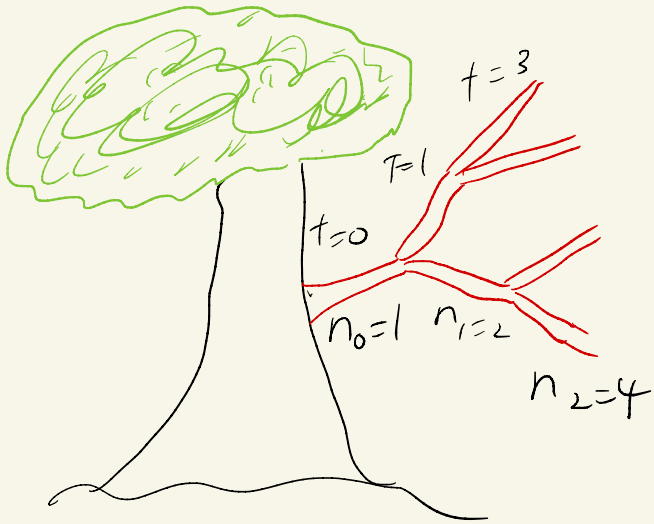
Tree Branching Model



Variables

State var

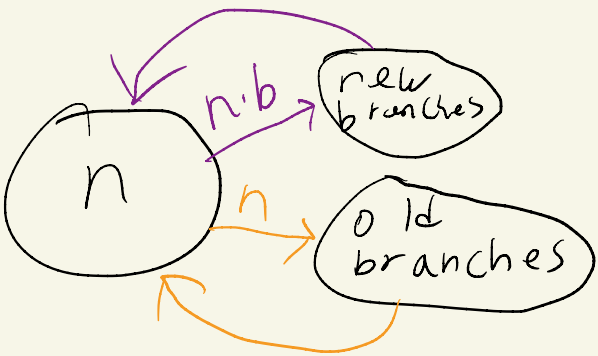
n_t = # branches

t = time

Parameters

b = chance branch splits

Flow Diagram



Assumptions

- no branches die
- All branches act the same

$$n_{t+1} = n_t + n_t \cdot b = n_t(1+b)$$

Recursion EQ

$$n_{t+1} = n_t(1+b)$$

Difference EQ

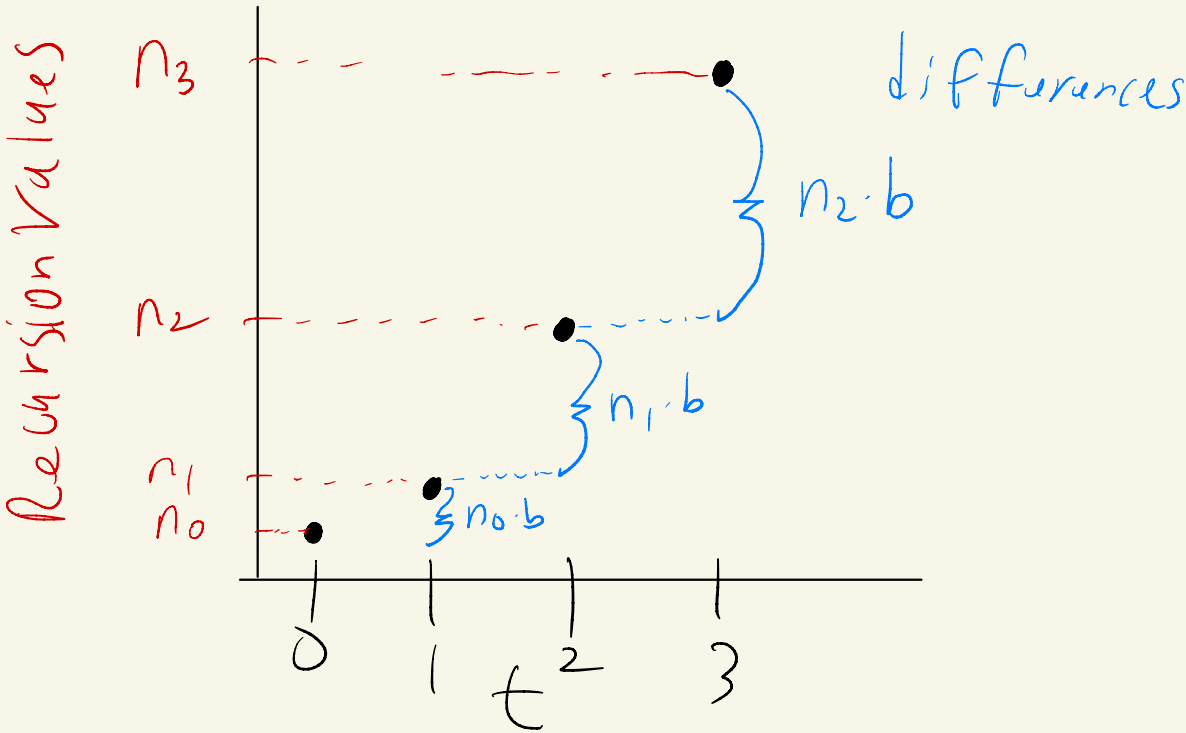
$$n_{t+1} - n_t = ?$$

$$n_{t+1} = n_t + n_t \cdot b$$

$$-n_t \Downarrow -n_t$$

$$n_{t+1} - n_t = n_t \cdot b$$

Plot n over time



n_t in the future

$$n_{t+1} = n_t(1+b)$$

start w/ n_0

$$n_1 = n_0(1+b)$$

$$n_2 = n_1(1+b)$$ what is n_1?

