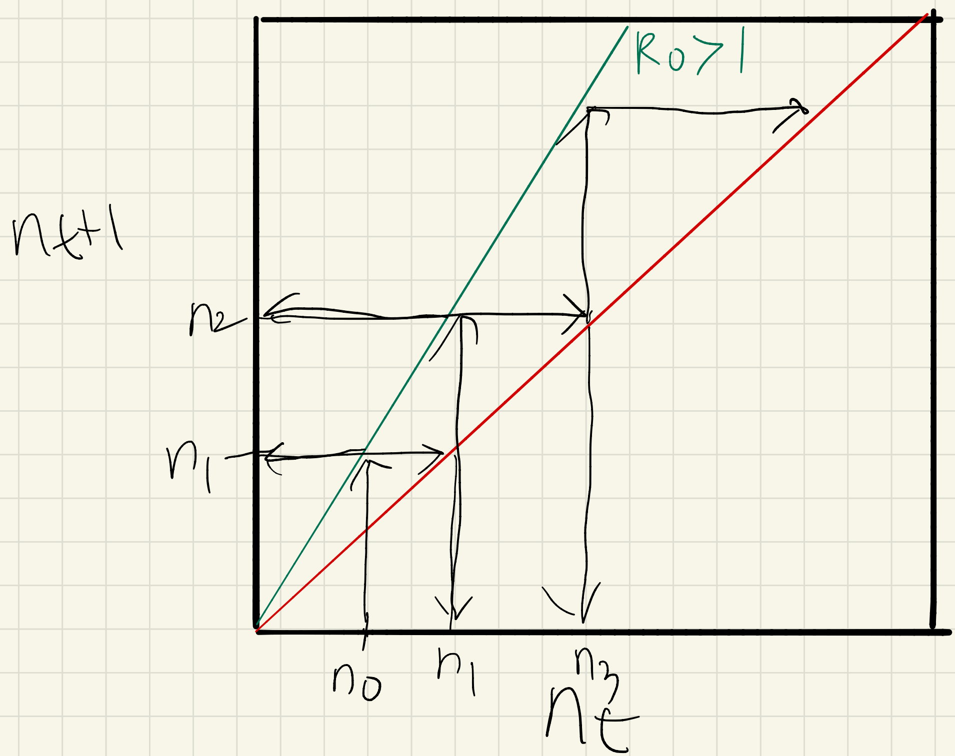
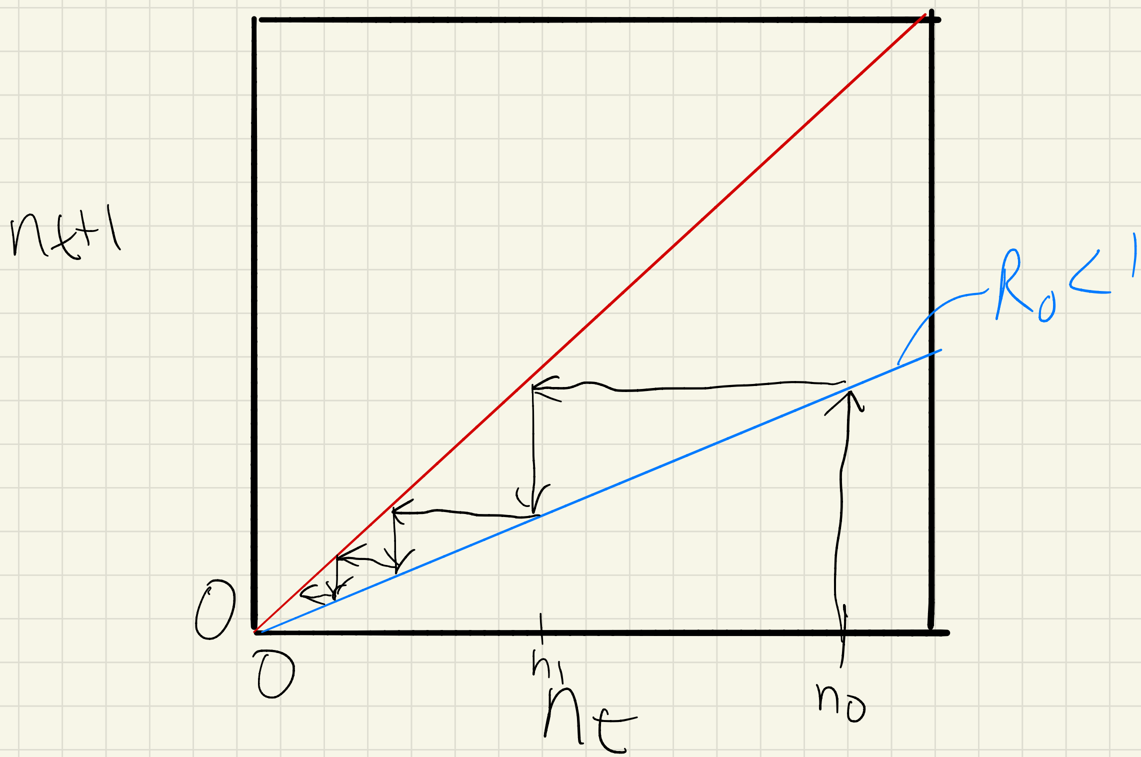
Graphing Recursions and Stability Analysis

