

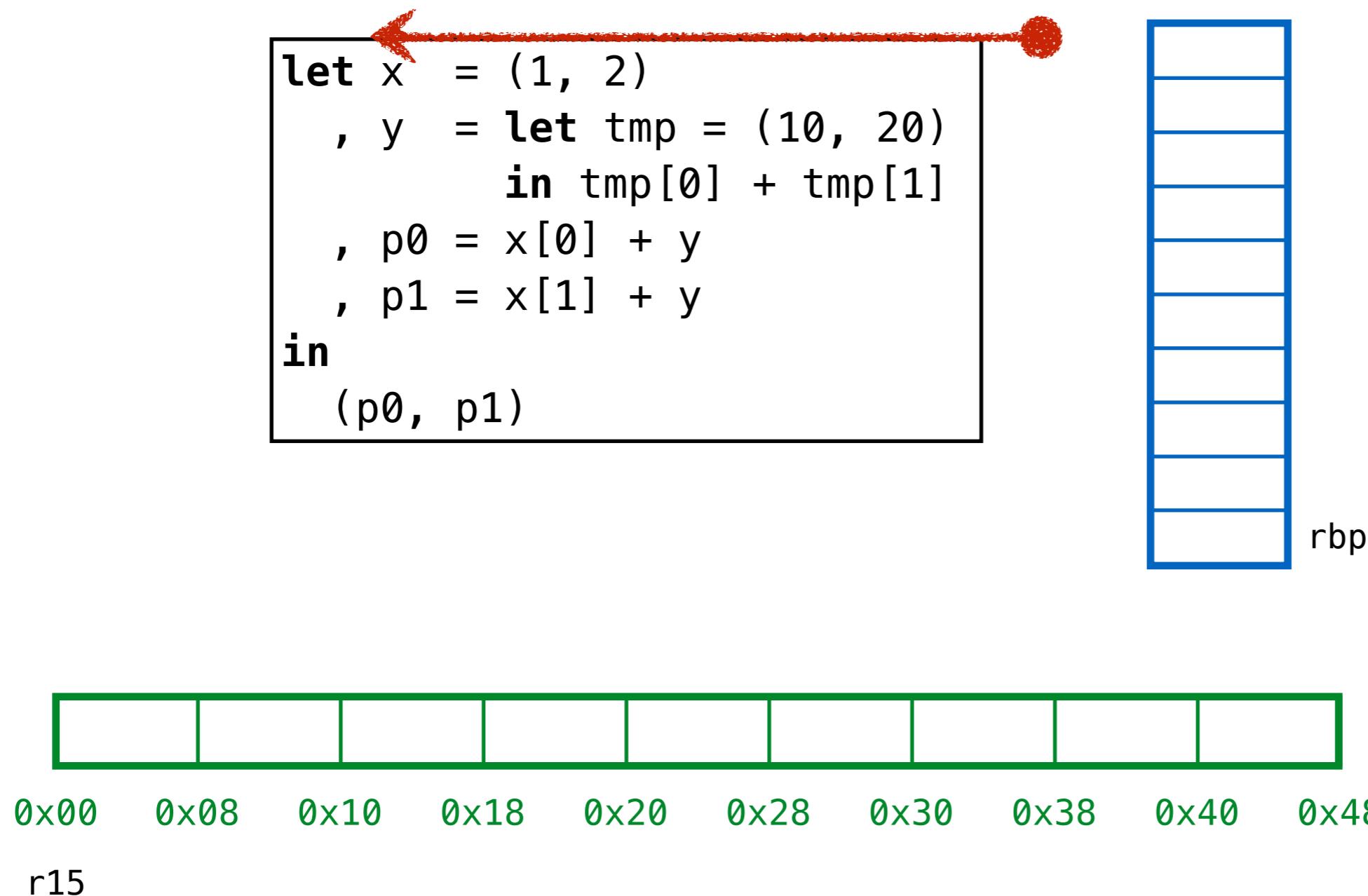
Garter

Garbage Collection

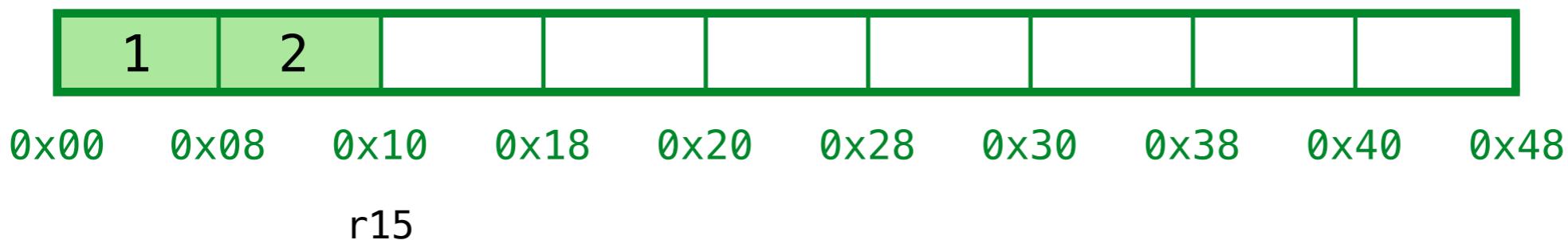
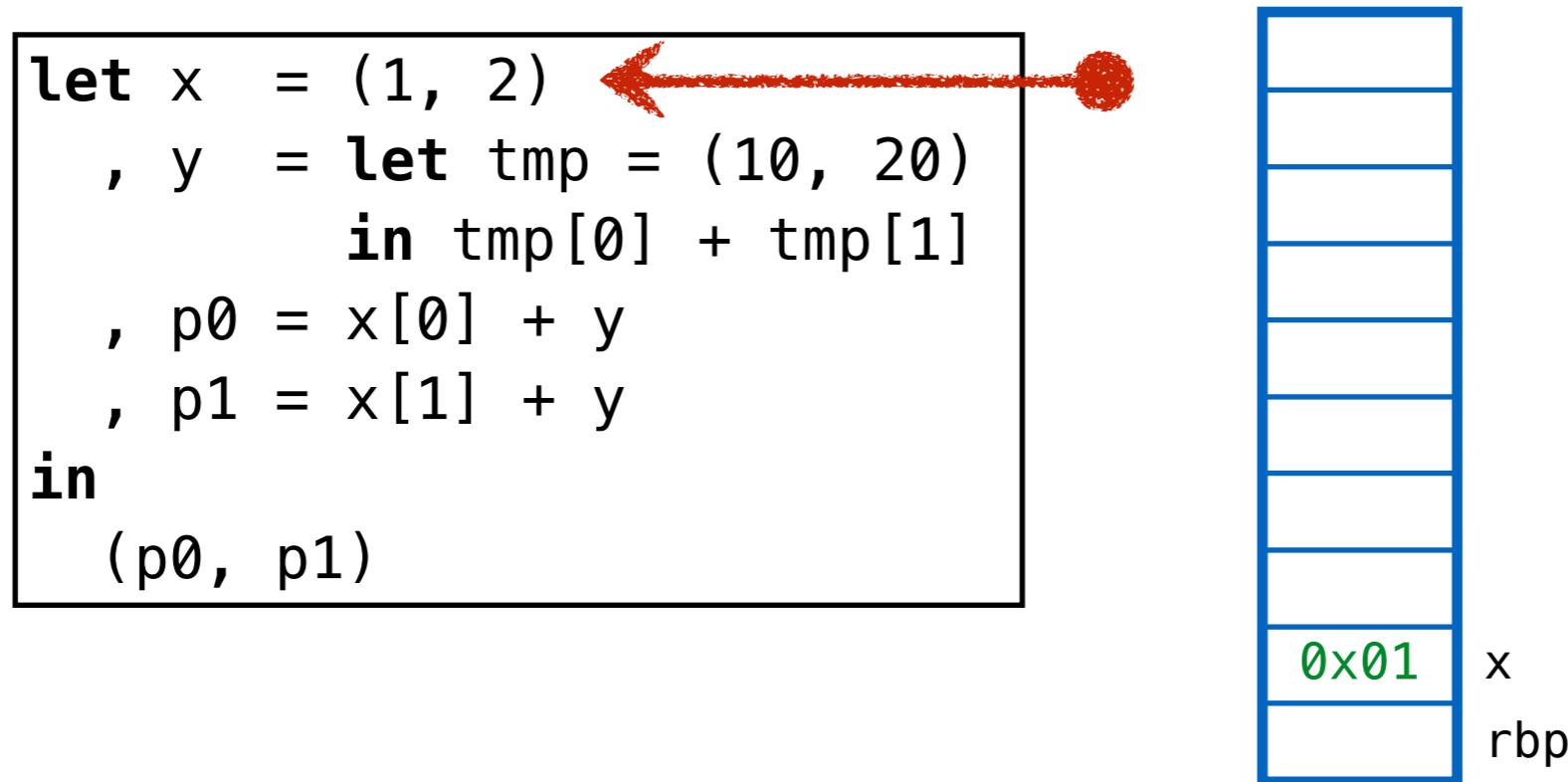
Garter / GC

Example 1

ex1: garbage at end

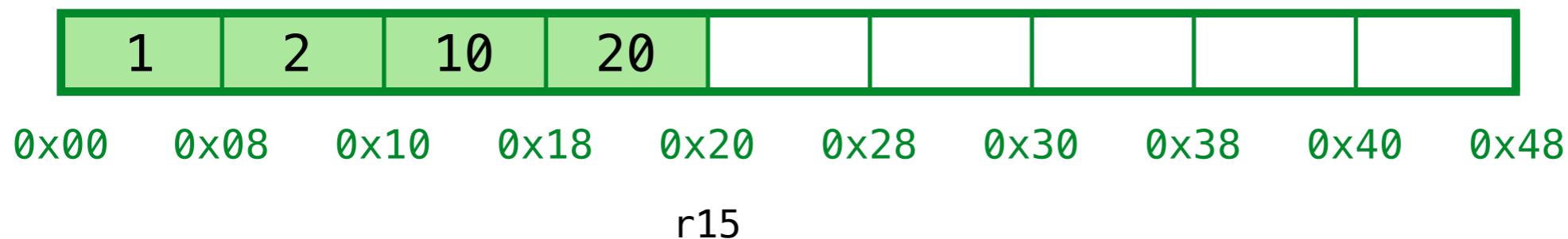
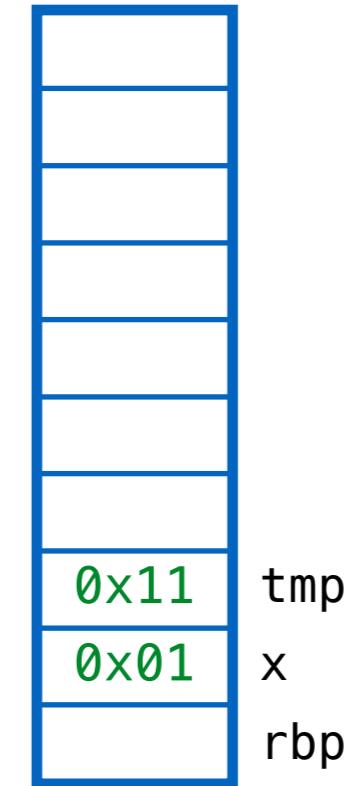


ex1: garbage at end



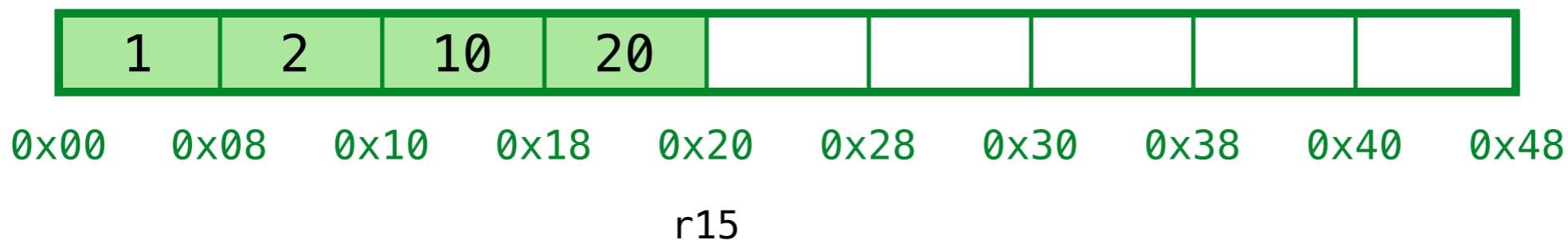
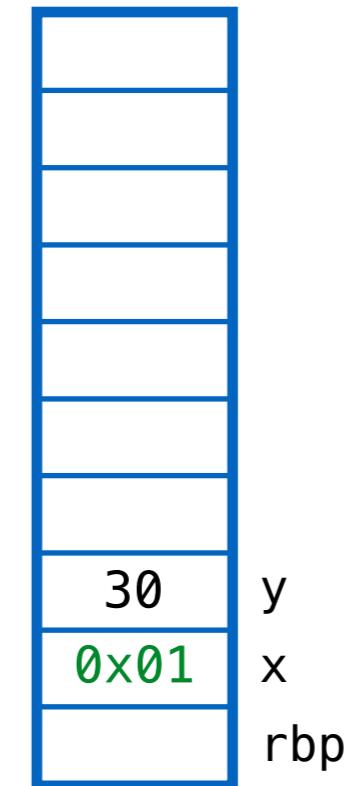
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20) ←
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



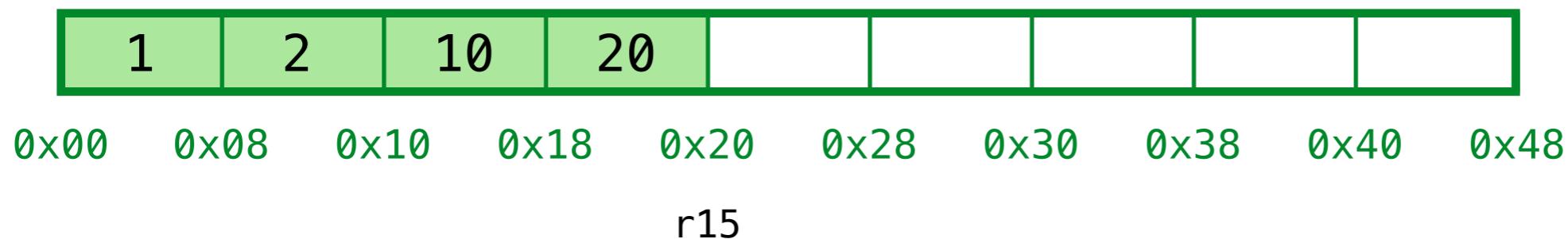
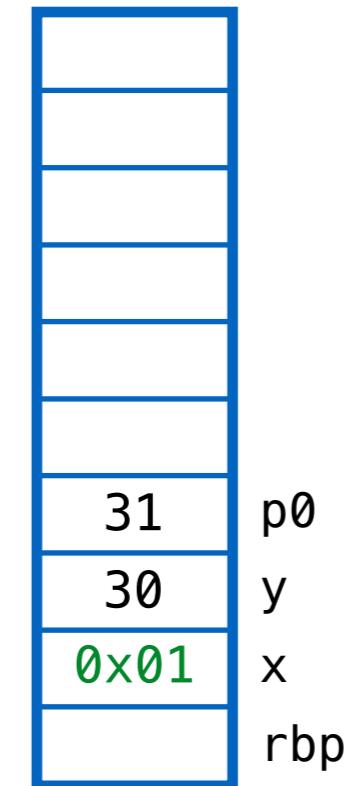
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



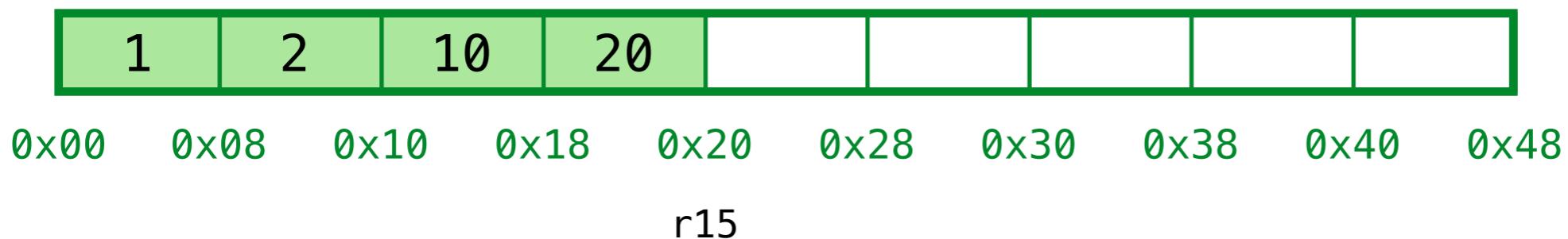
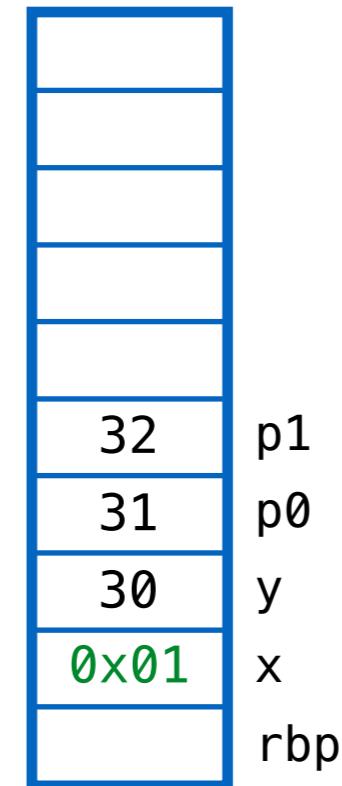
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



ex1: garbage at end

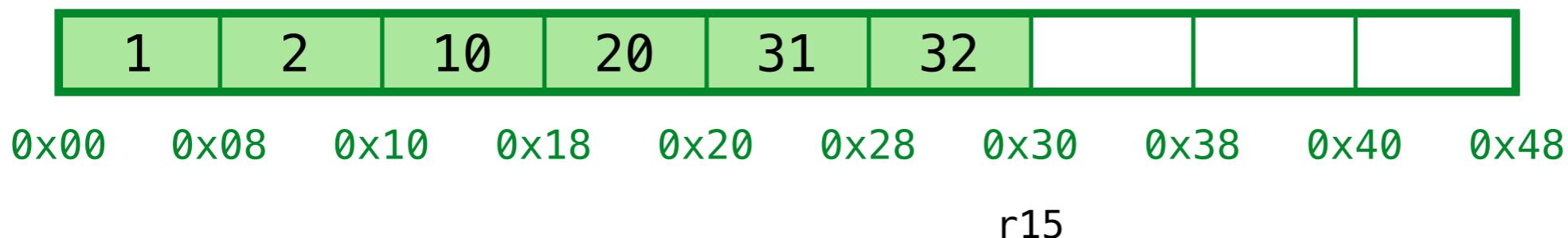
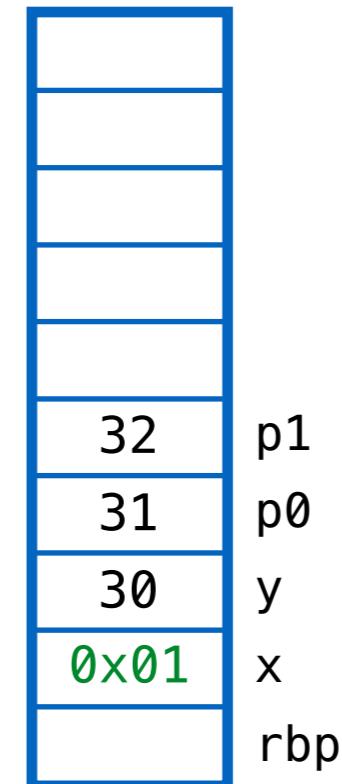
```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```



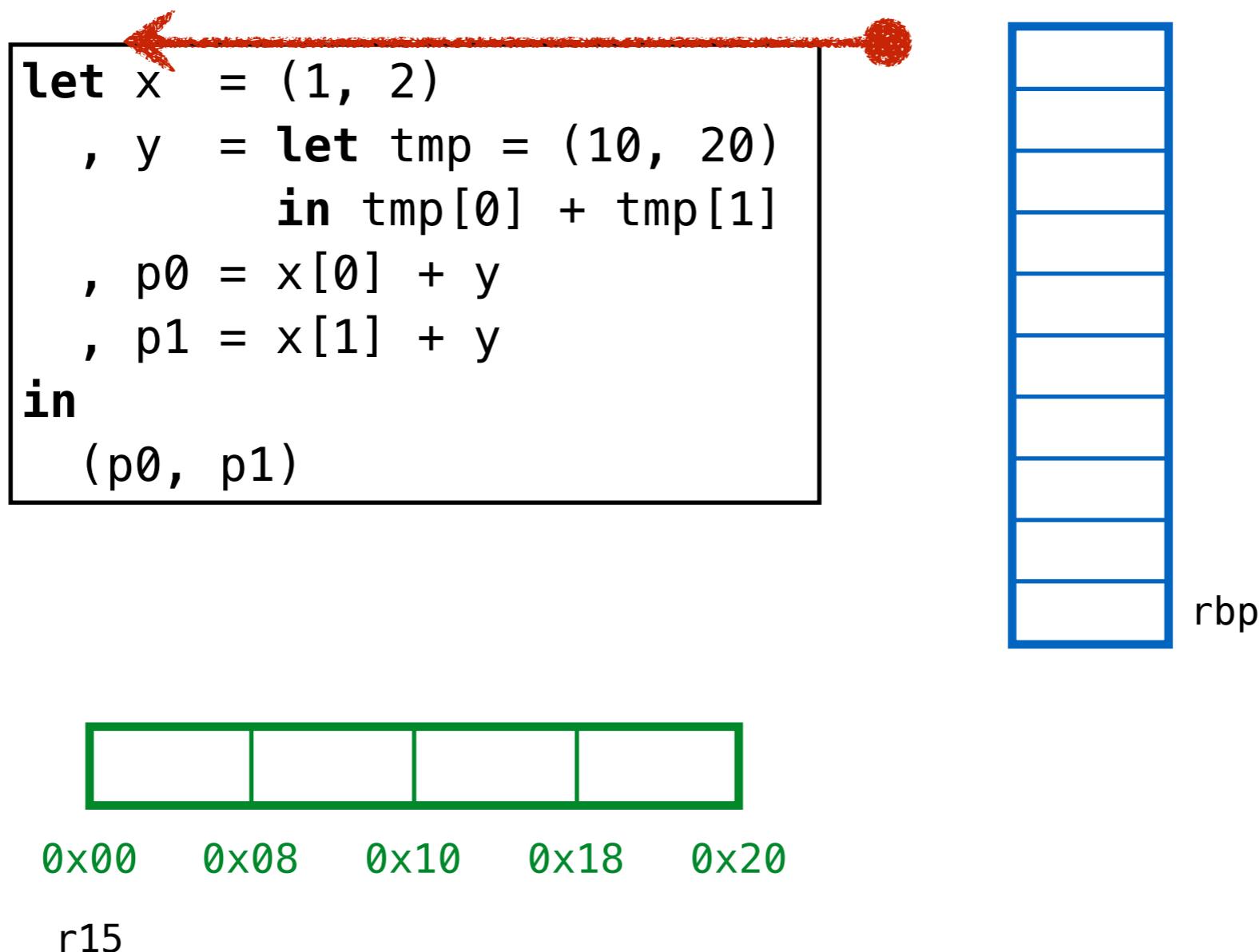
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```

Result (rax) = 0x21



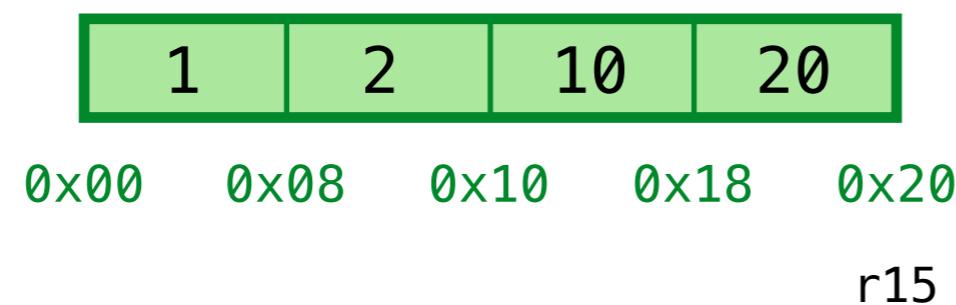
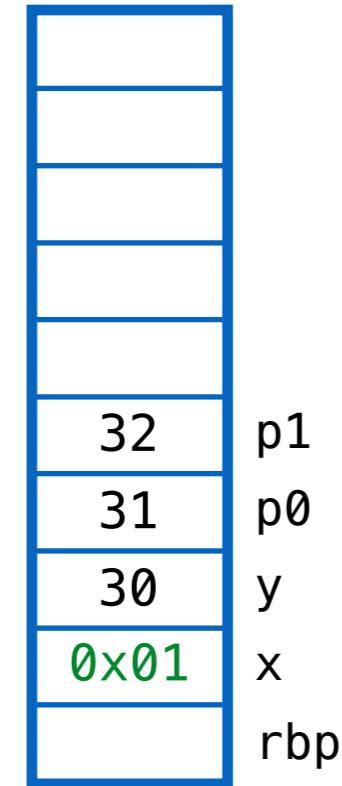
ex1: garbage at end



Suppose we had a smaller, 4-word heap

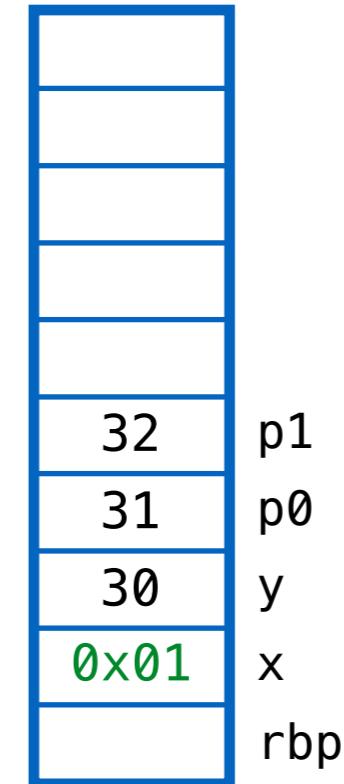
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```

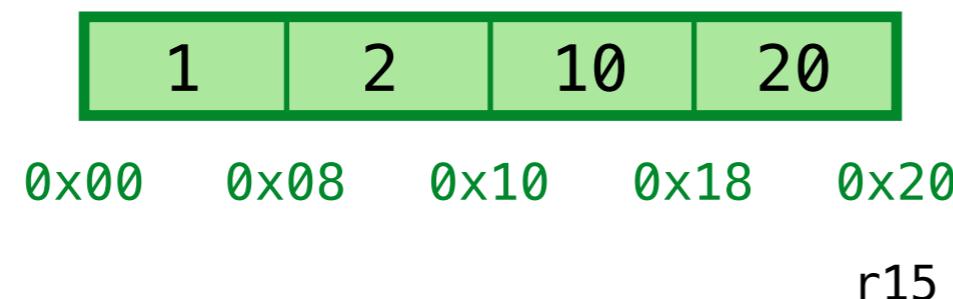


ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



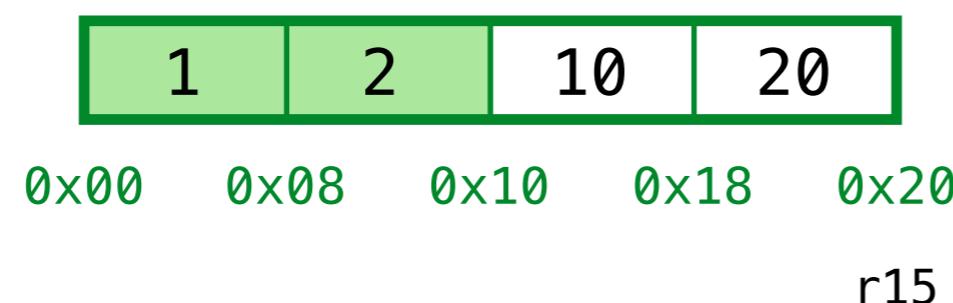
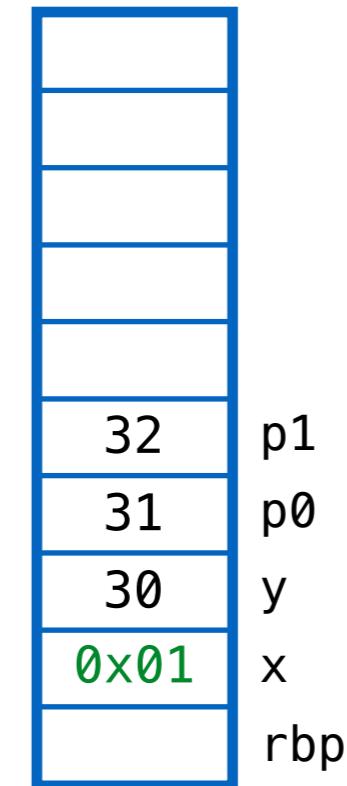
Out of memory!
Can't allocate (p0, p1)



ex1: garbage at end

