$. In other words, $-\operatorname{rms}(\mathbf{x}) \le \operatorname{avg}(\mathbf{x}) \le \operatorname{rms}(\mathbf{x})$.

What are the conditions on $\mathbf{x}$ to have equality in the upper bound? When do we have equality in the lower bound?

2 Q2. AM-HM inequality

Use the Cauchy-Schwartz inequality to prove that

$$\frac{1}{n}\sum_{i=1}^n x_i \ge \left(\frac{1}{n}\sum_{i=1}^n \frac{1}{x_i}\right)^{-1}$$

for any $\mathbf{x} \in \mathbb{R}^n$ with positive entries x_i.

The left hand side is called the arithmetic mean (AM) and the right hand side is called the harmonic mean (HM). You may wonder what can be a practical use of such a simple inequality. See this paper, which uses the AM-HM inequality to derive a class of optimization algorithms for geometric and signomial programming.

3 Q3. Bias-variance tradeoff

Prove the formula

$$\operatorname{avg}(\mathbf{x})^2 + \operatorname{std}(\mathbf{x})^2 = \operatorname{rms}(\mathbf{x})^2$$

using the vector notation and do BV 3.15.

4 BV exercises

1.7, 1.9, 1.13, 1.16, 1.20, 3.4, 3.5, 3.12.