Lecture 1: Introduction

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Expectations and prerequisites

- Comfortable with fundamentals of likelihood-based inference
- Familiarity with (generalized) linear models (220A)
- Comfortable with R programming

Class Resources

Course Pages

- Class website on canvas

Required Textbook

- Bayesian Data Analysis (Third Edition), available for free as pdf

Grades

- 50% - weekly homework
- 20% - Final project (More info to come)
- 30% - Final Exam

Homework

- There will be 5 homeworks (50% of your grade total)
- You will typically have 1-2 weeks to complete the homeworks
  - In general: one theory problem, one computational problem and one applied problem
- Homework turned in within 24 hrs after the deadline without prior approval will receive a 10 pt deduction (out of 100)
- Homework will not be accepted more than 24 hrs late.

Homework submission format

- All code must be reproducible as a Quarto document
- All derivations can be done in any format of your choosing (latex, written by hand) but must be legible and _must be integrated into your Quarto document
- All files must be zipped together and submitted to Gradescope

Software and Deliverables

Software

- R studio. Latest R studio version needed for Quarto.

Homeworks submission format

- Electronic submission via Gauchospace
- Must turn in a pdf and the Quarto document used to generate it
- Any supplementary files

Class topics

- Conjugate priors for EF models (e.g binomial, poisson, and multivariate normal)
- Hierarchical modeling
- Monte Carlo Methods
- Basics of Decision Theory
- Advanced topics as time permits

RStudio Cloud Service

- Log on to pstat215.lsit.ucsb.edu
  - Cloud based rstudio service
  - Log in with your UCSB NetID
- Use https://tinyurl.com/pstat215a to sync new class material to your account.
- Make sure you can write and compile a Quarto document (`qmd`)

R resources

- https://r4ds.had.co.nz/
- Cheatsheets: https://www.rstudio.com/resources/cheatsheets/
- Most relevant cheat sheets are uploaded, in `resources` folder

Artificial intelligence

- LLMs (ChatGPT etc) are allowed BUT…
- The less you know the more likely you are to be convinced by misinformation
- It’s not about getting the right answer
  - “It’s the journey, not the destination”
- Ask yourself “Am I using it to avoid work? Or am I using it to help me develop an understanding?”

