# Weird Rejection Sampler

$$\theta \sim \pi(\theta)$$

1) $X \sim \pi(\theta)$ 3) $u \sim \text{uni}(0,1)$

2) $\theta^* = f(X)$

$$q(\theta) = \pi(\overbrace{f^{-1}(\theta^*)}^{X})\,|\nabla f^{-1}|$$

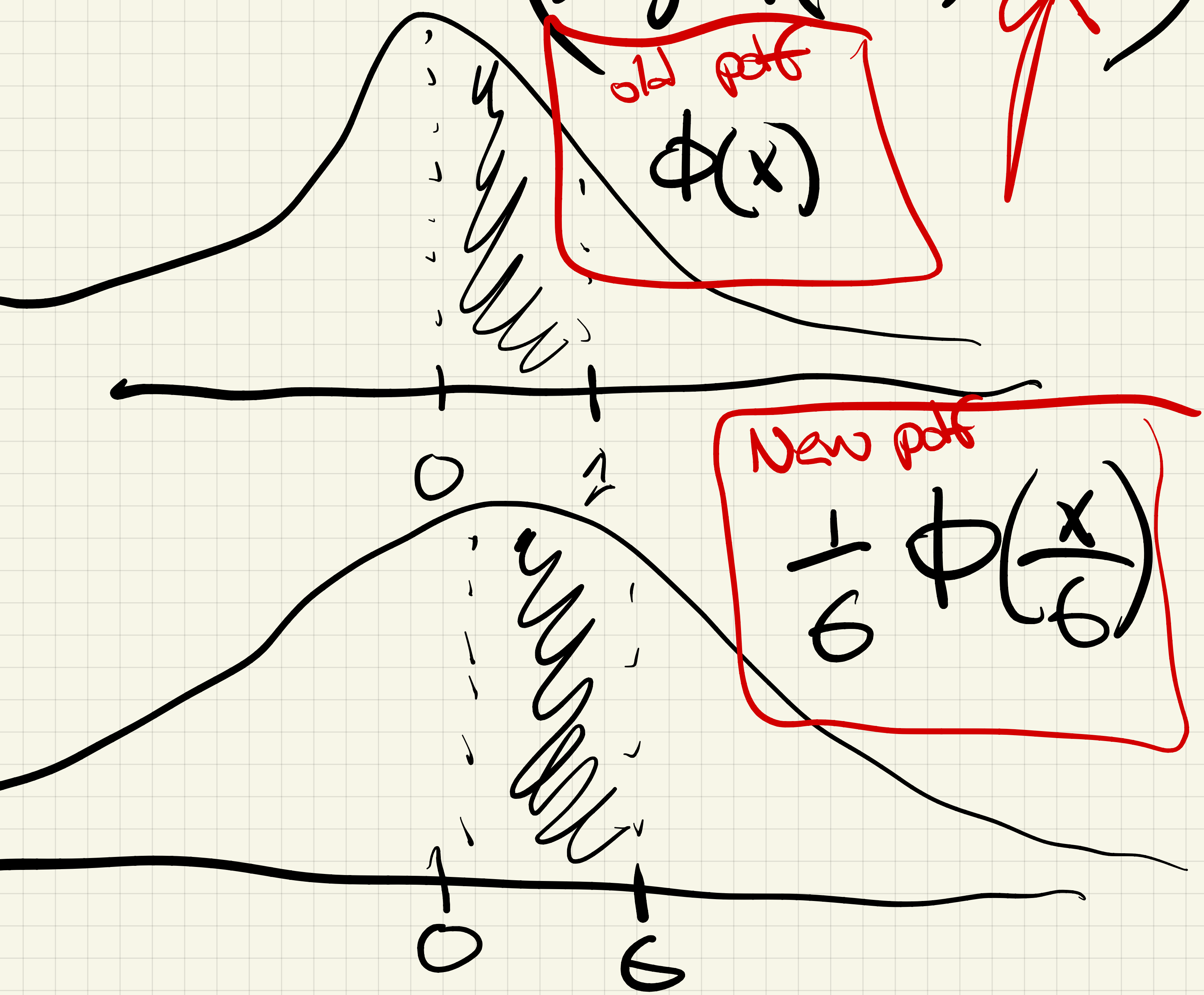
If

$$u < \frac{\pi(\theta^*)\,|\nabla f|}{\pi(X)\,M}$$

Then $\theta \leftarrow \theta^*$ $\theta \sim \pi(\theta)$

# Metropolis-Hastings-Green

$$a = \min\left(1, \frac{\pi(\theta^*)}{\pi(\theta^{(s-1)})} * |\nabla f|\right)$$



