HEAPS

Problem Solving with Computers-II

Reminders

- PA02 released, due Wed of Week 10 (06/02).
- Lab05 due this Wed (05/19)
- zyBook, Chapter 7 activities due this Friday (05/21)
- Quiz 4 next week Mon (05/24): zybook chapters 5, 6, 7 (Run Time Analysis, Stack, Queue, STL)

Heaps

- Clarification
    - *heap*, the data structure is not related to *heap,* the region of memory
- What are the operations supported?
- What are the running times?

Heaps

	Min-Heaps	Max-Heap	BST
• Insert :			
• Min:			
• Delete Min:			
• Max			
• Delete Max			

Applications:

- Efficient sort
- Finding the median of a sequence of numbers
- Compression codes

Choose heap if you are doing repeated insert/delete/(min OR max) operations

std::priority_queue (STL's version of heap)

A C++ `priority_queue` is a generic container, and can store any data type on which an ordering can be defined: for example `ints, structs (Card),` pointers etc.

#include <queue>

```
priority_queue<int> pq;
```

Methods:

```
* push()   //insert
* pop()    //delete max priority item
* top()    //get max priority item
* empty()   //returns true if the priority queue is empty
```

- You can extract object of highest priority in O(log N)
- To determine priority: objects in a priority queue must be comparable to each other

STL Heap implementation: Priority Queues in C++

What is the output of this code?

```
priority_queue<int> pq;
pq.push(10);
pq.push(2);
pq.push(80);
cout<<pq.top();
pq.pop();
cout<<pq.top();
pq.pop();
cout<<pq.top();
pq.pop();
```

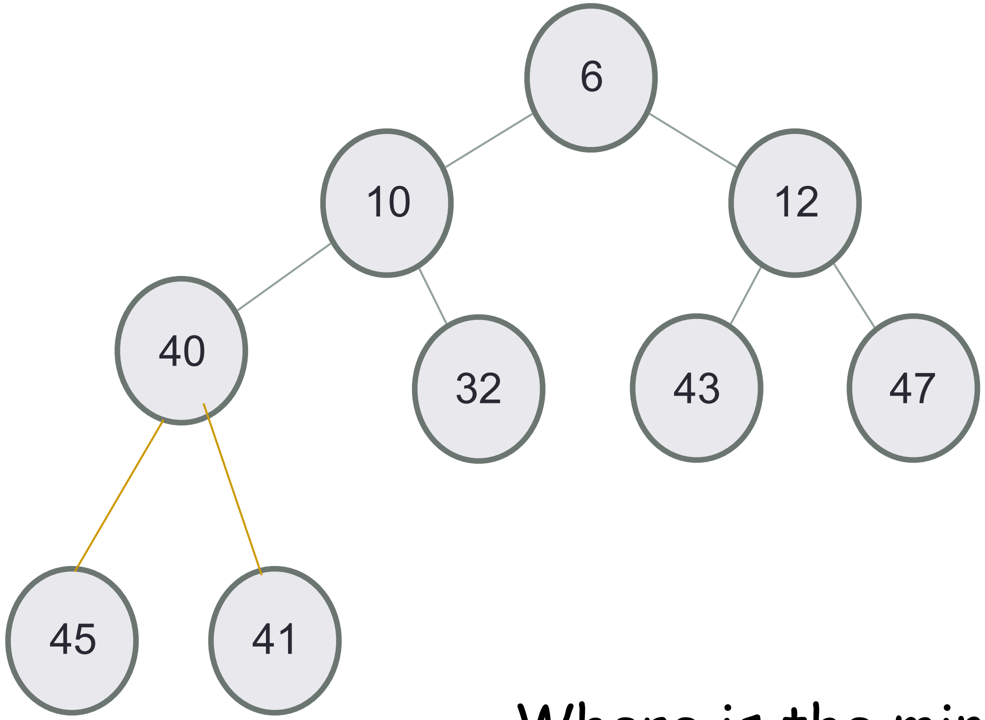
A. 10 2 80
B. 2 10 80
C. 80 10 2
D. 80 2 10
E. None of the above