$$n_2 = n_0(1+b)(1+b) = n_0(1+b)^2$$

$$n_3 = n_2(1+b) = n_0(1+b)^3$$

so

$$\boxed{n_T = n_0(1+b)^T}$$

Geometric Growth

if $(1+b) > 1$ then $n_T \to \infty$

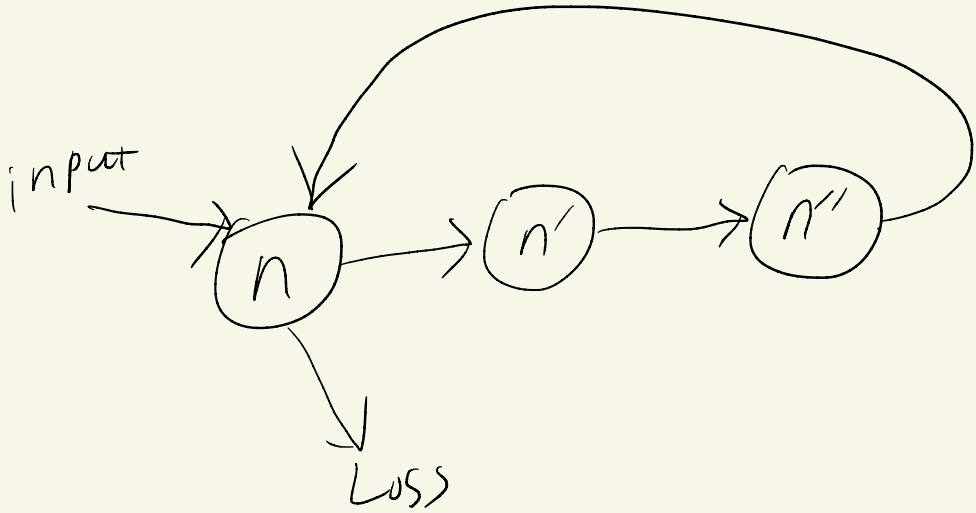
if $(1+b) = 1$ then $n_T = n_0$

Flow Diagrams

○ State Variables

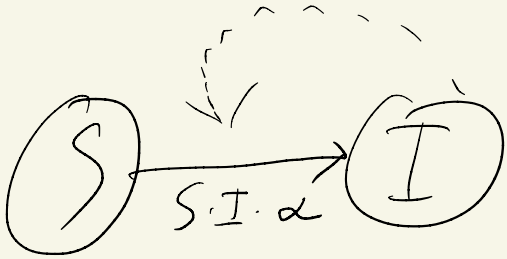
→ Flow of #'s

