Simplifying Digital Logic Functions
Introduction to Combinatorial Logic

CS 64: Computer Organization and Design Logic
Lecture #13

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Administrative

- Lab# 6 is due tomorrow
- Remaining on the calendar… ***This supersedes anything on the syllabus***

DATE	TOPIC	ASSIGNMENTS
Thu. 3/1	Simplifying Digital Logic Functions	Lab 6 (due Fri. 3/2)
Tue. 3/6	Combinatorial Logic	
Thu. 3/8	Sequential Logic	Lab 7 (due Fri. 3/9)
Tue. 3/13	Finite State Machines	
Thu. 3/15	Ethics	Labs 8 and 9 (due Fri. 3/16)

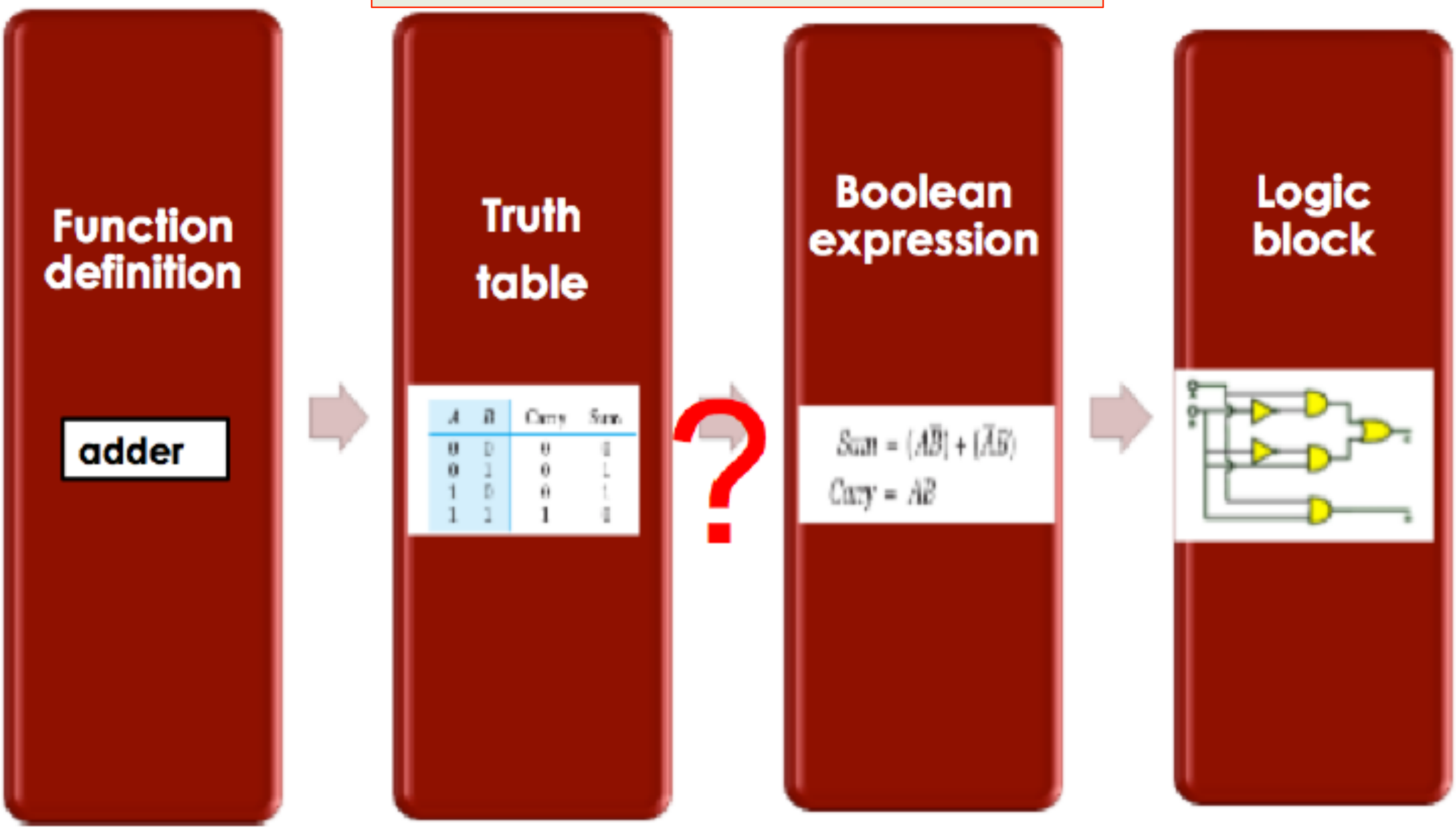
Lecture Outline

- Logic Functions and their Simplifications:

    Truth Table Use *vs.* **Karnaugh Maps**

Digital Circuit Design Process

CAN THIS PROCESS BE REVERSED?



Boolean Logic Laws

Circuit Equivalence - each law has 2 forms that are duals of each other.