```
let x  = (1, 2)
    , y  = let tmp = (10, 20)
            in tmp[0] + tmp[1]
    , p0 = x[0] + y
    , p1 = x[1] + y
in ←
    (p0, p1)
```

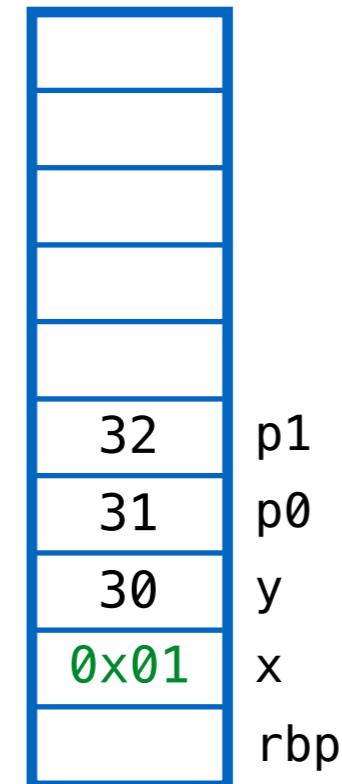
(10, 20) is “garbage”



Q: How to determine if cell is garbage?

ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```



(10, 20) is “garbage”

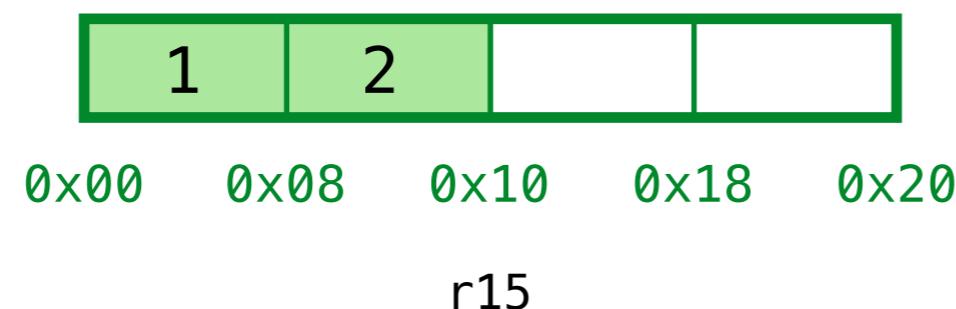
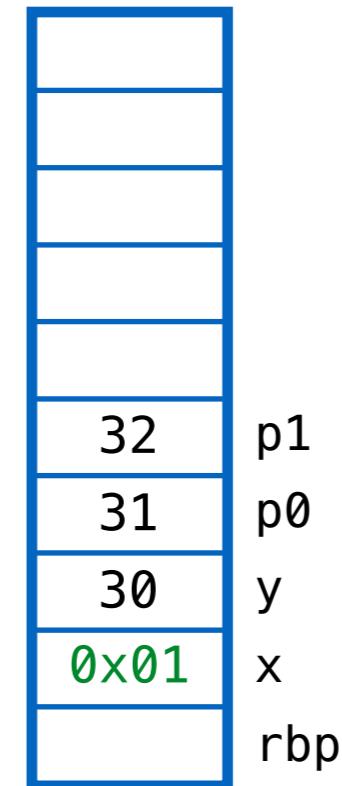


0x00 0x08 0x10 0x18 0x20

r15

ex1: garbage at end

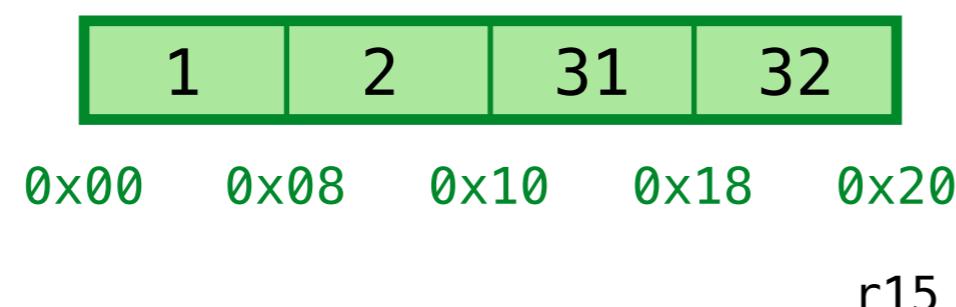
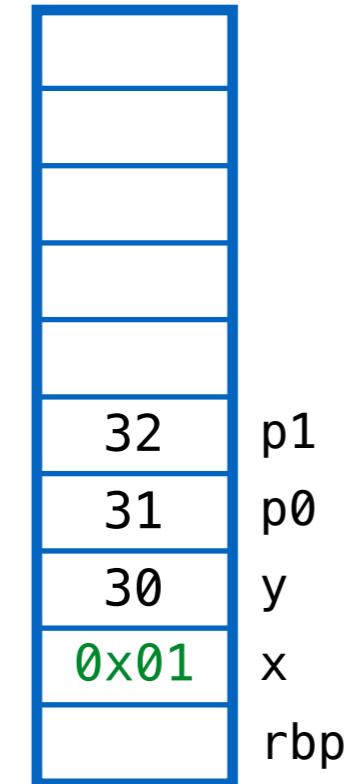
```
let x  = (1, 2)
    , y  = let tmp = (10, 20)
            in tmp[0] + tmp[1]
    , p0 = x[0] + y
    , p1 = x[1] + y
in ←
    (p0, p1)
```



ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```

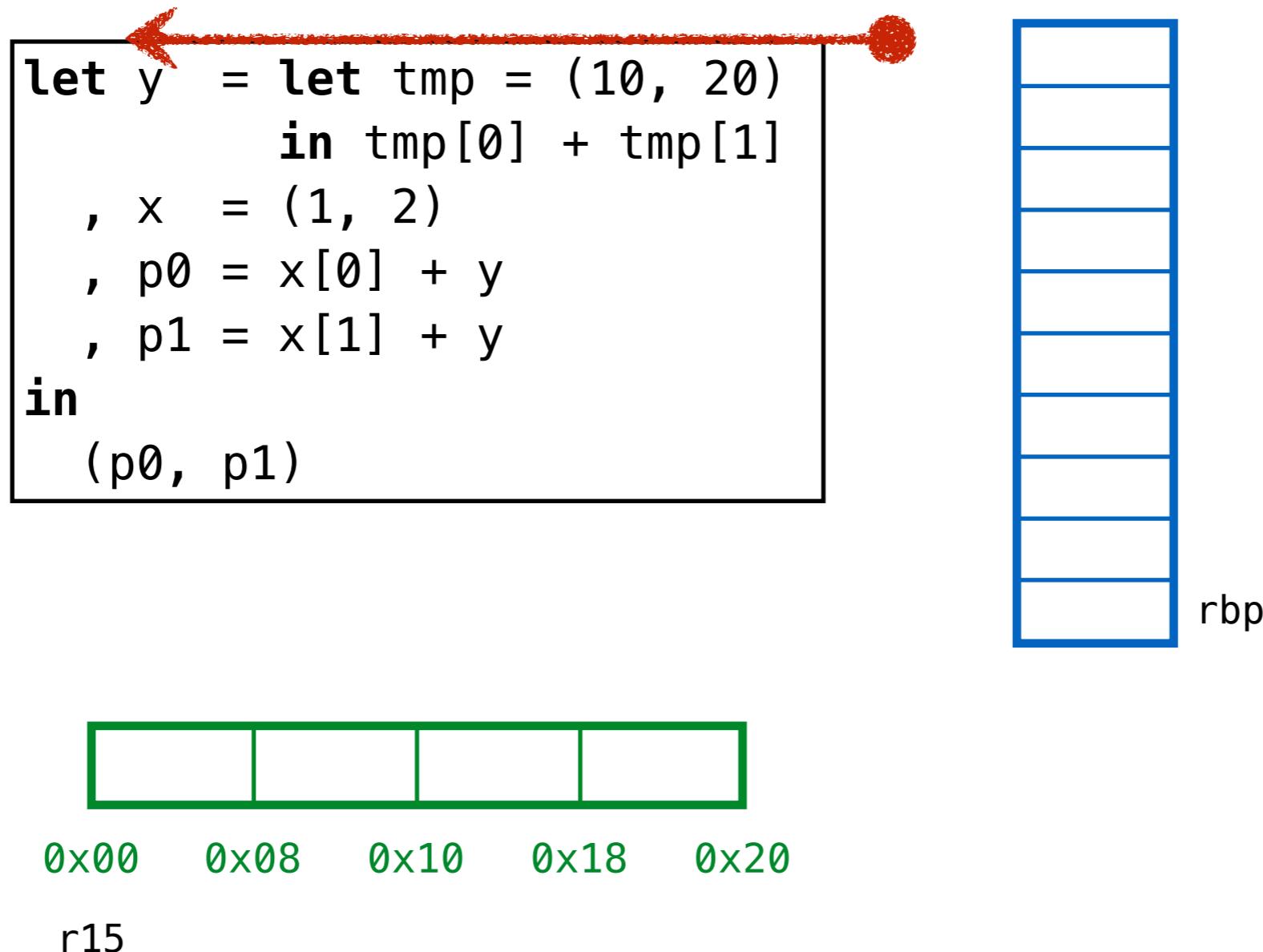
Result (rax) = 0x11



Garter / GC

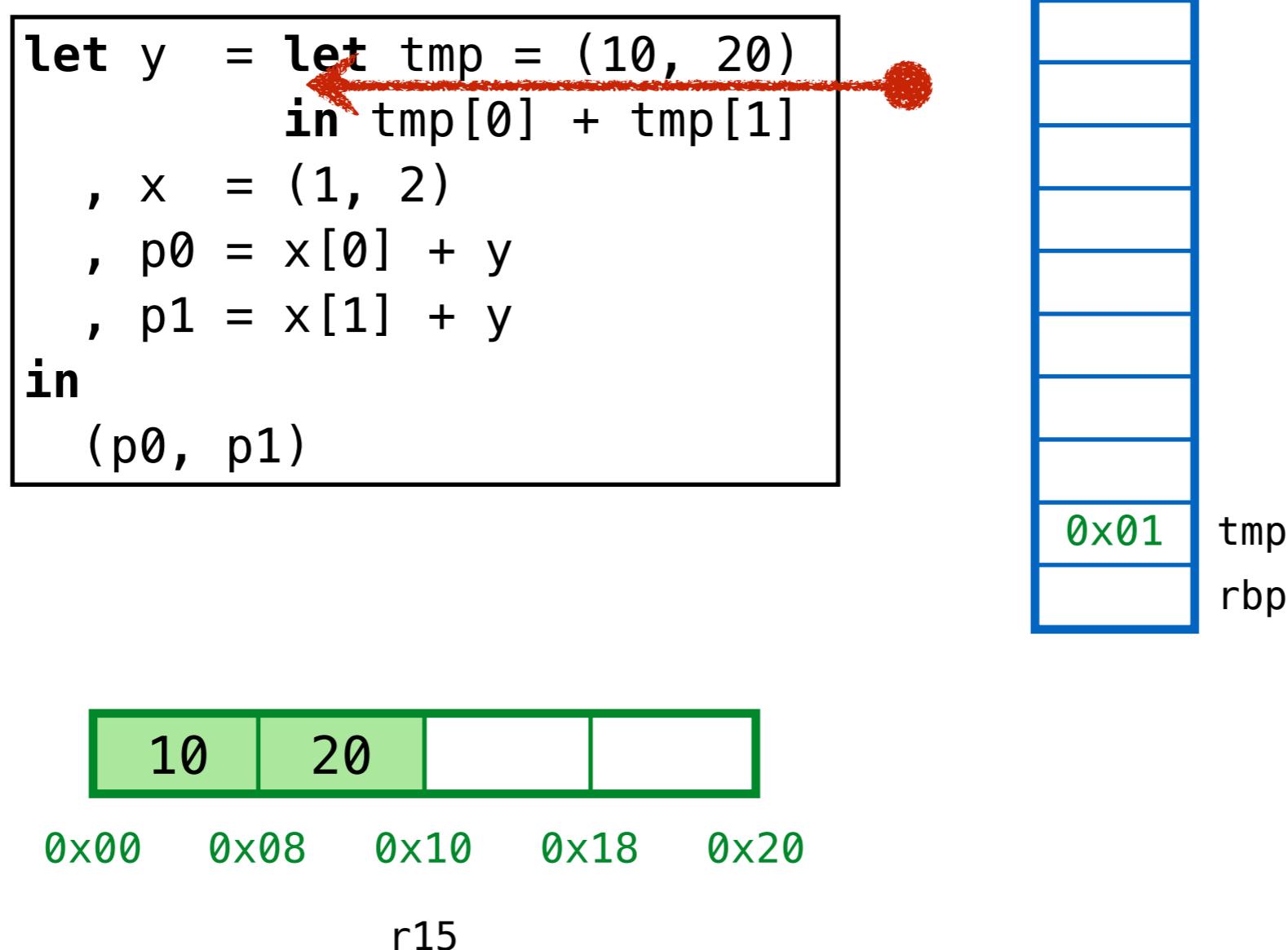
Example 2

ex2: garbage in the middle

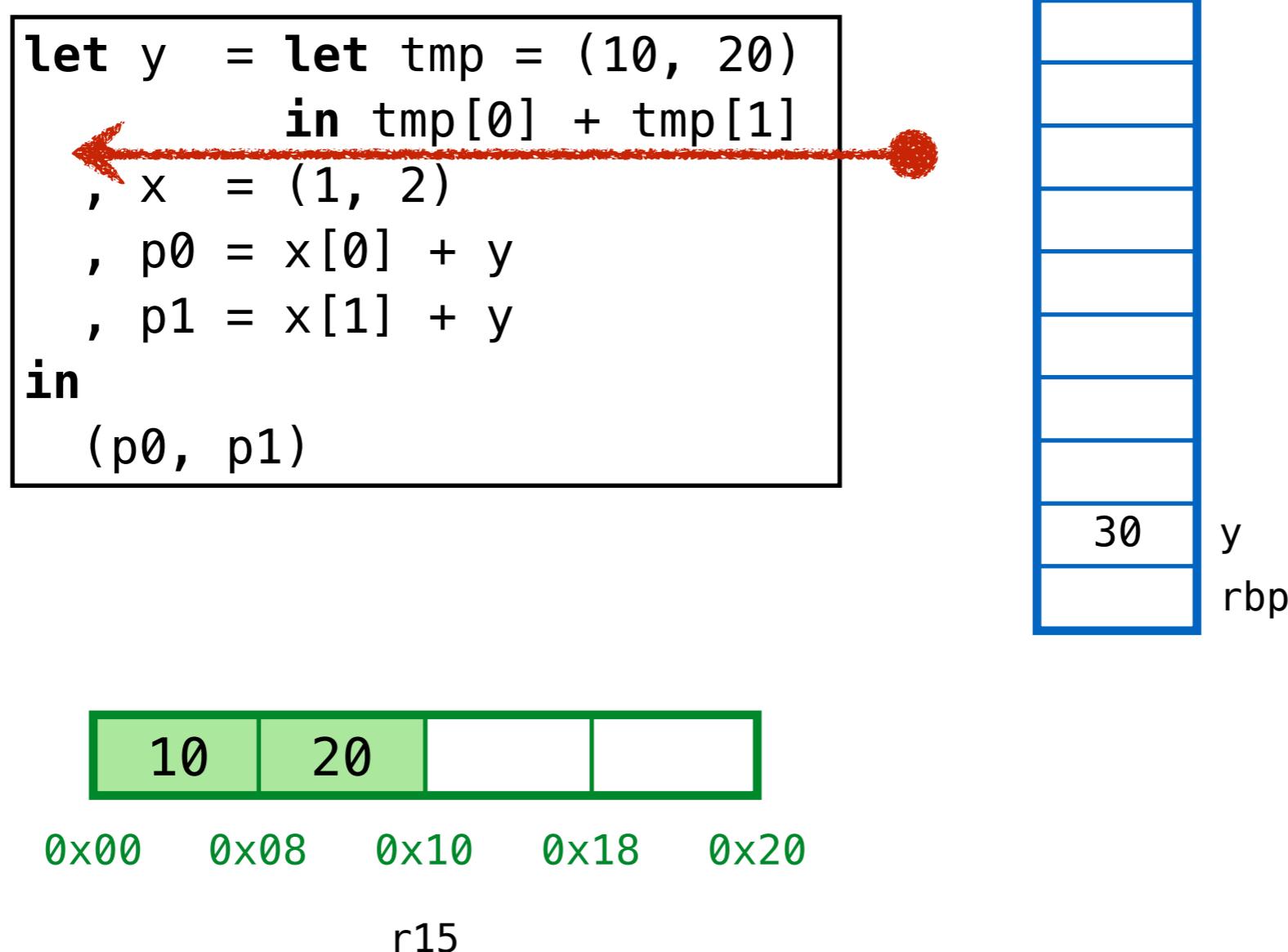


Start with a 4-word heap

ex2: garbage in the middle

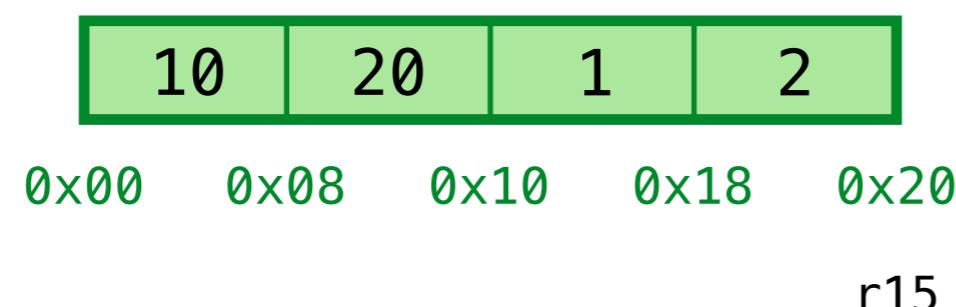
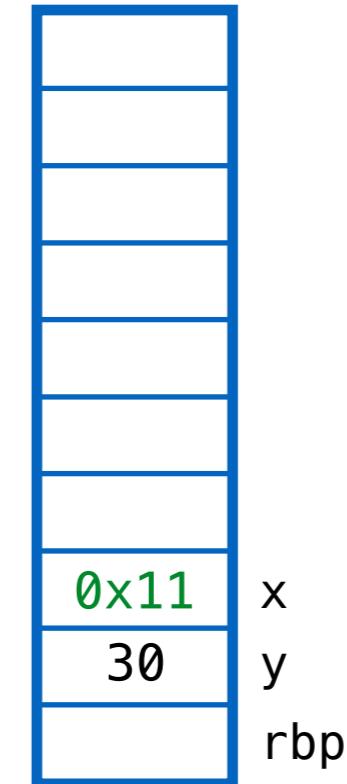


ex2: garbage in the middle



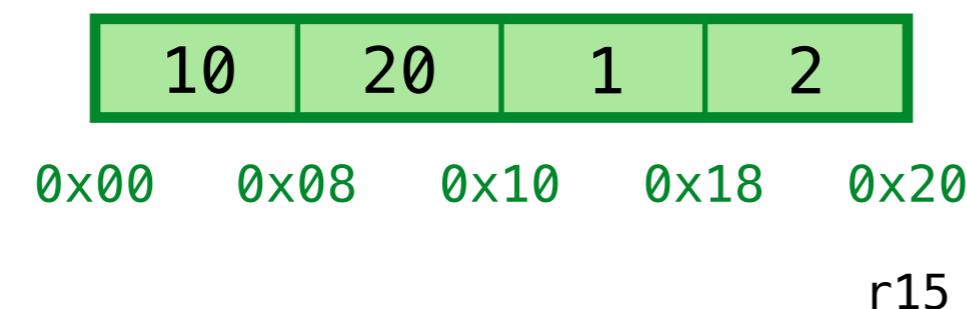
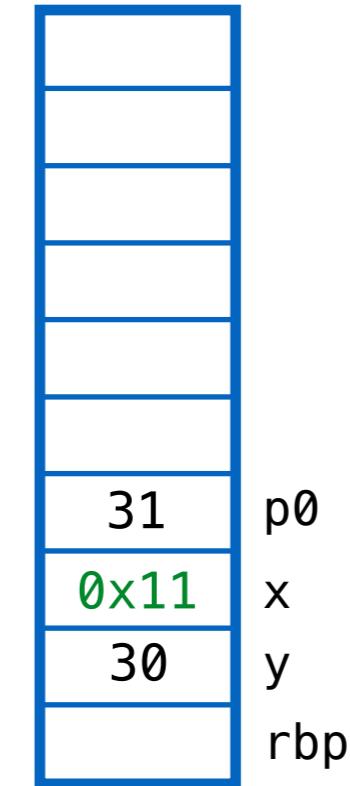
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
        , x = (1, 2)
        , p0 = x[0] + y
        , p1 = x[1] + y
in
  (p0, p1)
```



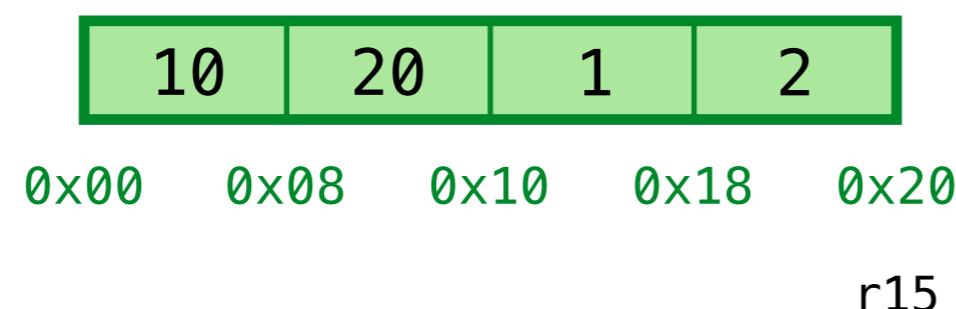
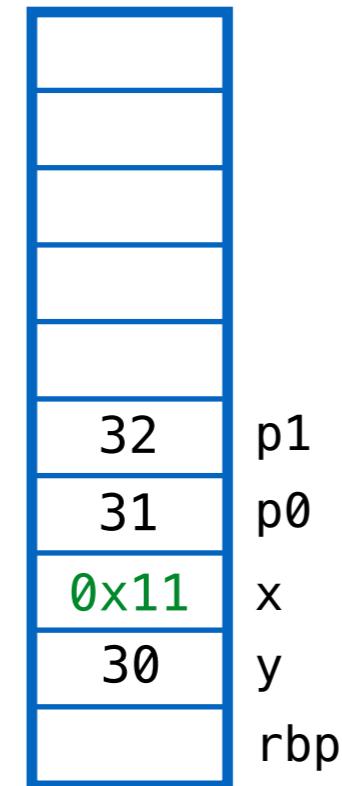
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



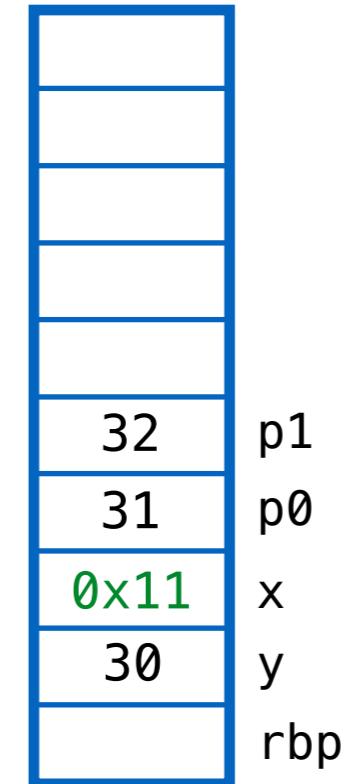
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
      , x  = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```



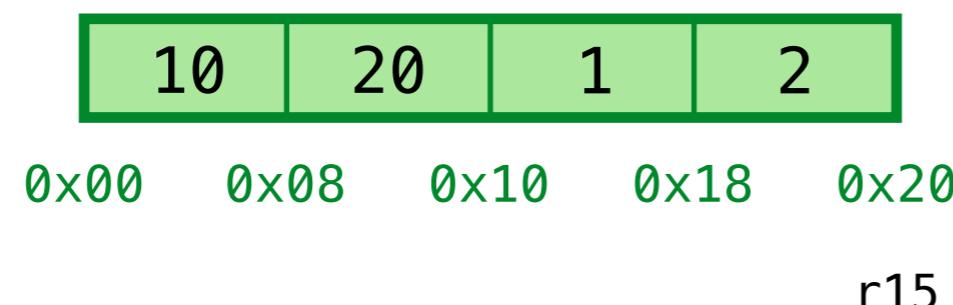
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
      , x  = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```



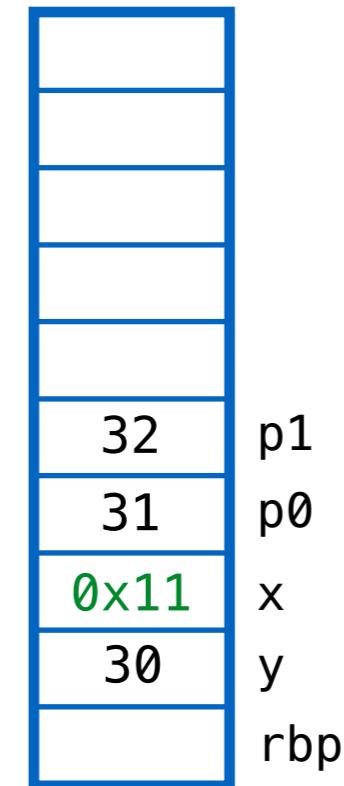
Out of memory!