⇢ Indirect effect

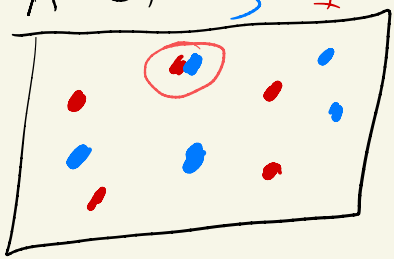


$$n_{t+1} = n_t - \text{Loss} + \text{input} + n''$$

Example: Infection Model



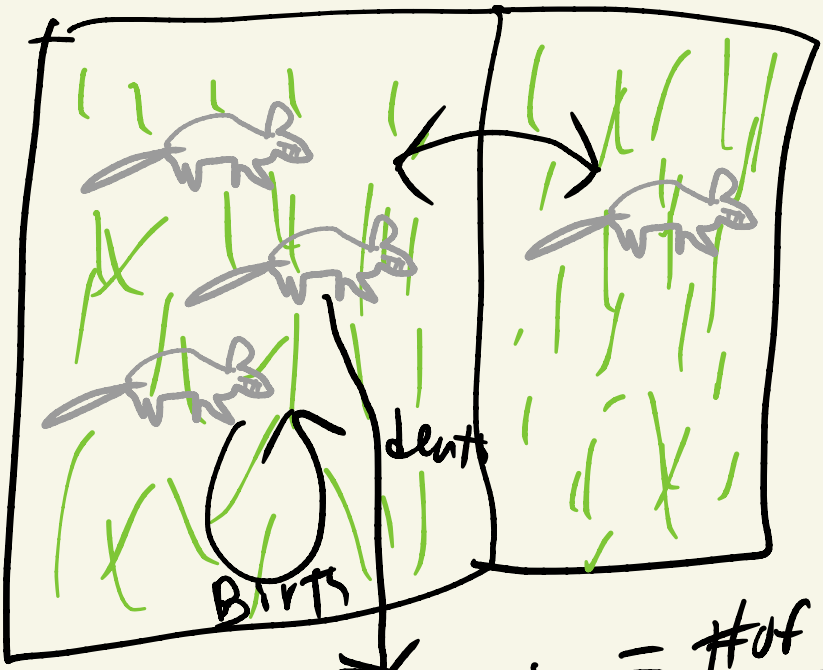
Mass Action



$$\frac{dS}{dt} = -SI\alpha$$

$$\frac{dI}{dt} = SI\alpha$$

Backyard Mice

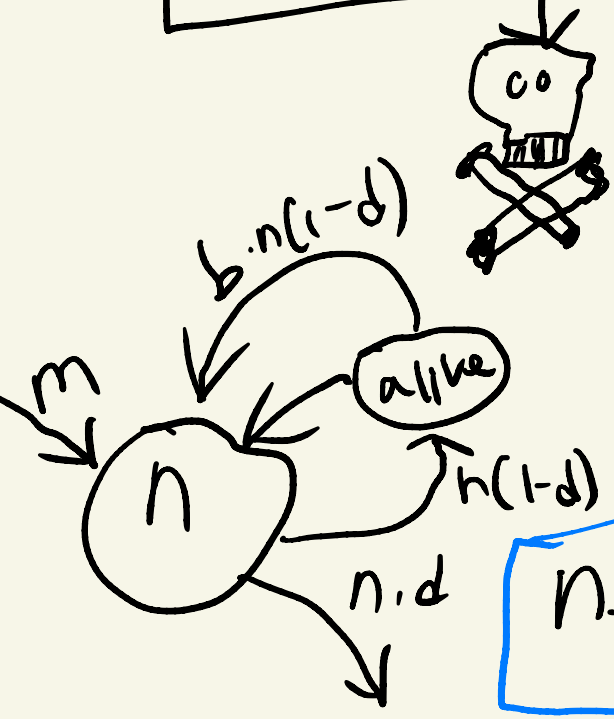