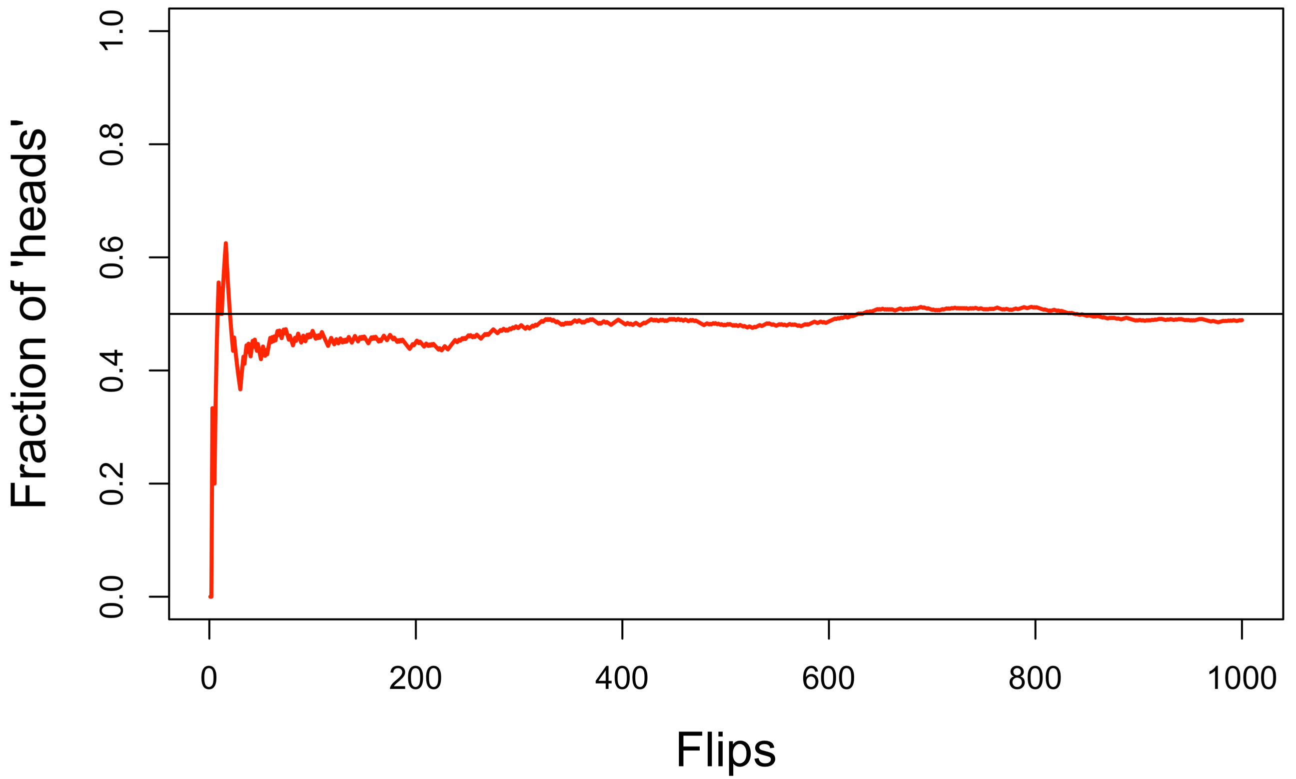
Bayesian Statistics

Frequentist statistics

- Associated with the *frequentist* interpretation of probability
  - For any given event, only one of two possibilities may hold: it occurs or it does not.
  - The *frequency* of an event (in repeated experiments) is the *probability* of the event
  - Focus on finding estimators with well established properties (consistent, unbiased, low variance, coverage etc)
  - Premised on imaginary resampling of data
- Example: Null Hypothesis Significant Testing (NHST)
  - If the null model is true, and I re-run the experiment many times, how often will I reject?

Frequentist probability

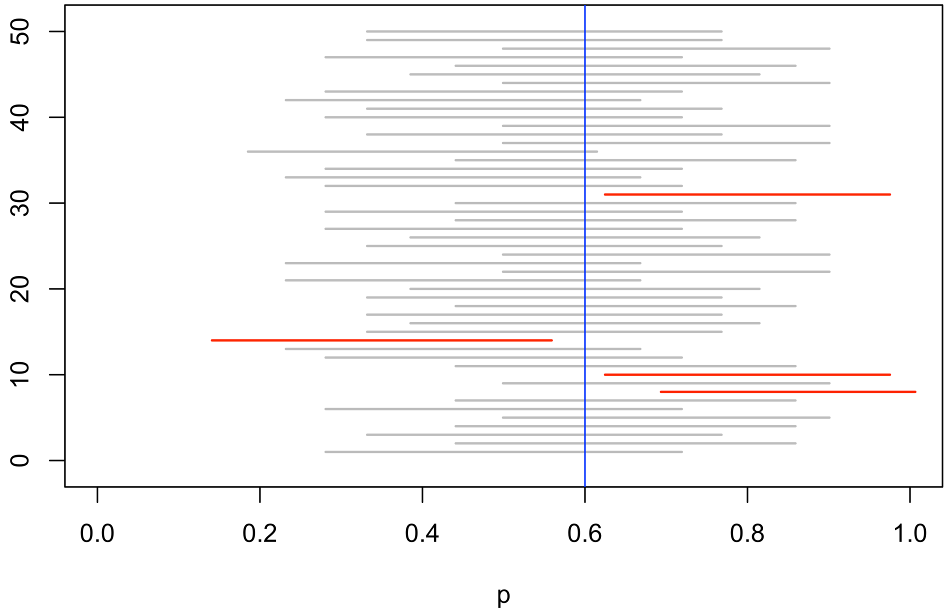
The probability of a coin landing on heads is 50%



