CSE 152: Computer Vision

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Lecture 11: Camera Models

Credit: CS231a, Stanford, Silvio Savarese

Agenda

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

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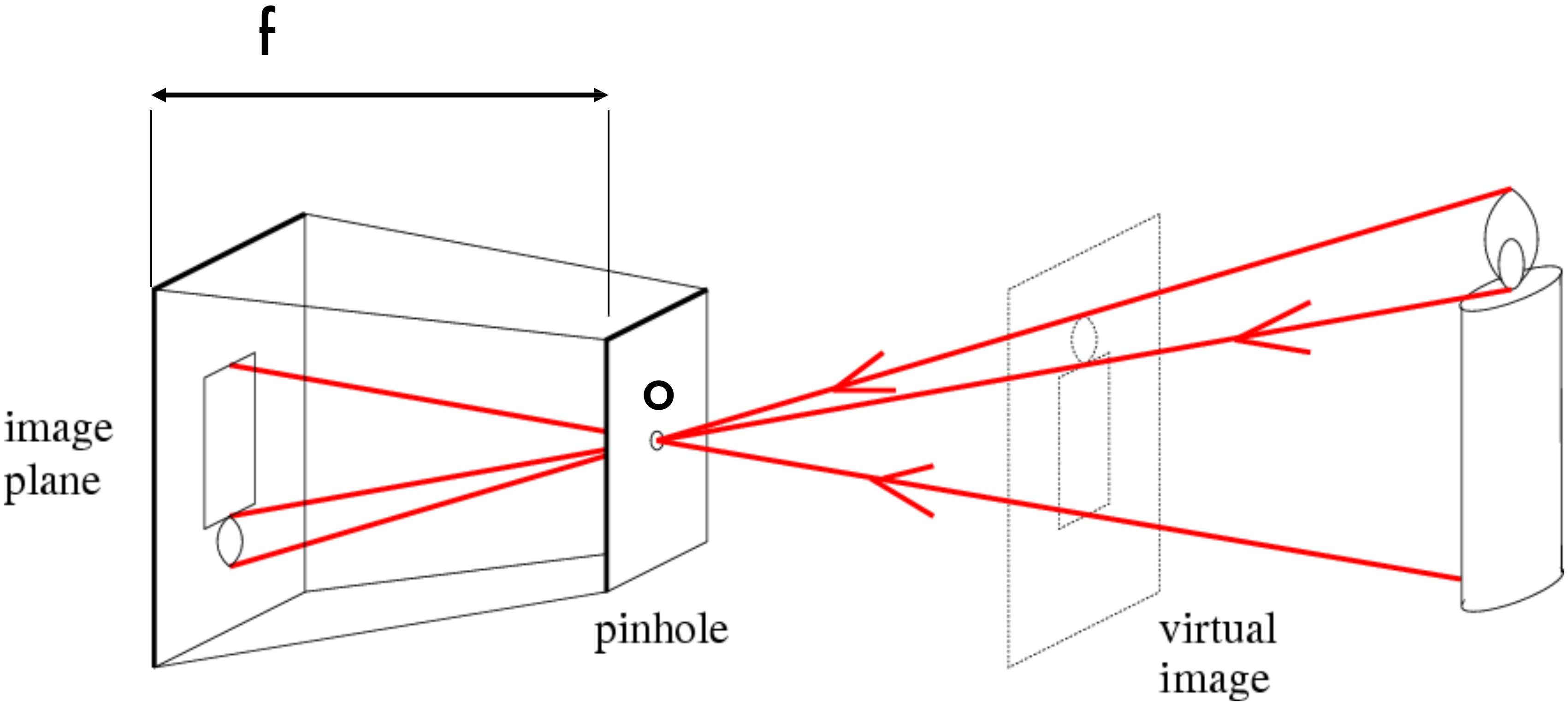
- **Pinhole cameras**
- Cameras & lenses
- The geometry of pinhole cameras

