

Getting Started with Uclid5

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1. Introduction

UCLID5 is a software toolkit for the formal modeling, specification, verification, and synthesis of computational systems. The UCLID5 toolchain aims to:

1. Enable compositional (modular) modeling of finite and infinite state transition systems across a range of concurrency models and background logical theories;
2. Verification of a range of properties, including assertions, invariants, and temporal properties, and
3. Integrate modeling and verification with algorithmic and inductive synthesis.

UCLID5 draws inspiration from the earlier UCLID system for modeling and verification of systems [?, ?], in particular the idea of modeling concurrent systems in first-order logic with a range of background theories, and the use of proof scripts within the model. However, the UCLID5 modeling language and verification capabilities go beyond the original modeling language, and the planned integration with synthesis is novel.

This document serves as introduction to the UCLID5 modeling language and toolchain. With the UCLID5 system under active development, we expect this document to undergo several changes as the system and its applications evolve.

1.1. Getting Started: A Simple Uclid5 Model

A simple UCLID5 module that computes the Fibonacci sequence is shown in Example 1.1. We will now walk through each line in this model to understand the basics of UCLID5.

The top-level syntactic structure in UCLID5 is a `module`. All modeling, verification and synthesis code in UCLID5 is contained within modules. In Example 1.1, we have defined one `module` named `main`. This module starts on line 1 and ends on line 18. The module can be conceptually split into three parts: a system model, a specification and proof script.

In the example, these three conceptual parts are also kept separate in the code.¹ The following subsections will describe each of these sections of the module.

¹This is not required by UCLID5 syntax, as `invariant` declarations and assumptions can be interleaved with `init`, `next`, `var` declarations as well other types of declarations. However, keeping these conceptually different parts separate is good design practice. UCLID5 does require that if a `control` block is specified, then it is the very last element of a module.

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```
1 module main {
2   // Part 1: System description.
3   var a, b : integer;
4
5   init {
6     a = 0;
7     b = 1;
8   }
9   next {
10    a', b' = b, a + b;
11  }
12
13  // Part 2: System specification.
14  invariant a_le_b: a <= b;
15
16  // Part 3: Proof script.
17  control {
18    bmc (3);
19    check;
20    print_results;
21  }
22 }
```

Example 1.1.: A UCLID5 model that computes the Fibonacci sequence

The System Model

This part of a UCLID5 module describes the functionality of the transition system that is being modeled: it tells us *what the system does*.

The first item of interest within the module `main` are *state variables*. These are declared using the `var` keyword. The module `main` declares two state variables: `a` and `b` on line 3. These are both of type `integer`, which corresponds to mathematical integers.²

The `init` block appears next and spans lines 5 to 8. It defines the initial values of the state variables in the module. The notation `a'` refers to the value of the state variable `a` at the end of the current “step”, which in this case refers to initial state. The model is specifying that after the `init` block is executed, `a` and `b` have the values 0 and 1 respectively.

The `next` block appears after this and it defines the transition relation of the module. In the figure, the next statement spans from lines 9 to 11; `a` is assigned to the (old) value of `b`, while `b` is assigned to the value `a + b`.

The System Specification

The specification answers the question: *what is the system supposed to do?*

²Mathematical integer types, as opposed to the machine integer types present in languages like C/C++ and Java, do not have a fixed bit-width and do not overflow.

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In our example, we have a single `invariant` that comprises that entire specification. Line 14 defines this `invariant`. It is named `a_le_b` and as the name suggests, it states that `a` must be less than or equal to `b` for every reachable state of the system.

The Proof Script

The third and final part of the UCLID5 module is a set of commands to the UCLID5 verification engine. These tell how we should go about proving³ that the system satisfies its specification.

The proof script is contained within the `control` block. The commands here execute the system for 3 steps and check whether all of the systems properties (in this case, we only have one invariant: `a_le_b`) are satisfied for each of these steps.

The command `bmc` executes the system for 3 steps. This execution generates four *proof obligations*. These proof obligations ask whether the system satisfies the invariant `a_le_b` in the initial state and in each of the 3 states reached next. The `check` command *checks* whether these proof obligations are satisfied and the `print_results` prints out the results of these checks.

1.2. Installing Uclid5

Public releases of the UCLID5 can be obtained at: <https://github.com/uclid-org/uclid/releases>. For the impatient, the short version of the installation instructions is: download the archive with the latest release, unzip the archive and add the ‘bin/’ subdirectory to your PATH.

More detailed instructions for installation are as follows.

1.2.1. Prerequisites

UCLID5 has two prerequisites.

1. UCLID5 requires that the Java™ Runtime Environment be installed on your machine. You can download the latest Java Runtime Environment for your platform from <https://www.java.com>.
2. UCLID5 uses the Z3 SMT solver. You can install Z3 from: <https://github.com/Z3Prover/z3/releases>. Make sure the ‘z3’ or ‘z3.exe’ binary is in your path after Z3 installed. Also make sure, the shared libraries for libz3 and libz3java are in the dynamic library load path (LD_LIBRARY_PATH on Unix-like systems).

UCLID5 has been tested with Java™ SE Runtime Environment version 1.8.0 and Z3 versions 4.5.1 and 4.6.0.

³We are using a broad definition of the word “prove” here to refer to any systematic method that gives us assurance that the specification is satisfied.

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1.2.2. Detailed Installation Instructions

First, down the platform independent package from <https://github.com/uclid-org/uclid/releases>.

Next, follow these instructions which are provided for the bash shell running on a Unix-like platform. Operations for Microsoft Windows, or a different shell should be similar.

- Unzip the archive.

```
$ unzip uclid-0.9.5.zip.
```

- Add the uclid binary to your path.

```
$ export PATH=$PATH:$PWD/uclid-0.9.5/bin/
```

- Check that the uclid works.

```
$ uclid
```

This should produce output similar to the following.

```
$ uclid

Usage: uclid [options] filename [filenames]
Options:
  -h/--help : This message.
  -m/--main : Set the main module.
  -d/--debug : Debug options.

Error : Unable to find main module.
```

1.2.3. Running Uclid5

Invoke UCLID5 on a model is easy. Just run the `uclid` binary and provide a list of files containing the model as a command-line argument. When invoked, UCLID5 will parse each of these files and look for a module named `main` among them. It will execute the commands in the `main` module's control block. The `--main` command line argument can be used to specify a different name for the "main" module. Note only the `main` module's control blocks will be executed, even if the `main` module instantiates other modules with control blocks. If no `main` module is found, UCLID5 will exit with an error, as we saw in the previous section when `uclid` was invoked without arguments.

Example 1.1 is part of the UCLID5 distribution in the `examples/tutorial/` sub-directory. You can run UCLID5 on this model as:

```
$ uclid examples/tutorial/ex1.1-fib-model.ucl
```

This should produce the following output.

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```
Successfully parsed 1 and instantiated 1 module(s).  
4 assertions passed.  
0 assertions failed.  
0 assertions indeterminate.  
Finished execution for module: main.
```

1.3. Looking Forward

This chapter has provided an brief overview of UCLID5's features and toolchain. The rest of this tutorial will take a more detailed looked at more of UCLID5's features.

2. Basics: Types and Statements

This chapter will provide an overview of UCLID5's type system and modelling features. Let us start with Example 2.1, a model of a simple arithmetic logic unit (ALU).

2.1. Types in Uclid5

Types supported by UCLID5 are of the following kinds:

1. `integer`: the type of mathematical integers.
2. `boolean`: the Boolean type. This type has two values: `true` and `false`.
3. `bvW`: The family of bit-vector types parameterized by their width (W).
4. `enum`: enumerated types.
5. `float`: single precision floating point type with an 8 bit exponent and 23 bit significand and 1 sign bit.
6. Tuples and records.
7. Array types.
8. Uninterpreted types.
9. Groups: A finite group of variables of a single type. Groups are not modifiable once declared, i.e., they are a static size.

An enumerated type is used in line 2 of Example 2.1. This declares a *type synonym*: `cmd_t` is an alias for the enumerated type consisting of three values: `add`, `sub` and `mov_imm`. The input `cmd` is then declared to be of type `cmd_t` on line 6.

The input `valid` is of type `boolean`. Register indices `r1` and `r2` are bit-vectors of width 3 (`bv3`), while `immed`, `r1val` and `r2val` are bit-vectors of width 8 (`bv8`).

Line 3 declares a type synonym for a `record`. It declares `result_t` as consisting of two fields: a Boolean field `valid` and a bit-vector field `value`. The output `result` is declared to be of type `result_t` on Line 9.

The final point of interest in this example, type-wise, is line 10. The state variable `regs` is declared to be of type array: indices to the array are of type `bv3` and elements of the array are of type `bv8`. This is used to model an 8-entry register file, where each register is a bit-vector of width 8.

