

RUNNING TIME ANALYSIS - PART 2

BINARY SEARCH TREES

Problem Solving with Computers-II

The image shows the C++ logo in a large, blue, 3D-style font. Below the logo is a snippet of C++ code in a monospaced font, with some words highlighted in color (red for preprocessor directives, blue for keywords, green for strings, and cyan for return values).

```
#include <iostream>
using namespace std;

int main(){
    cout<<"Hola Facebook\n";
    return 0;
}
```

How is PA01 going?

- A. Done!
- B. On track to finish
- C. On track to finish but my code is a mess
- D. Stuck and struggling
- E. Haven't started

Midterm 2

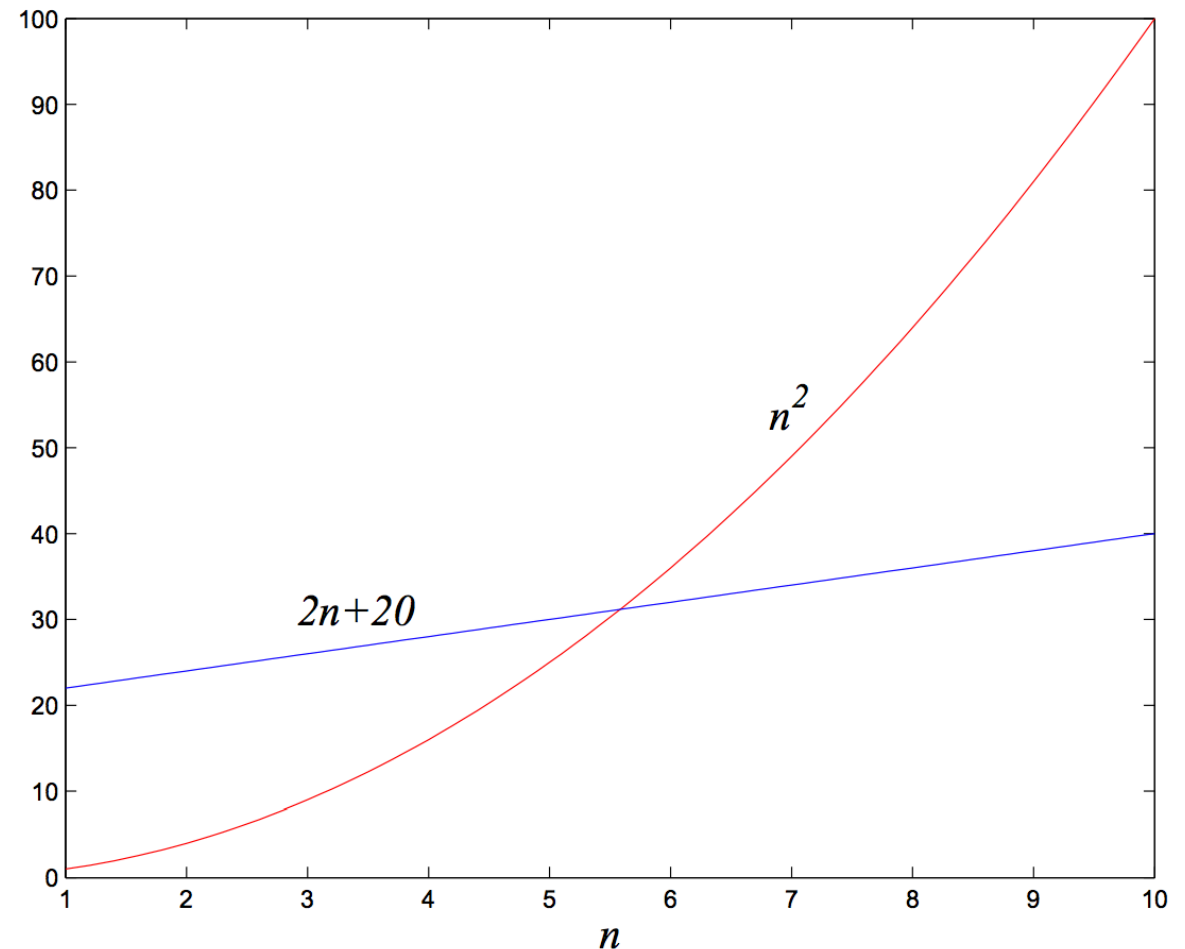
- Cumulative but the focus will be on
 - BST
 - Running time analysis

A more precise definition of Big-O

- $f(n)$ and $g(n)$: running times of two algorithms on inputs of size n .
- $f(n)$ and $g(n)$ map positive integer inputs to positive reals.