Name	AND form	OR form
Identity law	$1A = A$	$0 + A = A$
Null law	$0A = 0$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse law	$A\overline{A} = 0$	$A + \overline{A} = 1$
Commutative law	$AB = BA$	$A + B = B + A$
Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption law	$A(A + B) = A$	$A + AB = A$
De Morgan's law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\,\overline{B}$

More Simplification Examples

Simplify the Boolean expression:

- **(A+B+C)(D+E)' + (A+B+C)(D+E)**

Simplify the Boolean expression and write it out on a truth table as proof

- **XZ + Z(X'+ XY)**

Use DeMorgan’s Theorm to re-write the expression below using at least one OR operation

- **NOT(X + YZ)**

Scaling Up Simplification

- When we get to *more* than 3 variables, it becomes challenging to use truth tables
- We can instead use ***Karnaugh Maps*** to make it immediately apparent as to what can be simplified

Example of a K-Map

	A	B	f(A,B)
0	0	0	a
1	0	1	b
2	1	0	c
3	1	1	d

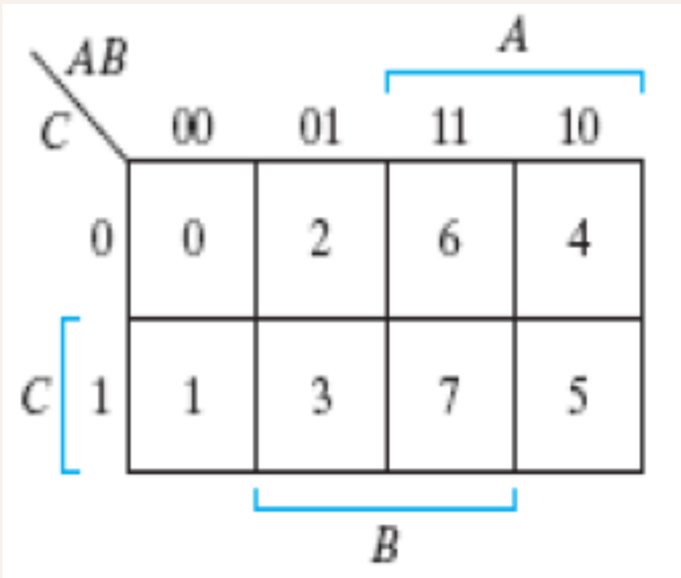
B \ A	0	1
0	a	c
1	b	d

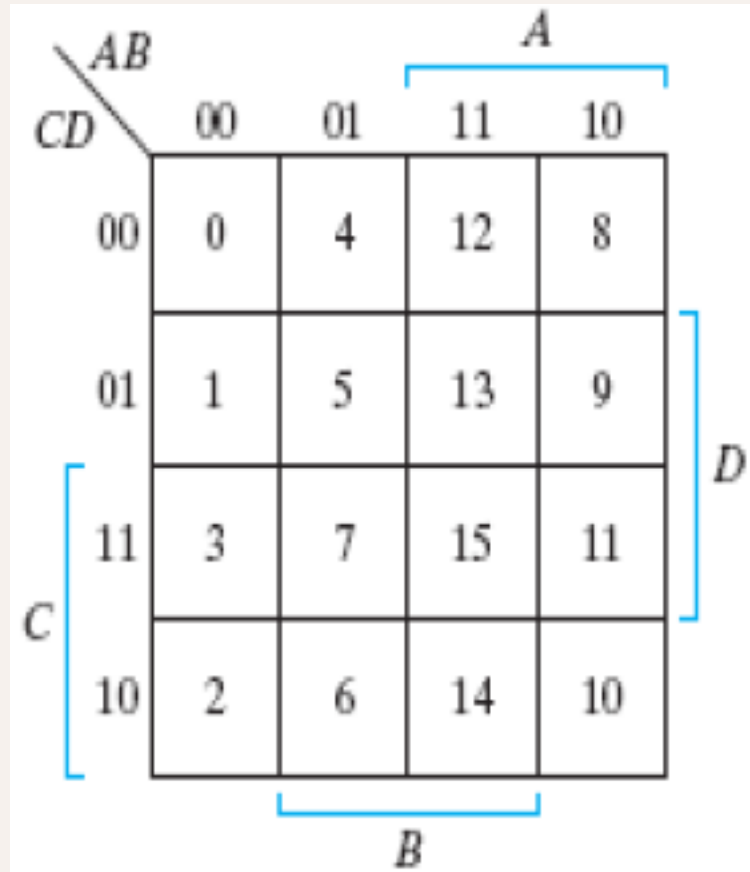
B \ A	0	1
0	0	2
1	1	3

A	B	f(A,B)
0	0	0
0	1	1
1	0	1
1	1	1

B \ A	0	1
0	0	1
1	1	1

K-Maps with 3 or 4 Variables





Note the adjacent placement of:

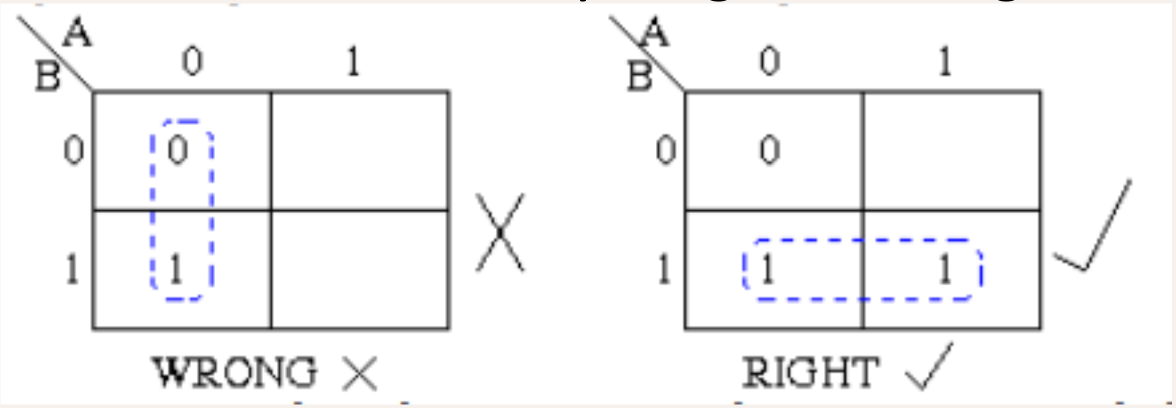
00 01 11 10

It's NOT:

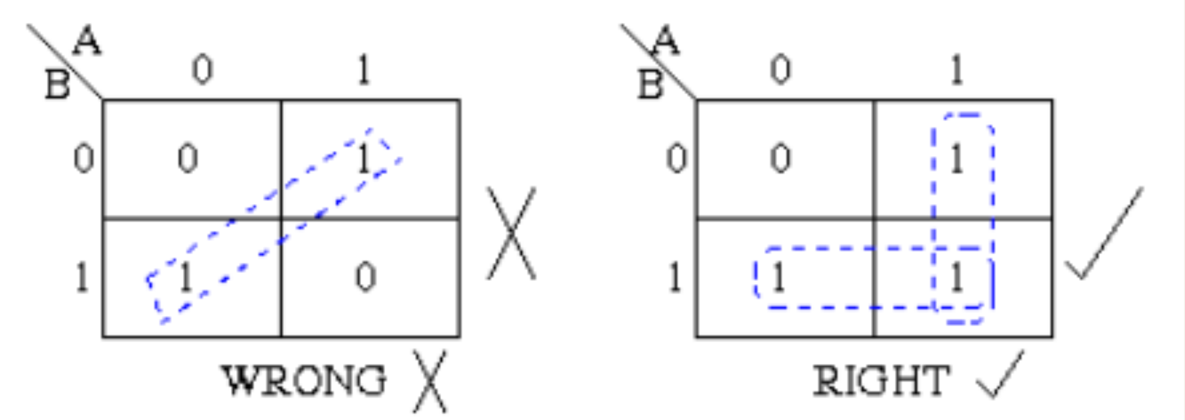
00 01 10 11

Rules for Using K-Maps for Simplification

1. Group together **adjacent cells** containing “1”
2. Groups should **not include** anything containing “0”

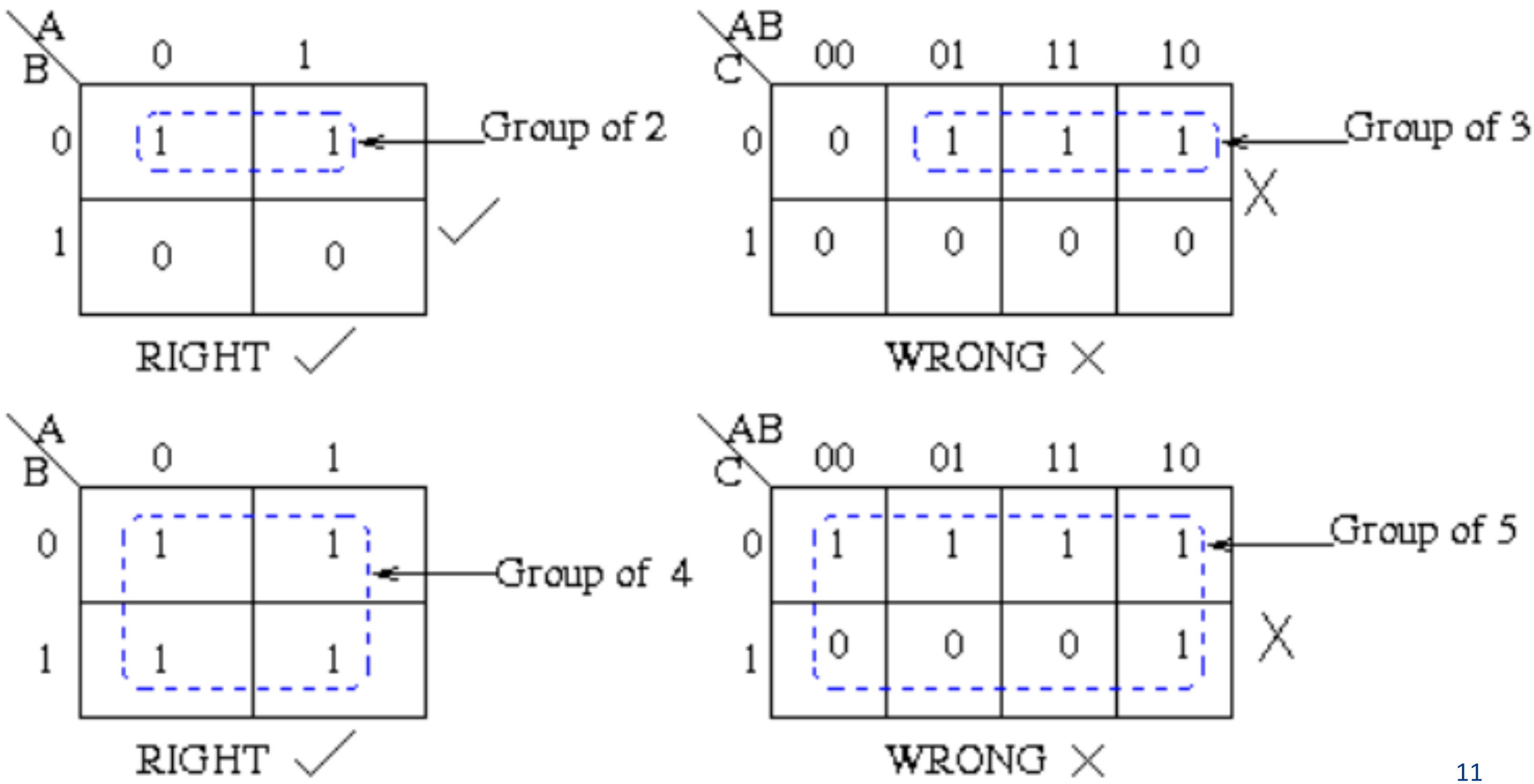


3. Groups may be horizontal or vertical, but **not diagonal**



Rules for Using K-Maps for Simplification

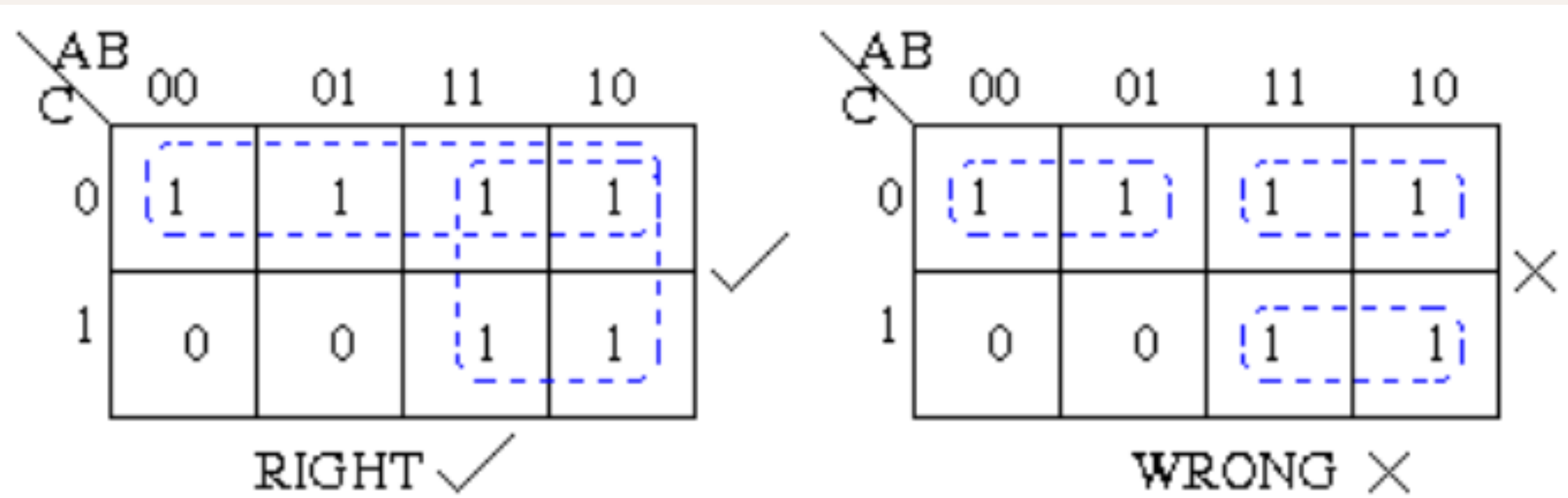
4. Groups must contain 1, 2, 4, 8, or in general 2^n cells.