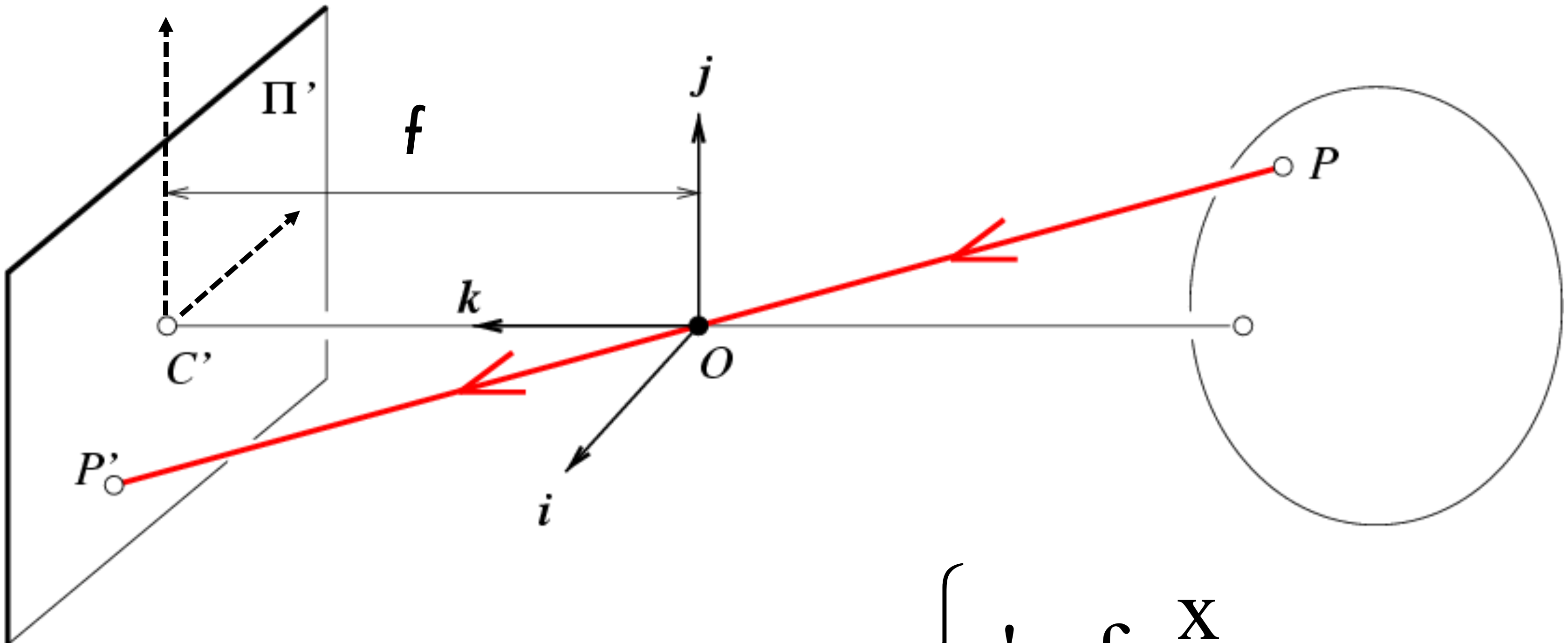
Pinhole camera



f = focal length

o = aperture = pinhole = center of the camera

Pinhole camera

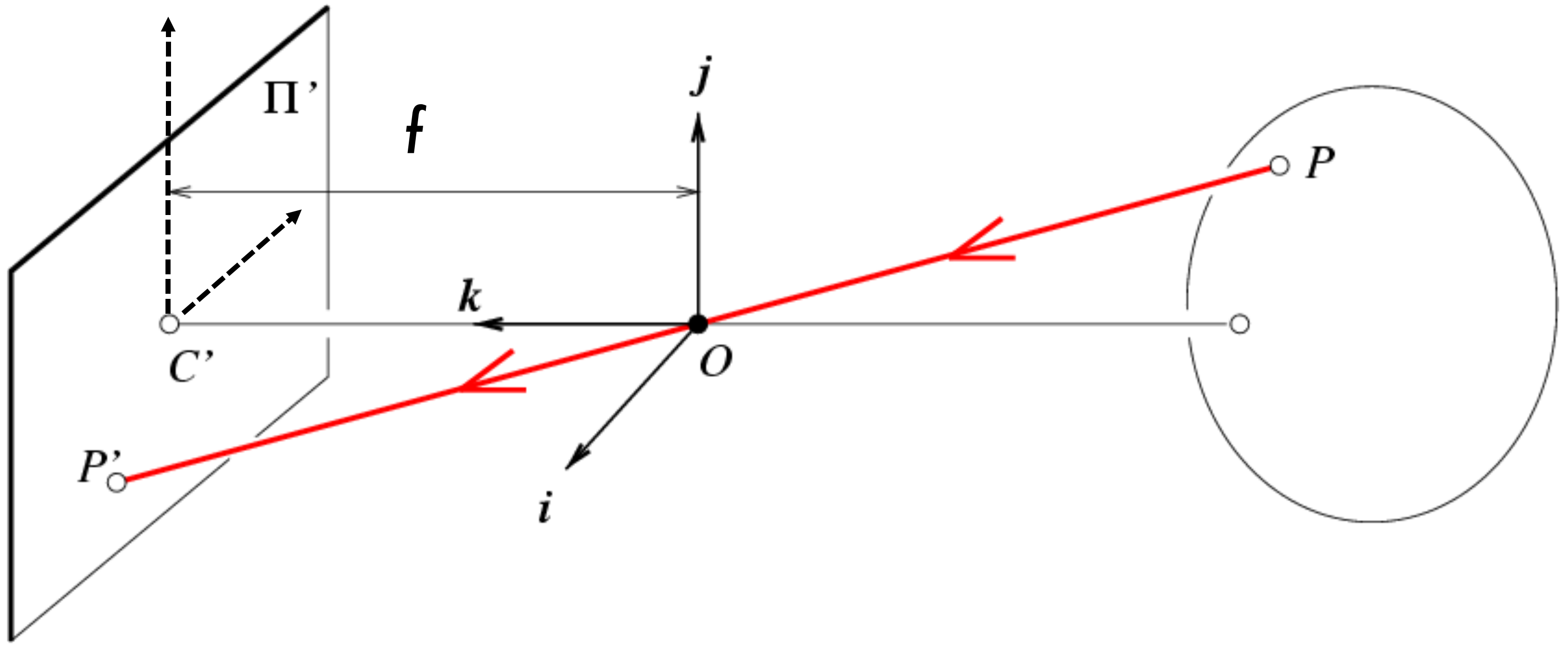


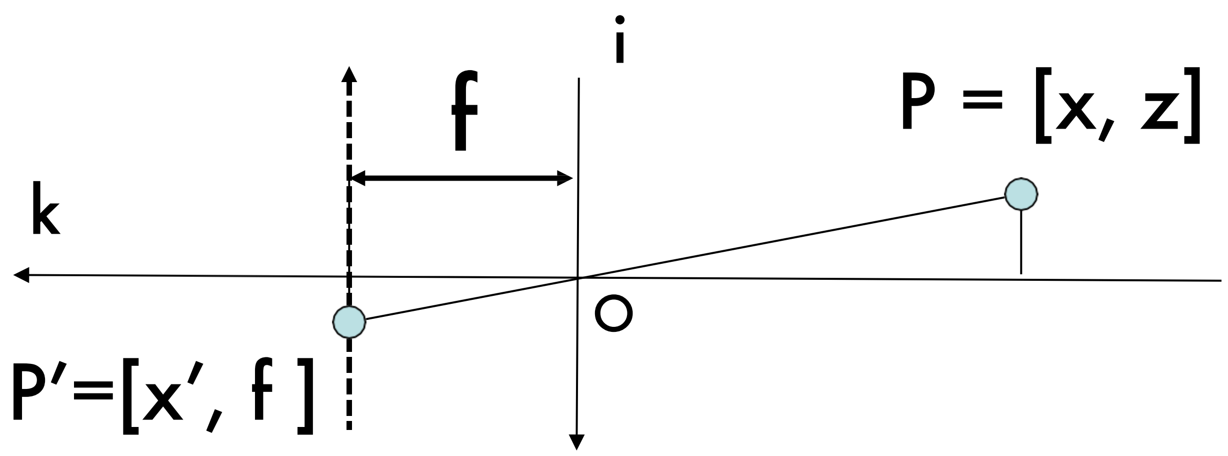
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$ [Eq. 1]

Derived using similar triangles

Pinhole camera



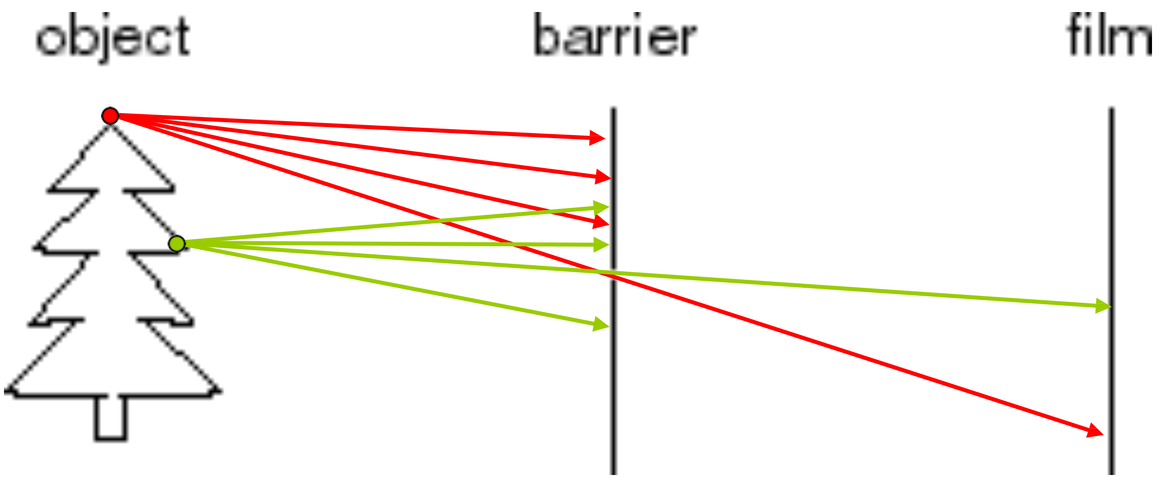


[Eq. 2]