We say $f = O(g)$ if there is a constant $c > 0$ and $k > 0$ such that
 $f(n) \leq c \cdot g(n)$ for all $n \geq k$.

$f = O(g)$
means that “ f grows no faster than g ”



What is the Big-O running time of algoX?

- Assume **dataA** is some data structure that supports the following operations with the given running times, where N is the number of keys stored in the data structure:
 - insert: $O(\log N)$
 - min: $O(1)$
 - delete: $O(\log N)$
- A. $O(N^2)$
 - ☒ B. $O(N \log N)$
 - C. $O(N)$
 - D. $O(\log N)$
 - E. Not enough information to compute

```
void algoX(int arr[], int N)
{
    dataA ds; // ds contains no keys
    for(int i=0; i < N; i=i++)
        ds.insert(arr[i]);
    for(int i=0; i < N; i=i++)
        arr[i] = ds.min();
        ds.delete(arr[i]);
}
```

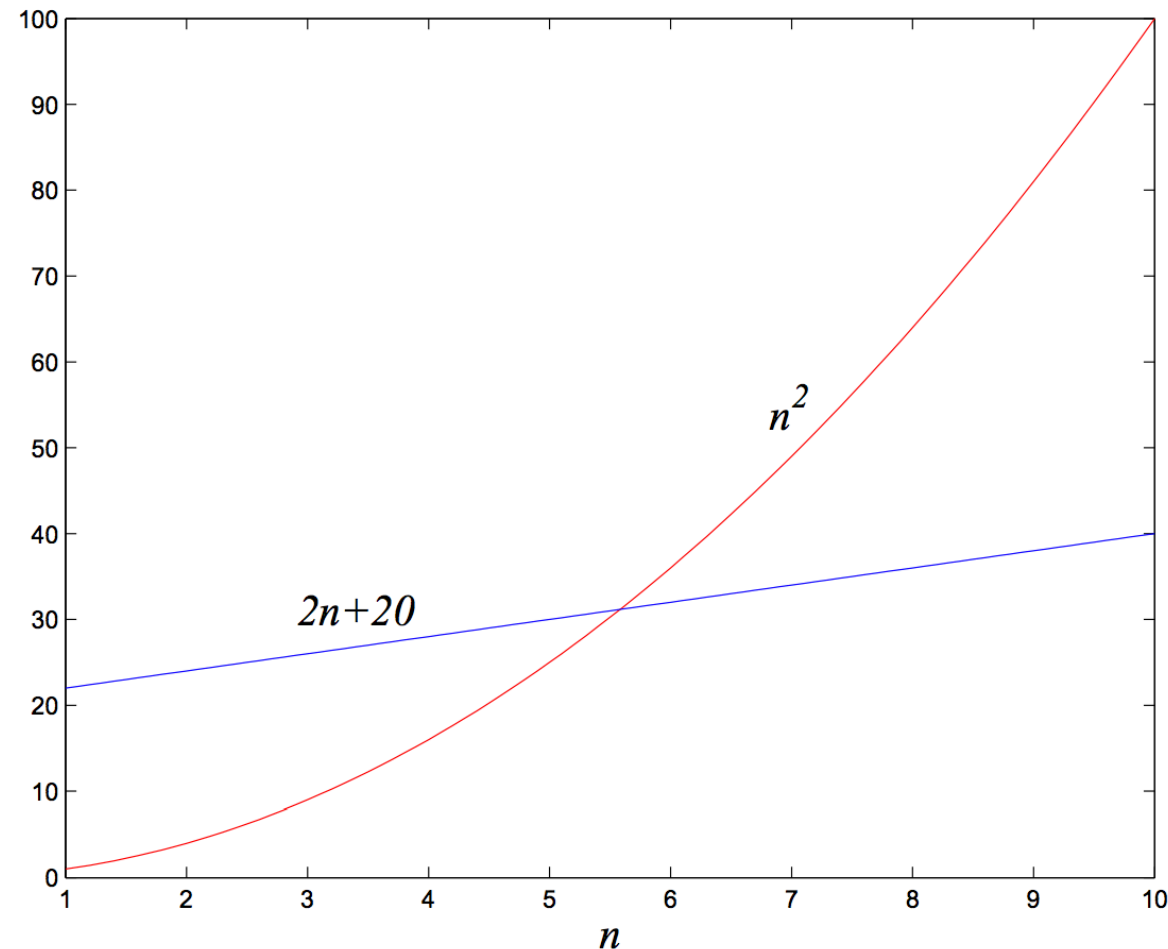
Big-Omega

- $f(n)$ and $g(n)$: running times of two algorithms on inputs of size n .
- $f(n)$ and $g(n)$ map positive integer inputs to positive reals.