Rules for Using K-Maps for Simplification

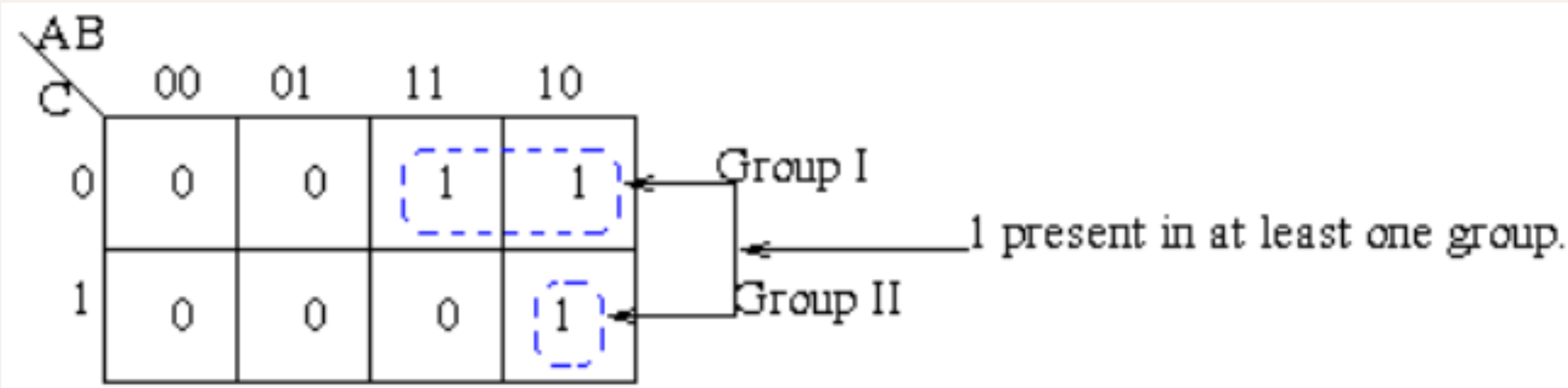
5. **Each group must be as large as possible**

   (Otherwise we're not being as minimal as we can be,
   even though we're not breaking any Boolean rules)



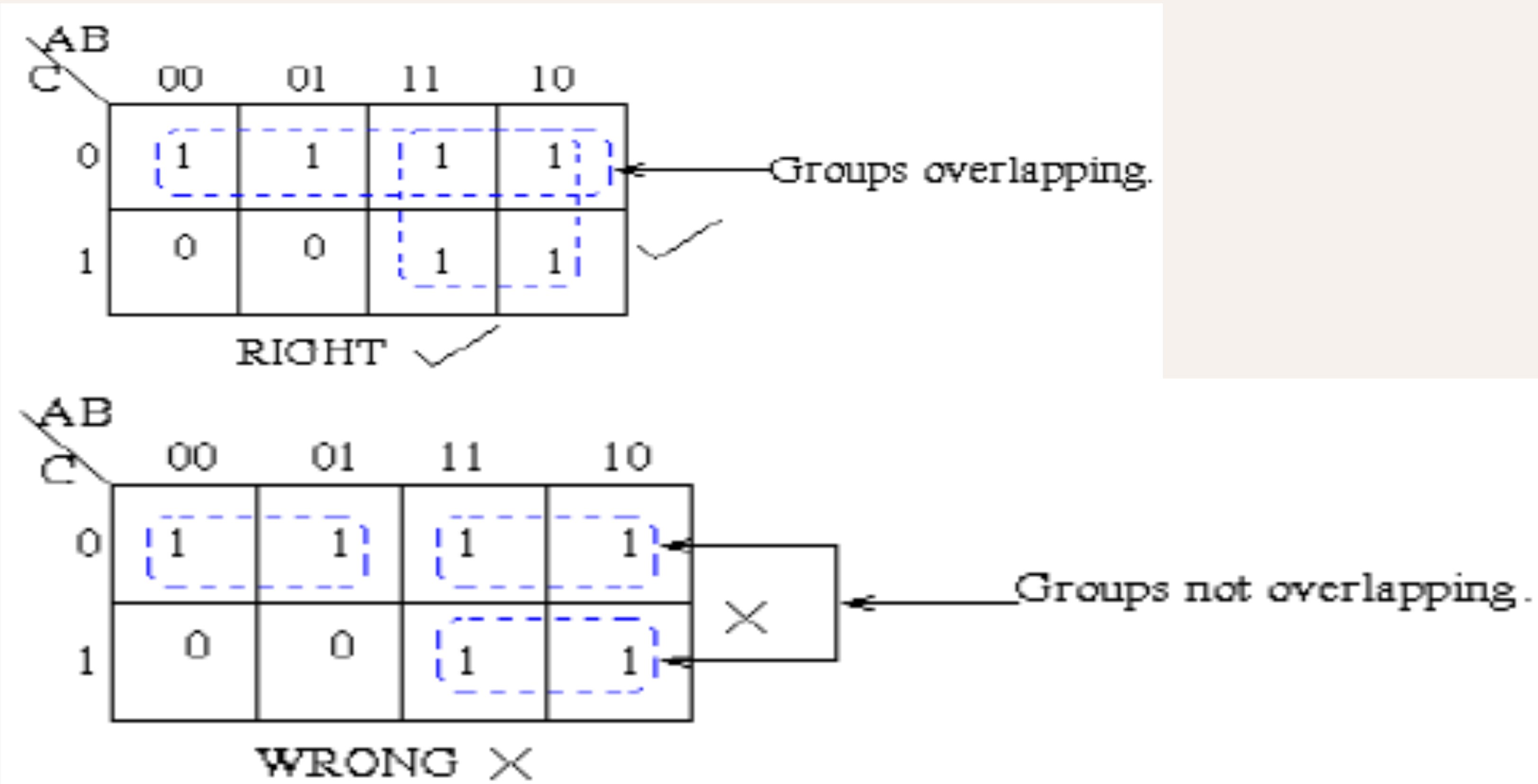
Rules for Using K-Maps for Simplification