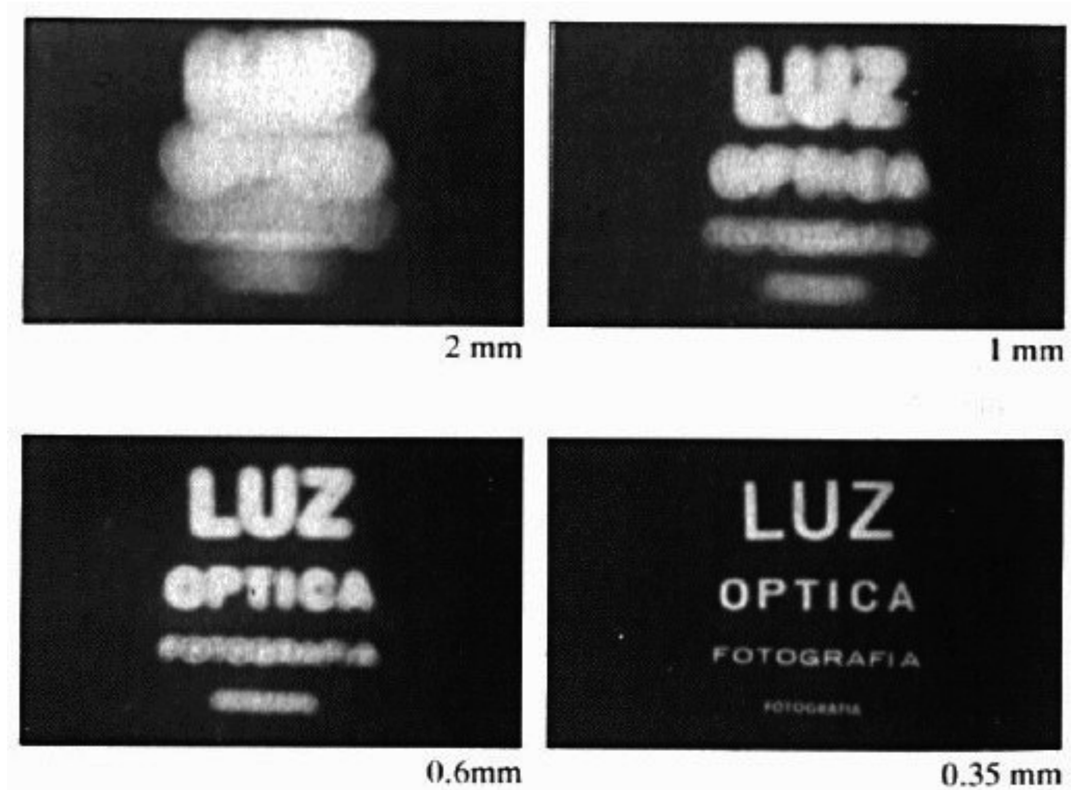
$$\frac{x'}{f} = \frac{x}{z}$$

Pinhole camera

Is the size of the aperture important?



Shrinking aperture size



-What happens if the aperture is too small?

-Less light passes through

Adding lenses!

Agenda

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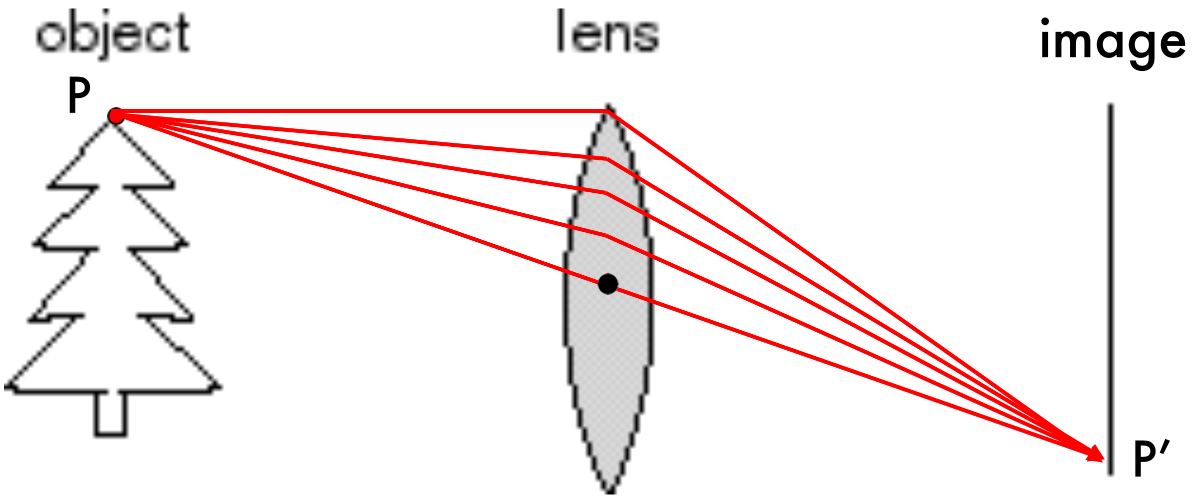
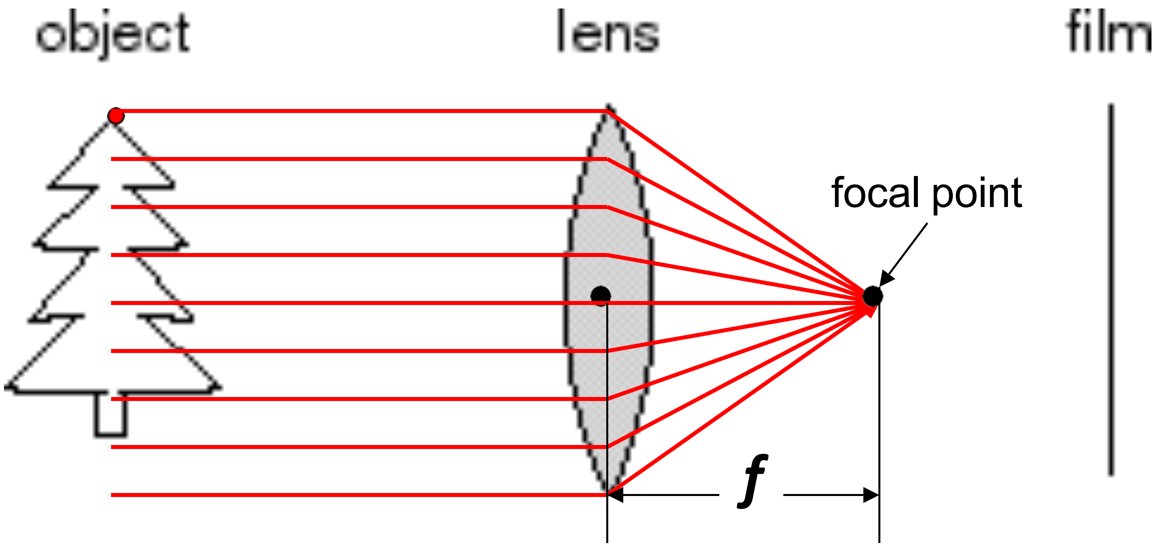
- **Cameras & lenses**
- The geometry of pinhole cameras

Cameras & Lenses



- A lens focuses light onto the film

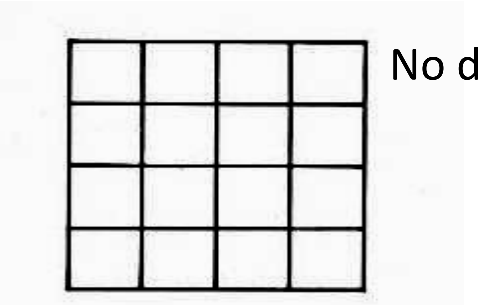
Cameras & Lenses



- A lens focuses light onto the film
  - All rays parallel to the optical (or principal) axis converge to one point (the *focal point*) on a plane located at the *focal length f* from the center of the lens.
  - Rays passing through the center are not deviated

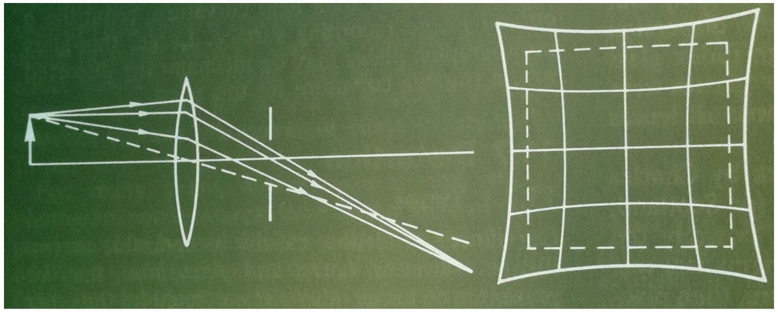
Issues with lenses: Radial Distortion

– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion



Barrel (fisheye lens)

