PSTAT 115 - Section One

Winter 2023

120AB Review

Definition of a Random Variable

Definition (Random Variable)**.** A random variable X represents an ___________ from a *random experiment*. Let Ω represent the collection of _____ outcomes from an experiment. Then, $X : \Omega \to \mathbb{R}$ (is a _____ from sample space to _____)

Examples.

- Experiment: number of heads after tossing a fair coin twice. $\Omega =$ ________________ and $X = 1 \Rightarrow$ ________________.
- Experiment: Time it takes for a buss to arrive to bus stop. $\Omega = [0, \infty)$ and $X = 5$ mins means ___________________________

Notation. Random variables are always represented with ________________ letters (i.e. $X, Y \ldots$). Their observed values are represented with ___________________________ (i.e. $x, y, \ldots$).

Population vs Sample

Definition (Population)**.** A _____________________ represents our <u>entire</u> set of interest from which we wish to draw _____________________ on!

Definition. Our _____________________ is the data points from our population of interest. A sequence (or sample) is given by $(X_1, \ldots, X_n)$.

Examples.

(i) Research Question: Most popular 2020 song. Population: ________________; Sample $(X_1, \ldots, X_n)$: ________________.

(ii) Research Question: Average height of college male. Population: ________________; Sample $(X_1, \ldots, X_n)$: ________________

Estimator vs Estimate

Definition (Statistic)**.** Given a sequence of random variables $(X_1, X_2, \ldots, X_n)$ a **statistic** is any function h of those random variables.

Special Case: an ________________ is a statistic used is <u>used</u> to *estimate* a parameter from ________________ of the RVs.

Note. <u>Estimators</u> are <u>RVs</u>..

Example. Suppose we have a sample $(X_1, X_2, \ldots X_n)$ from an unknown distribution $\sim N($_____$, 9)$. Then, an <u>estimator</u> for μ equals ___________.

Definition. The values that the estimator can take is the _____________________. For example, $h(X) = \frac{1}{n}\sum_{i=1}^{n} X_i$ then ________________.

Likelihood Function

Problem. Let $X_1, \dots, X_n \sim N(\mu, 1)$. The dataset obtained is $x_1, \dots, x_n$. Compute the likelihood function, log-likelihood and proportional simplification.

Notes

Problem. Let $X_1, X_2, \dots, X_n \sim N(\mu, \sigma^2)$. The dataset obtained is $x_1, x_2, \dots, x_n$.

(i) Find the MLE of μ, σ^2.

(ii) Find the MLE of μ^3.

Notes