6. **Each cell containing a “1” must be at least in one group**



Rules for Using K-Maps for Simplification

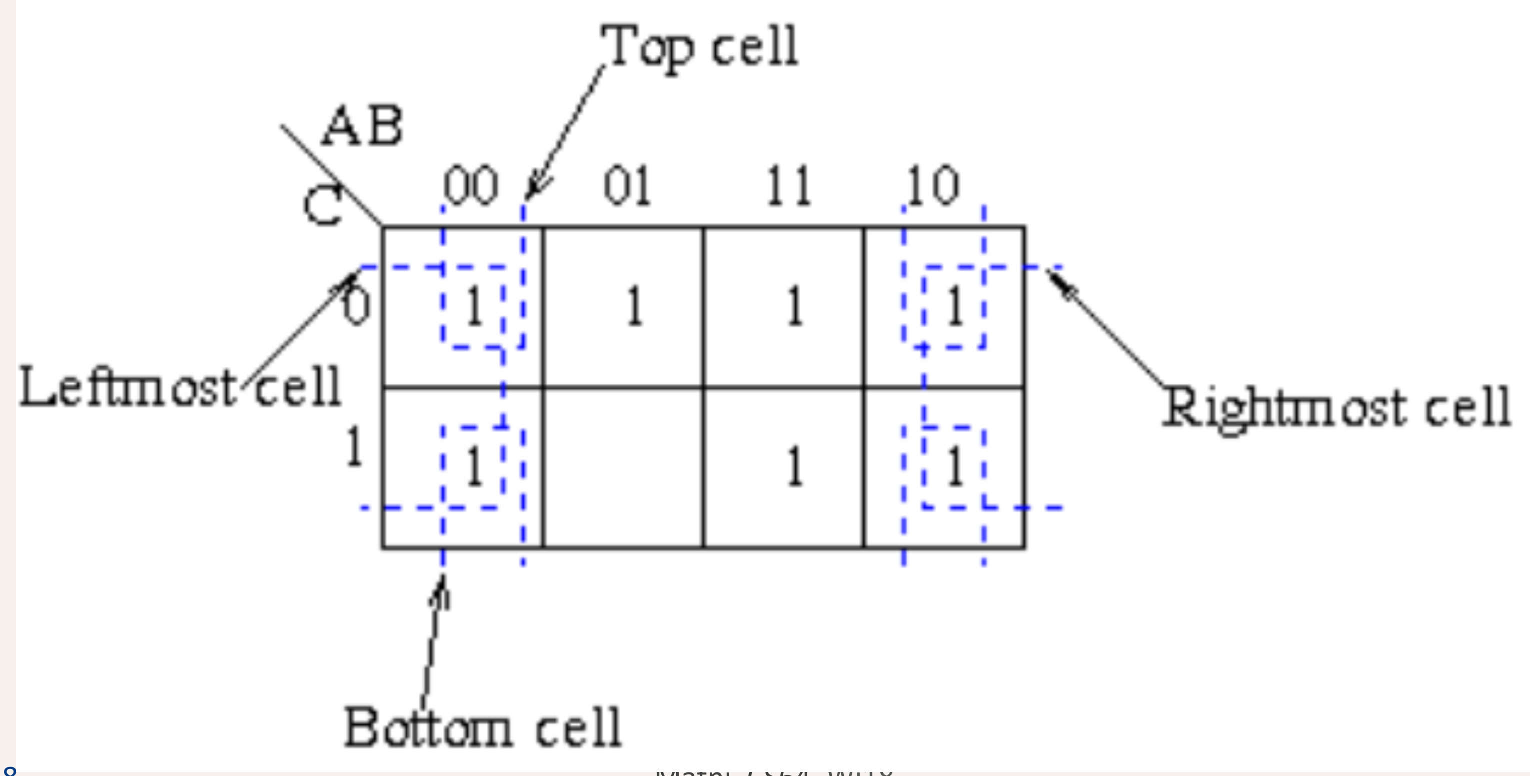
7. Groups may overlap esp. to maximize group size



Rules for Using K-Maps for Simplification

8. Groups may wrap around the table.

The leftmost cell in a row may be grouped with the rightmost cell **and** the top cell in a column may be grouped with the bottom cell.