The long run fraction of heads is 50%

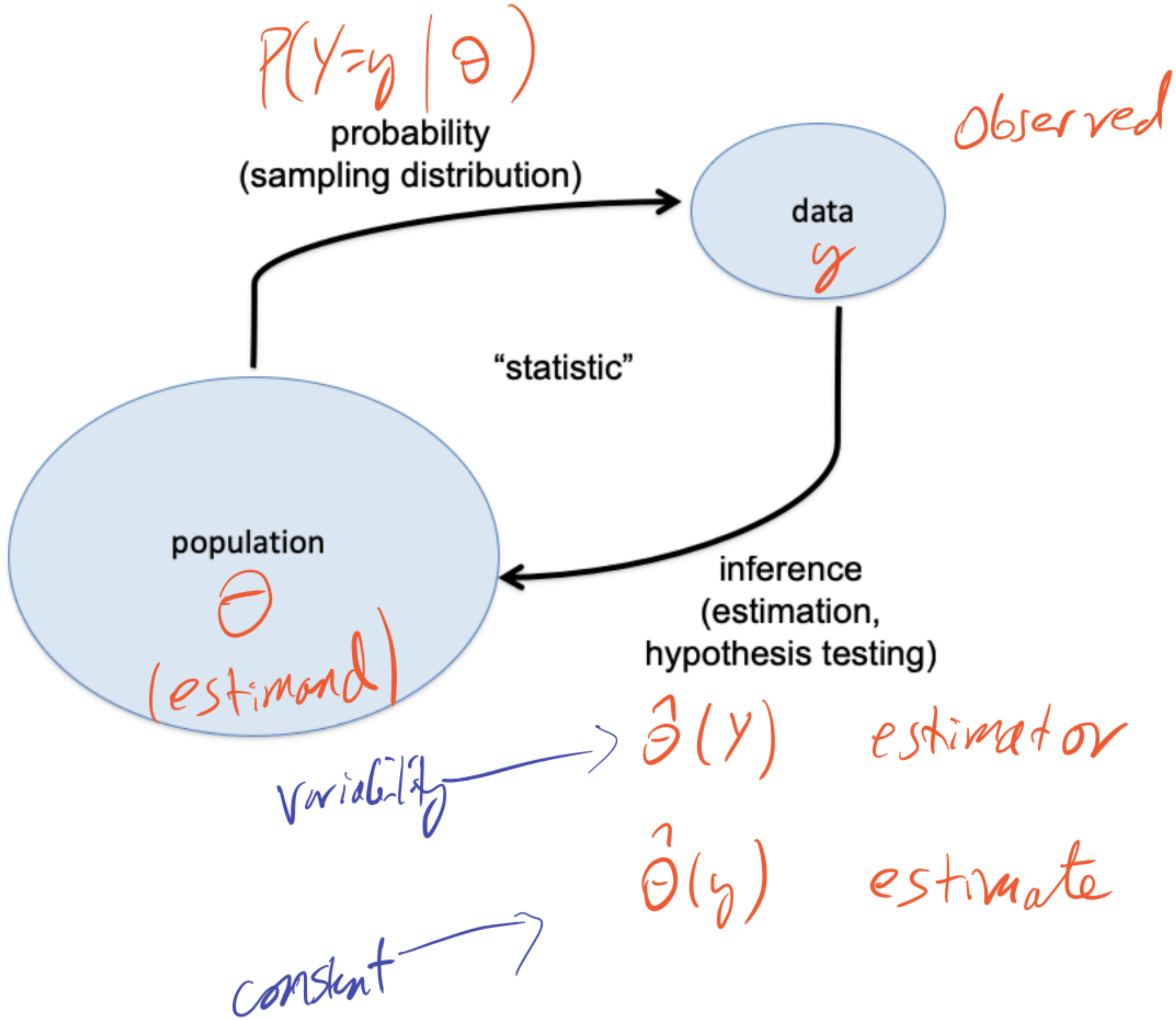
Confidence intervals

I have a 95% confidence interval for a parameter θ. What does this mean?



We expect $0.05 \times 50 = 2.5$ of the intervals to *not* cover the true parameter, $p = 0.6$, on average

