ECON0106: Microeconomics Take-home Exam

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Question 1

(i)

If σ_i admits an APU representation then we know that for a utility function $u_i : A_i \rightarrow \mathbb{R}$ and a cost function $c_i : [0,1] \rightarrow \mathbb{R} \cup \{\infty\}$ such that for every $S_i \in \mathscr{S}_i$:

$$\sigma_i(.|S_i) = arg \max_{p_i \in \Delta(S_i)} \sum_{s_i \in S_i} p_i(s_i)u_i(s_i) - c_i(p_i(s_i))$$

The first-order condition with respect to $p_i(s_i)$ is satisfied if and only if:

$$u_i(s_i) + \lambda(S_i) = c_i'(p_i(s_i|S_i))$$

Where $\lambda(S_i)$ is the Lagrangian multiple given the set of menus S_i The first order condition implies for each $s_i, s_i' \in S_i \cap S_i'$,

$$c_i'(p_i(s_i|S_i)) - c_i'(p_i(s_i'|S_i)) = c_i'(p_i(s_i|S_i')) - c_i'(p_i(s_i'|S_i'))$$

Given this, a strictly increasing function $\phi : (0,1) \rightarrow \mathbb{R}_+$ by $\phi(x) := \exp(c'(x))$. Which means it satisfies Ordinal IIA.

(ii)

A best response correspondence for player i, $b_i : S_{-i} \rightrightarrows S_i$ is defined as

Question 3

For any arbitrary value of $c \in [0,1]$ we know the seller will accept price p from the buyer if and only if $p \geq \alpha + (1-\alpha)c$, where $alpha > 0$ is a constant.
As the buyer also observes the probability distribution of c then it believes that the lowest possible reservation value the seller could have is λ where $\lambda \in [0,1]$ and offers the price $p(\lambda) = \beta + (1-\beta)\lambda$, where $1 > \beta > \alpha$.

If the buyer's offer is rejected in period 1, then the buyer will update his or her belief about the c and updates λ_t. Based on Bayes' rule and the seller's strategy that if $\lambda_2 = 0$ if $p_1 < \alpha$ and $\lambda_2 = c^*(p_1)$ if $\alpha < p_1 < 1$ where:

$$c * (p) = \frac{p - \alpha}{1 - \alpha}$$

And the buyer will know that:

$$F_2(c) = \frac{c - \lambda_1}{1 - \lambda_1}$$

So the buyer's equilibrium posterior belief at the beginning of period $t = 2$ can be defined by a number based on what was offered in the previous period.

So at time 1, the equilibrium price offered is $p_1 = \beta$ which the seller accepts if $c^\beta \geq c$. So at period 2, the equilibrium price offer will be $p_2 = \beta + (1 - \beta)c^*(\beta)$ as $c^*(\beta)$ is the lowest possible reservation price that the buyer believes the seller will accept.
The seller will only accept p_1 if $c^*(p_1) \geq c\, c^*(\beta) > 0$. From this we know that $p_0 - c^*(\beta) = \delta[p_1 - c^*(\beta)]$, which given how we defined p_1 and p_0 means $\alpha = \delta\beta$.

At period t=1, the buyers' payoff given price p is:

$$W_B(p, \lambda) = [\frac{c^*(p) - \lambda}{1 - \lambda}](1 - p) + [\frac{1 - c^*(p)}{1 - \lambda}]\delta U_B(c^*(p))$$

Where $U_B(c^*(p)$ is the buyer's payoff at the beginning of $t = 2$ when the lowest possible reservation value the buyer believes the sekker could have is λ.
As for any $\lambda < 1$ and $p \notin \alpha + (1 - \alpha), 1$, $W_B(p, \lambda) \leq 0$ then $\forall \lambda < 1$:

$$U_B(\lambda) = \max_{1>p>\alpha+(1-\alpha)\lambda} [\frac{c^*(p) - \lambda}{1 - \lambda}](1 - p) + [\frac{1 - c^*(p)}{1 - \lambda}]\delta U_B(c^*(p))$$

For simplification let $V_B(\lambda) = (1 - \lambda)U_B(\lambda)$ so:

$$V_B(\lambda) = \max_{1>p>\alpha+(1-\alpha)\lambda} [c^*(p) - \lambda](1 - p)\delta U_B(c^*(p))$$

The first order condition is:

$$\frac{1 - p^*}{1 - \alpha} - \frac{p^* - \alpha}{1 - \alpha} + \lambda + \frac{\delta}{1 - \alpha} V'_B(\frac{p^* - \alpha}{1 - \alpha}) = 0$$

So $p(\lambda)$ only satisfies the first order condition if $p(\lambda) = p^*$ which if substituted into the equation above results in:

$$(1 - \lambda)(1 - 2\beta + \alpha) + \delta V"_B(\frac{\beta + (1 - \beta)\lambda - \alpha}{1 - \alpha}) = 0$$

Now the Envelope theorem implies that, $V_B'(\lambda) = -(1-p^*)$. This combined with $p^* = p(\lambda)$ then $V_B'(\lambda) = -(1-\beta)(1-\lambda)$. So:

$$V_B'(\frac{\beta + (1-\beta)\lambda - \alpha}{1-\alpha}) = -\frac{(1-\beta)^2(1-\lambda)}{1-\alpha}$$

Using this, we then know that:

$$(1-\alpha)(1-2\beta+\alpha) - \delta(1-\beta)^2 = 0$$

Using the fact that $\alpha = \beta\delta$, this equation can be solved and the solutions are $\alpha = \sqrt{1-\delta}$ and $\beta = \frac{1-\sqrt{1-\delta}}{\delta}$. So there exists a unique stationary and linear weak perfect Bayesian equilibrium given the values. Given this we know that $p_1 = 1 + (1-\beta)\beta$ and $p_0 = \beta$. Given the value we found for β under a wPBE we know that $p_1 \geq p_0$ where $\delta \in [0,1]$.