Applied Algebra

Modeling and functions

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compiled by Sean Laverty on behalf of others
University of Central Oklahoma

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Preface

Mathematics, as we all know, is the language of science, and fluency in algebraic skills has always been necessary for anyone aspiring to disciplines based on calculus. But in the information age, increasingly sophisticated mathematical methods are used in all fields of knowledge, from archaeology to zoology. Consequently, there is a new focus on the courses before calculus. The availability of calculators and computers allows students to tackle complex problems involving real data, but requires more attention to analysis and interpretation of results. All students, not just those headed for science and engineering, should develop a mathematical viewpoint, including critical thinking, problem-solving strategies, and estimation, in addition to computational skills. *Modeling, Functions and Graphs* employs a variety of applications to motivate mathematical thinking.

Modeling. The ability to model problems or phenomena by algebraic expressions and equations is the ultimate goal of any algebra course. Through a variety of applications, we motivate students to develop the skills and techniques of algebra. Each chapter includes an interactive Investigation that gives students an opportunity to explore an openended modeling problem. These Investigations can be used in class as guided explorations or as projects for small groups. They are designed to show students how the mathematical techniques they are learning can be applied to study and understand new situations.

Functions. The fundamental concept underlying calculus and related disciplines is the notion of function, and students should acquire a good understanding of functions before they embark on their study of college-level mathematics. While the formal study of functions is usually the content of precalculus, it is not too early to begin building an intuitive understanding of functional relationships in the preceding algebra courses. These ideas are useful not only in calculus but in practically any field students may pursue. We begin working with functions in Chapter 1 and explore the different families of functions in subsequent chapters.

In all our work with functions and modeling we employ the "Rule of Four," that all problems should be considered using algebraic, numerical, graphical, and verbal methods. It is the connections between these approaches that we have endeavored to establish in this course. At this level it is crucial that students learn to write an algebraic expression from a verbal description, recognize trends in a table of data, and extract and interpret information from the graph of a function.

Graphs. No tool for conveying information about a system is more powerful than a graph. Yet many students have trouble progressing from a point-wise understanding of graphs to a more global view. By taking advantage of graphing calculators, we examine a large number of examples and study them in more detail than is possible when every graph is plotted by hand. We can consider more realistic models in which calculations by more traditional methods are difficult or impossible.

We would like to thank Roy Simpson and his colleagues at Cosumnes River College, especially Min Zeng and Phuong Le, for their careful reading of the text and superior error-spotting skills. We also thank Tom Judson and the faculty at Stephen F. Austin State University for their help designing WebWork exercises for the text.

Sean Laverty (with others, via Katherine Yoshiwara)
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1 Functions and Their Graphs

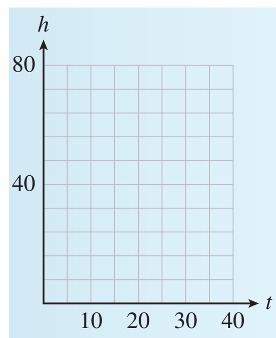
1.1 Linear Models

1.1.1 Tables, Graphs and Equations

Checkpoint 1.1.4

Frank plants a dozen corn seedlings, each 6 inches tall. With plenty of water and sunlight they will grow approximately 2 inches per day. Complete the table of values for the height, h, of the seedlings after t days.

t	0	5	10	15	20
h					



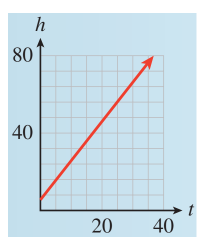
a Write an equation for the height of the seedlings in terms of the number of days since they were planted.

b Graph the equation.

Answer.

a $h = 6 + 2t$

b



Checkpoint 1.1.5 Use your equation from Checkpoint 1.1.4 to answer the questions. Illustrate each answer on the graph.

a How tall is the corn after 3 weeks?

b How long will it be before the corn is 6 feet tall?

Hint. For part (b), convert feet to inches.

Answer.

a 48 inches tall

b 33 days

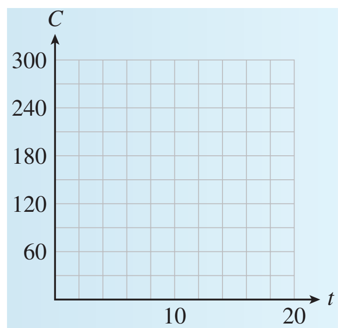
1.1.2 Choosing Scales for the Axes

Checkpoint 1.1.8 Silver Lake has been polluted by industrial waste products. The concentration of toxic chemicals in the water is currently 285 parts per million (ppm). Environmental officials would like to reduce the concentration by 15 ppm each year.

a Complete the table of values showing the desired concentration, C, of toxic chemicals t years from now. For each t-value, calculate the corresponding value for C. Write your answers as ordered pairs.

t	C		(t, C)
0		$C = 285 - 15(\mathbf{0})$	(0,)
5		$C = 285 - 15(\mathbf{5})$	(5,)
10		$C = 285 - 15(\mathbf{10})$	(10,)
15		$C = 285 - 15(\mathbf{15})$	(15,)

b To choose scales for the axes, notice that the value of C starts at 285 and decreases from there. We'll scale the vertical axis up to 300, and use 10 tick marks at intervals of 30. Graph the ordered pairs on the grid, and connect them with a straight line. Extend the graph until it reaches the horizontal axis, but no farther. Points with negative C-coordinates have no meaning for the problem.



c Write an equation for the concentration, C, of toxic chemicals t years from now.

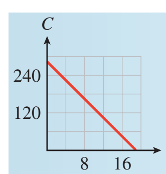
Hint. For part (c): The concentration is initially 285 ppm, and we subtract 15 ppm for each year that passes, or $15 \times t$.

Answer.

a

(t, C)
$(0, 285)$
$(5, 210)$
$(10, 135)$
$(15, 60)$

b



c $C = 285 - 15t$

Checkpoint 1.1.11

a Solve the equation $2y - 1575 = 45x$ for y in terms of x.

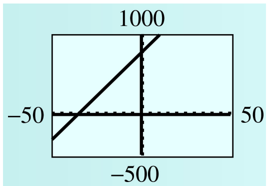
b Graph the equation on a graphing calculator. Use the window

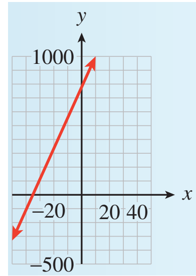
Xmin$= -50$	Xmax$= 50$	Xscl$= 5$
Ymin$= -500$	Ymax$= 1000$	Yscl$= 100$

c Sketch the graph on paper. Use the window settings to choose appropriate scales for the axes.

Answer.

a $y = (1575 + 45x)/2$

b 

c 

1.1.3 Linear Equations

Checkpoint 1.1.13 In central Nebraska, each acre of corn requires 25 acre-inches of water per year, and each acre of winter wheat requires 18 acre-inches of water. (An acre-inch is the amount of water needed to cover one acre of land to a depth of one inch.) A farmer can count on 9000 acre-inches of water for the coming year. (Source: Institute of Agriculture and Natural Resources, University of Nebraska)

a Write an equation relating the number of acres of corn, x, and the number of acres of wheat, y, that the farmer can plant.

b Complete the table.

x	50	100	150	200
y				

Answer.

a $25x + 18y = 9000$

b

x	50	100	150	200
y	430.6	361.1	291.7	222.2

1.1.4 Intercepts

Checkpoint 1.1.15

a Find the intercepts of the graph in Example 1.1.12, about the advertising budget for Albert's Appliances: $150x + 50y = 3000$.

b What do the intercepts tell us about the problem?

Answer. $(20, 0)$: The manager can buy 20 television ads if she buys no radio ads. $(0, 60)$: The manager can buy 60 radio ads if she buys no television ads.

1.1.5 Intercept Method for Graphing Lines

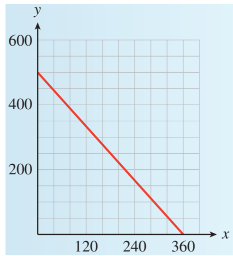
Checkpoint 1.1.17

a In Checkpoint 1.1.13, you wrote an equation about crops in Nebraska. Find the intercepts of the graph.

b Use the intercepts to help you choose appropriate scales for the axes, and then graph the equation.

c What do the intercepts tell us about the problem?

Answer. a., c. $(360, 0)$: If he plants no wheat, the farmer can plant 360 acres of corn. $(0, 500)$: If he plants no corn, the farmer can plant 500 acres of wheat.

b. 

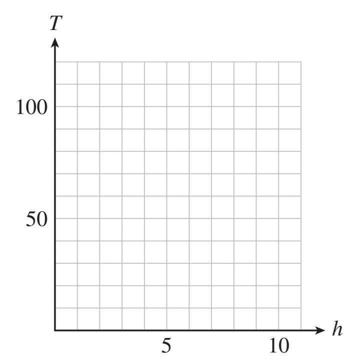
1.1.7 Homework 1.1

1.1.7.1. The temperature in the desert at 6 a.m., just before sunrise, was 65°F. The temperature rose 5 degrees every hour until it reached its maximum value at about 5 p.m. Complete the table of values for the temperature, T, at h hours after 6 a.m.

h	0	3	6	9	10
T					

a Write an equation for the temperature, T, in terms of h.

b Graph the equation.



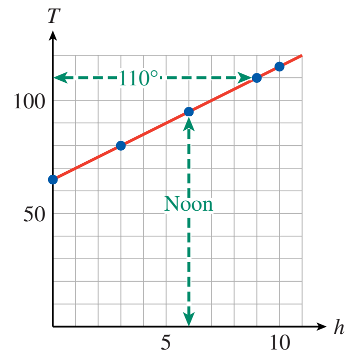
c How hot is it at noon? Illustrate the answer on your graph.

d When will the temperature be 110°F? Illustrate the answer on your graph.

Answer.

h	0	3	6	9	10
T	65	80	95	110	115

a $T = 65 + 5h$

b 

c 95°

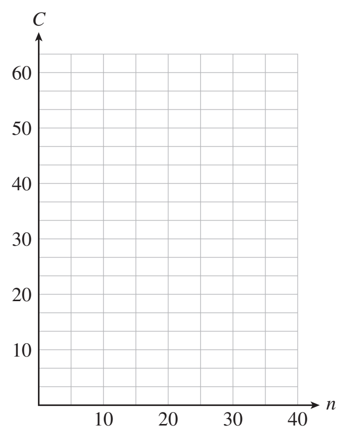
d 3 p.m.

1.1.7.2. The taxi out of Dulles Airport charges a traveler with one suitcase an initial fee of \$2.00, plus \$1.50 for each mile traveled. Complete the table of values showing the charge, C, for a trip of n miles.

n	0	5	10	15	20	25
C						

a Write an equation for the charge, C, in terms of the number of miles traveled, n.

b Graph the equation.



c What is the charge for a trip to Mount Vernon, 40 miles from the airport? Illustrate the answer on your graph.

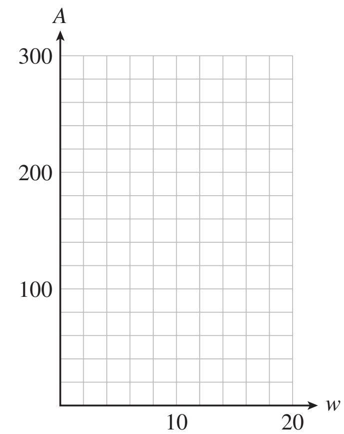
d If a ride to the National Institutes of Health (NIH) costs \$39.50, how far is it from the airport to the NIH? Illustrate the answer on your graph.

1.1.7.3. On October 31, Betty and Paul fill their 250-gallon oil tank for their heater. Beginning in November, they use an average of 15 gallons of oil per week. Complete the table of values for the amount of oil, A, left in the tank after w weeks.

w	0	4	8	12	16
A					

a Write an equation that expresses the amount of oil, A, in the tank in terms of the number of weeks, w, since October 31.

b Graph the equation.



c How much did the amount of fuel oil in the tank decrease between the third week and the eighth week? Illustrate this amount on the graph.

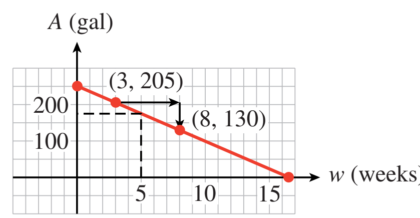
d When will the tank contain more than 175 gallons of fuel oil? Illustrate on the graph.

Answer.

w	0	4	8	12	16
A	250	190	130	70	10

a $A = 250 - 15w$

b



c 75 gallons

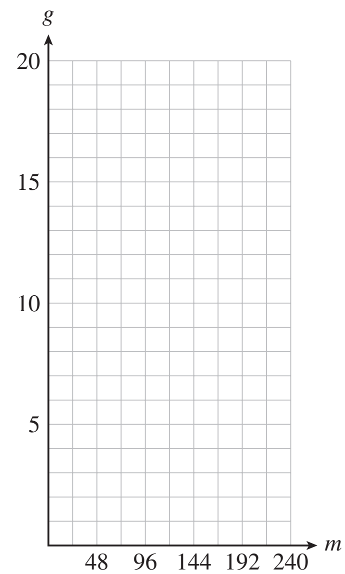
d Until the fifth week

1.1.7.4. Leon's camper has a 20-gallon gas tank, and he gets 12 miles to the gallon. (That is, he uses $\frac{1}{12}$ gallon per mile.) Complete the table of values for the amount of gas, g, left in Leon's tank after driving m miles.

m	0	48	96	144	192
g					

a Write an equation that expresses the amount of gas, g, in Leon's fuel tank in terms of the number of miles, m, he has driven.

b Graph the equation.



c How much gas will Leon use between 8 a.m., when his odometer reads 96 miles, and 9 a.m., when the odometer reads 144 miles? Illustrate on the graph.

d If Leon has less than 5 gallons of gas left, how many miles has he driven? Illustrate on the graph.

1.1.7.5. Phil and Ernie buy a used photocopier for \$800 and set up a copy service on their campus. For each hour that the copier runs, Phil and Ernie make \$40.

a Write an equation that expresses Phil and Ernie's profit (or loss), P, in terms of the number of hours, t, they run the copier.

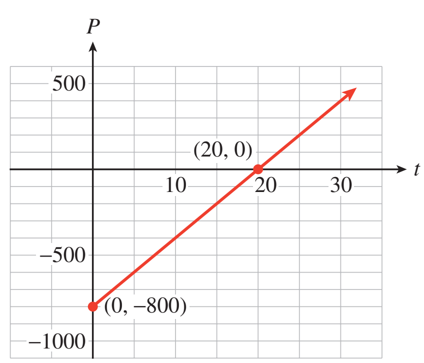
b Find the intercepts and sketch the graph. (Suggestion: Scale the horizontal axis from 0 to 40 in increments of 5, and scale the vertical axis from -1000 to 400 in increments of 100.)

c What do the intercepts tell us about the profit?

Answer.

a $P = -800 + 40t$

b $(0, -800)$, $(20, 0)$



c The P-intercept, -800, is the initial ($t = 0$) value of the profit. Phil and Ernie start out \$800 in debt. The t-intercept, 20, is the number of hours required for Phil and Ernie to break even.

1.1.7.6. A deep-sea diver is taking some readings at a depth of 400 feet. He begins rising at 20 feet per minute.

a Write an equation that expresses the diver's altitude, h, in terms of the number of minutes, m, elapsed. (Consider a depth of 400 feet as an altitude of -400 feet.)

b Find the intercepts and sketch the graph. (Suggestion: Scale the horizontal axis from 0 to 24 in increments of 2, and scale the vertical axis from -500 to 100 in increments of 50.)

c What do the intercepts tell us about the diver's depth?

1.1.7.7. There are many formulas for estimating the annual cost of driving. The Automobile Club estimates that fixed costs for a small car—including insurance, registration, depreciation, and financing—total about \$5000 per year. The operating costs for gasoline, oil, maintenance, tires, and so forth are about 12.5 cents per mile. (Source: Automobile Association of America)

a Write an equation for the annual driving cost, C, in terms of d, the number of miles driven.

b Complete the table of values.

Miles Driven	4000	8000	$12,000$	$16,000$	$20,000$
Cost (\$)					

c Choose scales for the axes and graph the equation.

d How much does the annual cost of driving increase when the mileage increases from 8000 to $12,000$ miles? Illustrate this amount on the graph.

e How much mileage will cause the annual cost to exceed \$7000? Illustrate on the graph.

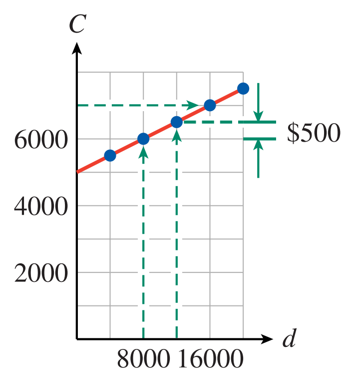
Answer.

a $C = 5000 + 0.125d$

b Complete the table of values.

Miles Driven	4000	8000	$12,000$	$16,000$	$20,000$
Cost (\$)	5500	6000	6500	7000	7500

c



d \$500

e More than 16,000 miles

1.1.7.8. The boiling point of water changes with altitude. At sea level, water boils at 212°F, and the boiling point diminishes by approximately 0.002°F for each 1-foot increase in altitude.

a Write an equation for the boiling point, B, in terms of a, the altitude in feet.

b Complete the table of values.

Altitude (ft)	-500	0	1000	2000	3000	4000	5000
Boiling point (°F)							

c Choose scales for the axes and graph the equation.

d How much does the boiling point decrease when the altitude increases from 1000 to 3000 feet? Illustrate this amount on the graph.

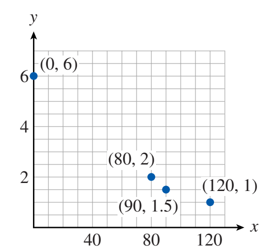
e At what altitudes is the boiling point less than 204°F? Illustrate on the graph.

For each table, choose appropriate scales for the axes and plot the given points.

1.1.7.9.

x	0	80	90	120
y	6	2	1.5	1

Answer.



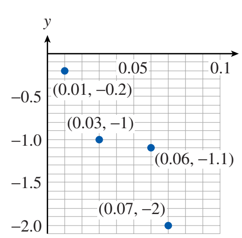
1.1.7.10.

x	300	500	800	1100
y	1.2	1.3	1.5	1.9

1.1.7.11.

x	0.01	0.03	0.06	0.07
y	-0.2	-1	-1.1	-2

Answer.



1.1.7.12.

x	0.003	0.005	0.008	0.011
y	6	2	1.5	1

For Problems 13-18,

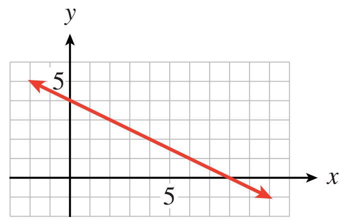
(a) Find the intercepts of the graph.

(b) Graph the equation by the intercept method.

1.1.7.13. $x + 2y = 8$

Answer.

a $(8, 0), (0, 4)$

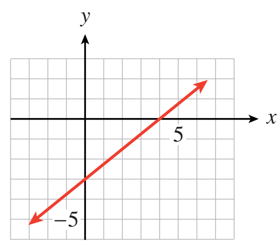
b 

1.1.7.14. $2x - y = 6$

1.1.7.15. $3x - 4y = 12$

Answer.

a $(4, 0), (0, -3)$

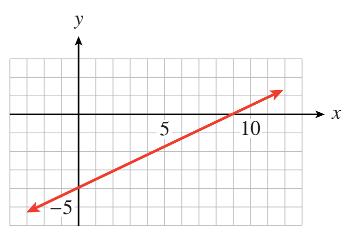
b 

1.1.7.16. $2x + 6y = 6$

1.1.7.17. $\dfrac{x}{9} - \dfrac{y}{4} = 1$

Answer.

a $(9, 0), (0, -4)$

b 

1.1.7.18. $\dfrac{x}{5} + \dfrac{y}{8} = 1$

For Problems 19-24,

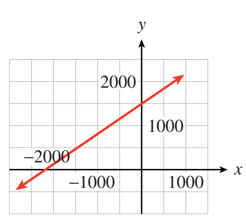
(a) Find the intercepts of the graph.

(b) Use the intercepts to choose scales for the axes, and then graph the equation by the intercept method.

1.1.7.19. $20x = 30y - 45,000$

Answer.

a $(-2250, 0), (0, 1500)$

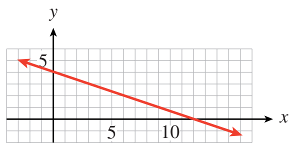
b 

1.1.7.20. $30x = 45y + 60,000$

1.1.7.21. $0.4x + 1.2y = 4.8$

Answer.

a $(12, 0), (0, 4)$

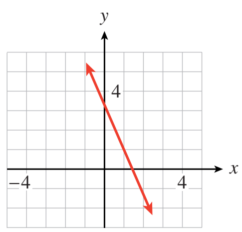
b 

1.1.7.22. $3.2x - 0.8y = 12.8$

1.1.7.23. $\dfrac{2x}{3} + \dfrac{3y}{11} = 1$

Answer.

a $\left(\dfrac{3}{2}, 0\right), \left(0, \dfrac{11}{3}\right)$

b 

1.1.7.24. $\dfrac{8x}{7} - \dfrac{2y}{7} = 1$

1.1.7.25. The owner of a gas station has $19,200 to spend on unleaded gas this month. Regular unleaded costs him $2.40 per gallon, and premium unleaded costs $3.20 per gallon.

a How much do x gallons of regular cost? How much do y gallons of premium cost?

b Write an equation in general form that relates the amount of regular unleaded gasoline, x, the owner can buy and the amount of premium unleaded, y.

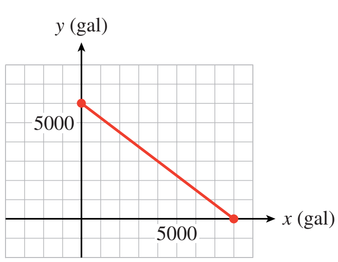
c Find the intercepts and sketch the graph.

d What do the intercepts tell us about the amount of gasoline the owner can purchase?

Answer.

a $\$2.40x$, $\$3.20y$

b $2.40x + 3.20y = 19,200$

c 

d The y-intercept, 6000 gallons, is the amount of premium that the gas station owner can buy if he buys no regular. The x-intercept, 8000 gallons, is the amount of regular he can buy if he buys no premium.

1.1.7.26. Five pounds of body fat is equivalent to $16,000$ calories. Carol can burn 600 calories per hour bicycling and 400 calories per hour swimming.

a How many calories will Carol burn in x hours of cycling? How many calories will she burn in y hours of swimming?

b Write an equation in general form that relates the number of hours, x, of cycling and the number of hours, y, of swimming Carol needs to perform in order to lose 5 pounds.

c Find the intercepts and sketch the graph.

d What do the intercepts tell us about Carol's exercise program?

1.1.7.27. Delbert must increase his daily potassium intake by 1800 mg. He decides to eat a combination of figs and bananas, which are both low in sodium. There are 9 mg potassium per gram of fig, and 4 mg potassium per gram of banana.

a How much potassium is in x grams of fig? How much potassium is in y grams of banana?

b Write an equation in general form that relates the number of grams, x, of fig and the number of grams, y, of banana Delbert needs to get 1800 mg of potassium.

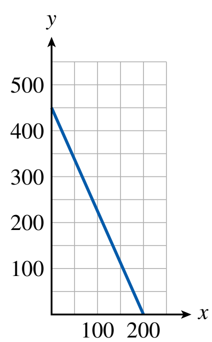
c Find the intercepts and sketch the graph.

d What do the intercepts tell us about Delbert's diet?

Answer.

a $9x$ mg, $4y$ mg

b $9x + 4y = 1800$

c 

d The x-intercept, 200 grams, tells how much fig Delbert should eat if he has no bananas, and the y-intercept, 450 grams, tells how much banana he should eat if he has no figs.

1.1.7.28. Leslie plans to invest some money in two CD accounts. The first account pays 3.6% interest per year, and the second account pays 2.8% interest per year. Leslie would like to earn \$500 per year on her investment.

a If Leslie invests x dollars in the first account, how much interest will she earn? How much interest will she earn if she invests y dollars in the second account?

b Write an equation in general form that relates x and y if Leslie earns \$500 interest.

c Find the intercepts and sketch the graph.

d What do the intercepts tell us about Leslie's investments?

1.1.7.29. Find the intercepts of the graph for each equation.

a $\dfrac{x}{3}+\dfrac{y}{5}=1$

b $2x-4y=1$

c $\dfrac{2x}{5}-\dfrac{2y}{3}=1$

d $\dfrac{x}{p}+\dfrac{y}{q}=1$

e. Why is the equation $\dfrac{x}{a}+\dfrac{y}{b}=1$ called the **intercept form** for a line?

Answer.

a $(3,0),(0,5)$

b $\left(\dfrac{1}{2},0\right),\left(0,\dfrac{-1}{4}\right)$

c $\left(\dfrac{5}{2},0\right),\left(0,\dfrac{-3}{2}\right)$

d $(p,0),(0,q)$

e The value of a is the x-intercept, and the value of b is the y-intercept.

1.1.7.30. Write an equation in intercept form (see Problem 29) for the line with the given intercepts. Then write the equation in general form.

a $(6,0),(0,2)$

b $(-3,0),(0,8)$

c $\left(\dfrac{3}{4},0\right),\left(0,\dfrac{-1}{4}\right)$

d $(v,0),(0,-w)$

e $\left(\dfrac{1}{H},0\right),\left(0,\dfrac{1}{T}\right)$

1.1.7.31.

a Find the y-intercept of the line $y=mx+b$.

b Find the x-intercept of the line $y=mx+b$.

Answer.

a $(0,b)$

b $\left(\dfrac{-b}{m},0\right)$, if $m\neq 0$

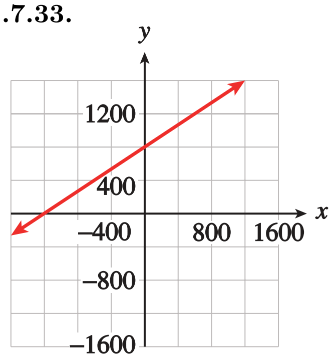
1.1.7.32.

a Find the y-intercept of the line $Ax+By=C$.

b Find the x-intercept of the line $Ax+By=C$.

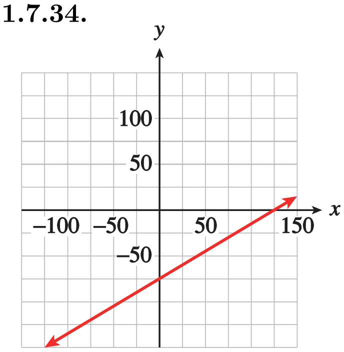
Write an equation in general form for each line.

1.1.7.33.

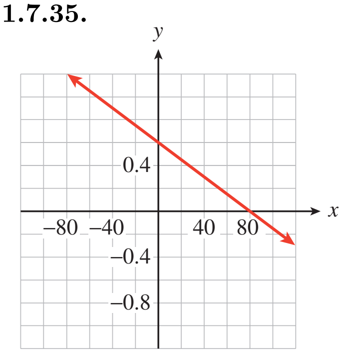


Answer. $-2x+3y=2400$

1.1.7.34.

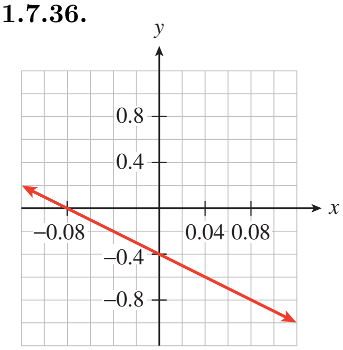


1.1.7.35.



Answer. $3x + 400y = 240$

1.1.7.36.



For Problems 37–44,

a Solve each equation for y in terms of x. (See the Algebra Skills Refresher Section A.2 to review this skill.)

b Graph the equation on your calculator in the specified window.

c Make a pencil and paper sketch of the graph. Label the scales on your axes, and the coordinates of the intercepts.

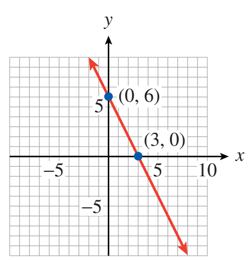
1.1.7.37. $2 + y = 6$

$$\begin{aligned} \text{Xmin} &= -10 & \text{Ymin} &= -10 \\ \text{Xmax} &= 10 & \text{Ymax} &= 10 \\ \text{Xscl} &= 1 & \text{Yscl} &= 1 \end{aligned}$$

Answer.

a $y = 6 - 2x$

b



1.1.7.38. $8 - y + 3x = 0$

$$\begin{aligned} \text{Xmin} &= -10 & \text{Ymin} &= -10 \\ \text{Xmax} &= 10 & \text{Ymax} &= 10 \\ \text{Xscl} &= 1 & \text{Yscl} &= 1 \end{aligned}$$

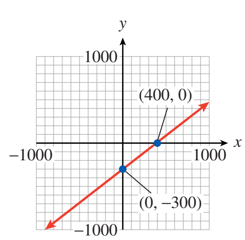
1.1.7.39. $3x - 4y = 1200$

$$\begin{aligned} \text{Xmin} &= -1000 & \text{Ymin} &= -1000 \\ \text{Xmax} &= 1000 & \text{Ymax} &= 1000 \\ \text{Xscl} &= 100 & \text{Yscl} &= 100 \end{aligned}$$

Answer.

a $y = \dfrac{3}{4}x - 300$

b



1.1.7.40. $x + 2y = 500$

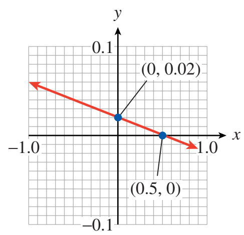
$$\begin{aligned} \text{Xmin} &= -1000 & \text{Ymin} &= -1000 \\ \text{Xmax} &= 1000 & \text{Ymax} &= 1000 \\ \text{Xscl} &= 100 & \text{Yscl} &= 100 \end{aligned}$$

1.1.7.41. $0.2x + 5y = 0.1$

$$\begin{aligned} \text{Xmin} &= -1 & \text{Ymin} &= -0.1 \\ \text{Xmax} &= 1 & \text{Ymax} &= 0.1 \\ \text{Xscl} &= 0.1 & \text{Yscl} &= 0.01 \end{aligned}$$

Answer.

a $y = 0.02 - 0.04x$

b 

1.1.7.42. $1.2x - 4.2y = 3.6$

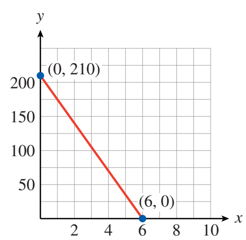
$$\begin{aligned} \text{Xmin} &= -1 & \text{Ymin} &= -1 \\ \text{Xmax} &= 4 & \text{Ymax} &= 1 \\ \text{Xscl} &= 0.2 & \text{Yscl} &= 0.1 \end{aligned}$$

1.1.7.43. $70x + 3y = y + 420$

$$\begin{aligned} \text{Xmin} &= 0 & \text{Ymin} &= 0 \\ \text{Xmax} &= 10 & \text{Ymax} &= 250 \\ \text{Xscl} &= 1 & \text{Yscl} &= 25 \end{aligned}$$

Answer.

a $y = 210 - 35x$

b 

1.1.7.44. $40y - 5x = 780 - 20y$

$$\begin{aligned} \text{Xmin} &= -200 & \text{Ymin} &= 0 \\ \text{Xmax} &= 0 & \text{Ymax} &= 20 \\ \text{Xscl} &= 20 & \text{Yscl} &= 2 \end{aligned}$$

For Problems 45–52,

a Find the x- and y-intercepts.

b Solve the equation for y.

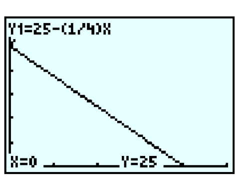
c Choose a graphing window in which both intercepts are visible, and graph the equation on your calculator.

1.1.7.45. $x + 4y = 100$

Answer.

a $(100, 0), (0, 25)$

b $y = 25 - \dfrac{1}{4}x$

c 

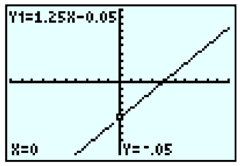
1.1.7.46. $2x - 3y = -72$

1.1.7.47. $25x - 20y = 1$

Answer.

a $(0.04, 0), (0, -0.05)$

b $y = 1.25x - 0.05$

c 

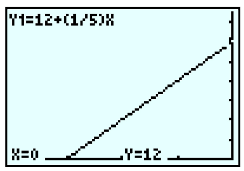
1.1.7.48. $4x + 75y = 60,000$

1.1.7.49. $\dfrac{y}{12} - \dfrac{x}{60} = 1$

Answer.

a $(-60, 0), (0, 12)$

b $y = 12 + \dfrac{1}{5}x$

c 

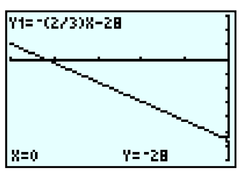
1.1.7.50. $\dfrac{x}{80} + \dfrac{y}{400} = 1$

1.1.7.51. $-2x = 3y + 84$

Answer.

a $(-42, 0), (0, -28)$

b $y = \dfrac{-2}{3}x - 28$

c 

1.1.7.52. $7x = 91 - 13y$

1.2 Functions

1.2.1 Definition of Function

Checkpoint 1.2.3

a As part of a project to improve the success rate of freshmen, the counseling department studied the grades earned by a group of students in English and algebra. Do you think that a student's grade in algebra is a function of his or her grade in English? Explain why or why not.

b Phatburger features a soda bar, where you can serve your own soft drinks in any size. Do you think that the number of calories in a serving of Zap Kola is a function of the number of fluid ounces? Explain why or why not.

Answer.

a No, students with the same grade in English can have different grades in algebra.

b Yes, the number of calories is proportional to the number of fluid ounces.

1.2.2 Functions Defined by Tables

Checkpoint 1.2.5 Decide whether each table describes y as a function of x. Explain your choice.

a

x	3.5	2.0	2.5	3.5	2.5	4.0	2.5	3.0
y	2.5	3.0	2.5	4.0	3.5	4.0	2.0	2.5

b

x	-3	-2	-1	0	1	2	3
y	17	3	0	-1	0	3	17

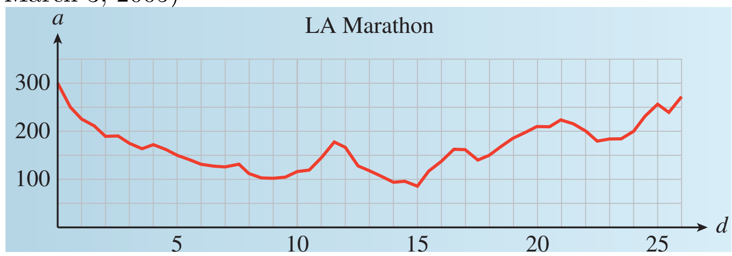
Answer.

a No, for example, $x = 3.5$ corresponds both to $y = 2.5$ and also to $y = 4.0$.

b Yes, each value of x has exactly one value of y associated with it.

1.2.3 Functions Defined by Graphs

Checkpoint 1.2.7 The graph shows the elevation in feet, a, of the Los Angeles Marathon course at a distance d miles into the race. (Source: *Los Angeles Times*, March 3, 2005)



a Which variable is the input, and which is the output?

b What is the elevation at mile 20?

c At what distances is the elevation 150 feet?

d What are the maximum and minimum values of a, and when do these values occur?

e The runners pass by the Los Angeles Coliseum at about 4.2 miles into the race. What is the elevation there?

Answer.

a The input variable is d, and the output variable is a.

b Approximately 210 feet

c Approximately where $d \approx 5$, $d \approx 11$, $d \approx 12$, $d \approx 16$, $d \approx 17.5$, and $d \approx 18$

d The maximum value of 300 feet occurs at the start, when $d = 0$. The minimum of 85 feet occurs when $d \approx 15$.

e Approximately 165 feet

1.2.4 Functions Defined by Equations

Checkpoint 1.2.9 Write an equation that gives the volume, V, of a sphere as a function of its radius, r.

Answer. $V = \dfrac{4}{3}\pi r^3$

1.2.5 Function Notation

Checkpoint 1.2.13 Let F be the name of the function defined by the graph in Example 1.2.6, the number of hours of daylight in Peoria.

a Use function notation to state that H is a function of t.

b What does the statement $F(15) = 9.7$ mean in the context of the problem?

Answer.

a $H = F(t)$

b The sun is above the horizon in Peoria for 9.7 hours on January 16.

1.2.6 Evaluating a Function

Checkpoint 1.2.15 When you exercise, your heart rate should increase until it reaches your target heart rate. The table shows target heart rate, $r = f(a)$, as a function of age.

a	20	25	30	35	40	45	50	55	60	65	70
r	150	146	142	139	135	131	127	124	120	116	112

a Find $f(25)$ and $f(50)$.

b Find a value of a for which $f(a) = 135$.

Answer.

a $f(25) = 146, \quad f(50) = 127$

b $a = 40$

Checkpoint 1.2.17 Complete the table displaying ordered pairs for the function $f(x) = 5 - x^3$. Evaluate the function to find the corresponding $f(x)$-value for each value of x.

x	$f(x)$	
-2		$f(\mathbf{-2}) = 5 - (\mathbf{-2})^3 =$
0		$f(\mathbf{0}) = 5 - \mathbf{0}^3 =$
1		$f(\mathbf{1}) = 5 - \mathbf{1}^3 =$
3		$f(\mathbf{3}) = 5 - \mathbf{3}^3 =$

Answer.

x	$f(x)$
-2	13
0	5
1	4
3	-22

Checkpoint 1.2.19 The volume of a sphere of radius r centimeters is given by

$$V = V(r) = \frac{4}{3}\pi r^3$$

Evaluate $V(10)$ and explain what it means.

Answer. $V(10) = 4000\pi/3 \approx 4188.79$ cm^3 is the volume of a sphere whose radius is 10 cm.

1.2.7 Operations with Function Notation

Checkpoint 1.2.21 A spherical balloon has a radius of 10 centimeters.

a If we increase the radius by h centimeters, what will the new volume be?

b If $h = 2$, how much did the volume increase?

Answer.

a $V(10 + h) = \dfrac{4}{3}\pi(10 + h)^3$ cm^3

b By 3049.44 cm^3

Checkpoint 1.2.24 Let $f(x) = x^3 - 1$ and evaluate each expression.

a $f(2) + f(3)$

b $f(2 + 3)$

c $2f(x) + 3$

Answer.

a 33

b 124

c $2x^3 + 1$

1.2.10 Homework 1.2

For which of Problems 1-6 is the second quantity a function of the first? Explain your answers.

1.2.10.1. Price of an item; sales tax on the item at 4%

Answer. Function; the tax is determined by the price of the item.

1.2.10.2. Time traveled at constant speed; distance traveled

1.2.10.3. Number of years of education; annual income

Answer. Not a function; incomes may differ for same number of years of education.

1.2.10.4. Distance flown in an airplane; price of the ticket

1.2.10.5. Volume of a container of water; the weight of the water

Answer. Function; weight is determined by volume.

1.2.10.6. Amount of a paycheck; amount of Social Security tax withheld

Each of the objects in Problems 7-14 establishes a correspondence between two variables. Suggest appropriate input and output variables and decide whether the relationship is a function.

1.2.10.7. An itemized grocery receipt

Answer. Input: items purchased; output: price of item. Yes, a function because each item has only one price.

1.2.10.8. An inventory list

1.2.10.9. An index

Answer. Input: topics; output: page or pages on which topic occurs. No, not a function because the same topic may appear in more than one page.

1.2.10.10. A will

1.2.10.11. An instructor's grade book

Answer. Input: students' names; output: students' scores on quizzes, tests, etc. No, not a function because the same student can have different grades on different tests.

1.2.10.12. An address book

1.2.10.13. A bathroom scale

Answer. Input: person stepping on scales; output: person's weight. Yes, a function because a person cannot have two different weights at the same time.

1.2.10.14. A radio dial

Which of the tables in Problems 15-26 define the second variable as a function of the first variable? Explain why or why not.

1.2.10.15.

x	t
-1	2
0	9
1	-2
0	-3
-1	5

Answer. No

1.2.10.16.

y	w
0	8
1	12
3	7
5	-3
7	4

1.2.10.17.

x	y
-3	8
-2	3
-1	0
0	-1
1	0
2	3
3	8

Answer. Yes

1.2.10.18.

s	t
2	5
4	10
6	15
8	20
6	25
4	30
2	35

1.2.10.19.

r	-4	-2	0	2	4
v	6	6	3	6	8

Answer. Yes

1.2.10.20.

p	-5	-4	-3	-2	-1
d	-5	-4	-3	-2	-1

1.2.10.21.

Pressure (p)	Volume (v)
15	100.0
20	75.0
25	60.0
30	50.0
35	42.8
40	37.5
45	33.3
50	30.0

Answer. Yes

1.2.10.22.

Frequency (f)	Wavelength (w)
5	60.0
10	30.0
20	15.0
30	10.0
40	7.5
50	6.0
60	5.0
70	4.3

1.2.10.23.

Temperature (T)		Humidity (h)
Jan. 1	34°F	42%
Jan. 2	36°F	44%
Jan. 3	35°F	47%
Jan. 4	29°F	50%
Jan. 5	31°F	52%
Jan. 6	35°F	51%
Jan. 7	34°F	49%

Answer. No

1.2.10.24.

Inflation rate (I)		Unemployment rate (U)
1972	5.6%	5.1%
1973	6.2%	4.5%
1974	10.1%	4.9%
1975	9.2%	7.4%
1976	5.8%	6.7%
1977	5.6%	6.8%
1978	6.7%	7.4%

1.2.10.25.

Adjusted gross income (I)	Tax bracket (T)
$0 – 2479	0%
$2480 – 3669	4.5%
$3670 – 4749	12%
$4750 – 7009	14%
$7010 – 9169	15%
$9170 – 11,649	16%
$11,650 – 13,919	18%

Answer. Yes

1.2.10.26.

Cost of merchandise (M)	Shipping charge (C)
$0.01 – 10.00	$2.50
10.01 – 20.00	3.75
20.01 – 35.00	4.85
35.01 – 50.00	5.95
50.01 – 75.00	6.95
75.01 – 100.00	7.95
Over 100.00	8.95

1.2.10.27. The function described in Problem 21 is called g, so that $v = g(p)$. Find the following:

a $g(25)$

b $g(40)$

c x so that $g(x) = 50$

Answer.

a 60

b 37.5

c 30

1.2.10.28. The function described in Problem 22 is called h, so that $w = h(f)$. Find the following:

a $h(20)$

b $h(60)$

c x so that $h(x) = 10$

1.2.10.29. The function described in Problem 25 is called T, so that $T = T(I)$. Find the following:

a $T(8750)$

b $T(6249)$

c x so that $T(x) = 15\%$

Answer.

a 15%

b 14%

c $7010–$9169

1.2.10.30. The function described in Problem 26 is called C, so that $C = C(M)$. Find the following:

a $C(11.50)$

b $C(47.24)$

c x so that $C(x) = 7.95$

1.2.10.31. Data indicate that U.S. women are delaying having children longer than their counterparts 50 years ago. The table shows $f(t)$ the percent of 20–24-year-old women in year t who had not yet had children. (Source: U.S. Dept of Health and Human Services)

Year (t)	1960	1965	1970	1975	1980	1985	1990	1995	2000
Percent of women	47.5	51.4	47.0	62.5	66.2	67.7	68.3	65.5	66.0

a Evaluate $f(1985)$ and explain what it means.

b Estimate a solution to the equation $f(t) = 68$ and explain what it means.

c In 1997, 64.9% of 20–24-year-old women had not yet had children. Write an equation with function notation that states this fact.

Answer.

a 67.7: In 1985, 67.7% of 20–24 year old women had not yet had children.

b 1987: Approximately 68% of 20–24 year old women had not yet had children in 1987.

c $f(1997) = 64.9$

1.2.10.32. The table shows $f(t)$, the death rate (per 100,000 people) from HIV among 15–24-year-olds, and $g(t)$, the death rate from HIV among 25–34-year-olds, for selected years from 1997 to 2002. (Source: U.S. Dept of Health and Human Services)

Year	1987	1988	1989	1990	1992	1994	1996	1998	2000	2002
15–24-year-olds	1.3	1.4	1.6	1.5	1.6	1.8	1.1	0.6	0.5	0.4
25–34-year-olds	11.7	14.0	17.9	19.7	24.2	28.6	19.2	8.1	6.1	4.6

a Evaluate $f(1995)$ and explain what it means.

b Find a solution to the equation $g(t) = 28.6$ and explain what it means.

c In 1988, the death rate from HIV for 25–34-year-olds was 10 times the corresponding rate for 15–24-year-olds. Write an equation with function notation that states this fact.

1.2.10.33. When you exercise, your heart rate should increase until it reaches your target heart rate. The table shows target heart rate, $r = f(a)$, as a function of age.

a	20	25	30	35	40	45	50	55	60	65	70
r	150	146	142	139	135	131	127	124	120	116	112

a Does $f(50) = 2f(25)$?

b Find a value of a for which $f(a) = 2a$. Is $f(a) = 2a$ for all values of a?

c Is $r = f(a)$ an increasing function or a decreasing function?

Answer.

a No

b 60; no

c Decreasing

1.2.10.34. The table shows $M = f(d)$, the men's Olympic record time, and $W = g(d)$, the women's Olympic record time, as a function of the length, d, of the race. For example, the women's record in the 100 meters is 10.62 seconds, and the men's record in the 800 meters is 1 minute, 42.58 seconds. (Source: www.hickoksports.com)

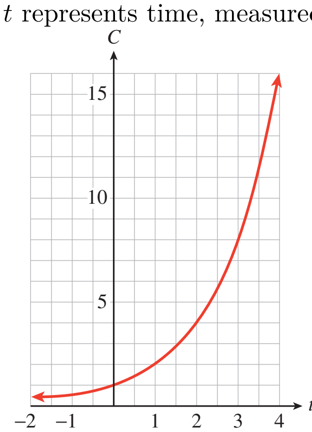
Distance (meters)	100	200	400	800	1500	5000	10,000
Men	9.63	19.30	43.03	1 : 40.91	3 : 32.07	12 : 57.82	27 : 01.17
Women	10.62	21.34	48.25	1 : 53.43	3 : 53.96	14 : 26.17	29 : 17.45

a Does $f(800) = 2f(400)$? Does $g(400) = 2g(200)$?

b Find a value of d for which $f(2d) < 2f(d)$. Is there a value of d for which $g(2d) < 2g(d)$?

In Problems 35—40, use the graph of the function to answer the questions.

1.2.10.35. The graph shows C as a function of t. C stands for the number of students (in thousands) at State University who consider themselves computer literate, and t represents time, measured in years since 1990.



a When did 2000 students consider themselves computer literate?

b How long did it take that number to double?

c How long did it take for the number to double again?

d How many students became computer literate between January 1992 and June 1993?

Answer.

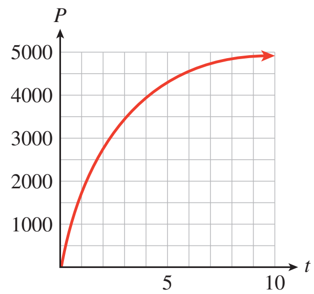
a 1991

b 1 yr

c 1 yr

d About 7300

1.2.10.36. The graph shows P as a function of t. P is the number of people in Cedar Grove who owned a portable DVD player t years after 2000.



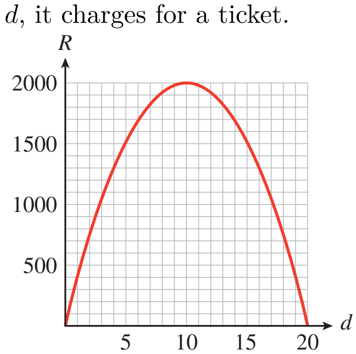
a When did 3500 people own portable DVD players?

b How many people owned portable DVD players in 2005?

c The number of owners of portable DVD players in Cedar Grove seems to be leveling off at what number?

d How many people acquired portable DVD players between 2001 and 2004?

1.2.10.37. The graph shows the revenue, R, a movie theater collects as a function of the price, d, it charges for a ticket.



a What is the revenue if the theater charges \$12.00 for a ticket?

b What should the theater charge for a ticket in order to collect \$1500 in revenue?

c For what values of d is $R > 1875$?

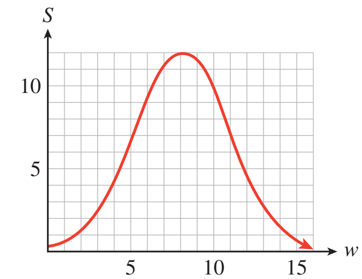
Answer.

a Approximately \$1920

b \$5 or \$15

c $7.50 < d < 12.50$

1.2.10.38. The graph shows S as a function of w. S represents the weekly sales of a best-selling book, in thousands of dollars, w weeks after it is released.



a In which weeks were sales over $7000?

b In which week did sales fall below $5000 on their way down?

c For what values of w is $S > 3.4$?

1.2.10.39. The graph shows the federal minimum wage, M, as a function of time, t, adjusted for inflation to reflect its buying power in 2004 dollars. (Source: www.infoplease.com)

a When did the minimum wage reach its highest buying power, and what was it worth in 2004 dollars?

b When did the minimum wage fall to its lowest buying power after its peak, and what was its worth at that time?

c Give two years in which the minimum wage was worth $8 in 2004 dollars.

Answer.

a 1968, about $8.70

b 1989, about $5.10

c 1967, approximately 1970

1.2.10.40. The graph shows the U.S. unemployment rate, U, as a function of time, t, for the years 1985–2004. (Source: U.S. Bureau of Labor Statistics)

a When did the unemployment rate reach its highest value, and what was its highest value?

b When did the unemployment rate fall to its lowest value, and what was its lowest value?

c Give two years in which the unemployment rate was 4.5%.

In Problems 41–48, evaluate each function for the given values, if possible. If not, state why.

1.2.10.41. $f(x) = 6 - 2x$

a $f(3)$

b $f(-2)$

c $f(12.7)$

d $f\left(\dfrac{2}{3}\right)$

Answer.

a 0

b 10

c -19.4

d $\dfrac{14}{3}$

1.2.10.42. $g(t) = 5t - 3$

a $g(1)$

b $g(-4)$

c $g(14.1)$

d $g\left(\dfrac{3}{4}\right)$

1.2.10.43. $h(v) = 2v^2 - 3v + 1$

a $h(0)$

b $h(-1)$

c $h\left(\dfrac{1}{4}\right)$

d $h(-6.2)$

Answer.

a 1

b 6

c $\dfrac{3}{8}$

d 96.48

1.2.10.44. $r(s) = 2s - s^2$

a $r(2)$

b $r(-4)$

c $r\left(\dfrac{1}{3}\right)$

d $r(-1.3)$

1.2.10.45. $H(z) = \dfrac{2z-3}{z+2}$

a $H(4)$

b $H(-3)$

c $H\left(\dfrac{4}{3}\right)$

d $H(4.5)$

Answer.

a $\dfrac{5}{6}$

b 9

c $\dfrac{-1}{10}$

d $\dfrac{12}{13} \approx 0.923$

1.2.10.46. $F(x) = \dfrac{1-x}{2x-3}$

a $F(0)$

b $F(-3)$

c $F\left(\dfrac{5}{2}\right)$

d $F(\dfrac{3}{2})$

1.2.10.47. $E(t) = \sqrt{t-4}$

a $E(16)$

b $E(4)$

c $E(7)$

d $E(4.2)$

Answer.

a $\sqrt{12}$

b 0

c $\sqrt{3}$

d $\sqrt{0.2} \approx 0.447$

1.2.10.48. $D(r) = \sqrt{5-r}$

a $D(4)$

b $D(-3)$

c $D(9)$

d $D(4.6)$

1.2.10.49. A sport utility vehicle costs \$28,000 and depreciates according to the formula

$$V(t) = 28,000(1 - 0.08t)$$

where V is the value of the vehicle after t years.

a Evaluate $V(12)$ and explain what it means.

b Solve the equation $V(t) = 0$ and explain what it means.

c If this year is $t = n$, what does $V(n + 2)$ mean?

Answer.

a $V(12) = 1120$: After 12 years, the SUV is worth \$1120.

b $t = 12.5$: The SUV has zero value after $12\frac{1}{2}$ years.

c The value 2 years later

1.2.10.50. In a profit-sharing plan, an employee receives a salary of

$$S(x) = 20,000 + 0.01x$$

where x represents the company's profit for the year.

a Evaluate $S(850,000)$ and explain what it means.

b Solve the equation $S(x) = 30,000$ and explain what it means.

c If the company made a profit of p dollars this year, what does $S(2p)$ mean?

1.2.10.51. The number of compact cars that a large dealership can sell at price p is given by

$$N(p) = \frac{12,000,000}{p}$$

a Evaluate $N(6000)$ and explain what it means.

b As p increases, does $N(p)$ increase or decrease? Why is this reasonable?

c If the current price for a compact car is D, what does $2N(D)$ mean?

Answer.

a $N(6000) = 2000$: 2000 cars will be sold at a price of \$6000.

b $N(p)$ decreases with increasing p because fewer cars will be sold when the price increases.

c $2N(D)$ represents twice the number of cars that can be sold at the current price.

1.2.10.52. A department store finds that the market value of its Christmas-related merchandise is given by

$$M(t) = \frac{600,000}{t}, \quad t \leq 30$$

where t is the number of weeks after Christmas.

a Evaluate $M(2)$ and explain what it means.

b As t increases, does $M(t)$ increase or decrease? Why is this reasonable?

c If this week $t = n$, what does $M(n+1)$ mean?

1.2.10.53. The velocity of a car that brakes suddenly can be determined from the length of its skid marks, d, by

$$v(d) = \sqrt{12d}$$

where d is in feet and v is in miles per hour.

a Evaluate $v(250)$ and explain what it means.

b Estimate the length of the skid marks left by a car traveling at 100 miles per hour.

c Write your answer to part (b) with function notation.

Answer.

a $v(250) = 54.8$ is the speed of a car that left 250-foot skid marks.

b $833\frac{1}{3}$ feet

c $v\left(833\frac{1}{3}\right) = 100$

1.2.10.54. The distance, d, in miles that a person can see on a clear day from a height, h, in feet is given by

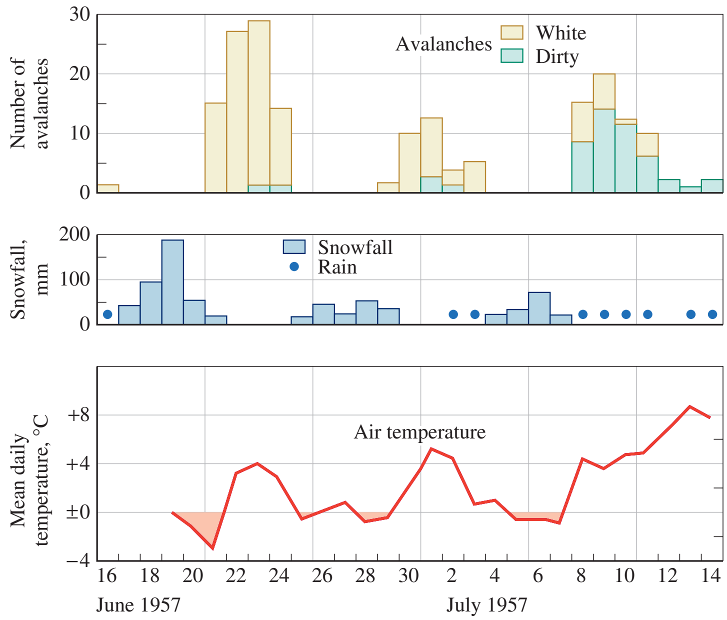
$$d(h) = 1.22\sqrt{h}$$

a Evaluate $d(20,320)$ and explain what it means.

b Estimate the height you need in order to see 100 miles.

c Write your answer to part (b) with function notation.

1.2.10.55. The figure gives data about snowfall, air temperature, and number of avalanches on the Mikka glacier in Sarek, Lapland, in 1957. (Source: Leopold, Wolman, Miller, 1992)



a During June and July, avalanches occurred over three separate time intervals. What were they?

b Over what three time intervals did snow fall?

c When was the temperature above freezing (0°C)?

d Using your answers to parts (a)–(c), make a conjecture about the conditions that encourage avalanches.

Answer.

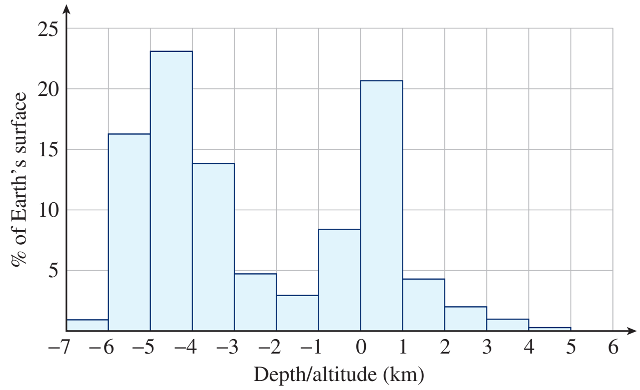
a June 21–24, June 29–July 3, July 8–14

b June 17–21, June 25–29, July 4–7

c June 22–24, June 27, June 29–July 4, July 8–14

d Avalanches occur when temperatures rise above freezing immediately after snowfall.

1.2.10.56. The bar graph shows the percent of Earth's surface that lies at various altitudes or depths below the surface of the oceans. (Depths are given as negative altitudes.) (Source: Open University)



a Read the graph and complete the table.

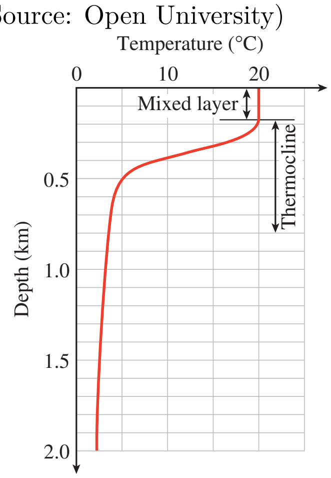
Altitude (km)	Percent of Earth's surface
−7 to −6	
−6 to −5	
−5 to −4	
−4 to −3	
−3 to −2	
−2 to −1	
−1 to 0	
0 to 1	
1 to 2	
2 to 3	
3 to 4	
4 to 5	

b What is the most common altitude? What is the second most common altitude??

c Approximately what percent of the Earth's surface is below sea level?

d The height of Mt. Everest is 8.85 kilometers. Can you think of a reason why it is not included in the graph?

1.2.10.57. The graph shows the temperature of the ocean at various depths. (Source: Open University)



a Is depth a function of temperature?

b Is temperature a function of depth?

c The axes are scaled in an unusual way. Why is it useful to present the graph in this way?

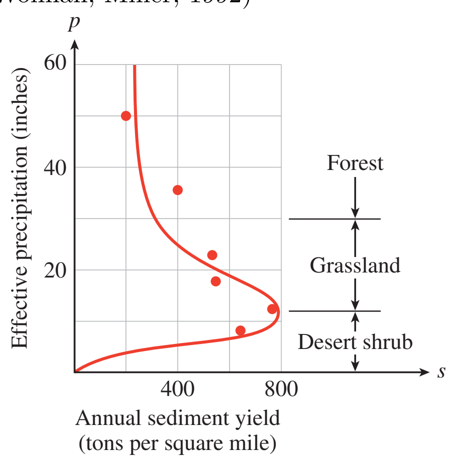
Answer.

a No

b Yes

c Moving downwards on the graph corresponds to moving downwards in the ocean.

1.2.10.58. The graph shows the relationship between annual precipitation, p, in a region and the amount of erosion, measured in tons per square mile, s. (Source: Leopold, Wolman, Miller, 1992)



a Is the amount of erosion a function of the amount of precipitation?

b At what annual precipitation is erosion at a maximum, and what is that maximum?

c Over what interval of annual precipitation does erosion decrease?

d An increase in vegetation inhibits erosion, and precipitation encourages vegetation. What happens to the amount of erosion as precipitation increases in each of these three environments?

desert shrub:	$0 < p < 12$
grassland:	$12 < p < 30$
forest:	$30 < p < 60$

In Problems 59—64, evaluate the function and simplify.

1.2.10.59. $G(s) = 3s^2 - 6s$

a $G(3a)$ c $G(a) + 2$

b $G(a+2)$ d $G(-a)$

Answer.

a $27a^2 - 18a$ c $3a^2-6a+2$

b $3a^2 + 6a$ d $3a^2 + 6a$

1.2.10.60. $h(x) = 2x^2 + 6x - 3$

a $h(2a)$ c $h(a) + 3$

b $h(a+3)$ d $h(-a)$

1.2.10.61. $g(x) = 8$

a $g(2)$ c $g(a+1)$

b $g(8)$ d $g(-x)$

Answer.

a 8 c 8

b 8 d 8

1.2.10.62. $f(t) = -3$

a $f(4)$ c $f(b-2)$

b $f(-3)$ d $f(-t)$

1.2.10.63. $P(x) = x^3 - 1$

a $P(2x)$ c $P(x^2)$

b $2P(x)$ d $[P(x)]^2$

Answer.

a $8x^3 - 1$ c $x^6 - 1$

b $2x^3 - 2$ d x^6-2x^3+1

1.2.10.64. $Q(t) = 5t^3$

a $Q(2t)$ c $Q(t^2)$

b $2Q(t)$ d $[Q(t)]^2$

In Problems 65—68, evaluate the function for the given expressions and simplify.

1.2.10.65. $f(x) = x^3$

a $f(a^2)$ c $f(ab)$

b $a^3 \cdot f(a^3)$ d $f(a+b)$

Answer.

a a^6 c a^3b^3

b a^{12} d $a^3+3a^2b+3ab^2+b^3$

1.2.10.66. $g(x) = x^4$

a $g(a^3)$ c $g(ab)$

b $a^4 \cdot g(a^4)$ d $g(a+b)$

1.2.10.67. $F(x) = 3x^5$

a $F(2a)$ c $F(a^2)$

b $2F(a)$ d $[F(a)]^2$

Answer.

a $96a^5$ c $3a^{10}$

b $6a^5$ d $9a^{10}$

1.2.10.68. $G(x) = 4x^3$

a $G(3a)$ c $G(a^4)$

b $3G(a)$ d $[G(a)]^4$

For the functions in Problems 69–76, compute the following:

a $f(2) + f(3)$ b $f(2+3)$ c $f(a) + f(b)$ d $f(a+b)$

For which functions does $f(a+b) = f(a) + f(b)$ for all values of a and b?

1.2.10.69.
$f(x) = 3x - 2$

Answer.

- a 11
- b 13
- c $3a+$ $3b-$
- 4
- d $3a+$ $3b-$ 2

This function does NOT satisfy $f(a+b) = f(a)+f(b)$.

1.2.10.70.
$f(x) = 1 - 4x$

1.2.10.71.
$f(x) = x^2 + 3$

Answer.

- a 19
- b 28
- c a^2+ b^2+
- 6
- d a^2+ $2ab+$ b^2+ 3

This function does NOT satisfy $f(a+b) = f(a)+f(b)$.

1.2.10.72.
$f(x) = x^2 - 1$

1.2.10.73.
$f(x) = \sqrt{x+1}$

Answer.

- a $\sqrt{3}+$ 2
- b $\sqrt{6}$
- c $\sqrt{a+1}+$ $\sqrt{b+1}$
- d $\sqrt{a+b+1}$

This function does NOT satisfy $f(a+b) = f(a)+f(b)$.

1.2.10.74.
$f(x) = \sqrt{6-x}$

1.2.10.75.
$f(x) = \dfrac{-2}{x}$

Answer.

- a $\dfrac{-5}{3}$
- b $\dfrac{-2}{5}$
- c $\dfrac{-2}{a}-$
- $\dfrac{-2}{b}$
- d $\dfrac{-2}{a+b}$

This function does NOT satisfy $f(a+b) = f(a)+f(b)$.

1.2.10.76. $f(x) = \dfrac{3}{x}$

1.2.10.77. Use a table of values to estimate a solution to

$$f(x) = 800 + 6x - 0.2x^2 = 500$$

as follows:

a Make a table starting at $x = 0$ and increasing by $\Delta x = 10$, as shown in the accompanying tables. Find two x-values a and b so that $f(a) > 500 > f(b)$.

x	0	10	20	30	40	50	60	70	80	90	100
$f(x)$											

b Make a new table starting at $x = a$ and increasing by $\Delta x = 1$. Find two x-values, c and d, so that $f(c) > 500 > f(d)$.

c Make a new table starting at $x = c$ and increasing by $\Delta x = 0.1$. Find two x-values, p and q, so that $f(p) > 500 > f(q)$.

d Take the average of p and q, that is, set $s = \dfrac{p+q}{2}$. Then s is an approximate solution that is off by at most 0.05.

e Evaluate $f(s)$ to check that the output is approximately 500.

Answer.

a

x	0	10	20	30	40	50	60	70	80	90
$f(x)$	800	840	840	800	720	600	440	240	0	-280
-600										

$a = 50$ and $b = 60$

b

x	50	51	52	53	54	55	56	57	58
$f(x)$	600	585.8	571.2	556.2	540.8	525	508.8	492.2	475.2

$c = 56$ and $d = 57$

c

x	56	56.1	56.2	56.3	56.4	56.5	56.6
$f(x)$	508.8	507.158	505.512	503.862	502.208	500.55	498.888

$p = 56.5$ and $q = 56.6$

d $s = 56.55$

e $f(56.55) = 499.7195$

1.2.10.78. Use a table of values to estimate a solution to

$$f(x) = x^3 - 4x^2 + 5x = 18,000$$

as follows:

a Make a table starting at $x = 0$ and increasing by $\Delta x = 10$, as shown in the accompanying tables. Find two x-values a and b so that $f(a) < 18,000 < f(b)$.

x	0	10	20	30	40	50	60	70	80	90	100
$f(x)$											

b Make a new table starting at $x = a$ and increasing by $\Delta x = 1$. Find two x-values, c and d, so that $f(c) < 18,000 < f(d)$.

c Make a new table starting at $x = c$ and increasing by $\Delta x = 0.1$. Find two x-values, p and q, so that $f(p) < 18,000 < f(q)$.

d Take the average of p and q, that is, set $s = \dfrac{p+q}{2}$. Then s is an approximate solution that is off by at most 0.05.

e Evaluate $f(s)$ to check that the output is approximately 18,000.

1.2.10.79. Use tables of values to estimate the positive solution to

$$f(x) = x^2 - \frac{1}{x} = 9000,$$

accurate to within 0.05.

Answer. 94.85

1.2.10.80. Use tables of values to estimate the positive solution to

$$f(x) = \frac{8}{x} + 500 - \frac{x^2}{9} = 300,$$

accurate to within 0.05.

1.2.10.81. Let $f(x) = -5x - 5$ and $g(x) = 2x^2 + 1$. Evaluate each of the following.

a $g(f(4))$

b $f(g(-4))$

c $f(f(2))$

d $g(g(4))$

e $(g - f)(2)$

f $(fg)(-4)$

1.2.10.82. Answer "True" or "False": $f(g(x))$ must always equal $g(f(x))$.

1.2.10.83. Suppose $f(x) = 3x + 1$ and $g(x) = |x|$. Evaluate each of the following.

a $f(x) + g(x)$

b $g(x) - f(x)$

c $f(x)g(x)$

d $f(x)/g(x)$

e $f(g(x))$

f $g(f(x))$

1.2.10.84. Let $f(x) = x^2 - 2$ and $g(x) = \sqrt{x} + 6$. Find $f(g(x))$ and $g(f(x))$.

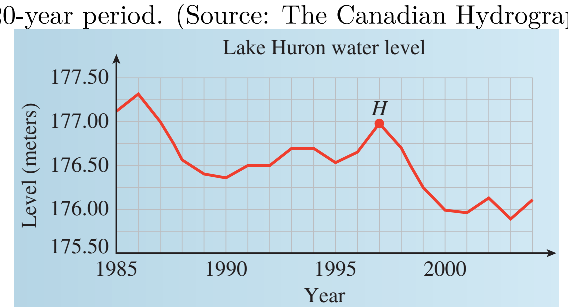
a $f(g(x))$

b $g(f(x))$

1.3 Graphs of Functions

1.3.1 Reading Function Values from a Graph

Checkpoint 1.3.2 The water level in Lake Huron alters unpredictably over time. The graph below gives the average water level, $L(t)$, in meters in the year t over a 20-year period. (Source: The Canadian Hydrographic Service)



a The coordinates of point H on the graph are $(1997, 176.98)$. What do the coordinates tell you about the function L?

b The average water level in 2004 was 176.11 meters. Write this fact in function notation. What can you say about the graph of L?

Answer.

a $L(1997) = 176.98$; the average water level was 176.98 meters in 1997.

b $L(2004) = 176.11$. The point $(2004, 176.11)$ lies on the graph of L.

Checkpoint 1.3.4 Refer to the graph of the function g shown in Example 1.3.3.

a Find $g(0)$.

b For what value(s) of t is $g(t) = 0$?

c What is the smallest, or minimum, value of $g(t)$? For what value of t does the function take on its minimum value?

d On what intervals is g decreasing?

Answer.

a 3

b $-2, 2, 4$

c -3; $t = -4$

d $(-5, -4)$ and $(1, 3)$

1.3.2 Constructing the Graph of a Function

Checkpoint 1.3.6 $f(x) = x^3 - 2$

a Complete the table of values and sketch a graph of the function.

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$f(x)$							

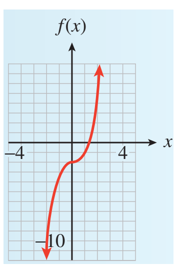
b Use your calculator to make a table of values and graph the function.

Answer.

a

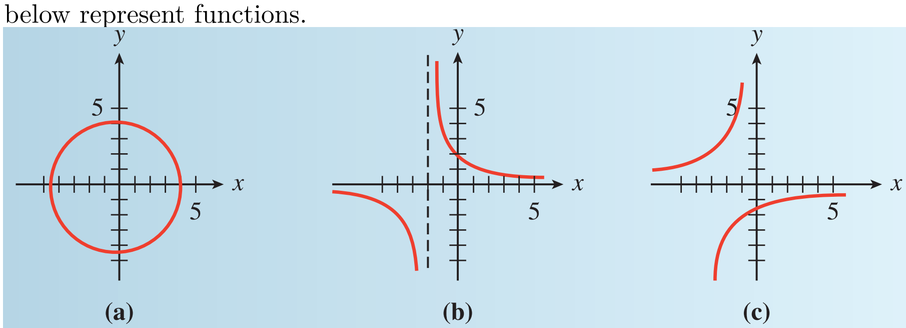
x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$f(x)$	-10	-3	$\frac{-17}{8}$	-2	$\frac{-15}{8}$	-1	6

b



1.3.3 The Vertical Line Test

Checkpoint 1.3.8 Use the vertical line test to determine which of the graphs below represent functions.



Answer. Only (b) is a function.

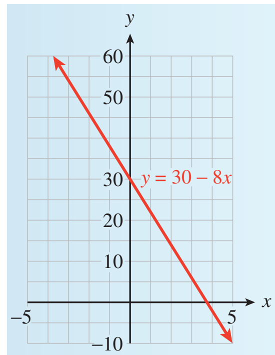
1.3.4 Graphical Solution of Equations and Inequalities

Checkpoint 1.3.11

a Use the graph of $y = 30 - 8x$ shown in the figure to solve the equation

$$30 - 8x = 50$$

b Verify your solution algebraically.



Answer. -2.5

Checkpoint 1.3.13

a Use the graph of $y = 30 - 8x$ in the previous Checkpoint to solve the inequality

$$30 - 8x \leq 50$$

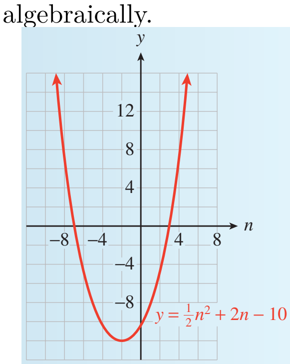
b Solve the inequality algebraically.

Answer. $x \geq -2.5$

Checkpoint 1.3.15 Use the graph of $y = \frac{1}{2}n^2 + 2n - 10$ shown below to solve

$$\frac{1}{2}n^2 + 2n - 10 = 6$$

and verify your solutions algebraically.



Answer. $-8, 4$

Checkpoint 1.3.17 Use the graph in Checkpoint 1.3.15 to solve the inequality

$$\frac{1}{2}n^2 + 2n - 10 < 6$$

Answer. $(-8, 4)$

1.3.6 Homework 1.3

In Problems 1–8, use the graphs to answer the questions about the functions.

1.3.6.1.

a Find $h(-3)$, $h(1)$, and $h(3)$.

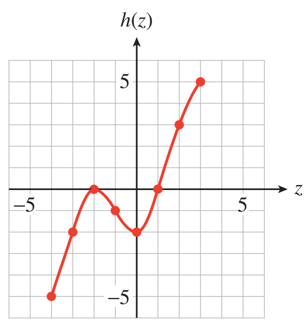
b For what value(s) of z is $h(z) = 3$?

c Find the intercepts of the graph. List the function values given by the intercepts.

d What is the maximum value of $h(z)$?

e For what value(s) of z does h take on its maximum value?

f On what intervals is the function increasing? Decreasing?



Answer.

a $-2, 0, 5$

b 2

c $h(-2) = 0,\ h(1) = 0,\ h(0) = -2$

d 5

e 3

f Increasing: $(-3, 0)$ and $(1, 3)$; decreasing: $(0, 1)$ and $(3, 5)$

1.3.6.2.

a Find $G(-3)$, $G(-1)$, and $G(2)$.

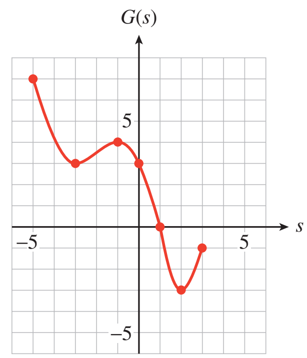
b For what value(s) of s is $G(s) = 3$?

c Find the intercepts of the graph. List the function values given by the intercepts.

d What is the minimum value of $G(s)$?

e For what value(s) of s does G take on its minimum value?

f On what intervals is the function increasing? Decreasing?



1.3.6.3.

a Find $R(1)$ and $R(3)$.

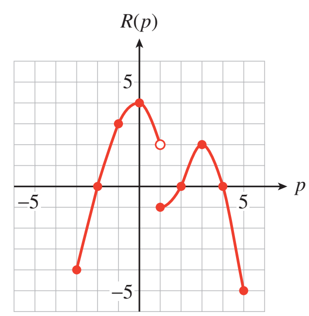
b For what value(s) of p is $R(p) = 2$?

c Find the intercepts of the graph. List the function values given by the intercepts.

d Find the maximum and minimum values of $R(p)$.

e For what value(s) of p does R take on its maximum and minimum values?

f On what intervals is the function increasing? Decreasing?



Answer.

a $-1, 2$

b $3, -1.3$

c $R(-2) = 0,\ R(2) = 0,\ R(4) = 0,\ R(0) = 4$

d Max: 4; min: -5

e Max at $p = 0$; min at $p = 5$

f Increasing: $(-3, 0)$ and $(1, 3)$; decreasing: $(0, 1)$ and $(3, 5)$

1.3.6.4.

a Find $f(-1)$ and $f(3)$.

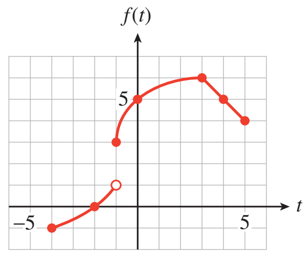
b For what value(s) of t is $f(t) = 5$?

c Find the intercepts of the graph. List the function values given by the intercepts.

d Find the maximum and minimum values of $f(t)$.

e For what value(s) of t does f take on its maximum and minimum values?

f On what intervals is the function increasing? Decreasing?



1.3.6.5.

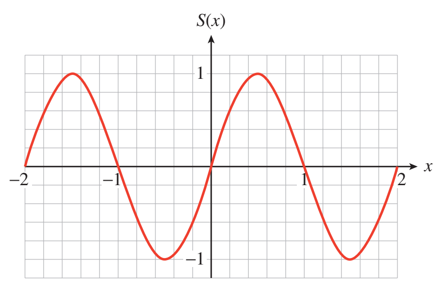
a Find $S(0)$, $S\left(\frac{1}{6}\right)$, and $S(-1)$.

b Estimate the value of $S\left(\frac{1}{3}\right)$ from the graph.

c For what value(s) of x is $S(x) = -\frac{1}{2}$?

d Find the maximum and minimum values of $S(x)$.

e For what value(s) of x does S take on its maximum and minimum values?



Answer.

a $0, \frac{1}{2}, 0$

b 0.9

c $\frac{-5}{6}, \frac{-1}{6}, \frac{7}{6}, \frac{11}{6}$

d Max: 1; min: -1

e Max at $x = -1.5, 0.5$; min at $x = -0.5, 1.5$

1.3.6.6.

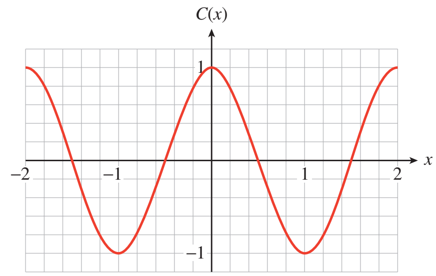
a Find $C(0)$, $C\left(-\frac{1}{3}\right)$, and $C(1)$.

b Estimate the value of $C\left(\frac{1}{6}\right)$ from the graph.

c For what value(s) of x is $C(x) = \frac{1}{2}$?

d Find the maximum and minimum values of $C(x)$.

e For what value(s) of x does C take on its maximum and minimum values?



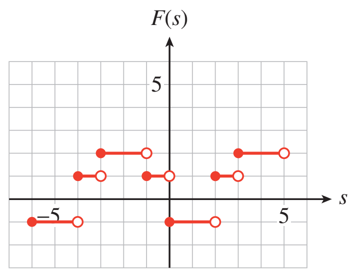
1.3.6.7.

a Find $F(-3)$, $F(-2)$, and $F(2)$.

b For what value(s) of s is $F(s) = -1$?

c Find the maximum and minimum values of $F(s)$.

d For what value(s) of s does F take on its maximum and minimum values?



Answer.

a $2, 2, 1$

b $-6 \le s < -4$ or $0 \le s < 2$

c Max: 2; min: -1

d Max for $-3 \le s < -1$ or $3 \le s < 5$; min for $-6 \le s < -4$ or $0 \le s < 2$

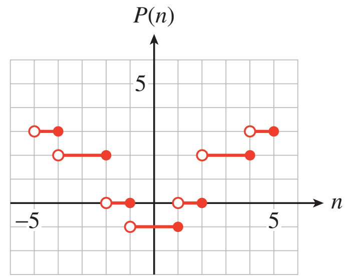
1.3.6.8.

a Find $P(-3)$, $P(-2)$, and $P(1)$.

b For what value(s) of n is $P(n) = 0$?

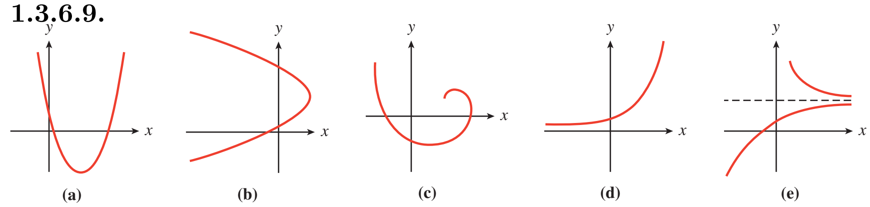
c Find the maximum and minimum values of $P(n)$.

d For what value(s) of n does P take on its maximum and minimum values?



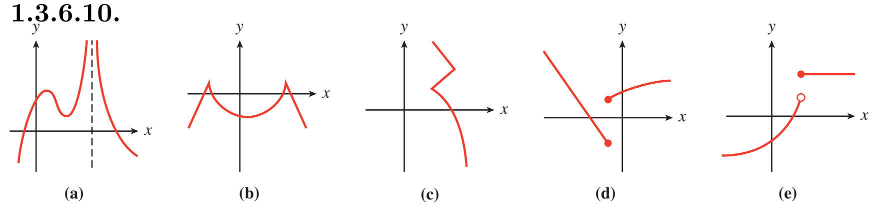
Which of the graphs in Problems 9 and 10 represent functions?

1.3.6.9.



Answer. (a) and (d)

1.3.6.10.



In Problems 11–16,

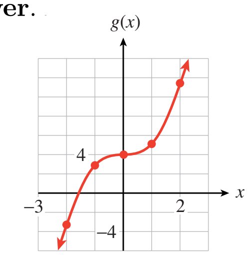
a Make a table of values and sketch a graph of the function by plotting points. (Use the suggested x-values.)

b Use your calculator to graph the function.

Compare the calculator's graph with your sketch.

1.3.6.11. $g(x) = x^3 + 4$;
$x = -2, -1, \ldots, 2$

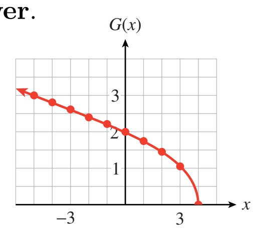
Answer.



1.3.6.12. $h(x) = 2 + \sqrt{x}$;
$x = 0, 1, \ldots, 9$

1.3.6.13. $G(x) = \sqrt{4 - x}$;
$x = -5, -4, \ldots, 4$

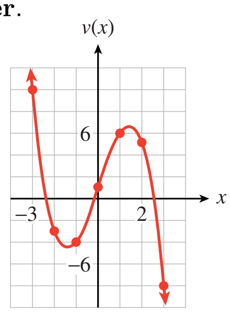
Answer.



1.3.6.14. $F(x) = \sqrt{x - 1}$;
$x = 1, 2, \ldots, 10$

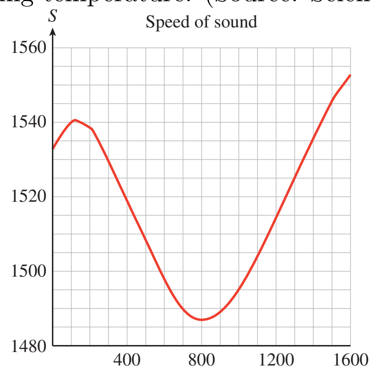
1.3.6.15. $v(x) = 1 + 6x - x^3$;
$x = -3, -2, \ldots, 3$

Answer.



1.3.6.16. $w(x) = x^3 - 8x$;
$x = -4, -3, \ldots, 4$

1.3.6.17. The graph shows the speed of sound in the ocean as a function of depth, $S = f(d)$. The speed of sound is affected both by increasing water pressure and by dropping temperature. (Source: Scientific American)



a Evaluate $f(1000)$ and explain its meaning.

b Solve $f(d) = 1500$ and explain its meaning.

c At what depth is the speed of sound the slowest, and what is the speed? Write your answer with function notation.

d Describe the behavior of $f(d)$ as d increases.

Answer.

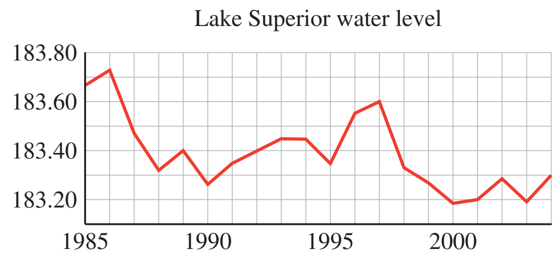
a $f(1000) = 1495$: The speed of sound at a depth of 1000 meters is approximately 1495 meters per second.

b $d = 570$ or $d = 1070$: The speed of sound is 1500 meters per second at both a depth of 570 meters and a depth of 1070 meters.

c The slowest speed occurs at a depth of about 810 meters and the speed is about 1487 meters per second, so $f(810) = 1487$.

d f increases from about 1533 to 1541 in the first 110 meters of depth, then drops to about 1487 at 810 meters, then rises again, passing 1553 at a depth of about 1600 meters.

1.3.6.18. The graph shows the water level in Lake Superior as a function of time, $L = f(t)$. (Source: The Canadian Hydrographic Service)



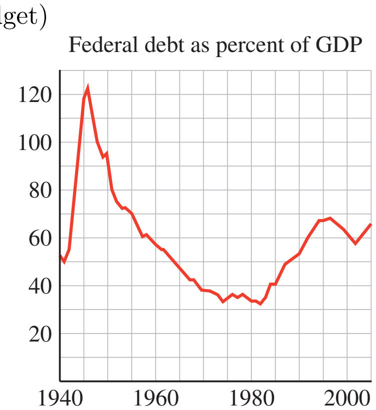
a Evaluate $f(1997)$ and explain its meaning.

b Solve $f(t) = 183.5$ and explain its meaning.

c In which two years did Lake Superior reach its highest levels, and what were those levels? Write your answers with function notation.

d Over which two-year period did the water level drop the most?

1.3.6.19. The graph shows the federal debt as a percentage of the gross domestic product (GDP), as a function of time, $D = f(t)$. (Source: Office of Management and Budget)



a Evaluate $f(1985)$ and explain its meaning.

b Solve $f(t) = 70$ and explain its meaning.

c When did the federal debt reach its highest level since 1960, and what was that level? Write your answer with function notation.

d What is the longest time interval over which the federal debt was decreasing?

Answer.

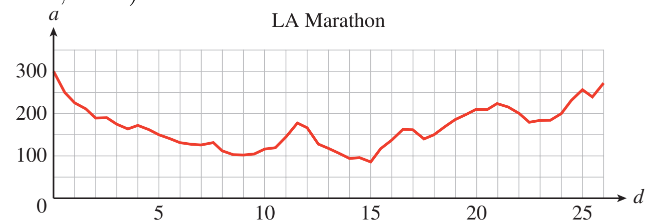
a $f(1985) = 41$: The federal debt in 1985 was about 41% of the gross domestic product.

b $t = 1942$ or $t = 1955$: The federal debt was 70% of the gross domestic product in 1942 and 1955.

c In about 1997, the debt was about 67% of the gross domestic product, so $f(1997) \approx 67.3$.

d The percentage basically dropped from 1946 to 1973, but there were small rises around 1950, 1954, 1958, and 1968, so the longest time interval was from 1958 to 1967.

1.3.6.20. The graph shows the elevation of the Los Angeles Marathon course as a function of the distance into the race, $a = f(t)$. (Source: Los Angeles Times, March 3, 2005)



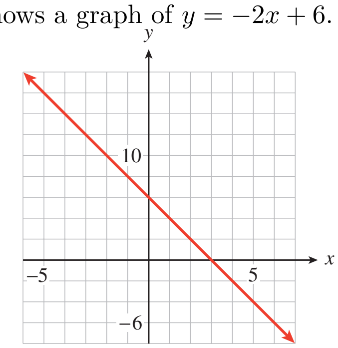
a Evaluate $f(5)$ and explain its meaning.

b Solve $f(d) = 200$ and explain its meaning.

c When does the marathon course reach its lowest elevation, and what is that elevation? Write your answer with function notation.

d Give three intervals over which the elevation is increasing.

1.3.6.21. The figure shows a graph of $y = -2x + 6$.



a Use the graph to find all values of x for which

I $y = 12$

II $y > 12$

III $y < 12$

b Use the graph to solve

I $-2x + 6 = 12$

II $-2x + 6 > 12$

III $-2x + 6 < 12$

c Explain why your answers to parts (a) and (b) are the same.

Answer.

a
i $x = -3$

ii $x < -3$

iii $x > -3$

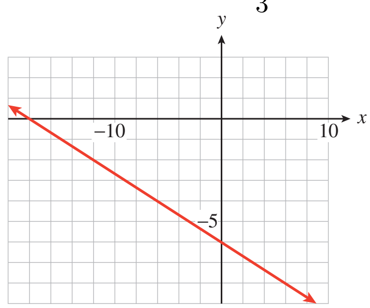
b
I $x = -3$

II $x < -3$

III $x > -3$

c On the graph of $y = -2x + 6$, a value of y is the same as a value of $-2x + 6$, so parts (a) and (b) are asking for the same x's.

1.3.6.22. The figure shows a graph of $y = \dfrac{-x}{3} - 6$.



a Use the graph to find all values of x for which

i $y = -4$

ii $y > -4$

iii $y < -4$

b Use the graph to solve

i $\dfrac{-x}{3} - 6 = -4$

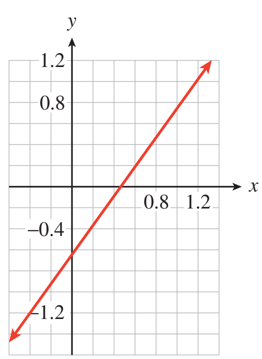
ii $\dfrac{-x}{3} - 6 > -4$

iii $\dfrac{-x}{3} - 6 < -4$

c Explain why your answers to parts (a) and (b) are the same.

In Problems 23 and 24, use the graph to solve the equation or inequality, and then solve algebraically. (To review solving linear inequalities algebraically, see Algebra Skills Refresher A.2.)

1.3.6.23. The figure shows the graph of $y = 1.4x - 0.64$. Solve the following:



a $1.4x - 0.64 = 0.2$

b $-1.2 = 1.4x - 0.64$

c $1.4x - 0.64 > 0.2$

d $-1.2 > 1.4x - 0.64$

Answer.

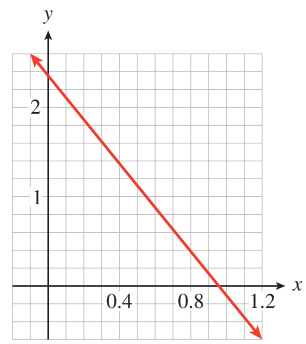
a $x = 0.6$

b $x = -0.4$

c $x > 0.6$

d $x < -0.4$

1.3.6.24. The figure shows the graph of $y = -2.4x + 2.32$. Solve the following:



a $1.6 = -2.4x + 2.32$

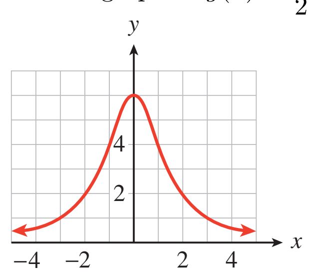
b $-2.4x + 2.32 = 0.4$

c $-2.4x + 2.32 \geq 1.6$

d $0.4 \geq -2.4x + 2.32$

For Problems 25–30, use the graphs to estimate solutions to the equations and inequalities.

1.3.6.25. The figure shows the graph of $g(x) = \dfrac{12}{2 + x^2}$.



a Solve $\dfrac{12}{2 + x^2} = 4$

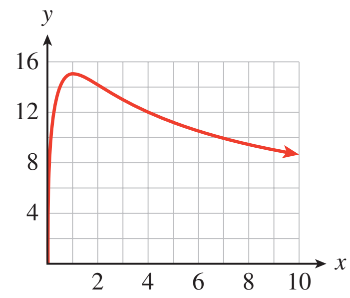
b Solve $1 \leq \dfrac{12}{2 + x^2} \leq 2$

Answer.

a $x = -1$ or $x = 1$

b Approximately $-3 \leq x \leq -2$ or $2 \leq x \leq 3$

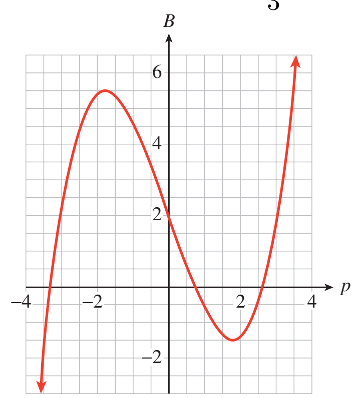
1.3.6.26. The figure shows the graph of $f(x) = \dfrac{30\sqrt{x}}{1 + x}$.



a Solve $\dfrac{30\sqrt{x}}{1+x} = 15$

b Solve $\dfrac{30\sqrt{x}}{1+x} < 12$

1.3.6.27. The figure shows a graph of $B = \dfrac{1}{3}p^3 - 3p + 2$.



a Solve $\dfrac{1}{3}p^3 - 3p + 2 = 6$

b Solve $\dfrac{1}{3}p^3 - 3p + 2 = 5$

c Solve $\dfrac{1}{3}p^3 - 3p + 2 < 1$

d What range of values does B have for p between -2.5 and 0.5?

e For what values of p is B increasing?

Answer.

a 3.5

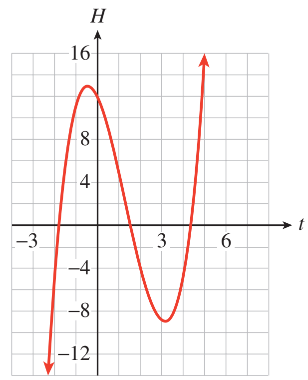
b $-2.2, -1.2, 3.4$

c $p < -3.1$ or $0.3 < p < 2.8$

d $0.5 < B < 5.5$

e $p < -1.7$ or $p > 1.7$

1.3.6.28. The figure shows a graph of $H = t^3 - 4t^2 - 4t + 12$.



a Solve $t^3 - 4t^2 - 4t + 12 = -4$

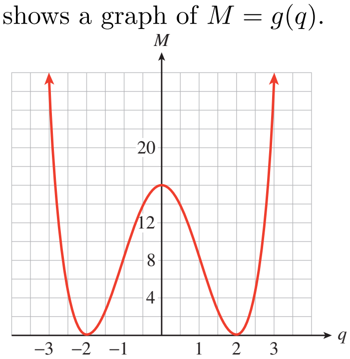
b Solve $t^3 - 4t^2 - 4t + 12 = 16$

c Solve $t^3 - 4t^2 - 4t + 12 > 6$

d Estimate the horizontal and vertical intercepts of the graph.

e For what values of t is H increasing?

1.3.6.29. The figure shows a graph of $M = g(q)$.



a Find all values of q for which

I $g(q) = 0$

II $g(q) = 16$

III $g(q) < 6$

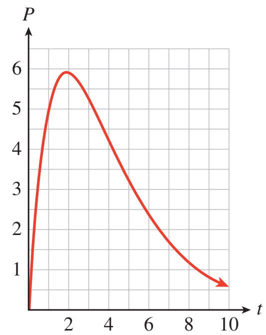
b For what values of q is $g(q)$ increasing?

Answer.

a
- i $-2, 2$
- ii $-2.8, 0, 2.8$
- iii $-2.5 < q < -1.25$ or $1.25 < q < 2.5$

b $-2 < q < 0$ or $q > 2$

1.3.6.30. The figure shows a graph of $P = f(t)$.



a Find all values of t for which

I $f(t) = 3$

II $f(t) > 4.5$

III $2 \leq f(t) \leq 4$

b For what values of t is $f(t)$ decreasing?

1.3.6.31.

a Delbert reads the following values from the graph of a function:

$$f(-3) = 5,\ f(-1) = 2,\ f(1) = 0,$$

$$f(-1) = -4,\ f(-3) = -2$$

Can his readings be correct? Explain why or why not.

b Francine reads the following values from the graph of a function:

$$g(-2) = 6,\ g(0) = 0,\ g(2) = 6,$$

$$g(4) = 0,\ g(6) = 6$$

Can her readings be correct? Explain why or why not.

Answer.

a He has an error: $f(-3)$ cannot have both the value 5 and also the value -2, and $f(-1)$ cannot have both values 2 and -4.

b Her readings are possible for a function: each input has only one output.

1.3.6.32.

a Sketch the graph of a function that has the following values:

$$F(-2) = 3,\ F(-1) = 3,\ F(0) = 3,$$

$$F(1) = 3,\ F(2) = 3$$

b Sketch the graph of a function that has the following values:

$$G(-2) = 1,\ G(-1) = 0,\ G(0) = -1,$$

$$G(1) = 0,\ G(2) = 1$$

For Problems 33–36, graph each function in the friendly window

Xmin $= -9.4$ Xmax $= 9.4$

Ymin = -10 Ymax = 10

Then answer the questions about the graph. (See Appendix B for an explanation of friendly windows.)

1.3.6.33. $g(x) = \sqrt{36 - x^2}$

a Complete the table. (Round values to tenths.)

x	-4	-2	3	5
$g(x)$				

b Find all points on the graph for which $g(x) = 3.6$.

Answer.

a

x	-4	-2	3	5
$g(x)$	4.5	5.7	5.2	3.3

b $-4.8, 4.8$

1.3.6.34. $g(x) = \sqrt{x^2} - 6$

(a) Complete the table. (Round values to tenths.)

x	-8	-2	3	6
$f(x)$				

(b) Find all points on the graph for which $f(x) = -2$.

1.3.6.35. $F(x) = 0.5x^3 - 4x$

a Estimate the coordinates of the turning points of the graph, that is, where the graph changes from increasing to decreasing or vice versa.

b Write an equation of the form $F(a) = b$ for each turning point.

Answer.

a $(-1.6, 4.352), (1.6, -4.352)$

b $F(-1.6) = 4.352$; $F(1.6) = -4.352$

1.3.6.36. $G(x) = 2 + 4x - x^3$

a Estimate the coordinates of the turning points of the graph, that is, where the graph changes from increasing to decreasing or vice versa.

b Write an equation of the form $G(a) = b$ for each turning point.

For Problems 37–40, graph the function

a First using the standard window.

b Then using the suggested window. Explain how the window alters the appearance of the graph in each case.

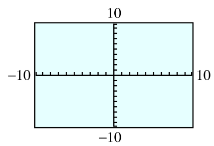
1.3.6.37. $h(x) = \dfrac{1}{x^2+10}$

$\text{Xmin} = -5 \qquad \text{Xmax} = 5$

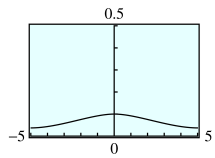
$\text{Ymin} = 0 \qquad \text{Ymax} = 0.5$

Answer.

a



b



The curve cannot be distinguished from the x-axis in the standard window because the values of y are closer to zero than the resolution of the calculator can display. The second window provides sufficient resolution to see the curve.

1.3.6.38. $H(x) = \sqrt{1-x^2}$

$\text{Xmin} = -2 \qquad \text{Xmax} = 2$

$\text{Ymin} = -2 \qquad \text{Ymax} = 2$

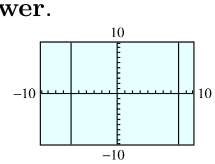
1.3.6.39.
$P(x) = (x-8)(x+6)(x-15)$

$\text{Xmin} = -10 \qquad \text{Xmax} = 20$

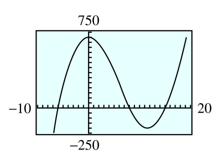
$\text{Ymin} = -250 \qquad \text{Ymax} = 750$

Answer.

a



b



The curve looks like two vertical lines in the standard window because that window covers too small a region of the plane. The second window allows us to see the turning points of the curve.

1.3.6.40. $p(x) = 200x^3$

$\text{Xmin} = -5 \qquad \text{Xmax} = 5$

$\text{Ymin} = -10,000 \qquad \text{Ymax} = 10,000$

For Problems 41–44, graph the equation with the ZInteger setting. (Press `ZOOM` 6,then `ZOOM` 8 `ENTER`.) Use the graph to answer each question. Use the equation to verify your answers.

1.3.6.41. Graph $y = 2x - 3$

a For what value of x is $y = 5$?

b For what value of x is $y = -13$?

c For what values of x is $y > -1$?

d For what values of x is $y < 25$?

Answer.

a $x = 4$ b $x = -5$ c $x > 1$ d $x < 14$

1.3.6.42. Graph $y = 4 - 2x$

a For what value of x is $y = 6$?

b For what value of x is $y = -4$?

c For what values of x is $y > -12$?

d For what values of x is $y < 18$?

1.3.6.43. Graph $y = 6.5 - 1.8x$

a For what value of x is $y = -13.3$?

b For what value of x is $y = 24.5$?

c For what values of x is $y \leq 15.5$?

d For what values of x is $y \geq -7.9$?

Answer.

a $x = 11$ b $x = -10$ c $x \geq -5$ d $x \leq 8$

1.3.6.44. Graph $y = 0.2x + 1.4$

a For what value of x is $y = -5.2$?

b For what value of x is $y = 2.8$?

c For what values of x is $y \leq -3.2$?

d For what values of x is $y \geq 4.4$?

For Problems 45–48, graph the equation with the ZInteger setting. Use the graph to solve each equation or inequality. Check your solutions algebraically.

1.3.6.45. Graph $y = -0.4x + 3.7$

a Solve $-0.4x + 3.7 = 2.1$

b Solve $-0.4x + 3.7 > -5.1$

Answer.

a $x = 4$ b $x < 22$

1.3.6.46. Graph $y = 0.4(x - 1.5)$

a Solve $0.4(x - 1.5) = -8.6$

b Solve $0.4(x - 1.5) < 8.6$

1.3.6.47. Graph $y = \frac{2}{3}x - 24$

a Solve $\frac{2}{3}x - 24 = -10\frac{2}{3}$

b Solve $\frac{2}{3}x - 24 \leq -19\frac{1}{3}$

Answer.

a $x = 20$

b $x \leq 7$

1.3.6.48. Graph $y = \frac{80 - 3x}{5}$.

a Solve $\frac{80 - 3x}{5} = 22\frac{3}{5}$.

b Solve $\frac{80 - 3x}{5} \leq -9\frac{2}{5}$.

1.3.6.49. Graph $y = 0.01x^3 - 0.1x^2 - 2.75x + 15$.

a Use your graph to solve $0.01x^3 - 0.1x^2 - 2.75x + 15 = 0$.

b Press `Y=` and enter $Y_2 = 10$. Press `GRAPH`, and you should see the horizontal line $y = 10$ superimposed on your previous graph. How many solutions does the equation

$$0.01x^3 - 0.1x^2 - 2.75x + 15 = 10$$

have? Estimate each solution to the nearest whole number.

Answer.

a $-15, 5, 20$

b $-13, 2, 22$

1.3.6.50. Graph $y = 2.5x - 0.025x^2 - 0.005x^3$.

a Use your graph to solve $2.5x - 0.025x^2 - 0.005x^3 = 0$.

b Press `Y=` and enter $Y_2 = -5$. Press `GRAPH`, and you should see the horizontal line $y = -5$ superimposed on your previous graph. How many solutions does the equation

$$2.5x - 0.025x^2 - 0.005x^3 = -5$$

have? Estimate each solution to the nearest whole number.

1.4 Slope and Rate of Change

1.4.1 Using Ratios for Comparison

Checkpoint 1.4.2 Delbert traveled 258 miles on 12 gallons of gas, and Francine traveled 182 miles on 8 gallons of gas. Compute the ratio $\frac{\text{miles}}{\text{gallon}}$ for each car. Whose car gets the better gas mileage?

Answer. Delbert: 21.5 mpg, Francine: 22.75 mpg. Francine gets better mileage.

1.4.2 Measuring Steepness

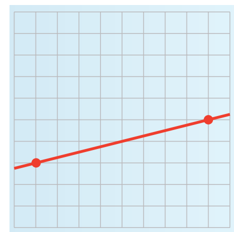
Checkpoint 1.4.4 Which is steeper, a staircase that rises 10 feet over a horizontal distance of 4 feet, or the steps in the football stadium, which rise 20 yards over a horizontal distance of 12 yards?

Answer. The staircase is steeper.

1.4.3 Definition of Slope

Checkpoint 1.4.6

Compute the slope of the line through the indicated points on the graph at right. On both axes, one square represents one unit.



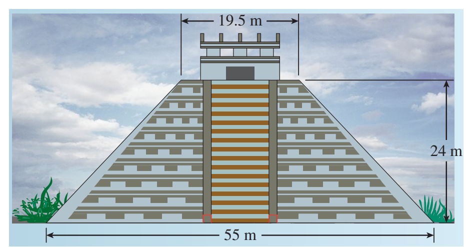
$$\frac{\text{change in } y\text{-coordinate}}{\text{change in } x\text{-coordinate}} =$$

Answer. $\frac{1}{4}$

1.4.4 Notation for Slope

Checkpoint 1.4.9 The Kukulcan Pyramid at Chichen Itza in Mexico was built around 800 A.D. It is 24 meters high, with a temple built on its top platform, as shown below.

The square base is 55 meters on each side, and the top platform is 19.5 meters on each side. Calculate the slope of the sides of the pyramid. Which pyramid is steeper, Kukulcan or the Great Pyramid?



Answer. 1.35; Kukulcan is steeper.

1.4.5 Lines Have Constant Slope

Checkpoint 1.4.11

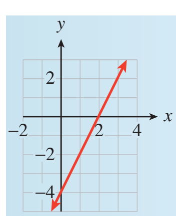
a Graph the line $4x - 2y = 8$ by finding the x- and y-intercepts

b Compute the slope of the line using the x-intercept and y-intercept.

c Compute the slope of the line using the points $(4, 4)$ and $(1, -2)$.

Answer.

a



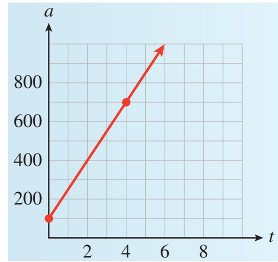
b 2

c 2

1.4.6 Meaning of Slope

Checkpoint 1.4.14

The graph shows the altitude, a (in feet), of a skier t minutes after getting on a ski lift.



a Choose two points and compute the slope (including units).

b What does the slope tell us about the problem?

Answer.

a 150

b Altitude increases by 150 feet per minute.

1.4.7 A Formula for Slope

Checkpoint 1.4.16

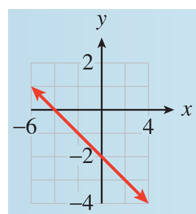
a Find the slope of the line passing through the points $(2, -3)$ and $(-2, -1)$.

b Sketch a graph of the line by hand.

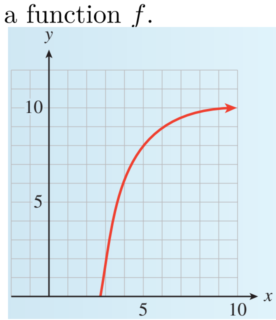
Answer.

a $\dfrac{-1}{2}$

b



Checkpoint 1.4.18 The figure shows the graph of a function f.



a Find $f(3)$ and $f(5)$.

b Compute the slope of the line segment joining the points at $x = 3$ and $x = 5$.

c Write an expression for the slope of the line segment joining the points at $x = a$ and $x = b$.

Answer.

a $f(3) = 2,\ f(5) = 8$

b 3

c $\dfrac{f(b) - f(a)}{b - a}$

1.4.9 Homework 1.4

Compute ratios to answer the questions in Problems 1–4.

1.4.9.1. Carl runs 100 meters in 10 seconds. Anthony runs 200 meters in 19.6 seconds. Who has the faster average speed?

Answer. Anthony

1.4.9.2. On his 512-mile round trip to Las Vegas and back, Corey needed 16 gallons of gasoline. He used 13 gallons of gasoline on a 429-mile trip to Los Angeles. On which trip did he get better fuel economy?

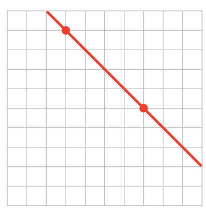
1.4.9.3. Grimy Gulch Pass rises 0.6 miles over a horizontal distance of 26 miles. Bob's driveway rises 12 feet over a horizontal distance of 150 feet. Which is steeper?

Answer. Bob's driveway

1.4.9.4. Which is steeper, the truck ramp for Acme Movers, which rises 4 feet over a horizontal distance of 9 feet, or a toy truck ramp, which rises 3 centimeters over a horizontal distance of 7 centimeters?

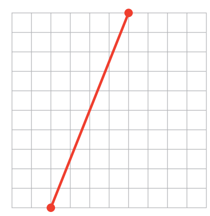
In Problems 5-8, compute the slope of the line through the indicated points. On both axes, one square represents one unit.

1.4.9.5.

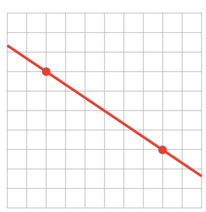


Answer. -1

1.4.9.6.

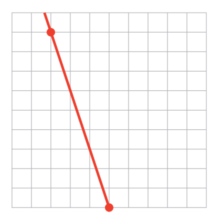


1.4.9.7.



Answer. $\dfrac{-2}{3}$

1.4.9.8.



For Problems 9–14,

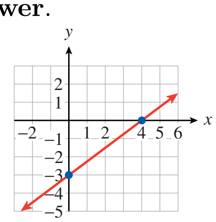
a Graph each line by the intercept method.

b Use the intercepts to compute the slope.

1.4.9.9. $3x - 4y = 12$

Answer.

a

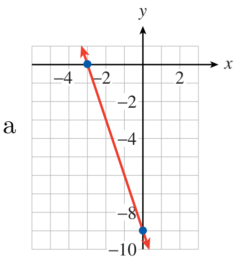


b $\dfrac{3}{4}$

1.4.9.10. $2y - 5x = 10$

1.4.9.11. $2y + 6x = -18$

Answer.

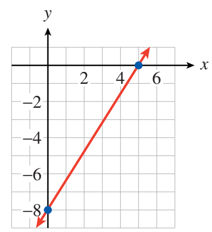
a 

b -3

1.4.9.12. $9x + 12y = 36$

1.4.9.13. $\dfrac{x}{5} - \dfrac{y}{8} = 1$

Answer.

a 

b $\dfrac{8}{5}$

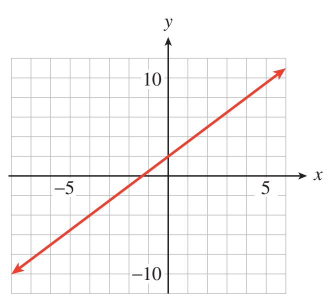
1.4.9.14. $\dfrac{x}{7} - \dfrac{y}{4} = 1$

1.4.9.15.

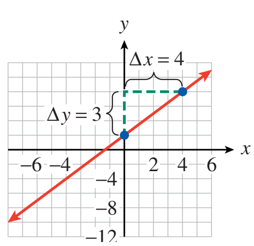
a Use the points $(0, 2)$ and $(4, 8)$ to compute the slope of the line. Illustrate Δy and Δx on the graph.

b Use the points $(-4, -4)$ and $(4, 8)$ to compute the slope of the line. Illustrate Δy and Δx on the graph.

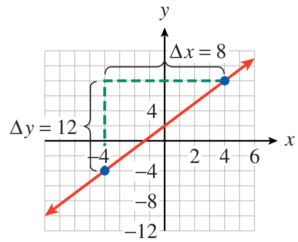
c Use the points $(0, 2)$ and $(-6, -7)$ to compute the slope of the line. Illustrate Δy and Δx on the graph.



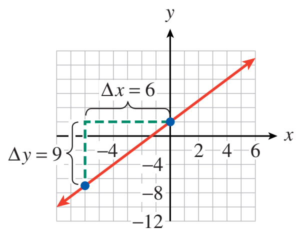
Answer.

a 

b



c

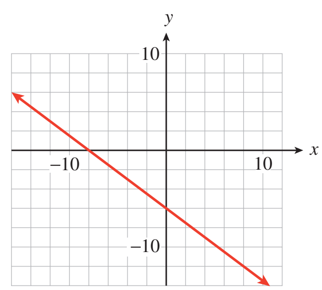


1.4.9.16.

a Use the points $(0, -6)$ and $(8, -12)$ to compute the slope of the line. Illustrate Δy and Δx on the graph.

b Use the points $(-8, 0)$ and $(4, -9)$ to compute the slope of the line. Illustrate Δy and Δx on the graph.

c Use the points $(4, -9)$ and $(0, -6)$ to compute the slope of the line. Illustrate Δy and Δx on the graph.



For Problems 17–20, use the formula $m = \dfrac{\Delta y}{\Delta x}$

1.4.9.17. A line has slope $\dfrac{-3}{4}$.

a Find the vertical change associated with each horizontal change along the line.

i $\Delta x = 4$	iii $\Delta x = 2$
ii $\Delta x = -8$	iv $\Delta x = -6$

b Find the horizontal change associated with each vertical change along the line.

i $\Delta y = 3$	iii $\Delta y = -2$
ii $\Delta y = -6$	iv $\Delta y = 1$

Answer.

a	i -3	ii 6	iii $\dfrac{-3}{2}$	iv $\dfrac{9}{2}$

b　i -4　ii 8　iii $\frac{8}{3}$　iv $\frac{4}{3}$

1.4.9.18. A line has slope $\frac{5}{3}$.

a Find the vertical change associated with each horizontal change along the line.

i $\Delta x = 3$
iii $\Delta x = 1$
ii $\Delta x = -6$
iv $\Delta x = -24$

b Find the horizontal change associated with each vertical change along the line.

i $\Delta y = -5$
iii $\Delta y = -1$
ii $\Delta y = -2.5$
iv $\Delta y = 3$

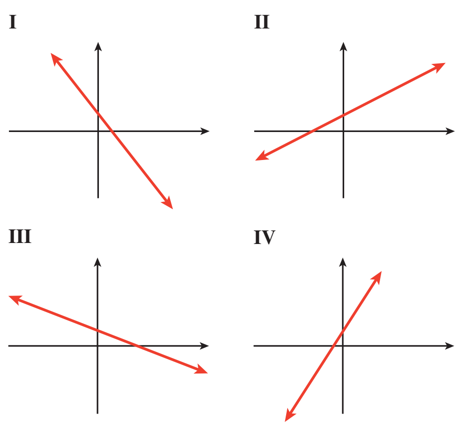
1.4.9.19. Residential staircases are usually built with a slope of 70%, or $\frac{7}{10}$. If the vertical distance between stories is 10 feet, how much horizontal space does the staircase require?

Answer. $\frac{100}{7}$ ft ≈ 14.286 ft ≈ 14 ft 3.4 in

1.4.9.20. A straight section of highway in the Midwest maintains a grade (slope) of 4%, or $\frac{1}{25}$, for 12 miles. How much does your elevation change as you travel the road?

1.4.9.21. Choose the line with the correct slope. The scales are the same on both axes.

a $m = 2$　b $m = -\frac{1}{2}$　c $m = \frac{2}{3}$　d $m = -\frac{5}{3}$

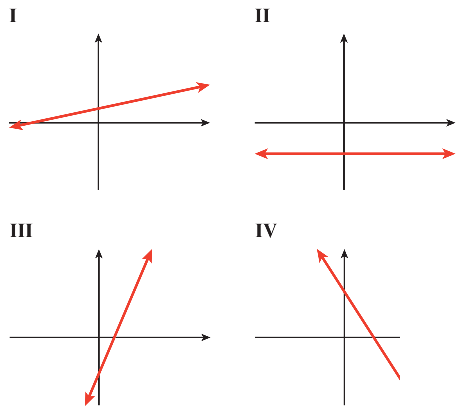


Answer.

a IV　b III　c II　d I

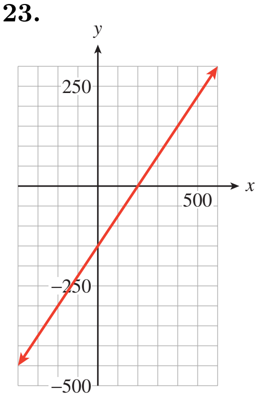
1.4.9.22. Choose the line with the correct slope. The scales are the same on both axes.

a $0 < m < 1$　b $m < -1$　c $m => 1$　d $m = 0$



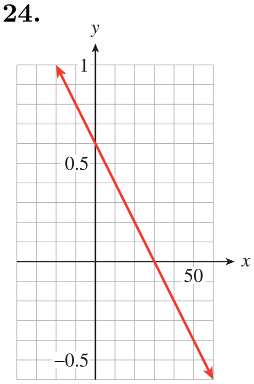
Compute the slope of the line in Problems 23-26. Note the scales on the axes.

1.4.9.23.

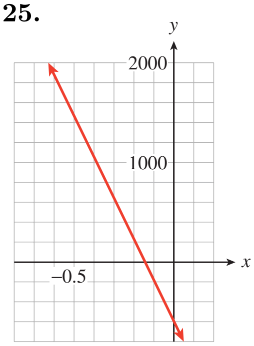


Answer. $\frac{3}{4}$

1.4.9.24.

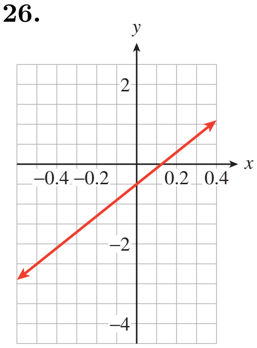


1.4.9.25.



Answer. -4000

1.4.9.26.



Each table in Problems 27–30 gives the coordinates of points on a line.

a Find the slope of the line.

b Fill in the missing table entries.

1.4.9.27.

x	y
-4	-14
-2	-9
2	1
3	
	11

Answer.

a $\frac{5}{2}$

b

x	y
3	$\frac{7}{2}$
6	11

1.4.9.28.

x	y
-5	-3.8
-1	-0.6
2	1.8
	4.2
7	

1.4.9.29.

x	y
-3	36
-1	
	12
6	9
10	-3

Answer.

a -3

b

x	y
-1	**30**
5	12

1.4.9.30.

x	y
-10	800
-2	
5	440
	368
16	176

1.4.9.31. A temporary typist's paycheck (before deductions) is given, in dollars, by $S = 8t$, where t is the number of hours she worked.

(a) Make a table of values for the function.

t	4	8	20	40
S				

(b) Graph the function.

(c) Using two points on the graph, compute the slope $\frac{\Delta S}{\Delta t}$, including units.

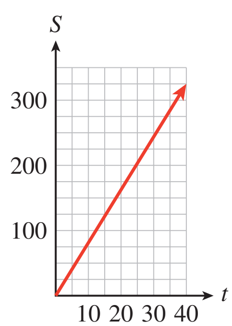
(d) What does the slope tell us about the typist's paycheck?

Answer.

a

t	4	8	20	40
S	32	64	160	320

b



c 8 dollars/hour

d The typist is paid \$8 per hour.

1.4.9.32. The distance (in miles) covered by a cross-country competitor is given by $d = 6t$, where t is the number of hours she runs.

(a) Make a table of values for the function.

t	2	4	6	8
d				

(b) Graph the function.

(c) Using two points on the graph, compute the slope $\frac{\Delta d}{\Delta t}$, including units.

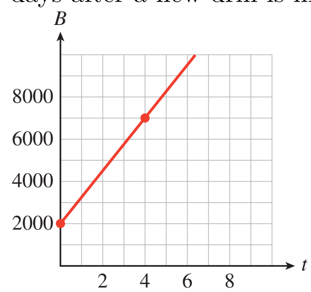
(d) What does the slope tell us about the cross-country runner?

In Problems 33–40,

a Choose two points and compute the slope of the graph (including units).

b Explain what the slope measures in the context of the problem.

1.4.9.33. The graph shows the number of barrels of oil, B, that has been pumped at a drill site t days after a new drill is installed.

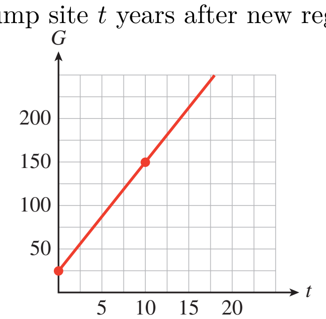


Answer.

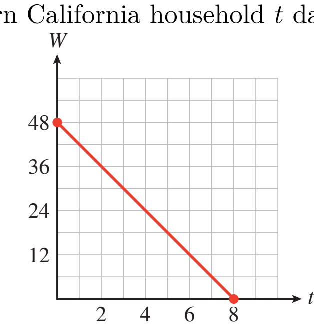
a 1250 barrels/day

b The slope indicates that oil is pumped at a rate of 1250 barrels per day.

1.4.9.34. The graph shows the amount of garbage, G (in tons), that has been deposited at a dump site t years after new regulations go into effect.



1.4.9.35. The graph shows the amount of emergency water, W (in liters), remaining in a southern California household t days after an earthquake.

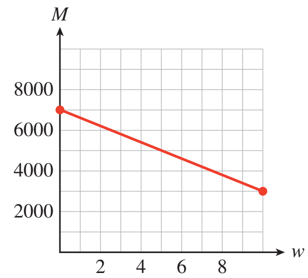


Answer.

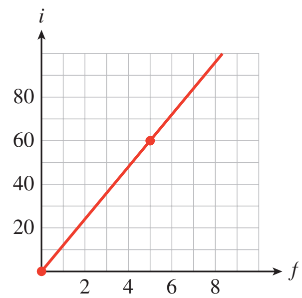
a -6 liters/day

b The slope indicates that the water is diminishing at a rate of 6 liters per day.

1.4.9.36. The graph shows the amount of money, M (in dollars), in Tammy's bank account w weeks after she loses all sources of income.



1.4.9.37. The graph shows the length in inches, i, corresponding to various lengths in feet f.

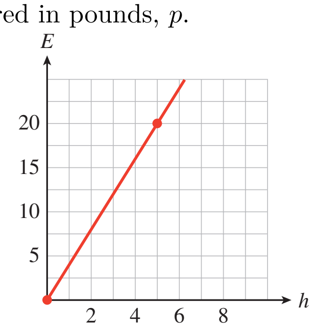


Answer.

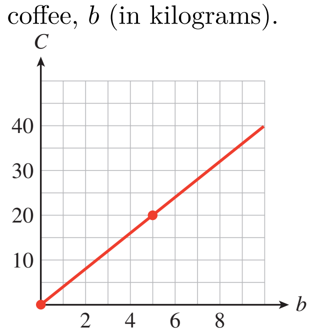
a 12 inches/foot

b The slope gives the conversion rate of 12 inches per foot.

1.4.9.38. The graph shows the number of ounces, z, that correspond to various weights measured in pounds, p.



1.4.9.39. The graph shows the cost, C (in dollars), of coffee beans in terms of the amount of coffee, b (in kilograms).

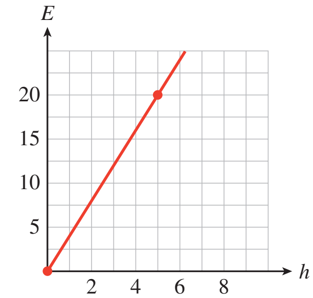


Answer.

a 4 dollars/kilogram

b The slope gives the unit price of $4 per kilogram

1.4.9.40. The graph shows Tracey's earnings, E (in dollars), in terms of the number of hours, h, that she babysits.



Which of the tables in Problems 41 and 42 represent variables that are related by a linear function? (Hint: Which relationships have constant slope?)

1.4.9.41.

a

x	y
2	12
3	17
4	22
5	27
6	32

b

t	P
2	4
3	9
4	16
5	25
6	36

Answer. (a)

1.4.9.42.

a

h	w
-6	20
-3	18
0	16
3	14
6	12

b

t	d
5	0
10	3
15	6
20	12
25	24

1.4.9.43. The table shows the amount of ammonium chloride salt, in grams, that can be dissolved in 100 grams of water at different temperatures.

Temperature,°C	10	12	15	21	25	40	52
Grams of salt	33	34	35.5	38.5	40.5	48	54

a If you plot the data, will the points lie on a straight line? Why or why not?

b Calculate the rate of change of salt dissolved with respect to temperature.

Answer.

a Yes, the slope between any two points is $\frac{1}{2}$.

b 0.5 grams of salt per degree Celsius

1.4.9.44. A spring is suspended from the ceiling. The table shows the length of the spring, in centimeters, as it is stretched by hanging various weights from it.

Weight, kg	3	4	8	10	12	15	22
Length, cm	25.87	25.88	26.36	26.6	26.84	27.2	28.04

a If you plot the data, will the points lie on a straight line? Why or why not?

b Calculate the rate of change of length with respect to weight.

1.4.9.45. The table gives the radius and circumference of various circles, rounded to three decimal places.

r	C
4	25.133
6	37.699
10	62.832
15	94.248

a If we plot the data, will the points lie on a straight line?

b What familiar number does the slope turn out to be? (Hint: Recall a formula from geometry.)

Answer.

a Yes

b 2π

1.4.9.46. The table gives the side and the diagonal of various squares, rounded to three decimal places.

s	d
3	4.243
6	8.485
8	11.314
10	14.142

a If we plot the data, will the points lie on a straight line?

b What familiar number does the slope turn out to be? (Hint: Draw a picture of one of the squares and use the Pythagorean theorem to compute its diagonal.)

1.4.9.47. Geologists can measure the depth of the ocean at different points using a technique called echo-sounding. Scientists on board a ship send a pulse of sound toward the ocean floor and measure the time interval until the echo returns to the ship. The speed of sound in seawater is about 1500 meters per second.

a Write the speed of sound as a ratio.

b If the echo returns in 4.5 seconds, what is the depth of the ocean at that point?

Answer.

a $\dfrac{1500 \text{ meters}}{1 \text{ second}}$

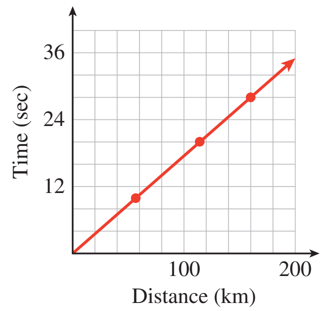
b 3375 meters

1.4.9.48. Niagara Falls was discovered by Father Louis Hennepin in 1682. In 1952, much of the water of the Niagara River was diverted for hydroelectric power, but until that time erosion caused the Falls to recede upstream by 3 feet per year.

a How far did the Falls recede from 1682 to 1952?

b The Falls were formed about $12,000$ years ago during the end of the last ice age. How far downstream from their current position were they then? (Give your answer in miles.)

1.4.9.49. Geologists calculate the speed of seismic waves by plotting the travel times for waves to reach seismometers at known distances from the epicenter. The speed of the wave can help them determine the nature of the material it passes through. The graph shows a travel-time graph for P-waves from a shallow earthquake.



a Why do you think the graph is plotted with distance as the input variable?

b Use the graph to calculate the speed of the wave.

Answer.

a The distances are known.

b 5.7 km per second

1.4.9.50. Energy (supplied by heat) is required to raise the temperature of a substance, and it is also needed to melt a solid substance to a liquid. The table shows data from heating a solid sample of stearic acid. Heat was applied at a constant rate throughout the experiment. (Source: J. A. Hunt and A. Sykes, 1984)

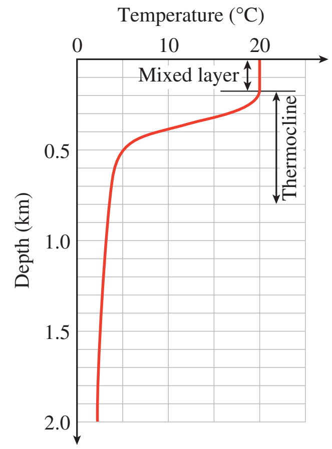
Time (minutes	0	0.5	1.5	2	2.5	3	4	5	6	7	8	8.5	9	9.5	10
Temperature, °C	19	29	40	48	53	55	55	55	55	55	55	64	70	73	74

a Did the temperature rise at a constant rate? Describe the temperature as a function of time.

b Graph temperature as a function of time.

c What is the melting point of stearic acid? How long did it take the sample to melt?

1.4.9.51. The graph shows the temperature of the ocean as a function of depth.



a What is the difference in temperature between the surface of the ocean and the deepest level shown?

b Over what depths does the temperature change most rapidly?

c What is the average rate of change of temperature with respect to depth in the region called the thermocline?

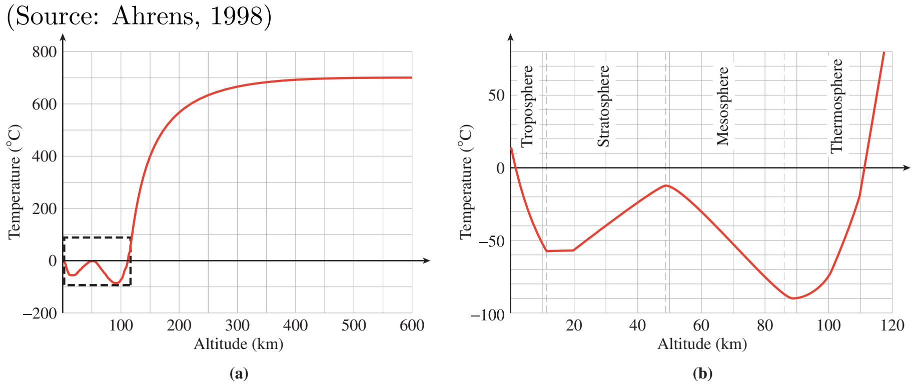
Answer.

a About 18°C

b 0.3 km to 0.4 km

c About -28°C per kilometer

1.4.9.52. The graph shows the average air temperature as a function of altitude. (Figure (b) is an enlargement of the indicated region of Figure (a).) (Source: Ahrens, 1998)



a Is temperature a decreasing function of altitude?

b The **lapse rate** is the rate at which the temperature changes with altitude. In which regions of the atmosphere is the lapse rate positive?

c The region where the lapse rate is zero is called the isothermal zone. Give an interval of altitudes that describes the isothermal zone.

d What is the lapse rate in the mesosphere?

e Describe the temperature for altitudes greater than 90 kilometers.

In Problems 53–56, evaluate the function at $x = a$ and $x = b$, and then find the slope of the line segment joining the two corresponding points on the graph. Illustrate the line segment on a graph of the function.

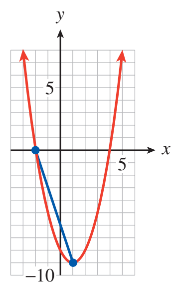
1.4.9.53. $f(x) = x^2 - 2x - 8$

a $a = -2, b = 1$

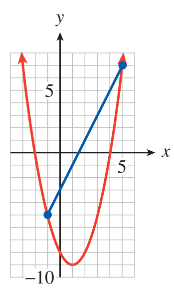
b $a = -1, b = 5$

Answer.

a -3



b 2



1.4.9.54. $g(x) = \sqrt{x+4}$

a $a = -2, b = 0$

b $a = 0, b = 5$

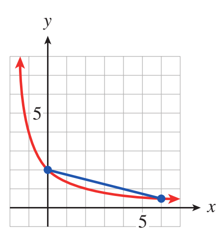
1.4.9.55. $h(x) = \dfrac{4}{x+2}$

a $a = 0, b = 6$

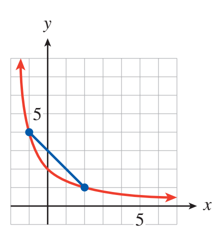
b $a = -1, b = 2$

Answer.

a $\dfrac{-1}{4}$



b -1



1.4.9.56. $q(x) = x^3 - 4x$

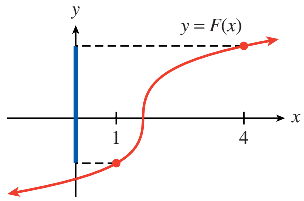
a $a = -1, b = 2$

b $a = -1, b = 3$

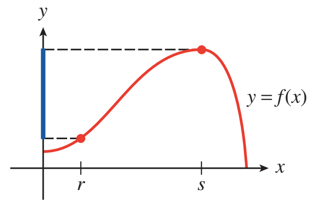
In Problems 57–62, find the coordinates of the indicated points, then write an algebraic expression using function notation for the indicated quantity.

1.4.9.57. The length of the vertical line segment on the y-axis

a



b



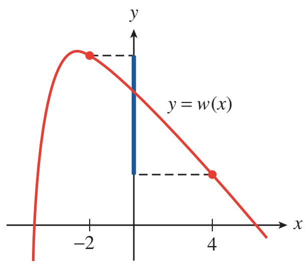
Answer.

a $(1, F(1)), (4, F(4));\qquad F(4) - F(1)$

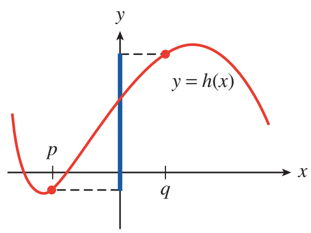
b $(r, f(r)), (s, f(s));\qquad f(s) - f(r)$

1.4.9.58. The length of the vertical line segment on the y-axis

a

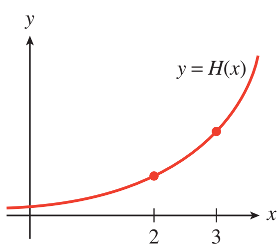


b

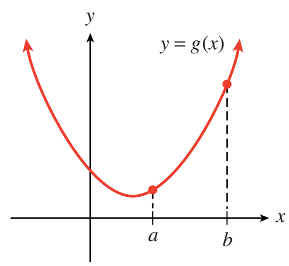


1.4.9.59.

a The increase in y as x increases from 2 to 3



b The increase in y as x increases from a to b



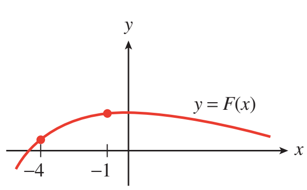
Answer.

a $(2, H(2)), (3, H(3));\qquad H(3) - H(2)$

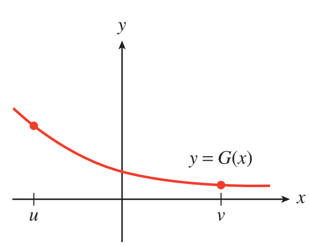
b $(a, g(a)), (b, g(b));\qquad g(b) - g(a)$

1.4.9.60.

a The increase in y as x increases from -4 to -1

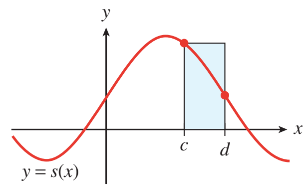


b The increase in y as x increases from u to v

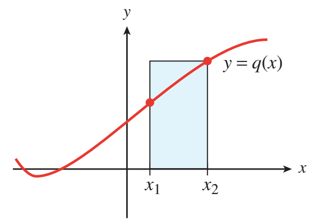


1.4.9.61. The shaded area

a



b



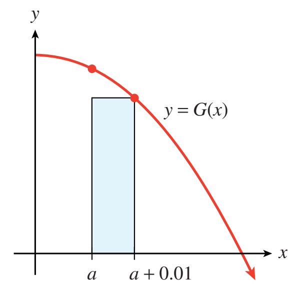
Answer.

a $(c, s(c)), (d, s(d));\quad s(c)(d-c)$

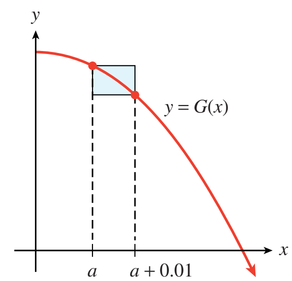
b $(x_1, q(x_1)), (x_2, q(x_2));\quad q(x_2)(x_2-x_1)$

1.4.9.62. The shaded area

a



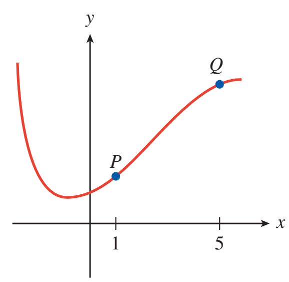
b



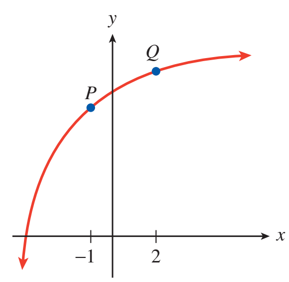
In Problems 63–66, find the coordinates of the indicated points on the graph of $y = f(x)$ and write an algebraic expression using function notation for the slope of the line segment joining points P and Q.

1.4.9.63.

a



b



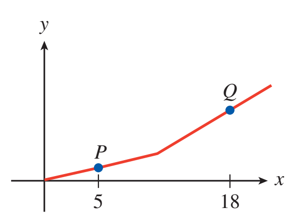
Answer.

a $(1, f(1)), (5, f(5));\quad \dfrac{f(5)-f(1)}{4}$

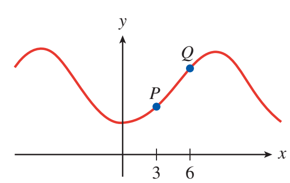
b $(-1, f(-1)), (2, f(2));\quad \dfrac{f(2)-f(-1)}{3}$

1.4.9.64.

a

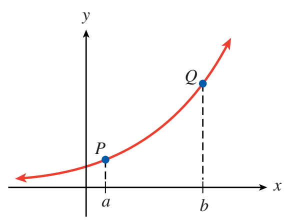


b

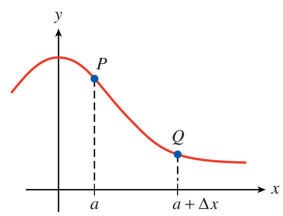


1.4.9.65.

a



b



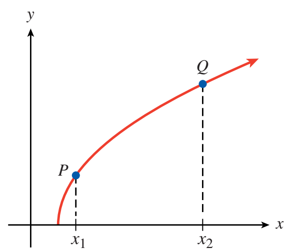
Answer.

a $(a, f(a)), (b, f(b))$; $\dfrac{f(b) - f(a)}{b - a}$

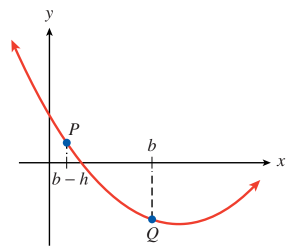
b $(a, f(a)), (a + \Delta x, f(a + \Delta x))$; $\dfrac{f(a + \Delta x) - f(a)}{\Delta x}$

1.4.9.66.

a



b



1.5 Linear Functions

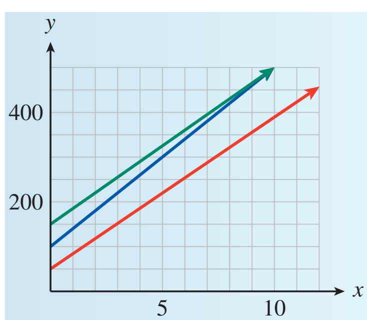
1.5.1 Slope-Intercept Form

Checkpoint 1.5.2 Delbert decides to use DSL for his Internet service.

- Earthlink charges a \$99 activation fee and \$39.95 per month,
- DigitalRain charges \$50 for activation and \$34.95 per month,
- and FreeAmerica charges \$149 for activation and \$34.95 per month.

a Write a formula for Delbert's Internet costs under each plan.

b Match Delbert's Internet cost under each company with its graph shown below.



Answer.

a Earthlink: $f(x) = 99 + 39.95x$; DigitalRain: $g(x) = 50 + 34.95x$; FreeAmerica: $h(x) = 149 + 34.95x$

b DigitalRain: I; Earthlink: II; FreeAmerica: III

1.5.2 Slope-Intercept Method of Graphing

Checkpoint 1.5.6

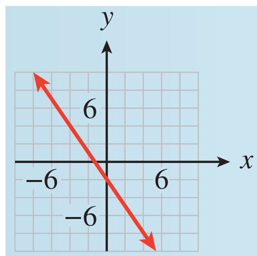
a Write the equation $2y + 3x + 4 = 0$ in slope-intercept form.

b Use the slope-intercept method to graph the line.

Answer.

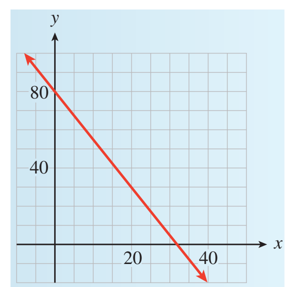
a $y = -2 - \dfrac{3}{2}x$

b



1.5.3 Finding a Linear Equation from a Graph

Checkpoint 1.5.8



Find an equation for the line shown at right.
$b =$
$m =$
$y =$

Answer. $b = 80,\ m = \dfrac{-5}{2},\ y = 80 - \dfrac{5}{2}x$

1.5.4 Point-Slope Form

Checkpoint 1.5.10 Use the point-slope form to find the equation of the line that passes through the point $(-3, 5)$ and has slope -1.4.

$y = y_1 + m(x - x_1)$ Substitute $\boldsymbol{-1.4}$ for $\boldsymbol{m}$ and $\boldsymbol{(-3, 5)}$ for $\boldsymbol{(x_1, y_1)}$.
Simplify: Apply the distributive law.

Answer. $y = 0.8 - 1.4x$

Checkpoint 1.5.12 A healthy weight for a young woman of average height, 64 inches, is 120 pounds. To calculate a healthy weight for a woman taller than 64 inches, add 5 pounds for each inch of height over 64.

a Write a linear equation in point-slope form for the healthy weight, W, for a woman of height, H, in inches.

b Write the equation in slope-intercept form.

Answer.

a $W = 120 + 5(H - 64)$

b $W = -200 + 5H$

1.5.6 Homework 1.5

In Problems 1–10,

a Write each equation in slope-intercept form.

b State the slope and y-intercept of the line.

1.5.6.1. $3x + 2y = 1$

Answer.

a $y = \frac{1}{2} - \frac{3}{2}x$

b Slope $\frac{-3}{2}$, y-intercept $\frac{1}{2}$

1.5.6.2. $5x - 4y = 0$

1.5.6.3. $\frac{1}{4}x + \frac{3}{2}y = \frac{1}{6}$

Answer.

a $y = \frac{1}{9} - \frac{1}{6}x$

b Slope $\frac{-1}{6}$, y-intercept $\frac{1}{9}$

1.5.6.4. $\frac{7}{6}x - \frac{2}{9}y = 3$

1.5.6.5. $4.2x - 0.3y = 6.6$

Answer.

a $y = -22 + 14x$

b Slope 14, y-intercept -22

1.5.6.6. $0.8x + 0.004y = 0.24$

1.5.6.7. $y + 29 = 0$

Answer.

a $y = -29$

b Slope 0, y-intercept -29

1.5.6.8. $0.7x - 12 = 0$

1.5.6.9. $250x + 150y = 2450$

Answer.

a $y = \frac{49}{3} - \frac{5}{3}x$

b Slope $\frac{-5}{3}$, y-intercept $\frac{49}{3}$

1.5.6.10. $80x - 360y = 6120$

In Problems 11–14,

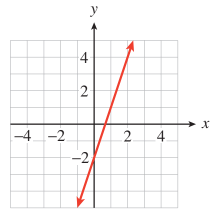
a Sketch by hand the graph of the line with the given slope and y-intercept.

b Write an equation for the line.

c Find the x-intercept of the line.

1.5.6.11. $m = 3$ and $b = -2$

Answer.

a 

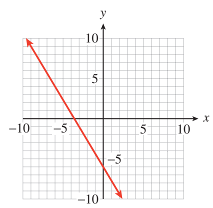
b $y = -2 + 3x$

c $\dfrac{2}{3}$

1.5.6.12. $m = -4$ and $b = 1$

1.5.6.13. $m = -\dfrac{5}{3}$ and $b = -6$

Answer.

a 

b $y = -6 + \dfrac{5}{3}x$

c $\dfrac{-18}{5}$

1.5.6.14. $m = \dfrac{3}{4}$ and $b = -2$

1.5.6.15. The point $(2, -1)$ lies on the graph of $f(x) = -3x + b$. Find b.

Answer. 5

1.5.6.16. The point $(-3, -8)$ lies on the graph of $f(x) = \dfrac{2}{3}x + b$. Find b.

1.5.6.17. The point $(8, -5)$ lies on the graph of $f(x) = mx - 3$. Find m.

Answer. $\dfrac{-1}{4}$

1.5.6.18. The point $(-5, -6)$ lies on the graph of $f(x) = mx + 2$. Find m.

1.5.6.19. Find the slope and intercepts of the line $Ax + By = C$

Answer. $m = \dfrac{-A}{B}$, x-intercept $\left(\dfrac{C}{A}, 0\right)$, y-intercept $\left(0, \dfrac{C}{B}\right)$

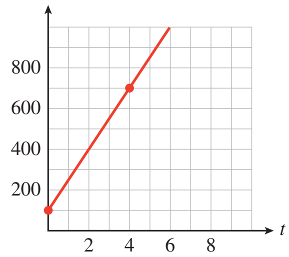
1.5.6.20. Find the slope and intercepts of the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$

In Problems 21–26,

a Find a formula for the function whose graph is shown.

b Say what the slope and the vertical intercept tell us about the problem.

1.5.6.21. The graph shows the altitude, a (in feet), of a skier t minutes after getting on a ski lift.

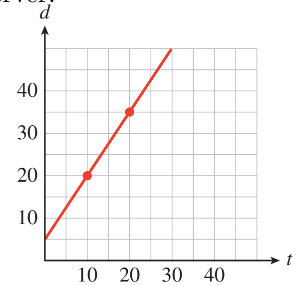


Answer.

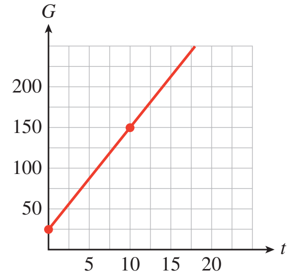
a $a = 100 + 150t$

b The slope tells us that the skier's altitude is increasing at a rate of 150 feet per minute, the vertical intercept that the skier began at an altitude of 200 feet.

1.5.6.22. The graph shows the distance, d (in meters), traveled by a train t seconds after it passes an observer.



1.5.6.23. The graph shows the amount of garbage, G (in tons), that has been deposited at a dump site t years after new regulations go into effect.

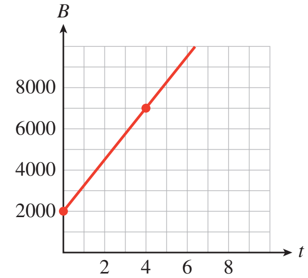


Answer.

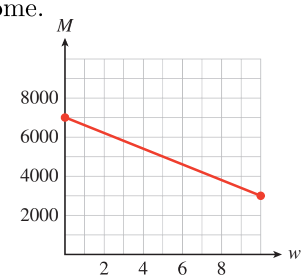
a $G = 25 + 12.5t$

b The slope tells us that the garbage is increasing at a rate of 12.5 tons per year, the vertical intercept that the dump already had 25 tons (when the new regulations went into effect).

1.5.6.24. The graph shows the number of barrels of oil, B, that has been pumped at a drill site t days after a new drill is installed.



1.5.6.25. The graph shows the amount of money, M (in dollars), in Tammy's bank account w weeks after she loses all sources of income.

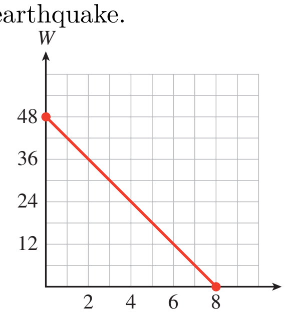


Answer.

a $M = 7000 - 400w$

b The slope tells us that Tammy's bank account is diminishing at a rate of \$400 per week, the vertical intercept that she had \$7000 (when she lost all sources of income).

1.5.6.26. The graph shows the amount of emergency water, W (in liters), remaining in a southern California household t days after an earthquake.



1.5.6.27. The formula $F = \dfrac{9}{5}C + 32$ defines a function that converts the temperature in degrees Celsius to degrees Fahrenheit.

a What is the Fahrenheit temperature when it is 10° Celsius?

b What is the Celsius temperature when it is $-4°$ Fahrenheit?

c Choose appropriate `WINDOW` settings and graph the equation $y = \dfrac{9}{5}x + 32$.

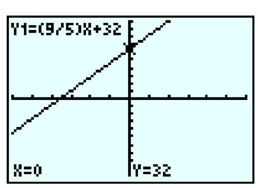
d Find the slope and explain its meaning for this problem.

e Find the intercepts and explain their meanings for this problem.

Answer.

a 50°F

b -20°C

c 

d The slope, $\frac{9}{5} = 1.8$, tells us that Fahrenheit temperatures increase by 1.8° for each increase of 1° Celsius.

e C-intercept $\left(-17\frac{7}{9}, 0\right)$: $-17\frac{7}{9}^{\circ}$ C is the same as 0°F; F-intercept $(0, 32)$: 0°C is the same as 32°F.

1.5.6.28. If the temperature on the ground is 70° Fahrenheit, the formula $T = 70 - \dfrac{3}{820}h$ defines a function that gives the temperature at an altitude of h feet.

a What is the temperature at an altitude of 4100 feet?

b At what altitude is the temperature 34° Fahrenheit?

c Choose appropriate `WINDOW` settings and graph the equation $y = 70 - \dfrac{3}{820}x$.

d Find the slope and explain its meaning for this problem.

e Find the intercepts and explain their meanings for this problem.

1.5.6.29. In England, oven cooking temperatures are often given as Gas Marks rather than degrees Fahrenheit. The table shows the equivalent oven temperatures for various Gas Marks.

