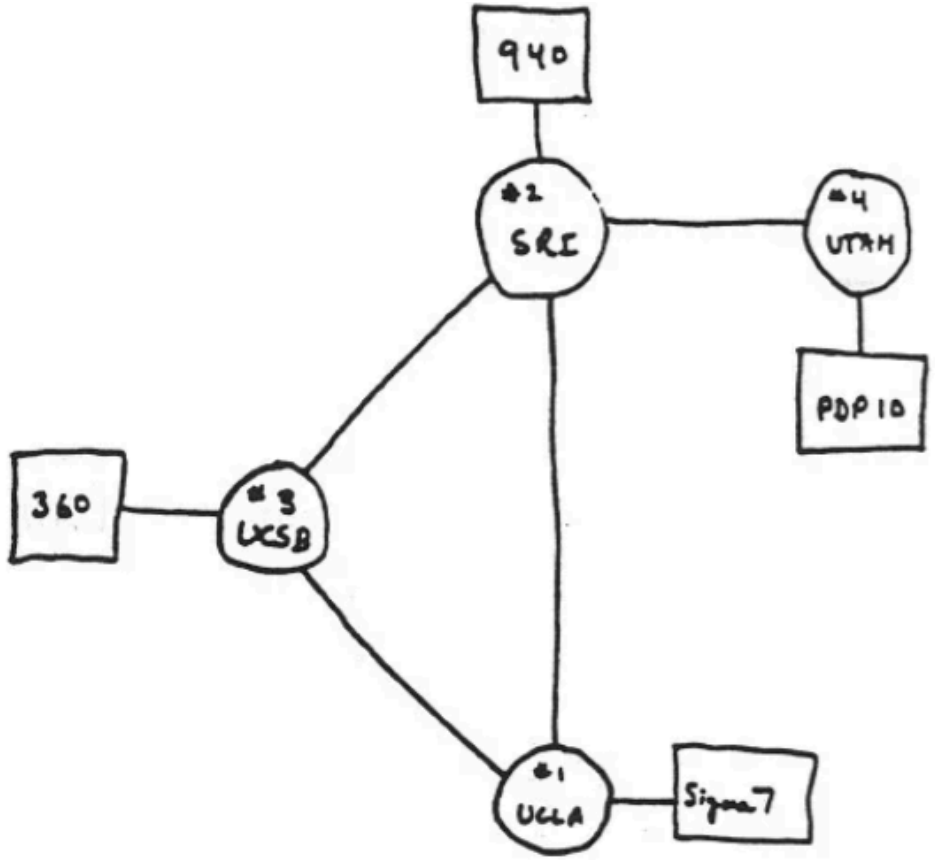
GRAPHS



THE ARPA NETWORK

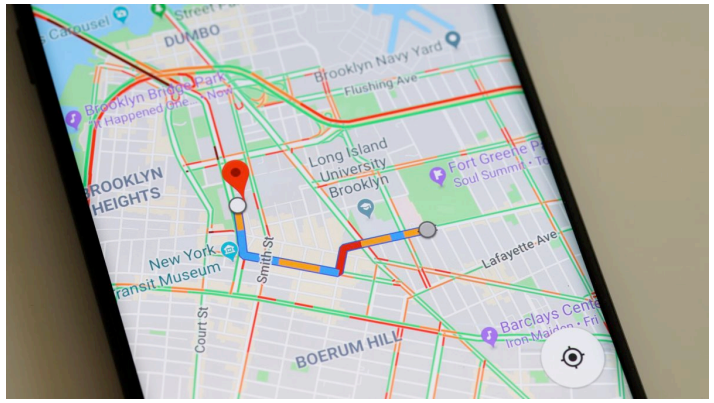
DEC 1969

4 NODES

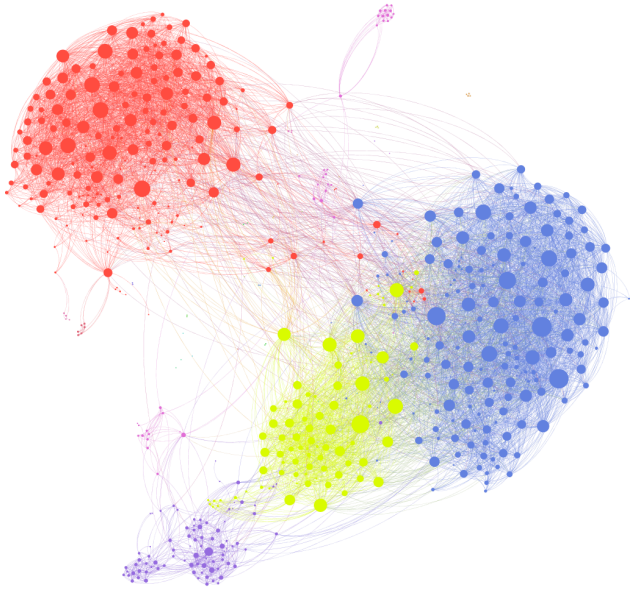
*The IBM 360, the IMP, and the workstations were all located in North Hall. https://jeweledplatypus.org/news/text/ucsbnet.html

Graphs: terminology