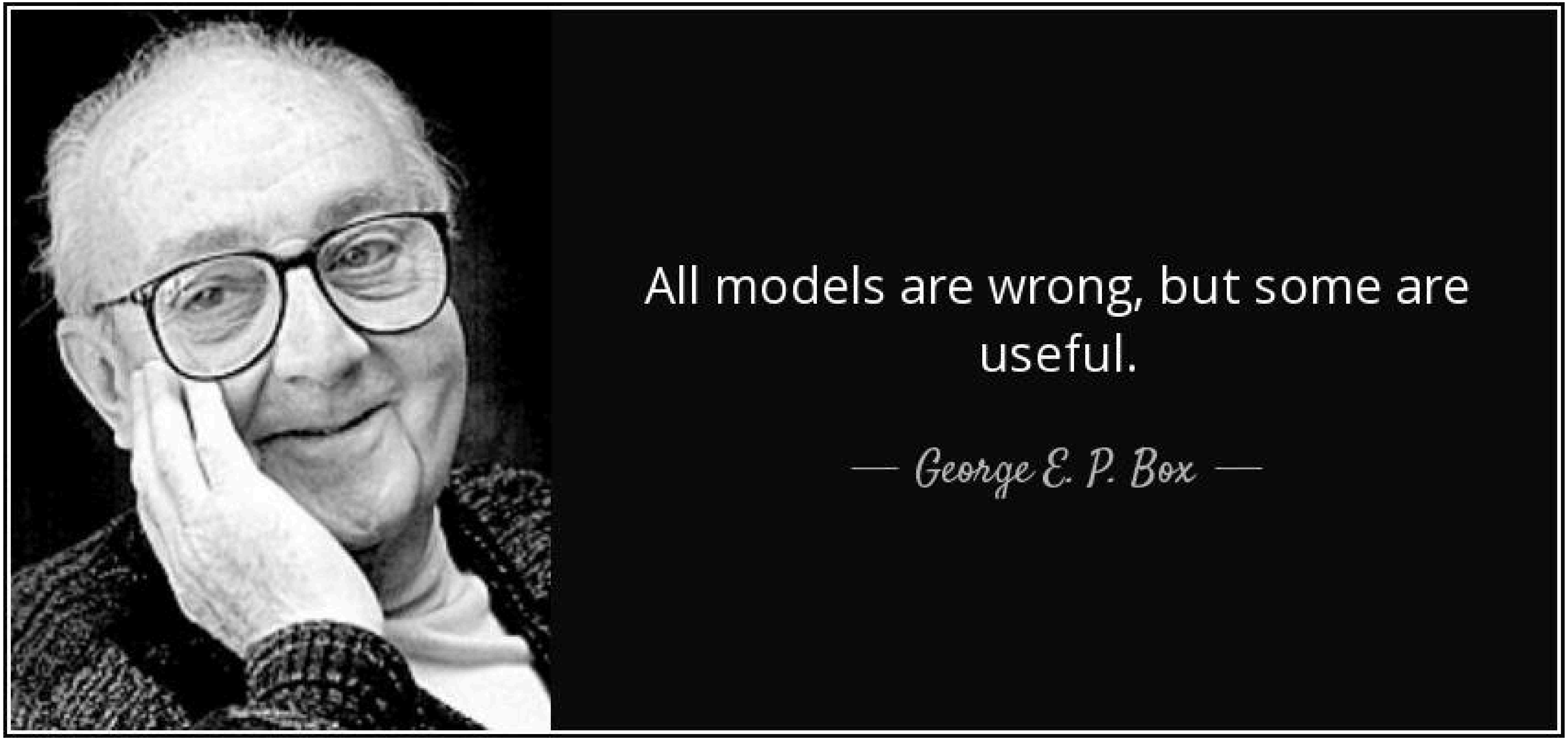
Frequentist Statistics



This class

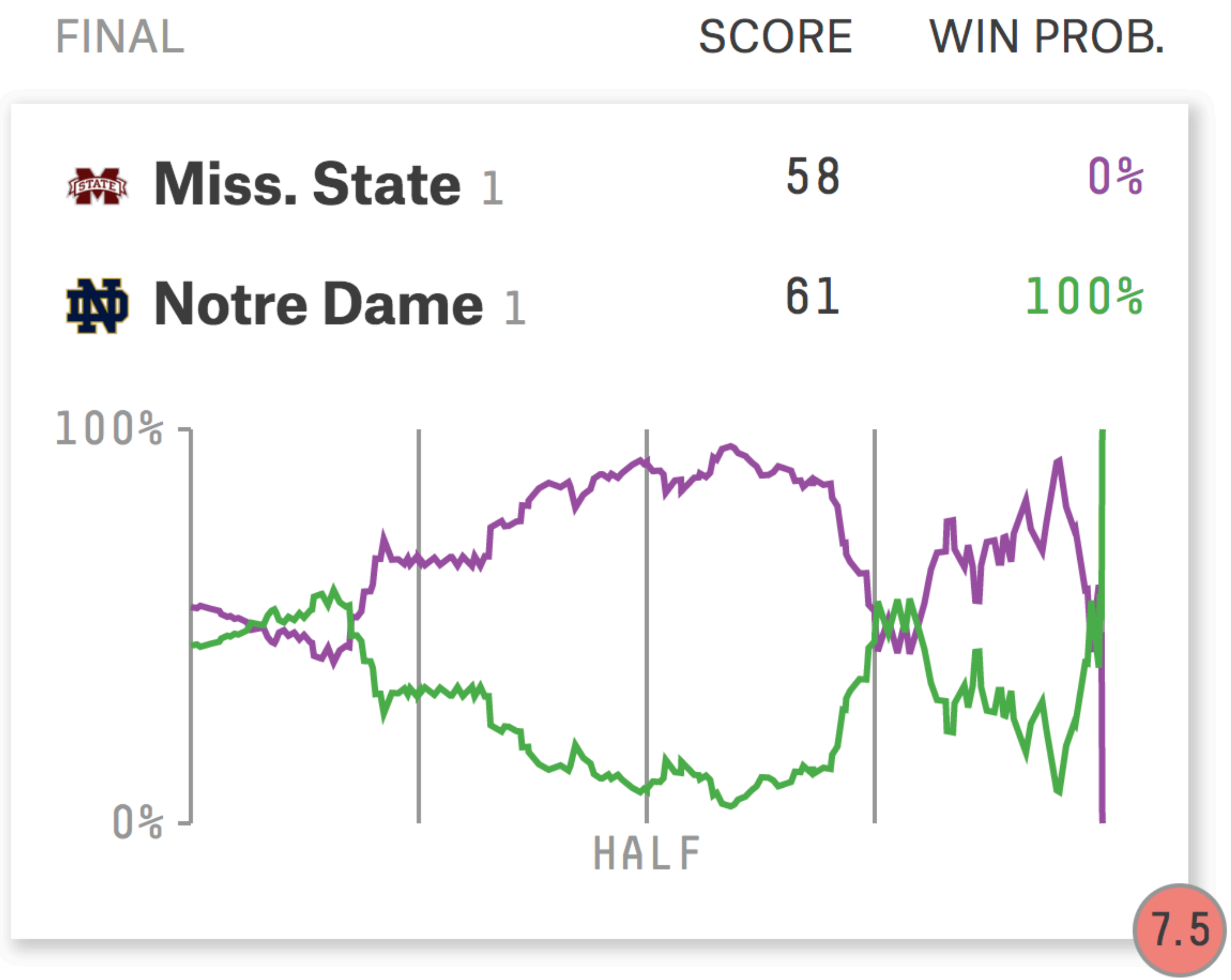
- Build statistical models representing a set of assumption about how the data was generated.
- Use models to develop statistical tests, predictions and forecasts
- Can (and should) be continuously refined and extended!
- Incorporate prior knowledge and condition on what we observe
- Can still consider frequentist properties of estimators derived from Bayesian principles

