CSE12 - Lecture 10

Friday, October 20, 2023 8:00 AM

PA3 → hidden tests
feedback on what/check off tests

Lecture 10

Categorizing Runtimes

Let f(n) = 100

Which of the following is **NOT** a correct bound?

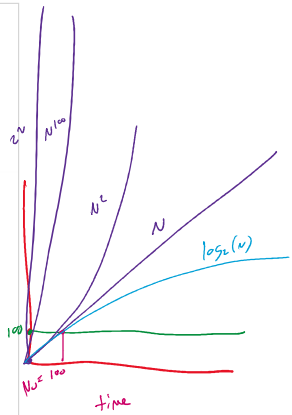
A. f(n) is O(2^n)
B. f(n) is O(n^2)
C. f(n) is O(n)
D. f(n) is O(n^{100})
E. None of these

Big-O ← upper bound

f(n) = O(g(n)), f(n) ≤ c * g(n)
for all n ≥ n0

For each function in the list below, it is related to the function below it by O, and the reverse is not true. That is, n is O(n^2) but n^2 is not O(n).

- f(n) = 1/(n^2)
- f(n) = 1/n
- f(n) = 1
- f(n) = log(n)
- f(n) = sqrt(n)
- f(n) = n
- f(n) = n^2
- f(n) = n^3
- f(n) = n^4
- … and so on for constant polynomials …
- f(n) = 2^n
- f(n) = n!
- f(n) = n^n



f(n) ≤ 12 n^3

Let f(n) = $3n^3$ + 2n + 7

Which of the following is a correct bound?

A. f(n) is O(log(n))
B. f(n) is O(n^2)
C. f(n) is O(n)
D. f(n) is O(n^3)
E. None of these

$3n^3 + 2n^3 + 7n^3 = 12n^3$ $g(N) = N^3$
$C = 12$
$N_0 = 0$

$3n^3 + 2n^3 + 7$ → $5n^3 + 7$ $C = 5$ $N_0 = 7$ $g(N) = N^3$

```
void printAllElementOfArray(int[] arr) {
    for (int i = 0; i < arr.length; i++) {
        printf("%d\n", arr[i]);
    }
}
```

$1 + (N+1) + N$ → $3N + 2$
$C = 3$ $g(N) = N$
$N_0 = 2$

Which of the following is a correct bound?

A. f(n) is O(log(n))
B. f(n) is O(n^2)
C. f(n) is O(n)
D. f(n) is O(n^3)
E. None of these

Name: _____ PID: _____ Code: 6365

Which of the following is a correct bound?

A. f(n) is O(log(n))
B. f(n) is $O(n^2)$
C. f(n) is O(n)
D. f(n) is $O(n^3)$
E. None of these

```
void printAllPossibleOrderedPairs(int arr[]) {
    for (int i = 0; i < arr.length; i++) {
        for (int j = 0; j < arr.length; j++) {
            printf("%d = %d\n", arr[i], arr[j]);
        }
    }
}
```

$1 + (N+1) + N$
$1 + (N+1) + N$ ×N
$2N+2$ +
$(3N+2) \times N$

$3N^2 + 2N + 7N + 2$

$f(N) = 3N^2 + 4N + 2$

$3N^2 + 4N^2 + 2$

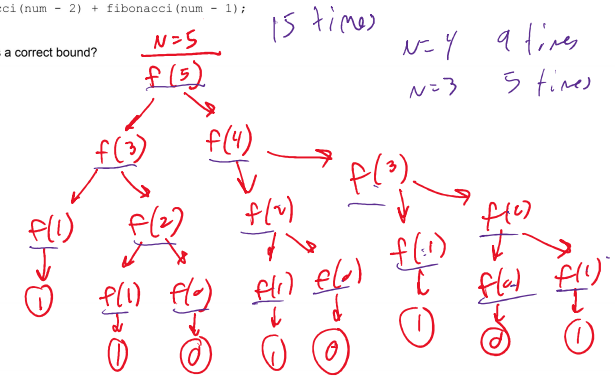
$7N^2 + 2$

$C = 7 \quad g(N) = N^2$
$N_0 = 2 \quad O(g(n))$
$O(N^2)$

Which of the following is a correct bound?

