RUNNING TIME ANALYSIS - PART 2
BINARY SEARCH TREES

Problem Solving with Computers-II

How is PA01 going?

A. Done!
B. On track to finish
C. On track to finish but my code is a mess
D. Stuck and struggling
E. Haven’t started

Midterm 2

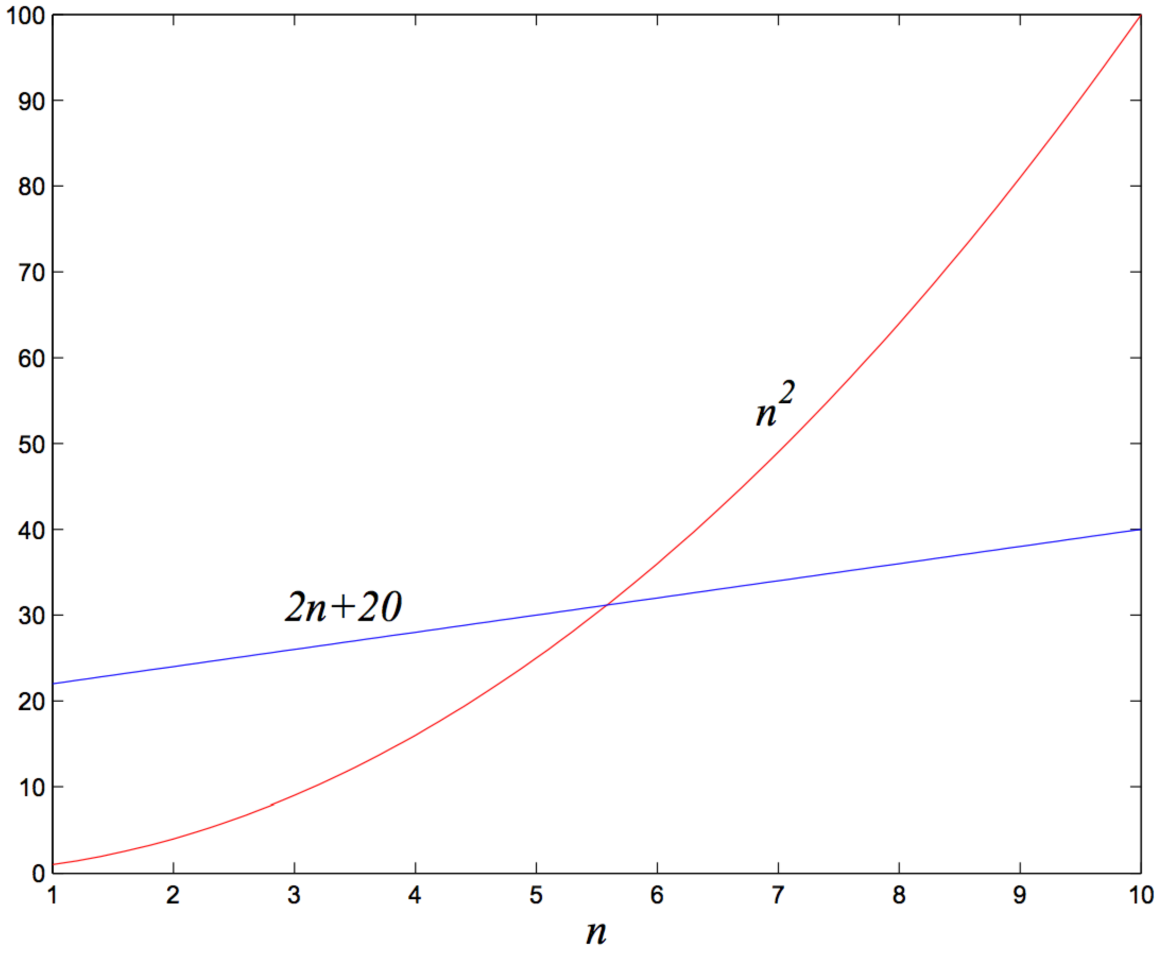
- Cumulative but the focus will be on
  - BST
  - Running time analysis

A more precise definition of Big-O

- f(n) and g(n): running times of two algorithms on inputs of size n.
- f(n) and g(n) map positive integer inputs to positive reals.