Can't allocate (p0, p1)

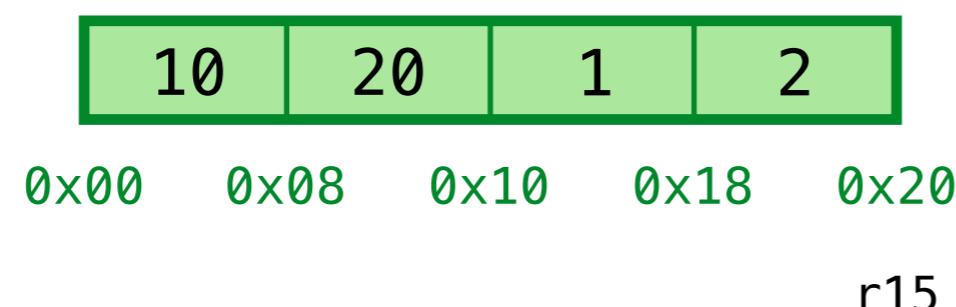


ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
      (p0, p1)
```

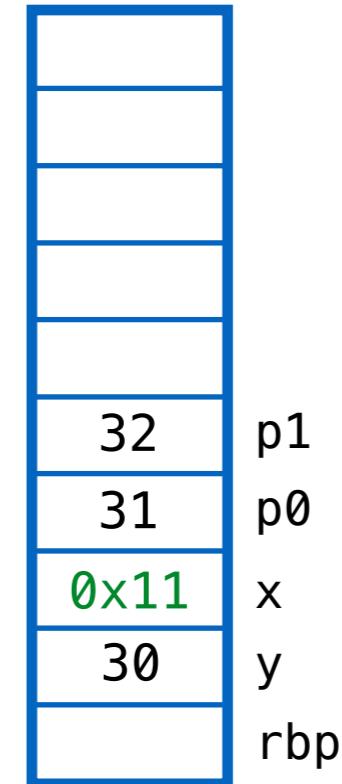


Lets reclaim & recycle garbage!

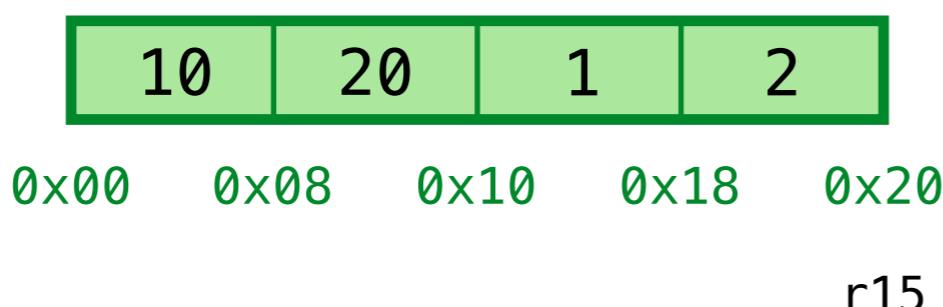


ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
      (p0, p1)
```



Lets reclaim & recycle garbage!

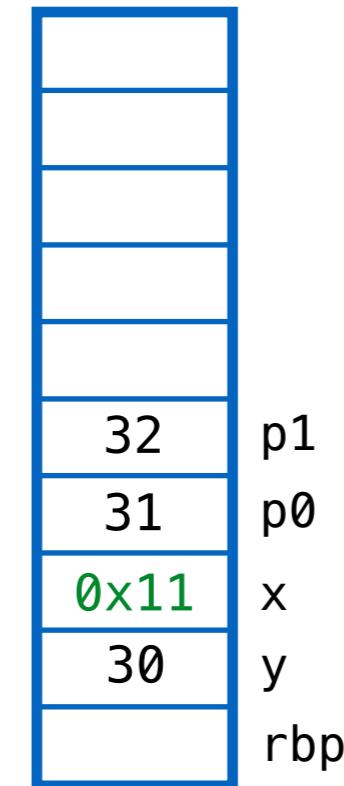


QUIZ: Which cells are garbage?

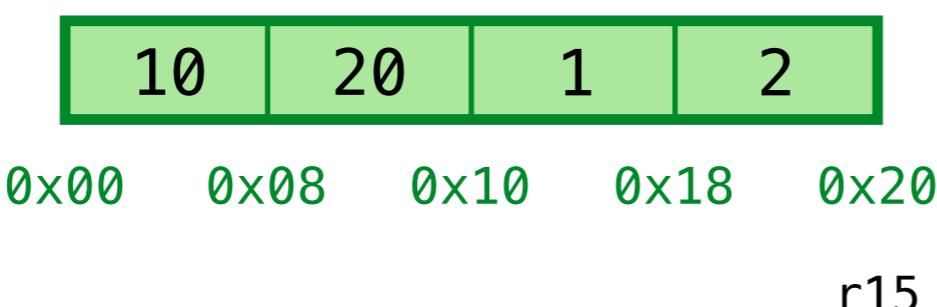
- (A) 0x00, 0x08 (B) 0x08, 0x10 (C) 0x18, 0x20 (D) None (E) All

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
      (p0, p1)
```



Lets reclaim & recycle garbage!



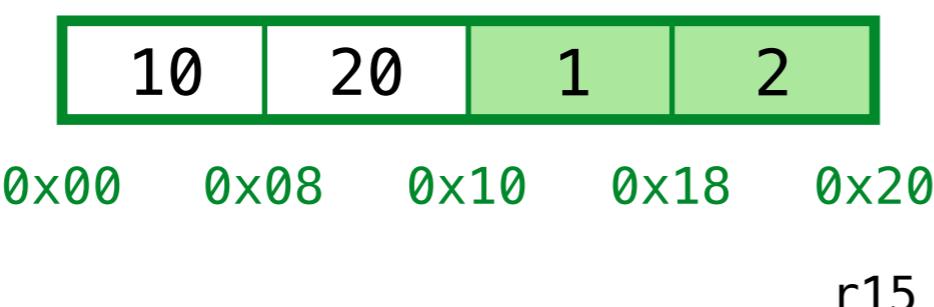
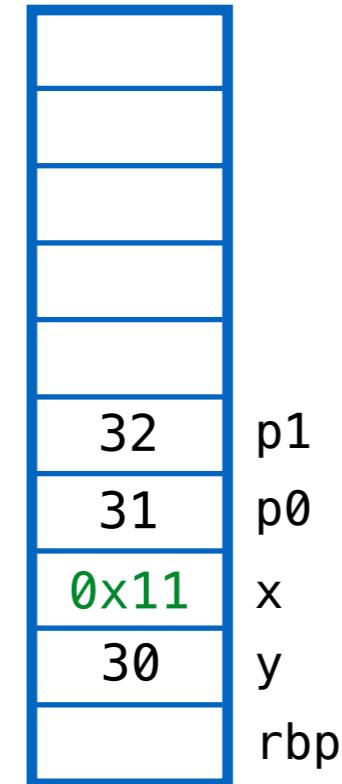
QUIZ: Which cells are garbage?

Those that are *not reachable from stack*

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```

Lets reclaim & recycle garbage!

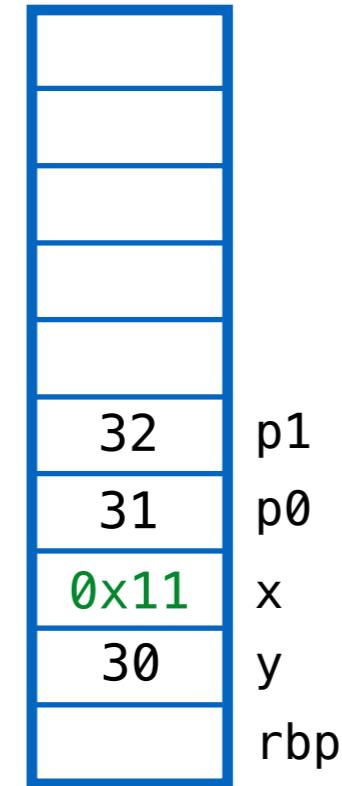


QUIZ: Which cells are garbage?

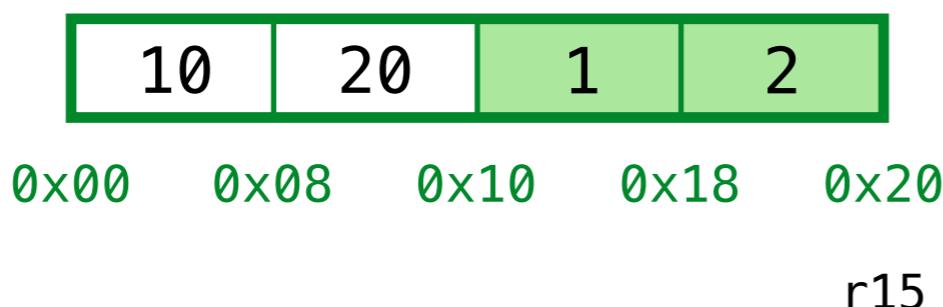
Those that are *not reachable from stack*

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

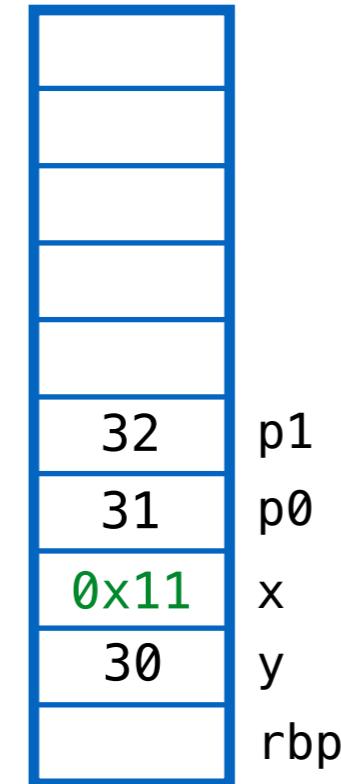


Q: How to reclaim space?

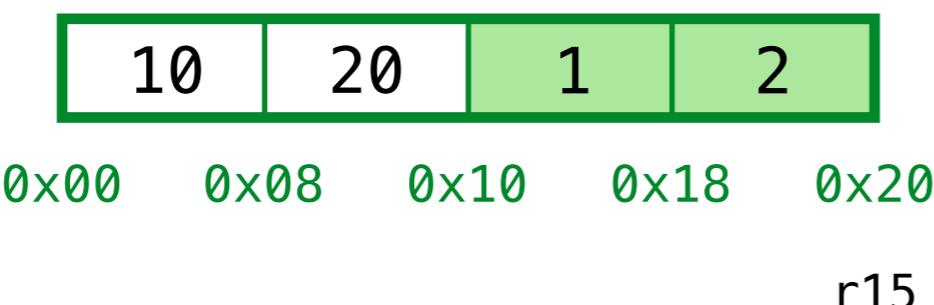
Why is it not enough to rewind r15?

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

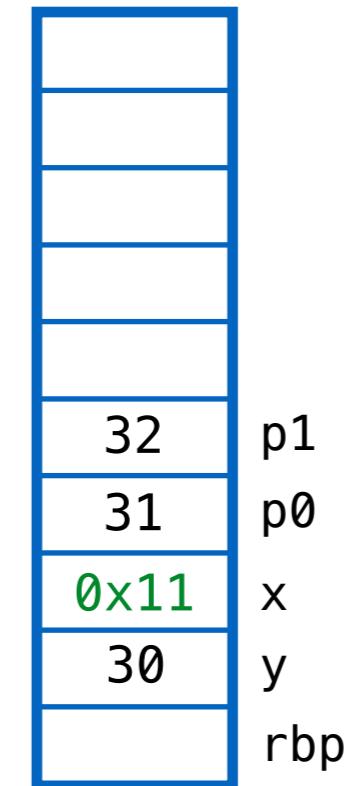


Why is it not enough to rewind r15?

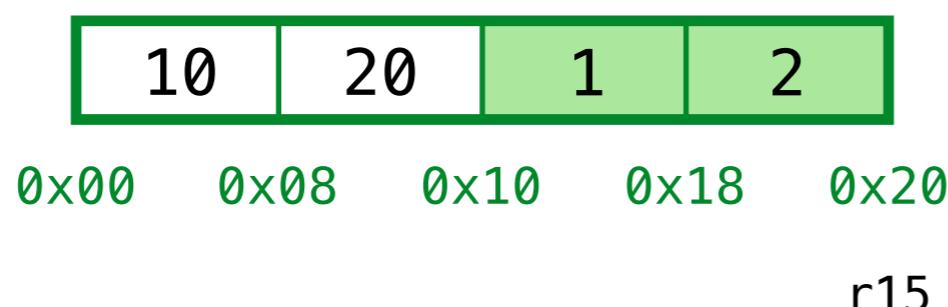
Want free space to be *contiguous* (i.e. go to end of heap)

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

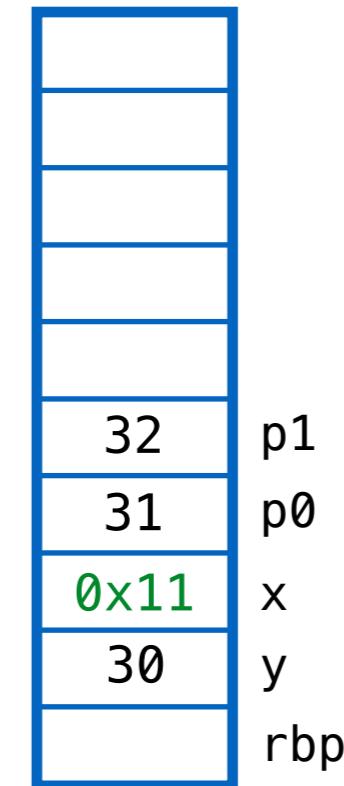


Solution: Compaction

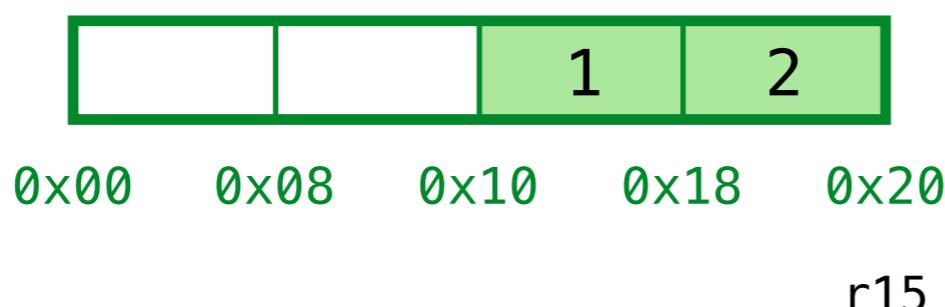
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

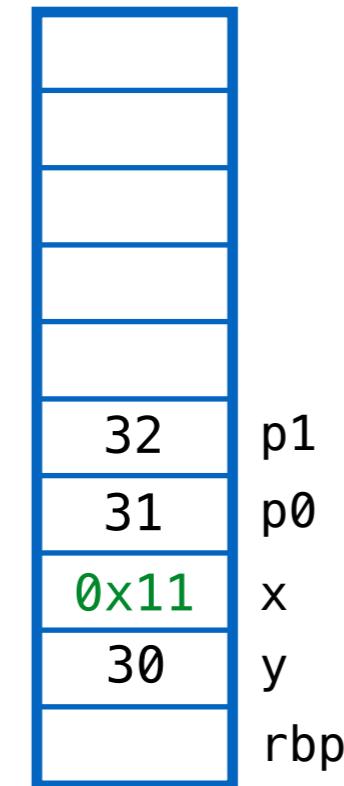


Solution: Compaction

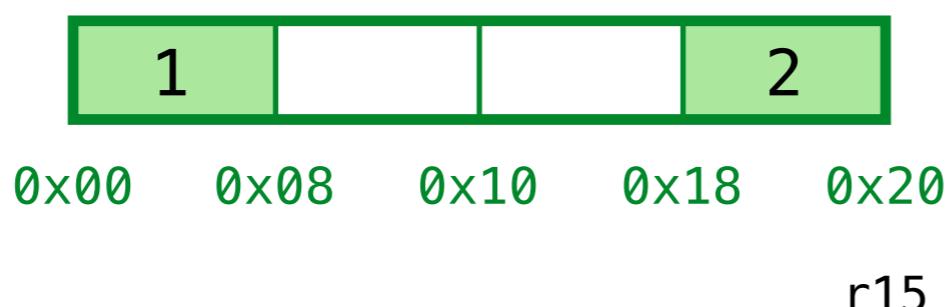
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

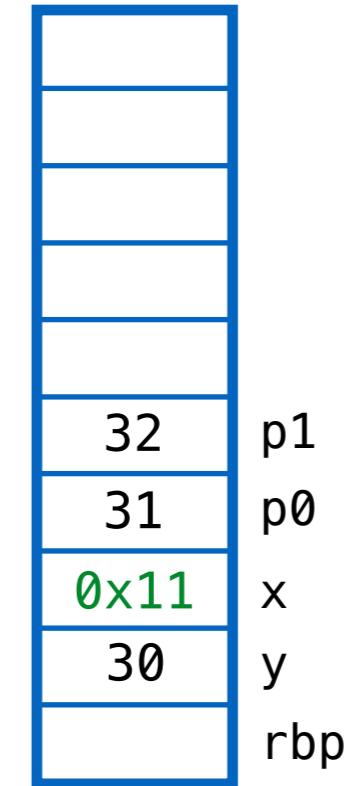


Solution: Compaction

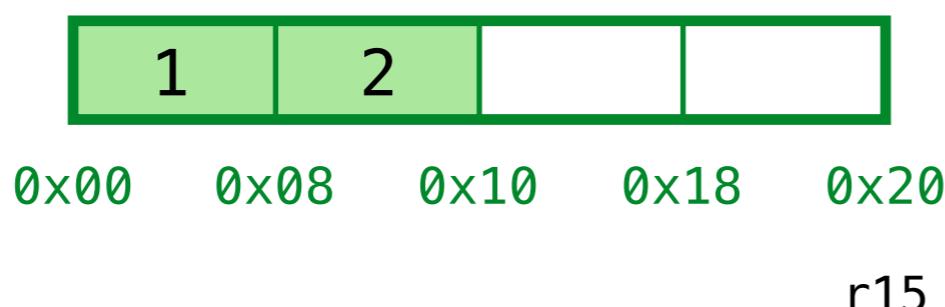
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

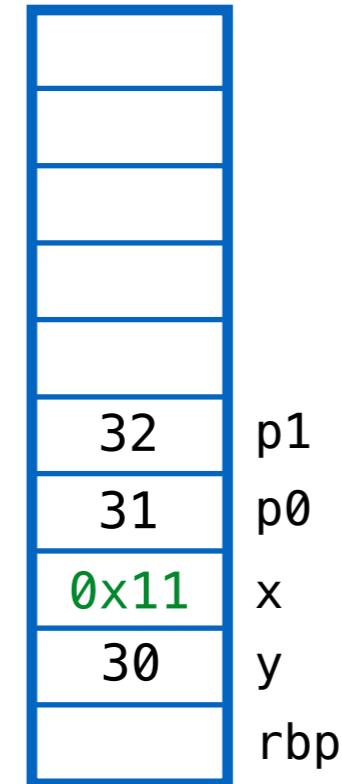


Solution: Compaction

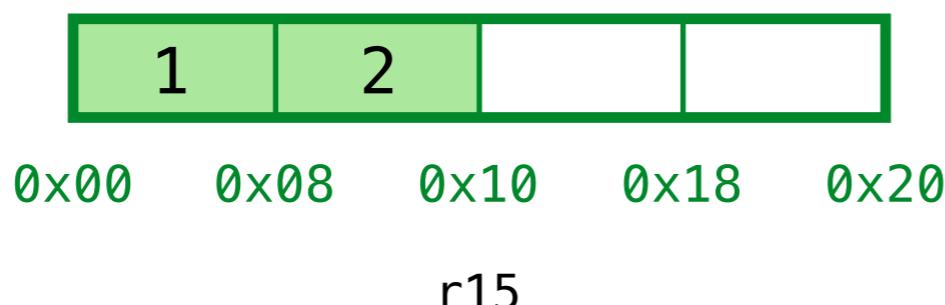
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

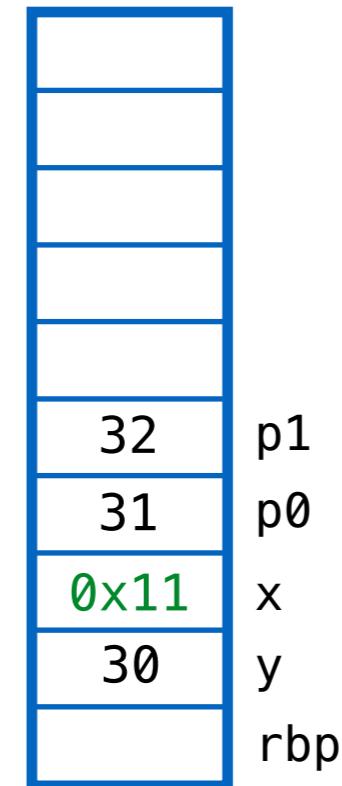


Solution: Compaction

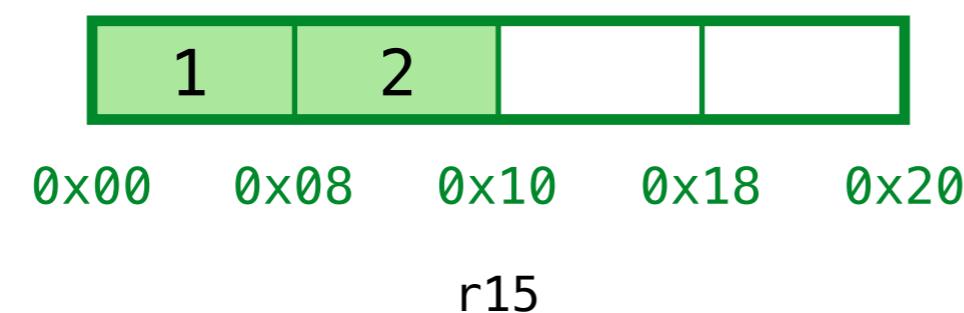
Copy “live” cells into “garbage” ... *and then* ... rewind r15!

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
      , x  = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```

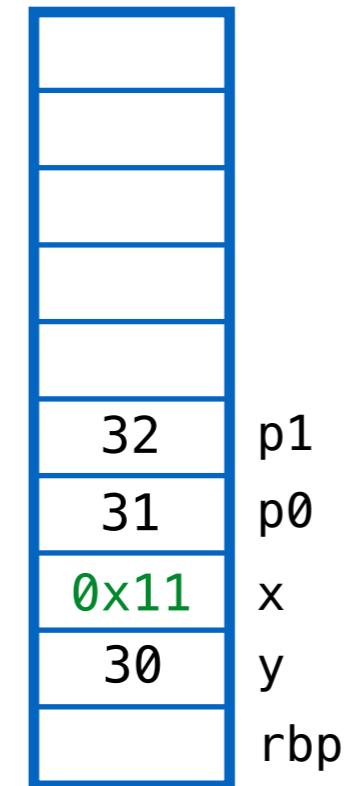


Yay! Have space for (p0, p1)

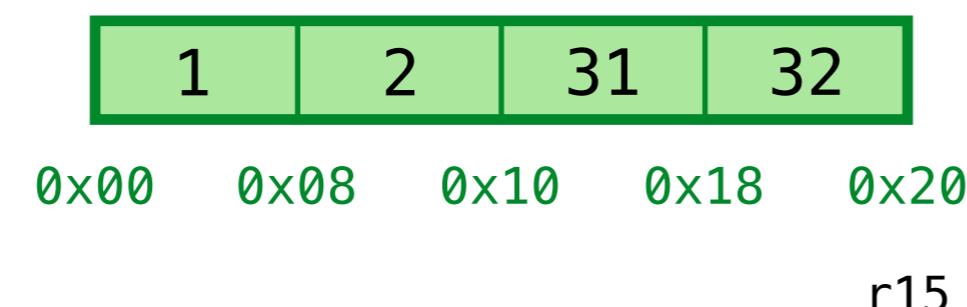


ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```



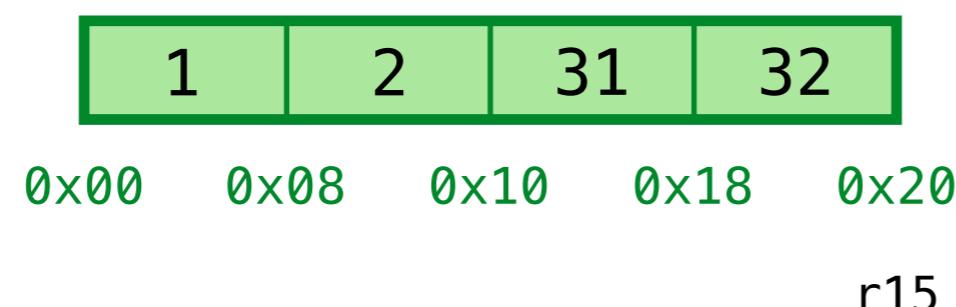
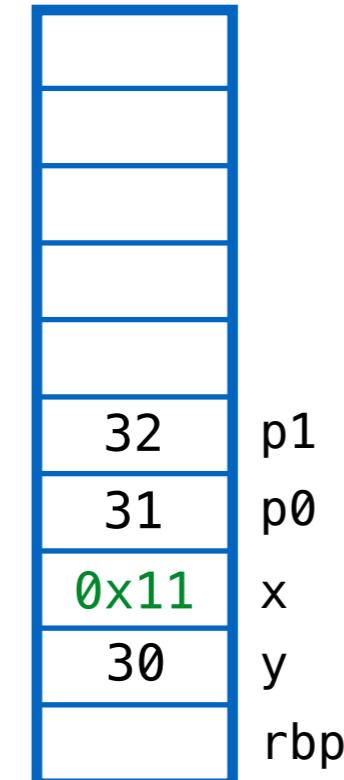
Yay! Have space for (p0, p1)



ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```

Result (rax) = 0x09

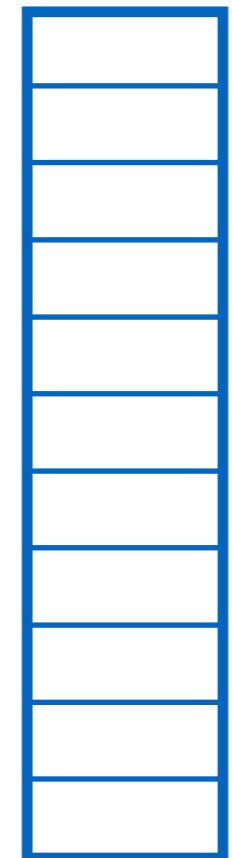


Garter / GC

Example 3

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]
←
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + y + z
```



