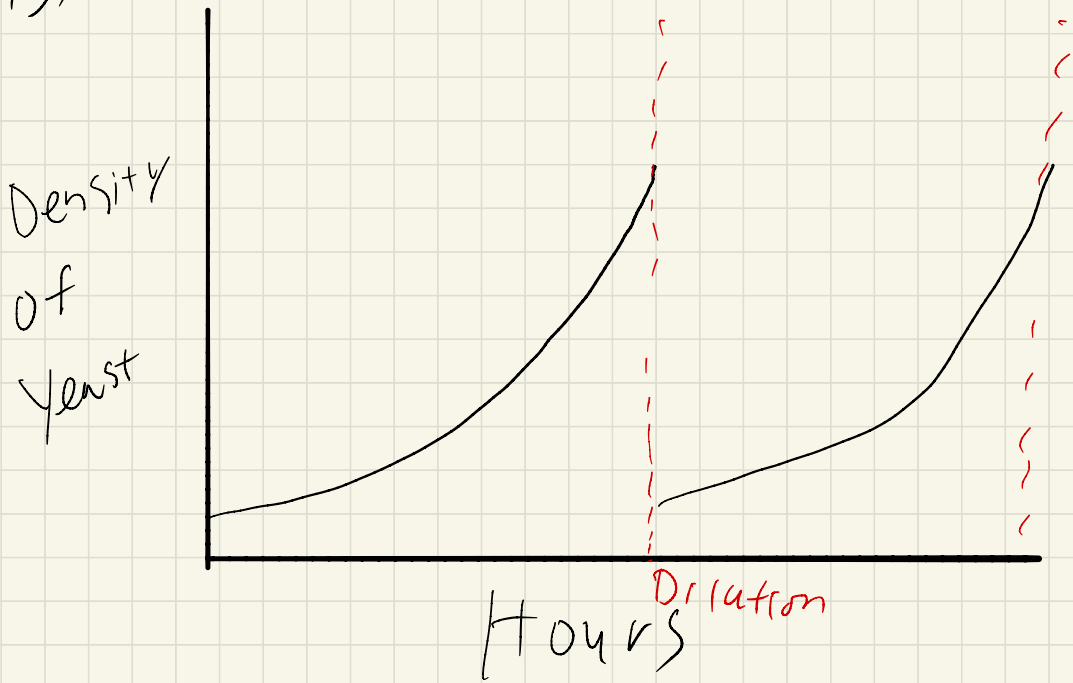
4/13/2020

Microbial selection, haploid evolution

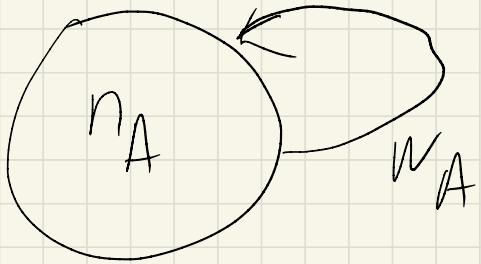


Assumptions

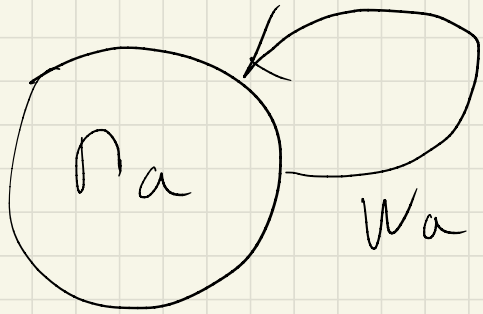
1. No Sex
2. No new mutations
3. No cell-to-cell competition
4. Discrete generations
5. Model without Density Dependence

Darwin's 4 Assumptions

1. Variation in traits
2. trait affects fitness
3. Heritability
   offspring are similar to parents
4. Not enough resources for all offspring



$$n_{A\,t+1} = n_{A\,t} \cdot W_A$$



$$n_{a\,t+1} = n_{a\,t} W_a$$

$$n_{A\,t+1} = n_{A\,t} W_A \Rightarrow n_{A\,T} = n_{A_0} W_A^T$$

$$n_{a\,t+1} = n_{a\,t} W_a \Rightarrow n_{a\,T} = n_{a_0} W_a^T$$

$$p \equiv \frac{\#A}{\#A + \#a}$$

$$p_{t+1} = \frac{n_{At+1}}{n_{A_{t+1}} + n_{at+1}} = \frac{n_{At} \cdot w_A}{n_{At} w_A + n_{at} w_a}$$

$$\frac{\frac{1}{n_{At}+n_{at}}}{\frac{1}{n_{At}+n_{at}}} \cdot \frac{n_{At} w_A}{n_{At} w_A + n_{at} w_a}$$

$$\frac{n_{At}}{n_{At}+n_{at}} = p_t \qquad \frac{n_{at}}{n_{At}+n_{at}} = 1 - p_t$$

$$p_{t+1} = \frac{p_t \cdot w_A}{p_t w_A + (1-p_t) w_a}$$

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$$w \equiv \frac{w_A}{w_a}$$

$$p_{t+1} = \frac{\frac{1}{w_a}}{\frac{1}{w_a}} \frac{p_t w_A}{p_t w_A + (1-p_t) w_a} = \frac{p_t \cdot w}{p_t w + (1-p_t)}$$

$$p_{t+1} = \frac{p_t w}{p_t w + (1-p_t)}$$

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$p_0 \equiv p$ at time 0

$$p_1 = \frac{p_0 w}{p_0 w + (1-p_0)}$$

$$p_2 = \frac{p_1 w}{p_1 w + (1-p_1)} =$$

$$\frac{\frac{p_0 w}{p_0 w + (1-p_0)} w}{\frac{p_0 w}{p_0 w + (1-p_0)} w + \left(1 - \frac{p_0 w}{p_0 w + (1-p_0)}\right)}$$

$$\frac{\frac{1}{P_0 W + (1-P_0)}}{\frac{1}{P_0 W + (1-P_0)}} : \frac{P_0 W^2}{P_0 W^2 + (1-P_0)}$$

$$P_2 = \frac{P_0 W^2}{P_0 W^2 + (1-P_0)}$$

$$P_T = \frac{P_0 W^T}{P_0 W^T + (1-P_0)}$$

P_∞ ?

if $W > 1$ $\quad P_\infty = \frac{P_0 \infty}{P_0 \infty + (1-P_0)} \approx \frac{P_0}{P_0} = 1$

if $W < 1$; $\quad P_\infty = \frac{P_0 \cdot 0}{P_0 0 + (1-P_0)} \approx 0$

$W_A > W_a \Rightarrow W > 1$ and $P_T \to 1$

$W_A < W_a \Rightarrow W < 1$ and $P_T \to 0$