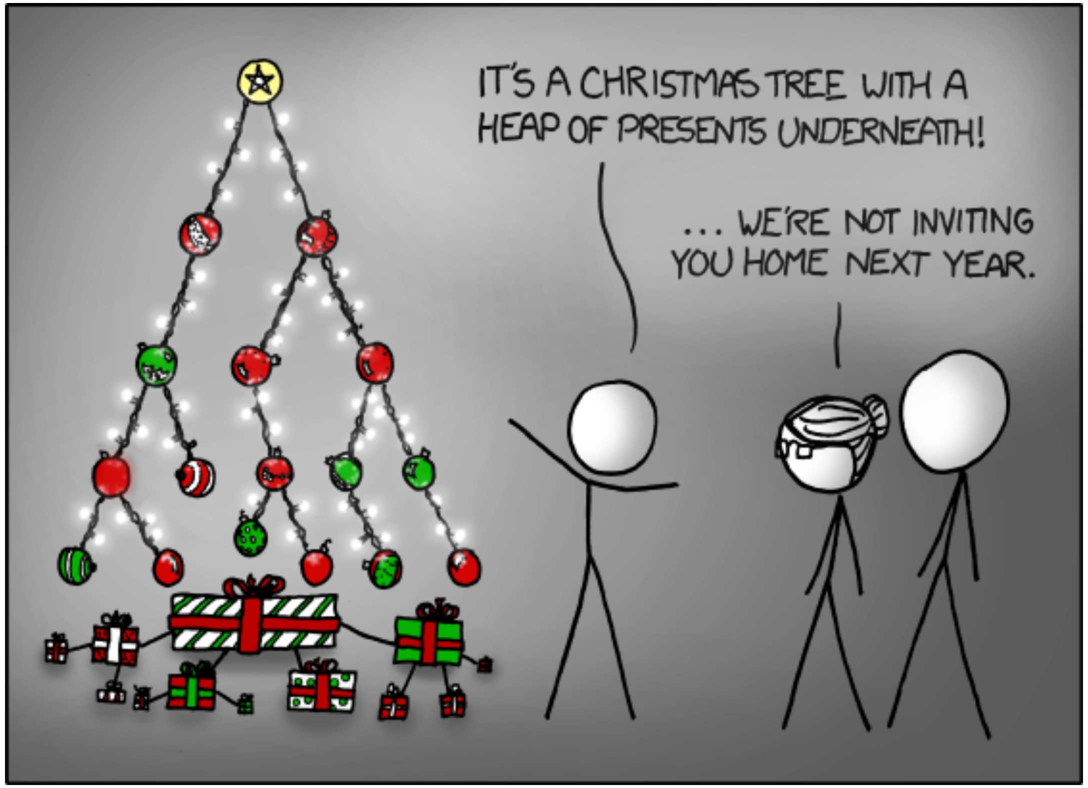
Heaps as binary trees

- **Rooted binary tree that is as complete as possible**
- **In a min-Heap, each node satisfies the following heap property:**

key(x)<= key(children of x)

Min Heap with 9 nodes



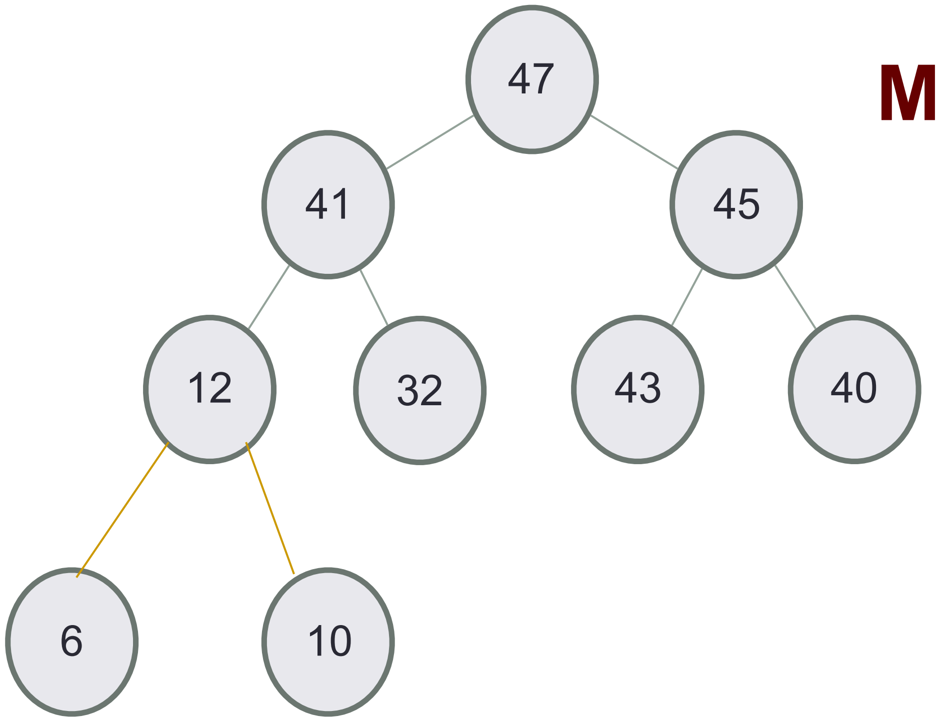


Where is the minimum element?