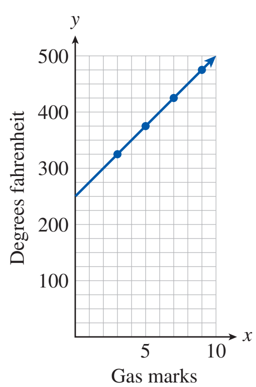
Gas Mark	3	5	7	9
Degrees (F)	325	375	425	475

a Plot the data and draw a line through the data points.

b Calculate the slope of your line. Estimate the y-intercept from the graph.

c Find an equation that gives the temperature in degrees Fahrenheit in terms of the Gas Mark.

Answer.

a 

b $m = 25,\ b = 250$

c $y = 250 + 25x$

1.5.6.30. European shoe sizes are scaled differently than American shoe sizes. The table shows the European equivalents for various American shoe sizes.

American shoe size	5.5	6.5	7.5	8.5
European shoe size	37	38	39	40

a Plot the data and draw a line through the data points.

b Calculate the slope of your line. Estimate the y-intercept from the graph.

c Find an equation that gives the European shoe size in terms of American shoe size.

1.5.6.31. A spring is suspended from the ceiling. The table shows the length of the spring in centimeters as it is stretched by hanging various weights from it.

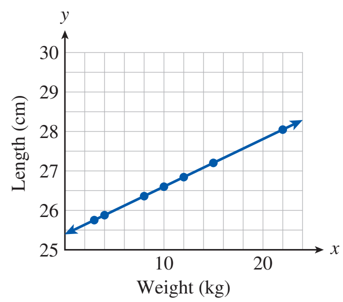
Weight, kg	3	4	8	10	12	15	22
Length, cm	25.76	25.88	26.36	26.6	26.84	27.2	28.04

a Plot the data on graph paper and draw a straight line through the points. Estimate the y-intercept of your graph.

b Find an equation for the line.

c If the spring is stretched to 27.56 cm, how heavy is the attached weight?

Answer.

a 

b $y = 0.12x + 25.4$

c 18 kg

1.5.6.32. The table shows the amount of ammonium chloride salt, in grams, that can be dissolved in 100 grams of water at different temperatures.

Temperature, °C	10	12	15	21	25	40	52
Grams of salt	33	34	35.5	38.5	40.5	48	54

a Plot the data on graph paper and draw a straight line through the points. Estimate the y-intercept of your graph.

b Find an equation for the line.

c At what temperature will 46 grams of salt dissolve?

In Problems 33–36,

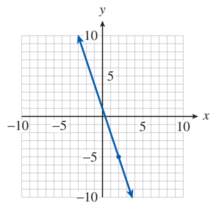
a Sketch by hand the graph of the line that passes through the given point and has the given slope.

b Write an equation for the line in point-slope form.

c Put your equation from part (b) into slope-intercept form.

1.5.6.33. $(2, -5)$; $m = -3$

Answer.

a 

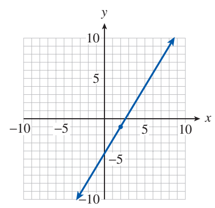
b $y + 5 = -3(x - 2)$

c $y = 1 - 3x$

1.5.6.34. $(-6, -1)$; $m = 4$

1.5.6.35. $(2, -1)$; $m = \dfrac{5}{3}$

Answer.

a 

b $y + 1 = \frac{5}{3}(x - 2)$

c $y = \frac{-13}{3} + \frac{5}{3}x$

1.5.6.36. $(-1, 2)$; $m = -\dfrac{3}{2}$

For Problems 37–40,

a Write an equation in point-slope form for the line that passes through the given point and has the given slope.

b Put your equation from part (a) into slope-intercept form.

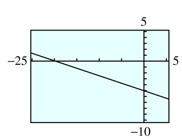
c Use your graphing calculator to graph the line.

1.5.6.37. $(-6.4, -3.5)$, $m = -0.25$

Answer.

a $y + 3.5 = -0.25(x + 6.4)$

b $y = -5.1 - 0.25x$

c 

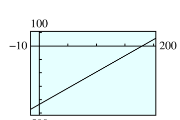
1.5.6.38. $(7.2, -5.6)$, $m = 1.6$

1.5.6.39. $(80, -250)$, $m = 2.4$

Answer.

a $y + 250 = 2.4(x - 80)$

b $y = -442 + 2.4x$

c 

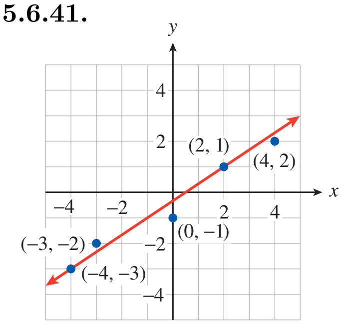
1.5.6.40. $(-150, 1800)$, $m = -24$

For Problems 41 and 42,

a Find the slope of the line. (Note that not all the labeled points lie on the line.)

b Find an equation for the line.

1.5.6.41.

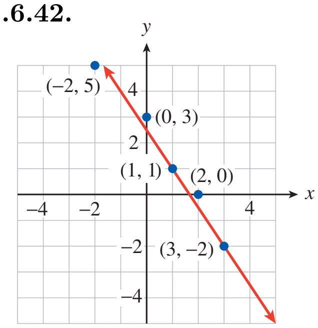


Answer.

a $m = \dfrac{2}{3}$

b $y = \dfrac{-1}{3} + \dfrac{2}{3}x$

1.5.6.42.



For Problems 43 and 44, the equation of line l_1 is $y = q + px$, and the equation of line l_2 is $y = v + tx$.

a Decide whether the coordinates of each labeled point are

I a solution of $y = q + px$,

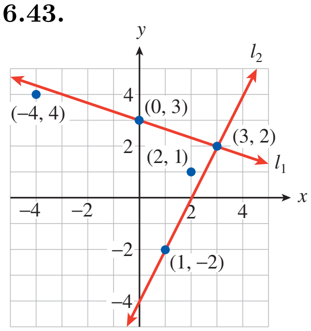
II a solution of $y = v + tx$,

III a solution of both equations, or

IV a solution of neither equation.

b Find p, q, t, and v.

1.5.6.43.

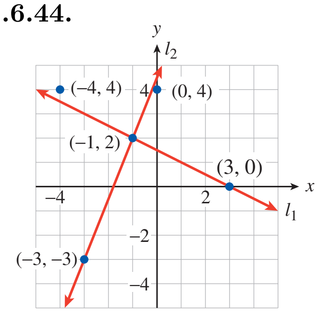


Answer.

a $(-4, 4)$: neither; $(0, 3)$: $y = px + q$; $(3, 2)$: both; $(2, 1)$: neither; $(1, -2)$: $y = tx + v$

b $p = \dfrac{-1}{3}$, $q = 3$, $t = 2$, $v = -4$

1.5.6.44.

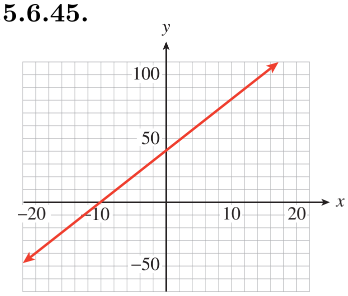


For Problems 45–50,

a Estimate the slope and vertical intercept of each line. (Hint: To calculate the slope, find two points on the graph that lie on the intersection of grid lines.)

b Using your estimates from (a), write an equation for the line.

1.5.6.45.

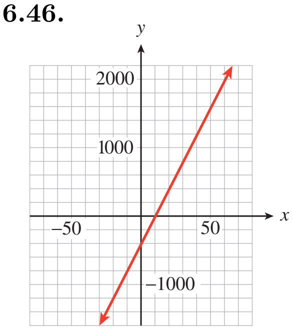


Answer.

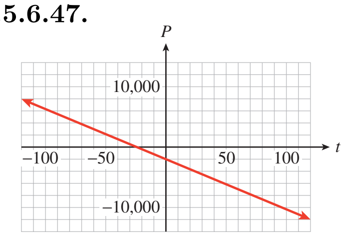
a $m = 4,\ b = 40$

b $y = 40 + 4x$

1.5.6.46.



1.5.6.47.

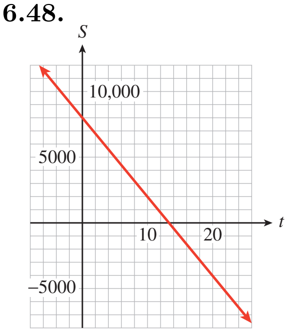


Answer.

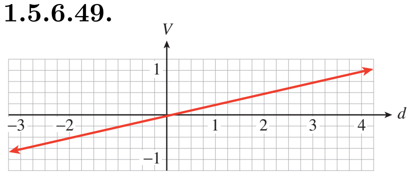
a $m = -80,\ b = -2000$

b $P = -2000 - 80t$

1.5.6.48.



1.5.6.49.

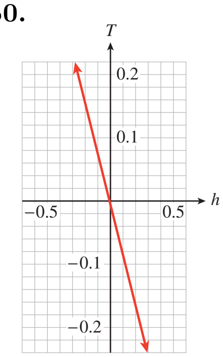


Answer.

a $m = \dfrac{1}{4},\ b = 0$

b $V = \dfrac{1}{4}d$

1.5.6.50.



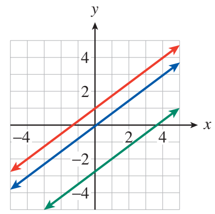
1.5.6.51.

a Write equations for three lines with slope $m = \dfrac{3}{4}$. (Many answers are possible.)

b Graph all three lines on the same axis. What do you notice about the lines?

Answer.

a $y = \dfrac{3}{4}x,\ y = 1 + \dfrac{3}{4}x,\ y = -2.7 + \dfrac{3}{4}x$

b 

The lines are parallel.

1.5.6.52.

a Write equations for three lines with slope $m = 0$. (Many answers are possible.)

b Graph all three lines in the same window. What do you notice about the lines?

In Problems 53–56, choose the correct graph for each equation. The scales on both axes are the same.

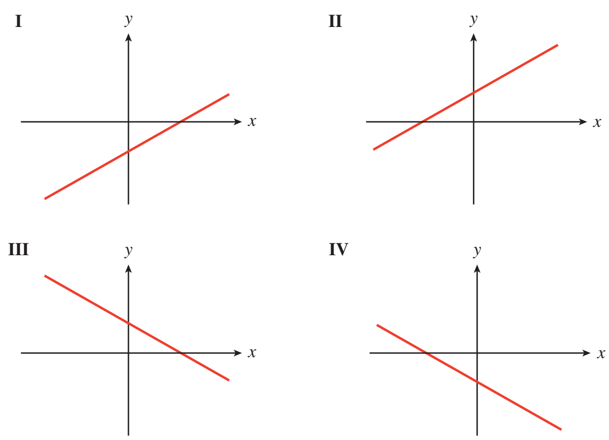
1.5.6.53.

a $y = \dfrac{3}{4}x + 2$

b $y = \dfrac{-3}{4}x + 2$

c $y = \dfrac{3}{4}x - 2$

d $y = \dfrac{-3}{4}x - 2$



Answer.

a II

b III

c I

d IV

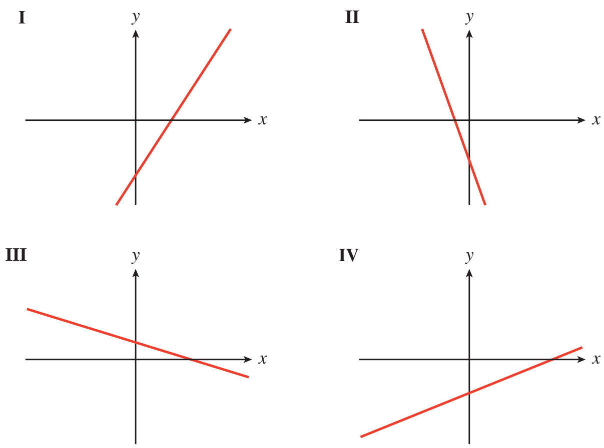
1.5.6.54.

a $m < 0$, $b > 0$

b $m > 1$, $b < 0$

c $0 < m < 1$, $b < 0$

d $m < -1$, $b < 0$



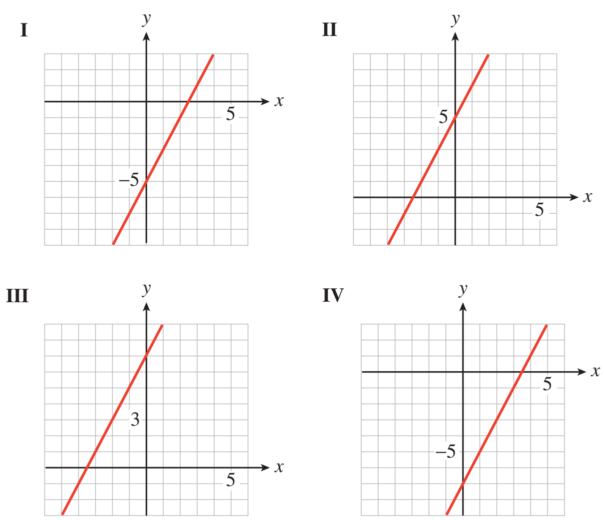
1.5.6.55.

a $y = 1 + 2(x + 3)$

b $y = -1 + 2(x - 3)$

c $y = -1 + 2(x + 3)$

d $y = 1 + 2(x - 3)$



Answer.

a III

b IV

c II

d I

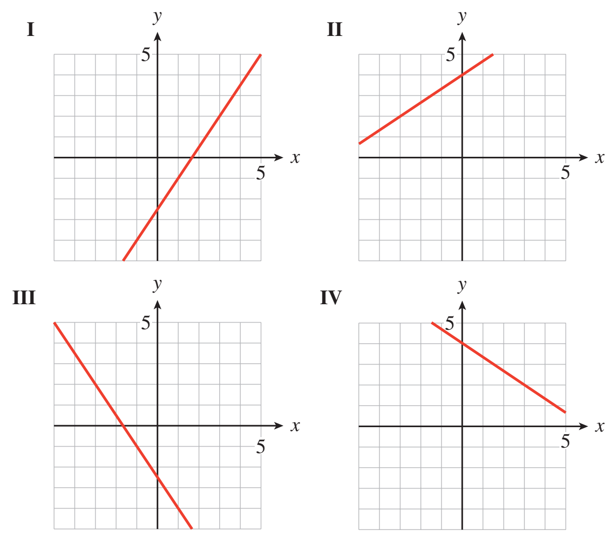
1.5.6.56.

a $y = 2 - \dfrac{2}{3}(x - 3)$

b $y = 2 - \dfrac{3}{2}(x + 3)$

c $y = 2 + \dfrac{3}{2}(x - 3)$

d $y = 2 + \dfrac{2}{3}(x + 3)$



In Problems 57–60, find the slope of each line and the coordinates of one point on the line. (No calculation is necessary!)

1.5.6.57. $y + 1 = 2(x - 6)$

Answer. $m = 2$; $(6, -1)$

1.5.6.58. $2(y - 8) = 5(x + 2)$

1.5.6.59. $y = 3 - \frac{4}{3}(x + 5)$

Answer. $m = \frac{-4}{3}$; $(-5, 3)$

1.5.6.60. $7x = -3y$

1.5.6.61.

a Draw a set of coordinate axes with a square grid (i.e., with units the same size in both directions). Sketch four lines through the point (0, 4) with the following slopes:

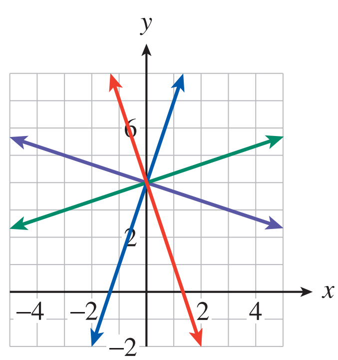
$$m = 3, \quad m = -3, \quad m = \frac{1}{3}, \quad m = \frac{-1}{3}$$

b What do you notice about these lines?

Hint. Look for perpendicular lines.

Answer.

a



b The lines with slope 3 and $\frac{-1}{3}$ are perpendicular to each other, and the lines with slope -3 and $\frac{1}{3}$ are perpendicular to each other.

1.5.6.62.

a Draw a set of coordinate axes with a square grid (see Problem 61). Sketch four lines through the point (0, -3) with the following slopes:

$$m = \frac{2}{5}, \quad m = \frac{-2}{5}, \quad m = \frac{5}{2}, \quad m = \frac{-5}{2}$$

b What do you notice about these lines?

1.5.6.63. The boiling point of water changes with altitude and is approximated by the formula

$$B = f(h) = 212 - 0.0018h$$

where B is in degrees Fahrenheit and h is in feet. State the slope and vertical intercept of the graph, including units, and explain their meaning in this context.

Answer. $m = -0.0018$ degree/foot, so the boiling point drops with altitude at a rate of 0.0018 degree per foot. $b = 212$, so the boiling point is 212° at sea level (where the elevation $h = 0$).

1.5.6.64. The height of a woman in centimeters is related to the length of her femur (in centimeters) by the formula

$$H = f(x) = 2.47x + 54.10$$

State the slope and the vertical intercept of the graph, including units, and explain their meaning in this context.

1.6 Chapter Summary and Review

1.6.2 Chapter 1 Review Problems

Write and graph a linear equation for each situation. Then answer the questions.

1.6.2.1. Last year, Pinwheel Industries introduced a new model calculator. It cost \$2000 to develop the calculator and \$20 to manufacture each one.

a Complete the table of values showing the total cost, C, of producing n calculators.

n	100	500	800	1200	1500
C					

b Write an equation that expresses C in terms of n.

c Graph the equation by hand.

d What is the cost of producing 1000 calculators? Illustrate this as a point on your graph.

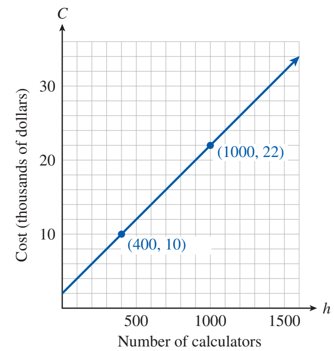
e How many calculators can be produced for \$10,000? Illustrate this as a point on your graph.

Answer.

a

n	100	500	800	1200	1500
C	4000	12,000	18,000	26,000	32,000

b $C = 20n + 2000$

c 

d $\$22,000$

e 400

1.6.2.2. Megan weighed 5 pounds at birth and gained 18 ounces per month during her first year.

a Complete the table of values for Megan's weight, w, in terms of her age, m, in months.

m	2	4	6	9	12
w					

b Write an equation that expresses 2 in terms of m.

c Graph the equation by hand.

d How much did Megan weigh at 9 months? Illustrate this as a point on your graph.

e When did Megan weigh 9 pounds? Illustrate this as a point on your graph.

1.6.2.3. The total amount of oil remaining in 2005 is estimated at 2.1 trillion barrels, and total annual consumption is about 28 billion barrels.

a Assuming that oil consumption continues at the same level, write an equation for the remaining oil, R, as a function of time, t (in years since 2005).

b Find the intercepts and graph the equation by hand.

c What is the significance of the intercepts to the world's oil supply?

Answer.

a $R = 2100 - 28t$

b $(75, 0)$, $(0, 2100)$

c t-intercept: The oil reserves will be gone in 2080; R-intercept: There were 2100 billion barrels of oil reserves in 2005.

1.6.2.4. The world's copper reserves were 950 million tons in 2004; total annual consumption was 16.8 million tons.

a Assuming that copper consumption continues at the same level, write an equation for the remaining copper reserves, R, as a function of time, t (in years since 2004).

b Find the intercepts and graph the equation by hand.

c What is the significance of the intercepts to the world's copper supply?

1.6.2.5. The owner of a movie theater needs to bring in \$1000 at each screening in order to stay in business. He sells adult tickets at \$5 apiece and children's tickets at \$2 each.

a Write an equation that relates the number of adult tickets, A, he must sell and the number of children's tickets, C.

b Find the intercepts and graph the equation by hand.

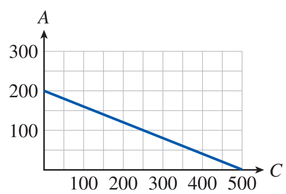
c If the owner sells 120 adult tickets, how many children's tickets must he sell?

d What is the significance of the intercepts to the sale of tickets?

Answer.

a $2C + 5A = 1000$

b $(500, 0)$, $(0, 200)$



c C-intercept: If no adult tickets are sold, he must sell 500 children's tickets; A-intercept: If no children's tickets are sold, he must sell 200 adult tickets.

1.6.2.6. Alida plans to spend part of her vacation in Atlantic City and part in Saint-Tropez. She estimates that after airfare her vacation will cost \$60 per day in Atlantic City and \$100 per day in Saint-Tropez. She has \$1200 to spend after airfare.

a Write an equation that relates the number of days, C, Alida can spend in Atlantic City and the number of days, T, in Saint-Tropez.

b Find the intercepts and graph the equation by hand.

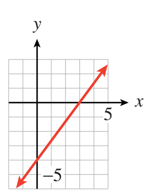
c If Alida spends 10 days in Atlantic City, how long can she spend in Saint-Tropez?

d What is the significance of the intercepts to Alida's vacation?

Graph each equation on graph paper. Use the most convenient method for each problem.

1.6.2.7. $4x - 3y = 12$

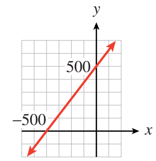
Answer.



1.6.2.8. $\dfrac{x}{6} - \dfrac{y}{12} = 1$

1.6.2.9. $50x = 40y - 20,000$

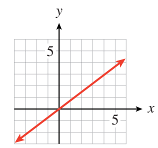
Answer.



1.6.2.10. $1.4x + 2.1y = 8.4$

1.6.2.11. $3x - 4y = 0$

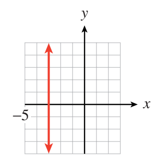
Answer.



1.6.2.12. $x = -4y$

1.6.2.13. $4x = -12$

Answer.



1.6.2.14. $2y - x = 0$

Which of the following tables describe functions? Explain.

1.6.2.15.

x	-2	-1	0	1	2	3
y	6	0	1	2	6	8

Answer. A function: Each x has exactly one associated y-value.

1.6.2.16.

p	3	-3	2	-2	-2	0
q	2	-1	4	-4	3	0

1.6.2.17.

Student	Score on IQ test	Score on SAT test
(A)	118	649
(B)	98	450
(C)	110	590
(D)	105	520
(E)	98	490
(F)	122	680

Answer. Not a function: The IQ of 98 has two possible SAT scores.

1.6.2.18.

Student	Correct answers on math quiz	Quiz grade
(A)	13	85
(B)	15	89
(C)	10	79
(D)	12	82
(E)	16	91
(F)	18	95

1.6.2.19. The total number of barrels of oil pumped by the AQ oil company is given by the formula

$$N(t) = 2000 + 500t$$

where N is the number of barrels of oil t days after a new well is opened. Evaluate $N(10)$ and explain what it means.

Answer. $N(10) = 7000$: Ten days after the new well is opened, the company has pumped a total of 7000 barrels of oil.

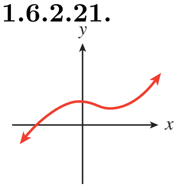
1.6.2.20. The number of hours required for a boat to travel upstream between two cities is given by the formula

$$H(v) = \frac{24}{v-8}$$

where v represents the boat's top speed in miles per hour. Evaluate $H(16)$ and explain what it means.

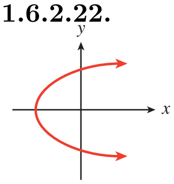
Which of the following graphs represent functions?

1.6.2.21.

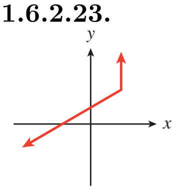


Answer. Function

1.6.2.22.

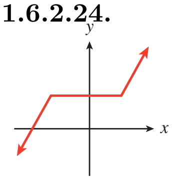


1.6.2.23.



Answer. Not a function

1.6.2.24.



Evaluate each function for the given values.

1.6.2.25. $F(t) = \sqrt{1+4t^2}, \quad F(0) \quad \text{and} \quad F(-3)$

Answer. $F(0) = 1, \quad F(-3) = \sqrt{37}$

1.6.2.26. $G(x) = \sqrt[3]{x-8}, \quad G(0) \quad \text{and} \quad G(20)$

1.6.2.27. $h(v) = 6 - |4-2v|, \quad h(8) \quad \text{and} \quad h(-8)$

Answer. $h(8) = -6, \quad h(-8) = -14$

1.6.2.28. $m(p) = \dfrac{120}{p+15}, \quad m(5) \quad \text{and} \quad m(-40)$

Refer to the graphs shown for Problems 29 and 30.

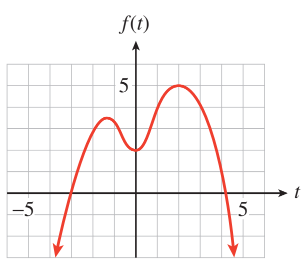
1.6.2.29.

a Find $f(-2)$ and $f(2)$.

b For what value(s) of t is $f(t) = 4$?

c Find the t- and $f(t)$-intercepts of the graph.

d What is the maximum value of f? For what value(s) of t does f take on its maximum value?



Answer.

a $f(-2) = 3, \quad f(2) = 5$

b $t = 1, \quad t = 3$

c t-intercepts $(-3, 0), (4, 0)$; $f(t)$-intercept: $(0, 2)$

d Maximum value of 5 occurs at $t = 2$

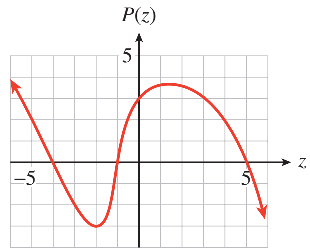
1.6.2.30.

a Find $P(-3)$ and $P(3)$.

b For what value(s) of z is $P(z) = 2$?

c Find the z- and $P(z)$-intercepts of the graph.

d What is the minimum value of P? For what value(s) of z does P take on its minimum value?



Graph the given function on a graphing calculator. Then use the graph to solve the equations and inequalities. Round your answers to one decimal place if necessary.

1.6.2.31. $y = \sqrt[3]{x}$

a Solve $\sqrt[3]{x} = 0.8$

b Solve $\sqrt[3]{x} = 1.5$

c Solve $\sqrt[3]{x} > 1.7$

d Solve $\sqrt[3]{x} \leq 1.26$

Answer.

a $x = \dfrac{1}{2} = 0.5$

b $x = \dfrac{27}{8} \approx 3.4$

c $x > 4.9$

d $x \leq 2.0$

1.6.2.32. $y = \dfrac{1}{x}$

a Solve $\dfrac{1}{x} = 2.5$

b Solve $\dfrac{1}{x} = 0.3125$

c Solve $\dfrac{1}{x} \geq 0.\overline{2}$

d Solve $\dfrac{1}{x} < 5$

1.6.2.33. $y = \dfrac{1}{x^2}$

a Solve $\dfrac{1}{x^2} = 0.03$

b Solve $\dfrac{1}{x^2} = 6.25$

c Solve $\dfrac{1}{x^2} > 0.16$

d Solve $\dfrac{1}{x^2} \leq 4$

Answer.

a $x \approx \pm 5.8$

b $x = \pm 0.4$

c $-2.5 < x < 0$ or $0 < x < 2.5$

d $x \leq -0.5$ or $x \geq 0.5$

1.6.2.34. $y = \sqrt{x}$

a Solve $\sqrt{x} = 0.707$

b Solve $\sqrt{x} = 1.7$

c Solve $\sqrt{x} < 1.5$

d Solve $\sqrt{x} \geq 1.3$

Evaluate each function.

1.6.2.35. $H(t) = t^2 + 2t, \quad H(2a) \quad \text{and} \quad H(a+1)$

Answer. $H(2a) = 4a^2 + 4a, \quad H(a+1) = a^2 + 4a + 3$

1.6.2.36. $F(x) = 2 - 3x, \quad F(2) + F(3) \quad \text{and} \quad F(2+3)$

1.6.2.37. $f(x) = 2x^2 - 4, \quad f(a) + f(b) \quad \text{and} \quad f(a+b)$

Answer. $f(a) + f(b) = 2a^2 + 2b^2 - 8, \quad f(a+b) = 2a^2 + 4ab + 2b^2 - 4$

1.6.2.38. $G(t) = 1 - t^2, \quad G(3w) \quad \text{and} \quad G(s+1)$

1.6.2.39. A spiked volleyball travels 6 feet in 0.04 seconds. A pitched baseball travels 66 feet in 0.48 seconds. Which ball travels faster?

Answer. The volleyball

1.6.2.40. Kendra needs $4\frac{1}{2}$ gallons of Luke's Brand primer to cover 1710 square feet of wall. She uses $5\frac{1}{3}$ gallons of Slattery's Brand primer for 2040 square feet of wall. Which brand covered more wall per gallon?

1.6.2.41. Which is steeper, Stone Canyon Drive, which rises 840 feet over a horizontal distance of 1500 feet, or Highway 33, which rises 1150 feet over a horizontal distance of 2000 feet?

Answer. Highway 33

1.6.2.42. The top of Romeo's ladder is on Juliet's window sill that is 11 feet above the ground, and the bottom of the ladder is 5 feet from the base of the wall. Is the incline of this ladder as steep as a firefighter's ladder that rises a height of 35 feet over a horizontal distance of 16 feet?

1.6.2.43. The table shows the amount of oil, B (in thousands of barrels), left in a tanker t minutes after it hits an iceberg and springs a leak.

t	0	10	20	30
B	800	750	700	650

a Write a linear function for B in terms of t.

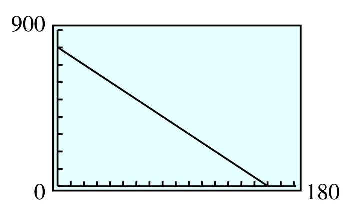
b Choose appropriate window settings on your calculator and graph your function.

c Give the slope of the graph, including units, and explain the meaning of the slope in terms of the oil leak.

Answer.

a $B = 800 - 5t$

b



c $m = -5$ thousand barrels/minute: The amount of oil in the tanker is decreasing by 5000 barrels per minute.

1.6.2.44. A traditional first experiment for chemistry students is to make 98 observations about a burning candle. Delbert records the height, h, of the candle in inches at various times t minutes after he lit it.

t	0	10	30	45
h	12	11.5	10.5	9.75

a Write a linear function for h in terms of t.

b Choose appropriate window settings on your calculator and graph your function.

c Give the slope of the graph, including units, and explain the meaning of the slope in terms of the candle.

1.6.2.45. An interior decorator bases her fee on the cost of a remodeling job. The accompanying table shows her fee, F, for jobs of various costs, C, both given in dollars.

C	5000	10,000	20,000	50,000
F	1000	1500	2500	5500

a Write a linear function for F in terms of C.

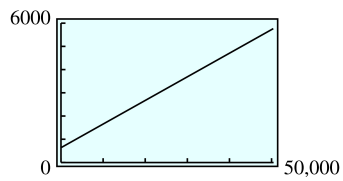
b Choose appropriate window settings on your calculator and graph your function.

c Give the slope of the graph, including units, and explain the meaning of the slope in terms of the the decorator's fee.

Answer.

a $F = 500 + 0.10C$

b



c $m = 0.10$: The fee increases by \$0.10 for each dollar increase in the remodeling job.

1.6.2.46. Auto registration fees in Connie's home state depend on the value of the automobile. The table below shows the registration fee, R, for a car whose value is V, both given in dollars.

V	5000	10,000	15,000	20,000
R	135	235	335	435

a Write a linear function for R in terms of V.

b Choose appropriate window settings on your calculator and graph your function.

c Give the slope of the graph, including units, and explain the meaning of the slope in terms of the registration fee.

Find the slope of the line segment joining each pair of points.

1.6.2.47. $(-1, 4),\ (3, -2)$

Answer. $\dfrac{-3}{2}$

1.6.2.48. $(5, 0),\ (2, -6)$

1.6.2.49. $(6.2, 1.4),\ (-2.1, 4.8)$

Answer. $\dfrac{-34}{83} \approx -0.4$

1.6.2.50. $(0, -6.4),\ (-5.6, 3.2)$

1.6.2.51. The planners at AquaWorld want the small water slide to have a slope of 25%. If the slide is 20 feet tall, how far should the end of the slide be from the base of the ladder?

Answer. 80 ft

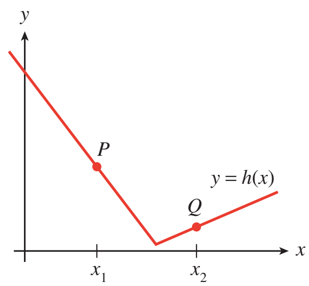
1.6.2.52. In areas with heavy snowfall, the pitch (or slope) of the roof of an A-frame house should be at least 1.2. If a small ski chalet is 40 feet wide at its base, how tall is the center of the roof?

Find the coordinates of the indicated points, and then write an algebraic expression using function notation for the indicated quantities.

1.6.2.53.

a Δy as x increases from x_1 to x_2

b The slope of the line segment joining P to Q



Answer.

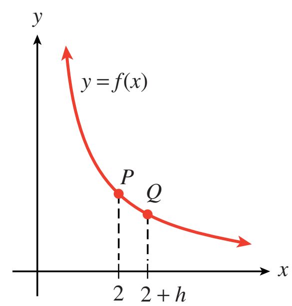
a $h(x_2) - h(x_1)$

b $\dfrac{h(x_2) - h(x_1)}{x_2 - x_1}$

1.6.2.54.

a Δy as x increases from 2 to $2 + h$

b The slope of the line segment joining P to Q



Which of the following tables could represent linear functions?

1.6.2.55.

a

r	E
1	5
2	$\frac{5}{2}$
3	$\frac{5}{3}$
4	$\frac{5}{4}$
5	1

b

s	t
10	6.2
20	9.7
30	12.6
40	15.8
50	19.0

Answer. Neither

1.6.2.56.

a

w	A
2	−13
4	−23
6	−33
8	−43
10	−53

b

x	C
0	0
2	5
4	10
8	20
16	40

Each table gives values for a linear function. Fill in the missing values.

1.6.2.57.

d	V
-5	-4.8
-2	-3
	-1.2
6	1.8
10	

Answer.

d	V
-5	-4.8
-2	-3
	-1.2
6	1.8
10	4.2

1.6.2.58.

q	S
-8	-8
-4	56
3	
	200
9	264

Find the slope and y-intercept of each line.

1.6.2.59. $2x - 4y = 5$

Answer. $m = \dfrac{1}{2},\ b = \dfrac{-5}{4}$

1.6.2.60. $\dfrac{1}{2}x + \dfrac{2}{3}y = \dfrac{5}{6}$

1.6.2.61. $8.4x + 2.1y = 6.3$

Answer. $m = -4,\ b = 3$

1.6.2.62. $y - 3 = 0$

For Problems 63 and 64,

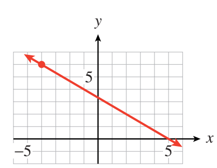
a Graph by hand the line that passes through the given point with the given slope.

b Find an equation for the line.

1.6.2.63. $(-4, 6)$; $m = \dfrac{-2}{3}$

Answer.

a



b $y = \dfrac{10}{3} - \dfrac{2}{3}x$

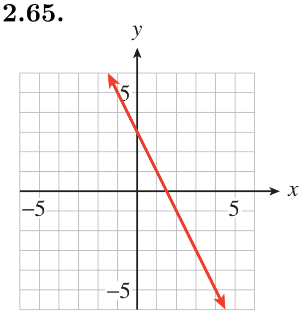
1.6.2.64. $(2, -5)$; $m = \dfrac{3}{2}$

For Problems 65 and 66,

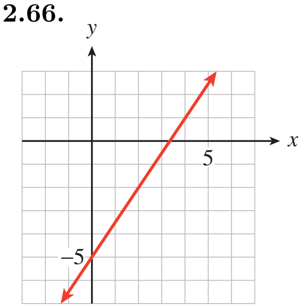
a Find the slope and y-intercept of each line.

b Write an equation for the line.

1.6.2.65.



1.6.2.66.



Answer.

a $m = -2,\ b = 3$

b $y = 3 - 2x$

1.6.2.67. What is the slope of the line whose intercepts are $(-5, 0)$ and $(0, 3)$?

Answer. $\dfrac{3}{5}$

1.6.2.68.

a Find the x- and y-intercepts of the line $\dfrac{x}{4} - \dfrac{y}{6} = 1$.

b What is the slope of the line in part (a)?

1.6.2.69.

a What is the slope of the line $y = 2 + \dfrac{3}{2}(x - 4)$?

b Find the point on the line whose x-coordinate is 4. Can there be more than one such point?

c Use your answers from parts (a) and (b) to find another point on the line.

Answer.

a $\dfrac{3}{2}$

b $(4, 2)$, no

c $(6, 5)$

1.6.2.70. A line passes through the point $(-5, 3)$ and has slope $\dfrac{2}{3}$. Find the coordinates of two more points on the line.

1.6.2.71. A line passes through the point $(-2, -6)$ and has slope $-\dfrac{8}{5}$. Find the coordinates of two more points on the line.

Answer. $(3, -14)$, $(-7, 2)$

1.6.2.72. Find an equation in point-slope form for the line of slope $\dfrac{6}{5}$ that passes through $(-3, -4)$.

1.6.2.73. The rate at which air temperature decreases with altitude is called the lapse rate. In the troposphere, the layer of atmosphere that extends from the Earth's surface to a height of about 7 miles, the lapse rate is about 3.6°F for every 1000 feet. (Source: Ahrens, 1998)

a If the temperature on the ground is 62°F, write an equation for the temperature, T, at an altitude of h feet.

b What is the temperature outside an aircraft flying at an altitude of 30,000 feet? How much colder is that than the ground temperature?

c What is the temperature at the top of the troposphere?

Answer.

a $T = 62 - 0.0036h$ b $-46°F$; $108°F$ c $-71°F$

1.6.2.74. In his television program *Notes from a Small Island*, aired in February 1999, Bill Bryson discussed the future of the British aristocracy. Because not all families produce an heir, 4 or 5 noble lines die out each year. At this rate, Mr. Bryson says, if no more peers are created, there will be no titled families left by the year 2175.

a Assuming that on average 4.5 titled families die out each year, write an equation for the number, N, of noble houses left in year t, where $t = 0$ in the year 1999.

b Graph your equation.

c According to your graph, how many noble families existed in 1999? Which point on the graph corresponds to this information?

Find an equation for the line passing through the two given points.

1.6.2.75. $(3, -5)$, $(-2, 4)$

Answer. $y = \frac{2}{5} - \frac{9}{5}x$

1.6.2.76. $(0, 8)$, $(4, -2)$

For Problems 77 and 78,

a Make a table of values showing two data points.

b Find a linear equation relating the variables.

c State the slope of the line, including units, and explain its meaning in the context of the problem.

1.6.2.77. The population of Maple Rapids was 4800 in 1990 and had grown to 6780 by 2005. Assume that the population increases at a constant rate. Express the population, P, of Maple Rapids in terms of the number of years, t, since 1990.

Answer.

a

t	0	15
P	4800	6780

b $P = 4800 + 132t$

c $m = 132$ people/year: the population grew at a rate of 132 people per year.

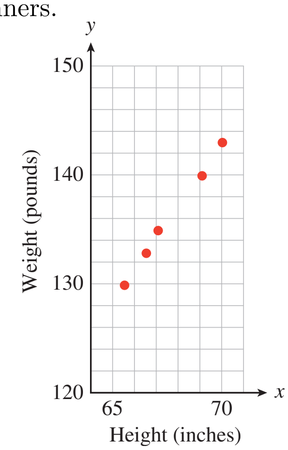
1.6.2.78. Cicely's odometer read 112 miles when she filled up her 14-gallon gas tank and 308 when the gas gauge read half full. Express her odometer reading, m, in terms of the amount of gas, g, she used.

1.6.2.79. In 1986, the space shuttle Challenger exploded because of O-ring failure on a morning when the temperature was about 30°F. Previously, there had been one incident of O-ring failure when the temperature was 70°F and three incidents when the temperature was 54°F. Use linear extrapolation to estimate the number of incidents of O-ring failure you would expect when the temperature is 30°F.

Answer. 6

1.6.2.80. Thelma typed a 19-page technical report in 40 minutes. She required only 18 minutes for an 8-page technical report. Use linear interpolation to estimate how long Thelma would require to type a 12-page technical report.

1.6.2.81. The scatterplot shows weights (in pounds) and heights (in inches) for a team of distance runners.



a Use a straightedge to draw a line that fits the data.

b Use your line to predict the weight of a 65-inch-tall runner and the weight of a 71-inch-tall runner.

c Use your answers from part (b) to approximate the equation of a regression line.

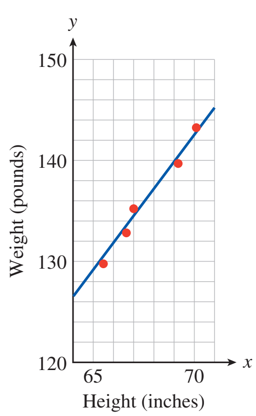
d Use your answer to part (c) to predict the weight of a runner who is 68 inches tall.

e The points on the scatterplot are $(65.5, 130)$, $(66.5, 133)$, $(67, 135)$, $(69, 140)$, and $(70, 143)$. Use your calculator to find the least squares regression line.

f Use the regression line to predict the weight of a runner who is 68 inches tall.

Answer.

a



b 129 lb, 145 lb

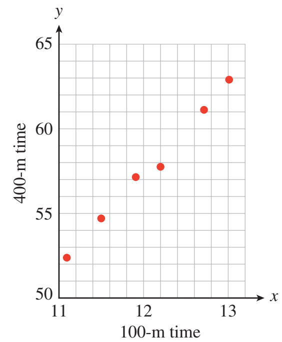
c $y = 2.\overline{6}x - 44.\overline{3}$

d 137 lb

e $y = 2.84x - 55.74$

f 137.33 lb

1.6.2.82. The scatterplot shows best times for various women running 400 meters and 100 meters.



a Use a straightedge to draw a line that fits the data.

b Use your line to predict the 400-meter time of a woman who runs the 100-meter dash in 11.2 seconds and the 400-meter time of a woman who runs the 100-meter dash in 13.2 seconds.

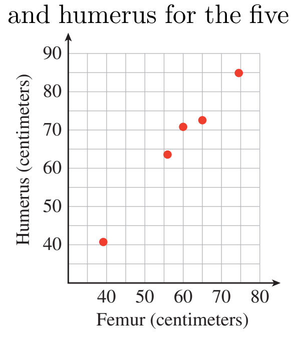
c Use your answers from part (b) to approximate the equation of a regression line.

d Use your answer to part (c) to predict the 400-meter time of a woman who runs the 100-meter dash in 12.1 seconds.

e The points on the scatterplot are $(11.1, 52.4)$, $(11.5, 54.7)$, $(11.9, 57.4)$, $(12.2, 57.9)$, $(12.7, 61.3)$, and $(13.0, 63.0)$. Use your calculator to find the least squares regression line.

f Use the regression line to predict the 400-meter time of a woman who runs the 100-meter dash in 12.1 seconds.

1.6.2.83. **Archaeopteryx** is an extinct creature with characteristics of both birds and reptiles. Only six fossil specimens are known, and only five of those include both a femur (leg bone) and a humerus (forearm bone) The scatterplot shows the lengths of femur and humerus for the five Archaeopteryx specimens.



a Use a straightedge to draw a line that fits the data.

b Predict the humerus length of an Archaeopteryx whose femur is 40 centimeters

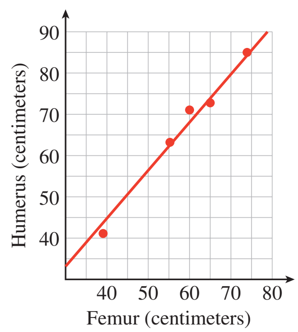
c Predict the humerus length of an Archaeopteryx whose femur is 75 centimeters

d Use your answers from parts (b) and (c) to approximate the equation of a regression line.

e Use your answer to part (d) to predict the humerus length of an Archaeopteryx whose femur is 60 centimeters.

f Use your calculator and the given points on the scatterplot to find the least squares regression line. Compare the score this equation gives for part (d) with what you predicted earlier. The ordered pairs defining the data are $(38, 41)$, $(56, 63)$, $(59, 70)$, $(64, 72)$, $(74, 84)$.

Answer.

a 

b 45 cm

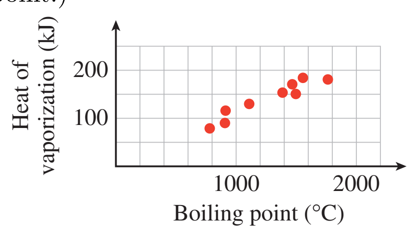
c 87 cm

d $y = 1.2x - 3$

e 69 cm

f $y = 1.197x - 3.660$; 68.16 cm

1.6.2.84. The scatterplot shows the boiling temperature of various substances on the horizontal axis and their heats of vaporization on the vertical axis. (The heat of vaporization is the energy needed to change the substance from liquid to gas at its boiling point.)



a Use a straightedge to estimate a line of best fit for the scatterplot.

b Use your line to predict the heat of vaporization of silver, whose boiling temperature is 2160°C.

c Find the equation of the regression line.

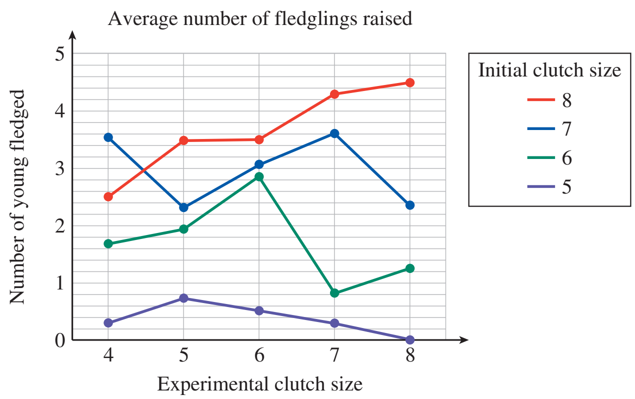
d Use the regression line to predict the heat of vaporization of potassium bromide, whose boiling temperature is 1435°C.

1.7 Projects for Chapter 1

Project 1.7.1 Optimal clutch size. The number of eggs (clutch size) that a bird lays varies greatly. Is there an optimal clutch size for birds of a given species, or does it depend on the individual bird?

In 1980, biologists in Sweden conducted an experiment on magpies as follows: They reduced or enlarged the natural clutch size by adding or removing eggs from the nests. They then computed the average number of fledglings successfully raised by the parent birds in each case.

The graph shows the results for magpies that initially laid 5, 6, 7, or 8 eggs. (Source: Högstedt, 1980, via Krebs as developed in Davies, 1993)



a Use the graph to fill in the table of values for the number of fledglings raised in each situation.

Initial clutch size laid	Experimental clutch size				
	4	5	6	7	8
5					
6					
7					
8					

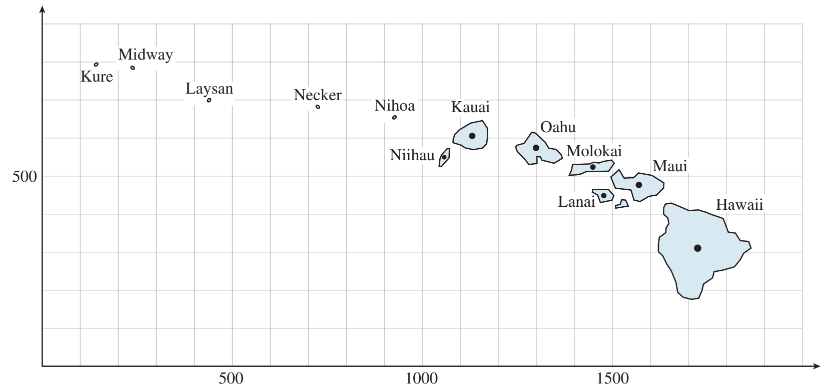
b For each initial clutch size, which experimental clutch size produced the most fledglings? Record your answers in the table.

Initial clutch size	5	6	7	8
Optimum clutch size				

c What conclusions can you draw in response to the question in the problem?

Project 1.7.2 Drift of Pacific tectonic plate. The Big Island of Hawaii is the last island in a chain of islands and submarine mountain peaks that stretch almost 6000 kilometers across the Pacific Ocean. All are extinct volcanoes except for the Big Island itself, which is still active.

The ages of the extinct peaks are roughly proportional to their distance from the Big Island. Geologists believe that the volcanic islands were formed as the tectonic plate drifted across a hot spot in the Earth's mantle. The figure shows a map of the islands, scaled in kilometers. (Source: Open University, 1998)



a The tables give the ages of the islands, in millions of years. Estimate the distance from each island to the Big Island, along a straight-line path through their centers. Fill in the third row of the tables.

Island Kauai	Hawaii	Maui	Lanai	Molokai	Oahu
Age 5.1	0.5	0.8	1.3	1.8	3.8
Distance					

Island	Nihau	Nihoa	Necker	Laysan	Midway
Age	4.9	7.5	10	20	27
Distance					

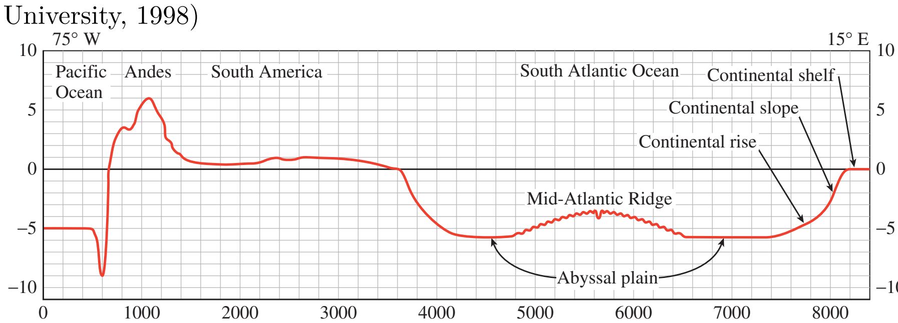
b Make a scatterplot showing the age of each island along the horizontal axis and its distance from Hawaii on the vertical axis.

c Draw a line of best fit through the data.

d Calculate the slope of the line of best fit, including units.

e Explain why the slope provides an estimate for the speed of the Pacific plate.

Project 1.7.3 Cross section of earth's surface. The graph shows a cross section of Earth's surface along an east-west line from the coast of Africa through the Atlantic Ocean to South America. Both axes are scaled in kilometers. Use the figure to estimate the distances in this problem. (Source: Open University, 1998)



a What is the highest land elevation shown in the figure? What is the lowest ocean depth shown? Give the horizontal coordinates of these two points, in kilometers west of the 75°W longitude line.

b How deep is the Atlantic Ocean directly above the crest of the Mid-Atlantic Ridge? How deep is the ocean above the abyssal plain on either side of the ridge?

c What is the height of the Mid-Atlantic Ridge above the abyssal plain? What is the width of the Mid-Atlantic Ridge?

d Using your answers to part (c), calculate the slope from the abyssal plain to the crest of the Mid-Atlantic Ridge, rounded to five decimal places

e Estimate the slopes of the continental shelf, the continental slope, and the continental rise. Use the coordinates of the points indicated on the figure

f Why do these slopes look much steeper in the accompanying figure than their numerical values suggest?

Project 1.7.4 Mid-Atlantic Range. The Mid-Atlantic Ridge is a mountain range on the sea floor beneath the Atlantic Ocean. It was discovered in the late nineteenth century during the laying of transatlantic telephone cables. The ridge is volcanic, and the ocean floor is moving away from the ridge on either side.

Geologists have estimated the speed of this sea-floor spreading by recording the age of the rocks on the sea floor and their distance from the ridge. (The age of the rocks is calculated by measuring their magnetic polarity. At known intervals over the last four million years, the Earth reversed its polarity, and this information is encoded in the rocks.) (Source: Open University, 1998)

a According to the table, rocks that are 0.78 million years old have moved 17 kilometers from the ridge. What was the speed of spreading over the past 0.78 million years? (This is the rate of spreading closest to the ridge.)

b Plot the data in the table, with age on the horizontal axis and separation distance on the vertical axis. Draw a line of best fit through the data.

c Calculate the slope of the regression line. What are the units of the slope?

d The slope you calculated in part (c) represents the average spreading rate over the past 3.58 million years. Is the average rate greater or smaller than the rate of spreading closest to the ridge?

e Convert the average spreading rate to millimeters per year

Age (millions of years)	0.78	0.99	1.07	1.79	1.95	2.60	3.04	3.11	3.22	3.33	3.58
Distance (km)	17	18	21	32	39	48	58	59	62	65	66

Project 1.7.5 Naismith's rule. Naismith's rule is used by runners and walkers to estimate journey times in hilly terrain. In 1892, Naismith wrote in the *Scottish Mountaineering Club Journal* that a person "in fair condition should allow for easy expeditions an hour for every three miles on the map, with an additional hour for every 2000 feet of ascent." (Source: Scarf, 1998)

a According to Naismith, one unit of ascent requires the same time as how many units of horizontal travel? (Convert miles to feet.) This is called **Naismith's number**. Round your answer to one decimal place b A walk in the Brecon Beacons in Wales covers 3.75 kilometers horizontally and climbs 582 meters. What is the equivalent flat distance?

c If you can walk at a pace of 15 minutes per kilometer over flat ground, how long will the walk in the Brecon Beacons take?

Project 1.7.6 Improved Naismith's number. Empirical investigations have improved Naismith's number (see Problem 5) to 8.0 for men and 9.5 for women. Part of the Karrimor International Mountain Marathon in the Arrochar Alps in Scotland has a choice of two routes. Route A is 1.75 kilometers long with a 240-meter climb, and route B is 3.25 kilometers long with a 90-meter climb. (Source: Scarf, 1998)

a Which route is faster for women?

b Which route is faster for men?

c At a pace of 6 minutes per flat kilometer, how much faster is the preferred route for women?

d At a pace of 6 minutes per flat kilometer, how much faster is the preferred route for men?

2 Modeling with Functions

2.1 Nonlinear Models

2.1.1 Solving Nonlinear Equations

Checkpoint 2.1.4

a Solve by extracting roots $\dfrac{3x^2 - 8}{5} = 10$.

First, isolate $\boldsymbol{x^2}$.

Take the square root of both sides.

b Give exact answers; then give approximations rounded to two decimal places.

Answer. $x = \pm\sqrt{\dfrac{58}{3}} \approx \pm 4.40$

2.1.2 Solving Formulas

Checkpoint 2.1.6 Find a formula for the radius of a circle in terms of its area.

Hint. Start with the formula for the area of a circle: $A =$ ________
Solve for r in terms of A.

Answer. $r = \sqrt{A/\pi}$

2.1.3 More Extraction of Roots

Checkpoint 2.1.8 Solve $2(5x + 3)^2 = 38$ by extracting roots.

a Give your answers as exact values.

b Find approximations for the solutions to two decimal places.

Answer.

a $x = \dfrac{-3 \pm \sqrt{19}}{5}$

b $x \approx -1.47$ or $x \approx 0.27$

2.1.4 Compound Interest and Inflation

Checkpoint 2.1.10 Two years ago, the average cost of dinner and a movie was $24. This year the average cost is $25.44. What was the rate of inflation over the past two years?

Answer. $r \approx 2.96\%$

2.1.5 Other Nonlinear Equations

Checkpoint 2.1.12 Solve $2\sqrt{x+4} = 6$

Answer. $x = 5$

Checkpoint 2.1.13 Use the intersect feature to solve the equation $2x^2 - 5 = 7$. Round your answers to three decimal places.

Answer. $x = \pm 2.449$

2.1.7 Homework 2.1

For Problems 1-6, solve by extracting roots. Give exact values for your answers.

2.1.7.1. $9x^2 = 25$

Answer. $\pm\dfrac{5}{3}$

2.1.7.2. $4x^2 = 9$

2.1.7.3. $4x^2 - 24 = 0$

Answer. $\pm\sqrt{6}$

2.1.7.4. $3x^2 - 9 = 0$

2.1.7.5. $\dfrac{2x^2}{3} = 4$

Answer. $\pm\sqrt{6}$

2.1.7.6. $\dfrac{3x^2}{5} = 6$

For Problems 7-12, solve by extracting roots. Round your answers to two decimal places.

2.1.7.7. $2x^2 = 14$

Answer. ± 2.65

2.1.7.8. $3x^2 = 15$

2.1.7.9. $1.5x^2 = 0.7x^2 + 26.2$

Answer. ± 5.72

2.1.7.10. $0.4x^2 = 2x^2 - 8.6$

2.1.7.11. $5x^2 - 97 = 3.2x^2 - 38$

Answer. ± 5.73

2.1.7.12. $17 - \dfrac{x^2}{4} = 43 - x^2$

>For Problems 13-16, solve the formulas for the specified variable.

2.1.7.13. $F = \dfrac{mv^2}{r}$, for v

Answer. $\pm\sqrt{\dfrac{Fr}{m}}$

2.1.7.14. $A = \dfrac{\sqrt{3}}{4}s^2$, for s

2.1.7.15. $s = \dfrac{1}{2}gt^2$, for t

Answer. $\pm\sqrt{\dfrac{2s}{g}}$

2.1.7.16. $S = 4\pi r^2$, for r

For Problems 17 and 18, refer to the geometric formulas in Appendix E.

2.1.7.17. A conical coffee filter is 8.4 centimeters tall.

a Write a formula for the filter's volume in terms of its widest radius (at the top of the filter).

b Complete the table of values for the volume equation. If you double the radius of the filter, by what factor does the volume increase?

r	1	2	3	4	5	6	7	8
V								

c If the volume of the filter is 302.4 cubic centimeters, what is its radius?

d Use your calculator to graph the volume equation. Locate the point on the graph that corresponds to the filter in part (c).

Answer.

a $V = 2.8\pi r^2 \approx 8.8r^2$

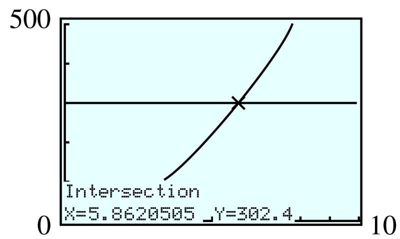
b

r	1	2	3	4	5	6	7	8
V	8.8	35.2	79.2	140.7	219.9	316.7	431.0	563.0

The volume increases by a factor of 4.

c 5.86 cm

d



2.1.7.18. A large bottle of shampoo is 20 centimeters tall and cylindrical in shape.

a Write a formula for the volume of the bottle in terms of its radius.

b Complete the table of values for the volume equation. If you halve the radius of the bottle, by what factor does the volume decrease?

r	1	2	3	4	5	6	7	8
V								

c What radius should the bottle have if it must hold 240 milliliters of shampoo? (One milliliter is equal to 1 cubic centimeter.)

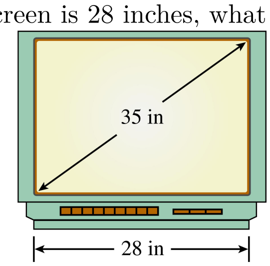
d Use your calculator to graph the volume equation. Locate the point on the graph that corresponds to the bottle in part (c).

For Problems 19–24,

a Make a sketch of the situation described, and label a right triangle.

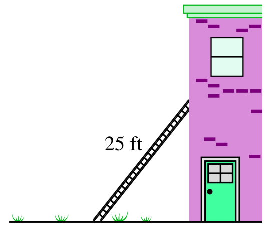
b Use the Pythagorean theorem to solve each problem. (See Algebra Skills Refresher Section A.11 to review the Pythagorean theorem.)

2.1.7.19. The size of a TV screen is the length of its diagonal. If the width of a 35-inch TV screen is 28 inches, what is its height?

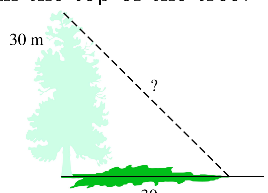


Answer. 21 in.

2.1.7.20. How high on a building will a 25-foot ladder reach if its foot is 15 feet away from the base of the wall?

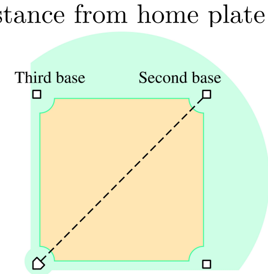


2.1.7.21. If a 30-meter pine tree casts a shadow of 30 meters, how far is the tip of the shadow from the top of the tree?

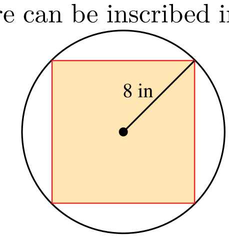


Answer. $\sqrt{1800} \approx 42.4$ m

2.1.7.22. A baseball diamond is a square whose sides are 90 feet in length. Find the straight-line distance from home plate to second base.

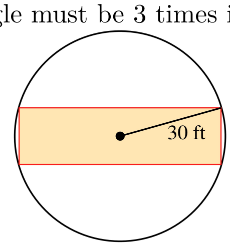


2.1.7.23. What size square can be inscribed in a circle of radius 8 inches?



Answer. $\sqrt{128}$ in. by $\sqrt{128}$ in. ≈ 11.3 in. $\times 11.3$ in.

2.1.7.24. What size rectangle can be inscribed in a circle of radius 30 feet if the length of the rectangle must be 3 times its width?



For Problems 25–30,

a Use a calculator or computer to graph the function in the suggested window.

b Use your graph to find two solutions for the given equation. (See Section 1.3 to review graphical solution of equations.)

c Check your solutions algebraically, using mental arithmetic.

2.1.7.25.

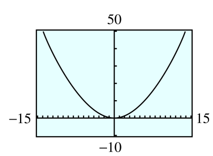
a $y = \frac{1}{4}x^2$

$$\text{Xmin} = -15 \quad \text{Xmax} = 15$$
$$\text{Ymin} = -10 \quad \text{Ymax} = 40$$

b $\frac{1}{4}x^2 = 36$

Answer.

a



b $x = \pm 12$

2.1.7.26.

a $y = 8x^2$

$$\text{Xmin} = -15 \quad \text{Xmax} = 15$$
$$\text{Ymin} = -50 \quad \text{Ymax} = 450$$

b $8x^2 = 392$

2.1.7.27.

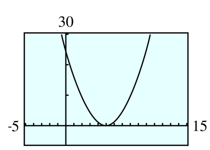
a $y = (x - 5)^2$

$$\text{Xmin} = -5 \quad \text{Xmax} = 15$$
$$\text{Ymin} = -5 \quad \text{Ymax} = 25$$

b $(x - 5)^2 = 16$

Answer.

a



b $x = 1$ or $x = 9$

2.1.7.28.

a $y = (x + 2)^2$

$$\text{Xmin} = -10 \quad \text{Xmax} = 10$$
$$\text{Ymin} = -2 \quad \text{Ymax} = 12$$

b $(x + 2)^2 = 9$

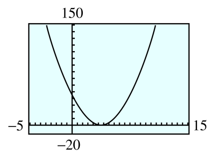
2.1.7.29.

a $y = 3(x-4)^2$

$$\begin{array}{ll} \text{Xmin} = -5 & \text{Xmax} = 15 \\ \text{Ymin} = -20 & \text{Ymax} = 130 \end{array}$$

b $3(x-4)^2 = 108$

Answer.

a 

b $x = 10$ or $x = -2$

2.1.7.30.

a $y = \dfrac{1}{2}(x+3)^2$

$$\begin{array}{ll} \text{Xmin} = -15 & \text{Xmax} = 5 \\ \text{Ymin} = -5 & \text{Ymax} = 15 \end{array}$$

b $\dfrac{1}{2}(x+3)^2 = 8$

For Problems 31-42, solve by extraction of roots.

2.1.7.31.
$(x-2)^2 = 9$
Answer. $5, -1$

2.1.7.32.
$(x+3)^2 = 4$

2.1.7.33.
$(2x-1)^2 = 16$
Answer. $\dfrac{5}{2}, \dfrac{-3}{2}$

2.1.7.34.
$(3x+1)^2 = 25$

2.1.7.35.
$4(x+2)^2 = 12$
Answer. $-2 \pm \sqrt{3}$

2.1.7.36.
$6(x-5)^2 = 42$

2.1.7.37.
$\left(x - \dfrac{1}{2}\right)^2 = \dfrac{3}{4}$
Answer. $\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}$

2.1.7.38.
$\left(x - \dfrac{2}{3}\right)^2 = \dfrac{5}{9}$

2.1.7.39.
$81\left(x + \dfrac{1}{3}\right)^2 = 1$
Answer. $\dfrac{-2}{9}, \dfrac{-4}{9}$

2.1.7.40.
$16\left(x + \dfrac{1}{2}\right)^2 = 1$

2.1.7.41.
$3(8x-7)^2 = 24$
Answer. $\dfrac{7}{8} \pm \dfrac{\sqrt{8}}{8}$

2.1.7.42.
$-2(5x-12)^2 = 48$

For Problems 43–54,

a Solve algebraically.

b Use the **intersect** feature on a graphing calculator to solve.

2.1.7.43.
$4x^3 - 12 = 852$
Answer. 6

2.1.7.44.
$\dfrac{8x^3+6}{3} = 74$

2.1.7.45.
$5\sqrt{x} - 9 = 31$
Answer. 64

2.1.7.46.
$25 - 2\sqrt{x} = 1$

2.1.7.47.
$\dfrac{1}{2x-3} = \dfrac{3}{4}$
Answer. $\dfrac{13}{6}$

2.1.7.48. $\dfrac{15}{x+16} = 3$

2.1.7.49.
$8 - 6\sqrt[3]{x} = -4$
Answer. 8

2.1.7.50.
$\dfrac{4\sqrt[3]{x}}{5} + 3 = 7$

2.1.7.51.
$\sqrt{3x-2} + 3 = 8$
Answer. 9

2.1.7.52. $6\sqrt{1-2x} = 30$

2.1.7.53. $\dfrac{2}{\sqrt{4x-2}} = 8$

Answer. $\dfrac{33}{64}$

2.1.7.54. $\dfrac{1}{\sqrt{x+2}} = \dfrac{3}{4}$

2.1.7.55. Cyril plans to invest $5000 in a money market account that pays interest compounded annually.

a Write a formula for the balance, B, in Cyril's account after two years as a function of the interest rate, r.

b If Cyril would like to have $6250 in two years, what interest rate must the account pay?

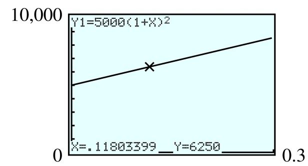
c Use your calculator to graph the formula for Cyril's account balance. Locate the point on the graph that corresponds to the amount in part (b).

Answer.

a $B = 5000(1+r)^2$

b 11.8%

c



2.1.7.56. You plan to deposit your savings of $1600 in an account that compounds interest annually.

a Write a formula for the amount in your savings account after two years as a function of the interest rate, r.

b To the nearest tenth of a percent, what interest rate will you require if you want your $1600 to grow to $2000 in two years?

c Use your calculator to graph the formula for the account balance. Locate the point on the graph that corresponds to the amount in part (b).

2.1.7.57. Carol's living expenses two years ago were $1200 per month. This year, the same items cost Carol $1400 per month. What was the annual inflation rate for the past two years?

Answer. 8%

2.1.7.58. Two years ago, the average price of a house in the suburbs was $188,600. This year, the average price is $203,700. What was the annual percent increase in the cost of a house?

2.1.7.59. A machinist wants to make a metal section of pipe that is 80 millimeters long and has an interior volume of 9000 cubic millimeters. If the pipe is 2 millimeters thick, its interior volume is given by the formula

$$V = \pi(r-2)^2 h$$

where h is the length of the pipe and r is its radius. What should the radius of the pipe be?

Answer. 7.98 mm

2.1.7.60. A storage box for sweaters is constructed from a square sheet of corrugated cardboard measuring x inches on a side. The volume of the box, in cubic inches, is

$$V = 10(x - 20)^2$$

If the box should have a volume of 1960 cubic inches, what size cardboard square is needed?

2.1.7.61. The area of an equilateral triangle is given by the formula $A = \frac{\sqrt{3}}{4}s^2$, where s is the length of the side.

a Find the areas of equilateral triangles with sides of length 2 centimeters, 4 centimeters, and 10 centimeters. First give exact values, then approximations to hundredths.

b Graph the area equation in the window

Xmin = 0	Xmax = 14.1
Ymin = 0	Ymax = 60

Use the `TRACE` or **value** feature to verify your answers to part (a).

c What does the coordinate $(5.1, 11.26266)$ represent?

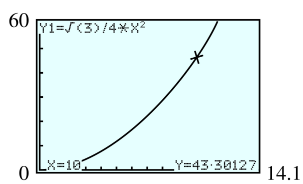
d Use your graph to estimate the side of an equilateral triangle whose area is 20 square centimeters.

e Write and solve an equation to answer part (d).

f If the area of an equilateral triangle is $100\sqrt{3}$ square centimeters, what is the length of its side?

Answer.

a $\sqrt{3} \approx 1.73$ sq cm, $4\sqrt{3} \approx 6.93$ sq cm, $25\sqrt{3} \approx 43.3$ sq cm

b 

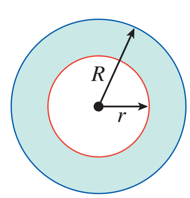
c An equilateral triangle with side 5.1 cm has area 11.263 cm^2.

d side ≈ 6.8 cm

e $\frac{\sqrt{3}}{4}s^2 = 20$; $s \approx 6.8$

f ≈ 20 cm

2.1.7.62. The area of the ring in the figure is given by the formula $A = \pi R^2 - \pi r^2$, where R is the radius of the outer circle and r is the radius of the inner circle.



a Suppose the inner radius of the ring is kept fixed at $r = 4$ centimeters, but the radius of the outer circle, R, is allowed to vary. Find the area of the ring when the outer radius is 6 centimeters, 8 centimeters, and 12 centimeters. First give exact values, then approximations to hundredths.

b Graph the area equation, with r = 4, in the window

Xmin = 0	Xmax = 14.1
Ymin = 0	Ymax = 400

Use the `TRACE` feature to verify your answers to part (a).

c **Trace** along the curve to the point $(9.75, 248.38217)$. What do the coordinates of this point represent?

d Use your graph to estimate the outer radius of the ring when its area is 100 square centimeters.

e Write and solve an equation to answer part (d).

f If the area of the ring is 9π square centimeters, what is the radius of the outer circle?

For Problems 63–68, solve for x in terms of a, b, and c.

2.1.7.63. $\dfrac{ax^2}{b} = c$

Answer. $\pm\sqrt{\dfrac{bc}{a}}$

2.1.7.64. $\dfrac{bx^2}{c} - a = 0$

2.1.7.65. $(x - a)^2 = 16$

Answer. $a \pm 4$

2.1.7.66. $(x + a)^2 = 36$

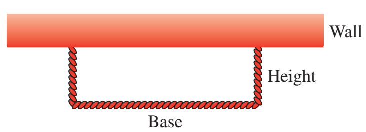
2.1.7.67. $(ax + b)^2 = 9$

Answer. $\dfrac{-b \pm 3}{a}$

2.1.7.68. $(ax - b)^2 = 25$

2.1.7.69. You have 36 feet of rope and you want to enclose a rectangular display area against one wall of an exhibit hall. The area enclosed depends on the dimensions of the rectangle you make. Because the wall makes one side of the rectangle, the length of the rope accounts for only three sides. Thus

$$\text{Base} + 2(\text{Height}) = 36$$



a Complete the table showing the base and the area of the rectangle for the given heights.

Height	Base	Area
1	34	34
2	32	64
3		
4		
5		
6		
7		
8		
9		

Height	Base	Area
10		
11		
12		
13		
14		
15		
16		
17		
18		

b Make a graph with *Height* on the horizontal axis and *Area* on the vertical axis. Draw a smooth curve through your data points.

c What is the area of the largest rectangle you can enclose in this way? What are its dimensions? On your graph, label the point that corresponds to this rectangle with the letter M.

d Let x stand for the height of a rectangle and write algebraic expressions for the base and the area of the rectangle.

e Enter your algebraic expression for the area in your calculator, then use the **Table** feature to verify the entries in your table in part (a).

f Graph your formula for area on your graphing calculator. Use your table of values and your handdrawn graph to help you choose appropriate `WINDOW` settings.

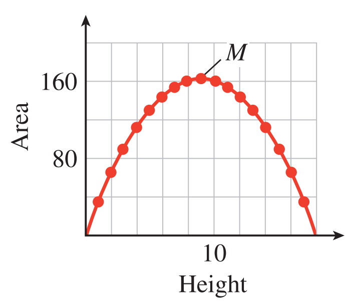
g Use the **intersect** command to find the height of the rectangle whose area is 149.5 square feet.

Answer.

a

Height	Base	Area
1	34	34
2	32	64
3	30	90
4	28	112
5	26	130
6	24	144
7	22	154
8	20	160
9	18	162

Height	Base	Area
10	16	160
11	14	154
12	12	144
13	10	130
14	8	112
15	6	90
16	4	64
17	2	34
18	0	0

b 

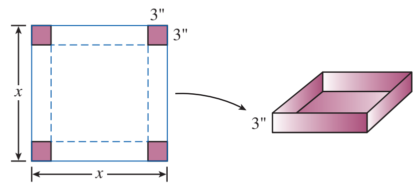
c 162 sq ft, with base 18 ft, height 9 ft

d Base: $36 - 2x$; area: $x(36 - 2x)$

e See (a)

f 6.5 ft or 11.5 ft

2.1.7.70. We are going to make an open box from a square piece of cardboard by cutting 3-inch squares from each corner and then turning up the edges as shown in the figure.



a Complete the table showing the side of the original sheet of cardboard, the dimensions of the box created from it, and the volume of the box.

Side	Length of box	Width of box	Height of box	Volume of box
7	1	1	3	3
8	2	2	3	12
9				
10				
11				
12				
13				
14				
15				

Explain why the side of the cardboard square cannot be smaller than 6 inches. What happens if the cardboard is exactly 6 inches on a side?

b Make a graph with *Side* on the horizontal axis and *Volume* on the vertical axis. Draw a smooth curve through your data points. (Use your table to help you decide on appropriate scales for the axes.)

c Let x represent the side of the original sheet of cardboard. Write algebraic expressions for the dimensions of the box and for its volume.

d Enter your expression for the volume of the box in your calculator; then use the **Table** feature to verify the values in your table in part (a).

e Graph your formula for volume on your graphing calculator. Use your table of values and your handdrawn graph to help you choose appropriate `WINDOW` settings.

f Use the **intersect** command to find out how large a square of cardboard you need to make a box with volume 126.75 cubic inches.

g Does your graph have a highest point? What happens to the volume of the box as you increase x?

2.1.7.71. The jump height, J, in meters, achieved by a pole vaulter is given approximately by $J = v^2/(2g)$, where v is the vaulter's speed in meters per second at the end of his run, and $g = 9.8$ is the gravitational acceleration. (Source: Alexander, 1992)

a Fill in the table of values for jump heights achieved with values of v from 0 to 11 meters per second.

v	0	1	2	3	4	5	6	7	8	9	10	11
J												

b Graph the jump height versus final speed. (Use the table values to help you choose a window for the graph.)

c The jump height should be added to the height of the vaulter's center of gravity (at about hip level) to give the maximum height, H, he can clear. For a typical pole vaulter, his center of gravity at the end of the run is 0.9 meters from the ground. Complete the table of values for maximum heights, H, and graph H on your graph of J.

v	0	1	2	3	4	5	6	7	8	9	10	11
H												

d A good pole vaulter can reach a final speed of 9.5 meters per second. What height will he clear?

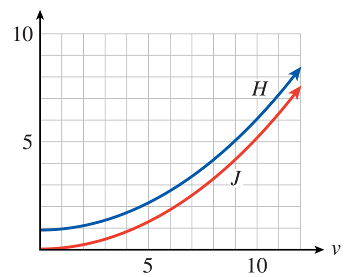
e In 2016, the world record in pole vaulting, established by Renaud Lavillenie in 2014, was 6.16 meters. What was the vaulter's speed at the end of his run?

Answer.

a

v	0	1	2	3	4	5	6	7	8	9	10	11
J	0	0.05	0.2	0.46	0.82	1.28	1.84	2.5	3.27	4.13	5.1	6.17

b



c

v	0	1	2	3	4	5	6	7	8	9	10	11
H	0.9	0.95	1.1	1.36	1.72	2.18	2.74	3.4	4.17	5.03	6.0	7.07

d 5.5 meters

e 10.15 meters per second

2.1.7.72. To be launched into space, a satellite must travel fast enough to escape Earth's gravity. This escape velocity, v, satisfies the equation

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

where m is the mass of the satellite, M is the mass of the Earth, R is the radius of the Earth, and G is the universal gravitational constant.

a Solve the equation for v in terms of the other variables.

b The equation

$$mg = \frac{GMm}{R^2}$$

gives the force of gravity at the Earth's surface. We can use this equation to simplify the expression for v: First, multiply both sides of the equation by $\frac{R}{m}$. You now have an expression for $\frac{GM}{R}$. Substitute this new expression into your formula for v.

c The radius of the Earth is about 6400 km, and $g = 0.0098\frac{\text{km}}{\text{s}^2}$. Calculate the escape velocity from Earth in kilometers per second. Convert your answer to miles per hour. (One kilometer is 0.621 miles.)

d The radius of the moon is 1740 km, and the value of g at the moon's surface is 0.0016. Calculate the escape velocity from the moon in kilometers per second and convert to miles per hour.

2.2 Some Basic Functions

2.2.1 Absolute Value

Checkpoint 2.2.3 Simplify each expression.

a $12 - 3\,|-6|$

b $-7 - 3\,|2 - 9|$

Answer.

a -6

b -28

2.2.2 Examples of Models

Investigation 2.2.1 Eight Basic Functions. Part I Some Powers

1. Complete the table of values for the squaring function, $f(x) = x^2$, and the cubing function, $g(x) = x^3$. Then sketch each function on graph paper, using the table values to help you scale the axes.
2. Verify both graphs with your graphing calculator.
3. State the intervals on which each graph is increasing.
4. Write a few sentences comparing the two graphs. The graph of $y = x^2$ is called a **parabola**, and the graph of $y = x^3$ is called a **cubic**.

x	$f(x) = x^2$	$g(x) = x^3$
-3		
-2		
-1		
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		
3		

Part II Some Roots

1. Complete the tables for the square root function, $f(x) = \sqrt{x}$, and the cube root function, $g(x) = \sqrt[3]{x}$. (Round your answers to two decimal places.) Then sketch each function on graph paper, using the table values to help you scale the axes.
2. Verify both graphs with your graphing calculator.
3. State the intervals on which each graph is increasing.
4. Write a few sentences comparing the two graphs.

x	$f(x) = \sqrt{x}$	x	$g(x) = \sqrt[3]{x}$
0		-8	
$\frac{1}{2}$		-4	
1		-1	
2		$-\frac{1}{2}$	
3		0	
4		$\frac{1}{2}$	
5		1	
7		4	
9		8	

Part III Asymptotes

1. Complete the table for the functions

$$f(x) = \frac{1}{x} \quad \text{and} \quad g(x) = \frac{1}{x^2}$$

What is true about $f(0)$ and $g(0)$?

x	$f(x) = \frac{1}{x}$	$g(x) = \frac{1}{x^2}$
-4		
-3		
-2		
-1		
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		
3		
4		

2. Prepare a grid on graph paper, scaling both axes from -5 to 5. Plot the points from the table and connect them with smooth curves.

3. As x increases through larger and larger values, what happens to the values of $f(x)$? Extend your graph to reflect your answer.

4. What happens to $f(x)$ as x decreases through larger and larger negative values (that is, for $x = -5, -6, -7, \ldots$)? Extend your graph for these x-values.

As the values of x get larger in absolute value, the graph approaches the x-axis. However, because $\frac{1}{x}$ never *equals* zero for any x-value, the graph never actually touches the x-axis. We say that the x-axis is a **horizontal asymptote** for the graph.

Repeat step (3) for the graph of $g(x)$.

Next we'll examine the graphs of f and g near $x = 0$.

1. Use your calculator to evaluate f for several x-values close to zero and record the results in the tables below.

x	$f(x) = \frac{1}{x}$	$g(x) = \frac{1}{x^2}$
-2		
-1		
-0.1		
-0.01		
-0.001		

x	$f(x) = \frac{1}{x}$	$g(x) = \frac{1}{x^2}$
2		
1		
0.1		
0.01		
0.001		

What happens to the values of $f(x)$ as x approaches zero? Extend your graph of f to reflect your answer.

As x approaches zero from the left (through negative values), the function values decrease toward $-\infty$. As x approaches zero from the right (through positive values), the function values increase toward ∞. The graph approaches but never touches the vertical line $x = 0$ (the y-axis.)We say that the graph of f has a **vertical asymptote** at $x = 0$.

2. Repeat step (1) for the graph of $g(x)$.

3. The functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ are examples of **rational functions**, so called because they are fractions, or ratios. Verify both graphs with your graphing calculator. Use the window

$$\begin{array}{ll} \text{Xmin} = -4 & \text{Xmax} = 4 \\ \text{Ymin} = -4 & \text{Ymax} = 4 \end{array}$$

4. State the intervals on which each graph is increasing.

5. Write a few sentences comparing the two graphs.

Part IV Absolute Value

1. Complete the table for the two functions $f(x) = x$ and $g(x) = |x|$. Then sketch each function on graph paper, using the table values to help you scale the axes.

x	$f(x) = x$	$g(x) = \lvert x\rvert$
-4		
-3		
-2		
-1		
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		
3		
4		

2. Verify both graphs with your graphing calculator. Your calculator uses the notation $abs(x)$ instead of $|x|$ for the absolute value of x. First, position the cursor after $Y_1 =$ in the graphing window. Now access the absolute value function by pressing `2nd` 0 for *CATALOG*; then `ENTER` for *abs(*. Don't forget to press X if you want to graph $y = |x|$.

3. State the intervals on which each graph is increasing.

4. Write a few sentences comparing the two graphs.

2.2.4 Properties of the Basic Functions

Checkpoint 2.2.6 Verify the multiplicative property of absolute value for the three cases in Example 2.2.4.

Answer.

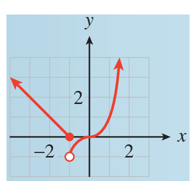
- $|3|\,|5| = 15 = |3 \cdot 5|$
- $|-3|\,|-5| = 15 = \big|(-3) \cdot (-5)\big|$
- $|3|\,|-5| = 15 = \big|3(-5)\big|$

2.2.5 Functions Defined Piecewise

Checkpoint 2.2.8 Graph the piecewise defined function

$$g(x) = \begin{cases} -1 - x & \text{if } x \le -1 \\ x^3 & \text{if } x > -1 \end{cases}$$

Answer.



Checkpoint 2.2.10

a Use your calculator to graph $g(x) = |x - 3|$ and $h(x) = |x| + |-3|$. Are the graphs the same?

b Explain why the functions $f(x) = |x + k|$ and $g(x) = |x| + |k|$ are not the same if $k \neq 0$.

Answer.

a No

b Because $|x + k| \neq |x| + |k|$ when x and k have opposite signs.

2.2.7 Homework 2.2

For problems 1-10, simplify the expression according to the order of operations.

2.2.7.1.

a $-|-9|$

b $-(-9)$

Answer.

a -9 b 9

2.2.7.2.

a $2 - (-6)$

b $2 - |-6|$

2.2.7.3.

a $|-8| - |12|$

b $|-8 - 12|$

Answer.

a -4 b 20

2.2.7.4.

a $|-3| + |-5|$

b $\left|-3 + (-5)\right|$

2.2.7.5. $4 - 9\,|2 - 8|$

Answer. -50

2.2.7.6. $2 - 5\,|-6 - 3|$

2.2.7.7. $|-4 - 5|\,\left|1 - 3(-5)\right|$

Answer. 144

2.2.7.8. $|-3 + 7|\,\left|-2(6 - 10)\right|$

2.2.7.9. $\left||-5| - |-6|\right|$

Answer. 1

2.2.7.10. $\left||4| - |-6|\right|$

In Problems 11–14, show how to use the graphs to find the values. Estimate your answers to one decimal point. Compare your estimates to values obtained with a calculator.

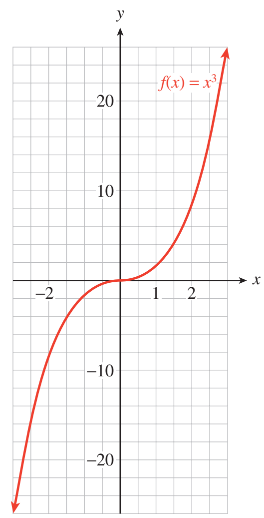
2.2.7.11. Refer to the graph of $f(x) = x^3$.

a Estimate the value of $(1.4)^3$.

b Find all numbers whose cubes are -20.

c Find all solutions of the equation $x^3 = 6$.

d Estimate the value of $\sqrt[3]{24}$.



Answer.

a 2.7

b -2.7

c 1.8

d 2.9

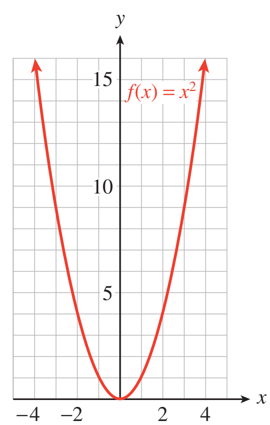
2.2.7.12. Refer to the graph of $f(x) = x^2$.

a Estimate the value of $(-2.5)^2$.

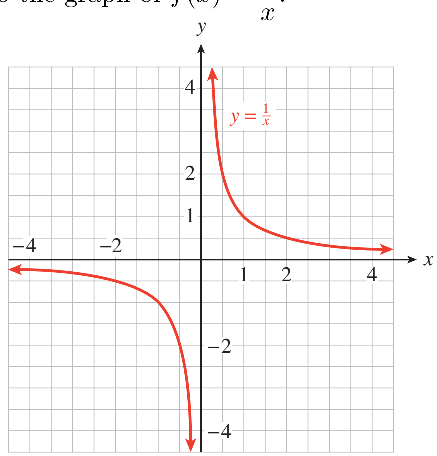
b Find all numbers whose squares are 12.

c Find all solutions of the equation $x^2 = 15$.

d Estimate the value of $\sqrt{10.5}$.



2.2.7.13. Refer to the graph of $f(x) = \dfrac{1}{x}$.



a Estimate the value of $\dfrac{1}{3.4}$.

b Find all numbers whose reciprocals are -2.5.

c Find all solutions of the equation $\dfrac{1}{x} = 4.8$.

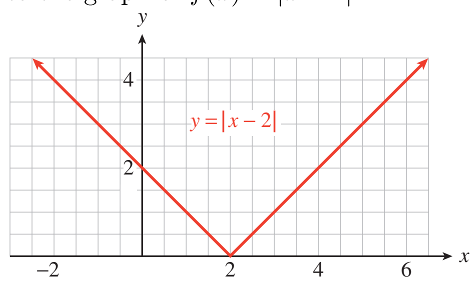
Answer.

a 0.3

b -0.4

c 0.2

2.2.7.14. Refer to the graph of $f(x) = |x - 2|$.



a Estimate the value of $|1.6 - 2|$.

b Find all values of x for which $|x - 2| = 3$.

c Find all solutions of the equation $|x - 2| = 0.4$.

For Problems 15–18,

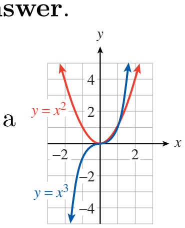
a Sketch both functions on the same grid, paying attention to the shape of the graph. Plot at least three guidepoints for each graph to ensure accuracy.

b Use the graph to find all solutions of the equation $f(x) = g(x)$.

c On what intervals is $f(x) > g(x)$?

2.2.7.15. $f(x) = x^2,\ g(x) = x^3$

Answer.

a 

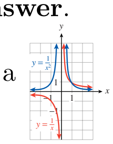
b $x = 0,\ x = 1$

c $(-\infty, 0)$ and $(0, 1)$

2.2.7.16. $f(x) = \sqrt{x},\ g(x) = \sqrt[3]{x}$

2.2.7.17. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x^2}$

Answer.

a 

b $x = 1$

c $(1, +\infty)$

2.2.7.18. $f(x) = x,\ g(x) = |x|$

For Problems 19-24, graph each set of functions together in the **ZDecimal** window. Describe how graphs (b) and (c) are different from the basic graph.

2.2.7.19.

a $f(x) = x^3$

b $g(x) = x^3 - 2$

c $h(x) = x^3 + 1$

Answer. Graph (b) is the basic graph shifted 2 units down; graph (c) is the basic graph shifted 1 unit up.

2.2.7.20.

a $f(x) = |x|$

b $g(x) = |x - 2|$

c $h(x) = |x + 1|$

2.2.7.21.

a $f(x) = \dfrac{1}{x}$

b $g(x) = \dfrac{1}{x + 1.5}$

c $h(x) = \dfrac{1}{x - 1}$

Answer. Graph (b) is the basic graph shifted 1.5 units left; graph (c) is the basic graph shifted 1 unit right.

2.2.7.22.

a $f(x) = \dfrac{1}{x^2}$

b $g(x) = \dfrac{1}{x^2} + 2$

c $h(x) = \dfrac{1}{x^2} - 1$

2.2.7.23.

a $f(x) = \sqrt{x}$

b $g(x) = -\sqrt{x}$

c $h(x) = \sqrt{-x}$

Answer. Graph (b) is the basic graph reflected about the x-axis; graph (c) is the basic graph reflected about the y-axis.

2.2.7.24.

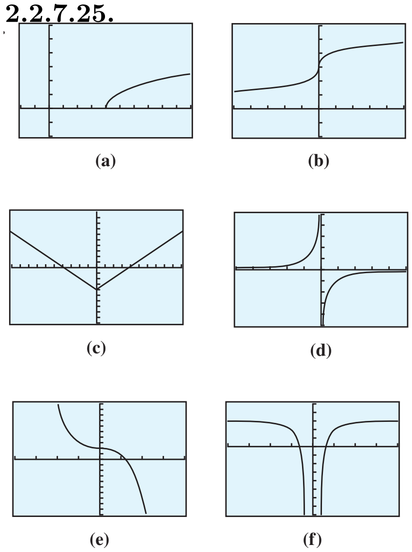
a $f(x) = \sqrt[3]{x}$

b $g(x) = -\sqrt[3]{x}$

c $h(x) = \sqrt[3]{-x}$

Each graph in Problems 25-26 is a variation of one of the eight basic graphs of Investigation 2.2.1. Identify the basic graph for each problem.

2.2.7.25.

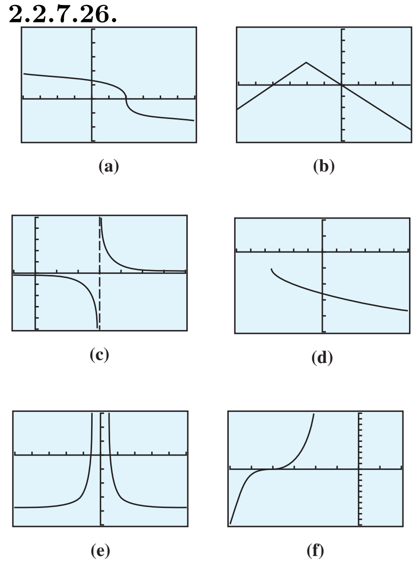


Answer.

a $\sqrt{x}$ c $|x|$ e x^3

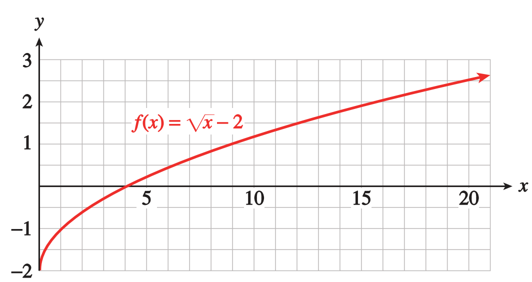
b $\sqrt[3]{x}$ d $\dfrac{1}{x}$ f $\dfrac{1}{x^2}$

2.2.7.26.



In Problems 27–30, use the graph to estimate the solution to the equation or inequality. Show the solution or solutions on the graph.

2.2.7.27. The figure shows a graph of $f(x) = \sqrt{x} - 2$, for $x > 0$. Solve the following:



a $\sqrt{x} - 2 = 1.5$

b $\sqrt{x} - 2 = 2.25$

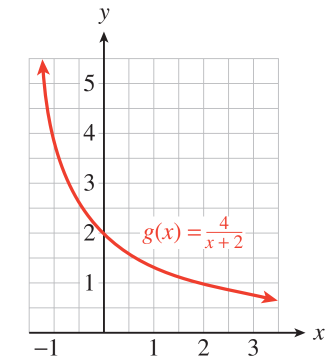
c $\sqrt{x} - 2 < 1$

d $\sqrt{x} - 2 > -0.25$

Answer.

a $x \approx 12$ b $x \approx 18$ c $x < 9$ d $x > 3$

2.2.7.28. The figure shows a graph of $g(x) = \dfrac{4}{x+2}$, for $x > -2$. Solve the following:



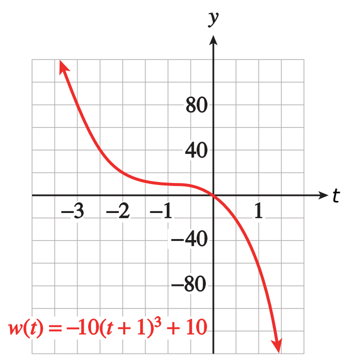
a $\dfrac{4}{x+2} = 4$

b $\dfrac{4}{x+2} = 0.8$

c $\dfrac{4}{x+2} > 1$

d $\dfrac{4}{x+2} < 3$

2.2.7.29. The figure shows a graph of $w(t) = -10(t+1)^3 + 10$. Solve the following:



a $-10(t+1)^3 + 10 = 100$

b $-10(t+1)^3 + 10 = -140$

c $-10(t+1)^3 + 10 > -50$

d $-20 < -10(t+1)^3 + 10 < 40$

Answer.

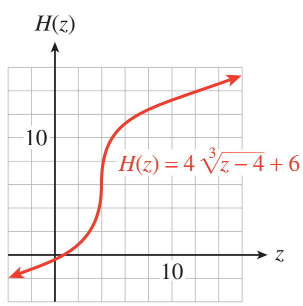
a $t \approx -3.1$

b $t \approx 1.5$

c $t < 0.8$

d $-2.4 < t < 0.4$

2.2.7.30. The figure shows a graph of $H(z) = 4\sqrt[3]{z-4} + 6$. Solve the following:



a $4\sqrt[3]{z-4} + 6 = 2$

b $4\sqrt[3]{z-4} + 6 = 12$

c $4\sqrt[3]{z-4} + 6 > 14$

d $4\sqrt[3]{z-4} + 6 < 6$

For Problems 31-34, gaph the function with the **ZInteger** setting. Use the graph to solve each equation or inequality. Check your solutionsalgebraically.

2.2.7.31. Graph $F(x) = 4\sqrt{x-25}$.

a Solve $4\sqrt{x-25} = 16$

b Solve $8 < 4\sqrt{x-25} \le 24$

Answer.

a $x = 41$

b $29 < x < 61$

2.2.7.32. Graph $G(x) = 15 - 0.01(x-2)^3$.

a Solve $15 - 0.01(x-2)^3 = -18.75$

b Solve $15 - 0.01(x-2)^3 \le 25$

2.2.7.33. Graph $H(x) = 24 - 0.25(x-6)^2$.

a Solve $24 - 0.25(x-6)^2 = -6.25$

b Solve $24 - 0.25(x-6)^2 > 11.75$

Answer.

a $x = -5$ or $x = 17$

b $-1 < x < 13$

2.2.7.34. Graph $R(x) = 0.1(x+12)^2 - 18$.

a Solve $0.1(x+12)^2 - 18 = 14.4$

b Solve $0.1(x+12)^2 - 18 < 4.5$

For Problems 35–40,

a Graph the equation by completing the table and plotting points.

b Does the equation define y as a function of x? Why or why not?

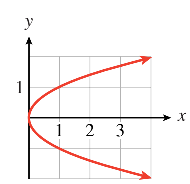
2.2.7.35. $x = y^2$

x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

Answer.

a

x	4	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	4
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2



b no

2.2.7.36. $x = y^3$

x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

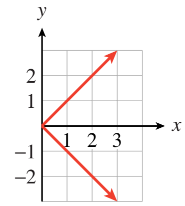
2.2.7.37. $x = |y|$

x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

Answer.

a

x	2	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2



b no

2.2.7.38. $|x| = |y|$

x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

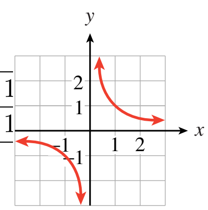
2.2.7.39. $x = \dfrac{1}{y}$

x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

Answer.

a

x	$-\frac{1}{2}$	-1	-2	undefined	2	1
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1



b yes

2.2.7.40. $x = \dfrac{1}{y^2}$

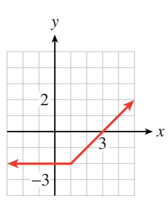
x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

For Problems 41–52, graph the following piecewise defined functions. Indicate whether the endpoints of each piece are included on the graph.

2.2.7.41.

$$f(x) = \begin{cases} -2 & \text{if } x \le 1 \\ x - 3 & \text{if } x > 1 \end{cases}$$

Answer.



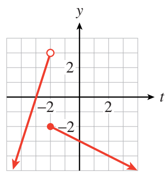
2.2.7.42.

$$h(x) = \begin{cases} -x + 2 & \text{if } x \le -1 \\ 3 & \text{if } x > -1 \end{cases}$$

2.2.7.43.

$$G(t) = \begin{cases} 3t + 9 & \text{if } t < -2 \\ -3 - \dfrac{1}{2}t & \text{if } t \geq -2 \end{cases}$$

Answer.



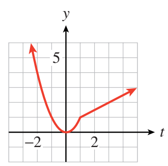
2.2.7.44.

$$F(s) = \begin{cases} \dfrac{1}{3}s + 3 & \text{if } s < 3 \\ 2s - 3 & \text{if } s \geq 3 \end{cases}$$

2.2.7.45.

$$H(t) = \begin{cases} t^2 & \text{if } t \leq 1 \\ \dfrac{1}{2}t + \dfrac{1}{2} & \text{if } t > 1 \end{cases}$$

Answer.

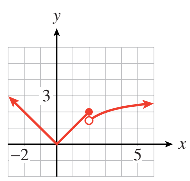


2.2.7.46.

$$g(t) = \begin{cases} \dfrac{3}{2}t + 7 & \text{if } t \leq -2 \\ t^2 & \text{if } t > -2 \end{cases}$$

2.2.7.47. $k(x) = \begin{cases} |x| & \text{if } x \leq 2 \\ \sqrt{x} & \text{if } x > 2 \end{cases}$

Answer.

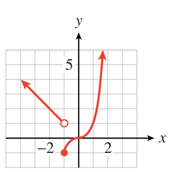


2.2.7.48. $S(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 1 \\ |x| & \text{if } x \geq 1 \end{cases}$

2.2.7.49.

$$D(x) = \begin{cases} |x| & \text{if } x < -1 \\ x^3 & \text{if } x \geq -1 \end{cases}$$

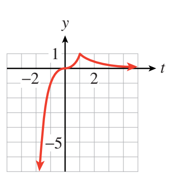
Answer.



2.2.7.50. $m(x) = \begin{cases} x^2 & \text{if } x \leq \dfrac{1}{2} \\ |x| & \text{if } x > \dfrac{1}{2} \end{cases}$

2.2.7.51. $P(t) = \begin{cases} t^3 & \text{if } t \leq 1 \\ \dfrac{1}{t^2} & \text{if } t > 1 \end{cases}$

Answer.



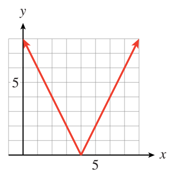
2.2.7.52. $Q(t) = \begin{cases} t^2 & \text{if } t \leq -1 \\ \sqrt[3]{t} & \text{if } t > -1 \end{cases}$

For Problems 53-58, write a piecewise definition for the function and sketch its graph.

2.2.7.53. $f(x) = |2x - 8|$

Answer.

$$f(x) = \begin{cases} 8 - 2x & x < 4 \\ 2x - 8 & x \geq 4 \end{cases}$$

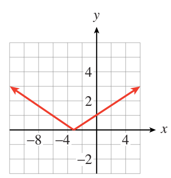


2.2.7.54. $g(x) = |3x + 6|$

2.2.7.55. $g(t) = \left|1 + \dfrac{t}{3}\right|$

Answer.

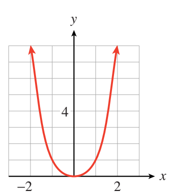
$$g(t) = \begin{cases} -1 - \dfrac{t}{3} & t < -3 \\ 1 + \dfrac{t}{3} & t \geq -3 \end{cases}$$



2.2.7.56. $f(t) = \left|\dfrac{1}{2}t - 3\right|$

2.2.7.57. $F(x) = \left|x^3\right|$

Answer. $F(x) = \begin{cases} -x^3 & x < 0 \\ x^3 & x \geq 0 \end{cases}$



2.2.7.58. $G(x) = \left|\dfrac{1}{x}\right|$

In Problems 59–64, decide whether each statement is true for all values of a and b. If the statement is true, give an algebraic justification. If it is false, find values of a and b to disprove it.

a $f(a + b) = f(a) + f(b)$

b $f(ab) = f(a)f(b)$

2.2.7.59. $f(x) = x^2$

Answer.

a Not always true: $f(1 + 2) \neq f(1) + f(2)$ because $9 \neq 5$.

b True: $(ab)^2 = a^2b^2$

2.2.7.60. $f(x) = x^3$

2.2.7.61. $f(x) = \dfrac{1}{x}$

Answer.

a Not always true:
$f(1+2) \neq f(1) + f(2)$
because $\frac{1}{3} \neq \frac{3}{2}$.

b True: $\dfrac{1}{ab} = \dfrac{1}{a} \cdot \dfrac{1}{b}$

2.2.7.62. $f(x) = \sqrt{x}$

2.2.7.63. $f(x) = mx + b$

Answer.

a Not always true (unless $b = 0$):
$f(1+2) \neq f(1) + f(2)$
because $3m + b \neq 3m + 2b$.

b Not always true:
$f(1 \cdot 2) \neq f(1) \cdot f(2)$ because
$2m + b \neq 2m^2 + 3mb + b^2$.

2.2.7.64. $f(x) = kx$

2.2.7.65. Verify that $|a - b|$ gives the distance between a and b on a number line.

a $a = 3,\ b = 8$

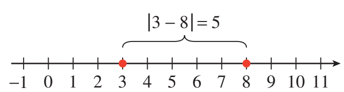
b $a = -2,\ b = -6$

c $a = 4,\ b = -3$

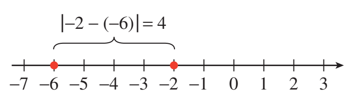
d $a = -2,\ b = 5$

Answer.

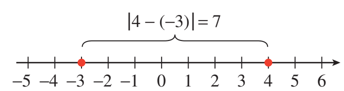
a



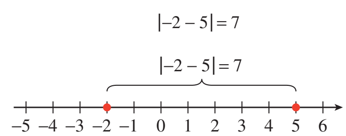
b



c



d



2.2.7.66. Which of the following statements is true for all values of a and b?

1 $|a - b| = |a| + |b|$

2 $|a - b| \leq |a| + |b|$

3 $|a - b| \geq |a| + |b|$

2.2.7.67. Explain how the distributive law, $a(b + c) = ab + ac$, is different from the equation $f(a + b) = f(a) + f(b)$.

Answer. The distributive law shows a relationship between multiplication and addition that always holds. The equation $f(a + b) = f(a) + f(b)$ is not about multiplication and may or may not be true.

2.2.7.68. For each function, decide whether $f(kx) = kf(x)$ for all $x \neq 0$, where $k \neq 0$ is a constant.

a $f(x) = x^2$

b $f(x) = \dfrac{1}{x}$

c $f(x) = \sqrt{x}$

d $f(x) = |x|$

For Problems 69-70,find the indicated value.

2.2.7.69. Use the function $F(s) = \begin{cases} \frac{1}{3}s + 3 & \text{if } s < 3 \\ 2s - 3 & \text{if } s \geq 3 \end{cases}$ (from Problem 44) and add the indicated values

a $F(0)$

b $F(3)$

c $F(9)$

2.2.7.70. Use the function $k(x) = \begin{cases} |x| & \text{if } x \leq 2 \\ \sqrt{x} & \text{if } x > 2 \end{cases}$ (from Problem 47) and add the indicated values

a $k(2)$

b $k(1)$

c $k(-3)$

d $k(4.5)$

2.3 Transformations of Graphs

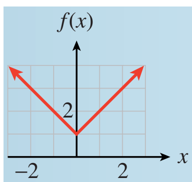
2.3.1 Vertical Translations

Checkpoint 2.3.2

a Graph the function $f(x) = |x| + 1$.

b How is the graph of f different from the graph of $y = |x|$?

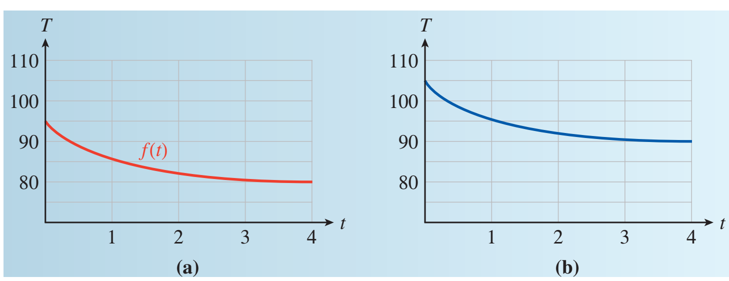
Answer.

a 

b Translate $y = |x|$ one unit up.

Checkpoint 2.3.4 An evaporative cooler, or swamp cooler, is an energy-efficient type of air conditioner used in dry climates. A typical swamp cooler can reduce the temperature inside a house by 15 degrees.

Figure (a) shows the graph of $T = f(t)$, the temperature inside Kate's house t hours after she turns on the swamp cooler. Write a formula in terms of f for the function g shown in figure (b), and give a possible explanation of its meaning.



Answer. $g(t) = f(t) + 10$. The outside temperature was 10° hotter.

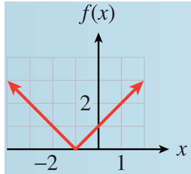
2.3.2 Horizontal Translations

Checkpoint 2.3.7

a Graph the function $f(x) = |x + 1|$.

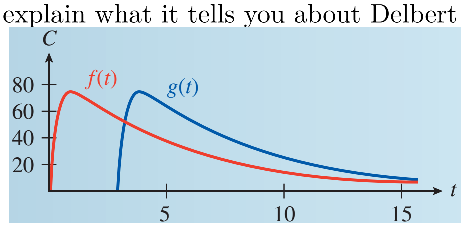
b How is the graph of f different from the graph of $y = |x|$?

Answer.

a 

b Translate $y = |x|$ one unit left.

Checkpoint 2.3.9 The function $C = f(t)$ shown below gives the caffeine level in Delbert's bloodstream at time t hours after he drinks a cup of coffee, and $g(t)$ gives the caffeine level in Francine's bloodstream. Write a formula for g in terms of f, and explain what it tells you about Delbert and Francine.



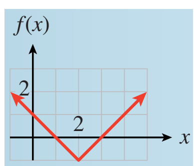
Answer. $g(t) = f(t-3)$. Francine drank her coffee 3 hours after Delbert drank his.

Checkpoint 2.3.11

a Graph the function $f(x) = |x-2| - 1$.

b How is the graph of f different from the graph of $y = |x|$?

Answer.

a 

b Translate $y = |x|$ one unit down and two units right.

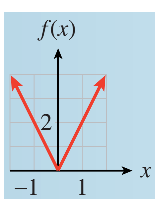
2.3.3 Scale Factors

Checkpoint 2.3.13

a Graph the function $f(x) = 2\,|x|$.

b How is the graph of f different from the graph of $y = |x|$?

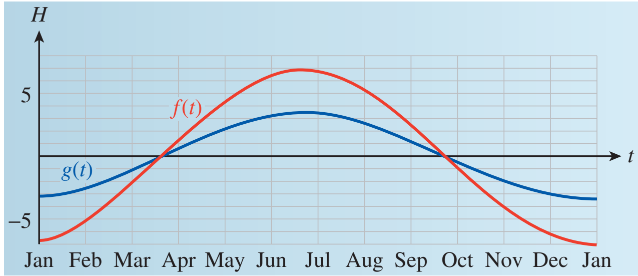
Answer.

a 

b Stretch $y = |x|$ vertically by a factor of 2.

Checkpoint 2.3.15 If the Earth were not tilted on its axis, there would be 12 daylight hours every day all over the planet. But in fact, the length of a day in a particular location depends on the latitude and the time of year.

The graph below shows $H = f(t)$, the length of a day in Helsinki, Finland, t days after January 1, and $R = g(t)$, the length of a day in Rome. Each is expressed as the number of hours greater or less than 12. Write a formula for f in terms of g. What does this formula tell you?

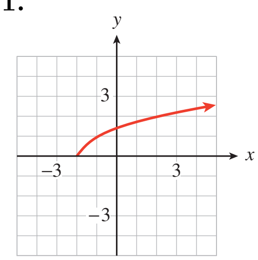


Answer. $f(t) \approx 2g(t)$. On any given day, the number of daylight hours varies from 12 hours about twice as much in Helsinki as it does in Rome.

2.3.5 Homework 2.3

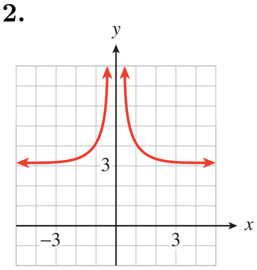
In Problems 1–6, identify the graph as a translation of a basic function, and write a formula for the graph.

2.3.5.1.

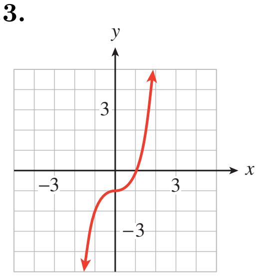


Answer. $y = \sqrt{x+2}$

2.3.5.2.

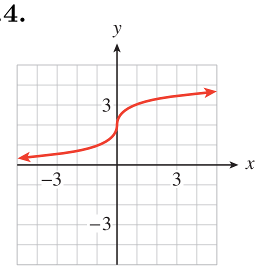


2.3.5.3.

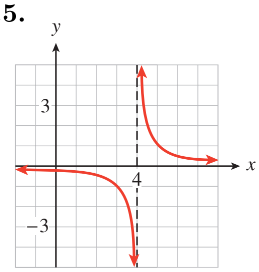


Answer. $y = x^3 - 1$

2.3.5.4.

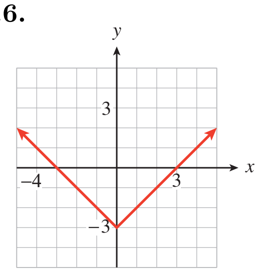


2.3.5.5.



Answer. $y = \dfrac{1}{x-4}$

2.3.5.6.



For Problems 7–18,

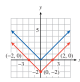
a Describe how to transform one of the basic graphs to obtain the graph of the given function.

b Using guidepoints, sketch the basic graph and the graph of the given function on the same axes. Label the coordinates of three points on the graph of the given function.

2.3.5.7.
$f(x) = |x| - 2$

Answer.

a Translate $y = |x|$ by 2 units down.

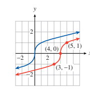
b 

2.3.5.8.
$g(x) = (x+1)^3$

2.3.5.9.
$g(s) = \sqrt[3]{s-4}$

Answer.

a Translate $y = \sqrt[3]{s}$ by 4 units right.

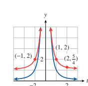
b 

2.3.5.10.
$f(s) = s^2 + 3$

2.3.5.11.
$F(t) = \dfrac{1}{t^2} + 1$

Answer.

a Translate $y = \dfrac{1}{t^2}$ by 1 unit up.

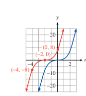
b 

2.3.5.12.
$G(t) = \sqrt{t-2}$

2.3.5.13.
$G(r) = (r+2)^3$

Answer.

a Translate $y = r^3$ by 2 units left.

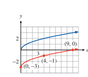
b 

2.3.5.14.
$F(r) = \dfrac{1}{r-4}$

2.3.5.15.
$H(d) = \sqrt{d} - 3$

Answer.

a Translate $y = \sqrt{d}$ by 3 units down.

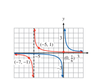
b 

2.3.5.16.
$h(d) = \sqrt[3]{d} + 5$

2.3.5.17.
$h(v) = \dfrac{1}{v+6}$

Answer.

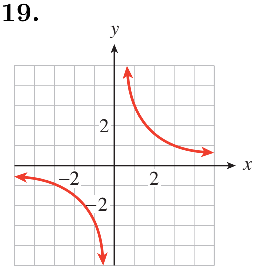
a Translate $y = \dfrac{1}{v}$ by 6 units left.

b 

2.3.5.18.
$H(v) = \dfrac{1}{v^2} - 2$

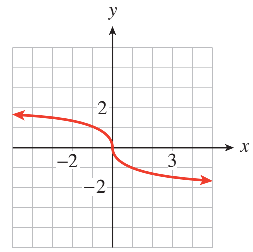
For Problems 19-22, identify the graph as a stretch, compression, or reflection of a basic function, and write a formula for the graph.

2.3.5.19.

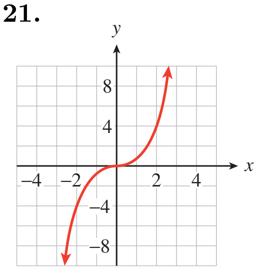


Answer. A vertical stretch by a factor of 3: $y = \dfrac{3}{x}$

2.3.5.20.

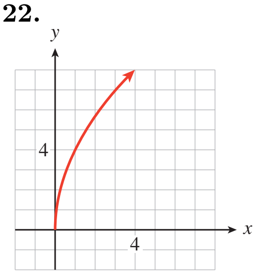


2.3.5.21.



Answer. A vertical compression, the scale factor is $\dfrac{1}{2}$: $y = \dfrac{1}{2}x^3$

2.3.5.22.



For Problems 23–32,

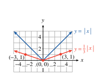
a Identify the scale factor for each function and describe how it affects the graph of the corresponding basic function.

b Using guidepoints, sketch the basic graph and the graph of the given function on the same axes. Label the coordinates of three points on the graph of the given function.

2.3.5.23. $f(x) = \dfrac{1}{3}|x|$

Answer.

a Scale factor $\frac{1}{3}$; $y = |x|$ is compressed vertically by the scale factor.

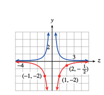
b 

2.3.5.24. $H(x) = -3|x|$

2.3.5.25. $h(z) = \dfrac{-2}{z^2}$

Answer.

a Scale factor -2; $y = \frac{1}{z^2}$ is reflected over the z-axis and stretched vertically by a factor of 2.

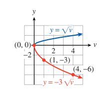
b 

2.3.5.26. $g(z) = \dfrac{2}{z}$

2.3.5.27. $G(v) = -3\sqrt{v}$

Answer.

a Scale factor -3; $y = \sqrt{v}$ is reflected over the v-axis and stretched vertically by a factor of 3.

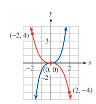
b 

2.3.5.28. $F(v) = -4\sqrt[3]{v}$

2.3.5.29. $g(s) = \dfrac{-1}{2}s^3$

Answer.

a Scale factor $\frac{-1}{2}$; $y = s^3$ is reflected over the s-axis and compressed vertically by a factor of $\frac{1}{2}$.

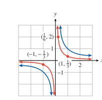
b 

2.3.5.30. $f(s) = \dfrac{1}{8}s^3$

2.3.5.31. $H(x) = \dfrac{1}{3x}$

Answer.

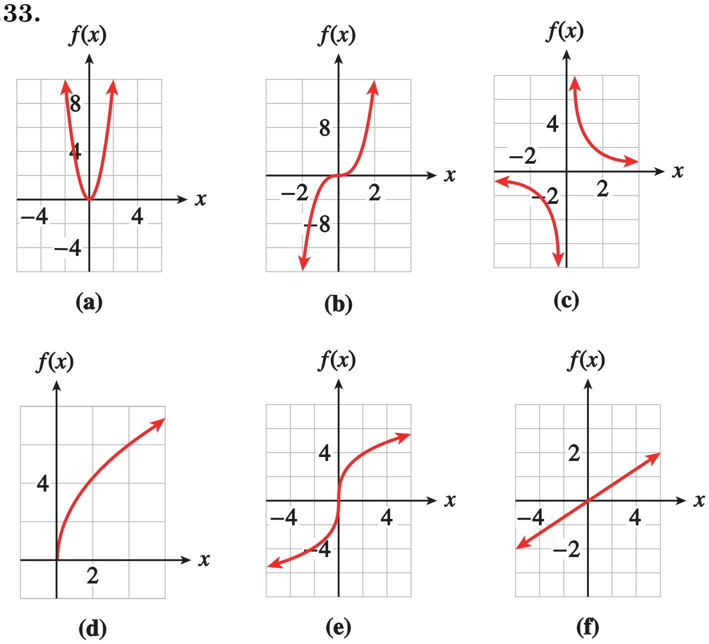
a Scale factor $\frac{1}{3}$; $y = \frac{1}{x}$ is compressed vertically by the scale factor.

b 

2.3.5.32. $h(x) = \dfrac{-1}{4x^2}$

In Problems 33 and 34, match each graph with its equation.

2.3.5.33.



i $f(x) = 3\sqrt{x}$

ii $f(x) = 2x^3$

iii $f(x) = \dfrac{x}{3}$

iv $f(x) = \dfrac{3}{x}$

v $f(x) = 2\sqrt[3]{x}$

vi $f(x) = 3x^2$

Answer.

a vi

b ii

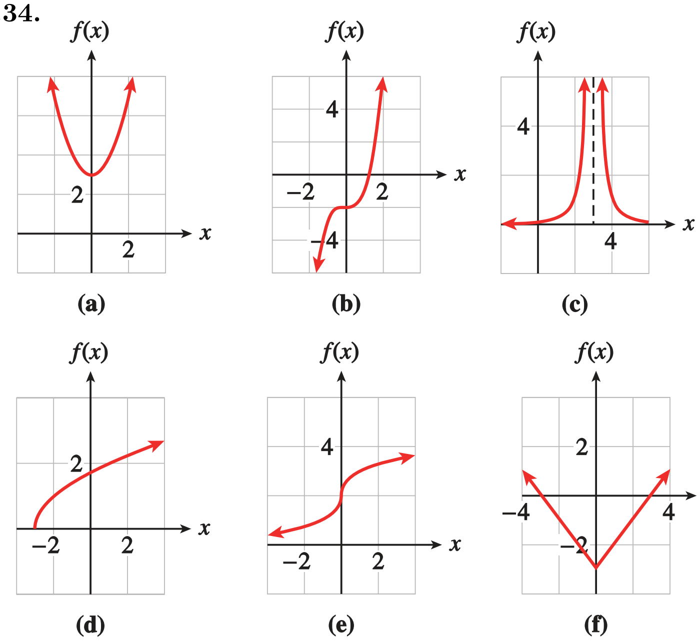
c iv

d i

e v

f iii

2.3.5.34.



i $f(x) = x^3 - 2$

ii $f(x) = \sqrt[3]{x} + 2$

iii $f(x) = \dfrac{1}{(x-3)^2}$

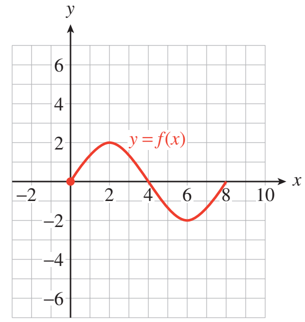
iv $f(x) = |x| - 3$

v $f(x) = x^2 + 3$

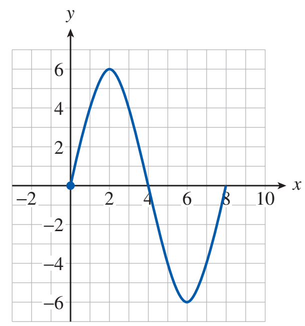
vi $f(x) = \sqrt{x-3}$

In Problems 35–38, the graph of a function is shown. Describe each transformation of the graph; then give a formula for each in terms of the original function.

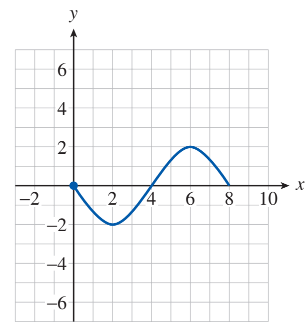
2.3.5.35.



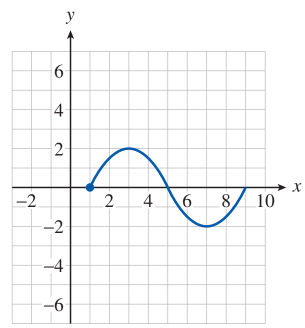
(a)



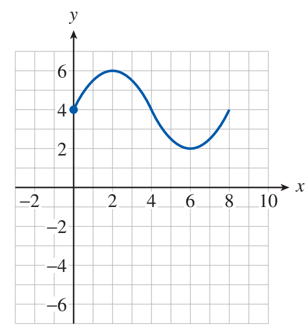
(b)



(c)



(d)



Answer.

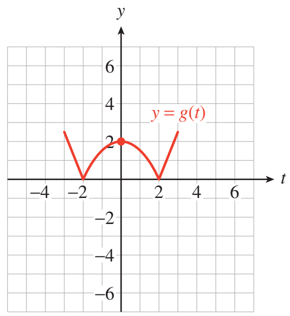
a Vertical stretch by a factor of 3: $y = 3f(x)$

b Reflection about the x-axis: $y = -f(x)$

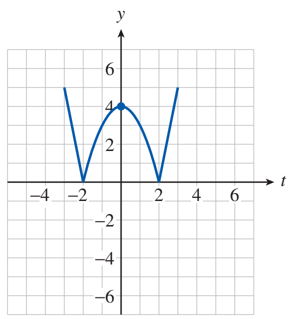
c Translation 1 unit right: $y = f(x - 1)$

d Translation 4 units up: $y = f(x) + 4$

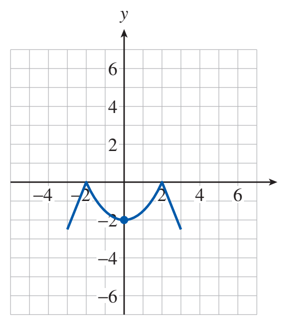
2.3.5.36.



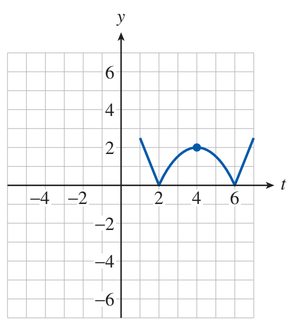
(a)



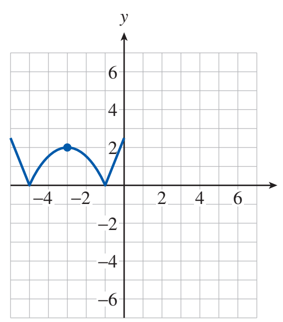
(b)



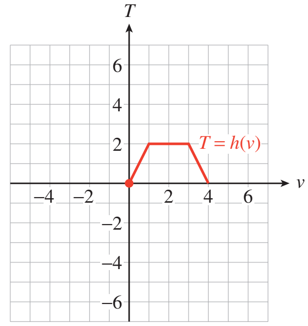
(c)



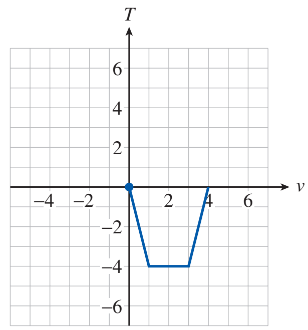
(d)



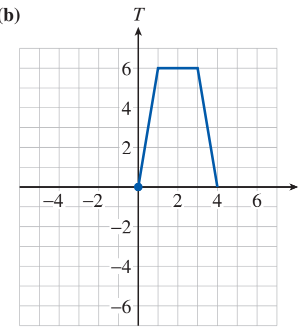
2.3.5.37.



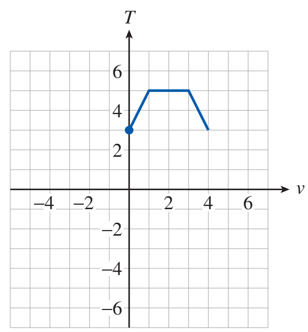
(a)



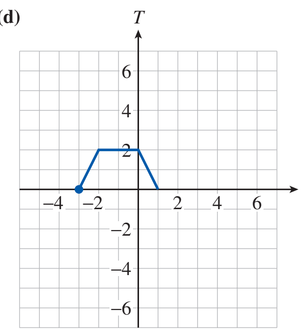
(b)



(c)



(d)



Answer.

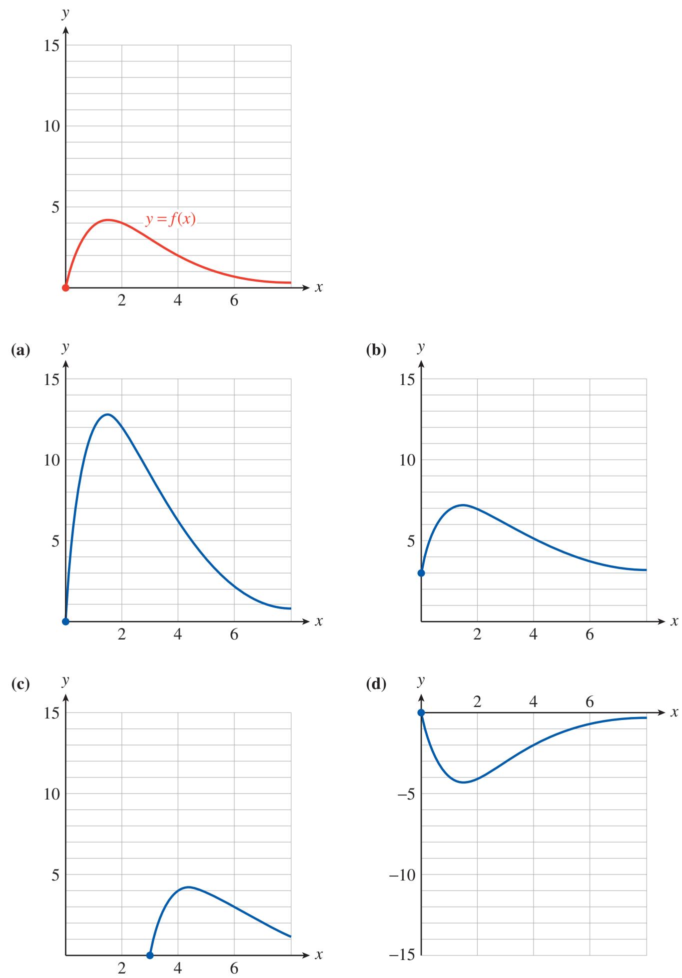
a Reflection about the v-axis and vertical stretch by a factor of 2: $T = -2h(v)$

b Vertical stretch by a factor of 3: $T = 3h(v)$

c Translation 3 units up: $T = h(v) + 3$

d Translation 3 units left: $T = h(v + 3)$

2.3.5.38.



In Problems 39–42, each table in parts (a)–(d) describes a transformation of $f(x)$. Identify the transformation and write a formula for the new function in terms of f.

2.3.5.39.

x	1	2	3	4	5	6
$f(x)$	8	6	4	2	0	2

a

x	1	2	3	4	5	6
y	10	8	6	4	2	4

b

x	1	2	3	4	5	6
y	4	2	0	−2	−4	−2

c

x	1	2	3	4	5	6
y	4	3	2	1	0	1

d

x	1	2	3	4	5	6
y	10	8	6	4	2	0

Answer.

a Translation 2 units up: $y = f(x) + 2$

b Translation 4 units down: $y = f(x) - 4$

c Vertical compression by a factor of $\frac{1}{2}$: $y = \frac{1}{2}f(x)$

d Translation 1 unit right: $y = f(x - 1)$

2.3.5.40.

x	$f(x)$
− 3	13
− 2	3
− 1	−3
0	−5
1	−3
2	3

a

x	− 3	− 2	− 1	0	1	2
y	−26	−6	6	10	6	−6

b

x	− 3	− 2	− 1	0	1	2
y	18	8	2	0	2	8

c

x	− 3	− 2	− 1	0	1	2
y	−3	−5	−3	3	13	27

d

x	− 3	− 2	− 1	0	1	2
y	2.6	0.6	−0.6	−1	−0.6	0.6

2.3.5.41.

x
-2
-1
0
1
2
3
$f(x)$
-9
-8
-7
-6
1
20

a

x	-2	-1	0	1	2	3
y	-34	-9	-8	-7	-6	1

b

x	-2	-1	0	1	2	3
y	-4	21	22	23	24	31

c

x	-2	-1	0	1	2	3
y	18	16	14	12	-2	-40

d

x	-2	-1	0	1	2	3
y	8	6	4	2	-12	-50

Answer.

a Translation 1 unit right: $y = f(x-1)$

b Part (a) is translated 30 units up: $y = f(x-1) + 30$

c f is reflected about the x-axis and stretched vertically by a factor of 2: $y = -2f(x)$

d Part (c) is translated 10 units down: $y = -2f(x) - 10$

2.3.5.42.

x
1
2
3
4
5
6
$f(x)$
60
30
20
15
12
10

a

x	1	2	3	4	5	6
y	30	15	10	7.5	6	5

b

x	1	2	3	4	5	6
y	35	20	15	12.5	11	10

c

x	1	2	3	4	5	6
y	-12	-6	-4	-3	-2.4	-2

d

x	1	2	3	4	5	6
y	-10	-4	-2	-1	1.4	0

For Problems 43-50, write the function in the form $y = kf(x)$, where $f(x)$ is one of the basic functions. Describe how the graph differs from that of the basic function.

2.3.5.43.
$y = \dfrac{1}{2x^2}$

Answer.
$y = \dfrac{1}{2} \cdot \dfrac{1}{x^2}$ is a vertical compression with factor $dfrac12$ of $y = \dfrac{1}{x^2}$.

2.3.5.44.
$y = \sqrt{9x}$

2.3.5.45.
$y = \sqrt[3]{8x}$

Answer.
$y = 2\sqrt[3]{x}$ is a vertical stretch with factor 2 of $y = \sqrt[3]{x}$.

2.3.5.46.
$y = \dfrac{1}{4x}$

2.3.5.47.
$y = |3x|$

Answer.
$y = 3|x|$ is a vertical stretch with factor 3 of $y = |x|$.

2.3.5.48.
$y = \left(\dfrac{x}{2}\right)^2$

2.3.5.49.
$y = \left(\dfrac{x}{2}\right)^3$

Answer.
$y = \dfrac{1}{8}x^3$ is a vertical compression with factor $\dfrac{1}{8}$ of $y = x^3$.

2.3.5.50.
$y = \left|\dfrac{x}{5}\right|$

For Problems 51–62,

a The graph of each function can be obtained from one of the basic graphs by two or more transformations. Describe the transformations.

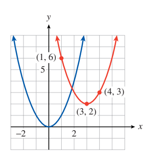
b Sketch the basic graph and the graph of the given function by hand on the same axes. Label the coordinates of three points on the graph of the given function.

2.3.5.51. $f(x) = 2 + (x - 3)^2$

Answer.

a Translation by 2 units up and 3 units right

b

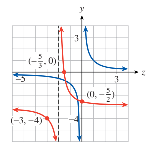


2.3.5.52. $f(x) = (x + 4)^2 + 1$

2.3.5.53. $g(z) = \dfrac{1}{z+2} - 3$

Answer.

a Translation by 2 units left and 3 units down.

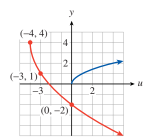
b 

2.3.5.54. $g(z) = \dfrac{1}{z-1} + 1$

2.3.5.55. $F(u) = -3\sqrt{u+4} + 4$

Answer.

a Reflection across the u-axis, vertical stretch by a factor of 3, translation by 4 units left and 4 units up

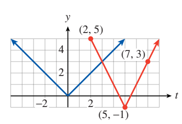
b 

2.3.5.56. $F(u) = 4\sqrt{u-3} - 5$

2.3.5.57. $G(t) = 2\,|t-5| - 1$

Answer.

a Vertical stretch by a factor of 2, translation by 5 units right and 1 down

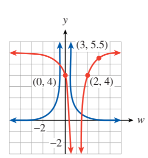
b 

2.3.5.58. $G(t) = 2 - |t+4|$

2.3.5.59. $H(w) = 6 - \dfrac{2}{(w-1)^2}$

Answer.

a Reflection across the w-axis, vertical stretch by a factor of 2, translation by 6 units up and 1 unit right

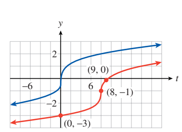
b 

2.3.5.60. $H(w) = \dfrac{3}{(w+2)^2} - 1$

2.3.5.61. $f(t) = \sqrt[3]{t-8} - 1$

Answer.

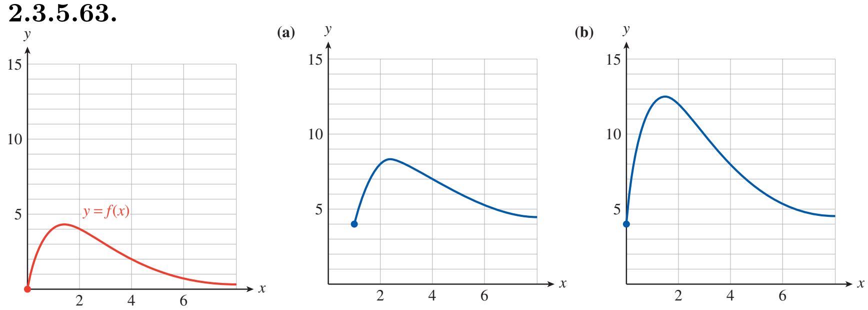
a Translation by 8 units right and 1 unit down

b 

2.3.5.62. $f(t) = \sqrt[3]{t+1} + 8$

In Problems 63 and 64, each graph can be obtained by two transformations of the given graph. Describe the transformations and write a formula for the new graph in terms of f.

2.3.5.63.

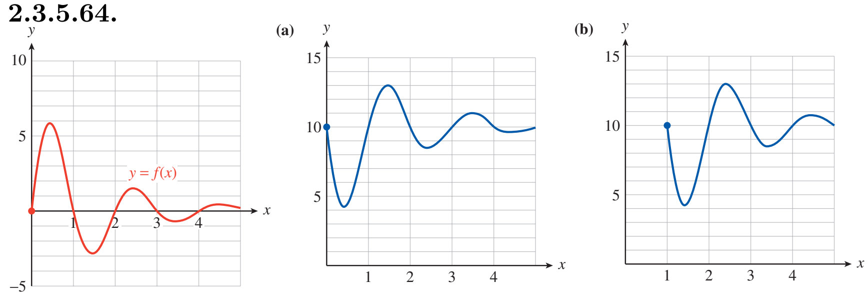


Answer.

a Translation by 4 units up and 1 unit right: $y = f(x-1)+4$

b Vertical stretch by a factor of 2 and a translation by 4 units up: $y = 2f(x)+4$

2.3.5.64.

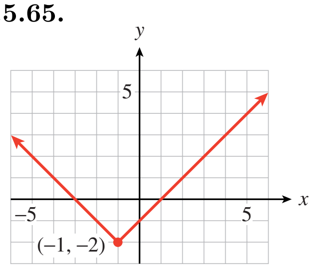


For Problems 65–70,

a Describe the graph as a transformation of a basic function.

b Give an equation for the function shown.

2.3.5.65.

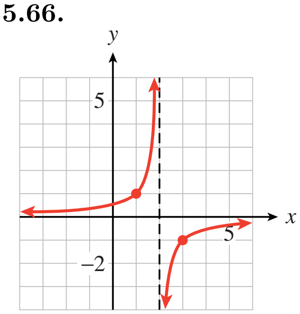


Answer.

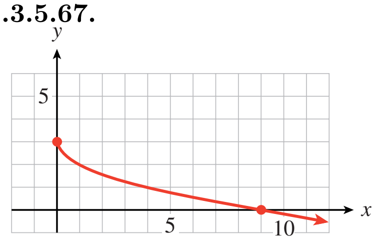
a $y = |x|$ translated by 1 unit left and 2 units down

b $y = |x+1|-2$

2.3.5.66.



2.3.5.67.

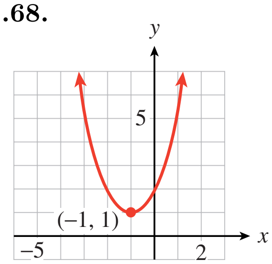


Answer.

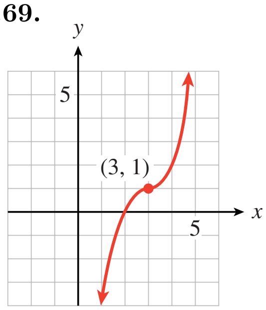
a $y = \sqrt{x}$ reflected about the x-axis and shifted 3 units up

b $y = -\sqrt{x} + 3$

2.3.5.68.



2.3.5.69.

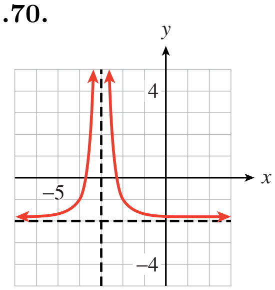


Answer.

a $y = x^3$ translated by 3 units right and 1 unit up

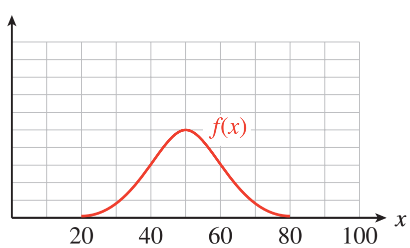
b $y = (x-3)^3 + 1$

2.3.5.70.

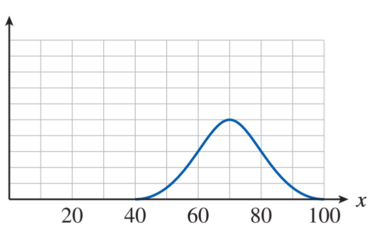


2.3.5.71.

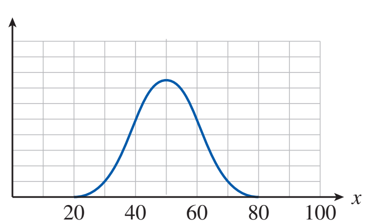
The graph of $f(x)$ shows the number of students in Professor Hilbert's class who scored x points on a quiz. Write a formula for each transformation. Explain how the quiz results given in (a) and (b) compare to the results in Professor Hilbert's class.



(a)



(b)



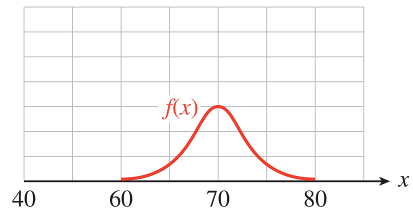
Answer.

a $y = f(x-20)$: Students scored 20 points higher than Professor Hilbert's class.

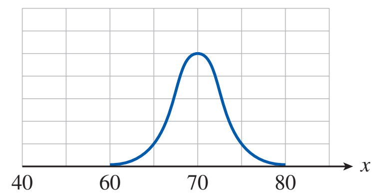
b $y = 1.5f(x)$: The class is about 50% larger than Hilbert's, but the classes scored the same.

2.3.5.72.

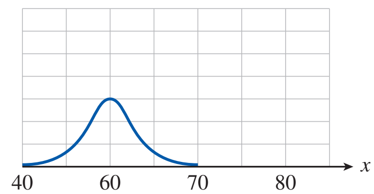
The graph of $f(x)$ shows the number of men at Tyler College who are x inches tall. Write a formula for each transformation of f ; then explain how the heights in that population compare to the Tyler College men.



(a)

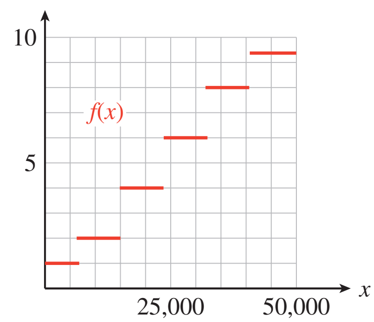


(b)

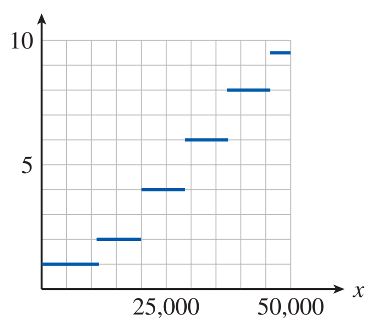


2.3.5.73.

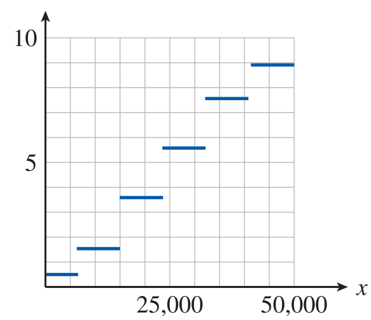
The graph of $f(x)$ shows the California state income tax rate, in percent, for a single taxpayer whose annual taxable income is x dollars. Write a formula for each transformation of f; then explain what it tells you about the income tax scheme in that state.



(a)



(b)



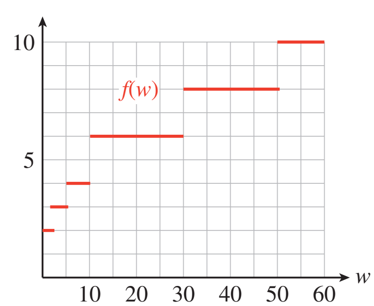
Answer.

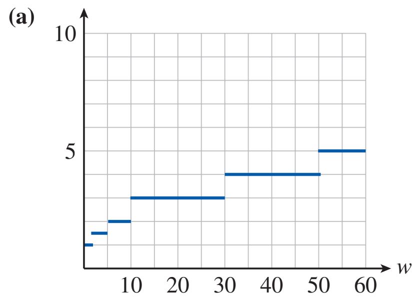
a $y = f(x - 5000)$: Taxpayers earn \$5000 more than Californians in each tax rate

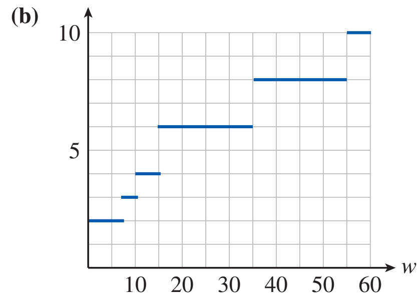
b $y = f(x) - 0.2$: Taxpayers pay 0.2% less tax than Californians on the same income.

2.3.5.74.

The graph of $f(w)$ shows the shipping rate at SendIt for a package that weighs w pounds. Write a formula for each transformation of f and explain how the shipping rates compare to the rates at SendIt.

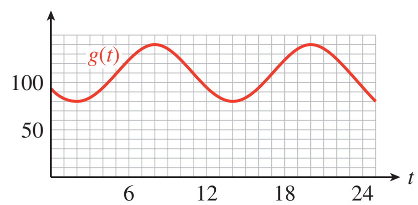


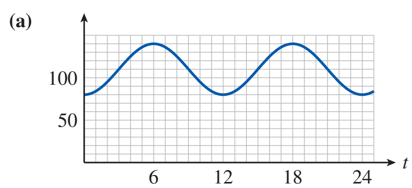


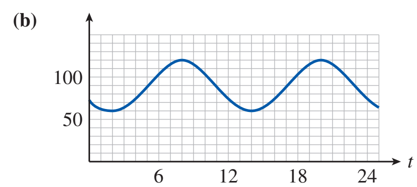


2.3.5.75.

The graph of $g(t)$ shows the population of marmots in a national park t months after January 1. Write a formula for each transformation of g and explain how the population of that species compares to the population of marmots.







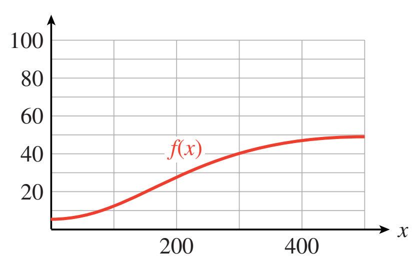
Answer.

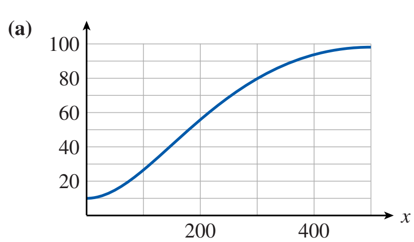
a $y = g(t+2)$: This population has its maximum and minimum two months before the marmots.

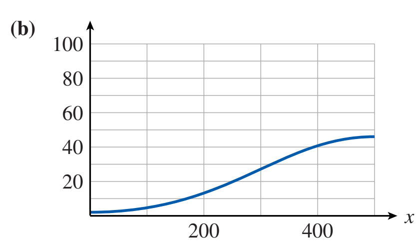
b $y = g(t)-20$: This population remains 20 fewer than that of the marmots.

2.3.5.76.

The graph of $f(x)$ is a dose-response curve. It shows the intensity of the response to a drug as a function of the dosage x milligrams administered. The intensity is given as a percentage of the maximum response. Write a formula for each transformation of f and explain what it tells you about the response to that drug





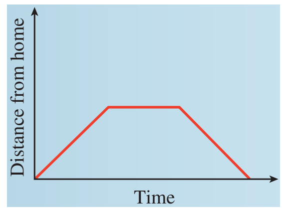


2.4 Functions as Mathematical Models

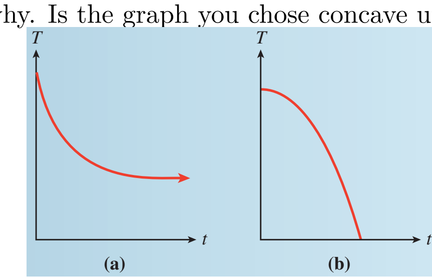
2.4.1 The Shape of the Graph

Checkpoint 2.4.2 Erin walks from her home to a convenience store, where she buys some cat food, and then walks back home. Sketch a possible graph of her distance from home as a function of time.

Answer.



Checkpoint 2.4.4 Francine bought a cup of cocoa at the cafeteria. The cocoa cooled off rapidly at first, and then gradually approached room temperature. Which graph more accurately reflects the temperature of the cocoa as a function of time? Explain why. Is the graph you chose concave up or concave down?



Answer. (a): The graph has a steep negative slope at first, corresponding to an initial rapid drop in the temperature of the cocoa. The graph becomes closer to a horizontal line, corresponding to the cocoa approaching room temperature. The graph is concave up.

2.4.2 Using the Basic Functions as Models

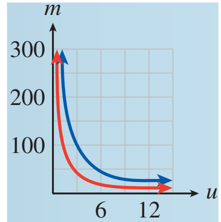
Checkpoint 2.4.6 The ultraviolet index (UVI) is issued by the National Weather Service as a forecast of the amount of ultraviolet radiation expected to reach Earth around noon on a given day. The data show how much exposure to the sun people can take before risking sunburn.

UVI	2	3	4	5	6	8	10	12
Minutes to burn (more sensitive)	30	20	15	12	10	7.5	6	5
Minutes to burn (more sensitive)	150	100	75	60	50	37.5	30	25

a Plot m, the minutes to burn, against u, the UVI, to obtain two graphs, one for people who are more sensitive to sunburn, and another for people less sensitive to sunburn. Which of the basic functions do your graphs most resemble?

b For each graph, find a value of k so that $m = kf(u)$ fits the data.

Answer.

a 

The graphs resemble $f(x) = \dfrac{1}{x}$.

b More sensitive: $k = 60$, Less sensitive: $k = 300$

2.4.3 Modeling with Piecewise Functions

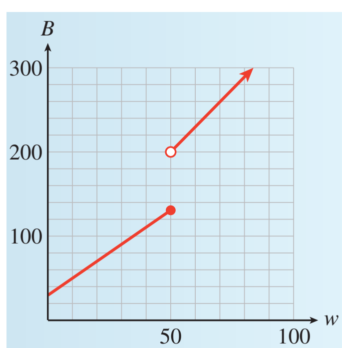
Checkpoint 2.4.8 As part of a water conservation program, the utilities commission in Arid, New Mexico, establishes a two-tier system of monthly billing for residential water usage: The commission charges a \$30 service fee plus \$2 per hundred cubic feet (HCF) of water if you use 50 HCF or less, and a \$50 service fee plus \$3 per HCF of water if you use over 50 HCF (1 HCF of water is about 750 gallons).

a Write a piecewise formula for the water bill, $B(w)$, as a function of the amount of water used, w, in HCF.

b Graph the function B.