We say f = O(g) if there is a constant $c > 0$ and $k>0$ such that $f(n) \leq c \cdot g(n)$ for all $n >= k$.

f = O(g)
means that "f grows no faster than g"



What is the Big-O running time of algoX?

- Assume **dataA** is some data structure that supports the following operations with the given running times, where N is the number of keys stored in the data structure:
  - insert: O(log N)
  - min: O(1)
  - delete: O(log N)

A. $O(N^2)$
B. O(N logN)
C. O(N)
D. O(log N)
E. Not enough information to compute

```
void algoX(int arr[], int N)
{
        dataA ds;//ds contains no keys
        for(int i=0; i < N; i=i++)
                ds.insert(arr[i]);
        for(int i=0; i < N; i=i++)
                arr[i] = ds.min();
                ds.delete(arr[i]);
}
```

Big-Omega

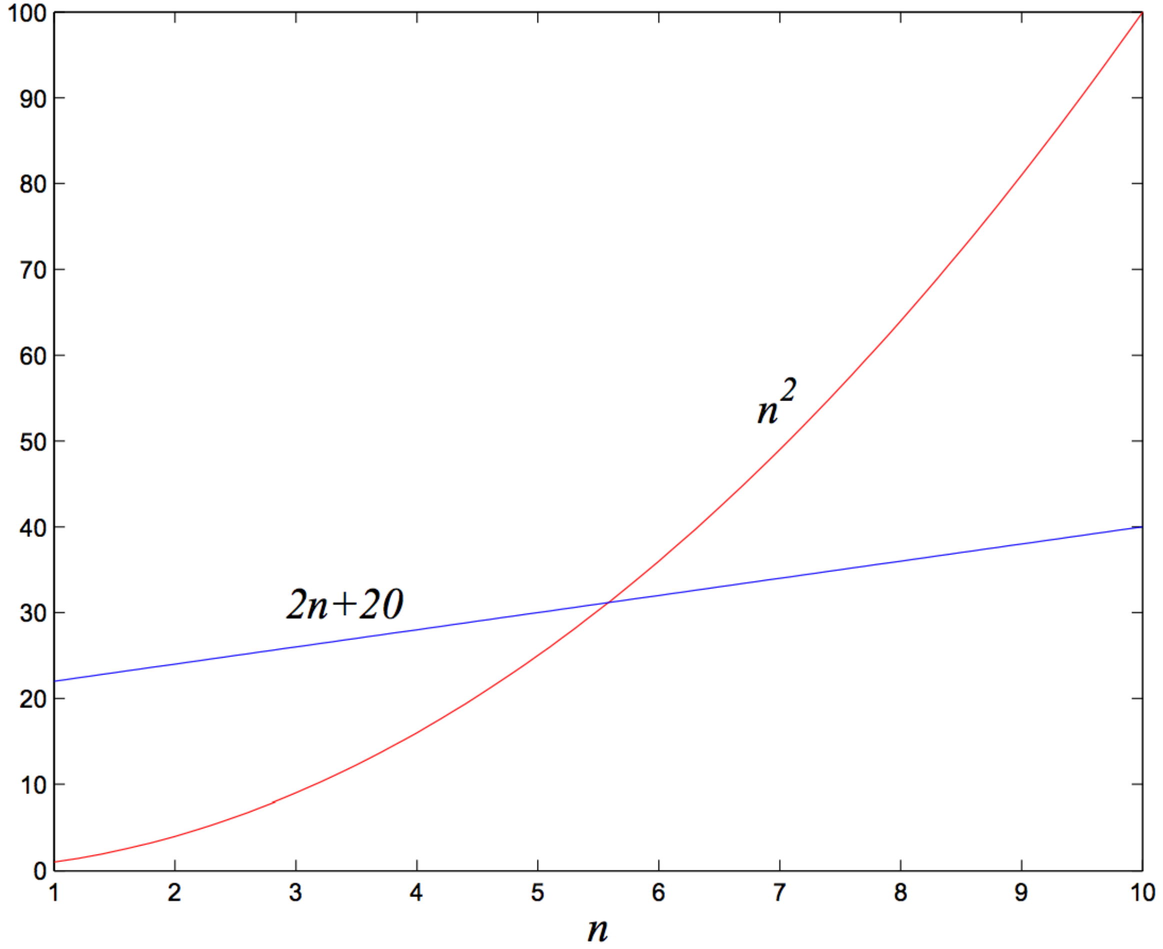
- f(n) and g(n): running times of two algorithms on inputs of size n.
- f(n) and g(n) map positive integer inputs to positive reals.

