

Continuous time population growth

$$\frac{dN}{dt} = b \cdot N - d \cdot N$$

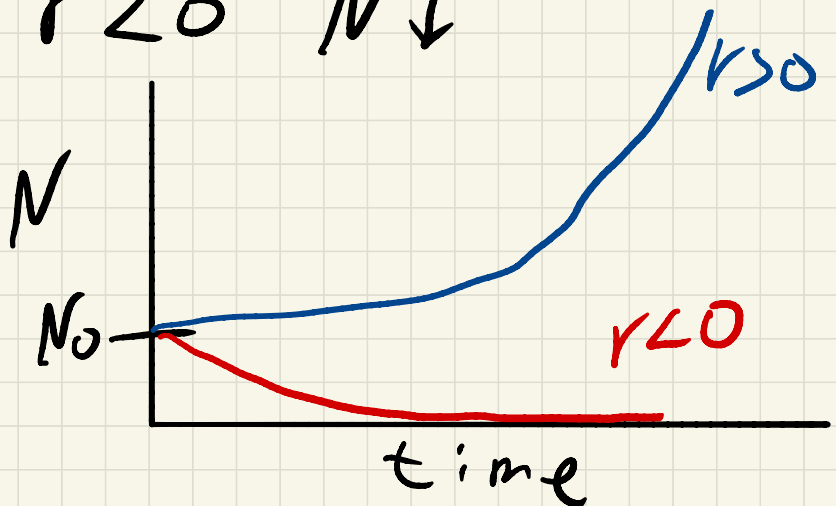
$$\frac{dN}{dt} = N(b-d)$$

Call $r \equiv b-d$

$$\frac{dN}{dt} = Nr$$

if $r > 0$ $N \uparrow$

if $r < 0$ $N \downarrow$

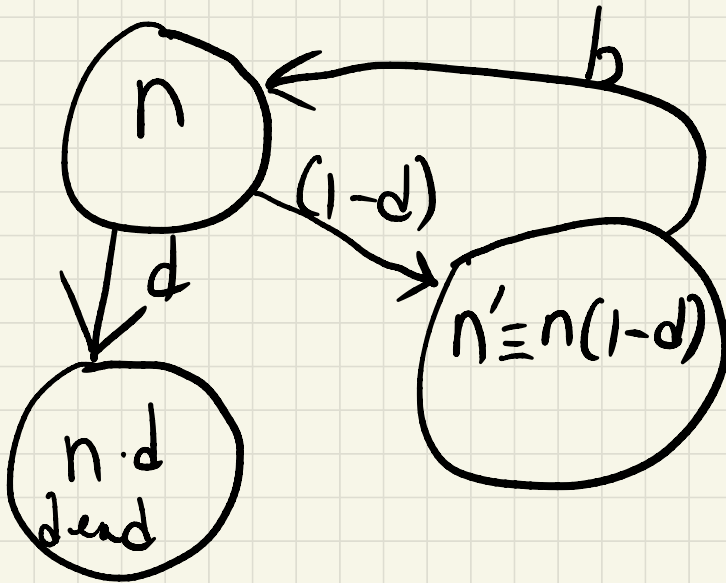


Discrete time population growth

$n \equiv$ # of individuals *State Variable*

$b \equiv$ % birth

$d \equiv$ % die



$$n_{t+1} = n_t(1-d) + n' \cdot b$$

$$n_{t+1} = n_t(1-d) + n_t(1-d) \cdot b$$

$$n_{t+1} = n_t(1-d)(1+b)$$

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$$\text{Call } R_0 = (1-d)(1+b)$$

$$n_{t+1} > n_t? \text{ if } R_0 > 1 \quad n \uparrow$$

$$\text{if } R_0 < 1 \quad n \downarrow$$

$$R_0 = (1-d)(1+b)$$

$$R_0 = 1 + b - bd - d$$

$$R_0 = 1 + b(1-d) - d$$

$$\text{is } R_0 > 1? \text{ if } b(1-d) - d > 0$$

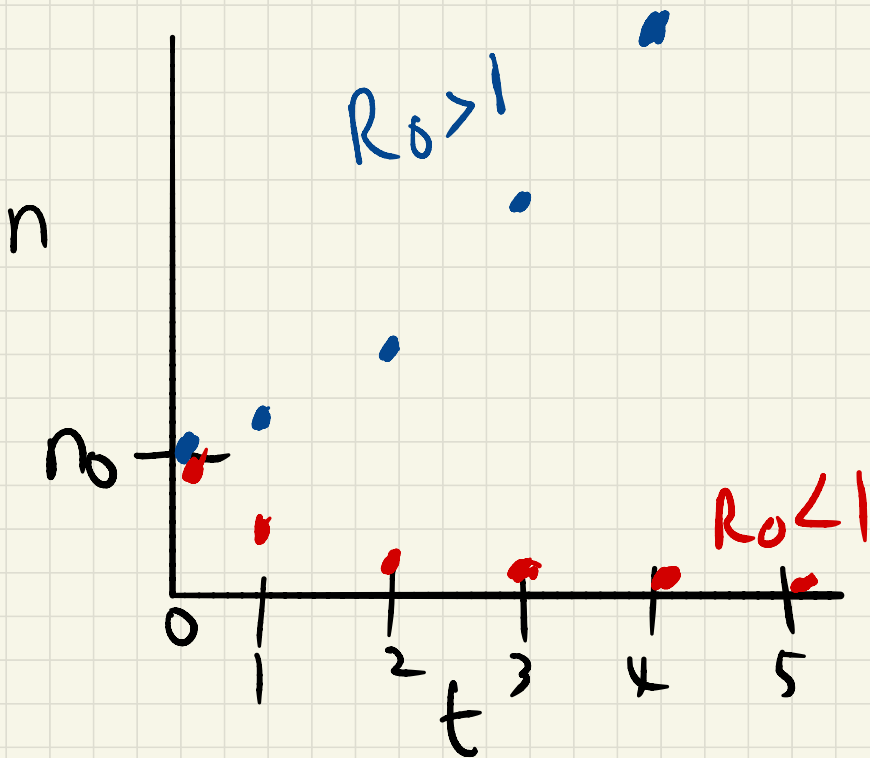
$$\underbrace{b(1-d)}_{\substack{\text{\# new} \\ \text{births}}} - \underbrace{d}_{\substack{\text{\# new} \\ \text{deaths}}}$$

$$n_{t+1} = n_t \cdot R_0$$

$$n_1 = n_0 \cdot R_0$$

$$n_2 = n_1 \cdot R_0 = n_0 \cdot R_0^2$$

$$n_T = n_0 R_0^T$$



$R_0 > 1?$