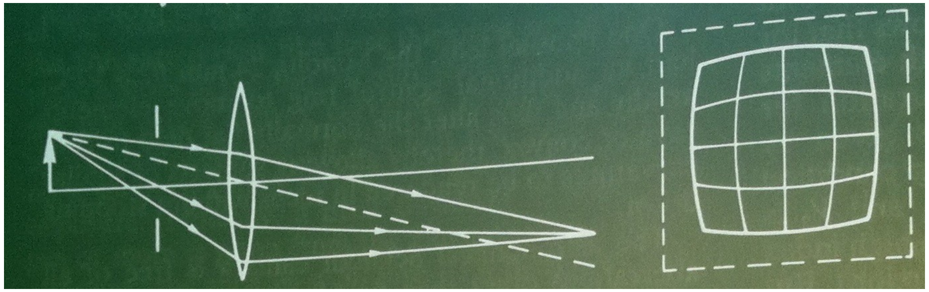
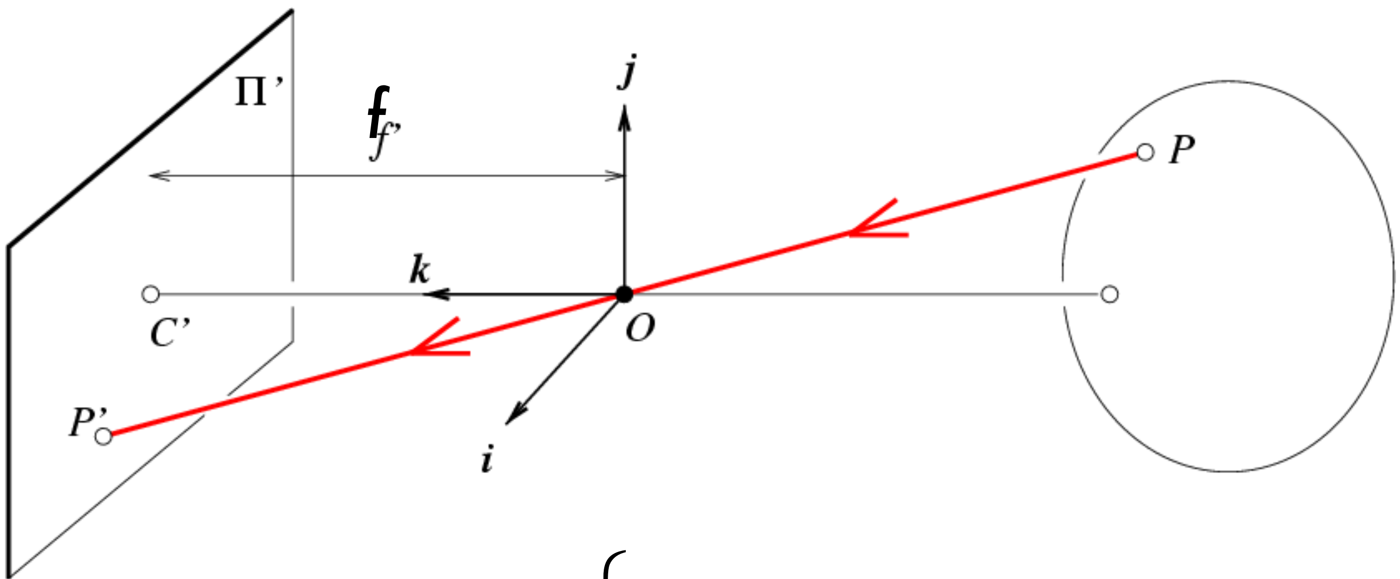


Image magnification decreases with distance from the optical axis

Agenda

- Pinhole cameras
- Cameras & lenses
- **The geometry of pinhole cameras**
  - Intrinsic
  - Extrinsic

Pinhole camera



$$
\begin{cases} x' = f\,\dfrac{x}{z} \\[2ex] y' = f\,\dfrac{y}{z} \end{cases}
$$

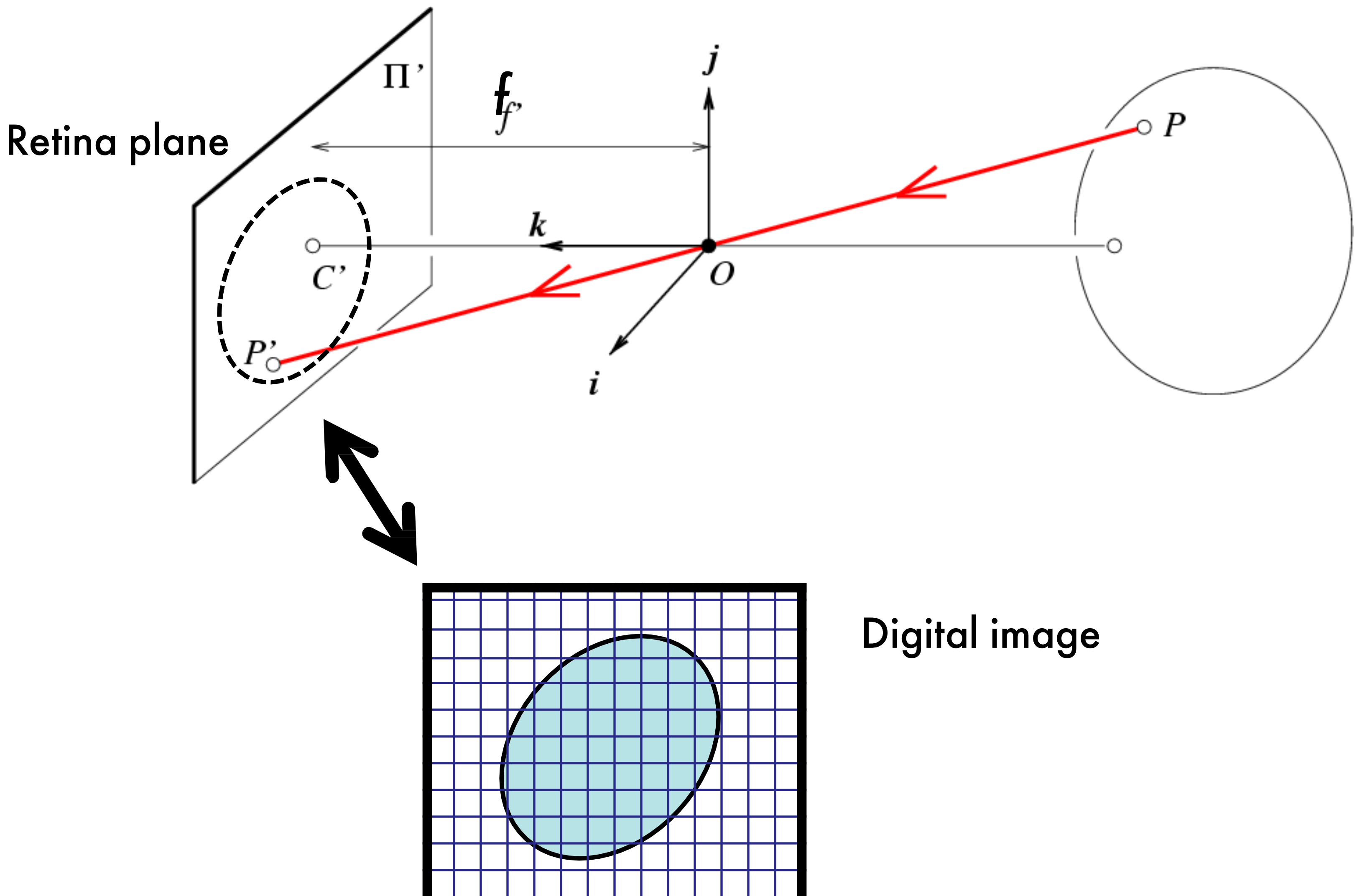
$$
P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}
$$

