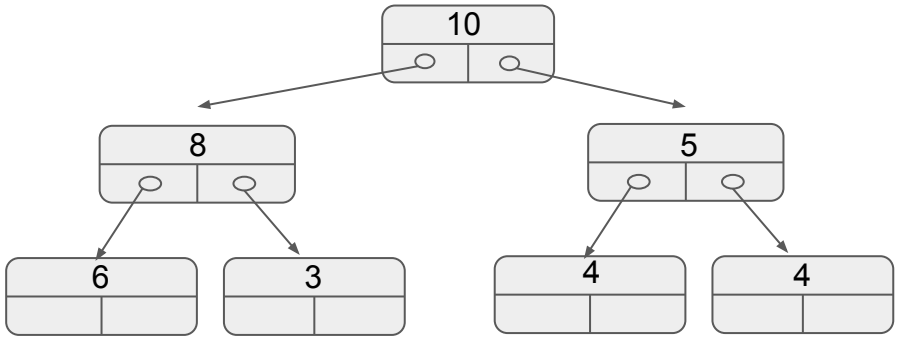
Definition: A tree is a **complete tree** if every level but the last level is completely full, and the last level has its nodes all the way to the **left**.

Property: A complete tree's size and height are related by: *height ~ log(size)*

Definition: A tree is in **max** (min) **heap order** if every node's key is **greater** (less) than or equal to all of its childrens' keys.

Definition: A **max** (min) **heap** is a **complete tree** that is in **max** (min) **heap order**.



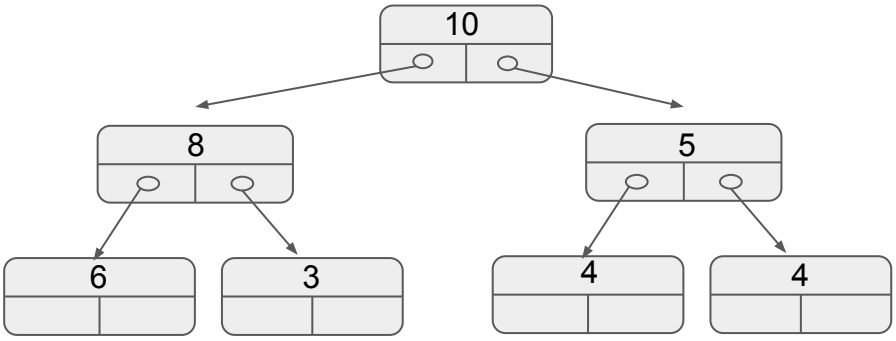
10	8	5	6	3	4	4

Key	index	Parent	Left	Right
10	0	N/A	1	2
8	1	0	3	4
5	2			
6	3	1	N/A	N/A
3	4			
4	5	2	N/A	N/A
4	6			

```
class BT<K,V> {

Node<K,V> root;

...


V get(Node<K,V> node, K key) {
  if(node == null) { return null; }

  if(node.key.equals(key)) { return node.value; }

  V leftResult = get(node.left, key);
  V rightResult = get(node.right, key);
  if(leftResult != null) { return leftResult; }
  if(rightResult != null) { return rightResult; }
  return null;
}

}
```

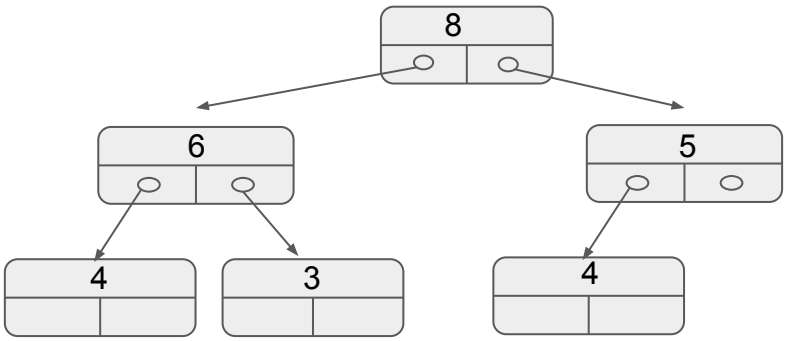
```
class Heap<K,V> {

List<Entry<K,V>> entries;

int left(int index) {



}

int right(int index) {



}

V get(int index, K key) {
  if(index >= this.entries.size()) { return null; }
  Entry<K,V> entry = entries.get(index);

  if(entry.key.equals(key)) { return entry.value; }

  V leftResult = get(left(index), key);
  V rightResult = get(right(index), key);
  if(leftResult != null) { return leftResult; }
  if(rightResult != null) { return rightResult; }
  return null;
}

}
```

```
K poll()

// remove and return largest element
```