Example 1
2 vars

F(X,Y)

= XY + Y

= Y (X + 1)

= Y

Y = 1 column

X \ Y	0	1
0		1
1		1

F(X,Y) = Y

Example 2
3 vars

F(X,Y,Z)

= XZ + Z(X'+ XY)

= XZ + ZX’ + ZXY

= Z (X + X’ + XY)

= Z (1 + XY)

= Z

Y = 1   *X = 1*

Z \ XY	00	01	11	10
0				
1	1	1	1	1

F(X,Y,Z) = Z

Example 3
3 vars

Class Ex.

!A!B!C + !A!BC + !ABC + !AB!C + A!B!C + AB!C

C \ AB	00	01	11	10
0	1	1	1	1
1	1	1		

F(X,Y,Z) = !C + !A

Example 4
4 vars

F(A,X,Y,Z)

= AX + Z(X+A'+Y)

= AX + ZX+ ZA'+ ZY

AZ \ XY	00	01	11	10
00				
01	1	1	1	1
11		1	1	1
10			1	1

Y = 1 *X = 1*

Z = 1 *A = 1*

F(A,X,Y,Z) = ZA' + AX + ZY

Example 4
4 vars

Class Ex.

F(A,B,C,D)

= ABCD’ + ABC’D + CD + A’B’ + C’D

B = 1 *A = 1*

CD \ AB	00	01	11	10
00	1			
01	1	1	1	1
11	1	1	1	1
10	1		1	

D = 1

C = 1

F(A,B,C,D) = A’B’ + D + ABC

K-Map Rules Summary

- Groups can contain only 1s
- Only 1s in adjacent groups are allowed (no diagonals)
- The number of 1s in a group must be a power of two (1, 2, 4, 8...)
- Groups must be as large AND as few in no.s as "legally" possible
- All 1s must belong to a group, even if it's a group of one element
- Overlapping groups are permitted
- Wrapping around the map is permitted

Exploiting “Don’t Cares”

- An output variable that’s designated “don’t care” (symbol = X) means that it could be a **0** or a **1** (i.e. we “don’t care” which)
  - That is, it is **unspecified**,
    usually because of invalid inputs

Example of a Don't Care Situation

- Consider coding all decimal digits (say, for a digital clock app):
  - 0 thru 9 --- requires how many bits?
    - 4 bits
  - But! 4 bits convey more numbers than that!
    - Don't forget A thru F!

- Not all binary values map to decimal

Example Continued…

Binary	Decimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7

Binary	Decimal
1000	8
1001	9
1010	X
1011	X
1100	X
1101	X
1110	X
1111	X

Don't Care: So What?

- Recall that in a K-map, we can only group 1s

- Because the value of a don't care is irrelevant, we can treat it as a 1 ***if it is convenient to do so*** (or a 0 if that would be more convenient)

Example

- A circuit that calculates if the 4-bit binary coded single digit decimal **input % 2 == 0**

- So, although 4-bits will give me numbers from 0 to 15, I don't care about the ones that yield 10 to 15.

I3	I2	I1	I0	R
0	0	0	0	**1**
0	0	0	1	**0**
0	0	1	0	**1**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**0**
0	1	1	0	**1**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**0**
1	0	1	0	**X**
1	0	1	1	**X**
1	1	0	0	**X**
1	1	0	1	**X**
1	1	1	0	**X**
1	1	1	1	**X**

Example as a K-Map

I_3I_2 \ I_1I_0	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

If We Don’t Exploit “Don’t Cares”

$$R = !I_1!I_0!I_3 + I_1I_0!I_3 + !I_0!I_1!I_2$$

$I_3I_2 \backslash I_1I_0$	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

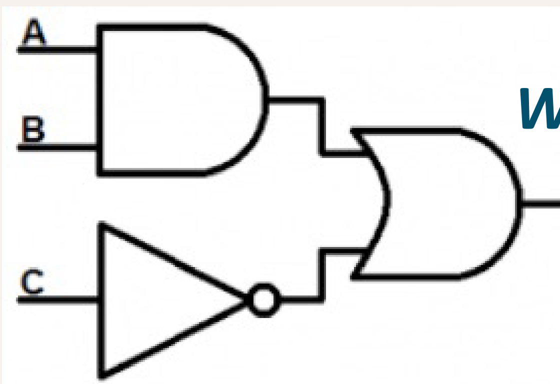
If We **DO** Exploit "Don't Cares"

$R = !I_1!I_0 + I_1I_0$

I_3I_2 \ I_1I_0	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

Combinatorial Logic Designs

- When you *combine* multiple logic blocks together to form a more complex logic function/circuit



What is the output?

$A.B + \overline{C}$

What is its truth table?

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

What is its K-Map?