[Eq. 1]

$$
\Re^3 \xrightarrow{E} \Re^2
$$

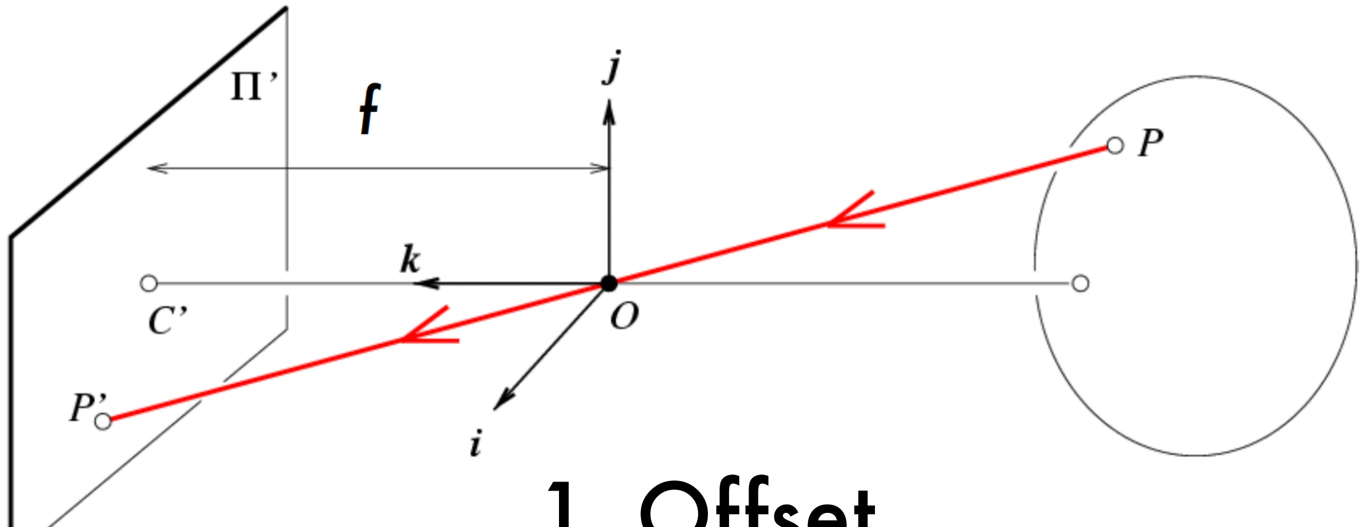
f = focal length
o = center of the camera

From retina plane to images

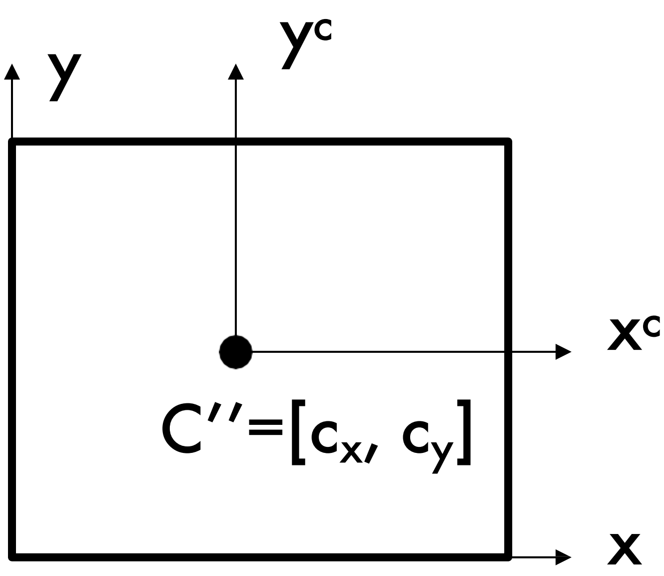


Pixels, bottom-left coordinate systems

Coordinate systems



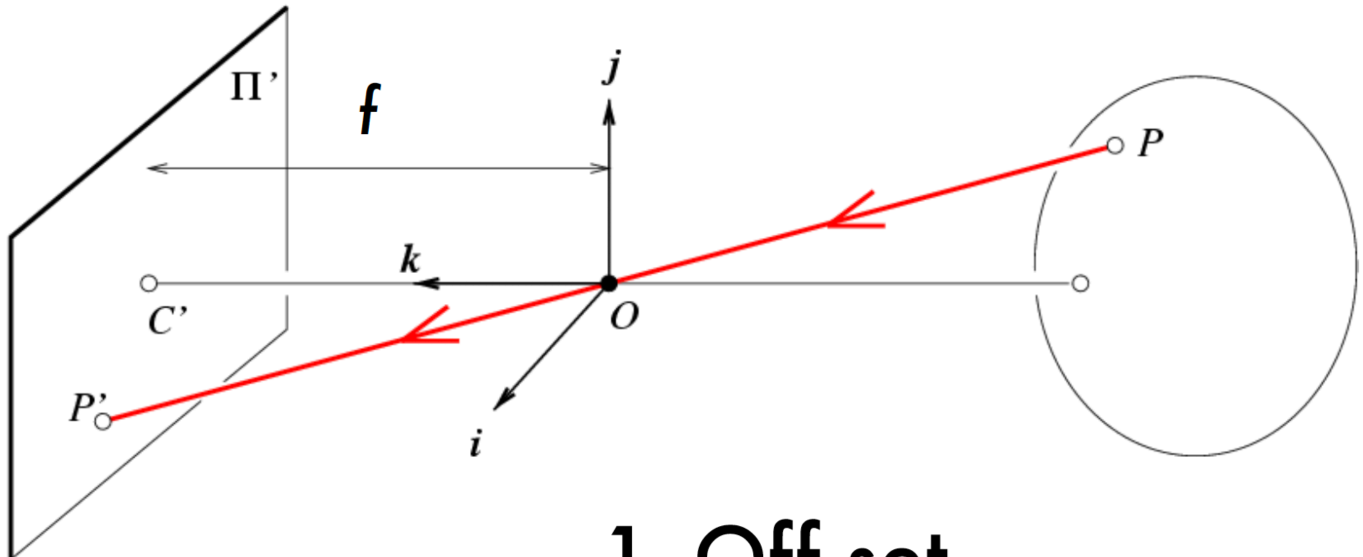
1. Offset



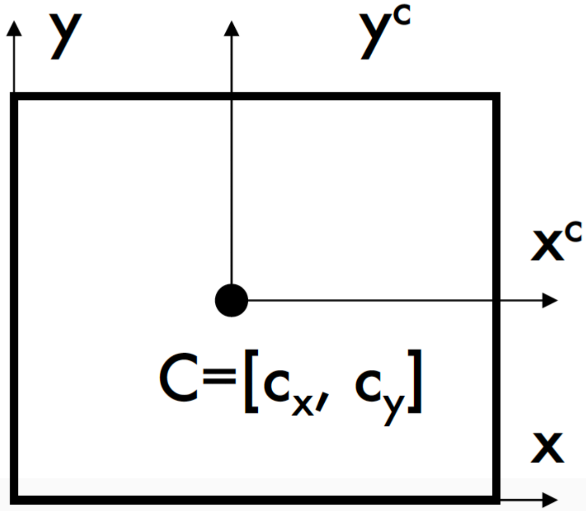
$$(x, y, z) \rightarrow (f\frac{x}{z} + c_x, f\frac{y}{z} + c_y)$$

[Eq. 5]

Converting to pixels



1. Off set
2. From metric to pixels



$$(x, y, z) \rightarrow (\underbrace{\boxed{f\,k}}_{\alpha}\frac{x}{z} + c_x, \underbrace{\boxed{f\,l}}_{\beta}\frac{y}{z} + c_y)$$

[Eq. 6]