We say $f = \Omega(g)$ if there are constants $c > 0$, $k>0$ such that $c \cdot g(n) \leq f(n)$ for $n >= k$

$f = \Omega(g)$

means that "f grows at least as fast as g"

g is a lower bound

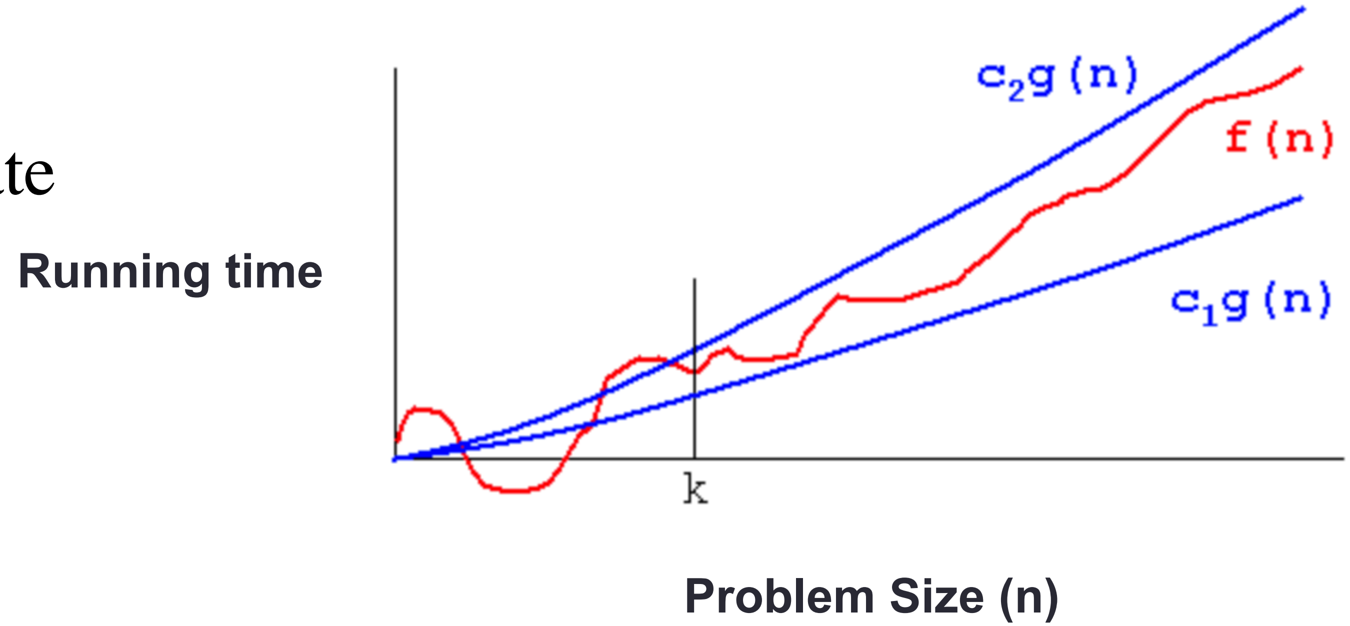


Big-Theta

- f(n) and g(n): running times of two algorithms on inputs of size n.
- f(n) and g(n) map positive integer inputs to positive reals.