We say $f = \Omega(g)$ if there are constants $c > 0$, $k > 0$ such that $c \cdot g(n) \leq f(n)$ for $n \geq k$

$f = \Omega(g)$
means that “ f grows at least as fast as g ”

g is a lower bound



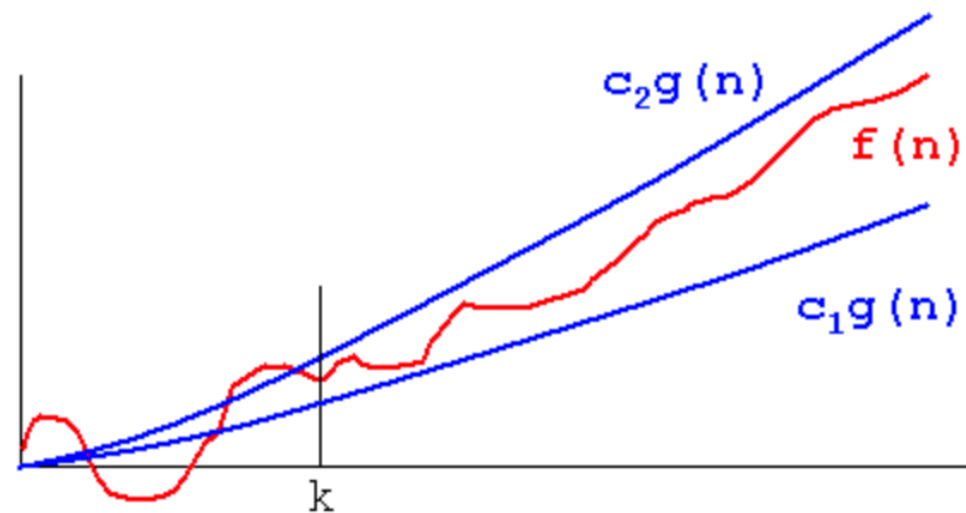
Big-Theta

- $f(n)$ and $g(n)$: running times of two algorithms on inputs of size n .
- $f(n)$ and $g(n)$ map positive integer inputs to positive reals.

We say $f = \Theta(g)$ if there are constants c_1, c_2, k such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$, for $n \geq k$

f and g grow at the same rate

Running time



Problem Size (n)

Worst case analysis of binary search

```
bool binarySearch(int arr[], int element, int N){  
    //Precondition: input array arr is sorted in ascending order  
    int begin = 0;  
    int end = N-1;  
    int mid;  
    while (begin <= end){  
        mid = (end + begin)/2;  
        if(arr[mid]==element){  
            return true;  
        }else if (arr[mid]< element){  
            begin = mid + 1;  
        }else{  
            end = mid - 1;  
        }  
    }  
    return false;  
}
```


Binary Search Trees

- WHAT are the operations supported?
- HOW do we implement them?
- WHAT are the (worst case) running times of each operation?

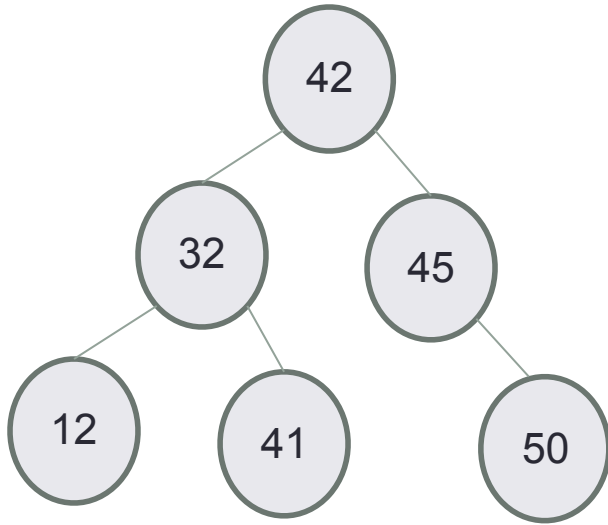
Height of the tree



- Path – a sequence of nodes and edges connecting a node with a descendant.
- A path starts from a node and ends at another node or a leaf
- Height of node – The height of a node is the number of edges on the longest downward path between that node and a leaf.