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```
1 module main {
2   type cmd_t = enum { add, sub, mov_imm };
3   type result_t = record { valid : boolean, value : bv8 };
4
5   input  valid  : boolean;
6   input  cmd    : cmd_t;
7   input  r1, r2 : bv3;
8   input  immed  : bv8;
9   output result : result_t;
10  var    regs   : [bv3]bv8;
11  var    cnt    : bv8;
12
13  init {
14    for i in range(0bv3, 7bv3) {
15      regs[i] = 1bv8;
16    }
17    cnt, result.value = 1bv8, 1bv8;
18  }
19
20  define double(arg : bv8) : bv8 = (arg + arg);
21
22  procedure exec_cmd()
23    returns (r : result_t)
24    modifies regs;
25  {
26    var r1val, r2val : bv8;
27    if (valid) {
28      r1val, r2val = regs[r1], regs[r2];
29      case
30        (cmd == add)      : { regs[r1] = r1val + r2val; }
31        (cmd == sub)      : { regs[r1] = r1val - r2val; }
32        (cmd == mov_imm) : { regs[r1] = immed; }
33      esac
34      r.valid, r.value = true, regs[r1];
35    } else { r.valid = false; }
36  }
37
38  next {
39    call (result') = exec_cmd();
40    cnt' = double(cnt);
41  }
42
43  assume regindex_zero : (r1 == 0bv3 && r2 == 0bv3);
44  assume cmd_is_add    : (cmd == add) && valid;
45  invariant result_eq_cnt : (cnt == result.value);
46
47  control {
48    f = bmc (5);
49    check;
50    print_results;
51  }
52 }
```

Example 2.1.: Model of a simple ALU

2. Basics: Types and Statements

Uninterpreted functions can be declared in UCLID5 using the `function` declaration. These functions are typed, mapping a tuple of typed arguments to a return type. For example, the following uninterpreted function models the mapping of an instruction to an opcode in a simple CPU model given later in this tutorial as Example 3.2.

```
1  function inst2op    (i : word_t) : op_t;
```

Two further types of functions without definitions can be declared in UCLID5 using the `function` declaration. First a synthesis function, declared using the `synthesis` keyword. Given a SyGuS solver using the command line option `-y "solver path"`, UCLID5 will construct a SyGuS-IF file that synthesises bodies for any synthesis function in the UCLID5 file

```
1  synthesis function inst2op    (i : word_t) : op_t;
```

The second function extension is an oracle function, declared using the `oracle` keyword. An oracle function is declared along with the name of an external executable binary which implements the function. Given an Satisfiability Modulo Theories and Oracles (SMTO) solver using the command line option `-s "solver path"`, or a Synthesis Modulo Oracles (SyMO) solver using the command line option `-y "solver path"`, UCLID5 will construct an SMTO/SyMO file for the verification/synthesis query including the oracle function. For more details on Synthesis and Satisfiability Modulo Oracles see <https://arxiv.org/abs/2107.13477>.

```
1  oracle function inst2op [binaryName] (i : word_t) : op_t;
```

Symbolic constants can be declared using a `const` declaration, as follows:

```
1  const w0 : word_t;
```

Finally, a useful feature of UCLID is groups. Groups are declared using the keyword `group`. All members of a group must have the same type, but they can be literals or variables that are already declared and in scope at the point of the group declaration.

```
1  group MyGroup : integer = {0, 1, 2, 3};  
2  group AnotherGroup: integer = {a, b, c, 7};
```

2.2. Statements in Uclid5

Computation in UCLID5 can be either procedural (sequential) or parallel (concurrent). Procedural computation is performed by defining a `procedure` (and in the `init` block) while parallel computation occurs in the `next` block.

2.2.1. Parallel vs. Procedural Assignments

Assignments inside procedures and the `init` block are called **procedural assignments** and must be of the form `variable = expression`; Assignments inside `next` block

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are **parallel assignments** and must be of the form `variable' = expression;`. Mathematically, parallel assignments compute the next state of the transition system described by the model.

An example showing the use of sequential assignments is the following:

```
1  x = 1;
2  x = x + 2;
3  x = x + 3;
```

In this example, `x` is assigned sequentially. Recall that these procedural assignments *must* appear inside a procedure or in the `init` block. Executing these three statements will result `x` having the value 6.

In contrast, the following sequence of parallel assignments is **not** allowed and will result in a compiler error.

```
1  // Error, will not compile.
2  x' = 1;
3  x' = x + 2;
4  x' = x + 3;
```

Only a single parallel assignment to a state/output variable is allowed in a code block. Furthermore, since parallel assignments are computed in data-flow order, the order in which they are specified does not matter. This means that the following two snippets of code are equivalent:

```
1  next {
2    x' = x + 1;
3    y' = x' + 1;
4  }
```

```
1  next {
2    y' = x' + 1;
3    x' = x + 1;
4  }
```

UCLID5 will determine that since `y'` depends on the value of `x'`, `x'` has to be computed first. This value is then used in the computation of `y'`. This is regardless of the order in which these assignments appear in the `next` block.

Note also that the assignment to `x'` uses the value of the variable `x` *at the beginning of the current step* of the transition system (i.e., the “old” value of `x`). In contrast the assignment to `y'` uses the “new” value of `x`, which is the value of `x` at the *end* of this step of the transition system. It is important to think carefully about which version of a variable (`var` or `var'`) must be used in a particular assignment.

2.2.2. Procedures

Example 2.1 demonstrates how sequential computation is used in concert with parallel computation. In Example 2.1, consider procedure `exec_cmd` which executes a single ALU command and returns (line 23) a single value of type `result_t`. The procedure

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is invoked on line 39 in the next block, and its return value is assigned to the output variable `result`. Note we are again using the notation `result'` to refer to parallel assignment. Since this procedure updates the `reg` state variable, a `modifies` clause must be used to declare this fact (line 24).

We emphasize that assignments inside procedures do not assign primed variables. However, if a state variable is defined to be modified by a procedure (mentioned in its `modifies` clause), then its next-state value is the value that variable has upon return from that procedure. Put another way, the post-state of the procedure determines the next-state assignment of all state variables modified by it. In our example, procedure `exec_cmd` modifies the state variable `regs`, and thus determines its next-state value.

2.2.3. Macro Definitions

UCLID5 also supports the definition of macro expressions using the `define` statement. Line 20 of Example 2.1 illustrates this construct. Macro definitions are useful, as in C, to define expressions over arguments that are instantiated in multiple places, or which help make the code more readable.

2.2.4. For Loops

The procedure `set_init_state` uses a `for` loop to initialize each value in the array `regs` to the bit-vector value 1.¹ The loop iterates over the values between 0 and 7 (both-inclusive).

The range over which a `for` loop iterates must be defined by two numeric literals.

Alternatively, the loop iterates may be defined by two macro definitions whose expressions are numeric literals. However, this requires the `for` statement to be declared with a typed iterator.

For example, if the following macro definitions are at the module level,

```
1  define begin() : bv3 = 0bv3;  
2  define end()   : bv3 = 7bv3;
```

we may alternatively write the `for` loop in `set_init_state` in the following way:

```
1  for (i : bv3) in (begin(), end()) { regs[i] = 1bv8; }
```

2.2.5. If and Case Statements

Also worth pointing out are the `if` statement that appears on line 27, and the `case` statement that appears on line 29. Syntax for `if` statements should be familiar.

`case` statements are delimited by `case` and `esac` and contain within them a list of boolean expressions and associated statement blocks. These expressions are evaluated in the order in which appear, and if any of them evaluate to `true`, the corresponding block is executed. If none of the case-expressions evaluate to `true`, nothing is executed. The keyword `default` can be used as a “catch-all” case like in C/C++.

¹1bv8 here refers to the bit-vector value 1 of width 8.

2.2.6. Expressions

The syntax for expressions in UCLID5 is similar to languages like C/C++/Java. Index `i` of array `regs` is accessed using the syntax `regs[i]`. Field `value` in the record `result` is accessed as `result.value`.

2.2.7. Quantifiers

UCLID5 supports universal and existential quantifiers, which can be used in any place where a predicate would be permitted, for instance, in assumptions:

```
1  assume (forall (ri : robind_t) :: ops[ri] == no_op);
2  assume (exists (i : integer) :: (i > 0) && (i < 11) && numbers[i] !=
    some_int);
```

UCLID5 Also supports finite universal quantification, where we universally quantify over a finite group type. These quantifiers are expanded to a conjunction before being passed to the underlying SMT solver.

```
1  group myGroup : integer = {0, 1, 2, 3};
2  property test : finite_forall (int : integer) in myGroup :: int < 4;
```

2.3. An Illustrative Example

This section briefly describes the execution semantics of Example 2.1.

2.3.1. Initialization

Execution of the model in Example 2.1 starts with the `init` block. This block invokes `set_init_state` and assigns initial values to `regs`, `cnt` and `result.value`. The other variables (e.g. `r1val` and `r2val`) are not assigned to in the `init` block and will be initialized non-deterministically.

2.3.2. Next State Computation

The next state of each state variable in the model is computed according to the `next` block. Any variables not assigned to in the `next` block retain their “old” values.

The `input` variables of the model are assigned (possibly different) non-deterministic values for each step of the transition system. These values can be controlled by using assumptions. Indeed, the model uses the two assumptions on lines 43–44 to constrain the input to the ALU to always be an add operation, where both operands refer to register index 0.

2.3.3. Verification

As in Example 1.1, the verification script in Example 2.1 unrolls the transition system for 5 steps and checks if the `invariant` on line 45 is violated in any of these steps.

2.3.4. Running Uclid5

Running UCLID5 on Example 2.1 produces the following output.