All models are wrong



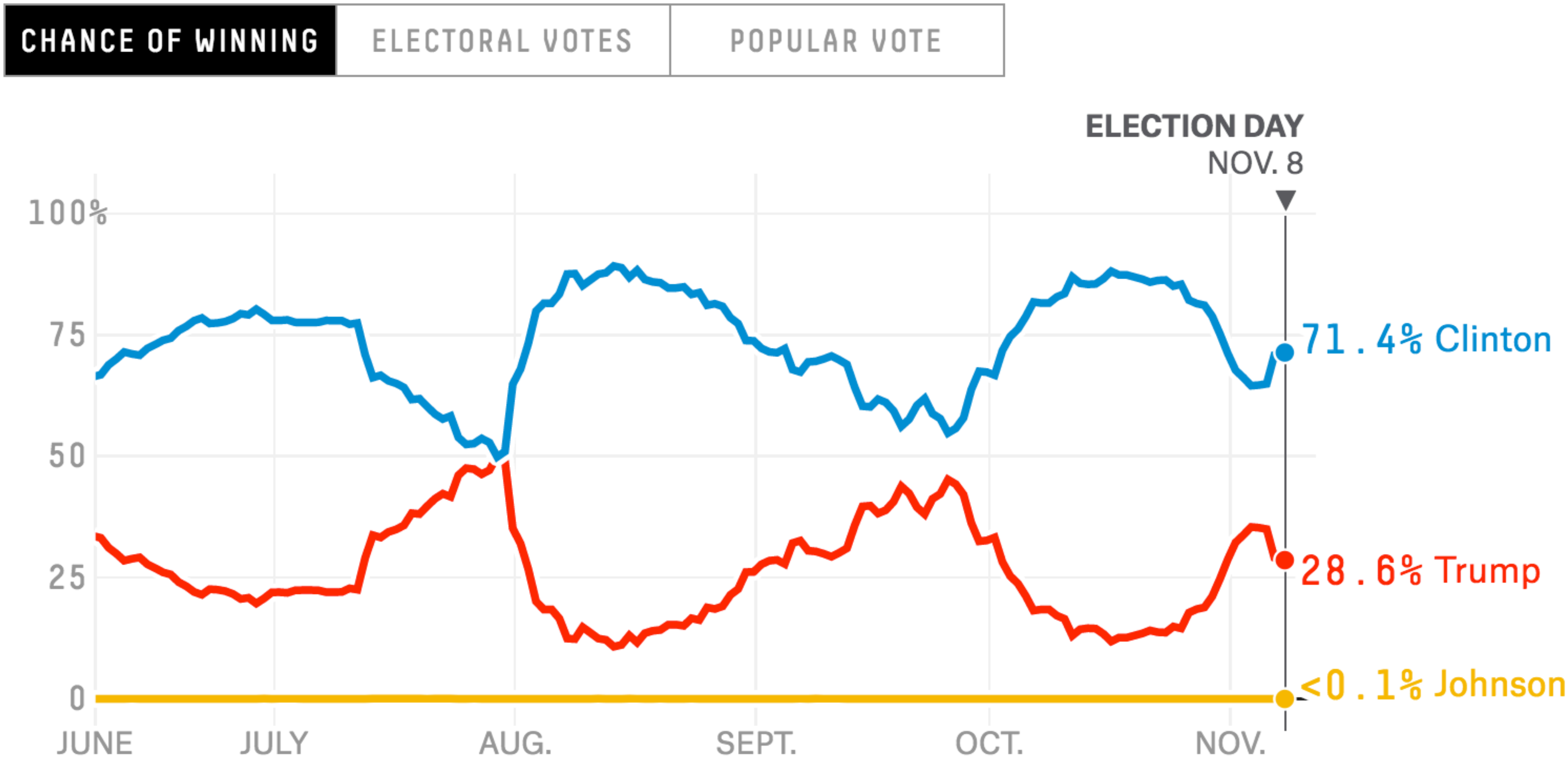
All models are wrong (wikipedia)

Win probability



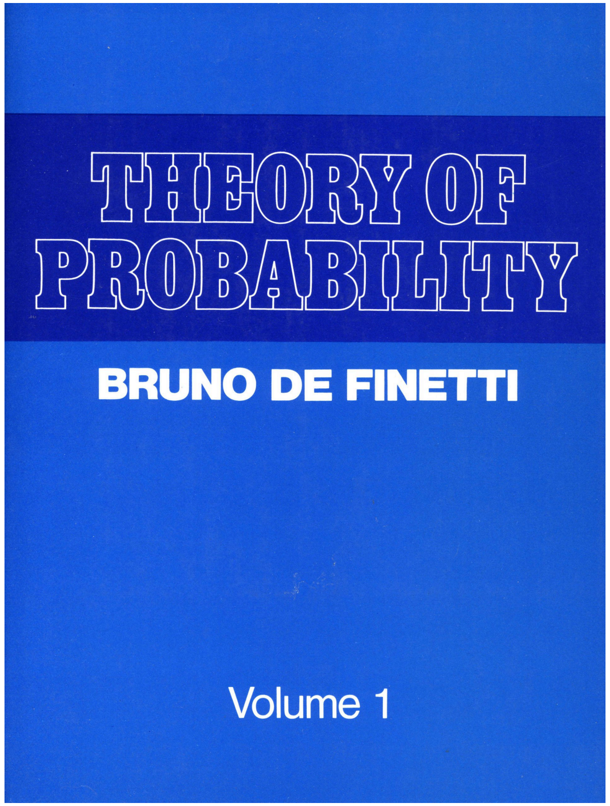
source: fivethirtyeight.com

Win probability



source: fivethirtyeight.com

Bayesian probability



Bruno de Finetti began his book on probability with:
"PROBABILITY DOES NOT EXIST"

