Binary Arithmetic 2
Introduction to Assembly Language

CS 64: Computer Organization and Design Logic
Lecture #3
Fall 2018

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Administrative Stuff

- The class is full – I will not be adding more ppl ☹

- Did you check out the syllabus?
- Did you check out the class website?
- Did you check out Piazza (and get access to it)?
- Did you go to lab yesterday?

Any Questions From Last Lecture?

5-Minute Pop Quiz!!!

YOU MUST SHOW YOUR WORK!!!

1. Calculate and give your answer in hexadecimal:

   ~(0x3E | 0xFC)

2. Convert from binary to decimal AND to hexadecimal. Use any technique(s) you like:

   a) 1001001

   b) 10010010

Answers…

1. Calculate and give your answer in hexadecimal:

   ~(0x3E | 0xFC) **= ~(0xFE) = 0x01**

2. Convert from binary to decimal AND hexadecimal. Use any technique you like:

   a) 1001001 **= 0100 1001 = 0x49**
   = 1 + 8 + 64 = 73

   b) 10010010 **= 1001 0010 = 0x92**
   I see that it's (1001001) x 2 = 146

Lecture Outline

- Two's complement
- Addition and subtraction in binary
- Carry Out and Overflow bits

- Intro to Assembly

Twos Complement Method

- This is how Twos Complement fixes this.
- Let's write out **$-6_{(10)}$** in 2s-Complement binary in **4 bits**:

First take the unsigned (abs) value (i.e. 6) and convert to binary: **0110**

Then negate it (i.e. do a "NOT" function on it): **1001**

Now add 1: **1010**

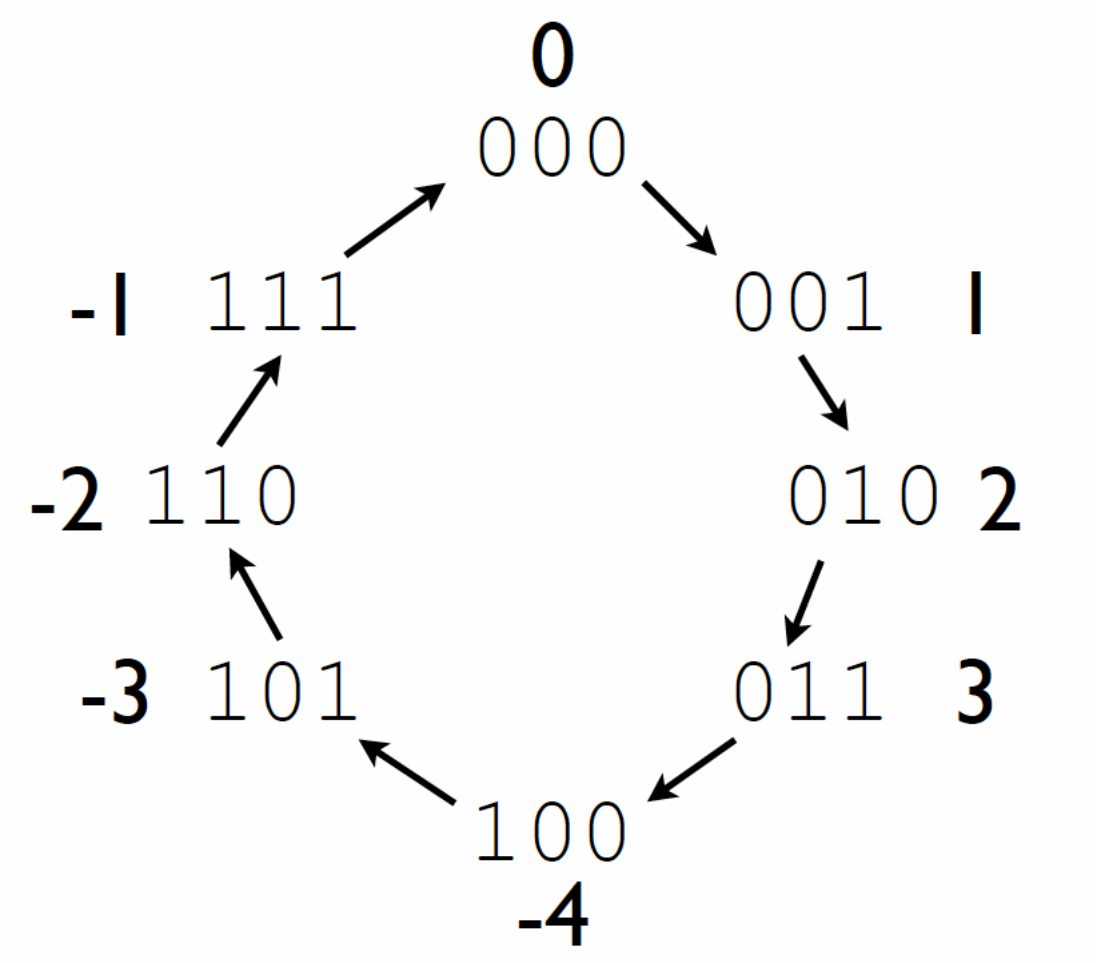
So, $-6_{(10)} = 1010_{(2)}$ according to this rule