We say $f = \Theta(g)$ if there are constants $c_1, c_{2\ ,k}$ such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$, for $n >= k$

f and g grow at the same rate



Best case, worst case, average case running times

Operations on sorted arrays

- Min :
- Max:
- Median:
- Successor:
- Predecessor:
- Search**:**
- Insert :
- Delete:

6	**13**	**14**	**25**	**33**	**43**	**51**	**53**	**64**	**72**	**84**	**93**	**95**	**96**	**97**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

lo **hi**

Worst case analysis of binary search

```
bool binarySearch(int arr[], int element, int N){
//Precondition: input array arr is sorted in ascending order
  int begin = 0;
  int end = N-1;
  int mid;
  while (begin <=  end){
    mid = (end + begin)/2;
    if(arr[mid]==element){
      return true;
    }else if (arr[mid]< element){
      begin = mid + 1;
    }else{
      end = mid - 1;

    }
  }
  return false;
}
```

Binary Search Trees

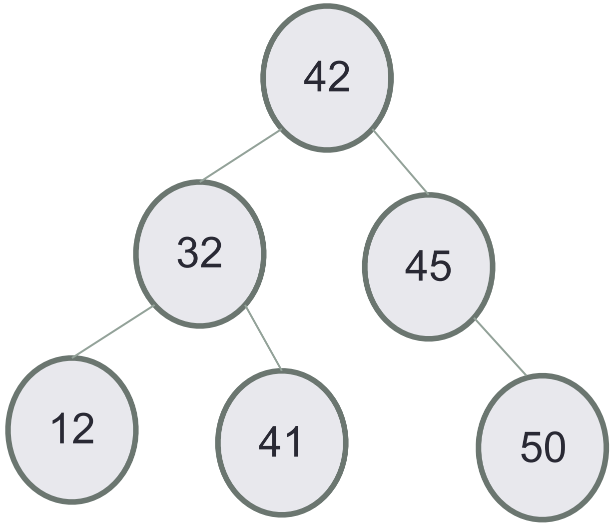
- WHAT are the operations supported?
- HOW do we implement them?
- WHAT are the (worst case) running times of each operation?

Height of the tree

- Path – a sequence of nodes and edges connecting a node with a descendant.
- A path starts from a node and ends at another node or a leaf
- Height of node – The height of a node is the number of edges on the longest downward path between that node and a leaf.

BSTs of different heights are possible with the same set of keys
Examples for keys: 12, 32, 41, 42, 45

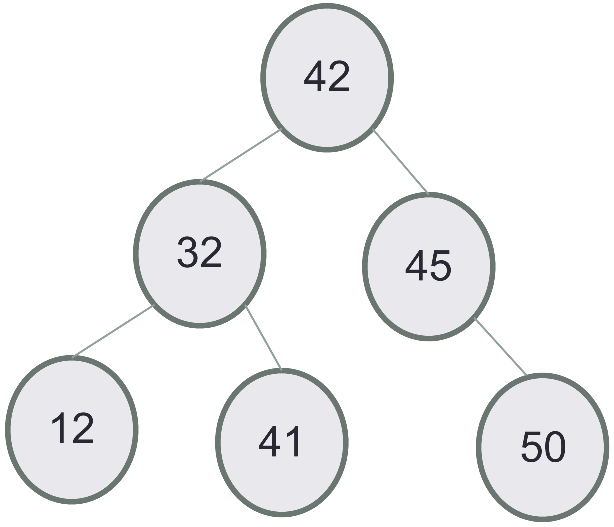
Worst case Big-O of search



- Given a BST of height H with N nodes, what is the worst case complexity of searching for a key?

