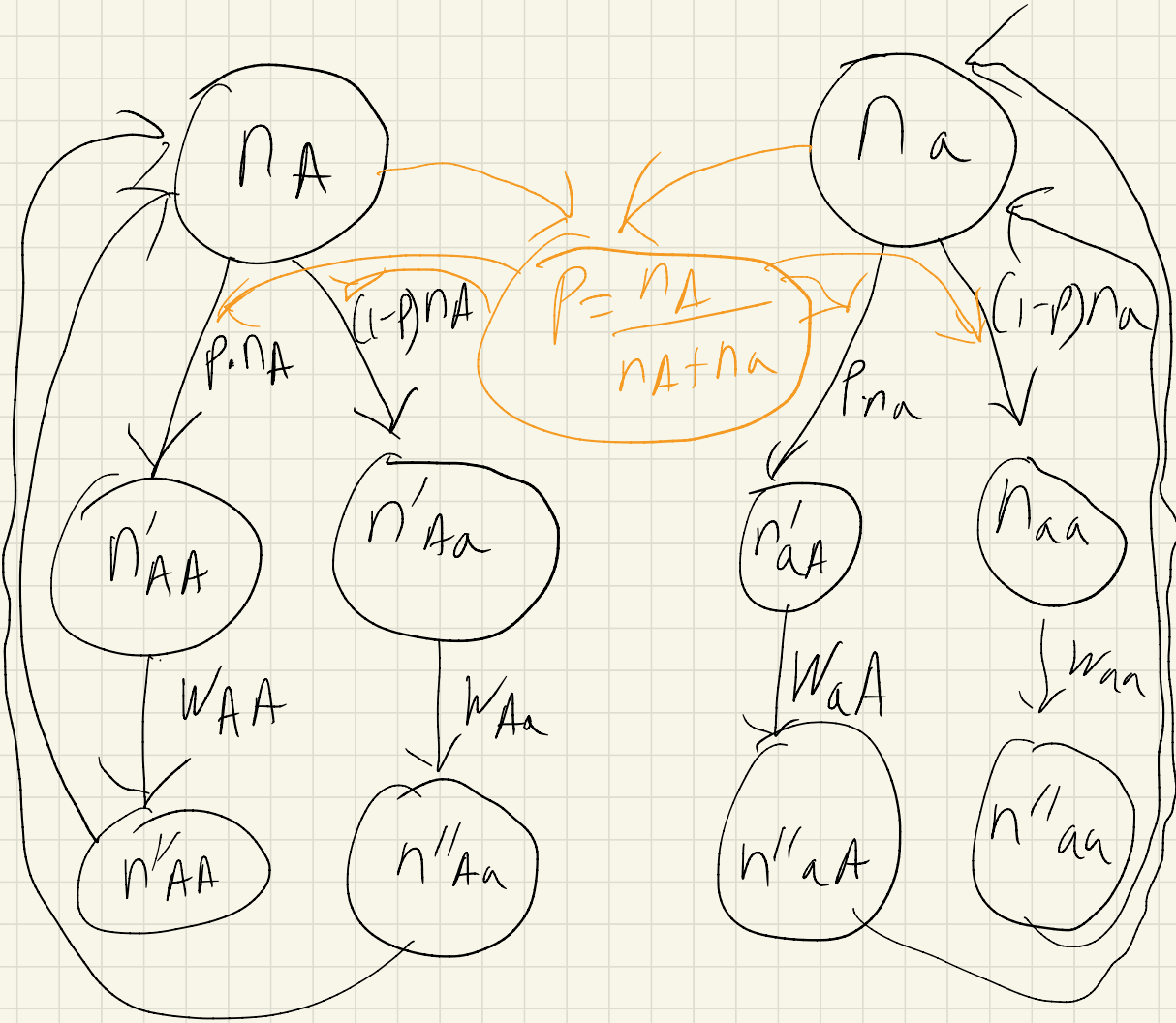
Census at Gamete Stage

$n_A \equiv \#A$ Gametes $\qquad n_a \equiv \#a$ Gametes

$$p \equiv \frac{n_A}{n_A + n_a} \; ; \quad q \equiv 1 - p$$



$$w_{Aa} = w_{aA}$$

$$n'_{AA} = n_A \cdot p$$

$$n'_{Aa} = n_A (1-p)$$

$$n''_{AA} = n'_{AA} \cdot W_{AA} = n_A W_{AA}\, p$$

$$n''_{Aa} = n'_{Aa} W_{Aa} = n_A W_{Aa}\, p$$

$$n_{A\,t+1} = n''_{AA} + n''_{Aa}$$

$$n_{A\,t+1} = n_A W_{AA}\, p + n_A W_{Aa}(1-p)$$

$$n_{A\,t+1} = n_A \left(W_{AA} \cdot p + W_{Aa}(1-p) \right)$$

$$n_{A_{t+1}} = n_A \left(W_{AA} \cdot p + W_{Aa}(1-p) \right)$$

$$n_{a_{t+1}} = n_a \left(W_{Aa} \cdot p + W_{aa}(1-p) \right)$$

$$\overline{W_A} = W_{AA}p + W_{Aa}(1-p)$$

$$\overline{W_a} = W_{Aa}p + W_{aa}(1-p)$$