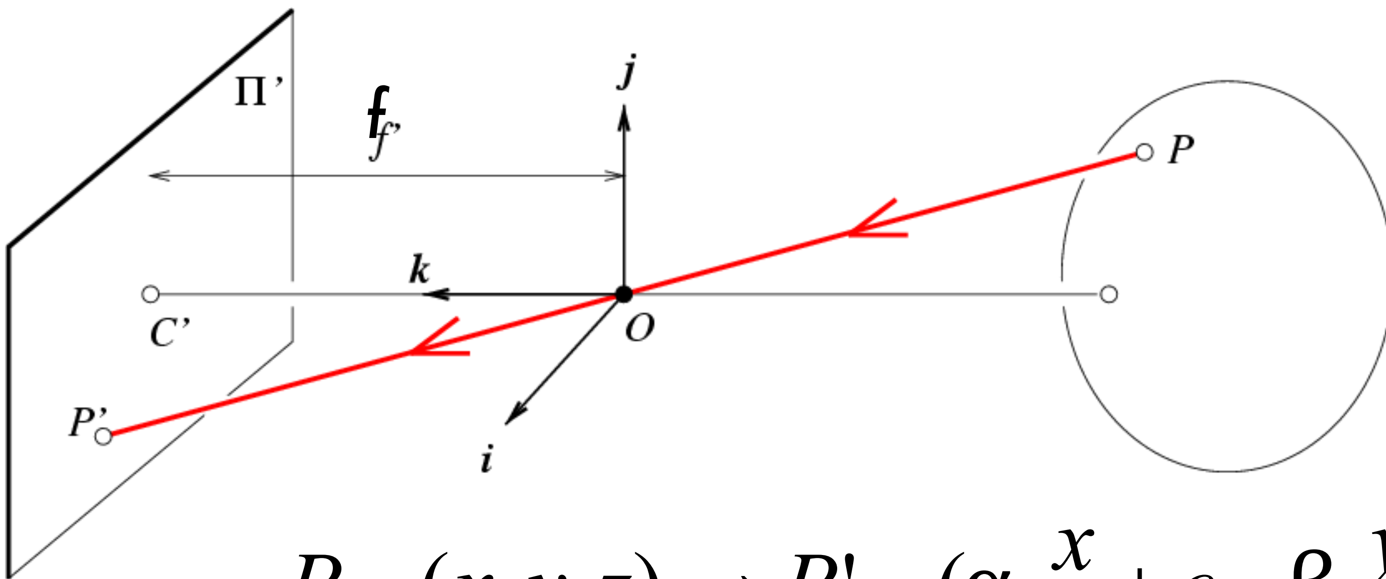
Units: k,l : pixel/m

f : m

Non-square pixels

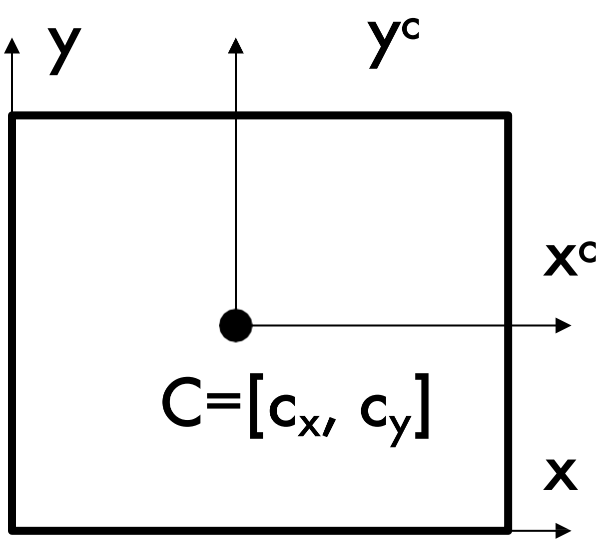
α, β : pixel

Is this projective transformation linear?



$$P = (x, y, z) \rightarrow P' = (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$

[Eq. 7]



- Is this a linear transformation?

  No — division by z is nonlinear

- Can we express it in a matrix form?

Homogeneous coordinates

E→H

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting back *from* homogeneous coordinates

H→E

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Projective transformation in the homogenous coordinate system

$$P_h' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}_{P_h}$$

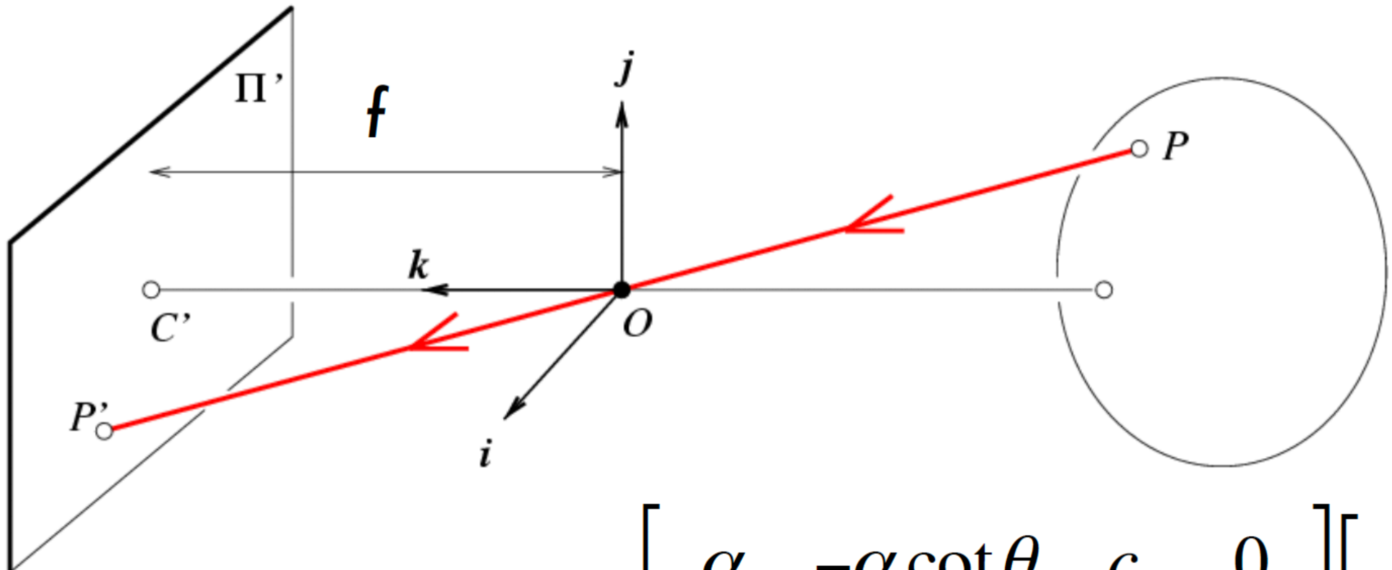
[Eq.8]

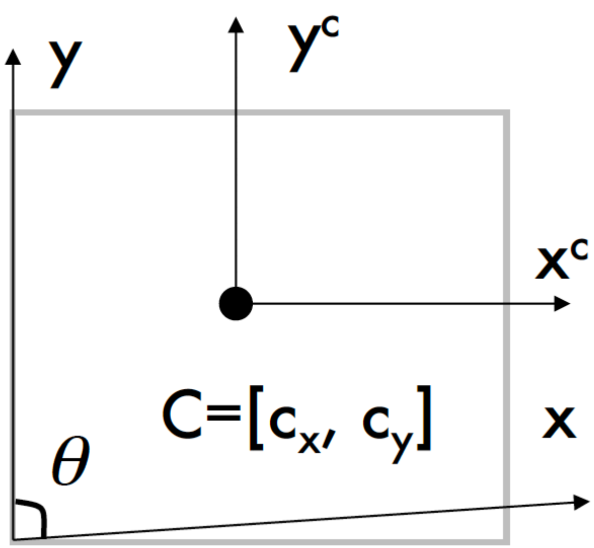
$$M = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\overbrace{P_h'}^{\text{Homogenous}} \rightarrow \overbrace{P' = (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)}^{\text{Euclidian}}$$