Heaps as binary trees

- **Rooted binary tree that is as complete as possible**
- **In a max-Heap, each node satisfies the following heap property:**

  key(x)>= key(children of x)

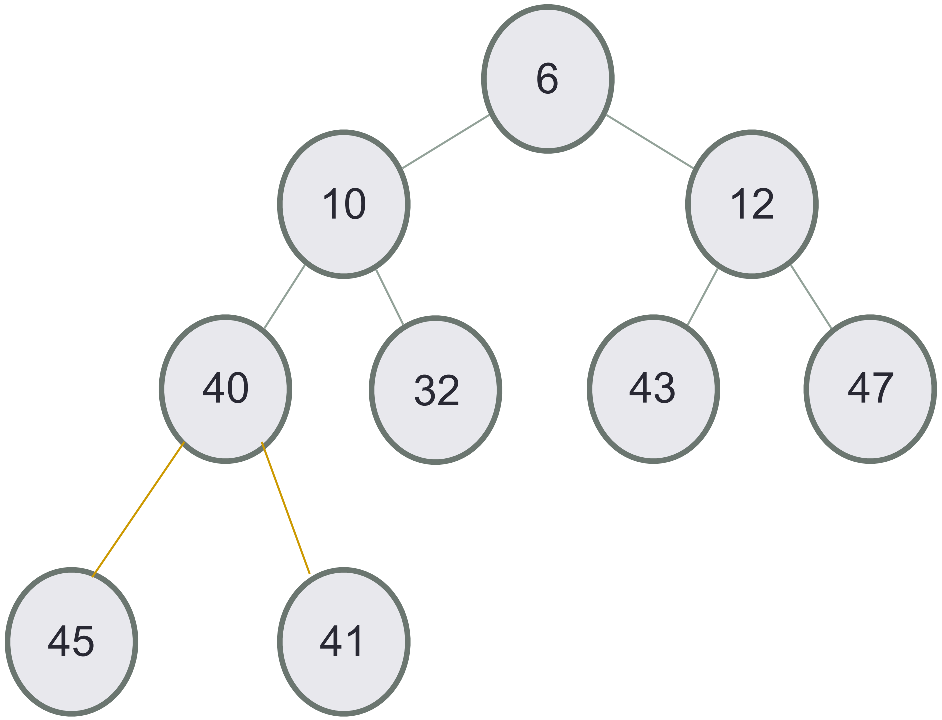


Max Heap with 9 nodes

Where is the maximum element?

Structure: Complete binary tree

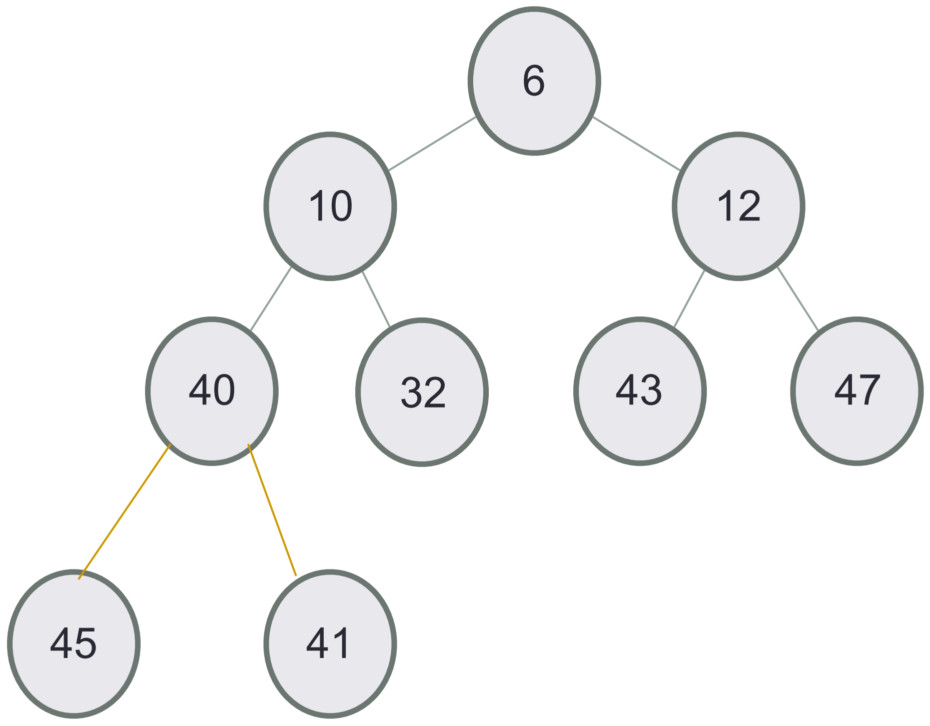
A heap is a complete binary tree: Each level is as full as possible.
Nodes on the bottom level are placed as far left as possible