BSTs of different heights are possible with the same set of keys
Examples for keys: 12, 32, 41, 42, 45

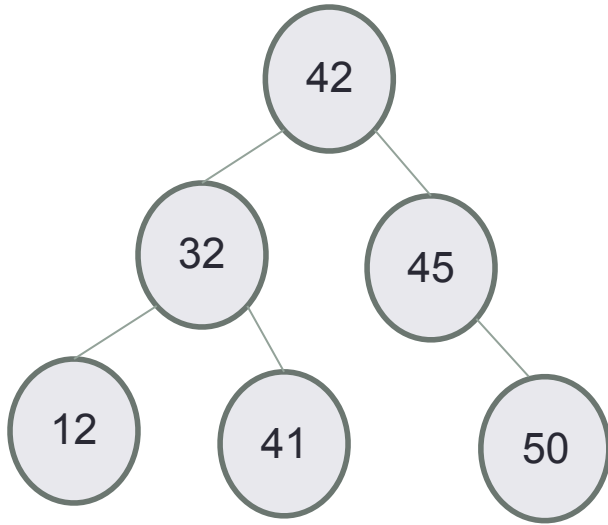
Worst case Big-O of search



- Given a BST of height H with N nodes, what is the worst case complexity of searching for a key?

- A. $O(1)$
- B. $O(\log H)$
- C. $O(H)$**
- D. $O(H \cdot \log H)$
- E. $O(N)$

Worst case Big-O of insert



- Given a BST of height H and N nodes, what is the worst case complexity of inserting a key?

A. $O(1)$

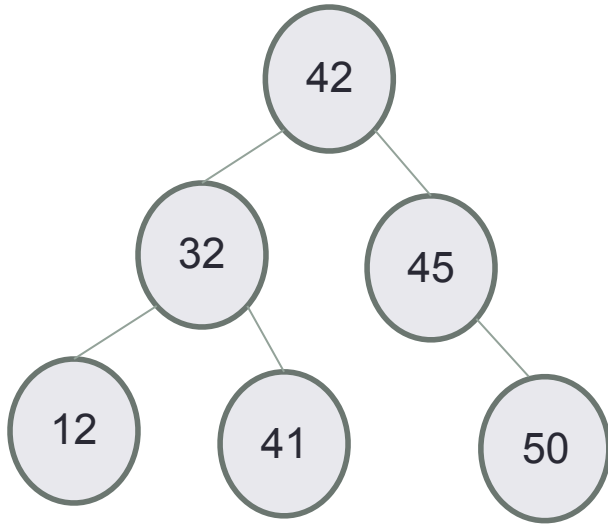
B. $O(\log H)$

C. $O(H)$

D. $O(H \cdot \log H)$

E. $O(N)$

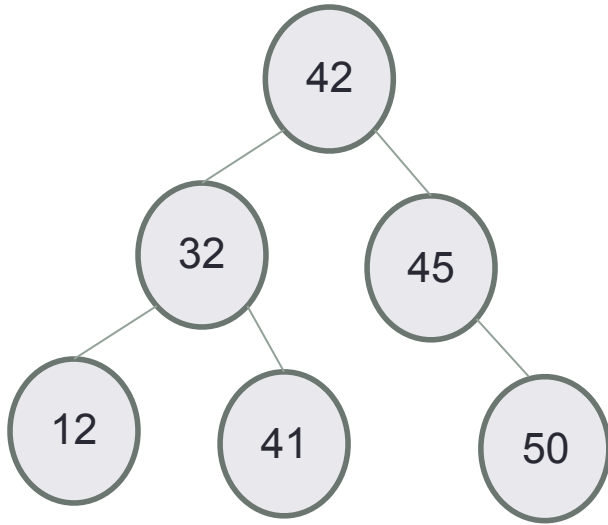
Worst case Big-O of min/max



- Given a BST of height H and N nodes, what is the worst case complexity of finding the minimum or maximum key?

- A. $O(1)$
- B. $O(\log H)$
- C. $O(H)$**
- D. $O(H \cdot \log H)$
- E. $O(N)$

Worst case Big-O of predecessor/successor



- Given a BST of height H and N nodes, what is the worst case complexity of finding the predecessor or successor key?