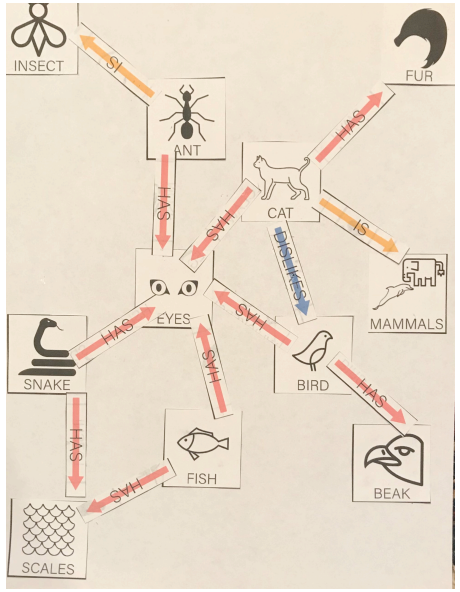
Graph applications



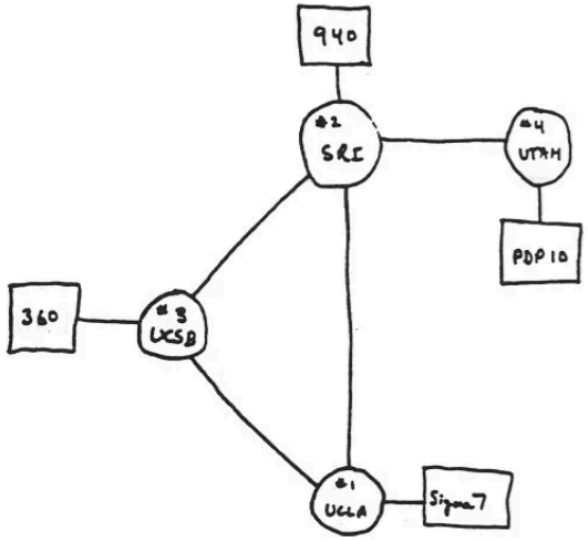
Road networks



Social networks



Semantic networks



Computer networks*

Remember: If your problem maps to a well-known graph problem, it usually means you can solve it blazingly fast!

How do we discuss running time on graphs?