```
$ uclid examples/tutorial/ex2.1-alu.ucl
Successfully parsed 1 and instantiated 1 module(s).
6 assertions passed.
0 assertions failed.
0 assertions indeterminate.
Finished execution for module: main.
```

UCLID5 is able to prove that the *invariant* on line 45 holds for all states reachable within 5 steps of the initial state, under the assumptions specified in lines 43–44.

3. Compositional Modeling and Abstraction

This chapter describes UCLID5’s features for compositional and modular verification, and the use of abstraction.

We will use a running example of a CPU model constructed in UCLID5 and use bounded unrolling of the model’s transition relation to prove that the execution of this CPU is deterministic: i.e. we show that given two identical instruction memories, the state updates performed by this CPU will be identical.

3.1. Common Type Definitions Across Modules

Example 3.1 shows a module that defines only type synonyms. Such a module can be used to share type definitions across other modules. The types declared in Example 3.1 are *imported* in lines 2-5 of module `cpu` declared in Example 3.2.

```
1 // This module declares types that are used in the rest of the model.
2 module common {
3   // addresses are uninterpreted types.
4   type addr_t = bv8;
5   type word_t = bv8;
6   // memory
7   type mem_t = [addr_t]word_t;
8   // CPU operation.
9   type op_t   = enum { op_mov, op_add, op_sub, op_load, op_store };
10 }
```

Example 3.1.: Module `common` of the CPU model

Isolating commonly used types into a single module in this manner allows the construction of large models parameterized by this types. These common types can be changed and the ramifications of these changes on the model’s behavior can be studied easily.

3.2. Uninterpreted Functions and Types

A convenient mechanism for abstraction in UCLID5 is through the use of uninterpreted functions and types. This is one of the novel modeling aspects for transition systems introduced by the original UCLID system [?].

3. Compositional Modeling and Abstraction

```
1 module cpu {
2   type addr_t = common.addr_t;
3   type mem_t  = common.mem_t;
4   type word_t = common.word_t;
5   type op_t   = common.op_t;
6   type regindex_t; // type of register file.
7   type regs_t = [regindex_t]word_t;
8
9   input imem      : mem_t; // program memory.
10  var dmem        : mem_t; // data memory.
11  var regs        : regs_t;
12  var pc          : addr_t;
13  var inst, result : word_t;
14
15  function inst2op  (i : word_t) : op_t;
16  function inst2reg0 (i : word_t) : regindex_t;
17  function inst2reg1 (i : word_t) : regindex_t;
18  function inst2imm  (i : word_t) : word_t;
19  function inst2addr (i : word_t) : addr_t;
20
21  procedure exec_inst(inst : word_t, pc : addr_t)
22    returns (result : word_t, pc_next : addr_t)
23    modifies regs, dmem;
24  {
25    var op      : op_t;
26    var r0ind, r1ind : regindex_t;
27    var r0, r1    : word_t;
28
29    op = inst2op(inst);
30    r0ind, r1ind = inst2reg0(inst), inst2reg1(inst);
31    r0, r1 = regs[r0ind], regs[r1ind];
32    case
33      (op == op_mov)      : { result = inst2imm(inst); }
34      (op == op_add)      : { result = r0 + r1; }
35      (op == op_sub)      : { result = r0 - r1; }
36      (op == op_load)     : { result = dmem[inst2addr(inst)]; }
37      (op == op_store)    : { result = r0; dmem[inst2addr(inst)] = r0; }
38    esac
39    pc_next = pc + 1bv8;
40    regs[r0ind] = result;
41  }
42
43  init {
44    assume (forall (r : regindex_t) :: regs[r] == 0bv8);
45    assume (forall (a : addr_t) :: dmem[a] == 0bv8);
46    pc, inst = 0bv8, 0bv8;
47  }
48
49  next {
50    inst' = imem[pc];
51    call (result', pc') = exec_inst(inst, pc);
52  }
53 }
```

Example 3.2.: The cpu module in the CPU model

3. Compositional Modeling and Abstraction

```
1 module main {
2
3   // Import types
4   type addr_t      = common.addr_t;
5   type mem_t       = common.mem_t;
6   type word_t      = common.word_t;
7   type op_t        = common.op_t;
8   type regindex_t  = cpu.regindex_t;
9
10  // instruction memory is the same for both CPUs.
11  var imem : mem_t;
12
13  // Create two instances of the CPU module.
14  instance cpu_i_1 : cpu(imem : (imem));
15  instance cpu_i_2 : cpu(imem : (imem));
16
17  init {
18  }
19
20  next {
21    // Invoke CPU 1 and CPU 2.
22    next (cpu_i_1);
23    next (cpu_i_2);
24  }
25
26  // These are our properties.
27  invariant eq_regs :
28    (forall (ri : regindex_t) :: cpu_i_1.regs[ri] == cpu_i_2.regs[ri]);
29  invariant eq_mem :
30    (forall (a : addr_t) :: cpu_i_1.dmem[a] == cpu_i_2.dmem[a]);
31  invariant eq_pc : (cpu_i_1.pc == cpu_i_2.pc);
32  invariant eq_inst : (cpu_i_1.inst == cpu_i_2.inst);
33
34  // Proof script.
35  control {
36    bmc(3);
37    check;
38    print_results;
39  }
40 }
```

Example 3.3.: Module main in the CPU model

3.2.1. Uninterpreted Types

Example 3.2 shows the use of the *uninterpreted type*: `regindex_t` on line 6. The index type to the register is an *abstract* type, as opposed to a specific type (e.g. `bv3`). This allows us to reason about an abstract register file that has an undefined (and unbounded) number of entries, as opposed to proving facts about some specific register file implementation, potentially enabling more general proofs about system behavior.

3.2.2. Uninterpreted Functions

Values belonging to an uninterpreted type can be created using *uninterpreted functions*. The functions `inst2op`, `inst2reg0`, `inst2reg1`, `inst2imm` and `inst2addr` on lines 15-19 of Example 3.2 are all examples of uninterpreted functions. An uninterpreted function f is a symbol about which we know nothing, except that it is a function; i.e. $\forall x_1, x_2. x_1 = x_2 \implies f(x_1) = f(x_2)$.

As an example, in the context of processor verification, uninterpreted functions allow us to reason about an abstract CPU model without considering specific instruction encodings or decoder models. This could potentially lead to more general proofs as well as more scalable automated proofs.

3.3. Module Instantiation and Scheduling

Modules are instantiated using the `instance` keyword. Lines 14 and 15 of Example 3.3 show two instantiations of the module `cpu`. For each instance, the module input `imem` is mapped to the state variable `imem` of module `main`.

Scheduling of instantiated modules is explicit and synchronous. The two `next` statements on lines 22 and 23 of Example 3.3 invoke the next state transitions of the two instances of the `cpu` module.

Asynchronous and partially synchronous composition can be modeled by explicitly encoding a scheduler into the UCLID5 model that specifies when the next block of each module is executed.

3.3.1. Accessing Members of Instances

The state variables, types, etc. of an instantiated module are accessed using the `.` operator. The four invariants on lines 27-32 of Example 3.3, refer to the registers, memory, pc and instruction variables of the two instantiated modules. These invariants state that both instances must have identical values for these state variables.

3.4. Running Uclid5

Executing UCLID5 on the complete CPU model shows that CPU is in fact deterministic.

3. Compositional Modeling and Abstraction

```
$ uclid ex3.1-cpu.ucl ex3.2-cpu.ucl ex3.3-cpu.ucl
Successfully parsed 3 and instantiated 1 module(s).
16 assertions passed.
0 assertions failed.
0 assertions indeterminate.
Finished execution for module: main.
```

Note the files `ex3.1-cpu.ucl`, `ex3.2-cpu.ucl` and `ex3.3-cpu.ucl` contains three modules. Each of these modules could potentially have `control` blocks. Which UCLID5 is invoked on this model, it executes only the `control` block of the main module. If we had included a `control` block for the `alu` module and wished to verify properties of this module, we would have to invoke UCLID5 on this specific module using the `--main` command-line option.

4. Verification Techniques

In the examples covered thus far, we have only used UCLID5 for bounded model checking of invariants. It can also be used to perform bounded model checking of linear temporal logic properties. In addition, UCLID5 can be used to do unbounded inductive proofs and also provides support for debugging counterexamples. It can be used for checking simulation (refinement) between two transition systems, such as the technique of correspondence checking between two processor models. Finally, UCLID5 can be used to check certain hyperproperties, such as two-safety properties, by the technique of self-composition. This chapter will describe these features of UCLID5. Further features are being implemented and will be described in a future version of this document.

4.1. Inductive Proofs

Let us revisit the model from Example 1.1. This is now shown again in Example 4.1, but with a different proof script. Instead of using the `bmc` command for bounded model checking, we are using the `induction` command to attempt an inductive proof.

```
1 module main {
2   // Part 1: System description.
3   var a, b : integer;
4
5   init {
6     a = 0;
7     b = 1;
8   }
9   next {
10    a', b' = b, a + b;
11  }
12
13  // Part 2: System specification.
14  invariant a_le_b: a <= b;
15
16  // Part 3: Proof script.
17  control {
18    induction;
19    check;
20    print_results;
21  }
22 }
```

Example 4.1.: UCLID5 Fibonacci model using induction in the proof script

4.1.1. Debugging Counterexamples

Let us try running UCLID5 on Example 4.1 with the new proof script.

