ECON0106: **Microeconomics**

Problem Set 2

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Question 1. 2. Structural Properties of Preferences and Utility Representations: Exercise 1

Question 2. 2. Structural Properties of Preferences and Utility Representations: Exercises 3 and 4

Question 3. There is an intuitive notion of what it means for an object x to be more similar to y than to z. This is an important notion to think about, e.g., whether choosing between more similar alternatives is easier or harder. Let us use what we've learn to model such an intuition.

For simplicity, let X be the unit interval, $[0,1]$. Let's define a binary relation S on X and consider the following properties:

Property 1: For all $x \in X$, xSx.

Property 2: For all $x, y \in X$, if xSy, then ySx.

Property 3: The graph of the relation S in $X \times X$ is a closed set (continuity).

Property 4: If $z \geq x \geq y \geq w$ and zSw, then xSy (betweenness).

Property 5: For any $x \in X$, there is an open interval around x such that xSy for all y in the interval.

Property 6: Let $M(x) := \max\{y | ySx\}$ and $m(x) := \min\{y | ySx\}$. Then M and m are nondecreasing functions, and strictly increasing whenever they don't have the values 0 or 1.

Question 3.(i) Do these properties capture your intuition about the concept of "approximately the same"?

Question 3.(ii) For $\epsilon > 0$, show that S_ϵ, defined by $xS_\epsilon y$ if $|x - y| \leq \epsilon$ satisfies all the the properties.

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Question 3.(iii) Now suppose S satisfy all the properties above and let $\epsilon > 0$. Show there is a strictly increasing and continuous function $H : X \to \mathbb{R}$ such that xSy if and only if $|H(x) - H(y)| \leq \epsilon$. **This part is hard.** Let's split the questions into multiple subquestions to guide you through the construction.

(iii).1 Let $\{x_n\}$ be a sequence such that $x_{n+1} = m(x_n)$ and $x_0 = 1$. Prove that this sequence is well-defined and there exists a natural number N such that $\forall n \geq N$, $x_n = 0$.

(iii).2 Given a real number $\lambda > 0$, construct a strictly increasing function G on $[0,1]$ such that $\frac{1}{\lambda} \leq \frac{G(x)}{G(y)} \leq \lambda$ iff xSy. (*Hint: You may want to use the sequence in the previous question and construct the function recursively. You could first define G on* $[x_1, x_0]$, *then extend it to the rest of* $[0,1]$.)

(iii).3 Given a real number $\epsilon > 0$. Construct a fuction H such that $|H(x) - H(y)| < \epsilon$ iff xSy and show that H is well-defined and strictly increasing.

(iii).4 Prove that $M(x)$ is continuous on the interval $[0, m(1)]$. (*Hint: if a function f is bijective and strictly increasing, f is continuous.*)

(iii).5 Prove that H is continuous on $[0,1]$.