Size of a graph:

Connected

Disconnected

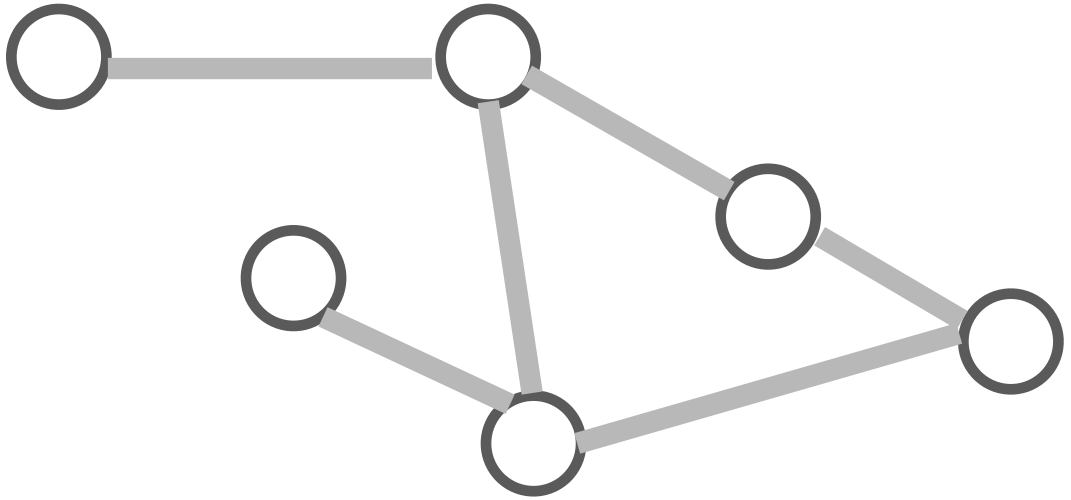
Fully connected

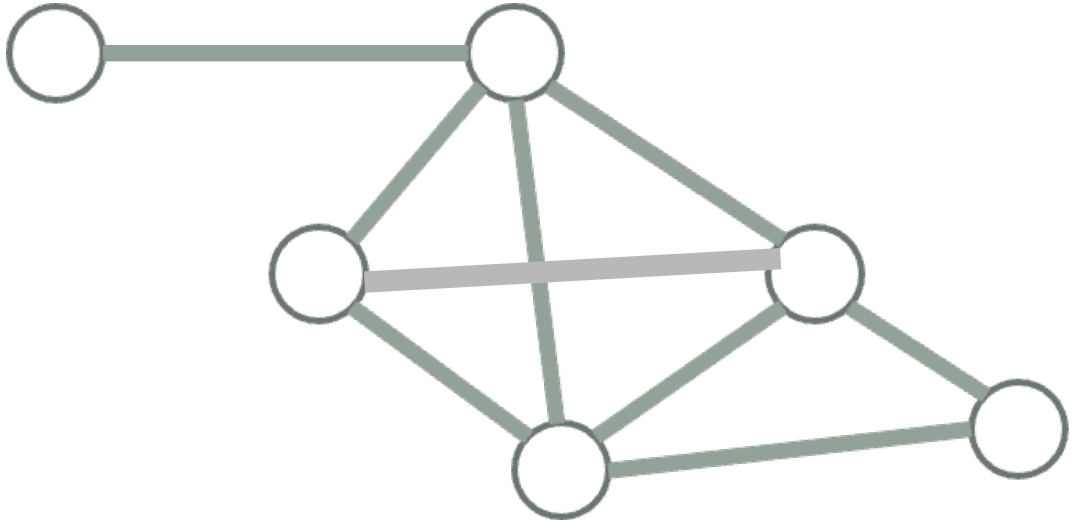
Concept Question

What is minimum and maximum number of edges in a connected graph with n vertices?