```
$ uclid examples/tutorial/ex4.1-fib-induction.ucl
Successfully parsed 1 and instantiated 1 module(s).
1 assertions passed.
1 assertions failed.
0 assertions indeterminate.
  FAILED -> induction (step) [Step #1]
    property a_le_b @ ex4.1-fib-induction.ucl, line 14
Finished execution for module: main.
```

Uh oh, we seem to have a problem! UCLID5 is telling us that the inductive proof failed. We can try to examine why the proof failed by using the `print_cex` command to examine the counterexample to the proof.

```
1 module main {
2   // Part 1: System description.
3   var a, b : integer;
4
5   init {
6     a = 0;
7     b = 1;
8   }
9   next {
10    a', b' = b, a + b;
11  }
12
13  // Part 2: System specification.
14  invariant a_le_b: a <= b;
15
16  // Part 3: Proof script.
17  control {
18    vobj = induction;
19    check;
20    print_results;
21    vobj.print_cex(a, b);
22  }
23 }
```

Example 4.2.: UCLID5 Fibonacci model with induction and `print_cex`

The only changes between Example 4.1 and Example 4.2 are on lines 18 and 21. `vobj` on line 18 is a reference to the verification conditions generated by the `induction` command. On line 21, we pass this reference to the `print_cex` command which prints out the values of `a` and `b` for the counterexample.

Running UCLID5 on Example 4.2 produces the following.

4. Verification Techniques

```
Successfully parsed 1 and instantiated 1 module(s).
1 assertions passed.
1 assertions failed.
0 assertions indeterminate.
  FAILED -> vobj: induction (step) [Step #1]
    property a_le_b @ ex4.2-fib-induction-cex.ucl, line 14
CEX for vobj: induction (step) [Step #1]
property a_le_b @ ex4.2-fib-induction-cex.ucl, line 14
=====
Step #0
  a : -1
  b : 0
=====
=====
Step #1
  a : 0
  b : -1
=====
Finished execution for module: main.
```

To understand the counterexample, it is helpful to review how the inductive proof engine works. When inductively proving the *invariant* `a_le_b`, UCLID5 considers some arbitrary state that satisfies this property, executes the *next* block, and checks whether `a_le_b` holds on the resultant state.

The counterexample shows us that we do start in a state where $a \leq b$ with $a = -1$ and $b = 0$. We execute the *next* block and now `a` gets the value of `b`, becoming 0 and `b` gets the value $a + b$, becoming -1. This new state does not satisfy the invariant!

What is the real problem here? Taking a closer look at Example 4.2, we see that this specific counterexample can never occur in our model because `a` and `b` are always ≥ 0 . But UCLID5 does not know this when attempting the inductive proof. Therefore, we have to strengthen the inductive argument with this information in order to help UCLID5's proof.

4.1.2. Inductive Proof for the Fibonacci Model

Example 4.3 shows the same model as Example 4.2, but with a stronger induction hypothesis. UCLID5's inductive engine will now start in an arbitrary state that assumes that both invariants `a_le_b` and `a_b_ge_0` hold and attempt to prove that both of these still hold after the *next* block is executed.

Let us now run UCLID5 on this new model.

4. Verification Techniques

```
1 module main {  
2   // Part 1: System description.  
3   var a, b : integer;  
4  
5   init {  
6     a = 0;  
7     b = 1;  
8   }  
9   next {  
10    a', b' = b, a + b;  
11  }  
12  
13  // Part 2: System specification.  
14  invariant a_le_b: a <= b;  
15  invariant a_b_ge_0: (a >= 0 && b >= 0);  
16  
17  // Part 3: Proof script.  
18  control {  
19    vobj = induction;  
20    check;  
21    print_results;  
22    vobj.print_cex(a, b);  
23  }  
24 }
```

Example 4.3.: Inductive proof for the Fibonacci model

4. Verification Techniques

```
Successfully parsed 1 and instantiated 1 module(s).
$ uclid examples/tutorial/ex4.3-fib-induction-proof.ucl
4 assertions passed.
0 assertions failed.
0 assertions indeterminate.
Finished execution for module: main.
```

Success! We have shown that our system model satisfies its specification.

4.1.3. Exercise: Inductive Proof of CPU model

Prove determinism of the CPU model in Examples 3.2 and 3.3 using induction rather than bounded model checking. You will need to add strengthening inductive invariants relating the two CPU instances.

4.2. Bounded Model Checking

Let us return to the model of Example 1.1 which is reproduced as Example 4.4 with a few changes. We used the `bmc` command for verification. This command performs bounded model checking and takes a single argument – the number of steps to unroll the model for. In Example 4.4, we are unrolling the model for 3 steps. The `bmc` command will check all properties (LTL and non-LTL), and supercedes the deprecated `unroll` command. To check only the non-LTL properties, use `bmc_noLTL`. We have introduced the constant `flag` on line 4. A constant holds a symbolic value that does not change during computation. The initial value of the constant is assigned non-deterministically and can be controlled using assumptions.

4.2.1. Embedded assume and assert statements

A second difference with between Example 1.1 and Example 4.4 is on lines 12–14, 24 and 25. Instead of using a module-level assumption declarations as in Example 1.1, we have three embedded assumptions in the `set_init` procedure on lines 12–14, and two embedded assertions in the `next` block on lines 23 and 25. A module-level assumption is assumed to hold for the solver at every step of execution, while an embedded assumption is assumed “instantaneously.” In particular, the assumptions on lines 12–14 tells the solver to assume that $a \leq b$, $a \geq 0$ and $b \geq 0$ at the end of the `set_init` procedure. Notice that we are not assigning specific values to `a` and `b`, instead we are asking UCLID5 to consider potential values of `a` and `b` such that $a \leq b$, $a \geq 0$ and $b \geq 0$.

Similarly the assertions on lines 23 and 25 are evaluated at that specific location in the code. In particular the assertion on line 23 is only checked when `flag` is `true`, while the assertion one line 25 is checked when `flag` is `false`. Since `flag` is always `true` in our model, the assertion on line 25 will never fire. In contrast, note that a module-level

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```
1 module main {
2   // System description.
3   var a, b : integer;
4   const flag : boolean;
5
6   procedure set_init()
7     modifies a, b;
8   {
9     havoc a;
10    havoc b;
11    // embedded assumptions.
12    assume (a <= b);
13    assume (a >= 0 && b >= 0);
14    assume (flag);
15  }
16
17  init {
18    call set_init();
19  }
20  next {
21    a', b' = b, a + b;
22    if (flag) {
23      assert (a' <= b');
24    } else {
25      assert (false);
26    }
27  }
28
29  // Proof script.
30  control {
31    bmc (3);
32    check;
33    print_results;
34  }
35 }
```

Example 4.4.: Revisiting the Fibonacci model from Example 1.1.

4. Verification Techniques

assertion would be evaluated after the `init` block and after each execution of the `next` block.

4.2.2. Running Uclid5

Running UCLID5 on Example 4.4 shows that the embedded assertions do indeed hold for all states reachable within 3 steps of the initial state.

```
$ uclid examples/tutorial/ex4.4-fib-model-revisted.ucl
Successfully parsed 1 and instantiated 1 module(s).
6 assertions passed.
0 assertions failed.
0 assertions indeterminate.
Finished execution for module: main.
```

4.3. Specifications in Linear Temporal Logic

UCLID5 supports the specification of module behavior using linear temporal logic (LTL).

Example 4.5 shows a UCLID5 model of an intersection with two traffic lights. Lines 3–39 define the functionality of the traffic light; this part of the model should be familiar. The current state of the lights are stored in the variables `light1` and `light2`, and these switch from red to green to yellow and back to red. The variables `step1` and `step2` can be thought of timers, and ensure that each light stays red for three transitions, green for two transitions and stays yellow for a single transition.

The LTL properties are on lines 41, 42 and 43. The property `always_one_red` specifies a safety property which states that at least one of the two lights must be red in every particular cycle. The notation $G(\phi)$ refers to the LTL globally operator, while the notation $F(\phi)$ refers to the LTL eventually (future) operator. Other supported operators include next-time: $X(\phi)$, (strong-)until: $U(\phi_1, \phi_2)$ and weak-until: $W(\phi_1, \phi_2)$. `always_one_red` is a safety property. The property `eventually_green` is an example of liveness property, and specifies that both lights become green infinitely often.

The command `bmc` performs bounded verification of all properties, including LTL properties. This is invoked on line 46 and specifies which properties must be checked within the square brackets. (If no properties are specified, and the square brackets are omitted `bmc` checks all properties in the module.) To check only the LTL properties use the `bmc_LTL` command.

4.3.1. Running Uclid5

Running UCLID5 on Example 4.5 produces the following output.