Identifying heaps

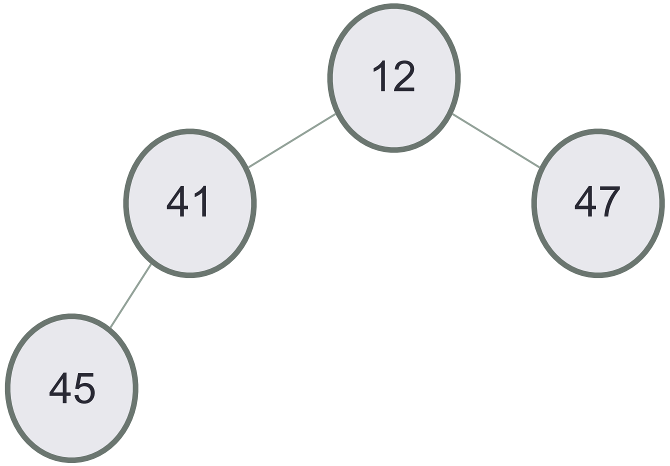
Starting with the following min-Heap which of the following operations will result in something that is NOT a min Heap

A. Swap the nodes 40 and 32
B. Swap the nodes 32 and 43
C. Swap the nodes 43 and 40
D. Insert 50 as the left child of 45
E. C&D