A. $O(1)$

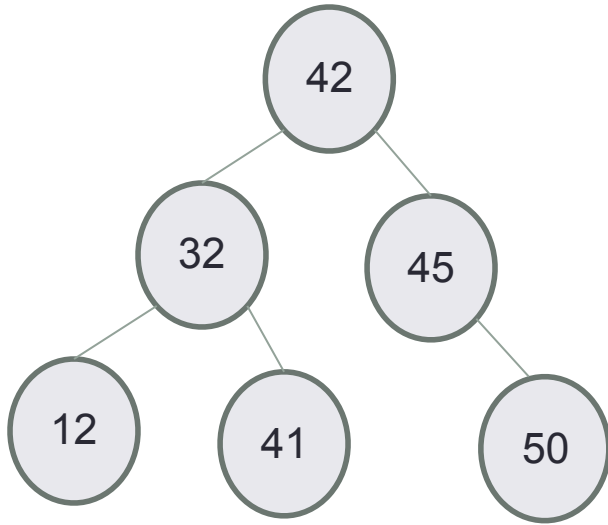
B. $O(\log H)$

C. $O(H)$

D. $O(H \cdot \log H)$

E. $O(N)$

Worst case Big-O of delete

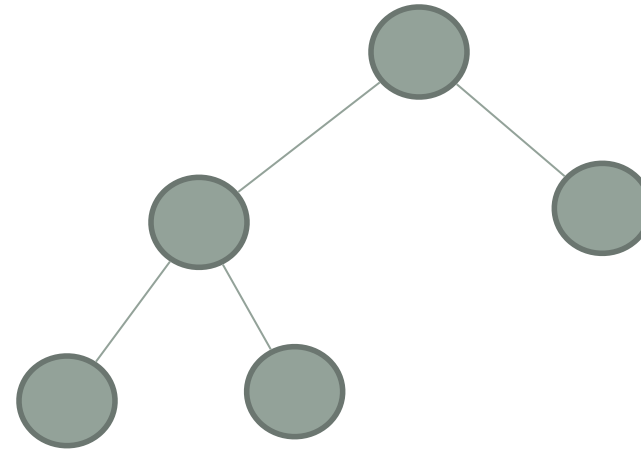


- Given a BST of height H and N nodes, what is the worst case complexity of deleting the key (assume no duplicates)?
 - A. $O(1)$
 - B. $O(\log H)$
 - C. $O(H)$
 - D. $O(H \cdot \log H)$
 - E. $O(N)$

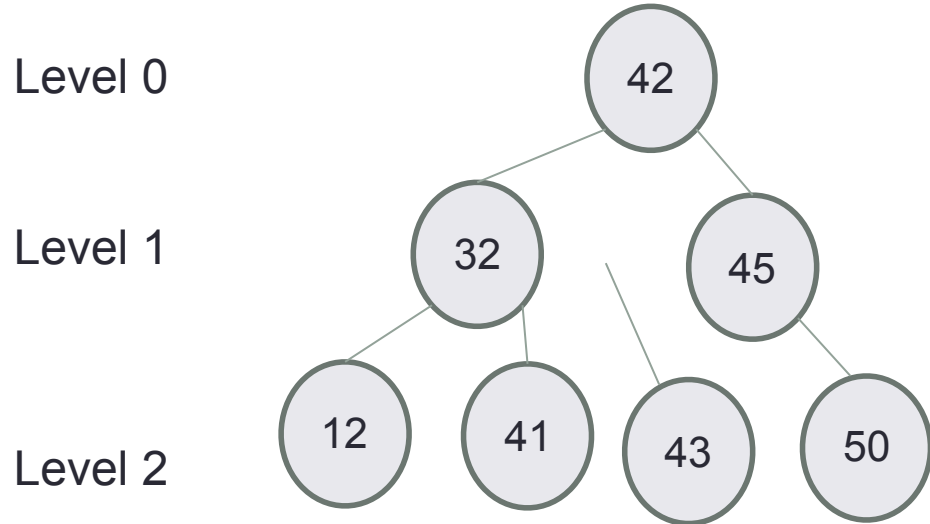
Worst case analysis

Are binary search trees *really* faster than linked lists for finding elements?