A. 0 and n
B. (n - 1) and n (n - 1) / 2
C. (n - 1) and n^2
D. (n - 1) and 2^n

Sparse vs. Dense Graphs

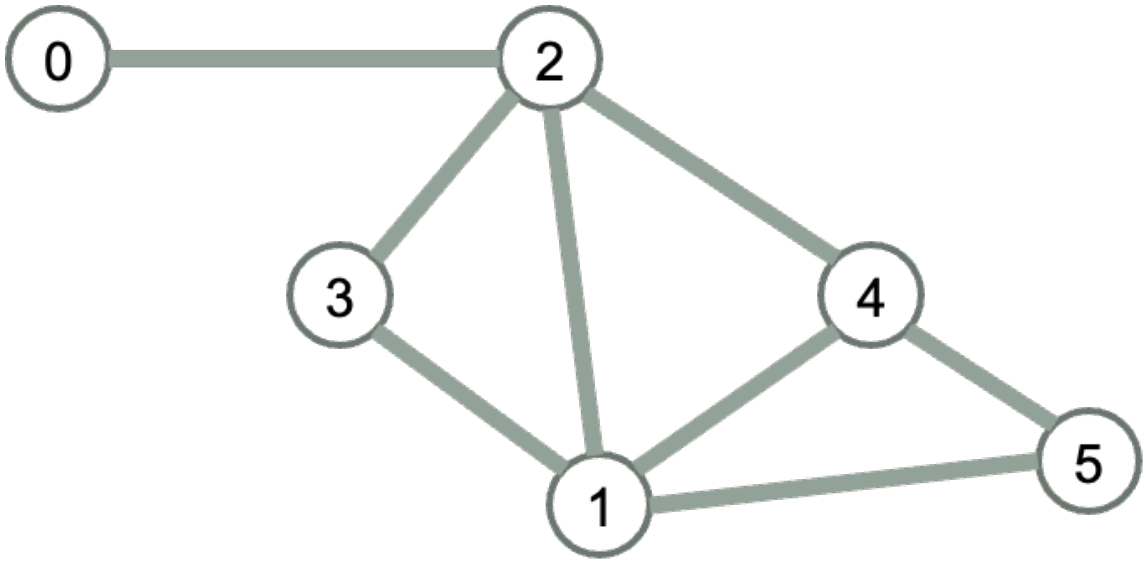




A dense graph is one where |E| is “close to” $|V|^2$.
A sparse graph is one where |E| is “closer to” |V|.

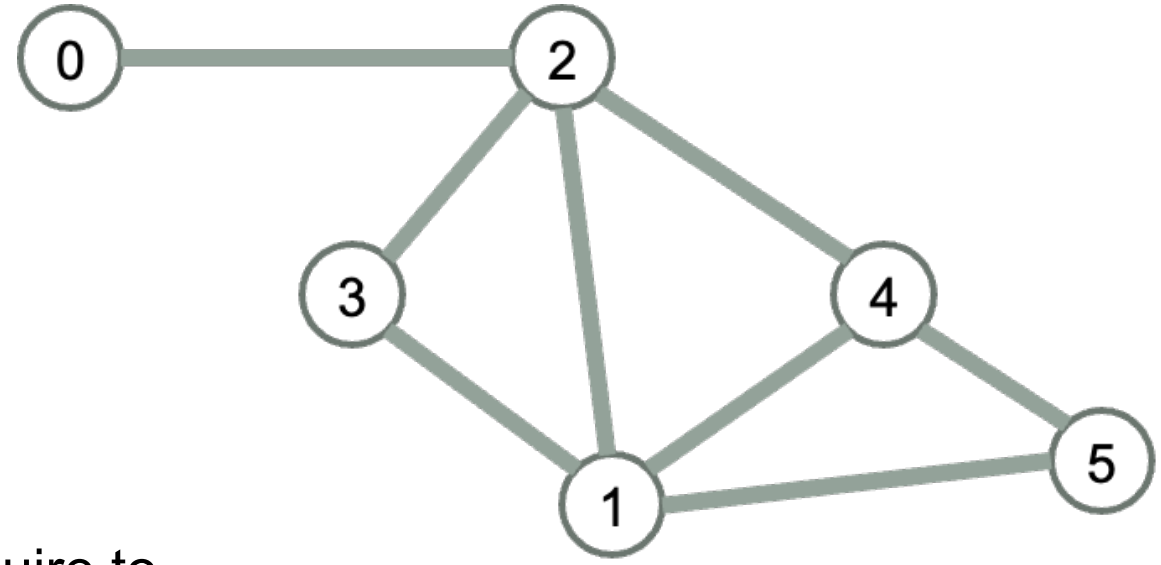
Adjacency Matrix Representation of a Graph