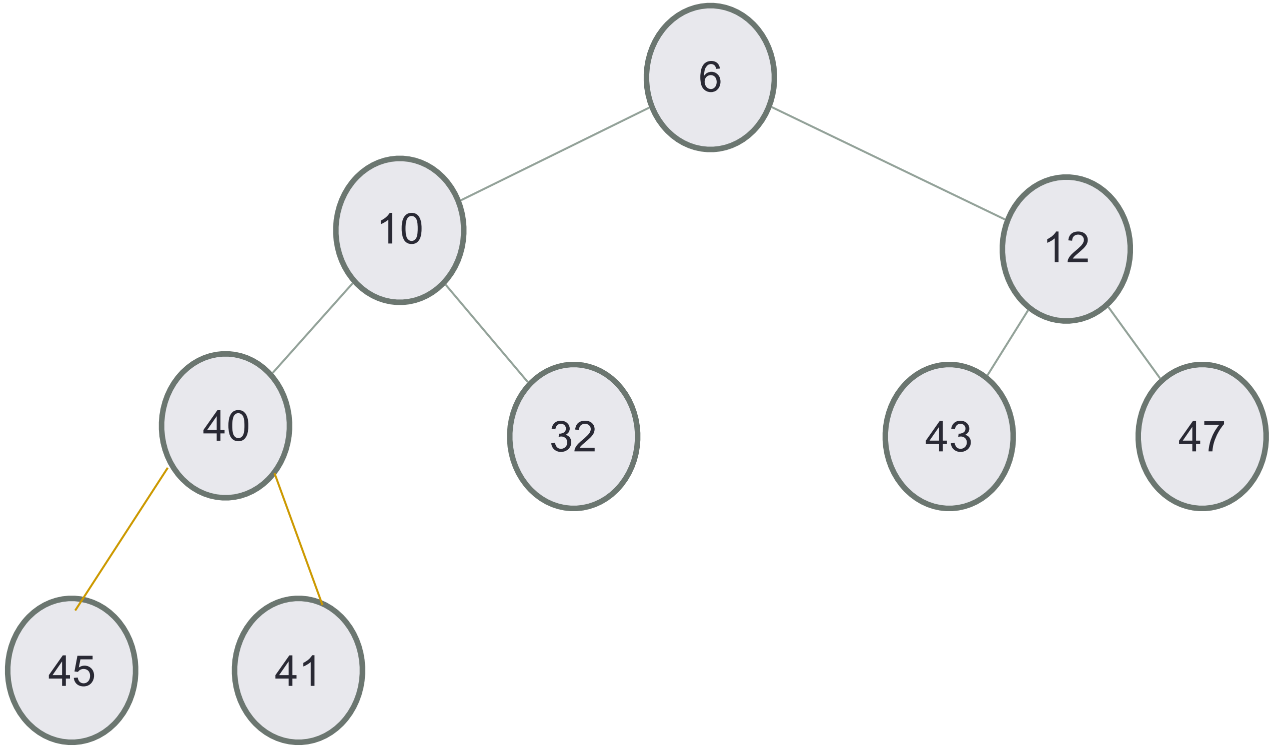


Insert 50 into a heap

- **Insert key(x) in the first open slot at the last level of tree (going from left to right)**
- **If the heap property is not violated - Done**
- **Else: while(key(parent(x))>key(x)) swap the key(x) with key(parent(x))**

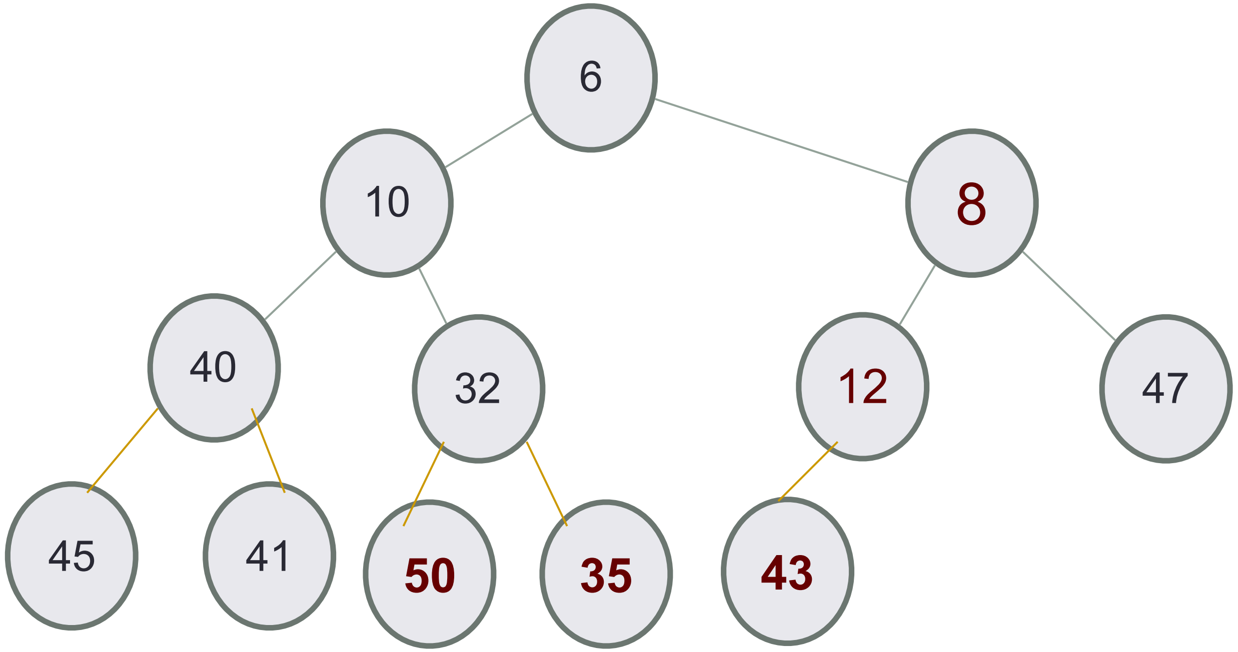


Insert 50, then 35, then 8



Delete min

- **Replace the root with the rightmost node at the last level**
- **“Bubble down”- swap node with one of the children until the heap property is restored**