Let's do it Backwards… By doing it THE SAME EXACT WAY!

- 2s-Complement to Decimal method **is the same!**

- Take **1010** from our previous example
- Negate it and it becomes **0101**
- Now add 1 to it & it becomes **0110**, which is $6_{(10)}$

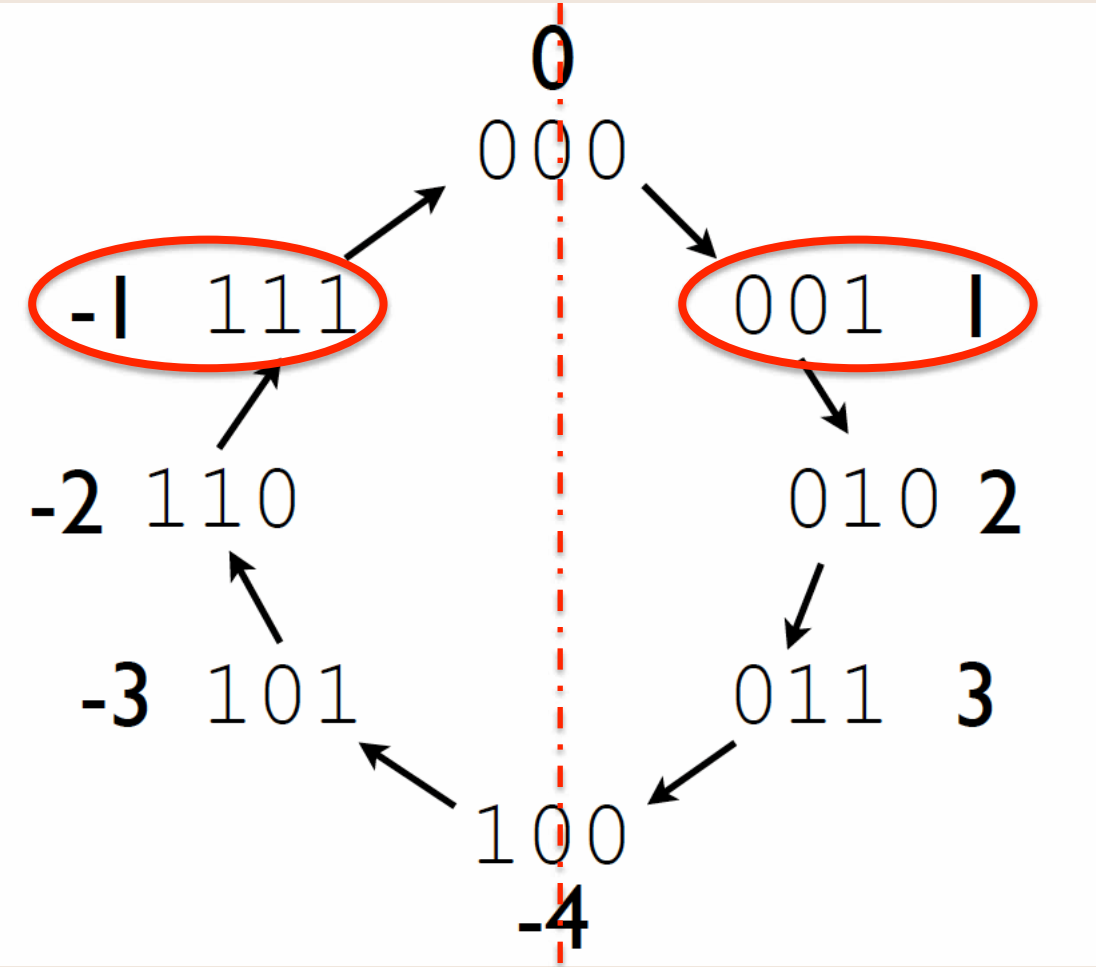
Another View of 2s Complement



NOTE:

In Two's Complement, if the number's MSB is "1", then that means it's a negative number and if it's "0" then the number is positive.