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```
1 module main
2 {
3   type light_t = enum { red, yellow, green };
4   var step1, step2 : integer;
5   var light1, light2 : light_t;
6
7   init {
8     light1, step1 = red, 2;
9     light2, step2 = green, 1;
10  }
11
12  next {
13    call (light1', step1') = next_light(light1, step1);
14    call (light2', step2') = next_light(light2, step2);
15  }
16
17  procedure next_light(light : light_t, step : integer)
18    returns (lightP : light_t, stepP : integer)
19  {
20    if (step == 0) {
21      case
22        (light == green) : {
23          lightP = yellow;
24          stepP = step;
25        }
26        (light == yellow) : {
27          lightP = red;
28          stepP = 2;
29        }
30        (light == red) : {
31          lightP = green;
32          stepP = 1;
33        }
34      esac
35    } else {
36      lightP = light;
37      stepP = step - 1;
38    }
39  }
40
41  property[LTL] always_one_red: G(light1 == red || light2 == red);
42  property[LTL] eventually_green:
43    G(F(light1 == green)) && G(F(light2 == green));
44
45  control {
46    v = bmc[properties=[always_one_red, eventually_green]](10);
47    check;
48    print_results;
49    v.print_cex(light1, step1, light2, step2);
50  }
51 }
```

Example 4.5.: Example of using LTL specifications in UCLID5.

```
$ uclid run examples/traffic-light.ucl
Running (fork) uclid.UclidMain examples/traffic-light.ucl
Successfully parsed 1 and instantiated 1 module(s).
44 assertions passed.
0 assertions failed.
0 assertions indeterminate.
Finished execution for module: main.
```

The output shows that all properties are verified.

Exercises

1. Does the property `always_one_red` hold if the assignment to `step2` on line 9 is changed to 2 (from 1)? Why or why not? Make this change, print-out and understand the counterexample if one exists.
2. Find a way to modify the model so that the property `eventually_green` is violated. Examine and understand the counter-example generated by UCLID5 when this happens.

4.4. Correspondence/Simulation Checking

UCLID5 provides constructs to check whether one transition system simulates another, where both are modeled as UCLID5 modules. In other words, we can check whether one module can simulate steps of another. Correspondence checking, a special case of simulation checking, is based on constructing a commutative diagram (see Fig. 4.4) via symbolic simulation and checking the validity of a property of interest at the end [?].

Thus, the outline of the verification task, as specified in the control section of a module, involves simulating the two sides of the commutative (simulation) diagram and checking an assertion at the end. An example of correspondence checking, the verification of a simple pipelined datapath, is illustrated in detail in the code included below (also in the distribution in `examples/simple-datapath.ucl`). This example is drawn from the original UCLID system [?, ?].

```
1 // Simple pipelined datapath, based on old uclid model in the old UCLID user
  manual
2
3 module common {
4   // Types
5   // Words -- Addresses, Data, etc.
6   type word_t = bv32;
7   // Registers
8   type reg_t = bv8; // arbit choice
9   // Register file
10  type rf_t = [reg_t]word_t;
11  // OpCode
```

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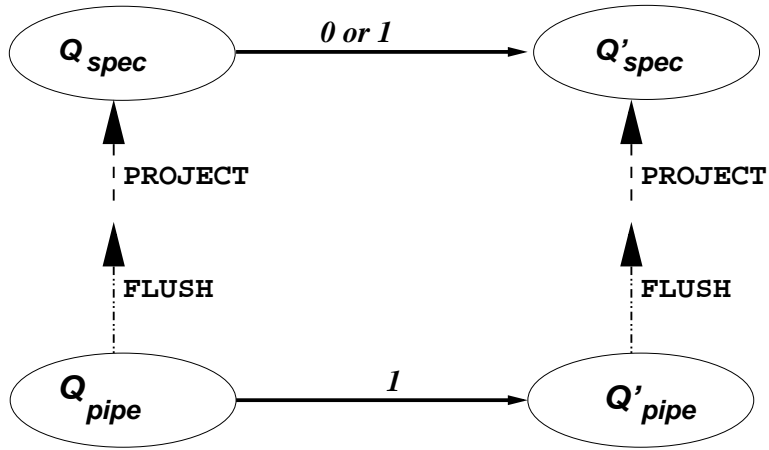


Figure 4.1.: Commutative diagram to check an implementation (pipelined processor) is simulated by its specification.

```

12  type op_t = bv8; // arbit choice
13
14  // Uninterpreted functions
15  function newPC(a: word_t) : word_t;
16  const pc0 : word_t;
17  function rf0 (r : reg_t) : word_t;
18  function src1 (i : word_t) : reg_t;
19  function src2 (i : word_t) : reg_t;
20  function dest (i : word_t) : reg_t;
21  function op (i : word_t) : op_t;
22  function alu (op : op_t, arg1: word_t, arg2 : word_t) : word_t;
23
24  }
25
26  // Specification/ISA model
27  module spec {
28
29      input proj_impl : boolean;
30      input impl_RF : common.rf_t;
31      input impl_PC : common.word_t;
32
33      var sPC : common.word_t;
34      var sRF : common.rf_t;
35
36      procedure update_sRF()
37          modifies sRF;
38      {
39          sRF[common.dest(sPC)] = common.alu(common.op(sPC),
40                                             sRF[common.src1(sPC)],
41                                             sRF[common.src2(sPC)]);
42      }
43

```

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```

44 | init {
45 |     sPC = common.pc0;
46 |     assume (forall (i : common.reg_t) :: (sRF[i] == common.rf0(i)));
47 | }
48 |
49 | next {
50 |     if (proj_impl) {
51 |         sPC' = impl_PC;
52 |         sRF' = impl_RF;
53 |     }
54 |     else {
55 |         sPC' = common.newPC(sPC);
56 |         call update_sRF();
57 |     }
58 | }
59 | }
60 |
61 | // Pipeline model
62 | module impl {
63 |
64 |     input flush : boolean;
65 |     input reinit : boolean;
66 |
67 |     var pPC : common.word_t;
68 |     var pRF : common.rf_t;
69 |     var eOP : common.op_t;
70 |     var eSRC2 : common.reg_t;
71 |     var eDEST : common.reg_t;
72 |     var eARG1 : common.word_t;
73 |     var eARG2 : common.word_t;
74 |     var eWRT : boolean;
75 |     var wVAL : common.word_t;
76 |     var wDEST : common.reg_t;
77 |     var wWRT : boolean;
78 |
79 |     const op0 : common.op_t;
80 |     const s0, d0, d1 : common.reg_t;
81 |     const a1, a2, x0 : common.word_t;
82 |     const w0, w1 : boolean;
83 |
84 |     procedure update_pRF()
85 |     modifies pRF;
86 |     {
87 |         if (wWRT) { pRF[wDEST] = wVAL; }
88 |     }
89 |
90 |     define stall() : boolean = eWRT && (common.src1(pPC) == eDEST);
91 |
92 |     init {
93 |         // initialize PC and RF same as spec module
94 |         pPC = common.pc0;
95 |         assume (forall (i : common.reg_t) :: (pRF[i] == common.rf0(i)));
96 |         // all other state variables get arbit symbolic initialization
97 |         eOP = op0;

```


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```

98   eSRC2 = s0;
99   eARG1 = a1;
100  eARG2 = a2;
101  eDEST = d0;
102  eWRT = w0;
103  wVAL = x0;
104  wDEST = d1;
105  wWRT = w1;
106  }
107
108
109  next {
110    if (reinit) {
111      pPC' = common.pc0;
112      havoc pRF;
113      assume (forall (i : common.reg_t) :: (pRF'[i] == common.rf0(i)));
114      // all other state variables get same arbit symbolic initialization as
115      before
116      eOP' = op0;
117      eSRC2' = s0;
118      eARG1' = a1;
119      eARG2' = a2;
120      eDEST' = d0;
121      eWRT' = w0;
122      wVAL' = x0;
123      wDEST' = d1;
124      wWRT' = w1;
125    }
126    else {
127      // updates to PC and RF
128      if ((!flush) && (!stall())) { pPC' = common.newPC(pPC); }
129      call update_pRF();
130      // Execute stage
131      eOP' = common.op(pPC);
132      eSRC2' = common.src2(pPC);
133      eARG1' = pRF'[common.src1(pPC)];
134      eARG2' = pRF'[common.src2(pPC)];
135      eDEST' = common.dest(pPC);
136      eWRT' = (!stall()) && (!flush);
137      // Writeback stage
138      wDEST' = eDEST;
139      wWRT' = eWRT;
140      if (wWRT && (wDEST == eSRC2)) // fwding logic
141      {
142        wVAL' = common.alu(eOP, eARG1, wVAL);
143      }
144      else {
145        wVAL' = common.alu(eOP, eARG1, eARG2);
146      }
147    }
148  }
149 }
150
```

4. Verification Techniques

