

Timing channels/thinking attacks

	Insertion	Selection	Merge	Quick
Best case time	Sorted array $\Theta(n)$	$\Theta(n^2)$	$\Theta(n \log_2(n))$	$\Theta(n \log_2(n))$
Worst case time	Reverse-sorted array $\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n \log_2(n))$	$\Theta(n^2)$ *avg $\Theta(n \log(n))$
Key operations	swap(a, j, j-1) (until in the right place)	swap(a, i, indexOfMin) (after finding minimum value)	l = copy(a, 0, len/2) r = copy(a, len/2, len) ls = sort(l) rs = sort(r) merge(ls, rs)	p = partition(a, l, h) sort(a, l, p) sort(a, p + 1, h)

{ 1, 4, 3, 2 }

{ 1, 4, 5, 2 }

- keep sorted flag

- if i = minIndex, break

```

import java.util.Arrays;
public class Sort {
    static void selectionSort(int[] arr) {
        for(int i = 0; i < arr.length; i += 1) {
            int minIndex = i;
            for(int j = i; j < arr.length; j += 1) {
                if(arr[minIndex] > arr[j]) { minIndex = j; }
            }
            else { break; } // X
            int temp = arr[i];
            arr[i] = arr[minIndex];
            arr[minIndex] = temp;
        }
    }

    static void insertionSort(int[] arr) {
        for(int i = 0; i < arr.length; i += 1) {
            for(int j = i; j > 0; j -= 1) {
                if(arr[j] < arr[j-1]) {
                    int temp = arr[j-1];
                    arr[j-1] = arr[j];
                    arr[j] = temp;
                }
            }
            else { break; } // new! exit inner loop early
        }
    }
}

```

- 2 indices

$l=1$
 $7 < 8$
 $i=2$
 $6 < 8$
 $6 < 7$
 $i=3$
 $5 < 8$
 $5 < 7$
 $5 < 6$

8 7 6 5 3

 3 5 6 7 8

$i=0$
 $i=1$
 $5 < 3$ ✓
 $break$
 $i=2$
 $6 < 5$ ✓
 $break$
 $i=3$
 $7 < 6$ ✓
 $break$

Best case?

- A: Array in decreasing order
- B: Array in increasing order
- C: Array of all equal elements
- D: Something else
- E: More than one of A-C

Divide and Conquer

```

import java.util.Arrays;
public class SortFaster {
    merge
    static int[] combine(int[] p1, int[] p2) {...}

    static int[] mergeSort(int[] arr) {
        int len = arr.length
        if(len <= 1) { return arr; }
        else {
            int[] p1 = Arrays.copyOfRange(arr, 0, len / 2);
            int[] p2 = Arrays.copyOfRange(arr, len / 2, len);
            int[] sortedPart1 = mergeSort(p1);
            int[] sortedPart2 = mergeSort(p2);
            int[] sorted = combine(sortedPart1, sortedPart2);
            return sorted;
        }
    }
}

```

Pivot:

- median value → can you do median in $O(n)$
- random index
- use min/max
- $(high - low) / 2 + low$

```

static int partition(String[] array, int l, int h) {...}

```

```

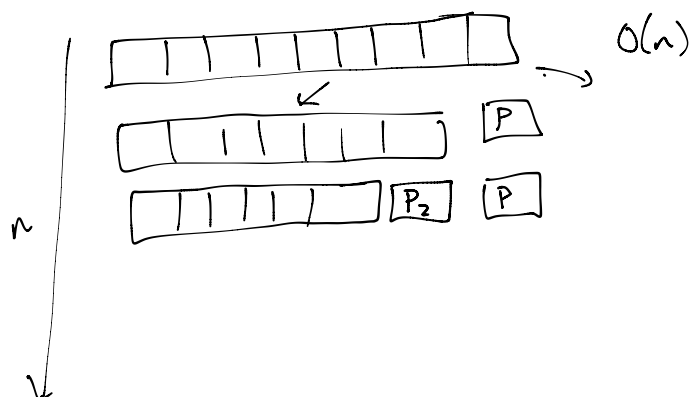
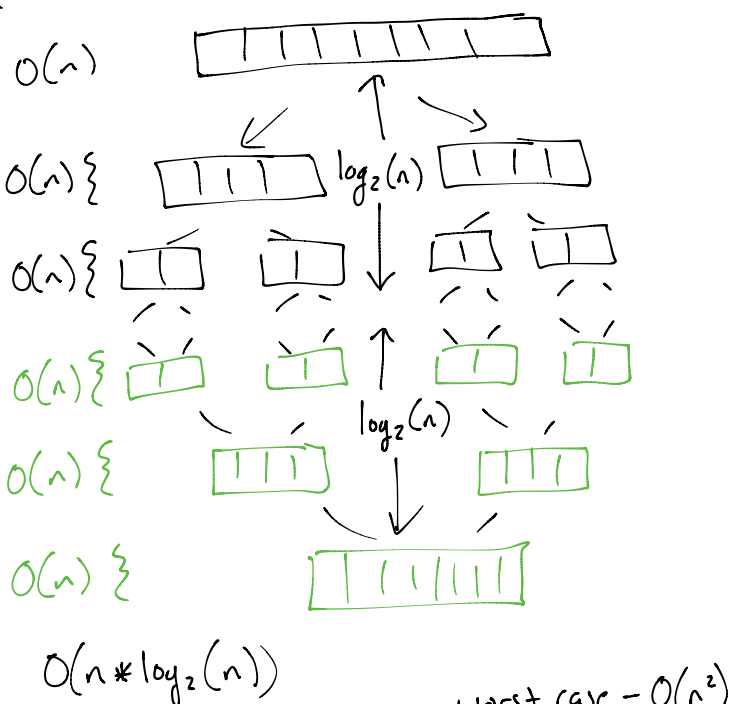
static void qsort(String[] array, int low, int high) {
    if(high - low <= 1) { return; }
    int splitAt = partition(array, low, high);
    qsort(array, low, splitAt);
    qsort(array, splitAt + 1, high);
}

```

```

public static void sort(String[] array) {
    qsort(array, 0, array.length);
}

```



```

interface Map<Key, Value> {

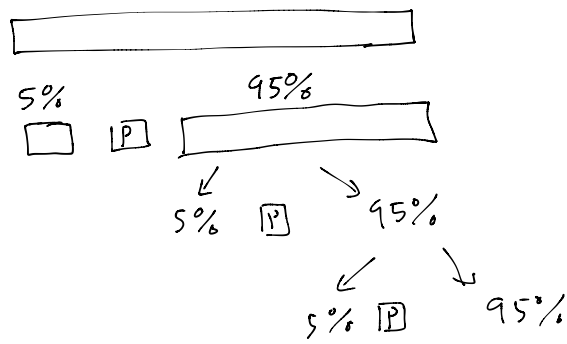
```

```

}

```

Randomized
Algorithm



$$n * .95 \quad \log_{\frac{1}{.95}}(n)$$

$$n * 0.5$$

$$\log_{\frac{1}{0.5}}(n) \quad \log_2(n)$$