

## CSE 152 Background Check

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This quiz contains 180 points. Please write down the question numbers and answer questions on the blank paper provided. This quiz should take less than 45 minutes. **You do not need to show intermediate steps unless you are asked to “prove” or “explain”.**

### 1 Python [5pt]

1. Have you used Python before? Have you used NumPy before? [0pt]
2. Which programming language do you use the most? [0pt]
3. Write a Python snippet that prints numbers from 1 to 100. If you have not coded in Python, write in any language. [5pt]

### 2 Linear Algebra [15pt]

Consider the following vectors,

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

1. What is the dot product  $\mathbf{v}_1 \cdot \mathbf{v}_2$ ? [5pt]
2. What is the cross product  $\mathbf{v}_1 \times \mathbf{v}_2$ ? [10pt]

### 3 Linear Algebra, matrices [55pt]

Consider the following matrices.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

1. What is the transpose of matrix  $A$ ? [5pt]
2. What is the rank of matrix  $A$ ? [5pt]
3. What is the determinant of matrix  $A$ ? [5pt]
4. What is the rank of matrix  $B$ ? [5pt]
5. Is matrix  $B$  invertible? explain. [5pt]
6. What is the null space of matrix  $B$ ? [5pt]
7. If  $R$  is a  $3 \times 3$  rotation matrix. What is  $R^T R$ ? [5pt]
8. If  $R$  is a  $3 \times 3$  rotation matrix. What is the determinant of  $R$ ? [5pt]
9. If the linear system  $B\mathbf{x} = \mathbf{b}$  can be solved, what is the form of vector  $\mathbf{b}$ ? [5pt]

10. Definition: a transformation  $T$  is linear if and only if given any vectors  $\mathbf{x}_1, \mathbf{x}_2$  and scalar  $a$ ,  $T(a\mathbf{x}_1 + \mathbf{x}_2) = aT(\mathbf{x}_1) + T(\mathbf{x}_2)$ . Now, given linear transformations  $T_1$  and  $T_2$ , prove  $T_1 \circ T_2$  is linear. [10pt]

hint.  $T_1 \circ T_2(\mathbf{x}) = T_1(T_2(\mathbf{x}))$

#### 4 Linear Algebra, calculation.

[30pt]

Consider the following matrix,

$$C = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix}$$

1. Solve for  $x_1, x_2$ , where

$$\begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

[10pt]

2. What are the eigenvalues and eigenvectors of matrix  $C$ ?

[20pt]

#### 5 Calculus

[35pt]

Consider the following functions

$$f(x) = x^2, \quad g(x) = \sin(x), \quad h(x, y) = x^2y$$

1. What is the derivative of  $f(x)$ ? [5pt]
2. Sketch a graph for  $g(x)$ . [5pt]
3. Sketch a graph for the derivative of  $g(x)$ . [5pt]
4. What is the partial derivative of  $h$  with respect to  $x$ ,  $\frac{\partial}{\partial x}h(x, y)$ ? [5pt]
5. What is the mixed second order partial derivative  $\frac{\partial^2}{\partial x \partial y}h(x, y)$ ? [5pt]
6. What is the gradient of  $h(x, y)$ ,  $\nabla h(x, y)$ ? [5pt]
7. State the chain rule of differentiation. [5pt]

#### 6 Probability

[40pt]

1. Given  $P(A) = 0.7, P(B) = 0.2, P(A \cap B) = 0.1$ , what is  $P(A \cup B)$ ? [5pt]
2. Given the above, what is  $P(A|B)$ ? [5pt]
3. Express  $P(A|B)$  in terms of  $P(A), P(B)$  and  $P(B|A)$ . [10pt]
4. (**hard**) Given random variable  $X$ , prove or disprove  $\mathbb{E}[X]^2 \leq \mathbb{E}[X^2]$ . [10pt]
5. (**hard**) Given random variables  $A, B, C$ .  $A$  and  $B$  are independent,  $B$  and  $C$  are independent. Are  $A$  and  $C$  independent? If not, give an example. [10pt]