Bayesian probability

- de Finetti is implicitly arguing that probability is about *belief*
  - Claim: probability doesn't exist in an *objective* sense
  - "The coin is fair" means *I believe* that its equally likely to be heads or tails.
  - "Hillary Clinton has a 71% chance to win" reflects a belief, since the election happens only once
- Rarely, if ever, get *true* replications to estimate frequentist probabilities
- Bayesian idea: focus statistical practice around belief about parameters

Bayesian probability

> “The terms *certain* and *probable* describe the various degrees of rational belief about a proposition which different amounts of knowledge authorise us to entertain. All propositions are true or false, but the knowledge we have of them depends on our circumstances
>
> — John M Keynes

Why Bayesian statistics?

- Philosophy: quantify degrees of belief rather than reason about counterfactuals
  - Can easily “share information” across related observations
  - Ability to incorporate real prior knowledge
  - Particularly effective with small samples
- A variety of powerful tools for inference with computer simulation
- Can still characterize frequentist properties of Bayesian procedures

Setup

- The *sample space* $\mathcal{Y}$ is the set of all possible datasets.
  - Y is a random variable with support in $\mathcal{Y}$
  - We observe one dataset y from which we hope to learn about the world.
- The *parameter space* Θ is the set of all possible parameter values θ
- θ encodes the population characteristics that we want to learn about!

Three steps of Bayesian data analysis

1. Construct a plausible probability model governed by parameters θ
    - This includes specifying your belief about θ before seeing data (*the prior*)
2. Condition on the observed data and compute *the posterior* distribution for θ
3. Evaluate the model fit, revise and extend. Then repeat.

Bayesian Inference in a Nutshell

1. The *prior distribution* $p(\theta)$ describes our belief about the true population characteristics, for each value of $\theta \in \Theta$.
2. Our *sampling model* $p(y \mid \theta)$ describes our belief about what data we are likely to observe if θ is true.
3. Once we actually observe data, y, we update our beliefs about θ by computing *the posterior distribution* $p(\theta \mid y)$. We do this with Bayes' rule!

Key difference: θ is random!

Bayes' Rule for Bayesian Statistics

$$P(\theta \mid y) = \frac{P(y \mid \theta)P(\theta)}{P(y)}$$

- $P(\theta \mid y)$ is the posterior distribution
- $P(y \mid \theta)$ is the likelihood
- $P(\theta)$ is the prior distribution
- $P(y) = \int_\Theta p(y \mid \tilde{\theta})p(\tilde{\theta})d\tilde{\theta}$ is the model evidence