3 local vars x, y, z

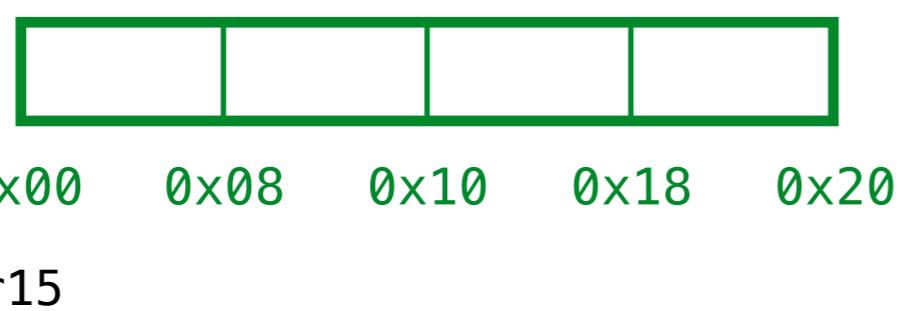
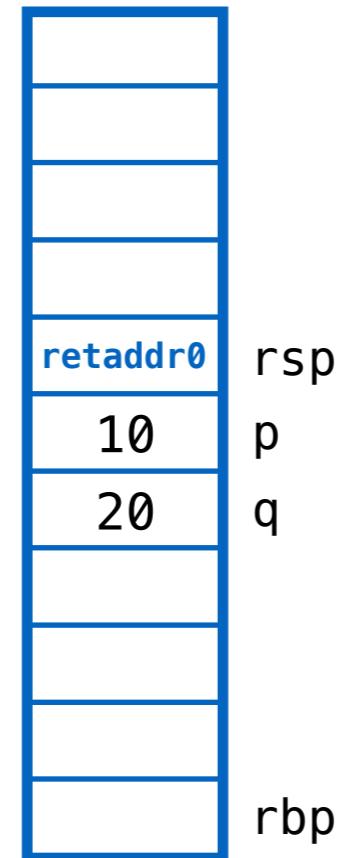


0x00 0x08 0x10 0x18 0x20
r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

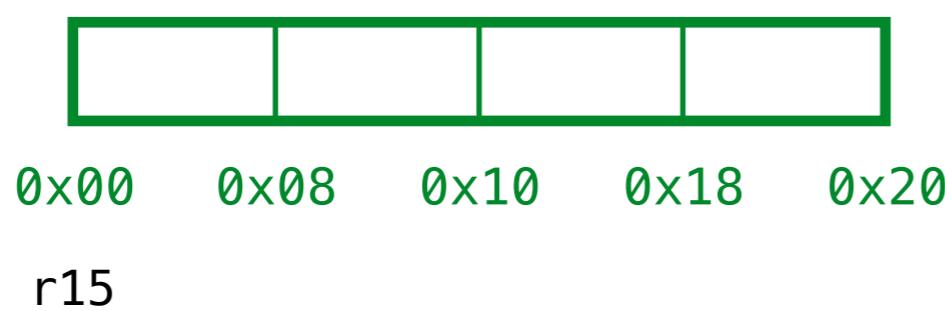
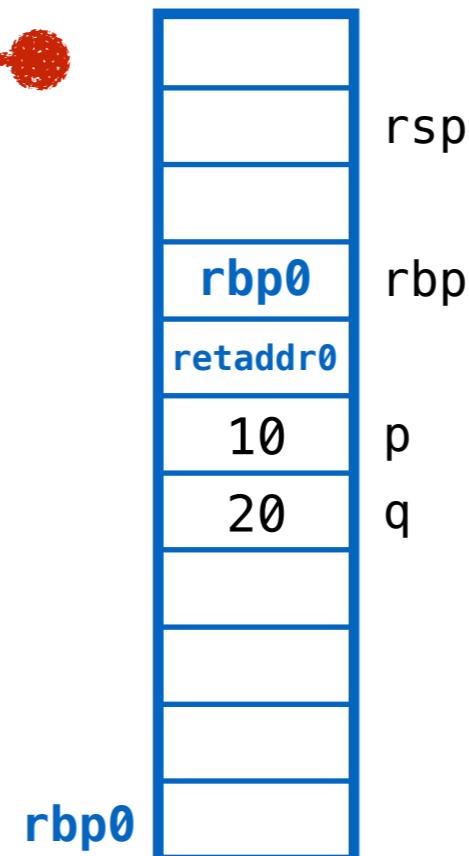
let y  = foo(10, 20) ←
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + y + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

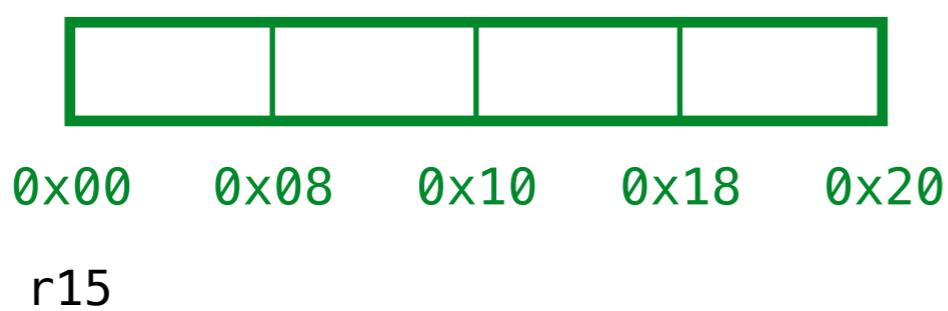
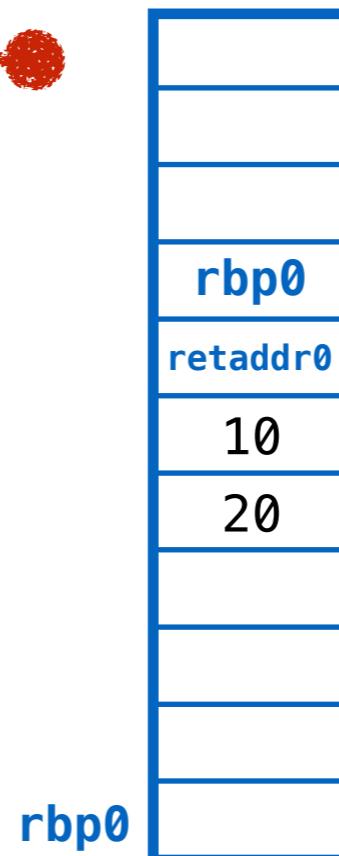
let y = foo(10, 20)
    , x = (y, y + 1)
    , z = foo(100, 200)
in
    x[0] + y + z
```



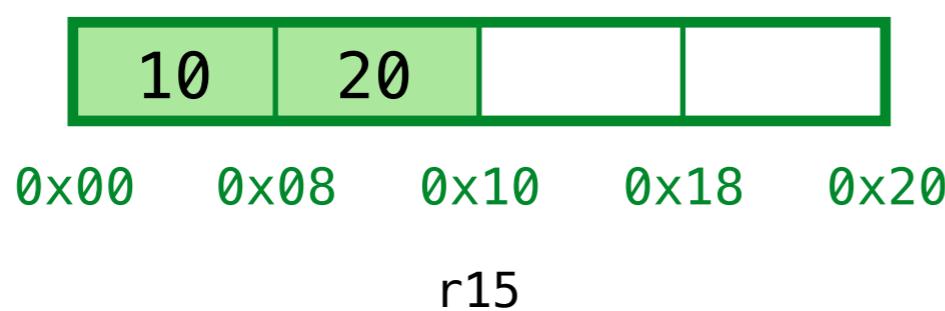
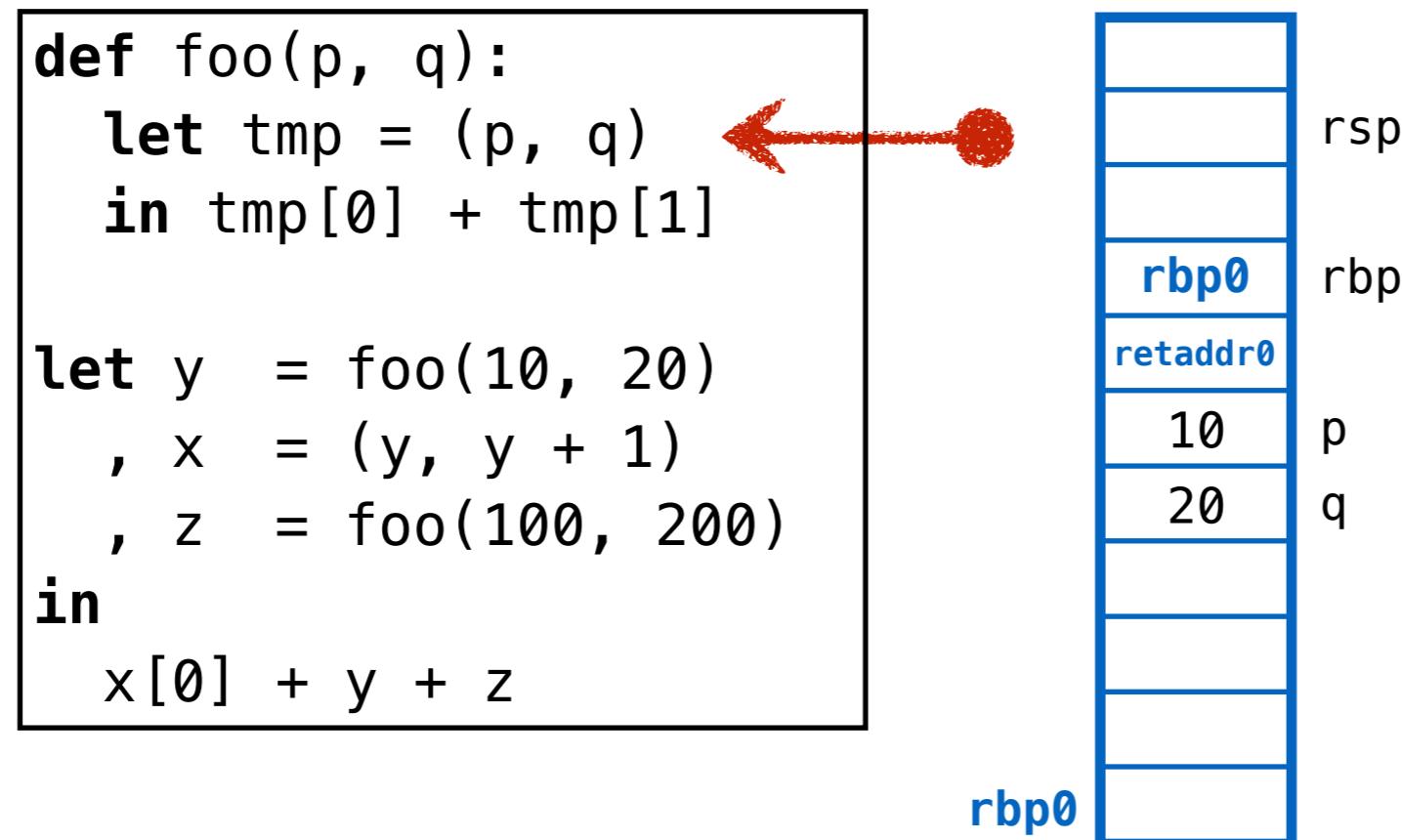
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + y + z
```

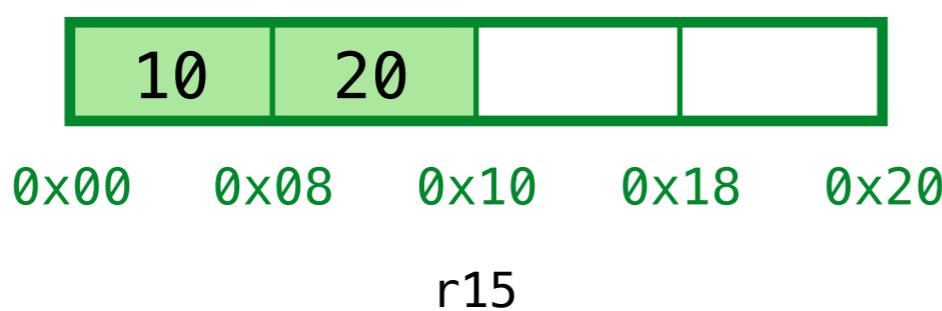
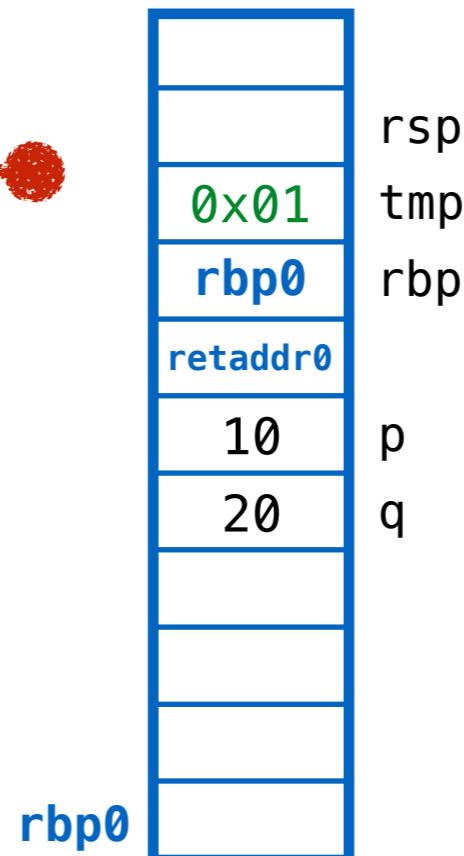


ex3: garbage in the middle (with stack)



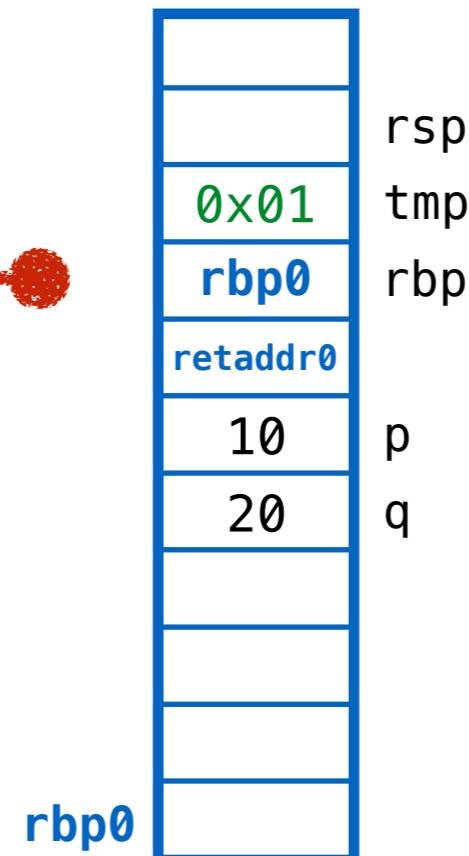
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + y + z
```

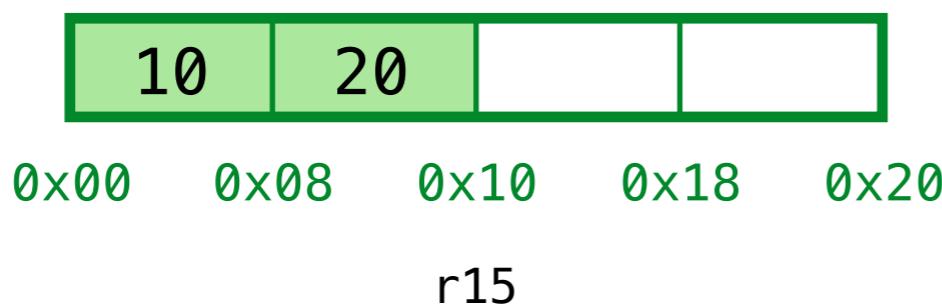


ex3: garbage in the middle (with stack)

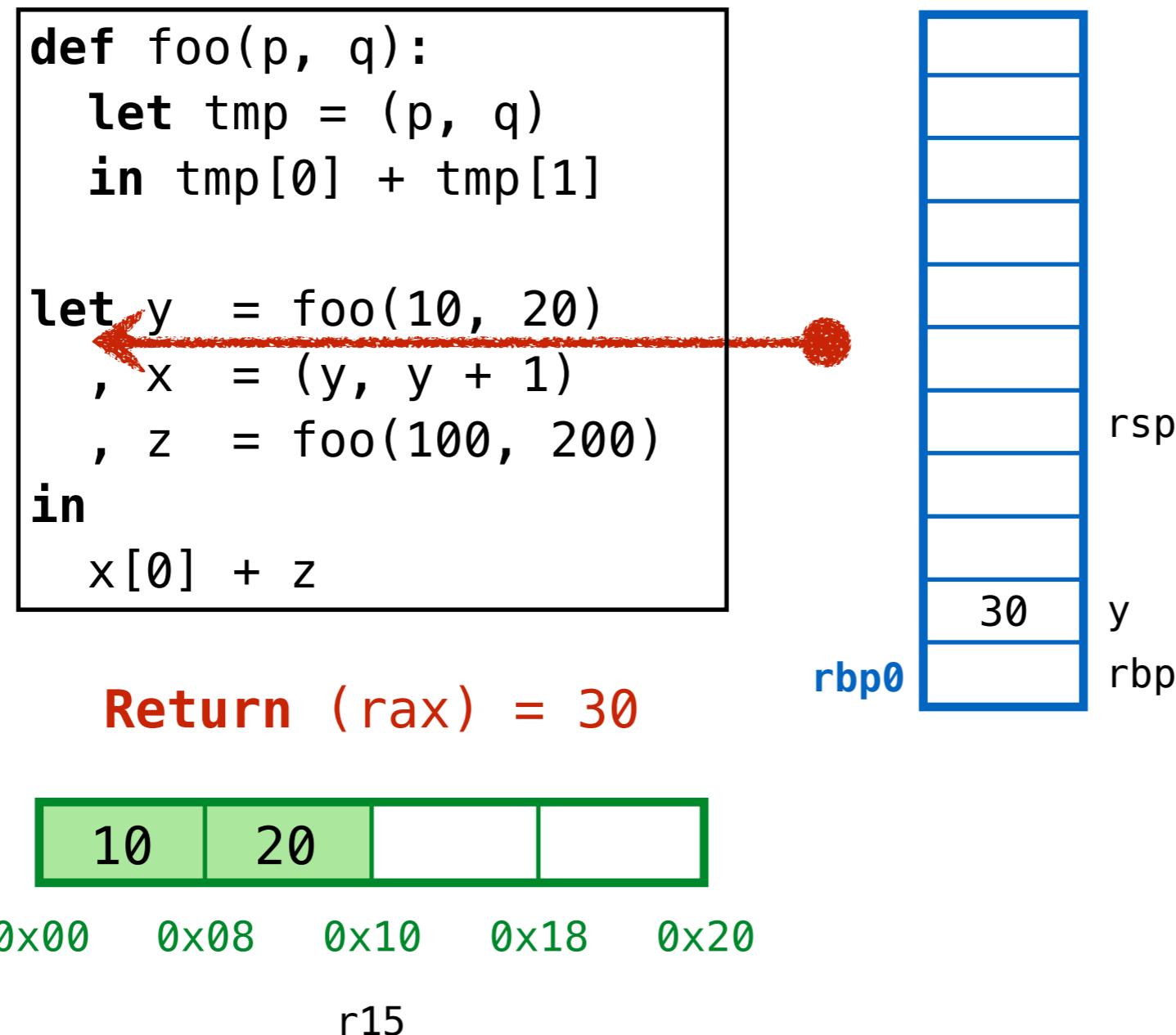
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + y + z
```



Return (rax) = 30



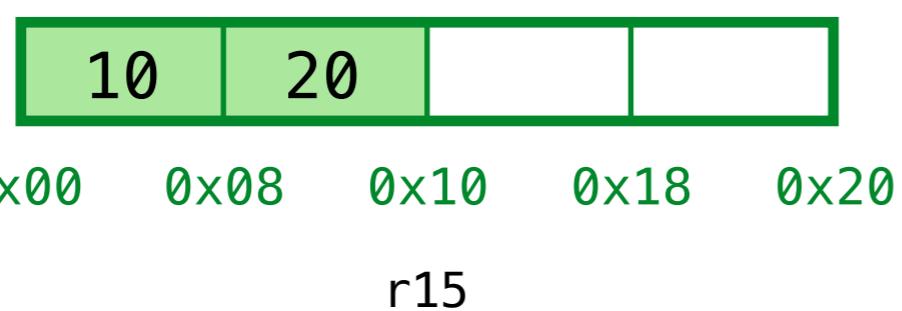
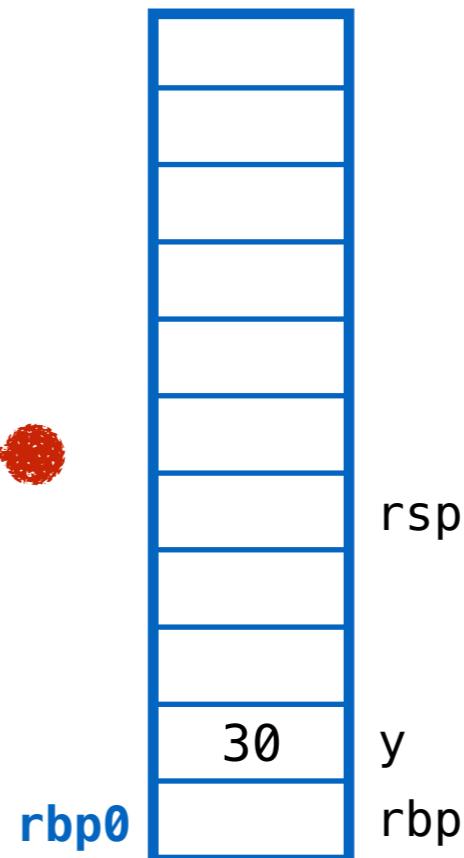
ex3: garbage in the middle (with stack)



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

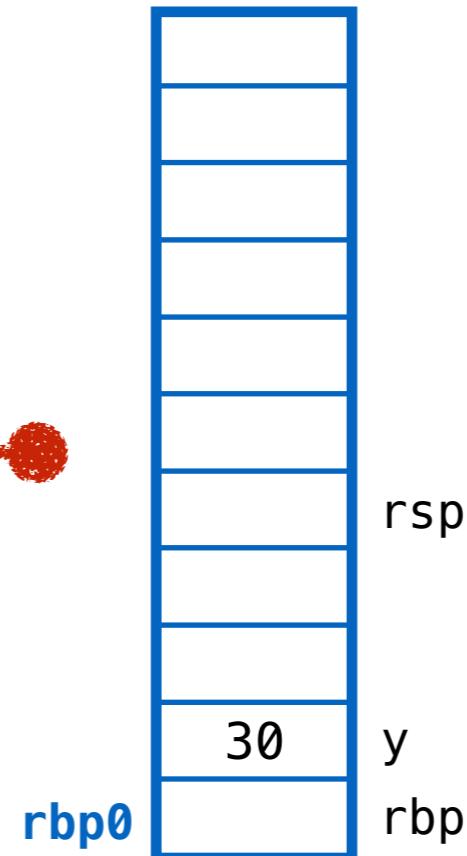
let y  = foo(10, 20)
, x  = (y, y + 1) ←
, z  = foo(100, 200)
in
x[0] + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1) ←
, z  = foo(100, 200)
in
x[0] + z
```



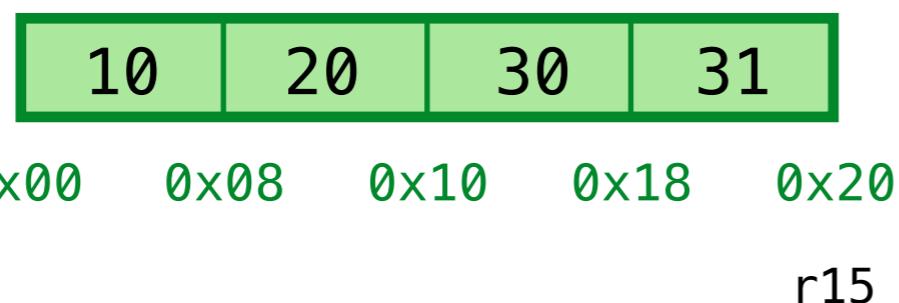
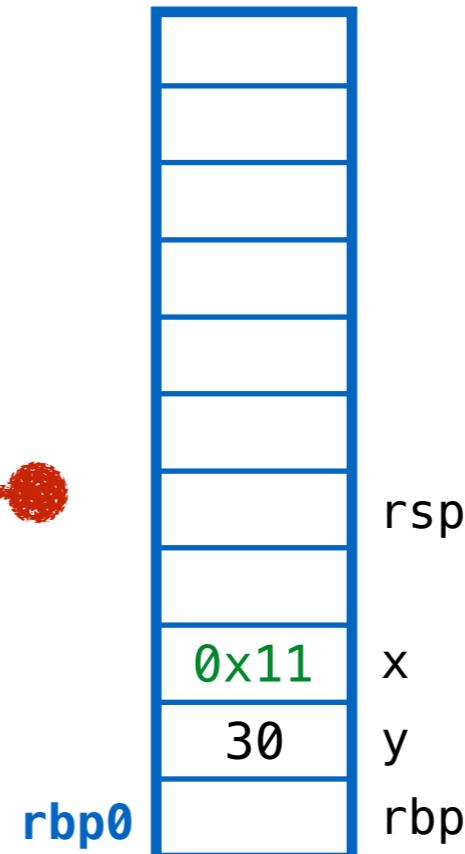
0x00 0x08 0x10 0x18 0x20

r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

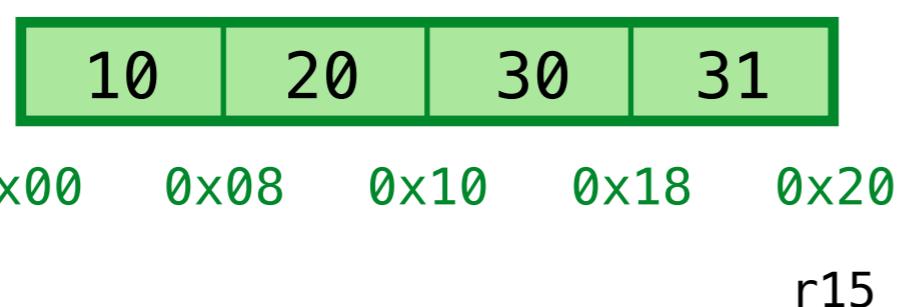
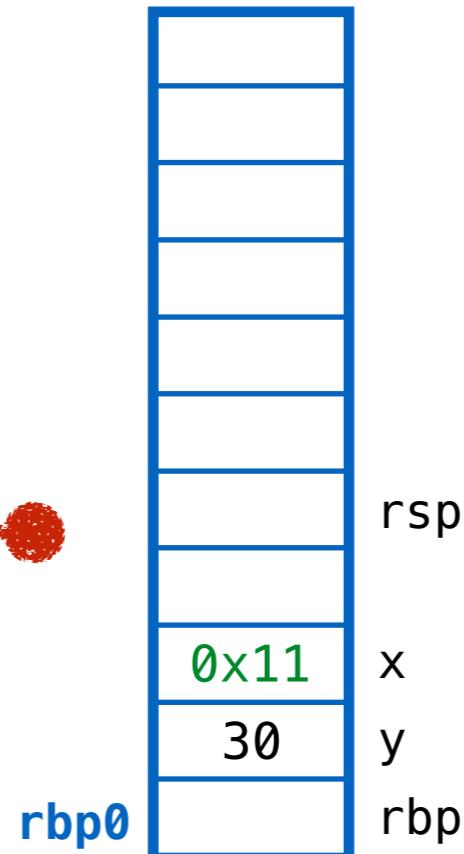
let y  = foo(10, 20)
      ← x = (y, y + 1)
      , z = foo(100, 200)
in
    x[0] + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

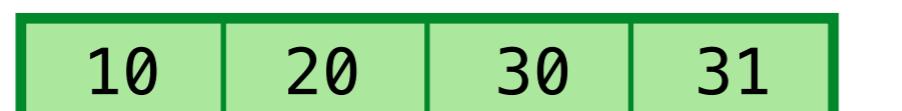
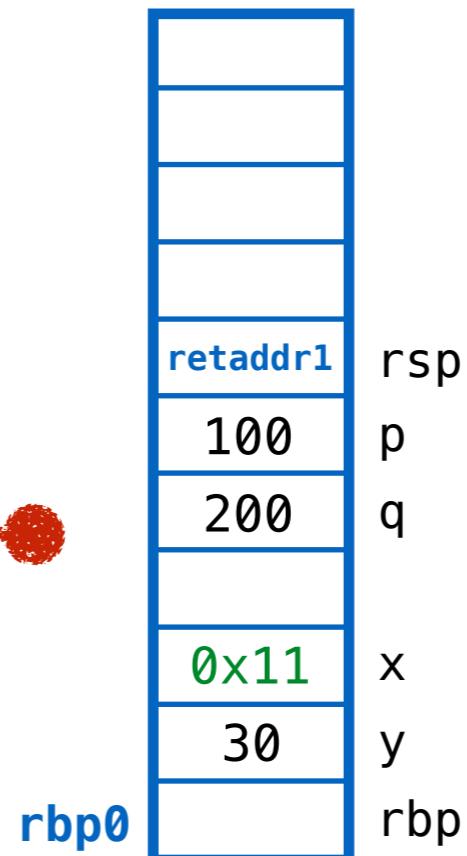
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200) ←
in
x[0] + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200) ←
in
x[0] + z
```