Answer.

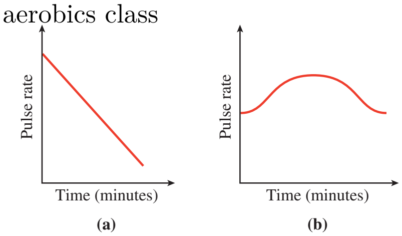
a $B(w) = \begin{cases} 30 + 2w & 0 \leq w \leq 50 \\ 50 + 3w & w > 50 \end{cases}$

b 

2.4.5 Homework 2.4

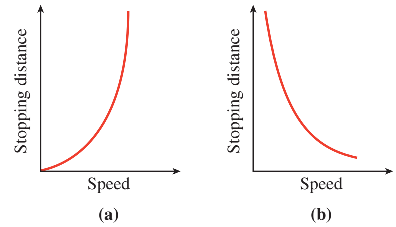
In Problems 1–4, which graph best illustrates each of the following situations?

2.4.5.1. Your pulse rate during an aerobics class

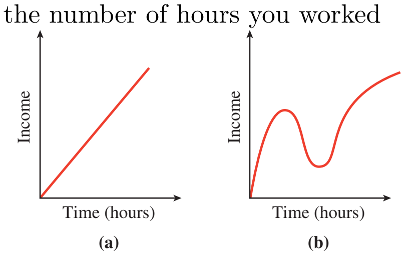


Answer. (b)

2.4.5.2. The stopping distances for cars traveling at various speeds

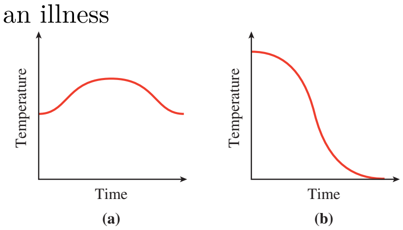


2.4.5.3. Your income in terms of the number of hours you worked



Answer. (a)

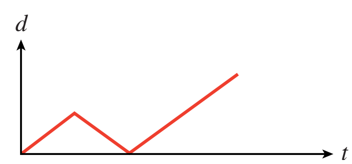
2.4.5.4. Your temperature during an illness



In Problems 5–8, sketch graphs to illustrate the following situations

2.4.5.5. Halfway from your English class to your math class, you realize that you left your math book in the classroom. You retrieve the book, then walk to your math class. Graph the distance between you and your English classroom as a function of time, from the moment you originally leave the English classroom until you reach the math classroom.

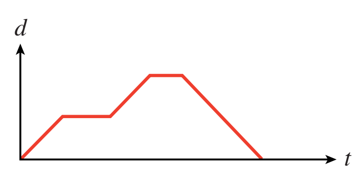
Answer.



2.4.5.6. After you leave your math class, you start off toward your music class. Halfway there you meet an old friend, so you stop and chat for a while. Then you continue to the music class. Graph the distance between you and your math classroom as a function of time, from the moment you leave the math classroom until you reach the music classroom.

2.4.5.7. Toni drives from home to meet her friend at the gym, which is halfway between their homes. They work out together at the gym; then they both go to the friend's home for a snack. Finally Toni drives home. Graph the distance between Toni and her home as a function of time, from the moment she leaves home until she returns.

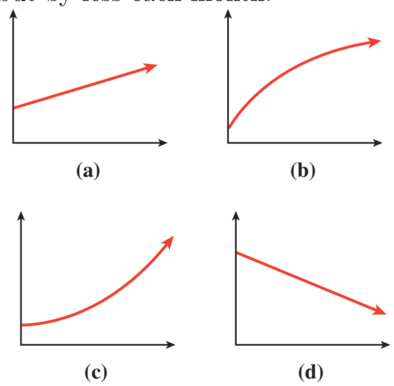
Answer.



2.4.5.8. While bicycling from home to school, Greg gets a flat tire. He repairs the tire in just a few minutes but decides to backtrack a few miles to a service station, where he cleans up. Finally, he bicycles the rest of the way to school. Graph the distance between Greg and his home as a function of time, from the moment he leaves home until he arrives at school.

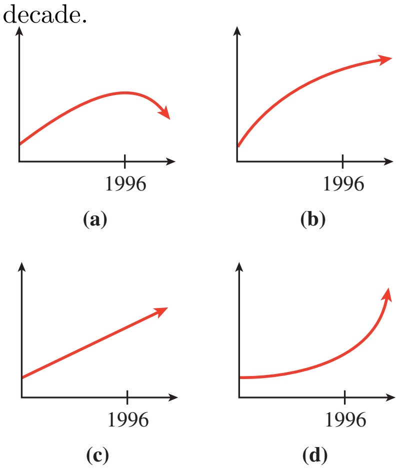
Choose the graph that depicts the function described in Problems 9 and 10.

2.4.5.9. Inflation is still rising, but by less each month.



Answer. (b)

2.4.5.10. The price of wheat was rising more rapidly in 1996 than at any time during the previous decade.



In Problems 11 and 12, match each graph with the function it illustrates.

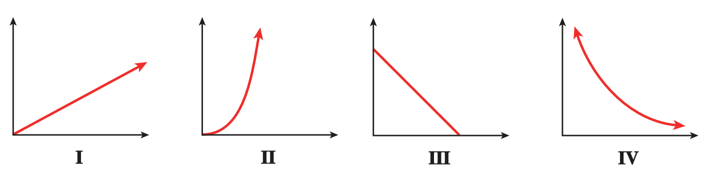
2.4.5.11.

a The volume of a cylindrical container of constant height as a function of its radius

b The time it takes to travel a fixed distance as a function of average speed

c The simple interest earned at a given interest rate as a function of the investment

d The number of Senators present versus the number absent in the U.S. Senate



Answer.

a II b IV c I d III

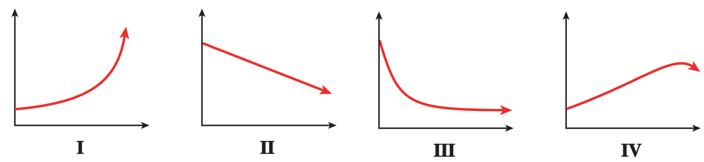
2.4.5.12.

a Unemployment was falling but is now steady.

b Inflation, which rose slowly until last month, is now rising rapidly.

c The birthrate rose steadily until 1990 but is now beginning to fall.

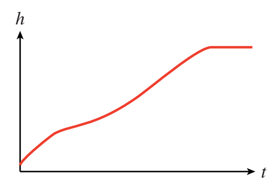
d The price of gasoline has fallen steadily over the past few months.



Sketch possible graphs to illustrate the situations described in Problems 13–18.

2.4.5.13. The height of a man as a function of his age, from birth to adulthood

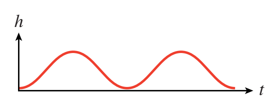
Answer.



2.4.5.14. The number of people willing to buy a new high-definition television, as a function of its price

2.4.5.15. The height of your head above the ground during a ride on a Ferris wheel

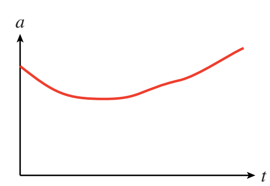
Answer.



2.4.5.16. The height above the ground of a rubber ball dropped from the top of a 10-foot ladder

2.4.5.17. The average age at which women first marry decreased from 1940 to 1960, but it has been increasing since then

Answer.

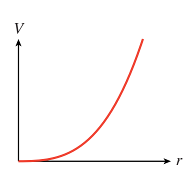


2.4.5.18. When you learn a foreign language, the number of vocabulary words you know increases slowly at first, then increases more rapidly, and finally starts to level off.

Each situation in Problems 19–24 can be modeled by a transformation of a basic function. Name the basic function and sketch a possible graph.

2.4.5.19. The volume of a hot air balloon, as a function of its radius

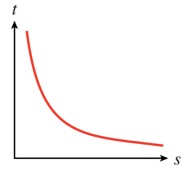
Answer. $y = x^3$ stretched or compressed vertically



2.4.5.20. The length of a rectangle as a function of its width, if its area is 24 square feet

2.4.5.21. The time it takes you to travel 600 miles, as a function of your average speed

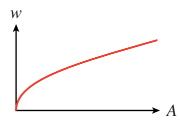
Answer. $y = \dfrac{1}{x}$ stretched or compressed vertically



2.4.5.22. The sales tax on a purchase, as a function of its price

2.4.5.23. The width of a square skylight, as a function of its area

Answer. $y = \sqrt{x}$



2.4.5.24. The sales tax on a purchase, as a function of its price

In Problems 25–28, use the table of values to answer the questions.

a Based on the given values, is the function increasing or decreasing?

b Could the function be concave up, concave down, or linear?

2.4.5.25.

x	$f(x)$
0	1
1	1.5
2	2.25
3	3.375
4	5.0625

Answer.

a Increasing

b Concave up

2.4.5.26.

x	$g(x)$
0	1
1	0.8
2	0.64
3	0.512
4	0.4096

2.4.5.27.

x	$f(x)$
0	0
1	0.174
2	0.342
3	0.5
4	0.643

Answer.

a Increasing

b Concave down

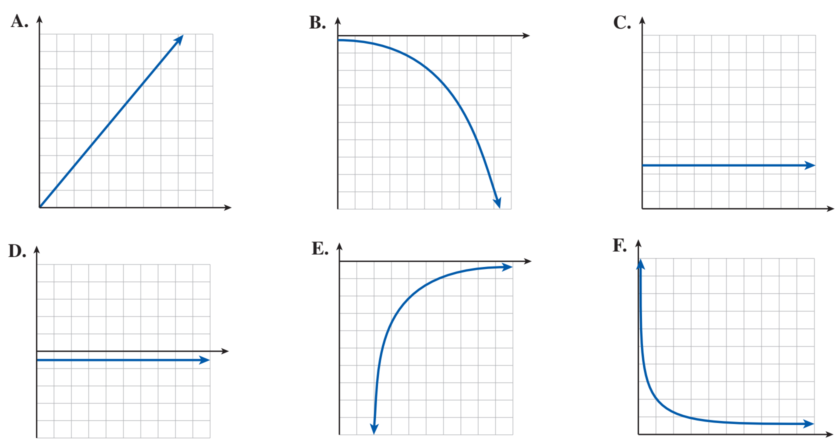
2.4.5.28.

x	$c(x)$
0	1
1	0.985
2	0.940
3	0.866
4	0.766

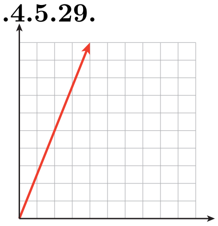
In Problems 29–34,

a Is the graph increasing or decreasing, concave up or concave down?

b Match the graph of the function with the graph of its rate of change, shown in Figures A–F.



2.4.5.29.

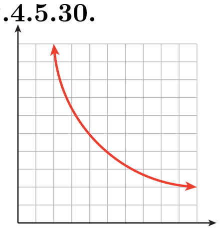


Answer.

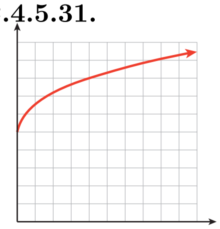
a Increasing, linear (neither concave up nor down)

b C

2.4.5.30.



2.4.5.31.

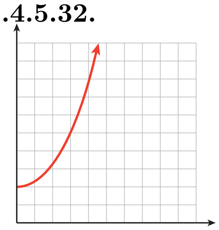


Answer.

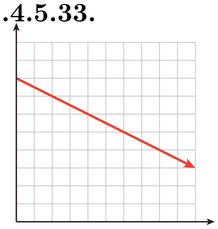
a Increasing, concave down

b F

2.4.5.32.



2.4.5.33.

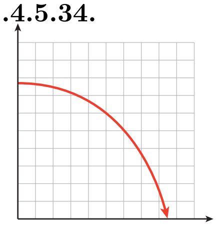


Answer.

a Decreasing, linear (neither concave up nor down)

b D

2.4.5.34.

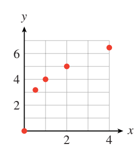


For Problems 35–40, plot the data; then decide which of the basic functions could describe the data.

2.4.5.35.

x	0	0.5	1	2	4
y	0	3.17	4	5.04	6.35

Answer.



$y = 4\sqrt[3]{x}$

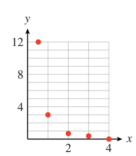
2.4.5.36.

x	0	0.5	1	2	4
y	0	5.66	8	11.31	16

2.4.5.37.

x	0.5	1	2	3	4
y	12	3	0.75	0.33	0.1875

Answer.



$y = 3 \cdot \dfrac{1}{x^2}$

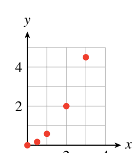
2.4.5.38.

x	0.5	1	2	3	4
y	12	6	3	2	1.5

2.4.5.39.

x	0	0.5	1	2	3
y	0	0.125	0.5	2	4.5

Answer.



$y = 0.5x^2$

2.4.5.40.

x	0	0.5	1	2	3
y	0	0.0125	0.1	0.8	2.7

2.4.5.41. Four different functions are described below. Match each description with the appropriate table of values and with its graph.

a As a chemical pollutant pours into a lake, its concentration is a function of time. The concentration of the pollutant initially increases quite rapidly, but due to the natural mixing and self-cleansing action of the lake, the concentration levels off and stabilizes at some saturation level.

b An overnight express train travels at a constant speed across the Great Plains. The train's distance from its point of origin is a function of time.

c The population of a small suburb of a Florida city is a function of time. The population began increasing rather slowly, but it has continued to grow at a faster and faster rate.

d The level of production at a manufacturing plant is a function of capital outlay, that is, the amount of money invested in the plant. At first, small increases in capital outlay result in large increases in production, but eventually the investors begin to experience diminishing returns on their money, so that although production continues to increase, it is at a disappointingly slow rate.

1

x	1	2	3	4	5	6	7	8
y	60	72	86	104	124	149	179	215

2

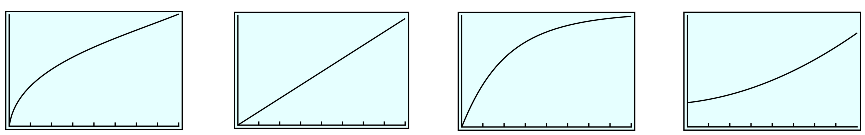
x	1	2	3	4	5	6	7	8
y	60	85	103	120	134	147	159	169

3

x	1	2	3	4	5	6	7	8
y	60	120	180	240	300	360	420	480

4

x	1	2	3	4	5	6	7	8
y	60	96	118	131	138	143	146	147



(A) (B) (C) (D)

Answer.

a Table (4), Graph (C)

b Table (3), Graph (B)

c Table (1), Graph (D)

d Table (2), Graph (A)

2.4.5.42. Four different functions are described below. Match each description with the appropriate table of values and with its graph.

a Fresh water flowing through Crystal Lake has gradually reduced the phosphate concentration to its natural level, and it is now stable.

b The number of bacteria in a person during the course of an illness is a function of time. It increases rapidly at first, then decreases slowly as the patient recovers.

c A squirrel drops a pine cone from the top of a California redwood. The height of the pine cone is a function of time, decreasing ever more rapidly as gravity accelerates its descent.

d Enrollment in Ginny's Weight Reduction program is a function of time. It began declining last fall. After the holidays, enrollment stabilized for a while but soon began to fall off again.

1

x	0	1	2	3	4
y	160	144	96	16	0

2

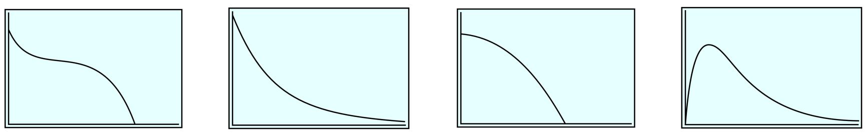
x	0	1	2	3	4
y	20	560	230	90	30

3

x	0	1	2	3	4
y	480	340	240	160	120

4

x	0	1	2	3	4
y	250	180	170	150	80

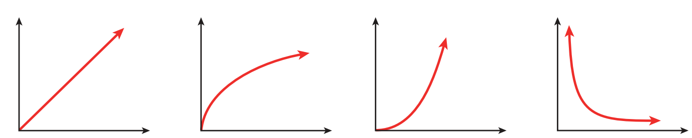


(A) (B) (C) (D)

2.4.5.43. The table shows the radii, r, of several gold coins, in centimeters, and their value, v, in dollars.

Radius
0.5
1
1.5
2
2.5
Value
200
800
1800
3200
5000

a Which graph represents the data?



I II III IV

b Which equation describes the function?

1 $v = k\sqrt{r}$ 2 $v = kr$ 3 $v = kr^2$ 4 $v = \dfrac{k}{r}$

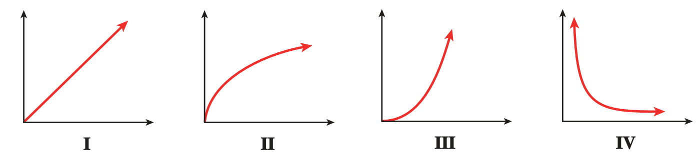
Answer.

a III b 3

2.4.5.44. The table shows how the amount of water, A, flowing past a point on a river is related to the width, W, of the river at that point.

Width (feet)
11
23
34
46
Amount of water (ft^3/sec)
23
34
41
47

a Which graph represents the data?



b Which equation describes the function?

1 $A = k\sqrt{W}$ 2 $A = kW$ 3 $A = kW^2$ 4 $A = \dfrac{k}{W}$

2.4.5.45. If you order from Coldwater Creek, the shipping charges are given by the following table.

Purchase amount	Shipping charge
Up to \$25	\$5.95
\$25.01 to \$50	\$7.95
\$50.01 to \$75	\$9.95
\$75.01 to \$100	\$10.95

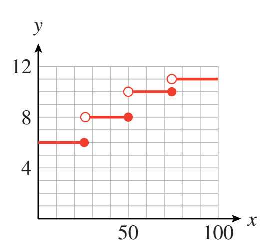
a Write a piecewise formula for $S(x)$, the shipping charge as a function of the purchase amount, x.

b Graph $S(x)$.

Answer.

a $S(x) = \begin{cases} 5.95 & x \le 25 \\ 7.95 & 25 < x \le 50 \\ 9.95 & 50 < x \le 75 \\ 10.95 & 75 < x \le 100 \end{cases}$

b



2.4.5.46. The Bopp-Busch Tool and Die Company markets its products to individuals, to contractors, and to wholesale distributors. The company offers three different price structures for its toggle bolts. If you order 20 or fewer boxes, the price is \$2.50 each. If you order more than 20 but no more than 50 boxes, the price is \$2.25 each. If you order more than 50 boxes, the price is \$2.10 each.

a Write a piecewise formula for $C(x)$, the cost of ordering x boxes of toggle bolts.

b Graph $C(x)$.

2.4.5.47. Bob goes skydiving on his birthday. The function $h(t)$ approximates Bob's altitude t seconds into the trip.

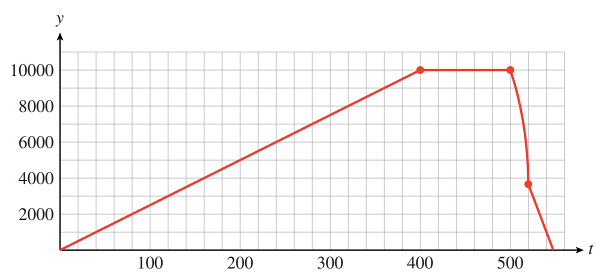
$$h(t) = \begin{cases} 25t & 0 \le t < 400 \\ 10,000 & 400 \le t < 500 \\ 10,000 - 16(t-500)^2 & 500 \le t < 520 \\ 3600 - 120(t-520) & 520 \le t \le 550 \end{cases}$$

a Graph $h(t)$. Describe what you think is happening during each piece of the graph.

b Find two times when Bob is at an altitude of 6000 feet.

Answer.

a



During the first 400 seconds Bob's altitude is climbing with the aircraft; then the aircraft maintains a constant altitude of 10,000 feet for the next 100 seconds; after jumping from the plane, Bob falls for 20 seconds before opening the parachute; he falls at a constant rate after the chute opens.

b 240 seconds (4 minutes) and $500 + \sqrt{250} \approx 515.8$

2.4.5.48. Jenni lives in the San Fernando Valley, where it is hot during summer days but cools down at night. Jenni the air conditioner as little as possible. The function $T(h)$ approximates the temperature in Jenni's house h hours after midnight.

$$T(h) = \begin{cases} 65 & 0 \le h < 8 \\ 25 + 5h & 8 \le h < 14 \\ \dfrac{2240}{h} - 65 & 14 \le h < 16 \\ 75 & 16 \le h < 20 \\ 125 - 2.5h & 20 \le h < 24 \end{cases}$$

a Graph $T(h)$. Describe what you think is happening during each piece of the graph.

b Find two times when the temperature inside the house is 85° Fahrenheit.

2.4.5.49. Lead nitrate and potassium iodide react in solution to produce lead iodide, which settles out, or precipitates, as a yellow solid at the bottom of the container. As you add more lead nitrate to the solution, more lead iodide is produced until all the potassium iodide is used up. The table shows the height of the precipitate in the container as a function of the amount of lead nitrate added. (Source: Hunt and Sykes, 1984)

Lead nitrate solution (cc)	Height of precipitate (mm)
0.5	2.8
1.0	4.8
1.5	6.2
2.0	7.4
2.5	9.5
3.0	9.6
3.5	9.6
4.0	9.6

a Plot the data. Sketch a piecewise linear function with two parts to fit the data points

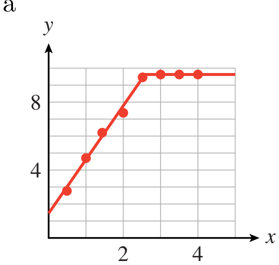
b Calculate the slope of the increasing part of the graph, including units. What is the significance of the slope?

c Write a formula for your piecewise function.

d Interpret your graph in the context of the problem.

Answer.

a

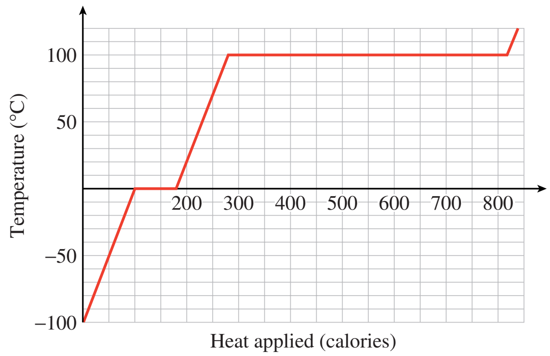


b $m \approx 3.2$ mm/cc: The height of precipitate increases by 1 mm for each additional cc of lead nitrate

c $f(x) = \begin{cases} 1.34 + 3.2x & x < 2.6 \\ 9.6 & x \geq 2.6 \end{cases}$

d The increasing portion of the graph corresponds to the period when the reaction was occurring, and the horizontal section corresponds to when the potassium iodide is used up.

2.4.5.50. The graph shows the temperature of 1 gram of water as a function of the amount of heat applied, in calories. Recall that water freezes at 0°C and boils at 100°C.



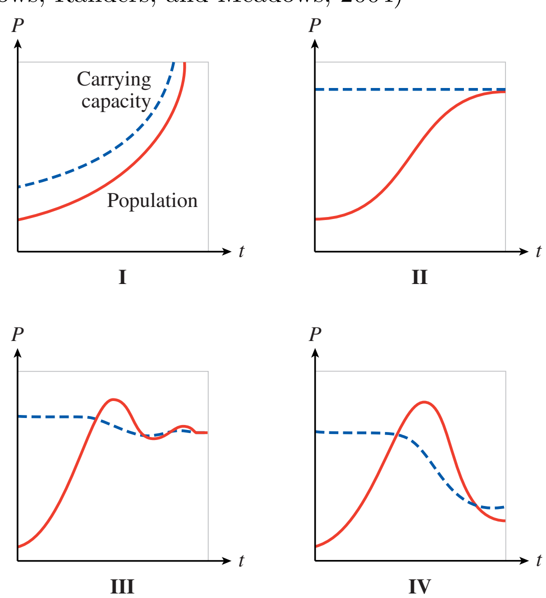
a How much heat is required to raise the temperature of 1 gram of water by 1 degree?

b How much heat is required to convert 1 gram of ice to water?

c How much heat is required to convert 1 gram of water to steam?

d Write a piecewise function to describe the graph.

2.4.5.51. As the global population increases, many scientists believe it is approaching, or has already exceeded, the maximum number the Earth can sustain. This maximum number, or carrying capacity, depends on the finite natural resources of the planet—water, land, air, and materials—but also on how people use and preserve the resources. The graphs show four different ways that a growing population can approach its carrying capacity over time. (Source: Meadows, Randers, and Meadows, 2004)



Match each graph to one of the scenarios described in (a)–(d) and explain your choice.

a Sigmoid growth: The population levels off smoothly below the carrying capacity.

b Overshoot and collapse: The population exceeds the carrying capacity with severe damage to the resource base and is forced to decline rapidly to achieve a new balance with a reduced carrying capacity

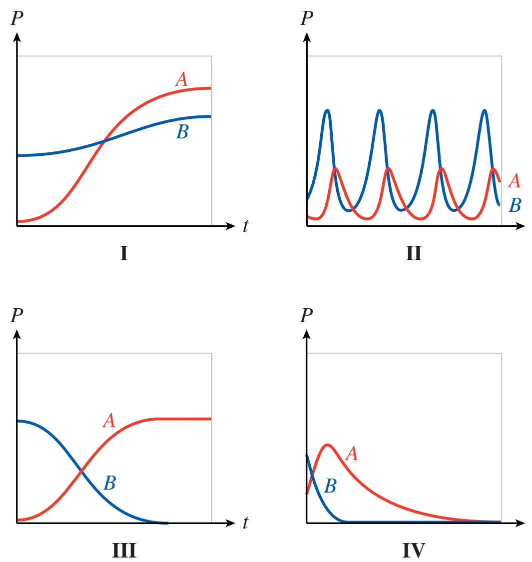
c Continued growth: The carrying capacity is far away, or growing faster than the population.

d Overshoot and oscillation: The population exceeds the carrying capacity without inflicting permanent damage, then oscillates around the limit before leveling off.

Answer.

a II b IV c I d III

2.4.5.52. The introduction of a new species into an environment can affect the growth of an existing species in various ways. The graphs show four hypothetical scenarios after Species A is introduced into an environment where Species B is established.



Match each graph to one of the scenarios described in (a)–(d) and explain your choice.

a Predator-prey (sustained): Species A becomes a predator population that grows when its prey, Species B, is abundant, but declines when the prey population is small. The prey population grows when predators are scarce but shrinks when predators are abundant.

b Predator-prey (extinction): Species A becomes a predator population that annihilates Species B, but then Species A itself declines toward extinction.

c Competition: Species A and B have a common food source, and the Species A replaces Species B in the environment.

d Symbiosis: Species A and B help each other to grow.

2.4.5.53. The Java Stop uses paper cups at a rate of 300 per day. At opening on Tuesday morning Java Stop has on hand 1200 paper cups. On Friday mornings Java Stop takes delivery of a week's worth of cups.

a Write a piecewise function for the number of cups Java Stop has on hand for one week, starting Tuesday morning.

b Graph the function.

c State the domain and range of the function.

2.5 The Absolute Value Function

2.5.1 Introduction

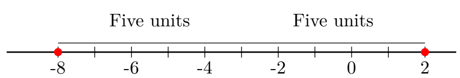
Checkpoint 2.5.2 Write each statement using absolute value notation; then illustrate the solutions on a number line.

a x is five units away from -3.

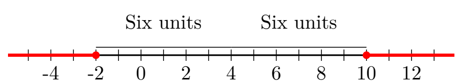
b x is at least six units away from 4.

Answer.

a $|x+3| = 5$



b $|x-4| \geq 6$



2.5.2 Absolute Value Equations

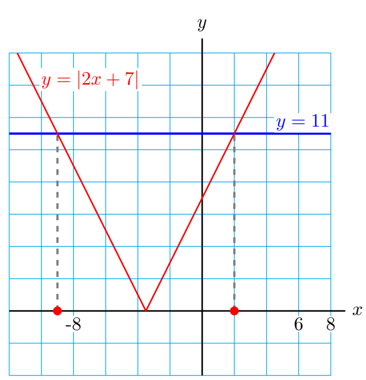
Checkpoint 2.5.4

a Graph $y = |2x+7|$ for $-12 \leq x \leq 8$.

b Use your graph to solve the equation $|2x+7| = 11$.

Answer.

a



b $x = 9$ or $x = 2$

Checkpoint 2.5.6 Solve $|2x+7| = 11$ algebraically.

Answer. $x = -9$ or $x = 2$

2.5.3 Absolute Value Inequalities

Checkpoint 2.5.8

a Solve the inequality $|2x + 7| < 11$

b Solve the inequality $|2x + 7| > 11$

Answer.

a $(-9, 2)$

b $(-\infty, -9) \cup (2, \infty)$

2.5.4 Using the Absolute Value in Modeling

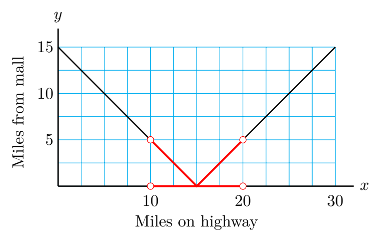
Checkpoint 2.5.10

a Use the graph in Example 2.5.9 to determine how far Marlene has driven when she is within 5 miles of the mall. Write and solve an absolute value inequality to verify your answer.

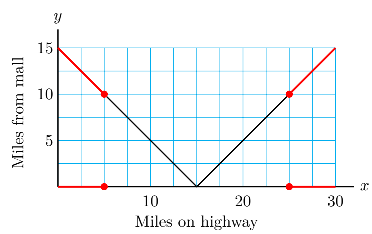
b Write and solve an absolute value inequality to determine how far Marlene has driven when she is at least 10 miles from the mall.

Answer.

a $|x - 15| < 5;\ \ 10 < x < 20$



b $|x - 15| \geq 10;\ \ x \leq 5\ \text{ or }\ x \geq 25$



2.5.5 Measurement Error

Checkpoint 2.5.12 The temperature, T, in a laboratory must remain between 9°C and 12°C.

a Write the error tolerance as an absolute value inequality.

b For a special experiment, the temperature in degrees celsius must satisfy $|T - 6.7| \leq 0.03$. Give the interval of possible temperatures.

Answer.

a $|T - 10.5| < 1.5$

b $6.67 \leq T \leq 6.73$

2.5.7 Homework 2.5

In Problems 1–8,

a Use absolute value notation to write each expression as an equation or an inequality. (It may be helpful to restate each sentence using the word *distance*.)

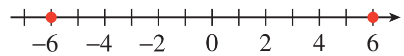
b Illustrate the solutions on a number line.

2.5.7.1. x is six units from the origin.

Answer.

a $|x| = 6$

b



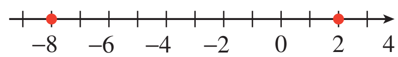
2.5.7.2. a is seven units from the origin.

2.5.7.3. The distance from p to -3 is five units.

Answer.

a $|p + 3| = 5$

b



2.5.7.4. The distance from q to -7 is two units.

2.5.7.5. t is within three units of 6.

Answer.

a $|t - 6| < 3$

b

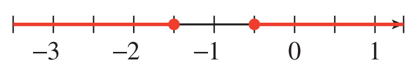
2.5.7.6. w is no more than one unit from -5.

2.5.7.7. b is at least 0.5 unit from -1.

Answer.

a $|b + 1| \geq 0.5$

b

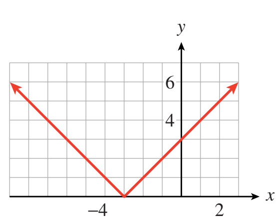


2.5.7.8. m is more than 0.1 unit from 8.

2.5.7.9. Graph $y = |x + 3|$. Use your graph to solve the following equations and inequalities.

a $|x + 3| = 2$ b $|x + 3| \leq 4$ c $|x + 3| > 5$

Answer.



a $x = -5$ or $x = -1$

b $-7 \leq x \leq 1$

c $x < -8$ or $x > 2$

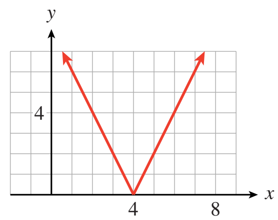
2.5.7.10. Graph $y = |x-2|$. Use your graph to solve the following equations and inequalities.

a $|x-2| = 5$ b $|x-2| < 8$ c $|x-2| \geq 4$

2.5.7.11. Graph $y = |2x-8|$. Use your graph to solve the following equations and inequalities.

a $|2x-8| = 0$ b $|2x-8| = -2$ c $|2x-8| < -6$

Answer.



a $x = 4$

b No solution

c No solution

2.5.7.12. Graph $y = |4x+8|$. Use your graph to solve the following equations and inequalities.

a $|4x+8| = 0$ b $|4x+8| < 0$ c $|4x+8| > -3$

For Problems 13-24, solve.

2.5.7.13. $|2x-1| = 4$

Answer. $x = \dfrac{-3}{2}$ or $x = \dfrac{5}{2}$

2.5.7.14. $|3x-1| = 5$

2.5.7.15. $0 = |7+3q|$

Answer. $q = \dfrac{-7}{3}$

2.5.7.16. $|-11-5t| = 0$

2.5.7.17. $4 = \dfrac{|b+2|}{3}$

Answer. $b = -14$ or $b = 10$

2.5.7.18. $6\,|n+2| = 9$

2.5.7.19. $\big|2(w-7)\big| = 1$

Answer. $w = \dfrac{13}{2}$ or $w = \dfrac{15}{2}$

2.5.7.20. $2 = \left|\dfrac{a-4}{5}\right|$

2.5.7.21. $|c-2| + 3 = 1$

Answer. No solution

2.5.7.22. $5 = 4 - |h+3|$

2.5.7.23. $-7 = |2m+3|$

Answer. No solution

2.5.7.24. $|5r-3| = -2$

For Problems 25-36, solve.

2.5.7.25. $|2x+6| < 3$

Answer. $\dfrac{-9}{2} < x < \dfrac{-3}{2}$

2.5.7.26. $|5-3x| \leq 1$

2.5.7.27. $7 \leq |3-2d|$

Answer. $d \leq -2$ or $d \geq 5$

2.5.7.28.
$10 < |3r + 2|$

2.5.7.29.
$|6s + 15| > -3$

Answer. All real numbers

2.5.7.30.
$|8b - 12| < -4$

2.5.7.31.
$|t - 1.5| < 0.1$

Answer.
$1.4 < t < 1.6$

2.5.7.32.
$|z - 2.6| \le 0.1$

2.5.7.33.
$|T - 3.25| \ge 0.05$

Answer. $T \le 3.2$ or $T \ge 3.3$

2.5.7.34.
$|P - 0.6| > 0.01$

2.5.7.35.
$-1 \ge \left|\dfrac{n-3}{2}\right|$

Answer. No solution

2.5.7.36.
$-0.1 \le \left|9(p + 2)\right|$

In Problems 37–40, give an interval of possible values for the measurement.

2.5.7.37. The length, l, of a rod is given by $|l - 4.3| < 0.001$, in centimeters.

Answer. $4.299 < l < 4.301$

2.5.7.38. The mass, m, of the device shall be $|m - 450| < 4$, in grams.

2.5.7.39. The candle will burn for t minutes, where $|t - 300| \le 50$.

Answer. $250 \le t \le 350$

2.5.7.40. The ramp will have angle of inclination α, and $|\alpha - 10°| \le 0.5°$.

In Problems 41–44, write the error tolerance using absolute values.

2.5.7.41. The chemical compound must be maintained at a temperature, T, between 4.7° and 5.3°C.

Answer. $|T - 5| < 0.3$

2.5.7.42. The diameter, d, of the hole shall be in the range of 24.98 to 25.02 centimeters.

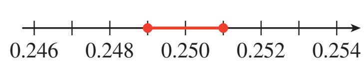
2.5.7.43. The subject will receive a dosage D from 95 to 105 milligrams of the drug.

Answer. $|D - 100| \le 5$

2.5.7.44. The pendulum swings out and back in a time period t between 0.9995 and 1.0005 seconds.

2.5.7.45. An electrical component of a high-tech sensor requires 0.25 ounce of gold. Assume that the actual amount of gold used, g, is not in error by more than 0.001 ounce. Write an absolute value inequality for the possible error and show the possible values of g on a number line.

Answer. $|g - 0.25| \le 0.001$



2.5.7.46. In a pasteurization process, milk is to be irradiated for 10 seconds. The actual period t of irradiation cannot be off by more than 0.8 second. Write an absolute value inequality for the possible error and show the possiblev alues of t on a number line.

2.5.7.47. In a lab assignment, a student reports that a chemical reaction required 200 minutes to complete. Let t represent the actual time of the reaction.

a Write an absolute value inequality for t, assuming that the student rounded his answer to the nearest 100 minutes. Give the smallest and largest possible value for t.

b Write an absolute value inequality for t, assuming that the student rounded his answer to the nearest minute. Give the smallest and largest possible value for t.

c Write an absolute value inequality for t, assuming that the student rounded his answer to the nearest 0.1 minute. Give the smallest and largest possible value for t.

Hint. What is the shortest time that would round to 200 minutes? The greatest time?

Answer.

a $|t - 200| < 50,\ \ 150 \le t < 250$

b $|t - 200| < 0.5,\ \ 199.5 \le t < 200.5$

c $|t - 200| < 0.05,\ \ 199.95 \le t < 200.05$

2.5.7.48. An espresso machine has a square metal plate. The side of the plate is 2 ± 0.01 cm.

a Write an absolute value inequality for the length of the side, x. Give the smallest and largest possible value for s.

b Compute the smallest and largest possible area of the plate, including units.

c Write an absolute value inequality for the area, A.

2.5.7.49.

a Write the piecewise definition for $|3x - 6|$.

b Use your answer to part (a) to write two inequalities that together are equivalent to $|3x - 6| < 9$.

c Solve the inequalities in part (b) and check that the solutions agree with the solutions of $|3x - 6| < 9$.

d Show that $|3x - 6| < 9$ is equivalent to the compound inequality $-9 < 3x - 6 < 9$.

Answer.

a $|3x - 6| = \begin{cases} -(3x-6) & \text{if } x < 2 \\ 3x - 6 & \text{if } x \ge 2 \end{cases}$

b $-(3x - 6) \le 9,\ \ 3x - 6 < 9$

c $-1 < x < 5$

d The solutions are the same.

2.5.7.50.

a Write the piecewise definition for $|3x - 6|$.

b Use your answer to part (a) to write two inequalities that together are equivalent to $|3x - 6| > 9$.

c Solve the inequalities in part (b) and check that the solutions agree with the solutions of $|3x-6|>9$.

d Show that $|3x-6|>9$ is equivalent to the compound inequality $3x-6<-9$ or $3x-6>9$.

2.5.7.51.

a Write the piecewise definition for $|2x+5|$.

b Use your answer to part (a) to write two inequalities that together are equivalent to $|2x+5|>7$.

c Solve the inequalities in part (b) and check that the solutions agree with the solutions of $|2x+5|>7$.

d Show that $|2x+5|>7$ is equivalent to the compound inequality $2x+5<-7$ or $2x+5>7$.

Answer.

a $|2x+5| = \begin{cases} -(2x+5) & \text{if } x < \dfrac{-5}{2} \\ 2x+5 & \text{if } x \geq \dfrac{-5}{2} \end{cases}$

b $-(2x+5)>7,\ \ 2x+5>7$

c $x<-6$ or $x>1$

d The solutions are the same.

2.5.7.52.

a Write the piecewise definition for $|2x+5|$.

b Use your answer to part (a) to write two inequalities that together are equivalent to $|2x+5|<7$.

c Solve the inequalities in part (b) and check that the solutions agree with the solutions of $|2x+5|<7$.

d Show that $|2x+5|<7$ is equivalent to the compound inequality $-7<2x+5<7$.

For Problems 53–56, graph the function and answer the questions.

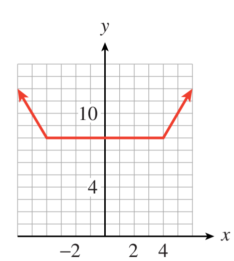
2.5.7.53. $f(x)=|x+4|+|x-4|$

a Using your graph, write a piecewise formula for $f(x)$.

b Experiment by graphing $g(x)=|x+p|+|x-q|$ for different positive values of p and q. Make a conjecture about how the graph depends on p and q.

c Write a piecewise formula for $g(x)=|x+p|+|x-q|$.

Answer.



a $f(x) = \begin{cases} -2x, & x < -4 \\ 8, & -4 \le x \le 4 \\ 2x, & x > 4 \end{cases}$

b The graphs looks like like a trough. The middle horizontal section is $y = p + q$ for $-p \le x \le q$, the left side, $x < -p$, has slope -2 and the right side, $x > q$, has slope 2.

c $g(x) = \begin{cases} -2x + q - p, & x < -p \\ p + q, & -p \le x \le q \\ 2x + p - q, & x > q \end{cases}$

2.5.7.54. $f(x) = |x + 4| - |x - 4|$

a Using your graph, write a piecewise formula for $f(x)$.

b Experiment by graphing $g(x) = |x + p| - |x - q|$ for different positive values of p and q. Make a conjecture about how the graph depends on p and q.

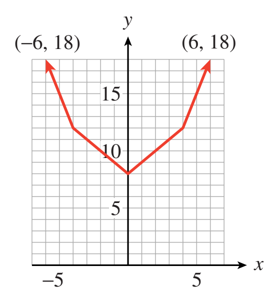
2.5.7.55. $f(x) = |x + 4| + |x| + |x - 4|$

a Using your graph, write a piecewise formula for $f(x)$.

b What is the minimum value of $f(x)$?

c If $p, q \ge 0$, what is the minimum value of $g(x) = |x + p| + |x| + |x - q|$?

Answer.



a $f(x) = \begin{cases} -3x, & x < -4 \\ -x + 8, & -4 \le x \le 0 \\ x + 8, & 0 < x < 4 \\ 3x, & x \ge 4 \end{cases}$

b 8

c $p + q$

2.5.7.56. $f(x) = |x + 4| - |x| + |x - 4|$

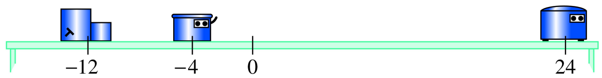
a Using your graph, write a piecewise formula for $f(x)$.

b What is the minimum value of $f(x)$?

c If $p, q \ge 0$, what is the minimum value of $g(x) = |x + p| - |x| + |x - q|$?

Problems 57–60 use the absolute value function to model distance. Use the strategy outlined in Problems 57 and 58 to solve Problems 59 and 60.

2.5.7.57. A small pottery is setting up a workshop to produce mugs. Three machines are located on a long table, as shown in the figure. The potter must use each machine once in the course of producing a mug. Let x represent the coordinate of the potter's station.



a Write expressions for the distance from the potter's station to each of the machines.

b Write a function that gives the sum of the distances from the potter's station to the three machines.

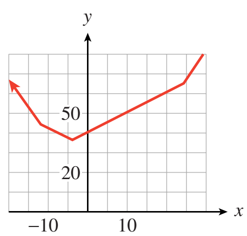
c Graph your function for $-20 \le x \le 30$. Where should the potter stand in order to minimize the distance she must walk to the machines?

Answer.

a $|x+12|$, $|x+4|$, $|x-24|$

b $f(x) = |x+12| + |x+4| + |x-24|$

c



At x-coordinate -4

2.5.7.58. Suppose the pottery in Problem 57 adds a fourth machine to the procedure for producing a mug, located at $x = 16$ in the figure.

a Write and graph a new function for the sum of the potter's distances to the four machines.

b Where should the potter stand now to minimize the distance she has to walk while producing a mug?

2.5.7.59. Richard and Marian are moving to Parkville to take jobs after they graduate. The main road through Parkville runs east and west, crossing a river in the center of town. Richard's job is located 10 miles east of the river on the main road, and Marian's job is 6 miles west of the river. There is a health club they both like located 2 miles east of the river. If they plan to visit the health club every workday, where should Richard and Marian look for an apartment to minimize their total daily driving distance?

Answer. 2 miles east of the river

2.5.7.60. Romina's Bakery has just signed contracts to provide baked goods for three new restaurants located on Route 28 outside of town. The Coffee Stop is 2 miles north of town center, Sneaky Pete's is 8 miles north, and the Sea Shell is 12 miles south. Romina wants to open a branch bakery on Route 28 to handle the new business. Where should she locate the bakery in order to minimize the distance she must drive for deliveries?

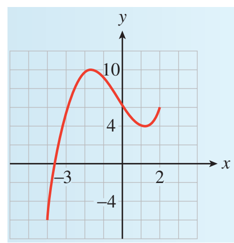
2.6 Domain and Range

2.6.2 Finding Domain and Range from a Graph

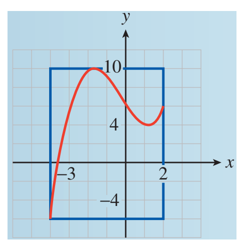
Checkpoint 2.6.2

a Draw the smallest viewing window possible around the graph shown below.

b Find the domain and range of the function.

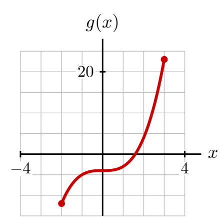


Answer. domain: $[-4,2]$; range: $[-6,10.1]$



Checkpoint 2.6.4 Graph the function $g(x) = x^3 - 4$ on the domain $[-2,3]$ and give its range.

Answer. range: $[-12,23]$



Checkpoint 2.6.6 In Checkpoint 2.4.8 of Section 2.4, you wrote a formula for residential water bills, $B(w)$, in Arid, New Mexico:

$$B(w) = \begin{cases} 30 + 2w, & 0 \le w \le 50 \\ 50 + 3w, & w > 50 \end{cases}$$

If the utilities commission imposes a cap on monthly water consumption at 120 HCF, find the domain and range of the function $B(w)$.

Answer. domain: $[0,120]$; range: $[30,130] \cup (200,410]$

2.6.3 Finding the Domain from a Formula

Checkpoint 2.6.8

a Find the domain of the function $h(x) = \dfrac{1}{(x-4)^2}$.

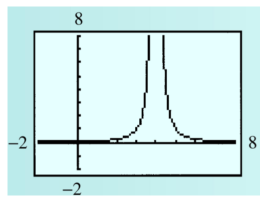
b Graph the function in the window

$$\begin{aligned} \text{Xmin} &= -2 & \text{Xmax} &= 8 \\ \text{Ymin} &= -2 & \text{Ymax} &= 8 \end{aligned}$$

Use your graph and the function's formula to find its range.

Answer.

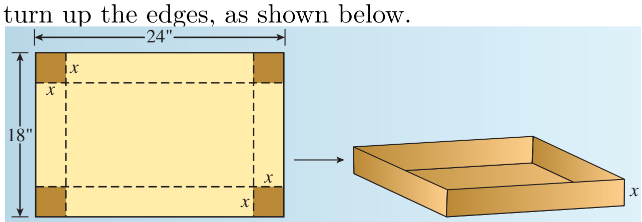
a Domain: $x \neq 4$



b Range: $y > 0$

2.6.4 Restricting the Domain

Checkpoint 2.6.10 The children in Francine's art class are going to make cardboard boxes. Each child is given a sheet of cardboard that measures 18 inches by 24 inches. To make a box, the child will cut out a square from each corner and turn up the edges, as shown below.



a Write a formula $V = f(x)$ for the volume of the box in terms of x, the side of the cut-out square. (See the geometric formulas inside the front cover for the formula for the volume of a box.)

b What is the domain of the function? (What are the largest and smallest possible values of x?)

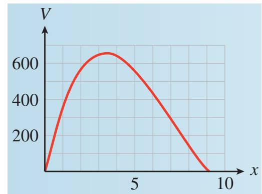
c Graph the function and estimate its range.

Answer.

a $V = f(x) = x(24-2x)(18-2x)$

b $(0, 9)$

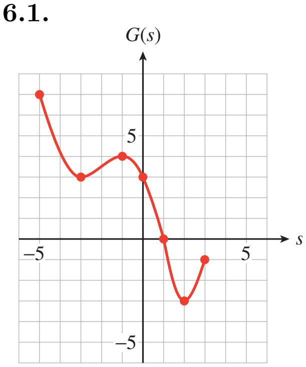
c $(0, 655)$



2.6.6 Homework 2.6

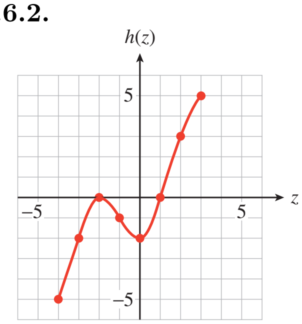
For Problems 1-8, find the domain and range of the function from its graph. Write answers in interval notation.

2.6.6.1.

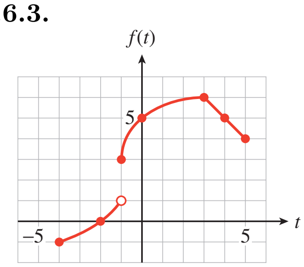


Answer. Domain: $[-5, 3]$; Range: $[-3, 7]$

2.6.6.2.

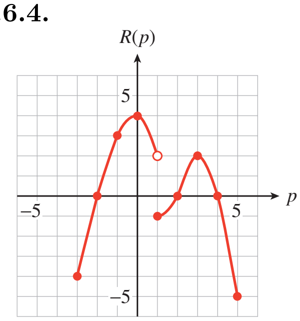


2.6.6.3.

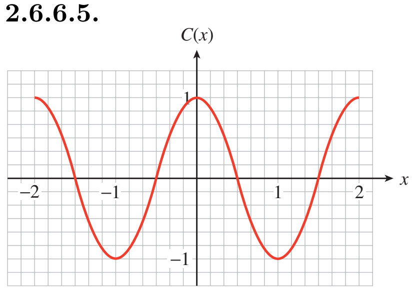


Answer. Domain: $[-4, 5]$; Range: $[-1, 1) \cup [3, 6]$

2.6.6.4.

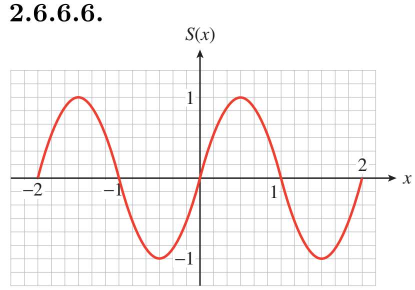


2.6.6.5.

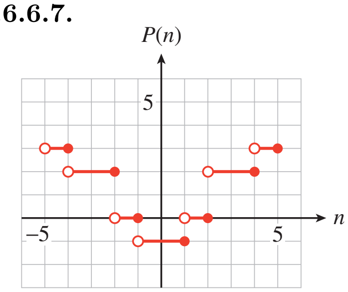


Answer. Domain: $[-2, 2]$; Range: $[-1, 1]$

2.6.6.6.

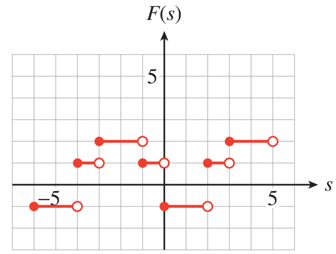


2.6.6.7.



Answer. Domain: $(-5, 5]$;
Range: $\{-1, 0, 2, 3\}$

2.6.6.8.



For Problems 9–2, state the domain and range of the basic function.

2.6.6.9.

a $f(x) = x^3$

b $g(x) = x^2$

Answer.

a Domain: all real numbers;
Range: all real numbers

b Domain: all real numbers;
Range: $[0, \infty)$

2.6.6.10.

a $F(x) = |x|$

b $G(x) = x$

2.6.6.11.

a $H(x) = \dfrac{1}{x^2}$

b $M(x) = \dfrac{1}{x}$

Answer.

a Domain: all real numbers except zero; Range: $(0, \infty)$

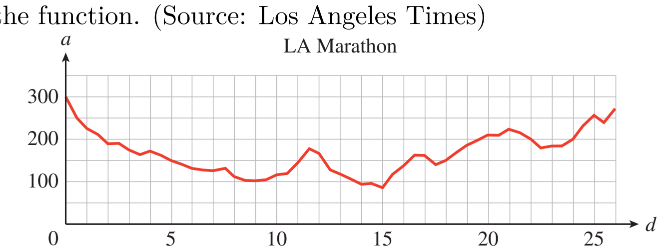
b Domain: all real numbers except zero; Range: all real numbers except zero

2.6.6.12.

a $p(x) = \sqrt[3]{x}$

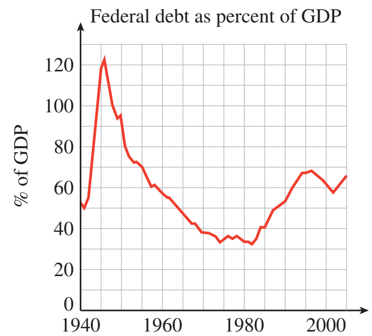
b $q(x) = \sqrt{x}$

2.6.6.13. The graph shows the elevation of the Los Angeles Marathon course as a function of the distance into the race, $a = f(d)$. Estimate the domain and range of the function. (Source: Los Angeles Times)

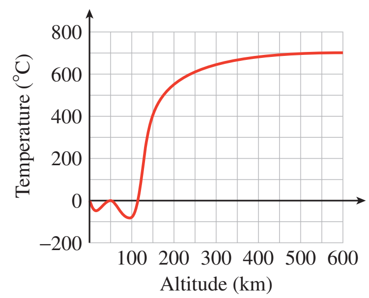


Answer. Domain: $[0, 26.2]$; Range: $[90, 300]$

2.6.6.14. The graph shows the federal debt as a percentage of the gross domestic product, as a function of time, $D = f(t)$. Estimate the domain and range of the function. (Source: Office of Management and Budget)

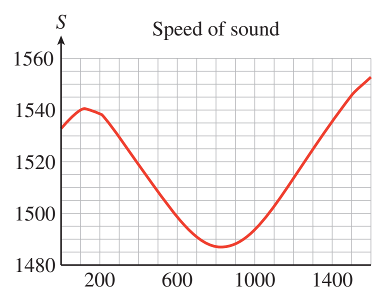


2.6.6.15. The graph shows the average air temperature as a function of altitude, $T = f(h)$. Estimate the domain and range of the function. (Source: Ahrens, 1998)



Answer. Domain: $[0, 600]$; Range: $[-90, 700]$

2.6.6.16. The graph shows the speed of sound in the ocean as a function of depth, $S = f(d)$. Estimate the domain and range of the function. (Source: Scientific American)



2.6.6.17. Clinton purchases \$6000 of photographic equipment to set up his studio. He estimates a salvage value of \$500 for the equipment in 10 years, and for tax purposes he uses straight-line depreciation.

a Write a formula for the value of the equipment, $V(t)$, after t years.

b State the domain and range of the function $V(t)$.

Answer.

a $V(t) = 6000 - 550t$

b Domain: $[0, 10]$; Range: $[500, 6000]$

2.6.6.18. Leslie plans to invest some money in two CD accounts. The first account pays 3.6% interest per year, and the second account pays 2.8% interest per year. Leslie would like to earn \$500 per year on her investment.

a Write a linear equation in general form that relates x, the amount Leslie invests at 3.6%, and y, the amount she invests at 2.8%.

b Use your equation from part (a) to write y as a function of x, $y = f(x)$.

c Find the domain and range of f.

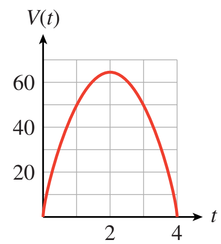
2.6.6.19. The height of a golfball, in feet, t seconds after being hit is given by the function $h = f(t) = -16(t-2)^2 + 64$.

a Graph the function.

b State the domain and range of the function and explain what they tell us about the golfball.

Answer.

a



b Domain: $[0, 4]$; Range: $[0, 64]$. The ball reaches a height of 64 feet and hits the ground 4 seconds after being hit.

2.6.6.20. Gameworld is marketing a new boardgame called Synaps. If Gameworld charges p dollars for the game, their revenue is given by the function $R = f(p) = -50(p-10)^2 + 5000$.

a Graph the function.

b State the domain and range of the function and explain what they tell us about the revenue.

2.6.6.21. In New York City, taxi cabs charge \$2.50 for distances up to $\dfrac{1}{3}$ mile, plus \$0.40 for each additional $\dfrac{1}{5}$ mile or portion thereof. (Source: www.visitnyc.com)

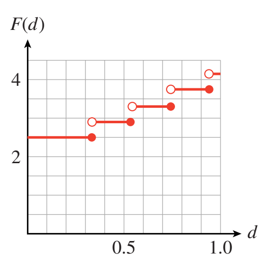
a Sketch a graph of $F(d)$, which gives taxi fare as a function of distance traveled, on the domain $0 < d < 1$.

b State the range of $F(d)$ on that domain.

c How much will it cost Renee to travel by taxi from Columbia University to Rockefeller Center, a distance of 5.7 miles?

Answer.

a



b Range: $\{2.50, 2.90, 3.30, 3.70, 4.10\}$

c \$13.30

2.6.6.22. If you order from Coldwater Creek, the shipping charges are given by the following table.

Purchase amount	Shipping charge
Up to \$25	\$5.95
\$25.01 to \$50	\$7.95
\$50.01 to \$75	\$9.95
\$75.01 to 4100	\$10.95

State the domain and range of $S(x)$, the shipping charge as a function of the purchase amount, x.

2.6.6.23. The Bopp-Busch Tool and Die Company markets its products to individuals, to contractors, and to wholesale distributors. The company offers three different price structures for its toggle bolts. If you order 20 or fewer boxes, the price is \$2.50 each. If you order more than 20 but no more than 50 boxes, the price is \$2.25 each. If you order more than 50 boxes, the price is \$2.10 each. State the domain and range of $C(x)$, the cost of ordering x boxes of toggle bolts.

Answer. Domain: nonnegative integers; The range includes all whole number multiples of 2.50 up to $20 \times 2.50 = 50$, all integer multiples of 2.25 from $21 \times 2.25 = 47.25$ to $50 \times 2.25 = 112.50$ and all integer multiples of 2.10 from $51 \times 2.10 = 107.10$ onwards: $0, 2.50, 5.00, 7.50, \ldots, 50$, $47.25, 49.50, 51.75, \ldots, 112.50$, $107.10, 109.20, 111.30, \ldots$

2.6.6.24. The Java Stop uses paper cups at a rate of 300 per day. At opening on Tuesday morning Java Stop has on hand 1200 paper cups. On Friday mornings Java Stop takes delivery of a week's worth of cups.

a Write a piecewise function for the number of cups Java Stop has on hand for one week, starting Tuesday morning.

b Graph the function.

c State the domain and range of the function.

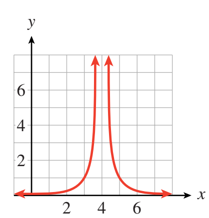
For Problems 25-30, find the domain of each function algebraically. Then graph the function, and use the graph to help you find the range.

2.6.6.25.

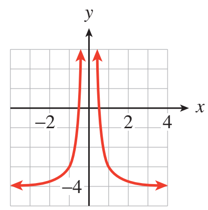
a $f(x) = \dfrac{1}{(x-4)^2}$ b $h(x) = \dfrac{1}{x^2} - 4$

Answer.

a $f(x)$ domain: $x \neq 4$; Range: $(0, \infty)$



b $h(x)$ domain: $x \neq 0$; Range: $(-4, \infty)$



2.6.6.26.

a $g(t) = \dfrac{1}{t} + 2$

b $F(t) = \dfrac{1}{t+2}$

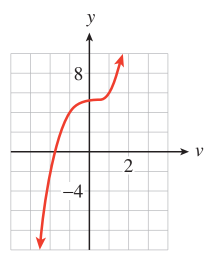
2.6.6.27.

a $G(v) = v^3 + 35$

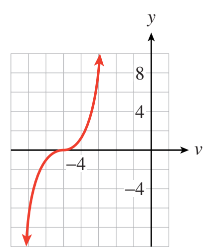
b $H(v) = (v+5)^3$

Answer.

a $G(v)$ domain: all real numbers; Range: all real numbers



b $H(v)$ domain: all real numbers; Range: all real numbers



2.6.6.28.

a $h(n) = 3 + (n-1)^2$

b $g(n) = 3 - (n+1)^2$

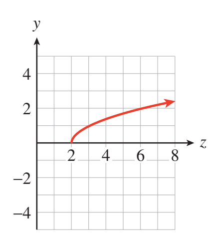
2.6.6.29.

a $T(z) = \sqrt{z-2}$

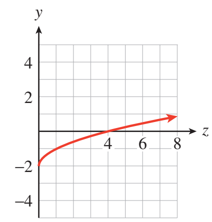
b $S(z) = \sqrt{z} - 2$

Answer.

a $G(v)$ domain: $[2, \infty)$; Range: $[0, \infty)$



b $H(v)$ domain: $[0, \infty)$; Range: $[-2, \infty)$



2.6.6.30.

a $Q(x) = 4 - |x|$

b $P(x) = |4 - x|$

For Problems 31-38, decide whether the given value is in the range of the function. If so, find the domain value(s) that produce each range value.

2.6.6.31. $f(x) = 6 - |2x + 4|$

a $f(x) = 8$

b $f(x) = -2$

Answer.

a Not in range

b $x = -6$ or $x = 2$

2.6.6.32. $g(x) = (x - 5)^3 + 1$

a $g(x) = 0$

b $g(x) = -7$

2.6.6.33. $h(t) = 4 + 2\sqrt[3]{t}$

a $h(t) = -4$

b $h(t) = 0$

Answer.

a $t = -64$

b $t = -8$

2.6.6.34. $F(t) = 12 + 0.5(t - 2)^2$

a $F(t) = 10$

b $F(t) = 20$

2.6.6.35. $G(w) = 3 + \dfrac{2}{w - 1}$

a $G(w) = -1$

b $G(w) = 3$

Answer.

a $w = \dfrac{1}{2}$

b Not in range

2.6.6.36. $H(n) = \dfrac{4}{(n + 2)^2} - 5$

a $H(n) = -6$

b $H(n) = -1$

2.6.6.37. $Q(h) = 2 + \sqrt{h + 5}$

a $Q(h) = 1$

b $Q(h) = 5$

Answer.

a Not in range

b $h = 4$

2.6.6.38. $P(q) = 8 - \sqrt{4 - q}$

a $P(q) = 4$

b $P(q) = 12$

For Problems 39–50,

a Use a graphing calculator to graph each function on the given domain.

Using the TRACE key, adjust **Ymin** and **Ymax** until you can estimate the range of the function.

b Verify your answer algebraically by evaluating the function. State the domain and range in interval notation.

2.6.6.39.
$f(x) = x^2 - 4x; \quad -2 \leq x \leq 5$

Answer. Domain: $[-2, 5]$; Range: $[-4, 12]$

2.6.6.40.
$g(x) = 6x - x^2; \quad -1 \leq x \leq 5$

2.6.6.41.
$g(t) = -t^2 - 2t; \quad -5 \leq t \leq 3$

Answer. Domain: $[-5, 3]$; Range: $[-15, 1]$

2.6.6.42.
$f(t) = -t^2 - 4t; \quad -6 \leq t \leq 2$

2.6.6.43.
$h(x) = x^3 - 1; \quad -2 \leq x \leq 2$

Answer. Domain: $[-2, 2]$; Range: $[-9, 7]$

2.6.6.44.
$q(x) = x^3 + 4; \quad -3 \leq x \leq 2$

2.6.6.45.
$F(t) = \sqrt{8 - t}; \quad -1 \leq t \leq 8$

Answer. Domain: $[-1, 8]$; Range: $[0, 3]$

2.6.6.46.
$G(t) = \sqrt{t + 6}; \quad -6 \leq t \leq 3$

2.6.6.47.
$G(x) = \dfrac{1}{3 - x}; \quad -1.25 \leq x \leq 2.75$

Answer. Domain: $[-1.25, 2.75]$; Range: $\left[\dfrac{4}{17}, 4\right]$

2.6.6.48.
$H(x) = \dfrac{1}{x - 1}; \quad -3.25 \leq x \leq -1.25$

2.6.6.49.
$G(x) = \dfrac{1}{3 - x}; \quad 3 < x \leq 6$

Answer. Domain: $(3, 6]$; Range: $\left[-\infty, \dfrac{-1}{3}\right]$

2.6.6.50.
$H(x) = \dfrac{1}{x - 1}; \quad 1 < x \leq 4$

2.6.6.51.

a Show that the graph of $y = \sqrt{16 - x^2}$ is a semicircle.

b State the domain and range of the function.

c Graph the function in the window

Xmin $= -6$	Xmax $= 6$
Ymin $= 0$	Ymax $= 8$

In what way is the calculator's graph misleading?

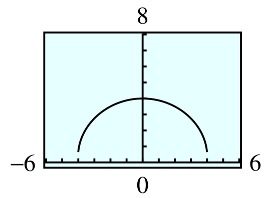
Hint. (Hint: Write the equation in the form $x^2 + y^2 = r^2$. See Algebra Skills Refresher Section A.11 to review circles.)

Answer.

a Squaring both sides of the equation gives the equation of the circle centered on the origin with radius 4, but the points in the third and fourth quadrants are extraneous solutions introduced by squaring. (The original equation allowed only $y \geq 0$.)

b Domain: $[-4, 4]$; Range: $[0, 4]$

c



The calculator does not show the graph extending down to the x-axis.

2.6.6.52.

a For what values of x is the function $y = \dfrac{2x-8}{x-2}$ undefined?

b Graph the function in the standard window. In what way is the calculator's graph misleading?

c Graph the function in the window

Xmin $= -9.4$	Xmax $= 9.4$
Ymin $= -10$	Ymax $= 10$

State the domain and range of the function.

In Problems 53–60, find the domain and range of each transformation of the given function.

2.6.6.53. $f(x) = \dfrac{1}{x^2}$

a $y = f(x-2)$

b $y = f(x) - 2$

c $y = f(x-3) - 5$

Answer.

a Domain: $x \neq 2$; Range: $(0, \infty)$

b Domain: $x \neq 0$; Range: $(-2, \infty)$

c Domain: $x \neq 3$; Range: $(-5, \infty)$

2.6.6.54. $f(x) = \sqrt{x}$

a $y = -f(x)$

b $y = 4 + f(x)$

c $y = 4 - f(x)$

2.6.6.55. $f(x) = x^2$

a $y = -2f(x)$

b $y = 6 - 2f(x)$

c $y = 6 - 2f(x + 3)$

Answer.

a Domain: all real numbers; Range: $(-\infty, 0)$

b Domain: all real numbers; Range: $(-\infty, 6]$

c Domain: all real numbers; Range: $(-\infty, 6]$

2.6.6.56. $f(x) = \dfrac{1}{x}$

a $y = 3f(x)$

b $y = 3 + f(x - 1)$

c $y = 3 - f(x - 1)$

2.6.6.57. The domain of f is $[0, 10]$ and the range is $[-2, 2]$.

a $y = f(x - 3)$

b $y = 3f(x)$

c $y = 2f(x - 5)$

Answer.

a Domain: $[3, 13]$; Range: $[-2, 2]$

b Domain: $[0, 10]$; Range: $[-6, 6]$

c Domain: $[5, 15]$; Range: $[-4, 4]$

2.6.6.58. The domain of f is $[-4, 4]$ and the range is $[3, 10]$.

a $y = f(x) + 10$

b $y = f(x + 10)$

c $y = f(x - 1) + 4$

2.6.6.59. The domain of f is $(0, +\infty)$ and the range is $(0, 1)$.

a $y = 5f(x)$

b $y = 3f(x + 2)$

c $y = 2f(x - 3) + 2$

Answer.

a Domain: $(0, \infty)$; Range: $(0, 5)$

b Domain: $(-2, \infty)$; Range: $(0, 3)$

c Domain: $(3, \infty)$; Range: $(2, 4)$

2.6.6.60. The domain of f is $(-1, 1)$ and the range is $(-\infty, 0)$.

a $y = f(x + 1)$

b $y = 3 - f(x + 1)$

c $y = 4 + 2f(x - 1)$

In Problems 61–64, use a graphing calculator to explore some properties of the basic functions.

2.6.6.61.

a Graph $f(x) = x^2$ and $g(x) = x^3$ on the domain $[0, 1]$ and state the range of each function. On the interval $(0,1)$, which is greater, $f(x)$ or $g(x)$?

b Graph $f(x)=x^2$ and $g(x)=x^3$ on the domain $[1,10]$ and state the range of each function. On the interval $(1,100)$, which is greater, $f(x)$ or $g(x)$?

Answer.

a $f(x)$ b $g(x)$

2.6.6.62.

a Graph $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$ on the domain $[0,1]$ and state the range of each function. On the interval $(0,1)$, which is greater, $f(x)$ or $g(x)$?

b Graph $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$ on the domain $[1,100]$ and state the range of each function. On the interval $(1,100)$, which is greater, $f(x)$ or $g(x)$?

2.6.6.63.

a Graph $f(x)=\dfrac{1}{x}$ and $g(x)=\dfrac{1}{x^2}$ on the domain $[0.01,1]$ and state the range of each function. On the interval $(0,1)$, which is greater, $f(x)$ or $g(x)$?

b Graph $f(x)=\dfrac{1}{x}$ and $g(x)=\dfrac{1}{x^2}$ on the domain $[1,10]$ and state the range of each function. On the interval $(1,\infty)$, which is greater, $f(x)$ or $g(x)$?

Answer.

a $g(x)$ b $f(x)$

2.6.6.64.

a Graph $F(x)=\left|x^3\right|$ in the **ZDecimal** window. How does the graph compare to the graph of $y=x^3$?

b Graph $G(x)=\left|\dfrac{1}{x}\right|$ in the **ZDecimal** window. How does the graph compare to the graph of $y=\dfrac{1}{x}$?

2.6.6.65. The number of hours of daylight on the summer solstice is a function of latitude in the northern hemisphere. Give the domain and range of the function.

Answer. Domain: $[0°,90°]$; Range: $[12,24]$

2.6.6.66. A semicircular window has a radius of 2 feet. The area of a sector of the window (a pie-shaped wedge) is a function of the angle at the center of the circle. Give the domain and range of this function.

2.7 Chapter Summary and Review

2.7.2 Chapter 2 Review Problems

For Problems 1-4, solve by extraction of roots.

2.7.2.1. $(2x-5)^2=9$

Answer. $x=1$ or $x=4$

2.7.2.2. $(7x-1)^2=15$

2.7.2.3. $6\left(\dfrac{w-1}{3}\right)^2-4=2$

Answer. $w=-2$ or $w=4$

2.7.2.4. $\left(\dfrac{2p}{5}\right)^2=-3$

For problems 5-6, solve the formula for the specified variable.

2.7.2.5. $A=P(1+r)^2$, for r

Answer. $r=-1\pm\sqrt{\dfrac{A}{P}}$

2.7.2.6. $V=\dfrac{4}{3}\pi r^3$, for r

2.7.2.7. Lewis invested \$2000 in an account that compounds interest annually. He made no deposits or withdrawals after that. Two years later, he closed the account, withdrawing \$2464.20. What interest rate did Lewis earn?

Answer. 11%

2.7.2.8. Earl borrowed \$5500 from his uncle for two years with interest compounded annually. At the end of two years, he owed his uncle \$6474.74. What was the interest rate on the loan?

For Problems 9-14, solve.

2.7.2.9. $\sqrt[3]{P-1}=0.1$

Answer. $P=1.001$

2.7.2.10. $\dfrac{1}{1-t}=\dfrac{2}{3}$

2.7.2.11. $\dfrac{3}{\sqrt{m+7}}=\dfrac{1}{2}$

Answer. $m=29$

2.7.2.12. $15=3\sqrt{w+1}$

2.7.2.13. $4r^3-8=100$

Answer. $r=3$

2.7.2.14. $5s^2+6=3s^2+31$

For Problems 15-16, use the Pythagorean theorem to write and solve an equation.

2.7.2.15. A widescreen television measures 96 cm by 54 cm. How long is the diagonal?

Answer. $\sqrt{12,132}\approx 110$ cm

2.7.2.16. A 15-foot ladder leans to the top of a 12-foot fence. How far is the foot of the ladder from the base of the fence?

For Problems 17-20, simplify.

2.7.2.17. $|-18|-|20|$

Answer. -2

2.7.2.18. $\left|-2\cdot(3-18)\right|$

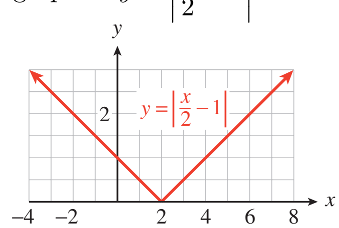
2.7.2.19. $|-2\cdot 3-18|$

Answer. 24

2.7.2.20. $-2\cdot|3-18|$

For Problems 21-24, use the graph to solve the equation or inequality.

2.7.2.21. Refer to the graph of $y=\left|\dfrac{x}{2}-1\right|$



(a) Solve $\left|\frac{x}{2}-1\right|=2$

(b) Solve $\left|\frac{x}{2}-1\right|<2$

(c) Solve $\left|\frac{x}{2}-1\right|\geq 2$

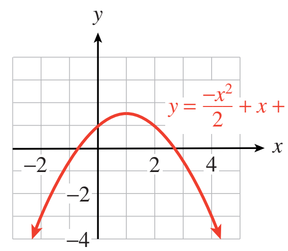
Answer.

a $x=-2$ or $x=6$

b $(-2,6)$

c $(-\infty,-2]\cup[6,+\infty)$

2.7.2.22. Refer to the graph of $y=\dfrac{-x^2}{2}+x+1$

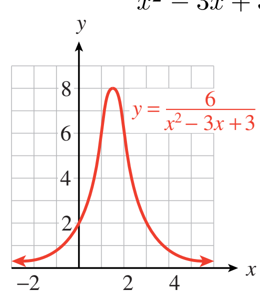


(a) Solve $\dfrac{-x^2}{2}+x+1=-3$

(b) Solve $\dfrac{-x^2}{2}+x+1\geq -3$

(c) Solve $\dfrac{-x^2}{2}+x+1\leq -3$

2.7.2.23. Refer to the graph of $y=\dfrac{6}{x^2-3x+3}$



(a) Solve $2=\dfrac{6}{x^2-3x+3}$

(b) Solve $2>\dfrac{6}{x^2-3x+3}$

(c) Solve $2<\dfrac{6}{x^2-3x+3}$

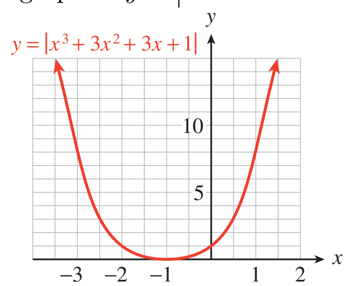
Answer.

a $x=0$ or $x=3$

b $(-\infty, 0) \cup (3, \infty)$

c $(0, 3)$

2.7.2.24. Refer to the graph of $y = \left|x^3 + 3x^2 + 3x + 1\right|$



(a) Solve $8 = \left|x^3 + 3x^2 + 3x + 1\right|$

(b) Solve $8 > \left|x^3 + 3x^2 + 3x + 1\right|$

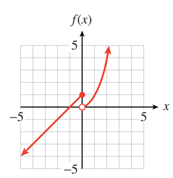
(c) Solve $8 < \left|x^3 + 3x^2 + 3x + 1\right|$

For Problems 25-30, graph the piecewise defined function.

2.7.2.25.

$$f(x) = \begin{cases} x+1 & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$$

Answer.

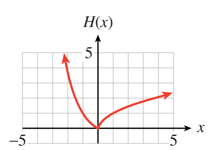


2.7.2.26.

$$g(x) = \begin{cases} x-1 & \text{if } x \le 1 \\ x^3 & \text{if } x > 1 \end{cases}$$

2.7.2.27. $H(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

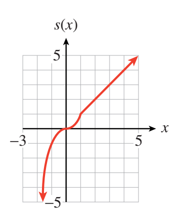
Answer.



2.7.2.28. $F(x) = \begin{cases} |x| & \text{if } x \le 0 \\ \dfrac{1}{x} & \text{if } x > 0 \end{cases}$

2.7.2.29. $S(x) = \begin{cases} x^3 & \text{if } x \le 1 \\ |x| & \text{if } x > 1 \end{cases}$

Answer.



2.7.2.30. $T(x) = \begin{cases} \dfrac{1}{x^2} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$

For Problems 31–38,

a Describe each function as transformation of a basic function.

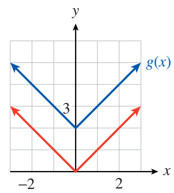
b Sketch a graph of the basic function and the given function on the same axes.

2.7.2.31. $g(x) = |x| + 2$

Answer.

a $y = |x|$ shifted up 2 units

b



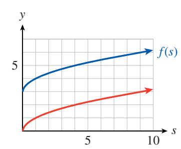
2.7.2.32. $F(t) = \dfrac{1}{t} - 2$

2.7.2.33. $f(s) = \sqrt{s} + 3$

Answer.

a $y = \sqrt{x}$ shifted up 3 units

b



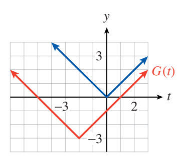
2.7.2.34. $g(u) = \sqrt{u+2} - 3$

2.7.2.35. $G(t) = |t + 2| - 3$

Answer.

a $y = |x|$ shifted left 2 units and down 3 units

b



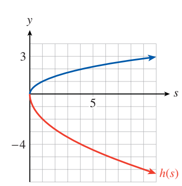
2.7.2.36. $H(t) = \dfrac{1}{(t-2)^2} + 3$

2.7.2.37. $h(s) = -2\sqrt{s}$

Answer.

a $y = \sqrt{x}$ reflected across the horizontal axis and stretched vertically by a factor of 2

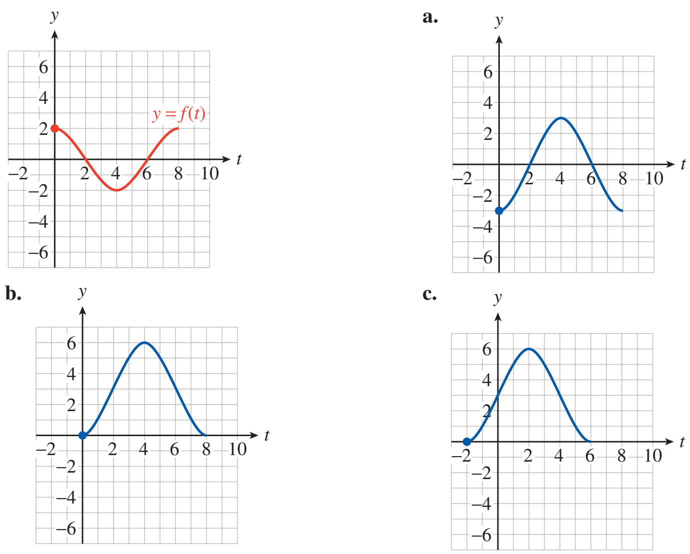
b



2.7.2.38. $H(t) = \dfrac{1}{2}\,|s|$

In Problems 39–42, write a formula for each transformation of the given function.

2.7.2.39.



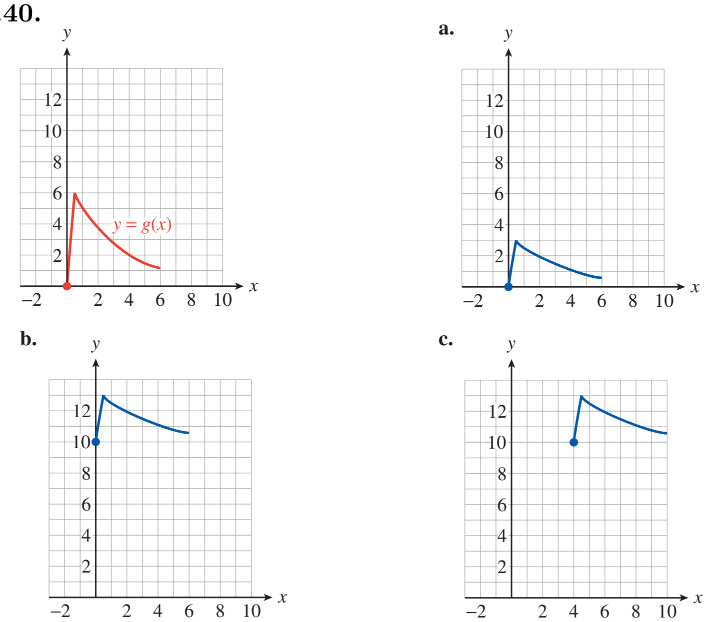
Answer.

a $y = \dfrac{-3}{2}f(t)$

b $y = \dfrac{-3}{2}f(t) + 3$

c $y = \dfrac{-3}{2}f(t+2) + 3$

2.7.2.40.



2.7.2.41.

t	0	1	2	3	4	5
$f(t)$	243	81	27	9	3	1

a

t	1	2	3	4	5	6
y	243	81	27	9	3	1

b

t	1	2	3	4	5	6
y	-243	-81	-27	-9	-3	-1

c

t	1	2	3	4	5	6
y	57	219	273	291	297	299

Answer.

a $y = f(t-1)$

b $y = -f(t-1)$

c $y = -f(t-1) + 300$

2.7.2.42.

x	1	2	3	4	5	6
$f(x)$	25	24	21	16	9	0

a

x	-1	0	1	2	3	4
y	25	24	21	16	9	0

b

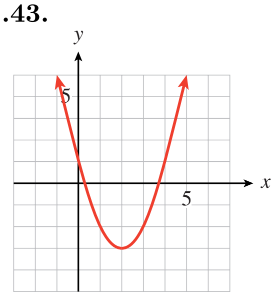
x	-1	0	1	2	3	4
y	50	48	42	32	18	0

c

x	-1	0	1	2	3	4
y	70	68	62	52	38	20

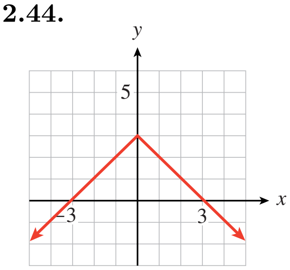
For Problems 43-44, give an equation for the function graphed.

2.7.2.43.



Answer. $y = (x-2)^2 - 4$

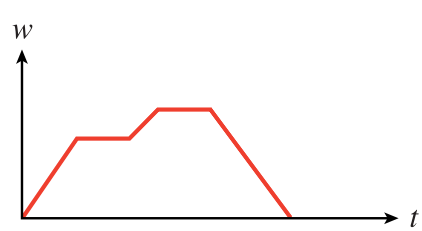
2.7.2.44.



Sketch graphs to illustrate the situations in Problems 45 and 46.

2.7.2.45. Inga runs hot water into the bathtub until it is about half full. Because the water is too hot, she lets it sit for a while before getting into the tub. After several minutes of bathing, she gets out and drains the tub. Graph the water level in the bathtub as a function of time, from the moment Inga starts filling the tub until it is drained.

Answer.



2.7.2.46. David turns on the oven and it heats up steadily until the proper baking temperature is reached. The oven maintains that temperature during the time David bakes a pot roast. When he turns the oven off, David leaves the oven door open for a few minutes, and the temperature drops fairly rapidly during that time. After David closes the door, the temperature continues to drop, but at a much slower rate. Graph the temperature of the oven as a function of time, from the moment David first turns on the oven until shortly after David closes the door when the oven is cooling.

For Problems 47-48, match each table with its graph.

2.7.2.47.

I

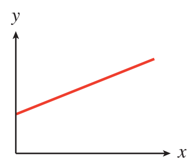
x	0	2	4	6	8
y	10	14	21	30	43

III

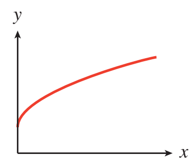
x	0	1	2	3	4
y	140	190	240	290	340

II

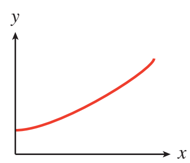
x	0	10	20	30	40
y	20	52	65	75	83



(a)



(b)



(c)

Answer. I (c), II (b), III (a)

2.7.2.48.

I

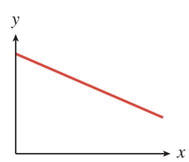
x	0	0.1	0.2	0.3	0.4
y	100	95	80	55	20

III

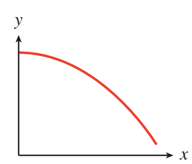
x	0	10	20	30	40
y	50	37	27	20	15

II

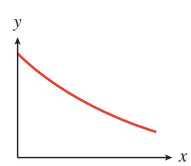
x	0	1	2	3	4
y	8.5	7.1	5.7	4.3	2.9



(a)



(b)

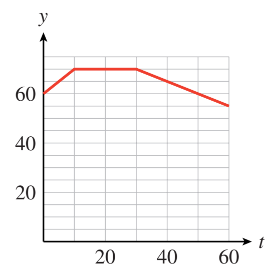


(c)

Write and graph a piecewise function for Problems 49 and 50.

2.7.2.49. The fluid level in a tank is a function of the number of days since the year began. The level was initially at 60 inches and rose an inch a day for 10 days, remained constant for the next 20 days, then dropped a half-inch each day for 30 days.

Answer. $g(t) = \begin{cases} 60 + t, & 0 \le t < 10 \\ 70, & 10 \le t < 30 \\ 70 - \frac{1}{2}(t-30), & 30 \le t \le 60 \end{cases}$



2.7.2.50. The temperature at different locations in a large room is a function of distance from the window. Within 2 feet of the window, the temperature is 66° Fahrenheit, but the temperature rises by 0.5° for each of the next 10 feet, then maintains the temperature at 12 feet for the rest of the room.

For Problems 51-54, use absolute value notation to write the expression as an equation or inequality.

2.7.2.51. x is four units from the origin.

Answer. $|x| = 4$

2.7.2.52. The distance from y to -5 is three units.

2.7.2.53. p is within four units of 7.

Answer. $|p - 7| < 4$

2.7.2.54. q is at least $\dfrac{3}{10}$ unit from -4.

For Problems 55-64, solve.

2.7.2.55. $|9 - 5t| = 3$

Answer. $t = \dfrac{6}{5}$ or $t = \dfrac{12}{5}$

2.7.2.56. $1 = |4q - 7|$

2.7.2.57. $-29 = |2w + 3|$

Answer. No solutions

2.7.2.58. $\left|\dfrac{8n+3}{5}\right| = -11$

2.7.2.59. $1 = \left|\dfrac{7-2p}{5}\right|$

Answer. $p = 1$ or $p = 6$

2.7.2.60. $\bigl|6(r - 10)\bigr| = 30$

2.7.2.61. $|3x - 2| < 4$

Answer. $\left(\dfrac{-2}{3}, 2\right)$

2.7.2.62. $|2x + 0.3| \le 0.5$

2.7.2.63. $|3y + 1.2| \ge 1.5$

Answer. $(-\infty, -0.9] \cup [0.1, \infty)$

2.7.2.64. $\left|3z + \dfrac{1}{2}\right| > \dfrac{1}{3}$

For Problems 65-66, express the error tolerance using absolute value.

2.7.2.65. The height, H, of a female trainee must be between 56 inches and 75 inches.

Answer. $|H - 65.5| < 9.5$

2.7.2.66. The time, t, in freefall must be at least 3.5 seconds but no more than 8.1 seconds.

For Problems 67-68, give an interval of possible values for the measurement.

2.7.2.67. The mass, M, of the sample must satisfy $|M - 2.1| \leq 0.05$.

Answer. $[2.05, 2.15]$

2.7.2.68. The temperature, T, of the refrigerator is specified by $|T - 4.0| < 0.5$.

In Problems 69 and 70,

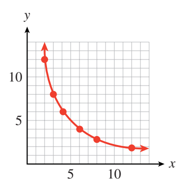
a Plot the points and sketch a smooth curve through them.

b Use your graph to help you discover the equation that describes the function.

2.7.2.69.

x	$g(x)$
2	12
3	8
4	6
6	4
8	3
12	2

Answer.

a 

b $g(x) = \dfrac{24}{x}$

2.7.2.70.

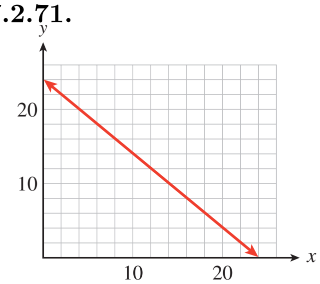
x	$F(x)$
-2	8
-1	1
0	0
1	-1
2	-8
3	-27

In Problems 71–76,

a Use the graph to complete the table of values.

b By finding a pattern in the table of values, write an equation for the graph.

2.7.2.71.



x	0	4	8		16	
y				10		2

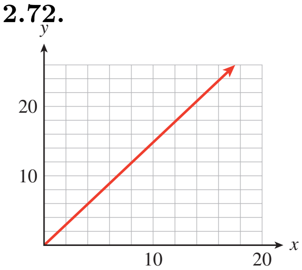
Answer.

a

x	0	4	8	14	16	22
y	24	20	16	10	8	2

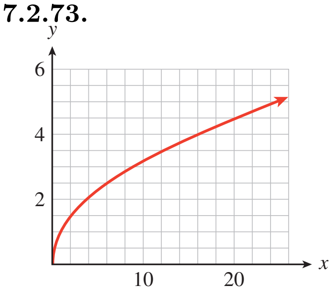
b $y = 24 - x$

2.7.2.72.



x	0	4	10		14	
y				18		24

2.7.2.73.



x	0		4		16	25
y		1		3		

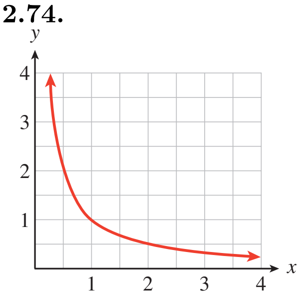
Answer.

a

x	0	1	4	9	16	25
y	0	1	2	3	4	5

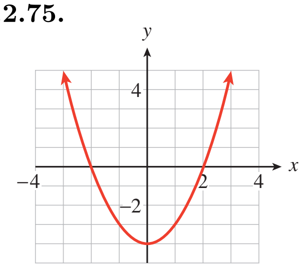
b $y = \sqrt{x}$

2.7.2.74.



x		0.5	1	1.5		4
y	4				0.5	

2.7.2.75.



x	-3	-2		0	1	2
y			-3			

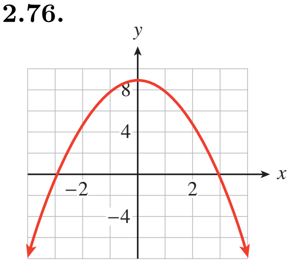
Answer.

a

x	-3	-2	-1	0	1	2
y	5	0	-3	-4	-3	0

b $y = x^2 - 4$

2.7.2.76.



x	-3	-2		0	1	
y			8			-7

For Problems 77-80, use a graphing calculator to graph the function on the given domain. Adjust **Ymin** and **Ymax** until you can determine the range of the function using the `TRACE` key. Then verify your answer algebraically by evaluating the function. State the domain and corresponding range in interval notation.

2.7.2.77. $f(t) = -t^2 + 3t$; $-2 \le t \le 4$

Answer. Domain: $[-2, 4]$; Range: $[-10, -4]$

2.7.2.78. $g(x) = \sqrt{s-2}$; $2 \le s \le 6$

2.7.2.79. $F(x) = \dfrac{1}{x+2}$; $-2 < x \le 4$

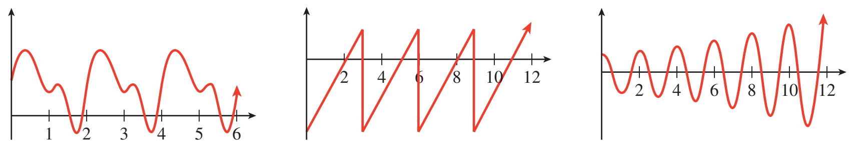
Answer. Domain: $(-2, 4]$; Range: $\left[\dfrac{1}{6}, \infty\right)$

2.7.2.80. $H(x) = \dfrac{1}{2-x}$; $-4 \le x < 2$

2.8 Projects for Chapter 2: Periodic Functions

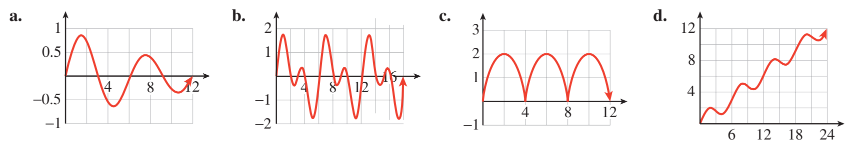
Project 2.8.1 Part I.

1 Which of the functions are periodic? If the function is periodic, give its period.

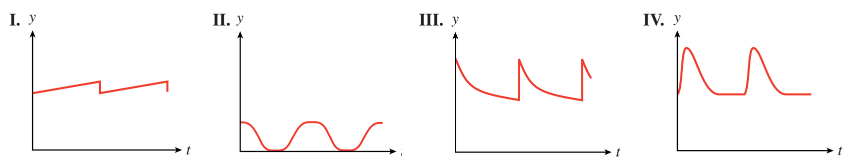


(a) (b) (c)

2 Which of the functions are periodic? If the function is periodic, give its period.



3 Match each of the following situations with the appropriate graph.



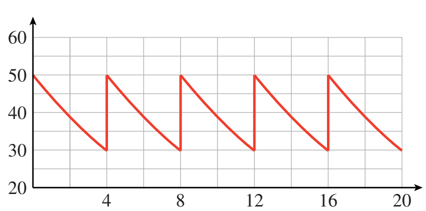
a When the heart contracts, blood pressure in the arteries rises rapidly to a peak (systolic blood pressure) and then falls off quickly to a minimum (diastolic blood pressure). Blood pressure is a periodic function of time.

b After an injection is given to a patient, the amount of the drug present in his bloodstream decreases exponentially. The patient receives injections at regular intervals to restore the drug level to the prescribed level. The amount of the drug present is a periodic function of time.

c The monorail shuttle train between the north and south terminals at Gatwick Airport departs from the south terminal every 12 minutes. The distance from the train to the south terminal is a periodic function of time.

d Delbert gets a haircut every two weeks. The length of his hair is a periodic function of time.

4 A patient receives regular doses of medication to maintain a certain level of the drug in his body. After each dose, the patient's body eliminates a certain percent of the medication before the next dose is administered. The graph shows the amount of the drug, in milliliters, in the patient's body as a function of time in hours.

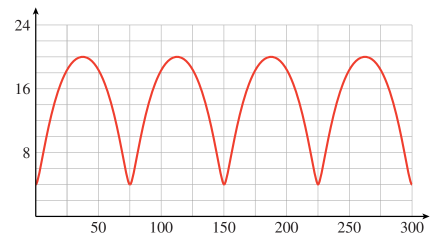


a How much of the medication is administered with each dose?

b How often is the medication administered?

c What percent of the drug is eliminated from the body between doses?

5 You are sitting on your front porch late one evening, and you see a light coming down the road tracing out the path shown below, with distances in inches. You realize that you are seeing a bicycle light, fixed to the front wheel of the bike.

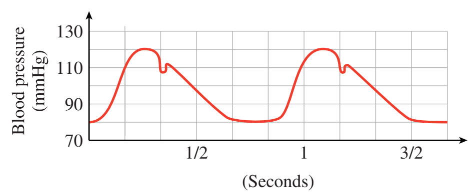


a Approximately what is the period of the graph?

b How far above the ground is the light?

c What is the diameter of the bicycle wheel?

6 The graph shows arterial blood pressure, measured in millimeters of mercury (mmHg), as a function of time.



a What are the maximum (systolic) and minimum (diastolic) pressures? The **pulse pressure** is the difference of systolic and diastolic pressures. What is the pulse pressure?

b The **mean arterial pressure** is the diastolic pressure plus one-third of the pulse pressure. Calculate the mean arterial pressure and draw a horizontal line on the graph at that pressure.

c The blood pressure graph repeats its cycle with each heartbeat. What is the heart rate, in beats per minute, of the person whose blood pressure is shown in the graph?

7 At a ski slope, the lift chairs take 5 minutes to travel from the bottom, at an elevation of 3000 feet, to the top, at elevation 4000 feet. The cable supporting the ski lift chairs is a loop turning on pulleys at a constant speed. At the top and bottom, the chairs are at a constant elevation for a few seconds to allow skiers to get on and off.

a Sketch a graph of $h(t)$, the height of one chair at time t. Show at least two complete up-and-down trips.

b What is the period of $h(t)$?

8 The heater in Paul's house doesn't have a thermostat; it runs on a timer. It uses 300 watts when it is running. Paul sets the heater to run from 6 a.m. to noon, and again from 4 p.m. to 10 p.m.

a Sketch a graph of $P(t)$, the power drawn by the heater as a function of time. Show at least two days of heater use.

b What is the period of $P(t)$?

9 Francine adds water to her fish pond once a week to keep the depth at 30 centimeters. During the week, the water evaporates at a constant rate of 0.5 centimeter per day.

a Sketch a graph of $D(t)$, the depth of the water, as a function of time. Show at least two weeks.

b What is the period of $D(t)$?

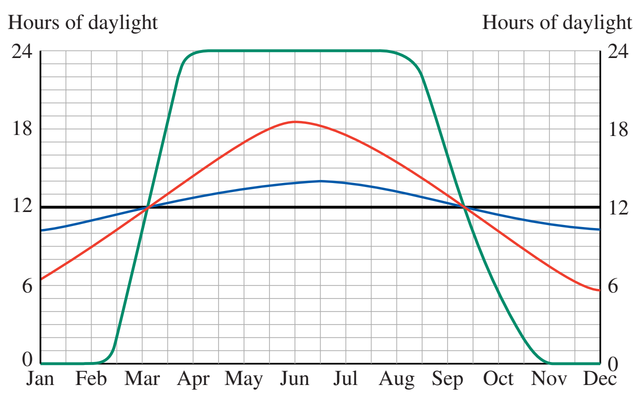
10 Erin's fox terrier, Casey, is very energetic and bounces excitedly at dinner time. Casey can jump 30 inches high,and each jump takes him 0.8 second.

a Sketch a graph of Casey's height, $h(t)$, as a function of time. Show at least two jumps.

b What is the period of $h(t)$?

Project 2.8.2 Part II.

1 The graph shows the number of daylight hours in Jacksonville, in Anchorage, at the Arctic Circle, and at the Equator.



a Which graph corresponds to each location?

b What are the maximum and minimum number of daylight hours in Jacksonville?

c For how long are there 24 hours of daylight per day at the Arctic Circle?

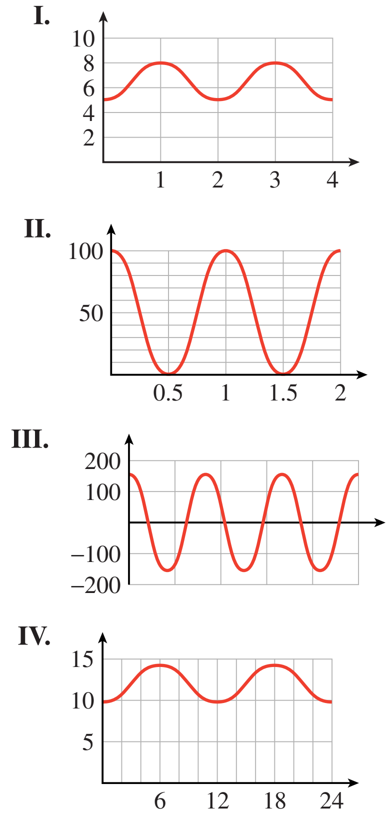
2 Match each of the following situations with the appropriate graph.

a The number of hours of daylight in Salt Lake City varies from a minimum of 9.6 hours on the winter solstice to a maximum of 14.4 hours on the summer solstice.

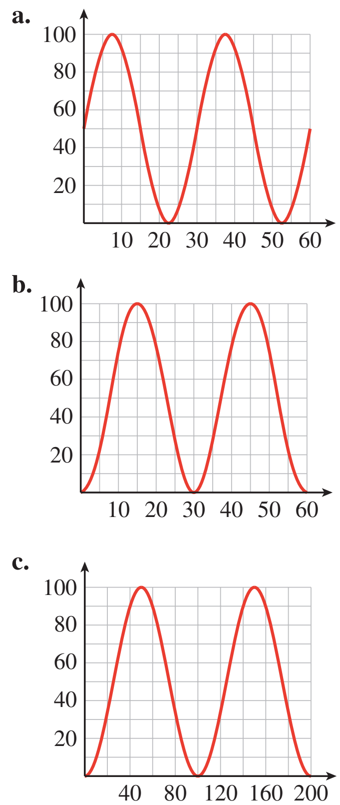
b A weight is 6.5 feet above the floor, suspended from the ceiling by a spring. The weight is pulled down to 5 feet above the floor and released, rising past 6.5 feet in 0.5 second before attaining its maximum height of 8 feet. Neglecting the effects of friction, the height of the weight will continue to oscillate between its minimum and maximum height.

c The voltage used in U.S. electrical current changes from 155 V to -155 V and back 60 times each second.

d Although the moon is spherical, what we can see from Earth looks like a (sometimes only partly visible) disk. The percentage of the moon's disk that is visible varies between 0 (at new moon) to 100 (at full moon).



3 As the moon revolves around the Earth, the percent of the disk that we see varies sinusoidally with a period of approximately 30 days. There are eight phases, starting with the new moon, when the moon's disk is dark, followed by waxing crescent, first quarter, waxing gibbous, full moon (when the disk is 100% visible), waning gibbous, last quarter, and waning crescent. Which graph best represents the phases of the moon?



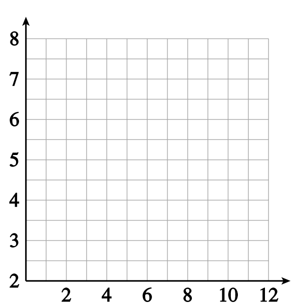
4 The table shows sunrise and sunset times in Los Angeles on the fifteenth of each month.

Month	Oct	Nov	Dec	Jan	Feb	Mar
Sunrise	5:58	6:26	6:51	6:59	6:39	6:04
Sunset	17:20	16:50	16:45	17:07	17:37	18:01

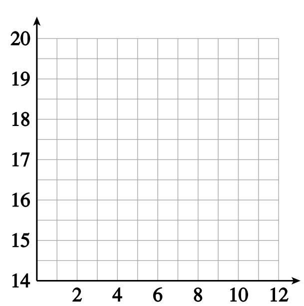
Month	Apr	May	Jun	Jul	Aug	Sep
Sunrise	5 : 22	4 : 52	4 : 42	4 : 43	5 : 15	5 : 37
Sunset	18 : 25	18 : 48	19 : 07	19 : 05	18 : 40	18 : 00

a Use the grid (a) to plot the sunrise times and sketch a sinusoidal graph through the points

b Use the grid (b) to plot the sunset times and sketch a sinusoidal graph through the points.



(a)



(b)

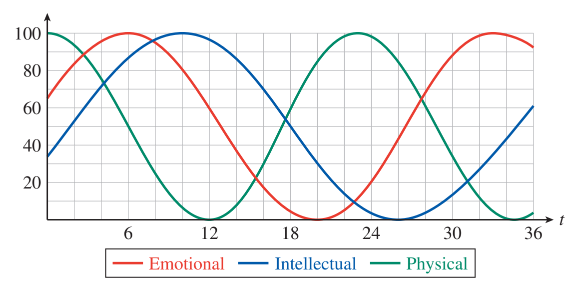
5 a Use the data from Problem 4 to complete the table with the hours of sunlight in Los Angeles on the fifteenth of each month.

Month	Oct	Nov	Dec	Jan	Feb	Mar
Hours of daylight						

Month	Apr	May	Jun	Jul	Aug	Sep
Hours of daylight						

b Plot the daylight hours and sketch a sinusoidal graph through the points.

6 Many people who believe in astrology also believe in biorhythms. The graph shows an individual's three biorhythms—physical, emotional, and intellectual—for 36 days, from $t = 0$ on September 30 to November 5.

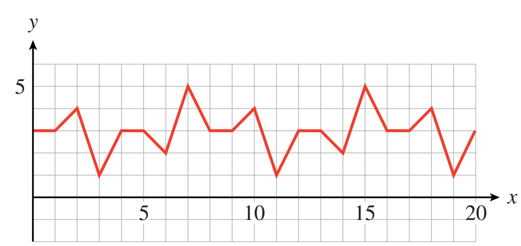


a Find the dates of highest and lowest activity for each biorhythm during the month of October.

b Find the period of each biorhythm in days

c On the day of your birth, all three biorhythms are at their maximum. How old will you be before all three are again at the maximum level?

7 a Is the function shown periodic? If so, what is its period? If not, explain why not.

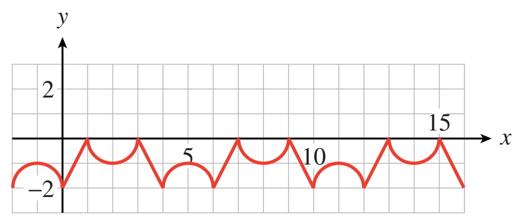


b Compute the difference between the maximum and minimum function values. Sketch in the midline of the graph.

c Find the smallest positive value of k for which $f(x) = f(x + k)$ for all x.

d Find the smallest positive values of a and b for which $f(b) - f(a)$ is a maximum.

8 a Find the period, the maximum and minimum values, and the midline of the graph of $y = f(x)$.

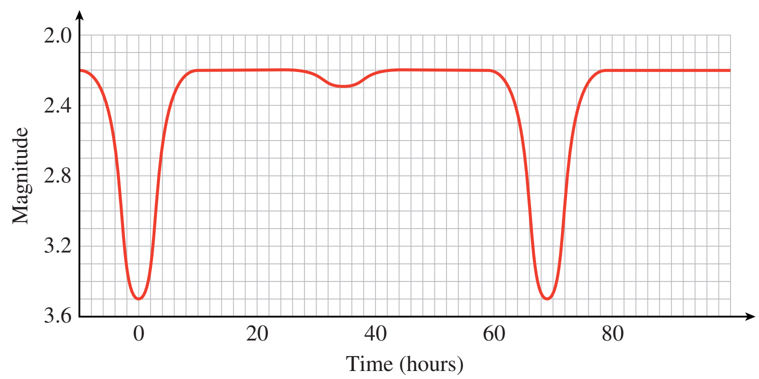


b Sketch a graph of $y = 2f(x)$.

c Sketch a graph of $y = 2 + f(x)$.

d Modify the graph of $f(x)$ so that the period is twice its current value.

9 The apparent magnitude of a star is a measure of its brightness as seen from Earth. Smaller values for the apparent magnitude correspond to brighter stars. The graph below, called a light curve, shows the apparent magnitude of the star Algol as a function of time. Algol is an eclipsing binary star, which means that it is actually a system of two stars, a bright principal star and its dimmer companion, in orbit around each other. As each star passes in front of the other, it eclipses some of the light that reaches Earth from the system. (Source: Gamow, 1965, Brandt & Maran, 1972)

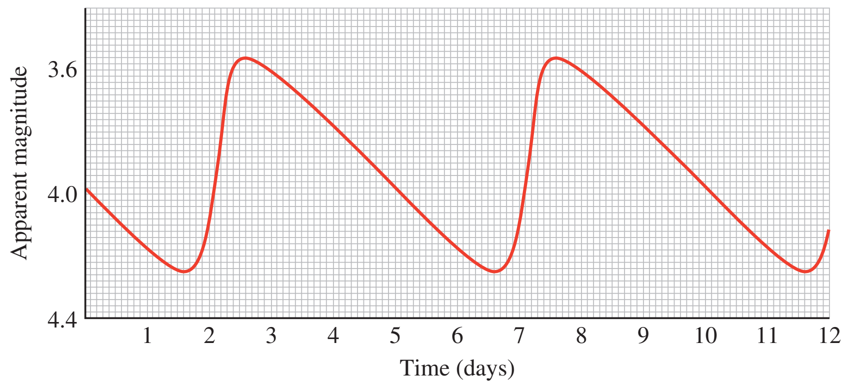


a The light curve is periodic. What is its period?

b What is the range of apparent magnitudes of the Algol system?

c Explain the large and small dips in the light curve. What is happening to cause the dips?

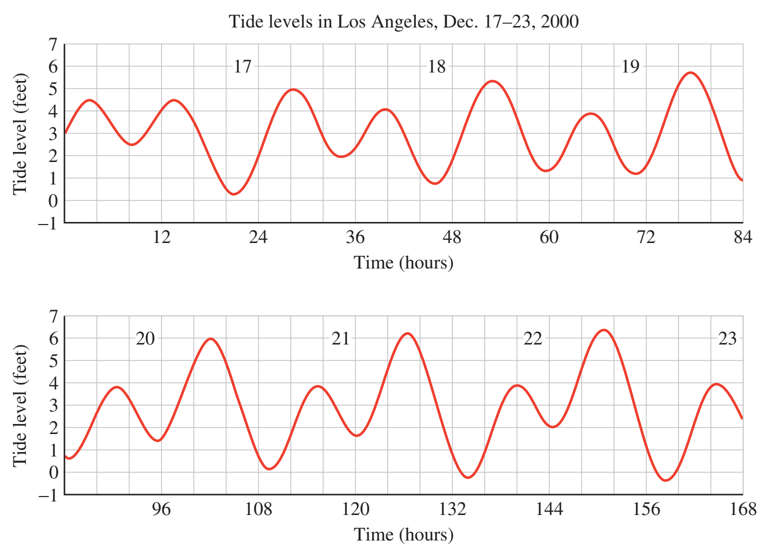
10 Some stars, called Cepheid variable stars, appear to pulse, getting brighter and dimmer periodically. The graph shows the light curve for the star Delta Cephei. (Source: Ingham, 1997)



a What is the period of the graph?

b What is the range of apparent magnitudes for Delta Cephei?

11 The figure is a tide chart for Los Angeles for the week of December 17–23, 2000. The horizontal axis shows time in hours, with $t = 12$ corresponding to noon on December 17. The vertical axis shows the height of the tide in feet above mean sea level.



a High tides occurred at 3:07 a.m. and 2:08 p.m. on December 17, and low tides at 8:41 a.m. and 9:02 p.m. Estimate the heights of the high and low tides on that day.

b Is tide height a periodic function of time? Use the information from part (a) to justify your answer.

c Make a table showing approximate times and heights for the high tides throughout the week. Make a similar table for the low tides

d Describe the trend in the heights of the high tides over the week. Describe the trend in the heights of the low tides.

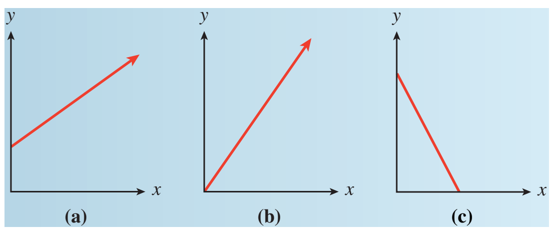
e What is the largest height difference between consecutive high and low tides during the week shown? When does it occur?

3 Power Functions

3.1 Variation

3.1.1 Direct Variation

Checkpoint 3.1.3 Which of the graphs below could represent direct variation? Explain why.



Answer. (b): The graph is a straight line through the origin.

3.1.2 The Scaling Property of Direct Variation

Checkpoint 3.1.5 Which table could represent direct variation? Explain why. (Hint: What happens to$y$ if you multiply x by a constant?)

a

x	1	2	3	6	8	9
y	2.5	5	7.5	15	20	22.5

b

x	2	3	4	6	8	9
y	2	3.5	5	7	8.5	10

Answer. (a): If we multiply x by c, y is also multiplied by c.

3.1.3 Finding a Formula for Direct Variation

Checkpoint 3.1.7 The volume of a bag of rice, in cups, is directly proportional to the weight of the bag. A 2-pound bag contains 3.5 cups of rice.

a Express the volume, V, of a bag of rice as a function of its weight, w.

b How many cups of rice are in a 15-pound bag?

Answer.

a $V = 1.75w$

b 26.25

3.1.4 Direct Variation with a Power of x

Checkpoint 3.1.9 The volume of a sphere varies directly with the *cube* of its radius. A balloon of radius 5 centimeters has volume $\dfrac{500\pi}{3}$ cubic centimeters, or about 524 cubic centimeters. Find a formula for the volume of a sphere as a function of its radius.

Answer. $V = \dfrac{4}{3}\pi r^3$

Checkpoint 3.1.12 Does B vary directly with the cube of r? Explain your decision.

r	0.1	0.3	0.5	0.8	1.2
B	0.072	1.944	9.0	16.864	124.416

Answer. Yes, $\dfrac{B}{r^3}$ is constant.

3.1.5 Scaling

Checkpoint 3.1.14 Use the formula for the area of a circle to show that doubling the diameter of a pizza quadruples its area.

Answer. $A = \pi r^2$, so $A_{\text{new}} = \pi(2r)^2 = 4\pi r^2 = 4A_{\text{old}}$

3.1.6 Inverse Variation

Checkpoint 3.1.17 The amount of force, F, (in pounds) needed to loosen a rusty bolt with a wrench is inversely proportional to the length, l, of the wrench. Thus,

$$F = \frac{k}{l}$$

If you increase the length of the wrench by 50% so that the new length is $1.5l$, what happens to the amount of force required to loosen the bolt?

Answer. $F_{\text{new}} = \dfrac{2}{3}F_{\text{old}}$

3.1.7 Finding a Formula for Inverse Variation

Checkpoint 3.1.19 Delbert's officemates want to buy a \$120 gold watch for a colleague who is retiring. The cost per person is inversely proportional to the number of people who contribute.

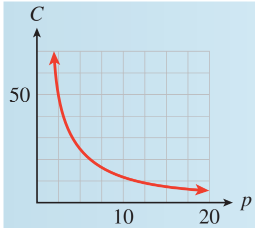
a Express the cost per person, C, as a function of the number of people, p, who contribute.

b Sketch the function on the domain $0 \le p \le 20$.

Answer.

a $C = \dfrac{120}{p}$

b



3.1.9 Homework 3.1

3.1.9.1. Delbert's credit card statement lists three purchases he made while on a business trip in the Midwest. His company's accountant would like to know the sales tax rate on the purchases.

Price of item	18	28	12
Tax	1.17	1.82	0.78
Tax/Price			

a Compute the ratio of the tax to the price of each item. Is the tax proportional to the price? What is the tax rate?

b Express the tax, T, as a function of the price, p, of the item.

c Sketch a graph of the function by hand, and label the scales on the axes.

Answer.

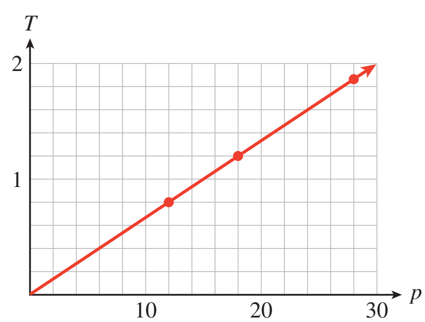
a

Price of item	18	28	12
Tax	1.17	1.82	0.78
Tax/Price	**0.065**	**0.065**	**0.065**

Yes; 6.5%

b $T = 0.065p$

c



3.1.9.2. At constant acceleration from rest, the distance traveled by a race car is proportional to the square of the time elapsed. The highest recorded road-tested acceleration is 0 to 60 miles per hour in 3.07 seconds, which produces the following data.

Time (seconds)	2	2.5	3
Distance (feet)	57.32	89.563	128.97
Distance/Time^2			

a Compute the ratios of the distance traveled to the square of the time elapsed. What was the acceleration, in feet per second squared?

b Express the distance traveled, d, as a function of time in seconds, t.

c Sketch a graph of the function by hand, and label the scales on the axes.

3.1.9.3. The marketing department for a paper company is testing wrapping paper rolls in various dimensions to see which shape consumers prefer. All the rolls contain the same amount of wrapping paper.

Width (feet)	2	2.5	3
Length (feet)	12	9.6	8
Length × width			

a Compute the product of the length and width for each roll of wrapping paper. What is the constant of inverse proportionality?

b Express the length, L, of the paper as a function of the width, w, of the roll.

c Sketch a graph of the function by hand, and label the scales on the axes.

Answer.

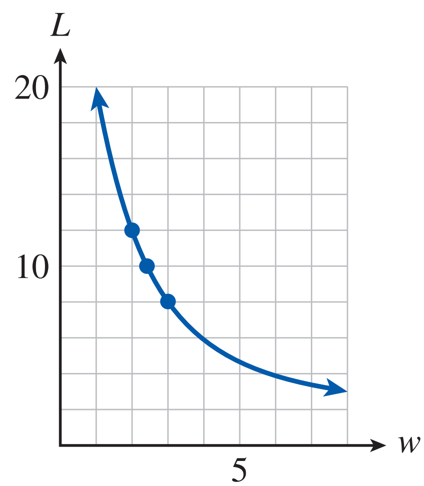
a

Width (feet)	2	2.5	3
Length (feet)	12	9.6	8
Length × width	**24**	**24**	**24**

24 square feet

b $L = \dfrac{24}{w}$

c



3.1.9.4. The force of gravity on a 1-kilogram mass is inversely proportional to the square of the object's distance from the center of the Earth. The table shows the force on the object, in newtons, at distances that are multiples of the Earth's radius.

Distance (Earth radii)	1	2	4
Force (newtons)	9.8	2.45	0.6125
Force × distance^2			

a Compute the products of the force and the square of the distance. What is the constant of inverse proportionality

b Express the gravitational force, F, on a 1-kilogram mass as a function of its distance, r, from the Earth's center, measured in Earth radii

c Sketch a graph of the function by hand, and label the scales on the axes.

3.1.9.5.

a How can you tell from a table of values whether y varies directly with x?

b How can you tell from a table of values whether y varies inversely with x?

Answer.

a The ratio $\frac{y}{x}$ is a constant.

b The product xy is a constant.

3.1.9.6.

a How can you tell from a table of values whether y varies directly with a power of x?

b How can you tell from a table of values whether y varies inversely with a power of x?

3.1.9.7. The length of a rectangle is 10 inches, and its width is 8 inches. Suppose we increase the length of the rectangle while holding the width constant.

a Fill in the table.

Length	Width	Perimeter	Area
10	8		
12	8		
15	8		
20	8		

b Does the perimeter vary directly with the length?

c Write a formula for the perimeter of the rectangle in terms of its length.

d Does the area vary directly with the length?

e Write a formula for the area of the rectangle in terms of its length.

Answer.

a

Length	Width	Perimeter	Area
10	8	**36**	**80**
12	8	**40**	**96**
15	8	**46**	**120**
20	8	**56**	**160**

b No

c $P = 16 + 2l$

d Yes

e $A = 8l$

3.1.9.8. The base of an isosceles triangle is 12 centimeters, and the equal sides have length 15 centimeters. Suppose we increase the base of the triangle while holding the sides constant.

a Fill in the table.

Base	Sides	Height	Perimeter	Area
12	15			
15	15			
18	15			
20	15			

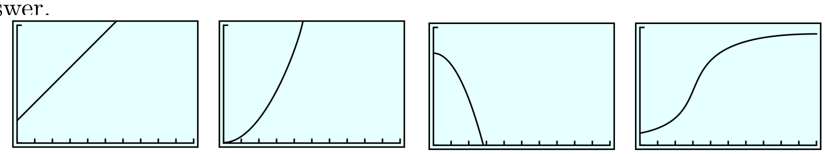
b Does the perimeter vary directly with the base?

c Write a formula for the perimeter of the triangle in terms of its base.

d Write a formula for the area of the triangle in terms of its base.

e Does the area vary directly with the base?

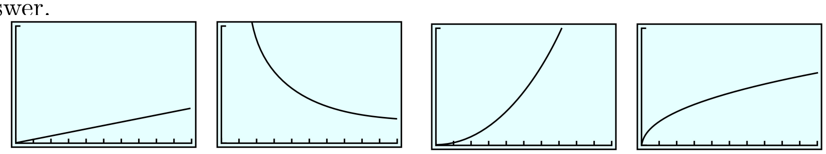
3.1.9.9. Which of the graphs could describe direct variation? Explain your answer.



(a) (b) (c) (d)

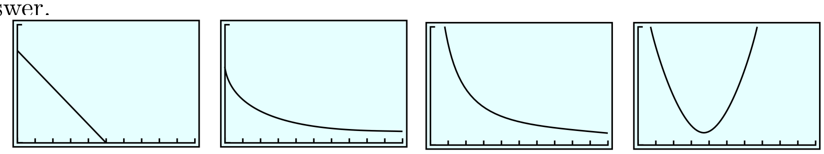
Answer. (b)

3.1.9.10. Which of the graphs could describe direct variation? Explain your answer.



(a) (b) (c) (d)

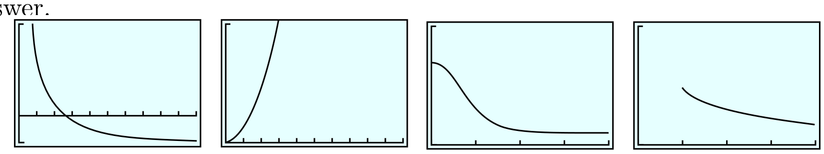
3.1.9.11. Which of the graphs could describe inverse variation? Explain your answer.



(a) (b) (c) (d)

Answer. (c)

3.1.9.12. Which of the graphs could describe inverse variation? Explain your answer.



(a) (b) (c) (d)

3.1.9.13. The weight of an object on the Moon varies directly with its weight on Earth. A person who weighs 150 pounds on Earth would weigh only 24.75 pounds on the Moon.

a Find a function that gives the weight m of an object on the Moon in terms of its weight w on Earth. Complete the table and graph your function in a suitable window.

w	50	100	200	400
m				

b How much would a person weigh on the Moon if she weighs 120 pounds on Earth?

c A piece of rock weighs 50 pounds on the Moon. How much will it weigh back on Earth?

d If you double the weight of an object on Earth, what will happen to its weight on the Moon?

Answer.

a $m = 0.165w$

w	50	100	200	400
m	**8.25**	**16.5**	**33**	**66**

b 19.8 lb

c 303.03 lb

d It will double.

3.1.9.14. Hubble's law says that distant galaxies are receding from us at a rate that varies directly with their distance. (The speeds of the galaxies are measured using a phenomenon called redshifting.) A galaxy in the constellation Ursa Major is 980 million light-years away and is receding at a speed of $15,000$ kilometers per second.

a Find a function that gives the speed, v, of a galaxy in terms of its distance, d, from Earth. Complete the table and graph your function in a suitable window. (Distances are given in millions of light-years.)

d	500	1000	2000	4000
m				

b How far away is a galaxy in the constellation Hydra that is receding at $61,000$ kilometers per second?

c A galaxy in Leo is 1240 million light-years away. How fast is it receding from us?

d If one constellation is twice as distant as another, how do their speeds compare?

3.1.9.15. The length, L, of a pendulum varies directly with the square of its period, T, the time required for the pendulum to make one complete swing back and forth. The pendulum on a grandfather clock is 3.25 feet long and has a period of 2 seconds.

a Express L as a function of T. Complete the table and graph your function in a suitable window.

T	1	5	10	20
L				

b How long is the Foucault pendulum in the Pantheon in Paris, which has a period of 17 seconds?

c A hypnotist uses a gold pendant as a pendulum to mesmerize his clients.

If the chain on the pendant is 9 inches long, what is the period of its swing?

d In order to double the period of a pendulum, how must you vary its length?

Answer.

a $L = 0.8125T^2$

T	1	5	10	20
L	**0.8125**	**20.3**	**81.25**	**325**

b 234.8125 ft

c 0.96 sec

d It must be four times as long.

3.1.9.16. The load, L, that a beam can support varies directly with the square of its vertical thickness, h. A beam that is 4 inches thick can support a load of 2000 pounds.

a Express L as a function of h. Complete the table and graph your function in a suitable window.

h	1	2	4	8
L				

b What size load can be supported by a beam that is 6 inches thick?

c How thick a beam is needed to support a load of 100 pounds?

d If you double the thickness of a beam, how will the load it can support change?

3.1.9.17. Computer monitors produce a magnetic field. The effect of the field, B, on the user varies inversely with his or her distance, d, from the screen. The field from a certain color monitor was measured at 22 milligauss 4 inches from the screen.

a Express the field strength as a function of distance from the screen. Complete the table and graph your function in a suitable window.

d	1	2	12	24
B				

b What is the field strength 10 inches from the screen?

c An elevated risk of cancer can result from exposure to field strengths of 4.3 milligauss. How far from the screen should the computer user sit to keep the field level below 4.3 milligauss?

d If you double your distance from the screen, how does the field strength change?

Answer.

a $B = \dfrac{88}{d}$

d	1	2	12	24
B	**88**	**44**	**7.3**	**3.7**

b 8.8 milligauss

c More than 20.47 in

d It is one half as strong.

3.1.9.18. The amount of current, I, that flows through a circuit varies inversely with the resistance, R, on the circuit. An iron with a resistance of 12 ohms draws 10 amps of current.

a Express the current as a function of the resistance. Complete the table and graph your function in a suitable window.

R	1	2	10	20
I				

b How much current is drawn by a light bulb with a resistance of 533.3 ohms?

c What is the resistance of a toaster that draws 12.5 amps of current?

d If the resistance of one appliance is double the resistance of a second appliance, how does the current they draw compare?

3.1.9.19. The amount of power, P, generated by a windmill varies directly with the cube of the wind speed, w. A windmill on Oahu, Hawaii, produces 7300 kilowatts of power when the wind speed is 32 miles per hour.

a Express the power as a function of wind speed. Complete the table and graph your function in a suitable window.

w	10	20	40	80
P				

b How much power would the windmill produce in a light breeze of 15 miles per hour?

c What wind speed is needed to produce $10,000$ kilowatts of power?

d If the wind speed doubles, what happens to the amount of power generated?

Answer.

a $P = \dfrac{1825}{8192}w^3 \approx 0.2228w^3$

w	10	20	40	80
P	**223**	**1782**	**$\mathbf{14,259}$**	**$\mathbf{114,074}$**

b 752 kilowatts

c 33.54 mph

d It is multiplied by 8.

3.1.9.20. A crystal form of pyrite (a compound of iron and sulfur) has the shape of a regular solid with 12 faces. Each face is a regular pentagon. This compound is called pyritohedron, and its mass, M, varies directly with the cube of the length, L, of one edge. If each edge is 1.1 centimeters, then the mass is 51 grams.

a Express the mass of pyritohedron as a function of the length of one edge.

Complete the table and graph your function in a suitable window.

L	0.5	1	2	4
M				

b What is the weight of a chunk of pyritohedron if each edge is 2.2 centimeters?

c How long would each edge be for a 1643-gram piece of pyritohedron?

d If one chunk has double the length of a second chunk, how do their weights compare?

For Problems 21–26,

a Use the values in the table to find the constant of variation, k, and write y as a function of x.

b Fill in the rest of the table with the correct values.

c What happens to y when you double the value of x?

3.1.9.21. y varies directly with x.

x	y
2	
5	1.5
	2.4
12	
	4.5

Answer.

a $y = 0.3x$

b

x	y
2	**0.6**
5	1.5
8	2.4
12	**3.6**
15	4.5

c y doubles.

3.1.9.22. y varies directly with x.

x	y
0.8	
1.5	54
	108
	126
6	

3.1.9.23. y varies directly with the square of x.

x	y
3	
6	24
	54
12	
	150

Answer.

a $y = \dfrac{2}{3}x^2$

b

x	y
3	**6**
6	24
9	
12	**96**
15	150

c y is quadrupled.

3.1.9.24. y varies directly with the cube of x.

x	y
2	120
3	
	1875
6	
	$15,000$

3.1.9.25. y varies inversely with x.

x	y
4	
	15
20	6
30	
	3

Answer.

a $y = \dfrac{120}{x}$

b

4	**30**
8	15
20	6
30	**4**
40	3

c y is halved.

3.1.9.26. y varies inversely with the square of x.

x	y
0.2	
	80
2	
4	1.25
	0.8

For Problems 27-30, decide whether

a y varies directly with x,

b y varies directly with x^2, or

c y does not vary directly with a power of x.

Explain why your choice is correct. If your choice is (a) or (b), find the constant of variation.

3.1.9.27.

x	y
2	2.0
3	4.5
5	12.5
8	32.0

Answer. (b) $y = 0.5x^2$

3.1.9.28.

x	y
2	12
4	28
6	44
9	68

3.1.9.29.

x	y
1.5	3.0
2.4	5.3
5.5	33
8.2	73.8

Answer. (c) $\frac{y}{x^p}$ is not constant for any exponent p.

3.1.9.30.

x	y
1.2	7.20
2.5	31.25
6.4	204.80
12	720.00

For Problems 31-34, decide whether

a y varies inversely with x,

b y varies inversely with x^2, or

c y does not vary inversely with a power of x.

Explain why your choice is correct. If your choice is (a) or (b), find the constant of variation.

3.1.9.31.

x	y
0.5	288
2.0	18
3.0	8
6.0	2

Answer. (b) $y = \frac{72}{x^2}$

3.1.9.32.

x	y
0.5	100.0
2.0	25.0
4.0	12.5
5.0	10.0

3.1.9.33.

x	y
1.0	4.0
1.3	3.7
3.0	2.0
4.0	1.0

Answer. (c) $x^p y$ is not constant for any exponent p.

3.1.9.34.

x	y
0.5	180.00
2.0	11.25
3.0	5.00
5.0	1.80

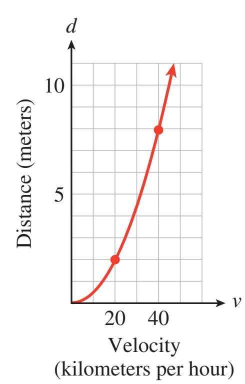
The functions described by a table of data or by a graph in Problems 35–42 are examples of direct or inverse variation.

a Find an algebraic formula for the function, including the constant of variation, k.

b Answer the question in the problem.

3.1.9.35.

The faster a car moves, the more difficult it is to stop. The graph shows the distance, d, required to stop a car as a function of its velocity, v, before the brakes were applied. What distance is needed to stop a car moving at 100 kilometers per hour?

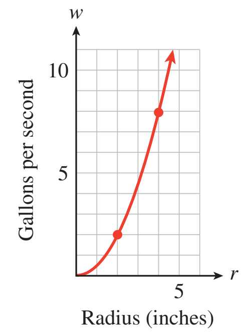


Answer.

a $d = 0.005v^2$ b 50 m

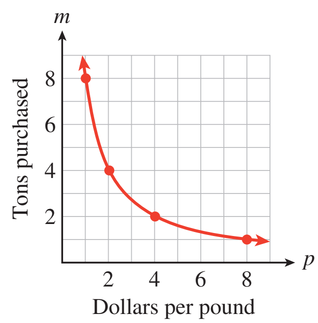
3.1.9.36.

A wide pipe can handle a greater water flow than a narrow pipe. The graph shows the water flow through a pipe, w, as a function of its radius, r. How great is the water flow through a pipe of radius of 10 inches?



3.1.9.37.

If the price of mushrooms goes up, the amount consumers are willing to buy goes down. The graph shows the number of tons of shiitake mushrooms, m, sold in California each week as a function of their price, p. If the price of shiitake mushrooms rises to \$10 per pound, how many tons will be sold?

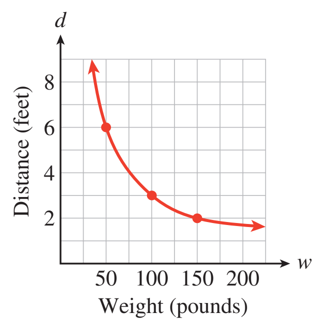


Answer.

a $m = \dfrac{8}{p}$ b 0.8 ton

3.1.9.38.

When an adult plays with a small child on a seesaw, the adult must sit closer to the pivot point to balance the seesaw. The graph shows this distance, d, as a function of the adult's weight, w. How far from the pivot must Kareem sit if he weighs 280 pounds?



3.1.9.39. Ocean temperatures are generally colder at the greater depths. The table shows the temperature of the water as a function of depth. What is the ocean temperature at a depth of 6 kilometers?

Depth (km)	Temperature (°C)
0.5	12
1	6
2	3
3	2

Answer.

a $T = \dfrac{6}{d}$ b 1°C

3.1.9.40. The shorter the length of a vibrating guitar string, the higher the frequency of the vibrations. The fifth string is 65 centimeters long and is tuned to A (with a frequency of 220 vibrations per second). The placement of the fret relative to the bridge changes the effective length of the guitar string. The table shows frequency as a function of effective length. How far from the bridge should the fret be placed for the note C (256 vibrations per second)?

Length (cm)	Frequency
55	260
57.2	250
65	220
71.5	200

3.1.9.41. The strength of a cylindrical rod depends on its diameter. The greater the diameter of the rod, the more weight it can support before collapsing. The table shows the maximum weight supported by a rod as a function of its diameter. How much weight can a 1.2-centimeter rod support before collapsing?

Diameter (cm)	Weight (newtons)
0.5	150
1.0	600
1.5	1350
2.0	2400

Answer.

a $W = 600d^2$

b 864 newtons

3.1.9.42. The maximum height attained by a cannonball depends on the speed at which it was shot. The table shows maximum height as a function of initial speed. What height is attained by a cannonball whose initial upward speed was 100 feet per second?

Speed (ft/sec)	Height (ft)
40	200
50	31.25
60	450
70	612.5

3.1.9.43. The wind resistance, W, experienced by a vehicle on the freeway varies directly with the square of its speed, v.

a If you double your speed, what happens to the wind resistance?

b If you drive one-third as fast, what happens to the wind resistance?

c If you decrease your speed by 10%, what happens to the wind resistance?

Answer.

a Wind resistance quadruples.

b It is one-ninth as great.

c It is decreased by 19% because it is 81% of the original.

3.1.9.44. The weight, w, of a bronze statue varies directly with the cube of its height, h.

a If you double the height of the statue, what happens to its weight?

b If you make the statue one-fourth as tall, what happens to its weight?

c If you increase the height of the statue by 50%, what happens to its weight?

3.1.9.45. The intensity of illumination, I, from a lamp varies inversely with the square of your distance, d, from the lamp.

a If you double your distance from a reading lamp, what happens to the illumination?

b If you triple the distance, what happens to the illumination?

c If you increase the distance by 25%, what happens to the illumination?

Answer.

a It is one-fourth the original illumination.

b It is one-ninth the illumination.

c It is 64% of the illumination.

3.1.9.46. The resistance, R, of an electrical wire varies inversely with the square of its diameter, d.

a If you replace an old wire with a new one whose diameter is half that of the old one, what happens to the resistance?

b If you replace an old wire with a new one whose diameter is two-thirds of the old one, what happens to the resistance?

c If you decrease the diameter of the wire by 30%, what happens to the resistance?

The quoted material in Problems 47–50 is taken from the article "Quantum Black Holes," by Bernard J. Carr and Steven B. Giddings, in the May 2005 issue of *Scientific American*. (See Algebra Skills Refresher A.1.4 to review scientific notation.)

3.1.9.47. "The density to which matter must be squeezed [to create a black hole] scales as the inverse square of the mass. For a hole with the mass of the Sun, the density is about 10^{19} kilograms per cubic meter, higher than that of an atomic nucleus."

a Recall that the density of an object is its mass per unit volume. Given that the mass of the sun is about 2×10^{30} kilograms, write a formula for the density, D, of a black hole as a function of its mass, m.

b "The known laws of physics allow for a matter density up to the so-called Planck value of 10^{97} kilograms per cubic meter." If a black hole with this density could be created, it would be the smallest possible black hole. What would its mass be?

c Assuming that a black hole is spherical, what would be the radius of the smallest possible black hole?

3.1.9.48. "A black hole radiates thermally, like a hot coal, with a temperature inversely proportional to its mass. For a solar-mass black hole, the temperature is around a millionth of a kelvin."

a The solar mass is given in Problem 47. Write a formula for the temperature, T, of a black hole as a function of its mass, m.

b What is the temperature of a black hole of mass 10^{12} kilograms, about the mass of a mountain?

3.1.9.49. "The total time for a black hole to evaporate away is proportional to the cube of its initial mass. For a solar-mass hole, the lifetime is an unobservably long 10^{64} years."

a The solar mass is given in Problem 47. Write a formula for the lifetime, L, of a black hole as a function of its mass, m.

b The present age of the universe is about 10^{10} years. What would be the mass of a black hole as old as the universe?

Answer.

a $L = (1.25 \times 10{-}27)m^3$

b 2×10^{12} kg

3.1.9.50. "String theory ... predicts that space has dimensions beyond the usual three. In three dimensions, the force of as strong." In three dimensions, the forcgravity quadruples as you halve the distance between two objects. But in nine dimensions, gravity would get 256 timese of gravity varies inversely with the square of distance. Write a formula for the force of gravity in nine dimensions.

Use algebra to support your answers to Problems 51–56. Begin with a formula for direct or inverse variation.

3.1.9.51. Suppose y varies directly with x. Show that if you multiply x by any constant c, then y will be multiplied by the same constant.

Answer. $y = kx$ implies that $k(cx) = c(kx) = cy$.

3.1.9.52. Suppose y varies inversely with x. Show that if you multiply x by any constant c, then y will be divided by the same constant.

3.1.9.53. Explain why the ratio $\dfrac{y}{x^2}$ is a constant when y varies directly with x^2.

Answer. If $y = kx^2$, then dividing both sides of the equation by x^2 gives $\dfrac{y}{x^2} = k$.

3.1.9.54. Explain why the product yx^2 is a constant when y varies inversely with x^2.

3.1.9.55. If x varies directly with y and y varies directly with z, does x vary directly with z?

Answer. Yes

3.1.9.56. If x varies inversely with y and y varies inversely with z, does x vary invesely with z?

3.2 Integer Exponents

3.2.1 Negative Exponents

Checkpoint 3.2.3 Write each expression without using negative exponents.

a 5^{-4}

b $5x^{-4}$

Answer.

a $\dfrac{1}{5^4}$

b $\dfrac{5}{x^4}$

Checkpoint 3.2.5 Solve the equation $0.2x^{-3} = 1.5$

Hint. Rewrite without a negative exponent.
Clear the fraction.
Isolate the variable.

Answer. $x = \sqrt[3]{\dfrac{2}{15}} \approx 0.51$

3.2.2 Power Functions

Checkpoint 3.2.7 Write each function as a power function in the form $y = kx^p$.

a $f(x) = \dfrac{12}{x^2}$

b $g(x) = \dfrac{1}{4x}$

c $h(x) = \dfrac{2}{5x^6}$

Answer.

a $f(x) = 12x^{-2}$

b $g(x) = \dfrac{1}{4}x^{-1}$

c $h(x) = \dfrac{2}{5}x^{-6}$

Checkpoint 3.2.9 Cell phone towers typically transmit signals at 10 watts of power. The signal strength varies inversely with the square of distance from the tower, and 1 kilometer away the signal strength is 0.8 picowatt. (A picowatt is 10^{-12} watt.) Cell phones can receive a signal as small as 0.01 picowatt. How far can you be from the nearest tower and still hope to have cell phone reception?

Answer. About 9 km

3.2.3 Working with Negative Exponents

Checkpoint 3.2.11 Simplify $\left(\dfrac{2}{x^2}\right)^{-4}$

Answer. $\dfrac{x^8}{16}$

Checkpoint 3.2.13 Write each expression without using negative exponents.

a $\left(\dfrac{3}{b^4}\right)^{-2}$

b $\dfrac{12}{x^{-6}}$

Answer.

a $\dfrac{b^8}{9}$

b $12x^6$

3.2.4 Laws of Exponents

Checkpoint 3.2.15 Simplify by applying the laws of exponents.

a $\left(2a^{-4}\right)\left(-4a^2\right)$

b $\dfrac{\left(r^2\right)^{-3}}{3r^{-4}}$

Answer.

a $\dfrac{-8}{a^2}$

b $\dfrac{1}{3r^2}$

3.2.6 Homework 3.2

3.2.6.1. Make a table showing powers of 3 from 3^{-5} to 3^5. Illustrate why defining $3^0 = 1$ makes sense.

Answer.

n	-5	-4	-3	-2	-1	0	1	2	3	4	5
3^n	$\frac{1}{243}$	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243

Each time n increases by 1, we multiply the power in the bottom row by 3.

3.2.6.2. Make a table showing powers of 5 from 5^{-4} to 5^4. Illustrate why defining $5^0 = 1$ makes sense.

For Problems 3–6, compute each power.

3.2.6.3.

a 2^3

b $(-2)^3$

c 2^{-3}

d $(-2)^{-3}$

Answer.

a 8

b -8

c $\dfrac{1}{8}$

d $\dfrac{-1}{8}$

3.2.6.4.

a 4^2

b $(-4)^2$

c 4^{-2}

d $(-4)^{-2}$

3.2.6.5.

a $\left(\dfrac{1}{2}\right)^3$

b $\left(-\dfrac{1}{2}\right)^3$

c $\left(\dfrac{1}{2}\right)^{-3}$

d $\left(-\dfrac{1}{2}\right)^{-3}$

Answer.

a $\dfrac{1}{8}$

b $\dfrac{-1}{8}$

c 8

d -8

3.2.6.6.

a $\left(\dfrac{1}{4}\right)^2$

b $\left(-\dfrac{1}{4}\right)^2$

c $\left(\dfrac{1}{4}\right)^{-2}$

d $\left(-\dfrac{1}{4}\right)^{-2}$

For Problems 7–12, write without negative exponents and simplify.

3.2.6.7.

a 2^{-1}

b $(-5)^{-2}$

c $\left(\dfrac{1}{3}\right)^{-3}$

d $\dfrac{1}{(-2)^{-4}}$

Answer.

a $\dfrac{1}{2^1} = \dfrac{1}{2}$

b $\dfrac{1}{(-5)^2} = \dfrac{1}{25}$

c $3^3 = 27$

d $(-2)^4 = 16$

3.2.6.8.

a 3^{-2}

b $(-2)^{-3}$

c $\left(\dfrac{3}{5}\right)^{-2}$

d $\dfrac{1}{(-3)^{-3}}$

3.2.6.9.

a $\frac{5}{4^{-3}}$ b $(2q)^{-5}$ c $-4x^{-2}$ d $\frac{8}{b^{-3}}$

Answer.

a $5 \cdot 4^3 = 320$

b $\frac{1}{(2q)^5} = \frac{1}{32q^5}$

c $\frac{-4}{x^2}$

d $8b^3$

3.2.6.10.

a $\frac{3}{2^{-6}}$ b $(4k)^{-3}$ c $-7x^{-4}$ d $\frac{5}{a^{-5}}$

3.2.6.11.

a $(m-n)^{-2}$

b $y^{-2} + y^{-3}$

c $2pq^{-4}$

d $\frac{-5y^{-2}}{x^{-5}}$

Answer.

a $\frac{1}{(m-n)^2}$

b $\frac{1}{y^2} + \frac{1}{y^3}$

c $\frac{2p}{q^4}$

d $\frac{-5x^5}{y^2}$

3.2.6.12.

a $(p+q)^{-3}$

b $z^{-1} - z^{-2}$

c $8m^{-2}n^2$

d $\frac{-6y^{-3}}{x^{-3}}$

Use your calculator to fill in the tables in Problems 13 and 14. Round your answers to two decimal places.

3.2.6.13. $f(x) = x^{-2}$

a

x	1	2	4	8	16
$f(x)$					

b What happens to the values of $f(x)$ as the values of x increase? Explain why.

c

x	1	0.5	0.25	0.125	0.0625
$f(x)$					

d What happens to the values of $f(x)$ as the values of x decrease toward 0? Explain why.

Answer.

(a)

x	1	2	4	8	16
x^{-2}	1	0.25	0.06	0.02	0.00

(b) The values of $f(x)$ decrease, because x^{-2} is the reciprocal of x^2.

(c)

x	1	0.5	0.25	0.125	0.0625
x^{-2}	1	4	16	64	256

(d) The values of $f(x)$ increase toward infinity, because x^{-2} is the reciprocal of x^2.

3.2.6.14. $g(x) = x^{-3}$

a

x	1	2	4.5	6.2	9.3
$g(x)$					

b What happens to the values of $g(x)$ as the values of x increase? Explain why.

c

x	1.5	0.6	0.1	0.03	0.002
$f(x)$					

d What happens to the values of $g(x)$ as the values of x decrease toward 0? Explain why.

3.2.6.15.

(a) Use your calculator to graph each of the following functions on the window

$$\text{Xmin} = -5 \qquad \text{Xmax} = 5$$
$$\text{Ymin} = -2 \qquad \text{Ymax} = 10$$

i. $f(x) = x^2$

ii. $f(x) = x^{-2}$

iii. $f(x) = \dfrac{1}{x^2}$

iv. $f(x) = \left(\dfrac{1}{x}\right)^2$

(b) Which functions have the same graph? Explain your results.

Answer. b. (ii), (iii), and (iv) have the same graph, because they represent the same function.

3.2.6.16.

(a) Use your calculator to graph each of the following functions on the window

$$\text{Xmin} = -3 \qquad \text{Xmax} = 5$$
$$\text{Ymin} = -5 \qquad \text{Ymax} = 5$$

i. $f(x) = x^3$

ii. $f(x) = x^{-3}$

iii. $f(x) = \dfrac{1}{x^3}$

iv. $f(x) = \left(\dfrac{1}{x}\right)^3$

(b) Which functions have the same graph? Explain your results.

For Problems 17–18, write each expression as a power function using negative exponents.

3.2.6.17.

(a) $F(r) = \dfrac{3}{r^4}$ (b) $G(w) = \dfrac{2}{5w^3}$ (c) $H(z) = \dfrac{1}{(3z)^2}$

Answer.

(a) $F(r) = 3r^{-4}$ (b) $G(w) = \dfrac{2}{5}w^{-3}$ (c) $H(z) = \dfrac{1}{9}z^{-2}$

3.2.6.18.

(a) $h(s) = \dfrac{9}{s^3}$ (b) $f(v) = \dfrac{3}{8v^6}$ (c) $g(t) = \dfrac{1}{(5t)^4}$

For Problems 19–24, olve.

3.2.6.19. $6x^{-2} = 3.84$

Answer. $x = -1.25$ or $x = 1.25$

3.2.6.20. $0.8w^{-2} = 1.25$

3.2.6.21. $12 + 0.04t^{-3} = 175.84$

Answer. $t = \dfrac{1}{16}$

3.2.6.22. $854 - 48z^{-3} = 104$

3.2.6.23. $100 - 0.15v^{-4} = 6.25$

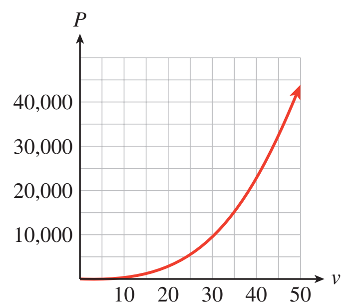
Answer. $v = \dfrac{1}{5}$ or $v = \dfrac{-1}{5}$

3.2.6.24.
$8100p^{-4} - 250 = 3656.25$

3.2.6.25. When an automobile accelerates, the power, P, needed to overcome air resistance varies directly with a power of the speed, v.

(a) Use the data and the graph to find the scaling exponent and the constant of variation. Then write a formula for P as a power function of v.

v (mph)	P (watts)
10	355
20	2840
30	9585
40	$22,720$



(b) Find the speed that requires $50,000$ watts of power.

(c) If you increase your speed by 50%, by what factor does the power requirement increase?

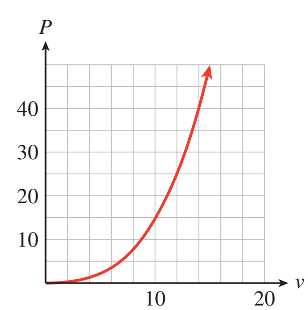
Answer.

(a) $P = 0.355v^3$ (b) $v \approx 52.03$ mph (c) 3.375

3.2.6.26. The power, P, generated by a windmill varies directly with a power of wind velocity, v.

(a) Use the data and the graph to find the scaling exponent and the constant of variation. Then write a formula for P as a power function of v.

v (mph)	P (watts)
10	15
20	120
30	405
40	960



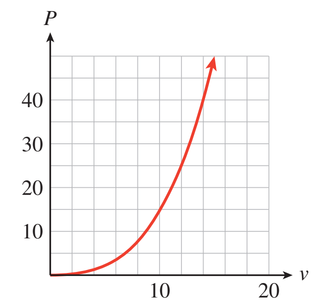
(b) Find the wind velocity needed to generate 500 watts of power.

(c) If the wind speed drops by half, what happens to the power generated?

3.2.6.27. The "Rule of 70" is used to estimate how long it takes an investment to double in value when interest is compounded annually. The doubling time, D, is inversely proportional to the interest rate, i. (Note that i is expressed as a percent, not as a decimal fraction. For example, if the interest rate is 8%, then $i = 8$.)