$$n_{t+1} = n_t R_0$$

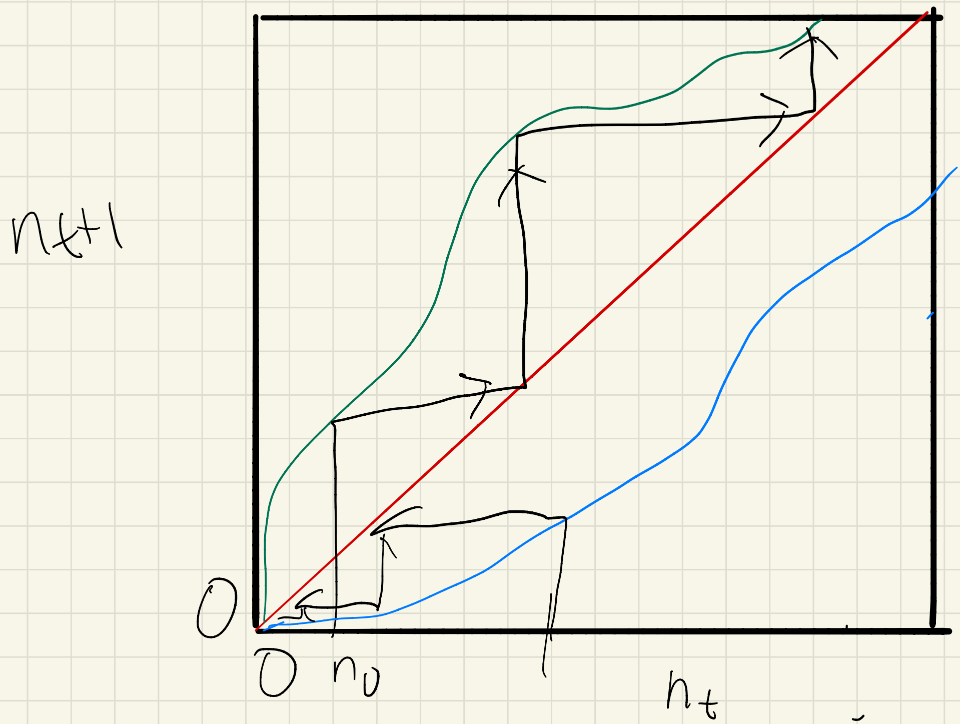
$$n_{t+1} - n_t = n_t \cdot R_0 - n_t = n_t(R_0 - 1)$$