A. f(n) is O(log(n))
B. f(n) is $O(n^2)$
C. f(n) is O(n)
D. f(n) is $O(n^3)$
E. None of these

```
int fibonacci(int num) {
    if (num <= 1) return num;
    return fibonacci(num - 2) + fibonacci(num - 1);
}
```



15 times

$N=4$ 9 times
$N=3$ 5 times

$2^N \rightarrow 2^5 = 32$
$2^4 \rightarrow 16$
$2^3 \rightarrow 8$

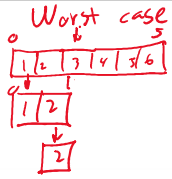
Which of the following is a correct bound?

A. f(n) is $O(2^n)$
B. f(n) is $O(n^2)$
C. f(n) is O(n)
D. f(n) is $O(n^3)$
E. None of these

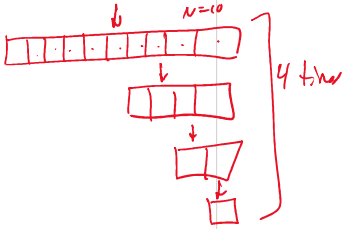
sorted

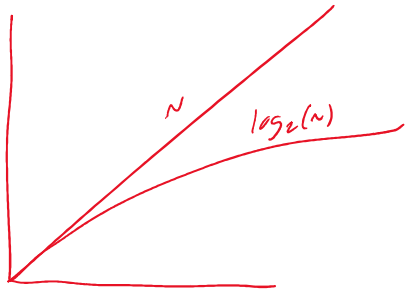
```
// A recursive binary search function. It returns location
// of x in given array arr[l..r] is present, otherwise -1
int binarySearch(int arr[], int l, int r, int x)
{
    if (r >= l) {
        int mid = l + (r - l) / 2;

        // If the element is present at the middle
        // itself
        if (arr[mid] == x)
            return mid;

        // If element is smaller than mid, then
        // it can only be present in left subarray
        if (arr[mid] > x)
            return binarySearch(arr, l, mid - 1, x);

        // Else the element can only be present
        // in right subarray
        return binarySearch(arr, mid + 1, r, x);
    }

    // We reach here when element is not
    // present in array
    return -1;
}
```

Worst case



$X = 2$
$N = 6$
3 times





$\log_2(N) \rightarrow$ # of times you divide by 2 to get 0 (or -1)

What are some correct bounds for binarySearch? What is the smallest correct bound?

$O(N), O(N^2), O(N^3), \ldots$ $O(\log_2(N))$

```
boolean isPrimeAll(int num) {
    // Check for divisors of num
    for (int i = 0; i < num; i += 1) {
        if (num % i == 0) {
            // Any divisor other than 1 or num means num is not prime
            return false;
        }
    }
    // No other divisors found means num is prime
    return true;
}

boolean isPrimeHalf(int num) {
    // Check for divisors of num
    for (int i = 0; i < num / 2; i++) {
        if (num % i == 0) {
            // Any divisor other than 1 or num means num is not prime
            return false;
        }
    }
    // No other divisors found means num is prime
```

What is the smallest correct bound?

$1 + (N+1) + N$
N
1

$3N + 3$

$C = 3 \quad g(N) = N \quad O(N)$
$N_0 = 3$

What is the smallest correct bound?

$1 + (\frac{N}{2} + 1) + \frac{N}{2}$
$\frac{N}{2}$
1

$\frac{3N}{2} + 3$

$C = 3/2 \quad g(N) = N \quad O(N)$
$N_0 = 3$

```
        if (num % i == 0) {
            // Any divisor other than 1 or num means num is not prime
            return false;
        }
    }
    // No other divisors found means num is prime
    return true;
}
```

```
void printAllItemsTwice(int arr[], int size)
{
    for (int i = 0; i < size; i++) {
        printf("%d\n", arr[i]);
    }

    for (int i = 0; i < size; i++) {
        printf("%d\n", arr[i]);
    }
}
```

What is the smallest correct bound?

```
void printFirstItemThenFirstHalfThenSayHi100Times(int arr[], int size)
{
    printf("First element of array = %d\n",arr[0]);

    for (int i = 0; i < size/2; i++) {
        printf("%d\n", arr[i]);
    }

    for (int i = 0; i < 100; i++) {
        printf("Hi\n");
    }
}
```

What is the smallest correct bound?

```
void printAllNumbersThenAllPairSums(int arr[], int size)
{
    for (int i = 0; i < size; i++) {
        printf("%d\n", arr[i]);
    }

    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            printf("%d\n", arr[i] + arr[j]);
        }
    }
}
```

What is the smallest correct bound?