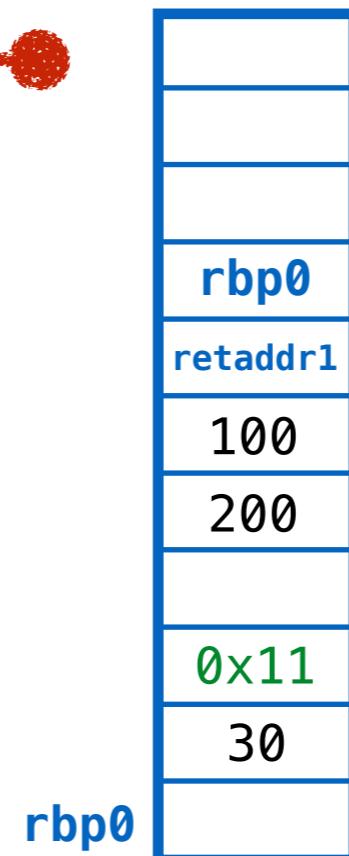
0x00 0x08 0x10 0x18 0x20

r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



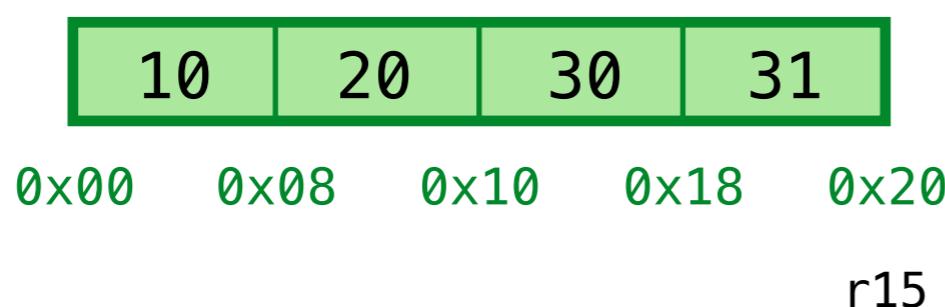
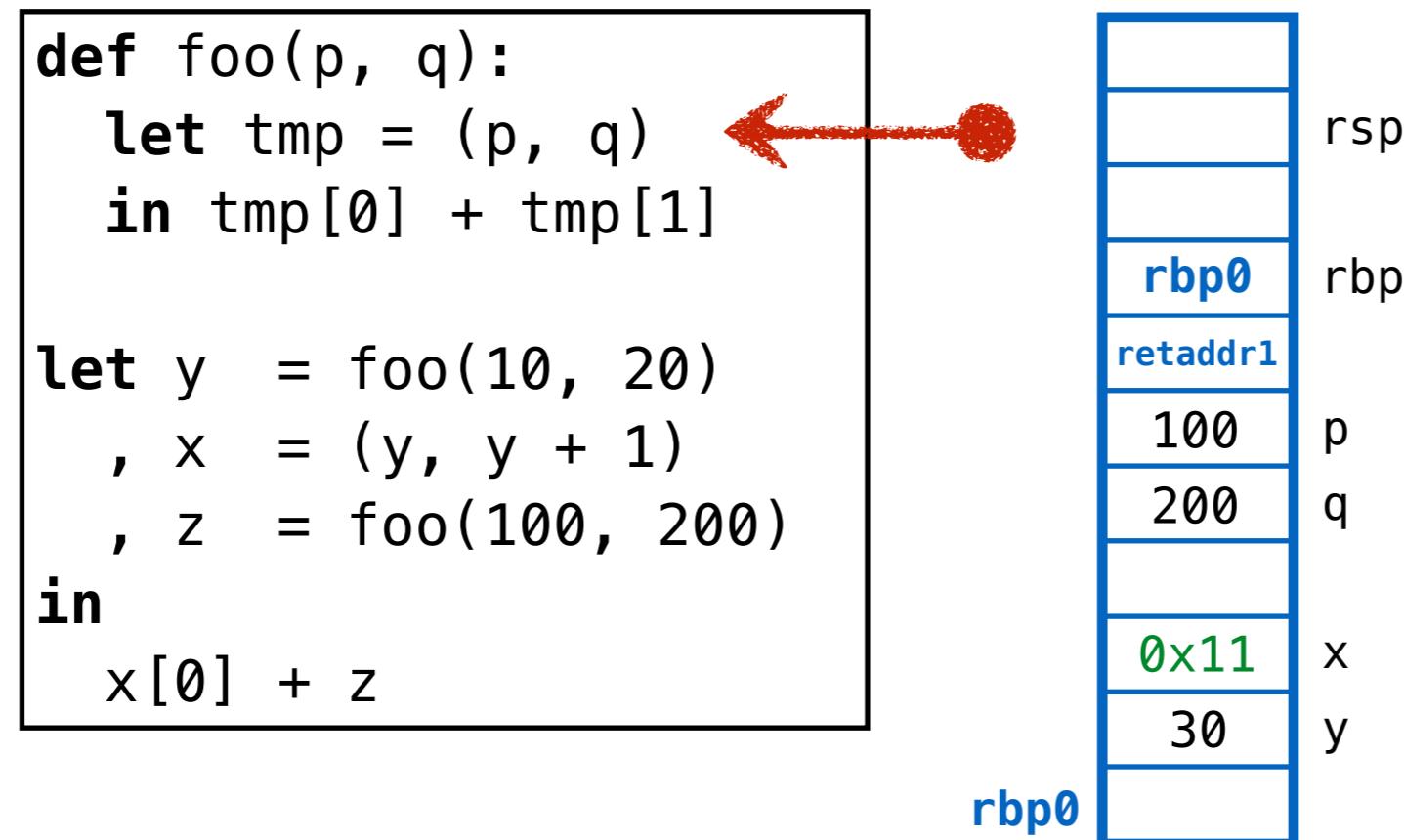
1 local var (tmp)



0x00 0x08 0x10 0x18 0x20

r15

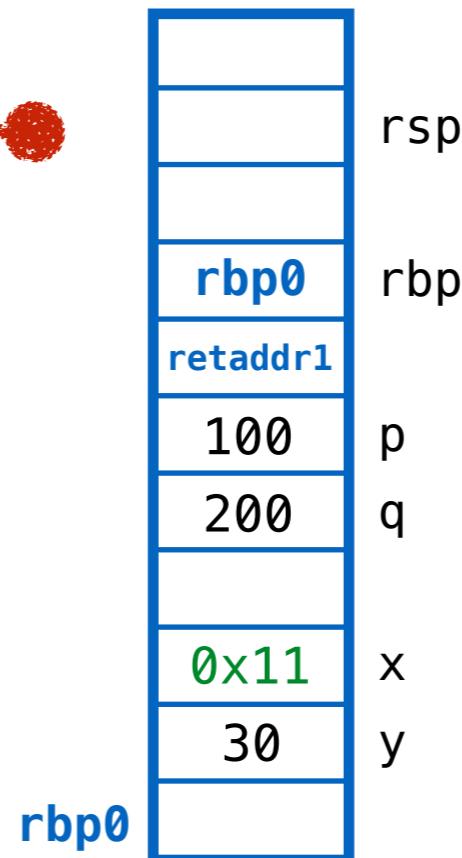
ex3: garbage in the middle (with stack)



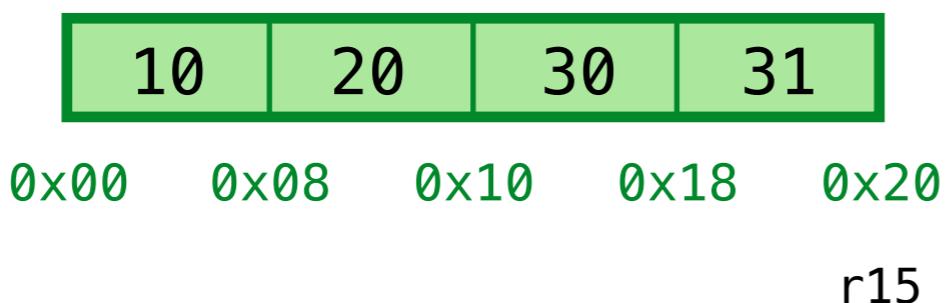
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

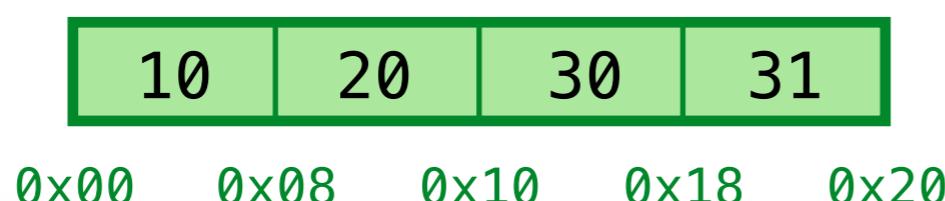
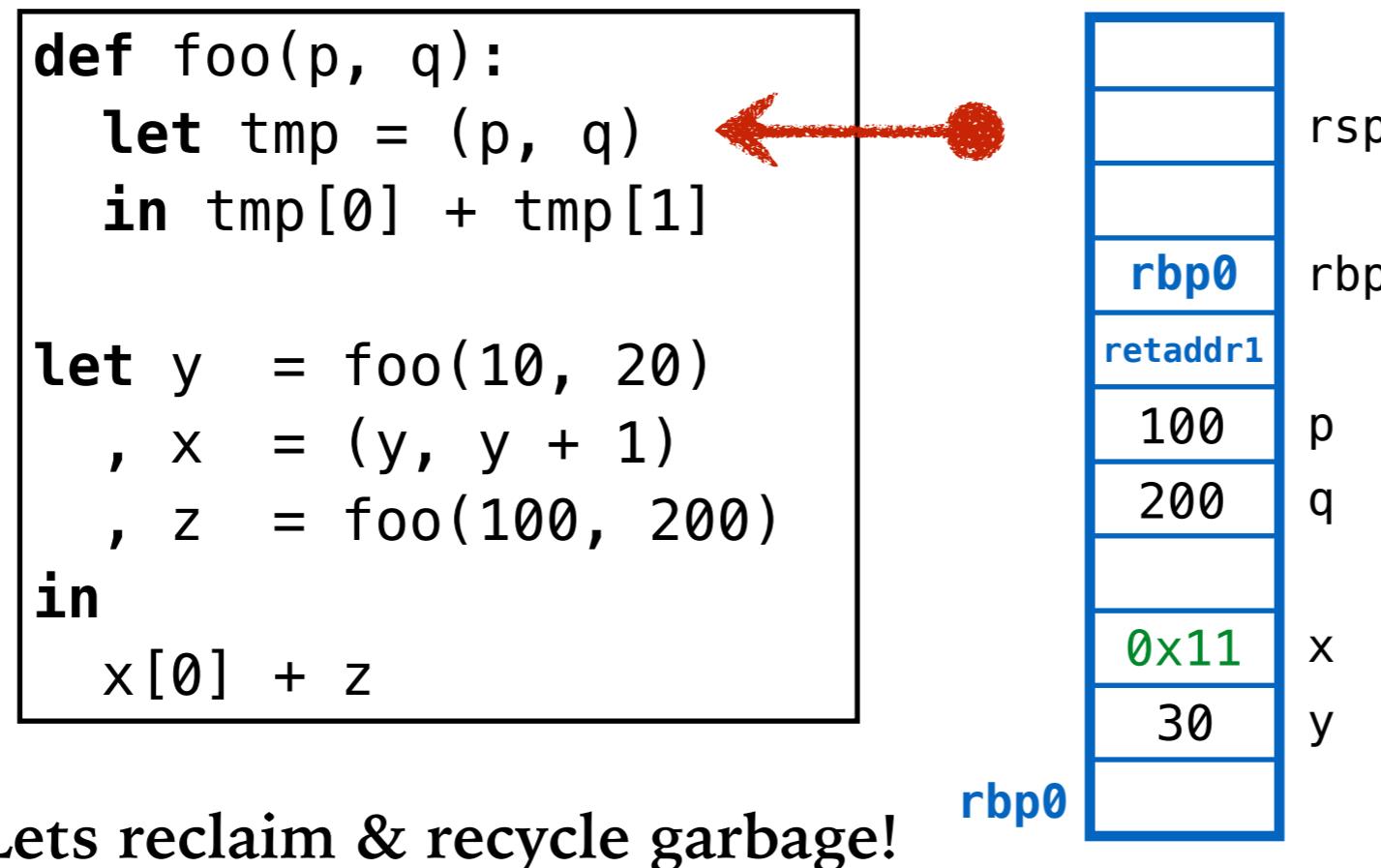
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



Lets reclaim & recycle garbage!



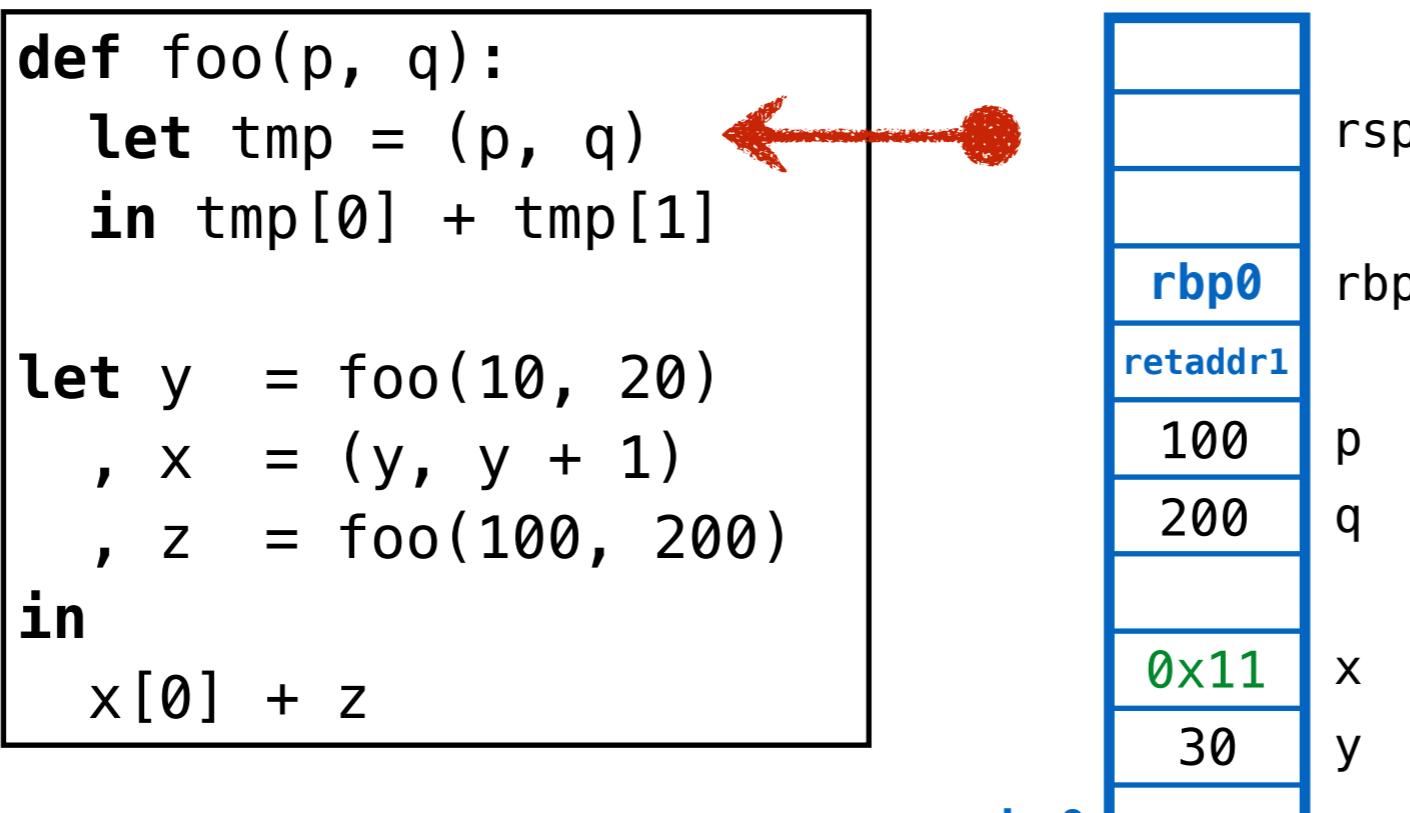
ex3: garbage in the middle (with stack)



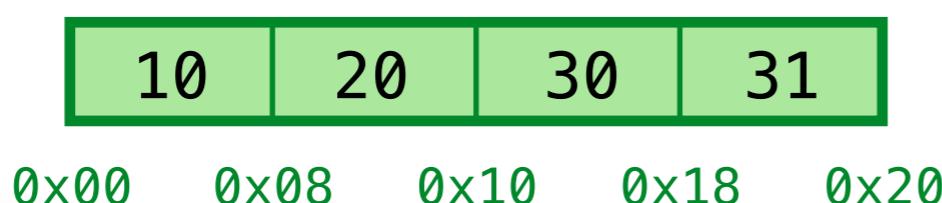
QUIZ: Which cells are garbage?

- (A) `0x00, 0x08` (B) `0x08, 0x10` (C) `0x10, 0x18` (D) None (E) All

ex3: garbage in the middle (with stack)



Lets reclaim & recycle garbage!



QUIZ: Which cells are garbage?

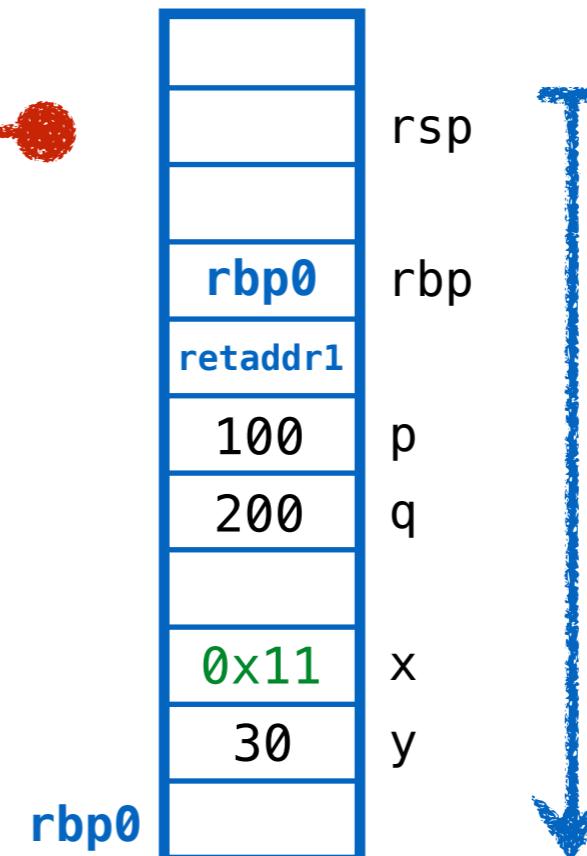
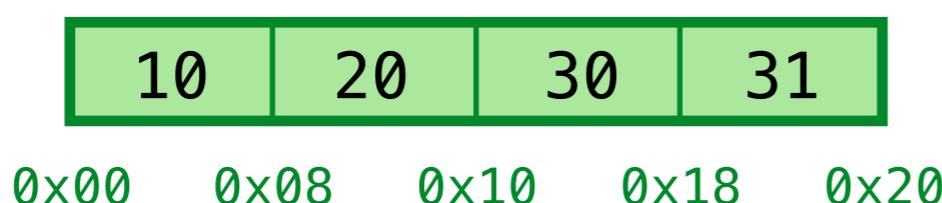
Those that are *not reachable from any stack frame*

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```

Lets reclaim & recycle garbage!



Traverse Stack
from top (rsp)
to bottom (rbp0)
to mark
reachable cells.

QUIZ: Which cells are garbage?

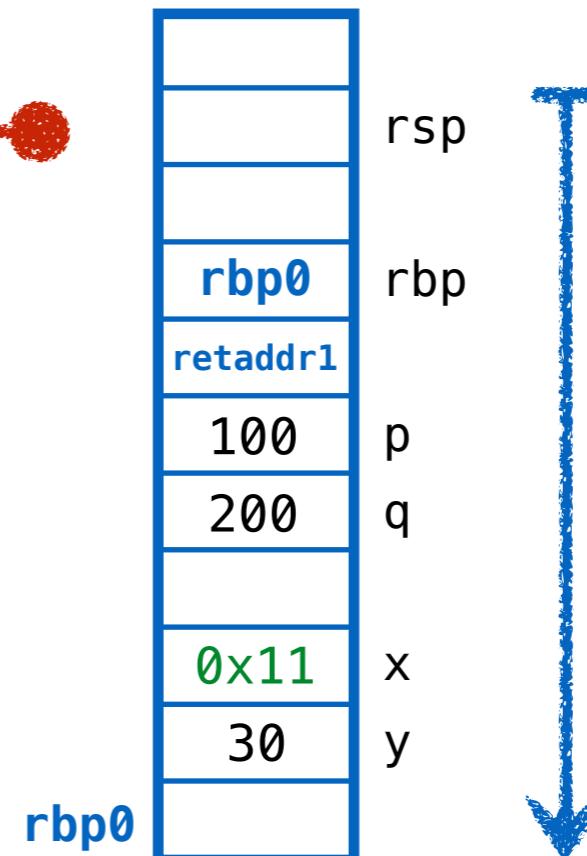
Those that are *not reachable from any stack frame*

ex3: garbage in the middle (with stack)

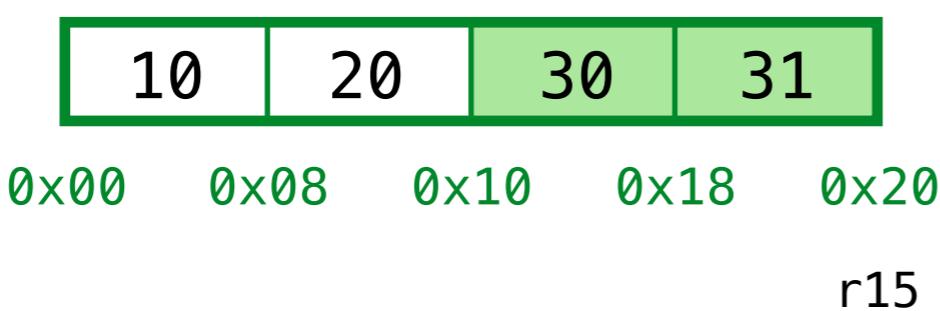
```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)

in
x[0] + z
```

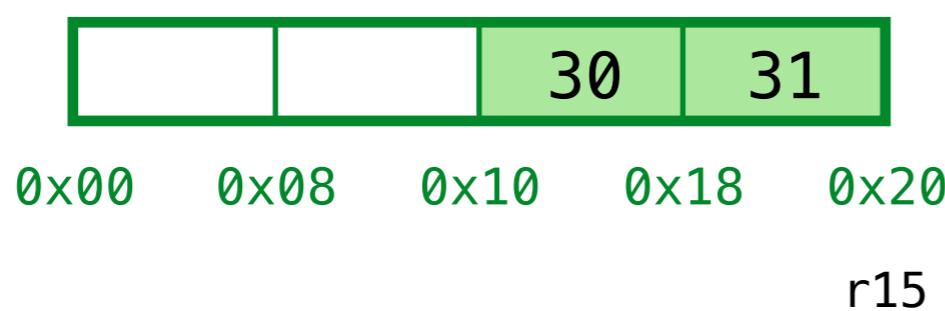
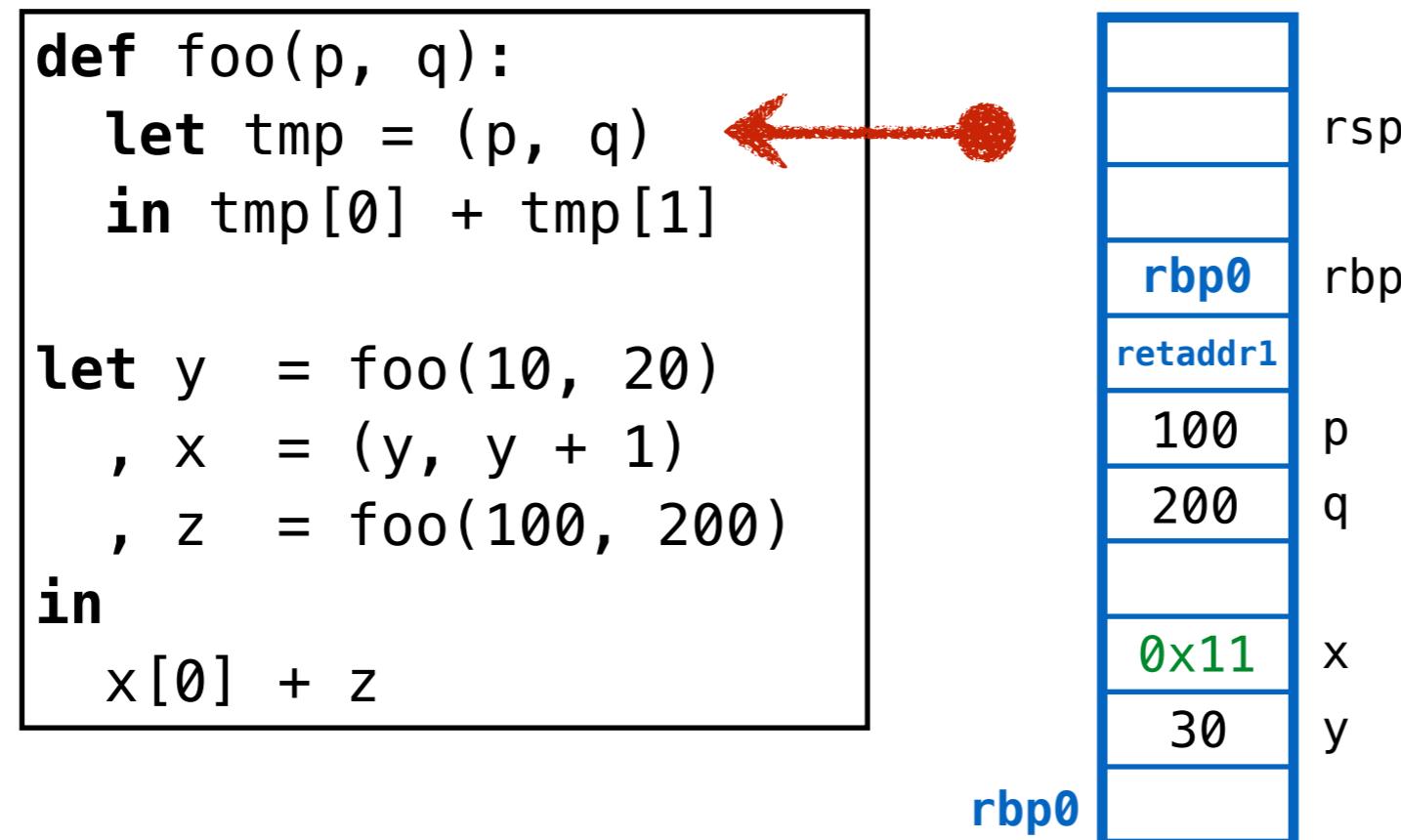


Lets reclaim & recycle garbage!