Represent the graph by a n x n binary valued adjacency matrix, A



Adjacency Matrix

Represent the graph by a n x n binary valued adjacency matrix, A
A[i, j] = 1, if there is an edge from I to j

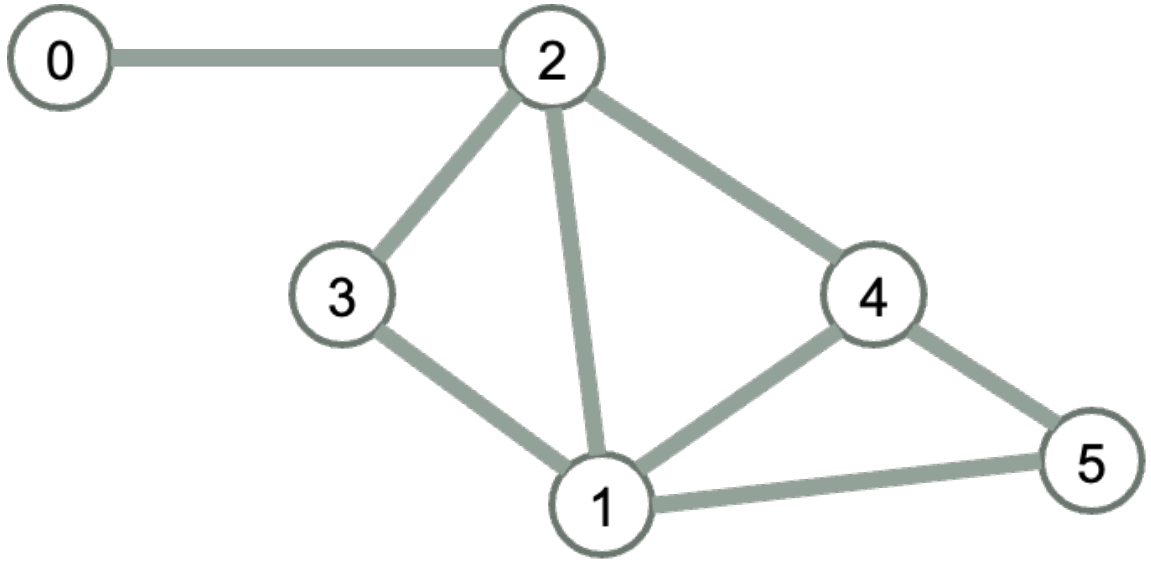


How much space does an adjacency matrix require to represent a graph with n vertices and m edges?

A. O(n)
B. O(m)
C. O(n + m)
D. O(n^2)
E. O(mn)

Adjacency List Representation of a Graph

Vertices and edges stored as lists
Each vertex points to all its edges



How much space does an adjacency list require to represent a graph with n vertices and m edges?