(a) Use the data and the graph to find the constant of proportionality and write D as a power function of i.

i (mph)	D (watts)
4	17.5
6	11.67
8	8.75
10	7



(b) If the interest rate increases from 5% to 6%, how will the doubling time change?

Answer.

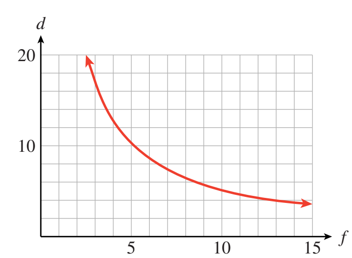
(a) $D = \dfrac{70}{i}$

(b) It decreases by about 2.3 years.

3.2.6.28. The f-stop setting on a camera regulates the size of the aperture and thus the amount of light entering the camera. The f-stop f is inversely proportional to the diameter, d, of the aperture.

(a) Use the data and the graph to find the constant of proportionality and write d as a power function of f. Values of d have been rounded to one decimal place.

f	d
2.8	17.9
4	12.5
5.6	8.9
8	6.3
11	4.5



(b) Why are the f-stop settings labeled with the values given in the table?

Hint. As you stop down the aperture from one f-value to the next, by what factor does d increase?

3.2.6.29. The Stefan-Boltzmann law relates the total amount of radiation emitted by a star to its temperature, T, in kelvins, by the following formula:

$$sT^4 = \frac{L}{4\pi R^2}$$

where R is the radius of the star, L is its luminosity, and $s = 5.7\times10^{-8}$ watt/m^2 is a constant governing radiation. (See Algebra Skills Refresher A.1.4 to review scientific notation.)

a Write a formula for luminosity as a power function of temperature for a fixed radius.

b The radius of the Sun is $R = 9.96 \times 10^8$ meters, and its luminosity is $L = 3.9 \times 10^{26}$ watts. Calculate the temperature of the Sun.

Answer.

(a) $L = \left(4\pi sR^2\right) T^4 \approx 7.2 \times 10^{-7} R^2T^4$

(b) 4840 K

3.2.6.30. Poiseuille's law for the flow of liquid through a tube can be used to describe blood flow through an artery. The rate of flow, F, in liters per minute is proportional to the fourth power of the radius, r, divided by the length, L, of the artery.

a Write a formula for the rate of flow as a power function of radius.

b If the radius and length of the artery are measured in centimeters, then the constant of variation, $k = 7.8 \times 10^5$, is determined by blood pressure and viscosity. If a certain artery is 20 centimeters long, what should its radius be in order to allow a blood flow of 5 liters per minute?

3.2.6.31. Airplanes use radar to detect the distances to other objects. A radar unit transmits a pulse of energy, which bounces off a distant object, and the echo of the pulse returns to the sender. The power, P, of the returning echo is inversely proportional to the fourth power of the distance, d, to the object. A radar operator receives an echo of 5×10^{-10} watts from an aircraft 2 nautical miles away.

a Express the power of the echo received in picowatts. (1 picowatt $= 10^{-12}$ watts.)

b Write a function that expresses P in terms of d using negative exponents. Use picowatts for the units of power.

c Complete the table of values for the power of the echo received from objects at various distances.

d (nautical miles)	4	5	7	10
P (picowatts)				

d Radar unit scan typically detect signals as low as 10^{-13} watts. How far away is an aircraft whose echo is 10^{-13} watts?

e Sketch a graph of P as a function of d. Use units of picowatts on the vertical axis

Hint. Convert 10^{-13} watts to picowatts.

Answer.

(a) 500 picowatts

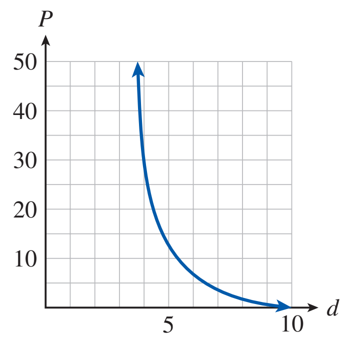
(b) $P = 8000d^{-4}$

(c)

d (nautical miles)	P (picowatts)
4	31.3
5	12.8
7	3.3
10	0.8

(d) 16.8 nautical miles

(e)



3.2.6.32. The lifetime of a star is roughly inversely proportional to the cube of its mass. Our Sun, which has a mass of one solar mass, will last for approximately 10 billion years.

(a) Write a power function for the lifetime, L, of a star in terms of its mass, m.

(b) Sketch a graph of the function using units of solar mass on the horizontal axis.

(c) How long will a star that is 10 times as massive as the Sun last?

(d) One solar mass is about 2×10^{30} kilograms. Rewrite your formula for L with the units of mass in kilograms.

(e) How long will a star that is half as massive as the Sun last?

3.2.6.33. The amount of force or thrust generated by the propeller of a ship is a function of two variables: the diameter of the propeller and its speed, in rotations per minute. The thrust, T, in pounds, is proportional to the square of the speed, r, and the fourth power of the diameter, d, in feet.

(a) Write a formula for the thrust in terms of the speed if the diameter of the propeller is 2 feet.

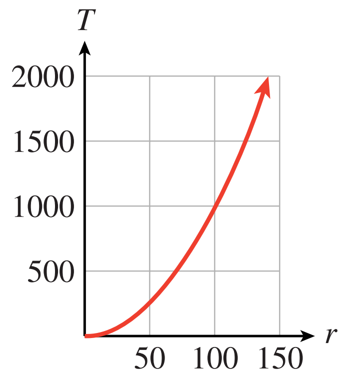
(b) A propeller of diameter 2 feet generates a thrust of 1000 pounds at 100 rotations per minute. Find the constant of variation in the formula for thrust.

(c) Sketch a graph of the thrust as a function of the propeller speed for a propellor of diameter 4 feet. If the speed of the propeller is doubled, by what factor does the thrust increase?

Answer.

(a) $T = 16kr^2$

(b) $T = 0.1r^2$

(c) 

3.2.6.34. Refer to Problem 33.

(a) Write a formula for the thrust, T, in terms of the diameter of the propeller if its speed is 100 rotations per minute.

(b) A propeller of diameter 4 feet generates a thrust of $32,000$ pounds at 100 rotations per minute. Find the constant of variation in the formula for thrust.

(c) Sketch a graph of the thrust as a function of the diameter of the propeller at a speed of 100 rotations per minute. If the diameter of the propeller is doubled, by what factor does the thrust increase?

For Problems 35–40, use the laws of exponents to simplify and write without negative exponents.

3.2.6.35.

(a) $a^{-3} \cdot a^8$

(b) $5^{-4} \cdot 5^{-3}$

(c) $\dfrac{p^{-7}}{p^{-4}}$

(d) $\left(7^{-2}\right)^5$

Answer.

(a) a^5

(b) $\dfrac{1}{5^7}$

(c) $\dfrac{1}{p^3}$

(d) $\dfrac{1}{7^{10}}$

3.2.6.36.

(a) $b^2 \cdot b^{-6}$

(b) $4^{-2} \cdot 4^{-6}$

(c) $\dfrac{w^{-9}}{w^2}$

(d) $\left(9^{-4}\right)^3$

3.2.6.37.

(a) $\left(4x^{-5}\right)\left(5x^2\right)$

(b) $\dfrac{3u^{-3}}{9u^9}$

(c) $\dfrac{5^6t^0}{5^{-2}t^{-1}}$

Answer.

(a) $\dfrac{20}{x^3}$ (b) $\dfrac{1}{3u^{12}}$ (c) $5^8 t$

3.2.6.38.

(a) $\left(3y^{-8}\right)\left(2y^4\right)$ (b) $\dfrac{4c^{-4}}{8c^{-8}}$ (c) $\dfrac{3^{10}s^{-1}}{3^{-5}s^0}$

3.2.6.39.

(a) $\left(3x^{-2}y^3\right)^{-2}$ (b) $\left(\dfrac{6a^{-3}}{b^2}\right)^{-2}$ (c) $\dfrac{5h^{-3}(h^4)^{-2}}{6h^{-5}}$

Answer.

(a) $\dfrac{x^4}{9y^6}$ (b) $\dfrac{a^6b^4}{36}$ (c) $\dfrac{5}{6h^6}$

3.2.6.40.

(a) $\left(2x^3y^{-4}\right)^{-3}$ (b) $\left(\dfrac{a^4}{4b^{-5}}\right)^{-3}$ (c) $\dfrac{4v^{-5}(v^{-2})^{-4}}{3v^{-8}}$

For Problems 41–44, write each expression as a sum of terms of the form kx^p.

3.2.6.41.

(a) $\dfrac{x}{3}+\dfrac{3}{x}$ (b) $\dfrac{x-6x^2}{4x^3}$

Answer.

(a) $\dfrac{1}{3}x+3x^{-1}$ (b) $\dfrac{1}{4}x^{-2}-\dfrac{3}{2}x^{-1}$

3.2.6.42.

(a) $\dfrac{2}{x^2}-\dfrac{x^2}{2}$ (b) $\dfrac{5x+1}{(3x)^2}$

3.2.6.43.

(a) $\dfrac{2}{x^4}\left(\dfrac{x^2}{4}+\dfrac{x}{2}-\dfrac{1}{4}\right)$ (b) $\dfrac{x^2}{3}\left(\dfrac{2}{x^4}-\dfrac{1}{3x^2}+\dfrac{1}{2}\right)$

Answer.

(a) $\dfrac{1}{2}x^{-2}+x^{-3}-\dfrac{1}{2}x^{-4}$ (b) $\dfrac{2}{3}x^{-2}-\dfrac{1}{9}+\dfrac{1}{6}x^2$

3.2.6.44.

(a) $\dfrac{9}{x^3}\left(\dfrac{x^3}{3}-1-\dfrac{1}{x^3}\right)$ (b) $\dfrac{x^2}{2}\left(\dfrac{3}{x}-\dfrac{5}{x^3}+\dfrac{7}{x^5}\right)$

For Problems 45–50, use the distributive law to write each product as a sum of power functions.

3.2.6.45. $x^{-1}(x^2-3x+2)$

Answer. $x-3+2x^{-1}$

3.2.6.46. $3x^{-2}(2x^4+x^2-4)$

3.2.6.47. $-3t^{-2}(t^2-2+4t^{-2})$

Answer. $-3+6t^{-2}+12t^{-4}$

3.2.6.48. $-t^{-3}(3t^2-1-t^{-2})$

3.2.6.49. $2u^{-3}(-2u^3-u^2+3u)$

Answer. $-4-2u^{-1}+6u^{-2}$

3.2.6.50. $2u^{-1}(-1-u-2u^2)$

For Problems 51–54, factor as indicated, writing the second factor with positive exponents only.

3.2.6.51.
$4x^2 + 16x^{-2} = 4x^{-2}(\ \ ?\ \ )$

Answer. $4x^{-2}(x^4 + 4)$

3.2.6.52.
$20y - 15y^{-1} = 5y^{-1}(\ \ ?\ \ )$

3.2.6.53.
$3a^{-3} - 3a + a^3 = a^{-3}(\ \ ?\ \ )$

Answer. $a^{-3}(3 - 3a^4 + a^6)$

3.2.6.54.
$2 - 4q^{-2} - 8q^{-4} = 2q^{-4}(\ \ ?\ \ )$

3.2.6.55.

(a) Is it true that $(x + y)^{-2} = x^{-2} + y^{-2}$? Explain why or why not.

(b) Give a numerical example to support your answer.

Answer.

(a) No, because $\frac{1}{(x+y)^2}$ is not $\frac{1}{x^2} + \frac{1}{y^2}$.

(b) Let $x = 1$, $y = 2$, then $(x+y)^{-2} = (1+2)^{-2} = 3^{-2} = \frac{1}{9}$, but $x^{-2}+y^{-2} = 1^{-2} + 2^{-2} = 1 + \frac{1}{4} = \frac{5}{4}$

3.2.6.56.

(a) Is it true that $(a - b)^{-1} = a^{-1} - b^{-1}$? Explain why or why not.

(b) Give a numerical example to support your answer.

3.2.6.57.

(a) Show that $x + x^{-1} = \dfrac{x^2 + 1}{x}$.

(b) Show that $x^3 + x^{-3} = \dfrac{x^6 + 1}{x^3}$.

(c) Write $x^n + x^{-n}$ as an algebraic fraction. Justify your answer.

Answer.

(a) $x + x^{-1} = x + \dfrac{1}{x} = \dfrac{x^2}{x} + \dfrac{1}{x} = \dfrac{x^2 + 1}{x}$

(b) $x^3 + x^{-3} = x^3 + \dfrac{1}{x^3} = \dfrac{x^6}{x^3} + \dfrac{1}{x^3} = \dfrac{x^6 + 1}{x^3}$

(c) $x^n + x^{-n} = x^n + \dfrac{1}{x^n} = \dfrac{x^{2n}}{x^n} + \dfrac{1}{x^n} = \dfrac{x^{2n} + 1}{x^2}$

3.2.6.58.

(a) Show that $x^{-m} + x^{-n} = \dfrac{x^n + x^m}{x^{n+m}}$.

(b) If $m < n$, show that $x^{-m} + x^{-n} = \dfrac{x^{n-m} + 1}{x^n}$.

By rewriting the expressions in Problems 59–62 as fractions, verify that the laws of exponents hold for negative exponents . Show where you apply the corresponding law for positive exponents. Here is the fourth law as an example:

$$\begin{aligned}(ab)^{-3} &= \frac{1}{(ab)^3} = \frac{1}{a^3b^3} && \text{By the fourth law of exponents.}\\ &= \frac{1}{a^3} \cdot \frac{1}{b^3} = a^{-3}b^{-3}\end{aligned}$$

3.2.6.59. $a^{-2}a^{-3} = a^{-5}$

Answer.

$$\begin{aligned} a^{-2}a^{-3} &= \frac{1}{a^2} \cdot \frac{1}{a^3} = \frac{1}{a^2 \cdot a^3} \\ &= \frac{1}{a^{2+3}} \\ &= \frac{1}{a^5} = a^{-5} \end{aligned}$$

3.2.6.60. $\dfrac{a^{-6}}{a^{-2}} = a^{-4}$

By the first law of exponents.

3.2.6.61. $\dfrac{a^{-2}}{a^{-6}} = a^4$

Answer.

$$\begin{aligned} \frac{a^{-2}}{a^{-6}} &= a^{-2} \div a^{-6} = \frac{1}{a^2} \div \frac{1}{a^6} \\ &= \frac{1}{a^2} \cdot \frac{a^6}{1} = \frac{a^6}{a^2} \\ &= a^{6-2} \\ &= a^4 \end{aligned}$$

3.2.6.62. $\left(a^{-2}\right)^{-3} = a^6$

By the second law of exponents.

3.3 Roots and Radicals

3.3.1 nth Roots

Checkpoint 3.3.2 Evaluate each radical.

a $\sqrt[4]{16}$ b $\sqrt[5]{243}$

Answer.

a 2 b 3

3.3.2 Exponential Notation for Radicals

Checkpoint 3.3.4 Evaluate each power.

a $4^{1/2}$ b 4^{-2} c $4^{-1/2}$ d $\left(\dfrac{1}{4}\right)^{1/2}$

Answer.

a 2 b $\dfrac{1}{16}$ c $\dfrac{1}{2}$ d $\dfrac{1}{2}$

Checkpoint 3.3.7 Write each power with radical notation, and then evaluate.

a $32^{1/5}$ b $625^{1/4}$

Answer.

a $\sqrt[5]{32} = 2$ b $\sqrt[4]{625} = 5$

Checkpoint 3.3.9 Write each power with radical notation, and then evaluate.

a $100,000^{0.2}$ b $81^{0.25}$

Answer.

a $\sqrt[5]{100,000} = 10$

b $\sqrt[4]{81} = 3$

3.3.4 Working with Fractional Exponents

Checkpoint 3.3.12 Convert each radical to exponential notation.

a $\dfrac{1}{\sqrt[5]{ab}}$

b $3\sqrt[6]{w}$

Answer.

a $(ab)^{-1/5}$

b $3w^{1/6}$

Checkpoint 3.3.15

a Convert $\dfrac{3}{\sqrt[4]{2x}}$ to exponential notation.

b Convert $-5b^{0.125}$ to radical notation.

Answer.

a $3(2x)^{-1/4}$

b $-5\sqrt[8]{b}$

3.3.5 Using Fractional Exponents to Solve Equations

Checkpoint 3.3.17 A spherical fish tank in the lobby of the Atlantis Hotel holds about 905 cubic feet of water. What is the radius of the fish tank?

Answer. About 6 feet

3.3.6 Power Functions

Checkpoint 3.3.19

a Complete the table of values for the power function $f(x) = x^{-1/2}$.

x	0.1	0.25	0.5	1	2	4	8	10	20	200
$f(x)$										

b Sketch the graph of $y = f(x)$.

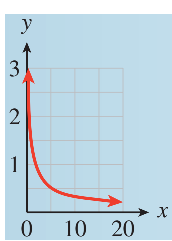
c Write the formula for $f(x)$ with a decimal exponent, and with radical notation.

Answer.

a

x	0.1	0.25	0.5	1	2	4	8	10	20	200
$f(x)$	3.2	2	1.4	1	0.71	0.5	0.35	0.32	0.22	0.1

b



c $f(x) = x^{-0.5}$, $f(x) = \dfrac{1}{\sqrt{x}}$

3.3.7 Solving Radical Equations

Checkpoint 3.3.22 In Example 3.3.18, we found the heart-rate function, $H(m) = 202.5m^{-1/4}$. What would be the mass of an animal whose heart rate is 120 beats per minute?

Answer. 81 kg

3.3.8 A Note on Roots of Negative Numbers

Checkpoint 3.3.24 Evaluate each power, if possible.

a $-81^{1/4}$

b $(-81)^{1/4}$

c $-64^{1/3}$

d $(-64)^{1/3}$

Answer.

a -3 b undefined c -4 d -4

3.3.10 Homework 3.3

Find the indicated root without using a calculator; then check your answers.

3.3.10.1.

(a) $\sqrt{121}$ (b) $\sqrt[3]{27}$ (c) $\sqrt[4]{625}$

Answer.

(a) 11 (b) 3 (c) 5

3.3.10.2.

(a) $\sqrt{169}$ (b) $\sqrt[3]{64}$ (c) $\sqrt[4]{81}$

3.3.10.3.

(a) $\sqrt[5]{32}$ (b) $\sqrt[4]{16}$ (c) $\sqrt[3]{729}$

Answer.

(a) 2 (b) 2 (c) 9

3.3.10.4.

(a) $\sqrt[5]{100,000}$ (b) $\sqrt[4]{1296}$ (c) $\sqrt[3]{343}$

Find the indicated power without using a calculator; then check your answers.

3.3.10.5.

(a) $9^{1/2}$ (b) $81^{1/4}$ (c) $64^{1/6}$

Answer.

(a) 3 (b) 3 (c) 2

3.3.10.6.

(a) $25^{1/2}$ (b) $16^{1/4}$ (c) $27^{1/3}$

3.3.10.7.

(a) $32^{0.2}$ (b) $8^{-1/3}$ (c) $64^{-0.5}$

Answer.

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{8}$

3.3.10.8.

(a) $625^{0.25}$ (b) $243^{-1/5}$ (c) $49^{-0.5}$

Write each expression in radical form.

3.3.10.9.

(a) $3^{1/2}$ (b) $4x^{1/3}$ (c) $(4x)^{0.2}$

Answer.

(a) $\sqrt{3}$ (b) $4\sqrt[3]{x}$ (c) $\sqrt[5]{4x}$

3.3.10.10.

(a) $7^{1/2}$ (b) $3x^{1/4}$ (c) $(3x)^{0.25}$

3.3.10.11.

(a) $6^{-1/3}$ (b) $3(xy)^{-0.125}$ (c) $(x-2)^{1/4}$

Answer.

(a) $\dfrac{1}{\sqrt[3]{6}}$ (b) $\dfrac{3}{\sqrt[8]{xy}}$ (c) $\sqrt[4]{x-2}$

3.3.10.12.

(a) $8^{-1/4}$ (b) $y(5x)^{-0.5}$ (c) $(y+2)^{1/3}$

Write each expression in exponential form.

3.3.10.13.

(a) $\sqrt{7}$ (b) $\sqrt[3]{2x}$ (c) $2\sqrt[5]{z}$

Answer.

(a) $7^{1/2}$ (b) $(2x)^{1/3}$ (c) $2z^{1/5}$

3.3.10.14.

(a) $\sqrt{5}$ (b) $\sqrt[3]{4y}$ (c) $5\sqrt[3]{x}$

3.3.10.15.

(a) $\dfrac{-3}{\sqrt[4]{6}}$ (b) $\sqrt[4]{x-3y}$ (c) $\dfrac{-1}{\sqrt[5]{1+3b}}$

Answer.

(a) $-3\cdot 6^{-1/4}$ (b) $(x-3y)^{1/4}$ (c) $-(1+3b)^{-1/5}$

3.3.10.16.

(a) $\dfrac{2}{\sqrt[5]{3}}$ (b) $\sqrt[3]{y+2x}$ (c) $\dfrac{-1}{\sqrt[4]{3a-2b}}$

Simplify.

3.3.10.17.

(a) $\left(\sqrt[3]{125}\right)^3$ (b) $\left(\sqrt[4]{2}\right)^4$ (c) $\left(3\sqrt{7}\right)^2$ (d) $\left(-x^2\sqrt[3]{2x}\right)^3$

Answer.

(a) 125 (b) 2 (c) 63 (d) $-2x^7$

3.3.10.18.
(a) $\left(\sqrt[4]{16}\right)^4$ (b) $\left(\sqrt[3]{6}\right)^3$ (c) $\left(2\sqrt[3]{12}\right)^3$ (d) $\left(-a^3\sqrt[4]{a^2}\right)^4$

Use a calculator to approximate each irrational number to the nearest thousandth.

3.3.10.19.
(a) $2^{1/2}$
(b) $\sqrt[3]{75}$
(c) $\sqrt[4]{1.6}$
(d) $365^{-1/3}$
(e) $0.006^{-0.2}$

Answer.

(a) 1.414 (b) 4.217 (c) 1.125 (d) 0.140 (e) 2.782

3.3.10.20.
(a) $3^{1/2}$
(b) $\sqrt[4]{60}$
(c) $\sqrt[3]{1.4}$
(d) $1058^{-1/5}$
(e) $1.05^{-0.1}$

Write each expression as a power function.

3.3.10.21.
(a) $g(x) = 3.7\sqrt[3]{x}$ (b) $H(x) = \sqrt[4]{85x}$ (c) $F(t) = \dfrac{25}{\sqrt[5]{t}}$

Answer.

(a) $g(x) = 3.7x^{1/3}$ (b) $H(x) = 85^{1/4}x^{1/4}$ (c) $F(t) = 25t^{-1/5}$

3.3.10.22.
(a) $h(v) = 12.7\sqrt{v}$ (b) $F(p) = \sqrt[3]{2.9p}$ (c) $G(w) = \dfrac{5}{8\sqrt[8]{w}}$

Solve.

3.3.10.23. $6.5x^{1/3} + 3.8 = 33.05$

Answer. $x = 91.125$

3.3.10.24. $9.8 - 76x^{1/4} + 15 = 9.6$

3.3.10.25. $4(x+2)^{1/5} = 12$

Answer. $x = 241$

3.3.10.26. $-9(x-3)^{1/5} = 18$

3.3.10.27. $(2x-3)^{-1/4} = \dfrac{1}{2}$

Answer. $x = \dfrac{19}{2}$

3.3.10.28. $(5x+2)^{-1/3} = \dfrac{1}{4}$

3.3.10.29. $\sqrt[3]{x^2-3} = 3$

Answer. $x = \pm\sqrt{30}$

3.3.10.30. $\sqrt[4]{x^3-7} = 2$

Solve each formula for the indicated variable.

3.3.10.31. $T = 2\pi\sqrt{\dfrac{L}{g}}$ for L.
Also solve for g.

Answer. $L = \dfrac{gT^2}{4\pi^2}$

3.3.10.32. $T = 2\pi\sqrt{\dfrac{m}{k}}$ for m

3.3.10.33. $r = \sqrt{t^2-s^2}$ for s.
Also solve for t.

Answer. $s = \pm\sqrt{t^2-r^2}$

3.3.10.34. $c = \sqrt{a^2-b^2}$ for b

3.3.10.35. $r = \sqrt[3]{\dfrac{3V}{4\pi}}$ for V

Answer. $v = \dfrac{4}{3}\pi r^3$

3.3.10.36. $d = \sqrt[3]{\dfrac{16Mr^2}{m}}$ for M

3.3.10.37. $R = \sqrt[4]{\dfrac{8Lvf}{\pi p}}$ for p

Answer. $p = \dfrac{8Lvf}{\pi R^4}$

3.3.10.36. $T = \sqrt[4]{\dfrac{E}{SA}}$ for A

3.3.10.39. The period of a pendulum is the time it takes for the pendulum to complete one entire swing, from left to right and back again. The greater the length, L, of the pendulum, the longer its period, T. In fact, if L is measured in feet, then the period is given in seconds by

$$T = 2\pi\sqrt{\frac{L}{32}}$$

(a) Write the formula for T as a power function in the form $f(x) = kx^p$.

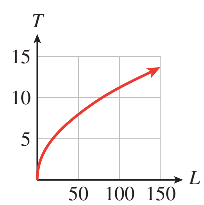
(b) Suppose you are standing in the Convention Center in Portland, Oregon, and you time the period of its Foucault pendulum (the longest in the world). Its period is approximately 10.54 seconds. How long is the pendulum?

(c) Choose a reasonable domain for the function $T = f(L)$ and graph the function.

Answer.

(a) $T = \dfrac{2\pi}{\sqrt{32}}L^{1/2}$

(b) 90 feet

(c) 

3.3.10.40. If you are flying in an airplane at an altitude of h miles, on a clear day you can see a distance of d miles to the horizon, where

$$d = \sqrt{7920h}.$$

(a) Write the formula for d as a power function in the form $f(x) = kx^p$.

(b) Choose a reasonable domain for the function $d = f(h)$ and graph the function.

(c) At what altitude will you be able to see for a distance of 100 miles? How high is that in feet?

3.3.10.41. If you walk in the normal way, your maximum speed, v, in meters per second, is limited by the length of your legs, r, according to the formula

$$v = \sqrt{gr}$$

where the constant g is approximately 10 meters per second squared. (Source: Alexander, 1992)

(a) A typical adult man has legs about 0.9 meter long. How fast can he walk?

(b) A typical four-year-old has legs 0.5 meter long. How fast can she walk?

(c) Graph maximum walking speed as a function of leg length.

(d) Race-walkers can walk as fast as 4.4 meters per second by rotating their hips so that the effective length of their legs is increased. What is that effective length?

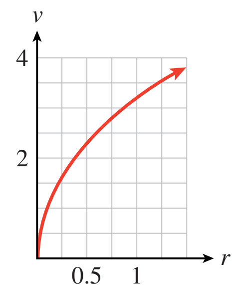
(e) On the Moon the value of g is 1.6 meters per second squared. How fast can a typical adult man walk on the Moon?

Answer.

(a) 3 meters per second

(b) $b \approx 2.2$ meters per second

(c)



(d) 1.9 meters

(e) 1.2meters per second

3.3.10.42. When a ship moves through the water, it creates waves that impede its own progress. Because of this resistance, there is an upper limit to the speed at which a ship can travel, given, in knots, by

$$v_{\text{max}} = 1.3\sqrt{L}$$

where L is the length of the vessel, in feet. (Source: Gilner, 1972)

(a) Graph maximum speed as a function of vessel length.

(b) The world's largest ship, the oil tanker *Jahre Viking*, is 1054 feet long. What is its top speed?

(c) As a ship approaches its maximum speed, the power required increases sharply. Therefore, most merchant ships are designed to cruise at speeds no higher than $v_c = 0.8\sqrt{L}$. Graph v_c on the same axes with v_{max}.

(d) What is the cruising speed of the *Jahre Viking*? What percent of its maximum speed is that?

3.3.10.43. A rough estimate for the radius of the nucleus of an atom is provided by the formula

$$r = kA^{1/3}$$

where A is the mass number of the nucleus and $k \approx 1.3 \times 10^{-13}$ centimeter.

(a) Estimate the radius of the nucleus of an atom of iodine-127, which has mass number 127. If the nucleus is roughly spherical, what is its volume?

(b) The nuclear mass of iodine-127 is 2.1×10^{-22} gram. What is the density of the nucleus? (Density is mass per unit volume.)

(c) Complete the table of values for the radii of various radioisotopes.

Element	Carbon	Potassium	Cobalt	Technetium	Radium
Mass number, A	14	40	60	99	226
Radius, r					

(d) Sketch a graph of r as a function of A. (Use units of 10^{-13} centimeter on the vertical axis.)

Answer.

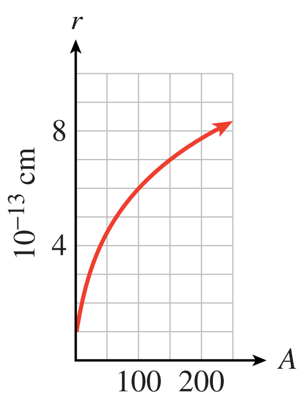
(a) 6.5×10^{-13} cm; 1.17×10^{-36} cm^3

(b) $1.8 \times 10^{14} g/\text{cm}^3$

(c)

Element Carbon Potassium Cobalt Technetium Radium
Mass number, A 14 40 60 99 226
Radius, r (10^{-13} cm) 3.1 4.4 5.1 6 7.9

(d)



3.3.10.44. In the sport of crew racing, the best times vary closely with the number of men in the crew, according to the formula

$$t = kn^{-1/9}$$

where n is the number of men in the crew and t is the winning time, in minutes, for a 2000-meter race.

(a) If the winning time for the 8-man crew was 5.73 minutes, estimate the value of k.

(b) Complete the table of values of predicted winning times for the other racing classes.

Size of crew, n	Winning time, t
1	
2	
4	
8	

(c) Sketch a graph of t as a function of n.

In Problems 45–48, one quantity varies directly with the square root of the other, that is, $y = k\sqrt{x}$.

a Find the value of k and write a power function relating the variables.

b Use your function to answer the question.

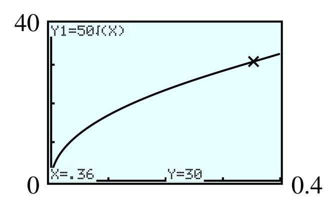
c Graph your function and verify your answer to part (b) on the graph.

3.3.10.45. The stream speed necessary to move a granite particle is a function of the diameter of the particle; faster river currents can move larger particles. The table shows the stream speed necessary to move particles of different sizes. What speed is needed to carry a particle with diameter 0.36 centimeter?

Diameter, d (cm)	Speed, s (cm/sec)
0.01	5
0.04	10
0.09	15
0.16	20

Answer.

(a) $s = 50\sqrt{d}$

(c) 

(b) 30 cm/sec

3.3.10.46. The speed at which water comes out of the spigot at the bottom of a water jug is a function of the water level in the jug; it slows down as the water level drops. The table shows different water levels and the resulting flow speeds. What is the flow speed when the water level is at 16 inches?

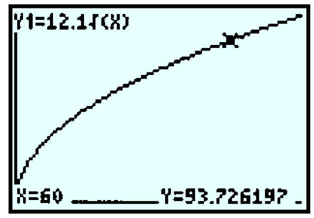
Level, L (in)	Speed, s (gal/min)
9	1.5
6.25	1.25
4	1
2.25	0.75

3.3.10.47. The rate, r, in feet per second, at which water flows from a fire hose is a function of the water pressure, P, in psi (pounds per square inch). What is the rate of water flow at a typical water pressure of 60 psi?

P (psi)	10	20	30	40
r (ft/sec)	38.3	54.1	66.3	76.5

Answer.

(a) $r = 12.1\sqrt{P}$

(c) 

(b) 94 ft/sec

3.3.10.48. When a layer of ice forms on a pond, the thickness of the ice, d, in centimeters, is a function of time, t, in minutes. How thick is the ice after 3 hours?

t (min)	10	30	40	60
d (cm)	0.50	0.87	1.01	1.24

3.3.10.49. Membership in the County Museum has been increasing since it was built in 1980. The number of members is given by the function

$$M(t) = 72 + 100t^{1/3}$$

where t is the number of years since 1980.

(a) How many members were there in 1990? In 2000?

(b) In what year will the museum have 400 members? If the membership continues to grow according to the given function, when will the museum have 500 members?

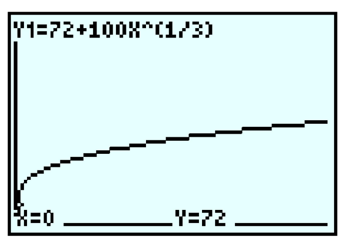
(c) Graph the function $M(t)$. How would you describe the growth of the membership over time?

Answer.

(a) 287; 343

(b) 2015; 2058

(c) The membership grows rapidly at first but is growing less rapidly with time.



3.3.10.50. Due to improvements in technology, the annual electricity cost of running most major appliances has decreased steadily since 1970. The average annual cost of running a refrigerator is given, in dollars, by the function

$$C(t) = 148 - 28t^{1/3}$$

where t is the number of years since 1970.

(a) How much did it cost to run a refrigerator in 1980? In 1990?

(b) When was the cost of running a refrigerator half of the cost in 1970? If the cost continues to decline according to the given function, when will it cost \$50 per year to run a refrigerator?

(c) Graph the function $C(t)$. Do you think that the cost will continue to decline indefinitely according to the given function? Why or why not?

3.3.10.51. Match each function with the description of its graph in the first quadrant.

I $f(x) = x^2$

II $f(x) = x^{-2}$

III $f(x) = x^{1/2}$

(a) Increasing and concave up

(b) Increasing and concave down

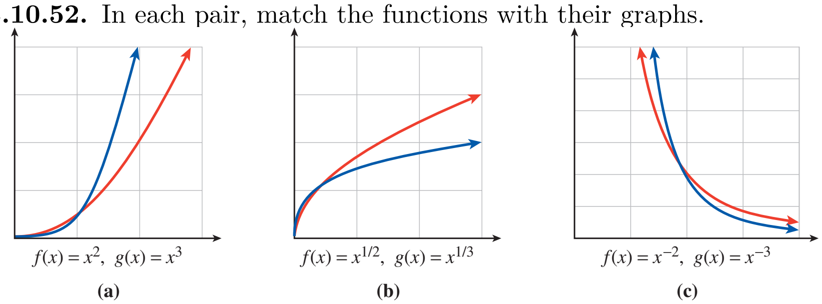
(c) Decreasing and concave up

(d) Decreasing and concave down

Answer.

(a) I (b) III (c) II (d) none

3.3.10.52. In each pair, match the functions with their graphs.



3.3.10.53.

(a) Graph the functions

$$y_1 = x^{1/2}, \quad y_2 = x^{1/3}, \quad y_3 = x^{1/4}, \quad y_4 = x^{1/5}$$

in the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 100 \\ \text{Ymin} = 0 & \text{Ymax} = 10 \end{array}$$

What do you observe?

(b) Use your graphs to evaluate $100^{1/2}$, $100^{1/3}$, $100^{1/4}$, and $100^{1/5}$.

(c) Use your calculator to evaluate $100^{1/n}$ for $n = 10$, $n = 100$, and $n = 1000$. What happens when n gets large?

Answer.

(a) The graphs of $x^{1/n}$ become closer and closer to horizontal when n increases (for $x > 1$).

(b) $10, \ 4.64, \ 3.16, \ 2.51$

(c) $1.58, 1.05, \ 1.005$; the values decrease towards 1.

3.3.10.54.

(a) Graph the functions

$$y_1 = x^{1/2}, \quad y_2 = x^{1/3}, \quad y_3 = x^{1/4}, \quad y_4 = x^{1/5}$$

in the window

$$\text{Xmin} = 0 \qquad \text{Xmax} = 1$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 1$$

What do you observe?

(b) Use your graphs to evaluate $0.5^{1/2}$, $0.5^{1/3}$, $0.5^{1/4}$, and $0.5^{1/5}$.

(c) Use your calculator to evaluate $0.5^{1/n}$ for $n = 10$, $n = 100$, and $n = 1000$. What happens when n gets large?

For Problems 55–58, graph each set of functions in the given window. What do you observe?

3.3.10.55. $y_1 = \sqrt{x}$, $y_2 = x^2$, $y_3 = x$

$$\text{Xmin} = 0 \qquad \text{Xmax} = 4$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 4$$

Answer. The graphs of y_1 and y_2 are symmetric about $y_3 = x$.

3.3.10.56. $y_1 = \sqrt[3]{x}$, $y_2 = x^3$, $y_3 = x$

$$\text{Xmin} = -4 \qquad \text{Xmax} = 4$$
$$\text{Ymin} = -4 \qquad \text{Ymax} = 4$$

3.3.10.57. $y_1 = \sqrt[5]{x}$, $y_2 = x^5$, $y_3 = x$

$$\text{Xmin} = -2 \qquad \text{Xmax} = 2$$
$$\text{Ymin} = -2 \qquad \text{Ymax} = 2$$

Answer. The graphs of y_1 and y_2 are symmetric about $y_3 = x$.

3.3.10.58. $y_1 = \sqrt[4]{x}$, $y_2 = x^4$, $y_3 = x$

$$\text{Xmin} = 0 \qquad \text{Xmax} = 2$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 2$$

3.3.10.59.

(a) Graph the functions $f(x) = 4\sqrt[3]{x-9}$ and $g(x) = 12$ in the window

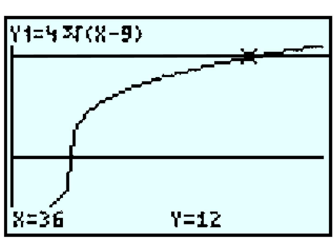
$$\text{Xmin} = 0 \qquad \text{Xmax} = 47$$
$$\text{Ymin} = -8 \qquad \text{Ymax} = 16$$

(b) Use the graph to solve the equation $4\sqrt[3]{x-9} = 12$.

(c) Solve the equation algebraically.

Answer.

(a)



(b) $x = 36$

3.3.10.60.

(a) Graph the functions $f(x) = 6 + 2\sqrt[4]{12 - x}$ and $g(x) = 10$ in the window

$$\begin{array}{ll} \text{Xmin} = -27 & \text{Xmax} = 20 \\ \text{Ymin} = 4 & \text{Ymax} = 12 \end{array}$$

(b) Use the graph to solve the equation $6 + 2\sqrt[4]{12 - x} = 10$.

(c) Solve the equation algebraically.

3.3.10.61.

(a) Write $\sqrt{x}$ with a fractional exponent.

(b) Write $\sqrt{\sqrt{x}}$ with a fractional exponents.

(c) Use the laws of exponents to show that $\sqrt{\sqrt{x}} = \sqrt[4]{x}$.

Answer.

(a) $x^{1/2}$

(b) $\left(x^{1/2}\right)^{1/2}$

(c)

$$\begin{aligned} \sqrt{\sqrt{x}} &= \left(x^{1/2}\right)^{1/2} && \text{By definition of fractional exponents.} \\ &= x^{1/4} && \text{By the third law of exponents.} \\ &= \sqrt[4]{x} && \text{By definition of fractional exponents.} \end{aligned}$$

3.3.10.62.

(a) Write $\sqrt[3]{x}$ with a fractional exponent.

(b) Write $\sqrt{\sqrt[3]{x}}$ with a fractional exponents.

(c) Use the laws of exponents to show that $\sqrt{\sqrt[3]{x}} = \sqrt[6]{x}$.

Write each expression as a sum of terms of the form kx^p.

3.3.10.63. $\dfrac{\sqrt{x}}{4} - \dfrac{2}{\sqrt{x}} + \dfrac{x}{\sqrt{2}}$

Answer. $\dfrac{1}{4}x^{1/2} - 2x^{-1/2} + \dfrac{1}{\sqrt{2}}x$

3.3.10.64. $\dfrac{\sqrt{3}}{x} + \dfrac{3}{\sqrt{x}} - \dfrac{\sqrt{x}}{3}$

3.3.10.65. $\dfrac{6 - \sqrt[3]{x}}{2\sqrt[3]{x}}$

Answer. $3x^{-1/3} - \dfrac{1}{2}$

3.3.10.66. $\dfrac{\sqrt[4]{x} + 2}{2\sqrt[4]{x}}$

3.3.10.67. $x^{-0.5}\left(x + x^{0.25} - x^{0.5}\right)$

Answer. $x^{0.5} + x^{-0.25} - x^{0}$

3.3.10.68. $x^{0.5}\left(x^{-1} + x^{-0.5} + x^{-0.25}\right)$

3.4 Rational Exponents

3.4.1 Powers of the Form $a^{m/n}$

Checkpoint 3.4.3 Evaluate each power.

a $32^{-3/5}$

b $-81^{1.25}$

Answer.

a $\dfrac{1}{8}$

b -243

3.4.2 Power Functions

Checkpoint 3.4.5

a Complete the table of values for the function $f(x) = x^{-3/4}$.

x	0.1	0.2	0.5	1	2	5	8	10
$f(x)$								

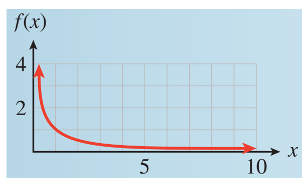
b Sketch the graph of the function.

Answer.

a

x	0.1	0.2	0.5	1	2	5	8	10
$f(x)$	5.623	3.344	1.682	1	0.595	0.299	0.210	0.178

b



3.4.3 More about Scaling

Checkpoint 3.4.7 A human being weighs about 70 kg, and 0.2 mg of LSD is enough to induce severe psychotic symptoms. Use these data and Kleiber's rule to predict what dosage would produce a similar effect in an elephant.

Answer. About 3.3 mg

3.4.4 Radical Notation

Checkpoint 3.4.9 Write each expression in radical notation.

a $5t^{1.25}$

b $3m^{-5/3}$

Answer.

a $5\sqrt[4]{t^5}$

b $\dfrac{3}{\sqrt[3]{m^5}}$

Checkpoint 3.4.11 Convert to exponential notation.

a $\sqrt[3]{6w^2}$

b $\sqrt[4]{\dfrac{v^3}{s^5}}$

Answer.

a $6^{1/3}w^{2/3}$

b $v^{3/4}s^{-5/4}$

3.4.5 Operations with Rational Exponents

Checkpoint 3.4.13 Simplify by applying the laws of exponents.

a $x^{1/3}\left(x+x^{2/3}\right)$

b $\dfrac{n^{9/4}}{4n^{3/4}}$

Answer.

a $x^{4/3}+x$

b $\dfrac{n^{3/2}}{4}$

3.4.6 Solving Equations

Checkpoint 3.4.15 Solve the equation $3.2z^{0.6}-9.7=8.7$. Round your answer to two decimal places.

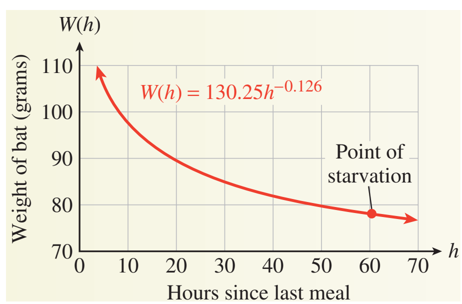
Hint. Isolate the power.
both sides to the reciprocal power.

Answer. 18.45

Investigation 3.4.1 Vampire Bats. Small animals such as bats cannot survive for long without eating. The graph below shows how the weight, W, of a typical vampire bat decreases over time until its next meal, until the bat reaches the point of starvation. The curve is the graph of the function

$$W(h)=130.25h^{-0.126}$$

where h is the number of hours since the bat's most recent meal. (Source: Wilkinson, 1984)



1. Use the graph to estimate answers to the following questions: How long can the bat survive after eating until its next meal? What is the bat's weight at the point of starvation?

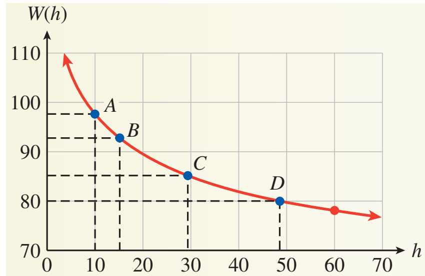
2. Use the formula for $W(h)$ to verify your answers.

3. Write and solve an equation to answer the question: When the bat's weight has dropped to 90 grams, how long can it survive before eating again?

4. Complete the table showing the number of hours since the bat last ate when its weight has dropped to the given values.

Weight (grams)	97.5	92.5	85	80
Hours since eating				
Point on graph	A	B	C	D

5. Compute the slope of the line segments from point A to point B, and from point C to point D. Include units in your answers.



6. What happens to the slope of the curve as h increases? What does this tell you about the concavity of the curve?

7. Suppose a bat that weighs 80 grams consumes 5 grams of blood. How many hours of life does it gain? Suppose a bat that weighs 97.5 grams gives up a meal of 5 grams of blood. How many hours of life does it forfeit?

8. Vampire bats sometimes donate blood (through regurgitation) to other bats that are close to starvation. Suppose a bat at point A on the curve donates 5 grams of blood to a bat at point D. Explain why this strategy is effective for the survival of the bat community.

3.4.8 Homework 3.4

Evaluate each power in Problems 1–4.

3.4.8.1.
(a) $81^{3/4}$ (b) $125^{2/3}$ (c) $625^{0.75}$

Answer.

(a) 27 (b) 25 (c) 125

3.4.8.2.
(a) $-8^{2/3}$ (b) $-64^{2/3}$ (c) $243^{0.4}$

3.4.8.3.
(a) $16^{-3/2}$ (b) $8^{-4/3}$ (c) $32^{-1.6}$

Answer.

(a) $\dfrac{1}{64}$ (b) $\dfrac{1}{16}$ (c) $\dfrac{1}{256}$

3.4.8.4.
(a) $-125^{-4/3}$ (b) $-32^{-3/5}$ (c) $100^{-2.5}$

For Problems 5–8, write each power in radical form.

3.4.8.5.
(a) $x^{4/5}$ (b) $b^{-5/6}$ (c) $(pq)^{-2/3}$

Answer.

(a) $\sqrt[5]{x^4}$ (b) $\dfrac{1}{\sqrt[6]{b^5}}$ (c) $\dfrac{1}{\sqrt[3]{(pq)^2}}$

3.4.8.6.
(a) $y^{3/4}$ (b) $a^{-2/7}$ (c) $(st)^{-3/5}$

3.4.8.7.
(a) $3x^{0.4}$ (b) $4z^{-4/3}$ (c) $-2x^{0.25}y^{0.75}$

Answer.

(a) $3\sqrt[5]{x^2}$ (b) $\dfrac{4}{\sqrt[3]{z^4}}$ (c) $-2\sqrt[4]{xy^3}$

3.4.8.8.
(a) $5y^{2/3}$ (b) $6w^{-1.5}$ (c) $-3x^{0.4}y^{0.6}$

For Problems 9–12, write each expression with fractional exponents.

3.4.8.9.
(a) $\sqrt[3]{x^2}$ (b) $2\sqrt[5]{ab^3}$ (c) $\dfrac{-4m}{\sqrt[6]{p^7}}$

Answer.

(a) $x^{2/3}$ (b) $2a^{1/5}b^{3/5}$ (c) $-4mp^{-7/6}$

3.4.8.10.
(a) $\sqrt{y^3}$ (b) $6\sqrt[5]{(ab)^3}$ (c) $\dfrac{-2n}{\sqrt[8]{q^{11}}}$

3.4.8.11.
(a) $\sqrt[3]{(ab)^2}$ (b) $\dfrac{8}{\sqrt[4]{x^3}}$ (c) $\dfrac{R}{3\sqrt{TK^5}}$

Answer.

(a) $(ab)^{2/3}$ (b) $8x^{-3/4}$ (c) $\dfrac{1}{3}RT^{-1/2}K^{-5/2}$

3.4.8.12.
(a) $\sqrt[3]{ab^2}$ (b) $\dfrac{5}{\sqrt[3]{y^2}}$ (c) $\dfrac{S}{4\sqrt{VH^3}}$

For Problems 13–16, evaluate each root without using a calculator.

3.4.8.13.
(a) $\sqrt[5]{32^3}$ (b) $-\sqrt[3]{27^4}$ (c) $\sqrt[4]{16y^{12}}$

Answer.

(a) 8 (b) -81 (c) $2y^3$

3.4.8.14.
(a) $\sqrt[4]{16^5}$ (b) $-\sqrt[3]{125^2}$ (c) $\sqrt[5]{243x^{10}}$

3.4.8.15.
(a) $-\sqrt{a^8b^{16}}$ (b) $\sqrt[3]{8x^9y^{27}}$ (c) $-\sqrt[4]{81a^8b^{12}}$

Answer.

(a) $-a^4b^8$ (b) $2x^3y^9$ (c) $-3a^2b^3$

3.4.8.16.
(a) $-\sqrt{a^{10}b^{35}}$ (b) $\sqrt[3]{64x^6y^{18}}$ (c) $\sqrt[5]{32x^{25}y^5}$

For Problems 17–18, use a calculator to approximate each power or root to the nearest thousandth.

3.4.8.17.

(a) $12^{5/6}$ (b) $\sqrt[3]{6^4}$ (c) $37^{-2/3}$ (d) $4.7^{2.3}$

Answer.

(a) 7.931

(b) 10.903

(c) 0.090

(d) 35.142

3.4.8.18.

(a) $20^{5/4}$ (b) $\sqrt[5]{8^3}$ (c) $128^{-3/4}$ (d) $16.1^{0.29}$

3.4.8.19. During a flu epidemic in a small town, health officials estimate that the number of people infected t days after the first case was discovered is given by

$$I(t) = 50t^{3/5}$$

(a) Make a table of values for $I(t)$ on the domain $0 \le t \le 20$. What is the range of the function on that domain?

t	5	10	15	20
$I(t)$				

(b) How long will it be before 300 people are ill?

(c) Graph the function $I(t)$ and verify your answer to part (b) on your graph.

Answer.

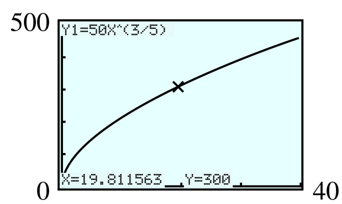
(a)

t	5	10	15	20
$I(t)$	131	199	254	302

Range: $[0, 302]$

(b) ≈ 19.812 or about 20 days

(c)



3.4.8.20. The research division of an advertising firm estimates that the number of people who have seen their ads t days after the campaign begins is given by the function

$$N(t) = 2000t^{5/4}$$

(a) Make a table of values for $N(t)$ on the domain $0 \le t \le 20$. What is the range of the function on that domain?

t	6	10	14	20
$N(t)$				

(b) How long will it be before 75,000 people have seen the ads?

(c) Graph the function $N(t)$ and verify your answer to part (b) on your graph.

In Problems 21–22, graph each set of power functions in the suggested window and compare the graphs.

3.4.8.21. $y_1 = x, \quad y_2 = x^{5/4}, \quad y_3 = x^{3/2}, \quad y_4 = x^2, \quad y_5 = x^{5/2}$
Xmin $= 0$, Xmax $= 6$, Ymin $= 0$, Ymax $= 10$

Answer. All the graphs are increasing and concave up. For $x > 1$, each graph increases more quickly than the previous one.

3.4.8.22. $y_1 = x^{2/5}, \quad y_2 = x^{1/2}, \quad y_3 = x^{2/3}, \quad y_4 = x^{3/4}, \quad y_5 = x$
Xmin $= 0$, Xmax $= 6$, Ymin $= 0$, Ymax $= 4$

3.4.8.23. The *surface to volume ratio* is important in studying how organisms grow and why animals of different sizes have different characteristics.

(a) Write formulas for the volume, V, and the surface area, A, of a cube in terms of its length, L.

(b) Express the length of the cube as a function of its volume. Express the length of the cube as a function of its surface area.

(c) Express the surface area of the cube as a function of its volume.

(d) Express the surface to volume ratio of a cube in terms of its length. What happens to the surface to volume ratio as L increases?

Answer.

(a) $V = L^3$, $A = 6L^2$

(b) $L = V^{1/3}$, $L = \left(\dfrac{A}{6}\right)^{1/2}$

(c) $A = 6V^{2/3}$

(d) $\frac{A}{V} = \frac{6}{L}$. As L increases, the surface-to-volume ratio decreases.

3.4.8.24. Repeat Problem 23 for the volume and surface area of a sphere in terms of its radius, R.

(a) Write formulas for the volume, V, and the surface area, A, of a sphere in terms of its radius, R.

(b) Express the radius of the sphere as a function of its volume. Express the radius of the sphere as a function of its surface area.

(c) Express the surface area of the sphere as a function of its volume.

(d) Express the surface to volume ratio of a sphere in terms of its radius. What happens to the surface to volume ratio as R increases?

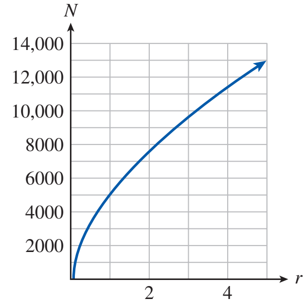
3.4.8.25. A brewery wants to replace its old vats with larger ones. To estimate the cost of the new equipment, the accountant uses the 0.6 rule for industrial costs, which states that the cost of a new container is approximately $N = Cr^{0.6}$, where C is the cost of the old container and r is the ratio of the capacity of the new container to the old one.

(a) If an old vat cost \$5000, graph N as a function of r.

(b) How much should the accountant budget for a new vat that holds 1.8 times as much as the old one?

Answer.

(a)



(b) \$7114.32

3.4.8.26. If a quantity of air expands without changing temperature, its pressure, in pounds per square inch, is given by $P = kV^{-1.4}$, where V is the volume of the air in cubic inches and $k = 2.79 \times 10^4$.

(a) Graph P as a function of V.

(b) Find the air pressure of an air sample when its volume is 50 cubic inches.

3.4.8.27. In the 1970s, Jared Diamond studied the number of bird species on small islands near New Guinea. He found that larger islands support a larger number of different species, according to the formula

$$S = 15.1A^{0.22}$$

where S is the number of species on an island of area A square kilometers. (Source: Chapman and Reiss, 1992)

(a) Fill in the table.

A	10	100	1000	5000	$10,000$
S					

(b) Graph the function on the domain $0 < A \leq 10,000$.

(c) How many species of birds would you expect to find on Manus Island, with an area of 2100 square kilometers? On Lavongai, which bird's area is 1140 square kilometers?

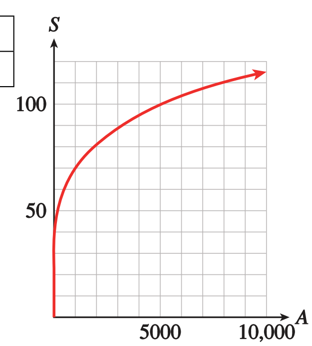
(d) How large must an island be in order to support 200 different species of bird?

Answer.

(a)

A	10	100	1000	5000	$10,000$
S	25	42	69	98	115

(b)



(c) 81, 71

(d) $126,000$ sq km

3.4.8.28. The drainage basin of a river channel is the area of land that contributes water to the river. The table gives the lengths in miles of some of the world's largest rivers and the areas of their drainage basins in square miles. (Source: Leopold, Wolman, and Miller 1992)

(a) Plot the data, using units of 100,000 on the horizontal axis and units of 500 on the vertical axis.

(b) The length, L, of the channel is related to the area, A, of its drainage basin according to the formula

$$L = 1.05A^{0.58}$$

Graph this function on top of the data points.

(c) The drainage basin for the Congo covers about 1,600,000 square miles. Estimate the length of the Congo River.

(d) The Rio Grande is 1700 miles long. What is the area of its drainage basin?

River	Area of drainage basin	Length
Amazon	2,700,000	4300
Nile	1,400,000	4200
Mississippi	1,300,000	4100
Yangtze	580,000	2900
Volga	480,000	2300
St. Lawrence	460,000	1900
Ganges	440,000	1400
Orinoco	380,000	1400
Indus	360,000	2000
Danube	350,000	1800
Colorado	250,000	1700
Platte	72,000	800
Rhine	63,000	900
Seine	48,000	500
Delaware	12,000	200

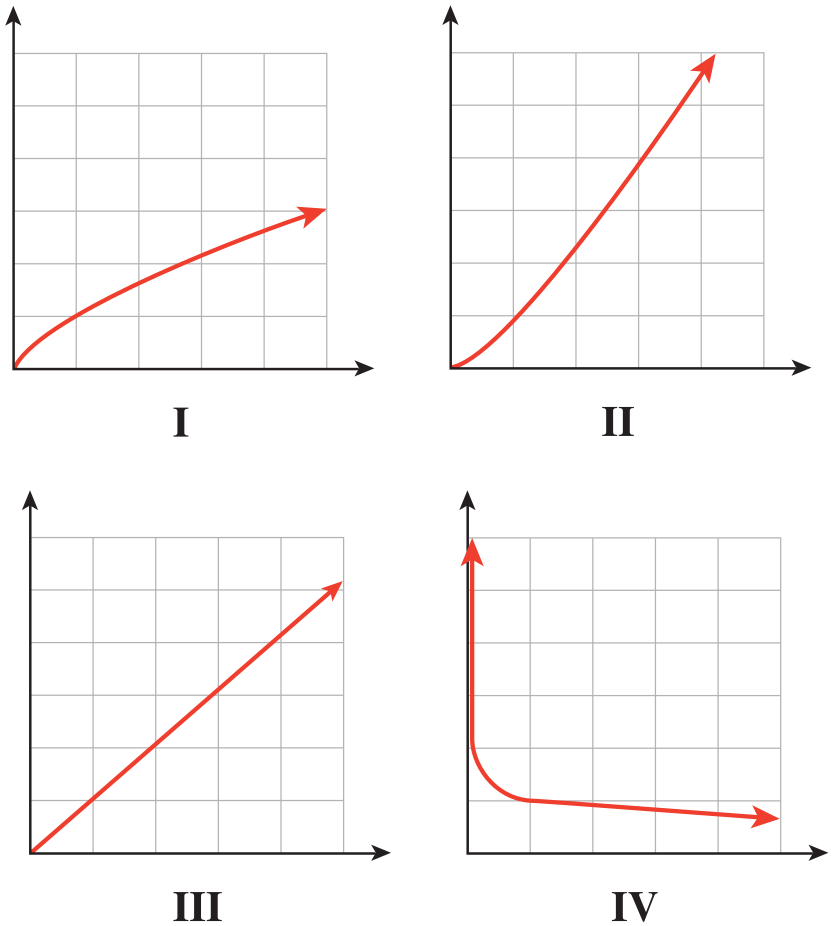
3.4.8.29.
The table at right shows the exponent, p, in the allometric equation

$$\text{variable} = k(\text{body mass})^p$$

for some variables related to mammals. (Source: Chapman and Reiss, 1992)

Variable	Exponent, p
Home range size	1.26
Lung volume	1.02
Brain mass	0.70
Respiration rate	−0.26

a Match each equation to one of the graphs shown in the figure.



b Explain how the value of p in the allometric equation determines the shape of the graph. Consider the cases $p > 1$, $0 < p < 1$, and $p < 0$.

Answer.

(a) Home range size: II, lung volume: III, brain mass: I, respiration rate: IV

(b) If $p > 1$, the graph is increasing and concave up. If $0 < p < 1$, the graph is increasing and concave down. If $p < 0$, the graph is decreasing and concave up.

3.4.8.30. The average body mass of a dolphin is about 140 kilograms, twice the body mass of an average human male.

(a) Using the allometric equations in Problem 29, calculate the ratio of the brain mass of a dolphin to that of a human.

(b) A good-sized brown bear weighs about 280 kilograms, twice the weight of a dolphin. Calculate the ratio of the brain mass of a brown bear to that of a dolphin.

(c) Use a ratio to compare the heartbeat frequencies of a dolphin and a human, and those of a brown bear and a dolphin. (See Example 3.3.18 of Section 3.3.)

3.4.8.31. The gourd species *Tricosanthes* grows according to the formula $L = ad^{2.2}$, where L is its length and d is its width. The species *Lagenaria* has the growth law $L = ad^{0.81}$. (Source: Burton, 1998)

(a) By comparing the exponents, predict which gourd grows into a long, thin shape, and which is relatively fatter. Which species is called the snake gourd, and which is the bottle gourd?

(b) The snake gourd reaches a length of 2 meters (200 cm), with a diameter of only 4 cm. Find the value of a in its growth law.

(c) The bottle gourd is 10 cm long and 7 cm in diameter at maturity. Find the value of a in its growth law.

(d) The giant bottle gourd grows to a length of 23 cm with a diameter of 20 cm. Does it grow according to the same law as standard bottle gourds?

Answer.

(a) Tricosanthes is the snake gourd and Lagenaria is the bottle gourd. Tricosanthes is thinner and Lagenaria is fatter.

(b) $a \approx 9.5$

(c) $a \approx 2$

(d) Yes

3.4.8.32. As a fiddler crab grows, one claw (called the chela) grows much faster than the rest of the body. The table shows the mass of the chela, C, versus the mass of the rest of the body, b, for a number of fiddler crabs. (Source: Burton, 1998)

b	65	110	170	205	300	360	615
C	6	15	30	40	68	110	240

(a) Plot the data.

(b) On the same axes, graph the function $C = 0.007b^{1.63}$. How well does the function fit the data?

(c) Using the function in part (b), predict the chela mass of a fiddler crab if the rest of its body weighs 400 mg.

(d) The chela from a fiddler crab weighs 250 mg. How much does the rest of its body weigh?

(e) As the body mass of a fiddler crab doubles from 100 mg to 200 mg, by what factor does the mass of its chela increase? As the body mass doubles from 200 mg to 400 mg?

3.4.8.33. The climate of a region has a great influence on the types of animals that can survive there. Extreme temperatures create difficult living conditions, so the diversity of wildlife decreases as the annual temperature range increases. Along the west coast of North America, the number of species of mammals, M, is approximately related to the temperature range, R, (in degrees Celsius) by the function $M = f(R) = 433.8R^{-0.742}$. (Source: Chapman and Reiss, 1992)

(a) Graph the function for temperature ranges up to 30°C.

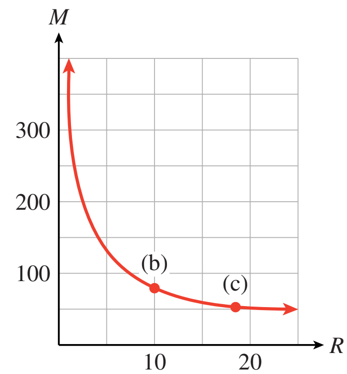
(b) How many species would you expect to find in a region where the temperature range is 10°C? Label the corresponding point on your graph.

(c) If 50 different species are found in a certain region, what temperature range would you expect the region to experience? Label the corresponding point on your graph.

(d) Evaluate the function to find $f(9)$, $f(10)$, $f(19)$, and $f(20)$. What do these values represent? Calculate the change in the number of species as the temperature range increases from 9°C to 10°C and from 19°C to 20°C. Which 1° increase results in a greater decrease in diversity? Explain your answer in terms of slopes on your graph.

Answer.

(a)

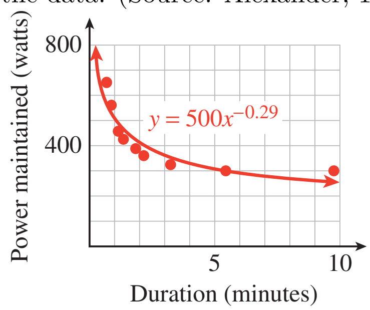


(b) ≈ 78.5 or about 79 species

(c) 18.4°C

(d) $f(9) \approx 85$, $f(10) \approx 79$, $f(19) \approx 49$, $f(20) \approx 47$; from 9°C to 10°C has the greater decrease, corresponding to the steeper slope. If the temperature range is 9°C, there will be approximately 85 species. If the temperature range is 10°C, there will be approximately 79 species. If the temperature range is 19°C, there will be approximately 49 species. If the temperature range is 20°C, there will be approximately 47 species.

3.4.8.34. A bicycle ergometer is used to measure the amount of power generated by a cyclist. The scatterplot shows how long an athlete was able to sustain various levels of power output. The curve is the graph of $y = 500x^{-0.29}$, which approximately models the data. (Source: Alexander, 1992)



(a) In this graph, which variable is independent and which is dependent?

(b) The athlete maintained 650 watts of power for 40 seconds. What power output does the equation predict for 40 seconds?

(c) The athlete maintained 300 watts of power for 10 minutes. How long does the equation predict that power output can be maintained?

(d) In 1979, a remarkable pedal-powered aircraft called the Gossamer Albatross was successfully flown across the English Channel. The flight took 3 hours. According to the equation, what level of power can be maintained for 3 hours?

(e) The Gossamer Albatross needed 250 watts of power to keep it airborne. For how long can 250 watts be maintained according to the given equation?

3.4.8.35. Investigation 3.0.1 at the start of this chapter gives data for the pressure inside April and Tolu's balloon as a function of its diameter. As the diameter of the balloon increases from 5 cm to 20 cm, the pressure inside decreases. Can we find a function that describes this portion of the graph?

(a) Pressure is the force per unit area exerted by the balloon on the air inside, or $P = \dfrac{F}{A}$. Because the balloon is spherical, its surface area, A, is given by $A = \pi d^2$. Because the force increases as the balloon expands, we will try a power function $F = kd^p$, where k and p are constants, and see if this fits the data. Combine the three equations, $P = \dfrac{F}{A}$, $A = \pi d^2$, and $F = kd^p$, to express P as a power function of d.

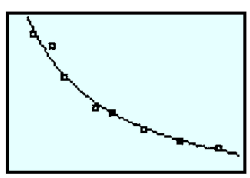
(b) Use your calculator's power regression feature to find a power function that fits the data. Graph the function $P = 211d^{-0.7}$ on top of the data. Do the data support the hypothesis that P is a power function of d?

(c) What is the value of the exponent p in $F = kd^p$?

Answer.

(a) $P = \dfrac{k}{\pi} d^{p-2}$

(b)



The power function is a good fit on this interval.

(c) 1.3

3.4.8.36.

The table shows the total number of frequent flyer miles redeemed by customers through the given year. (Source: www.hotelnewsresource.com)

(a) Plot the data, with $t = 0$ in 1980. What type of function might model the data?

(b) Graph the function $f(t) = 3.13t^{2.33}$ on top of the data.

(c) Evaluate $f(5)$ and $f(25)$. What do those values mean in this context?

(d) Use the regression equation to predict when the total number of miles redeemed will reach 10 trillion.

Year	Cumulative miles redeemed (billions)
1982	14.8
1984	85.3
1986	215.4
1988	387.5
1990	641.3
1992	975.2
1994	1455.9
1996	1996
1998	2670.8
2000	3379.1
2002	4123.6

Hint. (For part d): How many billions make a trillion?

For Problems 37–42, simplify by applying the laws of exponents. Write your answers with positive exponents only.

3.4.8.37.

(a) $4a^{6/5}a^{4/5}$

(b) $9b^{4/3}b^{1/3}$

Answer.

(a) $4a^2$

(b) $9b^{5/3}$

3.4.8.38.

(a) $(-2m^{2/3})^4$

(b) $(-5n^{3/4})^3$

3.4.8.39.

(a) $\dfrac{8w^{9/4}}{2w^{3/4}}$

(b) $\dfrac{12z^{11/3}}{4z^{5/3}}$

Answer.

(a) $4w^{3/2}$

(b) $3z^2$

3.4.8.40.

(a) $(-3u^{5/3})(5u^{-2/3})$

(b) $(-2v^{7/8})(-3v^{-3/8})$

3.4.8.41.

(a) $\dfrac{k^{3/4}}{2k}$

(b) $\dfrac{4h^{2/3}}{3h}$

Answer.

(a) $\dfrac{1}{2k^{1/4}}$

(b) $\dfrac{4}{3h^{1/3}}$

3.4.8.42.

(a) $c^{-2/3}\left(\dfrac{2}{3}c^2\right)$

(b) $\dfrac{r^3}{4}(r^{-5/2})$

3.4.8.43. The incubation time for a bird's egg is a function of the mass, m, of the egg, and has been experimentally determined as

$$I(m) = 12.0m^{0.217}$$

where m is measured in grams and I is in days. (Source: Burton, 1998)

(a) Calculate the incubation time (to the nearest day) for the wren, whose eggs weigh about 2.5 grams, and the greylag goose, whose eggs weigh 46 grams.

(b) During incubation, birds' eggs lose water vapor through their porous shells. The rate of water loss from the egg is also a function of its mass, and it appears to follow the rule

$$W(m) = 0.015m^{0.742}$$

in grams per day. Combine the functions $I(m)$ and $W(m)$ to calculate the fraction of the initial egg mass that is lost during the entire incubation period.

(c) Explain why your result shows that most eggs lose about 18% of their mass during incubation.

Answer.

(a) Wren: 15 days, greylag goose: 28 days

(b) $\dfrac{I(m) \cdot W(m)}{m} = 0.18m^{-0.041}$

(c) Because $m^{-0.041}$ is close to m^0, the fraction lost is close to 0.18.

3.4.8.44. The incubation time for birds' eggs is given by

$$I(m) = 12.0m^{0.217}$$

where m is the weight of the egg in grams, and I is in days. (See Problem 43.) Before hatching, the eggs take in oxygen at the rate of

$$O(m) = 22.2m^{0.77}$$

in milliliters per day. (Source: Burton, 1998)

(a) Combine the functions $I(m)$ and $O(m)$ to calculate the total amount of oxygen taken in by the egg during its incubation.

(b) Use your result from part (a) to explain why total oxygen consumption per unit mass is approximately inversely proportional to incubation time.

(c) Predict the oxygen consumption per gram of a herring gull's eggs, given that their incubation time is 26 days. (The actual value is 11 milliliters per day.)

For Problems 45–50, solve. Round your answers to the nearest thousandth if necessary.

3.4.8.45. $x^{2/3} - 1 = 15$

Answer. $x = 64$

3.4.8.46. $x^{3/4} + 3 = 11$

3.4.8.47. $x^{-2/5} = 9$

Answer. $x = \dfrac{1}{243}$

3.4.8.48. $x^{-3/2} = 8$

3.4.8.49. $2(5.2 - x^{5/3}) = 1.4$

Answer. $x \approx 2.466$

3.4.8.50. $3(8.6 - x^{5/2}) = 6.5$

3.4.8.51. Kepler's law gives a relation between the period, p, of a planet's revolution, in years, and its average distance, a, from the sun:

$$p^2 = Ka^3$$

where $K = 1.243 \times 10^{-24}$, a is measured in miles, and p is in years.

(a) Solve Kepler's law for p as a function of a.

(b) Find the period of Mars if its average distance from the sun is 1.417×10^8 miles.

Answer.

(a) $p = 1.115 \times 10^{-12}a^{3/2}$

(b) 1.88 years

3.4.8.52. Refer to Kepler's law, $p^2 = Ka^3$, in Problem 51.

(a) Solve Kepler's law for a as a function of p.

(b) Find the distance from Venus to the sun if its period is 0.615 years.

3.4.8.53. If $f(x) = (3x - 4)^{3/2}$, find x so that $f(x) = 27$.

Answer. $\dfrac{13}{3}$

3.4.8.54. If $g(x) = (6x - 2)^{5/3}$, find x so that $g(x) = 32$.

3.4.8.55. If $S(x) = 12x^{-5/4}$, find x so that $S(x) = 20$.

Answer. 0.665

3.4.8.56. If $T(x) = 9x^{-6/5}$, find x so that $T(x) = 15$.

For Problems 57–64, use the distributive law to find the product.

3.4.8.57. $2x^{1/2}(x - x^{1/2})$

Answer. $2x^{3/2} - 2x$

3.4.8.58. $x^{1/3}(2x^{2/3} - x^{1/3})$

3.4.8.59. $\dfrac{1}{2}y^{-1/3}(y^{2/3} + 3y^{-5/6})$

Answer. $\dfrac{1}{2}y^{1/3} + \dfrac{3}{2}y^{-7/6}$

3.4.8.60. $3y^{-3/8}\left(\dfrac{1}{4}y^{-1/4} + y^{3/4}\right)$

3.4.8.61. $(2x^{1/4} + 1)(x^{1/4} - 1)$

Answer. $2x^{1/2} - x^{1/4} - 1$

3.4.8.62. $(2x^{1/3} - 1)(x^{1/3} + 1)$

3.4.8.63. $(a^{3/4} - 2)^2$

Answer. $a^{3/2} - 4a^{3/4} + 4$

3.4.8.64. $(a^{2/3} + 3)^2$

For Problems 65–70, factor out the smallest power from each expression. Write your answers with positive exponents only.

3.4.8.65. $x^{3/2} + x = x(\ \ ?\ \ )$

Answer. $x(x^{1/2} + 1)$

3.4.8.66. $y - y^{2/3} = y^{2/3}(\ \ ?\ \ )$

3.4.8.67. $y^{3/4} - y^{-1/4} = y^{-1/4}(\ \ ?\ \ )$

Answer. $\dfrac{y - 1}{y^{1/4}}$

3.4.8.68. $x^{-3/2} + x^{-1/2} = x^{-3/2}(\ \ ?\ \ )$

3.4.8.69. $a^{1/3} + 3 - a^{-1/3} = a^{-1/3}(\ \ ?\ \ )$

Answer. $\dfrac{a^{2/3} + a^{1/3} - 1}{a^{1/3}}$

3.4.8.70. $3b - b^{3/4} + 4b^{-3/4} = b^{-3/4}(\ \ ?\ \ )$

3.5 Chapter Summary and Review

3.5.2 Chapter 3 Review Problems

3.5.2.1. The distance s a pebble falls through a thick liquid varies directly with the square of the length of time t it falls.

a If the pebble falls 28 centimeters in 4 seconds, express the distance it will fall as a function of time.

b Find the distance the pebble will fall in 6 seconds.

Answer.

a $d = 1.75t^2$

b 63 cm

3.5.2.2. The volume, V, of a gas varies directly with the temperature, T, and inversely with the pressure, P, of the gas.

a If $V = 40$ when $T = 300$ and $P = 30$, express the volume of the gas as a function of the temperature and pressure of the gas.

b Find the volume when $T = 320$ and $P = 40$.

3.5.2.3. The demand for bottled water is inversely proportional to the price per bottle. If Droplets can sell 600 bottles at \$8 each, how many bottles can the company sell at \$10 each?

Answer. 480 bottles

3.5.2.4. The intensity of illumination from a light source varies inversely with the square of the distance from the source. If a reading lamp has an intensity of 100 lumens at a distance of 3 feet, what is its intensity 8 feet away?

3.5.2.5. A person's weight, w, varies inversely with the square of his or her distance, r, from the center of the Earth.

a Express w as a function of r. Let k stand for the constant of variation.

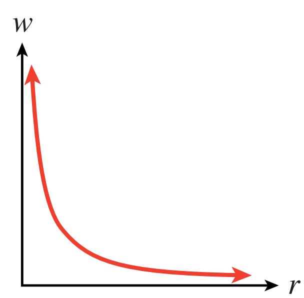
b Make a rough graph of your function.

c How far from the center of the Earth must Neil be in order to weigh one-third of his weight on the surface? The radius of the Earth is about 3960 miles.

Answer.

a $w = \dfrac{k}{r^2}$

b



c $3960\sqrt{3} \approx 6860$ miles

3.5.2.6. The period, T, of a pendulum varies directly with the square root of its length, L.

a Express T as a function of L. Let k stand for the constant of variation.

b Make a rough graph of your function.

c If a certain pendulum is replaced by a new one four-fifths as long as the old one, what happens to the period?

In Problems 7–10, y varies directly or inversely with a power of x. Find the power of x and the constant of variation, k. Write a formula for each function of the form $y = kx^n$ or $y = \dfrac{k}{x^n}$.

3.5.2.7.

x	y
2	4.8
5	30.0
8	76.8
11	145.2

Answer.
$y = 1.2x^2$

3.5.2.8.

x	y
1.4	75.6
2.3	124.2
5.9	318.6
8.3	448.2

3.5.2.9.

x	y
0.5	40.0
2.0	10.0
4.0	5.0
8.0	2.5

Answer.
$y = \dfrac{20}{x}$

3.5.2.10.

x	y
1.5	320.0
2.5	115.2
4.0	45.0
6.0	20.0

For Problems 11–16, write without negative exponents and simplify.

3.5.2.11.
a $(-3)^{-4}$ b 4^{-3}

Answer.

a $\dfrac{1}{81}$ b $\dfrac{1}{64}$

3.5.2.12.
a $\left(\dfrac{1}{3}\right)^{-2}$ b $\dfrac{3}{5^{-2}}$

3.5.2.13.
a $(3m)^{-5}$ b $-7y^{-8}$

Answer.

a $\dfrac{1}{243m^5}$ b $\dfrac{-7}{y^8}$

3.5.2.14.
a $a^{-1} + a^{-2}$ b $\dfrac{3q^{-9}}{r^{-2}}$

3.5.2.15.
a $6c^{-7} \cdot (3)^{-1}c^4$ b $\dfrac{11z^{-7}}{3^{-2}z^{-5}}$

Answer.

a $\dfrac{2}{c^3}$ b $\dfrac{99}{z^2}$

3.5.2.16.
a $\left(2d^{-2}k^3\right)^{-4}$ b $\dfrac{2w^3(w^{-2})^{-3}}{5w^{-5}}$

For Problems 17–20, write each power in radical form.

3.5.2.17.
a $25m^{1/2}$ b $8n^{-1/3}$

Answer.

a $25\sqrt{m}$ b $\dfrac{8}{\sqrt[3]{n}}$

3.5.2.18.
a $(13d)^{2/3}$ b $6x^{2/5}y^{3/5}$

3.5.2.19.
a $(3q)^{-3/4}$ b $7(uv)^{3/2}$

Answer.

a $\dfrac{1}{\sqrt[4]{27q^3}}$

b $7\sqrt{u^3v^3}$

3.5.2.20.

a $(a^2+b^2)^{0.5}$

b $(16-x^2)^{0.25}$

For Problems 21–24, write each radical as a power with a fractional exponent.

3.5.2.21.

a $2\sqrt[3]{x^2}$

b $\dfrac{1}{4}\sqrt[4]{x}$

Answer.

a $2x^{2/3}$

b $\dfrac{1}{4}x^{1/4}$

3.5.2.22.

a $z^2\sqrt{z}$

b $z\sqrt[3]{z}$

3.5.2.23.

a $\dfrac{6}{\sqrt[4]{b^3}}$

b $\dfrac{-1}{3\sqrt[3]{b}}$

Answer.

a $6b^{-3/4}$

b $\dfrac{-1}{3}b^{-1/3}$

3.5.2.24.

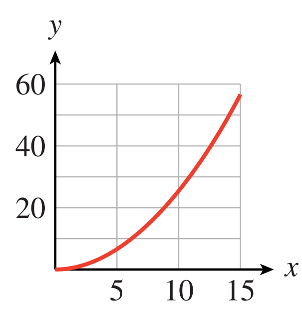
a $\dfrac{-4}{(\sqrt[4]{a})^2}$

b $\dfrac{2}{(\sqrt{a})^3}$

For Problems 25–28, sketch graphs by hand for each function on the domain $(0,\infty)$.

3.5.2.25. y varies directly with x^2. The constant of variation is $k=0.25$.

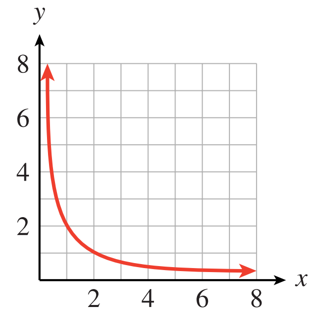
Answer.



3.5.2.26. y varies directly with x. The constant of variation is $k=1.5$.

3.5.2.27. y varies inversely with x. The constant of variation is $k=2$.

Answer.



3.5.2.28. y varies inversely with x^2. The constant of variation is $k = 4$.

For Problems 29–30, write each function in the form $y = kx^p$.

3.5.2.29. $f(x) = \dfrac{2}{3x^4}$

Answer. $f(x) = \dfrac{2}{3}x^{-4}$

3.5.2.30. $g(x) = \dfrac{8x^7}{29}$

For Problems 31–34,

(a) Evaluate each function for the given values.

(b) Graph the function.

3.5.2.31. $Q(x) = 4x^{5/2}$

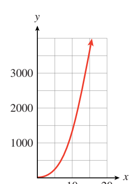
x	16	$\frac{1}{4}$	3	100
$Q(x)$				

Answer.

(a)

x	16	$\frac{1}{4}$	3	100
$Q(x)$	4096	$\frac{1}{8}$	$4\sqrt{3^5} \approx 62.35$	400,000

(b)



3.5.2.32. $T(w) = -3w^{2/3}$

w	27	$\frac{1}{8}$	20	1000
$T(w)$				

3.5.2.33. $f(x) = x^{0.3}$

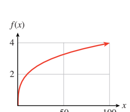
x	0	1	5	10	20	50	70	100
$f(x)$								

Answer.

(a)

x	0	1	5	10	20	50	70	100
$f(x)$	0	1	1.62	2.00	2.46	3.23	3.58	3.98

(b)



3.5.2.34. $g(x) = -x^{-0.7}$

x	0.1	0.2	0.5	1	2	5	8	10
$g(x)$								

3.5.2.35. According to the theory of relativity, the mass of an object traveling at velocity v is given by the function

$$m = \frac{M}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

where M is the mass of the object at rest and c is the speed of light. Find the mass of a man traveling at a velocity of $0.7c$ if his rest mass is 80 kilograms.

Answer. 112 kg

3.5.2.36. The cylinder of smallest surface area for a given volume has a radius and height both equal to $\sqrt[3]{\dfrac{V}{\pi}}$. Find the dimensions of the tin can of smallest surface area with volume 60 cubic inches.

3.5.2.37. Membership in the Wildlife Society has grown according to the function

$$M(t) = 30t^{3/4}$$

where t is the number of years since its founding in 1970.

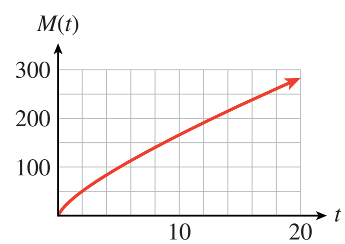
a Sketch a graph of the function $M(t)$.

b What was the society's membership in 1990?

c In what year will the membership be 810 people?

Answer.

a



b 283.7 or ≈ 284

c 2051

3.5.2.38. The heron population in Saltmarsh Refuge is estimated by conservationists at

$$P(t) = 360t^{-2/3}$$

where t is the number of years since the refuge was established in 1990.

a Sketch a graph of the function $P(t)$.

b How many heron were there in 1995?

c In what year will there be only 40 heron left?

3.5.2.39. Manufacturers of ships (and other complex products) find that the average cost of producing a ship decreases as more of those ships are produced. This relationship is called the **experience curve**, given by the equation

$$C = ax^{-b}$$

where C is the average cost per ship in millions of dollars and x is the number of ships produced. The value of the constant b depends on the complexity of the ship. (Source: Storch, Hammon, and Bunch, 1988)

a What is the significance of the constant of proportionality a?

b For one kind of ship, $b = \frac{1}{8}$, and the cost of producing the first ship is \$12 million. Write the equation for C as a function of x using radical notation.

c Compute the cost per ship when 2 ships have been built. By what percent does the cost per ship decrease? By what percent does the cost per ship decrease from building 2 ships to building 4 ships?

d By what percent does the average cost decrease from building n ships to building $2n$ ships? (In the shipbuilding industry, the average cost per ship usually decreases by 5 to 10% each time the number of ships doubles.)

Hint. What is the value of C if only one ship is built?

Answer.

a It is the cost of producing the first ship.

b $C = \dfrac{12}{\sqrt[8]{x}}$ million

c About \$11 million; about 8.3%

d About 8.3%

3.5.2.40. A population is in a period of **supergrowth** if its rate of growth, R, at any time is proportional to P^k, where P is the population at that time and k is a constant greater than 1. Suppose R is given by

$$R = 0.015P^{1.2}$$

where P is measured in thousands and R is measured in thousands per year.

a Find R when $P = 20$, when $P = 40$, and when $P = 60$.

b What will the population be when its rate of growth is 5000 per year?

c Graph R and use your graph to verify your answers to parts (a) and (b).

For Problems 41–50, solve

3.5.2.41. $6t^{-3} = \dfrac{3}{500}$

Answer. $t = 10$

3.5.2.42. $3.5 - 2.4p^{-2} = -6.1$

3.5.2.43. $\sqrt[3]{x+1} = 2$

Answer. $x = 7$

3.5.2.44. $x^{2/3} + 2 = 6$

3.5.2.45. $(x-1)^{-3/2} = \dfrac{1}{8}$

Answer. $x = 5$

3.5.2.46. $(2x+1)^{-1/2} = \dfrac{1}{3}$

3.5.2.47. $8\sqrt[4]{x+6} = 24$

Answer. $x = 75$

3.5.2.48. $9.8 = 7\sqrt[3]{z-4}$

3.5.2.49. $\dfrac{2}{3}(2y+1)^{0.2} = 6$

Answer. $y = 29,524$

3.5.2.50. $1.3w^{0.3} + 4.7 = 5.2$

For Problems 51–54, solve each formula for the indicated variable.

3.5.2.51. $t = \sqrt{\dfrac{2v}{g}}$, for g

Answer. $g = \dfrac{2v}{t^2}$

3.5.2.52. $q - 1 = 2\sqrt{\dfrac{r^2 - 1}{3}}$, for r

3.5.2.53. $R = \dfrac{1 + \sqrt{p^2 + 1}}{2}$, for p

Answer. $p = \pm 2\sqrt{R^2 - R}$

3.5.2.54. $q = \sqrt[3]{\dfrac{1 + r^2}{2}}$, for r

For Problems 55–60, simplify by applying the laws of exponents.

3.5.2.55. $(7t)^3(7t)^{-1}$

Answer. $49t^2$

3.5.2.56. $\dfrac{36r^{-2}s}{9r^{-3}s^4}$

3.5.2.57. $\dfrac{(2k^{-1})^{-4}}{4k^{-3}}$

Answer. $\dfrac{k^7}{64}$

3.5.2.58. $(2w^{-3})(2w^{-3})^5(-5w^2)$

3.5.2.59. $\dfrac{8a^{-3/4}}{a^{-11/4}}$

Answer. $8a^2$

3.5.2.60. $b^{2/3}(4b^{-2/3} - b^{1/3})$

3.5.2.61. When the Concorde landed at Heathrow Airport in London, the width, w, of the sonic boom felt on the ground is given in kilometers by the following formula:

$$w = 4\left(\frac{Th}{m}\right)^{1/2}$$

where T stands for the temperature on the ground in kelvins, h is the altitude of the Concorde when it breaks the sound barrier, and m is the drop in temperature for each gain in altitude of one kilometer.

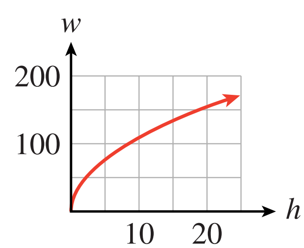
a Find the width of the sonic boom if the ground temperature was 293 K, the altitude of the Concorde was 15 kilometers, and the temperature drop was 4 K per kilometer of altitude.

b Graph w as a function of h if $T = 293$ and $m = 4$.

Answer.

(a) 132.6 km

(b)



3.5.2.62. The manager of an office supply store must decide how many of each item in stock she should order. The Wilson lot size formula gives the most cost-efficient quantity, Q, as a function of the cost, C, of placing an order, the number of items, N, sold per week, and the weekly inventory cost, I, per item (cost of storage, maintenance, and so on).

$$Q = \left(\frac{2CN}{I}\right)^{1/2}$$

a How many reams of computer paper should she order if she sells on average 80 reams per week, the weekly inventory cost for a ream is \$0.20, and the cost of ordering, including delivery charges, is \$25?

b Graph Q as a function of N if $C = 25$ and $I = 0.2$.

3.5.2.63. Two businesswomen start a small company to produce saddle bags for bicycles. The number of saddle bags, q, they can produce depends on the amount of money, m, they invest and the number of hours of labor, w, they employ, according to the Cobb-Douglas formula

$$q = 0.6m^{1/4}w^{3/4}$$

where m is measured in thousands of dollars.

a If the businesswomen invest \$100,000 and employ 1600 hours of labor in their first month of production, how many saddle bags can they expect to produce?

b With the same initial investment, how many hours of labor would they need in order to produce 200 saddle bags?

Answer.

(a) 480

(b) 498

3.5.2.64. A child who weighs w pounds and is h inches tall has a surface area (in square inches) given approximately by

$$S = 8.5h^{0.35}w^{0.55}$$

a What is the surface area of a child who weighs 60 pounds and is 40 inches tall?

b What is the weight of a child who is 50 inches tall and whose surface area is 397 square inches?

3.5.2.65. The cost, C, of insulating the ceiling in a building depends on the thickness of the insulation and the area of the ceiling. The table shows values of $C = f(t, A)$, where t is the thickness of the insulation and A is the area of the ceiling.

Cost of Insulation (dollars)						
	Area (sq m)					
Thickness (cm)	100	200	300	400	500	600
4	72	144	216	288	300	432
5	90	180	270	360	450	540
6	108	216	324	432	540	648
7	126	252	378	504	630	756
8	144	288	432	576	720	864
9	162	324	486	648	810	972

a What does it cost to insulate a ceiling with an area of 500 square meters with 5 cm of insulation? Write your answer in function notation.

b Solve the equation $864 = f(t, 600)$ and interpret your answer.

c Consider the row corresponding to a thickness of 4 cm. How does the cost of insulating the ceiling depend on the area of the ceiling?

d Consider the column corresponding to an area of 100 square meters. How does the cost depend on the thickness of the insulation?

e Given that the cost varies jointly with the thickness of the insulation and the area of the ceiling, write an equation for cost as a function of area and thickness of insulation.

f Use your formula from part (e) to determine the cost of insulating a building with 10 centimeters of insulation if the area of the ceiling is 800 square meters.

Answer.

a \$450

b $t = 8$: It costs \$864 to insulate a ceiling with 8 cm of insulation over an area of 600 square meters.

c $C = 0.72A$

d $C = 18T$

e $C = 0.18AT$

f \$1440

3.5.2.66. The volume, V, of a quantity of helium depends on both the temperature and the pressure of the gas. The table shows values of $V = f(P,T)$ for temperature in kelvins and pressure in atmospheres.

Volume (cubic meters)						
	Temperature (K)					
Pressure (atmospheres)	100	150	200	250	300	350
1	18	27	36	45	54	63
2	9	13.5	18	22.5	27	31.5
3	6	9	12	15	18	21
4	4.5	6.75	9	11.25	13.5	15.75

a What is the volume of helium when the pressure is 4 atmospheres and the temperature is 350 K? Write your answer in function notation.

b Solve the equation $15 = f(3,T)$ and interpret your answer.

c Consider the row corresponding to 2 atmospheres. How is the volume related to the absolute temperature?

d Consider the column corresponding to 300 K. How is the volume related to the pressure?

e Given that the volume of the gas varies directly with temperature and inversely with pressure, write an equation for volume as a function of temperature and pressure.

f Use your formula from part (e) to determine the volume of the helium at 50 K and pressure of 0.4 atmospheres.

3.5.2.67. In his hiking guidebook, *Afoot and Afield in Los Angeles County*, Jerry Schad notes that the number of people on a wilderness trail is inversely proportional to "the square of the distance and the cube of the elevation gain from the nearest road."

a Choose variables and write a formula for this relationship.

b On a sunny Saturday afternoon, you count 42 people enjoying the Rock Pool at Malibu Creek State Park. The Rock Pool is 1.5 miles from the main parking lot, and the trail includes an elevation gain of 250 feet. Calculate the constant of variation in your formula from part (a).**Hint**. Hint: Convert the elevation gain to miles.

c Lookout Trail leads 1.9 miles from the parking lot and involves an elevation gain of 500 feet. How many people would you expect to encounter at the end of the trail?

Answer.

a $N = \dfrac{k}{d^2E^3}$, where N is number of people, d is distance in miles from the road, E is the elevation gain, and k is the constant of variation.

b $k \approx 0.01$

c 3

3.5.2.68. A company's monthly production, P, depends on the capital, C, the company has invested and the amount of labor, L, available each month. The Cobb-Douglas model for production assumes that P varies jointly with C^a and L^b, where a and b are positive constants less than 1. The Aztech Chip Company invested 625 units of capital and hired 256 workers and produces 8000 computer chips each month.

a Suppose that $a = 0.25$, $b = 0.75$. Find the constant of variation and a formula giving P in terms of C and L.

b If Aztech increases its labor force to 300 workers, what production level can they expect?

c If Aztech maintains its labor force at 256 workers, what amount of capital outlay would be required for monthly production to reach $16,000$ computer chips?

3.6 Projects for Chapter 3

Project 3.6.1 Wien's Law. A hot object such as a light bulb or a star radiates energy over a range of wavelengths, but the wavelength with maximum energy is inversely proportional to the temperature of the object. If temperature is measured in kelvins, and wavelength in micrometers, the constant of proportionality is 2898. (One micrometer is one thousandth of a millimeter, or $1\mu m = 10^{-6}$ meter.)

a Write a formula for the wavelength of maximum energy, λ_{max}, as a function of temperature, T. This formula, called Wien's law, was discovered in 1894.

b Our sun's temperature is about 5765 K. At what wavelength is most of its energy radiated?

c The color of light depends on its wavelength, as shown in the table. Can you explain why the sun does not appear to be green? Use Wien's law to describe how the color of a star depends on its temperature.

Color	Wavelength (μm)
Red	0.64 – 0.74
Orange	0.59 – 0.64
Yellow	0.56 – 0.59
Green	0.50 – 0.56
Blue	0.44 – 0.50
Violet	0.39 – 0.44

d Astronomers cannot measure the temperature of a star directly, but they can determine the color or wavelength of its light. Write a formula for T as a function of λ_{max}.

e Estimate the temperatures of the following stars, given the approximate value of λ_{max} for each.

Star	λ_{max}	Temperature
R Cygni	1.115	
Betelgeuse	0.966	
Arcturus	0.725	
Polaris	0.414	
Sirius	0.322	
Rigel	0.223	

f Sketch a graph of T as a function of λ_{max} and locate each star on the graph.

Project 3.6.2 Halley's Comet. Halley's comet which orbits the sun every 76 years, was first observed in 240 B.C. Its orbit is highly elliptical, so that its closest approach to the Sun (**perihelion**) is only 0.587 AU, while at its greatest distance (**aphelion**) the comet is 34.39 AU from the Sun. (An AU, or astronomical unit, is the distance from the Earth to the Sun, 1.5×10^8 kilometers.)

a Calculate the distances in meters from the Sun to Halley's comet at perihelion and aphelion.

b Halley's comet has a volume of 700 cubic kilometers, and its density is about 0.1 gram per cubic centimeter. Calculate the mass of the comet in kilograms.

c The gravitational force (in newtons) exerted by the Sun on its satellites is inversely proportional to the square of the distance to the satellite in meters. The constant of variation is Gm_1m_2, where $m_1 = 1.99 \times 10^{30}$ kilograms is the mass of the Sun, m_2 is the mass of the satellite, and $G = 6.67 \times 10^{-11}$ is the gravitational constant. Write a formula for the force, F, exerted by the sun on Halley's comet at a distance of d meters.

d Calculate the force exerted by the sun on Halley's comet at perihelion and at aphelion.

Project 3.6.3 World Records. Are world record times for track events proportional to the length of the race? The table gives the men's and women's world records in 2005 for races from 1 kilometer to 100 kilometers in length.

Distance (km)	Men's record (min)	Women's record (min)
1	2.199	2.483
1.5	3.433	3.841
2	4.747	5.423
3	7.345	8.102
5	12.656	14.468
10	26.379	29.530
20	56.927	65.443
25	73.93	87.098
30	89.313	105.833

a On separate graphs, plot the men's and women's times against distance. Does time appear to be proportional to distance?

b Use slopes to decide whether the graphs of time versus distance are in fact linear.

c Both sets of data can be modeled by power functions of the form $t = kx^b$, where b is called the **fatigue index**. Graph the function $M(x) = 2.21x^{1.086}$ over the men's data points, and $W(x) = 2.46x^{1.099}$ over the women's data. Describe how the graphs of the two functions differ. Explain why b is called the fatigue index.

Project 3.6.4 Naismith's Number. Fell running is a popular sport in the hills, or fells, of the British Isles. Fell running records depend on the altitude gain over the course of the race as well as its length. The equivalent horizontal distance for a race of length x kilometers with an ascent of y kilometers is given by $x+Ny$, where N is Naismith's number (see Project 1.7.5). The record times for women's races are approximated in minutes by $t = 2.43(x + 9.5y)^{1.15}$, and men's times by $t = 2.18(x + 8.0y)^{1.14}$. (Source: Scarf, 1998)

a Whose times show a greater fatigue index, men or women? (See Project 3.6.3.)

b Whose times are more strongly affected by ascents?

c Predict the winning times for both men and women in a 56-kilometer race with an ascent of 2750 meters.

Project 3.6.5 Elasticity. Elasticity is the property of an object that causes it to regain its original shape after being compressed or deformed. One measure of elasticity considers how high the object bounces when dropped onto a hard surface,

$$e = \sqrt{\frac{\text{height bounced}}{\text{height dropped}}}$$

(Source: Davis, Kimmet, and Autry, 1986)

a The table gives the value of e for various types of balls. Calculate the bounce height for each ball when it is dropped from a height of 6 feet onto a wooden floor.

Type of ball	Bounce height	e
Baseball		0.50
Basketball		0.75
Golfball		0.60
Handball		0.80
Softball		0.55
Superball		0.90
Tennisball		0.74
Volleyball		0.75

b Write a formula for e in terms of H, the bounce height, for the data in part (a).

c Graph the function from part (b).

d If Ball A has twice the elasticity of Ball B, how much higher will Ball A bounce than Ball B?

Project 3.6.6 Mersenne's Laws. The tone produced by a vibrating string depends on the frequency of the vibration. The frequency in turn depends on the length of the string, its weight, and its tension. In 1636, Marin Mersenne quantified these relationships as follows. The frequency, f, of the vibration is

i inversely proportional to the string's length, L,

ii directly proportional to the square root of the string's tension, T, and

iii inversely proportional to the square root of the string's weight per unit length, w. (Source: Berg and Stork, 1982)

a Write a formula for f that summarizes Mersenne's laws.

b Sketch a graph of f as a function of L, assuming that T and w are constant. (You do not have enough information to put scales on the axes, but you can show the shape of the graph.)

c On a piano, the frequency of the highest note is about 4200 hertz. This frequency is 150 times the frequency of the lowest note, at about 28 hertz. Ideally, only the lengths of the strings should change, so that all the notes have the same tonal quality. If the string for the highest note is 5 centimeters long, how long should the string for the lowest note be?

d Sketch a graph of f as a function of T, assuming that L and w are constant

e Sketch a graph of f as a function of w, assuming that L and T are constant.

f The tension of all the strings in a piano should be about the same to avoid warping the frame. Suggest another way to produce a lower note.**Hint**. Look at a piano's strings.

g The longest string on the piano in part (c) is 133.5 cm long. How much heavier (per unit length) is the longest string than the shortest string?

Project 3.6.7 Damuth's Formula. In 1981, John Damuth collected data on the average body mass, m, and the average population density, D, for 307 species of herbivores. He found that, very roughly,

$$D = km^{-0.75}$$

(Source: Burton, 1998)

a Explain why you might expect an animal's rate of food consumption to be proportional to its metabolic rate. (See Example 3.4.4 in Section 3.4 for an explanation of metabolic rate.)

b Explain why you might expect the population density of a species to be inversely proportional to the rate of food consumption of an individual animal.

c Use Kleiber's rule and your answers to parts (a) and (b) to explain why Damuth's proposed formula for population density is reasonable.

d Sketch a graph of the function D. You do not have enough information to put scales on the axes, but you can show the shape of the graph.**Hint**. Graph the function for $k = 1$.

Project 3.6.8 Self-thinning Law. Studies on pine plantations in the 1930s showed that as the trees grow and compete for space, some of the die, so that the density of trees per unit area decreases. The average mass of an individual tree is a power function of the density, d, of the trees per unit area, given by

$$M(d) = kd^{-1.5}$$

This formula is known as the $\dfrac{-3}{2}$ **self-thinning law**. (Source: Chapman and Reiss, 1992)

a To simplify the calculations, suppose that a pine tree is shaped like a tall circular cone and that as it grows, its height is always a constant multiple of its base radius, r. Explain why the base radius of the tree is proportional to the square root of the area the tree covers. Write r as a power function of d.

b Write a formula for the volume of the tree in terms of its base radius, r. Use part (b) to write the volume as a power function of d.

c The mass (or weight) of a pine tree is roughly proportional to its volume, and the area taken up by a single tree is inversely proportional to the plant density, d. Use these facts to justify the self-thinning law.

d Sketch a graph of the function M. You do not have enough information to put scales on the axes, but you can show the shape of the graph.**Hint**. Graph the function for $k = 1$.

4 Exponential Functions

4.1 Exponential Growth and Decay

4.1.1 Exponential Growth

Checkpoint 4.1.3 A population of 24 fruit flies triples every month.

a Write a formula for the population of fruit flies after t weeks.

b How many fruit flies will there be after 6 months? After 3 weeks? (Assume that a month equals 4 weeks.)

Answer.

a $P(t) = 24 \cdot 3^t$

b $17,496$; 55

4.1.2 Growth Factors

Checkpoint 4.1.5 In 1999, analysts expected the number of Internet service providers to double in five years.

a What was the annual growth factor for the number of Internet service providers?

b If there were 5078 Internet service providers in April 1999, estimate the number of providers in April 2000 and in April 2001.

c Write a formula for $I(t)$, the number of Internet service providers t years after 1999.

Source: LA Times, Sept. 6, 1999

Answer.

a $2^{1/5}$

b 5833 and 6700

c $I(t) = 5078 \cdot 2^{t/5}$

4.1.3 Percent Increase

Checkpoint 4.1.8 In 1998, the average annual cost of attending a public college was $\$10,069$, and costs were climbing by 6% per year.

a Write a formula for $C(t)$, the cost of one year of college t years after 1998.

b Complete the table and sketch a graph of $C(t)$.

t	0	5	10	15	20	25
$C(t)$						

c If the percent growth rate remained steady, how much did a year of college cost in 2005?

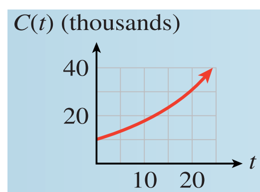
d If the percent growth rate continues to remain steady, how much will a year of college cost in 2020?

Answer.

a $C(t) = 10,069 \cdot 1.06^t$

b

t
0
5
10
15
20
25
$C(t)$
$10,069$
$13,475$
$18,032$
$24,131$
$32,293$
$43,215$



c $\$15,140$ per year

d $\$36,284$

4.1.4 Exponential Decay

Investigation 4.1.1 Exponential Decay.

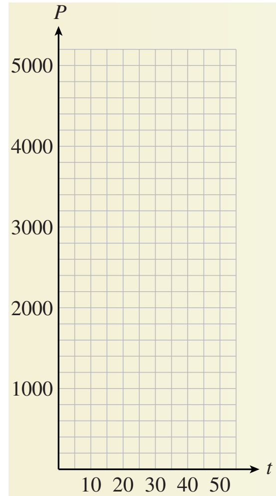
A A small coal-mining town has been losing population since 1940, when 5000 people lived there. At each census thereafter (taken at 10-year intervals), the population declined to approximately 0.90 of its earlier figure.

t	$P(t)$
0	5000
10	
20	
30	
40	
50	

$P(0) = 5000$
$P(10) = 5000 \cdot 0.90 =$
$P(20) = [5000 \cdot 0.90] \cdot 0.90 =$
$P(3) =$
$P(4) =$
$P(5) =$

1 Fill in the table showing the population $P(t)$ of the town t years after 1940.

2 Plot the data points and connect them with a smooth curve.



3 Write a function that gives the population of the town at any time t in years after 1940. **Hint**. Express the values you calculated in part (1) using powers of 0.90. Do you see a connection between the value of t and the exponent on 0.90?

4 Graph your function from part (3) using a calculator. (Use the table to choose an appropriate domain and range.) The graph should resemble your hand-drawn graph from part (2).

5 Evaluate your function to find the population of the town in 1995. What was the population in 2000?

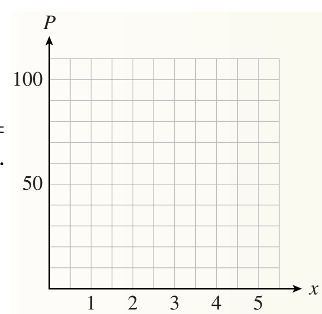
B A plastic window coating 1 millimeter thick decreases the light coming through a window by 25%. This means that 75% of the original amount of light comes through 1 millimeter of the coating. Each additional millimeter of coating reduces the light by another 25%.

1 Fill in the table showing the percent of the light, $P(x)$, that shines through x millimeters of the window coating.

2 Plot the data points and connect them with a smooth curve.

x	$P(x)$
0	100
1	
2	
3	
4	
5	

$P(0) = 100$
$P(1) = 100 \cdot 0.75 =$
$P(2) = [100 \cdot 0.75] \cdot$
$P(3) =$
$P(4) =$
$P(5) =$



3 Write a function that gives the percent of the light that shines through x millimeters of the coating. **Hint**. Express the values you calculated in part (1) using powers of 0.75. Do you see a connection between the value of x and the exponent on 0.75?

4 Graph your function from part (3) using a calculator. (Use your table of values to choose an appropriate domain and range.) The graph should resemble your hand-drawn graph from part (2).

5 Evaluate your function to find the percent of the light that comes through 6 millimeters of plastic coating. What percent comes through $\frac{1}{2}$ millimeter?

4.1.5 Decay Factors

Checkpoint 4.1.11 The number of butterflies visiting a nature station is declining by 18% per year. In 1998, 3600 butterflies visited the nature station.

a What is the decay factor in the annual butterfly count?

b Write a formula for $B(t)$, the number of butterflies t years after 1998.

c Complete the table and sketch a graph of $B(t)$.

t	0	2	4	6	8	10
$B(t)$						

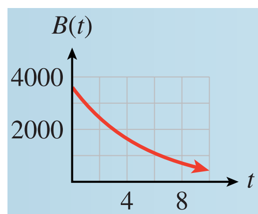
Answer.

a 0.82

b $B(t) = 3600 \cdot 0.82^t$

c

t	0	2	4	6	8	10
$B(t)$	3600	2421	1628	1094	736	495



4.1.6 Comparing Linear Growth and Exponential Growth

Checkpoint 4.1.13 A new car begins to depreciate in value as soon as you drive it off the lot. Some models depreciate linearly, and others depreciate exponentially. Suppose you buy a new car for $\$20,000$, and 1 year later its value has decreased to $\$17,000$.

a If the value decreased linearly, what was its annual rate of decrease?

b If the value decreased exponentially, what was its annual decay factor? What was its annual percent depreciation?

c Calculate the value of your car when it is 5 years old under each assumption, linear or exponential depreciation.

Answer.

a \$3000 per year

b 0.85; 15%

c Linear: \$5000; Exponential: \$8874

4.1.8 Homework 4.1

4.1.8.1.

(a) A parking permit at Huron College cost \$25 last year, but this year the price increased by 12%. What is the price this year?

(b) If the price of a parking permit increases by 12% again next year, what will the price be then?

Answer.

(a) \$28 (b) \$31.36

4.1.8.2.

(a) The computer you want cost \$1200 when it first came on the market, but after 3 months the price was reduced by 15%. What was the price then?

(b) If the price falls by another 15% next month, what will the price be then?

4.1.8.3. The value of your stock portfolio fell 10% last year, but this year it increased by 10%. How does the current value of your portfolio compare to what it was two years ago?

Answer. It is 99% of what it was 2 years ago.

4.1.8.4. You got a 5% raise in January, but then in March everyone took a pay cut of 5%. How does your new salary compare to what it was last December?

4.1.8.5. The population of Summerville is currently 12 hundred people.

(a) Write a formula for the population if it grows at a constant rate of 1.5 hundred people per year. What is the population after 3 years?

(b) Write a formula for the population if it has a constant growth factor of 1.5 per year. What is the population after 3 years?

Answer.

(a) $P = 1200 + 150t$; 1650 (b) $P = 1200 \cdot 1.5^t$; 4050

4.1.8.6. Delbert's sports car was worth \$45,000 when he bought it.

(a) Write a formula for the value of the car if it depreciates at a constant rate of \$7000 per year. What is the value of the car after 4 years?

(b) Write a formula for the value of the car if it has a constant depreciation factor of 0.70 per year. What is the value of the car after 4 years?

4.1.8.7. Francine's truck was worth \$18,000 when she bought it.

(a) Write a formula for the value of the truck if it depreciates by \$2000 per year. What is the value of the truck after 5 years?

(b) Write a formula for the value of the truck if it depreciates by 20% per year. What is the value of the truck after 5 years?

Answer.

(a) $V = 18,000 - 2000t$; \$8000 (b) $V = 18,000 \cdot 0.8^t$; \$5898.24

4.1.8.8. The population of Lakeview is currently 150,000 people.

(a) Write a formula for the population if it grows by 6000 people per year. What is the population after 2 years?

(b) Write a formula for the population if grows by 6% per year. What is the population after 2 years?

4.1.8.9. The table shows the growth factor for a number of different populations. For each population, find the percent growth rate.

Population	A	B	C	D	E
Growth factor	1.2	1.02	1.075	2.0	2.15
Percent growth rate					

Answer. A: 20%; B: 2%; C: 7.5%; D: 100%; E: 115%

4.1.8.10. The table shows the decay factor for a number of different populations. For each population, find the percent growth rate.

Population E	A	B	C	D
Decay factor 0.096	0.6	0.06	0.96	0.996
Percent decay rate				

For Problems 11–16,

a Write a function that describes exponential growth.

b Graph the function.

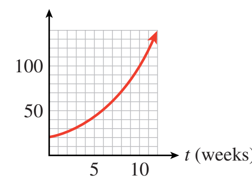
c Evaluate the function at the given values.

4.1.8.11. A typical beehive contains 20,000 insects. The population can increase in size by a factor of 2.5 every 6 weeks. How many bees could there be after 4 weeks? After 20 weeks?

Answer.

a $P = 20,000 \cdot 2.5^{t/6}$

b



c 36,840 bees; 424,128 bees

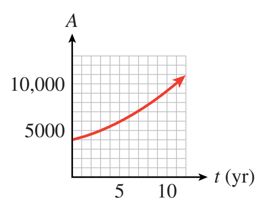
4.1.8.12. A rancher who started with 800 head of cattle finds that his herd increases by a factor of 1.8 every 3 years. How many head of cattle will he have after 1 year? After 10 years?

4.1.8.13. A sum of \$4000 is invested in an account that pays 8% interest compounded annually. How much is in the account after 2 years? After 10 years?

Answer.

a $A = 4000 \cdot 1.08^t$

b



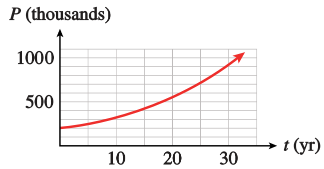
c \$4665.60; \$8635.70

4.1.8.14. Otto invests \$600 in an account that pays 7.3% interest compounded annually. How much is in Otto's account after 3 years? After 6 years?

4.1.8.15. Paul bought a house for \$200,000 in 2003. Since 2003, housing prices have risen an average of 5% per year. How much was the house worth in 2015? How much will it be worth in 2030?

Answer.

a $P = 200,000 \cdot 1.05^t$

b 

c $\$359,171$; $\$746,691$

4.1.8.16. Sales of Windsurfers have increased 12% per year since 2010. If Sunsails sold 1500 Windsurfers in 2010, how many did it sell in 2015? How many should it expect to sell in 2022?

For Problems 17–22,

a Write a function that describes exponential decay.

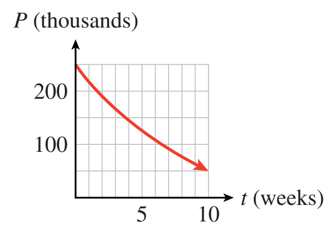
b Graph the function.

c Evaluate the function at the given values.

4.1.8.17. During a vigorous spraying program, the mosquito population was reduced to $\dfrac{3}{4}$ of its previous size every 2 weeks. If the mosquito population was originally estimated at $250,000$, how many mosquitoes remained after 3 weeks of spraying? After 8 weeks?

Answer.

a $P = 250,000 \cdot 0.75^{t/2}$

b 

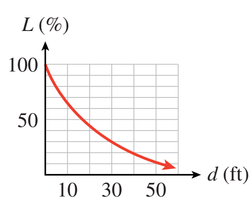
c $162,380$; $79,102$

4.1.8.18. The number of perch in Hidden Lake has declined to half of its previous value every 5 years since 1985, when the perch population was estimated at 8000. How many perch were there in 1995? In 2013?

4.1.8.19. Scuba divers find that the water in Emerald Lake filters out 15% of the sunlight for each 4 feet that they descend. How much sunlight penetrates to a depth of 20 feet? To a depth of 45 feet?

Answer.

a $L = 0.85^{d/4}$

b 

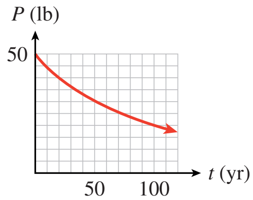
c 44%; 16%

4.1.8.20. Arch's motorboat cost $\$15,000$ in 2005 and has depreciated by 10% every 3 years. How much was the boat worth in 2014? In 2015?

4.1.8.21. Plutonium-238 is a radioactive element that decays over time into a less harmful element at a rate of 0.8% per year. A power plant has 50 pounds of plutonium-238 to dispose of. How much plutonium-238 will be left after 10 years? After 100 years?

Answer.

a $P = 50 \cdot 0.992^t$

b 

c 46.1 lb; 22.4 lb

4.1.8.22. Iodine-131 is a radioactive element that decays at a rate of 8.3% per day. How much of a 12-gram sample will be left after 1 week? After 15 days?

In Problems 23–26, use the laws of exponents to simplify.

4.1.8.23.

a $3^x\, 3^4$ b $(3^x)^4$ c $3^x\, 4^x$

Answer.

a 3^{x+4} b 3^{4x} c 12^x

4.1.8.24.

a $8^x\, 8^x$ b $8^{x+2}\, 8^{x-1}$ c $\dfrac{8^{2x}}{8^x}$

4.1.8.25.

a $b^{-4t}\, b^{2t}$ b $(b^t)^{1/2}$ c $b^{t-1}\, b^{1-t}$

Answer.

a b^{-2t} b $b^{t/2}$ c 1

4.1.8.26.

a $b^{t/2}\, b^{t/2}$ b $\dfrac{b^{2t}}{b}$ c $b^{1/t}\, b^t$

4.1.8.27. Let $P(t) = 12(3)^t$. Show that $P(t+1) = 3P(t)$.

Answer. $P(t+1) = 12(3)^{t+1} = 12(3)^t \cdot 3 = P(t) \cdot 3$

4.1.8.28. Let $N(t) = 8(5)^t$. Show that $\dfrac{N(t+k)}{N(t)} = 5^k$

4.1.8.29. Let $P(x) = P_0 a^x$. Show that $P(x+k) = a^k P(x)$.

Answer. $P(x+k) = P_0 a^{x+k} = P_0 a^x \cdot a^k = P(x) \cdot a^k$

4.1.8.30. Let $N(x) = N_0 b^x$. Show that $\dfrac{N(x+1)}{N(x)} = b$

4.1.8.31.

a Explain why $P(t) = 2 \cdot 3^t$ and $Q(t) = 6^t$ are not the same function.

b Complete the table of values for P and Q, showing that their values are not the same.

t	0	1	2
$P(t)$			
$Q(t)$			

Answer.

a In the expression $2 \cdot 3^t$, only the 3 is raised to a power t, and the result is doubled, but if both the 2 and the 3 were raised to the power t, the result would be 6^t.

b

t	0	1	2
$P(t)$	2	6	18
$Q(t)$	1	6	36

4.1.8.32.

a Explain why $P(t) = 4 \cdot \left(\dfrac{1}{2}\right)^t$ and $Q(t) = 2^t$ are not the same function.

b Complete the table of values for P and Q, showing that their values are not the same.

t	0	1	2
$P(t)$			
$Q(t)$			

Solve each equation. (See Section 3.3 to review solving equations involving powers of the variable.) Round your answer to two places if necessary.

4.1.8.33. $768 = 12b^3$

Answer. 4

4.1.8.34. $75 = 3b^4$

4.1.8.35. $14,929.92 = 5000b^6$

Answer. 1.2

4.1.8.36. $151,875 = 20,000b^5$

4.1.8.37. $1253 = 260(1 + r)^{12}$

Answer. $r \approx 0.14$

4.1.8.38. $116,473 = 48,600(1 + r)^{15}$

4.1.8.39. $56.27 = 78(1 - r)^8$

Answer. $r \approx 0.04$

4.1.8.40. $10.56 = 12.4(1 - r)^{20}$

4.1.8.41.

a Riverside County is the fastest growing county in California. In 2000, the population was $1,545,387$. Write a formula for the population of Riverside County. (You do not know the value of the growth factor, b, yet.)

b In 2004, the population had grown to $1,871,950$. Find the growth factor and the percent rate of growth, rounded to the nearest tenth of a percent.

c Estimate the population of Riverside County in 2010.

Answer.

a $P(t) = 1,545,387b^t$

b Growth factor 1.049; Percent rate of growth 4.9%

c $2,493,401$

4.1.8.42.

a In 2006, a new Ford Focus cost \$15,574. The value of a Focus decreases exponentially over time. Write a formula for the value of a Focus. (You do not know the value of the decay factor, b, yet.)

b A 2-year old Focus cost \$11,788. Find the decay factor and the percent rate of depreciation, rounded to the nearest tenth of a percent.

c About how much would a 4-year old Focus cost?

4.1.8.43. In the 1940s, David Lack undertook a study of the European robin. He tagged 130 one-year-old robins and found that on average 35.6% of the birds survived each year. (Source: Burton, 1998)

a According to the data, how many robins would have originally hatched to produce 130 one-year-olds?

b Write a formula for the number of the original robins still alive after t years.

c Graph your function.

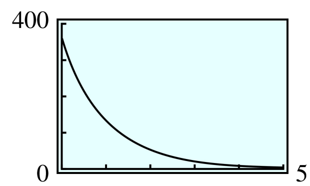
d One of the original robins actually survived for 9 years. How many robins does the model predict will survive for 9 years?

Answer.

a 365

b $N(t) = 365(0.356)^t$

c



d 0.03. (Therefore, none)

4.1.8.44. Many insects grow by discrete amounts each time they shed their exoskeletons. Dyar's rule says that the size of the insect increases by a constant ratio at each stage. (Source: Burton, 1998)

a Dyar measured the width of the head of a caterpillar of a swallowtail butterfly at each stage. The caterpillar's head was initially approximately 42 millimeters wide, and 63.84 millimeters wide after its first stage. Find the growth ratio.

b Write a formula for the width of the caterpillar's head at the nth stage.

c Graph your function.

d What head width does the model predict after 5 stages?

For Problems 45–54,

a Each table describes exponential growth or decay. Find the growth or decay factor.

b Complete the table. Round values to two decimal places if necessary.

4.1.8.45.

t	0	1	2	3	4
P	8	12	18		

Answer. The growth factor is 1.5.

t	0	1	2	3	4
P	8	12	18	27	40.5

4.1.8.46.

t	0	1	2	3	4
P	4	5	6.25		

4.1.8.47.

x	0	1	2	3	4
Q	20	24			

Answer. The growth factor is 1.2.

x	0	1	2	3	4
Q	20	24	28.8	34.56	41.47

4.1.8.48.

x	0	1	2	3	4
Q	100	105			

4.1.8.49.

w	0	1	2	3	4
N	120	96			

Answer. The decay factor is 0.8.

w	0	1	2	3	4
N	120	96	76.8	61.44	49.15

4.1.8.50.

w	0	1	2	3	4
N	640	480			

4.1.8.51.

t	0	1	2	3	4
C	10		6.4		

Answer. The decay factor is 0.8.

t	0	1	2	3	4
C	10	8	6.4	5.12	4.10

4.1.8.52.

t	0	1	2	3	4
C	20			2.5	

4.1.8.53.

n	0	1	2	3	4
B	200			266.2	

Answer. The growth factor is 1.1.

n	0	1	2	3	4
B	200	220	242	266.2	292.82

4.1.8.54.

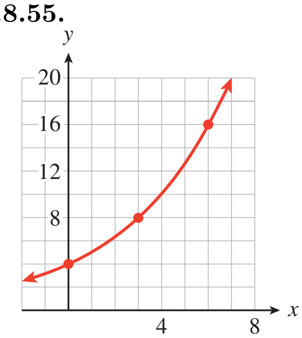
n	0	1	2	3	4
B	40		62.5		

Each graph in Problems 55–58 represents exponential growth or decay.

(a) Find the initial value and the growth or decay factor.

(b) Write a formula for the function.

4.1.8.55.

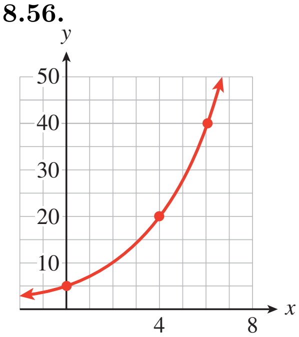


Answer.

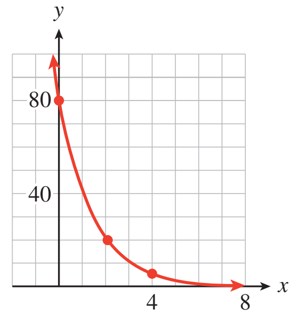
(a) Initial value 4, growth factor $2^{1/3}$

(b) $f(x) = 4 \cdot 2^{x/3}$

4.1.8.56.



4.1.8.57.

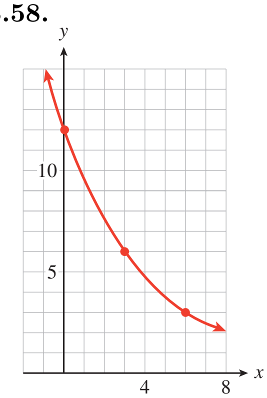


Answer.

(a) Initial value 80, decay factor $\frac{1}{2}$

(b) $f(x) = 80 \cdot \left(\frac{1}{2}\right)^x$

4.1.8.58.



4.1.8.59. If 8% of the air leaks out of Brian's bicycle tire every day, what percent of the air will be left after 2 days? After a week?

Answer. 84.6%, 55.8%

4.1.8.60. If housing prices are increasing by 15% per year, by what percent will they increase in 2 years? In 3 years?

4.1.8.61. Francine says that if a population grew by 48% in 6 years, then it grew by 8% per year. Is she correct? Either justify or correct her calculation.

Answer. No, an increase of 48% in 6 years corresponds to a growth factor of $1.48^{1/6} \approx 1.0675$, or an annual growth rate of about 6.75%.

4.1.8.62. Delbert says that if a population decreased by 60% in 5 years, then it decreased by 12% per year. Is he correct? Either justify or correct his calculation.

In Problems 63–66, assume that each population grows exponentially with constant annual percent increase, r.

4.1.8.63.

a The population of the state of Texas was $16,986,335$ in 1990. Write a formula in terms of r for the population in millions t years later. Round to the nearest hundredth.

b In 2000, the population was 20.85 million. Write an equation and solve for r. What was the annual percent increase to the nearest hundredth of a percent?

Answer.

a $P(t) = 16,986,335(1 + r)^t$ b 2.07%

4.1.8.64.

a The population of the state of Florida was $12,937,926$ in 1990. Write a formula in terms of r for the population of Florida t years later.

b In 2000, the population was $15,982,378$. Write an equation and solve for r. What was the annual percent increase to the nearest hundredth of a percent?

4.1.8.65.

a The population of Rainville was $10,000$ in 1990 and doubled in 20 years. What was the annual percent increase to the nearest hundredth percent?

b The population of Elmira was $350,000$ in 1990 and doubled in 20 years. What was the annual percent increase to the nearest hundredth of a percent?

c If a population doubles in 20 years, does the percent increase depend on the size of the original population?

d The population of Grayling doubled in 20 years. What was the annual percent increase to the nearest hundredth of a percent?

Answer.

a 3.53% b 3.53% c No d 3.53%

4.1.8.66.

a The population of Boomtown was 300 in 1908 and tripled in 7 years. What was the annual percent increase to the nearest hundredth of a percent?

b The population of Fairview was $15,000$ in 1962 and tripled in 7 years. What was the annual percent increase to the nearest hundredth of a percent?

c If a population triples in 7 years, does the percent increase depend on the size of the original population?

d The population of Pleasant Lake tripled in 7 years. What was the annual percent increase to the nearest hundredth of a percent?

4.1.8.67. A researcher starts 2 populations of fruit flies of different species, each with 30 flies. Species A increases by 30% in 6 days and species B increases by 20% in 4 days.

a What was the population of species A after 6 days? Find the daily growth factor for species A.

b What was the population of species B after 4 days? Find the daily growth factor for species B.

c Which species multiplies more rapidly?

Answer.

a 39; 1.045 b 35; 1.047 c Species B

4.1.8.68. A biologist isolates two strains of a particular virus and monitors the growth of each, starting with samples of 0.01 gram. Strain A increases by 10% in 8 hours and strain B increases by 12% in 9 hours.

a How much did the sample of strain A weigh after 8 hours? What was its hourly growth factor?

b How much did the sample of strain B weigh after 9 hours? What was its hourly growth factor?

c Which strain of virus grows more rapidly?

In Problems 69–72, we compare linear and exponential growth.

4.1.8.69. At a large university 3 students start a rumor that final exams have been canceled. After 2 hours, 6 students (including the first 3) have heard the rumor.

a Assuming that the rumor grows linearly, complete the table below for $L(t)$, the number of students who have heard the rumor after t hours. Then write a formula for the function $L(t)$. Graph the function.

t	0	2	4	6	8
$L(t)$					

b Complete the table below, assuming that the rumor grows exponentially. Write a formula for the function $E(t)$ and graph it on the same set of axes with $L(t)$.

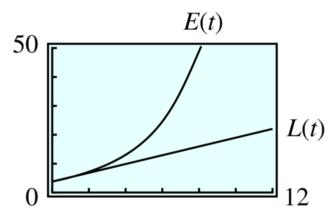
t	0	2	4	6	8
$E(t)$					

Answer.

a

t	0	2	4	6	8
$L(t)$	3	6	9	12	15

$L(t) = 3 + 1.5t$



b

t	0	2	4	6	8
$E(t)$	3	6	12	24	48

$E(t) = 3 \cdot 2^{t/2}$

4.1.8.70. Over the weekend the Midland Infirmary identifies four cases of 2009 H1N1 Pandemic flu. Three days later it has treated a total of ten cases.

a Assuming that the number of flu cases grows linearly, complete the table below for $L(t)$, the number of people infected after t days. Then write a formula for the function $L(t)$. Graph the function.

t	0	3	6	9	12
$L(t)$			16		

b Complete the table below, assuming that the flu grows exponentially. Write a formula for the function $E(t)$ and graph it on the same set of axes with $L(t)$.

t	0	3	6	9	12
$E(t)$				62.5	

4.1.8.71. The world's population of tigers declined from $10,400$ in 1980 to 6000 in 1998.

a If the population declined linearly, what was its annual rate of decrease?

b If the population declined exponentially, what was its annual decay factor? What was its annual percent decrease?

c Predict the number of tigers in 2010 under each assumption, linear or exponential decline.

Answer.

a 244 tigers per year

b 0.97; 3%

c Linear: 3067; Exponential: 4170

4.1.8.72. In 2003, the Center for Biological Diversity filed a lawsuit against the federal government for failing to protect Alaskan sea otters. The population of sea otters, which numbered between $150,000$ and $300,000$ before hunting began in 1741, declined from about $20,000$ in 1992 to 6000 in 2000. (Source: Center for Biological Diversity)

a If the population declined linearly after 1992, what was its annual rate of decrease?

b If the population declined exponentially after 1992, what was its annual rate of decrease?

c Predict the number of sea otters in 2010 under each assumption, linear or exponential decline

4.2 Exponential Functions

4.2.1 Graphs of Exponential Functions

Checkpoint 4.2.4

a State the ranges of the functions f and g in Example 4.2.2 on the domain $[-2, 2]$.

b State the ranges of the functions p and q shown in the Note above on the domain $[-2, 2]$. Round your answers to two decimal places.

Answer.

a $f : \left[\dfrac{1}{9}, 9\right]$; $g : \left[\dfrac{1}{16}, 16\right]$

b $p : [0.64, 1.56]$; $q : [0.25, 4]$

4.2.2 Transformations of Exponential Functions

Checkpoint 4.2.6 Which of the functions below have the same graph? Explain why.

a $f(x) = \left(\dfrac{1}{4}\right)^x$

b $g(x) = -4^x$

c $h(x) = 4^{-x}$

Answer. (a) and (c)

4.2.3 Comparing Exponential and Power Functions

Checkpoint 4.2.8 Which of the following functions are exponential functions, and which are power functions?

a $F(x) = 1.5^x$

b $G(x) = 3x^{1.5}$

c $H(x) = 3^{1.5x}$

d $K(x) = (3x)^{1.5}$

Answer. Exponential: (a) and (c); power: (b) and (d)

4.2.4 Exponential Equations

Checkpoint 4.2.10 Solve the equation $2^{x+2} = 128$.

Hint. Write each side as a power of 2.
Equate exponents.

Answer. $x = 5$

Checkpoint 4.2.12 During an advertising campaign in a large city, the makers of Chip-O's corn chips estimate that the number of people who have heard of Chip-O's increases by a factor of 8 every 4 days.

a If 100 people are given trial bags of Chip-O's to start the campaign, write a function, $N(t)$, for the number of people who have heard of Chip-O's after t days of advertising.

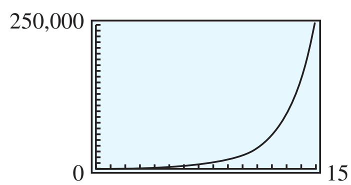
b Use your calculator to graph the function $N(t)$ on the domain $0 \leq t \leq 15$.

c How many days should the makers run the campaign in order for Chip-O's to be familiar to $51,200$ people? Use algebraic methods to find your answer and verify on your graph.

Answer.

a $N(t) = 100 \cdot 8^{t/4}$

b



c 12 days

Checkpoint 4.2.13 Use the graph of $y = 5^x$ to find an approximate solution to $5^x = 285$, accurate to two decimal places.

Answer. $x \approx 3.51$

4.2.6 Homework 4.2

Find the y-intercept of each exponential function and decide whether the graph is increasing or decreasing.

4.2.6.1.

(a) $f(x) = 26(1.4)^x$

(b) $g(x) = 1.2(0.84)^x$

(c) $h(x) = 75\left(\dfrac{4}{5}\right)^x$

(d) $k(x) = \dfrac{2}{3}\left(\dfrac{9}{8}\right)^x$

Answer.

(a) 26; increasing

(b) 1.2; decreasing

(c) 75; decreasing

(d) $\frac{2}{3}$; increasing

4.2.6.2.

(a) $M(x) = 1.5(0.05)^x$

(b) $N(x) = 0.05(1.05)^x$

(c) $P(x) = \left(\frac{5}{8}\right)^x$

(d) $Q(x) = \left(\frac{4}{3}\right)^x$

Sketch the functions on the same set of axis with a domain of $[-3, 3]$. Be sure to label your functions. Describe the similarities and differences between the two graphs.

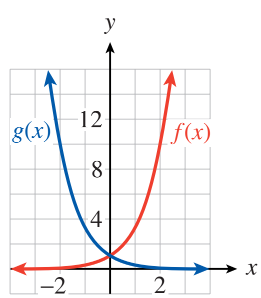
4.2.6.3.

a $f(x) = 3^x$

b $g(x) = \left(\frac{1}{3}\right)^x$

Answer.

x	-3	-2	-1	0	1	2	3
$f(x) = 3^x$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27
$g(x) = \left(\frac{1}{3}\right)^x$	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$



The two graphs are reflections of each other across the y-axis. f is increasing, g is decreasing. f has the negative x-axis as an asymptote, and g has the positive x-axis as its asymptote.

4.2.6.4.

a $F(x) = \left(\frac{1}{10}\right)^x$

b $G(x) = 10^x$

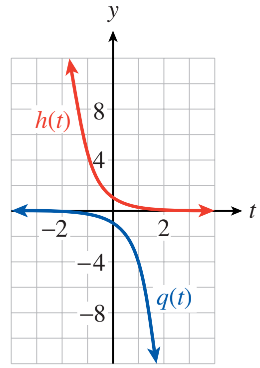
4.2.6.5.

a $h(t) = 4^{-t}$

b $q(t) = -4^t$

Answer.

t	-3	-2	-1	0	1	2	3
$h(t) = 4^{-t}$	64	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
$q(t) = -4^t$	$\frac{-1}{64}$	$\frac{-1}{16}$	$\frac{-1}{4}$	-1	-4	-16	-64



The graphs are reflections of each other across the origin. Both are decreasing, but h has the negative t-axis as an asymptote, and q has the positive t-axis as its asymptote.

4.2.6.6.

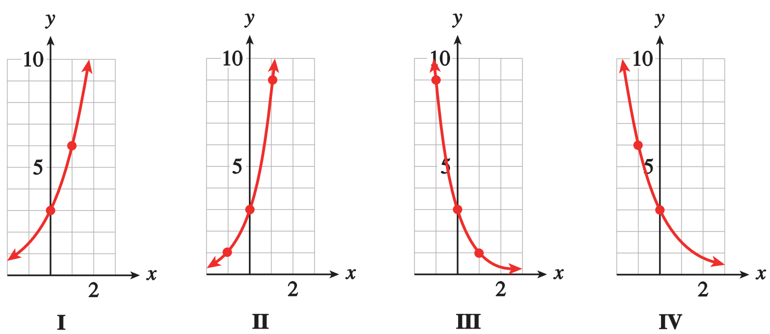
a $g(t) = 5^t$

b $P(t) = -5^t$

c $R(t) = 5^{-t}$

Match each function with its graph.

4.2.6.7.



a $f(x) = 3(2^x)$

b $f(x) = 3\left(\dfrac{1}{2}\right)^x$

c $f(x) = 3\left(\dfrac{1}{3}\right)^x$

d $f(x) = 3(3^x)$

Answer.

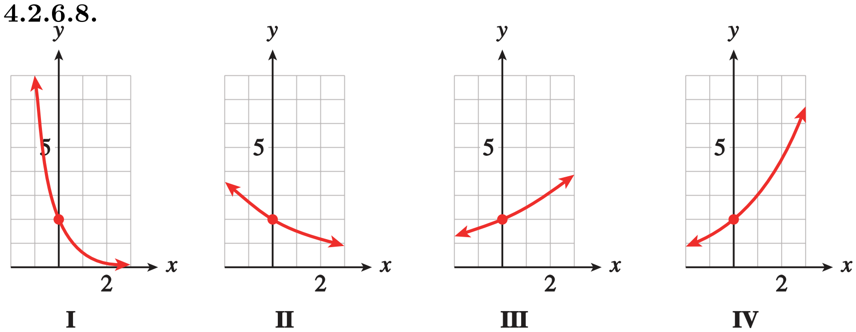
a I

b IV

c III

d II

4.2.6.8.



a $g(x) = 2(1.5^x)$

b $g(x) = 2\,(1.25)^x$

c $g(x) = 2\,(0.75)^x$

d $g(x) = 2(0.25)^x$

For Problems 9–12,

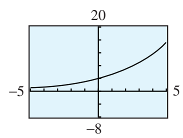
a Use a graphing calculator to graph the functions on the domain $[-5, 5]$.

b Give the range of the function on that domain, accurate to hundredths.

4.2.6.9. $g(t) = 4(1.3^t)$

Answer.

a



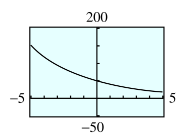
b $[1.08, 14.85]$

4.2.6.10. $h(t) = 3(2.4^t)$

4.2.6.11. $N(x) = 50(0.8^x)$

Answer.

a



b $[16.38, 152.59]$

4.2.6.12. $P(x) = 80(0.7^x)$

In each group of functions, which have identical graphs? Explain why using algebra and the properties of exponents.

4.2.6.13.

a $h(x) = 6^x$

b $k(x) = \left(\dfrac{1}{6}\right)^x$

c $m(x) = 6^{-x}$

d $n(x) = \dfrac{1}{6^x}$

Answer. Because they are defined by equivalent expressions, (b), (c), and (d) have identical graphs

4.2.6.14.

a $Q(t) = 5^t$

b $R(t) = \left(\dfrac{1}{5}\right)^t$

c $F(t) = \left(\dfrac{1}{5}\right)^{-t}$

d $G(t) = \dfrac{1}{5^{-t}}$

For Problems 15–18,

a Use the order of operations to explain why the two functions are different.

b Complete the table of values and graph both functions on the same set of axes.

c Describe each as a transformation of $y = 2^x$ or $y = 3^x$.

4.2.6.15. $f(x) = 2^{x-1}, \;\; g(x) = 2^x - 1$

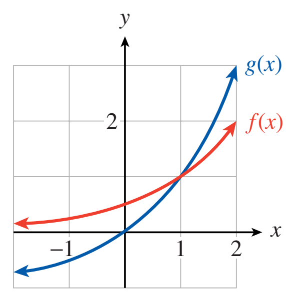
x	$y = 2^x$	$f(x)$	$g(x)$
-2			
-1			
0			
1			
2			

Answer.

a To evaluate f we subtract 1 from the input before evaluating the exponential function; to evaluate g we subtract 1 from the output of the exponential function.

b

x	$y = 2^x$	$f(x)$	$g(x)$
-2	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{-3}{4}$
-1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{-1}{2}$
0	1	$\frac{1}{2}$	0
1	2	1	1
2	4	2	3



c The graph of f is translated 1 unit to the right; the graph of g is shifted 1 unit down.

4.2.6.16. $f(x) = 3^x + 2, \;\; g(x) = 3^{x+2}$

x	$y = 3^x$	$f(x)$	$g(x)$
-2			
-1			
0			
1			
2			

4.2.6.17. $f(x) = -3^x, \;\; g(x) = 3^{-x}$

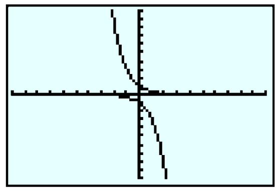
x	$y = 3^x$	$f(x)$	$g(x)$
-2			
-1			
0			
1			
2			

Answer.

a To evaluate f we take the negative of the output of the exponential function; to evaluate g we take the negative of the input.

b

x	$y = 3^x$	$f(x)$	$g(x)$
-2	$\frac{1}{9}$	$\frac{-1}{9}$	9
-1	$\frac{1}{3}$	$\frac{-1}{3}$	3
0	1	-1	1
1	3	-3	$\frac{1}{3}$
2	9	-9	$\frac{1}{9}$



c The graph of f is reflected about the x-axis; the graph of g is reflected about the y-axis.

4.2.6.18. $f(x) = 2^{-x}$, $g(x) = -2^x$

x	$y = 2^x$	$f(x)$	$g(x)$
-2			
-1			
0			
1			
2			

For the given function, evaluate each pair of expressions. Are they equivalent?

4.2.6.19. $f(x) = 3(5^x)$

a $f(a+2)$ and $9f(a)$

b $f(2a)$ and $2f(a)$

Answer.

a $3(5^{a+2})$ is not equivalent to $9 \cdot 3(5^a)$.

b $3(5^{2a})$ is not equivalent to $2 \cdot 3(5^a)$.

4.2.6.20. $g(x) = 1.8^x$

a $g(h+3)$ and $g(h)g(3)$

b $g(2h)$ and $[g(h)]^2$

4.2.6.21. $P(t) = 8^t$

a $P(w) - P(z)$ and $P(w - z)$

b $P(-x)$ and $\dfrac{1}{P(x)}$

Answer.

a $8^w - 8^z$ is not equivalent to 8^{w-z}.

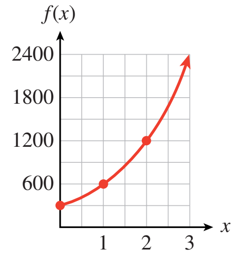
b 8^{-x} is equivalent to $\dfrac{1}{8^x}$.

4.2.6.22. $Q(t) = 5(0.2)^t$

a $Q(b-1)$ and $5Q(b)$

b $Q(a)Q(b)$ and $5Q(a+b)$

4.2.6.23. The graph of $f(x) = P_0 b^x$ is shown in the figure.



a Read the value of P_0 from the graph.

b Make a short table of values for the function by reading values from the graph. Does your table confirm that the function is exponential?

c Use your table to calculate the growth factor, b.

d Using your answers to parts (a) and (c), write a formula for $f(x)$.

Answer.

a $P_0 = 300$

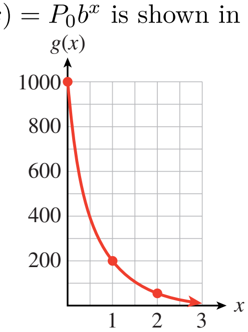
b

x	0	1	2
$f(x)$	300	600	1200

c $b = 2$

d $f(x) = 300(2)^x$

4.2.6.24. The graph of $g(x) = P_0 b^x$ is shown in the figure.



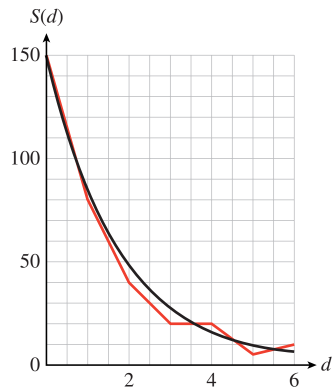
a Read the value of P_0 from the graph.

b Make a short table of values for the function by reading values from the graph. Does your table confirm that the function is exponential?

c Use your table to calculate the decay factor, b.

d Using your answers to parts (a) and (c), write a formula for $g(x)$.

4.2.6.25. For several days after the Northridge earthquake on January 17, 1994, the area received a number of significant aftershocks. The red graph shows that the number of aftershocks decreased exponentially over time. The graph of the function $S(d) = S_0 b^d$, shown in black, approximates the data. (Source: *Los Angeles Times*, June 27, 1995)



a Read the value of S_0 from the graph.

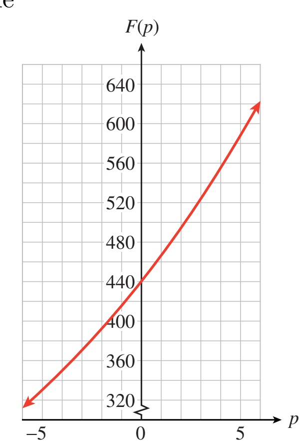
b Find an approximation for the decay factor, b, by comparing two points on the graph. (Some of the points on the graph of $S(d)$ are approximately $(1, 82)$, $(2, 45)$, $(3, 25)$, and $(4, 14)$.)

c Using your answers to (a) and (b), write a formula for $S(d)$.

Answer.

a $S_0 = 150$ b $b \approx 0.55$ c $S(d) = 150(0.55)^d$

4.2.6.26. The frequency of a musical note depends on its pitch. The graph shows that the frequency increases exponentially. The function $F(p) = F_0 b^p$ gives the frequency as a function of the number of half-tones, p, above the starting point on the scale



a Read the value of F_0 from the graph. (This is the frequency of the note A above middle C.)

b Find an approximation for the growth factor, b, by comparing two points on the graph. (Some of the points on the graph of $F(p)$ are approximately $(1, 466)$, $(2, 494)$, $(3, 523)$, and $(4, 554)$.)

c Using your answers to (a) and (b), write a formula for $F(p)$.

d The frequency doubles when you raise a note by one octave, which is equivalent to 12 half-tones. Use this information to find an exact value for b.

Solve each equation algebraically.

4.2.6.27. $5^{x+2} = 25$

Answer. $\dfrac{2}{3}$

4.2.6.28. $3^{x-1} = 27^{1/2}$

4.2.6.29. $3^{2x-1} = \dfrac{\sqrt{3}}{9}$

Answer. $\dfrac{-1}{4}$

4.2.6.30. $2^{3x-1} = \dfrac{\sqrt{2}}{16}$

4.2.6.31. $4 \cdot 2^{x-3} = 8$

Answer. $\dfrac{1}{7}$

4.2.6.32. $9 \cdot 3^{x+2} = 81^{-x}$

4.2.6.33. $27^{4x+2} = 81^{x-1}$

Answer. $\dfrac{-5}{4}$

4.2.6.34. $16^{2-3x} = 64^{x+5}$

4.2.6.35. $10^{x^2-1} = 1000$

Answer. ± 2

4.2.6.36. $5^{x^2-x-4} = 25$

4.2.6.37. Before the advent of antibiotics, an outbreak of cholera might spread through a city so that the number of cases doubled every 6 days.

a Twenty-six cases were discovered on July 5. Write a function for the number of cases of cholera t days later.

b Use your calculator to graph your function on the interval $0 \le t \le 90$.

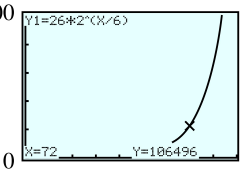
c When should hospitals expect to be treating 106, 496 cases? Use algebraic methods to find your answer, and verify it on your graph.

Answer.

a $N(t) = 26(2)^{t/6}$

b

500,000



0 90

c 72 days later

4.2.6.38. An outbreak of ungulate fever can sweep through the livestock in a region so that the number of animals affected triples every 4 days.

a A rancher discovers 4 cases of ungulate fever among his herd. Write a function for the number of cases of ungulate fever t days later.

b Use your calculator to graph your function on the interval $0 \le t \le 20$.

c If the rancher does not act quickly, how long will it be until 324 head are affected? Use algebraic methods to find your answer, and verify it on your graph.

4.2.6.39. A color television set loses 30% of its value every 2 years.

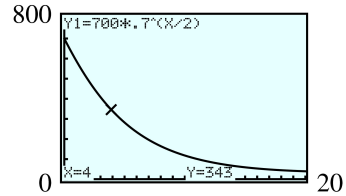
a Write a function for the value of a television set t years after it was purchased if it cost \$700 originally.

b Use your calculator to graph your function on the interval $0 \le t \le 20$.

c How long will it be before a \$700 television set depreciates to \$343? Use algebraic methods to find your answer, and verify it on your graph.

Answer.

a $V(t) = 700(0.7)^{t/2}$

b 

c 4 yr

4.2.6.40. A mobile home loses 20% of its value every 3 years.

a A certain mobile home costs \$20,000. Write a function for its value after t years.

b Use your calculator to graph your function on the interval $0 \le t \le 30$.

c How long will it be before a \$20,000 mobile home depreciates to \$12,800? Use algebraic methods to find your answer, and verify it on your graph.

Use a graph to find an approximate solution accurate to the nearest hundredth.

4.2.6.41.
$3^{x-1} = 4$

Answer.
$x = 2.26$

4.2.6.42.
$2^{x+3} = 5$

4.2.6.43.
$4^{-x} = 7$

Answer.
$x = -1.40$

4.2.6.44.
$6^{-x} = 3$

Decide whether each function is an exponential function, a power function, or neither.

4.2.6.45.

a $g(t) = 3t^{0.4}$

b $h(t) = 4(0.3)^t$

c $D(x) = 6x^{1/2}$

d $E(x) = 4x + x^4$

Answer.

a Power

b Exponential

c Power

d Neither

4.2.6.46.

a $R(w) = 5(5)^{w-1}$

b $Q(w) = 2^w - w^2$

c $M(z) = 0.2z^{1.3}$

d $N(z) = z^{-3}$

Decide whether the table could describe a linear function, a power function, an exponential function, or none of these.

4.2.6.47.

a

x	y
0	3
1	6
2	12
3	24
4	48

b

t	P
0	0
1	0.5
2	2
3	4.5
4	8

Answer.

a Exponential $y = 3 \cdot 2^x$

b Power $P = 0.5t^2$

4.2.6.48.

a

x	N
0	0
1	2
2	16
3	54
4	128

b

p	R
0	405
1	135
2	45
3	15
4	5

4.2.6.49.

a

t	y
1	100
2	50
3	$33\frac{1}{3}$
4	25
5	20

b

x	P
1	$\frac{1}{2}$
2	1
3	2
4	4
5	8

Answer.

a Power $y = 100x^{-1}$

b Exponential $P = \frac{1}{4} \cdot 2^x$

4.2.6.50.

a

h	a
0	70
1	7
2	0.7
3	0.07
4	0.007

b

t	Q
0	0
1	$\frac{1}{4}$
2	1
3	$\frac{9}{4}$
4	4

Fill in the tables. Graph each pair of functions in the same window. Then answer the questions below.

a Give the range of f and the range of g.

b For how many values of x does $f(x) = g(x)$?

c Estimate the value(s) of x for which $f(x) = g(x)$.

d For what values of x is $f(x) < g(x)$?