$q$ position $\quad d$

$p$ momentum $\quad 2d$

$H(q,p)$

$$\boxed{\frac{dq}{dt} = \frac{\partial H}{\partial p}, \quad \frac{dp}{dt} = \frac{-\partial H}{\partial q}}$$

1) $T_t(q_0, p_0) = (q_t, p_t)$ Reversible?

$$T_t^{-1}(q_t, p_t) = T_{-t}(q_t, p_t) = (q_0, p_0)$$

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2) $$\frac{dH}{dt} = \sum_{i=1}^{d} \frac{\partial H}{\partial q_i}\frac{dq_i}{dt} + \frac{\partial H}{\partial p_i}\frac{dp_i}{dt}$$

$$= \sum_{i=1}^{d} \frac{\partial H}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial H}{\partial q_i}$$

$= 0$

Conservation of energy

3) $|\nabla T| = 1$ Volume preservation

If divergence $= 0$
then volume is preserved

$$\nabla \cdot F = \sum_{i=1}^{d} \nabla F_i \quad \Big| \quad F = \sum_{i=1}^{d} \frac{dq_i}{dt} + \frac{dp_i}{dt}$$

$$\sum_{i=1}^{d} \frac{\partial}{\partial q_i}\frac{dq_i}{dt} + \frac{\partial}{\partial p_i}\frac{dp_i}{dt}$$

$$= \sum_{i=1}^{d} \frac{\partial}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial}{\partial p_i}\frac{\partial H}{\partial q_i}$$

$$= 0$$

$$q \sim \pi(q) \qquad q \perp p$$

$$p \sim \xi(p) \propto \exp\left(-\frac{p^T M^{-1} p}{2}\right)$$

joint distribution has pdf

$$\pi(q)\,\xi(p)$$

By fiat: $H(q,p) = -\log\left(\pi(q)\,\xi(p)\right)$

$$H(q,p) = -\log \pi(q)\,\xi(p)$$
$$= -\log \pi(q)$$
$$- \log \xi(p)$$

**Algorithm for sampling $q \sim \pi(q)$**

1) choose any $t$
2) Advance system according to
$$\frac{\partial q}{\partial t} = \frac{\partial H}{\partial p} = M^{1} p$$
$$\frac{\partial p}{\partial t} = -\frac{\partial H}{\partial q} = \nabla \underline{\log \pi(q)}$$
3) stop at time $t$

4)* MHG $\alpha = 1 \wedge \frac{\pi(q_t)\xi(p_t)}{\pi(q_0)\xi(p_0)} |\nabla \Pi_t|$

$H(q,p) = U(q) + K(p)$

potential kinetic

$K(p) = \frac{p^T M^{-1} p}{2}$ Gaussian

$K(p) = |p|_1$

Laplace

$$a = 1 \wedge \frac{\pi(q_t)\xi(p_t)}{\pi(q_0)\xi(p_0)}$$

$$= 1 \wedge \frac{\pi(q_t)\xi(p_t)}{\pi(q_0)\xi(p_0)}$$

$$\log(a) = 0 \wedge$$

$$\log(\pi_t \xi_t) - \log(\pi_0 \xi_0)$$

$$= 0 \wedge H(q_t, p_t) - H(p_0, \xi_0)$$

Conservation of Energy $\frac{\partial H}{\partial t} = 0 \Rightarrow \log(a) = 0$

$\Rightarrow a = 1$

$M$?

Think Newton's Method:

$$M = \nabla^2 \log \pi(q)$$

$$M(q)$$

If $q \sim$ Gaussian

$$\mathcal{N}(0, M^{-1})$$

Harmonic oscillator

$$\Rightarrow \frac{\partial q}{dt} = \frac{\partial H}{\partial p} = M^{-1} p$$

$$\frac{\partial p}{dt} = -\frac{\partial H}{\partial q} = M q$$