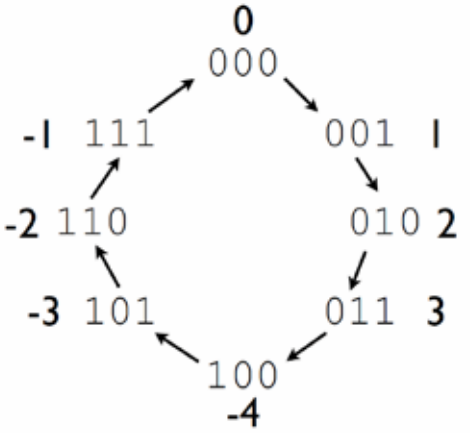
Another View of 2s Complement



NOTE:
Opposite numbers show up as symmetrically opposite each other in the circle.

VERY IMPORTANT:
When we talk of 2s complement, we must also mention the number of bits involved

Ranges



- The *range* represented by number of bits differs between positive and negative binary numbers

- Given **N** bits, the range represented is:

0 to **$+2^N - 1$** *for positive numbers*

and **-2^{N-1}** to **$+2^{N-1} - 1$**

for 2's Complement negative numbers

Addition

- We have an elementary notion of adding single digits, along with an idea of carrying digits
  - Example: when adding 3 to 9, we put forward 2 and carry the 1 (i.e. to mean 12)

- We can build on this notion to add numbers together that are more than one digit long

- Example:

```
    1 1   <-- carried digits
    1 2 3
+   3 8 9
-------
    5 1 2
```

Addition in Binary

- Same mathematical principal applies

Q: What's being assumed here???

A: That these are purely positive numbers

```
carry carry carry carry
  1     1     1     1
    0     0     1     1        3
  + 1     1     0     1      + 13
-------------------------    ----
  1     0     0     0     0    16
```

***Theoretically**, I can add any binary no. with N1 digits to any other binary no. with N2 digits.*

***Practically**, a CPU must have a defined no. of digits that it's working with.*

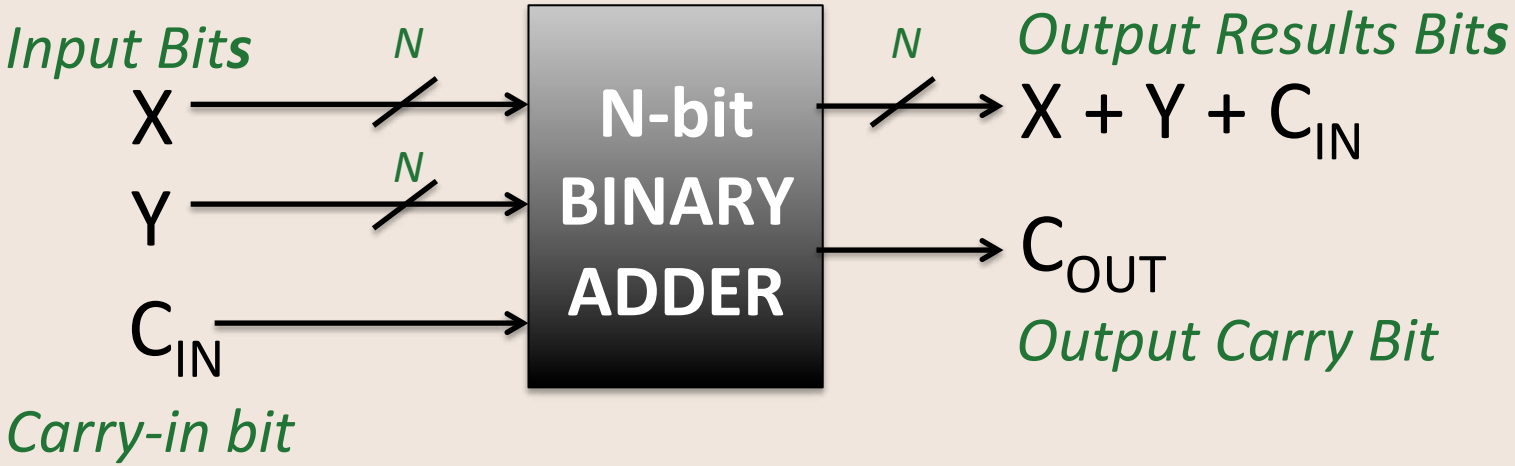
WHY???