A. O(n)
B. O(m)
C. O(n + m)
D. O(n^2)
E. O(m.n)

Assume each vertex is identified by an integer index

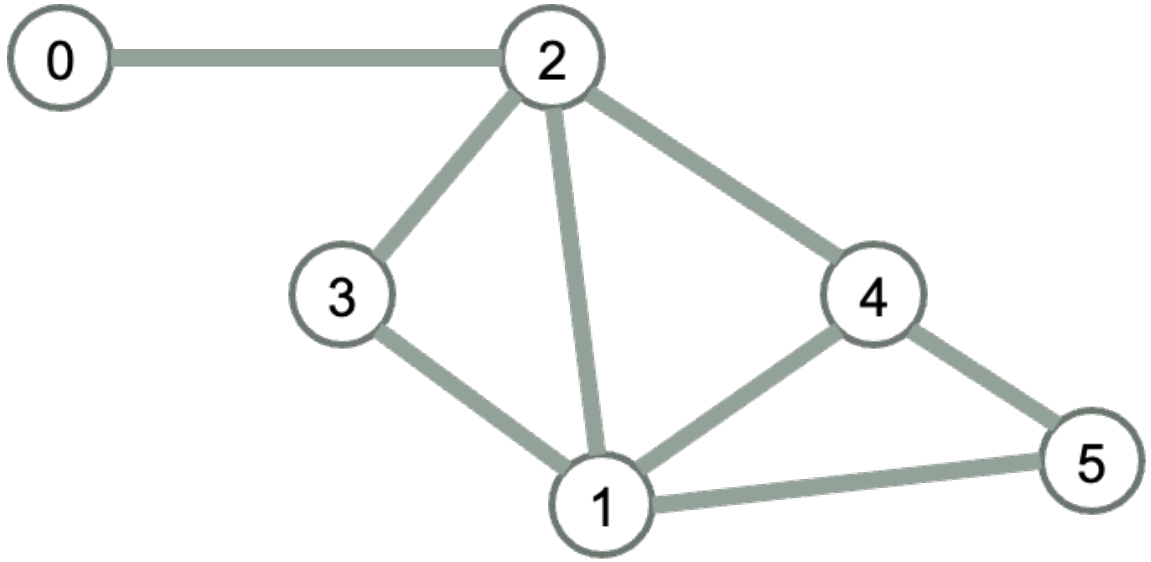
```
class graph{

  private:
  __________ adjlist;
};
```

Choose the ADT to represent the adjacency list

A. vector<int>
B. vector<vector<int>>
C. list<vector<int>>
D. vector<list<int>>
E. set<list<int>>

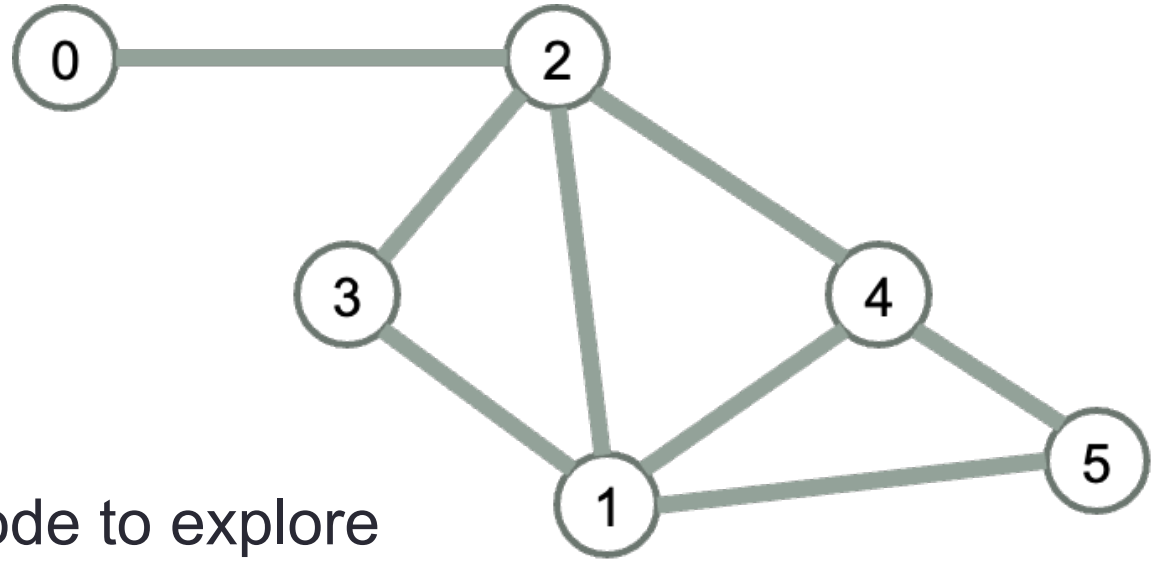
Graph search: general approach

Starting with a source node
- find everything that can be explored
- don’t explore anything twice