```
151 // Main module
152 module main {
153
154     var flush_pipeline : boolean;
155     var reinit : boolean;
156     var project_impl : boolean;
157
158     var step : integer;
159
160     // Variables to store impl state
161     var I_pc : common.word_t;
162     var I_rf : common.rf_t;
163     // Variables to store spec state
164     var S_pc0 : common.word_t; // after 0 steps
165     var S_rf0 : common.rf_t;
166     var S_pc1 : common.word_t; // after 1 step
167     var S_rf1 : common.rf_t;
168
169     // instantiate spec and impl modules
170     instance impl_i : impl(flush : (flush_pipeline), reinit : (reinit));
171     instance spec_i : spec(proj_impl : (project_impl), impl_RF : (impl_i.pRF),
172         impl_PC : (impl_i.pPC));
173
174     init {
175         step = 0;
176         flush_pipeline = false;
177         project_impl = false;
178         reinit = false;
179     }
180
181     next {
182         step' = step + 1;
183         case
184             (step == 0) : {
185                 flush_pipeline' = true;
186                 next(impl_i); // step impl module, normal step
187             }
188             (step == 1) : {
189                 flush_pipeline' = true;
190                 next(impl_i); // step impl module, flush
191             }
192             (step == 2) : {
193                 flush_pipeline' = false;
194                 reinit' = true;
195                 next(impl_i); // step impl module, flush
196             }
197             (step == 3) : {
198                 flush_pipeline' = true;
199                 reinit' = false;
200                 I_pc' = impl_i.pPC; // store impl state after pipeline flushed for 2
201                 steps
202                 I_rf' = impl_i.pRF;
203                 next(impl_i); // step impl module, reinitialize
204             }
205         }
206     }
```

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```
203     (step == 4) : {
204         flush_pipeline' = true;
205         next(impl_i); // step impl module, flush
206     }
207     (step == 5) : {
208         flush_pipeline' = false;
209         project_impl' = true;
210         next(impl_i); // step impl module, flush
211     }
212     (step == 6) : {
213         project_impl' = false;
214         next(spec_i); // step spec to project impl state onto spec
215     }
216     (step == 7) : { // step spec module
217         S_pc0' = spec_i.sPC; // initial state of spec
218         S_rf0' = spec_i.sRF;
219         next(spec_i); // step
220     }
221     (step == 8) : { // store spec state after one step
222         S_pc1' = spec_i.sPC;
223         S_rf1' = spec_i.sRF;
224     }
225     (step == 9) : {
226         // assert(false); // sanity check to make sure execution can get
here
227         // correspondence check
228         assert(((S_pc1 == I_pc) && (S_rf1 == I_rf)) || ((S_pc0 == I_pc) && (
S_rf0 == I_rf)));
229     }
230     esac;
231 }
232
233
234
235 control {
236     vobj = unroll(10);
237     check;
238     print_results;
239     vobj.print_cex(step, flush_pipeline, reinit, project_impl, I_pc, I_rf,
S_pc0, S_rf0, S_pc1, S_rf1, impl_i.pPC, impl_i.pRF, spec_i.sPC, spec_i.
sRF, impl_i.wVAL);
240 }
241
242
243 }
```

Running UCLID5 on the file given above produces the following output.

```

$ uclid examples/simple-datapath.ucl
Successfully parsed 4 and instantiated 1 module(s).
10 assertions passed.
0 assertions failed.
0 assertions indeterminate.
Finished execution for module: main.

```

4.5. Verifying Two-Safety Properties

Certain security properties involve reasoning about a system that is composed of modules modeling multiple interacting agents, some of whom can be malicious. For confidentiality properties, the goal is to ensure that secret data is not revealed to a malicious agent. For integrity properties, the goal is that actions of a malicious agent should not be able to influence the values of certain high-integrity variables. Such properties form a special class of the general category of *hyperproperties* [?] known as *2-safety properties*. The name arises from the way in which such properties are checked, by formulating them as a safety property on a “self-composed” model of the system — a model obtained by composing two copies of the system together synchronously.

With UCLID5, we verify 2-safety properties via self-composition and inductive invariant checking. Counterexamples comprise two traces and are found by bounded model checking. The code included below shows an illustrative UCLID5 model drawn from the chapter on Security from the textbook on embedded (cyber-physical) systems by Lee and Seshia [?].

```

1  /*
2   * Based on an example from Lee & Seshia, "Introduction to Embedded Systems",
3   *   Chapter 17 on Security
4   */
5  module process1
6  {
7
8     type pc_t = enum { A };
9
10    var pc : pc_t;
11    input s : boolean; // secret input
12    input x : boolean; // public input
13    output z : boolean; // public output
14
15    init {
16        pc = A;
17        z = false;
18    }
19
20    next {
21        z' = !x;
22    }
23 }

```

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```
23
24 module process2
25 {
26   type pc_t = enum { B, C };
27
28   var pc : pc_t;
29   input s : boolean; // secret input
30   input x : boolean; // public input
31   output z : boolean; // public output
32
33   procedure next_state()
34     modifies z;
35     modifies pc;
36   {
37     case
38       (pc == B) && s && !x : { pc = C; z = true; }
39       (pc == C) && s && !x : { pc = B; z = false; }
40     esac
41   }
42
43   init {
44     pc = B;
45     z = false;
46   }
47
48   next {
49     call next_state();
50   }
51 }
52
53
54 module main
55 {
56   type t1pc_t = process1.pc_t;
57   type t2pc_t = process2.pc_t;
58
59   var x1,x2 : boolean;
60   var s1,s2 : boolean;
61   var z1,z2 : boolean;
62
63   /*
64   instance p11 : process1(z: (z1), s : (s1), x : (x1));
65   instance p12 : process1(z: (z2), s : (s2), x : (x2));
66   */
67
68   /*
69   */
70   instance p21 : process2(z: (z1), s : (s1), x : (x1));
71   instance p22 : process2(z: (z2), s : (s2), x : (x2));
72
73   init {
74     havoc x1; havoc x2;
75     havoc s1; havoc s2;
76     assume(x1 == x2);
```

```

77   }
78
79   next {
80     havoc x1; havoc x2;
81     havoc s1; havoc s2;
82     assume(x1' == x2');
83     // next(p11); next(p12);
84     next(p21); next(p22);
85   }
86
87   invariant same_output_z : (z1 == z2);
88
89
90   control {
91     // *** BASIC ASSERTIONS/INVARIANT
92     /*
93     */
94     v = unroll(5);
95     check;
96     print_results;
97     //v.print_cex(x1,x2,s1,s2,z1,z2,p11.pc,p12.pc);
98     v.print_cex(x1,x2,s1,s2,z1,z2,p21.pc,p22.pc);
99
100    /*
101    // *** INDUCTION
102    v = induction(1);
103    check;
104    print_results;
105    v.print_cex(x1,x2,s1,s2,z1,z2,p11.pc,p12.pc);
106    */
107   }
108 }

```

4.6. Synthesis

UCLID5 has full support for Syntax-Guided Synthesis across all verification modes. If a synthesis function is inserted into the UCLID5 model, and the control block contains a verification command as per usual, UCLID5 will construct a synthesis query that searches for a body to any synthesis functions in the model.

The example in Figure ?? uses synthesis to strengthen the invariant for the Fibonacci example we have seen before. When run with the command line option `-y "cvc4 --lang sygus"` (or another SyGuS solver), UCLID5 will return the correct body for the synthesis function so that the verification condition passes.

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```
1 // Expected Output: h(x, y) := x >= 0
2 // Note: must run with a synthesis engine backend, like cvc4
3 // Note: format of output depends on synthesis engine backend
4
5 module main {
6   // Part 1: System Description.
7   var a, b : integer;
8   init {
9     a, b = 0, 1;
10  }
11  next {
12    a', b' = b, a + b;
13  }
14
15  // Part 2: System Specification.
16  invariant a_le_b: a <= b;
17
18  // Part 3: (NEW) Synthesis Integration
19  synthesis function
20    h(x : integer, y : integer): boolean;
21  invariant hole: h(a, b);
22
23  // Part 4: Proof Script.
24  control {
25    induction;
26    check;
27    print_results;
28  }
29 }
```

Example 4.6.: UCLID5 Fibonacci model using synthesis function

A. Appendix: Uclid5 Grammar

This appendix describes UCLID5's grammar.

A.1. Grammar of Modules and Declarations

A model consist of a list of modules. Each module consists of a list of declarations followed by an optional control block.

$\langle Model \rangle ::= \langle Module \rangle^*$

$\langle Module \rangle ::= \text{module } \langle Id \rangle \text{ '}' \langle Decl \rangle^* \langle ControlBlock \rangle? \text{ '}'$

Declarations can be of the following types.

$\langle Decl \rangle ::= \langle TypeDecl \rangle$
| $\langle InputsDecl \rangle$
| $\langle OutputsDecl \rangle$
| $\langle VarsDecl \rangle$
| $\langle SharedVarsDecl \rangle$
| $\langle DefineDecl \rangle$
| $\langle ConstsDecl \rangle$
| $\langle ConstLitDecl \rangle$
| $\langle FuncDecl \rangle$
| $\langle ProcedureDecl \rangle$
| $\langle InstanceDecl \rangle$
| $\langle InitDecl \rangle$
| $\langle NextDecl \rangle$
| $\langle AxiomDecl \rangle$
| $\langle SpecDecl \rangle$

Type declarations declare either a type synonym or an uninterpreted type.

$\langle TypeDecl \rangle ::= \text{type } \langle Id \rangle \text{ '=' } \langle Type \rangle \text{ ';'}$
| $\text{type } \langle Id \rangle \text{ ';'}$

Variable declarations can refer to inputs, outputs, state variables, symbolic constants, named constant literals, shared variables, group declarations, or define declarations.