A. O(1)
B. O(log H)
C. O(H)
D. O(H*log H)
E. O(N)

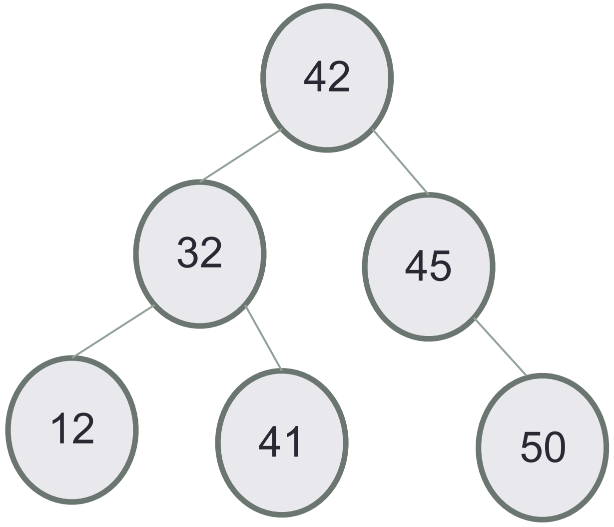
Worst case Big-O of insert



- Given a BST of height H and N nodes, what is the worst case complexity of inserting a key?

A. O(1)
B. O(log H)
C. O(H)
D. O(H*log H)
E. O(N)

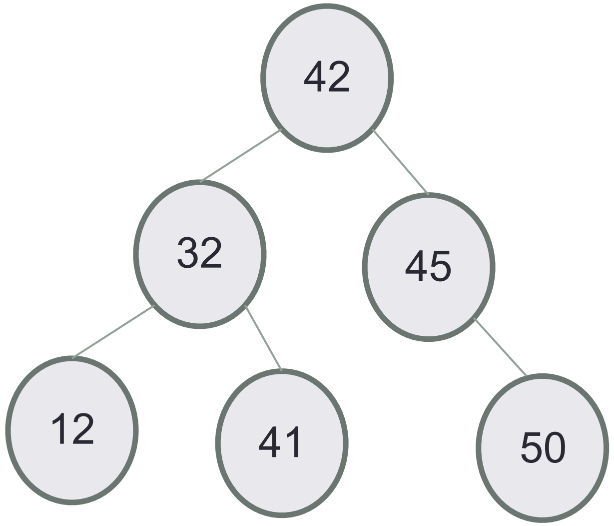
Worst case Big-O of min/max



- Given a BST of height H and N nodes, what is the worst case complexity of finding the minimum or maximum key?

A. O(1)
B. O(log H)
C. O(H)
D. O(H*log H)
E. O(N)

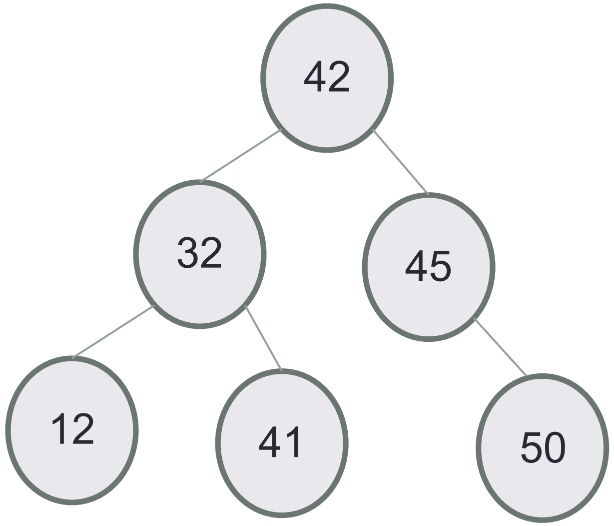
Worst case Big-O of predecessor/successor



- Given a BST of height H and N nodes, what is the worst case complexity of finding the predecessor or successor key?