C \ AB	00	01	11	10
0	1	1	1	1
1			1	

Exercise 1

- Given the following truth table, draw the resulting logic circuit

  – **STEP 1**: Draw the K-Map and simplify the function

  – **STEP 2**: Construct the circuit from the now simplified function

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

Exercise 1 – Step 1

Get the simplified function

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

B = 1 *A = 1*

D = 1 *C = 1*

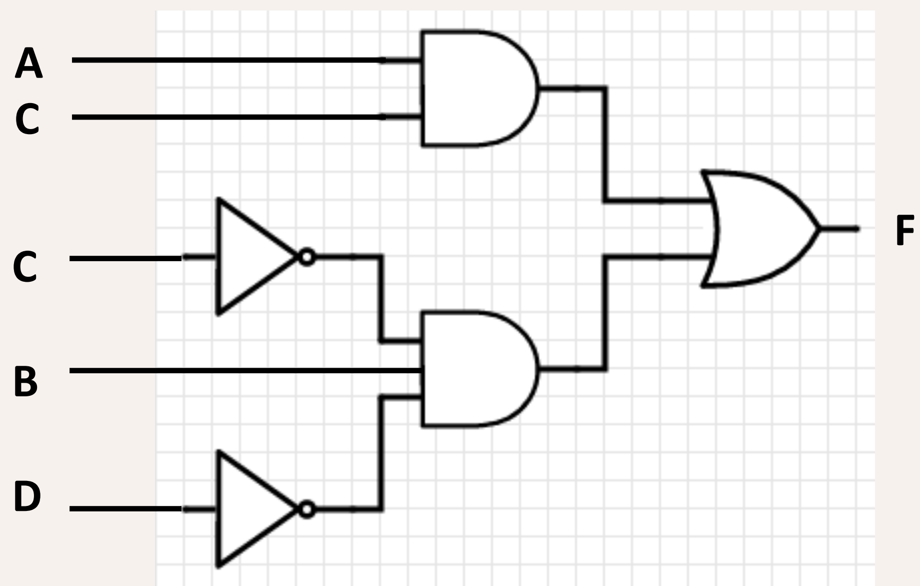
CD \ AB	00	01	11	10
00		1	1	
01				
11			1	1
10			1	1

F(A,B,C) = B.C’.D’ + A.C

Exercise 1 – Step 2

Draw the logic circuit diagram

F(A,B,C) = B.C’.D’ + A.C



Exercise 2

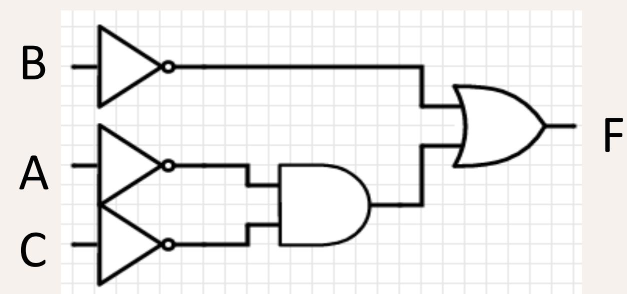
Class Ex.

- Given the following truth table, draw the resulting logic circuit

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

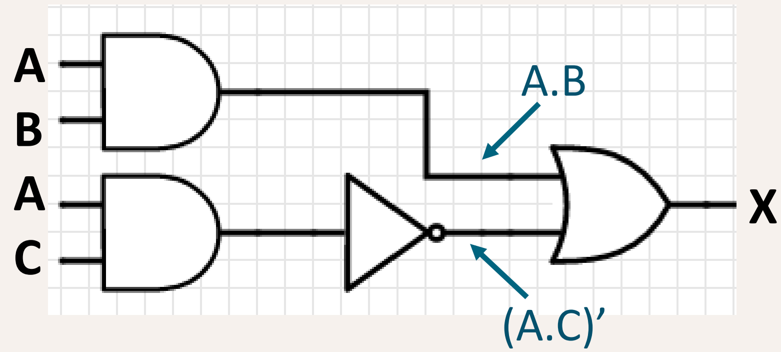
C \ AB	00	01	11	10
0	1	1		1
1	1			1

$F(A,B,C) = B' + A'.C'$



Exercise 3

- Given the following schematic of a circuit, (a) write the function and (b) fill out the truth table:



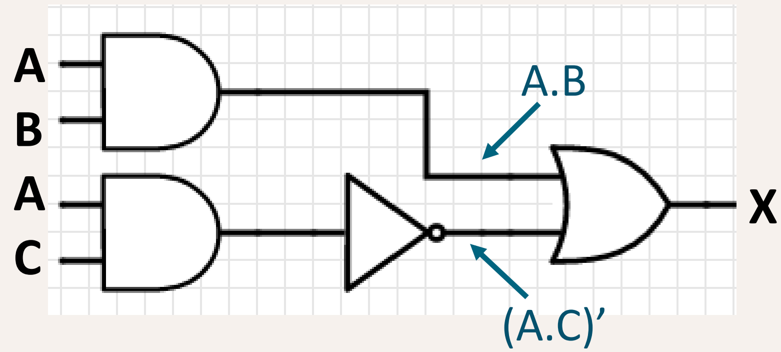
A	B	C	X
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

X = A.B + (A.C)'

(note that also means: **X = A.B + A' + C'**)

Exercise 3

- Given the following schematic of a circuit, (a) write the function and (b) fill out the truth table:



A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

X = A.B + (A.C)'

(note that also means: **X = A.B + A' + C'**)

Your To Dos

- Lab #6 is due end of day Friday

</LECTURE>