

1 (...)

2.1 -3

2.2 $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3.1 $\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$

3.2 2

3.3 2

3.4 1 (number of linearly independent rows/columns)

3.5 No. Not full rank / determinant is 0 .

3.6 $\text{span}\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)$ (solve for $B\mathbf{x} = \mathbf{0}$)

3.7 I (transpose of rotation matrix is its inverse)

3.8 1 (determinant measures “volume change”, rotation does not change volume)

3.9 $k \begin{bmatrix} 1 \\ 2 \end{bmatrix}, k \in \mathbb{R}$ (linear combination of columns)

3.10

$$T_1(T_2(a\mathbf{x}_1 + \mathbf{x}_2)) = T_1(aT_2(\mathbf{x}_1) + T_2(\mathbf{x}_2)) = aT_1(T_2(\mathbf{x}_1)) + T_1(T_2(\mathbf{x}_2))$$

4.1 $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

4.2 eigenvalues: $4, -1$, eigenvectors: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ (or their scalar multiples)

5.1 $2x$

5.2 <https://www.wolframalpha.com/input/?i=plot+sin%28x%29>

5.3 <https://www.wolframalpha.com/input/?i=plot+sin%27%28x%29>

5.4 $2xy$

5.5 $2x$

5.6 $\begin{bmatrix} 2xy & x^2 \end{bmatrix}$

5.7 $(g(f(x)))' = g'(f(x)) \cdot f'(x)$

6.1 0.8

6.2 0.5

6.3 $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

6.4

$$\begin{aligned} 0 \leq \mathbb{E}[(X - E[X])^2] &= \mathbb{E}[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2] \\ &= \mathbb{E}[X^2] + \mathbb{E}[\mathbb{E}[X]^2] - 2\mathbb{E}[X\mathbb{E}[X]] \\ &= \mathbb{E}[X^2] + \mathbb{E}[X]^2 - 2\mathbb{E}[X]\mathbb{E}[X] \\ &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \end{aligned}$$

6.5 No. e.g. A and C are the same, B is independent of anything.