Haploid

$$p_{t+1} = \frac{p_t \cdot W_A}{p_t W_A + (1-p_t) W_a}$$

Diploid

$$p_{t+1} = \frac{p_t\left(W_{AA}p_t + W_{Aa}(1-p_t)\right)}{p_t\left(W_{AA}p_t + W_{Aa}(1-p_t)\right) + (1-p_t)\left(W_{Aa}p_t + W_{aa}(1-p_t)\right)}$$

if $\overline{W_A} > \overline{W_a}$ then $p_{t+1} > p_t$

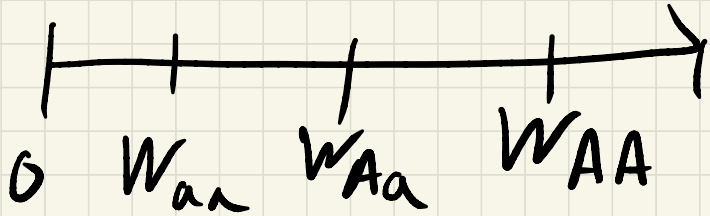
if $\overline{W_A} < \overline{W_a}$ then $p_{t+1} < p_t$

$$\overline{W_A} = p\,W_{AA} + q\,W_{Aa}$$

$$\overline{W_a} = p\,W_{Aa} + q\,W_{aa}$$

4 possible Orderings

1



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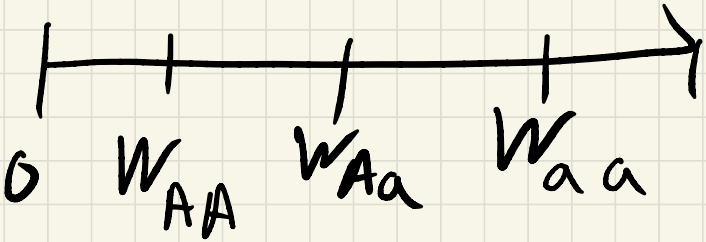

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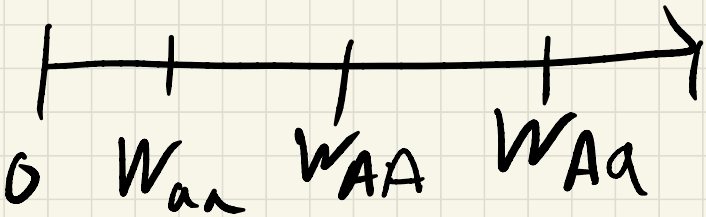

