VI : variational inference

$$p(z|x) \approx q_\psi(z|x)$$

$$\psi^* = \arg\min_\psi KL\left(q_\psi \| p(z|x)\right)$$

✓ $$KL(q_\psi \| p) = \int q_\psi(z|x) \log\left(\frac{q_\psi(z|x)}{p(z|x)}\right) dz$$

Bayes' rule

$$= \int q_\psi \log\left(\frac{q_\psi \cdot p(x)}{p(z,x)}\right) dz$$

$$= \int q_\psi \left[\log q_\psi - \log p(z,x)\right] dz \checkmark$$

$$+ \int q_\psi \log p(x)\, dz = \underbrace{\log p(x)}_{\text{log evidence}} \checkmark \underbrace{\int q_\psi(z|x)\, dz}_{1}$$

$$\log p(x) = KL - \int q_\psi \left[\log q_\psi - \log \underline{p(z,x)}\right] dx$$

$$= KL + \mathcal{L}(q,\psi) \quad \text{ELBO}$$

$$\mathcal{L}(q,\psi,\phi) = -\int q_\psi \log q_\psi\, dz + \int q_\psi \log\left[p(x|z,\phi)\, p(z)\right] dz$$

$$= -\int q_\psi \log\left(\frac{q_\psi}{p(z)}\right) dz + \int q_\psi \log p(x|z,\phi)\, dz$$