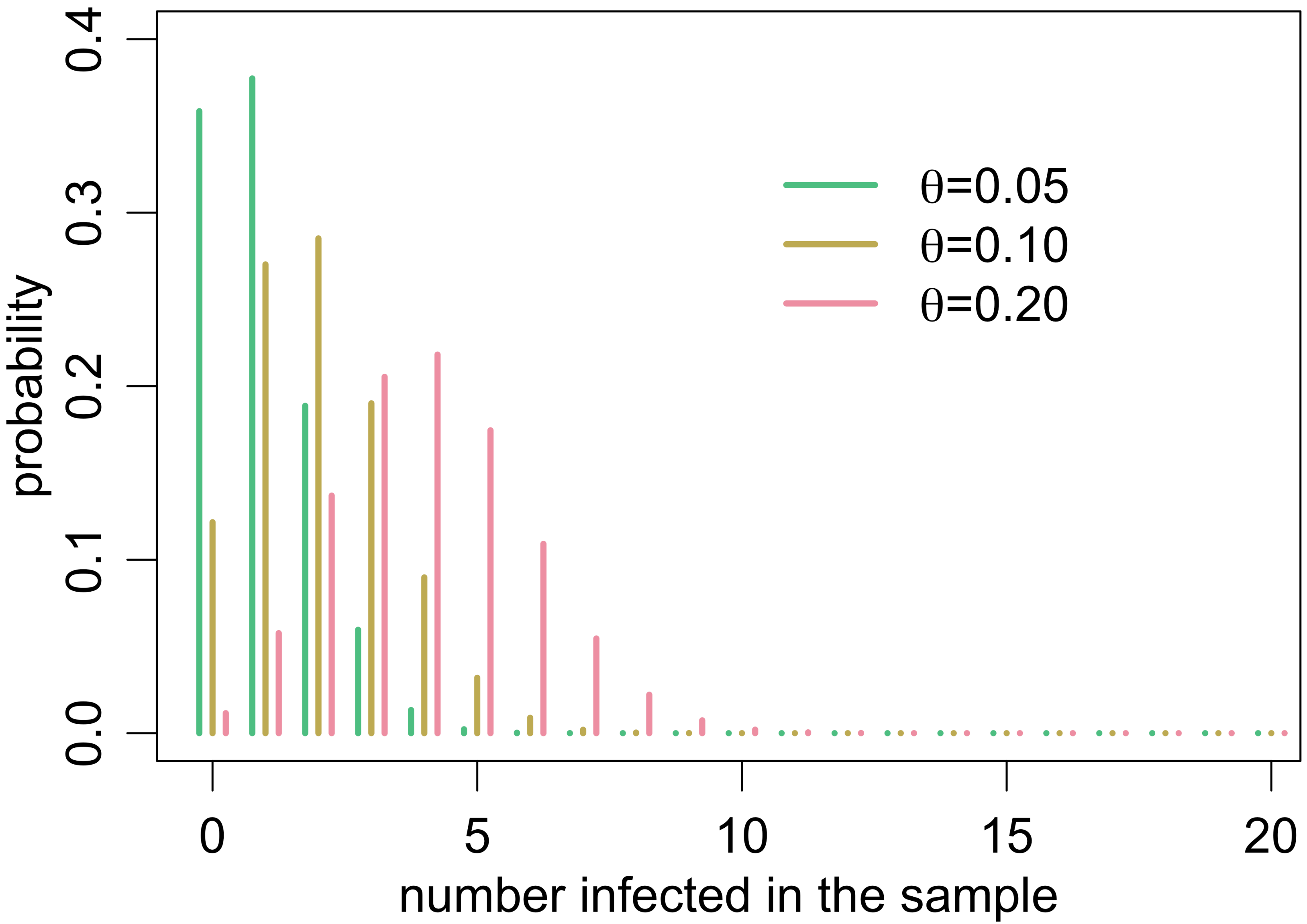
Example: Estimating COVID Infection Rates

- We need to estimate the prevalence of a COVID in Isla Vista
- Get a small random sample of 20 individuals to check for infection

Example: Estimating Infection Rates

- θ represents the population fraction of infected
- Y is a random variable reflecting the number of infected in the sample
- $\Theta = [0, 1] \quad \mathcal{Y} = \{0, 1, \ldots, 20\}$
- Sampling model: $Y \sim \text{Binom}(20, \theta)$

Example: Estimating Infection Rates



Example: Estimating Infection Rates

- Assume *a priori* that the population rate is low
  - The infection rate in comparable cities ranges from about 0.05 to 0.20
- Assume we observe $Y = 0$ infected in our sample
- What is our estimate of the true population fraction of infected individuals?

Bayesian vs Frequentist

- In frequentist inference, unknown parameters treated as constants
  - Estimators are random (due to sampling variability)
  - “What would I expect to see if I repeated the experiment?”
- In Bayesian inference, unknown parameters are random variables.
  - Need to specify a prior distribution for θ (not easy)
  - “What do I *believe* are plausible values for the unknown parameters **given the data**?”

Uncertainty Quantification: Bayes vs Frequentist

- $X_i \sim$ Rademacher
- 50/50 chance of being ± 1
- $Y_i = \theta + X_i$.
- Observe Y_1 and Y_2 and construct the following confidence "interval":

$$\begin{cases} \{(Y_1 + Y_2)/2\} & \text{if} \qquad Y_1 \neq Y_2 \\ \{Y_1 + 1\} & \text{else} \end{cases}$$

What fraction of time does this cover?

Assignment

- Read chapters 1 and 2 of BDA3
- Read: “Does Probability Exist?”
    - In Resources folder in rstudio environment