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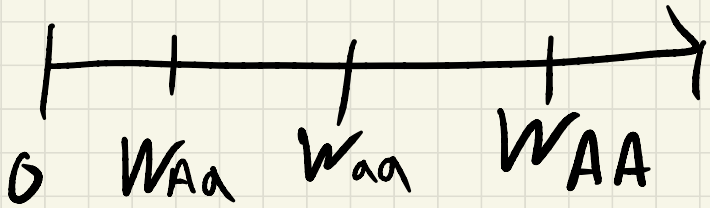
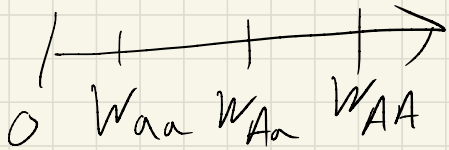
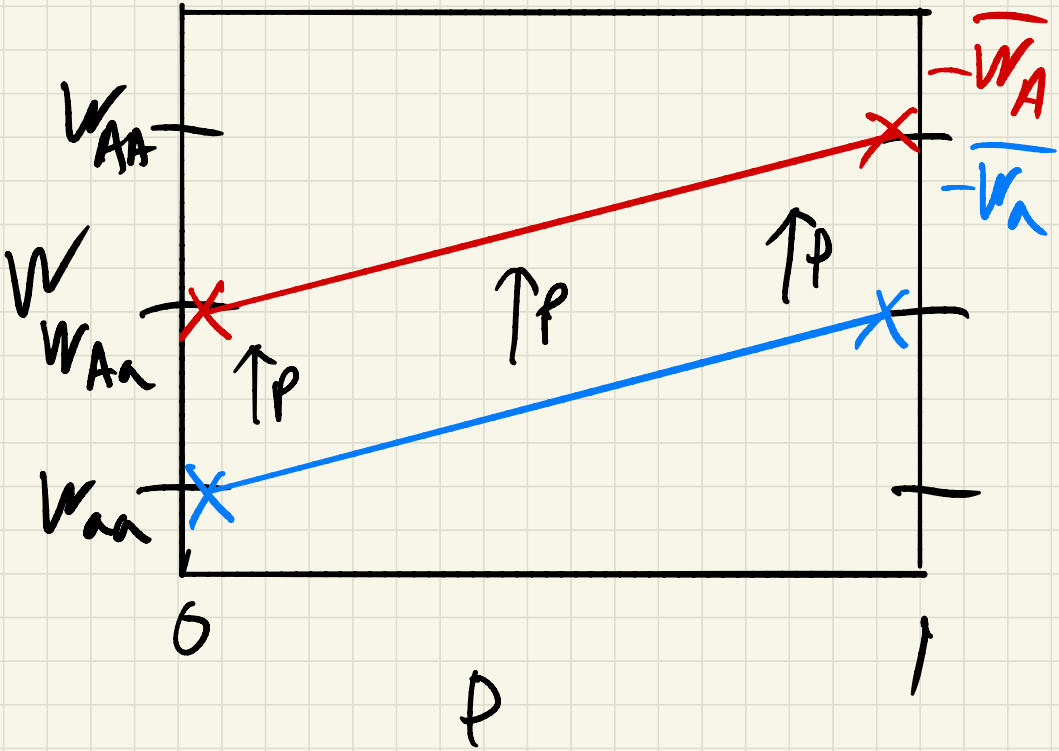