$\langle InputsDecl \rangle ::= \text{input } \langle IdList \rangle \text{ ':' } \langle Type \rangle \text{ ';'}$

$\langle OutputsDecl \rangle ::= \text{output } \langle IdList \rangle \text{ ':' } \langle Type \rangle \text{ ';'}$

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$\langle \text{VarsDecl} \rangle$	$::= \text{var } \langle \text{IdList} \rangle \text{ ':' } \langle \text{Type} \rangle \text{ ';'}$
$\langle \text{ConstsDecl} \rangle$	$::= \text{const } \langle \text{IdList} \rangle \text{ ':' } \langle \text{Type} \rangle \text{ ';'}$
$\langle \text{ConstLitDecl} \rangle$	$::= \text{const } \langle \text{Id} \rangle \text{ ':' } \langle \text{Type} \rangle = \langle \text{Number} \rangle \text{ ';'}$
$\langle \text{SharedVarsDecl} \rangle$	$::= \text{sharedvar } \langle \text{IdList} \rangle \text{ ':' } \langle \text{Type} \rangle \text{ ';'}$
$\langle \text{DefineDecl} \rangle$	$::= \text{define } \langle \text{Id} \rangle \text{ ' (' } \langle \text{IdTypeList} \rangle \text{ ') ' ':' } \langle \text{Type} \rangle = \langle \text{Expr} \rangle \text{ ';'}$
$\langle \text{GroupDecl} \rangle$	$::= \text{group } \langle \text{Id} \rangle \text{ ':' } \langle \text{Type} \rangle = \langle \text{IdList} \rangle \text{ ';'}$

Function declarations refer to uninterpreted functions, synthesis functions or oracle functions.

$\langle \text{FuncDecl} \rangle$	$::= \text{function } \langle \text{Id} \rangle \text{ ' (' } \langle \text{IdTypeList} \rangle \text{ ') ' ':' } \langle \text{Type} \rangle \text{ ';'}$
$\langle \text{SynthFuncDecl} \rangle$	$::= \text{synthesis function } \langle \text{Id} \rangle \text{ ' (' } \langle \text{IdTypeList} \rangle \text{ ') ' ':' } \langle \text{Type} \rangle \text{ ';'}$
$\langle \text{OracleFuncDecl} \rangle$	$::= \text{oracle function } \langle \text{Id} \rangle \langle \text{AnnotationList} \rangle \text{ ' (' } \langle \text{IdTypeList} \rangle \text{ ') ' ':' } \langle \text{Type} \rangle \text{ ';'}$

Procedure declarations consist of a formal parameter list, a list of return values and types, followed by optional pre-/post-conditions and the list of state variables modified by procedure.

$\langle \text{ProcedureDecl} \rangle$	$::= \text{procedure } \langle \text{Id} \rangle (\langle \text{AnnotationList} \rangle)^* \text{ ' (' } \langle \text{IdTypeList} \rangle \text{ ') '}$ $\quad \langle \text{ProcReturnArg} \rangle ?$ $\quad \langle \text{RequireExprs} \rangle \langle \text{EnsureExprs} \rangle \langle \text{ModifiesExprs} \rangle$ $\quad \langle \text{BlkStmt} \rangle$
$\langle \text{ProcReturnArg} \rangle$	$::= \text{returns ' (' } \langle \text{IdTypeList} \rangle \text{ ') '}$
$\langle \text{RequireExprs} \rangle$	$::= (\text{requires } \langle \text{Expr} \rangle \text{ ';' })^*$
$\langle \text{EnsureExprs} \rangle$	$::= (\text{ensures } \langle \text{Expr} \rangle \text{ ';' })^*$
$\langle \text{ModifiesExprs} \rangle$	$::= (\text{modifies } \langle \text{IdList} \rangle \text{ ';' })^*$

Instance declarations allow the instantiation (duh!) of other modules. They consist of the instance name, the name of the module being instantiated and the list of mappings for the instances' inputs, output and shared variables.

$\langle \text{InstanceDecl} \rangle$	$::= \text{instance } \langle \text{Id} \rangle \text{ ':' } \langle \text{Id} \rangle \langle \text{ArgMapList} \rangle \text{ ';'}$
$\langle \text{ArgMapList} \rangle$	$::= \text{' (' ' '}$ $\quad \text{' (' } \langle \text{ArgMap} \rangle \text{ ' , ' } \langle \text{ArgMapList} \rangle \text{ ') '}$

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$\langle ArgMap \rangle$::= $\langle Id \rangle ' : ' (' ') '$
| $\langle Id \rangle ' : ' (' \langle Expr \rangle ') '$

Axioms refer to assumptions while a **specification declaration** refers to design **invariants**. Note **axiom** and **assume** are synonyms, as are **property** and **invariant**.

$\langle AxiomDecl \rangle$::= $\langle AxiomKW \rangle \langle Id \rangle ' : ' \langle Expr \rangle ' ; '$
| $\langle AxiomKW \rangle \langle Expr \rangle ' ; '$

$\langle AxiomKW \rangle$::= **axiom** | **assume**

$\langle SpecDecl \rangle$::= $\langle PropertyKW \rangle \langle Id \rangle ' : ' \langle Expr \rangle ' ; '$
| $\langle PropertyKW \rangle \langle Expr \rangle ' ; '$

$\langle PropertyKW \rangle$::= **property** | **invariant**

Init and **next** blocks consist of lists of statements.

$\langle InitDecl \rangle$::= **init** $\langle BlkStmt \rangle$

$\langle NextDecl \rangle$::= **next** $\langle BlkStmt \rangle$

Assignment statements in the **next** block declaration must assign primed variables only, and are concurrently evaluated.

A.2. Statement Grammar

Statements are the following types, most of which should be familiar. Note the support for simultaneous assignment à la Python. The keyword **next** allows for synchronous scheduling of instantiated modules.

$\langle Statement \rangle$::= **skip** $' ; '$
| **assert** $\langle Expr \rangle ' ; '$
| **assume** $\langle Expr \rangle ' ; '$
| **havoc** $\langle Id \rangle ' ; '$
| $\langle LhsList \rangle ' = ' \langle ExprList \rangle ' ; '$
| **call** $' (' \langle LhsList \rangle ') ' = ' \langle Id \rangle \langle ExprList \rangle ' ; '$
| **call** $\langle Id \rangle ' (' \langle ExprList \rangle ') ' ; '$
| **next** $' (' \langle Id \rangle ') ' ; '$
| $\langle IfStmt \rangle$
| $\langle CaseStmt \rangle$
| $\langle ForLoop \rangle$
| $\langle WhileLoop \rangle$
| $\langle BlkStmt \rangle$

Block statements are a list of variables with local scope, and a list of statements.

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$\langle \text{BlkStmt} \rangle ::= \{ ' \langle \text{BlockVarDecl} \rangle^* \langle \text{Statement} \rangle^* \}$

$\langle \text{BlockVarDecl} \rangle ::= \text{var } \langle \text{IdList} \rangle \text{ : } \langle \text{Type} \rangle \text{ ;}$

Assignments and **call** statements refer to the nonterminal $\langle \text{LhsList} \rangle$. As the name suggests, this is a list of syntactic forms that can appear on the left hand side of an assignment. $\langle \text{Lhs} \rangle$ are of four types: (i) identifiers, bitvector slices within identifiers, (iii) array indices, and (iv) fields within records.

$\langle \text{LhsList} \rangle ::= \langle \text{Lhs} \rangle (, \langle \text{Lhs} \rangle)^*$

$\langle \text{Lhs} \rangle ::= \langle \text{Id} \rangle$
 $\quad | \langle \text{Id} \rangle [\text{''}]$
 $\quad | \langle \text{Id} \rangle [\langle \text{Expr} \rangle \text{ : } \langle \text{Expr} \rangle]$
 $\quad | \langle \text{Id} \rangle [\langle \text{ExprList} \rangle]$
 $\quad | \langle \text{Id} \rangle (. \langle \text{Id} \rangle) +$

If statements are as per usual. “Braceless” if statements are not permitted.

$\langle \text{IfStmt} \rangle ::= \text{if } (\langle \text{CondExpr} \rangle) \langle \text{BlkStmt} \rangle$
 $\quad \text{else } \langle \text{BlkStmt} \rangle$
 $\quad | \text{if } (\langle \text{CondExpr} \rangle) \langle \text{BlkStmt} \rangle$

$\langle \text{IfExpr} \rangle ::= \langle \text{Expr} \rangle \mid *$

Case statements are as follows.

$\langle \text{CaseStmt} \rangle ::= \text{case } \langle \text{CaseBlock} \rangle^* \text{ esac}$
 $\langle \text{CaseBlock} \rangle ::= \langle \text{Expr} \rangle \text{ : } \langle \text{BlkStmt} \rangle$
 $\quad | \text{default } \text{ : } \langle \text{BlkStmt} \rangle$

For loops allow iteration over a statically defined range of values.

$\langle \text{ForLoop} \rangle ::= \text{for } \langle \text{Id} \rangle \text{ in range } (\langle \text{Number} \rangle , \langle \text{Number} \rangle) \langle \text{BlkStmt} \rangle$

While loops allow unbounded iteration.