Exercises

Implementing an 8-bit adder:

- What is (0x52) + (0x4B) ?
  - Ans: 0x9D, output carry bit = 0
- What is (0xCA) + (0x67)?
  - Ans: 0x31, output carry bit = 1

Black Box Perspective of ANY N-Bit Binary Adder



This is a useful perspective for either writing
an N-bit adder function in code,
or for designing the actual digital circuit that does this!

Output Carry Bit Significance

- For unsigned (i.e. positive) numbers, C_{OUT} = 1 means that the result **did not fit into the number of bits allotted**

- Could be used as an error condition for software
  - For example, **you've designed a 16-bit adder** and during some calculation of positive numbers, your carry bit/flag goes to "1". Conclusion?
  - Your result is ***outside the maximum range allowed by 16 bits.***

Carry vs. Overflow

- The **carry** bit/flag works for – and is looked at – only for ***unsigned (positive)*** numbers

- A similar bit/flag works is looked at for if ***signed*** (two's complement) numbers are used in the addition: the **overflow** bit

Overflow: for Negative Number Addition

- What about if I'm adding two ***negative*** numbers? Like: 1001 + 1011?
  - Then, I get: 0100 with the extra bit set at 1
  - Sanity Check: That's adding (-7) + (-5), so I expected -12, so what's wrong here?
  - The answer is beyond the capability of 4 bits in 2's complement!!!

- The extra bit in this case is called **overflow** and it indicates that the addition of negative numbers has resulted in a number that's

***beyond the range of the given bits*.**

How Do We Determine if Overflow Has Occurred?

- When adding 2 *signed* numbers: ***x + y = s***

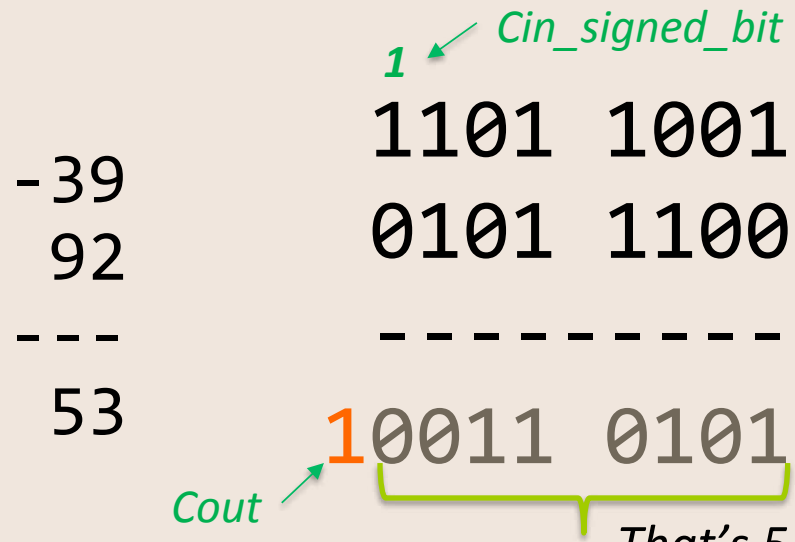
if **x, y > 0** AND **s < 0**

OR if **x, y < 0** AND **s > 0**

Then, overflow has occurred

Example 1

Add: -39 and 92 in *signed* 8-bit binary



Side-note:
What is the range of signed numbers w/ 8 bits?

-2^7 to $(2^7 - 1)$, or
-128 to 127

There's a carry-out (we don't care)
But there is no overflow (V)
Note that V = 0, while Cout = 1 and Cin_signed_bit = 1

Example 2

V = Cout⊕Cin_signed_bit

Add: 104 and 45 in *signed* 8-bit binary

Cin_signed_bit

```
  1
 104      0110 1000
  45      0010 1101
 ---      ---------
 149      1001 0101
```

Cout = 0

That's NOT 149 in signed 8-bits!

Side-note (again):
What is the range of signed numbers w/ 8 bits?

-2^7 to $(2^7 - 1)$, or
-128 to 127

There's no carry-out (again, we don't care)

But there **is** overflow!

Given that this binary result is not 149, but actually **–107** !

Note that V = 1, while Cout = 0 and Cin_signed_bit = 1

YOUR TO-DOs

- Assignment #2
  - Due on Friday!!!
- Next week: **Assembly Language!**
  - Do your readings!!

</LECTURE>