- A. Yes
- B. No

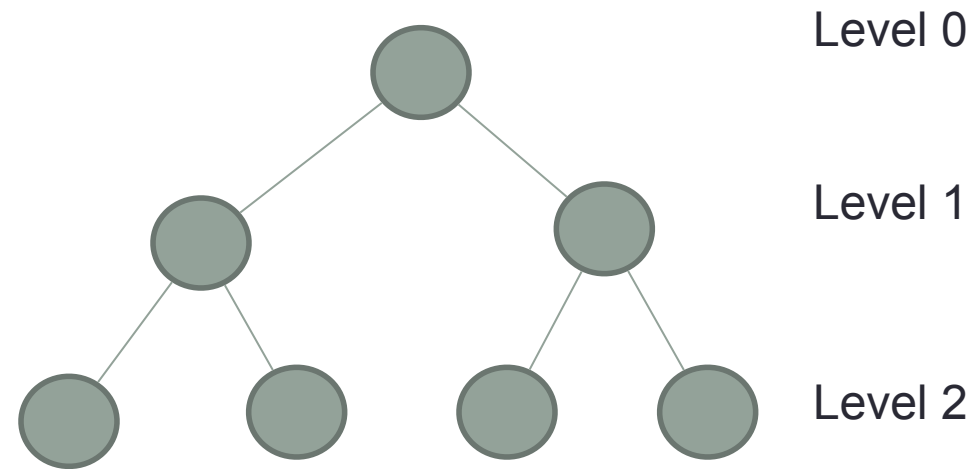


Completely filled binary tree



Nodes at each level have exactly two children, except the nodes at the last level

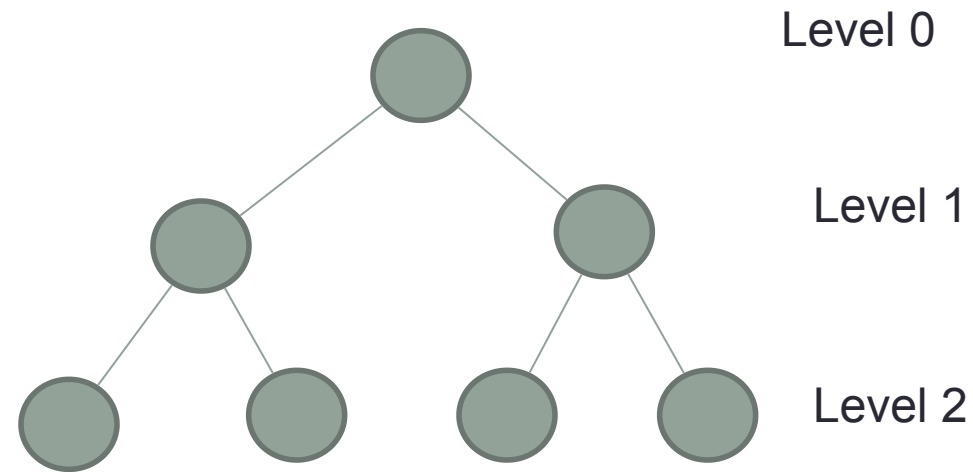
Relating H (height) and N (#nodes)
find is $O(H)$, we want to find a $f(N) = H$



How many nodes are on level L in a completely filled binary search tree?

- A. 2
- B. L
- C. $2 * L$
- D. 2^L

Relating H (height) and N (#nodes)
find is $O(H)$, we want to find a $f(N) = H$

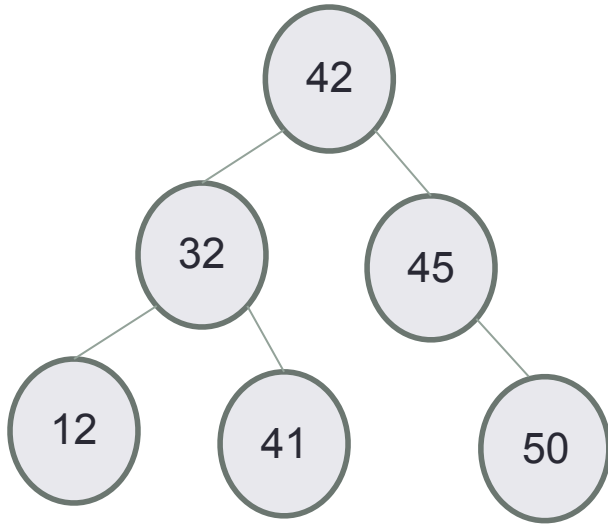


Finally, what is the height (exactly) of the tree in terms of N ?
...

Balanced trees

- Balanced trees by definition have a height of $O(\log N)$
- A completely filled tree is one example of a balanced tree
- Other Balanced BSTs include AVL trees, red black trees and so on
- Visualize operations on an AVL tree: <https://visualgo.net/bn/bst>

Big O of traversals



In Order:

Pre Order:

Post Order:

Summary of operations

Operation	Sorted Array	Balanced Binary Search Tree	Linked List
Min	$O(?)$	$O(?)$	$O(?)$
Max			
Median			
Successor			
Predecessor			
Search			
Insert			
Delete			