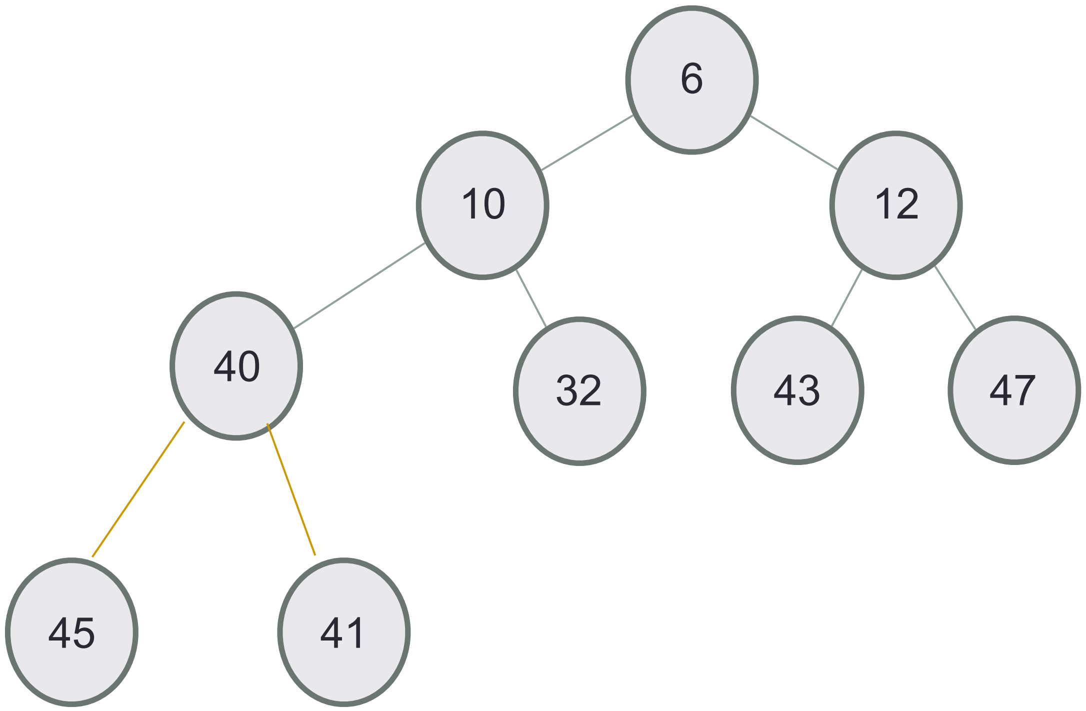


Under the hood of heaps

- An efficient way of implementing heaps is using vectors
- Although we think of heaps as trees, the entire tree can be efficiently represented as a vector!!