$n \equiv$ # of

$b \equiv$ birth

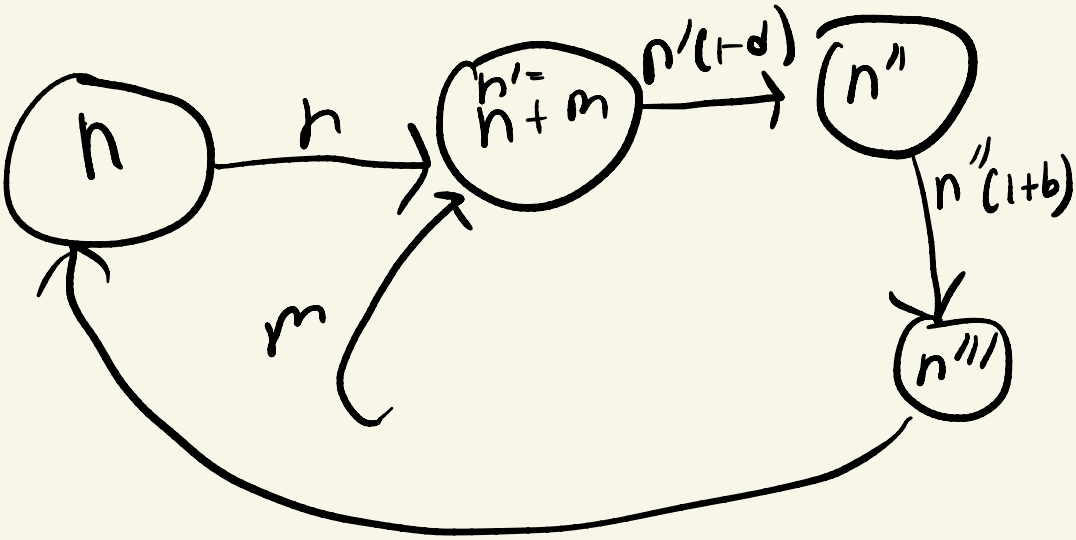
$m \equiv$ migration

$d \equiv$ deaths

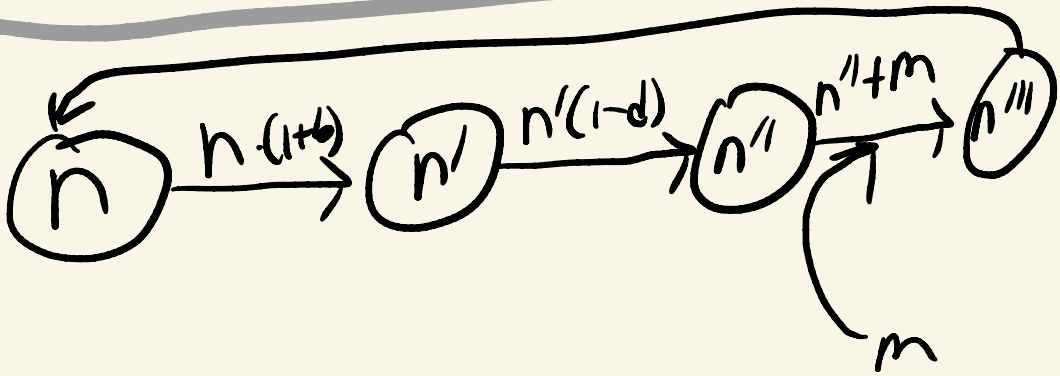


$$\boxed{n_{t+1} = n_t(1-d)(1+b)+m}$$

Other Order of Lifecycle



$$n_{t+1} = (n+m)(1-d)(1+b)$$



$$n_{t+1} = n'' + m = n'(1-d) + m = n(1+b)(1-d) + m$$