Case 1

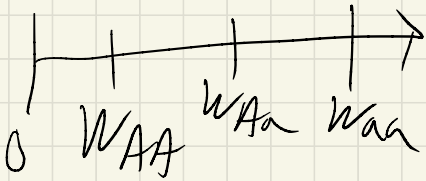


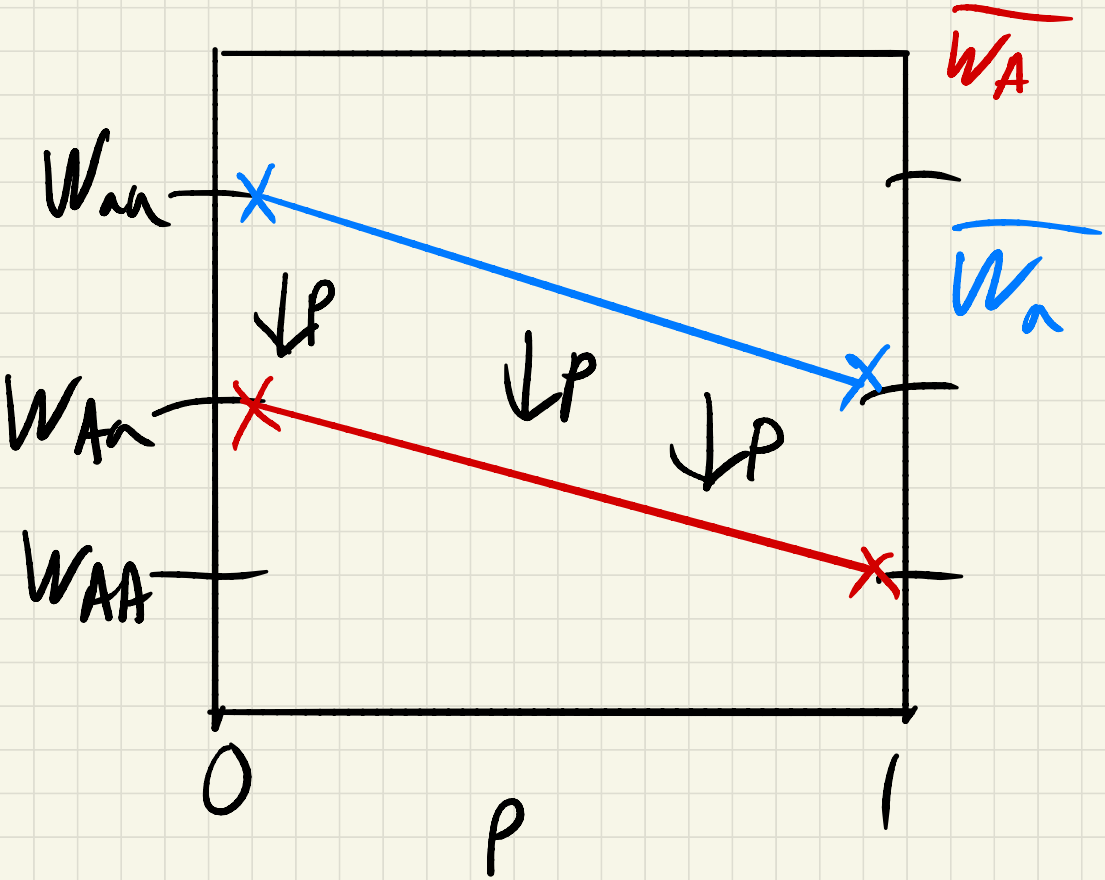


$p_{t+1} > p_t$ for any p

So A spreads

Case 2

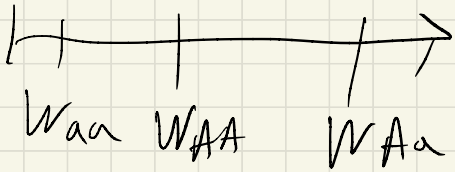


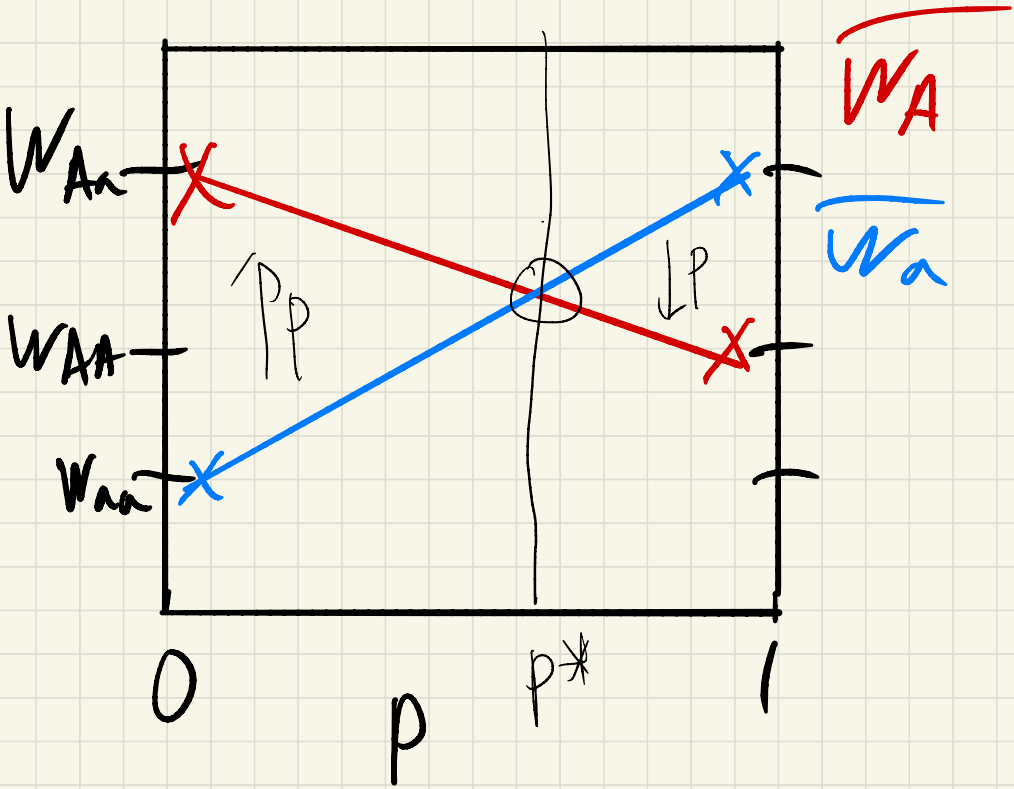


$p_{t+1} < p_t$ for any p

a spreads

Case 3





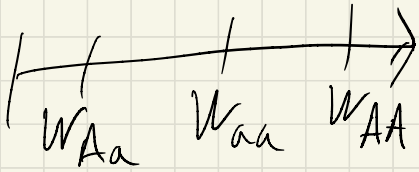
if p is low, $p \uparrow$

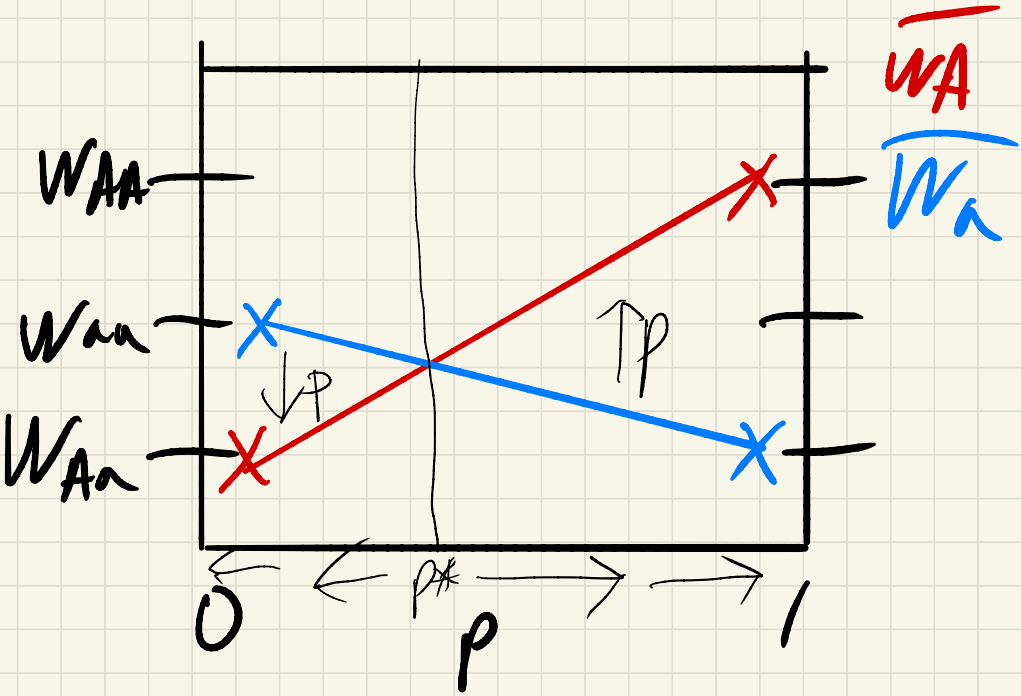
if p is high, $p \downarrow$

Both alleles persist

p^* is equilibrium

Case 4





if $p_0 < p^*$ $p \to 0$

if $p_0 > p^*$ $p \to 1$

only 1 allele is maintained

Case 1 and Case 2

Heterozygote intermediate

Allele with higher fitness becomes fixed

Case 3

Heterozygote Best

Both alleles maintained

Case 4

Heterozygote worst

only one allele maintained

rare allele goes extinct