n_{t+1}
$R_0 < 1$
0
0
n_1
n_0
n_t

$$n_{t+1} = f(n_t)$$

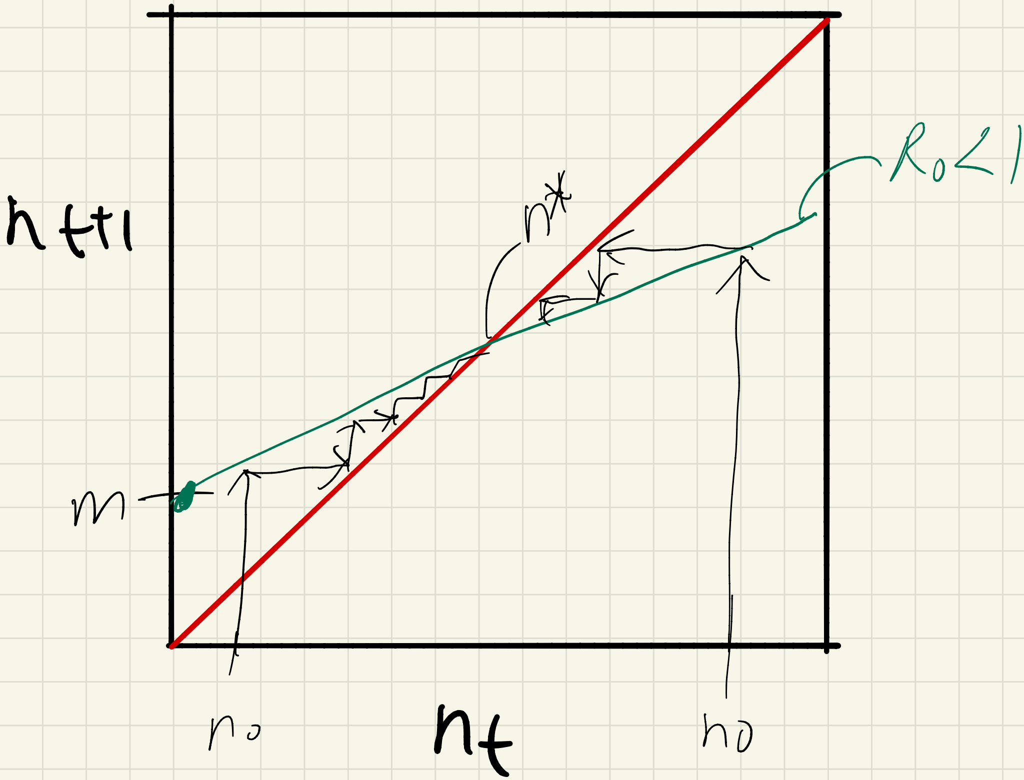


if $f'(0) > 1 \Rightarrow$ n goes up : unstable

if $0 < f'(0) < 1 \Rightarrow$ n goes down : Stable

$m \equiv$ # immigrants

$$n_{t+1} = n_t R_0 + m$$



$$n_t = n_t R_0 + m \Rightarrow n_t(1 - R_0) = m$$