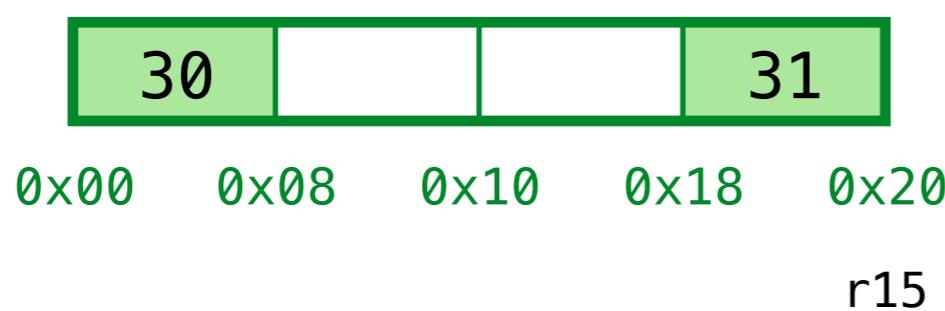
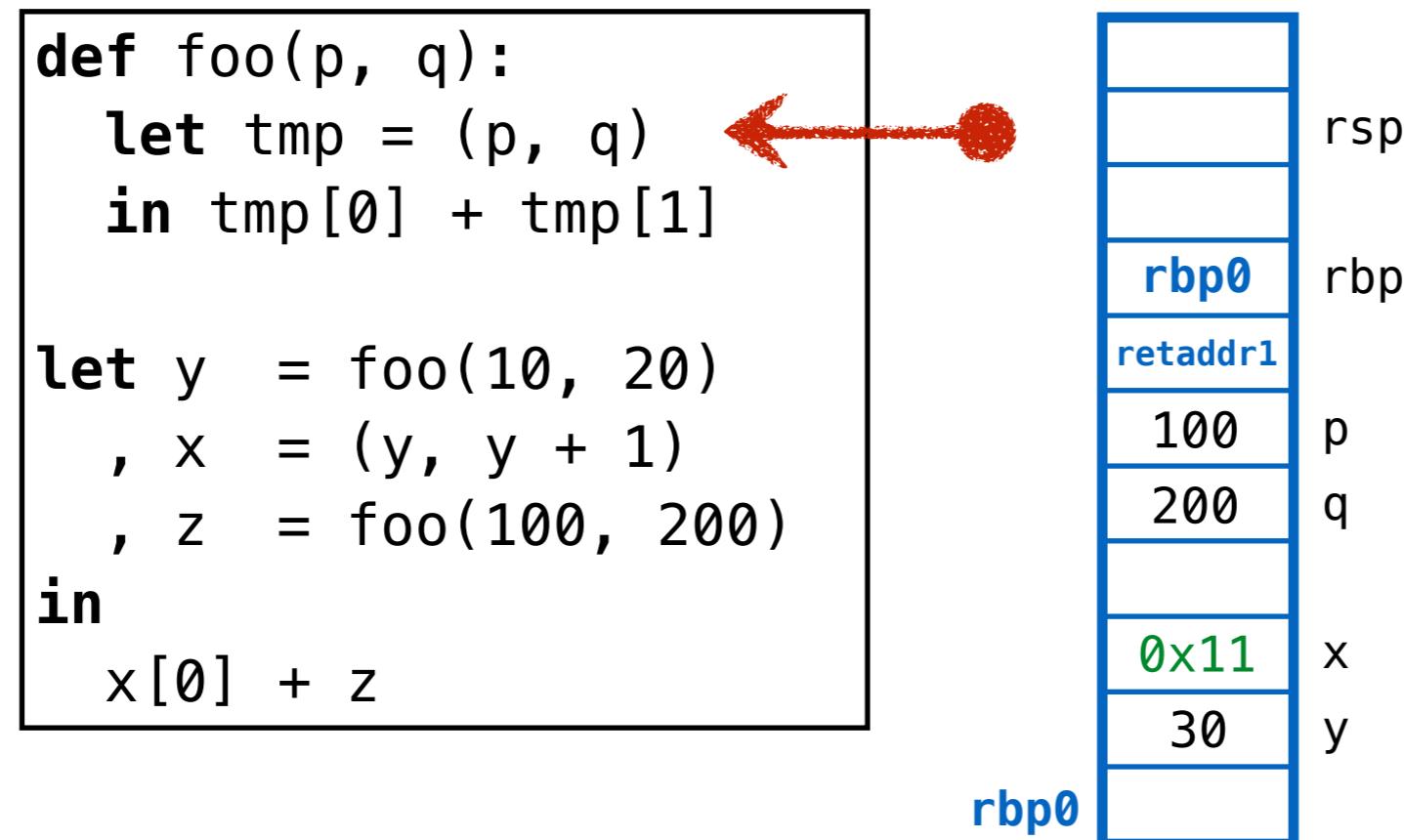
Which cells are garbage?

ex3: garbage in the middle (with stack)



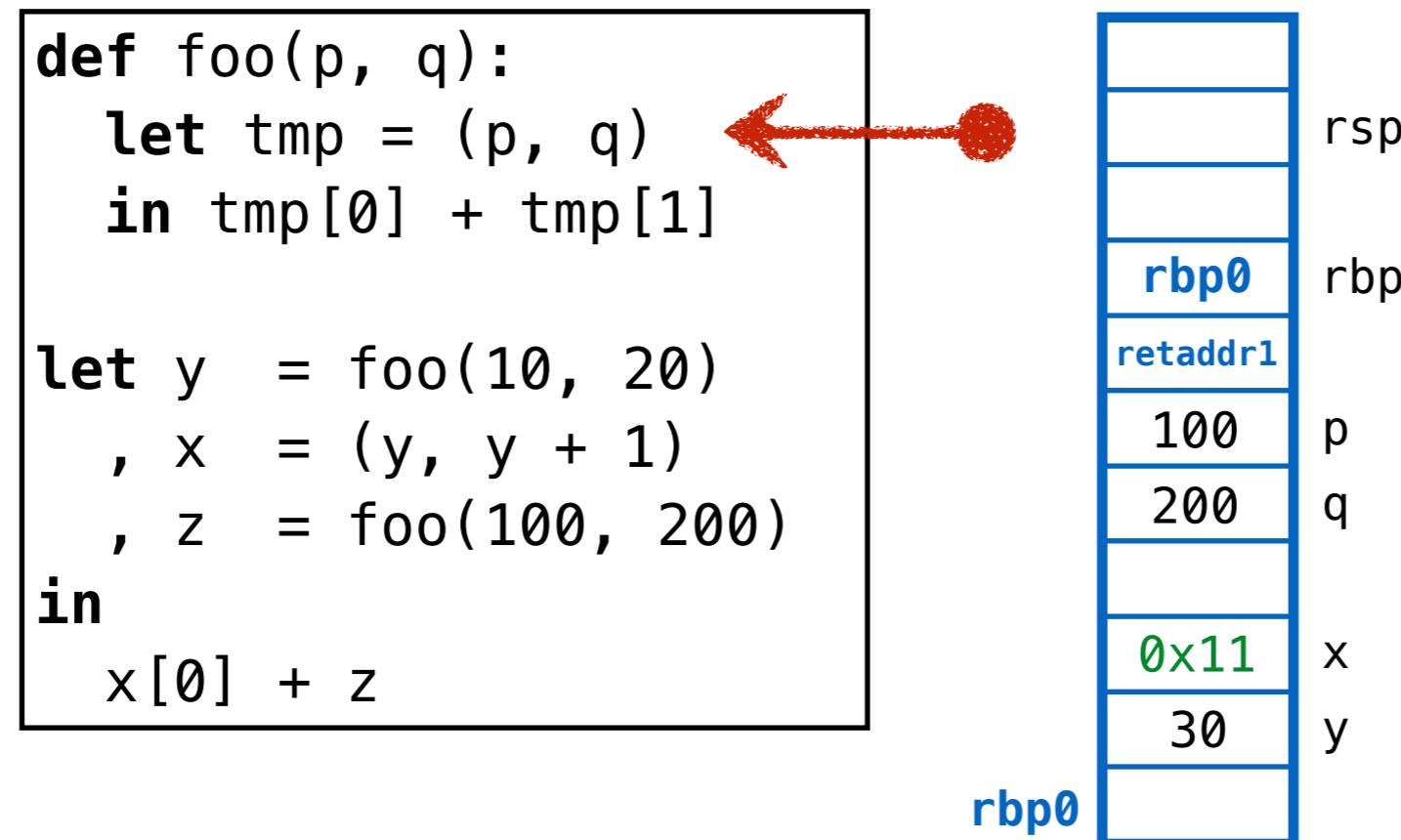
Compact the live cells

ex3: garbage in the middle (with stack)



Compact the live cells

ex3: garbage in the middle (with stack)

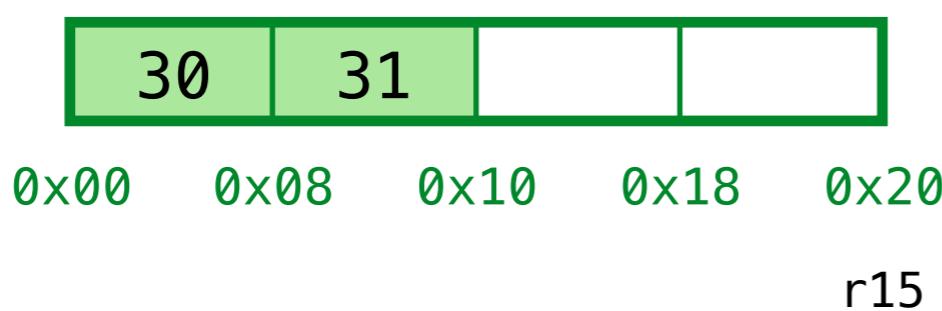
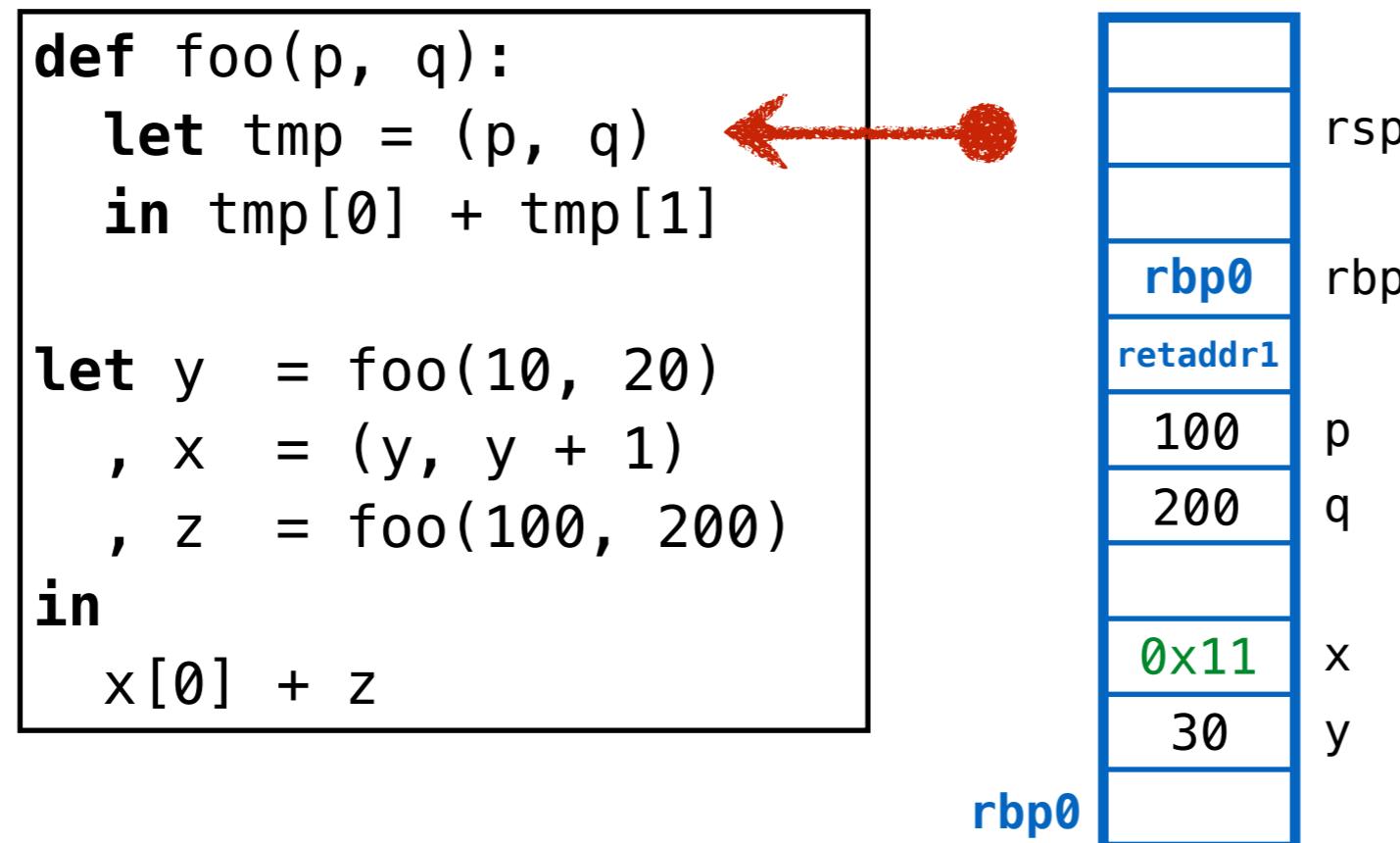


30	31		
----	----	--	--

0x00 0x08 0x10 0x18 0x20
r15

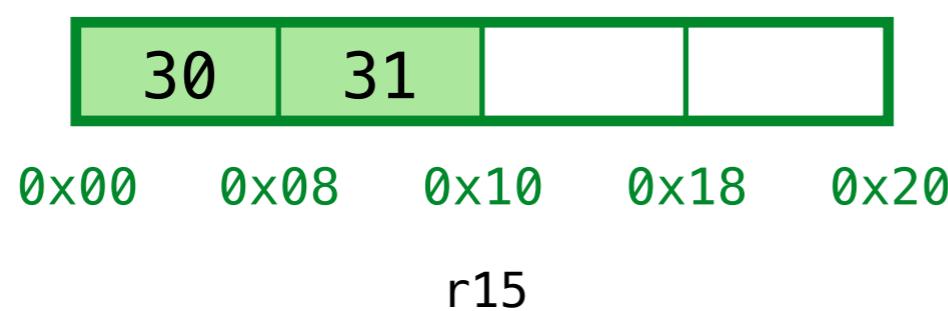
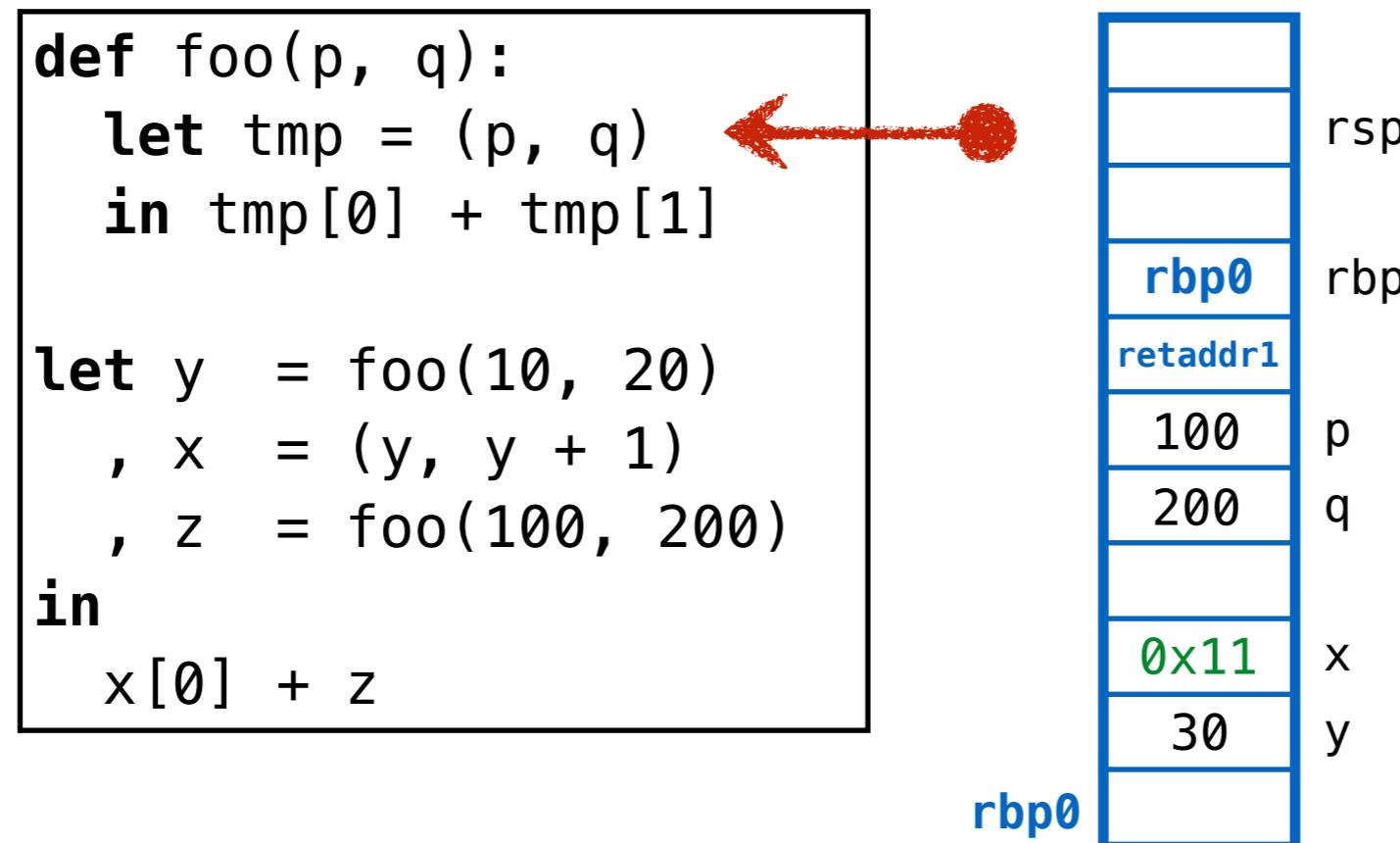
Compact the live cells

ex3: garbage in the middle (with stack)



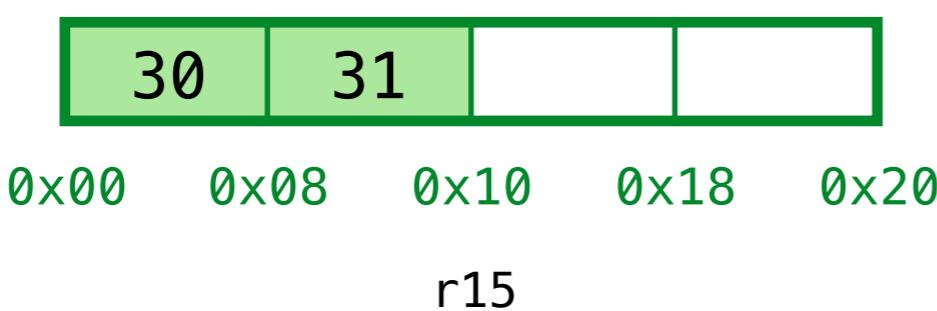
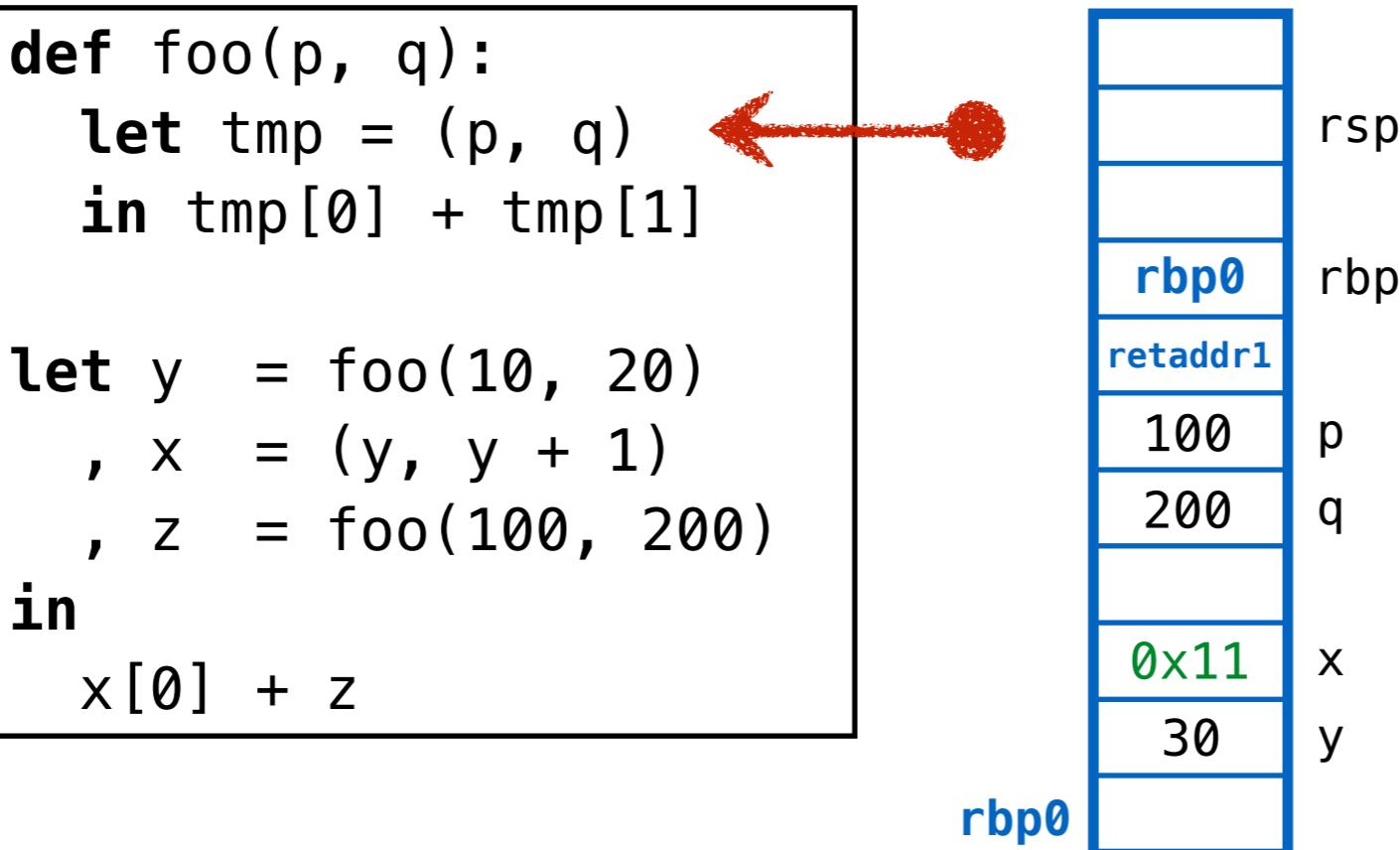
Compact the live cells ... then rewind r15

ex3: garbage in the middle (with stack)



Compact the live cells ... then rewind r15

ex3: garbage in the middle (with stack)