$\langle \text{WhileLoop} \rangle ::= \text{while } (\langle \text{CondExpr} \rangle)$
 $\quad \langle \text{InvariantClause} \rangle^*$
 $\quad \langle \text{BlkStmt} \rangle$

$\langle \text{InvariantClause} \rangle ::= \text{invariant } \langle \text{Expr} \rangle \text{ ;}$

A.3. Expression Grammar

Let us turn to **expressions**, which may be quantified (include finite forall quantifiers, which expand to a disjunction and quantify over a finite group).

$$\begin{aligned}
\langle Expr \rangle &::= \langle E1 \rangle \\
\langle E1 \rangle &::= \langle E2 \rangle \\
&| \text{'('} \text{forall} \text{'('} \langle IdTypeList \rangle \text{'')' ':' E1 '('} \\
&| \text{'('} \text{exists} \text{'('} \langle IdTypeList \rangle \text{'')' ':' E1 '('} \\
&| \text{'('} \text{finite_forall} \text{'('} \langle GroupDecl \rangle \text{'')' ':' E1 '('}
\end{aligned}$$

The usual logical and bitwise operators are allowed.

$$\begin{aligned}
\langle E2 \rangle &::= \langle E3 \rangle \text{'<==>'} \langle E2 \rangle | \langle E3 \rangle \\
\langle E3 \rangle &::= \langle E4 \rangle \text{'==>'} \langle E3 \rangle | \langle E4 \rangle \\
\langle E4 \rangle &::= \langle E5 \rangle \text{'\&\&'} \langle E4 \rangle | \langle E5 \rangle \text{'||'} \langle E4 \rangle | \\
&| \langle E5 \rangle \text{'\&'} \langle E4 \rangle | \langle E5 \rangle \text{'|'} \langle E4 \rangle | \langle E5 \rangle \text{'^'} \langle E4 \rangle \\
&| \langle E5 \rangle
\end{aligned}$$

As are relational operators, bitvector concatenation (++) and arithmetic.

$$\begin{aligned}
\langle E5 \rangle &::= \langle E6 \rangle \langle RelOp \rangle \langle E6 \rangle | \langle E6 \rangle \\
\langle RelOp \rangle &::= \text{'>'} | \text{'<'} | \text{'='} | \text{'!='} | \text{'>='} | \text{'<='} \\
\langle E6 \rangle &::= \langle E7 \rangle \text{'++'} \langle E6 \rangle | \langle E7 \rangle \\
\langle E7 \rangle &::= \langle E8 \rangle \text{'+'} \langle E7 \rangle | \langle E8 \rangle \\
\langle E8 \rangle &::= \langle E8 \rangle \text{'-'} \langle E9 \rangle | \langle E9 \rangle \\
\langle E9 \rangle &::= \langle E9 \rangle \text{'*'} \langle E10 \rangle | \langle E9 \rangle \text{'/'} \langle E10 \rangle | \langle E10 \rangle
\end{aligned}$$

The unary operators are arithmetic negation (unary minus), logical negation and bitwise negation of bitvectors.

$$\begin{aligned}
\langle E10 \rangle &::= \langle UnOp \rangle \langle E11 \rangle | \langle E11 \rangle \\
\langle UnOp \rangle &::= \text{'-'} | \text{'!'} | \text{'\sim'}
\end{aligned}$$

Array select, update and bitvector select operators are defined as follows.

$$\begin{aligned}
\langle E11 \rangle &::= \langle E12 \rangle \text{'['} \langle Expr \rangle \text{'('} \text{' , ' } \langle Expr \rangle \text{')* '['} \\
&| \langle E12 \rangle \text{'['} \langle Expr \rangle \text{'('} \text{' , ' } \langle Expr \rangle \text{')* '->' } \langle Expr \rangle \text{' '['} \\
&| \langle E12 \rangle \text{'['} \langle Expr \rangle \text{' ':' } \langle Expr \rangle \text{' '['} \\
&| \langle E12 \rangle
\end{aligned}$$

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Function invocation, record selection, and access to variables in instantiated modules is as follows.

$$\begin{aligned} \langle E12 \rangle & ::= \langle E13 \rangle ' (' \langle ExprList \rangle ') ' \\ & \quad | \langle E13 \rangle ' ('.' \langle Id \rangle) + \end{aligned}$$

And finally, we have the terminal symbols, identifiers, record field access, tuples and the if-then-else operator.

$$\begin{aligned} \langle E12 \rangle & ::= \text{false} | \text{true} | \langle Number \rangle | \langle String \rangle \\ & \quad | \langle Id \rangle | \langle Id \rangle ' .' \langle Id \rangle \\ & \quad | ' (' \langle Expr \rangle ' (',' \langle Expr \rangle) * ') ' \\ & \quad | \text{if} ' (' \langle Expr \rangle ' ') ' \text{then} \langle Expr \rangle \text{else} \langle Expr \rangle \end{aligned}$$

Strings can only be used as arguments to the `print` command in the control block.

A.4. Types

$$\begin{aligned} \langle Type \rangle & ::= \langle PrimitiveType \rangle \\ & \quad | \langle EnumType \rangle \\ & \quad | \langle TupleType \rangle | \langle RecordType \rangle \\ & \quad | \langle ArrayType \rangle \\ & \quad | \langle SynonymType \rangle \\ & \quad | \langle ExternalType \rangle \end{aligned}$$

Supported primitive types are Booleans, integers, single precision floats and bit-vectors. Bit-vector types are defined according the regular expression `'bv[0-9]+'` and the number following `'bv'` is the width of the bit-vector.

$$\langle PrimitiveType \rangle ::= \text{boolean} | \text{integer} | \text{float} | \langle BitVectorType \rangle$$

Enumerated types are defined using the `enum` keyword.

$$\langle EnumType \rangle ::= \text{enum} ' \{ ' \langle IdList \rangle ' \} '$$

Tuple types are declared using curly brace notation.

$$\langle TupleType \rangle ::= ' \{ ' \langle Type \rangle ' (',' \langle Type \rangle) * ' \} '$$

Record types use the keyword `record`.

$$\langle Recordtype \rangle ::= \text{record} ' \{ ' \langle IdTypeList \rangle ' \} '$$

Array types are defined using square brackets. The list of types within square brackets defined the array's index type.

$$\langle ArrayType \rangle ::= ' [' \langle Type \rangle ' (',' \langle Type \rangle) * '] ' \langle Type \rangle$$

Type synonyms are just identifiers, while external types refer to synonym types defined in a different module.

$$\langle SynonymType \rangle ::= \langle Id \rangle$$

$$\langle ExternalType \rangle ::= \langle Id \rangle ' .' \langle Id \rangle$$

A.5. Control Block

The **control block** consists of a list of commands. A command can have an optional result object, an optional argument object, an optional list of command parameters and finally an optional list of argument expressions.

$$\begin{aligned}\langle \textit{ControlBlock} \rangle &::= \textit{control} \ \{ \langle \textit{Cmd} \rangle^* \} \\ \langle \textit{Cmd} \rangle &::= (\langle \textit{Id} \rangle \text{'='})? (\langle \textit{Id} \rangle \text{'.'})? \langle \textit{CmdName} \rangle \\ &\quad (\text{'['} \langle \textit{IdList} \rangle \text{'}]')? \langle \textit{ExprList} \rangle? \text{';'}\end{aligned}$$

The following is a list of currently accepted commands.

$$\begin{aligned}\langle \textit{CmdName} \rangle &::= \text{'bmc'} \\ &\quad | \text{'bmc_LTL'} \\ &\quad | \text{'bmc_noLTL'} \\ &\quad | \text{'check'} \\ &\quad | \text{'clear_context'} \\ &\quad | \text{'induction'} \\ &\quad | \text{'print'} \\ &\quad | \text{'print_cex'} \\ &\quad | \text{'print_module'} \\ &\quad | \text{'print_results'} \\ &\quad | \text{'print_smt2'} \text{'unroll'} \\ &\quad | \text{'verify'}\end{aligned}$$

A.6. Miscellaneous Nonterminals

$\langle \textit{IdList} \rangle$, $\langle \textit{IdTypeList} \rangle$ and $\langle \textit{ExprList} \rangle$ are non-empty, comma-separated list of identifiers, identifier/type tuples and expressions respectively. $\langle \textit{AnnotationList} \rangle$ is a non-empty comma-separated list of strings, used as annotations to indicate, for example, whether a procedure should be inline or not.

$$\begin{aligned}\langle \textit{IdList} \rangle &::= \langle \textit{Id} \rangle \\ &\quad | \langle \textit{Id} \rangle \text{' , ' } \langle \textit{IdList} \rangle \\ \langle \textit{IdTypeList} \rangle &::= \langle \textit{Id} \rangle (\text{' , ' } \langle \textit{Id} \rangle)^* \text{' : ' } \langle \textit{Type} \rangle \\ &\quad | \langle \textit{Id} \rangle (\text{' , ' } \langle \textit{Id} \rangle)^* \text{' : ' } \langle \textit{Type} \rangle \text{' , ' } \langle \textit{IdTypeList} \rangle \\ \langle \textit{ExprList} \rangle &::= \langle \textit{Expr} \rangle \\ &\quad | \langle \textit{Expr} \rangle \text{' , ' } \langle \textit{ExprList} \rangle \\ \langle \textit{AnnotationList} \rangle &::= \text{' [' } \langle \textit{String} \rangle (\text{' , ' } \langle \textit{String} \rangle)^* \text{'] '}\end{aligned}$$