The Camera Matrix

Camera Skewness

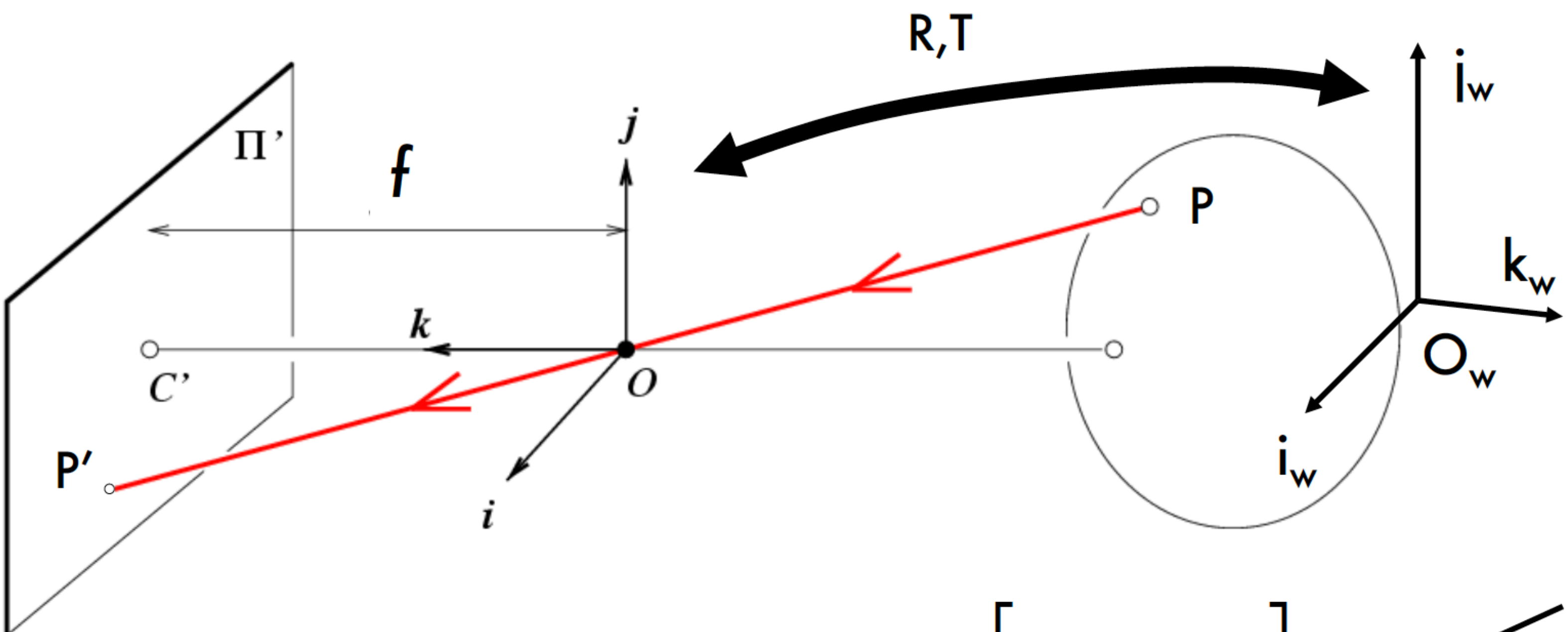




$$P' = \begin{bmatrix} \alpha & -\alpha\cot\theta & c_x & 0 \\ 0 & \dfrac{\beta}{\sin\theta} & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

How many degree does K have?
5 degrees of freedom!

World reference system



In 4D homogeneous coordinates: $P = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times4} P_w$, $P_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$

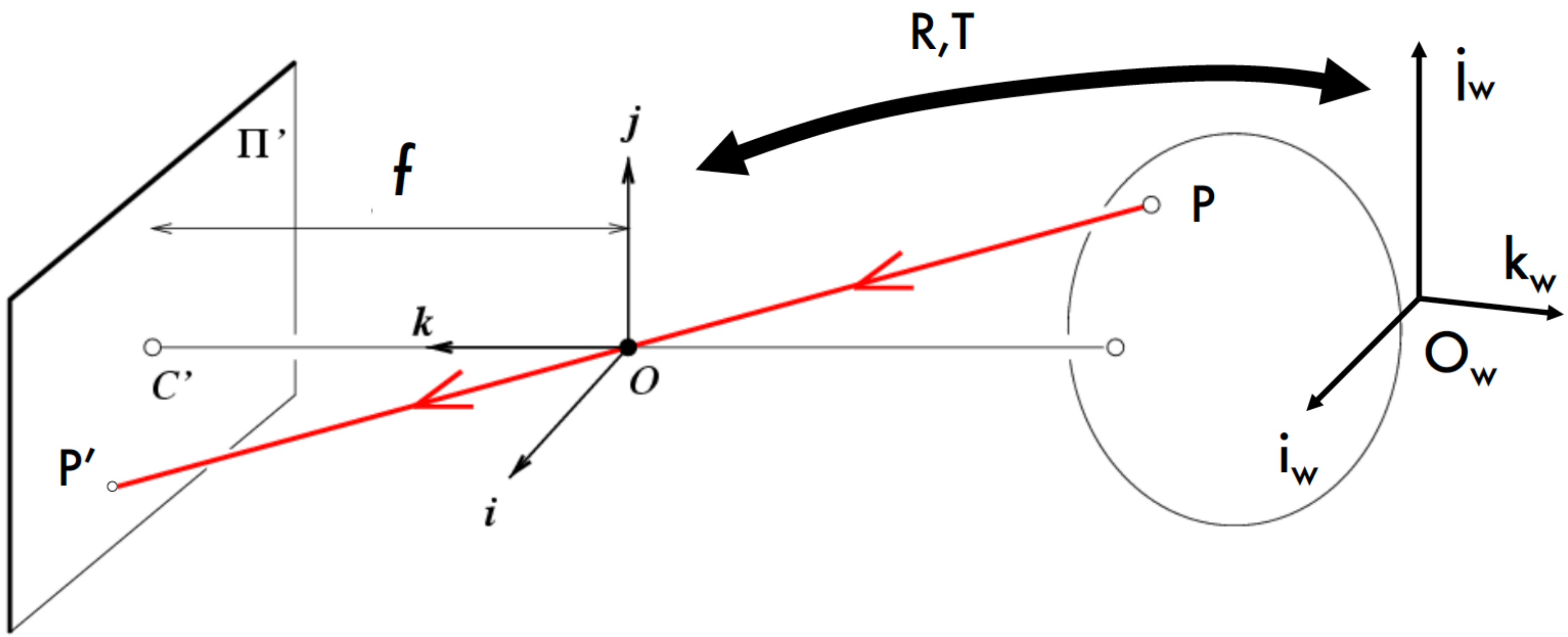
intrinsic parameters

extrinsic parameters

$$P' = K\begin{bmatrix} I & 0 \end{bmatrix} P = K\begin{bmatrix} I & 0 \end{bmatrix}\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times4} P_w = \underbrace{\boxed{K\begin{bmatrix} R & T \end{bmatrix}}}_{M} P_w$$

[Eq.11]

The projective transformation



$$P'_{3\times1} = M_{3x4}\,P_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} P_{w\,4\times1}$$

How many degrees of freedom?

5 + 3 + 3 =11!