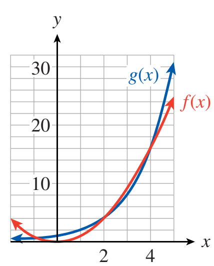
e Which function grows more rapidly for large values of x?

4.2.6.51.

x	$f(x) = x^2$	$g(x) = 2^x$
-2		
-1		
0		
1		
2		
3		
4		
5		

Answer.

x	$f(x) = x^2$	$g(x) = 2^x$
-2	4	$\frac{1}{4}$
-1	1	$\frac{1}{2}$
0	0	1
1	1	2
2	4	4
3	9	8
4	16	16
5	25	32



a Range of f: $[0, \infty)$; Range of g: $(0, \infty)$

b 3

c -0.7667, 2, 4

d $(-0.7667, 2)$ and $(4, \infty)$

e g

4.2.6.52.

x	$f(x) = x^3$	$g(x) = 3^x$
-2		
-1		
0		
1		
2		
3		
4		
5		

For Problems 53–60, state the domain and range of each transformation, its intercept(s), and any asymptotes.

4.2.6.53. $f(x) = 3^x$

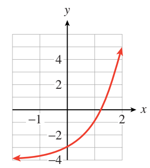
a $y = f(x) - 4$

b $y = f(x - 4)$

c $y = -4f(x)$

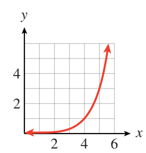
Answer.

a $y = 3^x - 4$



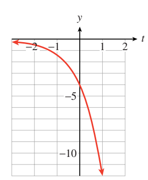
Domain: $(-\infty, \infty)$; range: $(-4, \infty)$, x-intercept $(1.26, 0)$; y-intercept $(0, -3)$; horizontal asymptote $y = -4$

b $y = 3^{x-4}$,



Domain: $(-\infty, \infty)$; range: $(0, \infty)$, no x-intercept; y-intercept $\left(0, \dfrac{1}{81}\right)$; the x-axis is the horizontal asymptote.

c $y = -4 \cdot 3^x$,



Domain: $(-\infty, \infty)$; range: $(-\infty, 0)$, no x-intercept; y-intercept $(0, -4)$; the x-axis is the horizontal asymptote.

4.2.6.54. $g(x) = 4^x$

a $y = g(x) + 2$

b $y = g(x + 2)$

c $y = 2g(x)$

4.2.6.55. $h(t) = 6^t$

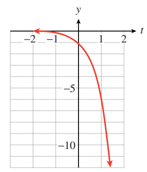
a $y = -h(t)$

b $y = h(-t)$

c $y = -h(-t)$

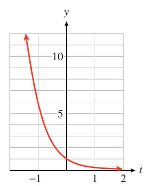
Answer.

a $y = -6^t$



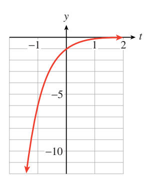
Domain: $(-\infty, \infty)$; range: $(-\infty, 0)$, no t-intercept; y-intercept $(0, -1)$; the t-axis is the horizontal asymptote.

b $y = 6^{-t}$,



Domain: $(-\infty, \infty)$; range: $(0, \infty)$, no t-intercept; y-intercept $(0, 1)$; the t-axis is the horizontal asymptote.

c $y = -6^{-t}$,



Domain: $(-\infty, \infty)$; range: $(-\infty, 0)$, no t-intercept; y-intercept $(0, -1)$; the t-axis is the horizontal asymptote.

4.2.6.56. $j(t) = \left(\frac{1}{3}\right)^t$

a $y = j(-t)$

b $y = -j(t)$

c $y = -j(-t)$

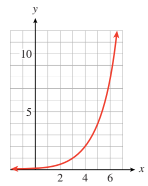
4.2.6.57. $g(x) = 2^x$

a $y = g(x - 3)$

b $y = g(x - 3) + 4$

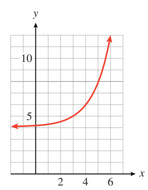
Answer.

a $y = 2^{x-3}$



Domain: $(-\infty, \infty)$; range: $(0, \infty)$, no x-intercept; y-intercept $\left(0, \frac{1}{8}\right)$; the x-axis is the horizontal asymptote.

b $y = 2^{x-3} + 4$,



Domain: $(-\infty, \infty)$; range: $(4, \infty)$, no x-intercept; y-intercept $> \left(0, \frac{33}{8}\right)$; horizontal asymptote $y = 4$

4.2.6.58. $f(x) = 10^x$

a $y = f(x + 5)$

b $y = f(x + 5) - 20$

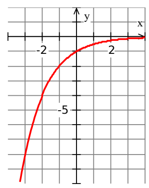
4.2.6.59. $N(t) = \left(\dfrac{1}{2}\right)^t$

a $y = -N(t)$

b $y = 6 - N(t)$

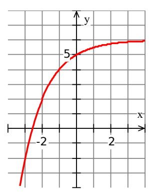
Answer.

a $y = -\left(\dfrac{1}{2}\right)^t$



Domain: $(-\infty, \infty)$; range: $(-\infty, 0)$, no t-intercept; y-intercept $(0, -1)$; the t-axis is the horizontal asymptote.

b $y = 6 - \left(\dfrac{1}{2}\right)^t$,



Domain: $(-\infty, \infty)$; range: $(-\infty, 6)$, t-intercept approximately $(-2.58, 0)$; y-intercept $(0, 5)$; horizontal asymptote is $y = 6$

4.2.6.60. $P(t) = 0.4^t$

a $y = -P(t)$

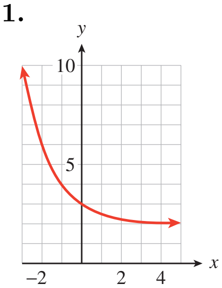
b $y = 8 - P(t)$

For Problems 61–64,

a Describe the graph as a transformation of $y = 2^x$.

b Give an equation for the function graphed.

4.2.6.61.

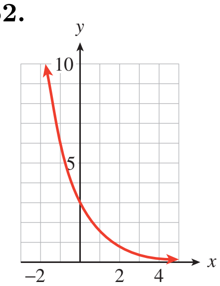


Answer.

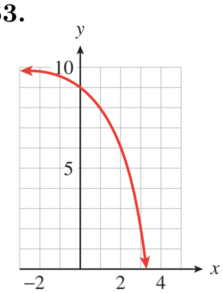
a The graph of $y = 2^x$ has been reflected about the y-axis and shifted up 2 units.

b $y = 2^{-x} + 2$

4.2.6.62.



4.2.6.63.

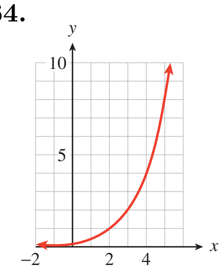


Answer.

a The graph of $y = 2^x$ has been reflected about the x-axis and shifted up 10 units.

b $y = -2^x + 10$

4.2.6.64.



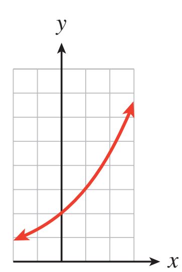
Match the graph of each function to its formula. In each formula, $a > 0$ and $b > 1$.

4.2.6.65.

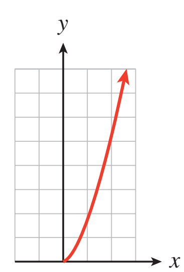
a $y = ab^x$

b $y = ab^{-x}$

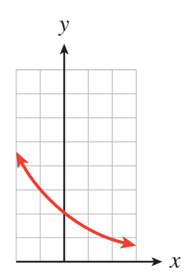
c $y = ax^b$



I



II



III

Answer.

a I

b III

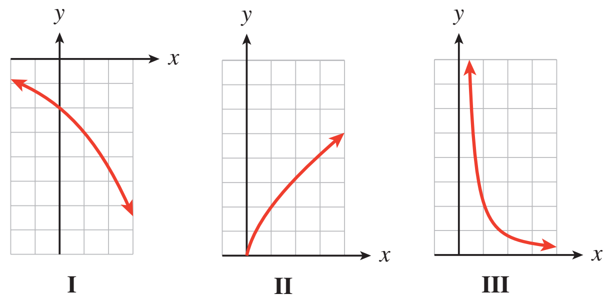
c II

4.2.6.66.

a $y = ax^{-b}$

b $y = -ab^x$

c $y = ax^{1/b}$



4.2.6.67. The function $f(t)$ describes a volunteer's heart rate during a treadmill test.

$$f(t) = \begin{cases} 100 & 0 \le t < 3 \\ 56t - 68 & 3 \le t < 4 \\ 186 - 500(0.5)^t & 4 \le t < 9 \\ 100 + 6.6(0.6)^{t-14} & 9 \le t < 20 \end{cases}$$

The heart rate is given in beats per minute and t is in minutes. (See Section 2.2 to review functions defined piecewise.) (Source: Davis, Kimmet, and Autry, 1986)

a Evaluate the function to complete the table.

t	3.5	4	8	10	15
$f(t)$					

b Sketch the graph of the function.

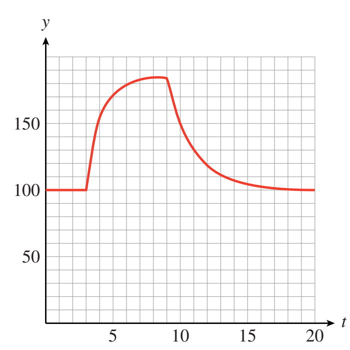
c The treadmill test began with walking at 5.5 kilometers per hour, then jogging, starting at 12 kilometers per hour and increasing to 14 kilometers per hour, and finished with a cool-down walking period. Identify each of these activities on the graph and describe the volunteer's heart rate during each phase.

Answer.

a

t	3.5	4	8	10	15
$f(t)$	128	154.75	184.05	150.93	103.96

b



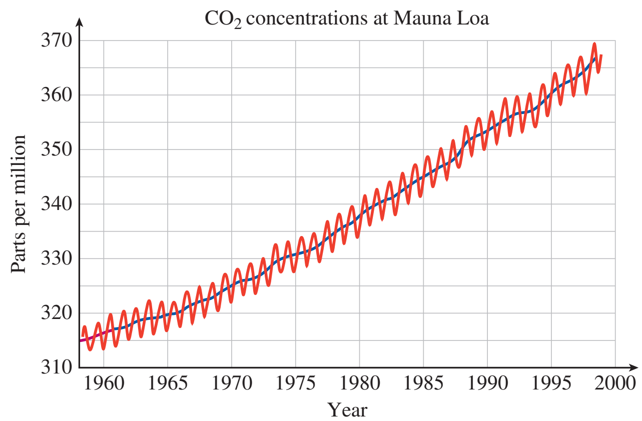
c From 0 to 3 minutes, the volunteer is walking with heart rate 100 beats per minute. The volunteer jogged at a steady pace from 3 to 4 minutes, and the heart rate increased to about 155 beats per minutes. From 4 to 9 minutes, the jogging pace increased, and the heart rate rose to about 185 beats per minute. The cooldown started at 9 minutes, and the heart rate decreased rapidly and leveled off to about 100 beats per minute.

4.2.6.68. Carbon dioxide (CO_2) is called a greenhouse gas because it traps part of the Earth's outgoing energy. Animals release CO_2 into the atmosphere, and plants remove CO_2 through photosynthesis. In modern times, deforestation and the burning of fossil fuels both contribute to CO_2 levels. The figure shows atmospheric concentrations of CO_2, in parts per million, measured at the Mauna Loa Observatory in Hawaii.

a The red curve shows annual oscillations in CO_2 levels. Can you explain why CO_2 levels vary throughout the year?

b The blue curve shows the average annual CO_2 readings. By approximately how much does the CO_2 level vary from its average value during the year?

c In 1960, the average CO_2 level was 316.75 parts per million, and the average level has been rising by 0.4% per year. If the level continues to rise at this rate, what CO_2 readings can we expect in the year 2100?



Hint. For part (a): Why would photosynthesis vary throughout the year?

4.3 Logarithms

4.3.1 Introduction

Checkpoint 4.3.3 Find each logarithm.

a $\log_3(81)$

b $\log\left(\dfrac{1}{1000}\right)$

Answer.

a 4

b -3

Checkpoint 4.3.5 Find each logarithm.

a $\log_n(1)$

b $\log_n(n^3)$

Answer.

a 0

b 3

4.3.2 Using the Conversion Equations

Checkpoint 4.3.7 Rewrite each equation in logarithmic form.

a $8^{-1/3} = \dfrac{1}{2}$

b $5^x = 46$

Answer.

a $\log_8\left(\dfrac{1}{2}\right) = \dfrac{-1}{3}$

b $\log_5(46) = x$

4.3.3 Base 10 Logarithms

Checkpoint 4.3.10

a Evaluate $\log(250)$, and round your answer to two decimal places. Check your answer using the conversion equations.

b Evaluate $\log(250)$, and round your answer to four decimal places. Check your answer using the conversion equations.

Answer.

a 2.40

b 2.3979

4.3.4 Solving Exponential Equations

Checkpoint 4.3.13 Solve $\quad 12 - 30(10^{-0.2x}) = 11.25$

Answer. 8.01

4.3.5 Application to Exponential Models

Checkpoint 4.3.15 The percentage of American homes with computers grew exponentially from 1994 to 1999. For $t = 0$ in 1994, the growth law was

$$P(t) = 25.85(10)^{0.052t}$$

[Source: Los Angeles Times, August 20, 1999]

a What percent of American homes had computers in 1994?

b If the percentage of homes with computers continued to grow at the same rate, when did 90% of American homes have a computer?

c Do you think that the function $P(t)$ will continue to model the percentage of American homes with computers? Why or why not?

Answer.

a 25.85%

b $t \approx 10.4$ (year 2004)

c No, the percent of homes with computers cannot exceed 100%.

4.3.7 Homework 4.3

For Problems 1-10, find each logarithm without using a calculator.

4.3.7.1.
(a) $\log_7(49)$ (b) $\log_2(32)$

Answer.

(a) 2 (b) 5

4.3.7.2.
(a) $\log_4(64)$ (b) $\log_3(27)$

4.3.7.3.
(a) $\log_3(\sqrt{3})$ (b) $\log_3\left(\dfrac{1}{3}\right)$

Answer.

(a) $\dfrac{1}{2}$ (b) -1

4.3.7.4.
(a) $\log_5\left(\dfrac{1}{5}\right)$ (b) $\log_5(\sqrt{5})$

4.3.7.5.
(a) $\log_4(4)$ (b) $\log_6(1)$

Answer.

(a) 1 (b) 0

4.3.7.6.
(a) $\log(1)$ (b) $\log(10^{-6})$

4.3.7.7.
(a) $\log_8(8^5)$ (b) $\log_7(7^6)$

Answer.

(a) 5 (b) 6

4.3.7.8.
(a) $\log(10^{-4})$ (b) $\log(10^{-6})$

4.3.7.9.
(a) $\log(0.1)$ (b) $\log(0.001)$

Answer.

(a) -1 (b) -3

4.3.7.10.
(a) $\log(10,000)$ (b) $\log(1000)$

For Problems 11-22, rewrite the equation in logarithmic form.

4.3.7.11. $2^{10} = 1024$

Answer.
$\log_2(1024) = 10$

4.3.7.12.
$11^4 = 14,641$

4.3.7.13. $10^{0.699} \approx 5$

Answer.
$\log(5) \approx 0.699$

4.3.7.14.
$10^{-0.602} \approx 0.25$

4.3.7.15. $t^{3/2} = 16$

Answer.
$\log_t(16) = \dfrac{3}{2}$

4.3.7.16. $v^{5/3} = 12$

4.3.7.17. $0.8^{1.2} = M$

Answer.
$\log_{0.8}(M) = 1.2$

4.3.7.18. $3.7^{2.5} = Q$

4.3.7.19.
$x^{5t} = W - 3$

Answer.
$\log_x(W - 3) = 5t$

4.3.7.20.
$z^{-3t} = 2P + 5$

4.3.7.21.
$3^{-0.2t} = 2N_0$

Answer.
$\log_3(2N_0) = -0.2t$

4.3.7.22.
$10^{1.3t} = 3M_0$

For Problems 23–26,

a Solve each equation, writing your answer as a logarithm.

b Use trial and error to approximate the logarithm to one decimal place.

4.3.7.23.
$4^x = 2.5$

Answer.

a $\log_4(2.5)$ b 0.7

4.3.7.24.
$2^x = 0.2$

4.3.7.25.
$10^x = 0.003$

Answer.

a $\log(0.003)$ b -2.5

4.3.7.26.
$10^x = 4500$

For Problems 27–30,

a By computing successive powers of the base, trap each log between two integers.

b Use a graph to approximate each logarithm to the nearest hundredth. (*Hint*: Use the conversion equations to rewrite $x = \log_b(y)$ as an appropriate exponential equation.)

4.3.7.27.
$\log(7)$

Answer.

a $0 < \log(7) < 1$

b 0.85

4.3.7.28.
$\log(50)$

4.3.7.29.
$\log_3(67.9)$

Answer.

a $3 < \log_3(67.9) < 4$

b 3.84

4.3.7.30.
$\log_5(86.3)$

For Problems 31-34, use a calculator to approximate each logarithm to four decimal places. Make a conjecture about logarithms based on the results of each problem.

4.3.7.31.

(a) $\log(5.43)$

(b) $\log(54.3)$

(c) $\log(543)$

(d) $\log(5430)$

Answer.

(a) 0.7348 (c) 2.7348

(b) 1.7348 (d) 3.7348

When the input to the common logarithm is multiplied by 10, the output is increased by 1.

4.3.7.32.

(a) $\log(0.625)$

(b) $\log(0.0625)$

(c) $\log(0.00625)$

(d) $\log(0.000625)$

4.3.7.33.

(a) $\log(2)$

(b) $\log(4)$

(c) $\log(8)$

(d) $\log(16)$

Answer.

(a) 0.3010

(b) 0.6021

(c) 0.9031

(d) 1.2041

When the input to the common logarithm is doubled, the output is increased by about 0.3010.

4.3.7.34.

(a) $\log(4)$

(b) $\log(0.25)$

(c) $\log(5)$

(d) $\log(0.2)$

For Problems 35-44, solve for x. Give both the exact answer and the solution rounded to the nearest hundredth.

4.3.7.35. $10^{-3x} = 5$

Answer. -0.23

4.3.7.36. $640 = 10x^3$

4.3.7.37. $25 \cdot 10^{0.2x} = 80$

Answer. 2.53

4.3.7.36. $8 \cdot 10^{1.6x} = 312$

4.3.7.39. $12.2 = 2\left(10^{1.4x}\right) - 11.6$

Answer. 0.77

4.3.7.40. $163 = 3\left(10^{0.7x}\right) - 49.3$

4.3.7.41.
$3\left(10^{-1.5x}\right) - 14.7 = 17.1$

Answer. -0.68

4.3.7.42.
$4\left(10^{-0.6x}\right) + 16.1 = 28.2$

4.3.7.43. $80\left(1 - 10^{-0.2x}\right) = 65$

Answer. 3.63

4.3.7.44. $250\left(1 - 10^{-0.3x}\right) = 100$

In Problems 45–48, each calculation contains an error.Identify the error and without simply correcting it, *explain* why it is a mistake.

4.3.7.45.

$$\begin{aligned} 2 \cdot 5^x &= 848 \\ 10^x &= 848 \\ x &= \log(848) \quad \textit{(Incorrect!)} \end{aligned}$$

Answer. $2 \cdot 5^x \neq 10^x$; the first step should be to divide both sides of the equation by 2;
$x = \log_5(424)$.

4.3.7.46.

$$\begin{aligned} 15 \cdot 10^x &= 20 \\ 10^x &= 5 \\ x &= \log(5) \quad \textit{(Incorrect!)} \end{aligned}$$

4.3.7.47.

$$\begin{aligned} 10^{4x} &= 20 \\ 10^x &= 5 \\ x &= \log(5) \quad \textcolor{blue}{(Incorrect!)} \end{aligned}$$

Answer. $\frac{10^{4x}}{4} \neq 10^x$; the first step should be to write $4x = \log(20)$; $x = \frac{\log(20)}{4}$.

4.3.7.48.

$$\begin{aligned} 12 + 6^x &= 42 \\ 6^x &= 30 \\ x &= 5 \quad \textcolor{blue}{(Incorrect!)} \end{aligned}$$

4.3.7.49. The population of the state of California increased during the years 1990 to 2000 according to the formula

$$P(t) = 29,760,021(10)^{0.0056t},$$

where t is measured in years since 1990.

(a) What was the population in 2000?

(b) Assuming the same rate of growth, estimate the population of California in the year 2015.

(c) When did the population of California reach $35,000,000$?

(d) When should the population reach 40 million?

(e) Graph the function P with a suitable domain and range, then verify your answers to parts (a) through (d).

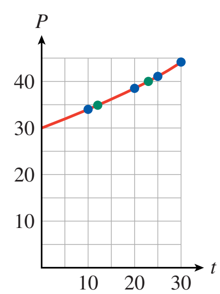
Answer.

(a) $33,855,812$

(b) $38,515,295$; $41,080,265$; $43,816,051$

(c) 2002

(d) 2012

(e) 

4.3.7.50. The population of the state of New York increased during the years 1990 to 2000 according to the formula

$$P(t) = 17.9905(10)^{0.0023t},$$

where t is measured in years since 1990.

(a) What was the population in 2000? Give units in your answer.

(b) Assuming the same rate of growth, estimate the population of New York in millions in the year 2015.

(c) When did the population of New York reach $20,000,000$?

(d) When should the population reach $30,000,000$?

(e) Graph the function P with a suitable domain and range, then verify your answers to parts (a) through (d).

4.3.7.51. The absolute magnitude, M, of a star is a measurement of its brightness. For example, our Sun, not a particularly bright star, has magnitude $M = 4.83$. The magnitude in turn is a measure of the luminosity, L, or amount of light energy emitted by the star, where

$$L = L_0 10^{-0.4M}$$

(a) The luminosity of a star is measured in solar units, so that our Sun has luminosity $L = 1$. Use the values of L and M for the Sun to calculate a value of L_0 in the equation above.

(b) Is luminosity an increasing or decreasing function of magnitude? Graph the function on the domain $[-3, 3]$. What is its range on that domain?

(c) The luminosity of Sirius is 22.5 times that of the Sun, or $L = 22.5$. Calculate the magnitude of Sirius.

(d) If two stars differ in magnitude by 5, what is the ratio of their luminosities?

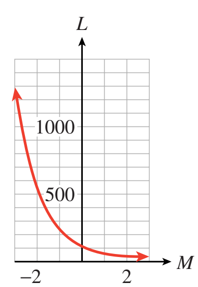
(e) A decrease in magnitude by 1 corresponds to an increase in luminosity by what factor? Give an exact value and an approximation to four decimal places.

(f) Normal stars have magnitudes between -10 and 19. What range of luminosities do stars exhibit?

Answer.

(a) 85.5

(b) Decreasing; range: $[5.4, 1355.2]$



(c) 1.45

(d) $\dfrac{1}{100}$

(e) $10^{0.4} \approx 2.5119$

(f) 2.15×10^{-6} to $855,067$

4.3.7.52. The loudness of a sound is a consequence of its intensity, I, or the amount of energy it generates, in watts per square meter. The intensity is related to the decibel level, D, which is another measure of loudness, by

$$I = 10^{-12+D/10}$$

(a) Is intensity an increasing or decreasing function of decibel level? The faintest sound a healthy human can hear is 0 decibels. What is the intensity of a 0 decibel sound?

(b) A whisper produces an energy intensity of 10^{-9} watts per square meter. What is the decibel level of a whisper?

(c) If two sounds differ in loudness by 10 decibels, what is the ratio of their intensities?

(d) An increase in loudness of 1 decibel produces a just noticeable difference to the human ear. By what factor does the intensity increase?

(e) Sounds of 130 decibels are at the threshold of pain for people. What is the range of the intensity function on the domain $[0, 130]$?

The atmospheric pressure decreases with altitude above the surface of the Earth. For Problems 53–58, use the function

$$P(h) = 30(10)^{-0.09h}$$

where altitude, h, is given in miles and atmospheric pressure, P, in inches of mercury. Graph this function in the window

Xmin $= 0$	Xmax $= 9.4$
Ymin $= 0$	Ymax $= 30$

Solve the problems below algebraically, and verify with your graph.

4.3.7.53. The elevation of Mount Everest, the highest mountain in the world, is $29,028$ feet. What is the atmospheric pressure at the top?

Hint. 1 mile = 5280 feet

Answer. 9.60 in

4.3.7.54. The elevation of Mount McKinley, the highest mountain in the United States, is $20,320$ feet. What is the atmospheric pressure at the top?

4.3.7.55. How high above sea level is the atmospheric pressure 20.2 inches of mercury?

Answer. 1.91 mi

4.3.7.56. How high above sea level is the atmospheric pressure 16.1 inches of mercury?

4.3.7.57. Find the height above sea level at which the atmospheric pressure is equal to one-half the pressure at sea level.

Hint. What is the altitude at sea level?

Answer. 3.34 mi

4.3.7.58. Find the height above sea level at which the atmospheric pressure is equal to one-fourth the pressure at sea level.

Hint. What is the altitude at sea level?

For Problems 59-66, simplify the expression.

4.3.7.59. $\log_2(\log_4(16))$

Answer. 1

4.3.7.60. $\log_5(\log_5(5))$

4.3.7.61. $\log[\log_3(\log_5(125))]$

Answer. 0

4.3.7.62. $\log\left(\log_2(\log_3(9))\right)$

4.3.7.63. $\log_2\left(\log_2(\log_3(81))\right)$

Answer. 1

4.3.7.64. $\log_4\left(\log_2(\log_3(81))\right)$

4.3.7.65. $\log_b(\log_b(b))$

Answer. 0

4.3.7.66. $\log_b(\log_a(a^b))$

4.4 Properties of Logarithms

4.4.1 Using the Properties of Logarithms

Checkpoint 4.4.2 Simplify $\log_b(xy^2)$.

Answer. $\log_b(x) - 2\log_b(y)$

Checkpoint 4.4.5 Express $2\log_b(x) + 4\log_b(x+3)$ as a single logarithm with a coefficient of 1.

Answer. $\log_b(x^2(x+3)^4)$

4.4.2 Solving Exponential Equations

Checkpoint 4.4.9 Solve $5(1.2)^{2.5x} = 77$

Hint. Divide both sides by 5.
the log of both sides.
Apply Property (3) to simplify the left side.
Solve for $\boldsymbol{x}$.

Answer. $x = \dfrac{\log(15.4)}{2.5\log(1.2)} \approx 5.999$

4.4.3 Applications

Checkpoint 4.4.11 Traffic on U.S. highways is growing by 2.7% per year. (Source: *Time*, Jan. 25, 1999)

a Write a formula for the volume, V, of traffic as a function of time, using V_0 for the current volume.

b How long will it take the volume of traffic to double? *Hint:* Find the value of t that gives $V = 2V_0$.

Answer.

a $V(t) = V_0(1.027)^t$

b about 26 years

4.4.4 Compound Interest

Checkpoint 4.4.14 Calculate the amount in Rashad's account after 5 years if the interest is compounded daily. (See Example 4.4.12. There are 365 days in a year.)

Answer. $1221.39

4.4.5 Solving Formulas

Checkpoint 4.4.16 Solve $A = P(1+r)^t$ for t.

Answer. $t = \dfrac{\log(A/P)}{\log(1+r)}$

4.4.7 Homework 4.4

4.4.7.1.

(a) Simplify $10^2 \cdot 10^6$.

(b) Compute $\log(10^2)$, $\log(10^6)$, and $\log(10^2 \cdot 10^6)$. How are they related?

Answer.

(a) 10^8

(b) $2;\;\; 6;\;\; 8;\;\; 2+6=8$

4.4.7.2.

(a) Simplify $\dfrac{10^9}{10^6}$.

(b) Compute $\log(10^9)$, $\log(10^6)$, and $\log\left(\dfrac{10^9}{10^6}\right)$. How are they related?

4.4.7.3.

(a) Simplify $\dfrac{b^8}{b^5}$.

(b) Compute $\log_b(b^8)$, $\log_b(b^5)$, and $\log_b\left(\dfrac{b^8}{b^5}\right)$. How are they related?

Answer.

(a) b^3

(b) $8;\;\; 5;\;\; 3;\;\; 8-5=3$

4.4.7.4.

(a) Simplify $b^4 \cdot b^3$.

(b) Compute $\log_b(b^4)$, $\log_b(b^3)$, and $\log_b(b^4 \cdot b^3)$. How are they related?

4.4.7.5.

(a) Simplify $\left(10^3\right)^5$.

(b) Compute $\log\left(10^3\right)^5$ and $\log\left(10^3\right)$. How are they related?

Answer.

(a) 10^{15}

(b) $15;\;\; 3;\;\; 15=3\cdot 5$

4.4.7.6.

(a) Simplify $\left(b^2\right)^6$.

(b) Compute $\log_b\left(b^2\right)^6$ and $\log_b\left(b^2\right)$. How are they related?

For Problems 7-14, use the properties of logarithms to expand each expression in terms of simpler logarithms. Assume that all variable expressions denote positive numbers.

4.4.7.7.

(a) $\log_b(2x)$ (b) $\log_b\left(\frac{x}{2}\right)$

Answer.

(a) $\log_b(2) + \log_b(x)$ (b) $\log_b(2) - \log_b(x)$

4.4.7.8.

(a) $\log_b\left(\frac{2x}{x-2}\right)$ (b) $\log_b(x(2x+3))$

4.4.7.9.

(a) $\log_3(3x^4)$ (b) $\log_5(1.1^{1/t})$

Answer.

(a) $1 + 4\log_3(x)$ (b) $\frac{1}{t}\log_5(1.1)$

4.4.7.10.

(a) $\log_b\left((4b)^t\right)$ (b) $\log_2\left(5(2^x)\right)$

4.4.7.11.

(a) $\log_b(\sqrt{bx})$ (b) $\log_3(\sqrt[3]{x^2+1})$

Answer.

(a) $\frac{1}{2} + \frac{1}{2}\log_b(x)$ (b) $\frac{1}{3}\log_3((x^2+1))$

4.4.7.12.

(a) $\log\left(\sqrt{\frac{2L}{R^2}}\right)$ (b) $\log\left(2\pi\sqrt{\frac{l}{g}}\right)$

4.4.7.13.

(a) $\log\left(P_0(1-m)^t\right)$

(b) $\log_4\left(\left(1+\frac{r}{4}\right)^{4t}\right)$

Answer.

(a) $\log(P_0) + t\log(1-m)$ (b) $4t[\log_4(4+r) - 1]$

4.4.7.14.

(a) $\log_3\left(\frac{a^2-2}{a^5}\right)$

(b) $\log\left(\frac{a^3b^2}{(a+b)^{3/2}}\right)$

For Problems 15-20, combine into one logarithm and simplify. Assume all expressions are defined.

4.4.7.15.

(a) $\log_b(8) - \log_b(2)$

(b) $2\log_4(x) + 3\log_4(y)$

Answer.

(a) $\log_b(4)$ (b) $\log_4(x^2y^3)$

4.4.7.16.

(a) $\log_b(5) + \log_b(2)$

(b) $\frac{1}{4}\log_5(x) - \frac{3}{4}\log_5(y)$

4.4.7.17.

(a) $\log(2x) + 2\log(x) - \log\left(\sqrt{x}\right)$

(b) $\log(t^2 - 16) - \log(t + 4)$

Answer.

(a) $\log(2x^{5/2})$ (b) $\log(t - 4)$

4.4.7.18.

(a) $\log(x^2) + \log(x^3) - 5\log(x)$

(b) $\log(x^2 - x) - \log\left(\sqrt{x^3}\right)$

4.4.7.19.

(a) $3 - 3\log(30)$

(b) $\dfrac{1}{3}\log_6(8w^6)$

Answer.

(a) $\log\left(\dfrac{1}{27}\right)$ (b) $\log_6(2w^2)$

4.4.7.20.

(a) $2 - \log_4(16z^2)$

(b) $1 - 2\log_3(x)$

For Problems 21-24, use the three logs below to find the value of each expression.

$$\log_b(2) = 1.6931, \quad \log_b(3) = 2.0986, \quad \log_b(5) = 3.6094$$

(*Hint*: For example, $\log_b(15) = \log_b(3) + \log_b(5)$.)

4.4.7.21.
(a) $\log_b(6)$ (b) $\log_b\left(\dfrac{2}{5}\right)$

Answer.

(a) 1.7917 (b) -0.9163

4.4.7.22.
(a) $\log_b(10)$ (b) $\log_b(\dfrac{3}{2})$

4.4.7.23.
(a) $\log_b(9)$ (b) $\log_b(\sqrt{50})$

Answer.

(a) 2.1972 (b) 1.9560

4.4.7.24.
(a) $\log_b(25)$ (b) $\log_b(75)$

For Problems 25-36, solve the equation by using logarithms base 10. “Give both the exact answer and the solution rounded to the nearest four decimal places.

4.4.7.25. $2^x = 7$

Answer. 2.8074

4.4.7.26. $3^x = 4$

4.4.7.27. $3^{x+1} = 8$

Answer. 0.8928

4.4.7.28. $2^{x-1} = 9$

4.4.7.29. $4^{x^2} = 15$

Answer. ±1.3977

4.4.7.30. $3^{x^2} = 21$

4.4.7.31.
$4.26^{-x} = 10.3$

Answer. -1.6092

4.4.7.32.
$2.13^{-x} = 8.1$

4.4.7.33.
$25 \cdot 3^{2.1x} = 47$

Answer. 0.2736

4.4.7.34.
$12 \cdot 5^{1.5x} = 85$

4.4.7.35.
$3600 = 20 \cdot 8^{-0.2x}$

Answer. -12.4864

4.4.7.36.
$0.06 = 50 \cdot 4^{-0.6x}$

4.4.7.37. If raw meat is allowed to thaw at 50°F, Salmonella grows at a rate of 9% per hour.

(a) Write a formula for the amount of Salmonella present after t hours, if the initial amount is S_0.

(b) Health officials advise that the amount of Salmonella initially present in meat should not be allowed to increase by more than 50%. How long can meat be left to thaw at 50°F?

Answer.

(a) $S(t) = S_0(1.09)^t$ (b) 4.7 hours

4.4.7.38. Starting in 1998, the demand for electricity in Ireland grew at a rate of 5.8% per year. In 1998, 20,500 gigawatts were used. (Source: Electricity Supply Board of Ireland)

(a) Write a formula for electricity demand in Ireland as a function of time.

(b) If demand continues to grow at the same rate, when would it reach 30,000 gigawatts?

4.4.7.39. The concentration of a certain drug injected into the bloodstream decreases by 20% each hour as the drug is eliminated from the body. The initial dose creates a concentration of 0.7 milligrams per milliliter.

(a) Write a function for the concentration of the drug as a function of time.

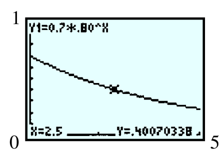
(b) The minimum effective concentration of the drug is 0.4 milligrams per milliliter. When should the second dose be administered?

(c) Verify your answer with a graph.

Answer.

(a) $C(t) = 0.7(0.80)^t$

(b) After 2.5 hours

(c) 

4.4.7.40. A small pond is tested for pollution and the concentration of toxic chemicals is found to be 80 parts per million. Clean water enters the pond from a stream, mixes with the polluted water, then leaves the pond so that the pollution level is reduced by 10% each month.

(a) Write a function for the concentration of toxic chemicals as a function of time.

(b) How long will it be before the concentration of toxic chemicals reaches a safe level of 25 parts per million?

(c) Verify your answer with a graph.

4.4.7.41. According to the National Council of Churches, the fastest growing denomination in the United States in 2004 was the Jehovah's Witnesses, with an annual growth rate of 1.82%.

(a) The Jehovah's Witnesses had 1,041,000 members in 2004. Write a formula for the membership in the Jehovah's Witnesses in millions as a function of time, assuming that the church continues to grow at the same rate.

(b) According to this model, when will the Jehovah's Witnesses have 2,000,000 members?

Answer.

(a) $J(t) = 1,041,000 \cdot 1.0182^t$ (b) In 2040

4.4.7.42. In 2004, the Presbyterian Church had 3,241,000 members, but membership was declining by 4.87% annually.

(a) Write a formula for the membership in the Presbyterian Church as a function of time, assuming that the membership continues to decline at the same rate.

(b) When will the Presbyterian Church have 2,000,000 members?

4.4.7.43. Sodium-24 is a radioactive isotope that is used in diagnosing circulatory disease. It decays into stable isotopes of sodium at a rate of 4.73% per hour.

(a) Technicians inject a quantity of sodium-24 into a patient's bloodstream. Write a formula for the amount of sodium-24 present in the bloodstream as a function of time.

(b) How long will it take for 75% of the isotope to decay?

Answer.

(a) $S(t) = S_0 \cdot 0.9527^t$ (b) 28.61 hours

4.4.7.44. The population of Afghanistan is growing at 2.6% per year.

(a) Write a formula for the population of Afghanistan as a function of time.

(b) In 2005, the population of Afghanistan was 29.9 million. At the given rate of growth, how long would it take the population to reach 40 million?

For Problems 46-52, evaluate each expression. Which (if any) are equal?

4.4.7.45.
(a) $\log_2(4 \cdot 8)$ (b) $(\log_2(4))(\log_2(8))$ (c) $\log_2(4) + \log_2(8)$

Answer.

(a) 5 (b) 6 (c) 5

(a) and (c) are equal.

4.4.7.46.
(a) $\log_2(16 + 16)$ (b) $\log_2(16) + \log_2(16)$ (c) $\log_2(2) + \log_2(16)$

4.4.7.47.
(a) $\log_3(27^2)$ (b) $(\log_3(27)^2)$ (c) $\log_3(27) + \log_3(27)$

Answer.

(a) 6 (b) 9 (c) 6

(a) and (c) are equal.

4.4.7.48.
(a) $\log_3(3 \cdot 27)$ (b) $\log_3(3) + \log_3(27)$ (c) $\log_3(3) \cdot \log_3(27)$

4.4.7.49.

(a) $\log\left(\dfrac{240}{10}\right)$ (b) $\dfrac{\log(240)}{\log(10)}$ (c) $\log(240)-\log(10)$

Answer.

(a) $\log(24) \approx 1.38$ (b) $\log(240) \approx 2.38$ (c) $\log(230) \approx 2.36$

None are equal.

4.4.7.50.

(a) $\log\left(\dfrac{1}{2}\cdot 80\right)$ (b) $\dfrac{1}{2}\log(80)$ (c) $\log\left(\sqrt{80}\right)$

4.4.7.51.

(a) $\log(75-15)$ (b) $\log(75)-\log(15)$ (c) $\dfrac{\log(75)}{\log(15)}$

Answer.

(a) $\log(60) \approx 1.78$ (b) $\log(5) \approx 0.70$ (c) $\dfrac{\log(75)}{\log(15)} \approx 1.59$

None are equal.

4.4.7.52.

(a) $\log(8\cdot 25)$ (b) $\log(25^8)$ (c) $\log(8+25)$

For Problems 53–58, use the formula for compound interest,

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

4.4.7.53. What rate of interest is required so that \$1000 will yield \$1900 after 5 years if the interest rate is compounded monthly?

Answer. 12.9%

4.4.7.54. What rate of interest is required so that \$400 will yield \$600 after 3 years if the interest rate is compounded quarterly?

4.4.7.55. How long will it take a sum of money to triple if it is invested at 10% compounded daily?

Answer. About 11 years

4.4.7.56. How long will it take a sum of money to increase by a factor of 5 if it is invested at 10% compounded quarterly?

4.4.7.57.

(a) Suppose you invest \$1000 at 12% annual interest for 5 years. In this problem, we will investigate how the number of compounding periods,n, affects the amount, A. Write A as a function of n, with $P = 1000$, $r = 0.12$, and $t = 5$.

(b) Use your calculator to make a table of values for A as a function of n. What happens to A as n increases?

(c) What value of n is necessary to produce an amount $A > 1818$? To produce $A > 1820$? To produce $A > 1822$?

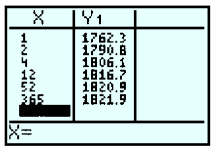
(d) Graph the function $A(n)$ in the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 52 \\ \text{Ymin} = 1750 & \text{Ymax} = 1850 \end{array}$$

Describe the graph: Is it increasing or decreasing? Concave up or down? Does it appear to have an asymptote? Give your best estimate for the asymptote.

Answer.

(a) $A = 1000\left(1 + \dfrac{0.12}{n}\right)^{5n}$

(b) 

X	Y1
1	1762.3
2	1790.8
4	1806.1
12	1816.7
52	1820.9
365	1821.9

X=

A increases.

(c) 16; 31; 553

(d) Increasing, concave down, asymptotically approaching $A \approx 1822.12$

4.4.7.58.

(a) In this problem we will repeat Problem 49 for 4% interest. Write A as a function of n, with $P = 1000$, $r = 0.04$, and $t = 5$.

(b) Use your calculator to make a table of values for A as a function of n. What happens to A as n increases?

(c) What value of n is necessary to produce an amount $A > 1218$? To produce $A > 1220$? To produce $A > 1221.40$?

(d) Graph the function $A(n)$ in the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 52 \\ \text{Ymin} = 1210 & \text{Ymax} = 1225 \end{array}$$

Describe the graph: Is it increasing or decreasing? Concave up or down? Does it appear to have an asymptote? Give your best estimate for the asymptote.

For Problems 59-64, solve the formula for the specified variable.

4.4.7.59. $N = N_0 a^{kt}$, for k

Answer. $k = \dfrac{1}{t}\dfrac{\log(N/N_0)}{\log(a)}$

4.4.7.60. $Q = Q_0 b^{t/2}$, for t

4.4.7.61. $A = A_0(10^{kt} - 1)$, for t

Answer. $t = \dfrac{1}{k}\log\left(\dfrac{A}{A_0} + 1\right)$

4.4.7.62. $B = B_0(1 - 10^{-kt})$, for t

4.4.7.63. $w = pv^q$, for q

Answer. $q = \dfrac{\log(w/p)}{\log(v)}$

4.4.7.64. $l = p^a q^b$, for b

In Problems 65–68 we use the laws of exponents to prove the properties of logarithms.

4.4.7.65. We will use the first law of exponents, $a^p \cdot a^q = a^{p+q}$, to prove the first property of logarithms.

(a) Let $m = \log_b(x)$ and $n = \log_b(y)$. Rewrite these equations in exponential form:

$$x = \underline{\hspace{3cm}} \quad \text{and} \quad y = \underline{\hspace{3cm}}$$

(b) Now consider the expression $\log_b(xy)$. Replace x and y by your answers to part (a).

(c) Apply the first law of exponents to your expression in part (b).

(d) Use the definition of logarithm to simplify your answer to part (c).

(e) Refer to the definitions of m and n in part (a) to finish the proof.

Answer.

(a) $x = b^m$, $y = b^n$

(b) $\log_b(b^m \cdot b^n)$

(c) $\log_b(b^m \cdot b^n) = \log_b(b^{m+n})$

(d) $\log_b(b^{m+n}) = m + n$

(e) $\log_b(b^{m+n}) = \log_b(x) + \log_b(y)$

4.4.7.66. We will use the second law of exponents, $\dfrac{a^p}{a^q} = a^{p-q}$, to prove the second property of logarithms.

(a) Let $m = \log_b(x)$ and $n = \log_b(y)$. Rewrite these equations in exponential form:

$$x = \underline{\hspace{3cm}} \quad \text{and} \quad y = \underline{\hspace{3cm}}$$

(b) Now consider the expression $\log_b\left(\frac{x}{y}\right)$. Replace x and y by your answers to part (a).

(c) Apply the second law of exponents to your expression in part (b).

(d) Use the definition of logarithm to simplify your answer to part (c).

(e) Refer to the definitions of m and n in part (a) to finish the proof.

4.4.7.67. We will use the third law of exponents, $(a^p)^q = a^{pq}$, to prove the third property of logarithms.

(a) Let $m = \log_b(x)$. Rewrite this equation in exponential form:

$$x = \underline{\hspace{3cm}}$$

(b) Now consider the expression $\log_b(x^k)$. Replace x by your answers to part (a).

(c) Apply the third law of exponents to your expression in part (b).

(d) Use the definition of logarithm to simplify your answer to part (c).

(e) Refer to the definitions of m in part (a) to finish the proof.

Answer.

(a) $x = b^m$

(b) $\log_b(b^m)^k$

(c) $\log_b(b^m)^k = \log_b(b^{mk})$

(d) $\log_b(b^{mk}) = mk$

(e) $\log_b(b^{mk}) = (\log_b(x)) \cdot k$

4.4.7.68.

(a) Use the laws of exponents to explain why $\log_b(1) = 0$.

(b) Use the laws of exponents to explain why $\log_b(b^x) = x$.

(c) Use the laws of exponents to explain why $b^{\log_b(x)} = x$.

4.5 Exponential Models

4.5.1 Fitting an Exponential Function through Two Points

Checkpoint 4.5.3 Use the ratio method to find an exponential function whose graph includes the points $(1, 20)$ and $(3, 125)$.

Answer. $f(x) = 8(2.5)^x$

Checkpoint 4.5.5 The number of earthquakes that occur worldwide is a decreasing exponential function of their magnitude on the Richter scale. Between 2000 and 2005, there were 7480 earthquakes of magnitude 5 and 793 earthquakes of magnitude 6. (Source: National Earthquake Information Center, U.S. Geological Survey)

a Find a formula for the number of earthquakes, $N(m)$, in terms of their magnitude.

b It is difficult to keep an accurate count of small earthquakes. Use your formula to estimate the number of magnitude 1 earthquakes that occurred between 2000 and 2005. How many earthquakes of magnitude 8 occurred?

Answer.

a $N(m) = 558,526,329(0.106)^m$

b $59,212,751$; 9

4.5.2 Doubling Time

Checkpoint 4.5.7 In 2005, the population of Uganda was 26.9 million people and was growing by 3.2% per year.

a Write a formula for the population of Uganda as a function of years since 2005.

b How long will it take the population of Uganda to double?

c Use your formula from part (a) to verify the doubling time for three doubling periods.

Answer.

a $P(t) = 26.9(1.032)^t$ million

b 22 years

c $P(0) = 26.9$; $P(22) \approx 53.8$, $P(44) \approx 107.6$, $P(66) \approx 215.1$

Checkpoint 4.5.9 At its current rate of growth, the population of Mexico will double in 36.8 years. What is its annual percent rate of growth?

Answer. 1.9%

4.5.3 Half-Life

Checkpoint 4.5.11 Alcohol is eliminated from the body at a rate of 15% per hour.

a Write a decay formula for the amount of alcohol remaining in the body.

b What is the half-life of alcohol in the body?

Answer.

a $A(t) = A_0(0.85)^t$

b 4.3 hours

Checkpoint 4.5.13 Cesium-137, with a half-life of 30 years, is one of the most dangerous by-products of nuclear fission. What is the annual decay rate for cesium-137?

Answer. 2.28%

4.5.4 Annuities and Amortization

Checkpoint 4.5.15 Rufus is saving for a new car. He puts \$2500 a year into an account that pays 4% interest compounded annually. How many years will it take him to accumulate \$20, 000? (Round up to the next whole year.)

Answer. 8 years

Checkpoint 4.5.17 Use the formula for the present value of an annuity to calculate your monthly mortgage payment on a home loan of \$250, 000 amortized over 30 years at 6% interest compounded monthly.

Answer. \$1498.88

4.5.6 Homework 4.5

For Problems 1-8, find an exponential function that has the given values.

4.5.6.1. $A(0) = 0.14,\ A(3) = 7$

Answer. $A(x) = 0.14(50)^{x/3}$

4.5.6.2. $B(0) = 28,\ B(5) = 0.25$

4.5.6.3. $f(7) = 12,\ f(8) = 9$

Answer. $f(x) = \dfrac{65,536}{729}\left(\dfrac{3}{4}\right)^x$

4.5.6.4. $g(2) = 2.6,\ g(3) = 3.9$

4.5.6.5. $M(4) = 100,\ M(7) = 0.8$

Answer. $M(x) = 62,500(0.2)^x$

4.5.6.6. $N(12) = 512,000,$
$N(14) = 1,024,000$

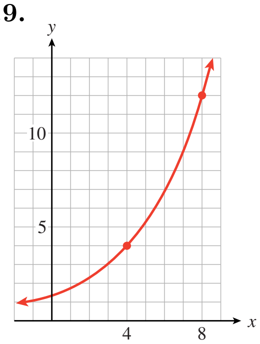
4.5.6.7. $s(3.5) = 16.2,$
$s(6) = 3936.6$

Answer. $s(x) = \dfrac{1}{135}(9)^x$

4.5.6.8. $T(1.2) = 15,$
$T(1.8) = 1.875$

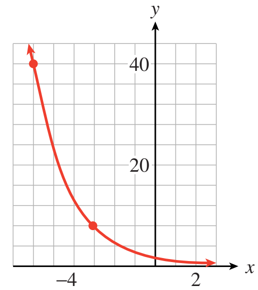
For Problems 9-12, find a formula for the exponential function shown.

4.5.6.9.

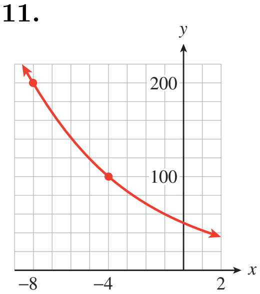


Answer. $y = \dfrac{4}{3}(3)^{x/4}$

4.5.6.10.

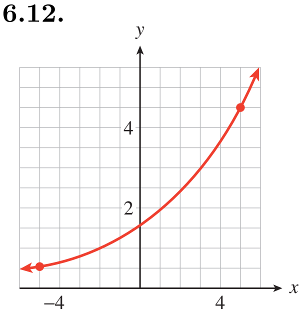


4.5.6.11.



Answer. $y = 50(2)^{-x/4}$

4.5.6.12.



For Problems 13–18,

a Fit a linear function to the points.

b Fit an exponential function to the points.

c Graph both functions on the same axes.

4.5.6.13. $(0, 2.6)$, $(1, 1.3)$

Answer.

(a) $y = 2.6 - 1.3x$

(b) $y = 2.6(0.5)^x$

(c)

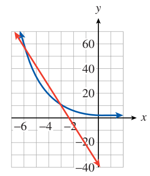
4.5.6.14. $(0, 0.48)$, $(1, 0.16)$

4.5.6.15. $(-6, 60),\ (-3, 12)$

Answer.

(a) $y = -36 - 16x$

(b) $y = \dfrac{12}{5}(5)^{-x/3}$

(c) 

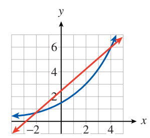
4.5.6.16. $(2, 1.5),\ (4, 4.5)$

4.5.6.17. $(-2, 0.75),\ (4, 6)$

Answer.

(a) $y = 2.5 + 0.875x$

(b) $y = 1.5(2)^{x/2}$

(c) 

4.5.6.18. $(-1, 0.5),\ (1, 1)$

4.5.6.19. Nevada was the fastest growing state in the nation between 1990 and 2000, with an annual growth rate of 5.2%.

a Write a function for the population of Nevada as a function of time. Let the initial population be P_0.

b How long will it take for the population to double?

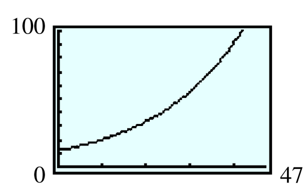
c In 1990, the population of Nevada was 12 hundred thousand. Write this in function notation.

d $P(5)$ represents the population of what year?

Answer.

(a) $P = P_0(1.052)^t$; t is the number of years since 1990.

(b) $\dfrac{\log(2)}{\log(1.052)} \approx 13.7$ years

(c) 

4.5.6.20. In 1986, the inflation rate in Bolivia was 8000% annually. The unit of currency in Bolivia is the boliviano.

a Write a formula for the price of an item as a function of time. Let P_0 be its initial price.

b How long did it take for prices to double? Give both an exact value and a decimal approximation rounded to two decimal places.

c Suppose $P_0 = 5$ bolivianos. Graph your function in the window Xmin = 0, Xmax = 0.94, Ymin = 0, Ymax = 100.

d Use **intersect** to verify that the price of the item doubles from 5 to 10 bolivianos, from 10 to 20, and from 20 to 40 in equal periods of time.

4.5.6.21. The gross domestic product (GDP) of the United Kingdom was 1 million pounds in the year 2000 and is growing at a rate of 2.8% per year. (The unit of currency in the U.K. is the pound, denoted by £.)

a Write a formula for the GDP as a function of years since 2000.

b How long will it take for the GDP to grow to 2 million pounds? Give both an exact value and a decimal approximation rounded to two decimal places.

c How long should it take for the GDP to 4 million pounds?

d Using your answers to (b) and (c), make a rough sketch of the function.

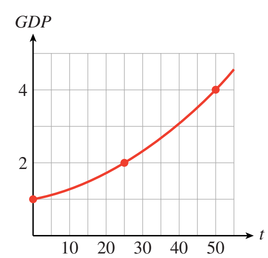
Answer.

(a) $GDP = 1.028^t$ million pounds

(b) $\dfrac{\log(2)}{\log(1.028)} \approx 25.1$ years

(c) 50.2 years

(d)



4.5.6.22. The number of phishing Web sites (fraudulent Web sites designed to trick victims into revealing personal financial information) is growing by 15% each month. In June 2005, there were 4000 phishing Web sites. (Source: www.itnews.com.au/newsstory)

a Write a formula for the number of phishing Web sites as a function of months since June 2005.

b How long will it take for the number of sites to reach 8000? Give both an exact value and a decimal approximation rounded to two decimal places.

c How long should it take for the number of sites to reach $16,000$?

d Using your answers to (b) and (c), make a rough sketch of the function.

4.5.6.23. Radioactive potassium-42, which is used by cardiologists as a tracer, decays at a rate of 5.4% per hour.

a Find the half-life of potassium-42.

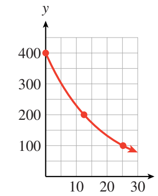
b How long will it take for three-fourths of the sample to decay? For seven-eighths of the sample?

c Suppose you start with 400 milligrams of potassium-42. Using your answers to (a) and (b), make a rough sketch of the decay function.

Answer.

a $\dfrac{\log(0.5)}{\log(0.946)} \approx 12.5$ hours

b 25 hours

c 

4.5.6.24. In October 2005, the *Los Angeles Times* published an article about efforts to save the endangered Channel Island foxes. "Their population declined by 95% to about 120 between 1994 and 2000, according to the park service."

a What was the fox population in 1994?

b Write a formula for the fox population as a function of time since 1994, assuming that their numbers declined exponentially.

c How long did it take for the fox population to be reduced to half its 1994 level? To one-quarter of the 1994 level?

d Using your answers to part (c), make a rough sketch of the decay function.

4.5.6.25. Caffeine leaves the body at a rate of 15.6% each hour. Your first cup of coffee in the morning has 100 mg of caffeine.

a How long will it take before you have 50 mg of that caffeine in your body?

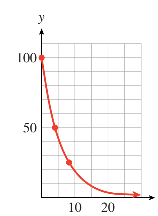
b How long will it take before you have 25 mg of that caffeine in your body?

c Using your answers to (a) and (b), make a rough sketch of the decay function.

Answer.

a $\dfrac{\log(0.5)}{\log(0.844)} \approx 4.1$ hours

b 8.2 hours

c 

4.5.6.26. Pregnant women should monitor their intake of caffeine, because it leaves the body more slowly during pregnancy and can be absorbed by the unborn child through the bloodstream. Caffeine leaves a pregnant woman's body at a rate of 6.7% each hour.

a How long will it take before the 100 mg of caffeine in a cup of coffee is reduced to 50 mg?

b How long will it take before the 100 mg of caffeine in a cup of coffee is reduced to 25 mg?

c Make a rough sketch of the decay function, and compare with the graph in Problem 25.

For Problems 27–30,

a Write a growth or decay formula for the exponential function.

b Find the percent growth or decay rate.

4.5.6.27. A population starts with 2000 and has a doubling time of 5 years.

Answer.

a $P = 2000(2)^{t/5}$

b 14.87%

4.5.6.28. You have 10 grams of a radioactive isotope whose half-life is 42 years.

4.5.6.29. A certain medication has a half-life of 18 hours in the body. You are given an initial dose of D_0 mg.

Answer.

a $D = D_0 \left(\frac{1}{2}\right)^{t/18}$

b 3.78%

4.5.6.30. The doubling time of a certain financial investment is 8 years. You invest an amount M_0.

4.5.6.31. The half-life of radium-226 is 1620 years.

a Write a decay law for radium-226.

b What is the annual decay rate for radium-226?

Answer.

(a) $A = A_0 \left(\frac{1}{2}\right)^{t/1620}$

(b) 0.043%

4.5.6.32. Dichloro-diphenyl-trichloroethane (DDT) is a pesticide that was used in the middle decades of the twentieth century to control malaria. After 1945, it was also widely used on crops in the United States, and as much as one ton might be sprayed on a single cotton field. However, after the toxic effects of DDT on the environment began to appear, the chemical was banned in 1972.

a A common estimate for the half-life of DDT in the soil is 15 years. Write a decay law for DDT in the soil.

b In 1970, many soil samples in the United States contained about 0.5 mg of DDT per kg of soil. The NOAA (National Oceanic and Atmospheric Administration) safe level for DDT in the soil is 0.008 mg/kg. When will DDT content in the soil be reduced to a safe level?

4.5.6.33. In 1798, the English political economist Thomas R. Malthus claimed that human populations, unchecked by environmental or social constraints, double every 25 years, regardless of the initial population size.

a Write a growth law for human populations under these conditions.

b What is the growth rate in unconstrained conditions?

Answer.

(a) $P = P_0(2)^{t/25}$

(b) 2.81%

4.5.6.34. David Sifry observed in 2005 that over the previous two years, the number of Weblogs, or blogs, was doubling every 5 months. (Source: www.sifry.com/alerts/archives)

a Write a formula for the number of blogs t years after January 2005, assuming it continues to grow at the same rate.

b What is the growth rate for the number of blogs?

4.5.6.35. Let $y = f(t) = ab^t$ be an exponential growth function, with $a > 0$ and $b > 1$.

a Suppose that the value of y doubles from $t = 0$ to $t = D$, so that $f(D) = 2 \cdot f(0)$. Rewrite this fact as an equation in terms of a, b, and D.

b What does your answer to (a) tell you about the value of b^D?

c Use the first law of exponents and your result from (b) to rewrite $f(t+D)$ in terms of $f(t)$.

d Explain why your result from (c) shows that the doubling time is constant.

Answer.

(a) $ab^D = 2 \cdot ab^0 = 2a$

(b) $b^D = 2$

(c) $f(t + D) = ab^{t+D} = a \cdot b^t \cdot b^D = ab^t \cdot 2 = 2f(t)$

(d) For any value of t, after D units of time, the new value of f is 2 times the old value.

4.5.6.36. Let $y = g(t) = ab^t$ be an exponential decay function, with $a > 0$ and $0 < b < 1$.

a Suppose that the value of y is halved from $t = 0$ to $t = H$, so that $g(H) = \dfrac{1}{2} \cdot g(0)$. Rewrite this fact as an equation in terms of a, b, and H.

b What does your answer to (a) tell you about the value of b^H?

c Use the first law of exponents and your result from (b) to rewrite $g(t+H)$ in terms of $g(t)$.

d Explain why your result from (c) shows that the half-life is constant.

4.5.6.37. Let $y = g(t) = ab^t$ be an exponential decay function, with $a > 0$ and $0 < b < 1$. In this problem, we will show that there is a fixed value R such that y is decreased by a factor of $\dfrac{1}{3}$ every R units.

a Suppose that $g(R) = \dfrac{1}{3} \cdot g(0)$. Rewrite this fact as an equation in terms of a, b, and R.

b What does your answer to (a) tell you about the value of b^R?

c Use the first law of exponents and your result from (b) to rewrite $g(t+R)$ in terms of $g(t)$.

d Explain why your result from (c) shows that an exponential decay function has a constant "one-third-life."

Answer.

(a) $ab^R = \frac{1}{3} \cdot ab^0 = \frac{1}{3}a$

(b) $b^R = \frac{1}{3}$

(c) $g(t + R) = ab^{t+R} = a \cdot b^t \cdot b^R = ab^t \cdot \frac{1}{3} = \frac{1}{3}g(t)$

(d) For any value of t, after R units of time, the new value of g is $\frac{1}{3}$ times the old value.

4.5.6.38. Let $y = f(t) = ab^t$ be an exponential decay function, with $a > 0$ and $b > 1$. In this problem, we will show that there is a fixed value T such that y triples every T units.

a Suppose that $f(T) = 3 \cdot f(0)$. Rewrite this fact as an equation in terms of a, b, and T.

b What does your answer to (a) tell you about the value of b^T?

c Use the first law of exponents and your result from (b) to rewrite $f(t+T)$ in terms of $f(t)$.

d Explain why your result from (c) shows that an exponential decay function has a constant tripling time.

In Problems 39–42,

a Write a decay law for the isotope.

b Use the decay law to answer the question. (Round to the nearest ten years.)

4.5.6.39. Carbon-14 occurs in living organisms with a fixed ratio to non-radioactive carbon-12. After a plant or animal dies, the carbon-14 decays into stable carbon with a halflife of 5730 years. When samples from the Shroud of Turin were analyzed in 1988, they were found to have 91.2% of their original carbon-14. How old were those samples in 1988?

Answer.

(a) $A = A_0 \left(\dfrac{1}{2}\right)^{t/5730}$

(b) About 760 years old

4.5.6.40. Rubidium-strontium radioactive dating is used in geologic studies to measure the age of minerals. Rubidium-87 decays into strontium-87 with a half-life of 48.8 billion years. Several meteors were found to have 93.7% of their original rubidium. How old are the meteors?

4.5.6.41. Americium-241 (Am-241) is used in residential smoke detectors. Particles emitted as Am-241 decays cause the air in a smoke alarm to ionize, allowing current to flow between two electrodes. If smoke absorbs the particles, the current changes and sets off the alarm. The half-life of Am-241 is 432 years. How long will it take for 30% of the Am-241 to decay?

Answer.

(a) $A = A_0 \left(\dfrac{1}{2}\right)^{t/432}$

(b) About 220 years

4.5.6.42. Doctors can measure the amount of blood in a patient by injecting a known volume of red blood cells tagged with chromium-51. After allowing the blood to mix, they measure the percentage of tagged cells in a sample of the patient's blood and use a proportion to compute the original blood volume. Chromium-51 has a half-life of 27.7 days. How much of the original chromium-51 will still be present after 2 days?

For Problems 43 and 44, use the formula for future value of an annuity.

4.5.6.43. You want to retire with a nest egg of one million dollars. You plan to make fixed monthly payments of \$1000 into a savings account until then. How long will you need to make payments if the account earns 6% interest compounded monthly? What if the annual interest rate is 5%?

Answer. ≈ 30 years; ≈ 33 years

4.5.6.44. Francine plans to make monthly payments into an account to save up for a cruise vacation. She wants to save \$25,000 for the trip. How many \$200 payments will she need if the account pays 3% interest compounded monthly? What if the rate is 4%?

For Problems 45 and 46, use the formula for present value of an annuity.

4.5.6.45. You want to finance \$25,000 to purchase a new car, and your financing institution charges an annual interest rate of 2.7%, compounded monthly. How large will your monthly payment be to pay off the loan in 5 years? In 6 years?

Answer. \$445.89; \$376.50

4.5.6.46. Delbert has accumulated \$5000 in credit card debt. The account charges an annual interest rate of 17%, compounded monthly. Delbert decides not to make any further charges to his account and to pay it off in equal monthly payments. What will the payment be if Delbert decides to pay off the entire amount in 5 years? In 10 years?

4.5.6.47. Moore's law predicts that the number of transistors per computer chip will continue to grow exponentially, with a doubling time of 18 months.

(a) Write a formula for Moore's law, with t in years and $M_0 = 2200$ in 1970.

(b) From 1970 to 1999, the number of transistors per chip was actually modeled approximately by $N(t) = 2200(1.356)^t$. How does this function compare with your answer to part (a)?

(c) Complete the table showing the number of transistors per chip in recent years, the number predicted by Moore's law, and the number predicted by $N(t)$.

Name of chip	Year	Moore's law	$N(t)$	Actual number
Pentium IV	2000			42,000,000
Pentium M (Banias)	2003			77,000,000
Pentium M (Dothan)	2004			140,000,000

(d) What is the doubling time for $N(t)$?

Answer.

(a) $N(t) = 2200(2)^{t/1.5}$

(b) The given model has a smaller growth factor, 1.356, than $2^{1/1.5} \approx 1.59$.

(c)

Name of chip	Year	Moore's law	$N(t)$	Actual number
Pentium IV	2000	2,306,867,200	20,427,413	42,000,000
Pentium M (Banias)	2003	9,227,468,800	50,932,200	77,000,000
Pentium M (Dothan)	2004	14,647,693,680	69,064,063	140,000,000

(d) About 2.3 years

4.5.6.48. If the population of a particular animal is very small, inbreeding will cause a loss of genetic diversity. In a population of N individuals, the percent of the species' original genetic variation that remains after t generations is given by

$$V = V_0 \left(1 - \frac{1}{2N}\right)^t$$

(Source: Chapman and Reiss, 1992)

(a) Assuming $V_0 = 100$, graph V as a function of t for three different values of N: $N = 1000$, 100, and 10.

(b) Fill in the table to compare the values of V after 5, 50, and 100 generations.

Population size	Number of generations		
	5	50	100
1000			
100			
10			

(c) Studies of the cheetah have revealed variation at only 3.2% of its genes. (Other species show variation at 10% to 43% of their genes.) The population of cheetah may be less than 5000. Assuming the population can be maintained at its current level, how many generations will it take before the cheetah's genetic variation is reduced to 1%?

4.6 Chapter Summary and Review

4.6.2 Chapter 4 Review Problems

For Problems 1–4,

a Write a function that describes exponential growth or decay.

b Evaluate the function at the given values.

4.6.2.1. The number of computer science degrees awarded by Monroe College has increased by a factor of 1.5 every 5 years since 1984. If the college granted 8 degrees in 1984, how many did it award in 1994? In 2005?

Answer.
a $D = 8(1.5)^{t/5}$ b 18; 44

4.6.2.2. The price of public transportation has been rising by 10% per year since 1975. If it cost \$0.25 to ride the bus in 1975, how much did it cost in 1985? How much will it cost in the year 2010 if the current trend continues?

4.6.2.3. A certain medication is eliminated from the body at a rate of 15% per hour. If an initial dose of 100 milligrams is taken at 8 a.m., how much is left at 12 noon? At 6 p.m.?

Answer.
a $M = 100(0.85)^t$ b 52.2 mg; 19.7 mg

4.6.2.4. After the World Series, sales of T-shirts and other baseball memorabilia decline 30% per week. If $\$200,000$ worth of souvenirs were sold during the Series, how much will be sold 4 weeks later? After 6 weeks?

For Problems 5-8, use the laws of exponents to simplify.

4.6.2.5. $(4n^{x+5})^2$

Answer. $16n^{2x+10}$

4.6.2.6. $9^x \cdot 3^{x-3}$

4.6.2.7. $\dfrac{m^{x+2}}{m^{2x+4}}$

Answer. $\dfrac{1}{m^{x+2}}$

4.6.2.8. $\sqrt[3]{8^{2x+1} \cdot 8^{x-2}}$

For Problems 5-8, find a growth or decay law for the function.

4.6.2.9.

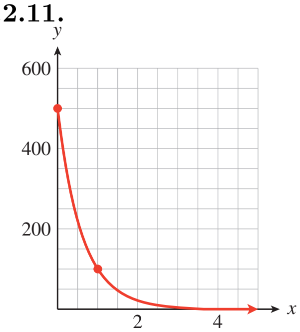
t	0	1	2	3
$g(t)$	16	13.6	11.56	9.83

Answer. $g(t) = 16(0.85)^t$

4.6.2.10.

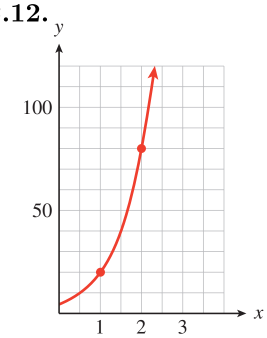
t	0	1	2	3
$f(t)$	12	19.2	30.72	49.15

4.6.2.11.



Answer. $f(x) = 500\left(\dfrac{1}{5}\right)^x$

4.6.2.12.



4.6.2.13. The president's approval rating increased by 12% and then decreased by 15%. What was the net change in his approval rating?

Answer. 4.8% loss

4.6.2.14. The number of students at Salt Creek Elementary School fell by 18% last year but increased by 26% this year. What was the net change in the number of students?

4.6.2.15. Enviroco's stock is growing exponentially in value and increased by 33.8% over the past 5 years. What was its annual rate of increase?

Answer. 6% loss

4.6.2.16. Sales of the software package Home Accountant 3.0 fell exponentially when the new version came out, decreasing by 60% over the past 3 months. What was the monthly rate of decrease?

For Problems 17–20,

(a) Graph the function.

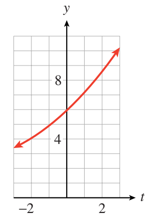
(b) List all intercepts and asymptotes.

(c) Give the range of the function on the domain $[-3, 3]$.

4.6.2.17. $f(t) = 6(1.2)^t$

Answer.

(a)



(b) y-intercept $(0, 6)$; asymptote: $y = 0$

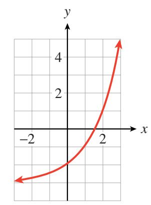
(c) $[3.472, 10.368]$

4.6.2.18. $g(t) = 35(0.6)^{-t}$

4.6.2.19. $P(x) = 2^x - 3$

Answer.

(a)



(b) x-intercept $\left(\frac{\log(3)}{\log(2)}, 0\right)$; y-intercept $(0, -2)$; asymptote: $y = -3$

(c) $[-2.875, 5]$

4.6.2.20. $R(x) = 2^{x+3}$

For Problems 21-24, solve the equation.

4.6.2.21. $3^{x+2} = 9^{1/3}$

Answer. $\dfrac{-4}{3}$

4.6.2.22. $2^{x-1} = 8^{-2x}$

4.6.2.23. $4^{2x+1} = 8^{x-3}$

Answer. -11

4.6.2.24. $3^{x^2-4} = 27$

For Problems 25–28,

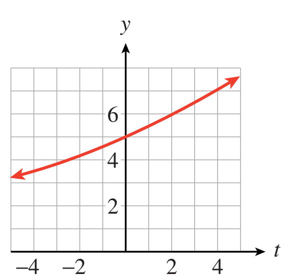
a Graph both functions in the same window. Are they equivalent?

b Justify your answer to part (a) algebraically.

4.6.2.25. $P(t) = 5(2^{t/8}), \quad Q(t) = 5(1.0905)^t$

Answer.

(a)



Not (quite) equivalent

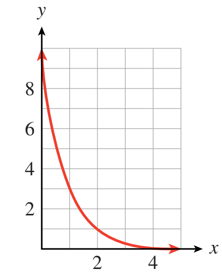
(b) $2^{1/8} \approx 1.090507733 > 1.0905$

4.6.2.26. $M(x) = 4(3^{x/5}), \quad N(x) = 4(1.2457)^x$

4.6.2.27. $H(x) = \left(\dfrac{1}{3}\right)^{x-2}, \quad G(x) = 9\left(\dfrac{1}{3}\right)^x$

Answer.

(a)



Equivalent

(b) $\left(\dfrac{1}{3}\right)^{x-2} = \left(\dfrac{1}{3}\right)^x \cdot \left(\dfrac{1}{3}\right)^{-2} = \left(\dfrac{1}{3}\right)^x \cdot 9$

4.6.2.28. $F(x) = \left(\dfrac{1}{2}\right)^{2x-3}, \quad L(x) = 8\left(\dfrac{1}{4}\right)^x$

For Problems 29–32, $f(x) = 2^x$.

(a) Write a formula for the function.

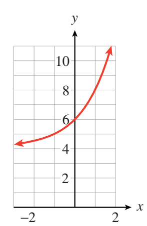
(b) Use transformations to sketch the graph, indicating any intercepts and asymptotes.

4.6.2.29. $y = 4 + f(x+1)$

Answer.

(a) $y = 4 + 2^{x+1}$

(b) Shift the graph of f 1 unit left, 4 units up.



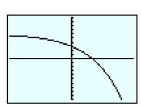
4.6.2.30. $y = -3 + f(x-2)$

4.6.2.31. $y = 6 - 3f(x)$

Answer.

(a) $y = 6 - 3 \cdot 2^x$

(b) Scale vertically by 3, reflect about x-axis, shift 6 units up.



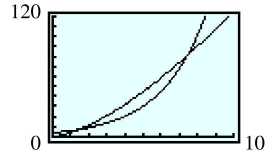
4.6.2.32. $y = 10 - 4f(x)$

In Problems 33–36, we compare power and exponential functions. Let

$$f(x) = 4x^{1.5}, \quad g(x) = 4(1.5)^x$$

4.6.2.33. Graph both functions in the window Xmin = 0, Xmax = 10, Ymin = 0, Ymax = 120. Which function grows more rapidly for large values of x?

Answer.



g eventually grows faster.

4.6.2.34. Estimate the solutions of $f(x) = g(x)$. For what values of x is $f(x) > g(x)$?

4.6.2.35. When x doubles from 2 to 4, $f(x)$ grows by a factor of ___, and $g(x)$ grows by a factor of ____.

Answer. $2^{1.5} \approx 2.83$; 2.25

4.6.2.36. What is the range of $f(x)$ on the domain $[0, 100]$? What is the range of $g(x)$ on the same domain?

4.6.2.37. "Within belts of uniform moisture conditions and comparable vegetation, the organic matter content of soil decreases exponentially with increasing temperature." Data indicate that the organic content doubles with each 10°C decrease in temperature. Write a formula for this function, stating clearly what each variable represents. (Source: Leopold, Wolman, Gordon, and Miller, 1992)

Answer. $M = M_0(2)^{t/10}$, where M is the organic content, M_0 is the organic content at 0°C, and t is the temperature in ° Celsius.

4.6.2.38. In 1951, a study of barley yields under diverse soil conditions led to the formula

$$Y = cV^a G^b$$

where V is a soil texture rating, G is a drainage rating, and a, b, and c are constants. In fields with similar drainage systems, the formula gives barley yields, Y, as a function of V, the soil texture. What type of function is it? If it is an increasing function, what can you say about a? (Source: Briggs and Courtney, 1985)

For Problems 39-44, find the logarithm.

4.6.2.39. $\log_2(16)$

Answer. 4

4.6.2.40. $\log_4(2)$

4.6.2.41. $\log_3\left(\dfrac{1}{3}\right)$

Answer. -1

4.6.2.42. $\log_7(7)$

4.6.2.43. $\log(10^{-3})$

Answer. -3

4.6.2.44. $\log(0.0001)$

For Problems 45-46, write the equation in logarithmic form.

4.6.2.45. $0.3^{-2} = x + 1$

Answer. $\log_{0.3}(x + 1) = -2$

4.6.2.46. $4^{0.3t} = 3N_0$

For Problems 47-50, solve.

4.6.2.47. $4 \cdot 10^{1.3x} = 20.4$

Answer. $\dfrac{\log(5.1)}{1.3} \approx 0.5433$

4.6.2.48. $127 = 2(10^{0.5x}) - 17.3$

4.6.2.49. $3(10^{-0.7x}) + 6.1 = 9$

Answer. $\dfrac{\log(2.9/3)}{-0.7} \approx 0.21$

4.6.2.50. $40(1 - 10^{-1.2x}) = 30$

For Problems 51-54, write the expression in terms of simpler logarithms. (Assume that all variables and variable expressions denote positive real numbers.)

4.6.2.51. $\log_b\left(\dfrac{xy^{1/3}}{z^2}\right)$

Answer.
$\log_b(x) + \dfrac{1}{3}\log_b(y) - 2\log_b(z)$

4.6.2.52. $\log_b\left(\sqrt{\dfrac{L^2}{2R}}\right)$

4.6.2.53. $\log\left(x\sqrt[3]{\dfrac{x}{y}}\right)$

Answer. $\dfrac{4}{3}\log(x) - \dfrac{1}{3}\log(y)$

4.6.2.54. $\log\left(\sqrt{(s-a)(s-g)^2}\right)$

For Problems 55-58, write the expression as a single logarithm with coefficient 1.

4.6.2.55. $\dfrac{1}{3}\left(\log(x) - 2\log(y)\right)$

Answer. $\log\left(\sqrt[3]{\dfrac{x}{y^2}}\right)$

4.6.2.56. $\dfrac{1}{2}\log(3x) - \dfrac{2}{3}\log(y)$

4.6.2.57.
$\dfrac{1}{3}\log(8) - 2\left(\log(8) - \log(2)\right)$

Answer. $\log\left(\dfrac{1}{8}\right)$

4.6.2.58.
$\dfrac{1}{2}(\log(9) + 2\log(4)) + 2\log(5)$

For Problems 59–62, solve the equation by using base 10 logarithms.

4.6.2.59. $3^{x-2} = 7$

Answer. $\dfrac{\log(63)}{\log(3)} \approx 3.77$

4.6.2.60. $4 \cdot 2^{1.2x} = 64$

4.6.2.61. $1200 = 24 \cdot 6^{-0.3x}$

Answer. $\dfrac{\log(50)}{-0.3\log(6)} \approx -7.278$

4.6.2.62. $0.08 = 12 \cdot 3^{-1.5x}$

4.6.2.63. Solve $N = N_0(10^{kt})$ for t.

Answer. $\dfrac{\log\left(N/N_0\right)}{k}$

4.6.2.64. Solve $Q = R_0 + R\log(kt)$ for t.

4.6.2.65. The population of Dry Gulch has been declining according to the function

$$P(t) = 3800 \cdot 2^{-t/20}$$

where t is the number of years since the town's heyday in 1910.

(a) What was the population of Dry Gulch in 1990?

(b) In what year did the population dip below 120 people?

Answer.

(a) 238

(b) 2010

4.6.2.66. The number of compact discs produced each year by Delta Discs is given by the function

$$N(t) = 8000 \cdot 3^{t/4}$$

where t is the number of years since discs were introduced in 1980.

(a) How many discs did Delta produce in 1989?

(b) In what year did Delta first produce over 2 million discs?

4.6.2.67.

(a) Write a formula for the cost of a camera t years from now if it costs \$90 now and the inflation rate is 6% annually.

(b) How much will the camera cost 10 months from now?

(c) How long will it be before the camera costs \$120?

Answer.

(a) $C = 90(1.06)^t$ (b) \$94.48 (c) 5 years

4.6.2.68.

(a) Write a formula for the cost of a sofa t years from now if it costs \$1200 now and the inflation rate is 8% annually.

(b) How much will the sofa cost 20 months from now?

(c) How long will it be before the sofa costs \$1500?

4.6.2.69. Francine inherited \$5000 and plans to deposit the money in an account that compounds interest monthly.

(a) If she can get 5.5% interest, how long will it take for the money to grow to \$7500?

(b) What interest rate will she need if she would like the money to grow to \$6000 in 3 years?

Answer.

(a) 7.4 years (b) 6.1

4.6.2.70. Delbert received a signing bonus of \$2500 and wants to invest the money in a certificate of deposit (CD) that compounds interest quarterly.

(a) If the CD pays 4.8% interest, how long will it take his money to grow to \$3000?

(b) What interest rate will he need if he would like the money to grow to \$3000 in 1 year?

For Problems 71-74, find an exponential growth or decay function that fits the data.

4.6.2.71.
$f(2) = 1714, \quad f(4) = 1836$

Answer. $f(x) \approx 1600(1.035)^x$

4.6.2.72.
$g(1) = 10,665, \quad g(6) = 24,920$

4.6.2.73.
$g(1) = 45, \quad g(5) = 0.00142$

Answer. $g(x) \approx 600(0.075)^x$

4.6.2.74.
$f(2) = 17,464, \quad f(5) = 16.690$

4.6.2.75. The population of Sweden is growing at 0.1% annually.

(a) What is the doubling time for Sweden's population?

(b) In 2005, the population of Sweden was 9 million. At the current rate of growth, how long will it take the population to reach 10 million?

Answer.

(a) $\dfrac{\log(2)}{\log(1.001)} \approx 693$ years

(b) 105 years

4.6.2.76. The bacteria *E. sakazakii* is found in powdered infant formula and can has a doubling time of 4.98 hours even if kept chilled to 50°F.

(a) What is the hourly growth rate for *E. sakazakii?*

(b) How long would it take a colony of *E. sakazakii* to increase by 50%?

4.6.2.77. Manganese-53 decays to chromium-53 with a half-life of 3.7 million years and is used to estimate the age of meteorites. What is the decay rate of manganese-53, with time expressed in millions of years?

Answer. 17%

4.6.2.78. The cold medication pseudoephedrine decays at a rate of 5.95% per hour in the body. What is the half-life of pseudoephedrine?

4.6.2.79. You would like to buy a house with a 20-year mortgage for $300,000, at an interest rate of 6.25%, compounded monthly. Use the formula for the present value of an annuity to calculate your monthly payment.

Answer. $2192.78

4.6.2.80. Rosalie's retirement fund pays 7% interest compounded monthly. Use the formula for the future value of an annuity to calculate how much should she contribute monthly in order to have $500,000 in 25 years.

4.6.2.81. An eccentric millionaire offers you a summer job for the month of June. She will pay you 2 cents for your first day of work and will double your wages every day thereafter. (Assume that you work every day, including weekends.)

(a) Make a table showing your wages on each day. Do you see a pattern?

(b) Write a function that gives your wages in terms of the number of days you have worked.

(c) How much will you make on June 15? On June 30?

Answer.

(a)

Day	1	2	3	$\cdots$	t	$\cdots$	30
Wage (cent)	2	4	8	$\cdots$	2^t	$\cdots$	2^{30}

(b) $W(t) = 2^t$ cents

(c) $327.68; $10,737,418.24

4.6.2.82. The king of Persia offered one of his subjects anything he desired in return for services rendered. The subject requested that the king give him an amount of grain calculated as follows: Place one grain of wheat on the first square of a chessboard, two grains on the second square, four grains on the third square, and so on, until the entire chessboard is covered.

(a) Make a table showing the number of grains of wheat on each square of the chessboard

(b) Write a function for the amount of wheat on each square.

(c) How many grains of wheat should be placed on the last (64th) square?

4.7 Projects for Chapter 4

Project 4.7.1 Bode's Law. In 1772, the astronomer Johann Bode promoted a formula for the orbital radii of the six planets known at the time. This formula calculated the orbital radius, r, as a function of the planet's position, n, in line from the Sun. (Source: Bolton, 1974)

a Evaluate Bode's law, $r(n) = 0.4 + 0.3(2^{n-1})$, for the values in the table. (Use a large negative number, such as $n = -100$, to approximate $r(-\infty)$.)

n	$-\infty$	1	2	3	4	5	6
$r(n)$							

b How do the values of $r(n)$ compare with the actual orbital radii of the planets shown in the table? (The radii are given in astronomical units (AU). One AU is the distance from the Earth to the Sun, about 149.6×10^6 kilometers.) Assign values of n to each of the planets so that they conform to Bode's law.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Orbitan radius (AU)	0.39	0.72	1.00	1.52	5.20	9.54
n						

c In 1781, William Herschel discovered the planet Uranus at a distance of 19.18 AU from the Sun. If $n = 7$ for Uranus, what does Bode's law predict for the orbital radius of Uranus?

d None of the planets' orbital radii corresponds to $n = 2$ in Bode's law. However, in 1801 the first of a group of asteroids between the orbits of Mars and Jupiter was discovered. The asteroids have orbital radii between 2.5 and 3.0 AU. If we consider the asteroids as one planet, what orbital radius does Bode's law predict?

e In 1846, Neptune was discovered 30.6 AU from the Sun, and Pluto was discovered in 1930 39.4 AU from the Sun. What orbital radii does Bode's law predict for these planets?

Project 4.7.2 Plague. In 1665, there was an outbreak of the plague in London. The table shows the number of people who died of plague during each week of the summer that year. (Source: Bolton, 1974)

Week	Deaths
0, May 9	9
1, May 16	3
2, May 23	14
3, May 30	17
4, June 6	43
5, June 13	112
6, June 20	168
7, June 27	267
8, July 4	470
9, July 11	725
10, July 18	1089
11, July 25	1843

Week	Deaths
12, August 1	2010
13, August 8	2817
14, August 15	3880
15, August 22	4237
16, August 29	6102
17, September 5	6988
18, September 12	6544
19, September 19	7165
20, September 26	5533
21, October 3	4929
22, October 10	4327

1. Scale horizontal and vertical axes for the entire data set, but plot only the data for the first 8 weeks of the epidemic, from May 9 through July 4. On the same axes, graph the function $f(x) = 2.18(1.83)^x$.

2. By what weekly percent rate did the number of victims increase during the first eight weeks?

3. Add data points for July 11 through October 10 to your graph. Describe the progress of the epidemic relative to the function f and offer an explanation.

4. Make a table showing the total number of plague victims at the end of each week and plot the data. Describe the graph.

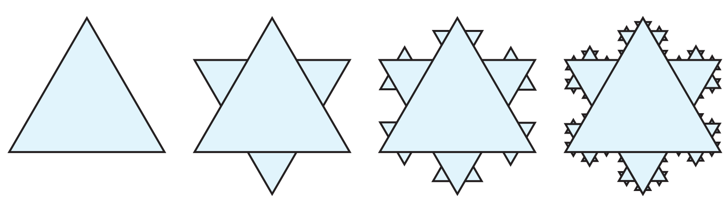
Project 4.7.3 Koch snowflake. The Koch snowflake is an example of a fractal. It is named in honor of the Swiss mathematician Niels Fabian Helge von Koch (1870–1924). Here is how to construct a Koch snowflake:

- Draw an equilateral triangle with sides of length 1 unit. This is stage $n = 0$.

- Divide each side into 3 equal segments and draw a smaller equilateral triangle on each middle segment, as shown in the figure. The new figure (stage $n = 1$) is a 6-pointed star with 12 sides.

- Repeat the process to obtain stage $n = 2$: Trisect each of the 12 sides and draw an equilateral triangle on each middle third.

- If you continue this process forever, the resulting figure is the **Koch snowflake**.



$n = 0$ $n = 1$ $n = 2$ $n = 3$

a We will consider several functions related to the Koch snowflake:

$S(n)$ is the length of each side in stage n
$N(n)$ is the number of sides in stage n
$P(n)$ is the perimeter of the snowflake at stage n

Fill in the table describing the snowflake at each stage.

Stage n	$S(n)$	$N(n)$	$P(n)$
0			
1			
2			
3			

b Write an expression for $S(n)$.

c Write an expression for $N(n)$.

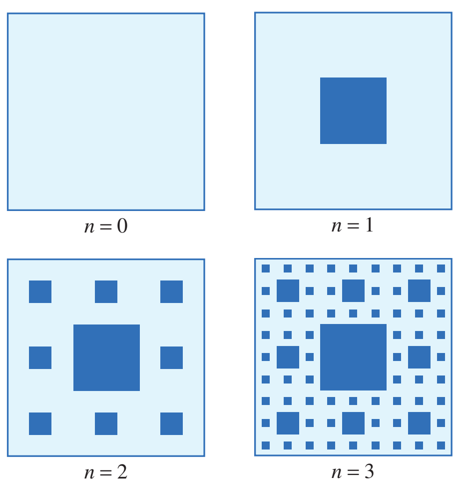
d Write an expression for $P(n)$.

e What happens to the perimeter as n gets larger and larger?

f As n increases, the area of the snowflake increases also. Is the area of the completed Koch snowflake finite or infinite?

Project 4.7.4 Sierpinski carpet. The Sierpinski carpet is another fractal. It is named for the Polish mathematician Waclaw Sierpinski (1882–1969). Here is how to build a Sierpinski carpet:

- Start with a unit square (sides of length 1 unit.)
- For stage $n = 1$, trisect each side and partition the square into 9 smaller squares. Remove the center square, leaving a hole surrounded by 8 squares, as shown in the figure.
- For stage $n = 2$, repeat the process on each of the remaining 8 squares.
- If you continue this process forever, the resulting is the **Sierpinski carpet**.



a We will consider several functions related to the Sierpinski carpet:

$S(n)$ is the side of a new square at stage n
$A(n)$ is the area of a new square at stage n
$N(n)$ is the number of new squares removed at stage n
$R(n)$ is the total area removed at stage n
$T(n)$ is the total area remaining at stage n

Fill in the table describing the carpet at each stage.

Stage n	$S(n)$	$A(n)$	$N(n)$	$R(n)$	$T(n)$
0					
1					
2					
3					

b Write an expression for $S(n)$.

c Write an expression for $A(n)$.

d Write an expression for $N(n)$.

e Write an expression for $R(n)$.

f Write an expression for $T(n)$.

g What happens to the area remaining as n approaches infinity?

Project 4.7.5 Stream order. The **order** of a stream or river is a measure of its relative size. A first-order stream is the smallest, one that has no tributaries. Second-order streams have only first-order streams as tributaries. Third-order streams may have first- and second-order streams as tributaries, and so on. The Mississippi River is an example of a tenth-order stream, and the Columbia River is ninth order.

Both the number of streams of a given order and their average length are exponential functions of their order. In this problem, we consider all streams in the United States. (Source: Leopold, Luna, Gordon, and Miller, 1992)

a Using the given values, find a function $N(x) = ab^{x-1}$ for the number of streams of a given order.

b Complete the column for number of streams of each order. (Round to the nearest whole number of streams for each order.)

c Find a function $L(x) = ab^{x-1}$ for the average lengthof streams of a given order, then complete that column.

d Find the total length of all streams of each order, hence estimating the total length of all stream channels in the United States.

Order	Number	Average Length	Total Length
1	$1,600,000$	1	
2	$339,200$	2.3	
3			
4			
5			
6			
7			
8			
9			
10			

Project 4.7.6 Species rank. Related species living in the same area often evolve in different sizes to minimize competition for food and habitat. Here are the masses of eight species of fruit pigeon found in New Guinea, ranked from smallest to largest. (Source: Burton, 1998)

Size rank	1	2	3	4
Mass (grams)	49	76	123	163

Size rank	5	6	7	8
Mass (grams)	245	414	592	802

a Plot the masses of the pigeons against their order of increasing size. What kind of function might fit the data?

b Compute the ratios of the masses of successive sizes of fruit pigeons. Are the ratios approximately constant? What does this information tell you about your answer to part (a)?

c Compute the average ratio to two decimal places. Using this ratio, estimate the mass of a hypothetical fruit pigeon of size rank 0.

d Using your answers to part (c), write an exponential function that approximates the data. Graph this function on top of the data and evaluate the fit.

Project 4.7.7 Future value. Suppose you deposit \$100 at the end of every 6 months into an account that pays 4% compounded annually. How much money will be in the account at the end of 3 years?

a During the 3 years, you will make 6 deposits. Use the formula $F = P(1 + \frac{r}{n})^{nt}$ to write an expression for the future value (principal plus interest) of each deposit. (Do not evaluate the expression!)

Deposit number	Amount deposited	Time in account	Future value
1	100	2.5	$100(1.02)^5$
2	100	2	
3	100	1.5	
4	100	1	
5	100	0.5	
6	100	0	

b Let S stand for the sum of the future values of all the deposits. Write out the sum, without evaluating the terms you found in part (a).

$$S =$$

c You could find S by working out all the terms and adding them up, but what if there were 100 terms, or more? We will use a trick to find the sum in an easier way. Multiply both sides of the equation in part (b) by 1.02. (Use the distributive law on the right side!)

$$1.02S =$$

d Now subtract the equation in part (b) from the equation in part (c). Be sure to line up like terms on the right side.

$$\begin{aligned} 1.02S &= \\ -S &= \\ 0.02S &= \end{aligned}$$

e Finally, solve for S. If you factor 100 from the numerator on the right side, your expression should look a lot like the formula for the future value of an annuity. (To help you see this, note that, for this example, $\frac{r}{n} = ?$ and $nt = ?$)

f Try to repeat the argument above, using letters for the parameters instead of numerical values.

Project 4.7.8 Present value. You would like to set up an account that pays 4% interest compounded semiannually so that you can withdraw \$100 at the end of every 6 months for the next 3 years. How much should you deposit now?

a During the 3 years, you will make 6 withdrawals. Use the formula $P = A(1 + \frac{r}{n})^{-nt}$ to write an expression for the present value of those withdrawals. (Do not evaluate the expression!)

Withdrawal number	Amount withdrawn	Time in account	Present value
1	100	0.5	$100(1.02)^{-1}$
2	100	1	
3	100	1.5	
4	100	2	
5	100	2.5	
6	100	3	

b Let S stand for the sum of the present values of all the withdrawals. Write out the sum, without evaluating the terms you found in part (a).

$$S =$$

c We will use a trick to evaluate the sum. Multiply both sides of the equation in part (b) by 1.02. (Use the distributive law on the right side!)

$$1.02S =$$

d Now subtract the equation in part (b) from the equation in part (c). Be sure to line up like terms on the right side.

$$\begin{aligned} 1.02S &= \\ -S &= \\ 0.02S &= \end{aligned}$$

e Finally, solve for S. If you factor 100 from the numerator on the right side, your expression should look a lot like the formula for the present value of an annuity. (To help you see this, note that, for this example, $\frac{r}{n} = ?$ and $nt = ?$)

f Try to repeat the argument above, using letters for the parameters instead of numerical values.

5 Logarithmic Functions

5.1 Inverse Functions

5.1.1 Introduction

Checkpoint 5.1.2 Suppose g is the inverse function for f, and suppose we know the following function values for f :

$$f(-1) = 0, \quad f(0) = 1, \quad f(1) = 2$$

Find $g(0)$ and $g(1)$.

Answer. $g(0) = -1, \; g(1) = 0$

5.1.2 Finding a Formula for the Inverse Function

Checkpoint 5.1.5 Carol can burn 600 calories per hour bicycling and 400 calories per hour swimming. She would like to lose 5 pounds, which is equivalent to $16,000$ calories.

a Write an equation relating the number of hours of cycling, x, and the number of hours swimming, y, that Carol must spend to lose 5 pounds.

b Write y as a function of x, $y = f(x)$. What does $f(10)$ tell you?

c Find the inverse function, $x = g(y)$. What does $g(10)$ tell you?

Answer.

a $600x + 400y = 16,000$

b $y = f(x) = 40 - 1.5x$; $f(10) = 25$; If Carol cycles for 10 hrs, she must swim for 25 hrs.

c $x = g(y) = 26\frac{2}{3} - \frac{2}{3}y$; $g(10) = 20$; If Carol swims for 10 hrs, she must cycle for 20 hrs.

5.1.3 Inverse Function Notation

Checkpoint 5.1.9

a If $\quad z = f(w) = \dfrac{1}{w+3}$, find $\quad f^{-1}(1)$.

b Write two equations about the value of $f^{-1}(1)$, one using f^{-1} and one using f.

c Show that $f^{-1}(1)$ is not equal to $\dfrac{1}{f(1)}$.

Answer.

a -2

b $f^{-1}(1) = -2$, $f(-2) = 1$

c $f^{-1}(1) = -2$, but $\dfrac{1}{f(1)} = 4$

Checkpoint 5.1.11

a Use the graph of h in Example 5.1.10 to find $h^{-1}(-10)$.

b Does $h^{-1}(-10) = -h^{-1}(10)$?

c Write two equations, one using h and one using h^{-1}, stating the Fahrenheit temperature when the Celsius temperature is $0°$.

Answer.

a -14

b No

c $h(32) = 0$, $h^{-1}(0) = 32$

5.1.4 Graph of the Inverse Function

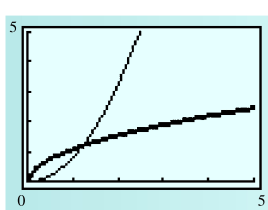
Checkpoint 5.1.13 The formula $T = f(L) = 2\pi\sqrt{\dfrac{L}{32}}$ gives the period in seconds, T, of a pendulum as a function of its length in feet, L.

a Graph the function on the domain $[0, 5]$.

b Find a formula for the inverse function, $L = f^{-1}(T)$. What is the meaning of the inverse function in this context?

c Sketch a graph of the inverse function.

Answer.

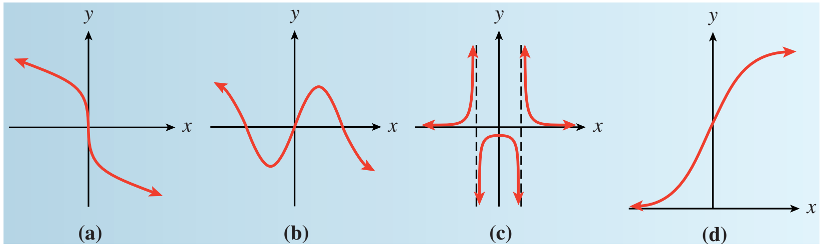
a 

b $L = f^{-1}(T) = \dfrac{8}{\pi^2}T^2$. f^{-1} gives the length of a pendulum as a function of its period.

c See graph for part(a).

5.1.5 When Is the Inverse a Function?

Checkpoint 5.1.15 Which of the functions whose graphs are shown below have inverses that are also functions?



Answer. (a) and (d)

5.1.6 Mathematical Properties of the Inverse Function

Checkpoint 5.1.18

a Find a formula for the inverse of the function $f(x) = \dfrac{2}{x-1}$

b Show that f^{-1} undoes the effect of f on $x = 3$.

c Show that f undoes the effect of f^{-1} on $y = -2$.

Answer.

a $f^{-1}(y) = 1 + \dfrac{2}{y}$

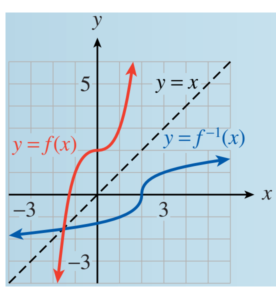
b $f(3) = 1$, and $f^{-1}(1) = 3$

c $f^{-1}(-2) = 0$ and $f(0) = -2$

5.1.7 Symmetry

Checkpoint 5.1.20 Graph the function $f(x) = x^3+2$ and its inverse $f^{-1}(x) = \sqrt[3]{x-2}$ on the same set of axes, along with the line $y = x$.

Answer.



5.1.8 Domain and Range

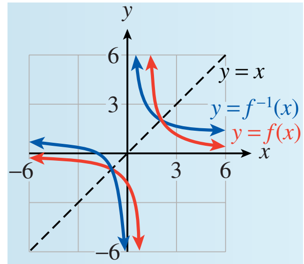
Checkpoint 5.1.22

a Graph the function $f(x) = \dfrac{2}{x-1}$ and its inverse function, f^{-1} (which you found in Checkpoint 5.1.18), on the same set of axes, along with the line $y = x$.

b State the domain and range of f, and of f^{-1}.

Answer.

a



b Domain of f: all real numbers except 1, Range of f: all real numbers except 0, Domain of f^{-1}: all real numbers except 0, Range of f^{-1}: all real numbers except 1

5.1.10 Homework 5.1

5.1.10.1. Let $f(-1) = 0$, $f(0) = 1$, $f(1) = -2$, and $f(2) = -1$.

(a) Make a table of values for $f(x)$ and another table for its inverse function.

(b) Find $f^{-1}(1)$

(c) Find $f^{-1}(-1)$

Answer.

(a)

x	-1	0	1	2
$f(x)$	0	1	-2	-1

y	0	1	-2	-1
$f^{-1}(y)$	-1	0	1	2

(b) $f^{-1}(1) = 0$

(c) $f^{-1}(-1) = 2$

5.1.10.2. Let $f(-1) = 1$, $f(-1) = -2$, $f(0) = 0$, and $f(1) = -1$.

(a) Make a table of values for $f(x)$ and another table for its inverse function.

(b) Find $f^{-1}(-1)$

(c) Find $f^{-1}(1)$

5.1.10.3. $f(x) = x^3 + x + 1$

(a) Make a table of values for $f(x)$ and another table for its inverse function.

(b) Find $f^{-1}(1)$

(c) Find $f^{-1}(3)$

Answer.

(a)

x	-1	0	1	2
$f(x)$	-1	1	3	11

y	-1	1	3	11
$f^{-1}(y)$	-1	0	1	2

(b) $f^{1}(1) = 0$

(c) $f^{-1}(3) = 1$

5.1.10.4. $f(x) = x^5 + x^3 + 7$

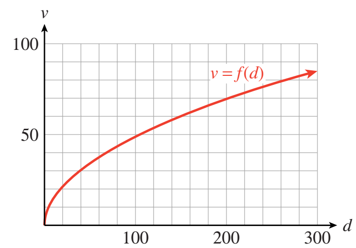
(a) Make a table of values for $f(x)$ and another table for its inverse function.

(b) Find $f^{-1}(7)$

(c) Find $f^{-1}(5)$

For Problems 5-8, use the graph to evaluate each expression.

5.1.10.5. An insurance investigator measures the length, d, of the skid marks at an accident scene, in feet. The graph shows the function $v = f(d)$, which gives the velocity, v (mph), at which a car was traveling when it hit the brakes.



(a) Use the graph to estimate $f(60)$ and explain its meaning in this context.

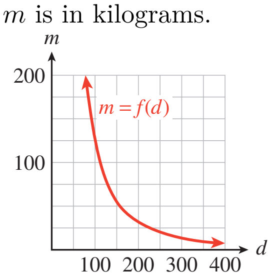
(b) Use the graph to estimate $f^{-1}(60)$ and explain its meaning in this context.

Answer.

(a) $f(60) \approx 38$. The car that left the 60-foot skid marks was traveling at 38 mph.

(b) $f^{-1}(60) \approx 150$. The car traveling at 60 mph left 150-foot skid marks

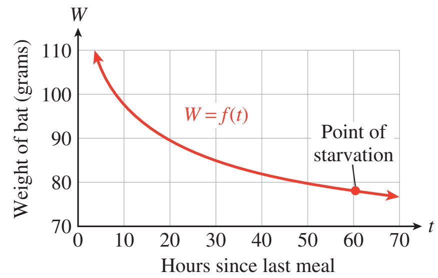
5.1.10.6. The weight, m, of a missile launched from a catapult is a function of the distance, d, to the target. The graph shows the function $m = f(d)$, where d is in meters and m is in kilograms.



(a) Use the graph to estimate $f(100)$ and explain its meaning in this context.

(b) Use the graph to estimate $f^{-1}(100)$ and explain its meaning in this context.

5.1.10.7. After eating, the weight of a vampire bat drops steadily until its next meal. The graph shows the function $W = f(t)$, which gives the weight, W, of the bat in grams t hours since its last meal.



(a) What are the coordinates of the point of starvation? Include units in your answer.

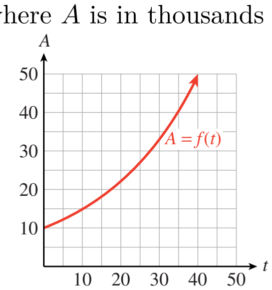
(b) Use the graph to estimate $f^{-1}(90)$ and explain its meaning in this context.

Answer.

(a) $(60\text{ hours}, 78\text{ grams})$

(b) $f^{-1}(90) \approx 19$, so that the vampire bat's weight has dropped to 90 grams about 19 hours after its last meal.

5.1.10.8. The amount of money, A, in an interest-bearing savings account is a function of the number of years, t, it remains in the account. The graph shows $A = f(t)$, where A is in thousands of dollars.



(a) Use the graph to estimate $f(30)$ and explain its meaning in this context.

(b) Use the graph to estimate $f^{-1}(30)$ and explain its meaning in this context.

5.1.10.9. The function $I = g(r) = (1 + r)^5 - 1$ gives the interest, I, that a dollar earns in 5 years in terms of the interest rate, r.

(a) Evaluate $g(0.05)$ and explain its meaning in this context.

(b) Find the interest rate needed to earn \$0.50 by substituting $I = 0.50$ in the formula and solving for r.

(c) Find a formula for the inverse function.

(d) Write your answer to part (b) with inverse function notation.

Answer.

(a) $g(0.05) = 0.28$. At 5% interest, \$1 earns \$0.28 interest in 5 years.

(b) 8.45%

(c) $g^{-1}(I) = (I + 1)^{1/5} - 1$

(d) $g^{-1}(0.50) \approx 0.0845$

5.1.10.10. The function $C = h(F) = \dfrac{5}{9}(F - 32)$ gives the Celsius temperature C in terms of the Fahrenheit temperature F.

(a) Evaluate $h(104)$ and explain its meaning in this context.

(b) Find the Fahrenheit temperature of 37° Celsius by substituting $C = 37$ in the formula and solving for F.

(c) Find a formula for the inverse function.

(d) Write your answer to part (b) with inverse function notation.

5.1.10.11. If you are flying in an airplane at an altitude of h miles, on a clear day you can see a distance of d miles to the horizon, where $d = f(h) = \sqrt{7920h}$.

(a) Evaluate $f(0.5)$ and explain its meaning in this context.

(b) Find the altitude needed in order to see a distance of 10 mile by substituting $d = 10$ in the formula and solving for h.

(c) Find a formula for the inverse function.

(d) Write your answer to part (b) with inverse function notation.

Answer.

(a) $f(0.5) \approx 62.9$. At an altitude of 0.5 miles, you can see 62.9 miles to the horizon.

(b) 0.0126 mile, or 66.7 feet

(c) $h = f^{-1}(d) = \dfrac{d^2}{7920}$

(d) $f^{-1}(10) \approx 0.0126$

5.1.10.12. A moving ship creates waves that impede its own speed. The function $v = f(L) = 1.3\sqrt{L}$ gives the ship's maximum speed in knots in terms of its length, L, in feet.

(a) Evaluate $f(400)$ and explain its meaning in this context.

(b) Find the length needed for a maximum speed of 35 knots by substituting $v = 35$ in the formula and solving for L.

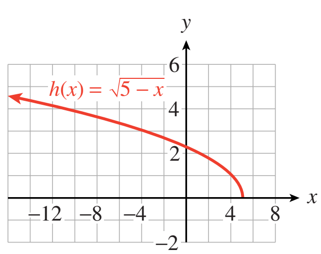
(c) Find a formula for the inverse function.

(d) Write your answer to part (b) with inverse function notation.

5.1.10.13.

(a) Use the graph of $h(x) = \sqrt{5 - x}$ to find $h^{-1}(3)$.

(b) Find a formula for $h^{-1}(x)$ and evaluate $h^{-1}(3)$.



Answer.

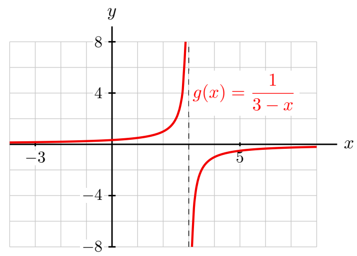
(a) $h^{-1}(3) \approx -4$

(b) $h^{-1}(x) = 5 - x^2$; $h^{-1}(3) = -4$

5.1.10.14.

(a) Use the graph of $g(x) = \dfrac{1}{3-x}$ to find $g^{-1}(-2)$.

(b) Find a formula for $g^{-1}(x)$ and evaluate $g^{-1}(-2)$.



5.1.10.15.

(a) Find f^{-1} for the function $f(x) = (x-2)^3$.

(b) Show that f^{-1} undoes the effect of f on $x = 4$.

(c) Show that f undoes the effect of f^{-1} on $x = -8$.

(d) Graph the function and its inverse on the same grid, along with the graph of $y = x$.

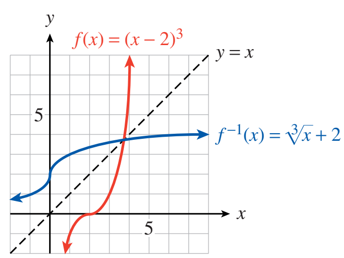
Answer.

(a) $f^{-1}(y) = 3\sqrt[3]{y} + 2$

(b) $f^{-1}(f(4)) = f^{-1}(8) = 4$

(c) $f(f^{-1}(-8)) = f(0) = -8$

(d)



5.1.10.16.

(a) Find f^{-1} for the function $f(x) = \dfrac{2}{x+1}$.

(b) Show that f^{-1} undoes the effect of f on $x = 3$.

(c) Show that f undoes the effect of f^{-1} on $x = -1$.

(d) Graph the function and its inverse on the same grid, along with the graph of $y = x$.

5.1.10.17. If $F(t) = \dfrac{2}{3}t + 1$, find $F^{-1}(5)$.

Answer. 6

5.1.10.18. If $G(s) = \dfrac{s-3}{4}$, find $G^{-1}(-2)$.

5.1.10.19. If $m(v) = 6 - \dfrac{2}{v}$, find $m^{-1}(-3)$.

Answer. $\dfrac{2}{9}$

5.1.10.20. If $p(z) = 1 - 2z^3$, find $p^{-1}(7)$.

5.1.10.21. If $f(x) = \dfrac{x+2}{x-1}$, find $f^{-1}(2)$.

Answer. 4

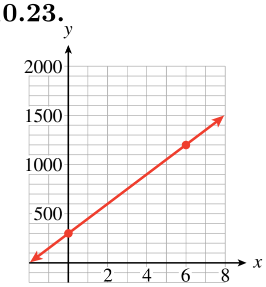
5.1.10.22. If $g(n) = \dfrac{3n+1}{n-3}$, find $g^{-1}(-2)$.

For Problems 23–26,

a Use the graph to make a table of values for the function $y = f(x)$.

b Make a table of values and a graph of the inverse function.

5.1.10.23.



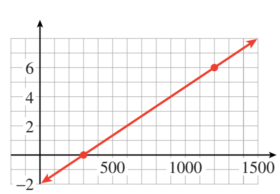
Answer.

(a)

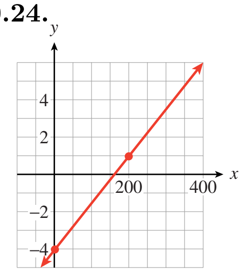
x	0	6
y	300	1200

(b)

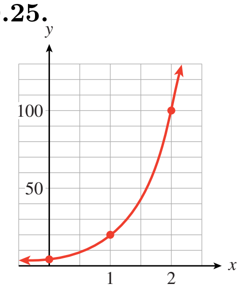
x	300	1200
y	0	6



5.1.10.24.



5.1.10.25.



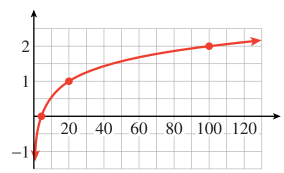
Answer.

(a)

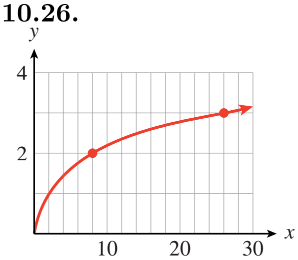
x	0	1	2
y	5	20	100

(b)

x	5	20	100
y	0	1	2



5.1.10.26.



For Problems 27–32,

a Find a formula for the inverse of the function.

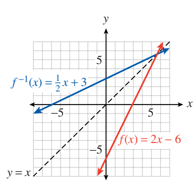
b Graph the function and its inverse on the same set of axes, along with the graph of $y = x$.

5.1.10.27.
$f(x) = 2x - 6$

Answer.

(a) $f^{-1}(x) = \dfrac{x+6}{2}$

(b)



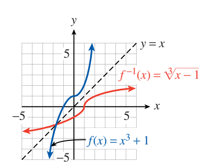
5.1.10.28.
$f(x) = 3x - 1$

5.1.10.29.
$f(x) = x^3 + 1$

Answer.

(a) $f^{-1}(x) = \sqrt[3]{x-1}$

(b)



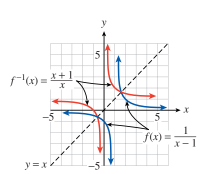
5.1.10.30.
$f(x) = \sqrt[3]{x+1}$

5.1.10.31.
$f(x) = \dfrac{1}{x-1}$

Answer.

(a) $f^{-1}(x) = \dfrac{1}{x} + 1$

(b)



5.1.10.32.
$f(x) = \dfrac{1}{x} - 3$

5.1.10.33.

(a) Find the domain and range of the function $g(x) = \sqrt{4 - x}$.

(b) Find a formula for $g^{-1}(x)$.

(c) State the domain and range of $g^{-1}(x)$.

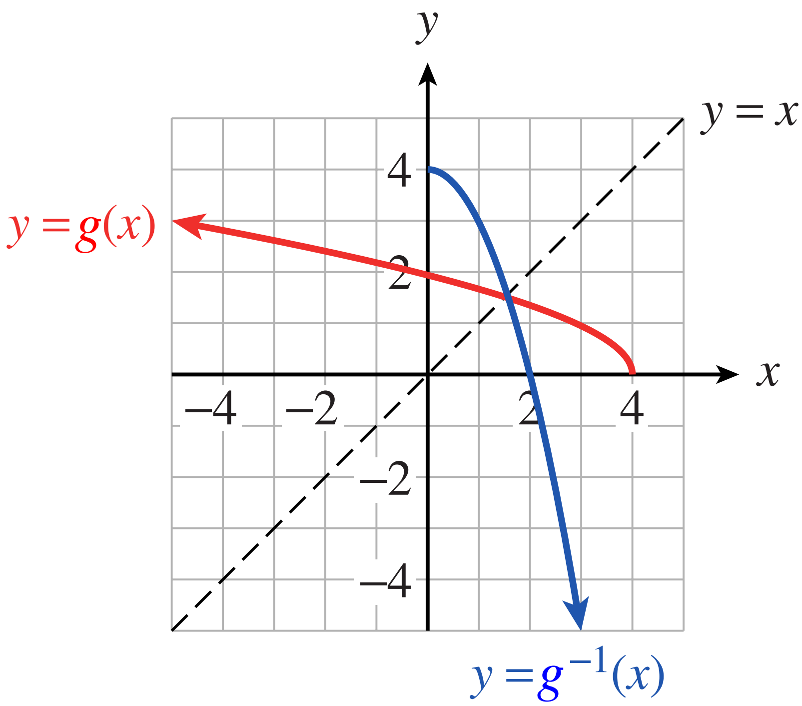
(d) Graph g and g^{-1} on the same grid.

Answer.

(a) Domain: $(-\infty, 4]$; Range: $[0, \infty)$

(b) $g^{-1}(x) = 4 - x^2$

(c) Domain: $[0, \infty)$; Range: $(-\infty, 4]$

(d) 

5.1.10.34.

(a) Find the domain and range of the function $g(x) = 8 - \sqrt{x}$.

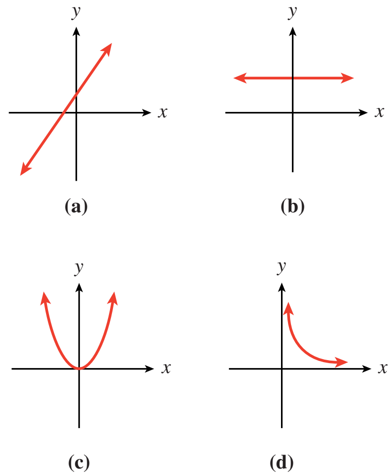
(b) Find a formula for $g^{-1}(x)$.

(c) State the domain and range of $g^{-1}(x)$.

(d) Graph g and g^{-1} on the same grid.

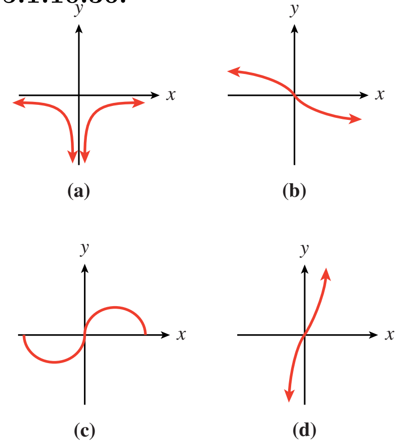
Which of the functions in Problems 35–42 have inverses that are also functions?

5.1.10.35.



Answer. (a) and (d)

5.1.10.36.



5.1.10.37.

(a) $f(x) = x$

(b) $f(x) = x^2$

Answer. (a)

5.1.10.38.

(a) $f(x) = x^3$

(b) $f(x) = |x|$

5.1.10.39.

(a) $f(x) = \dfrac{1}{x}$

(b) $f(x) = \dfrac{1}{x^2}$

Answer. (a)

5.1.10.40.

(a) $f(x) = \sqrt{x}$

(b) $f(x) = \sqrt[3]{x}$

5.1.10.41.

(a) $f(x) = 2^x$

(b) $f(x) = \left(\dfrac{1}{2}\right)^x$

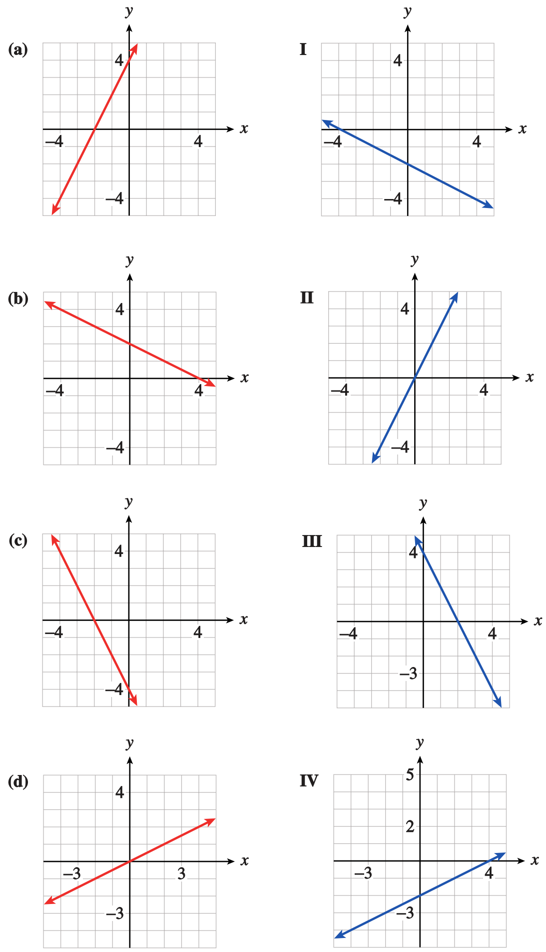
Answer. (a) and (b)

5.1.10.42.

(a) $f(x) = x^3 + x^2$

(b) $f(x) = x^3 + x$

5.1.10.43. Match each function with its inverse from I–IV.



Answer.

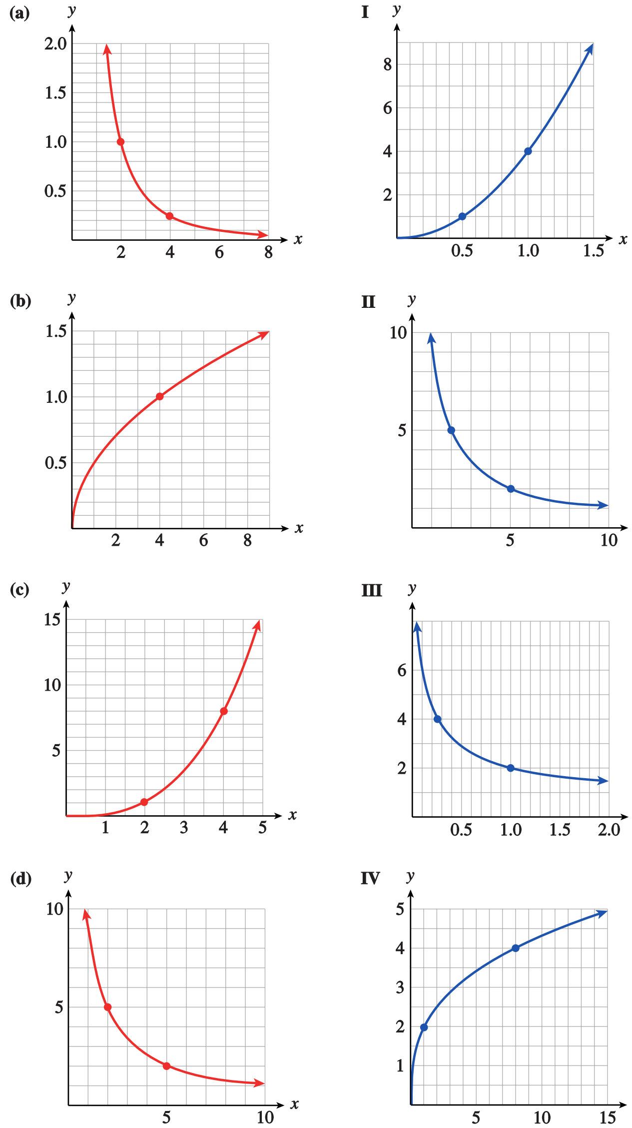
(a) $f(x) = 4 + 2x$; IV

(b) $f(x) = 2 - \dfrac{x}{2}$; III

(c) $f(x) = -4 - 2x$; I

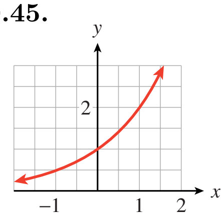
(d) $f(x) = \dfrac{x}{2}$; II

5.1.10.44. Find a formula for each function shown in (a)–(d). Then match each function with its inverse from I–IV.



For Problems 45 and 46, use the graph of f to match the other graphs with the appropriate function. (*Hint*: Look at the coordinates of some specific points.)

5.1.10.45.

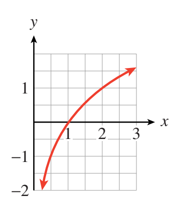


a $-f$

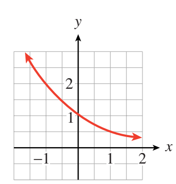
b $\dfrac{1}{f}$

c f^{-1}

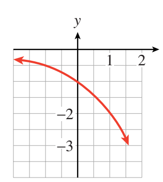
I



II



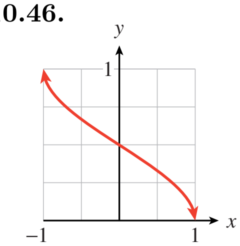
III



Answer.

(a) III (b) II (c) I

5.1.10.46.

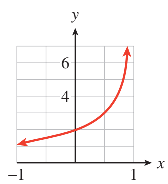


a $-f$

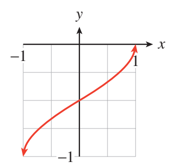
b $\dfrac{1}{f}$

c f^{-1}

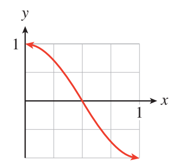
I



II



III



5.2 Logarithmic Functions

5.2.1 Inverse of the Exponential Function

Checkpoint 5.2.2 Simplify each expression.

a $\log(10^6)$

b $\log_w(w^{x+1})$, for $w > 0$, $w \neq 1$

Answer.

a 6 b $x + 1$

Checkpoint 5.2.4 Simplify each expression.

a $4^{\log_4(64)}$ b $2^{\log_2(x^2+1)}$

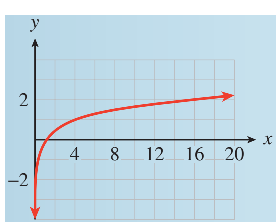
Answer.

a 64 b $x^2 + 1$

5.2.2 Graphs of Logarithmic Functions

Checkpoint 5.2.6 Make a table of values and graph the function $h(x) = \log_4(x)$.

Answer.



Checkpoint 5.2.8

a Find the inverse function for $f(x) = 2\log(x+1)$.

b Graph f and f^{-1} in the window

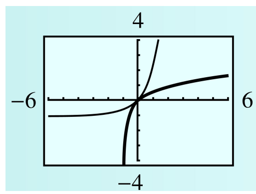
$$\text{Xmin} = -6 \quad \text{Xmax} = 6$$
$$\text{Ymin} = -4 \quad \text{Ymax} = 4$$

c State the domain and range of f and f^{-1}.

Answer.

a $f^{-1}(x) = 10^{x/2} - 1$

b



c Domain of f: $(-1, \infty)$, Range of f: all real numbers, Domain of f^{-1}: all real numbers, Range of f^{-1}: $(-1, \infty)$

5.2.3 Evaluating Logarithmic Functions

Checkpoint 5.2.10 The formula $T = \dfrac{\log(2 \cdot t_i)}{3\log\left(D_f/D_0\right)}$ is used by X-ray technicians to calculate the doubling time of a malignant tumor. D_0 is the diameter of the tumor when first detected, D_f is its diameter at the next reading, and t_i is the time interval between readings, in days. Calculate the doubling time of the following tumor: its diameter when first detected was 1 cm, and 7 days later its diameter was 1.05 cm.

Answer. 33 days

5.2.4 Logarithmic Equations

Checkpoint 5.2.12 Solve for the unknown value in each equation.

a $\log_b(2) = \dfrac{1}{2}$

b $\log_3(2x-1) = 4$

Answer.

a $b = 4$

b $x = 41$

Checkpoint 5.2.15 Imagine the graph of $f(x) = \log(x)$. How far must you travel along the x-axis until the y-coordinate reaches a height of 5.25?

Answer. $x = 177{,}827.941$

5.2.5 Using the Properties of Logarithms

Checkpoint 5.2.17 Solve $\log_2(x) + \log_2(x-2) = 3$.

Hint. Rewrite the left side as a single logarithm.
Rewrite the equation in exponential form.
Solve for x.
Check for extraneous solutions.

Answer. $x = 4$

5.2.7 Homework 5.2

In Problems 1–4,

a Make tables of values for each exponential function and its inverse logarithmic function.

b Graph both functions on the same set of axes.

5.2.7.1. $f(x) = 2^x$

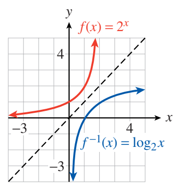
Answer.

(a)

x	-1	0	1	2
2^x	$\frac{1}{2}$	1	2	4

x	$\frac{1}{2}$	1	2	4
$\log_2(x)$	-1	0	1	2

(b)



5.2.7.2. $f(x) = 3^x$

5.2.7.3. $f(x) = \left(\dfrac{1}{3}\right)^x$

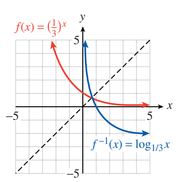
Answer.

(a)

x	-2	-1	0	1
$\left(\frac{1}{3}\right)^x$	9	3	1	$\frac{1}{3}$

x	9	3	1	$\frac{1}{3}$
$\log_{1/3}(x)$	-2	-1	0	1

(b)



5.2.7.4. $f(x) = \left(\dfrac{1}{2}\right)^x$

5.2.7.5.

(a) How large must x be before the graph of $y = \log(x)$ reaches a height of 4?

(b) How large must x be before the graph of $y = \log(x)$ reaches a height of 8?

Answer.

(a) $x = 10,000$ (b) $x = 10^8$

5.2.7.6.

(a) How large must x be before the graph of $y = \log_2(x)$ reaches a height of 5?

(b) How large must x be before the graph of $y = \log(x)$ reaches a height of 10?

5.2.7.7. For what values of x is $y = \log(x) < -2$?

Answer. $0 < x < 0.01$

5.2.7.8. For what values of x is $y = \log_2(x) < -3$?

In Problems 9–14, $f(x) = \log(x)$. Evaluate.

5.2.7.9.

(a) $f(487) + f(206)$

(b) $f(487 + 206)$

Answer.

(a) $\log(100,322) \approx 5.001$

(b) $\log(693) \approx 2.841$

5.2.7.10.

(a) $f(93) + f(1500)$

(b) $f(93 + 1500)$

5.2.7.11.

(a) $f(-7)$

(b) $6f(28)$

Answer.

(a) $\log(-7)$ is undefined.

(b) $6\log(28) \approx 8.683$

5.2.7.12.

(a) $f(0)$

(b) $f(-7)$

5.2.7.13.

(a) $18 - 5f(3)$

(b) $\dfrac{2}{5 + f(0.6)}$

Answer.

(a) 15.614

(b) 0.419

5.2.7.14.

(a) $15 - 4f(7)$

(b) $\dfrac{3}{2 + f(0.2)}$

5.2.7.15. Let $f(x) = 3^x$ and $g(x) = \log_3(x)$.

(a) Compute $f(4)$.

(b) Compute $g[f(4)]$.

(c) Explain why $\log_3(3^x) = x$ for any x.

(d) Compute $\log_3(3^{1.8})$.

(e) Simplify $\log_3(3^a)$.

Answer.

(a) 81

(b) 4

(c) Definition of logarithm base 3

(d) 1.8

(e) a

5.2.7.16. Let $f(x) = 2^x$ and $g(x) = \log_2(x)$.

(a) Compute $f(32)$.

(b) Compute $g[f(32)]$.

(c) Explain why $2^{\log_2(x)} = x$ for any $x > 0$.

(d) Compute $2^{\log_2(6)}$.

(e) Simplify $2^{\log_2(Q)}$.

5.2.7.17.

(a) If $h(r) = \log_2(r)$, find $h^{-1}(8)$.

(b) If $H(w) = 3^w$, find $H^{-1}\left(\dfrac{1}{9}\right)$.

Answer.

(a) 2^8

(b) -2

5.2.7.18.

(a) If $g(z) = \log_3(z)$, find $g^{-1}(-3)$.

(b) If $G(q) = 2^q$, find $G^{-1}(1)$.

For Problems 19–20, simplify.

5.2.7.19.

(a) $10^{\log(2k)}$

(b) $10^{3\log(x)}$

(c) $(\sqrt{10})^{\log(x)}$

(d) $\log(100^m)$

Answer.

(a) $2k$

(b) x^3

(c) $\sqrt{x}$

(d) $2m$

5.2.7.20.

(a) $\log(10^{(1-x)})$

(b) $100^{\log(2x)}$

(c) $(0.1)^{\log(x-1)}$

(d) $\log\left(10^{\log(10)}\right)$

5.2.7.21.

(a) What is the domain of the function $f(x) = 4 + \log_3(x - 9)$?

(b) Find a formula for $f^{-1}(x)$.

Answer.

(a) $(9, \infty)$

(b) $f^{-1}(x) = 3^{x-4} + 9$

5.2.7.22.

(a) What is the domain of the function $f(x) = 1 - \log_2(16 - 4x)$?

(b) Find a formula for $f^{-1}(x)$.

5.2.7.23.

(a) Find the inverse of the function $f(x) = 100 - 4^{x+2}$.

(b) Show that f^{-1} undoes the effect of f on $x = 1$.

(c) Show that f undoes the effect of f^{-1} on $x = 84$.

Answer.

(a) $f^{-1}(x) = \log_4(100 - x) - 2$

(b) $f^{-1}(f(1)) = f^{-1}(36) = \log_4(64) - 2 = 1$

(c) $f\left(f^{-1}(84)\right) = f(0) = 100 - 4^2 = 84$

5.2.7.24.

(a) Find the inverse of the function $f(x) = 5 + 2^{-x}$.

(b) Show that f^{-1} undoes the effect of f on $x = -2$.

(c) Show that f undoes the effect of f^{-1} on $x = 6$.

For Problems 25–26, match each graph to its equation.

5.2.7.25.

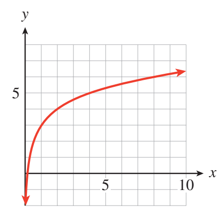
(a) $y = \log_2(x - 3)$

(b) $y = 3 + \log_2(x)$

(c) $y = 2 - \log_2(x)$

(d) $y = \log_2(x + 4) - 1$

I



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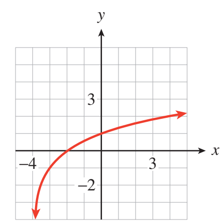

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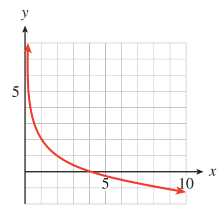

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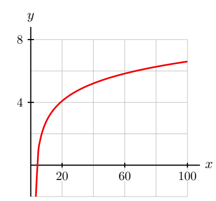


Answer.

(a) IV (b) I (c) II (d) III

5.2.7.26.

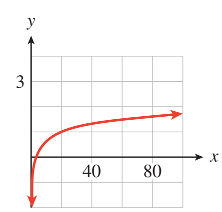
(a) $y = 5\log(x)$

(b) $y = \log\left(\dfrac{x}{2}\right)$

(c) $y = \log\left(\dfrac{1}{x}\right)$

(d) $y = \log(-x)$

I



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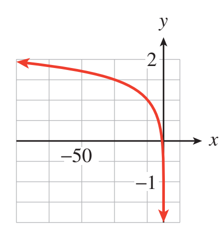

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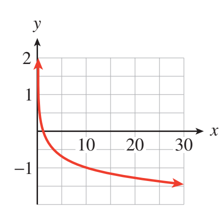

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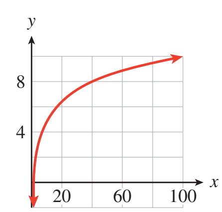


5.2.7.27. In a psychology experiment, volunteers were asked to memorize a list of nonsense words, then 24 hours later were tested to see how many of the words they recalled. On average, the subjects had forgotten 20% of the words. The researchers found that the more lists their volunteers memorized, the larger the fraction of words they were unable to recall. (Source: Underwood, *Scientific American*, vol. 210, no. 3)

Number of lists, n	1	4	8	12	16	20
Percent forgotten, F	20	40	55	66	74	80

(a) Plot the data. What sort of function seems to fit the data points?

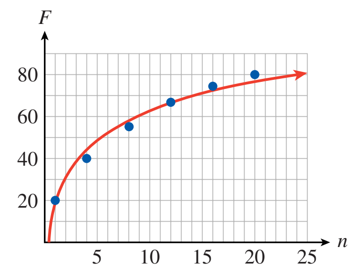
(b) Psychologists often describe rates of forgetting by logarithmic functions. Graph the function

$$f(n) = 16.6 + 46.3\log(n)$$

on the same graph with your data. Comment on the fit.

(c) What happens to the function $f(n)$ as n grows increasingly large? Does this behavior accurately reflect the situation being modeled?

Answer.

(a) 

(b) The graph resembles a logarithmic function. The (translated) log function is close to the points but appears too steep at first and not steep enough after $n = 15$. Overall, it is a good fit.

(c) f grows (more and more slowly) without bound. f will eventually exceed 100 per cent, but no one can forget more than 100% of what is learned.

5.2.7.28. The water velocity at any point in a stream or river is related to the logarithm of the depth at that point. For the Hoback River near Bondurant, Wyoming,

$$v = 2.63 + 1.03\log(d)$$

where v is the velocity of the water, in feet per second, and d is the vertical distance from the stream bed, in feet, at that point. For Pole Creek near Pinedale, Wyoming,

$$v = 1.96 + 0.65\log(d)$$

Both streams are 1.2 feet deep at the locations mentioned. (Source: Leopold, Luna, Wolman, and Gordon, 1992)

(a) Complete the table of values for each stream.

Distance from bed (feet)	0.2	0.4	0.6	0.8	1.0	1.2
Velocity, Hoback River, (ft/sec)						
Velocity, Pole Creek (ft/sec)						

(b) If you double the distance from the bed, by how much does the velocity increase in each stream?

(c) Plot both functions on the same graph.

(d) The average velocity of the entire stream can be closely approximated as follows: Measure the velocity at 20% of the total depth of the stream from the surface and at 80% of the total depth, then average these two values. Find the average velocity for the Hoback River and for Pole Creek.

In Problems 29–30, $f(x) = \log(x)$. Solve for x.

5.2.7.29.
(a) $f(x) = 1.41$ (b) $f(x) = -1.69$ (c) $f(x) = 0.52$

Answer.

(a) $10^{1.41} \approx 25.704$
(b) $10^{-1.69} \approx 0.020417$
(c) $10^{0.52} \approx 3.3113$

5.2.7.30.
(a) $f(x) = 2.3$ (b) $f(x) = -1.3$ (c) $f(x) = 0.8$

For Problems 31-38, convert the logarithmic equation to exponential form.

5.2.7.31. $\log_{16}(256) = w$

Answer. $16^w = 256$

5.2.7.32. $\log_9(729) = y$

5.2.7.33. $\log_b(9) = -2$

Answer. $b^{-2} = 9$

5.2.7.34. $\log_b(8) = -3$

5.2.7.35. $\log(A) = -2.3$

Answer. $10^{-2.3} = A$

5.2.7.36. $\log(C) = -4.5$

5.2.7.37. $\log_u(v) = w$

Answer. $u^w = v$

5.2.7.38. $\log_m(n) = p$

For Problems 39-46, solve for the unknown value.

5.2.7.39. $\log_b(8) = 3$

Answer. $b = 2$

5.2.7.40. $\log_b(625) = 4$

5.2.7.41. $\log_b(10) = \dfrac{1}{2}$

Answer. $b = 100$

5.2.7.42. $\log_b(0.1) = -1$

5.2.7.43. $\log_2(3x - 1) = 5$

Answer. $x = 11$

5.2.7.44. $\log_5(9 - 4x) = 3$

5.2.7.45. $3(\log_7(x)) + 5 = 7$

Answer. $x = 7^{2/3}$

5.2.7.46. $5(\log_2(x)) + 6 = -14$

For Problems 47-54, solve the logarithmic equation.

5.2.7.47. $\log(x) + \log(x + 21) = 2$

Answer. $x = 4$

5.2.7.48. $\log(x + 3) + \log(x) = 1$

5.2.7.49. $\log_8(x + 5) - \log_8(2) = 1$

Answer. $x = 11$

5.2.7.50. $\log(x - 1) - \log(4) = 2$

5.2.7.51. $\log(x + 2) + \log(x - 1) = 1$

Answer. $x = 3$

5.2.7.52. $\log_4(x + 8) - \log_4(x + 2) = 2$

5.2.7.53. $\log_3(x - 2) - \log_3(x + 1) = 3$

Answer. No solution

5.2.7.54. $\log(x + 3) - \log(x - 1) = 1$

For Problems 55-60, solve for the indicated variable.

5.2.7.55. $t = T\log\left(1+\dfrac{A}{k}\right)$, for A

Answer. $A = k(10^{t/T} - 1)$

5.2.7.56. $\log(R) = \log(R_0) + kt$, for R

5.2.7.57. $N = N_0 \log_b(ks)$, for s

Answer. $s = \dfrac{b^{N/N_0}}{k}$

5.2.7.58. $T = \dfrac{H\log\left(\dfrac{N}{N_0}\right)}{\log\left(\dfrac{1}{2}\right)}$, for N

5.2.7.59. $M = \sqrt{\dfrac{\log(H)}{k\log(H_0)}}$, for H

Answer. $H = (H_0)^{kM^2}$

5.2.7.60. $h = a - \sqrt{\dfrac{\log(B)}{t}}$, for B

5.2.7.61. Choose the graph for each function described below.

(a) The area, A, of a pentagon is a quadratic function of the length, l, of its side.

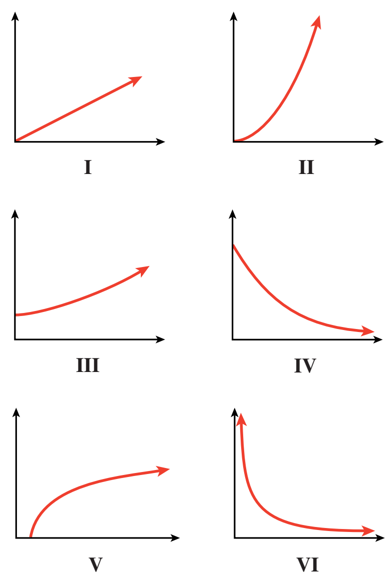
(b) The strength, F, of a hurricane varies inversely with its speed, s.

(c) The price of food has increased by 3% every year for a decade.

(d) The magnitude, M, of a star is a logarithmic function of its brightness, I.

(e) The speed of the train increased at a constant rate.

(f) If you do not practice a foreign language, you lose $\frac{1}{8}$ of the words in your working vocabulary, V, each year.



Answer.

(a) II

(b) VI

(c) III

(d) V

(e) I

(f) IV

5.2.7.62. For each of the functions $g(x)$ listed below, select the graph of its inverse function, if possible, from the figures labeled I–VI. (The inverse of one of the functions is not shown.)

(a) $g(x) = 2^x$

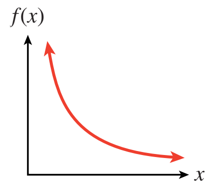
(b) $g(x) = x^2, \quad x \geq 0$

(c) $g(x) = \dfrac{2}{x}$

(d) $g(x) = \sqrt{x}$

(e) $g(x) = \log_2(x)$

(f) $g(x) = \left(\dfrac{1}{2}\right)^x$



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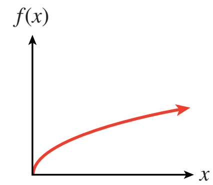

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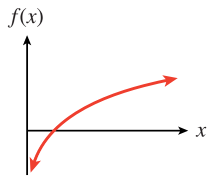

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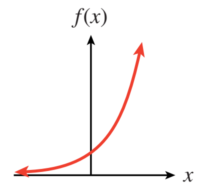

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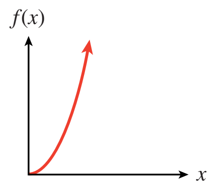

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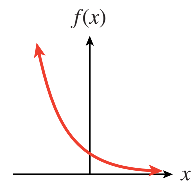


VI

For Problems 63-64, graph the function on the domain [-4, 4] and a suitable range. Which have inverses that are also functions?

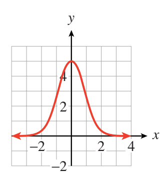
5.2.7.63.

(a) $f(x) = 5(2^{-x^2})$

(b) $f(x) = 2^x + 2^{-x}$

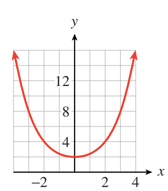
Answer.

(a)



No inverse function

(b)



No inverse function

5.2.7.64.

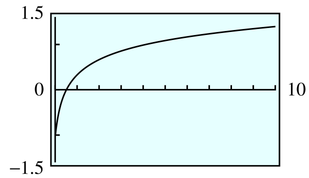
(a) $f(x) = 5(\log(x))^2 + 1$

(b) $f(x) = 5\log(x^2 + 1)$

For Problems 65-68, graph the pair of functions on your calculator. Explain the result.

5.2.7.65. $f(x) = \log(2x), \quad g(x) = \log(2) + \log(x)$

Answer.

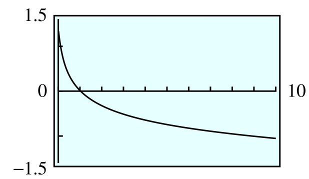


The functions are equal.

5.2.7.66. $f(x) = \log\left(\dfrac{x}{3}\right), \quad g(x) = \log(x) - \log(3)$

5.2.7.67. $f(x) = \log\left(\dfrac{1}{x}\right), \quad g(x) = -\log(x)$

Answer.



The functions are equal.

5.2.7.68. $f(x) = \log(x^3), \quad g(x) = 3\log(x)$

5.2.7.69.

(a) Complete the following table.

x	x^2	$\log(x)$	$\log(x^2)$
1			
2			
3			
4			
5			
6			

(b) Do you notice a relationship between $\log(x)$ and $\log(x^2)$? State the relationship as an equation.

Answer.

(a)

x	x^2	$\log(x)$	$\log(x^2)$
1	1	0	0
2	4	0.301	0.602
3	9	0.477	0.954
4	16	0.602	1.204
5	25	0.699	1.398
6	36	0.778	1.556

(b) $\log(x^2) = 2\log(x)$

5.2.7.70.

(a) Complete the following table.

x	$\frac{1}{x}$	$\log(x)$	$\log\left(\frac{1}{x}\right)$
1			
2			
3			
4			
5			
6			

(b) Do you notice a relationship between $\log(x)$ and $\log\left(\frac{1}{x}\right)$? State the relationship as an equation.

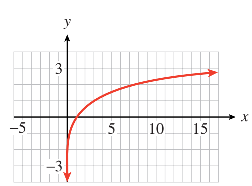
In Problems 69 and 70, you found relationships between $\log(x)$ and $\log(x^2)$, and between $\log(x)$ and $\log\left(\frac{1}{x}\right)$. Assumingthat those relationships hold for any base, complete the following tables and use them to graph the given functions.

5.2.7.71.

x	$y = \log_e(x)$
1	0
2	0.693
4	
16	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{16}$	

Answer.

x	$y = \log_e(x)$
1	0
2	0.693
4	1.386
16	2.772
$\frac{1}{2}$	-0.693
$\frac{1}{4}$	-1.386
$\frac{1}{16}$	-2.772



5.2.7.72.

x	$y = \log_f(x)$
1	0
2	0.431
4	
16	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{16}$	

5.2.8 Investigation

Investigation 5.2.1 Interest Compounded Continuously. We learned in Section 4.4 that the amount, A (principal plus interest), accumulated in an account with interest compounded n times annually is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where P is the principal invested, r is the interest rate, and t is the time period, in years.

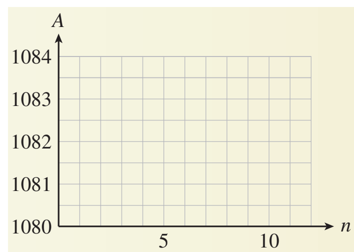
1 Suppose you keep \$1000 in an account that pays 8% interest. How much is the amount A after 1 year if the interest is compounded twice a year? Four times a year?

$$n = \mathbf{2} : A = 1000\left(1 + \frac{0.08}{\mathbf{2}}\right)^{\mathbf{2}(1)} =$$

$$n = \mathbf{4} : A = 1000\left(1 + \frac{0.08}{\mathbf{4}}\right)^{\mathbf{4}(1)} =$$

2 What happens to A as we increase n, the number of compounding periods per year? Fill in the table showing the amount in the account for different values of n.

n	A
1 (annually)	1080
2 (semiannually)	
4 (quarterly)	
6 (bimonthly)	
12 (monthly)	
365 (daily)	
1000	
$10,000$	



3 Plot the values in the table from $n = 1$ to $n = 12$, and connect them with a smooth curve. Describe the curve: What is happening to the value of A?

4 In part (2), as you increased the value of n, the other parameters in the formula stayed the same. In other words, A is a function of n, given by $A = 1000(1 + \dfrac{0.08}{n})^n$. Use your calculator to graph A on successively larger domains:

a $\text{Xmin} = 0, \text{Xmax} = 12; \text{Ymin} = 1080, \text{Ymax} = 1084$

b $\text{Xmin} = 0, \text{Xmax} = 50; \text{Ymin} = 1080, \text{Ymax} = 1084$

c $\text{Xmin} = 0, \text{Xmax} = 365; \text{Ymin} = 1080, \text{Ymax} = 1084$

5 Use the **Trace** feature or the **Table** feature to evaluate A for very large values of n. Rounded to the nearest penny, what is the largest value of A that you can find?

6 As n increases, the values of A approach a limiting value. Although A continues to increase, it does so by smaller and smaller increments and will never exceed \$1083.29. When the number of compounding periods increases without bound, we call the limiting result **continuous compounding**.

7 Is there an easier way to compute A under continues compounding? Yes! Compute $1000e^{0.08}$ on your calculator. (Press `2nd LN` to enter e^x.) Compare the value to your answer in part (5) for the limiting value. The number e is called the **natural base**. We'll compute its value shortly.

