CSE 152 Introduction to Computer Vision
Homework 0

Instructions:

- Total points: 100
- Please submit your solution to Gradescope.
- **Due: 11:59 pm, Thursday, Oct 11, 2018**

1. **[12 points]** Given two bases of $P(x)_3$: $1, x, x^2, x^3$ and $1, 1+x, (1+x)^2, (1+x)^3$.

   (a) **[4 points]** Find the invertible linear transformation matrix from basis $1, x, x^2, x^3$ to $1, 1+x, (1+x)^2, (1+x)^3$.

   (b) **[4 points]** Find the invertible linear transformation matrix from basis $1, 1+x, (1+x)^2, (1+x)^3$ to $1, x, x^2, x^3$.

   (c) **[4 points]** Find the coordinates of $a_3x^3+a_2x^2+a_1x+a_0$ with respect to the basis $1, 1+x, (1+x)^2, (1+x)^3$

2. **[12 points]** $\mathbf{A}$ is a 3×3 real symmetric matrix, and $\mathbf{A}^2 + 2\mathbf{A} = \mathbf{0}$. Given $rank(\mathbf{A}) = 2$, find all the eigenvalues of $\mathbf{A}$.

3. [**20 points**] Suppose that $\mathbf{u}$ is an n-dimensional column vector of unit length in $\boldsymbol{R}^n$, and let $\mathbf{u}^T$ be its transpose. Then $\mathbf{u}\mathbf{u}^T$ is a matrix. Consider the $n \times n$ matrix $\mathbf{A} = \mathbf{I} - \mathbf{u}\mathbf{u}^T$.

   (a) [**6 points**] Describe the action of the matrix $\mathbf{A}$ geometrically.

   (b) [**6 points**] Give the eigenvalues of $\mathbf{A}$.

   (c) [**4 points**] Describe the null space of $\mathbf{A}$.

   (d) [**4 points**] What is $\mathbf{A}^2$?

4. **[10 points]** Suppose $\mathbf{A}^+$ is the pseudo inverse of matrix $\mathbf{A} = [3\ 4]^T$. Find $\mathbf{A}^+$ and $\mathbf{A}^+\mathbf{A}$ and $\mathbf{A}\mathbf{A}^+$.

5. **[12 points]** In homogeneous system,

   (a) **[4 points]** Please write down the 4×4 matrix $\mathbf{S}$ that scales by a constant c.

   (b) **[4 points]** Multiply $\mathbf{ST}$ and also $\mathbf{TS}$, where $\mathbf{T}$ is translation by $(1, 4, 3)$.

   (c) **[4 points]** To blow up the picture around the center point $(1, 4, 3)$, would you use $\mathbf{vST}$ or $\mathbf{vTS}$?

6. **[10 points]** Suppose

$$\frac{\partial \mathbf{A}}{\partial \mathbf{X}} = \begin{bmatrix} 2xy & y^2 & y \\ x^2 & 2xy & x \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Find

$$\frac{\partial^2 \mathbf{A}}{\partial \mathbf{X}^2}$$

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7. [**10 points**] From the formula $\mathbf{A}\mathbf{C}^T = (\det \mathbf{A})\mathbf{I}$ show that $\det \mathbf{C} = (\det \mathbf{A})^{n-1}$.

8. **[14 points]** Suppose T is a linear transformation on linear space V. If $T^k(\mathbf{a}) \neq \mathbf{0}$, and $T^n(\mathbf{a}) = \mathbf{0}$ $(n > k)$. Show that $\mathbf{a}, T(\mathbf{a}), ..., T^k(\mathbf{a})$ are linearly independent.