10	8	5	6	3	4	4

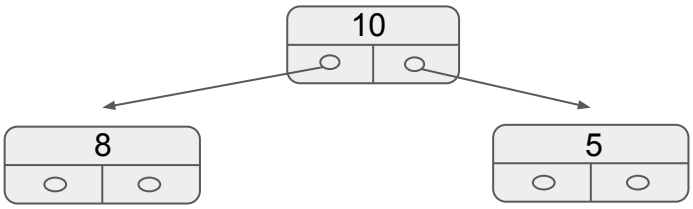
```
void add(K k)
// add the element, ensuring heap-ness

add(2)

add(9)
```

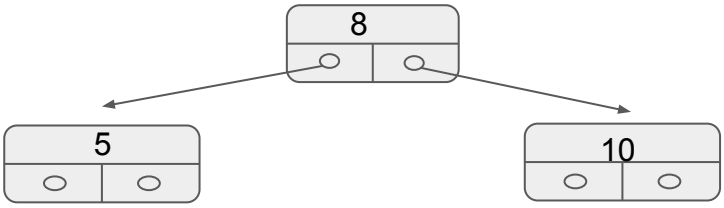


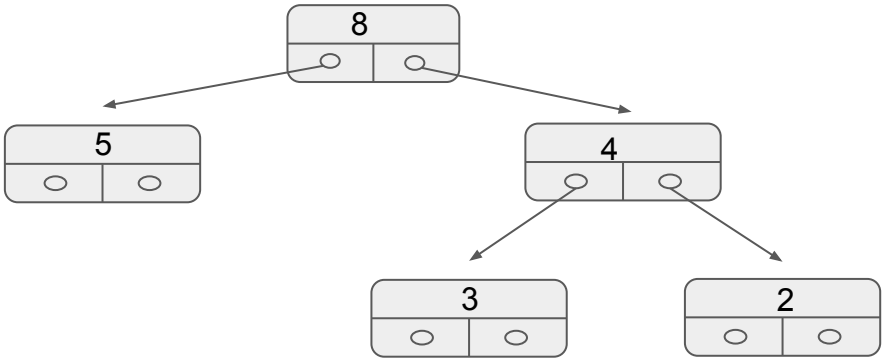
8	6	5	4	3	4	

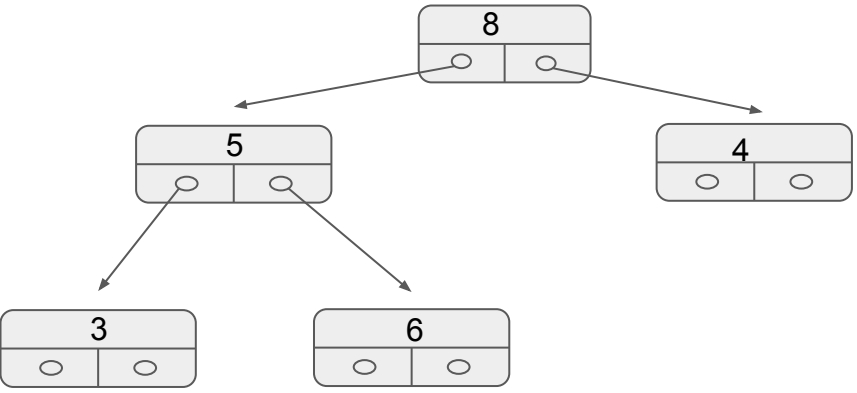
```
void bubbleDown(int index) {
  if(index >= this.entries.size()) { return; }
  int leftIndex = left(index);
  if(leftIndex >= this.entries.size()) { return; }
  int largerChildIndex = leftIndex;
  int rightIndex = right(index);
  if(existsAndGreater(rightIndex, leftIndex)) {
    largerChildIndex = rightIndex;
  }
  if(existsAndGreater(largerChildIndex, index)) {
    swap(index, largerChildIndex);
    bubbleDown(largerChildIndex);
  }
}
```

```
void bubbleUp(int index) {
  if(index <= 0) { return; }
  Entry<K,V> e = this.entries.get(index);
  Entry<K,V> parent = this.entries.get(parent(index));
  int comp = this.comparator.compare(e.key, parent.key);
  if(comp > 0) {
    swap(index, parent(index));
    bubbleUp(parent(index));
  }
  else {
    return;
  }
}
```


10
8
5


8
5
10


8
5
4
3
2


8
5
4
3
6