$$n_t = \frac{m}{1 - R_0} \Longrightarrow n^* = \frac{m}{1 - R_0}$$

$$n_{t+1} = f(n_t)$$

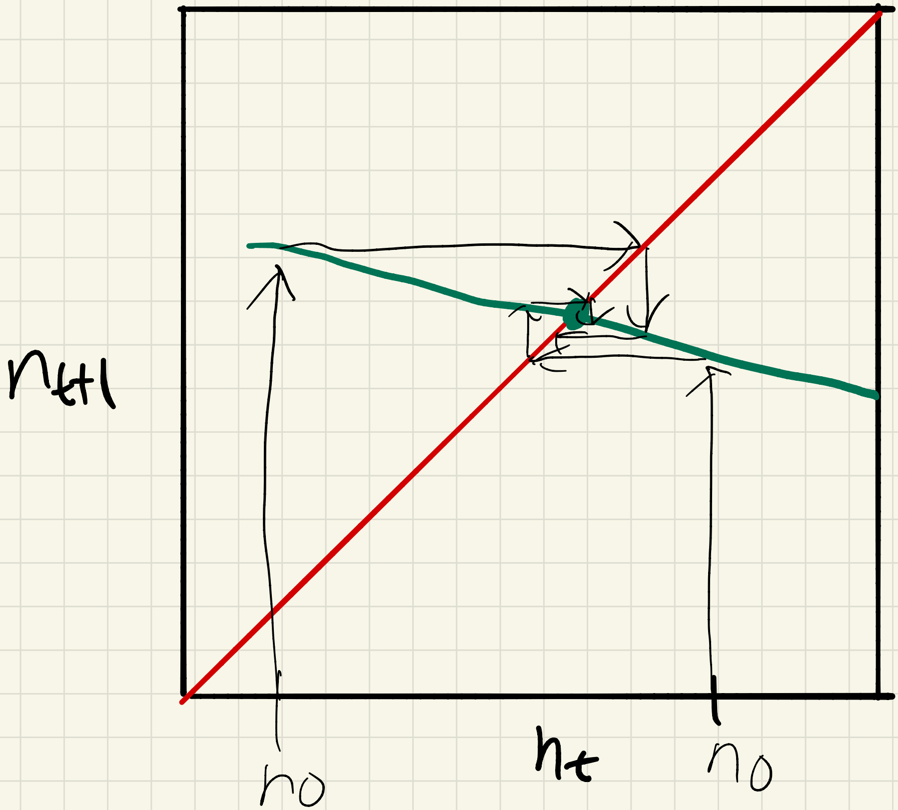
$$f(n^*) = n^*$$

$$\lambda = f'(n^*)$$

if $1 < \lambda < \infty$ then unstable

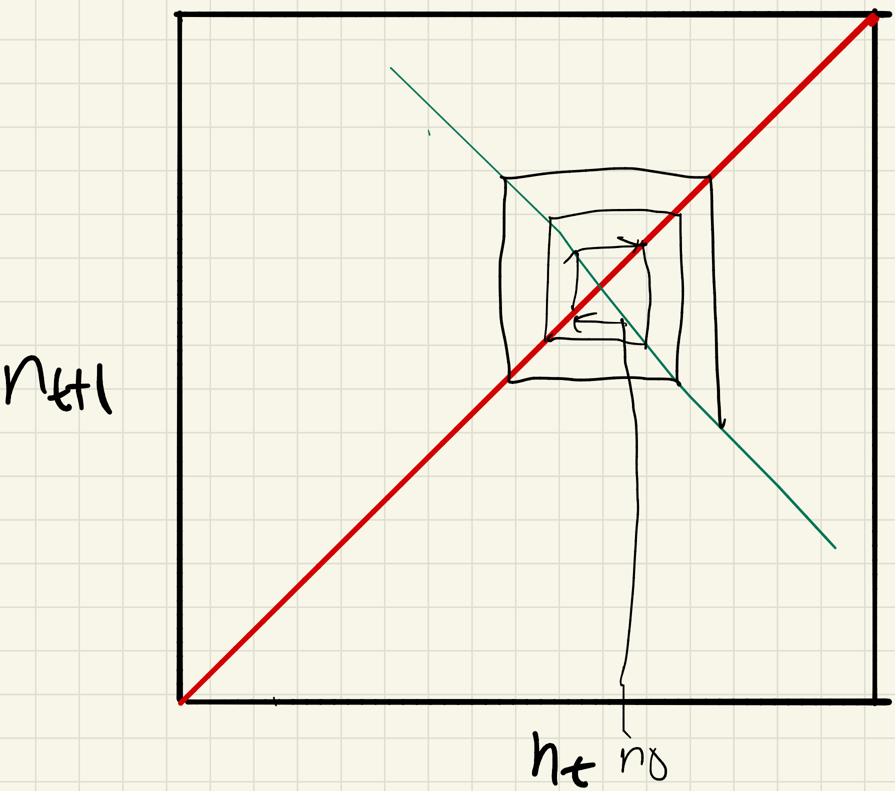
$0 < \lambda < 1$ then stable

$n_{t+1} = f(n_t)$



$-1 < \lambda < 0$ oscillate ; stable