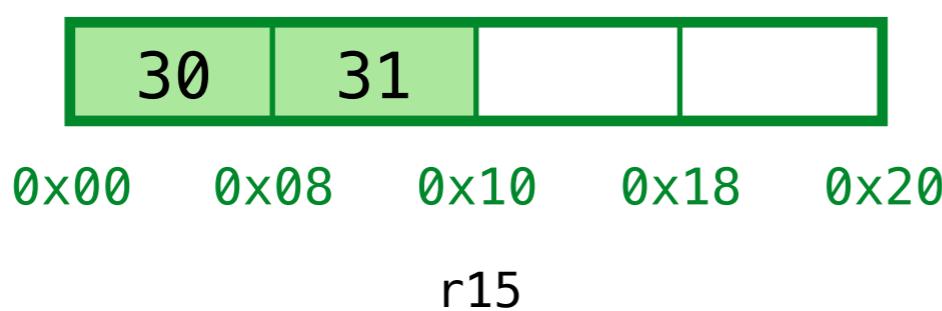
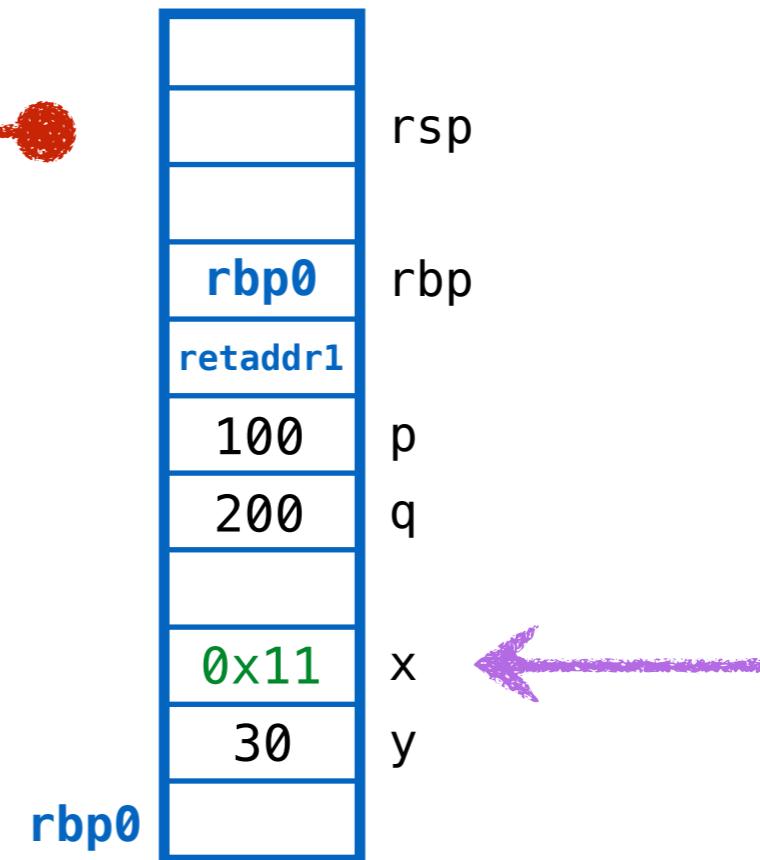


Problem???

ex3: garbage in the middle (with stack)

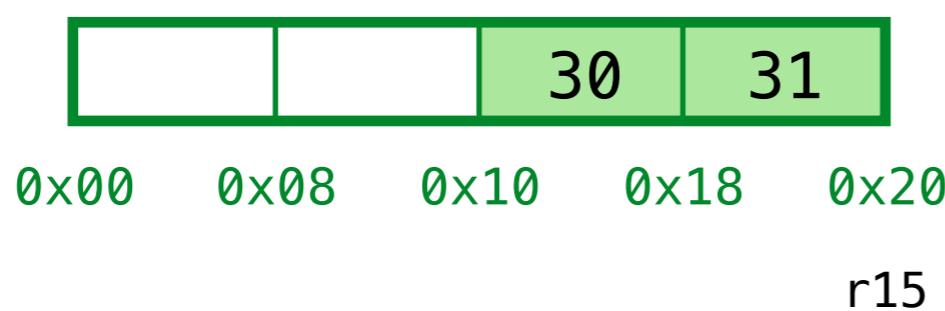
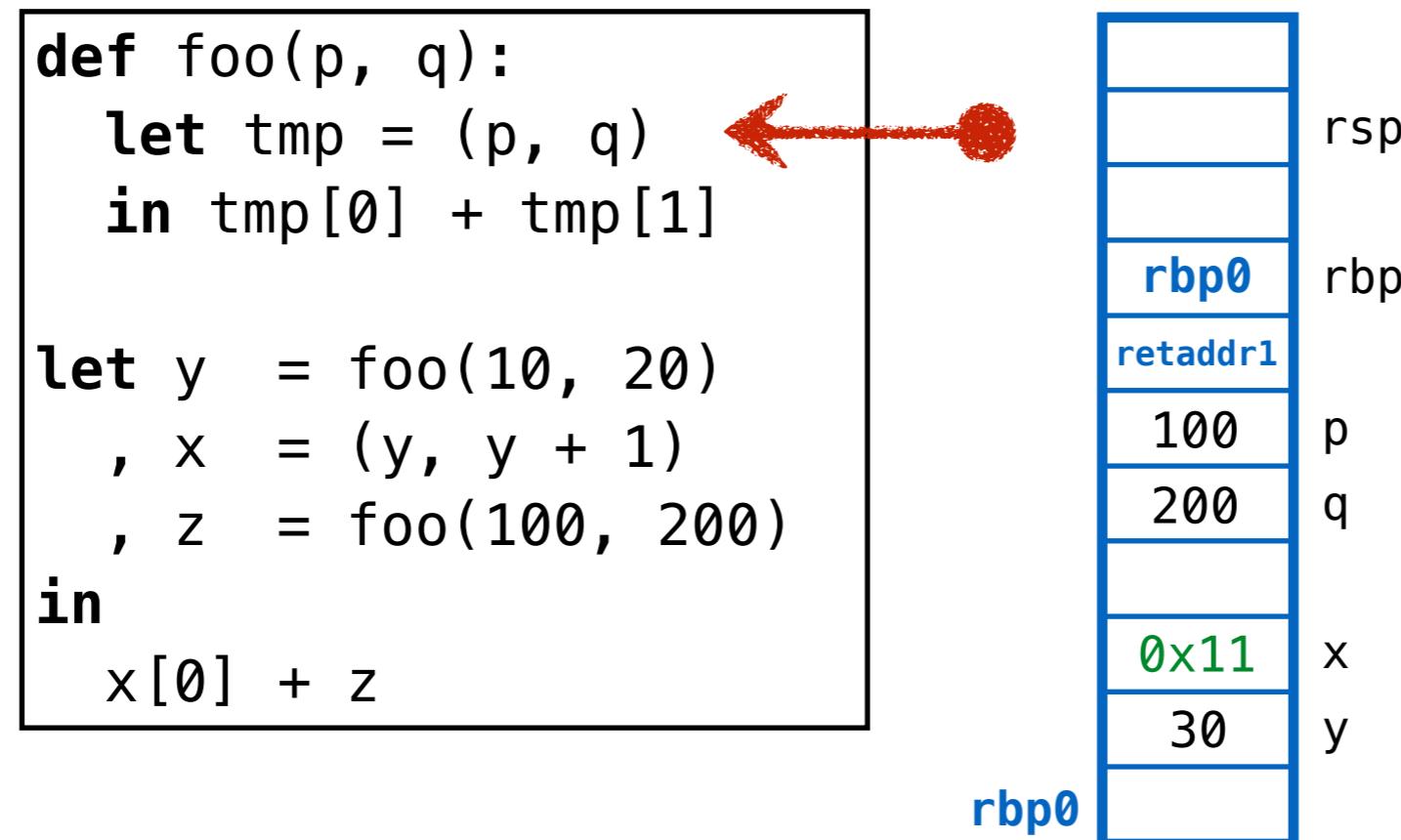
```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y = foo(10, 20)
    , x = (y, y + 1)
    , z = foo(100, 200)
in
    x[0] + z
```



Problem! Have to **REDIRECT** existing pointers

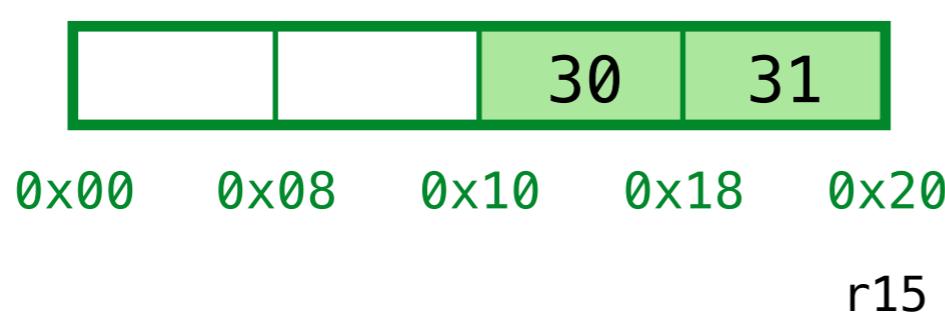
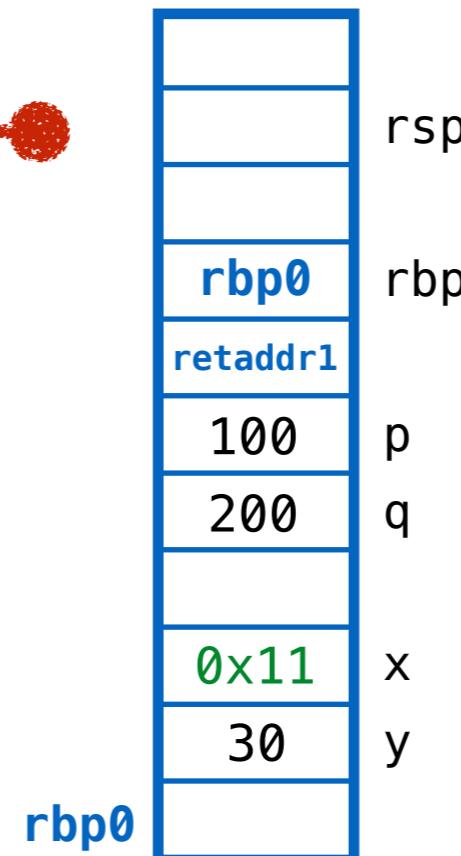
ex3: garbage in the middle (with stack)



ex3: garbage in the middle (with stack)

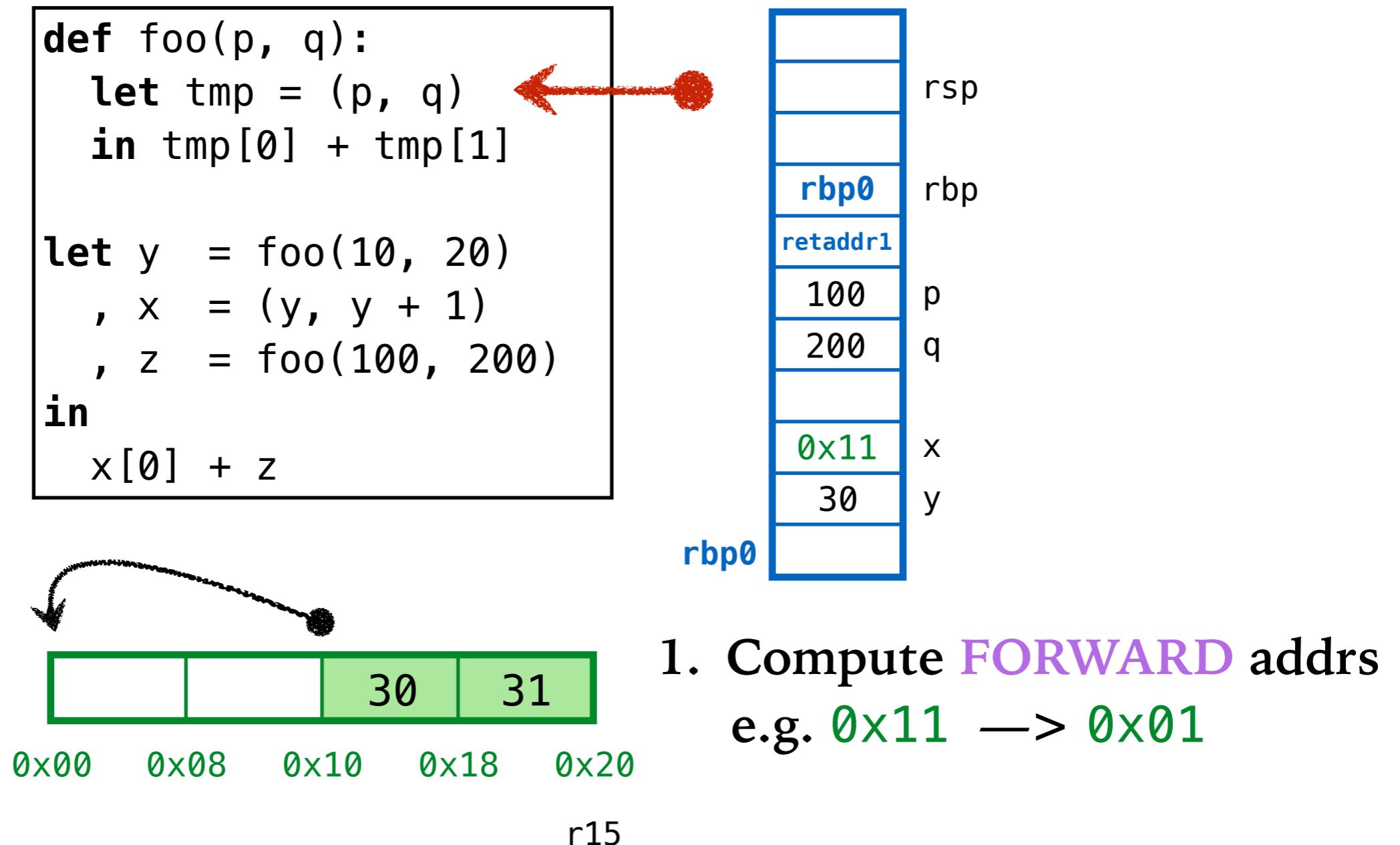
```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

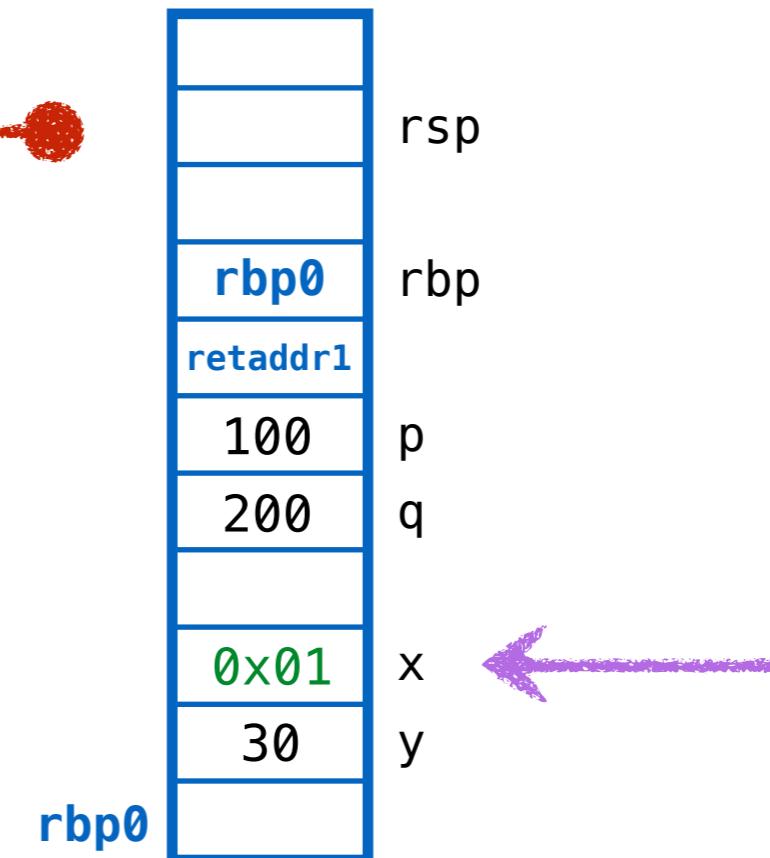
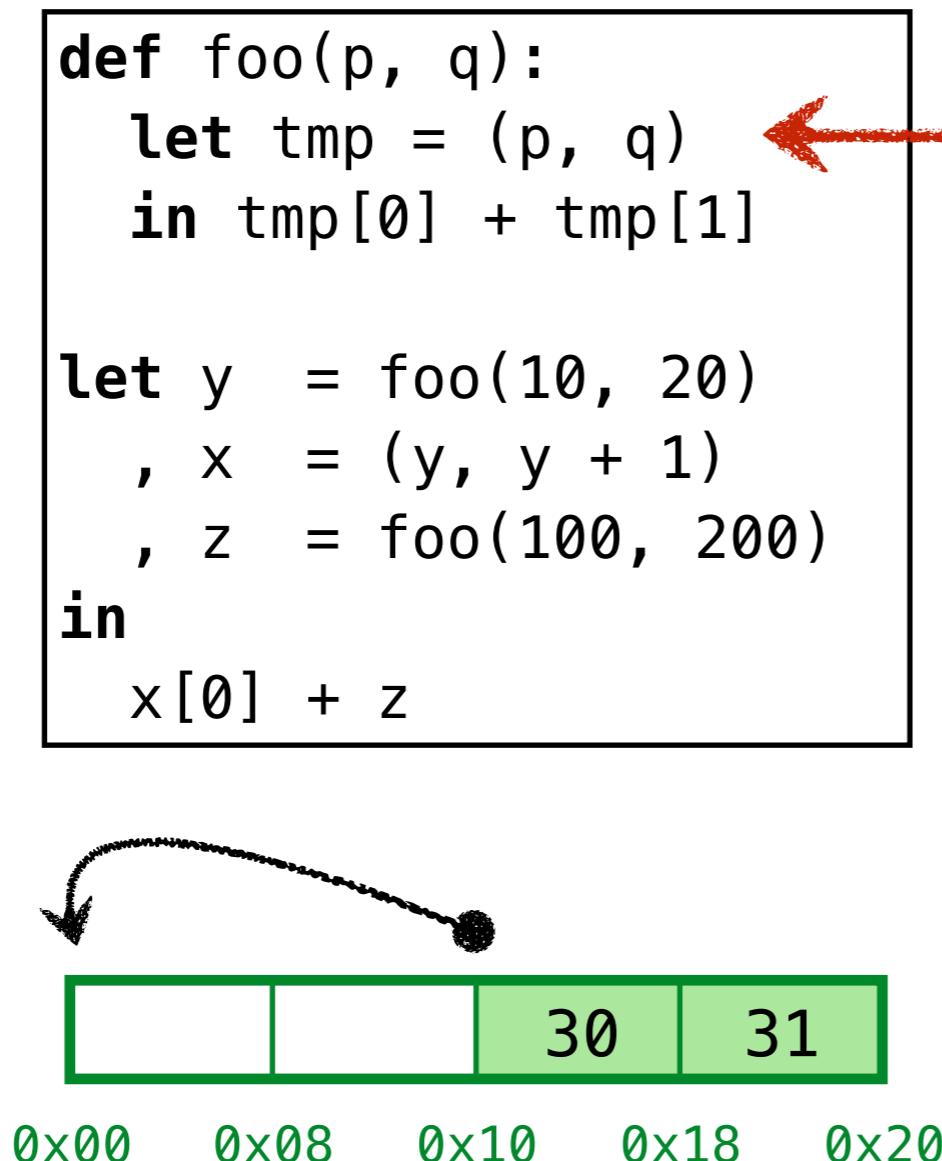


1. Compute **FORWARD** addrs
(i.e. new compacted addrs)

ex3: garbage in the middle (with stack)



ex3: garbage in the middle (with stack)

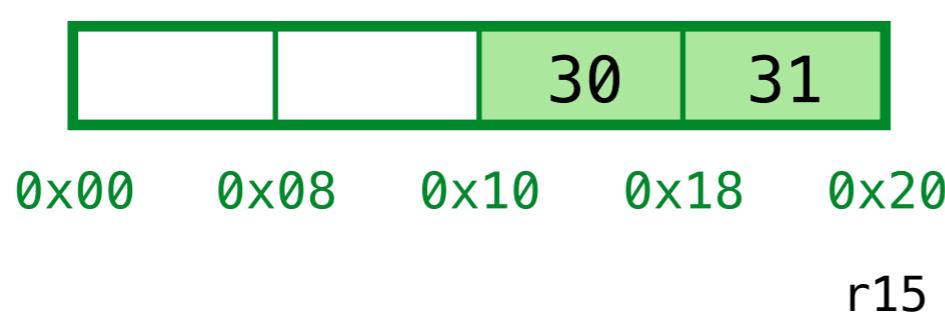
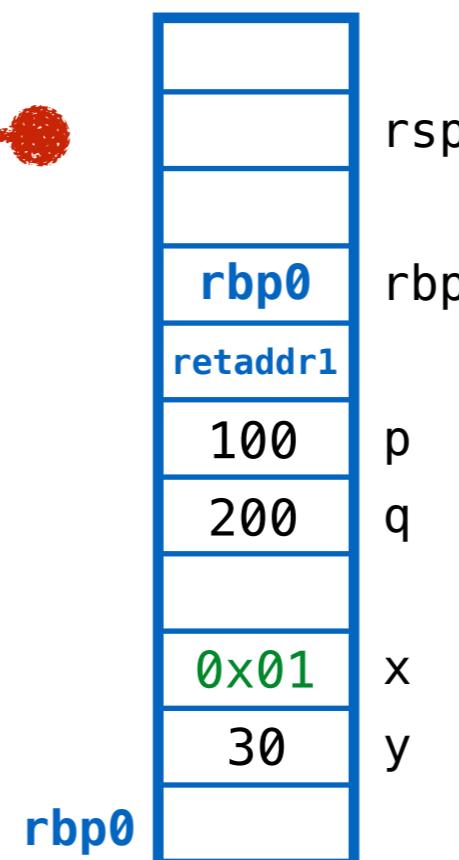


1. Compute **FORWARD** addrs
e.g. `0x11` \rightarrow `0x01`
2. **REDIRECT** addrs on stack

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

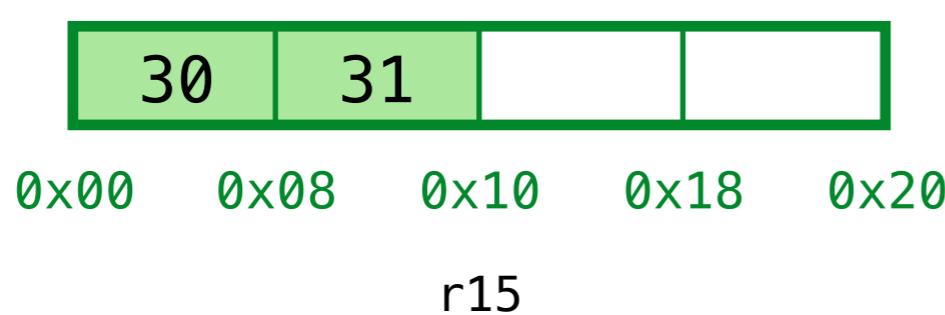
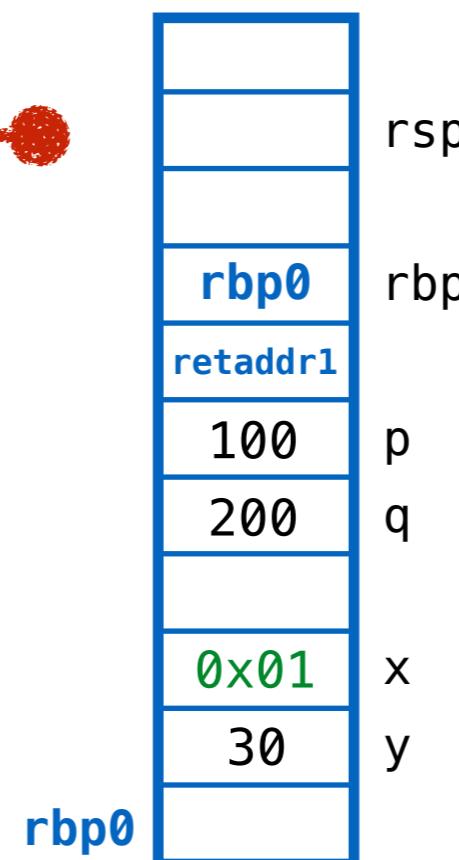


1. Compute **FORWARD** addrs
e.g. 0x11 → 0x01
2. **REDIRECT** addrs on stack
3. **COMPACT** cells on heap

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

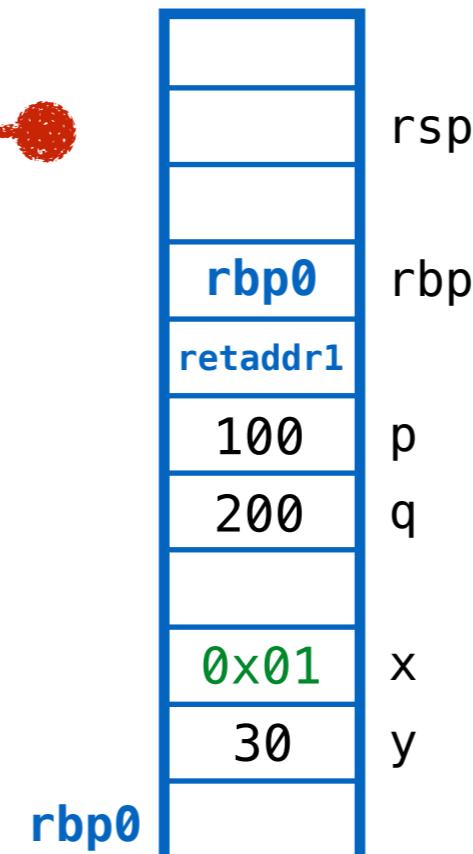


1. Compute **FORWARD** addrs
e.g. **0x11** → **0x01**
2. **REDIRECT** addrs on stack
3. **COMPACT** cells on heap

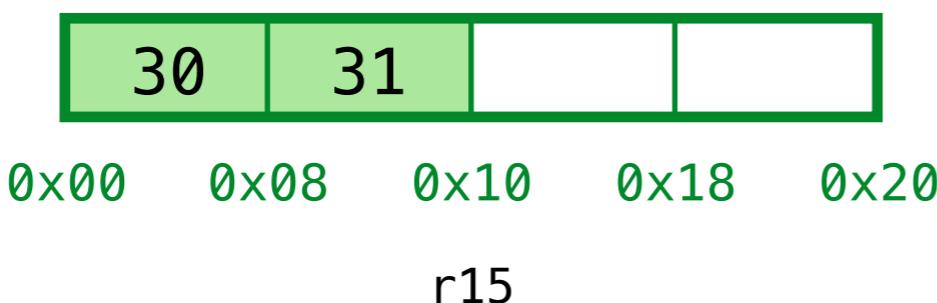
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



Yay! Have space for (p, q)

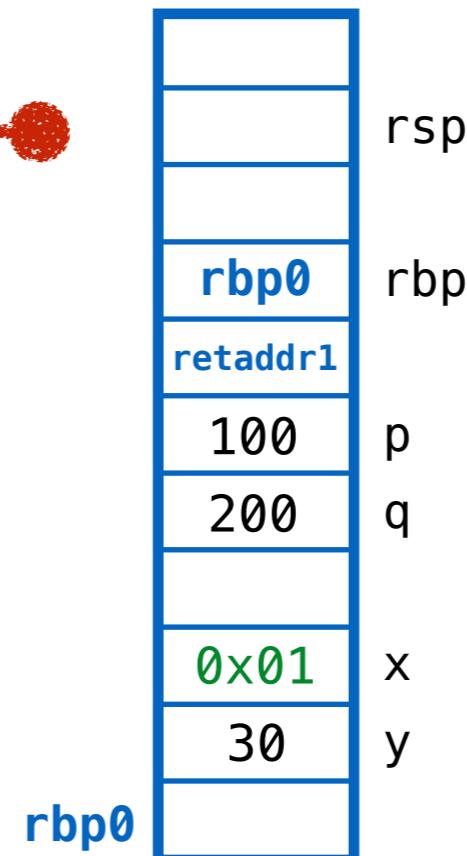


r15

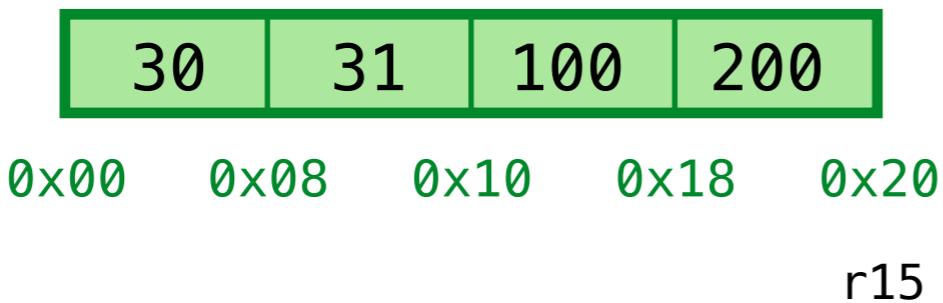
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



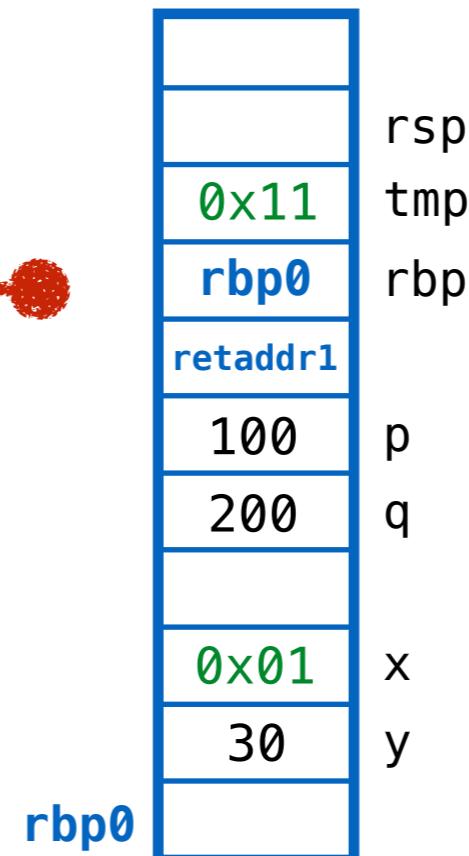
Yay! Have space for (p, q)



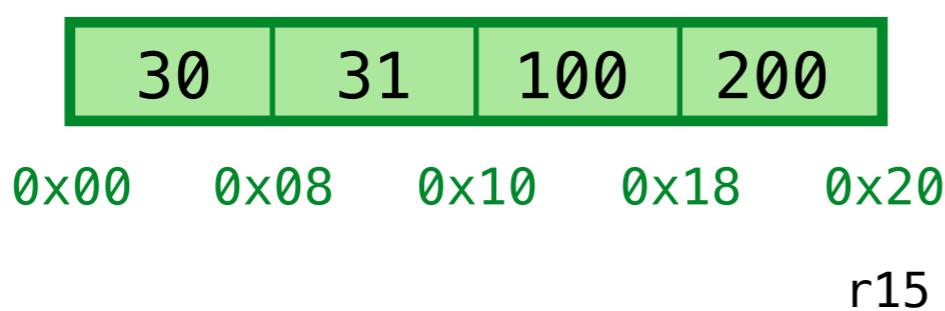
r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + z
```