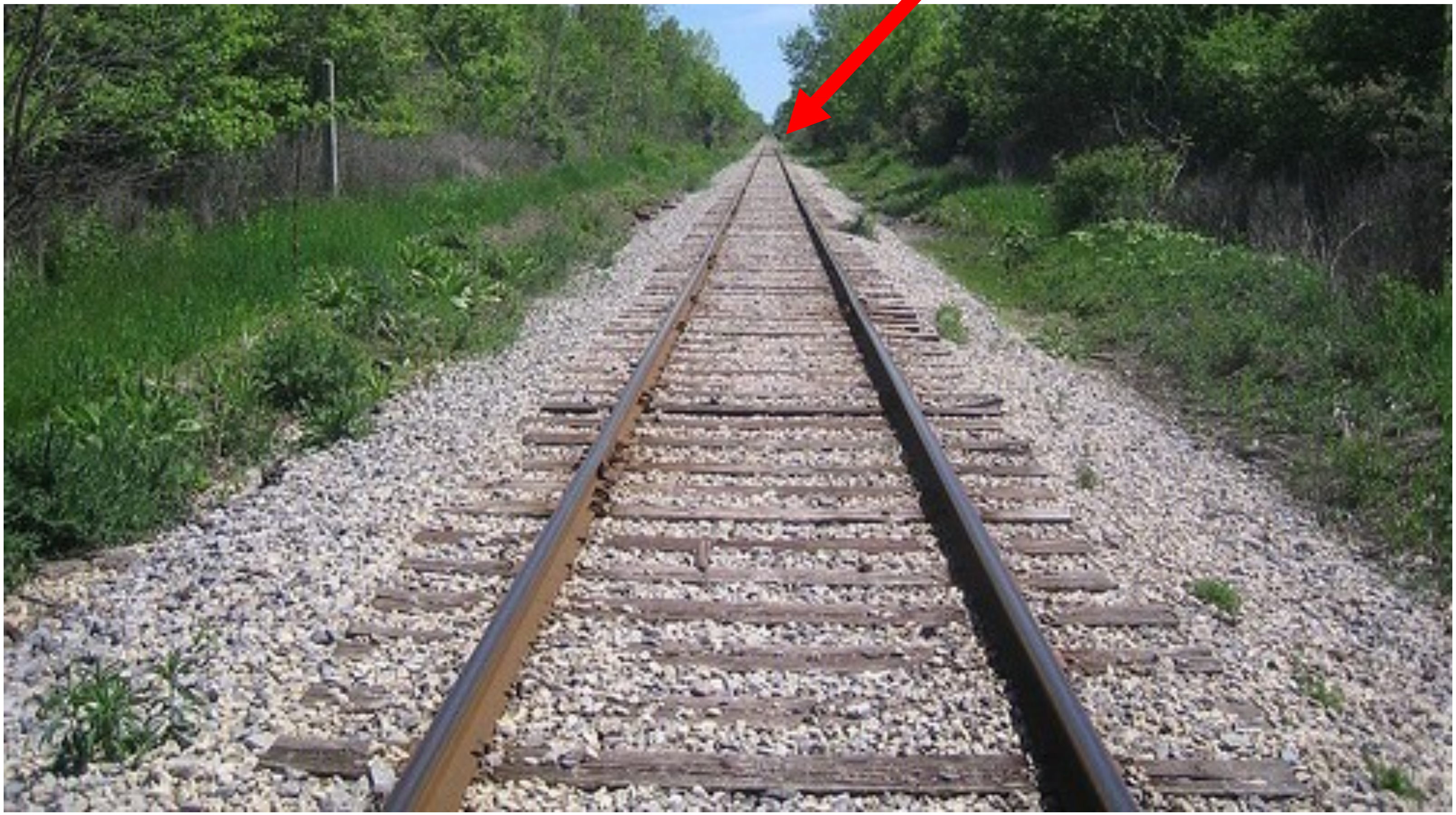
Properties of projective transformations

- Points project to points
- Lines project to lines
- Distant objects look smaller

Properties of Projection

- Angles are not preserved
- Parallel lines meet!

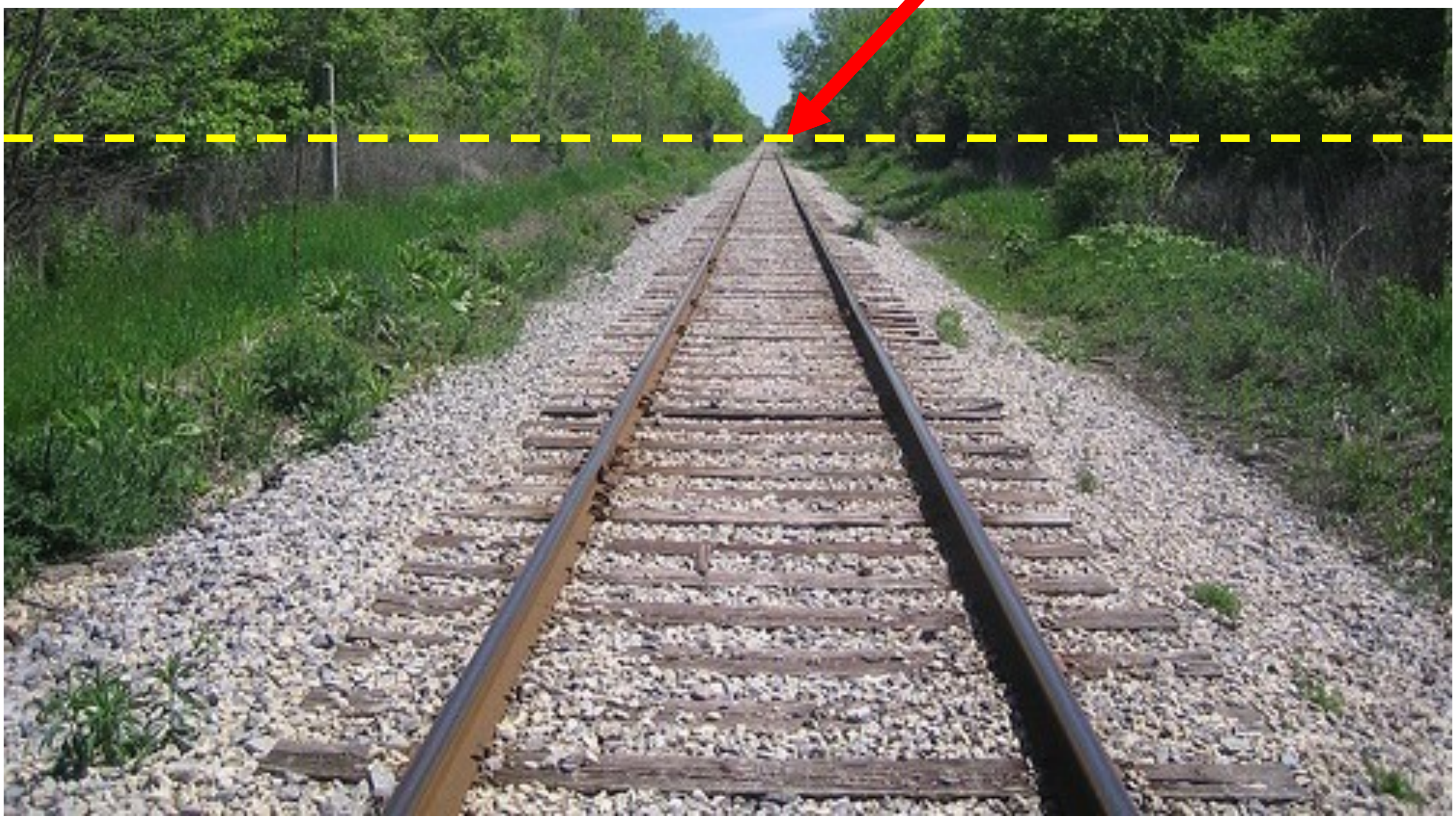
Parallel lines in the world intersect in the image at a "vanishing point"



Horizon line (vanishing line)

- Angles are not preserved
- Parallel lines meet!

Parallel lines in the world intersect in the image at a “vanishing point”



Horizon line (vanishing line)