Partial Differential Equations

- Equations that contain derivatives with respect to more than one variable
- Derivatives can be with respect to position in multiple dimensions of space
- Derivatives can be with respect to both space and time

Navier-Stokes Equations

$$\frac{\delta u}{\delta t} = \overset{\text{viscous drag}}{\boxed{\upsilon \nabla \cdot (\nabla u)}} - \overset{\text{convection}}{\boxed{(u \cdot \nabla) u}} + \overset{\text{gravity}}{\boxed{F_{body}}} - \overset{\text{pressure}}{\boxed{\frac{1}{\rho} \nabla p}}$$

velocity density

viscosity pressure

mass conservation

$$\boxed{\nabla \cdot u = 0}$$

The Navier-Stokes equations describe the flow of a fluid like air or water.

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} \qquad \text{(Gauss' Law)}$$

$$\nabla \cdot \mathbf{H} = 0 \qquad \text{(Gauss' Law for Magnetism)}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \text{(Ampere's Law)}$$

Maxwell's equations describe the behavior of an electromagnetic field.

Linear Elasticity Equations

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + F_r = \rho\,\frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2}{r}\sigma_{r\theta} + F_\theta = \rho\,\frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r}\sigma_{rz} + F_z = \rho\,\frac{\partial^2 u_z}{\partial t^2}$$

The linear elasticity equations describe vibration, stress, and strain in an elastic solid.

Schrödinger's Equation

Schrödinger's Equation

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t)$$

i is the imaginary number, $\sqrt{-1}$.
$\hbar$ is Planck's constant divided by 2π: 1.05459×10^{-34} joule·second.
$\psi(\mathbf{r},t)$ is the wave function, defined over space and time.
m is the mass of the particle.
∇^2 is the Laplacian operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.
$V(\mathbf{r},t)$ is the potential energy influencing the particle.

Schrödinger's equation of quantum dynamics describes the wave function of a particle.