A. O(1)
B. O(log H)
C. O(H)
D. O(H*log H)
E. O(N)

Worst case Big-O of delete



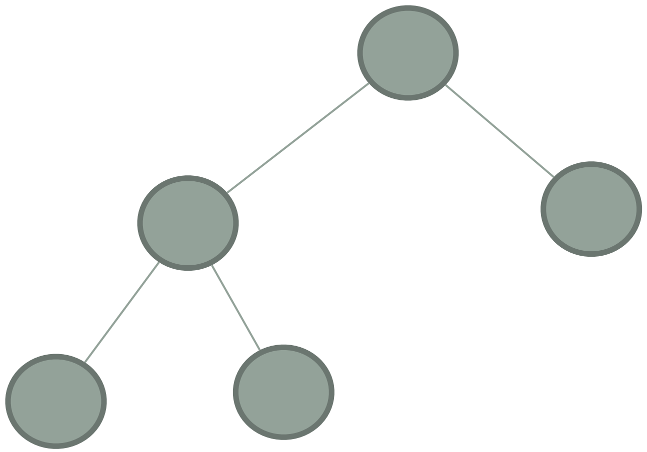
- Given a BST of height H and N nodes, what is the worst case complexity of deleting the key (assume no duplicates)?

A. O(1)
B. O(log H)
C. O(H)
D. O(H*log H)
E. O(N)

Worst case analysis

Are binary search trees *really* faster than linked lists for finding elements?

- A. Yes
- B. No

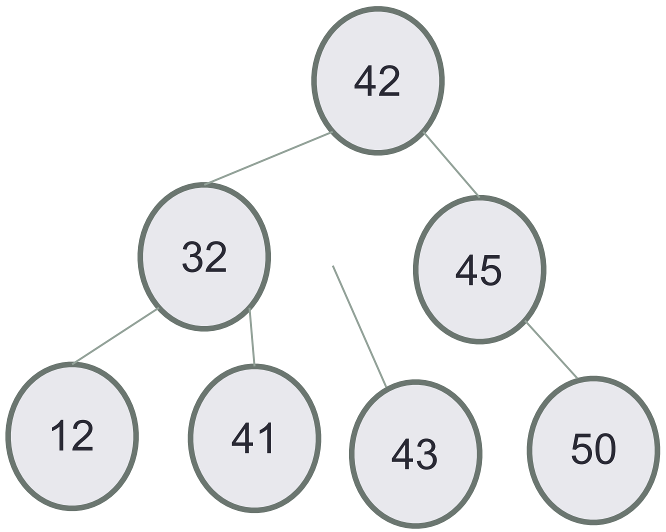


Completely filled binary tree

Level 0

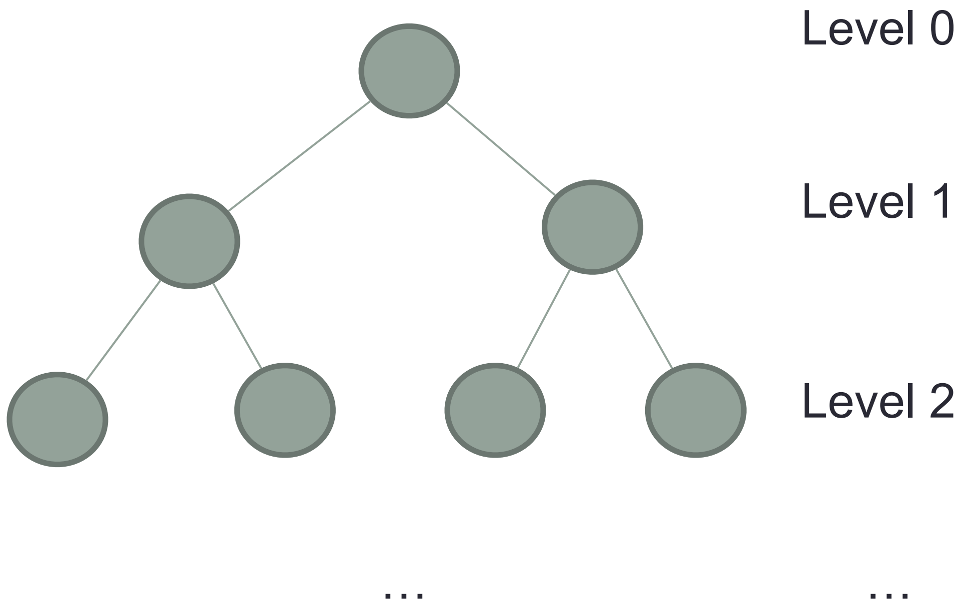
Level 1

Level 2



Nodes at each level have exactly two children, except the nodes at the last level