Graph search: breadth first (BFS)

Explore all the nodes reachable from a given node before moving on to the next node to explore

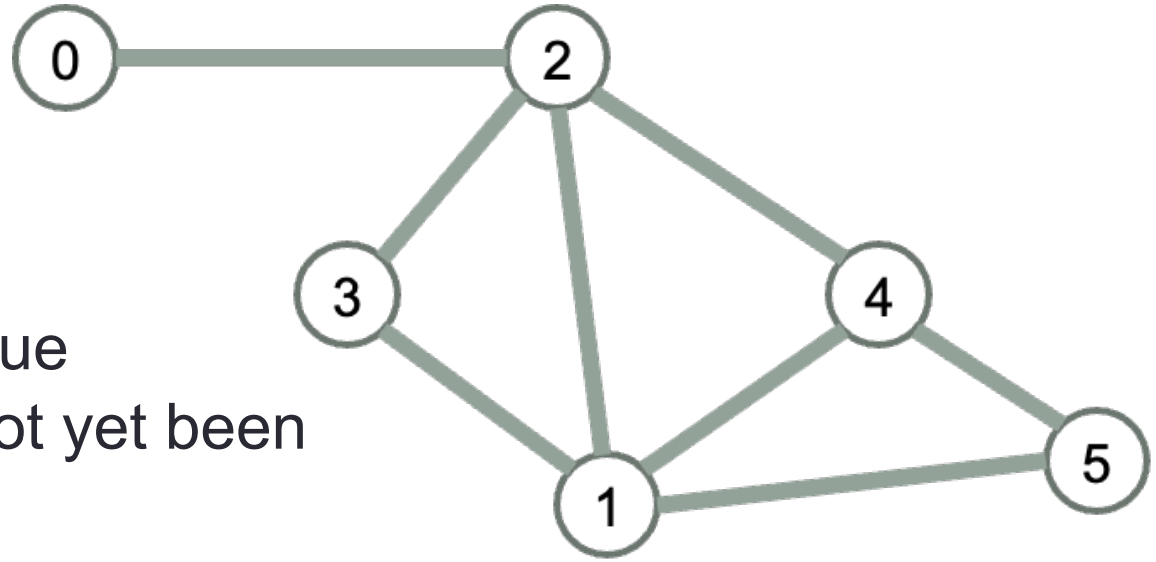


Assume BFS chooses the lower number node to explore first, in what order does BFS visit the nodes in this graph

A. 0, 1, 2, 3, 4, 5
B. 0, 1, 3, 2, 4, 5
C. 0, 2, 3, 1, 4, 5
D. 0, 2, 1, 3, 4, 5
E. Something else

BFS Traverse: Sketch of Algorithm

Start at source *s*;
Mark *s* as visited
push *s* into a queue
while the queue is not empty:
- pop the vertex *u* from the front of the queue
- for each of *u*'s adjacent nodes that has not yet been visited (*v*):
    - Mark *v* as visited
    - Push *v* in the queue



Questions:

- What difference(s) do you observe with the BSF we covered for trees?
- What data do you need to keep track of for each node?

Implement the graph ADT given in your handout

```
class graph{
  public:
    graph(int n = 0) { // n is the number of vertices
      _____________________________________;
    }
    void addEdge(int from, int to);
    bool hasEdge(int i, int j) const;
    vector<bool> bfs(int source);  // performs a breadth first search starting from the source and returns a vector with vertices that
were visited set to true
    bool isValidPath(const vector<int> & path) const; // returns true if the input path exists in the graph
    bool isReachable(int source, int dest);// returns true if a path exists from the source to the destination
  private:
    _____________________________________;
};
```

Link to hand out: https://bit.ly/CS24F23GraphsHandout