Implementing heaps using an array or vector

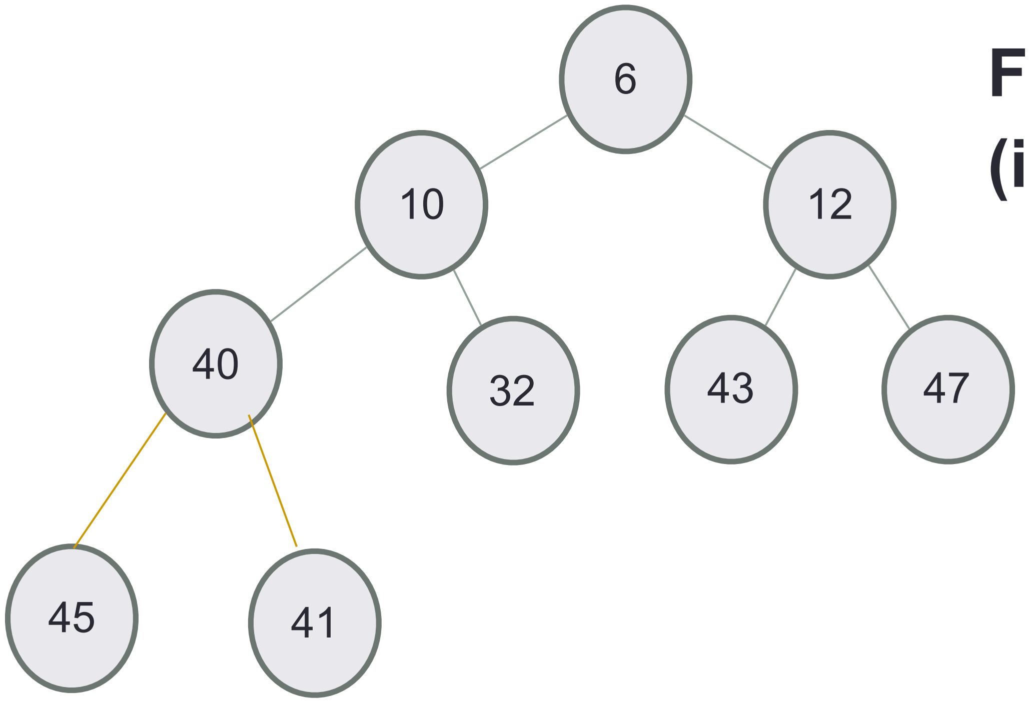
Value										
Index	0	1	2	3	4	5	6	7	8	



Using vector as the internal data structure of the heap has some advantages:

- **More space efficient than trees**
- **Easier to insert nodes into the heap**

Finding the "parent" of a "node" in the vector representation



For a key at index i, index of the parent is (i-1)/2

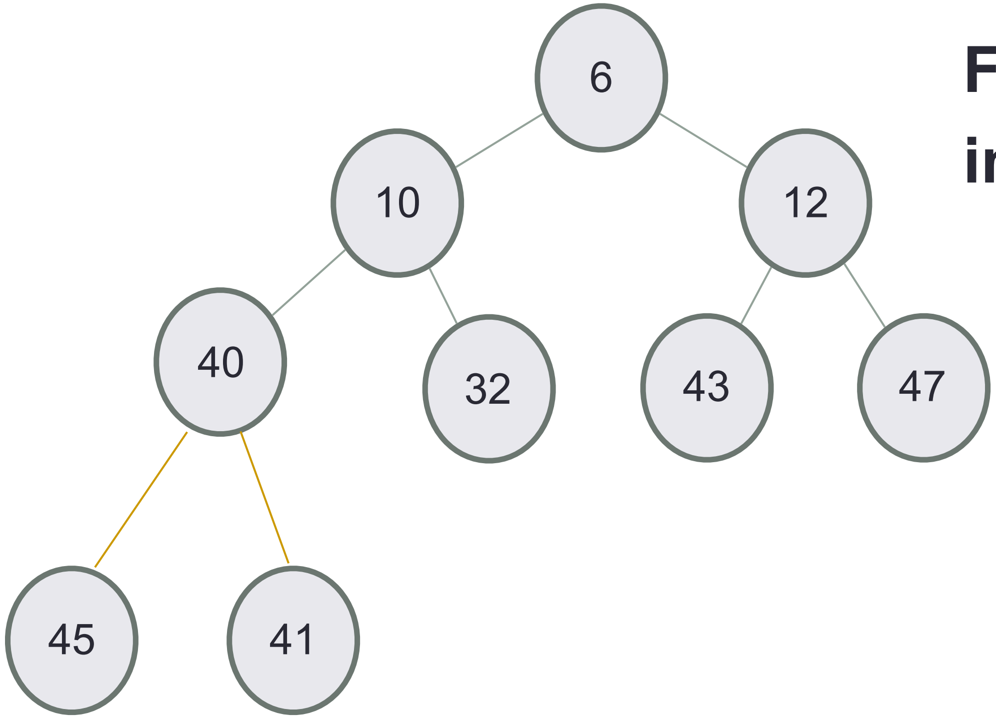
Value	6	10	12	40	32	43	47	45	41	
Index	0	1	2	3	4	5	6	7	8	

Insert into a heap

- **Insert key(x) in the first open slot at the last level of tree (going from left to right)**
- **If the heap property is not violated - Done**
- **Else….**

Insert the elements {12, 41, 47, 45, 32} in a min-Heap using the vector representation of the heap

Insert 50, then 35



For a node at index i, index of the parent is int(i-1/2)

Value	6	10	12	40	32	43	47	45	41	
Index	0	1	2	3	4	5	6	7	8	

Insert 8 into a heap

Value	6	10	12	40	32	43	47	45	41	50	35
Index	0	1	2	3	4	5	6	7	8	9	10

After inserting 8, which node is the parent of 8 ?

A. Node 6

B. Node 12

C. None 43

D. None - Node 8 will be the root

Next lecture

- **Under the hood of heaps**
- **More on STL implementation of heaps (priority queues)**