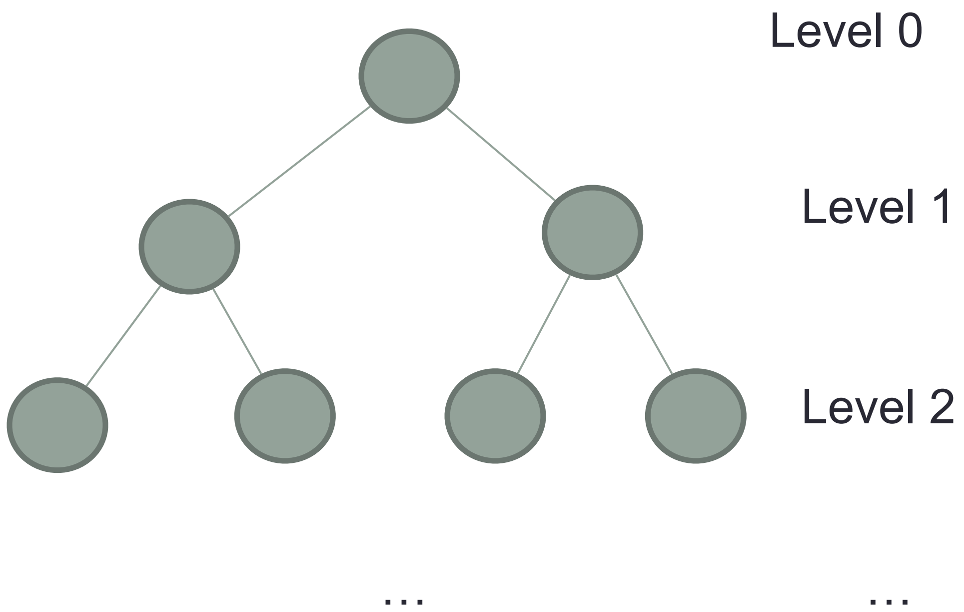
Relating H (height) and N (#nodes)
find is O(H), we want to find a f(N) = H



How many nodes are on level L in a completely filled binary search tree?

A. 2
B. L
C. 2*L
D. 2^L

Relating H (height) and N (#nodes)
find is O(H), we want to find a f(N) = H

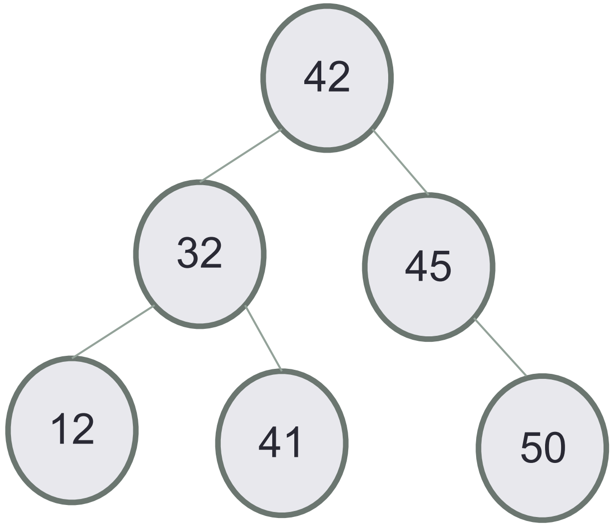


Finally, what is the height (exactly) of the tree in terms of N?

Balanced trees

- Balanced trees by definition have a height of O(log N)
- A completely filled tree is one example of a balanced tree
- Other Balanced BSTs include AVL trees, red black trees and so on
- Visualize operations on an AVL tree: https://visualgo.net/bn/bst

Big O of traversals



In Order:

Pre Order:

Post Order:

Summary of operations

Operation	Sorted Array	Balanced Binary Search Tree	Linked List
Min			
Max			
Median			
Successor			
Predecessor			
Search			
Insert			
Delete			