1 Classical Mechanics

- velocity of point in rotating object: $\boldsymbol{v} = \boldsymbol{v}_{com} + \boldsymbol{v}_{rot}$
- Power $P = IA = \frac{\mathrm{d}W}{\mathrm{d}t} = \mathbf{F} \cdot \mathbf{v}$ (for constant force)
- (flux of X) = X / (area × time)
- torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \dot{\mathbf{L}}$

$$\mathrm{d}W = \tau \, \mathrm{d}\theta \,, \qquad P = \boldsymbol{\tau} \cdot \boldsymbol{\omega} \tag{1}$$

- use energy cons: pot energy lost = kin energy gained
- but also: rate of E_p lost = rate of E_k gained
- constant acc: $v^2 = v_0^2 + 2a(x - x_0)$
- small oscillations about x_{eq}:

$$k_{\text{eff}} = \left.\frac{\mathrm{d}^2 U}{\mathrm{d}x^2}\right|_{x_{eq}} , \qquad \omega = \sqrt{\frac{k_{\text{eff}}}{m}} \tag{2}$$

2 Special Relativity

- four-vectors: $P = (\frac{E}{c}, \mathbf{p})$, $\quad P_A \cdot P_B = E_A E_B / c^2 - \mathbf{p}_A \cdot \mathbf{p}_B$

3 Electromagnetism

- Poynting vector $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$: EM energy flux = intensity
- Cyclotron: $F_L = qvB = mv^2/r = F_{ce}$
- conductor: $\mathbf{E} = \mathbf{0}$ inside, else dielectric / insulator
- self-inductance L: $\mathcal{E}_{\text{ind}} = -L \, \mathrm{d}I/\mathrm{d}t$
- force on wire: $F = BIl$

4 Optics

- refractive index η: wavevector $k \to k\eta$
- optical path length OPL $= \eta l$
- Phase shift: $\Delta\phi = k\Delta x = \omega \Delta t$
- magnification: $M = -\, v/u$ (deriv: ray through O)

5 Atomic Physics

General:

- Helium: 2 protons, 2 neutrons, $M = 4m_p$
- a molecule like $^{12}C^{16}O$ has $M = (12 + 16)m_p$
- Doppler broadening, Gaussian spread in v due to T

Selection Rules (electric dipole):

1. $\Delta n =$ anything
2. $\Delta l = \pm 1$
3. $\Delta m_l = \underbrace{\pm 1}_{\sigma}, \underbrace{0}_{\pi}$

6 Nuclear and Particle Physics

- stellar nucleosynth.: PP I cycle: $p + p \to d + e^+ + \nu_e$
  - does not need neutrons
  - slow: weak interaction & Coulomb barrier tunneling
- pions $\pi^{0,\pm}$ are mesons (e.g. $\pi^- = d\bar{u}$) $\Rightarrow$ bosons
- proton (uud) / neutron (udd) size: $\sim 1fm = 10^{-15}$m,
- branching fraction: $\mathcal{BR} = \Gamma_i / \Gamma = \lambda_i / \lambda$
- half-life: $\tau_{1/2} = \tau \ln 2$, $N(t) = N_0 e^{-t/\tau}$
- mean free path: $l = (\sigma n)^{-1}$, $N(x) = N_0 e^{-\sigma n x} = N_0 e^{-x/l}$
- solid angle element: $\mathrm{d}\Omega = \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$
- centre of mass energy:

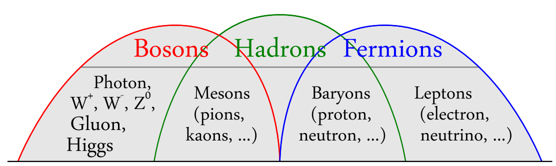
$$\sqrt{s} = E_{\text{cm}} = m_T c^2 = \sqrt{E_T^2 - p_T^2 c^2}$$

To find E_1 needed to create $\sqrt{s}$: Fixed target frame:

1. $E_T = E_1 + m_2 c^2$ and $p_T c = p_1 c = \sqrt{E_1^2 - m_1^2 c^4}$
2. insert this into $s = E_T^2 - p_T^2 c^2$

Centre-of-mass frame:

1. energies add: $\sqrt{s} = E_1 + E_2$
2. momenta are equal: $E_1^2 - m_1^2 c^4 = E_2^2 - m_2^2 c^4$
3. rearrange (using difference of squares) and eliminate E_2



7 Structure of Matter

- equipartition thm: equilibrium $E = \frac{1}{2} k_B T$ per d.o.f.
- heat capacity: $C = mc = \frac{\mathrm{d}Q}{\mathrm{d}T}$
  - E conservation: evaporation, heating, other work:

$$Q = Pt = mc\Delta T + mL + P\Delta V + F\Delta x + \dots \tag{3}$$

- ideal gas:

$$\begin{aligned} PV &= Nk_B T = nRT \\ U &= \frac{n_d}{2} N k_B T \\ \gamma &= \frac{n_d + 2}{n_d} = \frac{C_P}{C_V} \end{aligned} \tag{4}$$