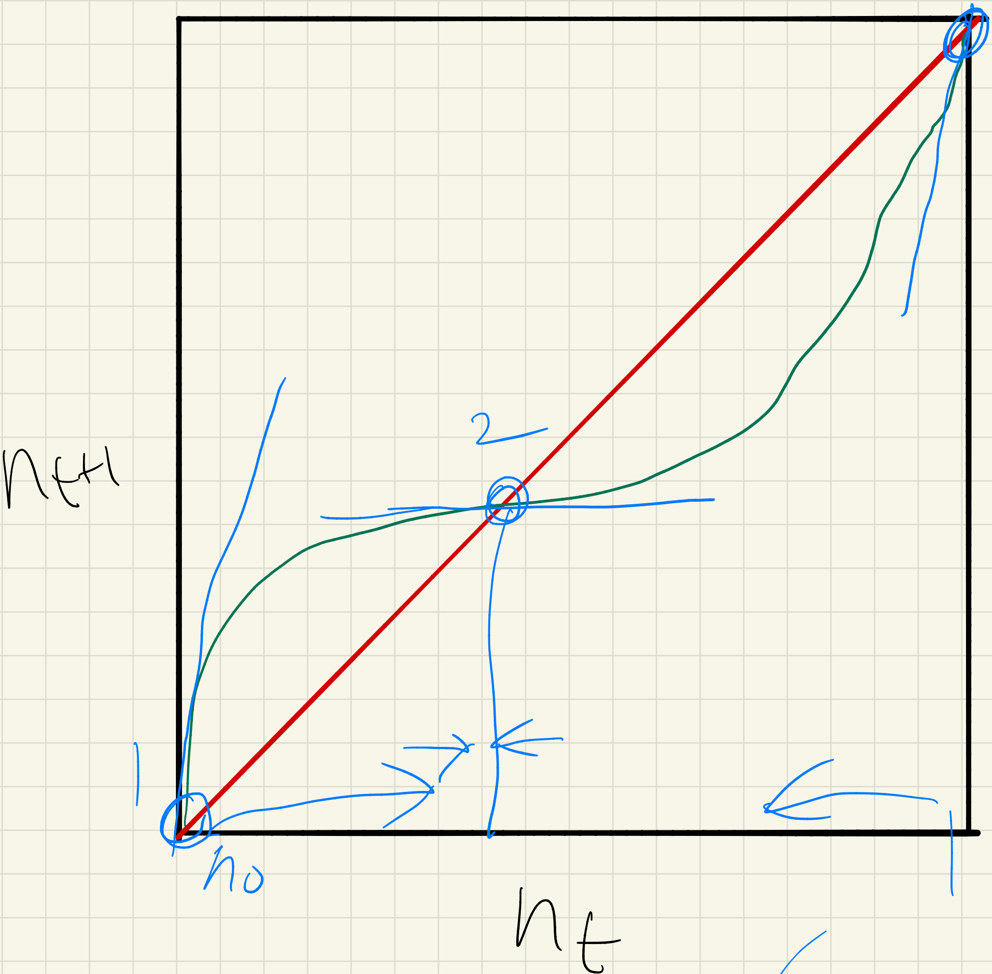
$$n_{t+1} = f(n_t)$$



$-\infty < \lambda < -1$ oscillates ; unstable

λ			
above	below	stable?	oscillates?
1	∞	NO	NO
0	1	Yes	NO
-1	0	Yes	Yes
$-\infty$	-1	NO	Yes

$n_{t+1} = f(n_t)$



Readoff the equilibria ✓

Check the slope

n_i^*	λ	Stable?
1	>1	No
2	$0<\lambda<1$	Yes
3	$\lambda>1$	No