8 Repeat your calculations for two other interest rates, 15% and (an extremely unrealistic) 100%, again for an investment of \$1000 for 1 year. In each case, compare the limiting value of A, and compare to the value of $1000e^r$.

a

$r = 0.15$	
n	A
1	115
2	
4	
6	
12	
3652	
1000	
$10,000$	

$1000e^{0.15} =$

b

$r = 1$	
n	A
1	200
2	
4	
6	
12	
3652	
1000	
$10,000$	

$1000e^1 =$

9 In part (8b), you have computed an approximation for $1000e$. What is the value of e, rounded to 5 decimal places?

10 Complete the table of values. What does $\left(1+\dfrac{1}{n}\right)^n$ appear to approach as n increases?

n	100	1000	$10,000$	$100,000$
$\left(1+\frac{1}{n}\right)^n$				

5.3 The Natural Base

5.3.1 The Natural Exponential Function

Checkpoint 5.3.2 Use your calculator to evaluate the following powers.

a e^2 b $e^{3.5}$ c $e^{-0.5}$

Answer.

a $e^2 \approx 7.389$ b $e^{3.5} \approx 33.115$ c $e^{-0.5} \approx 0.6065$

5.3.2 The Natural Logarithmic Function

Checkpoint 5.3.5 Use your calculator to evaluate each logarithm. Round your answers to four decimal places.

a $\ln(100)$ b $\ln(0.01)$ c $\ln(e^3)$

Answer.

a $\ln(100) \approx 4.6052$

b $\ln(0.01) \approx -4.6052$

c $\ln(e^3) = 3$

5.3.3 Properties of the Natural Logarithm

Checkpoint 5.3.7 Simplify each expression.

a $e^{(\ln(x))/2}$ b $\ln\left(\dfrac{1}{e^{4x}}\right)$

Answer.

a $\sqrt{x}$ b $-4x$

5.3.4 Solving Equations

Checkpoint 5.3.9 Solve each equation. Round your answers to four decimal places.

a $\ln(x) = -0.2$ b $e^x = 8$

Answer.

a 0.8187 b 2.0794

Checkpoint 5.3.12 Solve

$$80 - 16e^{-0.2x} = 70.3$$

Hint. Subtract from both sides and divide by $\boldsymbol{-16}$**.**
Take the natural log of both sides.
Divide by $\boldsymbol{-0.2}$**.**

Answer. $x = -5\ln\left(\dfrac{9.6}{16}\right) \approx 2.5023$

Checkpoint 5.3.14 Solve $N = Ae^{-kt}$ for k.

Hint. Divide both sides by $\boldsymbol{A}$**.**
Take the natural log of both sides.
Divide both sides by $\boldsymbol{-t}$**.**

Answer. $k = \dfrac{-\ln(N/A)}{t}$

5.3.5 Exponential Growth and Decay

Checkpoint 5.3.16 From 1994 to 1998, the number of personal computers connected to the Internet grew according to the formula $N(t) = 2.8e^{0.85t}$, where $t = 0$ in 1994 and N is in millions. (Source: Los Angeles Times, September 6, 1999) a Evaluate $N(1)$. By what percent did the number of Internet users grow in one year?

b Express the growth law in the form $N(t) = N_0(1+r)^t$.

Hint. $e^k = 1 + r$

Answer.

a $N(1) \approx 6.55$, 134%

b $N(t) \approx 2.8(1.3396)^t$

Checkpoint 5.3.18 A scientist isolates 25 grams of krypton-91, which decays according to the formula

$$N(t) = 25e^{-0.07t},$$

where t is in seconds.

a Complete the table of values showing the amount of krypton-91 left at 10-second intervals over the first minute.

t	0	10	20	30	40	50	60
$N(t)$							

b Use the table to choose a suitable window and graph the function $N(t)$.

c Write and solve an equation to answer the question: How long does it take for 60% of the krypton-91 to decay?

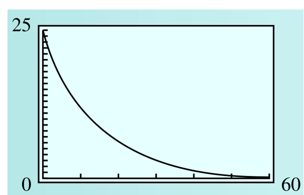
Hint. If 60% of the krypton-91 has decayed, 40% of the original 25 grams remains.

Answer.

a

t	0	10	20	30	40	50	60
$N(t)$	25	12.41	6.16	3.06	1.52	0.75	0.37

b



c $25e^{-0.07t} = 0.40(25)$; $t = \dfrac{\ln(0.4)}{-0.07} \approx 13.09$ seconds

5.3.6 Continuous Compounding

Checkpoint 5.3.20 Zelda invested \$1000 in an account that pays 4.5% interest compounded continuously. How long will it be before the account is worth \$2000?

Answer. About 15.4 years

5.3.8 Homework 5.3

For Problems 1-4, use your calculator to complete the table for each function. Then choose a suitable window and graph the function.

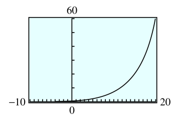
x	-10	-5	0	5	10	15	20
$f(x)$							

5.3.8.1. $f(x) = e^{0.2x}$

Answer.

x	-10	-5	0	5	10	15	20
$f(x)$	0.135	0.368	1	2.718	7.389	20.086	54.598

5.3.8.2. $f(x) = e^{0.6x}$

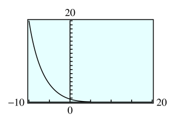


5.3.8.3. $f(x) = e^{-0.3x}$

Answer.

x	-10	-5	0	5	10	15	20
$f(x)$	20.086	4.482	1	0.223	0.05	0.011	0.00248

5.3.8.4. $f(x) = e^{-0.1x}$



For Problems 5-6, simplify.

5.3.8.5.

(a) $\ln(e^2)$ (b) $e^{\ln(5t)}$ (c) $e^{-\ln(x)}$ (d) $\ln(\sqrt{e})$

Answer.

(a) 2 (b) $5t$ (c) $\dfrac{1}{x}$ (d) $\dfrac{1}{2}$

5.3.8.6.

(a) $\ln(e^{x^4})$ (b) $e^{3\ln(x)}$ (c) $e^{\ln(x)-\ln(y)}$ (d) $\ln\left(\dfrac{1}{e^{2t}}\right)$

For Problems 7-10, solve for x. Give the exact solution and the solution rounded to the nearest 2 decimal places.

5.3.8.7.

(a) $e^x = 1.9$ (b) $e^x = 45$ (c) $e^x = 0.3$

Answer.

(a) 0.64 (b) 3.81 (c) -1.20

5.3.8.8.

(a) $e^x = 2.1$ (b) $e^x = -60$ (c) $e^x = 0.9$

5.3.8.9.

(a) $\ln(x) = 1.42$ (b) $\ln(x) = 0.63$ (c) $\ln(x) = -2.6$

Answer.

(a) 4.14 (b) 1.88 (c) 0.07

5.3.8.10.

(a) $\ln(x) = 2.03$ (b) $\ln(x) = 0.59$ (c) $\ln(x) = -3.4$

5.3.8.11. The number of bacteria in a culture grows according to the function

$$N(t) = N_0 e^{0.04t}$$

where N_0 is the number of bacteria present at time $t = 0$ and t is the time in hours.

(a) Write a growth law for a sample in which 6000 bacteria were present initially.

(b) Make a table of values for $N(t)$ in 5-hour intervals over the first 30 hours. Round to one decimal place.

(c) Graph $N(t)$.

(d) How many bacteria were present at $t = 24$ hours?

(e) How much time must elapse (to the nearest tenth of an hour) for the original 6000 bacteria to increase to $100,000$?

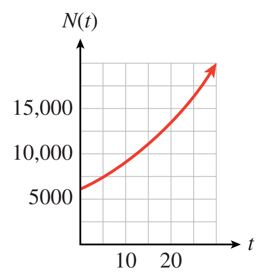
Answer.

(a) $N(t) = 6000e^{0.04t}$

(b)

t	0	5	10	15	20	25	30
$N(t)$	6000	7328	8951	$10,933$	$13,353$	$16,310$	$19,921$

(c)



(d) $15,670$

(e) 70.3 hrs

5.3.8.12. Hope invests \$2000 in a savings account that pays $5\frac{1}{2}\%$ annual interest compounded continuously.

(a) Write a formula that gives the amount of money $A(t)$ in Hope's account after t years.

(b) Make a table of values for $A(t)$ in 2-year intervals over the first 10 years.

(c) Graph $A(t)$.

(d) How much will Hope's account be worth after 7 years?

(e) How long will it take for the account to grow to \$5000?

5.3.8.13. The intensity, I (in lumens), of a light beam after passing through t centimeters of a filter having an absorption coefficient of 0.1 is given by the function

$$I(t) = 1000e^{-0.1t}$$

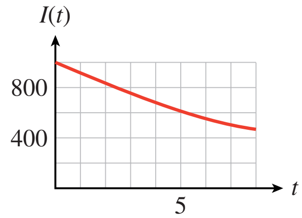
(a) Graph $I(t)$.

(b) What is the intensity (to the nearest tenth of a lumen) of a light beam that has passed through 0.6 centimeter of the filter?

(c) How many centimeters (to the nearest tenth) of the filter will reduce the illumination to 800 lumens?

Answer.

(a)



(b) 941.8 lumens

(c) 2.2 cm

5.3.8.14. X-rays can be absorbed by a lead plate so that

$$I(t) = I_0 e^{-1.88t}$$

where I_0 is the X-ray count at the source and $I(t)$ is the X-ray count behind a lead plate of thickness t inches.

(a) Graph $I(t)$.

(b) What percent of an X-ray beam will penetrate a lead plate $\frac{1}{2}$ inch thick?

(c) How thick should the lead plate be in order to screen out 70% of the X-rays?

For problems 15-18, express each exponential function in the form $P(t) = P_0 b^t$. Is the function increasing or decreasing? What is itsinitial value?

5.3.8.15. $P(t) = 20e^{0.4t}$

Answer.
$P(t) = 20\left(e^{0.4}\right)^t \approx 20 \cdot 1.492^t$; increasing; initial value 20

5.3.8.16. $P(t) = 0.8e^{1.3t}$

5.3.8.17. $P(t) = 6500e^{-2.5t}$

Answer. $P(t) = 6500\left(e^{-2.5}\right)^t \approx 6500 \cdot 0.082^t$; decreasing; initial value 6500

5.3.8.18. $P(t) = 1.7e^{-0.02t}$

5.3.8.19.

(a) Fill in the table, rounding your answers to four decimal places.

x	0	0.5	1	1.5	2	2.5
e^x						

(b) Compute the ratio of each function value to the previous one. Explain the result.

Answer.

(a)

x	0	0.5	1	1.5	2	2.5
e^x	1	1.6487	2.7183	4.4817	7.3891	12.1825

(b) Each ratio is $e^{0.5} \approx 1.6487$: Increasing x-values by a constant $\Delta x = 0.5$ corresponds to multiplying the y-values of the exponential function by a constant factor of $e^{\Delta x}$.

5.3.8.20.

(a) Fill in the table, rounding your answers to four decimal places.

x	0	2	4	6	8	10
e^x						

(b) Compute the ratio of each function value to the previous one. What do you notice about the ratios?

5.3.8.21.

(a) Fill in the table, rounding your answers to the nearest integer.

x	0	0.6931	1.3863	2.0794	2.7726	3.4657	4.1589
e^x							

(b) Subtract each x-value from the next one. Explain the result.

Answer.

(a)

x	0	0.6931	1.3863	2.0794	2.7726	3.4657	4.1589
e^x	1	2	4	8	16	32	64

(b) Each difference in x-values is approximately $\ln(2) \approx 0.6931$: Increasing x-values by a constant $\Delta x = \ln(2)$ corresponds to multiplying the y-values of the exponential function by a constant factor of $e^{\Delta x} = e^{\ln(2)} = 2$. That is, each function value is approximately equal to double the previous one.

5.3.8.22.

(a) Fill in the table, rounding your answers to the nearest integer.

x	0	1.0986	2.1972	3.2958	4.3944	5.4931	6.5917
e^x							

(b) Subtract each x-value from the next one. Explain the result.

For Problems 23–30, solve. Give the exact solution and the solution rounded to the nearest 2 decimal places.

5.3.8.23. $6.21 = 2.3e^{1.2x}$

Answer. 0.8277

5.3.8.24. $22.26 = 5.3e^{0.4x}$

5.3.8.25. $6.4 = 20e^{0.3x} - 1.8$

Answer. -2.9720

5.3.8.26. $4.5 = 4e^{2.1x} + 3.3$

5.3.8.27. $46.52 = 3.1e^{1.2x} + 24.2$

Answer. 1.6451

5.3.8.28. $1.23 = 1.3e^{2.1x} - 17.1$

5.3.8.29. $16.24 = 0.7e^{-1.3x} - 21.7$

Answer. -3.0713

5.3.8.30. $55.68 = 0.6e^{-0.7x} + 23.1$

For Problems 31-36, solve the equation for the specified variable.

5.3.8.31. $y = e^{kt}$, for t

Answer. $t = \dfrac{1}{k}\ln(y)$

5.3.8.32. $\dfrac{T}{R} = e^{t/2}$, for t

5.3.8.33. $y = k(1 - e^{-t})$, for t

Answer. $t = \ln\left(\dfrac{k}{k-y}\right)$

5.3.8.34. $B - 2 = (A + 3)e^{-t/3}$, for t

5.3.8.35. $T = T_0 \ln(k + 10)$, for k

Answer. $k = e^{T/T_0} - 10$

5.3.8.36. $P = P_0 + \ln(10k)$, for k

5.3.8.37.

(a) Fill in the table, rounding your answers to three decimal places.

n	0.39	3.9	39	390
$\ln(n)$				

(b) Subtract each natural logarithm in your table from the next one. (For example, compute $\ln(3.9) - \ln(0.39)$.) Explain the result.

Answer.

(a)

n	0.39	3.9	39	390
$\ln(n)$	-0.942	1.361	3.664	5.966

(b) Each difference in function values is approximately $\ln(10) \approx 2.303$: Multiplying x-values by a constant factor of 10 corresponds to adding a constant value of ln 10 to the y-values of the natural log function.

5.3.8.38.

(a) Fill in the table, rounding your answers to three decimal places.

n	0.64	6.4	64	640
$\ln(n)$				

(b) Subtract each natural logarithm in your table from the next one. (For example, compute $\ln(6.4) - \ln(0.64)$.) Explain the result.

5.3.8.39.

(a) Fill in the table, rounding your answers to three decimal places.

n	2	4	8	16
$\ln(n)$				

(b) Divide each natural logarithm in your table by $\ln(2)$. Explain the result.

Answer.

(a)

n	2	4	8	16
$\ln(n)$	0.693	1.386	2.079	2.773

(b) Each quotient equals k, where $n = 2^k$. Because $\ln(n) = \ln(2^k) = k \cdot \ln(2)$, $k = \dfrac{\ln(n)}{\ln(2)}$.

5.3.8.40.

(a) Fill in the table, rounding your answers to three decimal places.

n	5	25	125	625
$\ln(n)$				

(b) Divide each natural logarithm in your table by $\ln(5)$. Explain the result.

For Problems 41–46,

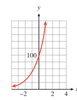
a Express each growth or decay law in the form $N(t) = N_0 e^{kt}$.

b Check your answer by graphing both forms of the function on the same axes. Do they have the same graph?

5.3.8.41.
$N(t) = 100 \cdot 2^t$

Answer.

(a) $N(t) = 100e^{(\ln(2))t} \approx 100e^{0.6931t}$

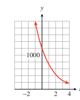
(b) 

5.3.8.42.
$N(t) = 50 \cdot 3^t$

5.3.8.43.
$N(t) = 1200(0.6)^t$

Answer.

(a) $N(t) = 1200e^{(\ln(0.6))t} \approx 1200e^{-0.5108t}$

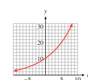
(b) 

5.3.8.44.
$N(t) = 300(0.8)^t$

5.3.8.45.
$N(t) = 10(1.15)^t$

Answer.

(a) $N(t) = 10e^{(\ln(1.15))t} \approx 10e^{0.1398t}$

(b) 

5.3.8.46.
$N(t) = 1000(1.04)^t$

5.3.8.47. The population of Citrus Valley was $20,000$ in 2000. In 2010, it was $35,000$.

(a) What is P_0 if $t = 0$ in 2000?

(b) Use the population in 2010 to find the growth factor e^k.

(c) Write a growth law of the form $P(t) = P_0 e^{kt}$ for the population of Citrus Valley.

(d) If it continues at the same rate of growth, what will the population be in 2030?

Answer.

(a) $20,000$

(b) $\left(\dfrac{35,000}{20,000}\right)^{1/10} \approx e^{0.056}$

(c) $P(t) = 20,000e^{0.056t}$

(d) $107,188$

5.3.8.48. A copy of *Time* magazine cost \$1.50 in 1981. In 1988, the cover price had increased to \$2.00.

(a) What is P_0 if $t = 0$ in 1981?

(b) Use the price in 1988 to find the growth factor e^k.

(c) Find a growth law of the form $P(t) = P_0 e^{kt}$ for the price of *Time*.

(d) In 1999, a copy of *Time* cost \$3.50. Did the price of the magazine continue to grow at the same rate from 1981 to 1999?

5.3.8.49. Cobalt-60 is a radioactive isotope used in the treatment of cancer. A 500-milligram sample of cobalt-60 decays to 385 milligrams after 2 years.

(a) Using $P_0 = 500$, find the decay factor e^k for cobalt-60.

(b) Write a decay law $N(t) = N_0 e^{kt}$ for cobalt-60.

(c) How much of the original sample will be left after 10 years?

Answer.

(a) $\left(\dfrac{385}{500}\right)^{1/2} \approx e^{-0.1307}$

(b) $N(t) = 500e^{-0.1307t}$

(c) 135.3 mg

5.3.8.50. Weed seeds can survive for a number of years in the soil. An experiment on cultivated land found 155 million weed seeds per acre, and in the following years the experimenters prevented the seeds from coming to maturity and producing new weeds. Four years later, there were 13.6 million seeds per acre. (Source: Burton, 1998)

(a) Find the annual decay factor e^k for the number of weed seeds in the soil.

(b) Write an exponential formula with base e for the number of weed seeds that survived after t years.

Problems 51–58 are about doubling time and half-life.

5.3.8.51. Delbert invests \$500 in an account that pays 9.5% interest compounded continuously.

(a) Write a formula for $A(t)$ that gives the amount of money in Delbert's account after t years.

(b) How long will it take Delbert's investment to double to \$1000?

(c) How long will it take Delbert's money to double again, to \$2000?

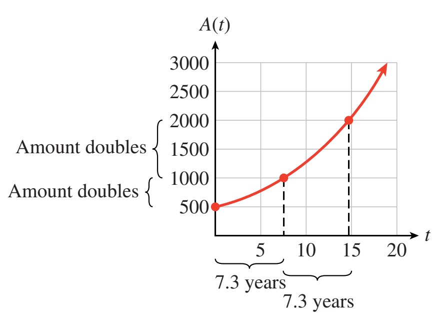
(d) Graph $A(t)$ and illustrate the doubling time on your graph.

(e) Choose any point (t_1, A_1) on the graph, then find the point on the graph with vertical coordinate $2A_1$. Verify that the difference in the t-coordinates of the two points is the doubling time.

Answer.

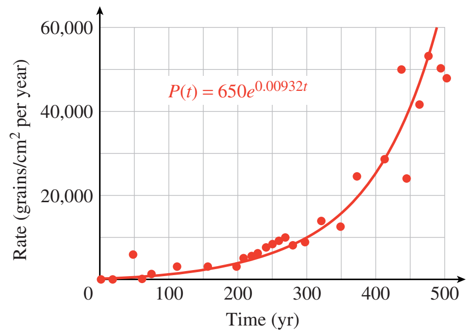
(a) $A(t) = 500e^{0.095t}$ (b) 7.3 years (c) 7.3 years

d–e



5.3.8.52.

The growth of plant populations can be measured by the amount of pollen they produce. The pollen from a population of pine trees that lived more than 9500 years ago in Norfolk, England, was deposited in the layers of sediment in a lake basin and dated with radiocarbon techniques.



The figure shows the rate of pollen accumulation plotted against time, and the fitted curve $P(t) = 650e^{0.00932t}$. (Source: Burton, 1998)

(a) What was the annual rate of growth in pollen accumulation?

(b) Find the doubling time for the pollen accumulation, that is, the time it took for the accumulation rate to double.

(c) By what factor did the pollen accumulation rate increase over a period of 500 years?

5.3.8.53. Technetium-99m (Tc-99m) is an artificially produced radionuclide used as a tracer for producing images of internal organs such as the heart, liver, and thyroid. A solution of Tc-99m with initial radioactivity of $10,000$ becquerels (Bq) decays according to the formula

$$N(t) = 10,000e^{-0.1155t}$$

where t is in hours.

(a) How long will it take the radioactivity to fall to half its initial value, or 5000 Bq?

(b) How long will it take the radioactivity to be halved again?

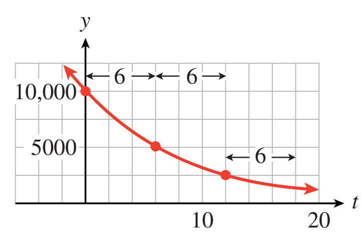
(c) Graph $N(t)$ and illustrate the half-life on your graph.

(d) Choose any point (t_1, N_1) on the graph, then find the point on the graph with vertical coordinate $0.5N_1$. Verify that the difference in the t-coordinates of the two points is the half-life.

Answer.

(a) 6 hours

(c)



(b) 6 hours

5.3.8.54. All living things contain a certain amount of the isotope carbon-14. When an organism dies, the carbon-14 decays according to the formula

$$N(t) = N_0e^{-0.000124t}$$

where t is measured in years. Scientists can estimate the age of an organic object by measuring the amount of carbon-14 remaining.

(a) When the Dead Sea scrolls were discovered in 1947, they had 78.8%

of their original carbon-14. How old were the Dead Sea scrolls then?

(b) What is the half-life of carbon-14, that is, how long does it take for half of an object's carbon-14 to decay?

5.3.8.55. The half-life of iodine-131 is approximately 8 days.

(a) If a sample initially contains N_0 grams of iodine-131, how much will it contain after 8 days? How much will it contain after 16 days? After 32 days?

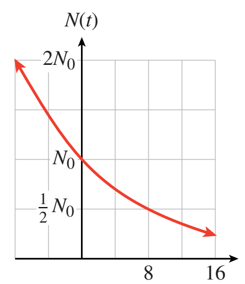
(b) Use your answers to part (a) to sketch a graph of $N(t)$, the amount of iodine-131 remaining, versus time. (Choose an arbitrary height for N_0 on the vertical axis.)

(c) Calculate k, and hence find a decay law of the form $N(t) = N_0e^{kt}$, where $k < 0$, for iodine-131.

Answer.

(a) $\frac{1}{2}N_0$, $\frac{1}{4}N_0$, $\frac{1}{16}N_0$

(b)



(c) $N(t) = N_0e^{-0.0866t}$

5.3.8.56. The half-life of hydrogen-3 is 12.5 years.

(a) If a sample initially contains N_0 grams of hydrogen-3, how much will it contain after 12.5 years? How much will it contain after 25 years?

(b) Use your answers to part (a) to sketch a graph of $N(t)$, the amount of hydrogen-3 remaining, versus time. (Choose an arbitrary height for N_0 on the vertical axis.)

(c) Calculate k, and hence find a decay law of the form $N(t) = N_0e^{kt}$, where $k < 0$, for hydrogen-3.

5.3.8.57. A Geiger counter measures the amount of radioactive material present in a substance. The table shows the count rate for a sample of iodine-128 as a function of time. (Source: Hunt and Sykes, 1984)

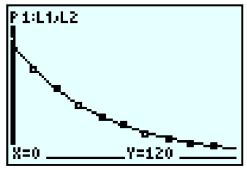
Time (min)	0	10	20	30	40	50	60	70	80	90
Counts/sec	120	90	69	54	42	33	25	19	15	13

(a) Graph the data and use your calculator's exponential regression feature to fit a curve to them.

(b) Write your equation in the form $G(t) = G_0e^{kt}$.

(c) Calculate the half-life of iodine-128.

Answer.

(a) 

$y = 116(0.975)^t$

(b) $G(t) = 116e^{-0.025t}$

(c) 28 minutes

5.3.8.58. The table shows the count rate for sodium-24 registered by a Geiger counter as a function of time. (Source: Hunt and Sykes, 1984)

Time (min)	0	10	20	30	40	50	60	70	80	90
Counts/sec	180	112	71	45	28	18	11	7	4	3

(a) Graph the data and use your calculator's exponential regression feature to fit a curve to them.

(b) Write your equation in the form $G(t) = G_0 e^{kt}$.

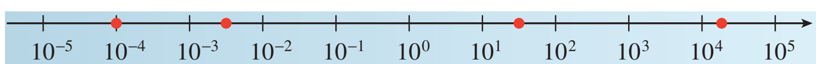
(c) Calculate the half-life of sodium-24.

5.4 Logarithmic Scales

5.4.1 Introduction

Checkpoint 5.4.2 Complete the table by estimating the logarithm of each point plotted on the log scale below. Then give a decimal value for each point.

$\log(x)$				
x				



Answer.

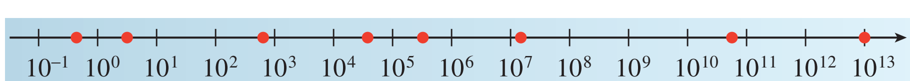
$\log(x)$	-4	-2.5	1.5	4.25
x	0.0001	0.0032	31.6	$17,782.8$

5.4.2 Using Log Scales

Checkpoint 5.4.4 Plot the following dollar values on a log scale.

Postage stamp	0.47
Medium cappuccino	3.65
Notebook computer	679
One year at Harvard	88,600
2016 Lamborghini	530,075
Kobe Bryant's salary	25,000,000
Bill Gates's financial worth	79,400,000,000
U.S. National debt	19,341,810,000

Answer.

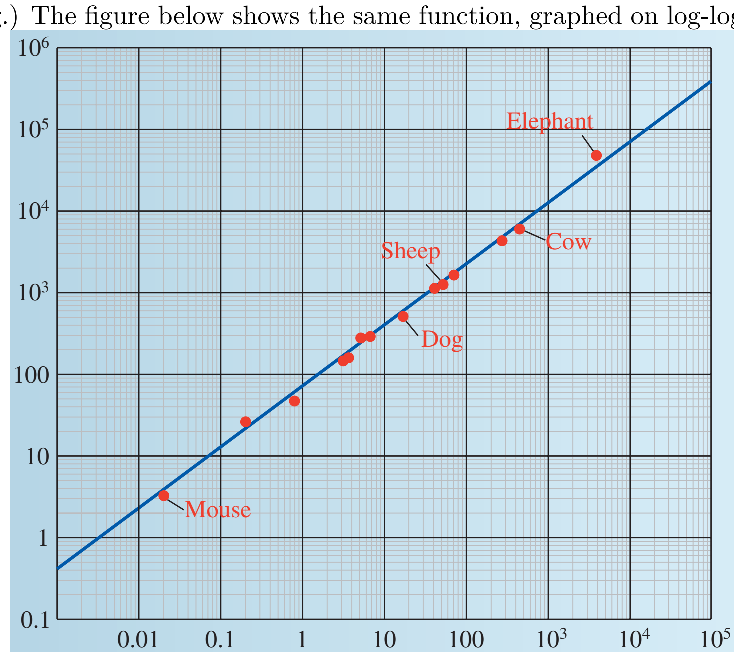


5.4.3 Equal Increments on a Log Scale

Checkpoint 5.4.6 What number is halfway between $10^{1.5}$ and 10^2 on a log scale?

Answer. 56.23

Checkpoint 5.4.8 The opening page of Chapter 3 shows the "mouse-to-elephant" curve, a graph of the metabolic rate of mammals as a function of their mass. (The elephant does not appear on that graph, because its mass is too big.) The figure below shows the same function, graphed on log-log paper.



Use this graph to estimate the mass and metabolic rate for the following animals, labeled on the graph.

Animal	Mouse	Dog	Sheep	Cow	Elephant
Mass (kg)					
Metabolic rate (kcal/day)					

Answer.

Animal	Mass (kg)	Metabolic rate (kcal/day)
Mouse	0.02	3.5
Dog	15	500
Sheep	50	1500
Cow	500	6000
Elephant	4000	$50,000$

5.4.4 Acidity and the pH Scale

Checkpoint 5.4.10 The pH of the water in a tide pool is 8.3. What is the hydrogen ion concentration of the water?

Answer. 5.01×10^{-9}

5.4.5 Decibels

Checkpoint 5.4.12 The noise of city traffic registers at about 70 decibels.

a What is the intensity of traffic noise, in watts per square meter?

b How many times more intense is traffic noise than conversation?

Answer.

a $I = 10^{-5}$ watts/m^2

b 1000

5.4.6 The Richter Scale

Checkpoint 5.4.15 In October 2005, a magnitude 7.6 earthquake struck Pakistan. How much more powerful was this earthquake than the 1989 San Francisco earthquake of magnitude 7.1?

Answer. 3.16

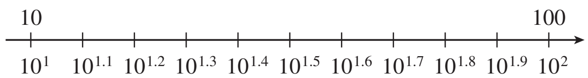
Checkpoint 5.4.18 Two points, labeled A and B, differ by 2.5 units on a log scale. What is the ratio of their decimal values?

Answer. 316.2

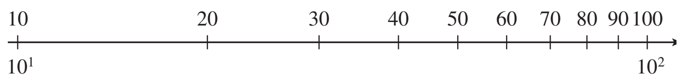
5.4.8 Homework 5.4

5.4.8.1.

(a) The log scale is labeled with powers of 10. Finish labeling the tick marks in the figure with their corresponding decimal values.

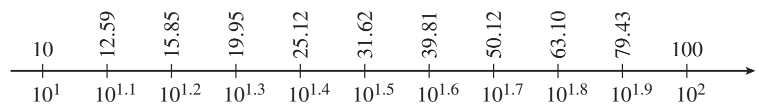


(b) The log scale is labeled with integer values. Label the tick marks in the figure with the corresponding powers of 10.

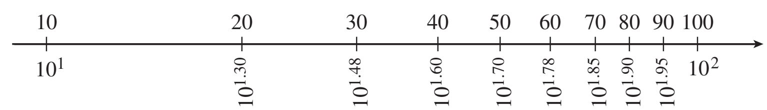


Answer.

(a)

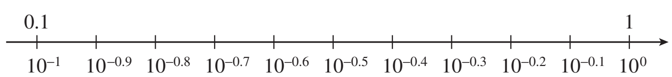


(b)

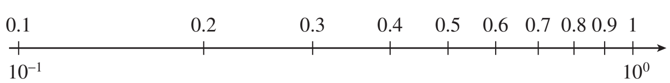


5.4.8.2.

(a) The log scale is labeled with powers of 10. Finish labeling the tick marks in the figure with their corresponding decimal values.



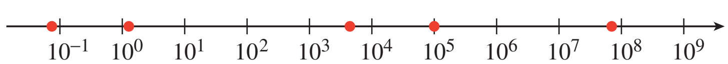
(b) The log scale is labeled with integer values. Label the tick marks in the figure with the corresponding powers of 10.



5.4.8.3. Plot the values on a log scale.

x	0.075	1.3	4200	87,000	6.5×10^7

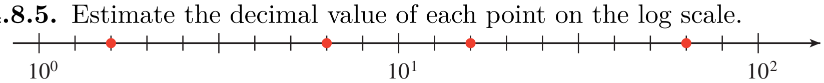
Answer.



5.4.8.4. Plot the values on a log scale.

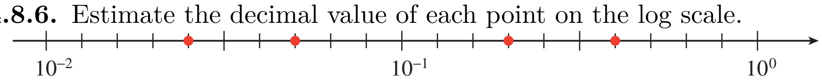
x	4×10^{-4}	0.008	0.9	27	90

5.4.8.5. Estimate the decimal value of each point on the log scale.

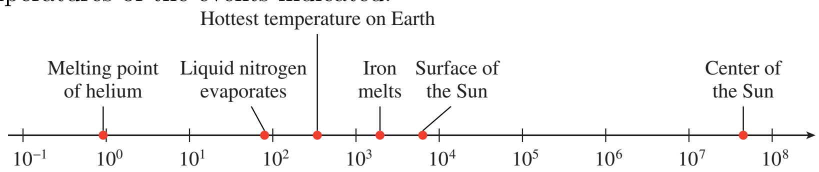


Answer. 1.58, 6.31, 15.8, 63.1

5.4.8.6. Estimate the decimal value of each point on the log scale.

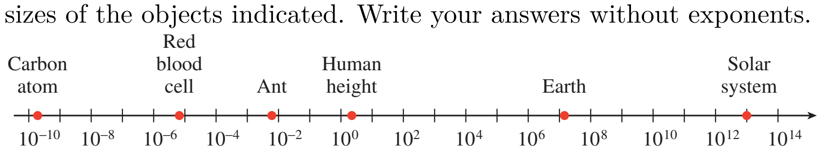


5.4.8.7. The log scale shows various temperatures in Kelvins. Estimate the temperatures of the events indicated.



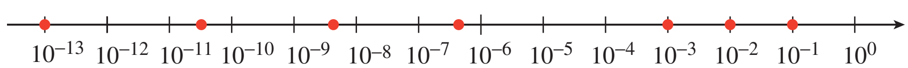
Answer. 1, 80, 330, 1600, 7000, 4×10^7

5.4.8.8. The log scale shows the size of various objects, in meters. Estimate the sizes of the objects indicated. Write your answers without exponents.



5.4.8.9. Plot the values of $[H^+]$ in the section "Acidity and the pH Scale" on a log scale.

Answer.

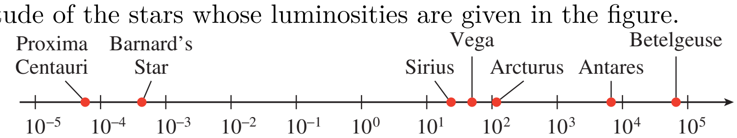


5.4.8.10. Plot the values of sound intensity in the section "Decibels" on a log scale.

5.4.8.11. The magnitude of a star is a measure of its brightness. It is given by the formula

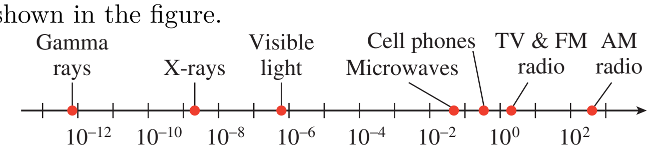
$$m = 4.83 - 2.5 \log(L)$$

where L is the luminosity of the star, measured in solar units. Calculate the magnitude of the stars whose luminosities are given in the figure.



Answer. Proxima Centauri: 15.5; Barnard: 13.2; Sirius: 1.4; Vega: 0.6; Arcturus: -0.4; Antares: -4.7; Betelgeuse: -7.2

5.4.8.12. Estimate the wavelength, in meters, of the types of electromagnetic radiation shown in the figure.



5.4.8.13. The **risk magnitude** of an event is defined by $R = 10 + \log(p)$, where p is the probability of the event occurring. Calculate the probability of each event.

(a) The sun will rise tomorrow, $R = 10$.

(b) The next child born in Arizona will be a boy, $R = 9.7$.

(c) A major hurricane will strike North Carolina this year, $R = 9.1$.

(d) A 100-meter asteroid will collide with Earth this year, $R = 8.0$.

(e) You will be involved in an automobile accident during a 10-mile trip, $R = 5.9$.

(f) A comet will collide with Earth this year, $R = 3.5$.

(g) You will die in an automobile accident on a 1000-mile trip, $R = 2.3$

(h) You will die in a plane crash on a 1000-mile trip, $R = 0.9$.

Answer.

(a) 1

(b) 0.5012

(c) 0.1259

(d) 0.01

(e) 0.000079

(f) 3.2×10^{-7}

(g) 2×10^{-8}

(h) 8×10^{-10}

5.4.8.14. Have you ever wondered why time seems to pass more quickly as we grow older? One theory suggests that the human mind judges the length of a long period of time by comparing it with its current age. For example, a year is 20% of a 5-year-old's lifetime, but only 5% of a 20-year-old's, so a year feels longer to a 5-year-old. Thus, psychological time follows a log scale, like the one shown in the figure.

(a) Label the tick marks with their base 10 logarithms, rounded to 3 decimal places. What do you notice about the values?

(b) By computing their logs, locate 18 and 22 on the scale

(c) Four years of college seems like a long time to an 18-year-old. What length of time feels the same to a 40-year-old?

(d) How long will the rest of your life feel? Let A be your current age, and let L be the age to which you think you will live. Compute the difference of their logs. Now move backward on the log scale an equal distance from your current age. What is the age at that spot? Call that age B. The rest of your life will feel the same as your life from age B until now.

(e) Compute B using a proportion instead of logs.

5.4.8.15.

(a) What number is halfway between $10^{1.5}$ and 10^2 on a log scale?

(b) What number is halfway between 20 and 30 on a log scale?

Answer.

(a) $10^{1.75} \approx 56.2341$

(b) $10^{(\log(600))/2} \approx 24.4949$

5.4.8.16.

(a) What number is halfway between $10^{3.0}$ and $10^{3.5}$ on a log scale?

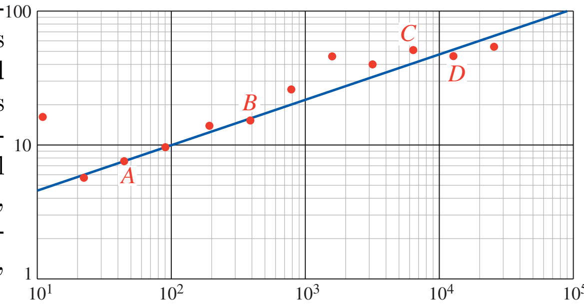
(b) Plot 500 and 600 on a log scale. What is halfway between them on this scale

5.4.8.17. The distances to two stars are separated by 3.4 units on a log scale. What is the ratio of their distances?

Answer. $10^{3.4} \approx 2512$

5.4.8.18. The populations of two cities are separated by 2.8 units on a log scale. What is the ratio of their populations?

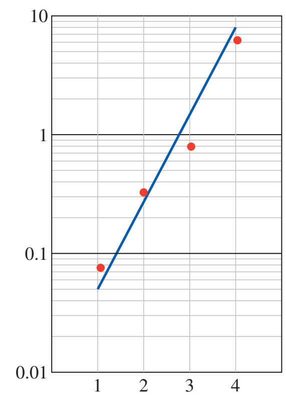
5.4.8.19. The probability of discovering an oil field increases with its diameter, defined to be the square root of its area. Use the graph to estimate the diameter of the oil fields at the labeled points, and their probability of discovery. (Source: Deffeyes, 2001)



Answer. A: $a \approx 45$, $p \approx 7.4\%$; B: $a \approx 400$, $p \approx 15\%$; C: $a \approx 6000$, $p \approx 50\%$; D: $a \approx 13000$, $p \approx 45\%$

5.4.8.20.

The **order** of a stream is a measure of its size. Use the graph to estimate the drainage area, in square miles, for streams of orders 1 through 4. (Source: Leopold, Wolman, and Miller)



In Problems 21–40, use the appropriate formulas for logarithmic models.

5.4.8.21. The hydrogen ion concentration of vinegar is about 6.3×10^{-4}. Calculate the pH of vinegar.

Answer. 3.2

5.4.8.22. The hydrogen ion concentration of spinach is about 3.2×10^{-6}. Calculate the pH of spinach.

5.4.8.23. The pH of lime juice is 1.9. Calculate its hydrogen ion concentration.

Answer. 0.0126

5.4.8.24. The pH of ammonia is 9.8. Calculate its hydrogen ion concentration.

5.4.8.25. A lawn mower generates a noise of intensity 10^{-2} watts per square meter. Find the decibel level of the sound of a lawn mower.

Answer. 100

5.4.8.26. A jet airplane generates 100 watts per square meter at a distance of 100 feet. Find the decibel level for a jet airplane.

5.4.8.27. The loudest sound emitted by any living source is made by the blue whale. Its whistles have been measured at 188 decibels and are detectable 500 miles away. Find the intensity of the blue whale's whistle in watts per square meter.

Answer. $6,309,573$ watts per square meter

5.4.8.28. The loudest sound created in a laboratory registered at 210 decibels. The energy from such a sound is sufficientto bore holes in solid material. Find the intensity of a210-decibel sound.

5.4.8.29. At a concert by The Who in 1976, the sound level 50 meters from the stage registered 120 decibels. How many times more intense was this than a 90-decibel sound (the threshold of pain for the human ear)?

Answer. 1000

5.4.8.30. The loudest scientifically measured shouting by a human being registered 123.2 decibels. How many times more intense was this than normal conversation at 40 decibels?

5.4.8.31. The pH of normal rain is 5.6. Some areas of Ontario have experienced acid rain with a pH of 4.5. How many times more acidic is acid rain than normal rain?

Answer. 12.6

5.4.8.32. The pH of normal hair is about 5, the average pH of shampoo is 8, and 4 for conditioner. Compare the acidityof normal hair, shampoo, and conditioner.

5.4.8.33. How much more acidic is milk than baking soda? (Refer to the table in this section..)

Answer. 100

5.4.8.34. Compare the acidity of lye and milk of magnesia. (Refer to the table in this section..)

5.4.8.35. In 1964, an earthquake in Alaska measured 8.4 on the Richter scale. An earthquake measuring 4.0 is consideredsmall and causes little damage. How many times stronger was the Alaska quake than one measuring 4.0?

Answer. $\approx 25,000$

5.4.8.36. On April 30, 1986, an earthquake in Mexico City measured 7.0 on the Richter scale. On September 21, a second earthquake occurred, this one measuring 8.1, hit Mexico City. How many times stronger was the September quake than the one in April?

5.4.8.37. A small earthquake measured 4.2 on the Richter scale. What is the magnitude of an earthquake three times as strong?

Answer. 4.7

5.4.8.38. Earthquakes measuring 3.0 on the Richter scale often go unnoticed. What is the magnitude of a quake 200 times as strong as a 3.0 quake?

5.4.8.39. The sound of rainfall registers at 50 decibels. What is the decibel level of a sound twice as loud?

Answer. 53

5.4.8.40. The magnitude, m, of a star is a function of its luminosity, L, given by

$$m = 4.83 - 2.5\log(L)$$

If one star is 10 times as luminous as another star, what is the difference in their magnitudes?

5.5 Chapter Summary and Review

5.5.2 Chapter 5 Review Problems

For Problems 1-4, make a table of values for the inverse function.

5.5.2.1. $f(x) = x^3 + x + 1$

Answer.

y	-1	1	3	11
$x = f^{-1}(y)$	-1	0	1	2

5.5.2.2. $g(x) = x + 6\sqrt[3]{x}$

5.5.2.3. $g(w) = \dfrac{1+w}{w-3}$

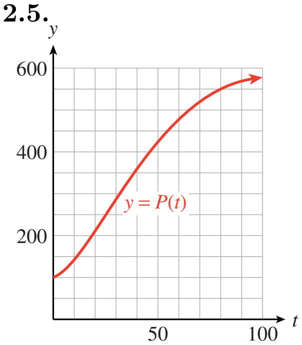
Answer.

y	0	$\frac{-1}{3}$	-1	-3
$w = g^{-1}(y)$	-1	0	1	2

5.5.2.4. $f(n) = \dfrac{n}{1+n}$

For Problems 5-6, use the graph to find the function values.

5.5.2.5.

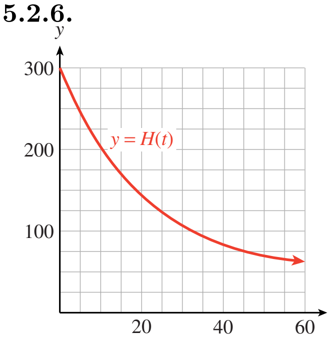


(a) $P^{-1}(350)$ (b) $P^{-1}(100)$

Answer.

(a) $P^{-1}(350) = 40$ (b) $P^{-1}(100) = 0$

5.5.2.6.



(a) $H^{-1}(200)$ (b) $H^{-1}(75)$

For Problems 7–12,

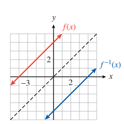
a Find a formula for the inverse f^{-1} of each function.

b Graph the function and its inverse on the same set of axes, along with the graph of $y = x$.

5.5.2.7. $f(x) = x + 4$

Answer.

(a) $f^{-1}(x) = x - 4$ (b)

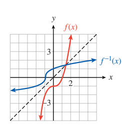


5.5.2.8. $f(x) = \dfrac{x-2}{4}$

5.5.2.9. $f(x) = x^3 - 1$

Answer.

(a) $f^{-1}(x) = \sqrt[3]{x+1}$ (b)

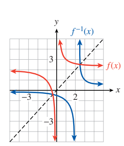


5.5.2.10. $f(x) = \dfrac{1}{x+2}$

5.5.2.11. $f(x) = \dfrac{1}{x} + 2$

Answer.

(a) $f^{-1}(x) = \dfrac{1}{x-2}$ (b)



5.5.2.12. $f(x) = \sqrt[3]{x} - 2$

5.5.2.13. If $F(t)=\dfrac{3}{4}t+2$, find $F^{-1}(2)$.

Answer. 0

5.5.2.14. If $G(x)=\dfrac{1}{x}-4$, find $G^{-1}(3)$.

5.5.2.15. The table shows the revenue, R, from sales of the Miracle Mop as a function of the number of dollars spent on advertising, A. Let f be the name of the function defined by the table, so $R=f(A)$.

A (thousands of dollars)	100	150	200	250	300
R (thousands of dollars)	250	280	300	310	315

(a) Evaluate $f^{-1}(300)$. Explain its meaning in this context.

(b) Write two equations to answer the following question, one using f and one using f^{-1}: How much should we spend on advertising to generate revenue of \$250,000?

Answer.

(a) $f^{-1}(300)=200$: \$200,000 in advertising results in \$300,000 in revenue.

(b) $f(A)=250$ or $A=f^{-1}(250)$

5.5.2.16. The table shows the systolic blood pressure, S, of a patient as a function of the dosage, d, of medication he receives. Let g be the name of the function defined by the table, so $S=g(d)$.

d (mg)	190	195	200	210	220
S (mm Hg)	220	200	190	185	183

(a) Evaluate $g^{-1}(200)$. Explain its meaning in this context.

(b) Write two equations to answer the following question, one using g and one using g^{-1}: What dosage results in systolic blood pressure of 220?

For Problems 17-24, write the equation in exponential form.

5.5.2.17. $\log(0.001)=z$

Answer. $10^z=0.001$

5.5.2.18. $\log_3(20)=t$

5.5.2.19. $\log_2(3)=x-2$

Answer. $2^{x-2}=3$

5.5.2.20. $\log_5(3)=6-2p$

5.5.2.21. $\log_b(3x+1)=3$

Answer. $b^3=3x+1$

5.5.2.22. $\log_m(8)=4t$

5.5.2.23. $\log_n(q)=p-1$

Answer. $n^{p-1}=q$

5.5.2.24. $\log_q(p+2)=w$

For Problems 25-28, simplify.

5.5.2.25. $10^{\log(6n)}$

Answer. $6n$

5.5.2.26. $\log(100^x)$

5.5.2.27. $\log_2\left(4^{x+3}\right)$

Answer. $2x+6$

5.5.2.28. $3^{2\log_3(t)}$

For Problems 29-36, solve.

5.5.2.29. $\log_3\left(\dfrac{1}{3}\right) = y$

Answer. -1

5.5.2.30. $\log_3(x) = 4$

5.5.2.31. $\log_2(y) = -1$

Answer. $\dfrac{1}{2}$

5.5.2.32. $\log_5(y) = -2$

5.5.2.33. $\log_b(16) = 2$

Answer. 4

5.5.2.34. $\log_b(9) = \dfrac{1}{2}$

5.5.2.35. $\log_4\left(\dfrac{1}{2}t + 1\right) = -2$

Answer. $\dfrac{-15}{8}$

5.5.2.36. $\log_2(3x - 1) = 3$

For Problems 37-40, solve.

5.5.2.37. $\log_3(x) + \log_3(4) = 2$

Answer. $\dfrac{9}{4}$

5.5.2.38. $\log_2(x + 2) - \log_2(3) = 6$

5.5.2.39. $\log(x - 1) + \log(x + 2) = 1$

Answer. 3

5.5.2.40. $\log(x + 2) - \log(x - 3) = 1$

For Problems 41-46, solve.

5.5.2.41. $e^x = 4.7$

Answer. $x \approx 1.548$

5.5.2.42. $e^x = 0.5$

5.5.2.43. $\ln(x) = 6.02$

Answer. $x \approx 411.58$

5.5.2.44. $\ln(x) = -1.4$

5.5.2.45. $4.73 = 1.2e^{0.6x}$

Answer. $x \approx 2.286$

5.5.2.46. $1.75 = 0.3e^{-1.2x}$

For Problems 47-50, simplify.

5.5.2.47. $e^{(\ln(x))/2}$

Answer. $\sqrt{x}$

5.5.2.48. $\ln\left(\dfrac{1}{e}\right)^{2n}$

5.5.2.49. $\ln\left(\dfrac{e^k}{e^3}\right)$

Answer. $k - 3$

5.5.2.50. $e^{\ln(e+x)}$

5.5.2.51. In 1970, the population of New York City was $7,894,862$. In 1980, the population had fallen to $7,071,639$.

(a) Write an exponential function using base e for the population of New York over that decade.

(b) By what percent did the population decline annually?

Answer.

(a) $P = 7,894,862e^{-0.011t}$

(b) 1.095%

5.5.2.52. In 1990, the population of New York City was $7,322,564$. In 2000, the population was $8,008,278$.

(a) Write an exponential function using base e for the population of New York over that decade.

(b) By what percent did the population increase annually?

5.5.2.53. You deposit \$1000 in a savings account paying 5% interest compounded continuously.

(a) Find the amount in the account after 7 years.

(b) How long will it take for the original principal to double?

(c) Find a formula for the time t required for the amount to reach A.

Answer.

(a) \$1419.07 (b) 13.9 years (c) $t = 20\ln\left(\dfrac{A}{1000}\right)$

5.5.2.54. The voltage, V, across a capacitor in a certain circuit is given by the function

$$V(t) = 100(1 - e^{-0.5t})$$

where t is the time in seconds.

(a) Make a table of values and graph $V(t)$ for $t = 0$ to $t = 10$.

(b) Describe the graph. What happens to the voltage in the long run?

(c) How much time must elapse (to the nearest hundredth of a second) for the voltage to reach 75 volts?

5.5.2.55. Solve for t: $y = 12e^{-kt} + 6$

Answer. $t = \dfrac{-1}{k}\ln\left(\dfrac{y-6}{12}\right)$

5.5.2.56. Solve for k: $N = N_0 + 4\ln(k + 10)$

5.5.2.57. Solve for M: $Q = \dfrac{1}{t}\left(\dfrac{\log(M)}{\log(N)}\right)$

Answer. $M = N^{Qt}$

5.5.2.58. Solve for t: $C_H = C_L \cdot 10^k t$

5.5.2.59. Express $P(t) = 750e^{0.32t}$ in the form $P(t) = P_0 b^t$.

Answer. $P(t) = 750(1.3771)^t$

5.5.2.60. Express $P(t) = 80e^{-0.6t}$ in the form $P(t) = P_0 b^t$.

5.5.2.61. Express $N(t) = 600(0.4)^t$ in the form $N(t) = N_0 e^{kt}$.

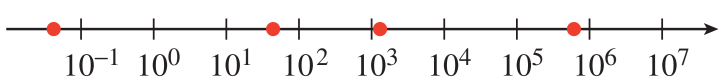
Answer. $N(t) = 600e^{-0.9163t}$

5.5.2.62. Express $N(t) = 100(1.06)^t$ in the form $N(t) = N_0 e^{kt}$.

5.5.2.63. Plot the values on a log scale.

x	0.04	45	1200	560,000

Answer.

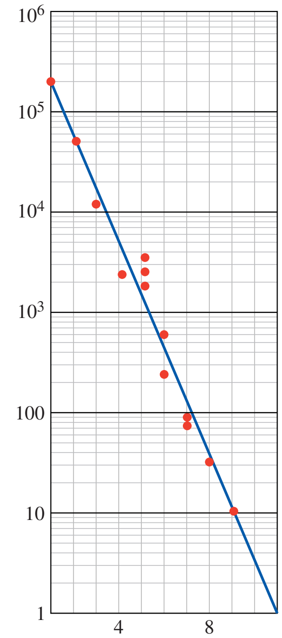


5.5.2.64. Plot the values on a log scale.

x	0.0007	0.8	3.2	2500

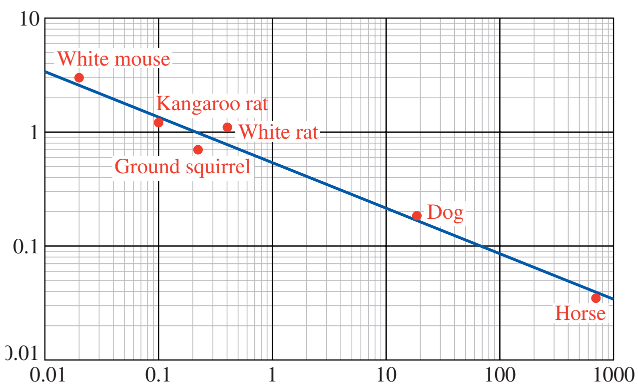
5.5.2.65.

The graph describes a network of streams near Santa Fe, New Mexico. It shows the number of streams of a given order, which is a measure of their size. Use the graph to estimate the number of streams of orders 3, 4, 8, and 9. (Source: Leopold, Wolman, and Miller)



Answer. Order 3: 17,000; Order 4: 5000; Order 8: 40; Order 9: 11

5.5.2.66. Large animals use oxygen more efficiently when running than small animals do. The graph shows the amount of oxygen various animals use, per gram of their body weight, to run 1 kilometer. Estimate the body mass and oxygen use for a kangaroo rat, a dog, and a horse. (Source: Schmidt-Neilsen, 1972)



5.5.2.67. The pH of an unknown substance is 6.3. What is its hydrogen ion concentration?

Answer. 5×10^{-7}

5.5.2.68. The noise of a leaf blower was measured at 110 decibels. What was the intensity of the sound waves?

5.5.2.69. A refrigerator produces 50 decibels of noise, and a vacuum cleaner produces 85 decibels. How much more intense are the sound waves from a vacuum cleaner than those from a refrigerator?

Answer. 3160

5.5.2.70. In 2004, a magnitude 9.0 earthquake struck Sumatra in Indonesia. How much more powerful was this quake than the 1906 San Francisco earthquake of magnitude 8.3?

5.6 Projects for Chapter 5

Project 5.6.1 The Logistic Function. In this project, we investigate the graph of the logistic function.

a Graph the **sigmoid function**, $s(t) = \dfrac{1}{1+e^{-t}}$, in the window

Xmin $= -4$	Xmax $= 4$
Ymin $= -1$	Ymax $= 2$

What are the domain and range of the function? List the intercepts of the graph, as well as any horizontal or vertical asymptotes. Estimate the coordinates of the **inflection point**, where the graph changes concavity.

b Graph the two functions $Y_1(t) = \dfrac{5}{1+4e^{-t}}$ and $Y_2 = \dfrac{10}{1+9e^{-t}}$ in the window

Xmin $= -2$	Xmax $= 10$
Ymin $= -1$	Ymax $= 11$

How do the graphs of these functions differ from the sigmoid function? State the domain and range, intercepts, and asymptotes of Y_1 and Y_2. Estimate the coordinates of their inflection points.

c The function $P(t) = \dfrac{KP_0}{P_0 + (K - P_0)e^{-rt}}$ is called a **logistic function**. It is used to model population growth, among other things. It has three parameters, K, P_0, and r. The parameter K is called the **carrying capacity**. The functions Y_1 and Y_2 in part (b) are logistic functions with $P_0 = 1$ and $r = 1$. What does the value of K tell you about the graph? What do you notice about the vertical coordinate of the inflection point?

d Graph the function $P(t) = \dfrac{10P_0}{P_0 + (10 - P_0)e^{-t}}$ for $P_0 = 3$, 4, and 5. What does the value of P_0 tell you about the graph?

e Graph the function $P(t) = \dfrac{20}{2+8e^{-rt}}$ for $r = 0.5$, 1, and 2. What does the value of r tell you about the graph?

Project 5.6.2 Bell-shaped Curve. In this project, we investigate the normal or bell-shaped curve.

a Graph the function $f(x) = e^{-x^2}$, in the window

Xmin $= -2$	Xmax $= 2$
Ymin $= -1$	Ymax $= 2$

What are the domain and range of the function? List the intercepts of the graph, as well as any horizontal or vertical asymptotes. Estimate the coordinates of the **inflection point**, where the graph changes concavity.

b Graph the function $f(x) = e^{-(x-m)^2}$ for $m = -1$, 0, 1, and 2. How does the value of m affect the graph?

c The function

$$N(x) = \frac{1}{s\sqrt{2\pi}} e^{-(x-m)^2/2s^2}$$

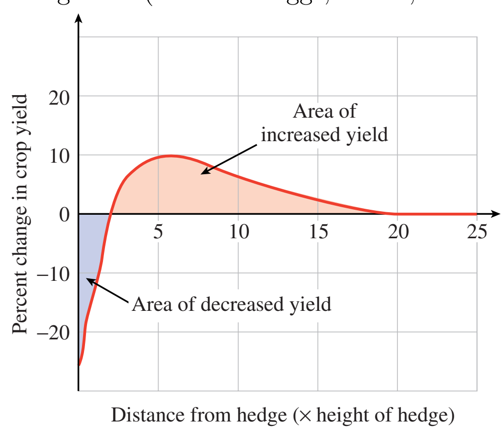
is called the **normal** curve. It is used in statistics to describe the distribution of a variable, such as height, among a population. The parameter m gives the **mean** of the distribution, and s gives the **standard deviation**. For example, the distribution of height among American women has a mean of 64 inches and a standard deviation of 2.5 inches. Graph $N(x)$ for these values.

d Graph the function

$$N(x) = \frac{1}{s\sqrt{2\pi}} e^{-(x-m)^2/2s^2}$$

for $s = 0.5$, 0.8, 1, and 1.2. (You may have to adjust the window to get a good graph.) How does the value of s affect the graph?

Project 5.6.3 Do hedgerows planted at the boundaries of a field have a good or bad effect on crop yields? Hedges provide some shelter for the crops and retain moisture, but they may compete for nutrients or create too much shade. Results of studies on the microclimates produced by hedges are summarized in the figure, which shows how crop yields increase or decrease as a function of distance from the hedgerow. (Source: Briggs, David, and Courtney, 1985)



a We will use trial-and-improvement to fit a curve to the graph. First, graph $y_1 = xe^{-x}$ in the window Xmin $= -2$, Xmax $= 5$, Ymin $= -1$, Ymax $= 1$ to see that it has the right shape.

b Graph $y_2 = (x-2)e^{-(x-2)}$ on the same axes. How is the graph of y_2 different from the graph of y_1?

c Next we'll find the correct scale by trying functions of the form $y = a(x-2)e^{-(x-2)/b}$. Experiment with different whole number values of a and b. How do the values of a and b affect the curve?

d Graph $y = 5(x-2)e^{-(x-2)/4}$ in the window Xmin $= -5$, Xmax $= 25$, Ymin $= -20$, Ymax $= 25$. This function is a reasonable approximation for the curve in the figure. Compare the area of decreased yield (below the x-axis) with the area of increased yield (above the x-axis). Which area is larger? Is the overall effect of hedgerows on crop yield good or bad?

e About how far from the hedgerow do the beneficial effects extend? If the average hedgerow is about 2.5 meters tall, how large should the field be to exploit their advantages?

Project 5.6.4 Carbon Content. Organic matter in the ground decomposes over time, and if the soil is cultivated properly, the fraction of its original organic carbon content is given by

$$C(t) = \frac{a}{b} - \frac{a-b}{b}e^{-bt}$$

where t is in years, and a and b are constants. (Source: Briggs, David, and Courtney, 1985)

a Write and simplify the formula for $C(t)$ if $a = 0.01$, $b = 0.028$.

b Graph $C(t)$ in the window Xmin $= 0$, Xmax $= 200$, Ymin $= 0$, Ymax $=$ 1.5.

c What value does $C(t)$ approach as t increases? Compare this value to $\frac{a}{b}$.

d The half-life of this function is the amount of time until $C(t)$ declines halfway to its limiting value, $\frac{a}{b}$. What is the half-life?

Project 5.6.5 Change of Base. This project derives the **change of base** formula.

a Follow the steps below to calculate $\log_8 20$.

Step 1 Let $x = \log_8 20$. Write the equation in exponential form.

Step 2 Take the logarithm base 10 of both sides of your new equation.

Step 3 Simplify and solve for x.

b Follow the steps in part (a) to calculate $\log_8 5$.

c Use part (a) to find a formula for calculating $\log_8 Q$, where Q is any positive number.

d Find a formula for calculating $\log_b Q$, where $b > 1$ and Q is any positive number.

e Find a formula for calculating $\ln Q$ in terms of $\log_{10} Q$.

f Find a formula for calculating $\log_{10} Q$ in terms of $\ln Q$.

Project 5.6.6 Log Equations. In this project, we solve logarithm equations with a graphing calculator. We have already used the **Intersect** feature to find approximate solutions for linear, exponential, and other types of equations in one variable. The same technique works for equations that involve common or natural logarithms.

a Solve $\log_{10}(x+1) + \log_{10}(x-2) = 1$ using the **Intersect** feature by setting $Y_1 = \log(x+1) + \log(x-2)$ and $Y_2 = 1$. What about logarithmic equations with other bases? The calculator typically does not have a log key for bases other than 10 or e. However, by using the change of base formula from Project 5, we can rewrite any logarithm in terms of a common or natural logarithm.

b Use the change of base formula to write $y = \log_2 x$ and $y = \log_2(x-2)$ in terms of common logarithms.

c Solve $\log_2 x + \log_2(x-2) = 3$ by using the **Intersect** feature on your calculator.

d Solve $\log_3(x-2) - \log_3(x+1) = 3$ by using the **Intersect** feature on your calculator.

6 Quadratic Functions

6.1 Factors and x-Intercepts

6.1.1 Zero-Factor Principle

Checkpoint 6.1.3 Graph the function

$$f(x) = (x-3)(2x+3)$$

on a calculator, and use your graph to solve the equation $f(x) = 0$. (UseXmin $=$ -9.4, Xmax $= 9.4$.) Check your answer with the zero-factor principle.

Answer. $x = -\dfrac{3}{2},\ x = 3$

6.1.2 Solving Quadratic Equations by Factoring

Checkpoint 6.1.6 Solve by factoring: $(t-3)^2 = 3(9-t)$

Answer. $x = -3,\ x = 6$

Checkpoint 6.1.8

a Solve $f(t) = 4t - t^2 = 0$ by factoring.

b Solve $g(t) = 20t - 5t^2 = 0$ by factoring.

c Graph $y = f(t)$ and $y = g(t)$ together in the window

$$\begin{array}{ll} \text{Xmin} = -2 & \text{Xmax} = 6 \\ \text{Ymin} = -20 & \text{Ymax} = 25 \end{array}$$

and locate the horizontal intercepts of each graph.

Answer.

a $t = 0,\ t = 4$

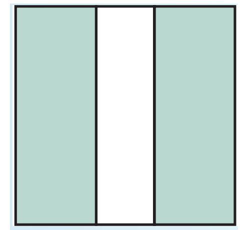
b $t = 0,\ t = 4$

c $(0,0),\ (4,0)$

6.1.3 Applications

Checkpoint 6.1.10

Francine is designing the layout for a botanical garden. The plan includes a square herb garden, with a path 5 feet wide through the center of the garden, as shown at right. To include all the species of herbs, the planted area must be 300 square feet. Find the dimensions of the herb garden.



Answer. 20 feet by 20 feet

6.1.4 Solutions of Quadratic Equations

Checkpoint 6.1.12 Find a quadratic equation with integer coefficients whose solutions are $\frac{2}{3}$ and -5.

Answer. $3x^2 + 13x - 10 = 0$

6.1.5 Equations Quadratic in Form

Checkpoint 6.1.15 Use the substitution $u = x^2$ to solve the equation $x^4 - 5x^2 + 6 = 0$.

Answer. $x = \pm\sqrt{2},\ \ x = \pm\sqrt{3}$

Checkpoint 6.1.17 Solve the equation $10^{2x} - 3 \cdot 10^x + 2 = 0$, and check the solutions.

Answer. $x = 0,\ \ x = \log(2)$

6.1.7 Homework 6.1

6.1.7.1. Delbert stands at the top of a 300-foot cliff and throws his algebra book directly upward with a velocity of 20 feet per second. The height of his book above the ground t seconds later is given by the equation

$$h = -16t^2 + 20t + 300$$

where h is in feet.

(a) Use your calculator to make a table of values for the height equation, with increments of 0.5 second.

(b) Graph the height equation on your calculator. Use your table of values to help you choose appropriate window settings.

(c) What is the highest altitude Delbert's book reaches? When does it reach that height? Use the `TRACE` feature to find approximate answers first. Then use the **Table** feature to improve your estimate.

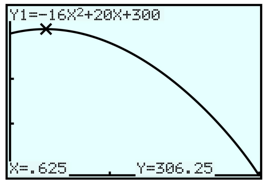
(d) When does Delbert's book pass him on its way down? (Delbert is standing at a height of 300 feet.) Use the **intersect** command.

(e) How long will it take Delbert's book to hit the ground at the bottom of the cliff?

Answer.

(a)

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
h	300	306	304	294	276	250	216	174	124	66	0

(b) 

(c) 306.25 ft at 0.625 sec

(d) 1.25 sec

(e) 5 sec

6.1.7.2. James Bond stands on top of a 240-foot building and throws a film canister upward to a fellow agent in a helicopter 16 feet above the building. The height of the film above the ground t seconds later is given by the formula

$$h = -16t^2 + 32t + 240$$

where h is in feet.

(a) Use your calculator to make a table of values for the height formula, with increments of 0.5 second.

(b) Graph the height formula on your calculator. Use your table of values to help you choose appropriate window settings.

(c) How long will it take the film canister to reach the agent in the helicopter? (What is the agent's altitude?) Use the `TRACE` feature to find approximate answers first. Then use the **Table** feature to improve your estimate.

(d) If the agent misses the canister, when will it pass James Bond on the way down? Use the intersect command.

(e) How long will it take it to hit the ground?

In Problems 3–10, use a graph to solve the equation $y = 0$. (Use Xmin $=$ -9.4, Xmax $= 9.4$.) Check your answers with the zero-factor principle.

6.1.7.3. $y = (2x+5)(x-2)$

Answer. $\dfrac{-5}{2},\ 2$

6.1.7.4. $y = (x+1)(4x-1)$

6.1.7.5. $y = x(3x+10)$

Answer. $0,\ \dfrac{-10}{3}$

6.1.7.6. $y = x(3x-7)$

6.1.7.7. $y = (4x+3)(x+8)$

Answer. $\dfrac{-3}{4},\ -8$

6.1.7.8. $y = (x-2)(x-9)$

6.1.7.9. $y = (x-4)^2$

Answer. 4

6.1.7.10. $y = (x+6)^2$

For Problems 11-24, solve by factoring. (See Algebra Skills Refresher Appendix A.8 to review factoring.)

6.1.7.11. $2a^2 + 5a - 3 = 0$

Answer. $\dfrac{1}{2},\ -3$

6.1.7.12. $3b^2 - 4b - 4 = 0$

6.1.7.13. $2x^2 = 6x$

Answer. 0, 3

6.1.7.14. $5z^2 = 5z$

6.1.7.15. $3y^2 - 6y = -3$

Answer. 1

6.1.7.16. $4y^2 + 4y = 8$

6.1.7.17. $x(2x-3) = -1$

Answer. $\dfrac{1}{2},\ 1$

6.1.7.18. $2x(x-2) = x+3$

6.1.7.19. $t(t-3) = 2(t-3)$

Answer. $2,\ 3$

6.1.7.20. $5(t+2) = t(t+2)$

6.1.7.21. $z(3z+2) = (z+2)^2$

Answer. $-1,\ 2$

6.1.7.22. $(z-1)^2 = 2z^2 + 3z - 5$

6.1.7.23. $(v+2)(v-5) = 8$

Answer. $-3,\ 6$

6.1.7.24. $(w+1)(2w-3) = 3$

In Problems 25–28, graph each set of functions in the standard window.What do you notice about the x-intercepts? Generalize your observation, and test your idea with examples.

6.1.7.25.

(a) $f(x) = x^2 - x - 20$

(b) $g(x) = 2(x^2 - x - 20)$

(c) $h(x) = 0.5(x^2 - x - 20)$

Answer. The 3 graphs have the same x-intercepts. In general, the graph of $y = ax^2 + bx + c$ has the same x-intercepts as the graph of $y = k(ax^2 + bx + c)$.

6.1.7.26.

(a) $f(x) = x^2 + 2x - 15$

(b) $g(x) = 3(x^2 + 2x - 15)$

(c) $h(x) = 0.2(x^2 + 2x - 15)$

6.1.7.27.

(a) $f(x) = x^2 + 6x - 16$

(b) $g(x) = -2(x^2 + 6x - 16)$

(c) $h(x) = -0.1(x^2 + 6x - 16)$

Answer. The 3 graphs have the same x-intercepts. In general, the graph of $y = ax^2 + bx + c$ has the same x-intercepts as the graph of $y = k(ax^2 + bx + c)$.

6.1.7.28.

(a) $f(x) = x^2 - 16$

(b) $g(x) = -1.5(x^2 - 16)$

(c) $h(x) = -0.4(x^2 - 16)$

In Problems 29–36, write a quadratic equation whose solutions are given. The equation should be in standard form with integer coefficients.

6.1.7.29. -2 and 1

Answer. $x^2 + x - 2 = 0$

6.1.7.30. -4 and 3

6.1.7.31. 0 and -5

Answer. $x^2 + 5x = 0$

6.1.7.32. 0 and 5

6.1.7.33. -3 and $\dfrac{1}{2}$

Answer. $2x^2 + 5x - 3 = 0$

6.1.7.34. $\dfrac{-2}{3}$ and 4

6.1.7.35. $\dfrac{-1}{4}$ and $\dfrac{3}{2}$

Answer. $8x^2 - 10x - 3 = 0$

6.1.7.36. $\dfrac{-1}{3}$ and $\dfrac{-1}{2}$

For problems 37-40, graph the function in the **ZInteger** window, and locate the x-intercepts of the graph. Use the x-intercepts to write the quadratic expression in factored form.

6.1.7.37.
$f(x) = 0.1(x^2 - 3x - 270)$

Answer.
$f(x) = 0.1(x - 18)(x + 15)$

6.1.7.38.
$h(x) = 0.1(x^2 + 9x - 360)$

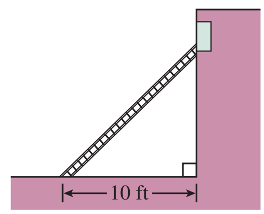
6.1.7.39.
$g(x) = -0.08(x^2 + 14x - 576)$

Answer.
$g(x) = -0.08(x - 18)(x + 32)$

6.1.7.40.
$F(x) = -0.06(x^2 - 22x - 504)$

Use the Pythagorean theorem to solve Problems 41 and 42. (See Algebra Skills Refresher Appendix A.11 to review the Pythagorean theorem.)

6.1.7.41.
One end of a ladder is 10 feet from the base of a wall, and the other end reaches a window in the wall. The ladder is 2 feet longer than the height of the window.



(a) Write a quadratic equation about the height of the window.

(b) Solve your equation to find the height of the window.

Answer.

(a) $10^2 + h^2 = (h + 2)^2$

(b) 24 ft

6.1.7.42. The diagonal of a rectangle is 20 inches. One side of the rectangle is 4 inches shorter than the other side.

(a) Write a quadratic equation about the length of the rectangle.

(b) Solve your equation to find the dimensions of the rectangle.

Use the following formula to answer Problems 43 and 44. If an object is thrown into the air from a height s_0 above the ground with an initial velocity v_0, its height t seconds later is given by the formula

$$h = -\frac{1}{2}gt^2 + v_0 t + s_0$$

where g is a constant that measures the force of gravity.

6.1.7.43. A tennis ball is thrown into the air with an initial velocity of 16 feet per second from a height of 8 feet. The value of g is 32.

(a) Write a quadratic equation that gives the height of the tennis ball at time t.

(b) Find the height of the tennis ball at $t = \frac{1}{2}$ second and at $t = 1$ second.

(c) Write and solve an equation to answer the question: At what time is the tennis ball 11 feet high?

(d) Use the Table feature on your calculator to verify your answers to parts (b) and (c). (What value of ΔTbl is useful for this problem?)

(e) Graph your equation from part (a) on your calculator. Use your table to help you choose an appropriate window.

(f) If nobody hits the tennis ball, approximately how long will it be in the air?

Answer.

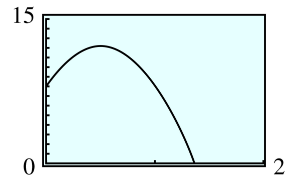
(a) $h = -16t^2 + 16t + 8$

(b) 12 ft; 8 ft

(c) $11 = -16t^2 + 16t + 8$; at $\dfrac{1}{4}$ sec and $\dfrac{3}{4}$ sec

(d) $\Delta\text{Tbl} = 0.25$

(e)



(f) 1.37 sec

6.1.7.44. A mountain climber stands on a ledge 80 feet above the ground and tosses a rope down to a companion clinging to the rock face below the ledge. The initial velocity of the rope is -8 feet per second, and the value of g is 32.

(a) Write a quadratic equation that gives the height of the rope at time t.

(b) What is the height of the rope after $\dfrac{1}{2}$ second? After 1 second?

(c) Write and solve an equation to answer the question: How long does it take the rope to reach the second climber, who is 17 feet above the ground?

(d) Use the Table feature on your calculator to verify your answers to parts (b) and (c). (What value of ΔTbl is useful for this problem?)

(e) Graph your equation from part (a) on your calculator. Use your table to help you choose an appropriate window.

(f) If the second climber misses the rope, approximately how long will the rope take to reach the ground?

For Problems 45 and 46, you may want to review Investigation 2.0.2, Perimeter and Area, in Chapter 2.

6.1.7.45. A rancher has 360 yards of fence to enclose a rectangular pasture. If the pasture should be 8000 square yards in area, what should its dimensions be? We will use 3 methods to solve this problem: a table of values, a graph, and an algebraic equation.

(a) Make a table by hand that shows the areas of pastures of various widths, as shown here.

Width	Length	Area
10	170	1700
⋮	⋮	⋮

(To find the length of each pasture, ask yourself, What is the sum of the length plus the width if there are 360 yards of fence?) Continue the table until you find the pasture whose area is 8000 square yards.

(b) Write an expression for the length of the pasture if its width is x. Next, write an expression for the area, A, of the pasture if its width is x. Graph the equation for A on your calculator, and use the graph to find the pasture of area 8000 square yards.

(c) Write an equation for the area, A, of the pasture in terms of its width x. Solve your equation algebraically for $A = 8000$. Explain why there are two solutions.

Answer.

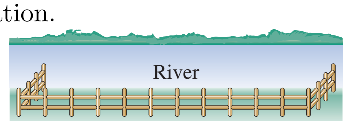
(a)

Width	Length	Area
10	170	1700
20	160	3200
30	150	4500
40	140	5600
50	130	6500
60	120	7200
70	110	7700
80	100	8000

(b) $l = 180 - x$, $A = 180x - x^2$; 80 yd by 100 yd

(c) $180x - x^2 = 8000$, 80 yd by 100 yd, or 100 yd by 80 yd. There are two solutions because the pasture can be oriented in two directions.

6.1.7.46. If the rancher in Problem 45 uses a riverbank to border one side of the pasture as shown in the figure, he can enclose 16,000 square yards with 360 yards of fence. What will the dimensions of the pasture be then? We will use three methods to solve this problem: a table of values, a graph, and an algebraic equation.



(a) Make a table by hand that shows the areas of pastures of various widths, as shown here.

Width	Length	Area
10	340	3400
20	320	6400
⋮	⋮	⋮

(Be careful computing the length of the pasture: Remember that one side of the pasture does not need any fence!) Continue the table until you find the pasture whose area is 16,000 square yards.

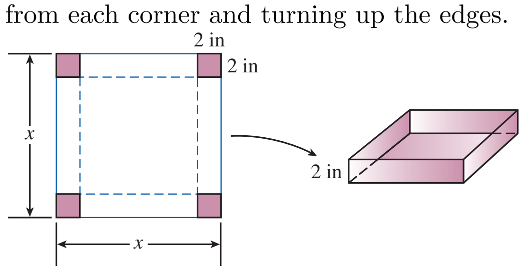
(b) Write an expression for the length of the pasture if its width is x.

Next, write an expression for the area, A, of the pasture if its width is x. Graph the equation for A, and use the graph to find the pasture of area $16,000$ square yards.

(c) Write an equation for the area, A, of the pasture in terms of its width x. Solve your equation algebraically for $A = 16,000$.

For Problems 47 and 48, you will need the formula for the volume of a box.

6.1.7.47. A box is made from a square piece of cardboard by cutting 2-inch squares from each corner and turning up the edges.



(a) If the piece of cardboard is x inches square, write expressions for the length, width, and height of the box. Then write an expression for the volume, V, of the box in terms of x.

(b) Use your calculator to make a table of values showing the volumes of boxes made from cardboard squares of side 4 inches, 5 inches, and so on.

(c) Graph your expression for the volume on your calculator. What happens to V as x increases?

(d) Use your table or your graph to find what size cardboard you need to make a box with volume 50 cubic inches.

(e) Write and solve a quadratic equation to answer part (d).

Answer.

(a) $l = x - 4, \;\; w = x - 4, \;\; h = 2, \;\; V = 2(x-4)^2$

(b)

x	4	5	6	7	8	9	10
V	0	2	8	18	32	50	72

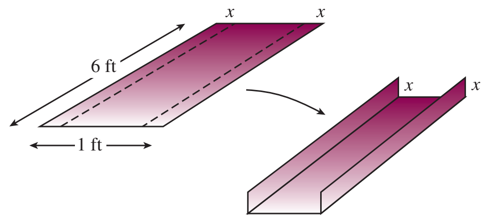
(c) As x increases, V increases.

(d) 9 inches by 9 inches.

(e) $2(x-4)^2 = 50, \;\; x = 9$

6.1.7.48. A length of rain gutter is made from a piece of aluminum 6 feet long and 1 foot wide.

(a) If a strip of width x is turned up along each long edge, write expressions for the length, width, and height of the gutter. Then write an expression for the volume, V, of the gutter in terms of x.



(b) Use your calculator to make a table of values showing the volumes of various rain gutters formed by turning up edges of 0.1 foot, 0.2 foot, and so on.

(c) Graph your expression for the volume. What happens to V as x increases?

(d) Use your table or your graph to discover how much metal should be turned up along each long edge so that the gutter has a capacity of $\frac{3}{4}$ cubic foot of rainwater.

(e) Write and solve a quadratic equation to answer part (d).

Problems 49 and 50 deal with wildlife management. The annual increase, I, in a population often depends on the size x of the population, according to the formula

$$I = kCx - kx^2$$

where k and C are constants related to the fertility of the population and the availability of food.

6.1.7.49. The annual increase, $f(x)$, in the deer population in a national park is given by

$$f(x) = 1.2x - 0.0002x^2$$

where x is the size of the population that year.

(a) Make a table of values for $f(x)$ for $0 \le x \le 7000$. Use increments of 500 in x.

(b) How much will a population of 2000 deer increase? A population of 5000 deer? A population of 7000 deer?

(c) Use your calculator to graph the annual increase, $f(x)$, versus the size of the population, x, for $0 \le x \le 7000$.

(d) What do the x-intercepts tell us about the deer population?

(e) Estimate the population size that results in the largest annual increase. What is that increase?

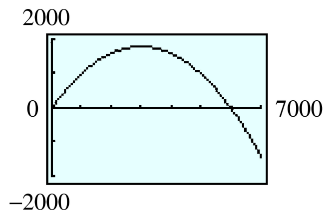
Answer.

(a)

x	0	500	1000	1500	2000	2500	3000	3500
I	0	550	1000	1350	1600	1750	1800	1750

x	4000	4500	5000	5500	6000	6500	7000
I	1600	1350	1000	550	0	-650	-1400

(b) 1600, 1000, -1400

(c) 

(d) No increase

(e) 3000; 1800

6.1.7.50. Commercial fishermen rely on a steady supply of fish in their area. To avoid overfishing, they adjust their harvest to the size of the population. The function

$$g(x) = 0.4x - 0.0001x^2$$

gives the annual rate of growth, in tons per year, of a fish population of biomass x tons.

(a) Make a table of values for $g(x)$ for $0 \leq x \leq 5000$. Use increments of 500 in x.

(b) How much will a population of 1000 tons increase? A population of 3000 tons? A population of 5000 tons?

(c) Use your calculator to graph the annual increase, $g(x)$, versus the size of the population, x, for $0 \leq x \leq 5000$.

(d) What do the x-intercepts tell us about the fish population?

(e) Estimate the population size that results in the largest annual increase. What is that increase?

For Problems 51-62, use a substitution to solve the equation.

6.1.7.51. $a^4 + a^2 - 2 = 0$

Answer. ± 1

6.1.7.52. $t^6 - t^3 - 6 = 0$

6.1.7.53. $4b^6 - 3 = b^3$

Answer. $\sqrt[3]{-3/4}$, 1

6.1.7.54. $3x^4 + 1 = 4x^2$

6.1.7.55. $c^{2/3} + 2c^{1/3} - 3 = 0$

Answer. -27, 1

6.1.7.56. $y^{1/2} - 3y^{1/4} - 4 = 0$

6.1.7.57. $10^{2w} - 5 \cdot 10^w + 6 = 0$

Answer. $\log(2)$, $\log(3)$

6.1.7.58. $e^{2x} - 5e^x + 4 = 0$

6.1.7.59. $5^{2t} - 30 \cdot 5^t + 125 = 0$

Answer. 1, 2

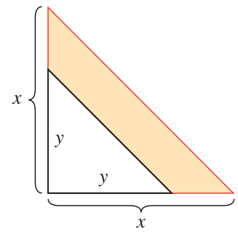
6.1.7.60. $e^{4r} - 3e^{2r} + 2 = 0$

6.1.7.61. $\dfrac{1}{m^2} + \dfrac{5}{m} - 6 = 0$

Answer. $\dfrac{-1}{6}$, 1

6.1.7.62. $\dfrac{1}{s^2} + \dfrac{4}{s} - 5 = 0$

6.1.7.63. The sail in the figure is a right triangle of base and height x. It has a colored stripe along the hypotenuse and a white triangle of base and height y in the lower corner.



(a) Write an expression for the area of the colored stripe.

(b) Express the area of the stripe in factored form.

(c) If the sail is $7\frac{1}{2}$ feet high and the white strip is $4\frac{1}{2}$ feet high, use your answer to (b) to calculate mentally the area of the stripe.

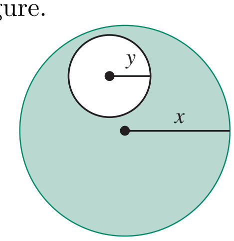
Answer.

(a) $A = \dfrac{1}{2}(x^2 - y^2)$

(b) $A = \dfrac{1}{2}(x - y)(x + y)$

(c) 18 sq ft

6.1.7.64. An hors d'oeuvres tray has radius x, and the dip container has radius y, as shown in the figure.



(a) Write an expression for the area for the chips (shaded region).

(b) Express the area in factored form.

(c) If the tray has radius $8\frac{1}{2}$ inches and the space for the dip has radius $2\frac{1}{2}$ inches, use your answer to part (b) to calculate mentally the area for chips. (Express your answer as a multiple of π.)

6.2 Solving Quadratic Equations

6.2.1 Quadratic Formula

Checkpoint 6.2.2 Use the quadratic formula to solve $x^2 - 3x = 1$.

Answer. $x = \dfrac{3 \pm \sqrt{13}}{2}$

6.2.2 Applications

Checkpoint 6.2.4 In Investigation 6.0.1, we considered the height of a baseball, given by the equation

$$h = -16t^2 + 64t + 4$$

Find two times when the ball is at a height of 20 feet. Give your answers to two decimal places.

Answer. 0.27 sec, 3.73 sec

Checkpoint 6.2.6 Solve $2x^2 + kx + k^2 = 1$ for x in terms of k.

Answer. $x = \dfrac{-k \pm \sqrt{8 - 7k^2}}{4}$

6.2.3 Introduction to complex numbers

Checkpoint 6.2.8 Solve the equation $x^2 - 6x + 13 = 0$ by using the quadratic formula.

Answer. $x = \dfrac{6 \pm \sqrt{-16}}{2}$

6.2.1 Imaginary Numbers

Checkpoint 6.2.11 Write each radical as an imaginary number.

a $\sqrt{-18}$

b $-6\sqrt{-5}$

Answer.

a $3i\sqrt{2}$

b $-6i\sqrt{5}$

6.2.2 Complex Numbers

Checkpoint 6.2.14 Use extraction of roots to solve $(2x + 1)^2 + 9 = 0$. Write your answers as complex numbers.

Answer. $x = \dfrac{-1}{2} \pm \dfrac{3}{2}i$

6.2.4 Arithmetic of Complex Numbers

Checkpoint 6.2.16 Subtract: $(-3 + 2i) - (-3 - 2i)$.

Answer. $4i$

6.2.8 Homework 6.2

For Problems 1-2, complete the square and write the result as the square of a binomial.

6.2.8.1.

(a) $x^2 + 8x$

(b) $x^2 - 7x$

(c) $x^2 + \dfrac{3}{2}x$

(d) $x^2 - \dfrac{4}{5}x$

Answer.

(a) $(x + 4)^2$

(b) $\left(x - \dfrac{7}{2}\right)^2$

(c) $\left(x + \dfrac{3}{4}\right)^2$

(d) $\left(x - \dfrac{2}{5}\right)^2$

6.2.8.2.

(a) $x^2 - 14x$

(b) $x^2 + 3x$

(c) $x^2 - \dfrac{5}{2}x$

(d) $x^2 + \dfrac{2}{3}x$

For Problems 3-18, solve by completing the square.

6.2.8.3. $x^2 - 2x + 1 = 0$

Answer. 1

6.2.8.4. $x^2 + 4x + 4 = 0$

6.2.8.5. $x^2 + 9x + 20 = 0$

Answer. $-4, \quad -5$

6.2.8.6. $x^2 - x - 20 = 0$

6.2.8.7. $x^2 = 3 - 3x$

Answer. $\dfrac{3}{2} \pm \sqrt{\dfrac{21}{4}} = \dfrac{-3 \pm \sqrt{21}}{2}$

6.2.8.8. $x^2 = 5 - 5x$

6.2.8.9. $2x^2 + 4x - 3 = 0$

Answer. $-1 \pm \sqrt{\dfrac{5}{2}}$

6.2.8.10. $3x^2 + 12x + 2 = 0$

6.2.8.11. $3x^2 + x = 4$

Answer. $\dfrac{-4}{3}, \ 1$

6.2.8.12. $4x^2 + 6x = 3$

6.2.8.13. $4x^2 - 3 = 2x$

Answer. $\dfrac{1}{4} \pm \sqrt{\dfrac{13}{16}} = \dfrac{1 \pm \sqrt{13}}{4}$

6.2.8.14. $2x^2 - 5 = 3x$

6.2.8.15. $3x^2 - x - 4 = 0$

Answer. $-1, \dfrac{4}{3}$

6.2.8.16. $2x^2 - x - 3 = 0$

6.2.8.17. $5x^2 + 8x = 4$

Answer. $-2, \dfrac{2}{5}$

6.2.8.18. $9x^2 - 12x - 5 = 0$

For Problems 19-24, solve by completing the square. Your answers will involve a, b, or c.

6.2.8.19.
$x^2 + 2x + c = 0$

Answer.
$-1 \pm \sqrt{1 - c}$

6.2.8.20.
$x^2 - 4x + c = 0$

6.2.8.21.
$x^2 + bx + 1 = 0$

Answer.
$-\dfrac{b}{2} \pm \sqrt{\dfrac{b^2 - 4}{4}} = \dfrac{-b \pm \sqrt{b^2 - 4}}{2}$

6.2.8.22.
$x^2 + bx - 4 = 0$

6.2.8.23.
$ax^2 + 2x - 4 = 0$

Answer.
$\dfrac{-1 \pm \sqrt{4a + 1}}{a}$

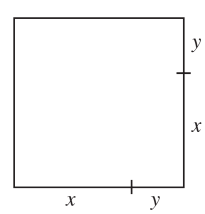
6.2.8.24.
$ax^2 - 4x + 9 = 0$

6.2.8.25.

(a) Write an expression for the area of the square in the figure.

(b) Express the area as a polynomial.

(c) Divide the square into four pieces whose areas are given by the terms of your answer to part (b).

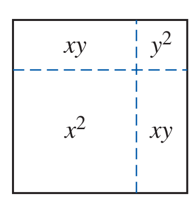


Answer.

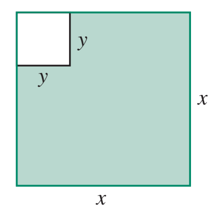
(a) $A = (x + y)^2$

(b) $A = x^2 + 2xy + y^2$

(c) $x^2, \, xy, \, xy, \, y^2$



6.2.8.26.



(a) Write an expression for the area of the shaded region in the figure.

(b) Express the area in factored form.

(c) By making one cut in the shaded region, rearrange the pieces into a rectangle whose area is given by your answer to part (b).

For Problems 23-36, solve using the quadratic formula. Round your answers to three decimal places.

6.2.8.27. $x^2 - x - 1 = 0$

Answer. 1.618, -0.618

6.2.8.28. $x^2 + x + 1 = 0$

6.2.8.29. $y^2 + 2y = 5$

Answer. 1.449, -3.449

6.2.8.30. $y^2 - 4y = -4$

6.2.8.31. $3z^2 = 4.2z + 1.5$

Answer. 1.695, -0.295

6.2.8.32. $2z^2 = 7.5z - 6.3$

6.2.8.33. $0 = x^2 - \dfrac{5}{3}x + \dfrac{1}{3}$

Answer. 1.434, 0.232

6.2.8.34. $0 = -x^2 + \dfrac{5}{2}x - \dfrac{1}{2}$

6.2.8.35.
$-5.2z^2 + 176z + 1218 = 0$

Answer. -5.894, 39.740

6.2.8.36. $15z^2 - 18z - 2750 = 0$

6.2.8.37. A car traveling at s miles per hour on a dry road surface requires approximately d feet to stop, where d is given by the function

$$d = f(s) = \frac{s^2}{24} + \frac{s}{2}$$

(a) Make a table showing the stopping distance, d, for speeds of 10, 20, ... , 100 miles per hour. (Use the **Table** feature of your calculator.)

(b) Graph the function for d in terms of s. Use your table values to help you choose appropriate window settings.

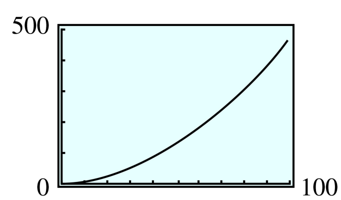
(c) Write and solve an equation to answer the question: If a car must be able to stop in 50 feet, what is the maximum safe speed it can travel?

Answer.

(a)

s	10	20	30	40	50	60	70	80	90	100
d	9	27	53	87	129	180	239	307	383	467

(b)



(c) $\dfrac{s^2}{24} + \dfrac{s}{2} = 50$; 29.16 mph

6.2.8.38. A car traveling at s miles per hour on a wet road surface requires approximately d feet to stop, where d is given by the function

$$d = f(s) = \frac{s^2}{12} + \frac{s}{2}$$

(a) Make a table showing the stopping distance, d, for speeds of 10, 20, ... , 100 miles per hour. (Use the **Table** feature of your calculator.)

(b) Graph the function for d in terms of s. Use your table values to help you choose appropriate window settings.

(c) Insurance investigators at the scene of an accident find skid marks 100 feet long leading up to the point of impact. Write and solve an equation to discover how fast the car was traveling when it put on the brakes. Verify your answer on your graph.

6.2.8.39. A skydiver jumps out of an airplane at $11,000$ feet. While she is in free-fall, her altitude in feet t seconds after jumping is given by the function

$$h = f(t) = -16t^2 - 16t + 11,000$$

(a) Make a table of values showing the skydiver's altitude at 5-second intervals after she jumps from the airplane. (Use the **Table** feature of your calculator.)

(b) Graph the function. Use your table of values to choose appropriate window settings.

(c) If the skydiver must open her parachute at an altitude of 1000 feet, how long can she free-fall? Write and solve an equation to find the answer.

(d) If the skydiver drops a marker just before she opens her parachute, how long will it take the marker to hit the ground? (*Hint*: The marker continues to fall according to the equation given above.)

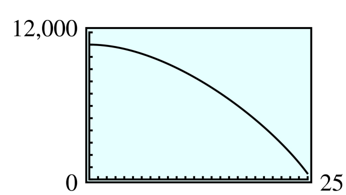
(e) Find points on your graph that correspond to your answers to parts (c) and (d).

Answer.

(a)

t	0	5	10	15	20	25
h	$11,000$	$10,520$	9240	7160	4280	600

(b)



(c) $-16t^2 - 16t + 11,000 = 1000$; 24.5 sec

(d) 1.2 sec

6.2.8.40. A high diver jumps from the 10-meter springboard. His height in meters above the water t seconds after leaving the board is given by the function

$$h = f(t) = -4.9t^2 + 8t + 10$$

(a) Make a table of values showing the diver's altitude at 0.25-second intervals after he jumps from the airplane. (Use the **Table** feature of your calculator.)

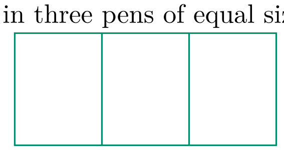
(b) Graph the function. Use your table of values to choose appropriate window settings.

(c) How long is it before the diver passes the board on the way down?

(d) How long is it before the diver hits the water?

(e) Find points on your graph that correspond to your answers to parts (c) and (d).

6.2.8.41. A dog trainer has 100 meters of chain link fence. She wants to enclose 250 square meters in three pens of equal size, as shown in the figure.



(a) Let l and w represent the length and width, respectively, of the entire area. Write an equation about the amount of chain link fence.

(b) Solve your equation for l in terms w.

(c) Write and solve an equation in w for the total area enclosed.

(d) Find the dimensions of each pen.

Answer.

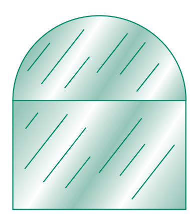
(a) $2l + 4w = 100$

(b) $l = 50 - 2w$

(c) $w(50 - 2w) = 250$; $w = 6.91$, 18.09

(d) 12.06 m by 6.91 m, or 4.61 m by 18.09 m

6.2.8.42. An architect is planning to include a rectangular window topped by a semicircle in his plans for a new house, as shown in the figure. In order to admit enough light, the window should have an area of 120 square feet. The architect wants the rectangular portion of the window to be 2 feet wider than it is tall.



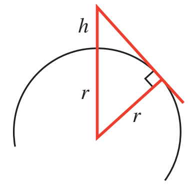
(a) Let x stand for the horizontal width of the window. Write expressions for the height of the rectangular portion and for the radius of the semicircular portion.

(b) Write an expression for the total area of the window.

(c) Write and solve an equation to find the width and overall height of the window.

6.2.8.43.

When you look down from a height, say a tall building or a mountain peak, your line of sight is tangent to the Earth at the horizon, as shown in the figure.



(a) Suppose you are standing on top of the Petronas Tower in Kuala Lumpur, 1483 feet high. How far can you see on a clear day? (You will need to use the Pythagorean theorem and the fact that the radius of the Earth is 3960 miles. Do not forget to convert the height of the Petronas Tower to miles.)

(b) How tall a building should you stand on in order to see 100 miles?

Answer.

(a) 47.2 mi

(b) 1.26 mi

6.2.8.44.

(a) If the radius of the Earth is 6370 kilometers, how far can you see from an airplane at an altitude of $10,000$ meters? (*Hint*: See Problem 43.)

(b) b. How high would the airplane have to be in order for you to see a distance of 10 kilometers?

For Problems 45-52, use the quadratic formula to solve each equation for the indicated variable.

6.2.8.45. $A = 2w^2 + 4lw, \quad \text{for } w$

Answer.
$w = \dfrac{-4l \pm \sqrt{16l^2 + 8A}}{4} = \dfrac{-2l \pm \sqrt{4l^2 + 2A}}{2}$

6.2.8.46. $A = \pi r^2 + \pi rs, \quad \text{for } r$

6.2.8.47. $h = 4t - 16t^2, \quad \text{for } t$

Answer.
$t = \dfrac{4 \pm \sqrt{16 + 64h}}{32} = \dfrac{1 \pm \sqrt{1 + 4h}}{8}$

6.2.8.48. $P = IE - RI^2, \quad \text{for } I$

6.2.8.49. $s = vt - \dfrac{1}{2}at^2, \quad \text{for } t$

Answer. $t = \dfrac{v \pm \sqrt{v^2 - 2as}}{a}$

6.2.8.50. $S = \dfrac{n^2 + n}{2}, \quad \text{for } n$

6.2.8.51. $3x^2 + xy + y^2 = 2, \quad \text{for } y$

Answer. $y = \dfrac{-x \pm \sqrt{8 - 11x^2}}{2}$

6.2.8.52. $y^2 - 3xy + x^2 = 3, \quad \text{for } x$

For Problems 53-60, solve for y in terms of x. Use whichever method of solution seems easiest.

6.2.8.53.
$x^2y - y^2 = 0$

Answer. $0, \ x^2$

6.2.8.54.
$x^2y^2 - y = 0$

6.2.8.55.
$(2y + 3x)^2 = 9$

Answer. $\dfrac{-3x \pm 3}{2}$

6.2.8.56.
$(3y - 2x)^2 = 4$

6.2.8.57.
$4x^2 - 9y^2 = 36$

Answer. $\dfrac{\pm\sqrt{4x^2-36}}{3} = \dfrac{\pm 2\sqrt{x^2-9}}{3}$

6.2.8.58.
$9x^2 + 4y^2 = 36$

6.2.8.59.
$4x^2 - 25y^2 = 0$

Answer. $\dfrac{\pm 2x}{5}$

6.2.8.60.
$(2x - 5y)^2 = 0$

For Problems 61-66, solve the formula for the indicated variable.

6.2.8.61. $V = \pi(r-3)^2 h,\quad$ for r

Answer. $3 \pm \sqrt{\dfrac{V}{\pi h}}$

6.2.8.62. $A = P(1+r)^2,\quad$ for P

6.2.8.63. $E = \dfrac{1}{2}mv^2 + mgh,\quad$ for v

Answer. $\pm\sqrt{\dfrac{2(E-mgh)}{m}}$

6.2.8.64. $h = \dfrac{1}{2}gt^2 + dl,\quad$ for t

6.2.8.65. $V = 2(s^2+t^2)w,\quad$ for t

Answer. $\pm\sqrt{\dfrac{V}{2w} - s^2}$

6.2.8.66. $V = \pi(r^2+R^2)h,\quad$ for R

6.2.8.67. What is the sum of the two solutions of the quadratic equation $ax^2 + bx + c = 0$?

Hint. The two solutions are given by the quadratic formula.

Answer. $\dfrac{-b}{2a}$

6.2.8.68. What is the product of the two solutions of the quadratic equation $ax^2 + bx + c = 0$?

Hint. Do *not* try to multiply the two solutions given by the quadratic formula! Think about the factored form of the equation

In Problems 69 and 70, we prove the quadratic formula.

6.2.8.69. Complete the square to find the solutions of the equation $x^2 + bx + c = 0$. (Your answers will be expressions in b and c.)

Answer. $\dfrac{-b \pm \sqrt{b^2-4c}}{2}$

6.2.8.70. Complete the square to find the solutions of the equation $ax^2 + bx + c = 0$. (Your answers will be expressions in a, b, and c.)

6.3 Graphing Parabolas

6.3.1 The Graph of $y = ax^2$

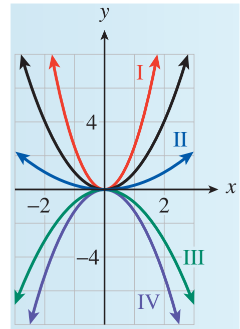
Checkpoint 6.3.2 Match each parabola in the figure at right with its equation. The basic parabola is shown in black.

a $y = -\dfrac{3}{4}x^2$

b $y = \dfrac{1}{4}x^2$

c $y = \dfrac{5}{2}x^2$

d $y = -\dfrac{5}{4}x^2$



Answer.

a III

b II

c I

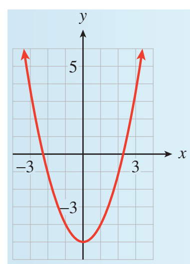
d IV

6.3.2 The Graph of $y = x^2 + c$

Checkpoint 6.3.4

a Find an equation for the parabola shown below.

b Give the x- and y-intercepts of the graph.



Answer.

a $y = x^2 - 5$

b $(-\sqrt{5}, 0), (\sqrt{5}, 0), (0, -5)$

6.3.3 The Graph of $y = ax^2 + bx$

Checkpoint 6.3.5

a Find the x-intercepts and the vertex of the parabola $y = 6x - x^2$.

b Verify your answers by graphing the function in the window

$$\begin{array}{ll} \text{Xmin} = -9.4 & \text{Xmax} = 9.4 \\ \text{Ymin} = -10 & \text{Ymax} = 10 \end{array}$$

Answer. x-intercepts: $(0, 0)$ and $(6, 0)$; vertex: $(3, 9)$

6.3.5 The Graph of $y = ax^2 + bx + c$

Checkpoint 6.3.8 Find the vertex of the graph of $f(x) = 3x^2 - 6x + 4$. Decide whether the vertex is a maximum point or a minimum point of the graph.

Answer. $(1, 1)$, minimum

6.3.6 Number of x-Intercepts

Checkpoint 6.3.11 Use the discriminant to discover how many x-intercepts the graph of each function has.

a $y = x^2 + 5x + 7$

b $y = -\dfrac{1}{2}x^2 + 4x - 8$

Answer.

a None

b One

6.3.7 Sketching a Parabola

Checkpoint 6.3.13

a Find the intercepts and the vertex of the graph of $f(x) = x^2 - 5x + 4$.

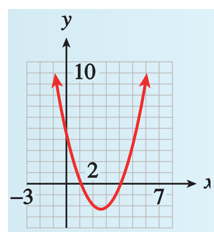
b Sketch the graph by hand.

c Use your calculator to verify your graph.

Answer.

a $(0, 4)$; $(1, 0)$, $(4, 0)$; vertex $\left(\dfrac{5}{2}, \dfrac{-9}{4}\right)$

b



6.3.9 Homework 6.3

For Problems 1–2, describe what each graph will look like compared to the basic parabola. Then sketch a graph by hand and label the coordinates of three points on the graph.

6.3.9.1.

(a) $y = 2x^2$

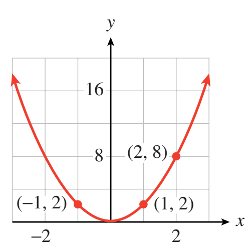
(b) $y = 2 + x^2$

(c) $y = (x + 2)^2$

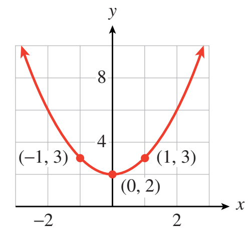
(d) $y = x^2 - 2$

Answer.

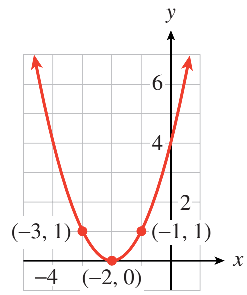
(a) The parabola opens up, twice as steep as the standard parabola.



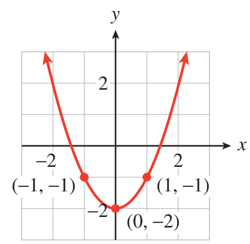
(b) The parabola is the standard parabola shifted 2 units up.



(c) The parabola is the standard parabola shifted 2 units left.



(d) The parabola is the standard parabola shifted 2 units down.



6.3.9.2.

(a) $y = -4x^2$

(b) $y = (x - 4)^2$

(c) $y = 4 - x^2$

(d) $y = x^2 - 4$

For problems 3–6, find the vertex and the x-intercepts (if there are any) of the graph. Then sketch the graph by hand.

6.3.9.3.

a $y = x^2 - 16$

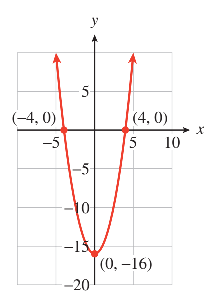
b $y = 16 - x^2$

c $y = 16x - x^2$

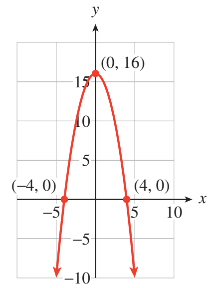
d $y = x^2 - 16x$

Answer.

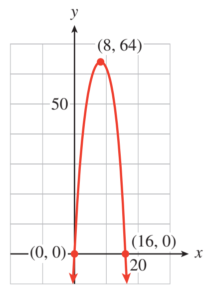
a Vertex $(0, -16)$; x-intercepts $(\pm 4, 0)$



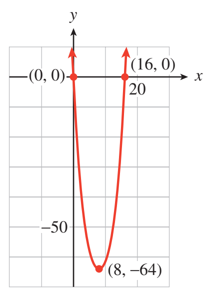
b Vertex $(0, 16)$; x-intercepts $(\pm 4, 0)$



c Vertex $(8, 64)$; x-intercepts $(0, 0)$ and $(16, 0)$



d Vertex $(8, -64)$; x-intercepts $(0, 0)$ and $(16, 0)$



6.3.9.4.

a $y = x^2 - 1$

b $y = 1 - x^2$

c $y = x^2 - x$

d $y = x - x^2$

6.3.9.5.

a $y = 3x^2 + 6x$

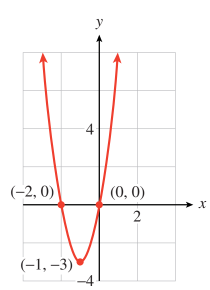
b $y = 3x^2 - 6x$

c $y = 3x^2 + 6$

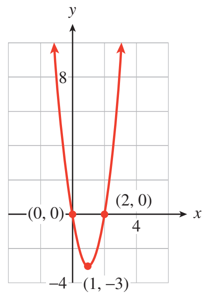
d $y = 3x^2 - 6$

Answer.

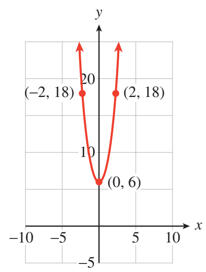
a Vertex $(1, -3)$; x-intercepts $(0, 0)$ and $(-2, 0)$



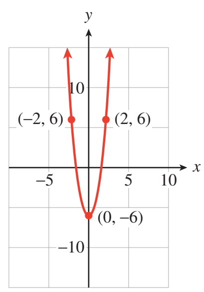
b Vertex $(1, -3)$; x-intercepts $(0, 0)$ and $(2, 0)$



c Vertex $(0, 6)$; no x-intercepts



d Vertex $(0, -6)$; x-intercepts $(\pm\sqrt{2}, 0)$



6.3.9.6.

a $y = 12x - 2x^2$

b $y = 12 - 2x^2$

c $y = 12 + 2x^2$

d $y = 12x + 2x^2$

6.3.9.7. Match each function with its graph. In each equation, $a > 0$.

(a) $y = x^2 + a$

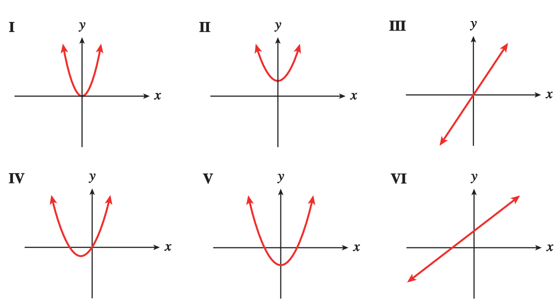
(b) $y = x^2 + ax$

(c) $y = ax^2$

(d) $y = ax$

(e) $y = x + a$

(f) $y = x^2 - a$



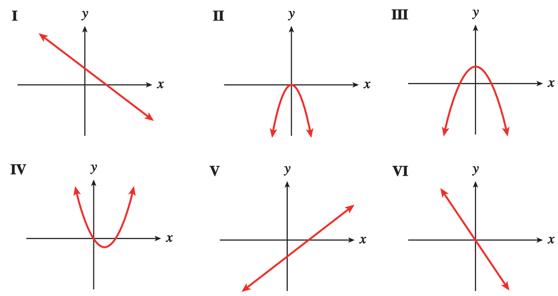
Answer.

(a) II (b) IV (c) I (d) III (e) VI (f) V

6.3.9.8. Match each function with its graph. In each equation, $b > 0$.

(a) $y = -bx$ (c) $y = b - x^2$ (e) $y = b - x$

(b) $y = -bx^2$ (d) $y = x - b$ (f) $y = x^2 - bx$



6.3.9.9. Commercial fishermen rely on a steady supply of fish in their area. To avoid overfishing, they adjust their harvest to the size of the population. The function

$$g(x) = 0.4x - 0.0001x^2$$

gives the annual rate of growth, in tons per year, of a fish population of biomass x tons.

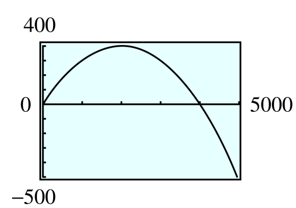
(a) Find the vertex of the graph. What does it tell us about the fish population?

(b) Sketch the graph for $0 \le x \le 5000$.

(c) For what values of x does the fish population decrease rather than increase? Suggest a reason why the population might decrease.

Answer.

(a) $(2000, 400)$; The largest annual increase in biomass, 400 tons, occurs when the biomass is 2000 tons.

(b) 

(c) $4000 < x \le 5000$; When there are too many fish, there will not be enough food to support all of them.

6.3.9.10. The annual increase, I, in the deer population in a national park depends on the size, x, of the population that year, according to the function

$$I = f(x) = 1.2x - 0.0002x^2$$

(a) Find the vertex of the graph. What does it tell us about the deer population?

(b) Sketch the graph for $0 \le x \le 7000$.

(c) For what values of x does the deer population decrease rather than increase? Suggest a reason why the population might decrease.

6.3.9.11. Many animals live in groups. A species of marmot found in Colorado lives in harems composed of a single adult male and several females with their young. The number of offspring each female can raise depends on the number of females in the harem. On average, if there are x females in the harem, each female can raise $y = 2 - 0.4x$ young marmots each year.

(a) Complete the table of values for the average number of offspring per female, and the total number of young marmots, A, produced by the entire harem in one year.

x	1	2	3	4	5
y					
A					

(b) Write a formula for A in terms of x.

(c) Graph A as a function of x.

(d) What is the maximum number of young marmots a harem can produce (on average)? What is the optimal number of female marmots per harem?

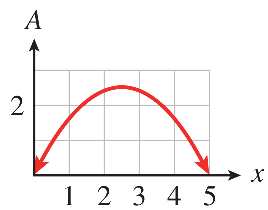
Answer.

(a)

x	1	2	3	4	5
y	1.6	1.2	0.8	0.4	0
A	1.6	2.4	2.4	1.6	0

(b) $A = x(2 - 0.4x)$ or $A = 2x - 0.4x^2$

(c)



(d) The maximum number of young marmots, on average, is 2.5; the optimal number of female marmots is 2.5.

6.3.9.12. Greenshield's model for traffic flow assumes that the average speed, u, of cars on a highway is a linear function of the traffic density, k, in vehicles per mile, given by

$$u = u_f\left(1 - \frac{k}{k_j}\right)$$

where u_f is the free-flow speed and k_j is the maximum density (the point when traffic jams). Then the traffic flow, q, in vehicles per hour, is given by $q = uk$.

(a) Write a formula for q as a function of k.

(b) If the free-flow speed is 70 mph and the maximum density is 240 vehicles per mile, graph q as a function of k.

(c) What value of k gives the maximum traffic flow? What is the average speed of vehicles at that density?

6.3.9.13. After touchdown, the distance the space shuttle travels is given by

$$d = vT + \frac{v^2}{2a}$$

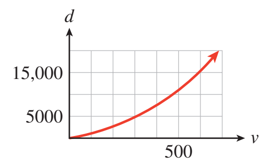
where v is the shuttle's velocity in ft/sec at touchdown, T is the pilot's reaction time before the brakes are applied, and a is the shuttle's deceleration.

(a) Graph $d = f(v)$ for $T = 0.5$ seconds and $a = 12$ ft/sec^2. Find the coordinates of the vertex and the horizontal intercepts. Explain their meaning, if any, in this context.

(b) The runway at Edwards Air Force base is 15,000 feet long. What is the maximum velocity the shuttle can have at touchdown and still stop on the runway?

Answer.

(a)



Vertex: $(-6, -1.5)$; Horizontal intercepts $(-12, 0)$ and $(0, 0)$. The point $(0, 0)$ means that no distance is required to stop a plane that is not moving.

(b) 594 ft/sec

6.3.9.14. When setting the pump pressure at the engine, firefighters must take into account the pressure loss due to friction inside the fire hose. For every 100 feet of hoseline, a hose of diameter 2.5 inches loses pressure according to the formula

$$L = \begin{cases} 2Q^2 + Q, & Q \geq 1 \\ 2Q^2 + \dfrac{1}{2}Q, & Q < 1 \end{cases}$$

where Q is the water flow in hundreds of gallos per minute. The friction loss, L, is measured in pounds per square inch (psi) (Source: www.hcc.hawaii.edu/~jkemmer)

(a) Graph $L = g(Q)$ on the domain $[0, 5]$.

(b) The firefighters have unrolled 600 feet of 2.5-inch-diameter hose, and they would like to deliver water at a rate of 200 gallons per minute, with nozzle pressure at 100 psi. They must add the friction loss to the nozzle pressure to calculate the engine pressure required. What should the engine pressure be?

For Problems 15–16, find the coordinates of the vertex. Decide whether the vertex is a maximum point or a minimum point on the graph and explain why.

6.3.9.15.

(a) $y = 2 + 3x - x^2$

(b) $y = \dfrac{1}{2}x^2 - \dfrac{2}{3}x + \dfrac{1}{3}$

(c) $y = 2.3 - 7.2x - 0.8x^2$

Answer.

(a) $\left(\dfrac{3}{2}, \dfrac{17}{4}\right)$, maximum

(b) $\left(\dfrac{2}{3}, \dfrac{1}{9}\right)$, minimum

(c) $(-4.5, 18.5)$, maximum

6.3.9.16.

(a) $y = 3 - 5x + x^2$

(b) $y = \dfrac{-3}{4}x^2 + \dfrac{1}{2}x - \dfrac{1}{4}$

(c) $y = -5.1 - 0.2x + 4.6x^2$

In Problems 17–26,

a Find the coordinates of the intercepts and the vertex.

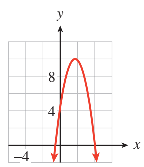
b Sketch the graph by hand.

c Use your calculator to verify your graph.

6.3.9.17. $y = -2x^2 + 7x + 4$

Answer.

(a) x-intercepts: $\left(\frac{-1}{2}, 0\right)$ and $(4, 0)$; y-intercept: $(0, 4)$; vertex: $\left(\frac{7}{4}, \frac{81}{8}\right)$

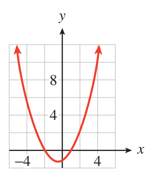
(b) 

6.3.9.18. $y = -3x^2 + 2x + 8$

6.3.9.19. $y = 0.6x^2 + 0.6x - 1.2$

Answer.

(a) x-intercepts: $(-2, 0)$ and $(1, 0)$; y-intercept: $(0, -1.2)$; vertex: $(-0.5, -1.35)$

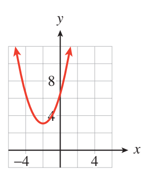
(b) 

6.3.9.20. $y = 0.5x^2 - 0.25x - 0.75$

6.3.9.21. $y = x^2 + 4x + 7$

Answer.

(a) No x-intercepts; y-intercept: $(0, 7)$; vertex: $(-2, 3)$

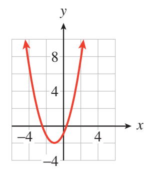
(b) 

6.3.9.22. $y = x^2 - 6x + 10$

6.3.9.23. $y = x^2 + 2x - 1$

Answer.

(a) x-intercepts: $\left(-1 \pm \sqrt{2}, 0\right)$;
y-intercept: $(0, -1)$; vertex: $(-1, -2)$

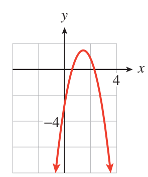
(b) 

6.3.9.24. $y = x^2 - 6x + 2$

6.3.9.25. $y = -2x^2 + 6x - 3$

Answer.

(a) x-intercepts: $\left(\dfrac{3 \pm \sqrt{3}}{2}, 0\right)$;
y-intercept: $(0, -3)$; vertex: $\left(\dfrac{3}{2}, \dfrac{3}{2}\right)$

(b) 

6.3.9.26. $y = -2x^2 - 8x - 5$

6.3.9.27.

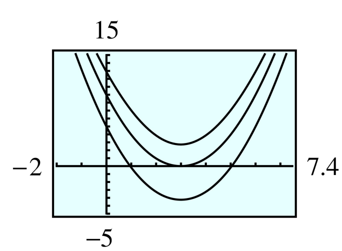
(a) Graph the three functions $f(x) = x^2 - 6x + 5$, $g(x) = x^2 - 6x + 9$, and $h(x) = x^2 - 6x + 12$ in the window

Xmin $= -2$	Xmax $= 7.4$
Ymin $= -5$	Ymax $= 15$

Use the **Trace** to locate the x-intercepts of each graph.

(b) Set $y = 0$ for each of the equations in part (a) and calculate the discriminant. What does the discriminant tell you about the solutions of the equation? How does your answer relate to the graphs in part (a)?

Answer.

(a) 

$f(x) = x^2 - 6x + 5$: x-intercepts $(1, 0)$ and $(5, 0)$; $g(x) = x^2 - 6x + 9$: x-intercept $(3, 0)$; $h(x) = x^2 - 6x + 12$: No x-intercept.

(b) $16, 0, -12$: $D = 16$ means that there are two rational x-intercepts, $D = 0$ means that there is exactly one x-intercept, $D = -12$ means that there is no x-intercept.

6.3.9.28.

(a) Graph the three functions $F(x) = 3 - 2x - x^2$, $G(x) = -1 - 2x - x^2$, and $H(x) = -4 - 2x - x^2$ in the window

Xmin $= -6.4$	Xmax $= 3$
Ymin $= -10$	Ymax $= 5$

Use the **Trace** to locate the x-intercepts of each graph.

(b) Set $y = 0$ for each of the equations in part (a) and calculate the discriminant. What does the discriminant tell you about the solutions of the equation? How does your answer relate to the graphs in part (a)?

For Problems 29–34, use the discriminant to determine the nature of the solutions of each equation.

6.3.9.29. $3x^2 + 26 = 17x$

Answer. Two complex solutions

6.3.9.30. $4x^2 + 23x = 19$

6.3.9.31. $16x^2 - 712x + 7921 = 0$

Answer. One repeated rational solution

6.3.9.32. $121x^2 + 1254x + 3249 = 0$

6.3.9.33. $65.2x = 13.2x^2 + 41.7$

Answer. Two distinct real solutions

6.3.9.34. $0.03x^2 = 0.05x - 0.12$

For problems 35–38, use the discriminant to decide if we can solve the equation by factoring.

6.3.9.35. $3x^2 - 7x + 6 = 0$

Answer. No

6.3.9.36. $6x^2 - 11x - 7 = 0$

6.3.9.37. $15x^2 - 52x - 32 = 0$

Answer. Yes

6.3.9.38. $17x^2 + 65x - 12 = 0$

For Problems 39–41,

a Given one zero of a quadratic equation with rational coefficients, find the other zero.

b Write a quadratic equation that has those zeros.

6.3.9.39. $2 + \sqrt{5}$

Answer.

(a) $2 - \sqrt{5}$

(b) $x^2 - 4x - 1 = 0$

6.3.9.40. $3 - 2i$

6.3.9.41. $4 - 3\sqrt{2}$

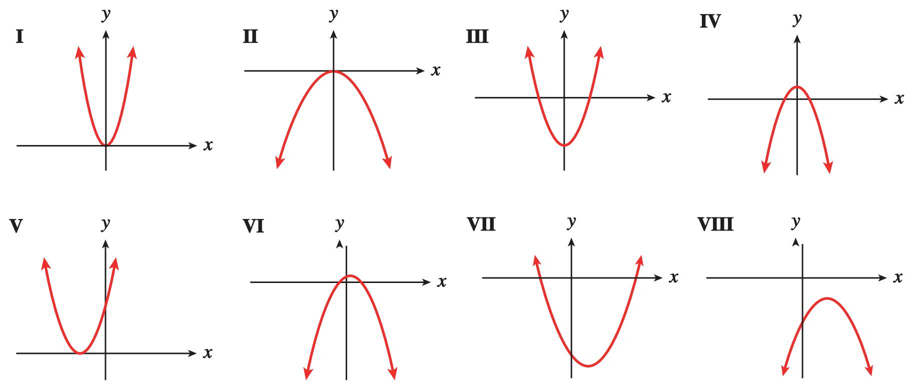
Answer.

(a) $4 + 3\sqrt{2}$

(b) $x^2 - 8x - 2 = 0$

6.3.9.42. If 5 is zero to a quadratic equation with rational coefficients, must -5 also be a solution?

For Problems 43 and 44, match each equation with one of the eight graphs shown.



6.3.9.43.

(a) $y = 1 - x^2$

(b) $y = (x + 2)^2$

(c) $y = 2x^2$

(d) $y = (x - 4)(x + 2)$

Answer.

(a) IV (b) V (c) I (d) VII

6.3.9.44.

(a) $y = -2 - (x - 2)^2$

(b) $y = x - x^2$

(c) $y = x^2 - 4$

(d) $y = -0.5x^2$

6.3.9.45.

(a) Write an equation for a parabola that has x-intercepts at $(2, 0)$ and $(-3, 0)$.

(b) Write an equation for another parabola that has the same x-intercepts.

Answer.

(a) $y = x^2 + x - 6;\;\; x = \dfrac{-1}{2}$

(b) $y = 2x^2 + 2x - 12;\;\; x = \dfrac{-1}{2}$

6.3.9.46.

(a) Write an equation for a parabola that opens upward and has x-intercepts at $(-1, 0)$ and $(4, 0)$. What is the equation of the parabola's axis of symmetry?

(b) Write an equation for a parabola that opens downward and has x-intercepts $(-1, 0)$ and $(4, 0)$. What is the equation of its axis of symmetry?

6.3.9.47.

(a) Graph the functions in the same window on your calculator:

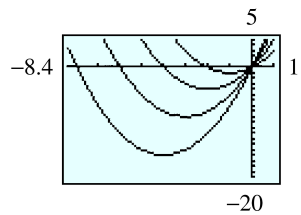
$$\begin{aligned} f(x) &= x^2 + 2x, \quad & g(x) &= x^2 + 4x, \\ h(x) &= x^2 + 6x, \quad & j(x) &= x^2 + 8x \end{aligned}$$

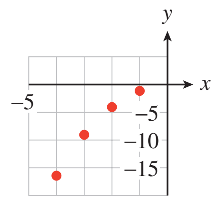
(b) Find the vertex of each graph in part (a) and plot the points.

(c) Find the equation of the curve in part (b).

(d) Show that the vertex of $y = x^2 + 2kx$ lies on the curve for any value of k.

Answer.

(a) 

(b) 

$(-1,-1),(-2,-4),(-3,-9),(-4,-16)$

(c) $y=-x^2$

(d) The vertex of $y=x^2+2kx$ is $(-k,-k^2)$

6.3.9.48.

(a) Graph the functions in the same window on your calculator:

$$F(x)=x-\frac{1}{2}x^2,\quad G(x)=3x-\frac{1}{2}x^2,$$
$$H(x)=5x-\frac{1}{2}x^2,\quad J(x)=7x-\frac{1}{2}x^2$$

(b) Find the vertex of each graph in part (a) and plot the points.

(c) Find the equation of the curve in part (b).

(d) Show that the vertex of $y=kx-\frac{1}{2}x^2$ lies on the curve for any value of k.

6.3.9.49. Because of air resistance, the path of a kicked soccer ball is not actually parabolic. However, both the horizontal and vertical coordinates of points on its trajectory can be approximated by quadratic functions. For a soccer ball kicked from the ground, these functions are

$$x=f(t)=12.8t-1.3t^2$$
$$y=g(t)=17.28t-4.8t^2$$

where x and y are given in meters and t is the number of seconds since the ball was kicked.

(a) Fill in the table.

t	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
x								
y								

(b) Plot the points (x,y) from your table and connect them with a smooth curve to represent the path of the ball.

(c) Use your graph to estimate the maximum height of the ball.

(d) Estimate the horizontal distance traveled by the ball before it strikes the ground

(e) Using the formula given for y, determine how long the ball is in the air.

(f) Use your answer from part (e) and the formula for x to find the horizontal distance traveled by the ball before it strikes the ground

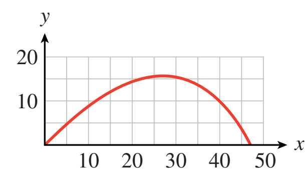
(g) Use the formula given for y to find the maximum height for the ball.

Answer.

(a)

t	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
x	0	6.075	11.5	16.275	20.4	23.875	26.7	28.875
y	0	7.44	12.48	15.12	15.36	13.2	8.64	1.68

(b)



(c) $y \approx 15.4$ m

(d) $x \approx 30$ m

(e) 3.6 sec

(f) $x \approx 29.2$ m

(g) $y \approx 15.55$ m

6.3.9.50. How far can you throw a baseball? The distance depends on the initial speed of the ball, v, and on the angle at which you throw it. For maximum range, you should throw the ball at 45°.

(a) If there were no air resistance, the height, x, of the ball t seconds after its release would be given in meters by the function

$$h = f(t) = \frac{vt}{\sqrt{2}} - \frac{gt^2}{2}$$

where g is the acceleration due to gravity. Find an expression for the total time the ball is in the air. (*Hint*: Set $h = 0$ and solve for t in terms of the other variables.)

(b) At time t, the ball has traveled a horizontal distance d given by

$$d = \frac{vt}{\sqrt{2}}$$

Find an expression for the range of the ball in terms of its velocity, v. (*Hint*: In part (a), you found an expression for t when $h = 0$. Use that value of t to calculate d when $h = 0$.)

(c) The fastest baseball pitch on record was 45 meters per second, or about 100 miles per hour. Use your formula from part (b) to calculate the theoretical range of such a pitch. The value of g is 9.8.

(d) The maximum distance a baseball has actually been thrown is 136 meters. Can you explain the discrepancy between this figure and your answer to part (c)?

6.4 Problem Solving

6.4.1 Maximum or Minimum Values

Checkpoint 6.4.3 The Metro Rail service sells $1200 - 80x$ tickets each day when it charges x dollars per ticket.

a Write an equation for the revenue, R, as a function of the price of a ticket.

b What ticket price will return the maximum revenue? What is the maximum revenue?

Answer.

a $R = 1200x - 80x^2$

b \$7.50, \$4500

6.4.2 The Vertex Form for a Parabola

Checkpoint 6.4.5

a Find the vertex of the graph of $y = 5 - \dfrac{1}{2}(x+2)^2$.

b Write the equation of the parabola in standard form.

Answer.

a $(-2, 5)$

b $y = -\frac{1}{2}x^2 - 2x + 3$

Checkpoint 6.4.7 Write the equation $y = 2x^2 + 12x + 13$ in vertex form, and find the vertex of its graph.

Hint.

1 Factor 2 from the variable terms.

2 Complete the square inside parentheses.

3 Subtract $2p^2$ outside parentheses.

4 Write the vertex form.

Answer. $y = 2(x+3)^2 - 5$; $(-3, -5)$

6.4.3 Graphing with the Vertex Form

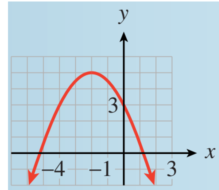
Checkpoint 6.4.9

a List the transformations of $y = x^2$ needed to graph $g(x) = 5 - \dfrac{1}{2}(x+2)^2$.

b Use transformations to sketch the graph.

Answer.

a Shift 2 units left, reflect about x-axis and compress by a factor of 2, shift 5 units up.

b 

6.4.4 Systems Involving Quadratic Equations

Checkpoint 6.4.11

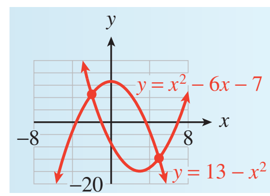
a Solve the system algebraically:

$$y = x^2 - 6x - 7$$
$$y = 13 - x^2$$

b Graph both equations, and show the solutions on the graph.

Answer.

a $(-2, 9), (5, -12)$

b 

6.4.6 Homework 6.4

6.4.6.1. The owner of a motel has 60 rooms to rent. She finds that if she charges \$0 per room per night, all the rooms will be rented. For every \$2 that she increases the price of a room, 3 rooms will stand vacant.

(a) Complete the table. The first two rows are filled in for you.

No. of price increases	Price of room	No. of rooms rented	Total revenue
0	20	60	1200
1	22	57	1254
2			
3			
4			
5			
6			
7			
8			
10			
12			
16			
20			

(b) Let x stand for the number of \$2 price increases the owner makes. Write algebraic expressions for the price of a room, the number of rooms that will be rented, and the total revenue earned at that price.

(c) Use your calculator to make a table of values for your algebraic expressions. Let Y_1 stand for the price of a room, Y_2 for the number of rooms rented, and Y_3 for the total revenue. Verify the values you calculated in part (a).

(d) Use your table to find a value of x that causes the total revenue to be zero.

(e) Use your graphing calculator to graph your formula for total revenue.

(f) What is the lowest price that the owner can charge for a room if she wants her revenue to exceed \$1296 per night? What is the highest price she can charge to obtain this revenue?

(g) What is the maximum revenue the owner can earn in one night? How much should she charge for a room to maximize her revenue? How many rooms will she rent at that price?

Answer.

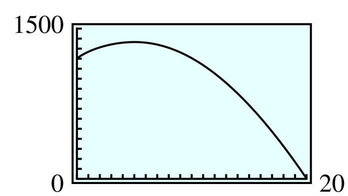
(a)

No. of price increases	Price of room	No. of rooms rented	Total revenue
0	20	60	1200
1	22	57	1254
2	24	54	1296
3	26	51	1326
4	28	48	1344
5	30	45	1350
6	32	42	1344
7	34	39	1326
8	36	36	1296
10	40	30	1200
12	44	24	1056
16	52	12	624
20	60	0	0

(b) Price of a room: $20+2x$; Rooms rented: $60-3x$; Revenue: $1200+60x-6x^2$

(c) 20

(d)



(e) \$24; \$36

(f) \$1350; \$30; 45 rooms

6.4.6.2. The owner of a video store sells 96 blank tapes per week if he charges \$6 per tape. For every \$0.50 he increases the price, he sells 4 fewer tapes per week.

(a) Complete the table. The first two rows are filled in for you.

No. of price increases	Price of tape	No. of tapes sold	Total revenue
0	6	96	576
1	6.50	92	598
2			
3			
4			
5			
6			
7			
8			
12			
16			
20			
24			

(b) Let x stand for the number of \$0.50 price increases the owner makes. Write algebraic expressions for the price of a tape, the number of tapes sold, and the total revenue.

(c) Use your calculator to make a table of values for your algebraic expressions. Let Y_1 stand for the price of a tape, Y_2 for the number of tapes sold, and Y_3 for the total revenue. Verify the values you calculated in part (a).

(d) Use your table to find a value of x that causes the total revenue to be zero.

(e) Use your graphing calculator to graph your formula for total revenue.

(f) How much should the owner charge for a tape in order to bring in \$630 per week from tapes? (You should have two answers.)

(g) What is the maximum revenue the owner can earn from tapes in one week? How much should he charge for a tape to maximize his revenue? How many tapes will he sell at that price?

6.4.6.3.

(a) Give the dimensions of two different rectangles with perimeter 60 meters. Compute the areas of the two rectangles.

(b) A rectangle has a perimeter of 60 meters. If the length of the rectangle is x meters, write an expression for its width.

(c) Write an expression for the area of the rectangle.

Answer.

(a) (For example) 10 m by 20 m with area 200 sq m; or 15 m by 15 m, area 225 sq m

(b) $30 - x$

(c) $30x - x^2$

6.4.6.4.

(a) Give the dimensions of two different rectangles with perimeter 48 inches.

Compute the areas of the two rectangles.

(b) A rectangle has a perimeter of 48 inches. If the width of the rectangle is w inches, write an expression for its length.

(c) Write an expression for the area of the rectangle.

For Problems 5–8,

(a) Find the maximum or minimum value algebraically.

(b) Obtain a good graph on your calculator and verify your answer. (Use the coordinates of the vertex and the vertical intercept to help you choose an appropriate window for the graph.)

6.4.6.5. Delbert launches a toy water rocket from ground level. Its distance above the ground t seconds after launch is given, in feet, by

$$d = 96t - 16t^2$$

When will the rocket reach its greatest height, and what will that height be?

Answer. 3 sec, 144 ft

6.4.6.6. Francine throws a wrench into the air from the bottom of a trench 12 feet deep. Its height t seconds later is given, in feet, by

$$h = -12 + 32t - 16t^2$$

When will the wrench reach its greatest height, and what will that height be?

6.4.6.7. The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing x baskets is

$$C = 0.01x^2 - 2x + 120$$

How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

Answer. 100 baskets, \$2000

6.4.6.8. A new electronics firm is considering marketing a line of telephones. The cost per phone for producing x telephones is

$$C = 0.001x^2 - 3x + 2270$$

How many telephones should the firm produce in order to minimize the cost per phone? What will the firm's total cost be at that production level?

6.4.6.9. As part of a collage for her art class, Sheila wants to enclose a rectangle with 100 inches of yarn.

(a) Let w represent the width of the rectangle, and write an expression for its length. Then write an expression that gives the area, A, of the rectangle as a function of its width, w.

(b) What is the area of the largest rectangle that Sheila can enclose with 100 inches of yarn?

Answer.

(a) Length: $50 - w$; Area: $50w - w^2$

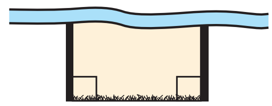
(b) 625 sq in

6.4.6.10. Gavin has rented space for a booth at the county fair. As part of his display, he wants to rope off a rectangular area with 80 yards of rope.

(a) Let w represent the width of the roped-off rectangle, and write an expression for its length. Then write an expression that gives the area, A, of the roped-off space as a function of its width, w.

(b) What is the largest area that Gavin can rope off? What will the dimensions of the rectangle be?

6.4.6.11. A farmer plans to fence a rectangular grazing area along a river with 300 yards of fence as shown in the figure.



(a) Write an expression that gives the area, A, of the grazing land as a function of the width, w, of the rectangle.

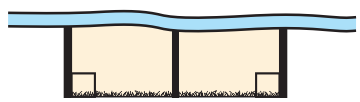
(b) What is the largest area the farmer can enclose?

Answer.

(a) $300w - 2w^2$

(b) $11,250$ sq yd

6.4.6.12. A breeder of horses wants to fence two rectangular grazing areas along a river with 600 meters of fence as shown in the figure.



(a) Write an expression that gives the area, A, of the grazing land as a function of the width, w, of the rectangles.

(b) What is the largest area the breeder can enclose?

6.4.6.13. A travel agent offers a group rate of \$2400 per person for a week in London if 16 people sign up for the tour. For each additional person who signs up, the price per person is reduced by \$100.

(a) Let x represent the number of additional people who sign up. Write expressions for the total number of people signed up, the price per person, and the total revenue.

(b) How many people must sign up for the tour in order for the travel agent to maximize her revenue?

Answer.

(a) Number of people: $16 + x$; Price per person: $2400 - 100x$; Total revenue: $38,400 + 800x - 100x^2$

(b) 20

6.4.6.14. An entrepreneur buys an apartment building with 40 units. The previous owner charged \$240 per month for a single apartment and on the average rented 32 apartments at that price. The entrepreneur discovers that for every \$20 he raises the price, another apartment stands vacant.

(a) Let x represent the number of \$20 price increases. Write expressions for the new price, the number of rented apartments, and the total revenue.

(b) What price should the entrepreneur charge for an apartment in order to maximize his revenue?

6.4.6.15. During a statistical survey, a public interest group obtains two estimates for the average monthly income of young adults aged 18 to 25. The first estimate is \$860 and the second estimate is \$918. To refine its estimate, the group will take a weighted average of these two figures:

$$I = 860a + 918(1-a) \quad \text{where} \quad 0 \le a \le 1$$

To get the best estimate, the group must choose a to minimize the function

$$V = 576a^2 + 5184(1-a)^2$$

(The numbers that appear in this expression reflect the **variance** of the data, which measures how closely the data cluster around the mean, or average.) Find the value of a that minimizes V, and use this value to get a refined estimate for the average income.

Answer. $a = 0.9$; $I = \$865.80$

6.4.6.16. The rate at which an antigen precipitates during an antigen-antibody reaction depends upon the amount of antigen present. For a fixed quantity of antibody, the time required for a particular antigen to precipitate is given in minutes by the function

$$t = 2w^2 - 20w + 54$$

where w is the quantity of antigen present, in grams. For what quantity of antigen will the reaction proceed most rapidly, and how long will the precipitation take?

For Problems 17-20, use transformations to graph the parabola. What is the vertex of each graph?

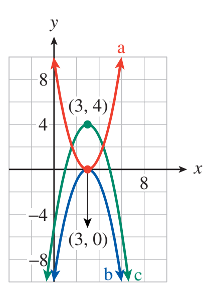
6.4.6.17.

(a) $y = (x-3)^2$

(b) $y = -(x-3)^2$

(c) $y = -(x-3)^2 + 4$

Answer.



6.4.6.18.

(a) $y = (x+1)^2$

(b) $y = 2(x+1)^2$

(c) $y = 2(x+1)^2 - 4$

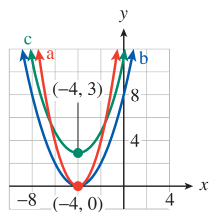
6.4.6.19.

(a) $y = (x+4)^2$

(b) $y = \frac{1}{2}(x+4)^2$

(c) $y = 3 + \frac{1}{2}(x+4)^2$

Answer.



6.4.6.20.

(a) $y = (x-2)^2$

(b) $y = -(x-2)^2$

(c) $y = -3 - (x-2)^2$

In Problems 21–24,

a Find the vertex of the parabola.

b Use transformations to sketch the graph.

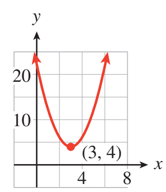
c Write the equation in standard form.

6.4.6.21. $y = 2(x-3)^2 + 4$

Answer.

(a) $(3, 4)$

(b)



(c) $y = 2x^2 - 12x + 22$

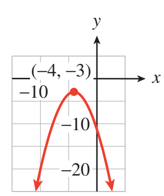
6.4.6.22. $y = -3(x+1)^2 - 2$

6.4.6.23. $y = -\frac{1}{2}(x+4)^2 - 3$

Answer.

(a) $(-4, -3)$

(b)



(c) $y = \frac{-1}{2}x^2 - 4x - 11$

6.4.6.24. $y = 4(x-2)^2 - 6$

For Problems 25–30,

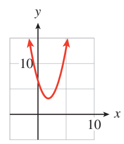
a Write each equation in the form $y = a(x-p)^2 + q$ by completing the square.

b Using horizontal and vertical translations, sketch the graph by hand.

6.4.6.25.
$y = x^2 - 4x + 7$

Answer.

(a) $y = (x - 2)^2 + 3$

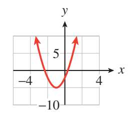
(b) 

6.4.6.26.
$y = x^2 - 2x - 1$

6.4.6.27.
$y = 3x^2 + 6x - 2$

Answer.

(a) $y = 3(x+1)^2 - 5$

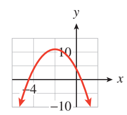
(b) 

6.4.6.28.
$y = \dfrac{1}{2}x^2 + 2x + 5$

6.4.6.29.
$y = -2x^2 - 8x + 3$

Answer.

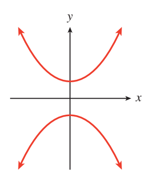
(a) $y = -2(x + 2)^2 + 11$

(b) 

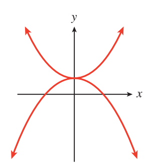
6.4.6.30.
$y = -x^2 + 5x + 2$

6.4.6.31. A system of two quadratic equations may have no solution, one solution, or two solutions. Sketch a system illustrating each case. In your sketches, one of the parabolas should open up, and the other down.

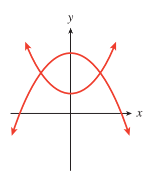
Answer. No solutions:



One solution:



Two solutions:



6.4.6.32. A system of two quadratic equations may have no solution, one solution, or two solutions. Sketch a system illustrating each case. In your sketches, both parabolas should open up.

For Problems 33–44, solve the system algebraically. Use your calculator to graph both equations and verify your solutions.

6.4.6.33. $y = x^2 - 4x + 7$
$y = 11 - x$

Answer. $(-1, 12), (4, 7)$

6.4.6.34. $y = x^2 + 6x + 4$
$y = 3x + 8$

6.4.6.35. $y = -x^2 - 2x + 7$
$y = 2x + 11$

Answer. $(-2, 7)$

6.4.6.36. $y = x^2 - 8x + 17$
$y + 4x = 13$

6.4.6.37. $y = x^2 + 8x + 8$
$3y + 2x = -36$

Answer. No solution

6.4.6.38. $y = -x^2 + 4x + 2$
$4y - 3x = 24$

6.4.6.39. $y = x^2 - 9$
$y = -2x^2 + 9x + 21$

Answer. $(-2, -5), (5, 16)$

6.4.6.40. $y = 4 - x^2$
$y = 3x^2 - 12x - 12$

6.4.6.41. $y = x^2 - 0.5x + 3.5$
$y = -x^2 + 3.5x + 1.5$

Answer. $(1, 4)$

6.4.6.42. $y = x^2 + 10x + 22$
$y = -0.5x^2 - 8x - 32$

6.4.6.43. $y = x^2 - 4x + 4$
$y = x^2 - 8x + 16$

Answer. $(3, 1)$

6.4.6.44. $y = 0.5x^2 + 3x + 5.5$
$y = 2x^2 + 12x + 4$

Problems 45–48 deal with wildlife management and sustainable yield.

6.4.6.45. In Problem 6.3.9.9 of Section 6.3, you graphed the annual growth rate of a population of fish,

$$R = f(x) = 0.4x - 0.0001x^2$$

where x is the current biomass of the population, in tons.

(a) Suppose that fishermen harvest 300 tons of fish each year. Sketch the graph of $H = 300$ on the same axes with your graph of y.

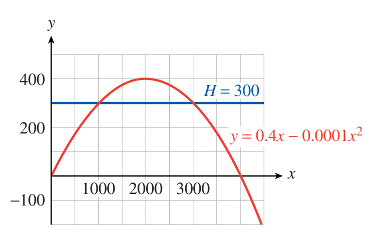
(b) If the biomass is currently 2500 tons and 300 tons are harvested, will the population be larger or smaller next year? By how much? What if the biomass is currently 3500 tons?

(c) What sizes of biomass will remain stable from year to year if 300 tons are harvested annually?

(d) If the biomass ever falls below 1000 tons, what will happen after several years of harvesting 300 tons annually?

Answer.

(a)



(b) Larger, by 75 tons. Smaller, by 125 tons.

(c) 1000 tons and 3000 tons

(d) The fish population will decrease each year until it is completely depleted.

6.4.6.46. In Problem 6.3.9.10 of Section 6.3, you graphed the annual increase, I, in the deer population in a national park,

$$I = g(x) = 1.2x - 0.0002x^2$$

where x is the current population.

(a) Suppose hunters are allowed to kill 1000 deer per year. Sketch the graph of $H = 1000$ on the same axes with a graph of y.

(b) What sizes of deer populations will remain stable from year to year if 1000 deer are hunted annually?

(c) Suppose 1600 deer are killed annually. What sizes of deer populations will remain stable?

(d) What is the largest annual harvest that still allows for a stable population? (This harvest is called the maximum sustainable yield.) What is the stable population?

(e) What eventually happens if the population falls below the stable value but hunting continues at the maximum sustainable yield?

6.4.6.47. The annual increase, N, in a bear population of size x is given by

$$N = F(x) = 0.0002x(2000 - x)$$

if the bears are not hunted. The number of bears killed each year by hunters is related to the bear population by the equation $K = 0.2x$. (Notice that in this model, hunting is adjusted to the size of the bear population.)

(a) Sketch the graphs of N and K on the same axes.

(b) When the bear population is 1200, which is greater, N or K? Will the population increase or decrease in the next year? By how many bears?

(c) When the bear population is 900, will the population increase or decrease in the next year? By how many bears?

(d) What sizes of bear population will remain stable after hunting?

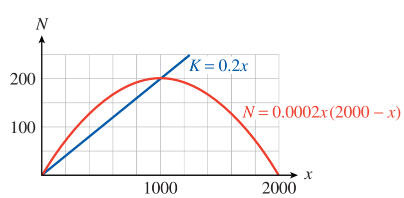
(e) What sizes of bear populations will increase despite hunting? What sizes of populations will decrease?

(f) Toward what size will the population tend over time?

(g) Suppose hunting limits are raised so that $K = 0.3x$. Toward what size will the population tend over time?

Answer.

(a)



(b) $K > N$. The population will decrease by 48 bears.

(c) The population will increase by 18 bears.

(d) 1000

(e) Populations between 0 and 1000 will increase; populations over 1000 will decrease.

(f) 1000 (unless the population is 0)

(g) 500 (unless the population is 0)

6.4.6.48. The annual increase in the biomass of a whale population is given in tons by

$$w = G(x) = 0.001x(1000 - x)$$

where x is the current population, also in tons.

(a) Sketch a graph of w for $0 \leq x \leq 1100$. What size biomass remains stable?

(b) Each year hunters are allowed to harvest a biomass given by $H = 0.6x$. Sketch H on the same graph with w. What is the stable biomass with hunting?

(c) What sizes of populations will increase despite hunting? What sizes will decrease?

(d) What size will the population approach over time? What biomass are hunters allowed to harvest for that size population?

(e) Find a value of k so that the graph of $H = kx$ will pass through the vertex of $w = 0.001x(1000 - x)$.

(f) For the value of k found in part (e), what size will the population approach over time? What biomass are hunters allowed to harvest for that size population?

(g) Explain why the whaling industry should prefer hunting quotas of kx rather than $0.6x$ for a long-term strategy, even though $0.6x > kx$ for any positive value of x.

For Problems 49–52,

a Find the break-even points by solving a system of equations.

b Graph the equations for Revenue and Cost in the same window and verify your solutions on the graph.

c Use the fact that

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

to find the value of x for which profit is maximum.

6.4.6.49. Writewell, Inc. makes fountain pens. It costs Writewell

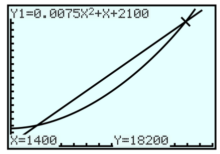
$$C = 0.0075x^2 + x + 2100$$

dollars to manufacture x pens, and the company receives $R = 13x$ dollars in revenue from the sale of the pens.

Answer.

(a) $(200,\ 2600), (1400,\ 18,200)$

(b) 20,000



0 1600

(c) $x = 800$

6.4.6.50. It costs The Sweetshop

$$C = 0.01x^2 + 1836$$

dollars to produce x pounds of chocolate creams. The company brings in $R = 12x$ dollars revenue from the sale of the chocolates.

6.4.6.51. It costs an appliance manufacturer

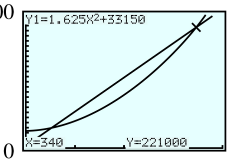
$$C = 1.625x^2 + 33,150$$

dollars to produce x front-loading washing machines, which will then bring in revenues of $R = 650x$ dollars.

Answer.

(a) $(60,\ 39,000), (340,\ 221,000)$

(b) 260,000



0 400

(c) $x = 200$

6.4.6.52. A company can produce x lawn mowers for a cost of

$$C = 0.125x^2 + 100,000$$

dollars. The sale of the lawn mowers will generate $R = 300x$ dollars in revenue.

Problems 53 and 54 prove that the vertical line $x = \dfrac{-b}{2a}$ is the axis of symmetry of the graph of $f(x) = ax^2 + bx + c$. A graph is **symmetric about the line** $x = h$ if the point $(h + d, v)$ lies on the graph whenever the point $(h - d, v)$ lies on the graph.

6.4.6.53.

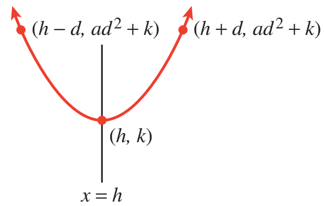
(a) Sketch a parabola $f(x) = a(x - h)^2 + k$ and the line $x = h$. We will show that the parabola is symmetric about the line $x = h$.

(b) Label a point on the parabola with x-coordinate $x = h + d$, where $d > 0$. What is the y-coordinate of that point?

(c) Label the point on the parabola with x-coordinate $x = h - d$. What is the y-coordinate of that point?

(d) Explain why your answers to parts (b) and (c) prove that the line $x = h$ is the axis of symmetry for the graph of $f(x) = a(x - h)^2 + k$.