  - isothermal $\Rightarrow dU = 0$

- linear thermal expansion coefficient:

$$\alpha_L = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T}, \qquad \frac{\Delta L}{L} \approx \alpha_L \Delta T \ll 1$$

- Fourier's Law: $\boldsymbol{q} = \kappa \boldsymbol{\nabla} T$
- always use Kelvin: $X°C = (X + 273.15)K$
- STP: $T = 0°C \approx 273K, \quad P = 1\text{atm} \approx 10^5\text{Pa}$
- vibrational dof (phonons) frozen out at low T

Ideal (non-viscous) Fluids:

- Buoyancy $= \rho g V_{\text{disp}}$
- continuity: $\mathbf{v} \cdot \mathbf{A} = \text{const.}$
- use volume of flow $\mathrm{d}V = A\,\mathrm{d}x = Au\,\mathrm{d}t$
  - e.g. work done: $\mathrm{d}W = P\,\mathrm{d}V = Pu\,\mathrm{d}t\,A$
  - mass $m = \rho V = \rho u\,\mathrm{d}t\,A$
  - work, potential and kinetic energy conserved:
- Bernoulli equation (energy conservation per volume)

$$B = P + \frac{1}{2}\rho v^2 + \rho g h = \text{const.} \tag{5}$$

- surface tension $\mathrm{d}W = \gamma\,\mathrm{d}A$

8 Statistical Physics

Partition function set up:

1. identify distinct states
2. energies $E(s)$ and degeneracies $g(s)$ of distinct energies
3. partition function is then

$$Z = \sum_n g(E_n) e^{-\beta E_n} \tag{6}$$

- average energy: $\bar{E} = \sum E_n p(E_n) = -\frac{\partial \ln(Z)}{\partial \beta}$
- partition fn for indep. systems: $Z = Z_a Z_b$
  - N distinguishable systems (solids):

$$\Omega = \frac{N!}{\prod n_i!}, \qquad Z_N = Z_1^N$$

  - N indistinguishable systems (gas):

$$Z_N = \frac{Z_1^N}{N!}$$

- Free energy $F = U - TS = -k_B T \ln Z$

9 Cosmology / Astronomy Questions

- Sun power sees Earth area as circle
- luminosity L = total radiated power

$$L = jA = \sigma A T^4 \tag{7}$$

- emittance j: intensity = power per area
- Radiation Pressure $P_{rad} = \langle S \rangle / c = L/Ac$
- At Sun's surface: $T_S = 6000K$ & core $T_c \sim$ millions

10 Thermodynamics

- Clausius inequality $\mathrm{d}Q \leqslant T\mathrm{d}S$

11 Quantum Mechanics

- particle in box: $\pm p$ equally likely $\Rightarrow \Delta p = 2p$
- box has width $L \Rightarrow$ uncertainty is $\Delta x = L$

12 Solid State

- mobility μ relates drift vel. to E-field: $v_d = \mu E$
- degenerate doping: semiconductor acts like a conducting metal
- Silicon: indirect BG (no emission); GaAs: direct BG

13 Electronics

- Electric field $E = V/d$
- Ohm's law: $J = \sigma E$
  - use to find resistivity $\rho = 1/\sigma = AR/d$
  - linear in temperature: $\rho = \rho_0(1 + \alpha \Delta T)$
- power $P = IV = I^2 R$
- capacitance $C = Q/V$

14 Mathematics / Statistics

Statistics:

- Error propagation (assumption: errors uncorrelated)

$$\sigma_f^2 = \sum_i (\frac{\partial f}{\partial x_i}\sigma_i)^2 \tag{8}$$

- FWHM $= 2\sqrt{2\ln 2}\sigma$
- Binomial: mean $\mu = Np$, variance $\sigma^2 = Np(1-p)$

$$B(n; p, N) = \binom{N}{n} p^n (1-p)^{N-n} \tag{9}$$

  - independent sets: $\mu = \mu_1 + \mu_2$ and $\sigma^2 = \sigma_1^2 + \sigma_2^2$
  - don't forget number of combinations $\binom{N}{n}$

- CLT: $N \gg 1$ uncorrelated / independent trials $\Rightarrow$ Gaussian with summed mean and variance (s.d. multiplied by $\sqrt{N}$)

- standard deviation of N independent events $\sigma = \sqrt{N}$
  - always calculate mean and variance / s.dev. for one trial first
  - then multiply by N or $\sqrt{N}$ respectively

- Gaussian / Normal: H_0 : prob. of result is 1% $\implies$ reject H_0 with 99% confidence
  - three-sigma rule: $68 - 95 - 99.7$

General:

- time average of wave $\left\langle \sin^2 \omega t \right\rangle = \left\langle \cos^2 \omega t \right\rangle = \frac{1}{2}$

- Integration: don't forget "$+C$", use BC to find C

- Jacobians

$$\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \mathrm{d}r\,r\,\mathrm{d}\phi\,r\sin\theta\,\mathrm{d}\theta = \mathrm{d}\rho\,\rho\,\mathrm{d}\phi\,\mathrm{d}z \tag{10}$$

- arclength $l = R\theta$ (don't forget to convert θ into radians!)

15 Exam Technique

- bullet points in explanations (~1 point per mark)