Answer.

(a) 

(b) See graph and (c)

(c) $ad^2 + k$

(d) The two points on the parabola that are the same horizontal distance from the line $x = h$ the axis of symmetry have the same y-coordinate, so they are symmetric about that line.

6.4.6.54. To find the axis of symmetry for the graph of $g(x) = ax^2+bx+c$, we will use the results of Problem 51 and the technique of completing the square.

(a) Write the equation $y = ax^2 + bx + c$ in vertex form by completing the square. (Follow the steps in Example 6.4.6.)

(b) Your answer to part (a) has the form $y = a(x - h)^2 + k$. What is your value of h? What is your value of k?

(c) What is the axis of symmetry for the parabola $g(x) = ax^2 + bx + c$?

6.5 Chapter Summary and Review

6.5.2 Chapter 6 Review Problems

For Problems 1-6, solve by factoring.

6.5.2.1. $x^2 + x = 4 - (x + 2)^2$

Answer. $0,\ \dfrac{-5}{2}$

6.5.2.2. $(n - 3)(n + 2) = 6$

6.5.2.3. $x(3x + 2) = (x + 2)^2$

Answer. $-1,\ 2$

6.5.2.4. $6y = (y + 1)^2 + 3$

6.5.2.5. $4x - (x + 1)(x + 2) = -8$

Answer. $-2,\ 3$

6.5.2.6. $3(x + 2)^2 = 15 + 12x$

For Problems 7-8, write a quadratic equation with integer coefficients and with the given solutions.

6.5.2.7. $\dfrac{-3}{4}$ and 8

Answer. $4x^2 - 29x - 24 = 0$

6.5.2.8. $\dfrac{5}{3}$ and $\dfrac{5}{3}$

For Problems 9-10, graph the equation using the **ZDecimal** setting. Locate the x-intercepts and use them to write the quadratic expression in factored form.

6.5.2.9. $y = x^2 - 0.6x - 7.2$

Answer. $y = (x - 3)(x + 2.4)$

6.5.2.10. $y = -x^2 + 0.7x + 2.6$

For Problems 11-14, use a substitution to solve.

6.5.2.11. $2^{2p} - 6 \cdot 2^p + 8 = 0$

Answer. $1,\ 2$

6.5.2.12. $3^{2r} - 6 \cdot 3^r + 5 = 0$

6.5.2.13. $\left(\frac{1}{b}\right)^2 - 3\left(\frac{1}{b}\right) - 4 = 0$

Answer. $-1,\ \frac{1}{4}$

6.5.2.14. $\left(\frac{1}{q}\right)^2 + \frac{1}{q} - 2 = 0$

For problems 15-18, solve by completing the square.

6.5.2.15. $x^2 - 4x - 6 = 0$

Answer. $2 \pm \sqrt{10}$

6.5.2.16. $x^2 + 3x = 3$

6.5.2.17. $2x^2 + 3 = 6x$

Answer. $\frac{3 \pm \sqrt{3}}{2}$

6.5.2.18. $3x^2 = 2x + 3$

For Problems 19-22, solve by using the quadratic formula.

6.5.2.19. $\frac{1}{2}x^2 + 1 = \frac{3}{2}x$

Answer. $1,\ 2$

6.5.2.20. $x^2 - 3x + 1 = 0$

6.5.2.21. $x^2 - 4x + 2 = 0$

Answer. $2 \pm \sqrt{2}$

6.5.2.22. $2x^2 + 2x = 3$

For Problems 23-26, solve the formula for the indicated variable.

6.5.2.23. $K = \frac{1}{2}mv^2$, for v

Answer. $\pm\sqrt{\frac{2K}{m}}$

6.5.2.24. $a^2 + b^2 = c^2$, for b

6.5.2.25. $h = 6t - 3t^2$, for t

Answer. $\frac{3 \pm \sqrt{9 - 3h}}{3}$

6.5.2.26. $D = \frac{n^2 - 3n}{2}$, for n

6.5.2.27. In a tennis tournament among n competitors, $\frac{n(n-1)}{2}$ matches must be played. If the organizers can schedule 36 matches, how many players should they invite?

Answer. 9

6.5.2.28. The formula $S = \frac{n(n+1)}{2}$ gives the sum of the first n positive integers. How many consecutive integers must be added to make a sum of 91?

6.5.2.29. Irene wants to enclose two adjacent chicken coops of equal size against the henhouse wall. She has 66 feet of chicken wire fencing and would like the total area of the two coops to be 360 square feet. What should the dimensions of the chicken coops be?

Answer. 10 ft by 18 ft or 12 ft by 15 ft

6.5.2.30. The base of an isosceles triangle is one inch shorter than the equal sides, and the altitude of the triangle is 2 inches shorter than the equal sides. What is the length of the equal sides?

6.5.2.31. A car traveling at 50 feet per second (about 34 miles per hour) can stop in 2.5 seconds after applying the brakes hard. The distance the car travels, in feet, t seconds after applying the brakes is $d = 50t - 10t^2$. How long does it take the car to travel 40 feet?

Answer. 1 sec

6.5.2.32. You have 300 feet of wire fence to mark off a rectangular Christmas tree lot with a center divider, using a brick wall as one side of the lot. If you would like to enclose a total area of 7500 square feet, what should be the dimensions of the lot?

6.5.2.33. The height, h, of an object t seconds after being thrown from ground level is given by

$$h = v_0 t - \frac{1}{2} g t^2$$

where v_0 is its starting velocity and g is a constant that depends on gravity. On the Moon, the value of g is approximately 5.6. Suppose you hit a golf ball on the Moon with an upward velocity of 100 feet per second.

(a) Write an equation for the height of the golf ball t seconds after you hit it.

(b) Graph your equation in the window

Xmin = 0	Xmax = 47
Ymin = 0	Ymax = 1000

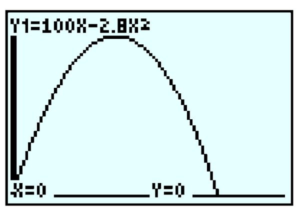
(c) Use the **Trace** to estimate the maximum height the golf ball reaches.

(d) Use your equation to calculate when the golf ball will reach a height of 880 feet.

Answer.

(a) $h = 100t - 2.8t^2$

(b)



(c) 893 ft

(d) $15\frac{5}{7}$ sec on the way up and 20 sec on the way down

6.5.2.34. An acrobat is catapulted into the air from a springboard at ground level. Her height, h, in meters is given by the formula

$$h = -4.9t^2 + 14.7t$$

where t is the time in seconds from launch. Use your calculator to graph the acrobat's height versus time. Use the window

Xmin = 0	Xmax = 4.7
Ymin = 0	Ymax = 12

(a) Use the **Trace** to find the coordinates of the highest point on the graph. When does the acrobat reach her maximum height, and what is that height?

(b) Use the formula to find the height of the acrobat after 2.4 seconds.

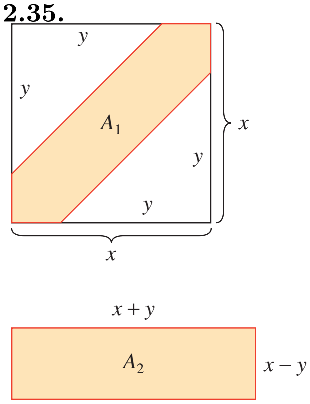
(c) Use the **Trace** to verify your answer to part (b). Find another time when the acrobat is at the same height.

(d) Use the formula to find two times when the acrobat is at a height of 6.125 meters. Verify your answers on the graph.

(e) What are the coordinates of the horizontal intercepts of your graph? What do these points have to do with the acrobat?

For problems 35-36, show that the shaded areas are equal.

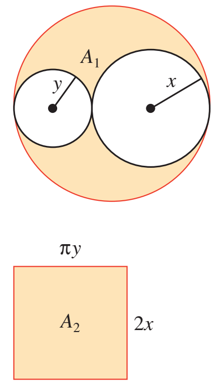
6.5.2.35.



Answer. A_1 is the area of a square minus the area of two triangles:

$$x^2 - 2\left(\frac{1}{2}y \cdot y\right) = x^2 - y^2$$

6.5.2.36.



For problems 37-46,

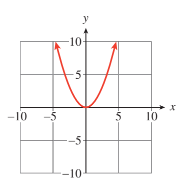
a Find the coordinates of the vertex and the intercepts.

b Sketch the graph.

6.5.2.37. $y = \dfrac{1}{2}x^2$

Answer.

(a) Vertex and intercepts are all $(0, 0)$.

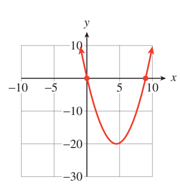
(b) 

6.5.2.38. $y = x^2 - 4$

6.5.2.39. $y = x^2 - 9x$

Answer.

(a) Vertex $(\dfrac{9}{2}, \dfrac{-81}{4})$; x-intercepts $(9, 0)$ and $(0, 0)$; y-intercept $(0, 0)$

(b) 

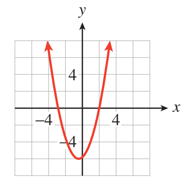
6.5.2.40. $y = -2x^2 - 4x$

6.5.2.41. $y = x^2 + x - 6$

Answer.

(a) Vertex $(\dfrac{-1}{2}, \dfrac{-25}{4})$; x-intercepts $(-3, 0)$ and $(2, 0)$; y-intercept $(0, -6)$

(b)



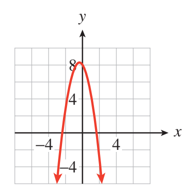
6.5.2.42. $y = x^2 - 3x + 4$

6.5.2.43. $y = 8 - x - 2x^2$

Answer.

(a) Vertex $(\dfrac{-1}{4}, \dfrac{65}{8})$; x-intercepts $\left(\dfrac{-1 \pm \sqrt{65}}{4}, 0\right)$; y-intercept $(0, 8)$

(b)



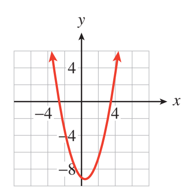
6.5.2.44. $y = -2x^2 + x - 4$

6.5.2.45. $y = x^2 - x - 9$

Answer.

(a) Vertex $(\dfrac{1}{2}, \dfrac{-37}{4})$; x-intercepts $\left(\dfrac{1 \pm \sqrt{37}}{2}, 0\right)$; y-intercept $(0, -9)$

(b)



6.5.2.46. $y = -x^2 + 2x + 4$

For problems 47-48, use the discriminant to determine how many x-intercepts the graph has.

6.5.2.47. $y = -2x^2 + 5x - 1$

Answer. Two

6.5.2.48. $y = -12 - 3x + 4x^2$

For Problems 49-52, use the discriminant to determine the nature of the solution of each equation.

6.5.2.49. $4x^2 - 12x + 9 = 0$

Answer. One rational solution

6.5.2.50. $2t^2 + 6t + 5 = 0$

6.5.2.51. $2y^2 = 3y - 4$

Answer. No real solutions

6.5.2.52. $\dfrac{x^2}{4} = x + \dfrac{5}{4}$

6.5.2.53. The total profit Kiyoshi makes from producing and selling x floral arrangements is

$$P(x) = -0.4x^2 + 36x - 400$$

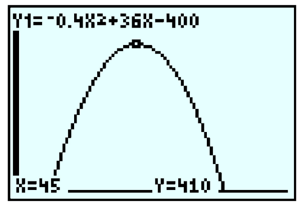
(a) How many floral arrangements should Kiyoshi produce and sell to maximize his profit? What is his maximum profit?

(b) Verify your answers on a graph.

Answer.

(a) 45; \$410

(b)



6.5.2.54. Lightning does about one billion dollars damage annually in the United States and kills 85 people. To study lightning, meteorologists fire small rockets at passing thunderclouds to induce lightning bolts. The rocket trails a thin copper wire that is vaporized by the lightning, leaving a plasma channel that carries the current to the grounding point. The rocket boosts the wire to a height of 250 meters, and t seconds later, its height is given in meters by

$$h(t) = -4.9t^2 + 32t + 250.$$

(a) When does the rocket reach its maximum height? What is the maximum height?

(b) Verify your answers on a graph.

6.5.2.55. A beekeeper has beehives distributed over 60 square miles of pastureland. When she places 4 hives per square mile, each hive produces about 32 pints of honey per year. For each additional hive per square mile, honey production drops by 4 pints per hive.

(a) Write a function for the total production of honey, in pints, in terms of the number of additional hives per square mile.

(b) How many additional hives per square mile should the beekeeper install in order to maximize honey production?

Answer.

(a) $y = 60(4 + x)(32 - 4x)$ (b) 2

6.5.2.56. A small company manufactures radios. When it charges \$20 for a radio, it sells 500 radios per month. For each dollar the price is increased, 10 fewer radios are sold per month.

(a) Write a function for the monthly revenue in terms of the price increase over \$20.

(b) What should the company charge for a radio in order to maximize its monthly revenue?

For Problems 57–60,

a Find all values of x for which $f(x) = 0$.

b Find all values of x for which $g(x) = 0$.

c Find all values of x for which $f(x) = g(x)$.

d Graph each pair of functions in the same window, then sketch the graph on paper. Illustrate your answers to (a)–(c) as points on the graph.

6.5.2.57. $f(x) = 2x^2 + 3x, \;\; g(x) = 5 - 6x$

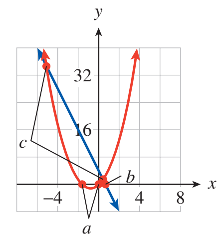
Answer.

(a) $0, \;\dfrac{-3}{2}$

(b) $\dfrac{5}{6}$

(c) $-5, \;\dfrac{1}{2}$

(d)



6.5.2.58. $f(x) = 3x^2 - 6x, \;\; g(x) = 8 + 4x$

6.5.2.59. $f(x) = 2x^2 - 2x, \;\; g(x) = x^2 + 3$

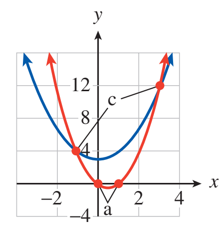
Answer.

(a) $0, \;1$

(b) None

(c) $-1, \;3$

(d)

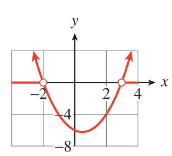


6.5.2.60. $f(x) = x^2 + 4x + 6, \;\; g(x) = 4 - x^2$

For Problems 61-66, solve the inequality algebraically, and give your answers in interval notation. Verify your solutions by graphing.

6.5.2.61. $(x - 3)(x + 2) > 0$

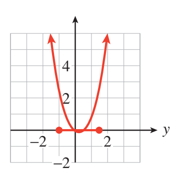
Answer. $(-\infty, -2) \cup (3, \infty)$



6.5.2.62. $y^2 - y - 12 \le 0$

6.5.2.63. $2y^2 - y \le 3$

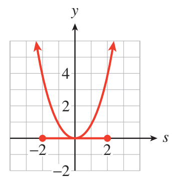
Answer. $\left[-1, \dfrac{3}{2}\right]$



6.5.2.64. $3z^2 - 5z > 2$

6.5.2.65. $s^2 \leq 4$

Answer. $[-2, 2]$



6.5.2.66. $4t^2 > 12$

6.5.2.67. The Sub Station sells $220 - \dfrac{1}{4}p$ submarine sandwiches at lunchtime if it sells them at p cents each.

(a) Write a function for the Sub Station's daily revenue in terms of p.

(b) What range of prices can the Sub Station charge if it wants to keep its daily revenue from subs over $480? (Remember to convert $480 to cents.)

Answer.

(a) $R = p\left(220 - \dfrac{1}{4}p\right)$

(b) Between $4.00 and $4.80

6.5.2.68. When it charges p dollars for an electric screwdriver, Handy Hardware will sell $30 - \dfrac{1}{2}p$ screwdrivers per month.

(a) Write a function in terms of p for Handy Hardware's monthly revenue from electric screwdrivers.

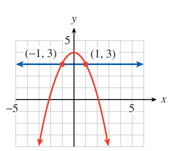
(b) How much should Handy charge per screwdriver if it wants the monthly revenue from the screwdrivers to be over $400?

For Problems 69-76, solve the system algebraically, and verify your solution with a graph.

6.5.2.69. $y + x^2 = 4$

$y = 3$

Answer. $(1, 3)$, $(-1, 3)$



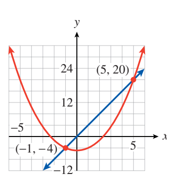
6.5.2.70. $y = 3 - x^2$

$5x + y = 7$

6.5.2.71. $y = x^2 - 5$

$y = 4x$

Answer. $(-1, -4)$, $(5, 20)$



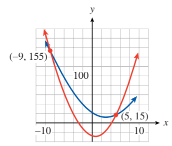
6.5.2.72. $y = x^2 - 2x + 1$

$y = 3 - x$

6.5.2.73. $y = x^2 - 6x + 20$

$y = 2x^2 - 2x - 25$

Answer. $(-9, 155),\ (5, 15)$



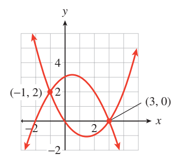
6.5.2.74. $y = x^2 - 5x - 28$

$y = -x^2 + 4x + 28$

6.5.2.75. $y = \frac{1}{2}x^2 - \frac{3}{2}x$

$y = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$

Answer. $(-1, 2),\ (3, 0)$



6.5.2.76. $y = 2x^2 + 5x - 3$

$y = x^2 + 4x - 1$

6.5.2.77. Find values of a, b, and c so that the graph of the parabola $y = ax^2 + bx + c$ contains the points $(-1, -4)$, $(0, -6)$, and $(4, 6)$.

Answer. $a = 1,\ b = -1,\ c = -6$

6.5.2.78.

(a) Find values of a, b, and c so that the graph of the parabola $y = ax^2+bx+c$ contains the points $(0, -2)$, $(-6, 1)$, and $(4, 6)$.

(b) Plot the data points and sketch the graph on the grid.

6.5.2.79. Find a parabola that fits the following data points.

x	-8	-4	2	4
y	10	18	0	-14

Answer. $p(x) = \frac{-1}{2}x^2 - 4x + 10$

6.5.2.80. Find a parabola that fits the following data points.

x	-3	0	2	4
y	-46	8	-6	-60

6.5.2.81. Find the equation for a parabola that has a vertex of $(15, -6)$ and passes through the point $(3, 22.8)$.

Answer. $y = 0.2(x - 15)^2 - 6$

6.5.2.82. Find the equation for a parabola that has a vertex of $(-3, -8)$ and passes through the point $(6, 12.25)$.

For Problems 83–86,

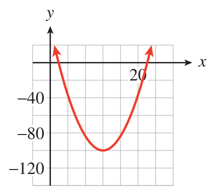
a Write the equation in vertex form.

b Use transformations to sketch the graph.

6.5.2.83. $f(x) = x^2 - 24x + 44$

Answer.

(a) $f(x) = (x - 12)^2 - 100$

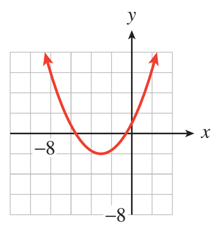
(b) 

6.5.2.84. $g(x) = x^2 + 30x + 300$

6.5.2.85. $y = \dfrac{1}{3}x^2 + 2x + 1$

Answer.

(a) $y = \dfrac{1}{3}(x + 3)^2 - 2$

(b) 

6.5.2.86. $y = -2x^2 + 4x + 3$

6.5.2.87. The height of a cannonball was observed at 0.2-second intervals after the cannon was fired, and the data were recorded in the table.

Time (seconds)	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Height (meters)	10.2	19.2	27.8	35.9	43.7	51.1	58.1	64.7	71.0	76.8

(a) Find the equation of the least-squares regression line for height in terms of time.

(b) Use the linear regression equation to predict the height of the cannonball at 3 seconds and at 4 seconds after it was fired.

(c) Make a scatterplot of the data and draw the regression line on the same axes.

(d) Find the quadratic regression equation for height in terms of time.

(e) Use the quadratic regression equation to predict the height of the cannonball at 3 seconds and at 4 seconds after it was fired.

(f) Draw the quadratic regression curve on the graph from part (c).

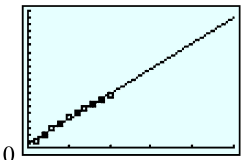
(g) Which model is more appropriate for the height of the cannonball, linear or quadratic? Why?

Answer.

(a) $h = 36.98t + 5.17$

(b) 116.1 m, 153.1 m

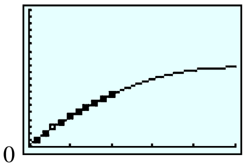
(c)



(d) $h = -4.858t^2 + 47.67t + 0.89$

(e) 100.2 m, 113.9 m

(f)



(g) Quadratic: Gravity will slow the cannonball, giving the graph a concave down shape.

6.5.2.88. Max took a sequence of photographs of an explosion spaced at equal time intervals. From the photographs, he was able to estimate the height and vertical velocity of some debris from the explosion, as shown in the table. (Negative velocities indicate that the debris is falling back to Earth.)

Velocity (meters/second)	67	47	27	8	-12	-31
Height (meters)	8	122	196	232	228	185

(a) Enter the data into your calculator and create a scatterplot. Fit a quadratic regression equation to the data, then graph the equation on the scatterplot.

(b) Use your regression equation to find the vertex of the parabola. What do the coordinates represent, in terms of the problem? What should the velocity of the debris be at the maximum height of the debris?

6.6 Projects for Chapter 6

Project 6.6.1 Optimum Feeding Rate. Starlings often feed in flocks, and their rate of feeding depends on the size of the flock. If the flock is too small, the birds are nervous and spend a lot of time watching for predators. If the flock is too large, the birds become overcrowded and fight each other, which interferes with feeding. Here are some data gathered at a feeding station. The data show the number of starlings in the flock and the total number of pecks per minute recorded at the station while the flock was feeding. (Source: Chapman & Reiss, 1992)

Number of starlings	Pecks per minute	Pecks per starling per minute
1	9	
2	26	
3	48	
4	80	
5	120	
6	156	
7	175	
8	152	
9	117	
10	180	
12	132	

a For each flock size, calculate the number of pecks per starling per minute. For purposes of efficient feeding, what flock size appears to be optimum? How many pecks per minute would each starling make in a flock of optimal size?

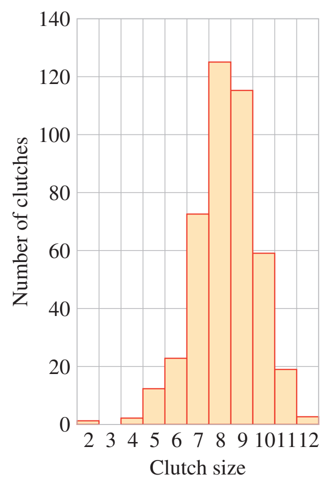
b Plot the number of pecks per starling per minute against flock size. Do the data points appear to lie on (or near) a parabola?

c The quadratic regression equation for the data is $y = -0.45x^2+5.8x+3.9$. Graph this parabola on the same axes with the data points.

d What are the optimum flock size and the maximum number of pecks per starling per minute predicted by the regression equation?

Project 6.6.2 Optimum clutch size.

Biologists conducted a four-year study of the nesting habits of the species **Parus major** in an area of England called Wytham Woods. The bar graph shows the clutch size (the number of eggs) in 433 nests. (Source: Perrins and Moss, 1975)



a Which clutch size was observed most frequently? Fill in the table, showing the total number of eggs produced in each clutch size.

Clutch size	2	3	4	5	6	7	8	9	10	11	12
Number of clutches	1	0	2	12	23	73	126	116	59	19	3
Number of eggs											

b The average weight of the nestlings declines as the size of the brood increases, and the survival of individual nestlings is linked to their weight. A hypothetical (and simplified) model of this phenomenon is described by the table below. Calculate the number of surviving nestlings for each clutch size. Which clutch size produces the largest average number of survivors?

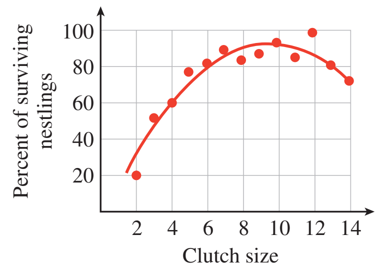
Clutch size	1	2	3	4	5	6	7	8	9	10
Percent survival	100	90	80	70	60	50	40	30	20	10
Number of survivors										

c

The figure shows the number of survivors for each clutch size in Wytham Woods, along with the curve of best fit. The equation for the curve is

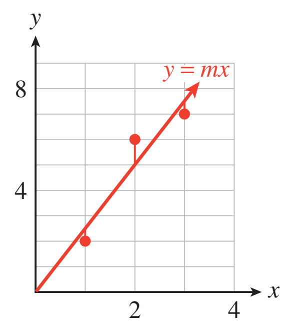
$$y = -0.0105x^2 + 0.2x - 0.035.$$

Find the optimal clutch size for maximizing the number of surviving nestlings. How does this optimum clutch size compare with the most frequently observed clutch size in part (a)?



Project 6.6.3 Line of best fit.

In this project, we minimize a quadratic expression to find the line of best fit. The figure shows a set of three data points and a line of best fit. For this example, the regression line passes through the origin, so its equation is $y = mx$ for some positive value of m. How shall we choose m to give the best fit for the data? We want the data points to lie as close to the line as possible. One way to achieve this is to minimize the sum of the squares of the vertical distances shown in the figure.



a The data points are $(1, 2)$, $(2, 6)$, and $(3, 7)$. Verify that the sum S we want to minimize is

$$\begin{aligned} S &= (2 - m)^2 + (6 - 2m)^2 + (7 - 3m)^2 \\ &= 14m^2 - 70m + 89 \end{aligned}$$

b Graph the formula for S in the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 9.4 \\ \text{Ymin} = 0 & \text{Ymax} = 100 \end{array}$$

c Find the vertex of the graph of S.

d Use the value of m to write the equation of the regression line $y = mx$.

e Graph the three data points and your regression line on the same axes.

Project 6.6.4 Quadratic growth rate.

The figure shows the typical weight of two species of birds each day after hatching. (Source: Perrins, 1979)

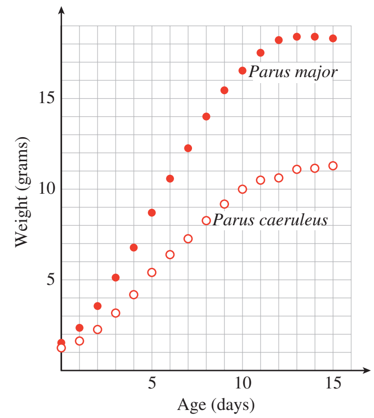


Figure 6.6.1

a Describe the rate of growth for each species over the first 15 days of life. How are the growth rates for the two species similar, and how are they different?

b Complete the tables showing the weight and the daily rate of growth for each species.

Parus major

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weight															
Growth rate															

Parus caeruleus

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weight															
Growth rate															

c Plot the rate of growth against weight in grams for each species. What type of curve does the growth rate graph appear to be?

d For each species, at what weight did the maximum growth rate occur? Locate the corresponding point on each original curve in Figure 6.6.1.

Project 6.6.5 Parus major growth rate.

a Find a quadratic regression equation for the growth rate of **Parus major** in terms of its weight using the data from Project 6.6.4.

b Make a scatterplot of the data and draw the regression curve on the same axes.

c Find the vertex of the graph of the regression equation. How does this estimate for the maximum growth rate compare with your estimate in Project 6.6.4?

Project 6.6.6 Parus caeruleus growth rate.

a Find a quadratic regression equation for the growth rate of **Parus caeruleus** in terms of its weight using the data from Project 6.6.4.

b Make a scatterplot of the data and draw the regression curve on the same axes.

c Find the vertex of the graph of the regression equation. How does this estimate for the maximum growth rate compare with your estimate in Project 6.6.4?

7 Polynomial and Rational Functions

7.1 Polynomial Functions

7.1.1 Products of Polynomials

Checkpoint 7.1.2 Multiply $(y+2)(y^2-2y+3)$.
Answer. y^3-y+6

Checkpoint 7.1.4 Find the coefficient of the fourth-degree term of the product of $f(x)=2x^6+2x^4-x^3+5x^2+1$ and $g(x)=x^5-3x^4+2x^3+x^2-4x-2$.
Answer. 2

7.1.2 Special Products

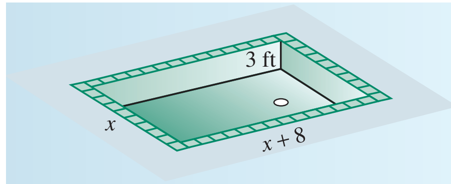
Checkpoint 7.1.6 Write $\left(5+x^2\right)^3$ as a polynomial.
Answer. $125+75x^2+15x^4+x^6$

7.1.3 Factoring Cubics

Checkpoint 7.1.8 Factor $125n^3-p^3$
Answer. $(5n-p)(25n^2+5np+p^2)$

7.1.4 Modeling with Polynomials

Checkpoint 7.1.10 An empty reflecting pool is 3 feet deep. It is 8 feet longer than it is wide, as illustrated below.



a Write a polynomial function $S(x)$ that gives the surface area of the empty pool.

b Write a polynomial function $V(x)$ for the volume of the pool.

Answer.

a $S(x) = x^2 + 20x + 48$

b $V(x) = 3x^2 + 24x$

Checkpoint 7.1.12 Leon is flying his plane to Au Gres, Michigan. He maintains a constant altitude until he passes over a marker just outside the neighboring town of Omer, when he begins his descent for landing. During the descent, his altitude, in feet, is given by

$$A(x) = 128x^3 - 960x^2 + 8000$$

where x is the number of miles Leon has traveled since passing over the marker in Omer.

a What is Leon's altitude when he begins his descent?

b Graph $A(x)$ in the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 5 \\ \text{Ymin} = 0 & \text{Ymax} = 8000 \end{array}$$

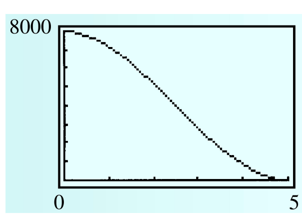
c Use the *Trace* feature to discover how far from Omer Leon will travel before landing. (In other words, how far is Au Gres from Omer?)

d Verify your answer to part (c) algebraically.

Answer.

a 8000 ft

b



c 5 mi

d $A(5) = 0$

7.1.6 Homework 7.1

For Problems 1-8, multiply.

7.1.6.1. $(3x - 2)(4x^2 + x - 2)$

Answer. $12x^3 - 5x^2 - 8x + 4$

7.1.6.2. $(2x + 3)(3x^2 - 4x + 2)$

7.1.6.3. $(x - 2)(x - 1)(x - 3)$

Answer. $x^3 - 6x^2 + 11x - 6$

7.1.6.4. $(z - 5)(z + 6)(z - 1)$

7.1.6.5.
$(2a^2-3a+1)(3a^2+2a-1)$

Answer. $6a^4-5a^3-5a^2+5a-1$

7.1.6.6. $(b^2-3b+5)(2b^2-b+1)$

7.1.6.7.
$(y-2)(y+2)(y+4)(y+1)$

Answer. $y^4+5y^3-20y-16$

7.1.6.8.
$(z+3)(z+2)(z-1)(z+1)$

For Problems 9-12, find the first three terms of the product in ascending powers. (Do not compute the entire product!)

7.1.6.9. $(2-x+3x^2)(3+2x-x^2+2x^4)$

Answer. $6+x+5x^2$

7.1.6.10. $(1+x-2x^2)(-3+2x-4x^3)$

7.1.6.11. $(1-2x^2-x^4)(4+x^2-2x^4)$

Answer. $4-7x^2-8x^4$

7.1.6.12. $(3+2x)(5-2x^2-3x^3-x^5+2x^6)$

For Problems 13-16, find the indicated term in each product. (Do not compute the entire product!)

7.1.6.13. $(4+2x-x^2)(2-3x+2x^2)$; x^2

Answer. $0x^2$

7.1.6.14. $(1-2x+3x^2)(6-x-x^3)$; x^3

7.1.6.15. $(3x+x^3-7x^5)(1+4x-3x^2)$; x^3

Answer. $-8x^3$

7.1.6.16. $(2+3x^2+2x^4)(2-x-x^2-x^4)$; x^4

For Problems 17-18, without performing the multiplication, give the degree of each product and the leading coefficient.

7.1.6.17.

(a) $(x^2-4)(3x^2-6x+2)$

(b) $(x-3)(2x-5)(x^3-x+2)$

(c) $(3x^2+2x)(x^3+1)(-2x^2+8)$

Answer.

(a) 4 (b) 5 (c) 7

7.1.6.18.

(a) $(6x^2-1)(4x^2-9)$

(b) $(3x+4)(3x+1)(2x^3+x^2-7)$

(c) $(x^2-3)(2x^3-5x^2+2)(-x^3-5x)$

For Problems 19-22, verify the following products discussed in the text.

7.1.6.19. $(x+y)^3=x^3+3x^2y+3xy^2+y^3$

Answer.
$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)^2\\ &= (x+y)(x^2+2xy+y^2)\\ &= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3\\ &= x^3+3x^2y+3xy^2+y^3\end{aligned}$$

7.1.6.20. $(x-y)^3 = x^3-3x^2y+3xy^2-y^3$

7.1.6.21. $(x+y)(x^2-xy+y^2) = x^3+y^3$

Answer.
$$\begin{aligned}(x+y)(x^2-xy+y^2) &= x^3-x^2y+xy^2+x^2y-xy^2+y^3\\ &= x^3+y^3\end{aligned}$$

7.1.6.22. $(x-y)(x^2+xy+y^2) = x^3-y^3$

7.1.6.23.

(a) As if you were addressing a classmate, explain how to remember the formula for expanding $(x+y)^3$. In particular, mention the exponents on each term and the numerical coefficients.

(b) Explain how to remember the formula for expanding $(x-y)^3$, assuming your listener already knows the formula for $(x+y)^3$.

Answer.

(a) The formula begins with x^3 and ends with y^3. As you proceed from term to term, the exponents on x decrease while the exponents on y increase, and on each term the sum of the exponents is 3. The coefficients of the two middle terms are both 3.

(b) The formula is the same as for $(x-y)^3$, except that the terms alternate in sign.

7.1.6.24.

(a) As if you were addressing a classmate, explain how to remember the formula for factoring a sum of two cubes. Pay particular attention to the placement of the variables and the signs of the terms.

(b) Explain how to remember the formula for factoring a difference of two cubes, assuming your listener already knows how to factor a sum of two cubes.

For Problems 25-28, use the formulas for the cube of a binomial to expand the products.

7.1.6.25. $(1+2z)^3$

Answer. $1+6z+12z^2+8z^3$

7.1.6.26. $(1-x^2)^3$

7.1.6.27. $(1-5\sqrt{t})^3$

Answer.
$1-15\sqrt{t}+75t-125t\sqrt{t}$

7.1.6.26. $\left(1-\dfrac{3}{a}\right)^3$

For Problems 29-34, write each product as a polynomial and simplify.

7.1.6.29. $(x-1)(x^2+x+1)$

Answer. x^3-1

7.1.6.30. $(x+2)(x^2-2x+4)$

7.1.6.31. $(2x+1)(4x^2-2x+1)$

Answer. $8x^3+1$

7.1.6.32. $(3x-1)(9x^2+3x+1)$

7.1.6.33. $(3a-2b)(9a^2+6ab+4b^2)$

Answer. $27a^3-8b^3$

7.1.6.34. $(2a+3b)(4a^2-6ab+9b^2)$

For Problems 35-46, factor completely.

7.1.6.35. x^3+27

Answer. $(x+3)(x^2-3x+9)$

7.1.6.36. y^3-1

7.1.6.37. a^3-8b^3

Answer. $(a-2b)(a^2+2ab+4b^2)$

7.1.6.38. $27a^3+b^3$

7.1.6.39. x^3y^6-1

Answer. $(xy^2-1)(x^2y^4+xy^2+1)$

7.1.6.40. $8+x^{12}y^3$

7.1.6.41. $27a^3+64b^3$

Answer. $(3a+4b)(9a^2-12ab+16b^2)$

7.1.6.42. $8a^3-125b^3$

7.1.6.43. $125a^3b^3-1$

Answer. $(5ab-1)(25a^2b^2+5ab+1)$

7.1.6.44. $64a^3b^3+1$

7.1.6.45. $64t^9+w^6$

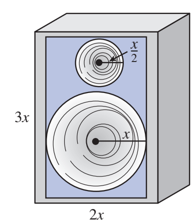
Answer. $(4t^3+w^2)(16t^6-4t^3w^2+w^4)$

7.1.6.46. $w^{15}-125t^9$

7.1.6.47.

(a) Write a polynomial function, $A(x)$, that gives the area of the front face of the speaker frame (the region in color) in the figure.

(b) If $x=8$ inches, find the area of the front face of the frame.



Answer.

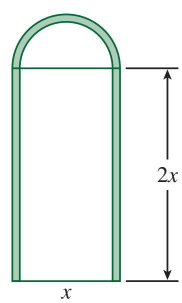
(a) $\left(6-\frac{5}{4}\pi\right)x^2$

(b) ≈ 132.67 square inches

7.1.6.48.

(a) A Norman window is shaped like a rectangle whose length is twice its width, with a semicircle at the top (see the figure). Write a polynomial, $A(x)$, that gives its area.

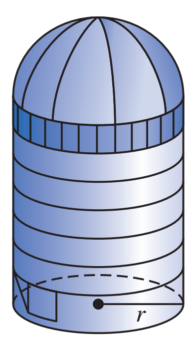
(b) If $x=3$ feet, find the area of the front face of the frame.



7.1.6.49.

(a) A grain silo is built in the shape of a cylinder with a hemisphere on top (see the figure). Write an expression for the volume of the silo in terms of the radius and height of the cylindrical portion of the silo.

(b) If the total height of the silo is five times its radius, write a polynomial function $V(r)$ in one variable for its volume.



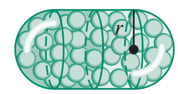
Answer.

(a) $\frac{2}{3}\pi r^3 + \pi r^2 h$

(b) $V(r) = \frac{14}{3}\pi r^3$

7.1.6.50.

(a) A cold medication capsule is shaped like a cylinder with a hemispherical cap on each end (see the figure). Write an expression for the volume of the capsule in terms of the radius and length of the cylindrical portion.



(b) If the radius of the capsule is one-fourth of its overall length, write a polynomial function $V(r)$ in one variable for its volume.

7.1.6.51. Jack invests \$500 in an account bearing interest rate r, compounded annually. This means that each year his account balance is increased by a factor of $1 + r$.

(a) Write expressions for the amount of money in Jack's account after 2 years, after 3 years, and after 4 years.

(b) Expand as polynomials the expressions you found in part (a).

(c) How much money will be in Jack's account at the end of 2 years, 3 years, and 4 years if the interest rate is 8%?

Answer.

(a) $500(1 + r)^2$; $500(1 + r)^3$; $500(1 + r)^4$

(b) $500r^2 + 1000r + 500$; $500r^3 + 1500r^2 + 1500r + 500$; $500r^4 + 2000r^3 + 3000r^2 + 2000r + 500$

(c) \$583.20, \$629.86, \$680.24

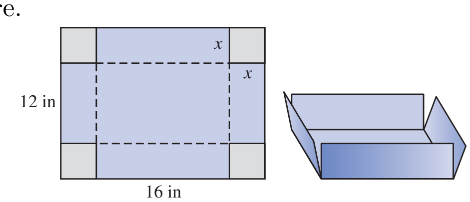
7.1.6.52. A small company borrows \$800 for start-up costs and agrees to repay the loan at interest rate r, compounded annually. This means that each year the debt is increased by a factor of $1 + r$.

(a) Write expressions for the amount of money the company will owe if it repays the loan after 2 years, after 3 years, or after 4 years.

(b) Expand as polynomials the expressions you found in part (a).

(c) How much money will the company owe after 2 years, after 3 years, or after 4 years at an interest rate of 12%?

7.1.6.53. A paper company plans to make boxes without tops from sheets of cardboard 12 inches wide and 16 inches long. The company will cut out four squares of side x inches from the corners of the sheet and fold up the edges as shown in the figure.



(a) Write expressions in terms of x for the length, width, and height of the resulting box.

(b) Write a formula for the volume, V, of the box as a function of x.

(c) What is the domain of the function V? (What are the largest and smallest reasonable values for x?)

(d) Make a table of values for $V(x)$ on its domain.

(e) Graph your function V in a suitable window.

(f) Use your graph to find the value of x that will yield a box with maximum possible volume. What is the maximum possible volume?

Answer.

(a) Length: $16 - 2x$; Width: $12 - 2x$; Height: x

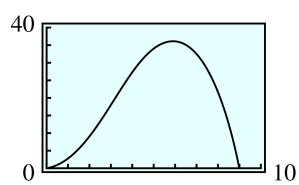
(b) $V = x(16 - 2x)(12 - 2x)$

(c) Real numbers between 0 and 6

(d)

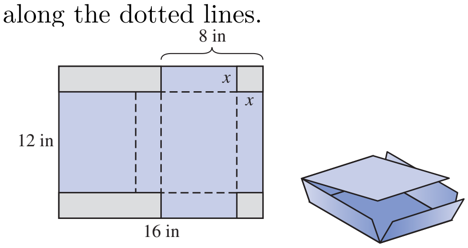
x	1	2	3	4	5
V	140	192	180	128	60

(e)



(f) 2.26 in, 194.07 cu in

7.1.6.54. The paper company also plans to make boxes with tops from 12-inch by 16-inch sheets of cardboard by cutting out the shaded areas shown in the figure and folding along the dotted lines.



(a) Write expressions in terms of x for the length, width, and height of the resulting box.

(b) Write a formula for the volume, V, of the box as a function of x.

(c) What is the domain of the function V? (What are the largest and smallest reasonable values for x?)

(d) Make a table of values for $V(x)$ on its domain.

(e) Graph your function V in a suitable window.

(f) Use your graph to find the value of x that will yield a box with maximum possible volume. What is the maximum possible volume?

Use your graphing calculator to help you answer the questions in Problems

55–62. Then verify your answers algebraically.

7.1.6.55. A doctor who is treating a heart patient wants to prescribe medication to lower the patient's blood pressure. The body's reaction to this medication is a function of the dose administered. If the patient takes x milliliters of the medication, his blood pressure should decrease by $R = f(x)$ points, where

$$f(x) = 3x^2 - \frac{1}{3}x^3$$

(a) For what values of x is $R = 0$?

(b) Find a suitable domain for the function and explain why you chose this domain.

(c) Graph the function f on its domain.

(d) How much should the patient's blood pressure drop if he takes 2 milliliters of medication?

(e) What is the maximum drop in blood pressure that can be achieved with this medication?

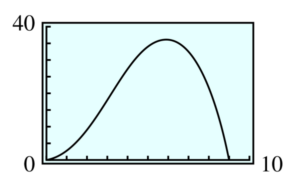
(f) There may be risks associated with a large change in blood pressure. How many milliliters of the medication should be administered to produce half the maximum possible drop in blood pressure?

Answer.

(a) $0, 9$

(b) $0 \leq x \leq 9$; $R \geq 0$ for these values

(c)

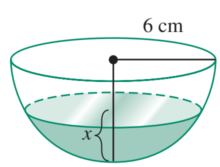


(d) $\frac{28}{3}$ points

(e) 36 points

(f) 3 ml or 8.2 ml

7.1.6.56. A soup bowl has the shape of a hemisphere of radius 6 centimeters. The volume of the soup in the bowl, $V = f(x)$, is a function of the depth, x, of the soup.



(a) What is the domain of f? Why?

(b) The function f is given by

$$f(x) = 6\pi x^2 - \frac{\pi}{3}x^3$$

Graph the function on its domain.

(c) What is the volume of the soup if it is 3 centimeters deep?

(d) What is the maximum volume of soup that the bowl can hold?

(e) Find the depth of the soup (to within 2 decimal places of accuracy) when the bowl is filled to half its capacity.

7.1.6.57. The population, $P(t)$, of Cyberville has been growing according to the formula

$$P(t) = t^3 - 63t^2 + 1403t + 900$$

where t is the number of years since 1970.

(a) Graph $P(t)$ in the window

$$\text{Xmin} = 0 \qquad \text{Xmax} = 47$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 20,000$$

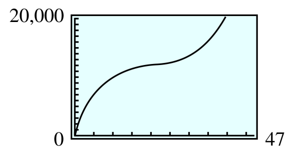
(b) What was the population in 1970? In 1985? In 2004?

(c) By how much did the population grow from 1970 to 1971? From 1985 to 1986? From 2004 to 2005?

(d) Approximately when was the population growing at the slowest rate, that is, when is the graph the least steep?

Answer.

(a)



(b) 900; 11, 145; 15, 078

(c) 1341; 171; 627

(d) Between 1990 and 1991

7.1.6.58. The annual profit, $P(t)$, of the Enviro Company, in thousands of dollars, is given by

$$P(t) = 2t^3 - 152t^2 + 3400t + 30$$

where t is the number of years since 1960, the first year that the company showed a profit.

(a) Graph $P(t)$ in the window

$$\text{Xmin} = 0 \qquad \text{Xmax} = 94$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 50,000$$

(b) What was the profit in 1960? In 1980? In 2000?

(c) How did the profit change from 1960 to 1961? From 1980 to 1981? From 2000 to 2001?

(d) During which years did the profit decrease from one year to the next?

7.1.6.59. The total annual cost of educating postgraduate research students at an Australian university, in thousands of dollars, is given by the function

$$C(x) = 0.0173x^3 - 0.647x^2 + 9.587x + 195.366$$

where x is the number of students, in hundreds. (Source: Creedy, Johnson, and Valenzuela, 2002)

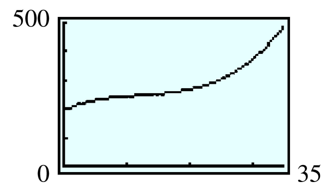
(a) Graph the function in a suitable window for up to 3500 students.

(b) Describe the concavity of the graph. For what value of x is the cost growing at the slowest rate?

(c) Approximately how many students can be educated for $350,000?

Answer.

(a)



(b) The graph is concave down until about $x = 12.5$ and is concave up afterwards. The cost is growing at the slowest rate at the inflection point at about $x = 12.5$, or 1250 students.

(c) About 2890

7.1.6.60. It has been proposed that certain cubic functions model the response of wheat and barley to nitrogen fertilizer. These functions exhibit a "plateau" that fits observations better than the standard quadratic model. (See Problem 8.2.5.36 of Section 8.2.) In trials in Denmark, the yield per acre was a function of the amount of nitrogen applied. A typical response function is

$$Y(x) = 54.45 + 0.305x - 0.001655x^2 + 2.935 \times 10^{-6}x^3$$

where x is the amount of fertilizer, in kilograms per acre.(Source: Beattie, Mortensen, and Knudsen, 2005)

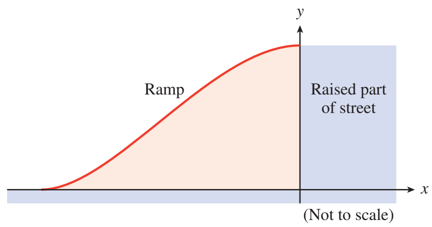
(a) Graph the function on the domain $[0, 400]$.

(b) Describe the concavity of the graph. In reality, the yield does not increase after reaching its plateau. Give a suitable domain for the model in this application.

(c) Estimate the maximum yield attainable and the optimum application of fertilizer.

7.1.6.61. During an earthquake, Nordhoff Street split in two, and one section shifted up several centimeters. Engineers created a ramp from the lower section to the upper section. In the coordinate system shown in the figure below, the ramp is part of the graph of

$$y = f(x) = -0.00004x^3 - 0.006x^2 + 20$$



(a) By how much did the upper section of the street shift during the earthquake?

(b) What is the horizontal distance from the bottom of the ramp to the raised part of the street?

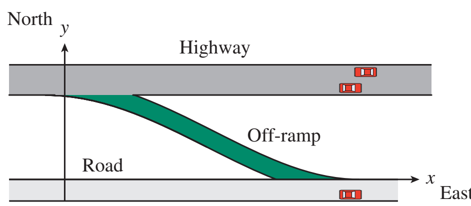
Answer.

(a) 20 cm

(b) 100 cm

7.1.6.62. The off-ramp from a highway connects to a parallel one-way road. The accompanying figure shows the highway, the off-ramp, and the road. The road lies on the x-axis, and the off-ramp begins at a point on the y-axis. The offramp is part of the graph of the polynomial

$$y = f(x) = 0.00006x^3 - 0.009x^2 + 30$$



(a) How far east of the exit does the off-ramp meet the one-way road?

(b) How far apart are the highway and the road?

7.1.6.63. The number of minutes of daylight per day in Chicago is approximated by the polynomial

$$H(t) = 0.000\,000\,525t^4 - 0.0213t^2 + 864$$

where t is the number of days since the summer solstice. The approximation is valid for $-74 < t < 74$. (A negative value of t corresponds to a number of days before the summer solstice.)

(a) Use a table of values with increments of 10 days to estimate the range of the function on its domain.

(b) Graph the polynomial on its domain.

(c) How many minutes of daylight are there on the summer solstice?

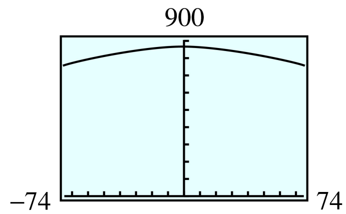
(d) How much daylight is there two weeks before the solstice?

(e) When are the days more than 14 hours long?

(f) When are the days less than 13 hours long?

Answer.

(a) $763.10 < H(t) < 864$

(b) 

(c) 864 min

(d) 859.8 min

(e) Within 34 days of the summer solstice

(f) More than 66 days from the summer solstice

7.1.6.64. The water level (in feet) at a harbor is approximated by the polynomial

$$W(t) = 0.00733t^4 - 0.332t^2 + 9.1$$

where t is the number of hours since the high tide. The approximation is valid for $-4 \le t \le 4$. (A negative value of t corresponds to a number of hours before the high tide.)

(a) Use a table of values to estimate the range of the function on its domain.

(b) Graph the polynomial on its domain.

(c) What is the water level at high tide?

(d) What is the water level 3 hours before high tide?

(e) When is the water level below 8 feet?

(f) When is the water level above 7 feet?

7.2 Graphing Polynomial Functions

7.2.2 Cubic Polynomials

Checkpoint 7.2.2

a Complete the table of values below for $C(x) = -x^3 - 2x^2 + 4x + 4$.

x	-4	-3	-2	-1	0	1	2	3	4
y									

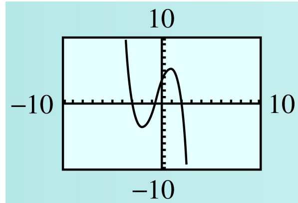
b Graph $y = C(x)$ in the standard window. Compare the graph to the graphs in Example 7.2.1: What similarities do you notice? What differences?

Answer.

a

x	-4	-3	-2	-1	0	1	2	3	4
y	20	1	-4	-1	4	5	-4	-29	-76

b



Both graphs have three x-intercepts, but the function in Example 7.2.1 has long-term behavior like $y = x^3$, and this function has long-term behavior like $y = -x^3$.

7.2.3 Quartic Polynomials

Checkpoint 7.2.4

a Complete the following table of values for $Q(x) = -x^4 - x^3 - 6x^2 + 2$.

x	-4	-3	-2	-1	0	1	2	3	4
y									

b Graph $y = Q(x)$ in the window

$$\text{Xmin} = -5 \quad \text{Xmax} = 5$$
$$\text{Ymin} = -15 \quad \text{Ymax} = 10$$

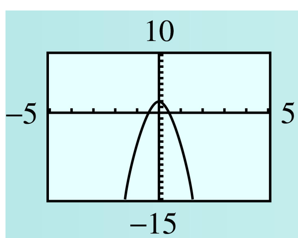
Compare the graph to the graphs in Example 7.2.3: What similarities do you notice? What differences?

Answer.

a

x	-4	-3	-2	-1	0	1	2	3	4
y	-286	-106	-30	-4	2	-6	-48	-160	-414

b



The graphs all have long-term behavior like a fourth degree power function, $y = ax^4$. The long-term behavior of the graphs in Example 7.2.3 is the same as that of $y = x^4$, but the graph here has long-term behavior like $y = -x^4$.

7.2.4 x-Intercepts and the Factor Theorem

Checkpoint 7.2.7

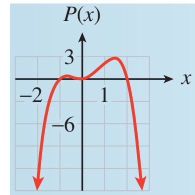
a Find the real-valued zeros of $P(x) = -x^4 + x^3 + 2x^2$ by factoring.

b Sketch a rough graph of $y = P(x)$ by hand.

Answer.

a $-1, 0, 2$

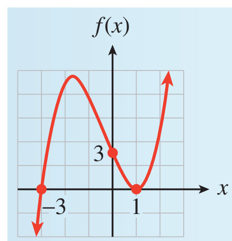
b



7.2.5 Zeros of Multiplicity Two or Three

Checkpoint 7.2.10 Sketch a rough graph of $f(x) = (x+3)(x-1)^2$ by hand. Label the x- and y-intercepts.

Answer.



7.2.7 Homework 7.2

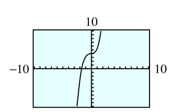
In Problems 1–8.

(a) Describe the long-term behavior of each graph. How does this behavior compare to that of the basic cubic? How does the sign of the lead coefficient affect the graph?

(b) What is the maximum number of x-intercepts? What is the maximum number of turning points? What is the maximum number of inflection points

7.2.7.1. $y = x^3 + 4$

Answer.



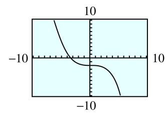
(a) The end behavior is the same as for the basic cubic because the lead coefficient is positive.

(b) There is one x-intercept, no turning points, one inflection point.

7.2.7.2. $y = x^3 - 8$

7.2.7.3. $y = -2 - 0.05x^3$

Answer.



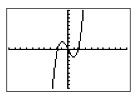
(a) The end behavior is the opposite to the basic cubic (the graph starts in the upper left and extends to the lower right) because the lead coefficient is negative.

(b) There is one x-intercept, no turning points, one inflection point.

7.2.7.4. $y = 5 - 0.02x^3$

7.2.7.5. $y = x^3 - 3x$

Answer.



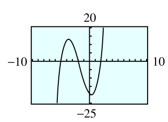
(a) The end behavior is the same as for the basic cubic because the lead coefficient is positive.

(b) There are three x-intercepts, two turning points, one inflection point.

7.2.7.6. $y = 9x - x^3$

7.2.7.7. $y = x^3 + 5x^2 - 4x - 20$

Answer.



(a) The end behavior is the same as for the basic cubic because the lead coefficient is positive.

(b) There are three x-intercepts, two turning points, one inflection point.

7.2.7.8. $y = -x^3 - 2x^2 + 5x + 6$

For Problems 9–10, use a calculator to graph each cubic polynomial.Which graphs are the same?

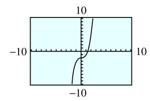
7.2.7.9.

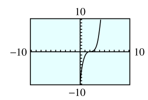
(a) $y = x^3 - 2$

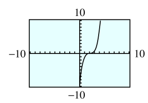
(b) $y = (x - 2)^3$

(c) $y = x^3 - 6x^2 + 12x - 8$

Answer.

(a) 

(b) 

(c) 

(b) and (c) are the same.

7.2.7.10.

(a) $y = x^3 + 3$

(b) $y = (x + 3)^3$

(c) $y = x^3 + 9x^2 + 27x + 27$

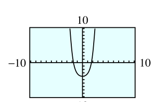
In Problems 11–18.

(a) Describe the long-term behavior of each graph. How does this behavior compare to that of the basic quartic? How does the sign of the lead coefficient affect the graph?

(b) What is the maximum number of x-intercepts? What is the maximum number of turning points? What is the maximum number of inflection points?"

7.2.7.11. $y = 0.5x^4 - 4$

Answer.



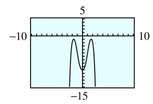
(a) The end behavior is the same as for the basic quartic because the lead coefficient is positive.

(b) There are two x-intercepts, one turning point, no inflection point.

7.2.7.12. $y = 0.3x^4 + 1$

7.2.7.13. $y = -x^4 + 6x^2 - 10$

Answer.



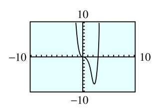
(a) The end behavior is the opposite of the basic quartic (the graph starts in the lower left and ends in the lower right) because the lead coefficient is negative.

(b) There are no x-intercepts, three turning points, two inflection points.

7.2.7.14. $y = x^4 - 8x^2 - 8$

7.2.7.15. $y = x^4 - 3x^3$

Answer.



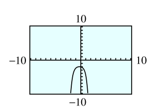
(a) The end behavior is the same as for the basic quartic because the lead coefficient is positive.

(b) There are two x-intercepts, one turning point, two inflection points.

7.2.7.16. $y = -x^4 - 4x^3$

7.2.7.17. $y = -x^4 - x^3 - 2$

Answer.



(a) The end behavior is the opposite of the basic quartic (the graph starts in the lower left and ends in the lower right) because the lead coefficient is negative.

(b) There are no x-intercepts, one turning point, two inflection points.

7.2.7.18. $y = x^4 + 2x^3 + 4x^2 + 10$

7.2.7.19. From your answers to Problems 1–8, what you can conclude about the graphs of cubic polynomials? Consider the long-term behavior, x-intercepts, turning points, and inflection points.

Answer. The graph of a cubic polynomial with a positive lead coefficient will have the same end behavior as the basic cubic, and a cubic with a negative lead coefficient will have the opposite end behavior. Each graph of a cubic polynomial has one, two, or three x-intercepts, it has two, one or no turning point, and it has exactly one inflection point.

7.2.7.20. From your answers to Problems 11–18, what you can conclude about the graphs of quartic polynomials? Consider the long-term behavior, x-intercepts, turning points, and inflection points.

For Problems 21–26,

a Use your calculator to graph each polynomial and locate the x-intercepts. Set Xmin $= -4.7$, Xmax $= 4.7$, and adjust **Ymin** and **Ymax** to get a good graph.

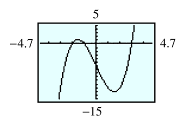
b Write the polynomial in factored form.

c Expand the factored form of the polynomial (that is, multiply the factors together). Do you get the original polynomial?

7.2.7.21. $P(x) = x^3 - 7x - 6$

Answer.

(a)



$(-2, 0), (-1, 0), (3, 0)$

(b) $P(x) = (x + 2)(x + 1)(x - 3)$

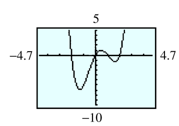
(c) Yes

7.2.7.22. $Q(x) = x^3 + 3x^2 - x - 3$

7.2.7.23.
$R(x) = x^4 - x^3 - 4x^2 + 4x$

Answer.

(a)



$(-2, 0), (0, 0), (1, 0), (2, 0)$

(b) $R(x) = (x+2)(x)(x-1)(x-2)$

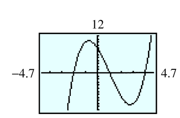
(c) Yes

7.2.7.24.
$S(x) = x^4 + 3x^3 - x^2 - 3x$

7.2.7.25. $p(x) = x^3 - 3x^2 - 6x + 8$

Answer.

(a)



$(-2, 0), (1, 0), (4, 0)$

(b) $p(x) = (x + 2)(x - 1)(x - 4)$

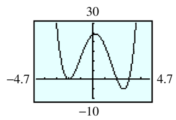
(c) Yes

7.2.7.26. $q(x) = x^3 + 6x^2 - x - 30$

7.2.7.27.
$r(x) = x^4 - x^3 - 10x^2 + 4x + 24$

Answer.

(a)



$(-2, 0), (2, 0), (3, 0)$

(b) $r(x) = (x + 2)^2(x - 2)(x - 3)$

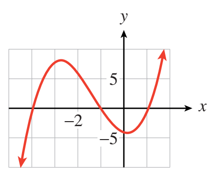
(c) Yes

7.2.7.28.
$s(x) = x^4 - x^3 - 12x^2 - 4x + 16$

For Problems 29–36, sketch a rough graph of each polynomial function by hand, paying attention to the shape of the graph near each x-intercept.

7.2.7.29.
$q(x) = (x + 4)(x + 1)(x - 1)$

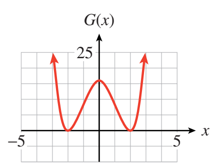
Answer.



7.2.7.30. $p(x) = x(x + 2)(x + 4)$

7.2.7.31. $G(x) = (x - 2)^2(x + 2)^2$

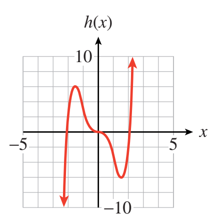
Answer.



7.2.7.32. $F(x) = (x - 1)^2(x - 3)^2$

7.2.7.33. $h(x) = x^3(x + 2)(x - 2)$

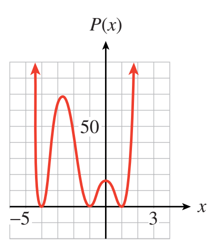
Answer.



7.2.7.34.
$H(x) = -(x + 1)^3(x - 2)^2$

7.2.7.35.
$P(x) = (x + 4)^2(x + 1)^2(x - 1)^2$

Answer.



7.2.7.36.
$Q(x) = x^2(x - 5)(x - 1)^2(x + 2)$

For Problems 37–46,

a Find the zeros of each polynomial by factoring.

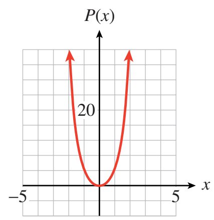
b Sketch a rough graph by hand.

7.2.7.37. $P(x) = x^4 + 4x^2$

Answer.

(a) 0 (multiplicity 2)

(b)



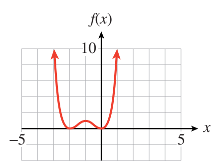
7.2.7.38. $P(x) = x^3 + 3x$

7.2.7.39. $f(x) = -x^4 + 4x^3 + 4x^2$

Answer.

(a) 0 (multiplicity 2), 2 (multiplicity 2)

(b)



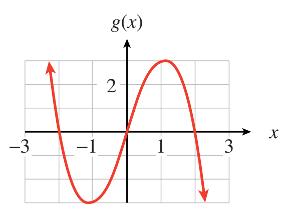
7.2.7.40. $g(x) = x^4 + 4x^3 + 3x^2$

7.2.7.41. $g(x) = 4x - x^3$

Answer.

(a) $0, \pm 2$

(b)



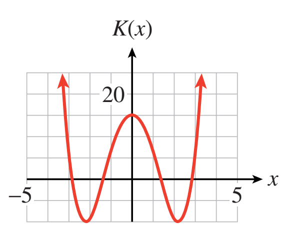
7.2.7.42. $f(x) = 8x - x^4$

7.2.7.43. $K(x) = x^4 - 10x^2 + 16$

Answer.

(a) $\pm\sqrt{2}, \pm\sqrt{8}$

(b)



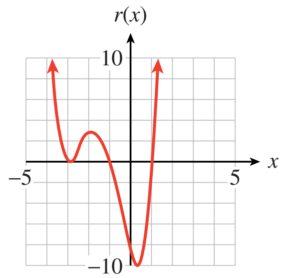
7.2.7.44. $m(x) = x^4 - 15x^2 + 36$

7.2.7.45. $r(x) = (x^2 - 1)(x + 3)^2$

Answer.

(a) $\pm 1, -3$ (multiplicity 2)

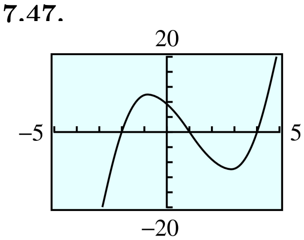
(b)



7.2.7.46. $s(x) = (x^2 - 9)(x - 1)^2$

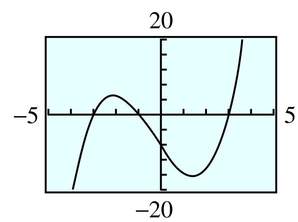
For Problems 47–52, find a possible equation of lowest possible degree for the polynomial whose graph is shown.

7.2.7.47.

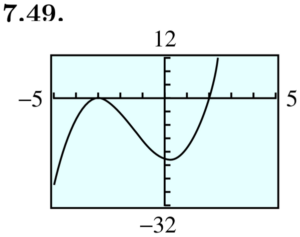


Answer.
$P(x) = (x + 2)(x - 1)(x - 4)$

7.2.7.48.

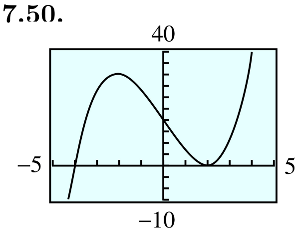


7.2.7.49.

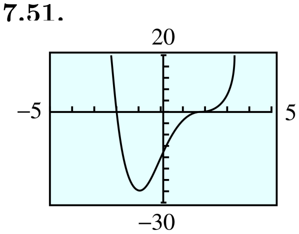


Answer. $P(x) = (x + 3)^2(x - 2)$

7.2.7.50.

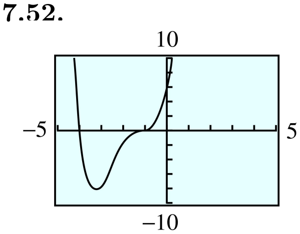


7.2.7.51.



Answer. $P(x) = (x - 2)^3(x + 2)$

7.2.7.52.



For Problems 53–56, write the formula for each function in parts (a) through (d) and graph with a calculator. Describe how the graph differs from the graph of $y = f(x)$.

7.2.7.53. $f(x) = x^3 - 4x$

(a) $y = f(x) + 3$

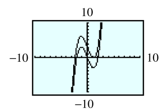
(b) $y = f(x) - 5$

(c) $y = f(x - 2)$

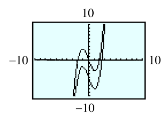
(d) $y = f(x + 3)$

Answer.

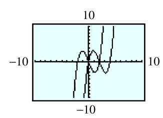
(a) $y = x^3 - 4x + 3$; The graph of $y = f(x)$ shifted 3 units up.



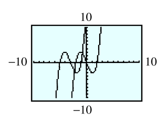
(b) $y = x^3 - 4x - 5$; The graph of $y = f(x)$ shifted 5 units down.



(c) $y = (x - 2)^3 - 4(x - 2)$; The graph of $y = f(x)$ shifted 2 units right.



(d) $y = (x + 3)^3 - 4(x + 3)$; The graph of $y = f(x)$ shifted 3 units left.



7.2.7.54. $f(x) = x^3 - x^2 + x - 1$

(a) $y = f(x) + 4$

(b) $y = f(x) - 4$

(c) $y = f(x - 3)$

(d) $y = f(x + 5)$

7.2.7.55. $f(x) = x^4 - 4x^2$

(a) $y = f(x) + 6$

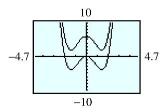
(b) $y = f(x) - 2$

(c) $y = f(x - 1)$

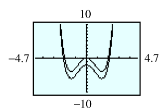
(d) $y = f(x + 2)$

Answer.

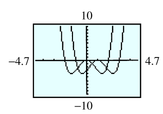
(a) $y = x^4 - 4x^2 + 6$; The graph of $y = f(x)$ shifted 6 units up.



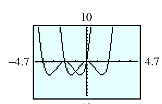
(b) $y = x^4 - 4x^2 - 2$; The graph of $y = f(x)$ shifted 2 units down.



(c) $y = (x - 1)^4 - 4(x - 1)^2$; The graph of $y = f(x)$ shifted 1 unit right.



(d) $y = (x + 2)^4 - 4(x + 2)^2$; The graph of $y = f(x)$ shifted 2 units left.



7.2.7.56. $f(x) = x^4 + 3x^3$

(a) $y = f(x) + 5$

(b) $y = f(x) - 3$

(c) $y = f(x - 2)$

(d) $y = f(x + 1)$

Division Algorithm for Polynomials.

If $f(x)$ and $g(x)$ are nonconstant polynomials with real coefficients, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = g(x)q(x) + r(x),$$

where $\deg r(x) < \deg g(x)$.

In Problems 57–60, use polynomial division to divide $f(x)$ by $g(x)$, and hence find the quotient, $q(x)$, and remainder, $r(x)$. (See Algebra Skills Refresher Section A.7 to review polynomial division.)

7.2.7.57. $f(x) = 2x^3 - 2x^2 - 19x - 11,\ g(x) = x - 3$

Answer. $q(x) = 2x^2 + 4x - 7$; $r(x) = -32$

7.2.7.58. $f(x) = 3x^3 + 12x^2 - 13x - 32,\ g(x) = x + 4$

7.2.7.59. $f(x) = x^5 + 2x^4 - 7x^3 - 12x^2 + 5,\ g(x) = x^2 + 2x - 1$

Answer. $q(x) = x^3 - 6x$; $r(x) = -6x + 5$

7.2.7.60. $f(x) = x^5 - 4x^4 + 11x^3 - 12x^2 + 5x + 2,\ g(x) = x^2 - x + 3$

7.2.7.61. The remainder theorem states: If $P(x)$ is a polynomial and a is any real number, there is a unique polynomial $Q(x)$ such that

$$P(x) = (x - a)Q(x) + P(a)$$

Follow the steps below to prove the remainder theorem.

(a) State the division algorithm applied to the polynomials $P(x)$ and $x - a$.

(b) What must be the degree of $r(x)$ in this case?

(c) Evaluate your expression from part (a) at $x = a$. What does this tell you about the remainder, $r(x)$?

Answer.

(a) If $P(x)$ is a nonconstant polynomial with real coefficients and a is any real number, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$P(x) = (x - a)q(x) + r(x)$$

where deg $r(x) <$ deg $(x - a)$.

(b) Zero

(c) $P(a) = (a - a)q(a) + r(a) = r(a)$. Because deg $r(x) = 0$, $r(x)$ is a constant. That constant value is $P(a)$, so $P(x) = (x - a)q(x) + P(a)$.

7.2.7.62. Verify the remainder theorem for the following:

(a) $P(x) = x^3 - 4x^2 + 2x - 1, a = 2$

(b) $P(x) = 3x^2 + x - 5,\ a = -3$

7.2.7.63. Use the remainder theorem to prove the factor theorem, stated earlier in this section. You will need to justify two statements:

(a) If $P(a) = 0$, show that $x - a$ is a factor of $P(x)$.

(b) If $x - a$ is a factor of $P(x)$, show that $P(a) = 0$.

Answer.

(a) From the remainder theorem,
$$\begin{aligned} P(x) &= (x - a)Q(x) + P(a) \\ &= (x - a)Q(x) + 0 \\ &= (x - a)Q(x) \end{aligned}$$

(b) By definition of a factor, if $x - a$ is a factor of $P(x)$, then $P(x) = (x - a)q(x)$, so $P(x) = (x - a)q(x) + 0$. The uniqueness guaranteed in the remainder theorem tells us that $P(a) = 0$.

7.2.7.64. Verify the factor theorem for the following:

(a) $P(x) = x^4 - 4x^3 - 11x^2 + 3x + 2,\ a = -2$

(b) $P(x) = x^3 + 2x^2 - 31x - 20,\ a = 5$

For Problems 65–68,

(a) Verify that the given value is a zero of the polynomial.

(b) Find the other zeros. (*Hint*: Use polynomial division to write $P(x) = (x-a)Q(x)$, then factor $Q(x)$.)

7.2.7.65. $P(x) = x^3 - 2x^2 + 1$; $a = 1$

Answer.

(a) $P(1) = 0$

(b) $\dfrac{1 \pm \sqrt{5}}{2}$

7.2.7.66. $P(x) = x^3 + 2x^2 - 1$; $a = -1$

7.2.7.67. $P(x) = x^4 - 3x^3 - 10x^2 + 24x$; $a = -3$

Answer.

(a) $P(-3) = 0$

(b) $0, 2, 4$

7.2.7.68. $P(x) = x^4 + 5x^3 - x^2 - 5x$; $a = -5$

In Problems 69–70, we use polynomials to approximate other functions.

7.2.7.69.

(a) Graph the functions $f(x) = e^x$ and

$$p(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

in the standard window. For what values of x does it appear that $p(x)$ would be a good approximation for $f(x)$?

(b) Change the window settings to

$$\text{Xmin} = -4.7 \qquad \text{Xmax} = 4.7$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 20$$

and fill in the table of values below. (You can use the **value** feature on your calculator.)

x	-1	-0.5	0	0.5	1	1.5	2
$f(x)$							
$p(x)$							

(c) The **error** in the approximation is the difference $f(x) - p(x)$. We can reduce the error by using a polynomial of higher degree. The nth degree polynomial for approximating e^x is

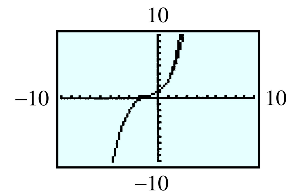
$$P_n(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n$$

where $n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$. Graph $f(x)$ and $P_5(x)$ in the same window as in part (b). What is the error in approximating $f(2)$ by $P_5(2)$?

(d) Graph $f(x) - P_5(x)$ in the same window as in part (b). What does the graph tell you about the error in approximating $f(x)$ by $P_5(x)$?

Answer.

(a) About $-1 < x < 2$

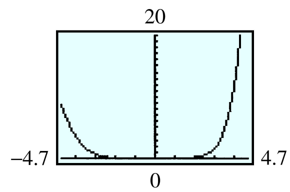


(b)

x	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	0.368	0.607	1	1.649	2.718	4.482	7.389
$p(x)$	0.333	0.604	1	1.646	2.667	4.188	6.333

(c) 0.122

(d)



The error is relatively small for values of x between -3 and 2.5.

7.2.7.70. In Projects for Chapter 2: Periodic Functions, we investigated periodic functions. The **sine function**, $f(x) = \sin(x)$, is a useful periodic function.

(a) Graph the functions

$$f(x) = \sin(x) \quad \text{and} \quad p(x) = x - \frac{1}{6}x^3$$

in the standard window. (Check that your calculator is set in **Radian** mode.) For what values of x does it appear that $p(x)$ would be a good approximation for $f(x)$?

(b) Change the window settings to

$$\text{Xmin} = -4.7 \qquad \text{Xmax} = 4.7$$
$$\text{Ymin} = -2 \qquad \text{Ymax} = 2$$

and fill in the table of values below. (You can use the **value** feature on your calculator.)

x	-1	-0.5	0	0.5	1	1.5	2
$f(x)$							
$p(x)$							

(c) Two more polynomials for approximating $f(x) = \sin(x)$ are

$$P_5(x) = 1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$
$$P_7(x) = 1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - +\frac{1}{7!}x^7$$

(See Problem 69 for the definition of $n!$.) Graph $f(x)$ and $P_5(x)$ in the same window as in part (b). What is the error in approximating $f(2)$ by $P_5(2)$?

(d) Graph $f(x) - P_5(x)$ in the same window as in part (b). What does the graph tell you about the error in approximating $f(x)$ by $P_5(x)$?

7.3 Complex Numbers

7.3.1 Introduction

Checkpoint 7.3.2 Solve the equation $x^2 - 6x + 13 = 0$ by using the quadratic formula.

Answer. $x = \dfrac{6 \pm \sqrt{-16}}{2}$

7.3.2 Imaginary Numbers

Checkpoint 7.3.5 Write each radical as an imaginary number.

a $\sqrt{-18}$

b $-6\sqrt{-5}$

Answer.

a $3i\sqrt{2}$

b $-6i\sqrt{5}$

7.3.3 Complex Numbers

Checkpoint 7.3.8 Use extraction of roots to solve $(2x + 1)^2 + 9 = 0$. Write your answers as complex numbers.

Answer. $x = \dfrac{-1}{2} \pm \dfrac{3}{2}i$

7.3.4 Arithmetic of Complex Numbers

Checkpoint 7.3.10 Subtract: $(-3 + 2i) - (-3 - 2i)$.

Answer. $4i$

7.3.5 Products of Complex Numbers

Checkpoint 7.3.12 Multiply $(-3 + 2i)(-3 - 2i)$.

Answer. 13

7.3.6 Quotients of Complex Numbers

Checkpoint 7.3.15 Divide $\dfrac{8 + 9i}{3i}$

Answer. $3 - \dfrac{8}{3}i$

Checkpoint 7.3.17 Write the quotient $\dfrac{4 - 2i}{1 + i}$ in the form $a + bi$.

Answer. $1 - 3i$

7.3.7 Zeros of Polynomials

Checkpoint 7.3.19 If $f(x) = x^2 - 6x + 13$, evaluate $f(3 + 2i)$.

Answer. 0

Checkpoint 7.3.21

a Let $z = -3 + 4i$. Compute $z\overline{z}$.

b Find a quadratic equation with one solution being $z = -3 + 4i$.

Answer.

a 25

b $x^2 + 6x + 25 = 0$

Checkpoint 7.3.24

a Find the zeros of the polynomial $f(x) = x^4 + 15x^2 - 16$.

b Write the polynomial in factored form.

Answer.

a $\pm 1, \quad \pm 4i$

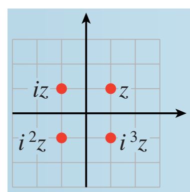
b $(x-1)(x+1)(x-4i)(x+4i)$

7.3.8 Graphing Complex Numbers

Checkpoint 7.3.26 Plot the following numbers as points on the complex plane.

- $z = 1 + i$
- $iz = i + i^2$
- $i^2 z = i^2 + i^3$
- $i^3 z = i^3 + i^4$

Answer.



7.3.10 Homework 7.3

For Problems 1–6, write the complex number in the form $a + bi$, where a and b are real numbers.

7.3.10.1. $\sqrt{-25} - 4$

Answer. $-4 + 5i$

7.3.10.2. $\sqrt{-9} + 3$

7.3.10.3. $\dfrac{-8 + \sqrt{-4}}{2}$

Answer. $-4 + i$

7.3.10.4. $\dfrac{6 - \sqrt{-36}}{2}$

7.3.10.5. $\dfrac{-5 - \sqrt{-2}}{6}$

Answer. $\dfrac{-5}{6} - \dfrac{\sqrt{2}}{6} i$

7.3.10.6. $\dfrac{7 + \sqrt{-3}}{4}$

For Problems 7–10, find the zeros of the quadratic polynomial. Write each in the form $a + bi$, where a and b are real numbers.

7.3.10.7. $x^2 + 6x + 13$

Answer. $-3 \pm 2i$

7.3.10.8. $x^2 - 2x + 10$

7.3.10.9. $3x^2 - x + 1$

Answer. $\dfrac{1}{6} \pm \dfrac{\sqrt{11}}{6} i$

7.3.10.10. $5x^2 + 2x + 2$

For Problems 11–14, add or subtract.

7.3.10.11. $(11 - 4i) - (-2 - 8i)$

Answer. $13 + 4i$

7.3.10.12. $(7i - 2) + (6 - 4i)$

7.3.10.13. $(2.1 + 5.6i) + (-1.8i - 2.9)$

Answer. $-0.8 + 3.8i$

7.3.10.14. $\left(\frac{1}{5}i - \frac{2}{5}\right) - \left(\frac{4}{5} - \frac{3}{5}i\right)$

For Problems 15–24, multiply.

7.3.10.15. $5i(2 - 4i)$

Answer. $20 + 10i$

7.3.10.16. $-7i(-1 + 4i)$

7.3.10.17. $(4 - i)(-6 + 7i)$

Answer. $-17 + 34i$

7.3.10.18. $(2 - 3i)(2 - 3i)$

7.3.10.19. $(7 + i\sqrt{3})^2$

Answer. $46 + 14i\sqrt{3}$

7.3.10.20. $(5 - i\sqrt{2})^2$

7.3.10.21. $(7 + i\sqrt{3})(7 - i\sqrt{3})$

Answer. 52

7.3.10.22. $(5 - i\sqrt{2})(5 + i\sqrt{2})$

7.3.10.23. $(1 - i)^3$

Answer. $-2 - 2i$

7.3.10.24. $(2 + i)^3$

For Problems 25–36, divide.

7.3.10.25. $\dfrac{12 + 3i}{-3i}$

Answer. $-1 + 4i$

7.3.10.26. $\dfrac{12 + 4i}{8i}$

7.3.10.27. $\dfrac{10 + 15i}{2 + i}$

Answer. $7 + 4i$

7.3.10.28. $\dfrac{4 - 6i}{1 - i}$

7.3.10.29. $\dfrac{5i}{2 - 5i}$

Answer. $\dfrac{-25}{29} + \dfrac{10}{29}i$

7.3.10.30. $\dfrac{-2i}{7 + 2i}$

7.3.10.31. $\dfrac{\sqrt{3}}{\sqrt{3} + i}$

Answer. $\dfrac{3}{4} - \dfrac{\sqrt{3}}{4}i$

7.3.10.32. $\dfrac{2\sqrt{2}}{1 - i\sqrt{2}}$

7.3.10.33. $\dfrac{1 + i\sqrt{5}}{1 - i\sqrt{5}}$

Answer. $\dfrac{-2}{3} + \dfrac{\sqrt{5}}{3}i$

7.3.10.34. $\dfrac{\sqrt{2} - i}{\sqrt{2} + i}$

7.3.10.35. $\dfrac{3 + 2i}{2 - 3i}$

Answer. i

7.3.10.36. $\dfrac{4 - 6i}{-3 - 2i}$

For Problems 37–42, evaluate the polynomial for the given values of the variable.

7.3.10.37. $z^2 + 9$

a $z = 3i$

b $z = -3i$

Answer.

(a) 0 (b) 0

7.3.10.38. $2y^2 - y - 2$

a $y = 2 - i$

b $y = -2 - i$

7.3.10.39. $x^2 - 2x + 2$

a $x = 1 - i$

b $x = 1 + i$

Answer.

(a) 0 (b) 0

7.3.10.40. $3w^2 + 5$

a $w = 2i$

b $w = -2i$

7.3.10.41. $q^2 + 4q + 13$

a $q = -2 + 3i$

b $q = -2 - 3i$

Answer.

(a) 0 (b) 0

7.3.10.42. $v^2 + 2v + 3$

a $v = 1 + i$

b $v = -1 + i$

For Problems 43–48, expand each product of polynomials.

7.3.10.43. $(2z + 7i)(2z - 7i)$

Answer. $4z^2 + 49$

7.3.10.44. $(5w + 3i)(5w - 3i)$

7.3.10.45. $[x + (3 + i)][x + (3 - i)]$

Answer. $x^2 + 6x + 10$

7.3.10.46. $[s - (1 + 2i)][s - (1 - 2i)]$

7.3.10.47. $[v - (4 + i)][v - (4 - i)]$

Answer. $v^2 - 8v + 17$

7.3.10.48. $[Z + (2 + i)][Z + (2 - i)]$

7.3.10.49. For what values of x will $\sqrt{x - 5}$ be real? Imaginary?

Answer. $x \geq 5$; $x < 5$

7.3.10.50. For what values of x will $\sqrt{x + 3}$ be real? Imaginary?

7.3.10.51. Simplify.

(a) i^6 (b) i^{12} (c) i^{15} (d) i^{102}

Answer.

(a) -1 (b) 1 (c) $-i$ (d) -1

7.3.10.52. Express with a positive exponent and simplify.

(a) i^{-1} (b) i^{-2} (c) i^{-3} (d) i^{-6}

In Problems 53–56,

a Given one solution of a quadratic equation with rational coefficients, find the other solution.

b Write a quadratic equation that has those solutions.

7.3.10.53. $2 + \sqrt{5}$

Answer.

(a) $2 - \sqrt{5}$

(b) $x^2 - 4x - 1$

7.3.10.54. $3 - \sqrt{2}$

7.3.10.55. $4 - 3i$

Answer.

(a) $4 + 3i$

(b) $x^2 - 8x + 25$

7.3.10.54. $5 + i$

Every polynomial factors into a product of a constant and linear factors of the form $(x - a)$, where a can be either real or complex. In Problems 57–58, how many linear factors are in the factored form of the given polynomial?

7.3.10.57.

(a) $x^4 - 2x^3 + 4x^2 + 8x - 6$

(b) $2x^5 - x^3 + 6x - 4$

Answer.

(a) 4

(b) 5

7.3.10.58.

(a) $x^6 - 6x$

(b) $x^3 + 3x^2 - 2x + 1$

For Problems 59–62, find a fourth-degree polynomial with real coefficients that has the given complex numbers as two of its zeros.

7.3.10.59. $1 + 3i,\ 2 - i$

Answer.
$x^4 - 6x^3 + 23x^2 - 50x + 50$

7.3.10.60. $5 - 4i,\ \ -i$

7.3.10.61. $\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i,\ 3 + 2i$

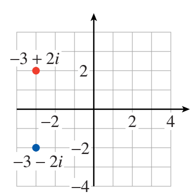
Answer.
$x^4 - 7x^3 + 20x^2 - 19x + 13$

7.3.10.62. $-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i,\ 4 - i$

For Problems 63–66, plot each number and its complex conjugate in the complex plane. What is the geometric relationship between complex conjugates?

7.3.10.63. $z = -3 + 2i$

Answer.

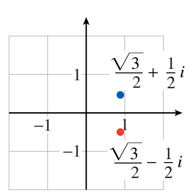


The complex conjugates are reflections of each other across the real axis.

7.3.10.64. $z = 4 - 3i$

7.3.10.65. $z = \dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$

Answer.



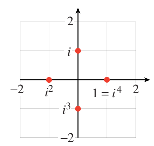
The complex conjugates are reflections of each other across the real axis.

7.3.10.66. $z = -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i$

For Problems 59–62, simplify and plot each complex number as a point on the complex plane.

7.3.10.67. $1, i, i^2, i^3$ and i^4

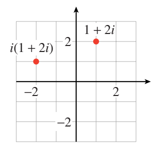
Answer.



7.3.10.68. $-1, -i, -i^2, -i^3$ and $-i^4$

7.3.10.69. $1 + 2i$ and $i(1 + 2i)$

Answer.



7.3.10.70. $3 - 4i$ and $i(3 - 4i)$

Problems 71–72 show that multiplication by i results in a rotation of $90°$.

7.3.10.71. Suppose that $z = a + bi$ and that the real numbers a and b are both nonzero.

(a) What is the slope of the segment in the complex plane joining the origin to z?

(b) What is the slope of the segment in the complex plane joining the origin to zi?

(c) What is the product of the slopes of the two segments from parts (a) and (b)? What can you conclude about the angle between the two segments?

Answer.

(a) $m = \dfrac{b}{a}$

(b) $m = \dfrac{a}{-b}$

(c) -1; The angle is $90°$.

7.3.10.72. Suppose that $z = a+bi$ and that a and b are both real numbers.

(a) If $a \neq 0$ and $b = 0$, then what is the slope of the segment in the complex plane joining the origin to z? What is the slope of the segment joining the origin to iz?

(b) If $a = 0$ and $b \neq 0$, then what is the slope of the segment in the complex plane joining the origin to z? What is the slope of the segment joining the origin to iz?

(c) What can you conclude about the angle between the two segments from parts (a) and (b)?

7.4 Graphing Rational Functions

7.4.1 Introduction

Checkpoint 7.4.2 Queueing theory is used to predict your waiting time in a line, or queue. For example, suppose the attendant at a toll booth can process 6 vehicles per minute. The average total time spent by a motorist negotiating the toll booth depends on the rate, r, at which vehicles arrive, according to the formula

$$T = g(r) = \frac{12 - r}{12(6 - r)}$$

a What is the average time spent at the toll booth if vehicles arrive at a rate of 3 vehicles per minute?

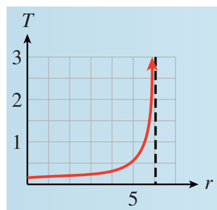
b Graph the function on the domain $[0, 6]$.

c What is the vertical asymptote of the graph? What does it tell you about the queue?

Answer.

a 0.25 min

b



c $r = 6$. The wait time becomes infinite as the arrival rate approaches 6 vehicles per minute.

Checkpoint 7.4.4 Delbert prepares a 20% glucose solution of by mixing 2 mL of glucose with 8 mL of water. If he adds x ml of glucose to the solution, its concentration is given by

$$C(x) = \frac{2 + x}{8 + x}$$

a How many milliliters of glucose should Delbert add to increase the concentration to 50%?

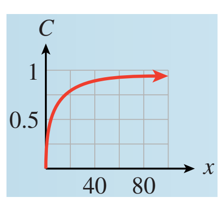
b Graph the function on the domain $[0, 100]$.

c What is the horizontal asymptote of the graph? What does it tell you about the solution?

Answer.

a 4 ml

b



c $C = 1$. As Delbert adds more glucose to the mixture, its concentration increases toward 100%.

7.4.2 Domain of a Rational Function

Checkpoint 7.4.7

a Find the domain of $F(x) = \dfrac{x-2}{x+4}$.

b Find the zeros of $F(x)$.

Answer.

a $x \neq -4$

b $x = 2$

7.4.3 Vertical Asymptotes

Checkpoint 7.4.10 Find the vertical asymptotes of $G(x) = \dfrac{4x^2}{x^2-4}$.

Answer. $x = -2$ and $x = 2$

Checkpoint 7.4.12

a Find the vertical asymptotes of $f(x) = \dfrac{1}{x^2-4}$. Locate any x-intercepts.

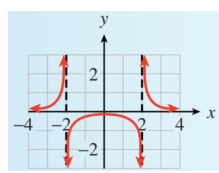
b Evaluate the function at $x = -3$, -1, 1, and 3. Sketch a graph of the function.

Answer.

a $x = -2$ and $x = 2$, no x-intercepts

b

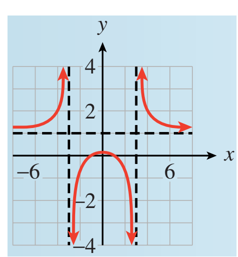
x	-3	-1	1	3
y	$\frac{1}{5}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{1}{5}$



7.4.4 Horizontal Asymptotes

Checkpoint 7.4.14 Locate the horizontal and vertical asymptotes and sketch the graph of $k(x) = \dfrac{x^2-1}{x^2-9}$. Label the x- and y-intercepts with their coordinates.

Answer. $y = 1$; $x = -3$, $x = 3$



7.4.5 Applications

Checkpoint 7.4.16 Navid took his outboard motorboat 20 miles upstream to a fishing site, returning downstream later that day. His boat travels 10 miles per hour in still water. Write an expression for the time Navid spent traveling, as a function of the speed of the current.

Answer. $\dfrac{400}{100-x^2}$ hrs

7.4.7 Homework 7.4

7.4.7.1. The eider duck, one of the world's fastest flying birds, can exceed an airspeed of 65 miles per hour. A flock of eider ducks is migrating south at an average airspeed of 50 miles per hour against a moderate headwind. Their next feeding grounds are 150 miles away.

(a) Express the ducks' travel time, t, as a function of the windspeed, v.

(b) Complete the table showing the travel time for various windspeeds.

v	0	5	10	15	20	25	30	35	40	45	50
t											

What happens to the travel time as the headwind increases?

(c) Use the table to choose an appropriate window and graph your function $t(v)$. Give the equations of any horizontal or vertical asymptotes. What does the vertical asymptote signify in the context of the problem?

Answer.

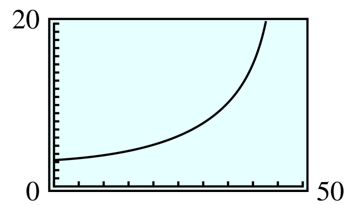
(a) $t=\dfrac{150}{50-v}$

(b)

v	0	5	10	15	20	25	30	35	40	45	50
t	3	3.33	3.75	4.29	5	6	7.5	10	15	30	—

The travel time increases as the headwind speed increases.

(c)



7.4.7.2. The fastest fish in the sea may be the bluefin tuna, which has been clocked at 43 miles per hour in short sprints. A school of tuna is migrating a distance of 200 miles at an average speed of 36 miles per hour in still water, but they have run into a current flowing against their direction of travel.

(a) Express the tuna's travel time, t, as a function of the current speed, v.

(b) Complete the table showing the travel time for various current speeds.

v	0	4	8	12	16	20	24	28	32	36
t										

What happens to the travel time as the current increases?

(c) Use the table to choose an appropriate domain and range for your function $t(v)$. Give the equations of any horizontal or vertical asymptotes. What does the vertical asymptote signify in the context of the problem?

7.4.7.3. The cost, in thousands of dollars, for immunizing p percent of the residents of Emporia against a dangerous new disease is given by the function

$$C(p) = \frac{72p}{100-p}$$

(a) What is the domain of C?

(b) Complete the table showing the cost of immunizing various percentages of the population.

p	0	15	25	40	50	75	80	90	100
C									

(c) Graph the function C. (Use Xmin = 6, Xmax = 100, and appropriate values of **Ymin** and **Ymax**.) What percentage of the population can be immunized if the city is able to spend \$108, 000?

(d) For what values of p is the total cost more than \$1, 728, 000?

(e) The graph has a vertical asymptote. What is it? What is its significance in the context of this problem?

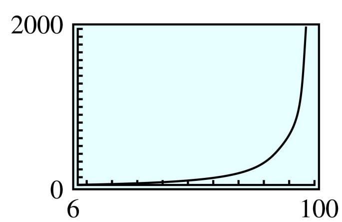
Answer.

(a) $0 \le p < 100$

(b)

p	0	15	25	40	50	75	80	90	100
C	0	12.7	24	48	72	216	288	648	—

(c) 60%



(d) $p > 96\%$

(e) $p = 100$; As the percentage immunized approaches 100, the cost grows without bound.

7.4.7.4. The cost, in thousands of dollars, for immunizing p percent of a precious ore from a mine is given by the equation

$$C(p) = \frac{360p}{100-p}$$

(a) What is the domain of C?

(b) Complete the table showing the cost of extracting various percentages of the ore.

p	0	15	25	40	50	75	80	90	100
C									

(c) Graph the function C. (Use Xmin = 6, Xmax = 100, and appropriate values of **Ymin** and **Ymax**.) What percentage of the ore can be extracted if \$540,000 can be spent on the extraction?

(d) For what values of p is the total cost less than \$1,440,000?

(e) The graph has a vertical asymptote. What is it? What is its significance in the context of this problem?

7.4.7.5. The total cost in dollars of producing n calculators is approximately $20,000 + 8n$.

(a) Express the cost per calculator, C, as a function of the number n of calculators produced.

(b) Complete the table showing the cost per calculator for various production levels.

n	100	200	400	500	1000	2000	4000	5000	8000
C									

(c) Graph the function $C(n)$ for the cost per calculator. Use the window

$$\text{Xmin} = 0 \qquad \text{Xmax} = 9400$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 50$$

(d) How many calculators should be produced so that the cost per calculator is \$18?

(e) For what values of n is the cost less than \$12 per calculator?

(f) Find the horizontal asymptote of the graph. What does it represent in this context?

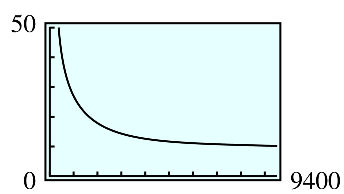
Answer.

(a) $C = 8 + \dfrac{20,000}{n}$

(b)

n	100	200	400	500	1000	2000	4000	5000	8000
C	208	108	58	48	28	18	13	12	10.5

(c)



(d) 2000

(e) $n > 5000$

(f) $C = 8$; As n increases, the average cost per calculator approaches \$8.

7.4.7.6. The number of loaves of Mom's Bread sold each day is approximated by the demand function

$$D(p) = \frac{100}{1 + (p - 1.10)^4}$$

where p is the price per loaf in dollars.

(a) Complete the table showing the demand for Mom's Bread at various prices per loaf. Round the values of $D(p)$ to the nearest whole number.

p	0.25	0.50	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
Demand											

(b) Graph the demand function $C(n)$ in the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 3.74 \\ \text{Ymin} = 0 & \text{Ymax} = 170 \end{array}$$

What happens to the demand for Mom's Bread as the price increases?

(c) Add a row to your table to show the daily revenue from Mom's Bread at various prices.

p	0.25	0.50	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
Demand											
Revenue											

(d) Using the formula for $D(p)$, write an expression $R(p)$ that approximates the total daily revenue as a function of the price, p.

(e) Graph the revenue function $R(p)$ in the same window with $D(p)$. Estimate the maximum possible revenue. Does the maximum for $D(p)$ occur at the same value of p as the maximum for $R(p)$?

(f) Find the horizontal asymptote of the graphs. What does it represent in this context?

7.4.7.7. A computer store sells approximately 300 of its most popular model per year. The manager would like to minimize her annual inventory cost by ordering the optimal number of computers, x, at regular intervals. If she orders x computers in each shipment, the cost of storage will be $6x$ dollars, and the cost of reordering will be $\dfrac{300}{x}(15x+10)$ dollars. The inventory cost is the sum of the storage cost and the reordering cost.

(a) Use the distributive law to simplify the expression for the reordering cost. Then express the inventory cost, C, as a function of x.

(b) Complete the table of values for the inventory cost for various reorder sizes.

x	10	20	30	40	50	60	70	80	90	100
C										

(c) Graph the function C for the cost per calculator. Use the window

$$\begin{array}{ll} \text{Xmin} = 0 & \text{Xmax} = 150 \\ \text{Ymin} = 4500 & \text{Ymax} = 5500 \end{array}$$

Estimate the minimum possible value for C.

(d) How many computers should the manager order in each shipment so as to minimize the inventory cost? How many orders will she make during the year?

(e) Graph the function $y = 6x + 4500$ in the same window with the function C. What do you observe?

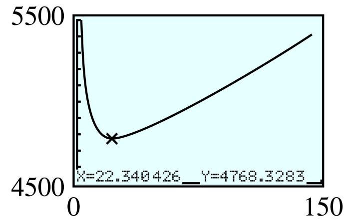
Answer.

(a) $4500 + \dfrac{3000}{x}$; $C(x) = 6x + 4500 + \dfrac{3000}{x}$

(b)

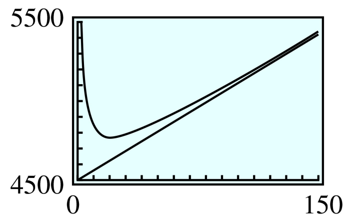
x	10	20	30	40	50	60	70	80	90	100
C	4860	4770	4780	4815	4860	4910	5018	5073	5130	

(c) \$4768.33



(d) 22; 14

(e)



The graph of C approaches the line as an asymptote.

7.4.7.8. A chain of electronics stores sells approximately 500 portable phones every year. The owner would like to minimize his annual inventory cost by ordering the optimal number of phones, x, at regular intervals. The cost of storing the phones will then be $2x$ dollars, and the cost of reordering will be $\dfrac{500}{x}(4x + 10)$. The total annual inventory cost is the sum of the storage cost and the reordering cost.

(a) Use the distributive law to simplify the expression for the reordering cost. Then express the inventory cost, C, as a function of x.

(b) Complete the table of values for the inventory cost for various reorder sizes.

x	10	20	30	40	50	60	70	80	90	100
C										

(c) Graph the function C in the window

Xmin $= 0$	Xmax $= 150$
Ymin $= 2000$	Ymax $= 2500$

Estimate the minimum possible value for C.

(d) How many portable phones should the manager order in each shipment so as to minimize the inventory cost? How many orders will he make during the year?

(e) Graph the function $y = 2x + 2000$ in the same window with the function C. What do you observe?

7.4.7.9. Francine wants to make a rectangular box. In order to simplify construction and keep her costs down, she plans for the box to have a square base and a total surface area of 96 square centimeters. She would like to know the largest volume that such a box can have.

(a) If the square base has length x centimeters, show that the height of the box is $h = \dfrac{24}{x} - \dfrac{x}{2}$ centimeters. (*Hint*: The surface area of the box is the sum of the areas of the six sides of the box.)

(b) Write an expression for the volume, V, of the box as a function of the length, x, of its base.

(c) Complete the table showing the heights and volumes of the box for various base lengths.

x	1	2	3	4	5	6	7
h							
V							

Explain why the values of h and V are negative when $x = 7$.

(d) Graph your expression for volume $V(x)$ in an appropriate window. Approximate the maximum possible volume for a box of surface area 96 square centimeters.

(e) What value of x gives the maximum volume?

(f) Graph the height, $h(x)$, in the same window with $V(x)$. What is the height of the box with greatest volume? (Find the height directly from your graph and verify by using the formula given for $h(x)$.)

Answer.

(a) The surface area is $2x^2+4xh = 96$. Solving for h, $h = \dfrac{96-2x^2}{4x} = \dfrac{24}{x} - \dfrac{x}{2}$.

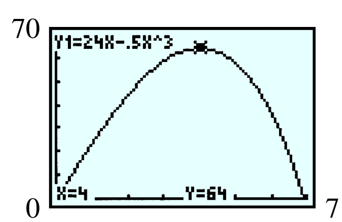
(b) $V = 24x - \dfrac{1}{2}x^3$

(c)

x	1	2	3	4	5	6	7
h	23.5	11	6.5	4	2.3	1	-0.07
V	23.5	44	58.5	64	57.5	36	-3.5

If the base is more than 7 cm, the top and bottom alone exceed the total area allowed.

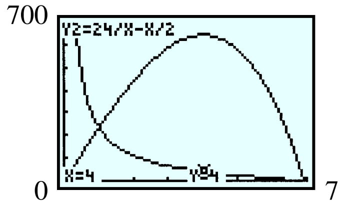
(d)



Maximum of 64 cu. cm

(e) 4 cm

(f) $h = 4$ cm



7.4.7.10. Delbert wants to make a box with a square base and a volume of 64 cubic centimeters. He would like to know the smallest surface area that such a box can have.

(a) If the square base has length x centimeters, show that the height of the box is $h = \dfrac{64}{x^2}$ centimeters.

(b) Write an expression for the surfacd area, S, of the box as a function of the length, x, of its base. (*Hint*: The surface area of the box is the sum of the areas of the six sides of the box.)

(c) Complete the table showing the heights and surface areas of the box for various base lengths.

x	1	2	3	4	5	6	7	8
h								
S								

(d) Graph your expression for surface area $S(x)$ in an appropriate window. Approximate the minimum possible surface area for Delbert's box.

(e) What value of x gives the minimum surface area?

(f) Graph the height, $h(x)$, in the same window with $S(x)$. What is the height of the box with smallest surface area? (Find the height directly from your graph and verify by using the formula given for $h(x)$.)

7.4.7.11. A train whistle sounds higher when the train is approaching you than when it is moving away from you. This phenomenon is known as the Doppler effect. If the actual pitch of the whistle is 440 hertz (this is the A note below middle C), then the note you hear will have the pitch

$$P(v) = \frac{440(332)}{332 - v}$$

where the velocity, v, in meters per second is positive as the train approaches and is negative when the train is moving away. (The number 332 that appears in this expression is the speed of sound in meters per second.)

(a) Complete the table of values showing the pitch of the whistle at various train velocities.

v	-100	-75	-50	-25	0	25	50	75	100
P									

(b) Graph the function P. (Use the window Xmin $= -94$, Xmax $= 94$, and appropriate values of **Ymin** and **Ymax**.)

(c) What is the velocity of the train if the note you hear has a pitch of 415 hertz (corresponding to the note A-flat)? A pitch of $553.\overline{3}$ hertz (C-sharp)?

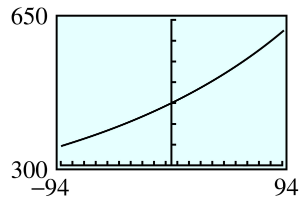
(d) For what velocities will the pitch you hear be greater than 456.5 hertz?

(e) The graph has a vertical asymptote (although it is not visible in the suggested window). Where is it and what is its significance in this context?

Answer.

(a)

v	-100	-75	-50	-25	0	25	50	75	100
P	338.15	358.92	382.41	409.19	440	475.83	518.01	568.4	629.66

(b) 

(c) -20 m/sec; 68 m/sec

(d) $v > 12$ m/sec

(e) $v = 332$; As v approaches 332 m per sec, the pitch increases without bound.

7.4.7.12. The maximum altitude (in meters) attained by a projectile shot from the surface of the Earth is

$$h(v) = \frac{6.4 \times 10^6 v^2}{19.6 \cdot 6.4 \times 10^6 - v^2}$$

where v is the speed (in meters per second) at which the projectile was launched. (The radius of the Earth is 6.4×10^6 meters, and the constant 19.6 is related to the Earth's gravitational constant.)

(a) Complete the table of values showing the maximum altitude for various launch velocities.

v	100	200	300	400	500	600	700	800	900	1000
h										

(b) Graph the function h. (Use the window Xmin = 0, Xmax = 940, and appropriate values of **Ymin** and **Ymax**.)

(c) Approximately what speed is needed to attain an altitude of 4000 meters? An altitude of 16 kilometers?

(d) For what velocities will the projectile attain an altitude exceeding 32 kilometers?

(e) The graph has a vertical asymptote (although it is not visible in the suggested window). Where is it and what is its significance in this context?

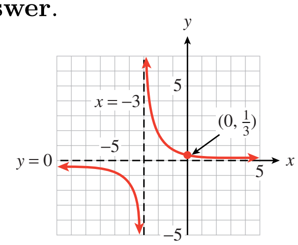
For Problems 13–30,

a Find the horizontal and vertical asymptotes for each function.

b Find the x- and y-intercepts for each function.

7.4.7.13. $y = \dfrac{1}{x+3}$

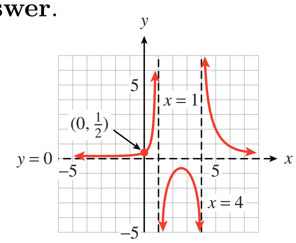
Answer.



7.4.7.14. $y = \dfrac{1}{x-3}$

7.4.7.15. $y = \dfrac{2}{x^2-5x+4}$

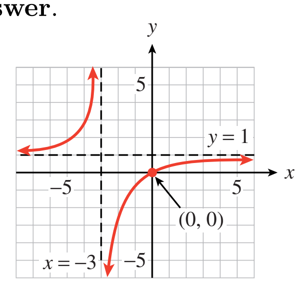
Answer.



7.4.7.16. $y = \dfrac{4}{x^2-x-6}$

7.4.7.17. $y = \dfrac{x}{x-3}$

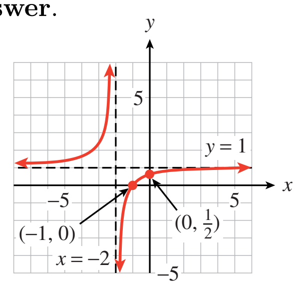
Answer.



7.4.7.18. $y = \dfrac{x}{x-2}$

7.4.7.19. $y = \dfrac{x+1}{x+2}$

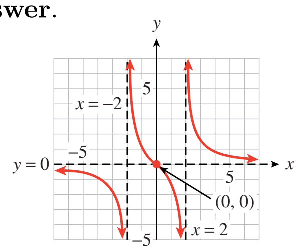
Answer.



7.4.7.20. $y = \dfrac{x-1}{x-3}$

7.4.7.21. $y = \dfrac{2x}{x^2-4}$

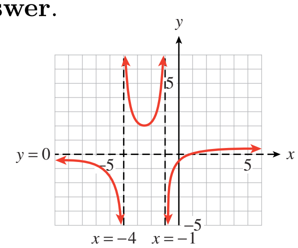
Answer.



7.4.7.22. $y = \dfrac{x}{x^2-9}$

7.4.7.23. $y = \dfrac{x-2}{x^2+5x+4}$

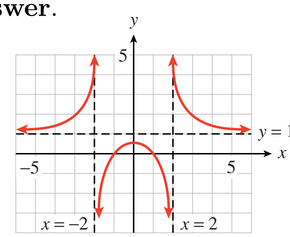
Answer.



7.4.7.24. $y = \dfrac{x+1}{x^2-x-6}$

7.4.7.25. $y = \dfrac{x^2-1}{x^2-4}$

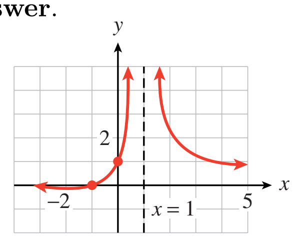
Answer.



7.4.7.26. $y = \dfrac{2x^2}{x^3-1}$

7.4.7.27. $y = \dfrac{x+1}{(x-1)^2}$

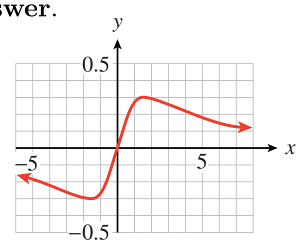
Answer.



7.4.7.28. $y = \dfrac{2(x^2-1)}{x^2+4}$

7.4.7.29. $y = \dfrac{x}{x^2+3}$

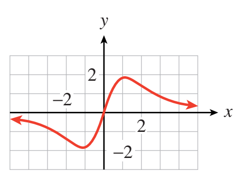
Answer.



7.4.7.30. $y = \dfrac{x^2+2}{x^2+4}$

7.4.7.31. Graph the curve known as Newton's Serpentine: $y = \dfrac{4x}{x^2+1}$.

Answer.



7.4.7.32. Graph the curve known as the Witch of Agnesi: $y = \dfrac{8}{x^2+4}$.

For Problems 33–38,

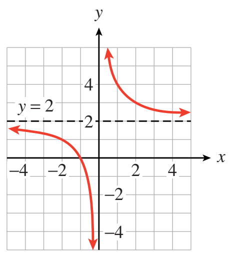
a Use polynomial division to write the fraction in the form $y = \dfrac{k}{p(x)} + c$, where k and c are constants.

b Use transformations to sketch the graph.

7.4.7.33. $y = \dfrac{2x+2}{x}$

Answer.

a $y = \dfrac{2}{x} + 2$

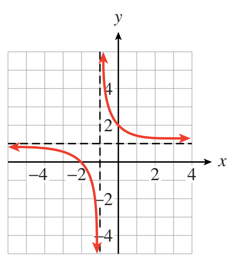
b 

7.4.7.34. $y = \dfrac{4x^2+3}{x^2}$

7.4.7.35. $y = \dfrac{x+2}{x+1}$

Answer.

a $y = \dfrac{1}{x+1} + 1$

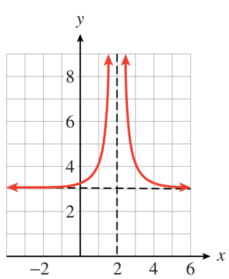
b 

7.4.7.36. $y = \dfrac{7-2x}{x-3}$

7.4.7.37. $y = \dfrac{3x^2-12x+13}{(x-2)^2}$

Answer.

a $y = \dfrac{1}{(x-2)^2} + 3$

b 

7.4.7.38. $y = \dfrac{-4x^2+8x-3}{(x-1)^2}$

Problems 39–45 involve operations on algebraci fractions. To review operations on algebraic fractions, see Algebra Skills Refresher Section A.9.

7.4.7.39. River Queen Tours offers a 50-mile round-trip excursion on the Mississippi River on a paddle wheel boat. The current in the Mississippi is 8 miles per hour.

(a) Express the time required for the downstream journey as a function of the speed of the paddle wheel boat in still water.

(b) Write a function for the time required for the return trip upstream.

(c) Write and simplify an expression for the time needed for the round trip as a function of the boat's speed.

Answer.

(a) $\dfrac{25}{s+8}$ (b) $\dfrac{25}{s-8}$ (c) $\dfrac{50s}{s^2-64}$

7.4.7.40. A rowing team can maintain a speed of 15 miles per hour in still water. The team's daily training session includes a 5-mile run up the Red Cedar River and the return downstream.

(a) Express the team's time on the upstream leg as a function of the speed of the current.

(b) Write a function for the team's time on the downstream leg.

(c) Write and simplify an expression for the total time for the training run as a function of the current's speed.

7.4.7.41. Two pilots for the Flying Express parcel service receive packages simultaneously. Orville leaves Boston for Chicago at the same time Wilbur leaves Chicago for Boston. Each selects an airspeed of 400 miles per hour for the 900-mile trip. The prevailing winds blow from east to west.

(a) Express Orville's flying time as a function of the windspeed.

(b) Write a function for Wilbur's flying time.

(c) Who reaches his destination first? By how much time (in terms of windspeed)?

Answer.

(a) $\dfrac{900}{400+w}$

(b) $\dfrac{900}{400-w}$

(c) Orville by $\dfrac{1800w}{160,000-w^2}$ hours

7.4.7.42. On New Year's Day, a blimp leaves its berth in Carson, California, and heads north for the Rose Bowl, 23 miles away. There is a breeze from the north at 6 miles per hour.

(a) Express the time required for the trip as a function of the blimp's airspeed.

(b) Write a function for the time needed for the return trip.

(c) Which trip takes longer? By how much time (in terms of the blimp's airspeed)?

7.4.7.43. The focal length of a lens is given by the formula

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

where f stands for the focal length, p is the distance from the object viewed to the lens, and q is the distance from the image to the lens. Suppose you estimate that the distance from your cat (the object viewed) to your camera lens is 60 inches greater than the distance from the lens to the film inside the camera, where the image forms.

(a) Express $1/f$ as a single fraction in terms of q.

(b) Write an expression for f as a function of q.

Answer.

(a) $\dfrac{1}{f} = \dfrac{2q+60}{q^2+60q}$ (b) $f = \dfrac{q^2+60q}{2q+60}$

7.4.7.44. If two resistors, R_1 and R_2, in an electrical circuit are connected in parallel, the total resistance R in the circuit is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

(a) Suppose that the second resistor, R_2, is 10 ohms greater than the first. Express $1/R$ as a single fraction in terms of R_1.

(b) Write an expression for R as a function of R_1.

7.4.7.45.

(a) Show that the equation $\dfrac{1}{y} - \dfrac{1}{x} = \dfrac{1}{k}$ is equivalent to $y = \dfrac{kx}{x+k}$ on their common domain.

(b) Graph the functions $y = \dfrac{kx}{x+k}$ for $k = 1, 2$, and 3 in the window

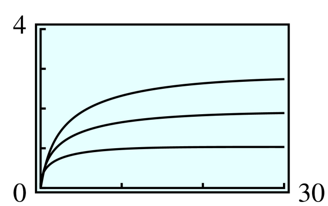
Xmin $= 0$	Xmax $= 30$
Ymin $= 0$	Ymax $= 4$

Describe the graphs.

Answer.

(a) $\dfrac{1}{y} = \dfrac{1}{x} + \dfrac{1}{k} = \dfrac{k+x}{xk}$, so by taking reciprocals, $y = \dfrac{kx}{x+k}$.

(b)



The graphs increase from the origin and approach a horizontal asymptote at $y = k$.

7.4.7.46. Consider the graph of $y = \dfrac{ax}{x+k}$, where a and k are positive constants.

(a) What is the horizontal asymptote of the graph?

(b) Show that for $x = k$, $y = \dfrac{a}{2}$.

(c) Sketch the graph of $y = \dfrac{ax}{x+k}$ for $a = 4$ and $k = 10$ in the window

Xmin = 0	Xmax = 60
Ymin = 0	Ymax = 5

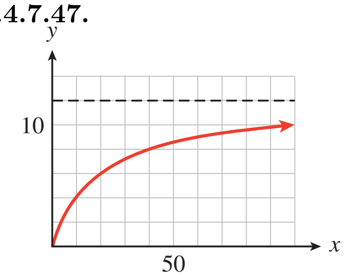
Illustrate your answers to parts (a) and (b) on the graph.

For Problems 47–48,

(a) Use your answers to Problem 46 to find equations of the form $y = \dfrac{ax}{x+k}$ for the graphs shown.

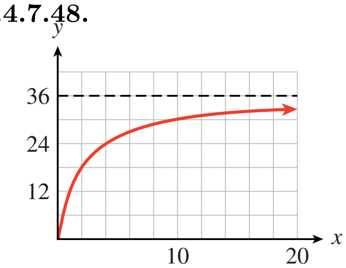
(b) Check your answer with a graphing calculator.

7.4.7.47.



Answer. $\dfrac{12x}{x+20}$

7.4.7.48.



7.4.7.49. The Michaelis-Menten equation is the rate equation for chemical reactions catalyzed by enzymes. The speed of the reaction v is a function of the initial concentration of the reactant s and is given by

$$v = f(s) = \frac{Vs}{s+K}$$

where V is the maximum possible reaction rate and K is called the Michaelis constant. (Source: Holme and Peck, 1993)

(a) What value does v approach as s increases?

(b) What is the value of v when $s = K$?

(c) The table gives data from reactions of the enzyme D-amino acid oxidase.

s	0.33	0.66	1.00	1.66	2.50	3.33	6.66
v	0.08	0.14	0.20	0.30	0.39	0.46	0.58

Plot the data and estimate the values of V and K from your graph.

(d) Graph the function $v = \dfrac{0.88s}{s+3.34}$ on top of your data points.

(e) For a fixed s and V, what happens to v if K is very big?

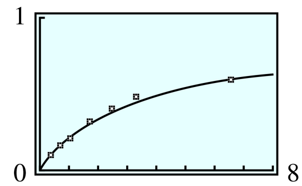
(f) For a fixed K and V, what happens to v if s is very big?

Answer.

(a) V

(b) $\dfrac{V}{2}$

(c)



$V \approx 0.7$, $K \approx 2.2$ (many answers are possible)

(d) (See figure.)

7.4.7.50. Show that

$$\frac{1}{v} = \frac{1}{V} + \frac{K}{Vs}$$

is another form of the Michaelis-Menten equation. (See Problem 49.)

7.4.7.51.

(a) Refer to the Michaelis-Menten equation in Problem 49. Solve for $\dfrac{1}{v}$, then write your new equation in the form $\dfrac{1}{v} = a \cdot \dfrac{1}{s} + b$. Express a and b in terms of V and K.

(b) Use the data from part (c) of Problem 49 to make a table of values for $\left(\dfrac{1}{s}, \dfrac{1}{v}\right)$.

(c) Plot the points $\left(\dfrac{1}{s}, \dfrac{1}{v}\right)$, then use linear regression to find the line of best fit.

(d) Use your values for a and b to solve for V and K.

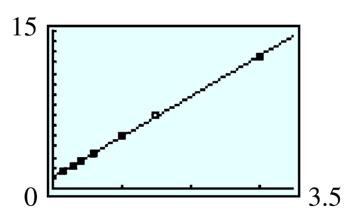
Answer.

(a) $\dfrac{1}{v} = \dfrac{K}{V} \cdot \dfrac{1}{s} + \dfrac{1}{V}$; Therefore, $a = \dfrac{K}{V}$ and $b = \dfrac{1}{V}$

(b)

$\frac{1}{s}$	3	1.5	1	0.6	0.4	0.3	0.15
$\frac{1}{v}$	12.5	7.1	5	3.3	2.6	2.2	1.7

(c)



$\dfrac{1}{v} = 3.8 \cdot \dfrac{1}{s} + 1.1$

(d) $V \approx 0.89$, $K \approx 3.37$

7.4.7.52.

(a) Refer to the Michaelis-Menten equation in Problem 49. Write an equation for $\dfrac{s}{v}$ in the form $\dfrac{s}{v} = cs + d$. Express c and d in terms of V and K.

(b) Use the data from part (c) of Problem 49 to make a table of values for

$\left(s, \dfrac{s}{v}\right)$.

(c) Plot the points $\left(s, \dfrac{s}{v}\right)$, then use linear regression to find the line of best fit.

(d) Use your values for c and d to solve for V and K.

Problems 53-56 give examples of functions whose graphs have holes.

a Find the domain of the function.

b Reduce the fraction to lowest terms.

c Graph the function. (*Hint*: The graph of the original function is identical to the graph of the function in part (b) except that certain points are excluded from the domain.) Indicate a hole in the graph by an open circle.

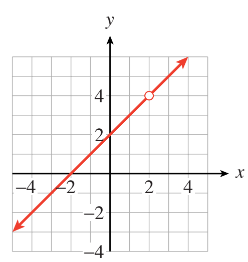
7.4.7.53. $y = \dfrac{x^2-4}{x-2}$

Answer.

(a) $x \neq 2$

(b) $x+2$

(c)



7.4.7.54. $y = \dfrac{x^2-1}{x+1}$

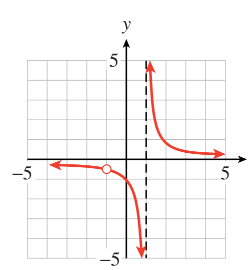
7.4.7.55. $y = \dfrac{x+1}{x^2-1}$

Answer.

(a) $x \neq \pm 1$

(b) $\dfrac{1}{x-1}$

(c)



7.4.7.56. $y = \dfrac{x-3}{x^2-9}$

7.5 Equations That Include Algebraic Fractions

7.5.1 Solving Equations with Fractions Algebraically

Checkpoint 7.5.2 Solve $\dfrac{x^2}{x+4} = 2$

Answer. $x = -2,\ x = 4$

Checkpoint 7.5.4 Solve $\dfrac{x}{6-x} = \dfrac{1}{2}$

Answer. $x = 2$

7.5.2 Extraneous Solutions

Checkpoint 7.5.6 Solve $\dfrac{9}{x^2+x-2} + \dfrac{1}{x^2-4} = \dfrac{4}{x-1}$

Answer. $x = \dfrac{-1}{2}$

7.5.3 Formulas

Checkpoint 7.5.8 Solve for a: $\dfrac{2ab}{a+h} = H$

Answer. $a = \dfrac{bH}{2b-H}$

7.5.5 Homework 7.5

For Problems 1-8, solve the equation algebraically.

7.5.5.1. $\dfrac{6}{w+2} = 4$

Answer. $\dfrac{-1}{2}$

7.5.5.2. $\dfrac{12}{r-7} = 3$

7.5.5.3. $9 = \dfrac{h-5}{h-2}$

Answer. $\dfrac{13}{8}$

7.5.5.4. $-3 = \dfrac{v+1}{v-6}$

7.5.5.5. $\dfrac{15}{s^2} = 8$

Answer. $\pm\sqrt{\dfrac{15}{8}}$

7.5.5.6. $\dfrac{3}{m^2} = 5$

7.5.5.7. $4.3 = \sqrt{\dfrac{18}{y}}$

Answer. $\dfrac{1800}{1849} \approx 0.97$

7.5.5.8. $6.5 = \dfrac{52}{\sqrt{z}}$

7.5.5.9. The total weight, S, that a beam can support is given in pounds by

$$S = \frac{182.6wh^2}{l}$$

where w is the width of the beam in inches, h is its height in inches, and l is the length of the beam in feet. A beam over the doorway in an interior wall of a house must support 1600 pounds. If the beam is 4 inches wide and 9 inches tall, how long can it be?

Answer. 37 ft

7.5.5.10. If two appliances are connected in parallel in an electrical circuit, the total resistance, R, in the circuit is given by

$$R = \frac{ab}{a+b}$$

where a and b are the resistances of the two appliances. If one appliance has a resistance of 18 ohms, and the total resistance in the circuit is measured at 12 ohms, what is the resistance of the second appliance?

7.5.5.11. A flock of eider ducks is making a 150-mile flight at an average airspeed of 50 miles per hour against a moderate headwind.

(a) Express the ducks' travel time, t, as a function of the windspeed, v, and graph the function in the window

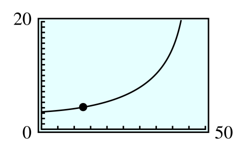
$$\text{Xmin} = 0 \qquad \text{Xmax} = 50$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 20$$

(See Problem 7.4.7.1 of Homework Section 7.4 7.4.7.)

(b) Write and solve an equation to find the windspeed if the flock makes its trip in 4 hours. Label the corresponding point on your graph.

Answer.

(a) $t = \dfrac{150}{50 - v}$



(b) $4 = \dfrac{150}{50 - v}$; $v = 12.5$ mph

7.5.5.12. Bluefin tuna swim at average speed of 36 miles per hour in still water. A school of tuna is making a 200-mile trip against a current.

(a) Express the tuna's travel time, t, as a function of the current speed, v, and graph the function in the window

$$\text{Xmin} = 0 \qquad \text{Xmax} = 36$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 50$$

(See Problem 7.4.7.2 of Homework Section 7.4 7.4.7.)

(b) Write and solve an equation to find the current speed if the school makes its trip in 8 hours. Label the corresponding point on your graph.

7.5.5.13. The cost, in thousands of dollars, for immunizing p percent of the residents of Emporia against a dangerous new disease is given by the function

$$C(p) = \frac{72p}{100 - p}$$

Write and solve an equation to determine what percent of the population can be immunized for $168,000.

Answer. $168 = \dfrac{72p}{100 - p}$; $p = 70\%$

7.5.5.14. The cost, in thousands of dollars, for extracting p percent of a precious ore from a mine is given by the function

$$C(p) = \frac{360p}{100 - p}$$

Write and solve an equation to determine what percentage of the ore can be extracted for $\$390,000$.

For Problems 15–18,

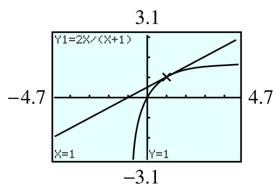
a Solve the equation graphically by graphing two functions, one for each side of the equation.

b Solve the equation algebraically.

7.5.5.15. $\dfrac{2x}{x+1} = \dfrac{x+1}{2}$

Answer.

(a)



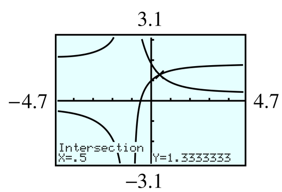
(b) $x = 1$

7.5.5.16. $\dfrac{3}{2x+1} = \dfrac{2x-3}{x}$

7.5.5.17. $\dfrac{2}{x+1} = \dfrac{x}{x+1} + 1$

Answer.

(a)



(b) $x = \dfrac{1}{2}$

7.5.5.18. $\dfrac{5}{x-3} = \dfrac{x+2}{x-3} + 3$

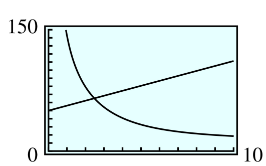
7.5.5.19. The manager of Joe's Burgers discovers that he will sell $\dfrac{160}{x}$ burgers per day if the price of a burger is x dollars. On the other hand, he can afford to make $6x + 49$ burgers if he charges x dollars apiece for them.

(a) Graph the **demand function**, $D(x) = \dfrac{160}{x}$, and the **supply function**, $S(x) = 6x + 49$. At what price x does the demand for burgers equal the number that Joe can afford to supply? This value for x is called the **equilibrium price**.

(b) Write and solve an equation to verify your equilibrium price.

Answer.

(a)



$\$2.50$

(b) $\dfrac{160}{x} = 6x + 49$; $x = 2.50$

7.5.5.20. A florist finds that she will sell $\dfrac{300}{x}$ dozen roses per week if she charges x dollars for a dozen. Her suppliers will sell her $5x - 55$ dozen roses if she sells them at x dollars per dozen.

(a) Graph the demand function, $D(x) = \dfrac{300}{x}$, and the supply function, $S(x) = 5x - 55$, in the same window. At what equilibrium price x will the florist sell all the roses she purchases?

(b) Write and solve an equation to verify your equilibrium price.

7.5.5.21. Francine wants to fence a rectangular area of 3200 square feet to grow vegetables for her family of three.

(a) Express the length of the garden as a function of its width.

(b) Express the perimeter, P, of the garden as a function of its width.

(c) Graph your function for perimeter and find the coordinates of the lowest point on the graph. Interpret those coordinates in the context of the problem.

(d) Francine has 240 feet of chain link to make a fence for the garden, and she would like to know what the width of the garden should be. Write an equation that describes this situation.

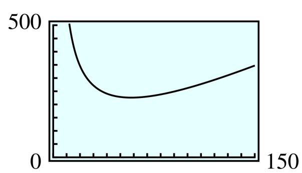
(e) Solve your equation and find the dimensions of the garden.

Answer.

(a) $L = \dfrac{3200}{w}$

(b) $P = \dfrac{6400}{w} + 2w$

(c)



Lowest point: $(56.6, 226)$; The minimum perimeter is 226 ft for a width of 56.6 ft.

(d) $240 = \dfrac{6400}{w} + 2w$

(e) 40 ft by 80 ft

7.5.5.22. The cost of wire fencing is \$7.50 per foot. A rancher wants to enclose a rectangular pasture of 1000 square feet with this fencing.

(a) Express the length of the pasture as a function of its width.

(b) Express the cost of the fence as a function of its width.

(c) Graph your function for the cost and find the coordinates of the lowest point on the graph. Interpret those coordinates in the context of the problem.

(d) The rancher has \$1050 to spend on the fence, and she would like to know what the width of the pasture should be. Write an equation to describe this situation.

(e) Solve your equation and find the dimensions of the pasture.

7.5.5.23. A proportion is an equation in which each side is a ratio: $\dfrac{a}{b} = \dfrac{c}{d}$. Show that this equation may be rewritten as $ad = bc$.

Answer. Multiply both sides of the equation by bd and simplify.

$$\frac{a}{b} \cdot \frac{bc}{1} = \frac{c}{d} \cdot \frac{bc}{1}, \text{ so } ac = bd$$

7.5.5.24. Suppose that y varies directly with x, and (a, b) and (c, d) are two points on the graph of y in terms of x. Show that $\dfrac{b}{a} = \dfrac{d}{c}$.

For Problems 25-28, solve the proportion using your result from Problem 23.

7.5.5.25. $\dfrac{3}{4} = \dfrac{y+2}{12-y}$

Answer. 4

7.5.5.26. $\dfrac{-3}{4} = \dfrac{y-7}{y+14}$

7.5.5.27. $\dfrac{50}{r} = \dfrac{75}{r+20}$

Answer. 40

7.5.5.28. $\dfrac{30}{r} = \dfrac{20}{r-10}$

For Problems 29-36, use your result from Problem 24 to write and solve a proportion for the problem.

7.5.5.29. Property taxes on a house vary directly with the value of the house. If the taxes on a house worth $\$120,000$ are $\$2700$, what would the taxes be on a house assessed at $\$275,000$?

Answer. $6187.50

7.5.5.30. The cost of electricity varies directly with the number of units (BTUs) consumed. If a typical household in the Midwest uses 83 million BTUs of electricity annually and pays $1236, how much will a household that uses 70 million BTUs annually spend for energy?

7.5.5.31. Distances on a map vary directly with actual distances. The scale on a map of Michigan uses $\frac{3}{8}$ inch to represent 10 miles. If Isle Royale is $1\frac{11}{16}$ inches long on the map, what is the actual length of the island?

Answer. 45 mi

7.5.5.32. The dimensions of an enlargement vary directly with the dimensions of the original. A photographer plans to enlarge a photograph that measures 8.3 centimeters by 11.2 centimeters to produce a poster that is 36 centimeters wide. How long will the poster be?

7.5.5.33. The Forest Service tags 200 perch and releases them into Spirit Lake. One month later, it captures 80 perch and finds that 18 of them are tagged. What is the Forest Service's estimate of the original perch population of the lake?

Answer. 689

7.5.5.34. The Wildlife Commission tags 30 Canada geese at one of its migratory feeding grounds. When the geese return, the commission captures 45 geese, of which 4 are tagged. What is the commission's estimate of the number of geese that use the feeding ground?

7.5.5.35. The highest point on Earth is Mount Everest in Tibet, with an elevation of 8848 meters. The deepest part of the ocean is the Challenger Deep in the Mariana Trench, near Indonesia, $11,034$ meters below sea level.

(a) What is the total height variation in the surface of the Earth?

(b) What percentage of the Earth's radius, 6400 kilometers, is this variation?

(c) If the Earth were shrunk to the size of a basketball, with a radius of 4.75 inches, what would be the corresponding height of Mount Everest?

Answer.

(a) $19,882$ m (b) 0.3% (c) 0.00657 in

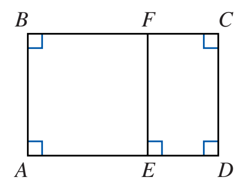
7.5.5.36. Shortly after the arrival of human beings at the Hawaiian islands around 400 A.D., many species of birds became extinct. Fossils of 29 different species have been found, but some species may have left no fossils for us to find. We can estimate the total number of extinct species using a proportion. Of 9 species that are still alive, biologists have found fossil evidence of 7. (Source: Burton, 1998)

(a) Assuming that the same fraction of extinct species have left fossil records, calculate the total number of extinct species

(b) Give two reasons why this estimate may not be completely accurate.

7.5.5.37. In the figure, the rectangle $ABCD$ is divided into a square and a smaller rectangle, $CDEF$. The two rectangles $ABCD$ and $CDEF$ are similar (their corresponding sides are proportional.) A rectangle $ABCD$ with this property is called a **golden rectangle**, and the ratio of its length to its width is called the golden ratio.

The golden ratio appears frequently in art and nature, and it is considered to give the most pleasing proportions to many figures. We will compute the golden ratio as follows.



(a) Let $AB = 1$ and $AD = x$. What are the lengths of AE, DE, and CD?

(b) Write a proportion in terms of x for the similarity of rectangles $ABCD$ and $CDEF$. Be careful to match up the corresponding sides.

(c) Solve your proportion for x. Find the golden ratio, $\dfrac{AD}{AB} = \dfrac{x}{1}$.

Answer.

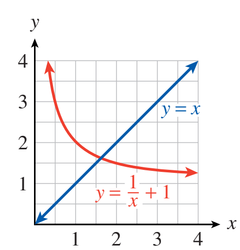
(a) $AE = 1,\ DE = x - 1,\ CD = 1$

(b) $\dfrac{1}{x} = \dfrac{x-1}{x}$

(c) $\dfrac{1+\sqrt{5}}{2}$

7.5.5.38.

The figure shows the graphs of two equations, $y = x$ and $y = \dfrac{1}{x} + 1$.



(a) Find the x-coordinate of the intersection point of the two graphs.

(b) Compare your answer to the golden ratio you computed in Problem 37.

For Problems 39-46, solve the formula for the specified variable and answer any additional questions.

7.5.5.39. $S = \dfrac{a}{1-r}$, for r

Answer. $r = \dfrac{S-a}{S}$

7.5.5.40. $I = \dfrac{E}{r+R}$, for R

7.5.5.41. $H = \dfrac{2xy}{x+y}$, for x

Answer. $x = \dfrac{Hy}{2y-H}$

7.5.5.42. $M = \dfrac{ab}{a+b}$, for b

7.5.5.43. $F = \dfrac{Gm_1m_2}{d^2}$, for d. What happens to F as d> gets big?

Answer. $d = \pm\sqrt{\dfrac{Gm_1m_2}{F}}$

7.5.5.44. $F = \dfrac{kq_1q_2}{r^2}$, for q_2

7.5.5.45. $\dfrac{1}{Q} + \dfrac{1}{I} = \dfrac{2}{r}$, for r

Answer. $r = \dfrac{2QI}{I+Q}$

7.5.5.46. $\dfrac{1}{R} = \dfrac{1}{A} + \dfrac{1}{B}$, for B

7.5.5.47. The sidereal period of a planet is the time for the planet to make one trip around the Sun (as seen from the Sun itself). The synodic period is the time between two successive conjunctions of the planet and the Sun, as seen from Earth. The relationship among the sidereal period, P, of a planet, the synodic period, S, of the planet, and the sidereal period of Earth, E, is given by

$$\frac{1}{P} = \frac{1}{S} + \frac{1}{E}$$

when the planet is closer to the Sun than the Earth is. Solve for P in terms of S and E.

Answer. $P = \dfrac{ES}{E+S}$

7.5.5.48. When a planet is farther from the Sun than Earth is,

$$\frac{1}{P} = \frac{1}{E} - \frac{1}{S}$$

where P, E, and S are as defined in Problem 47. Solve for P in terms of S and E.

For Problems 49-56, solve the equation algebraically.

7.5.5.49. $\dfrac{3}{x-2} = \dfrac{1}{2} + \dfrac{2x-7}{2x-4}$

Answer. 5

7.5.5.50. $\dfrac{2}{x+1} + \dfrac{1}{3x+3} = \dfrac{1}{6}$

7.5.5.51. $\dfrac{4}{x+2} - \dfrac{1}{x} = \dfrac{2x-1}{x^2+2x}$

Answer. 1

7.5.5.52. $\dfrac{1}{x-1} + \dfrac{2}{x+1} = \dfrac{x-2}{x^2-1}$

7.5.5.53. $\dfrac{x}{x+2} - \dfrac{3}{x-2} = \dfrac{x^2+8}{x^2-4}$

Answer. $\dfrac{-14}{5}$

7.5.5.54. $\dfrac{4}{2x-3} + \dfrac{4x}{4x^2-9} = \dfrac{1}{2x+3}$

7.5.5.55. $\dfrac{4}{3x} + \dfrac{3}{3x+1} + 2 = 0$

Answer. $\dfrac{-1}{6}, \dfrac{-4}{3}$

7.5.5.56. $-3 = \dfrac{-10}{x+2} + \dfrac{10}{x+5}$

7.5.5.57. A chartered sightseeing flight over the Grand Canyon is scheduled to return to its departure point in 3 hours. The pilot would like to cover a distance of 144 miles before turning around, and he hears on the Weather Service that there will be a headwind of 20 miles per hour on the outward journey.

(a) Express the time it takes for the outward journey as a function of the airspeed of the plane.

(b) Express the time it takes for the return journey as a function of the speed of the plane.

(c) Graph the sum of the two functions and find the point on the graph with y-coordinate 3. Interpret the coordinates of the point in the context of the problem.

(d) The pilot would like to know what airspeed to maintain in order to complete the tour in 3 hours. Write an equation to describe this situation.

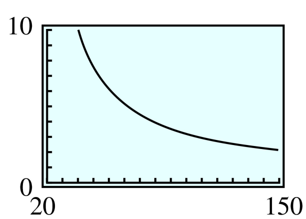
(e) Solve your equation to find the appropriate airspeed.

Answer.

(a) $t_1 = \dfrac{144}{s-20}$

(b) $t_2 = \dfrac{144}{s+20}$

(c)



If the airspeed is 100 mph, the round trip will take 3 hours.

(d) $\dfrac{144}{s-20} + \dfrac{144}{s+20} = 3$

(e) 100 mph

7.5.5.58. Two student pilots leave the airport at the same time. They both fly at an airspeed of 180 miles per hour, but one flies with the wind and the other flies against the wind.

(a) Express the time it takes the first pilot to travel 500 miles as a function of the windspeed.

(b) Express the time it takes the second pilot to travel 400 miles as a function of the windspeed.

(c) Graph the two functions in the same window, and find the coordinates of the intersection point. Interpret those coordinates in the context of the problem.

(d) Both pilots check in with their instructors at the same time, and the first pilot has traveled 500 miles while the second pilot has gone 400 miles. Write an equation to describe this situation.

(e) Solve your equation to find the speed of the wind.

7.5.5.59. Andy drives 300 miles to Lake Tahoe at 70 miles per hour and returns home at 50 miles per hour. What is his average speed for the round trip? (It is not 60 miles per hour!)

(a) Write expressions for the time it takes for each leg of the trip if Andy drives a distance, d, at speed r_1 and returns at speed r_2.

(b) Write expressions for the total distance and total time for the trip.

(c) Write an expression for the average speed for the entire trip.

(d) Write your answer to part (c) as a simple fraction.

(e) Use your formula to answer the question stated in the problem.

Answer.

(a) $t_1 = \dfrac{d}{r_1},\ t_2 = \dfrac{d}{r_2}$

(b) Total distance is $2d$; total time $\dfrac{d}{r_1} + \dfrac{d}{r_2}$.

(c) $\dfrac{2d}{\dfrac{d}{r_1} + \dfrac{d}{r_2}}$

(d) $\dfrac{2r_1r_2}{r_1 + r_2}$

(e) $58\frac{1}{3}$ mph

7.5.5.60. The owner of a print shop volunteers to produce flyers for his candidate's campaign. His large printing press can complete the job in 4 hours, and the smaller model can finish the flyers in 6 hours. How long will it take to print the flyers if he runs both presses simultaneously?

(a) Suppose that the large press can complete a job in t_1 hours and the smaller press takes t_2 hours. Write expressions for the fraction of a job that each press can complete in 1 hour.

(b) Write an expression for the fraction of a job that can be completed in 1 hour with both presses running simultaneously.

(c) Write an expression for the amount of time needed to complete the job with both presses running.

(d) Write your answer to part (c) as a simple fraction.

(e) Use your formula to answer the question stated in the problem.

7.6 Chapter Summary and Review

7.6.2 Chapter 7 Review Problems

For Problems 1–4, multiply.

7.6.2.1. $(2x - 5)(x^2 - 3x + 2)$
Answer. $2x^3 - 11x^2 + 19x - 10$

7.6.2.2. $(b^2 - 2b - 3)(2b^2 + b - 5)$

7.6.2.3. $(t + 4)(t^2 - t - 1)$
Answer. $t^3 + 3t^2 - 5t - 4$

7.6.2.4. $(b + 3)(2b - 1)(2b + 5)$

For Problems 5–8, find the indicated term.

7.6.2.5. $(1 - 3x + 5x^2)(7 + x - x^2)$; x^2
Answer. $31x^2$

7.6.2.6. $(-3 + x - 4x^2)(4 + 3x - 2x^3)$; x^3

7.6.2.7. $(4x - x^2 + 3x^3)(1 + 4x - 3x^2)$; x^3
Answer. $-13x^3$

7.6.2.8. $(3 - 2x + 2x^3)(5 + 3x - 2x^2 + 4x^4)$; x^4

For Problems 9–12, factor.

7.6.2.9. $8x^3 - 27z^3$
Answer.
$(2x - 3z)(4x^2 + 6xz + 9z^2)$

7.6.2.10. $1 + 125a^3b^3$

7.6.2.11. $y^3 + 27x^3$
Answer.
$(y + 3x)(y^2 - 3xy + 9x^2)$

7.6.2.12. $x^9 - 8$

For Problems 13–14, write as a polynomial.

7.6.2.13. $(v - 10)^3$
Answer. $v^3 - 30v^2 + 300v - 1000$

7.6.2.14. $(a + 2b^2)^3$

7.6.2.15. The expression $\dfrac{n}{6}(n-1)(n-2)$ gives the number of different 3-item pizzas that can be created from a list of n toppings.

(a) Write the expression as a polynomial.

(b) If Mitch's Pizza offers 12 different toppings, how many different combinations for 3-item pizzas can be made?

(c) Use a table or graph to determine how many different toppings are needed in order to be able to have more than 1000 possible combinations for 3-item pizzas.

Answer.

(a) $\dfrac{1}{6}n^3 - \dfrac{1}{2}n^2 + \dfrac{1}{3}n$

(b) 220

(c) 20

7.6.2.16. The expression $n(n-1)(n-2)$ gives the number of different triple-scoop ice cream cones that can be created from a list of n flavors.

(a) Write the expression as a polynomial.

(b) If Zanner's Ice Cream Parlor offers 21 flavors, how many different triple-scoop ice cream cones can be made?

(c) Use a table or graph to determine how many different flavors are needed in order to be able to have more than $10,000$ possible triple-scoop ice cream cones.

For Problems 17–18,

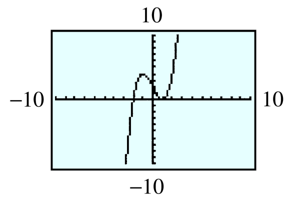
a Graph each polynomial in the standard window.

b Find the range of the function on the domain $[-10, 10]$.

7.6.2.17. $f(x) = x^3 - 3x + 2$

Answer.

(a)



(b) $[-968, 972]$

7.6.2.18. $g(x) = -0.1(x^4 - 6x^3 + x^2 + 24x + 16)$

For Problems 19–28,

a Find the zeros of the polynomial.

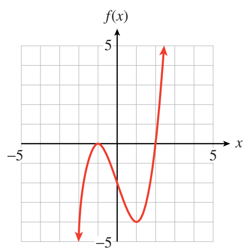
b Sketch the graph by hand.

7.6.2.19. $f(x) = (x-2)(x+1)^2$

Answer.

(a) $2, -1$

(b)



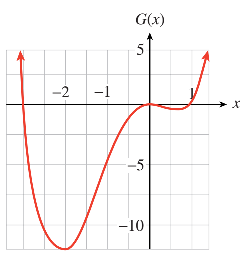
7.6.2.20. $g(x) = (x-3)^2(x+2)$

7.6.2.21. $G(x) = x^2(x-1)(x+3)$

Answer.

(a) $0, 1, -3$

(b)



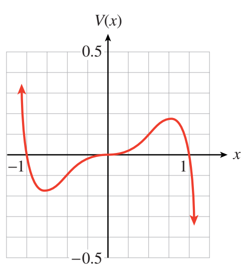
7.6.2.22. $F(x) = (x+1)^2(x-2)^2$

7.6.2.23. $V(x) = x^3 - x^5$

Answer.

(a) $0, 1, -1$

(b)



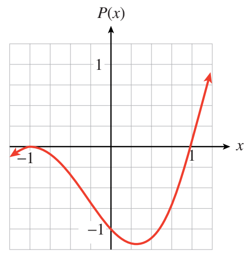
7.6.2.24. $H(x) = x^4 - 9x^2$

7.6.2.25. $P(x) = x^3 + x^2 - x - 1$

Answer.

(a) $-1, 1$

(b)



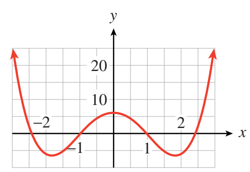
7.6.2.26. $y = x^3 + x^2 - 2x$

7.6.2.27. $y = x^4 - 7x^2 + 6$

Answer.

(a) $-1, 1, \pm\sqrt{6}$

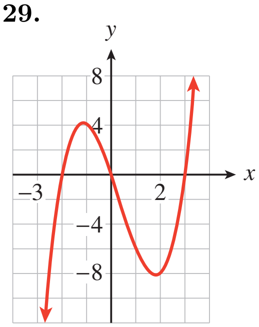
(b)



7.6.2.28. $y = x^4 + x^3 - 3x^2 - 3x$

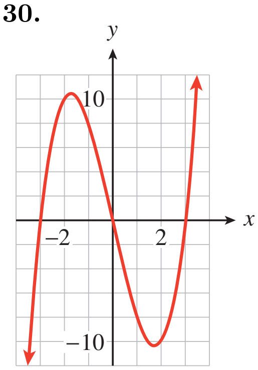
For Problems 29–34, find a possible formula for the polynomial, in factored form.

7.6.2.29.

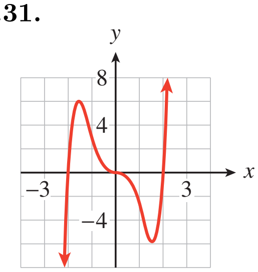


Answer. $x(x+2)(x-3)$

7.6.2.30.

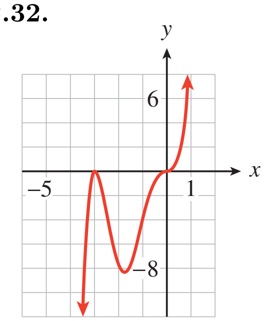


7.6.2.31.

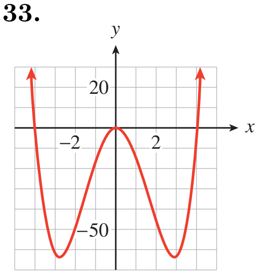


Answer. $x^3(x+2)(x-2)$

7.6.2.32.

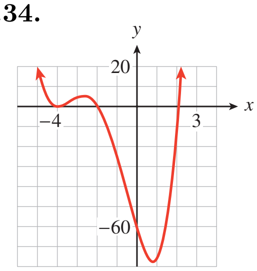


7.6.2.33.



Answer. $x^2(x+4)(x-4)$

7.6.2.34.



For Problems 35–36,

a Verify that the given value is a zero of the polynomial.

b Find the other zeros. (*Hint*: Use polynomial division to write $P(x) = (x-a)Q(x)$, then factor $Q(x)$.)

7.6.2.35. $P(x) = x^3 - x^2 - 7x - 2$; $a = -2$

Answer.
(a) $P(-2) = 0$
(b) $\dfrac{3 \pm \sqrt{13}}{2}$

7.6.2.36. $P(x) = 3x^3 - 11x^2 - 5x + 4$; $a = 4$

For Problems 37–40,

a Solve the quadratic equation, and write the solutions in the form $a + bi$.

b Check your solutions.

7.6.2.37. $x^2 + 4x + 10 = 0$

Answer. $-2 \pm i\sqrt{6}$

7.6.2.38. $x^2 - 2x + 7 = 0$

7.6.2.39. $3x^2 - 6x + 5 = 0$

Answer. $1 \pm \dfrac{\sqrt{6}}{3}i$

7.6.2.40. $2x^2 + 5x + 4 = 0$

For Problems 41–42, evaluate the polynomial for the given values of the variable.

7.6.2.41. $z^2 - 6z + 5$

(a) $z = 3 + 2i$

(b) $z = 3 - 2i$

Answer.

(a) -8 (b) -8

7.6.2.42. $w^2 + 4w + 7$

(a) $w = -1 - 3i$

(b) $w = -1 + 3i$

For Problems 43–44, find the quotient.

7.6.2.43. $\dfrac{2-5i}{3-i}$

Answer. $\dfrac{11}{10} - \dfrac{13}{10}i$

7.6.2.44. $\dfrac{1+i}{1-i}$

For Problems 45–46, find a fourth-degree polynomial with the given zeros.

7.6.2.45. $3i$, $1-2i$

Answer.
$x^4 - 2x^3 + 14x^2 - 18x + 45$

7.6.2.46. $2 - \sqrt{3}$, $2 + 3i$

For Problems 45–46, plot each complex number as a point on the complex plane.

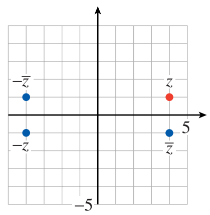
7.6.2.47.

(a) $z = 4 + i,\ \overline{z},\ -z,\ -\overline{z}$

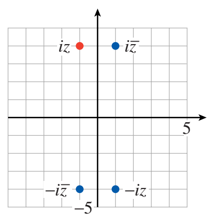
(b) $iz,\ i\overline{z},\ -iz,\ -i\overline{z}$

Answer.

(a)



(b)



7.6.2.48.

(a) $w = -2 + 3i,\ \overline{w},\ -w,\ -\overline{w}$

(b) $iw,\ i\overline{w},\ -iw,\ -i\overline{w}$

7.6.2.49. The radius, r, of a cylindrical can should be one-half its height, h.

(a) Express the volume, V, of the can as a function of its height.

(b) What is the volume of the can if its height is 2 centimeters? 4 centimeters?

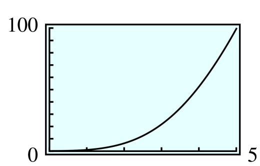
(c) Graph the volume as a function of the height and verify your results of part (b) graphically. What is the approximate height of the can if its volume is 100 cubic centimeters?

Answer.

(a) $V = \dfrac{\pi h^3}{4}$

(b) $2\pi\ \text{cm}^3 \approx 6.28\ \text{cm}^3$; $16\pi\ \text{cm}^3 \approx 50.27\ \text{cm}^3$

(c)



7.6.2.50. The Twisty-Freez machine dispenses soft ice cream in a cone-shaped peak with a height 3 times the radius of its base. The ice cream comes in a round bowl with base diameter d.

(a) Express the volume, V, of Twisty-Freez in the bowl as a function of d.

(b) How much Twisty-Freez comes in a 3-inch diameter dish? A 4-inch dish?

(c) Graph the volume as a function of the diameter and verify your results of part (b) graphically. What is the approximate diameter of a Twisty-Freez if its volume is 5 cubic inches?

7.6.2.51. A new health club opened up, and the manager kept track of the number of active members over its first few months of operation. The equation below gives the number, N, of active members, in hundreds, t months after the club opened.

$$N = \frac{44t}{40 + t^2}$$

(a) Use your calculator to graph the function N on a suitable domain.

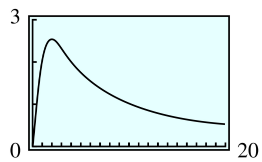
(b) How many active members did the club have after 8 months?

(c) In which months did the club have 200 active members?

(d) When does the health club have the largest number of active members? What happens to the number of active members as time goes on?

Answer.

(a)



(b) 338

(c) Months 2 and 20

(d) During month 6. The number of members eventually decreases to zero.

7.6.2.52. A small lake in a state park has become polluted by runoff from a factory upstream. The cost for removing p percent of the pollution from the lake is given, in thousands of dollars, by

$$C = \frac{25p}{100 - p}$$

(a) Use your calculator to graph the function C on a suitable domain.

(b) How much will it cost to remove 40% of the pollution?

(c) How much of the pollution can be removed for $\$100,000$?

(d) What happens to the cost as the amount of pollution to be removed increases? How much will it cost to remove all the pollution?

For Problems 53–54, state the domain of the function.

7.6.2.53. $h(x) = \dfrac{x^2 - 9}{x(x^2 - 4)}$

Answer. All numbers except $-2, 0, 2$.

7.6.2.54. $f(x) = \dfrac{x^2 - 3x + 10}{x^2(x^2 + 1)}$

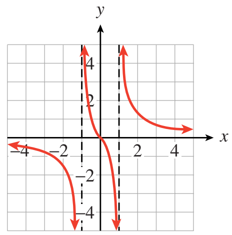
For Problems 55–56,

a Sketch the horizontal and vertical asymptotes for each function.

b Use the asymptotes to help you sketch the graph.

7.6.2.55. $F(x) = \dfrac{2x}{x^2 - 1}$

Answer.



7.6.2.56. $G(x) = \dfrac{2}{x^2 - 1}$

For Problems 57–62,

(a) Identify all asymptotes and intercepts.

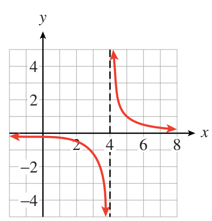
(b) Sketch the graph.

7.6.2.57. $y = \dfrac{1}{x - 4}$

Answer.

(a) Horizontal asymptote $y = 0$;
Vertical asymptote $x = 4$;
y-intercept $(0, \frac{-1}{4})$

(b)



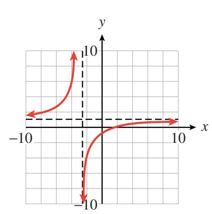
7.6.2.58. $y = \dfrac{2}{x^2 - 3x - 10}$

7.6.2.59. $y = \dfrac{x - 2}{x + 3}$

Answer.

(a) Horizontal asymptote $y = 1$;
Vertical asymptote $x = -3$;
x-intercept $(2, 0)$; y-intercept $(0, \frac{-2}{3})$

(b)

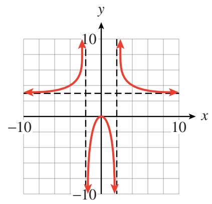


7.6.2.60. $y = \dfrac{x - 1}{x^2 - 2x - 3}$

7.6.2.61. $y = \dfrac{3x^2}{x^2 - 4}$

Answer.

(a) Horizontal asymptote $y = 3$; Vertical asymptote $x = \pm 2$; x-intercept $(0, 0)$; y-intercept $(0, 0)$

(b) 

7.6.2.62. $y = \dfrac{2x^2 - 2}{x^2 - 9}$

For Problems 63–66,

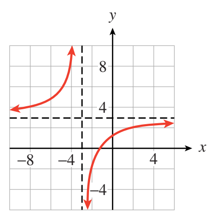
a Use polynomial division to write the fraction in the form $y = \dfrac{k}{p(x)} + c$, where k and c are constants.

b Use transformations to sketch the graph.

7.6.2.63. $y = \dfrac{3x + 4}{x + 3}$

Answer.

(a) $y = \dfrac{-5}{x + 3} + 3$

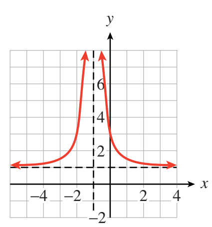
(b) 

7.6.2.64. $y = \dfrac{5x + 1}{x - 2}$

7.6.2.65. $y = \dfrac{x^2 + 2x + 3}{(x + 1)^2}$

Answer.

(a) $y = \dfrac{2}{(x + 1)^2} + 1$

(b) 

7.6.2.66. $y = \dfrac{x^2 - 4x + 3}{(x - 2)^2}$

7.6.2.67. The Explorer's Club is planning a canoe trip to travel 90 miles up the Lazy River and return in 4 days. Club members plan to paddle for 6 hours each day, and they know that the current in the Lazy River is 2 miles per hour.

(a) Express the time it will take for the upstream journey as a function of their paddling speed in still water.

(b) Express the time it will take for the downstream journey as a function of their paddling speed in still water.

(c) Graph the sum of the two functions and find the point on the graph with y-coordinate 24. Interpret the coordinates of the point in the context of the problem.

(d) The Explorer's Club would like to know what average paddling speed members must maintain in order to complete their trip in 4 days. Write an equation to describe this situation.

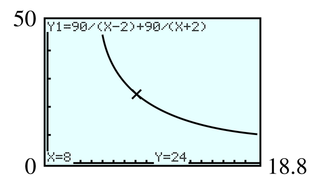
(e) Solve your equation to find the required paddling speed.

Answer.

(a) $t_1 = \dfrac{90}{v-2}$

(b) $t_2 = \dfrac{90}{v+2}$

(c)



(d) $\dfrac{90}{v-2} + \dfrac{90}{v+2} = 24$

(e) 8 mph

7.6.2.68. Pam lives on the banks of the Cedar River and makes frequent trips in her outboard motorboat. The boat travels at 20 miles per hour in still water.

(a) Express the time it takes Pam to travel 8 miles upstream to the gas station as a function of the speed of the current.

(b) Express the time it takes Pam to travel 12 miles downstream to Marie's house as a function of the speed of the current.

(c) Graph the two functions in the same window, then find the coordinates of the intersection point. Interpret those coordinates in the context of the problem.

(d) Pam traveled to the gas station in the same time it took her to travel to Marie's house. Write an equation to describe this situation.

(e) Solve your equation to find the speed of the current in the Cedar River.

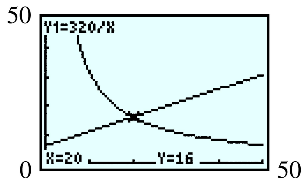
7.6.2.69. Mikala sells $\dfrac{320}{x}$ bottles of bath oil per week if she charges x dollars per bottle. Her supplier can manufacture $\dfrac{1}{2}x + 6$ bottles per week if she sells it at x dollars per bottle.

(a) Graph the demand function, $D(x) = \dfrac{320}{x}$, and the supply function, $S(x) = \dfrac{1}{2}x + 6$, in the same window.

(b) Write and solve an equation to find the equilibrium price, that is, the price at which the supply equals the demand for bath oil. Label this point on your graph.

Answer.

(a)



(b) $\dfrac{320}{x} = \dfrac{1}{2}x + 6$; \$20

7.6.2.70. Tomoko sells $\dfrac{4800}{x}$ exercise machines each month if the price of a machine is x dollars. On the other hand, her supplier can manufacture $2.5x+20$ machines if she charges x dollars apiece for them.

(a) Graph the demand function, $D(x) = \dfrac{4800}{x}$, and the supply function, $S(x) = 2.5x + 20$, in the same window.

(b) Write and solve an equation to find the equilibrium price, that is, the price at which the supply equals the demand for exercise machines. Label this point on your graph.

For Problems 71–72, write and solve a proportion.

7.6.2.71. A polling firm finds that 78 of the 300 randomly selected students at Citrus College play some musical instrument. Based on the poll, how many of the college's 1150 students play a musical instrument?

Answer. 299

7.6.2.72. Claire wants to make a scale model of Salem College. The largest building on campus, Lausanne Hall, is 60 feet tall, and her model of Lausanne Hall will be 8 inches tall. How tall should she make the model of Willamette Hall, which is 48 feet tall?

For Problems 73–80, solve.

7.6.2.73. $\dfrac{y+3}{y+5} = \dfrac{1}{3}$

Answer. -2

7.6.2.74. $\dfrac{z^2+2}{z^2-2} = 3$

7.6.2.75. $\dfrac{x}{x-2} = \dfrac{2}{x-2} + 7$

Answer. No solution

7.6.2.76. $\dfrac{3x}{x+1} - \dfrac{2}{x^2+x} = \dfrac{4}{x}$

7.6.2.77. $\dfrac{2}{a+1} + \dfrac{1}{a-1} = \dfrac{3a-1}{a^2-1}$

Answer. All a except -1 and 1

7.6.2.78. $\dfrac{2b-1}{b^2+2b} = \dfrac{4}{b+2} - \dfrac{1}{b}$

7.6.2.79. $\dfrac{-10}{u-2} = \dfrac{u-4}{u^2-u-2} + \dfrac{3}{u+1}$

Answer. 0

7.6.2.80. $\dfrac{1}{t^2+t} + \dfrac{1}{t} = \dfrac{3}{t+1}$

For Problems 81–84, solve for the indicated variable.

7.6.2.81. $V = C\left(1 - \dfrac{t}{n}\right)$, for n

Answer. $n = \dfrac{Ct}{C-V}$

7.6.2.82. $r = \dfrac{dc}{1-ec}$, for c

7.6.2.83. $\dfrac{p}{q} = \dfrac{r}{q+r}$, for q

Answer. $q = \dfrac{pr}{r-p}$

7.6.2.84. $I = \dfrac{E}{R + \dfrac{r}{n}}$, for R

7.7 Projects for Chapter 7

Project 7.7.1 Solving cubics: Part I. In this project, we solve cubic equations of the form

$$x^3 + mx = n$$

Note that there is no quadratic term. This special form was first solved by the Italian mathematicians Scipione del Ferro and Niccolò Fontana Tartaglia early in the sixteenth century. Tartaglia revealed the secret to solving the special cubic equation in a poem. He first found values u and v to satisfy the system

$$\begin{aligned} u - v &= n \\ uv &= \left(\frac{m}{3}\right)^3 \end{aligned}$$

a We will use Tartaglia's method to solve

$$x^3 + 6x = 7$$

What are the values of m and n?

b Substitute the values of m and n into Tartaglia's system, then use substitution to solve for u and v. You should find two possible solutions.

c For each solution of the system, compute $x = \sqrt[3]{u} - \sqrt[3]{v}$. You should get the same value of x for each (u, v).

d Check that your value for x is a solution of $x^3 + 6x = 7$.

Project 7.7.2 Solving cubics: Part II. Tartaglia's method always works to solve the special cubic equation, even when u and v are not convenient values. We will show why in this project.

a Expand the expression $(a - b)^3 + 3ab(a - b)$ and complete the identity.

$$(a - b)^3 + 3ab(a - b) = \underline{\qquad\qquad\qquad}$$

b Your answer to part (a) is actually Tartaglia's special cubic in disguise. Substitute $x = a - b$, $m = 3ab$, and $n = a^3 - b^3$ to see this. Therefore, if we can find numbers a and b that satisfy

$$\begin{aligned} 3ab &= m \\ a^3 - b^3 &= n \end{aligned}$$

then the solution to Tartaglia's cubic is $x = a - b$.

c Compare the system in part (b) to the system from Project 7.7.1,

$$\begin{aligned} u - v &= n \\ uv &= \left(\frac{m}{3}\right)^3 \end{aligned}$$

to show that $u = a^3$ and $v = b^3$.

d Use your answer to part (c) to show that Tartaglia's value, $x = \sqrt[3]{u} - \sqrt[3]{v}$, is a solution of $x^3 + mx = n$.

Project 7.7.3 Solving cubics: Part III. Use Tartaglia's method to solve the equation

$$x^3 + 3x = 2 \tag{0.0.1}$$

by carrying out the following steps.

a Identify the values of m and n from (7.7.1) and write two equations for u and v.

b Solve for values of u and v. You will need to use the quadratic formula.

c Take the positive values of u and v. Write the solution $x = \sqrt[3]{u} - \sqrt[3]{v}$. Do not try to simply the radical expression; instead, use your calculator to check the solution numerically.

Project 7.7.4 Solving cubics: Part IV. We can solve any cubic equation by first using a substitution to put the equation in Tartaglia's special form.

a Consider the equation $X^3 + bX^2 + cX + d = 0$. Make the substitution $X = x - \dfrac{b}{3}$, and expand the left side of the equation.

b What is the coefficient of x^2 in the resulting equation? What are the values of m and n?

c If you solve the special form in part (a) for x, how can you find the value of X that solves the original equation?

Project 7.7.5 Duration of eclipse. The time, T, it takes for the Moon to eclipse the Sun totally is given (in minutes) by the formula

$$T = \frac{1}{v}\left(\frac{rD}{R} - d\right)$$

where d is the diameter of the Moon, D is the diameter of the Sun, r is the distance from the Earth to the Moon, R is the distance from the Earth to the Sun, and v is the speed of the Moon.

a Solve the formula for v in terms of the other variables.

b It takes 2.68 minutes for the Moon to eclipse the Sun. Calculate the speed of the Moon given the following values:

$$d = 3.48 \times 10^3 \text{ km} \qquad D = 1.41 \times 10^6 \text{ km}$$
$$r = 3.82 \times 10^5 \text{ km} \qquad R = 1.48 \times 10^8 \text{ km}$$

Project 7.7.6 Optimal traffic flow. The stopping distance, s, for a car traveling at speed v meters per second is given (in meters) by

$$s = vT + \frac{v^2}{2a}$$

where T is the reaction time of the driver and a is the average deceleration as the car brakes. Suppose that all the cars on a crowded motorway maintain the appropriate spacing determined by the stopping distance for their speed. What speed allows the maximum flow of cars along the road per unit time? Using the formula time $= \dfrac{\text{distance}}{\text{speed}}$, we see that the time interval, t, between cars is

$$t = \frac{s}{v} + \frac{L}{v}$$

where L is the length of the car. To achieve the maximum flow of cars, we would like t to be as small as possible. (Source: Bolton, 1974)

a Substitute the expression for s into the formula for t, then simplify.

b A typical reaction time is $T = 0.7$ seconds, a typical car length is $L = 5$ meters, and $a = 7.5$ meters per second squared. With these values, graph t as a function of v in the window

Xmin = 0	Xmax = 20
Ymin = 0	Ymax = 3

c To one decimal place, what value of v gives the minimum value of t? Convert your answer to miles per hour.

Project 7.7.7 Effective population of endangered species. Many endangered species have fewer than 1000 individuals left. To preserve the species, captive breeding programs must maintain a certain effective population, N, given by

$$N = \frac{4FM}{F + M}$$

where F is the number of breeding females and M the number of breeding males. (Source: Chapman and Reiss, 1992)

a What is the effective population if there are equal numbers of breeding males and females?

b In 1972, a breeding program for Speke's gazelle was established with just three female gazelle. Graph the effective population, N, as a function of the number of males.

c What is the largest effective population that can be created with three females? How many males are needed to achieve the maximum value?

d With three females, for what value of M is $N = M$?

e The breeding program for Speke's gazelle began with only one male. What was the effective population?

Project 7.7.8 Biological half-life. When a drug or chemical is injected into a patient, biological processes begin removing that substance. If no more of the substance in introduced, the body removes a fixed fraction of the substance each hour. The amount of substance remaining in the body at time t is an exponential decay function, so there is a **biological half-life** to the substance denoted by T_b. If the substance is a radioisotope, it undergoes radioactive decay and so has a physical half-life as well, denoted T_p.

The **effective half-life**, denoted by T_e, is related to the biological and physical half-lives by the equation

$$\frac{1}{T_e} = \frac{1}{T_b} + \frac{1}{T_p}$$

The radioisotope ^{131}I is used as a label for the human serum albumin. The physical half-life of ^{131}I is 8 days. (Source: Pope, 1989)

a If ^{131}I is cleared from the body with a half-life of 21 days, what is the effective half-life of ^{131}I?

b The biological half-life of a substance varies considerably from person to person. If the biological half-life of ^{131}I is x days, what is the effective half life?

c Let $f(x)$ represent the effective half-life of ^{131}I when the biological half-life is x days. Graph $y = f(x)$.

d What would the biological half-life of ^{131}I need to be to produce an effective half-life of 6 days? Label the corresponding point on your graph.

e For what possible biological half-lives of ^{131}I will the effective half-life be less than 4 days?

Project 7.7.9 Rate of eating. Animals spend most of their time hunting or foraging for food to keep themselves alive. Knowing the rate at which an animal (or population of animals) eats can help us determine its metabolic rate or its impact on its habitat. The rate of eating is proportional to the availability of food in the area, but it has an upper limit imposed by mechanical considerations, such as how long it takes the animal to capture and ingest its prey. (Source: Burton, 1998)

a Sketch a graph of eating rate as a function of quantity of available food. This will be a qualitative graph only; you do not have enough information to put scales on the axes.

b Suppose that the rate at which an animal catches its prey is proportional to the number of prey available, or $r_c = ax$, where a is a constant and x is the number of available prey. The rate at which it handles and eats the prey is constant, $r_h = b$. Write expressions for T_c and T_h, the times for catching and handling N prey.

c Show that the rate of food consumption is given by

$$y = \frac{abx}{b + ax} = \frac{bx}{b/a + x}$$

Hint: $y = \dfrac{N}{T}$, where N is the number of prey consumed in the interval T, where $T = T_c + T_h$.

d In a study of ladybirds, it was discovered that larvae in their second stage of development consumed aphids at a rate y_2 aphids per day, given by

$$y_2 = \frac{20x}{x + 16}$$

where x is the number of aphids available. Larvae in the third stage ate at rate y_3, given by

$$y_3 = \frac{90x}{x + 79}$$

Graph both of these functions on the domain $0 \le x \le 140$.

e What is the maximum rate at which ladybird larvae in each stage of development can consume aphids?

Project 7.7.10 Buoyancy. A person will float in fresh water if his or her density is less than or equal to 1 kilogram per liter, the density of water. (Density is given by the formula density $= \dfrac{\text{weight}}{\text{volume}}$.) Suppose a swimmer weighs $50 + F$ kilograms, where F is the amount of fat her body contains. (Source: Burton, 1998)

a Calculate the volume of her nonfat body mass if its density is 1.1 kilograms per liter.

b Calculate the volume of the fat if its density is 0.901 kilograms per liter.

c The swimmer's lungs hold 2.6 liters of air. Write an expression for the total volume of her body, including the air in her lungs.

d Write an expression for the density of the swimmer's body.

e Write an equation for the amount of fat needed for the swimmer to float in fresh water.

f Solve your equation. What percent of the swimmer's weight is fat?

g Suppose the swimmer's lungs can hold 4.6 liters of air. What percent body fat does she need to be buoyant?

8 Extra!!

8.1 Linear Regression

8.1.1 Fitting a Line through Two Points

Checkpoint 8.1.2 In 1991, there were 64.6 burglaries per 1000 households in the United States. The number of burglaries reported annually declined at a roughly constant rate over the next decade, and in 2001 there were 28.7 burglaries per 1000 households. (Source: U.S. Department of Justice)

a Find a function for the number of burglaries, B, as a function of time, t, in years, since 1990.

b State the slope as a rate of change. What does the slope tell us about this problem?

Answer.

a $y = 68.19 - 3.59x$

b -3.59 burglaries per 1000 households per year. From 1991 to 2001, the burglary rate declined by 3.59 burglaries per 1000 households every year.

8.1.2 Scatterplots

Checkpoint 8.1.5

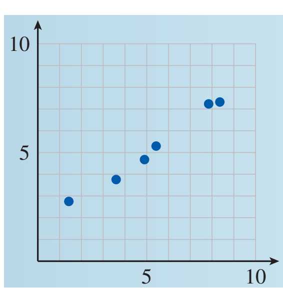
a Plot the data points. Do the points lie on a line?

b Draw a line that fits the data.

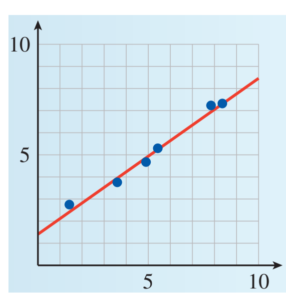
x	1.49	3.68	4.95	5.49	7.88	8.41
y	2.69	3.7	4.6	5.2	7.2	7.3

Answer.

a



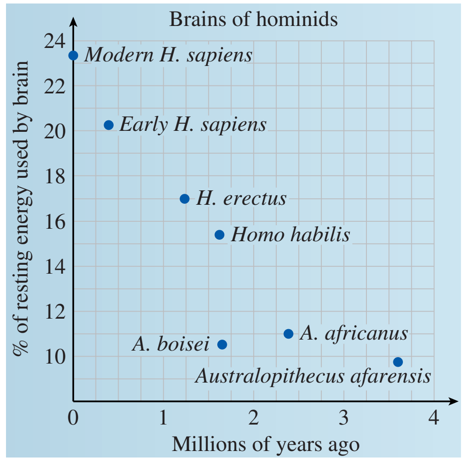
b



8.1.3 Linear Regression

Checkpoint 8.1.7 Human brains consume a large amount of energy, about 16 times as much as muscle tissue per unit weight. In fact, brain metabolism accounts for about 25% of an adult human's energy needs, as compared to about 5% for other mammals.

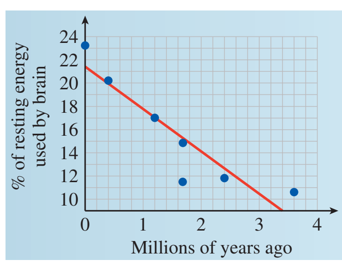
As hominid species evolved, their brains required larger and larger amounts of energy, as shown below. (Source: Scientific American, December 2002)



a Draw a line of best fit through the data points.

b Estimate the amount of energy used by the brain of a hominid species that lived three million years ago.

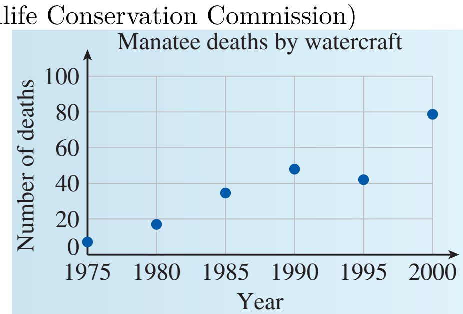
Answer.

a 

b About 10.5%

Checkpoint 8.1.9 The number of manatees killed by watercraft in Florida waters has been increasing since 1975. Data are given at 5-year intervals in the table. (Source: Florida Fish and Wildlife Conservation Commission)

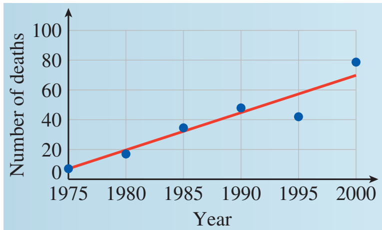
Year	Manatee deaths
1975	6
1980	16
1985	33
1990	47
1995	42
2000	78



a Draw a regression line through the data points shown in the figure.

b Use the regression equation to estimate the number of manatees killed by watercraft in 1998.

Answer.

a 

b $y = 4.7 + 2.6t$

c 65

8.1.4 Linear Interpolation and Extrapolation

Checkpoint 8.1.11 Emily was 82 centimeters tall at age 36 months and 88 centimeters tall at age 48 months.

a Find a linear equation that approximates Emily's height in terms of her age over the given time interval.

b Use linear interpolation to estimate Emily's height when she was 38 months old, and extrapolate to predict her height at age 50 months.

c Predict Emily's height at age 25 (300 months). Is your answer reasonable?

Answer.

a $y = 64 + 0.5x$ b 83 cm, 89 cm   c 214 cm; No

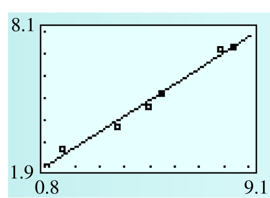
Checkpoint 8.1.14

a Use your calculator's statistics features to find the least squares regression equation for the data in Checkpoint 8.1.5.

b Plot the data and the graph of the regression equation.

Answer.

a $y = 1.34 + 0.71x$

b 

8.1.6 Homework 1.6

In Problems 1–6, we find a linear model from two data points.

a Make a table showing the coordinates of two data points for the model. (Which variable should be plotted on the horizontal axis?)

b Find a linear equation relating the variables.

c State the slope of the line, including units, and explain its meaning in the context of the problem.

8.1.6.1. It cost a bicycle company \$9000 to make 40 touring bikes in its first month of operation and \$15,000 to make 125 bikes during its second month. Express the company's monthly production cost, C, in terms of the number, x, of bikes it makes.

Answer.

a

x	50	125
y	9000	$15,000$

b $C = 5000 + 80x$

c $m = 80$ dollars/bike, so it costs the company \$80 per bike it manufactures.

8.1.6.2. Flying lessons cost \$645 for an 8-hour course and \$1425 for a 20-hour course. Both prices include a fixed insurance fee. Express the cost, C, of flying lessons in terms of the length, h, of the course in hours.

8.1.6.3. Under ideal conditions, Andrea's Porsche can travel 312 miles on a full tank (12 gallons of gasoline) and 130 miles on 5 gallons. Express the distance, d, Andrea can drive in terms of the amount of gasoline, g, she buys.

Answer.

a

g	12	5
d	312	130

b $d = 26g$

c $m = 26$ miles/gallon, so the Porche's fuel efficiency is 26 miles per gallon.

8.1.6.4. On an international flight, a passenger may check two bags each weighing 70 kilograms, or 154 pounds, and one carry-on bag weighing 50 kilograms, or 110 pounds. Express the weight, p, of a bag in pounds in terms of its weight, k, in kilograms.

8.1.6.5. A radio station in Detroit, Michigan, reports the high and low temperatures in the Detroit/Windsor area as 59°F and 23°F, respectively. A station in Windsor, Ontario, reports the same temperatures as 15°C and −5°C. Express the Fahrenheit temperature, F, in terms of the Celsius temperature, C.

Answer.

a

C	15
F	59
23	

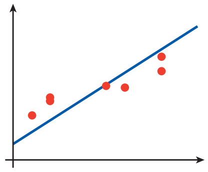
b $F = 32 + \frac{9}{5}C$

c $m = \frac{9}{5}$, so an increase of 1°C is equivalent to an increase of $\frac{9}{5}^{\circ}$ F.

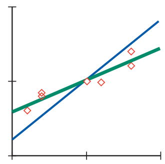
8.1.6.6. Ms. Randolph bought a used car in 2000. In 2002, the car was worth \$9000, and in 2005 it was valued at \$4500. Express the value, V , of Ms. Randolph's car in terms of the number of years, t, she has owned it.

Each regression line can be improved by adjusting either m or b. Draw a line that fits the data points more closely.

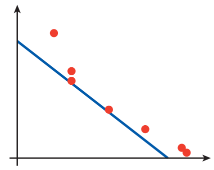
8.1.6.7.



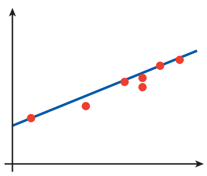
Answer.



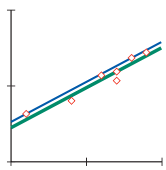
8.1.6.8.



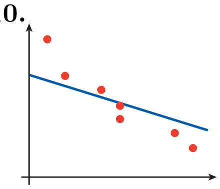
8.1.6.9.



Answer.

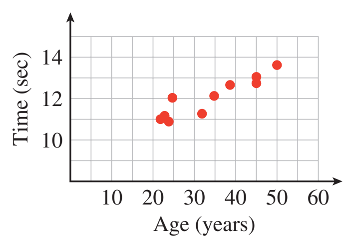


8.1.6.10.



In Problems 11 and 12, use information from the graphs to answer the questions.

8.1.6.11. The scatterplot shows the ages of 10 army drill sergeants and the time it took each to run 100 meters, in seconds.



a What was the hundred-meter time for the 25-year-old drill sergeant?

b How old was the drill sergeant whose hundred-meter time was 12.6 seconds?

c Use a straightedge to draw a line of best fit through the data points.

d Use your line of best fit to predict the hundred-meter time of a 28-year-old drill sergeant.

e Choose two points on your regression line and find its equation.

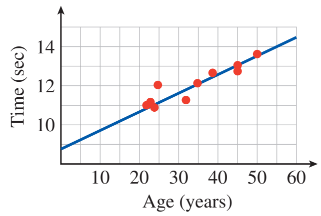
f Use the equation to predict the hundred-meter time of a 40-year-old drill sergeant and a 12 year-old drill sergeant. Are these predictions reasonable?

Answer.

a 12 seconds

b 39

c

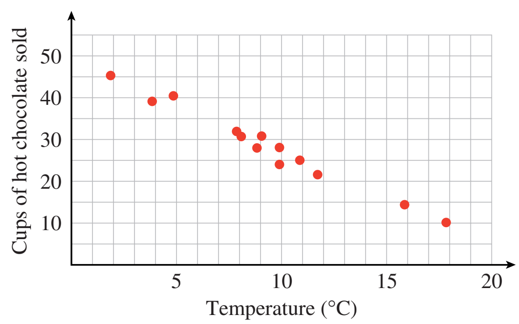


d 11.6 seconds

e $y = 8.5 + 0.1x$

f 12.7 seconds; 10.18 seconds; The prediction for the 40-year-old is reasonable, but not the prediction for the 12-year-old.

8.1.6.12. The scatterplot shows the outside temperature and the number of cups of cocoa sold at an outdoor skating rink snack bar on 13 consecutive nights.



a How many cups of cocoa were sold when the temperature was 2°C?

b What was the temperature on the night when 25 cups of cocoa were sold?

c Use a straightedge to draw a line of best fit through the data points

d Use your line of best fit to predict the number of cups of cocoa that will be sold at the snack bar if the temperature is 7°C.

e Choose two points on your regression line and find its equation.

f Use the equation to predict the number of cups of cocoa that will be sold when the temperature is 10°C and when the temperature is 24°C. Are these predictions reasonable?

8.1.6.13. With Americans' increased use of faxes, pagers, and cell phones, new area codes are being created at a steady rate. The table shows the number of area codes in the United States each year. (Source: USA Today, NeuStar, Inc.)

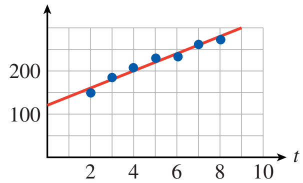
Year	1997	1998	1999	2000	2001	2002	2003
Number of area codes	151	186	204	226	239	262	274

a Let t represent the number of years after 1995 and plot the data. Draw a line of best fit for the data points.

b Find an equation for your regression line.

c How many area codes do you predict for 2010?

Answer.

a 

b $y = 121 + 19.86t$

c 419

8.1.6.14. The number of mobile homes in the United States has been increasing since 1960. The data in the table are given in millions of mobile homes. (Source: USA Today, U.S. Census Bureau)

Year	1960	1970	1980	1990	2000
Number of mobile homes	0.8	2.1	4.7	7.4	8.8

a Let t represent the number of years after 1960 and plot the data. Draw a line of best fit for the data points

b Find an equation for your regression line.

c How many mobile homes do you predict for 2010?

8.1.6.15. Teenage birth rates in the United States declined from 1991 to 2000. The table shows the number of births per 1000 women in selected years. (Source: U.S. National Health Statistics)

Year	1991	1993	1995	1996	1997	1998
Births	62.1	59.6	56.8	54.4	52.3	51.1

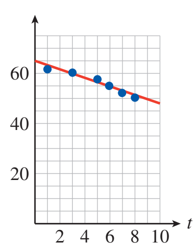
a Let t represent the number of years after 1990 and plot the data. Draw a line of best fit for the data points.

b Find an equation for your regression line.

c Estimate the teen birth rate in 1994.

d Predict the teen birth rate in 2010.

Answer.

a 

b $y = 64.2 - 1.63t$

c 58 births per 1000 women

d 32 births per 1000 women

8.1.6.16. The table shows the minimum wage in the United States at five-year intervals. (Source: Economic Policy Institute)

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000
Minimum wage	1.00	1.25	1.60	2.10	3.10	3.35	3.80	4.25	5.15

a Let t represent the number of years after 1960 and plot the data. Draw a line of best fit for the data points.

b Find an equation for your regression line.

c Estimate the minimum wage in 1972.

d Predict the minimum wage in 2010.

8.1.6.17. Life expectancy in the United States has been rising since the nineteenth century. The table shows the U.S. life expectancy in selected years. (Source: http://www.infoplease.com)

Year	1950	1960	1970	1980	1990	2000
Life expectancy at birth	68.2	69.7	70.8	73.7	75.4	77

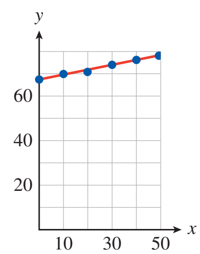
a Let t represent the number of years after 1950, and plot the data. Draw a line of best fit for the data points.

b Find an equation for your regression line.

c Estimate the life expectancy of someone born in 1987.

d Predict the life expectancy of someone born in 2010.

Answer.

a


b $y = 0.18t + 67.9$

c 74.9 years

d 79 years

8.1.6.18. The table shows the per capita cigarette consumption in the United States at five-year intervals. (Source: http://www.infoplease.com)

Year	1980	1985	1990	1995	2000
Per capita cigarette consumption	3851	3461	2827	2515	2092

a Let t represent the number of years after 1980, and plot the data. Draw a line of best fit for the data points.

b Find an equation for your regression line.

c Estimate the per capita cigarette consumption in 1998.

d Predict the per capita cigarette consumption in 2010.

8.1.6.19. "The earnings gap between high-school and college graduates continues to widen, the Census Bureau says. On average, college graduates now earn just over $\$51,000$ a year, almost twice as much as high-school graduates. And those with no high-school diploma have actually seen their earnings drop in recent years." The table shows the unemployment rate and the median weekly earnings for employees with different levels of education. (Source: Morning Edition, National Public Radio, March 28, 2005)

	Years of education	Unemployment rate	Weekly earnings ($)
Some high school no diploma	10	8.8	396
High-school graduate	12	5.5	554
Some college no degree	13	5.2	622
Associate's degree	14	4.0	672
Bachelor's degree	16	3.3	900
Master's degree	18	2.9	1064
Professional degree	20	1.7	1307

a Plot years of education on the horizontal axis and weekly earnings on the vertical axis.

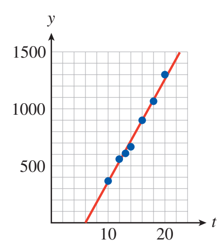
b Find an equation for the regression line.

c State the slope of the regression line, including units, and explain what it means in the context of the data.

d Do you think this model is useful for extrapolation or interpolation? For example, what weekly earnings does the model predict for someone with 15 years of education? For 25 years? Do you think these predictions are valid? Why or why not?

Answer.

a



b $y = 90.49t - 543.7$

c 90.49 dollars/year: Each additional year of education corresponds to an additional $90.49 in weekly earnings.

d No: The degree or diploma attained is more significant than the number of years. So, for example, interpolation for the years of education between a bachelor's and master's degree may be inaccurate because earnings with just the bachelor's degree will not change until the master's degree is attained. And the years after the professional degree will not add significantly to earnings, so extrapolation is inappropriate.

8.1.6.20. The table shows the birth rate (in births per woman) and the female literacy rate (as a percent of the adult female population) in a number of nations. (Source: UNESCO, The World Fact Book, EarthTrends)

Country	Literacy rate	Birth rate
Brazil	88.6	1.93
Egypt	43.6	2.88
Germany	99	1.39
Iraq	53	4.28
Japan	99	1.39
Niger	9.4	6.75
Pakistan	35.2	4.14
Peru	82.1	2.56
Philippines	92.7	3.16
Portugal	91	1.47
Russian Federation	99.2	1.27
Saudi Arabia	69.3	4.05
United States	97	2.08

a Plot the data with literacy rate on the horizontal axis. Draw a line of best fit for the data points.

b Find an equation for the regression line.

c What values for the input variable make sense for the model? What are the largest and smallest values predicted by the model for the output variable?

d State the slope of the regression line, including units, and explain what it means in the context of the data.

8.1.6.21. The table shows the amount of carbon released into the atmosphere annually from burning fossil fuels, in billions of tons, at 5-year intervals from 1950 to 1995. (Source: www.worldwatch.org)

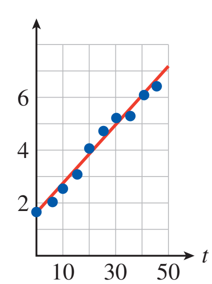
Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Carbon emissions	1.6	2.0	2.5	3.1	4.0	4.5	5.2	5.3	5.9	6.2

a Let t represent the number of years after 1950 and plot the data. Draw a line of best fit for the data points.

b Find an equation for your regression line.

c Estimate the amount of carbon released in 1992.

Answer.

a 

b $y = 1.6 + 0.11t$

c 6.2 billion tons

8.1.6.22. High-frequency radiation is harmful to living things because it can cause changes in their genetic material. The data below, collected by C. P. Oliver in 1930, show the frequency of genetic transmutations induced in fruit flies by doses of X-rays, measured in roentgens. (Source: C. P. Oliver, 1930)

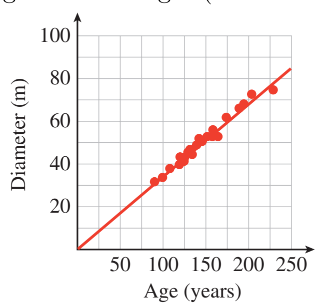
Dosage (roentgens)	285	570	1640	3280	6560
Percentage of mutated genes	1.18	2.99	4.56	9.63	15.85

a Plot the data and draw a line of best fit through the data points.

b Find an equation for your regression line.

c Use the regression equation to predict the percent of mutations that might result from exposure to 5000 roentgens of radiation.

8.1.6.23. Bracken, a type of fern, is one of the most successful plants in the world, growing on every continent except Antarctica. New plants, which are genetic clones of the original, spring from a network of underground stems, or rhizomes, to form a large circular colony. The graph shows the diameters of various colonies plotted against their age. (Source: Chapman et al.,1992)



a Calculate the rate of growth of the diameter of a bracken colony, in meters per year.

b Find an equation for the line of best fit. (What should the vertical intercept of the line be?)

c In Finland, bracken colonies over 450 meters in diameter have been found. How old are these colonies?

Answer.

a 0.34 meters per year

b $y = 0.34x$ ($b = 0$ because the plant has zero size until it begins.)

c Over 1300 years

8.1.6.24. The European sedge warbler can sing several different songs consisting of trills, whistles, and buzzes. Male warblers who sing the largest number of songs are the first to acquire mates in the spring. The data below show the number of different songs sung by several male warblers and the day on which they acquired mates, where day 1 is April 20. (Source: Krebs and Davies, 1993)

Number of songs	41	38	34	32	30	25	24	24	23	14
Pairing day	20	24	25	21	24	27	31	35	40	42

a Plot the data points, with number of songs on the horizontal axis. A regression line for the data is $y = -0.85x + 53$. Graph this line on the same axes with the data.

b What does the slope of the regression line represent?

c When can a sedge warbler that knows 10 songs expect to find a mate?

d What do the intercepts of the regression line represent? Do these values make sense in context?

8.1.6.25. One of the factors that determines the strength of a muscle is its cross-sectional area. The data below show the cross-sectional area of the arm flexor muscle for several men and women, and their strength, measured by the maximum force they exerted against a resistance. (Source: Davis, Kimmet, Autry, 1986)

Women	Area (sq cm)	11.5	10.8	11.7	12.0	12.5	12.7	14.4	14.4	15.7
	Strength (kg)	11.3	13.2	13.2	14.5	15.6	14.8	15.6	16.1	18.4
Men	Area (sq cm)	13.5	13.8	15.4	15.4	17.7	18.6	20.8	–	–
	Strength (kg)	15.0	17.3	19.0	19.8	20.6	20.8	26.3	–	–

a Plot the data for both men and women on the same graph using different symbols for the data points for men and the data points for women.

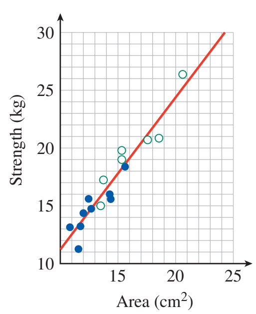
b Are the data for both men and women described reasonably well by the same regression line? Draw a line of best fit through the data.

c Find the equation of your line of best fit, or use a calculator to find the regression line for the data.

d What does the slope mean in this context?

Answer.

a



b Yes

c $y = 1.29x - 1.62$

d The slope, 1.29 kg/sq cm, tells us that strength increases by 1.29 kg when the muscle cross-sectional area increases by 1 sq cm.

8.1.6.26. Astronomers use a numerical scale called **magnitude** to measure the brightness of a star, with brighter stars assigned smaller magnitudes. When we view a star from Earth, dust in the air absorbs some of the light, making the star appear fainter than it really is. Thus, the observed magnitude of a star, m, depends on the distance its light rays must travel through the Earth's atmosphere. The observed magnitude is given by

$$m = m_0 + kx$$

where m_0 is the actual magnitude of the star outside the atmosphere, x is the air mass (a measure of the distance through the atmosphere), and k is a constant called the **extinction coefficient**. To calculate m_0, astronomers observe the same object several times during the night at different positions in the sky, and hence for different values of x. Here are data from such observations. (Source: Karttunen et al., 1987)

Altitude	Air mass, x	Magnitude, m
50°	1.31	0.90
35°	1.74	0.98
25°	2.37	1.07
20°	2.92	1.17

a Plot observed magnitude against air mass, and draw a line of best fit through the data.

b Find the equation of your line of best fit, or use a calculator to find the regression line for the data.

c Find the equation of your line of best fit, or use a calculator to find the regression line for the data.

d What is the value of the extinction coefficient? What is the apparent magnitude of the star outside Earth's atmosphere?

8.1.6.27. Six students are trying to identify an unknown chemical compound by heating the substance and measuring the density of the gas that evaporates. (Density = mass/volume.) The students record the mass lost by the solid substance and the volume of the gas that evaporated from it. They know that the mass lost by the solid must be the same as the mass of the gas that evaporated. (Source: Hunt and Sykes, 1984)

Student	A	B	C	D	E	F
Volume of gas (cm^3)	48	60	24	81	76	54
Loss in mass (mg)	64	81	32	107	88	72

a Plot the data with volume on the horizontal axis. Which student made an error in the experiment?

b Ignoring the incorrect data point, draw a line of best fit through the other points.

c Find an equation of the form $y = kx$ for the data. Why should you expect the regression line to pass through the origin?

d Use your equation to calculate the mass of 1000 cm^3 (one liter) of the gas.

e Here are the densities of some gases at room temperature:

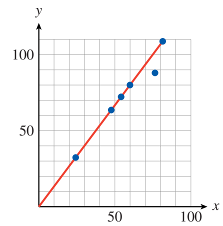
Hydrogen	8	mg/liter
Nitrogen	1160	mg/liter
Oxygen	1330	mg/liter
Carbon dioxide	1830	mg/liter

Which of these might have been the gas that evaporated from the unknown substance?

Hint. Use your answer to part (d) to calculate the density of the gas. 1 $\text{cm}^3 =$ 1 milliliter.

Answer.

a E

b 

c $y = 1.33x$; There should be no loss in mass when no gas evaporates.

d 1333 mg

e Oxygen

8.1.6.28. The formulas for many chemical compounds involve ratios of small integers. For example, the formula for water, H_20, means that two atoms of hydrogen combine with one atom of oxygen to make one water molecule. Similarly, magnesium and oxygen combine to produce magnesium oxide. In this problem, we will discover the chemical formula for magnesium oxide. (Source: Hunt and Sykes, 1984)

a Twenty-four grams of magnesium contain the same number of atoms as sixteen grams of oxygen. Complete the table showing the amount of oxygen needed if the formula for magnesium oxide is MgO, Mg_2O, or MgO_2.

Grams of Mg	Grams of O (if MgO)	Grams of O (if Mg_2O)	Grams of O (if MgO_2)
24	16		
48			
12			
6			

b Graph three lines on the same axes to represent the three possibilities, with grams of magnesium on the horizontal axis and grams of oxygen on the vertical axis.

c Here are the results of some experiments synthesizing magnesium oxide.

Experiment	Grams of Magnesium	Grams of oxygen
1	15	10
2	22	14
3	30	20
4	28	18
5	10	6

Plot the data on your graph from part (b). Which is the correct formula for magnesium oxide?

For Problems 29–32,

a Use linear interpolation to give approximate answers.

b What is the meaning of the slope in the context of the problem?

8.1.6.29. The temperature in Encino dropped from 81°F at 1 a.m. to 73°F at 5 a.m. Estimate the temperature at 4 a.m.

Answer.

a 75°F

b The slope of −2 degrees/hour says that temperatures are dropping at a rate of 2° per hour.

8.1.6.30. Newborn blue whales are about 24 feet long and weigh 3 tons. The young whale nurses for 7 months, at which time it is 53 feet long. Estimate the length of a 1-year-old blue whale.

8.1.6.31. A car starts from a standstill and accelerates to a speed of 60 miles per hour in 6 seconds. Estimate the car's speed 2 seconds after it began to accelerate.

Answer.

a 20 mph

b The slope of 10 mph/second says the car accelerates at a rate of 10 mph per second.

8.1.6.32. A truck on a slippery road is moving at 24 feet per second when the driver steps on the brakes. The truck needs 3 seconds to come to a stop. Estimate the truck's speed 2 seconds after the brakes were applied.

In Problems 33–36, use linear interpolation or extrapolation to answer the questions.

8.1.6.33. The temperature of an automobile engine is 9° Celsius when the engine is started and 51°C seven minutes later. Use a linear model to predict the engine temperature for both 2 minutes and 2 hours after it started. Are your predictions reasonable?

Answer. 2 min: 21°C; 2 hr: 729°C; The estimate at 2 minutes is reasonable; the estimate at 2 hours is not reasonable.

8.1.6.34. The temperature in Death Valley is 95° Fahrenheit at 5 a.m. and rises to 110° Fahrenheit by noon. Use a linear model to predict the temperature at 2 p.m. and at midnight. Are your predictions reasonable?

8.1.6.35. Ben weighed 8 pounds at birth and 20 pounds at age 1 year. How much will he weigh at age 10 if his weight increases at a constant rate?

Answer. 128 lb.

8.1.6.36. The elephant at the City Zoo becomes ill and loses weight. She weighed 10,012 pounds when healthy and only 9641 pounds a week later. Predict her weight after 10 days of illness.

8.1.6.37. Birds' nests are always in danger from predators. If there are other nests close by, the chances of predators finding the nest increase. The table shows the probability of a nest being found by predators and the distance to the nearest neighboring nest. (Source: Perrins, 1979)

Distance to nearest neighbor (meters)	20	40	60	80	100
Probability of predators (%)	47	34	32	17	1.5

a Plot the data and the least squares regression line.

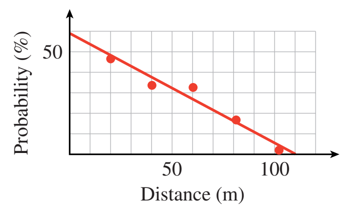
b Use the regression line to estimate the probability of predators finding a nest if its nearest neighbor is 50 meters away.

c If the probability of predators finding a nest is 10%, how far away is its nearest neighbor?

d What is the probability of predators finding a nest if its nearest neighbor is 120 meters away? Is your answer reasonable?

Answer.

a $y \approx -0.54x + 58.7$



b 31.7%

c 90 meters

d The regression line gives a negative probability, which is not reasonable.

8.1.6.38. A trained cyclist pedals faster as he increases his cycling speed, even with a multiple-gear bicycle. The table shows the pedal frequency, p (in revolutions per minute), and the cycling speed, c (in kilometers per hour), of one cyclist. (Source: Pugh, 1974)

Speed (km/hr)	8.8	12.5	16.2	24.4	31.9	35.0
Pedal frequency (rpm)	44.5	50.7	60.6	77.9	81.9	95.3

a Plot the data and the least squares regression line.

b Estimate the cyclist's pedal frequency at a speed of 20 kilometers per hour.

c Estimate the cyclist's speed when he is pedaling at 70 revolutions per minute.

d Does your regression line give a reasonable prediction for the pedaling frequency when the cyclist is not moving? Explain.

8.1.6.39. In this problem we will calculate the efficiency of swimming as a means of locomotion. A swimmer generates power to maintain a constant speed in the water. If she must swim against an opposing force, the power increases. The following table shows the power expended by a swimmer while working against different amounts of force. (A positive force opposes the swimmer, and a negative force helps her.) (Source: diPrampero et al., 1974, and Alexander, 1992)

Force (newtons)	−3.5	0	0	6	8	10	17	17
Metabolic power (watts)	100	190	230	320	380	450	560	600

a Plot the data on the grid, or use the **StatPlot** feature on your calculator. Use your calculator to find the least squares regression line. Graph the regression line on top of the data.

b Use your regression line to estimate the power needed for the swimmer to overcome an opposing force of 15 newtons.

c Use your regression line to estimate the power generated by the swimmer when there is no force either hindering or helping her.

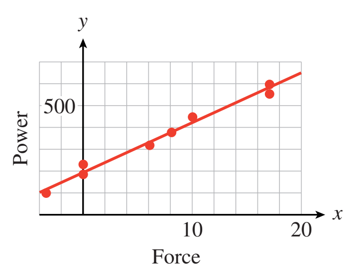
d Estimate the force needed to tow the swimmer at 0.4 meters per second while she rests. (If she is resting, she is not generating any power).

e The swimmer's **mechanical** power (or rate of work) is computed by multiplying her speed times the force needed to tow her at rest. Use your answer to part (d) to calculate the mechanical power she generates by swimming at 0.4 meters per second.

f The ratio of mechanical power to metabolic power is a measure of the swimmer's efficiency. Compute the efficiency of the swimmer when there is no external force opposing or helping her.

Answer.

a



$y \approx 22.8x + 198.5$

b ≈ 540 watts

c 198.5 watts

d ≈ -8.7 newtons

e 3.5 watts

f about 0.018 or 1.8%

8.1.6.40. In this problem, we calculate the amount of energy generated by a cyclist. An athlete uses oxygen slowly when resting but more quickly during physical exertion. In an experiment, several trained cyclists took turns pedaling on a bicycle ergometer, which measures their work rate. The table shows the work rate of the cyclists, in watts, measured against their oxygen intake, in liters per minute. (Source: Pugh, 1974)

Oxygen consumption (liters/min)	1	1.7	2	3.3	3.9	3.6	4.3	5
Work rate (watts)	40	100	180	220	280	300	320	410

a Plot the data on the grid, or use the **StatPlot** feature on your calculator. Use your calculator to find the least squares regression line. Graph the regression line on top of the data.

b Find the horizontal intercept of the regression line. What does the horizontal intercept tell you about this situation?

c Estimate the power produced by a cyclist consuming oxygen at 5.9 liters per minute.

d What is the slope of the regression line? The slope represents the amount of power, in watts, generated by a cyclist for each liter of oxygen consumed per minute. How many watts of power does a cyclist generate from each liter of oxygen?

e One watt of power represents an energy output of one joule per second. How many joules of energy does the cyclist generate in one minute?

f How many joules of energy can be extracted from each cubic centimeter of oxygen used? (One liter is equal to 1000 cubic centimeters.)

8.2 Curve Fitting

8.2.1 Introduction

Checkpoint 8.2.2 Follow the steps to solve the system

$$\begin{aligned} a + b + c &= 3 && (1)\\ 4a - b + c &= -4 && (2)\\ -3a + 2b + c &= 4 && (3) \end{aligned}$$

1 Eliminate c from Equations (1) and (2) to obtain a new Equation (4).

2 Eliminate c from Equations (2) and (3) to obtain a new Equation (5).

3 Solve the system of Equations (4) and (5).

4 Substitute the values of a and b into one of the original equations to find c.

Answer. $a = 1, b = 5, c = -3$

8.2.2 Finding a Quadratic Function through Three Points

Checkpoint 8.2.4

a Find the equation of a parabola

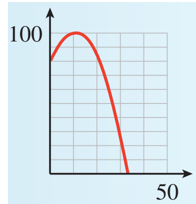
$$y = ax^2 + bx + c$$

that passes through the points $(0, 80)$, $(15, 95)$, and $(25, 55)$.

b Plot the data points and sketch the parabola.

Answer.

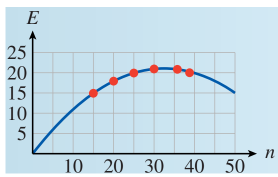
a $y = \frac{-1}{5}x^2 + 4x + 80$

b 

Checkpoint 8.2.6 Sara plans to start a side business selling eggs. She finds that the total number of eggs produced each day depends on the number of hens confined in the henhouse, as shown in the table. Use the first three data points to find a quadratic model $E = an^2 + bn + c$. Plot the data and sketch the curve on the same axes.

Number of hens, n	15	20	25	30	36	39
Number of eggs, E	15	18	20	21	21	20

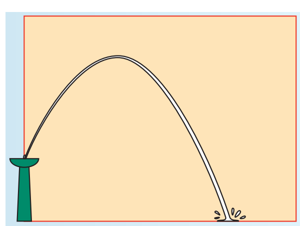
Answer. $E = -0.02n^2 + 1.3n$



8.2.3 Finding an Equation in Vertex Form

Checkpoint 8.2.8 Francine is designing a synchronized fountain display for a hotel in Las Vegas. For each fountain, water emerges in a parabolic arc from a nozzle 3 feet above the ground. Francine would like the vertex of the arc to be 8 feet high and 2 feet horizontally from the nozzle.

a Choose a coordinate system for the diagram below and write a function for the path of the water.



b How far from the base of the nozzle will the stream of water hit the ground?

Answer.

a With the origin on the ground directly below the nozzle, $y = \frac{-5}{4}x^2 + 5x + 3$.

b Approximately 4.53 feet

Checkpoint 8.2.10 To test the effects of radiation, a researcher irradiated male mice with various dosages and bred them with unexposed female mice. The table below shows the fraction of fertilized eggs that survived, as a function of the radiation dosage. (Source: Strickberger, Monroe W., 1976)

Radiation (rems)	100	300	500	700	900	1100	1500
Relative survival of eggs	0.94	0.700	0.544	0.424	0.366	0.277	0.195

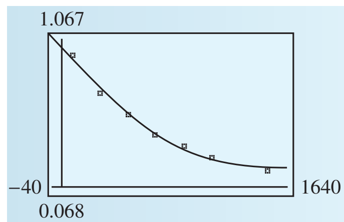
a Enter the data into your calculator and create a scatterplot. Does the graph appear to be linear? Does it appear to be quadratic?

b Fit a quadratic regression equation to the data and graph the equation on the scatterplot.

Answer.

a The graph appears to be quadratic.

b $y = 3.65x^2 - 0.001x + 1.02$



Checkpoint 8.2.13 A speeding motorist slams on the brakes when she sees an accident directly ahead of her. The distance she has traveled t seconds after braking is shown in the table.

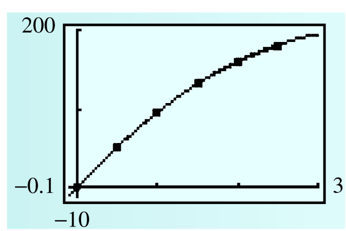
Time (seconds)	0	0.5	1.0	1.5	2.0	2.5
Distance (feet)	0	51	95	131	160	181

a Enter the data into your calculator and create a scatterplot. Fit a quadratic regression equation to the data and graph the equation on the scatterplot.

b Use your regression equation to find the vertex of the parabola. What do the coordinates represent in terms of the problem?

Answer.

a $y = -15x^2 + 110x - 0.07$



b $(3.67, 201)$: The car came to a stop in 3.67 seconds, after sliding 201 feet.

8.2.5 Homework 6.6

For Problems 1–4, solve the system by elimination. Begin by eliminating c.

8.2.5.1. $\begin{aligned} a + \ \ b + c &= -3 \\ a - \ \ b + c &= -9 \\ 4a + 2b + c &= -6 \end{aligned}$

Answer. $a = -2, b = 3, c = -4$

8.2.5.2. $\begin{aligned} a + \ \ b + c &= 10 \\ 4a + 2b + c &= 19 \\ 9a + 3b + c &= 38 \end{aligned}$

8.2.5.3. $\begin{aligned} a - \ \ b + c &= 12 \\ 4a - 2b + c &= 19 \\ 9a + 3b + c &= \ \ 4 \end{aligned}$

Answer. $a = 1, b = -4, c = 7$

8.2.5.4. $\begin{aligned} 4a + 2b + c &= \ \ 14 \\ 9a - 3b + c &= -41 \\ 16a - 4b + c &= -70 \end{aligned}$

For Problems 5–12, find a quadratic equation that fits the data points.

8.2.5.5. Find values for a, b, and c so that the graph of the parabola $y = ax^2 + bx + c$ includes the points $(-1, 0)$, $(2, 12)$, and $(-2, 8)$.

Answer. $a = 3, b = 1, c = -2$. The equation for the parabola is $y = 3x^2 + x - 2$

8.2.5.6. Find values for a, b, and c so that the graph of the parabola $y = ax^2 + bx + c$ includes the points $(-1, 2)$, $(1, 6)$, and $(2, 11)$.

8.2.5.7. A survey to determine what percent of different age groups regularly use marijuana collected the following data.

Age	15	20	25	30
Percent	4	13	11	7

(a) Use the percentages for ages 15, 20, and 30 to fit a quadratic function to the data, $P = ax^2 + bx + c$, where x represents age.

(b) What does your function predict for the percentage of 25-year-olds who use marijuana?

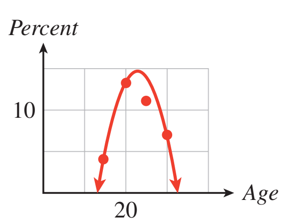
(c) Sketch the graph of your quadratic function and the given data on the same axes.

Answer.

(a) $P = -0.16x^2 + 7.4x - 71$

(b) 14%. It predicts that 14% of the 25-year old population use marijuana on a regular basis.

(c)



8.2.5.8. The following data show the number of people of certain ages who were the victims of homicide in a large city last year.

Age	10	20	30	40
Number of victims	12	62	72	40

(a) Use the first three data points to fit a quadratic function to the data, $N = ax^2 + bx + c$, where x represents age.

(b) What does your function predict for the number of 40-year-olds who were the victims of homicide?

(c) Sketch the graph of your quadratic function and the given data on the same axes.

8.2.5.9. The data below show Americans' annual per capita consumption of chicken for several years since 1985.

Year	1986	1987	1988	1989	1990
Pounds of chicken	51.3	55.5	57.4	60.8	63.6

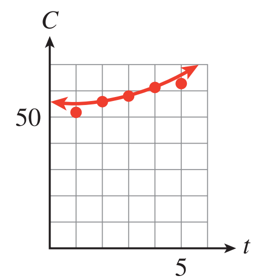
(a) Use the values for 1987 through 1989 to fit a quadratic function to the data, $C = at^2 + bt + c$, where t is measured in years since 1985.

(b) What does your function predict for per capita chicken consumption in 1990?

(c) Sketch the graph of your function and the given data.

Answer.

(a) $C = 0.75t^2 - 1.85t + 56.2$

(c) 

(b) 65.7 lb

8.2.5.10. The data show sales of in-line skates at a sporting goods store at the beach.

Year	1990	1991	1992	1993	1994
Skate sold	54	82	194	446	726

(a) Use the values for 1991 through 1993 to fit a quadratic function to the data, $S = at^2 + bt + c$, where t is measured in years since 1990.

(b) What does your function predict for the number of pairs of skates sold in 1994?

(c) Sketch the graph of your function and the given data.

8.2.5.11. Find a quadratic function for the number of diagonals that can be drawn in a polygon of n sides. Some data are provided.

Sides	4	5	6	7
Diagonals	2	5	9	14

Answer. $D = \dfrac{1}{2}n^2 - \dfrac{3}{2}n$

8.2.5.12. You are driving at 60 miles per hour when you step on the brakes. Find a quadratic function for the distance in feet that your car travels in t seconds after braking. Some data are provided.

Seconds	1	2	3	4
Feet	81	148	210	267

8.2.5.13.

(a) Write an equation for a parabola whose vertex is the point $(-2, 6)$. (Many answers are possible.)

(b) Find the value of a if the y-intercept of the parabola in part (a) is

18.

Answer.

(a) $y = a(x+2)^2 + 6$ (b) 3

8.2.5.14.

(a) Write an equation for a parabola whose vertex is the point $(5, -10)$. (Many answers are possible.)

(b) Find the value of a if the y-intercept of the parabola in part (a) is -5.

8.2.5.15.

(a) Write an equation for a parabola with vertex at $(0, -3)$ and one of its x-intercepts at $(2, 0)$.

(b) Write an equation for a parabola with vertex at $(0, -3)$ and no x-intercepts.

Answer.

(a) $y = \dfrac{3}{4}x^2 - 3$ (b) $y = ax^2 - 3$ for any $a < 0$

8.2.5.16. Write an equation for a parabola with vertex at $(4, 0)$ and y-intercept at $(0, 4)$. How many x-intercepts does the parabola have?

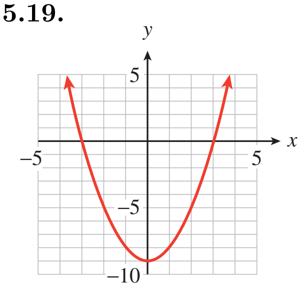
8.2.5.17. Find the equation for a parabola that has a vertex of $(30, 280)$ and passes through the point $(20, 80)$.

Answer. $y = -2(x - 30)^2 + 280$

8.2.5.18. Find the equation for a parabola that has a vertex of $(-12, -40)$ and passes through the point $(6, 68)$.

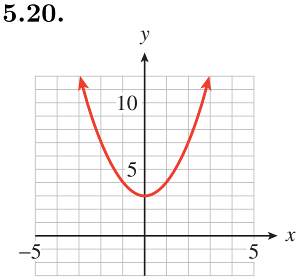
For Problems 19–26, find an equation for each parabola. Use the vertex form or the factored form of the equation, whichever is more appropriate.

8.2.5.19.

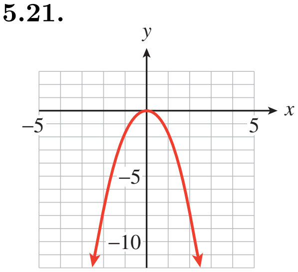


Answer. $y = x^2 - 9$

8.2.5.20.

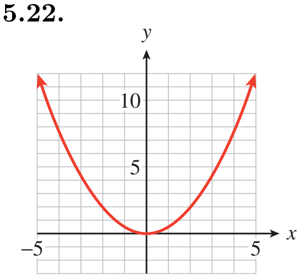


8.2.5.21.

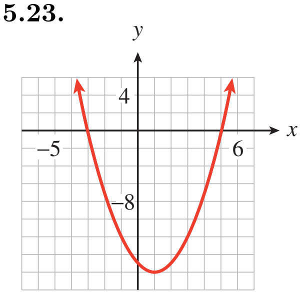


Answer. $y = -2x^2$

8.2.5.22.

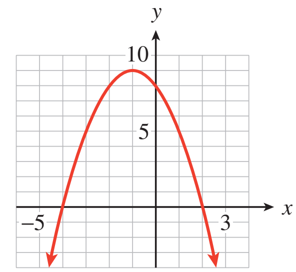


8.2.5.23.

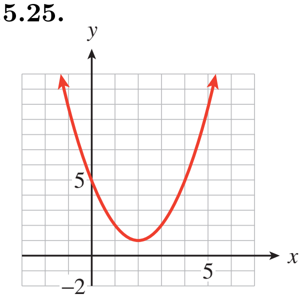


Answer. $y = x^2 - 2x - 15$

8.2.5.24.

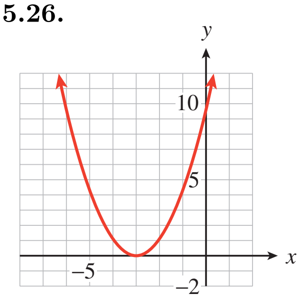


8.2.5.25.



Answer. $y = x^2 - 4x + 5$

8.2.5.26.



8.2.5.27. In skeet shooting, the clay pigeon is launched from a height of 4 feet and reaches a maximum height of 164 feet at a distance of 80 feet from the launch site.

(a) Write a function for the height of the clay pigeon in terms of the horizontal distance it has traveled.

(b) If the shooter misses the clay pigeon, how far from the launch site will it hit the ground?

Answer.

(a) $y = \dfrac{-1}{40}(x - 80)^2 + 164$

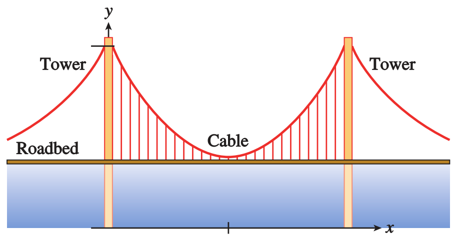
(b) 160.99 ft

8.2.5.28. The batter in a softball game hits the ball when it is 4 feet above the ground. The ball reaches the greatest height on its trajectory, 35 feet, directly above the head of the left-fielder, who is 200 feet from home plate.

(a) Write a function for the height of the softball in terms of its horizontal distance from home plate.

(b) Will the ball clear the left field wall, which is 10 feet tall and 375 feet from home plate?

The cables on a suspension bridge hang in the shape of parabolas. For Problems 29–30, imagine a coordinate system superimposed on a diagram of the bridge, as shown in the figure.



8.2.5.29. The Akashi Kaikyo bridge in Japan is the longest suspension bridge in the world, with a main span of 1991 meters. Its main towers are 297 meters tall. The roadbed of the bridge is 14 meters thick and clears the water below by 65 meters.

(a) Find the coordinates of the vertex and one other point on the cable.

(b) Use the points from part (a) to find an equation for the shape of the cable in vertex form.

Answer.

(a) Vertex: $\left(\frac{1991}{2}, 79\right)$; y-intercept: $(0, 297)$

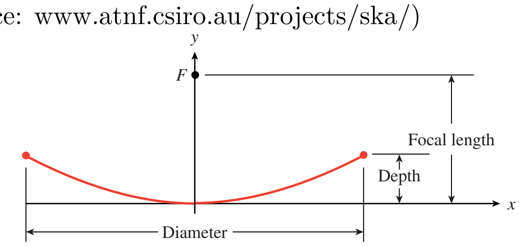
(b) $y = 0.00022(x - 995.5)^2 + 79$

8.2.5.30. A suspension bridge joining Sicily to the tip of Italy over the Straits of Messina has been planned planned and canceled multiple times. The main span of the bridge should be 3300 meters, and its main towers 375 meters tall. The roadbed should be 3 meters thick, clearing the water below by 65 meters.

(a) Find the coordinates of the vertex and one other point on the cable.

(b) Use the points from part (a) to find an equation for the shape of the cable in vertex form.

8.2.5.31. The Square Kilometre Array (SKA) is an international radio telescope project. Project members plan to build a telescope 30 times larger than the largest one currently available. The Australia Telescope National Facility held a workshop in 2005 to design an appropriate antenna. The antenna should be a parabolic dish with diameter from 12 to 20 meters, and the ratio of the focal length to the diameter should be 0.4. The figure shows a cross section of the dish. (Source: www.atnf.csiro.au/projects/ska/)



(a) You want to design a 20-meter-diameter parabolic antenna for the project. What will the focal length of your antenna be?

(b) The equation of the dish has the form $y = \dfrac{x^2}{4F}$, where F is the focal length. What is the equation of the parabola for your antenna?

(c) What is the depth of your parabolic antenna?

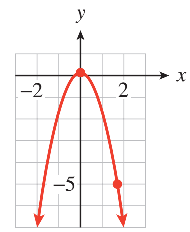
Answer.

(a) 8 m

(b) $y = \dfrac{x^2}{32}$

(c) 3.125 m

8.2.5.32.
Some comets move about the sun in parabolic orbits. In 1973, the comet Kohoutek passed within 0.14 AU (astronomical units), or 21 million kilometers, of the Sun. Imagine a coordinate system superimposed on a diagram of the comet's orbit, with the Sun at the origin, as shown in the figure. The units on each axis are measured in AU.



(a) The comet's closest approach to the Sun (called **perihelion**) occurred at the vertex of the parabola. What were the comet's coordinates at perihelion?

(b) When the comet was first discovered, its coordinates were $(1.68, -4.9)$. Find an equation for comet Kohoutek's orbit in vertex form.

Use your calculator's statistics features for Problems 33–38.

8.2.5.33. The table shows the height of a projectile at different times after it was fired.

Time (seconds)	2	4	6	8	10	12	14
Height (meters)	39.2	71.8	98.0	117.8	131.0	137.8	138.0

(a) Find the equation of the least-squares regression line for height in terms of time.

(b) Use the linear regression equation to predict the height of the projectile 15 seconds after it was fired.

(c) Make a scatterplot of the data and draw the regression line on the same axes.

(d) Find the quadratic regression equation for height in terms of time.

(e) Use the quadratic regression equation to predict the height of the projectile 15 seconds after it was fired.

(f) Draw the quadratic regression curve on the graph from part (c).

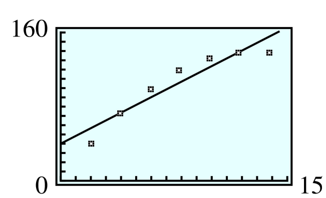
(g) Which model is more appropriate for the height of the projectile, linear or quadratic? Why?

Answer.

(a) $h = 8.24t + 38.89$

(b) 162.5 m

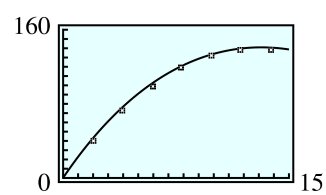
(c)



(d) $h = -0.81t^2 + 21.2t$

(e) 135.7 m

(f)



(g) Quadratic: Gravity will slow the projectile, giving the graph a concave down shape.

8.2.5.34. The table shows the height of a star-flare at different times after it exploded from the surface of a star.

Time (seconds)	0.2	0.4	0.6	0.8	1.0	1.2
Height (kilometers)	6.8	12.5	17.1	20.5	22.8	23.9

(a) Find the equation of the least-squares regression line for height of the flare in terms of time.

(b) Use the linear regression equation to predict the height of the flare 1.4 seconds after it exploded.

(c) Make a scatterplot of the data and draw the regression line on the same axes.

(d) Find the quadratic regression equation for height in terms of time.

(e) Use the quadratic regression equation to predict the height of the flare 1.4 seconds after it exploded.

(f) Draw the quadratic regression curve on the graph from part (c).

(g) Which model is more appropriate for the height of the star-flare, linear or quadratic? Why?

8.2.5.35. In the 1990s, an outbreak of mad cow disease (Creutzfeldt-Jakob disease) alarmed health officials in England. The table shows the number of deaths each year from the disease.

Year	’94	’95	’96	’97	’98	’99	2000	’01	’02	’03	’04
Deaths	0	3	10	10	18	15	28	20	17	19	9

(Source: www.cjd.ed.ac.uk/vcjdqsep05)

(a) The Health Protection Agency determined that a quadratic model was the best-fitting model for the data. Find a quadratic regression equation for the data.

(b) Use your model to estimate when the peak of the epidemic occurred and how many deaths from mad cow disease were expected in 2005.

Answer.

(a) $y = -0.587t^2 + 7.329t - 2.538$

(b) The predicted peak was in 2000, near the end of March. The model predicts 7 deaths for 2005.

8.2.5.36. The table shows the amount of nitrogen fertilizer applied to a crop of soybeans per hectare of land in a trial in Thailand and the resulting yield.

Nitrogen (kg)	0	15	30	60	120
Yield (tons)	2.12	2.46	2.65	2.80	2.60

(Source: www.arc-avrdc.org)

(a) Fit a quadratic regression equation to the data.

(b) Use your model to predict the maximum yield and the amount of nitrogen needed.

8.2.5.37. The number of daylight hours increases each day from the beginning of winter until the beginning of summer, and then begins to decrease. The table below gives the number of daylight hours in Delbert's hometown last year in terms of the number of days since January 1.

Days since January 1 300	0	50	100	150	200	250
Hours of daylight 10.7	9.8	10.9	12.7	14.1	13.9	12.5

(a) Find the equation of the least-squares regression line for the number of daylight hours in terms of the number of days since January 1.

(b) Use the linear regression equation to predict the number of daylight hours 365 days after January 1.

(c) Make a scatterplot of the data and draw the regression line on the same axes.

(d) Find the quadratic regression equation for the number of daylight hours in terms of the number of days since January 1.

(e) Use the quadratic regression equation to predict the number of daylight hours 365 days after January 1.

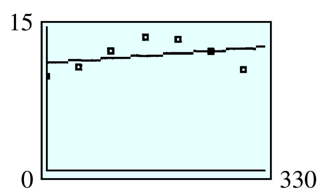
(f) Draw the quadratic regression curve on the graph from part (c).

(g) Predict the number of daylight hours 365 days since January 1 without using any regression equation. What does this tell you about the linear and quadratic models you found?

Answer.

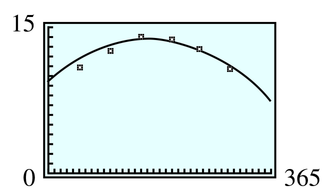
(a) $y = 0.0051t + 11.325$

(b) 13.2 hr

(c) 

(d) $y = -0.00016t^2 + 0.053t + 9.319$

(e) 7.4 hr

(f) 

(g) 9.8 hr (the same as the previous year); Neither model is appropriate.

8.2.5.38. To observers on Earth, the Moon looks like a disk that is completely illuminated at full moon and completely dark at new moon. The table below shows what fraction of the Moon is illuminated at 5-day interval after the last full moon.

Days since full moon	0	5	10	15	20	25
Fraction illuminated	1.000	0.734	0.236	0.001	0.279	0.785

(Source: www.arc-avrdc.org)

(a) Find the equation of the least-squares regression line for the fraction illuminated in terms of days.

(b) Use the linear regression equation to predict the fraction illuminated 30 days after the full moon.

(c) Make a scatterplot of the data and draw the regression line on the same axes.

(d) Find the quadratic regression equation for the fraction illuminated in terms of days.

(e) Use the quadratic regression equation to predict the fraction illuminated 30 days after the full moon.

(f) Draw the quadratic regression curve on the graph from part (c).

(g) Predict the fraction of the disk that is illuminated 30 days after the full moon without using any regression equation. What does this tell you about the linear and quadratic models you found?

Appendix A

Algebra Skills Refresher

Appendix B

Using a Graphing Calculator

This appendix provides instructions for TI-84 or TI-83 calculators from Texas Instruments, but most other calculators work similarly.We describe only the basic operations and features of the graphing calculator used in your textbook.

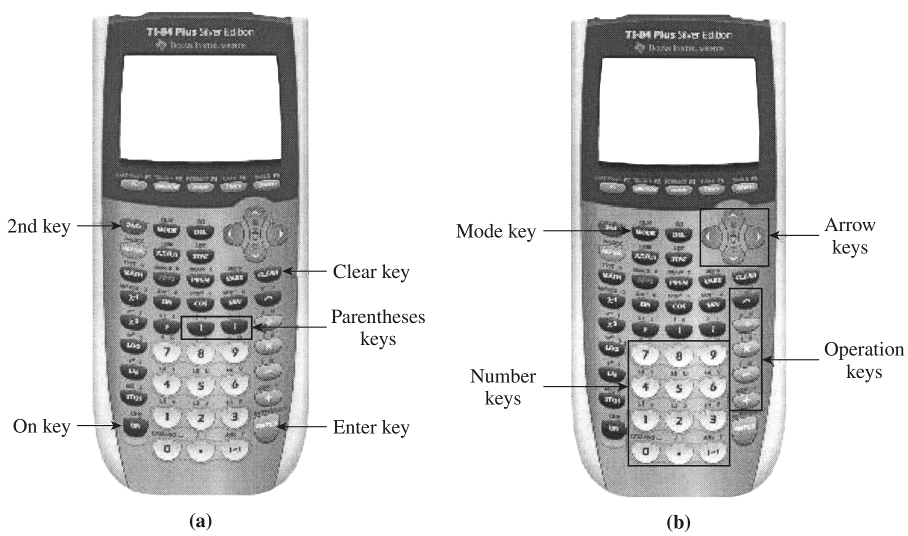


Figure B.0.1

Appendix C

Glossary

Abbreviations used in this glossary: **n** (noun), **v** (verb), **adj** (adjective)

A.

absolute value n, the distance on the number line from a number to 0. For example, the absolute value of -7 is 7. This fact is expressed by the equation $|-7| = 7$.

absolute value equation n, an equation in which the variable occurs between the absolute value bars.

absolute value inequality n, an inequality in which the variable occurs between the absolute value bars.

algebraic expression n, a meaningful combination of numbers, variables, and operation symbols. Also called an **expression**.

algebraic fraction n, a fraction whose numerator and denominator are polynomials. Also called a **rational expression**.

algebraic solution n, a method for solving equations (or inequalities) by manipulating the equations (or inequalities). Compare with **graphical solution** and **numerical solution**.

allometric equation n, an equation showing the (approximate) relationship between a living organism's body mass and another of the organism's properties or processes, usually given in the form $y = k(\text{mass})^p$.

altitude n, (i) the distance above the ground or above sea level; (ii) the vertical distance between the base and the opposite vertex of a triangle, pyramid, or cone; (iii) the distance between parallel sides of a parallelogram, trapezoid, or rectangle. Also called **height**.

amortization n, the payment of a debt through regular installments over a period of time.

amount (in an interest-bearing account), **n**, the sum of the principal that was invested and all the interest earned.

amplitude n, the vertical distance between the midline and the maximum value of a sinusoidal function.

annuity n, sequence of equal payments or deposits made at equal time intervals.

approximation n, an inexact result.

area n, a measure of the two-dimensional space enclosed by a polygon or curve, typically expressed in terms of square units, such as square meters or square feet, etc.

ascending powers n, an ordering of the terms of a polynomial so that the exponents on the variable are increasing, such as in the polynomial $1 + x + x^2$.

associative law of addition n, the property that when adding three or more terms, the grouping of terms does not affect the sum. We express this formally by saying that if a, b, and c are any numbers, then $(a + b) + c = a + (b + c)$.

associative law of multiplication n, the property that when multiplying three or more factors, the grouping of factors does not affect the product. We express this formally by saying that if a, b, and c are any numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

asymptote n, a reference line (or curve) towards which the graph of an equation tends as the value of x and/or y grows or diminishes without bound.

augmented matrix (for a linear systterm with n variables in standard form), n, the matrix obtained by making each row of the matrix correspond to an equation of the systterm, with the coefficients of the variables filling the first n columns, and the last (that is, the $n + 1$) column having the constants.

axis n, (**plural axes**), a line used as a reference for position and/or orientation.

axis of symmetry n, a line that cuts a plane figure into two parts, each a mirror image of the other.

B.

back substitution n, a technique for solving a triangular systterm of linear equations.

bar graph n, a picture of numerical information in which the lengths or heights of bars are used to represent the values of variables.

base n, (i) a number or algebraic expression that is used as a repeated factor, where an exponent indicates how many times the base is used as a factor. For example, when we write 3^5, the base is 3. (ii) The bottom side of a polygon. (iii) The bottom face of a solid.

base angles n, the angles opposite the equal sides in an isosceles triangle.

binomial n, a polynomial with exactly two terms.

binomial expression n, a sum of two unlike terms, such as $\sqrt{3} + \sqrt{2}$.

build (a fraction) v, to find an equivalent fraction by multiplying numerator and denominator by the same nonzero expression.

building factor n, an expression by which both numerator and denominator of a given fraction are multiplied (in order to build the fraction).

cartesian coordinate syststerm n, the grid that associates points in the coordinate plane to ordered pairs of numbers.

C.

cartesian plane n, a plane with a pair of coordinate axes. Also called a **coordinate plane**.

change in (a variable) n, the final value (of the variable) minus the starting value.

change of variables n, (i) a **transformation** of data, (ii) substitution of a new variable for a variable expression, for example, replacing t^2 with x so that the equation $y = at^2 + b$ becomes $y = ax + b$.

circle n, the set of all points in a plane at a fixed distance (the **radius**) from the center.

circumference n, the distance around a circle.

closed interval n, a set of numbers, denoted by $[a, b]$, which includes all the numbers between a and b as well as the numbers a and b thtermselves, where a and b are real numbers and $a < b$. Or the set of numbers denoted by $(-\infty, b]$, which includes the real number b and all numbers less than b, or the set of numbers denoted by $[a, \infty)$, which includes the real number a and all numbers greater than a.

coefficient n, the numerical factor in a term. For example, in the expression $32a + 7b$, the coefficient of a is 32 and the coefficient of b is 7.

coefficient matrix (for a linear syststerm with n variables in standard form) n, the matrix of n columns obtained by making each row of the matrix correspond to an equation of the syststerm, with the coefficients of the variables filling the n columns (and the constants are not represented in the matrix).

common factor (of two or more expressions) n, a quantity that divides evenly into each of the given expressions.

common log or common logarithm (of a given positive number x) n, the exponent, denoted by $\log(x)$ (or by $\log(x)$) for the number 10 to obtain the value x, that is, $10^{\log(x)} = x$.

commutative law of addition n, the property that when adding terms, the order of the terms does not affect the sum. We express this formally by saying that if a and b are any numbers, then $a + b = b + a$.

commutative law of multiplication n, the property that when multiplying factors, the order of the factors does not affect the product. We express this formally by saying that if a and b are any numbers, then $a \cdot b = b \cdot a$.

compltermentary angles n, two angles whose measures add up to 90°.

complete the square v, to determine the appropriate constant to add to a binomial of the form $ax^2 + bx$ so that the result can be written in the form $a(x + k)^2$.

complex conjugate (of a complex number) n, the complex number with the same real part and opposite imaginary part; for example, the complex conjugate of $1+i$ is $1-i$.

complex fraction n, a fraction that contains one or more fractions in its numerator and/or in its denominator.

complex plane n, a coordinate plane representing complex numbers, with the real parts corresponding to the values on the horizontal axis and imaginary parts corresponding to values on the vertical axis.

complex number n, a number that can be written in the form $a+bi$, where a and b are real numbers and $i^2 = -1$.

component n, one of the values of an ordered pair or ordered triple.

compound inequality n, a mathtermatical statterment involving two order symbols. For example, the compound inequality $1 < x < 2$ says that "1 is less than x, and x is less than 2."

compound interest (or compounded interest) n, an interest earning agreterment in which the interest payment at a given time is computed based on the sum of the original principal and any interest money already accrued.

compounding period n, the time interval between consecutive interest payments to an account that earns interest.

concave down (of a graph) adj, curving so that the ends of a flexible rod would need to be bent downward (compared with a straight rod) to lie along the graph. Or equivalently, curving so that a line segment tangent to the curve will lie above the curve.

concave up (of a graph) adj, curving so that the ends of a flexible rod would need to be bent upward (compared with a straight rod) to lie along the graph. Or equivalently, curving so that a line segment tangent to the curve will lie below the curve.

concavity n, a description of a curve as either concave up or concave down.

concentric (of circles or spheres) adj, having the same center.

conditional equation n, an equation that is true for some (but not all) values of the variable(s).

cone n, a three-dimensional object whose base is a circle and whose vertex is a point above the circle. The points on the segments joining the circle to the vertex make up the cone.

congruent adj, having all measure(s) matching exactly. For example, two line segments are congruent when they have the same length; two triangles are congruent if all three sides and all three angles of one match exactly with the corresponding parts of the other triangle.

conjugate n, (i) (of a complex number) the complex number with the same real part and opposite imaginary part; (ii) (of a binomial expression) the binomial expression with the same first term and opposite second term.

conjugate pair n, (i) (of a complex number) a complex number and its conjugate; (ii) (of a binomial expression) the binomial expression and its conjugate.

consistent (of a systterm of equations) adj, having at least one solution.

consistent and independent (of a systterm of linear equations) adj, having exactly one solution.

constant adj, unchanging, not variable. For example, we say that the product of two variables is constant if the product is always the same number, for any values of the variables.

constant n, a number (as opposed to a variable).

constant of proportionality n, the quotient of two directly proportional variables, or the product of two inversely proportional variables. Also called the **constant of variation**.

constant of variation *see* constant of proportionality.

constraint n, an equation or inequality involving one or more variables, typically specifying a condition that must be true in the given context.

continuous adj, without holes or gaps. For example, a curve is continuous if it can be drawn without lifting the pencil from the page, and a function is continuous if its graph can be drawn without lifting the pencil from the page.

continuous compounding n, an interest earning agreterment in which the amount in the account is Pe^{rt}, where P is the initial principal, r is the annual interest rate, and $e \approx 2.71828$ is the base of the natural logarithm.

conversion factor n, a ratio used to convert from one unit of measure to another.

coordinate n, a number used with a number line or an axis to designate position.

coordinate axis n, one of the two perpendicular number lines used to define the coordinates of points in the plane.

coordinate plane n, a plane with a pair of coordinate axes. Also called the **Cartesian plane** or ***xy*-plane**.

corollary n, a mathtermatical fact that is a consequence of a previously known fact.

costs n, money that an individual or group must pay out. For example, the costs of a company might include payments for wages, supplies, and rent.

counting number n, one of the numbers $1, 2, 3, 4, \ldots$.

cube n, (i) a three-dimensional box whose six faces all consist of squares; (ii) an expression raised to the power 3.

cube v, to raise an expression to the power 3. For example, to cube 2 means to form the product of three 2s: $2^3 = 2 \times 2 \times 2 = 8$.

cube root n, a number that when raised to the power 3 gives a desired value. For example, 2 is the cube root of 8 because $2^3 = 8$.

cubic adj, having to do with the third degree of a variable or with a polynomial of degree 3.

cylinder n, a three-dimensional figure in the shape of a soft drink can. The top and base are circles of identical size, and the line segments joining the two circles are perpendicular to the planes containing the two circles.

D.

decay factor n, the factor by which an initial value of a diminishing quantity is multiplied to obtain the final value.

decimal adj, having to do with a base-10 numeration systterm.

decimal place n, the position of a digit relative to the decimal point. For example, in the number 3.14159, the digit 4 is in the second decimal place, or hundredths place.

decimal point n, the mark "." that is written between the whole number part and the fractional part of a decimal number. For example, the decimal form of $1\frac{3}{10}$ is 1.3.

decreasing adj, (i) (of numbers) moving to the left on a number line: Positive numbers are decreasing when getting closer to zero, and negative numbers are decreasing when they move farther from 0; (ii) (of a graph) having decreasing values of y when moving along the graph from left to right; (iii) (of a function) having a decreasing graph.

degree n, a measure of angle equal to $\frac{1}{360}$ of a complete revolution.

degree n, (i) (of a monomial) the exponent on the variable, or if there are more than one variable, the sum of the exponents of all the variables; (ii) (of a polynomial) the largest degree of the monomials in the polynomial.

demand equation n, an equation that gives the quantity of some product that consumers are willing to purchase in terms of the price of that product.

denominator n, the expression below the fraction bar in a fraction.

dependent adj, (of a systterm of equations) having infinitely many solutions.

dependent variable n, a variable whose value is determined by specifying the value of the independent variable.

descending powers n, expressed with the term with the highest degree written first, then the term with the second highest degree, etc.

diagonal n, (i) a line segment joining one vertex of a quadrilateral to the opposite vertex; (ii) a line segment joining opposite corners of a box; (iii) the entries of a matrix whose row number match the column number, that is, the $(1,1),(2,2),\ldots,(n,n)$ entries

diameter n, (i) a line segment passing through the center of a circle (or sphere) with endpoints on the circle (sphere); (ii) the length of that line segment.

difference n, the result of a subtraction. For example, the expression $a-b$ represents the difference between a and b.

difference of squares n, an expression of the form a^2-b^2.

dimension n, (i) (of a matrix) the numbers of rows and columns respectively of the matrix, also called the **order** of the matrix. For example, a matrix with dimension 2 by 3 (or 2×3) has two rows and three columns; (ii) a measurterment defining a geometric figure, for example, the length and width are dimensions of a rectangle.

direct variation n, a relation between two variables in which one is a constant multiple of the other (so that the ratio between the two variables is the constant), or in which one is a constant multiple of a positive exponent power of the other variable.

directed distance n, the difference between the ending coordinate and the starting coordinate of points on a number line; the directed distance is negative if the ending value is smaller than the starting value. For example, the directed distance from 5 to 2 is $2 - 5 = -3$.

directly proportional adj, describing variables related by direct variation.

discriminant n, (for the quadratic polynomial $ax^2 + bx + c$) the quantity $b^2 - 4ac$.

distributive law n, the property that for any numbers a, b, and c, $a(b+c) = ab + ac$.

divisor n, a quantity that is divided into another quantity. For example, in the expression $a \div b$, the divisor is b.

domain n, the set of all acceptable inputs for a function or equation.

doubling time n, (of exponential growth) the time required for a quantity to double in size.

E.

elementary row operation n, one of the three following operations: (1) an exchange of two rows, (2) multiplying all entries of a row by a nonzero constant, (3) adding a multiple of any row to another row.

elimination n, a method for solving a system of equations that involves adding together the equations of the systterm or multiples of the equations of the systterm.

empirical model n, an equation whose graph (approximately) fits a given set of data (but gives no information about the physical processes involved).

entry n, a value in a matrix, often identified by specifying location by row and column.

equation n, a mathtermatical statterment that two expressions are equal, for example, $1 + 1 = 2$.

equation in two variables n, an equation that involves two variables.

equilateral adj, (of a polygon) having all sides of equal length.

equilibrium point n, the point where the graphs of the supply and dtermand equations intersect

equivalent adj, representing the same value.

equivalent equations n, equations that have the same solutions.

equivalent expressions n, expressions that have the same value for all permissible values of their variables.

error tolerance n, the allowable difference between an estimate and the actual value.

evaluate v, to determine the value of an expression when the variable in the expression is replaced by a number.

exact adj, not simply close, but with absolutely no deviation from an intended value.

exact solution n, the exact value of a solution, i.e., not an approximation.

exponent n, the expression that indicates how many times the base is used as a factor. For example, when we write 3^5, the exponent is 5, and $3^5 = 3 \times 3 \times 3 \times 3 \times 3$.

exponential decay n, a manner of decreasing characterized by a constant decay factor for any fixed specified interval of time, or equivalently, modeled by a function f with the form $f(t) = ab^t$, where a and b are positive constants and $0 < b < 1$.

exponential equation n, an equation containing a variable expression as an exponent.

exponential function n, a function f which can be put in the form $f(x) = ab^x$, where a is a nonzero constant and $b \neq 1$ is a positive constant.

exponential growth n, growth characterized by a constant growth factor for any fixed specified interval of time, or equivalently, modeled by a function f with the form $f(t) = ab^t$, where a and b are positive constants and $b > 1$.

exponential notation n, a way of writing an expression that involves radicals and/or reciprocals in terms of powers that have fractional and/or negative exponents. For example, the exponential notation for $\sqrt{3}$ is $3^{1/2}$.

expression *see* algebraic expression.

extraction of roots n, a method used to solve (quadratic) equations.

extraneous solution n, a value that is not a solution to a given equation but is a solution to an equation derived from the original.

extrapolate v, to estimate the value of a dependent variable for a value of the independent variable that is outside the range of the data.

F.

factor n, an expression that divides evenly into another expression. For example, 2 is a factor of 6.

factor v, to write as a product. For example, to factor 6 we write $6 = 2 \times 3$.

factored form n, (i) (of a polynomial or algebraic expression) an expression written as a product of two or more factors, where the algebraic factors cannot be further factored; (ii) (of an equation of a parabola) the form $y = a(x - r_1)(x - r_2)$.

feasible solution n, an ordered pair which satisfies the constraints of a linear programming problterm.

FOIL n, an acronym for a method for computing the product of two binomials: **F** stands for First terms, **O** for Outer terms, **I** for Inner terms, and **L** stands for Last terms.

formula n, an equation involving two or more variables.

fraction bar n, the line segment separating the numerator and denominator of a fraction. In the fraction $\frac{1}{2}$, the fraction bar is the short segment between the 1 and the 2.

function n, a relationship between two variables in which each value of the input variable determines a unique value of the output variable.

function of two variables n, a relationship between an output variable and an ordered pair of input variables in which each ordered pair of the input variables determines a unique value of the output variable.

function value n, an output value of a function.

fundamental principle of fractions n, the property that the value of a fraction is unchanged when both its numerator and denominator are multiplied by the same nonzero value. We express this formally by saying if a is any number, and b and c are nonzero numbers, then $\dfrac{a \cdot c}{b \cdot c} = \dfrac{a}{b}$.

G.

Gaussian reduction n, the process of performing eltermentary row operations on a matrix to obtain a matrix in echelon form.

geometrically similar adj, having the same shape (but possibly different size).

graph n, a visual representation of the values of a variable or variables, typically drawn on a number line or on the Cartesian plane.

graph v, to draw a graph.

graph of an equation (or inequality) n, a picture of the solutions of an equation (or inequality) using a number line or coordinate plane.

graphical solution n, a method for solving equations (or inequalities) by reading values off an appropriate graph. Compare with **algebraic solution** and **numerical solution**.

greatest common factor (GCF) of two or more expressions n, the largest factor that divides evenly into each expression.

growth factor n, the factor by which an initial value of a growing quantity is multiplied to obtain the final value.

guidepoints n, individual points that are plotted to help draw a graph (by hand).

H.

half-life n, (of exponential decay) the time required for a quantity to diminish to half its original size.

half-plane n, either of the two regions of a plane that has been divided into two regions by a straight line

height *see* altitude.

htermisphere n, half a sphere (on one side or the other of a plane passing through the center).

horizontal asymptote n, a line parallel to the x-axis toward which the graph of an equation tends as the value of x grows or diminishes without bound.

horizontal axis n, the horizontal coordinate axis. Often called the x-**axis**.

horizontal intercept n, where the graph meets the horizontal axis. Also called x-**intercept**.

horizontal line test n, a test to determine if a function has an inverse function: If no horizontal line intersects the graph of a function more than once, then the inverse is also a function.

horizontal translation (of a graph) n, the result of moving all points of the graph straight left (or all straight right) by the same distance.

hypotenuse n, the longest side of a right triangle. (It is always the side opposite the right angle.)

I.

identity n, an equation that is true for all permissible values of the variable(s).

imaginary axis n, the vertical axis in the complex plane.

imaginary number n, a complex number of the form bi, where b is a real number and $i^2 = -1$.

imaginary part n, (of a complex number) the coefficient of i when the complex number is written in the form $a+bi$, where a and b are real numbers. For example, the imaginary part of $4 - 7i$ is -7.

imaginary unit n, a nonreal number denoted by i and which satisfies $i^2 = -1$, that is, i is defined to be a square root of -1.

inconsistent adj, (of a system of equations) having no solution.

increasing adj, (i) (of numbers) moving to the right on a number line: Positive numbers are increasing when moving farther from zero, and negative numbers are increasing when they move closer to 0; (ii) (of a graph) having increasing values of y when moving along the graph from left to right; (iii) (of a function) having an increasing graph.

independent adj, (i) (of a system of 2 linear equations in 2 variables) having different graphs for the two equations; (ii) (of a system of n linear equations in n variables) having no one equation equal to a linear combination of the others.

independent variable n, a variable whose value determines the value of the dependent variable.

index n, (of a radical) the number at the left of the radical symbol that indicates the type of root involved; for example, the index of 3 in the expression $\sqrt[3]{x}$ indicates a cube root.

inequality n, a mathematical statement of the form $a < b$, $a \leq b$, $a > b$, $a \geq b$, or $a \neq b$.

inflation n, a persistent increase over time of consumer prices.

inflection point n, a point where a graph changes concavity.

initial value n, the starting value of a variable, often when $t = 0$.

input n, value of the independent variable.

integer n, a whole number or the negative of a whole number.

intercept n, a point where a graph meets a coordinate axis.

intercept method n, a method for graphing a line by finding its horizontal and vertical intercepts.

interest n, money paid for the use of money. For example, after borrowing money, the borrower must pay the lender not only the original amount of money borrowed (known as the **principal**) but also the interest on the principal.

interest rate n, the fraction of the principal that is paid as interest for one year. For example, an interest rate of 10% means that the interest for one year will be 10% of the principal.

interpolate v, to estimate the value of a dependent variable based on data that include both larger and smaller values of the independent variable.

intersection point n, a point in common to two graphs.

interval n, a set of numbers that includes all the numbers between a and b (possibly but not necessarily including a and/or b), where a and b are real numbers. Or the set of all numbers less than b (and possibly including b), or the set all numbers greater than a (and possibly including a).

interval notation n, notation used to designate an interval. For example, $[2, 3]$ is the interval notation to designate all the real numbers from 2 to 3, including both 2 and 3.

inverse function n, a function whose inputs are outputs of a given function f, and whose outputs are the corresponding inputs of f.

inverse square law n, a physical law that states that the magnitude of some quantity is inversely proportional to the square of the distance to the source of that quantity.

inverse variation n, a relation between two variables in which one is a constant divided by the other (so that the product of the two variables is the constant), or in which one is a constant divided by a positive exponent power of the other.

inversely proportional adj, describing variables related by inverse variation.

irrational number n, a number that is not rational but does correspond to a point on the number line.

isolate v, (a variable or expression) to create an equivalent equation (or inequality) in which the variable or expression is by itself on one side of the equation (or inequality).

isosceles triangle n, a triangle with two sides of equal length.

J.

joint variation n, a relationship among three or more variables in which whenever all but two variables are held constant, those remaining two variables vary directly or inversely with each other.

L.

law of exponents n, a basic property about powers and exponents.

lead coefficient n, (of a polynomial) the coefficient of term with highest degree.

leading entry n, (of a row in a matrix) the first nonzero entry of the row, when read from left to right.

leg n, one of the two shorter sides of a right triangle, or the length of that side.

like fractions n, fractions with equivalent denominators.

like terms n, terms with equivalent variable parts.

line segment n, the points on a single line that join two specified points (the **endpoints**) on that line.

linear combination n, (i) the sum of a nonzero constant multiple of one equation and a nonzero constant multiple of a second equation; (ii) the sum of constant multiples of quantities.

linear combinations n, a procedure for solving a linear system of equations which requires taking one or more linear combination of equations.

linear equation n, an equation such as $2x + 3y = 4$ or $x - 3y = 7$ in which each term has degree 0 or 1.

linear programming n, the study of optimizing functions with constraint equations and/or constraint inequalities.

linear regression n, the process of using a line to predict values of a (dependent) variable.

linear system n, a set of linear equations.

linear term n, a term that consists of a constant times a variable.

log *see* **logarithm**.

log scale n, a scale of measurement that uses the logarithm of a physical quantity rather than the quantity itself.

log-log paper n, a type of graph paper in which both horizontal and vertical axes use log scales.

logarithm n, (i) an exponent; (ii) a function whose outputs are exponents associated with a given base.

logarithmic equation n, an equation involving the logarithm of a variable expression.

logarithmic function n, a function of the form $f(x) = \log_b(x)$, where b is a positive constant different from 1.

lowest common denominator (LCD) n, (of two or more fractions) the smallest denominator that is a multiple of the denominators in the given fractions.

lowest common multiple (LCM) n, (of two or more counting numbers) the smallest counting number that the given numbers divide into evenly.

M.

mathematical model n, a representation of relationships among quantities using equations, tables, and/or graphs.

matrix n, a rectangular array of numbers.

maximum adj, largest or greatest.

maximum n, largest value.

maximum value n, (of a variable expression) the largest value that the expression can equal when the variable is allowed to assume all possible values.

mean n, the average of a set of numbers, computed by adding the numbers and dividing by how many are in the set. For example, the mean of 5, 2, and 11 is $\frac{5+2+11}{3} = 6$.

mechanistic model n, an equation whose graph (approximately) fits a given set of data and whose parameters are estimates about the physical properties involved.

median n, the middle number in a set of numbers when written in increasing order. For example, the median of 5, 2, and 11 is 5. If the set has two numbers in the middle when written in order, then the median of the set is the mean of those middle numbers. For example, the median of 6, 1, 9, and 27 is $\frac{6+9}{2} = 7.5$.

minimum adj, least or smallest.

minimum n, smallest value.

minimum value n, (of a variable expression) the smallest value that the expression can equal when the variable is allowed to assume all possible values.

mode n, the number that occurs most frequently in a set of numbers. For example, the mode of 1, 1, 2, and 3 is 1.

model n, a mathematical equation or graph or table used to represent a situation in the world or a situation described in words. For example, the equation $P = R - C$ is a model for the relationship among the variables of profit, revenue, and cost.

model v, to create a model.

monomial n, an algebraic expression with only one term.

monotonic adj, (of a function or graph) either never increasing or never decreasing.

multiplicative property (of absolute values) n, the property that $|a \cdot b| = |a| \cdot |b|$ for any real numbers a and b.

multiplicity n, (i) (of a zero of a polynomial) the number of times the corresponding linear factor appears as a factor of the polynomial. For example, -9 is a zero of multiplicity one and 7 is a zero of multiplicity two for the polynomial $p(x) = x^3 - 5x^2 - 77x + 441$ because $p(x)$ factors as $p(x) = (x+9)(x-7)^2$; (ii) (of a solution to a polynomial equation) the multiplicity of the zero of the corresponding polynomial. For example, -9 is a solution of multiplicity one and 7 is a solution of multiplicity two for the polynomial equation $x^3 = 5x^2 + 77x - 441$ because the equation can be written in the standard form $p(x) = 0$, where $p(x)$ factors as $p(x) = (x+9)(x-7)^2$.

N.

natural base n, the irrational number $e \approx 2.71828182846$, which is useful in calculus, statistics, and other mathematical topics.

natural exponential function n, the function $f(x) = e^x$, where e is the natural base.

natural log or natural logarithm n, the logarithm with base e, where e is the natural base.

natural number n, a counting number.

negative number n, a number that is less than zero.

negative of n, the opposite of.

net change n, the final value of a variable minus the initial value. For example, if an object's weight decreases from 15 pounds to 13 pounds, the net change in weight is -2 pounds.

nonstrict inequality n, a mathematical statement of the form $a \leq b$ or $a \geq b$.

normal adj, perpendicular.

n**th root n**, a number which when raised to the power n gives a desired value. When $b^n = a$, then b is an nth root of a.

number line n, a line with coordinates marked on it representing the real numbers.

numerator n, the expression in a fraction that is above the fraction bar.

numerical solution n, a method for solving equations by reading values from an appropriate table of values. Compare with **algebraic solution** and **graphical solution**.

O.

objective function n, (in linear programming) the function that is to be optimized.

one-to-one adj, (pertaining to a function) having the property that every output comes from one and only one input.

open interval n, a set of numbers denoted by (a, b), which includes all the numbers between a and b but not the numbers a and b themselves, where a and b are real numbers and $a \neq b$. Or the set of numbers denoted by $(-\infty b)$, which includes all numbers less than b, or the set of numbers denoted by (a, ∞), which includes all numbers greater than a.

operation n, addition, subtraction, multiplication, or division (or raising to a power or taking a root).

opposite n, the number on the number line that is on the other side of 0 and at the same distance. For example, 5 and -5 are opposites.

order n, (of a matrix) the numbers of rows and columns respectively of the matrix, also called the **dimension** of the matrix. For example, a matrix with order 2 by 3 (or 2×3) has two rows and three columns.

order of operations n, rules that prescribe the order in which to carry out the operations in an expression.

order symbol n, one of the four symbols $<$, or $\leq$, or $>$, or $\geq$.

ordered pair n, a pair of numbers enclosed in parentheses, like this: (x, y). Often used to specify a point or a location on the coordinate plane.

ordered triple n, three numbers enclosed in parentheses, like this: (x, y, z). Often used to specify a solution to a system of equations in three variables or a point in three-dimensional space.

origin n, the point where the coordinate axes meet. It has coordinates (0, 0).

output n, value of the dependent variable.

P.

parabola n, a curve with the shape of the graph of $y = ax^2$, where $a \neq 0$.

parallel lines n, lines that lie in the same plane but do not intersect, even if extended indefinitely.

parameter n, a constant in an equation that varies in other equations of the same form. For example, in the slope-intercept formula $y = b + mx$, the constants b and m are parameters.

percent n, a fraction with (an understood) denominator of 100. For example, to express the fraction $\frac{51}{100}$ as a percent, we write 51% or say "51 percent."

percent increase n, the change in some quantity, expressed as a percentage of the starting amount.

perfect square n, the square of an integer. For example, 9 is a perfect square because $9 = 3^2$.

perimeter n, the distance around the edge or boundary of a two-dimensional figure.

perpendicular lines n, lines that meet and form right angles with each other.

piecewise defined function n, a function defined by multiple expressions, one expression for each specified interval of the independent variable.

point-slope form n, one way of writing the equation for a line: $y - y_1 = m(x - x_1)$ or $\frac{y-y_1}{x-x_1} = m$.

polygon n, a simple closed geometric figure in the plane consisting of line segments (called sides) that meet only at their endpoints. For example, triangles are polygons with three sides.

polynomial n, a sum of terms, where each term is either a constant or a constant times a power of a variable, and the exponent is a positive integer.

polynomial function n, a function that can be written in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x_2 + a_1 x + a_0$ where $a_0, a_1, a_2, \ldots a_n$ are constants.

positive number n, a number greater than zero.

power n, an expression that consists of a base and an exponent.

power function n, a function of the form $f(x) = ax^p$, where a and p are constants.

prime (or prime number) n, an integer greater than 1 whose only whole number factors are itself and 1.

principal n, the original amount of money deposited in an account or borrowed from a lender. (Compare with **interest**.)

principal root *see* **principal square root**.

principal square root n, the nonnegative square root.

product n, the result of a multiplication. For example, the expression $a \cdot b$ represents the product of a and b.

profit n, the money left after counting all the revenue that came in and subtracting the costs that had to be paid out.

proportion n, an equation in which each side is a ratio.

proportional *see* **directly proportional**, **inversely proportional**.

pyramid n, a three-dimensional object like a cone except that the base is a polygon instead of a circle.

Pythagorean theorem: If the legs of a right triangle are a and b and the hypotenuse is c, then $a^2 + b^2 = c^2$.

Q.

quadrant n, any of the four regions into which the coordinate axes divide the plane. The **first quadrant** consists of the points where both coordinates are positive; the **second quadrant** where the first coordinate is negative and the second coordinate positive; the **third quadrant** consists of points where both coordinates are negative; and the **fourth quadrant** contains the points where the first coordinate is positive and the second coordinate is negative.

quadratic adj, relating to the square of a variable (or of an expression).

quadratic equation n, an equation that equates zero to a polynomial of degree 2 (or an equivalent equation).

quadratic formula n, the formula that gives the solutions of the quadratic equation $ax^2 + bx + c = 0$, namely $x = \frac{-b\text{ś}\sqrt{b^2-4ac}}{2a}$.

quadratic function n, a function of the form $f(x) = ax^2 + bx + c$.

quadratic polynomial n, a polynomial whose degree is 2.

quadratic regression n, the process of using a quadratic function to predict values of a (dependent) variable.

quadratic term n, a term whose degree is 2.

quadratic trinomial n, a polynomial of degree 2 and having exactly 3 terms.

quadrilateral n, a polygon with exactly 4 sides.

quartic adj, (pertaining to a polynomial) having degree 4.

quotient n, the result of a division. For example, the expression $a \div b$ represents the quotient of a and b.

R.

radical n, a root of a number, such as a square root or a cube root.

radical expression n, a square root, a cube root, or an nth root.

radical equation n, an equation in which the variable appears under a radical sign.

radical notation n, notation using the radical sign to indicate a root.

radical sign n, the symbol $\sqrt{\ }$, which is used to indicate the principal square root, or the symbol $\sqrt[3]{\ }$, which is used to indicate cube root, or the symbol $\sqrt[n]{\ }$, which is used to indicate nth root, where n is a counting number greater than 2.

radicand n, the expression under a radical sign.

radius n, (i) a line segment from the center of a circle (or sphere) to a point on the circle (sphere), (ii) the length of that line segment.

raise to a power v, use as a repeated factor, for example, to raise x to the power 2 is the same as multiplying $x \cdot x$.

range n, (i) the set of all output values for a function; (ii) the difference between the largest and smallest values in a set of data.

rate n, a ratio that compares two quantities (typically) with different units.

rate of change n, the ratio of change in the dependent variable to the corresponding change in the independent variable, measuring the change in the dependent variable per unit change in the independent variable.

ratio n, (i) a way to compare two quantities by division, (ii) a fraction. For example, "the ratio of 1 to 2" can be written as $\frac{1}{2}$.

rational adj, having to do with ratios.

rational exponent n, an exponent that is a rational number. For example, the expression $x^{1/3}$ has a rational exponent of 1/3, and $x^{1/3} = \sqrt[3]{x}$.

rational expression n, a ratio of two polynomials. Also called an **algebraic fraction**.

rational function n, a function of the form $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions.

rational number n, a number that can be expressed as the ratio of two integers.

rationalize the denominator v, to find an equivalent fraction that contains no radical in the denominator. For example, when we rationalize $\frac{1}{\sqrt{2}}$, we obtain $\frac{\sqrt{2}}{2}$.

real axis n, the horizontal axis in the complex plane.

real line *see* **number line**.

real part n, (of a complex number) the term which does not include i when the complex number is written in the form $a + bi$, where a and b are real numbers. For example, the real part of $4 - 7i$ is 4.

real number n, a number that corresponds to a point on a number line.

reciprocal (of a number) n, the result of dividing 1 by the given number. For example, the reciprocal of 2 is $\frac{1}{2}$. Two numbers are reciprocals of each other when their product is 1.

rectangle n, a four-sided figure (in the plane) with four right angles. The opposite sides are equal in length and parallel.

reduce a fraction v, to find an equivalent fraction whose numerator and denominator share no common factors (other than 1 and -1).

reduced row echelon form n, (of a matrix) a row echelon form matrix that also satisfies (1) the leading entry in each nonzero row is a 1, (2) each leading 1 is the only nonzero entry in its column.

reflection (of a point or graph across a line) n, the transformation that replaces each point of a graph with its mirror image on the other side of the line.

regression line n, the line used for linear regression.

regular polygon n, a polygon all of whose sides have equal length and all of whose angles are congruent.

restricted domain n, a domain of a function that does not include all real numbers.

revenue n, money that an individual or group receives. For example, a person might have revenues from both a salary and from earnings on investments.

right angle n, an angle of 90°.

right triangle n, a triangle that includes one right angle.

root n, the solution to an equation. See also **cube root**; ***n*th root**; **principal square root**; **square root**.

round v, to give an approximate value of a number by choosing the nearest number of a specified form. For example, to round 3.14159 to two decimal places, we use 3.14.

row echelon form n, (of a matrix) a matrix in which (1) only zeros occur below each nonzero leading entry, (2) the leading entry in any row is to the right of any leading entry above it, and (3) any row consisting entirely of zeros is below all rows with any nonzero entry.

S.

satisfy v, to make an equation true (when substituted for the variable or variables). For example, the number 5 satisfies the equation $x - 2 = 3$.

scale n, marked values on a number line or axes to establish how wide an interval of numbers is represented by a physical distance on the number line.

scale v, (i) to determine the scale on an axis or axes; (ii) to multiply (measurements) by a fixed number (the **scale factor**).

scale factor n, a fixed number by which measurements or values are multiplied.

scaling exponent n, the exponent defining direct variation or a power function. For example, if $y = 3x^4$, then the scaling exponent is 4.

scatterplot n, a type of graph used to represent pairs of data values. Each pair of data values provides the coordinates for one point on the scatterplot. Also called a **scatter diagram**.

scientific notation n, a standard method for writing very large or very small numbers that uses powers of 10. For example, the scientific notation for $12,000$ is 1.2×10^4.

semicircle n, half a circle (on one side or the other of a diameter).

signed number n, a positive or negative number.

significant digit n, (in the decimal form of a number) a digit warranted by the accuracy of the measuring device. When the decimal point is present, the significant digits are all those from the leftmost nonzero digit to the rightmost digit after the decimal point. When there is no decimal point, the significant digits are all those from the leftmost nonzero digit to the rightmost nonzero digit. For example, 123.40 has five significant digits, but $12,340$ has only four significant digits. Also called **significant figure**.

significant figure *see* **significant digit**.

similar *see* **geometrically similar**.

simplify v, to write in an equivalent but simpler or more convenient form. For example, we can simplify the expression $\sqrt{16}$ to 4.

sinusoidal adj, having the shape of a sine or cosine graph.

slope n, a measure of the steepness of a line or of the rate of change of one variable with respect to another.

slope-intercept form n, a standard form for the equation of a nonvertical line: $y = b + mx$.

slope-intercept method n, a method for graphing a line that uses the slope and the y-intercept.

solution n, a value for the variable that makes an equation or an inequality true. A solution to an equation in two variables is an ordered pair that satisfies the equation. A solution to a system is an ordered pair that satisfies each equation of the system.

solve v, (i) (an equation) to find any and all solutions to an equation, inequality, or system; (ii) (a formula) to write an equation for one variable in terms of any other variables, for example, when we solve $5x + y = 3$ for y to get $y = -5x + 3$; (iii) (a triangle) to find the measures of all three sides and of all three angles.

sphere n, a three-dimensional object in the shape of a ball. A sphere consists of all the points in space at a fixed distance (the radius) from the center of the sphere.

square n, (i) any expression times itself; (ii) a rectangle whose sides are all the same length.

square v, to multiply by itself, that is, to raise to the power 2.

square matrix n, a matrix with the same number of rows as columns.

square root n, a number that when squared gives a desired value. For example, 7 is a square root of 49 because $7^2 = 49$.

standard form n, (i) (of a linear, quadratic, or other polynomial equation) an equation in which the right side is 0, so the equation has the form $p(x) = 0$; (ii) (of a system of linear equations) a system in which the variables occur only on the left side of each equation and in alphabetic order.

strict inequality n, a mathematical statement of the form $a < b$ or $a > b$.

subscript n, a small number written below and to the right of a variable. For example, in the equation $x_1 = 3$, the variable x has the subscript 1.

substitution method n, a method for solving a system of equations that begins by expressing one variable in terms of the other.

sum n, the result of an addition. For example, the expression $a + b$ represents the sum of a and b.

surface area n, the total area of the faces or surfaces of a three-dimensional object.

supplementary angles n, two angles whose measures add up to $180°$.

supply equation n, an equation that gives the quantity of some product that producers are willing to produce in terms of the price of that product.

symmetry n, a geometric property of having sameness on opposite sides of a line (or plane) or about a point.

system of equations n, two or more equations involving the same variables.

T.

term n, (i) (in a sum) a quantity that is added to another. For example, in the expression $x + y - 4$, x, y, and -4 are the terms; (ii) an algebraic expression that is not a sum or difference, for example, 4x is one term.

test point n, (for an inequality) a point in the plane (or on a number line) used to determine which side of the plane (or number line) is included in the solution.

transform v, to apply a **transformation**.

transformation n, (i) (of data) applying a function to one or both components in a set of data, typically so that the resulting data becomes approximately linear; (ii) (of a graph) a change that occurs in the graph of an equation when one or more of the parameters defining that equation are altered.

translation n, (of a graph or geometric figure) sliding horizontally and/or vertically without rotating or changing any shapes.

trapezoid n, a four-sided figure in the plane with one pair of parallel sides.

triangle n, a three-sided figure in the plane.

triangle inequality n, the inequality $|a + b| \leq |a| + |b|$, which is true for any two real numbers a and b.

triangular form n, a system of linear equations in which the first variable does not occur in the second equation, the first two variables do not occur in the third equation (and the first three variables do not occur in the fourth equation if there are more than three variables, and so on).

trinomial n, a polynomial with exactly three terms.

turning point n, (of a graph) where the graph either changes from increasing to decreasing or vice versa.

U.

union n, the set obtained by collecting all the elements of one set along with all the elements of another set.

unit circle n, a circle of radius 1 unit (usually centered at the origin).

unlike fractions n, fractions whose denominators are not equivalent.

unlike terms n, terms with variable parts that are not equivalent.

upper triangular form n, (of a matrix) a matrix with all zeros in the lower left corner. More precisely, the entry in the ith row and jth column is 0 whenever $i > j$.

V.

variable adj, not constant, subject to change.

variable n, a numerical quantity that changes over time or in different situations.

variation *see* **direct variation**; **inverse variation**.

verify v, to prove the truth or validity of an assertion.

vertex n, (*plural* **vertices**), (i) a point where two sides of a polygon meet; (ii) a corner or extreme point of a geometric object; (iii) the highest or lowest point on a parabola.

vertex angle n, the angle between the equal sides in an isosceles triangle.

vertex form n, one way of writing a quadratic equation, $y = a(x - x_v)2 + y_v$, which displays the vertex, (x_v, y_v).

vertical asymptote n, a line $x = a$ parallel to the y-axis toward which the graph of an equation tends as the value of x approaches a.

vertical axis n, the vertical coordinate axis. Often called the y**-axis**.

vertical compression n, (of a graph) the result of replacing each point of the graph with the point obtained by scaling the y-coordinate by a fixed factor (when that factor is between 0 and 1).

vertical intercept n, where the graph meets the vertical axis. Also called the y**-intercept**.

vertical line test n, a test to decide whether a graph defines a function: A graph represents a function if and only if every vertical line intersects the graph in at most one point.

vertical stretch n, (of a graph) the result of replacing each point of the graph with the point obtained by scaling the y-coordinate by a fixed factor (when that factor is greater than 1).

vertical translation n, (of a graph) the result of moving all points of the graph straight up (or all straight down) by the same distance.

vertices n, the plural of **vertex**.

volume n, a measure of the three-dimensional space enclosed by a three-dimensional object, typically expressed in terms of cubic units, such as cubic meters or cubic feet.

W.

whole number n, one of the numbers $0, 1, 2, 3, \ldots$.

X.

x**-axis** *see* **horizontal axis**.

x**-intercept** *see* **horizontal intercept**.

xy**-plane** *see* **coordinate plane**.

Y.

y**-axis** *see* **vertical axis**.

y**-intercept** *see* **vertical intercept**.

Z.

zero **n**, (i) the number 0, with the property that when it is added to any other number, the resulting sum is equal to that second number; (ii) an input to a function which yields an output of 0.

zero-factor principle *see* Zero-Factor Principle below.

Properties of Numbers. Associative Laws.

Addition: If a, b, and c are any numbers, then $(a+b)+c = a+(b+c)$.

Multiplication If a, b, and c are any numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Associative Laws.

Addition: If a, b, and c are any numbers, then $(a+b)+c = a+(b+c)$.

Multiplication If a, b, and c are any numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Commutative Laws.

Addition: If a and b are any numbers, then $a + b = b + a$.

Multiplication If a and b are any numbers, then $a \cdot b = b \cdot a$.

Distributive Law.

$a(b + c) = ab + ac$ for any numbers a, b, and c.

Properties of Equality.

Addition: If $a = b$ and c is any number, then $a + c = b + c$.

Subtraction: If $a = b$ and c is any number, then $a - c = b - c$.

Multiplication If $a = b$ and c is any number, then $a \cdot c = b \cdot c$.

Division If $a = b$ and c is any nonzero number, then $\frac{a}{c} = \frac{b}{c}$.

Fundamental Principle of Fractions.

If a is any number, and b and c are nonzero numbers, then $\dfrac{a \cdot c}{b \cdot c} = \dfrac{a}{b}$.

Laws of Exponents.

1. $a^m \cdot a^n = a^{m+n}$
2. - $\dfrac{a^m}{a^n} = a^{m-n} \qquad (n < m)$
   - $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}} \qquad (n > m)$
3. $(a^m)^n = a^{m+n}$
4. $(ab)^n = a^n b^n$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Product Rule for Radicals.

If a and b are both nonnegative, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

Quotient Rule for Radicals.

If $a \geq 0$ and $b > 0$, then $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$.

Zero-Factor Principle.

If $ab = 0$ then either $a = 0$ or $b = 0$.

Properties of Absolute Value.

$\lvert a+b \rvert \leq \lvert a \rvert + \lvert b \rvert$	Triangle inequality
$\lvert ab \rvert = \lvert a \rvert \, \lvert b \rvert$	Multiplicative property

Appendix D

Technology (Graphing calculators)

Technology D.0.1 Graphing an Equation. We can use a graphing calculator to graph an equation. On most calculators, we follow three steps.

To Graph an Equation:

1. Press `Y=` and enter the equation you wish to graph.
2. Press `WINDOW` and select a suitable graphing window.
3. Press `GRAPH`

Technology D.0.2 Graphing an Equation. We can use a graphing calculator to graph an equation. On most calculators, we follow three steps.

To Graph an Equation:

1. Press `Y=` and enter the equation you wish to graph.
2. Press `WINDOW` and select a suitable graphing window.
3. Press `GRAPH`

Technology D.0.3 Choosing a Graphing Window. Knowing the intercepts can also help us choose a suitable window on a graphing calculator. We would like the window to be large enough to show the intercepts. For the graph in the example above, we can enter the equation

$$Y = (9000 - 150X)/-180$$

in the window

Xmin= -20	Xmax= 70
Ymin= -70	Ymax= 30

Technology D.0.4 Making a Table of Values with a Calculator. We can use a graphing calculator to make a table of values for a function defined by an equation. For the function in Example 1.2.8,

$$h = 1776 - 16t^2$$

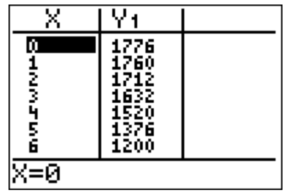
we follow the steps:

- Enter the equation: Press the `Y=` key, clear out any other equations, and define $Y_1 = 1776 - 16X^2$.

- Choose the x-values for the table. Press `2ndWINDOW` to access the *TblSet* (Table Setup) menu and set it to look like the figure at left below.

This setting will give us an initial x-value of 0 ($TblStart = 0$) and an increment of one unit in the x-values, ($\Delta Tbl = 1$). It also fills in values of both variables automatically.

- Press `2nd GRAPH` to see the table of values, as shown in the figure at right below. From this table, we can check the heights we found in Example 1.2.8.



Now try making a table of values with $TblStart = 0$ and $\Delta Tbl = 0.5$. Use the $\boxed{\uparrow}$ and $\boxed{\downarrow}$ arrow keys to scroll up and down the table.

Technology D.0.5 Evaluating a Function. We can use the table feature on a graphing calculator to evaluate functions. Consider the function of Checkpoint 1.2.17, $f(x) = 5 - x^3$.

- Press `Y=`, clear any old functions, and enter

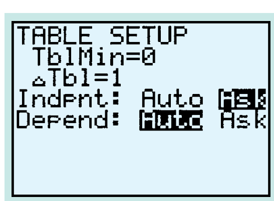
  $Y_1 = 5 - X^3$

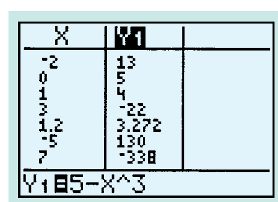
- Press `TblSet` (`2nd WINDOW`) and choose *Ask* after *Indpnt*, as shown in the figure at left below, and press `ENTER`. This setting allows you to enter any x-values you like.

- Press `TABLE` (using `2nd GRAPH`).

- To follow Checkpoint 1.2.17, key in `(-)` 2 `ENTER` for the x-value, and the calculator will fill in the y-value. Continue by entering 0, 1, 3, or any other x-values you choose.

One such table is shown in the figure at right below.





If you would like to evaluate a new function, you do not have to return to the `Y=` screen. Use the $\boxed{\rightarrow}$ and $\boxed{\uparrow}$ arrow keys to highlight Y_1 at the top of the second column. The definition of Y_1 will appear at the bottom of the display, as shown above. You can key in a new definition here, and the second column will be updated automatically to show the y-values of the new function.

Technology D.0.6 Making a Table of Values with a Calculator. We can use a graphing calculator to make a table of values for a function defined by an equation. For the function in Example 1.2.8,

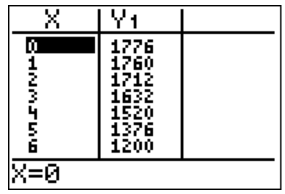
$$h = 1776 - 16t^2$$

we follow the steps:

- Enter the equation: Press the `Y=` key, clear out any other equations, and define $Y_1 = 1776 - 16X^2$.
- Choose the x-values for the table. Press `2ndWINDOW` to access the *TblSet* (Table Setup) menu and set it to look like the figure at left below.

This setting will give us an initial x-value of 0 ($TblStart = 0$) and an increment of one unit in the x-values, ($\Delta Tbl = 1$). It also fills in values of both variables automatically.

- Press `2nd GRAPH` to see the table of values, as shown in the figure at right below. From this table, we can check the heights we found in Example 1.2.8.

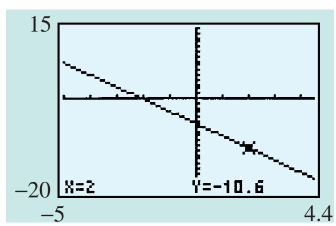


Now try making a table of values with $TblStart = 0$ and $\Delta Tbl = 0.5$. Use the $\boxed{\uparrow}$ and $\boxed{\downarrow}$ arrow keys to scroll up and down the table.

Technology D.0.7 Finding Coordinates with a Graphing Calculator. We can use the `TRACE` feature of the calculator to find the coordinates of points on a graph. For example, graph the equation $y = -2.6x - 5.4$ in the window

Xmin$= -5$ Xmax$= 4.4$

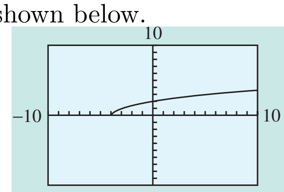
Ymin$= -20$ Ymax$= 15$



Press `TRACE`, and a "bug" begins flashing on the display. The coordinates of the bug appear at the bottom of the display, as shown in the figure. Use the left and right arrows to move the bug along the graph. You can check that the coordinates of the point $(2, -10.6)$ do satisfy the equation $y = -2.6x - 5.4$.

The points identified by the Trace bug depend on the window settings and on the type of calculator. If we want to find the y-coordinate for a particular x-value, we enter the x-coordinate of the desired point and press `ENTER`.

Technology D.0.8 Using a Calculator to Graph a Function. We can also use a graphing calculator to obtain a table and graph for the function in Example 1.3.5. We graph a function just as we graphed an equation. For this function, we enter $Y_1 = \sqrt{}(X + 4)$ and press `ZOOM` 6 for the standard window. The calculator's graph is shown below.



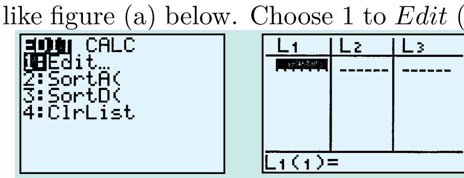
Technology D.0.9 Using the Trace Feature. You can use the Trace feature on a graphing calculator to approximate solutions to equations. Graph the function $f(x)$ in Example 1.3.14 in the window

$$\begin{array}{ll} \text{Xmin}= -4 & \text{Xmax}= 4 \\ \text{Ymin}= -20 & \text{Ymax}= 40 \end{array}$$

and trace along the curve to the point $(2.4680851, 15.512401)$. We are close to a solution, because the y-value is close to 15. Try entering x-values close to 2.4680851, for instance, $x = 2.4$ and $x = 2.5$, to find a better approximation for the solution.

We can use the intersect feature on a graphing calculator to obtain more accurate estimates for the solutions of equations.

Technology D.0.10 Using a Calculator for Linear Regression. You can use a graphing calculator to make a scatterplot, find a regression line, and graph the regression line with the data points. On the TI-83 calculator, we use the statistics mode, which you can access by pressing `STAT`. You will see a display that looks like figure (a) below. Choose 1 to *Edit* (enter or alter) data.



(a) (b)

Now follow the instructions in Example 8.1.12 for using your calculator's statistics features.

Technology D.0.11 Using the Intersect Feature. We can use the *intersect* feature on a graphing calculator to solve equations.

Example D.0.12 Use a graphing calculator to solve $\dfrac{3}{x-2} = 4$

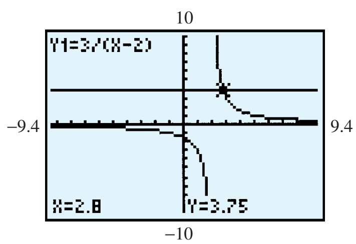
Solution. We would like to find the points on the graph of $y = \dfrac{3}{x-2}$ that have y-coordinate equal to 4. We graph the two functions

$$Y_1 = 3/(X-2)$$
$$Y_2 = 4$$

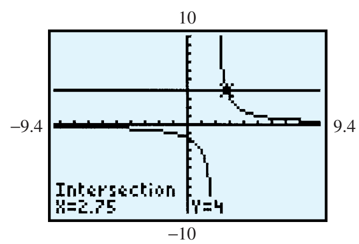
in the window

$$\text{Xmin} = -9.4 \quad \text{Xmax} = 9.4$$
$$\text{Ymin} = -10 \quad \text{Ymax} = 10$$

The point where the two graphs intersect locates the solution of the equation. If we trace along the graph of Y_1, the closest we can get to the intersection point is $(2.8, 3.75)$, as shown in figure (a). We get a better approximation using the *intersect* feature.



(a)



(b)

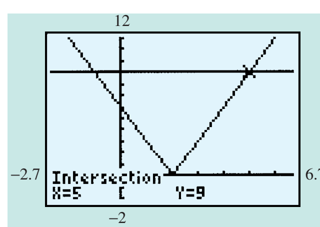
Use the arrow keys to position the Trace bug as close to the intersection point as you can. Then press `2nd` `TRACE` to see the Calculate menu. Press 5 for intersect; then respond to each of the calculator's questions, *First curve?*, *Second curve?*, and *Guess?* by pressing `ENTER`. The calculator will then display the intersection point, $x = 2.75$, $y = 4$, as shown in figure (b). The solution of the original equation is $x = 2.75$. □

Technology D.0.13 Solving Absolute Value Equations. We can use a graphing calculator to solve the equations in Example 2.5.3.

The graph shows the graphs of $Y_1 = \text{abs}(3X-6)$ and $Y_2 = 9$ in the window

$$\text{Xmin} = -2.7 \quad \text{Xmax} = 6.7$$
$$\text{Ymin} = -2 \quad \text{Ymax} = 12$$

We use the `Trace` or the *intersect* feature to locate the intersection points at $(-1, 9)$ and $(5, 9)$.

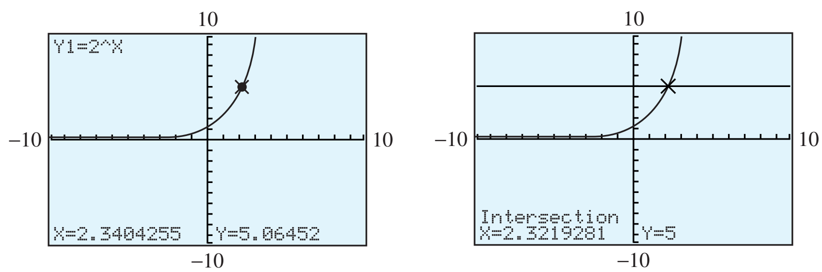


Technology D.0.14 Graphical Solution of Exponential Equations. It is not always so easy to express both sides of the equation as powers of the same base. In the following sections, we will develop more general methods for finding exact solutions to exponential equations. But we can use a graphing calculator to obtain approximate solutions.

Example D.0.15 Use the graph of $y = 2^x$ to find an approximate solution to the equation $2^x = 5$ accurate to the nearest hundredth.

Solution. Enter $Y_1 = 2$ ^ X and use the standard graphing window (`ZOOM` 6) to obtain the graph shown in figure (a). We are looking for a point on this graph with y-coordinate 5.

Using the TRACE feature, we see that the y-coordinates are too small when $x < 2.1$ and too large when $x > 2.4$. The solution we want lies somewhere between $x = 2.1$ and $x = 2.4$, but this approximation is not accurate enough.



(a) **(b)**

To improve our approximation, we will use the **intersect** feature. Set $Y_2 = 5$ and press `GRAPH`. The x-coordinate of the intersection point of the two graphs is the solution of the equation $2^x = 5$ Activating the **intersect** command results in figure (b), and we see that, to the nearest hundredth, the solution is 2.32.

We can verify that our estimate is reasonable by substituting into the equation:

$$2^{2.32} \stackrel{?}{=} 5$$

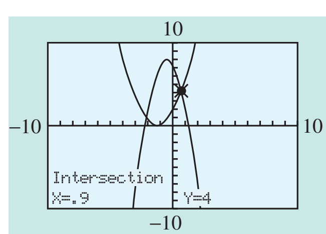
We enter 2 ^ 2.32 `ENTER` to get 4.993322196. This number is not equal to 5, but it is close, so we believe that $x = 2.32$ is a reasonable approximation to the solution of the equation $2^x = 5$. □

Technology D.0.16 Solving Systems with the Graphing Calculator. We can use the intersect feature of the graphing calculator to solve systems of quadratic equations. Consider the system

$$y = (x + 1.1)^2$$
$$y = 7.825 - 2x - 2.5x^2$$

We will graph these two equations in the standard window. The two intersection points are visible in the window, but we do not find their exact coordinates when we trace the graphs.

We can use the intersect command to locate one of the solutions, as shown below. You can check that the point $(0.9, 4)$ is an exact solution to the system by substituting $x = 0.9$ and $y = 4$ into each equation of the system. (The calculator is not always able to find the exact coordinates, but it usually gives a very good approximation.)



You can find the other solution of the system by following the same steps and moving the bug close to the other intersection point. You should verify that the other solution is the point $(-2.1, 1)$.

Technology D.0.17 Using a Calculator for Quadratic Regression. We can use a graphing calculator to find an approximate quadratic fit for a set of data. The procedure is similar to the steps for linear regression outlined in Section 1.2.

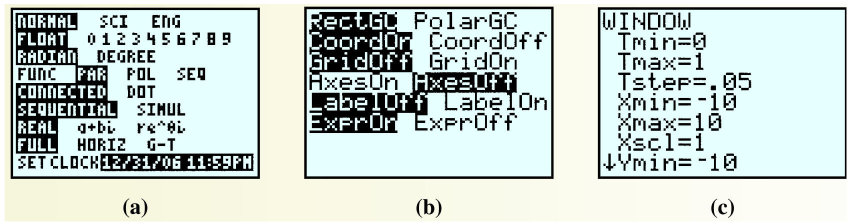
Technology D.0.18 Bézier Curves on the Graphing Calculator.

Investigation D.0.1

A We can draw Bézier curves on the graphing calculator using the parametric mode. First, press the `MODE` key and highlight *PAR*, as shown in figure (a). To remove the x- and y-axes from the display, press `2nd ZOOM` to get the *Format* menu, then choose *AxesOff* as shown in figure (b). Finally, we set the viewing window: Press `WINDOW` and set

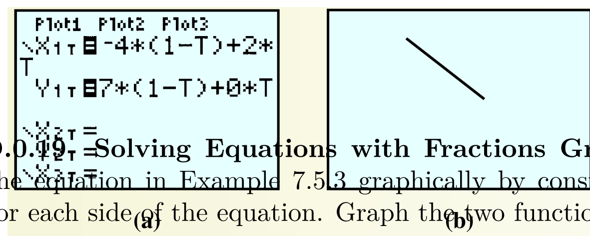
$$\text{Tmin} = 0 \quad \text{Tmax} = 1 \quad \text{TStep} = 0.05$$
$$\text{Xmin} = -10 \quad \text{Xmax} = 10 \quad \text{Ymin} = -10 \quad \text{Ymax} = 10$$

as shown in figure (c).



(a) (b) (c)

As an example, we will graph the linear curve from part (A). Press `Y=` and enter the definitions for $x(t)$ and $y(t)$, as shown below. Press `GRAPH` and the calculator displays the line segment joining $(-4, 7)$ and $(2, 0)$. Experiment by modifying the endpoints to see how the graph changes.



(a) (b)

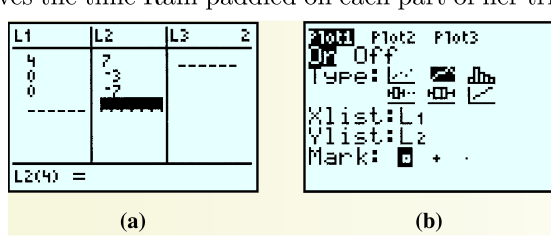
B Designing a Numeral 7

1 Press `2nd Y=` and enter the formulas for the quadratic Bézier curve defined by the endpoints $(4, 7)$ and $(0, -7)$, and the control point $(0, 5)$ under X_{1T} and Y_{1T}. (These are the same formulas you found in step B.1 of Investigation 7.0.1.)

2 Find the functions f and g for the linear Bézier curve joining the points $(-4, 7)$ and $(4, 7)$. Simplify the formulas for those functions and enter them into your calculator under X_{2T} and Y_{2T}. Press `GRAPH` to see the graph.

3 Now we will alter [illegible] Go back to X_{1T} and Y_{1T} and change the co[illegible]3). (These are the formulas you found in step [illegible]7.0.1.) How does the graph change?

4 We can see exactly how the control point affects the graph by connecting the three data points with line segments. Press `STAT ENTER` and enter the coordinates of $(4, 7)$, $(0, -3)$, and $(0, -7)$ in L_1 and L_2, as shown in figure (a). Press `2nd Y= ENTER`, turn on *Plot1*, and select the second plot type, as shown in figure (b). You should see the line segments superimposed on your numeral 7. How are those segments related to the curve?



(a) (b)

5 Now edit L_2 so that the control point is $(0, 5)$, and again define

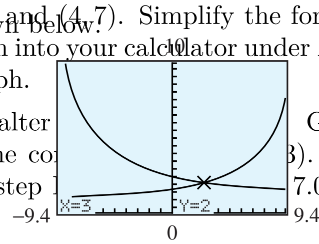
Technology D.0.19: Solving Equations with Fractions Graphically. We can solve the equation in Example 7.5.3 graphically by considering two functions, one for each side of the equation. Graph the two functions

$$Y_1 = \frac{30}{12+x} \qquad \text{and} \qquad Y_2 = \frac{30}{12-x}$$

in the window

$$\text{Xmin} = -9.4 \qquad \text{Xmax} = 9.4$$
$$\text{Ymin} = 0 \qquad \text{Ymax} = 10$$

to obtain the graph shown below.



The function Y_1 gives the time it takes Rani to paddle 30 miles downstream, and Y_2 gives the time it takes her to paddle 18 miles upstream. Both of these times depend on the speed of the current, x.

We are looking for a value of x that makes Y_1 and Y_2 equal. This occurs at the intersection point of the two graphs, $(3, 2)$. Thus, the speed of the current is 3 miles per hour, as we found in Example 7.5.3. The y-coordinate of the intersection point gives the time Rani paddled on each part of her trip: 2 hours each way.

Technology D.0.20 Using the Intersect Feature to Solve a System. Because the `TRACE` feature does not show every point on a graph, we may not find the exact solution to a system by tracing the graphs. In the next Example we demonstrate the *intersect* feature of the calculator.

Example D.0.21 Solve the system

$$\begin{aligned}3x - 2.8y &= 21.06\\ 2x + 1.2y &= 5.3\end{aligned}$$

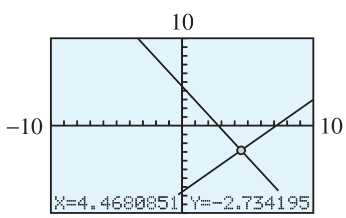
Solution. We can graph this system in the standard window by solving each equation for y. We enter

$$\begin{aligned}Y_1 &= (21.06 - 3X)/-2.8\\ Y_2 &= (5.3 - 2X)/1.2\end{aligned}$$

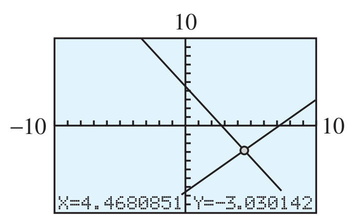
and then press `ZOOM` 6. (Don't forget the parentheses around the numerator of each expression.)

We Trace along the first line to find the intersection point. It appears to be at $x = 4.468051$, $y = -2.734195$, as shown in figure (a). However, if we press the up or down arrow to read the coordinates off the second line, we see that for the same x-coordinate we obtain a different y-coordinate, as in figure (b).

The different y-coordinates indicate that we have not found an intersection point, although we are close. The *intersect* feature can give us a better estimate, $x = 4.36$, $y = -2.85$.



(a)



(b)

We can substitute these values into the original system to check that they satisfy both equations.

$$\begin{aligned}3(\mathbf{4.36}) - 2.8(\mathbf{-2.85}) &= 21.06\\ 2(\mathbf{4.36}) + 1.2(\mathbf{-2.85}) &= 5.3\end{aligned}$$

□

Technology D.0.22 TI-84 and TI-83 calculators have a command for finding the reduced row echelon form of a matrix.

Example D.0.23 Solve the system

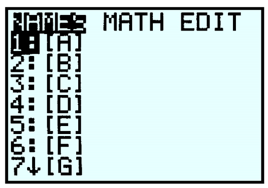
$$\begin{aligned} a + \ \ 2b + 4c + \ \ 8d &= 12 \\ -2a + \ \ 2b - 2c + \ \ 2d &= \ 1 \\ 6a + \ \ 6b + 6c + \ \ 6d &= 19 \\ 4a + 20b - 8c + 14d &= 41 \end{aligned}$$

by finding the reduced row echelon form of the augmented matrix.

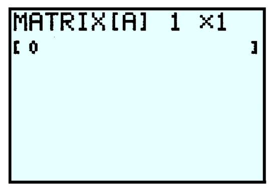
Solution. The augmented matrix is

$$\left[\begin{array}{cccc|c} 1 & 2 & 4 & 8 & 12 \\ -2 & 2 & -2 & 2 & 1 \\ 6 & 6 & 6 & 6 & 19 \\ 4 & 20 & -8 & 14 & 41 \end{array}\right]$$

We enter this matrix into the calculator as follows: First access the *MATRIX* menu by pressing `2nd` $\boxed{x^{-1}}$ on a TI-84 or`MATRX` on a TI-83. You will see the menu shown in figure (a). We will use matrix [A], which is already selected, and we presss $\boxed{\rightarrow}$ $\boxed{\rightarrow}$ `ENTER` to *EDIT* (or enter) the matrix, shown in figure(b).

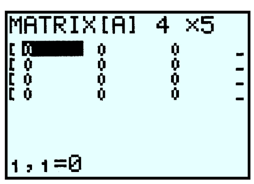


(a)

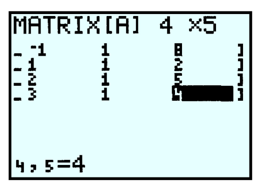


(b)

We want to enter a 4×5 matrix, so we press `4ENTER5ENTER`, and we see the display in figure (a) below. Now type in the first row of the matrix, pressing `ENTER` after each entry. The calculator automatically moves to the second row. Continue filling in the rest of the augmented matrix, as shown in figure (b).

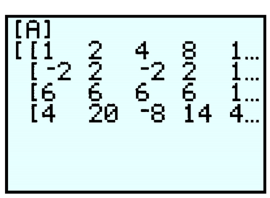


(a)

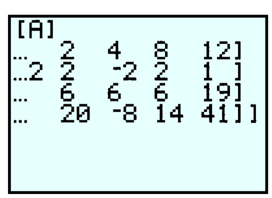


(b)

To make sure you have entered the values correctly, press `2ndMODE` to quit to the home screen, then open the matrix menu again. Press `1ENTER` to retrieve matrix [A]; the calculator display should look like figure (a) below. To check the rest of the matrix, press the right arrow key until you see the last column, as in figure (b).

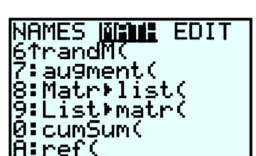


(a)



(b)

Now we are ready to compute the reduced row echelon form of the matrix. Access the matrix menu again, but this time press the right arrow once to highlight *MATH* as shown in figure (a). Scroll down until the rref(command is highlighted, as shown in figure (b), and press `ENTER`.



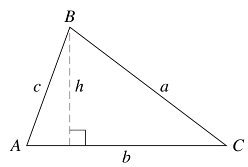
Appendix E

Geometry formulas

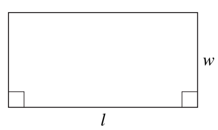
GEOMETRY

Perimeter and Area of Plane Figures

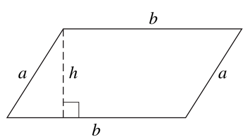
Triangle
$P = a + b + c$
$A = \frac{1}{2}bh$



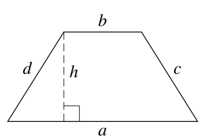
Rectangle
$P = 2l + 2w$
$A = lw$



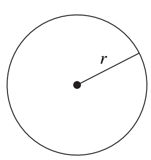
Parallelogram
$P = 2a + 2b$
$A = bh$



Trapezoid
$P = a + b + c + d$
$A = \frac{1}{2}h(a + b)$

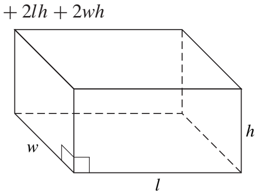


Circle
$C = 2\pi r$
$A = \pi r^2$

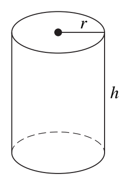


Volume and Surface Area of Solid Figures

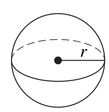
Rectangular Prism
$V = lwh$
$S = 2lw + 2lh + 2wh$



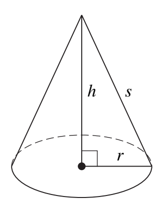
Right Circular Cylinder
$V = \pi r^2 h$
$S = 2\pi r^2 + 2\pi rh$



Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$



Right Circular Cone
$V = \frac{1}{3}\pi r^2 h$
$S = \pi r^2 + \pi rs$



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