Return (rax) = 300

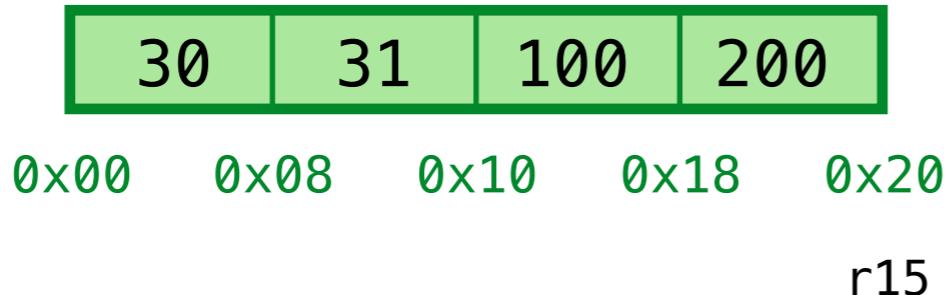
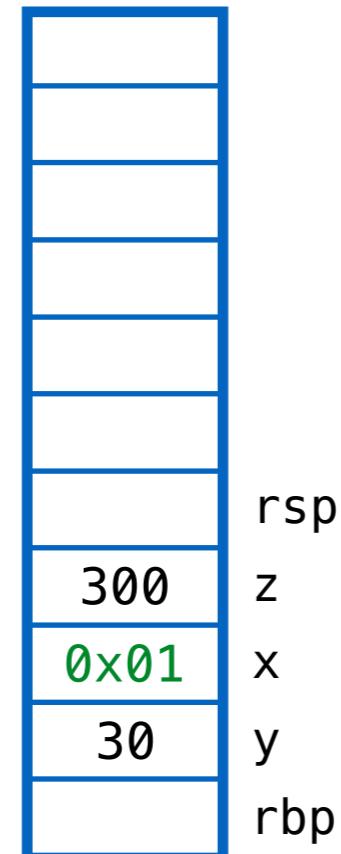


ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + z
```

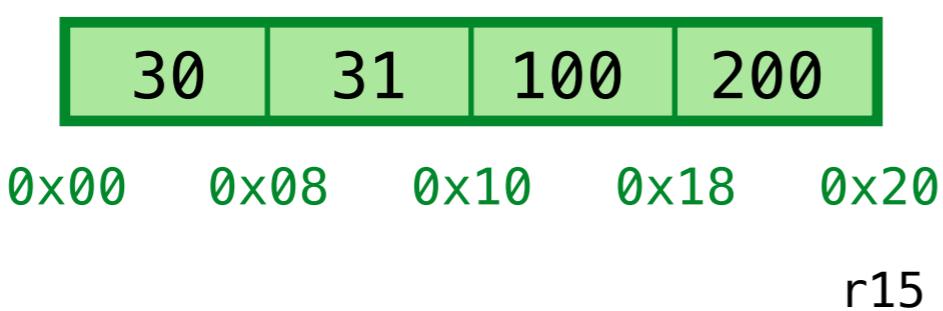
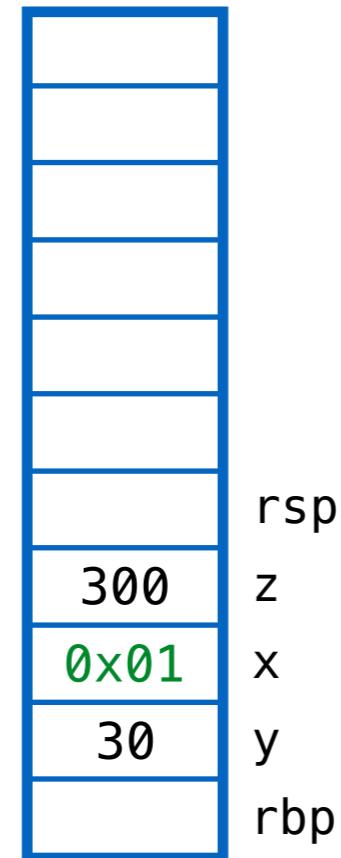
Return (rax) = 300



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

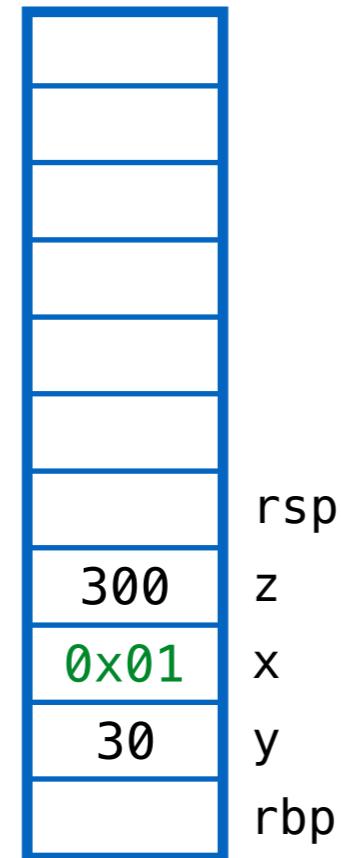
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



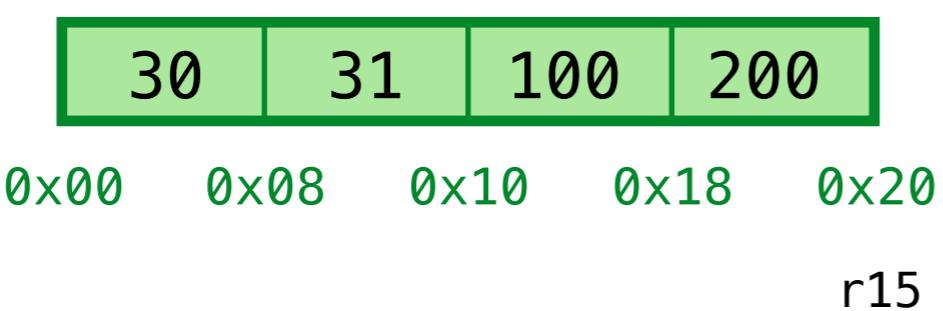
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



Return (rax) = 30+300 = 330



Garter / GC

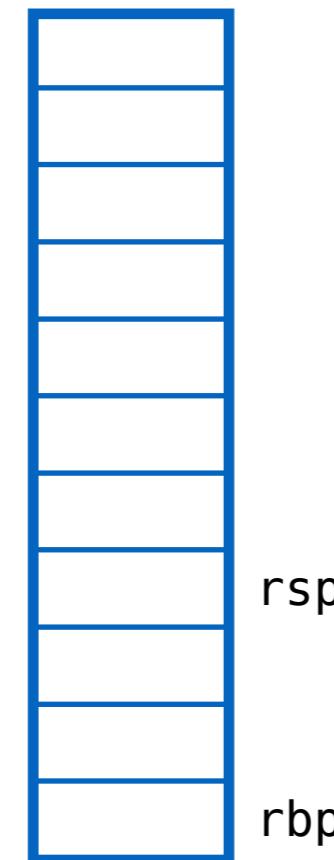
Example 4

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 = ←
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15



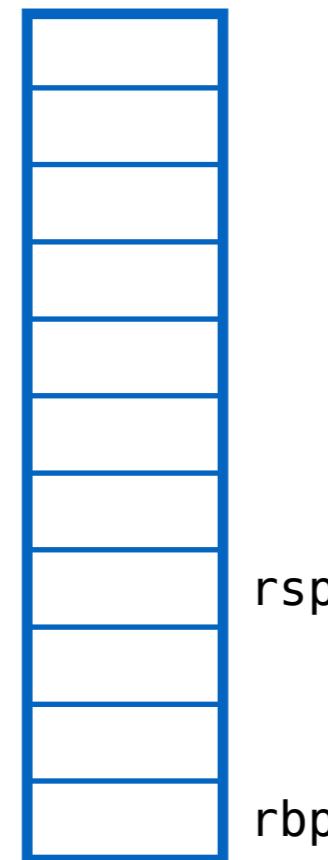
0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3) ←
        in sum(l1)
    , l  = range(t1, t1 + 3)
in
(1000, l)
```



call range(0, 3)

r15



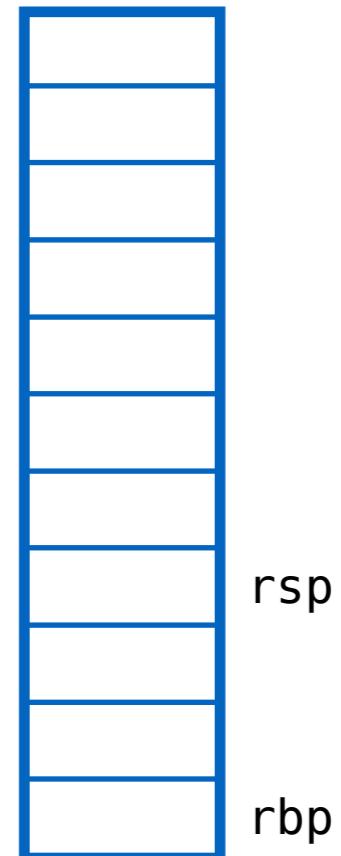
0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3) ←
        in sum(l1)
    , l  = range(t1, t1 + 3)
in
(1000, l)
```



QUIZ: What is heap when range(0,3) returns?

r15

(A)

0	0x11	1	0x21	2	false							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

r15

(B)

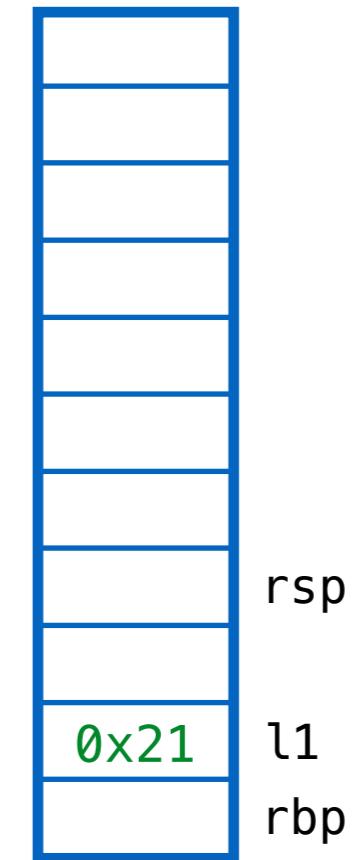
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



let l1 =
 let l1 = range(0, 3)
 in sum(l1)

r15

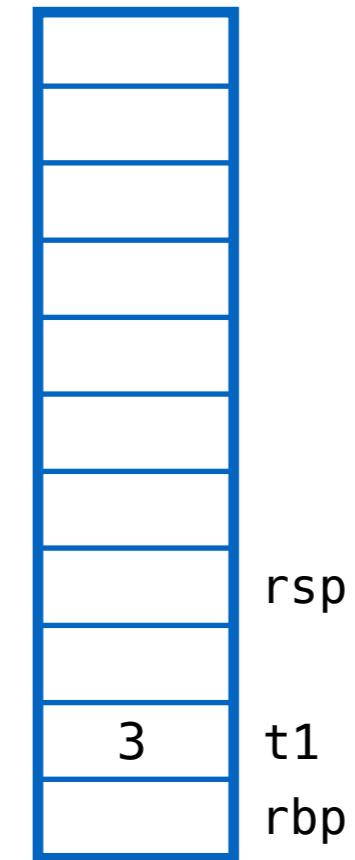
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1) ←
, l  = range(t1, t1 + 3)
in
(1000, l)
```



Result sum(0x11) = 3

r15

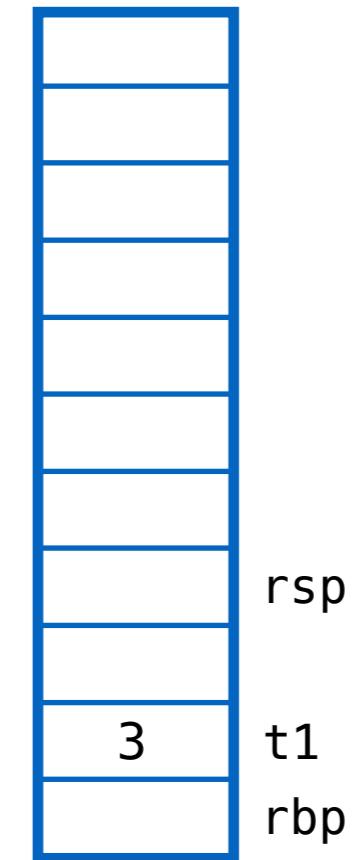
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, t = range(t1, t1 + 3)
in
(1000, l)
```



r15

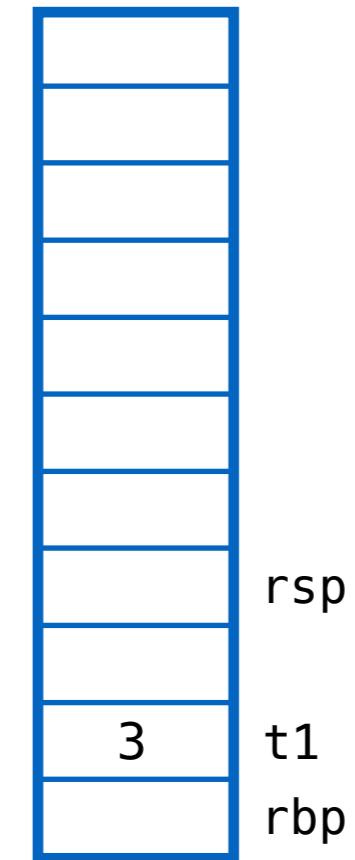
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3) ←
in
(1000, l)
```



call range(3,6)

r15

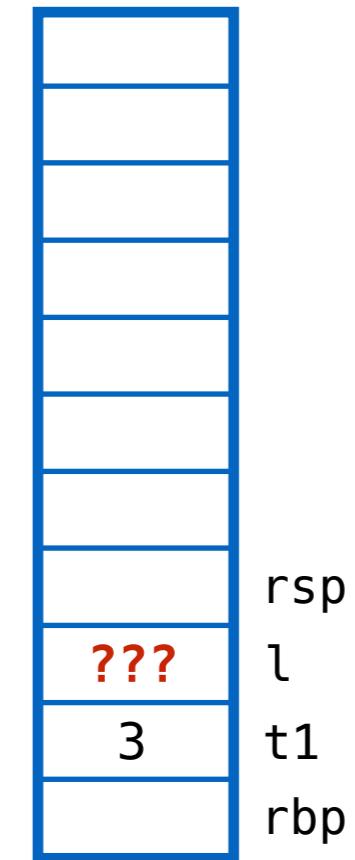
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



call range(3,6)

r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58

QUIZ: What is the value of l?

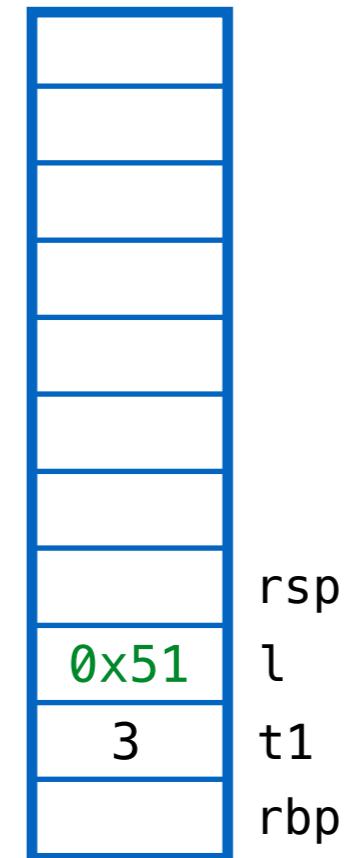
- (A) 0x30 (B) 0x31 (C) 0x50 (D) 0x51 (E) 0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



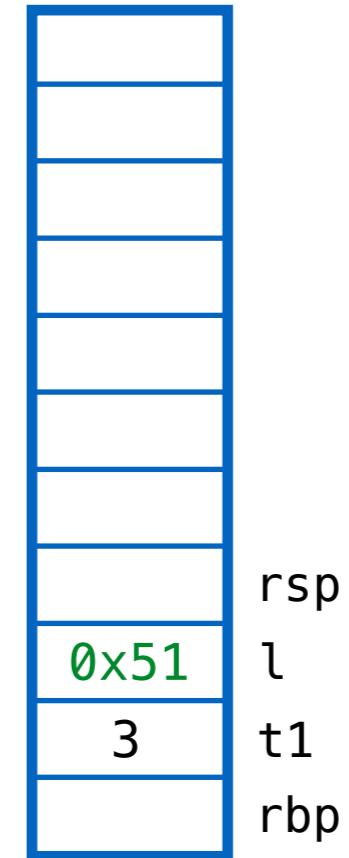
r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58

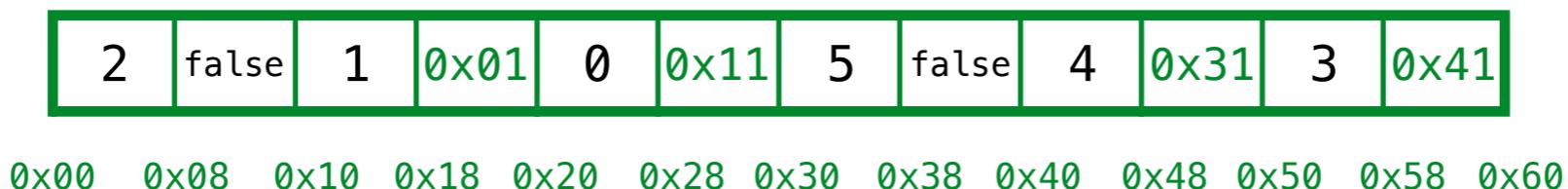
ex4: recursive data

QUIZ: Which cells are “live” on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50



r15

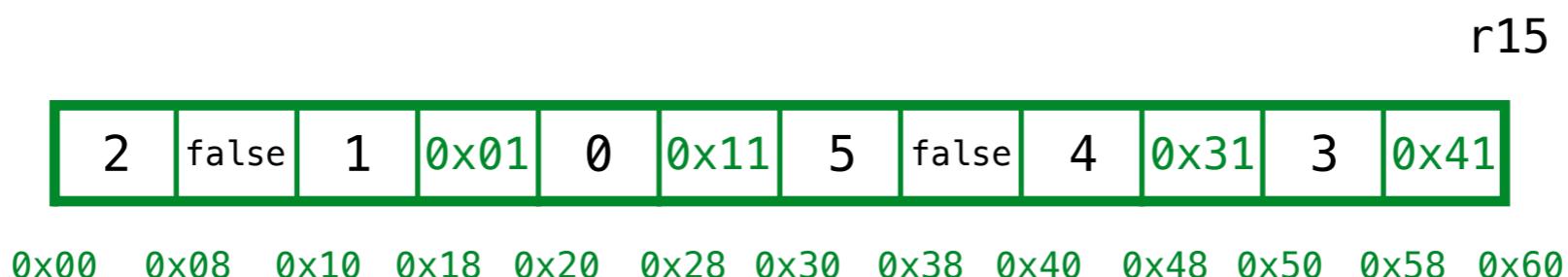
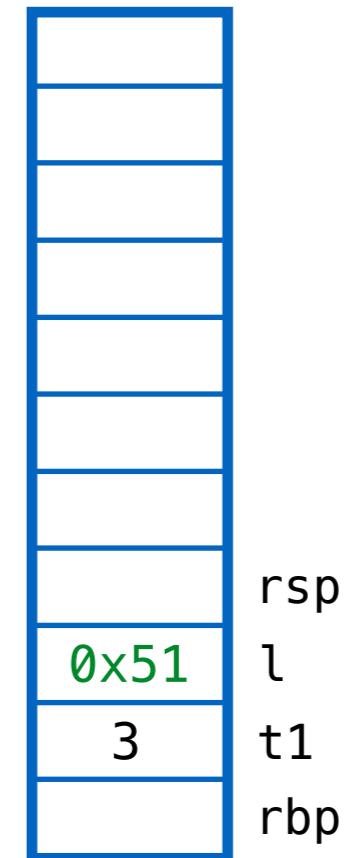


ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



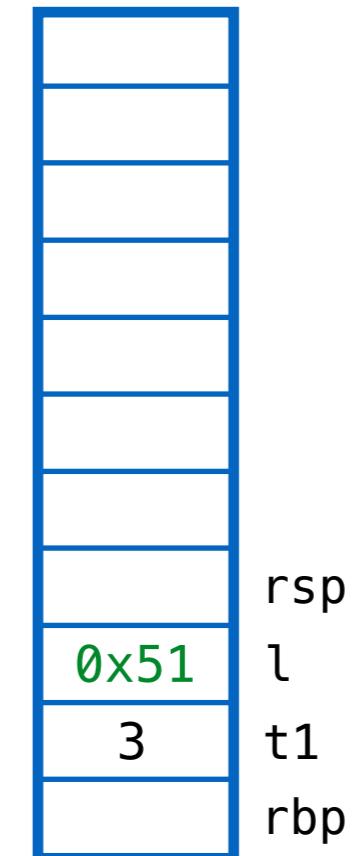
1. **MARK** live addrs
2. Compute **FORWARD** addrs
3. **REDIRECT** addrs on stack
4. **COMPACT** cells on heap

ex4: recursive data

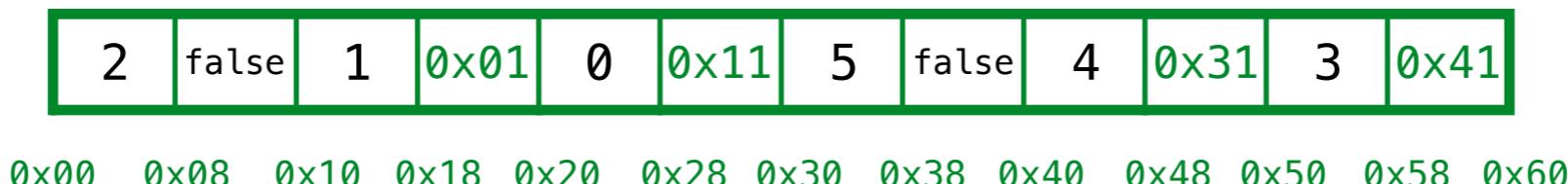
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15



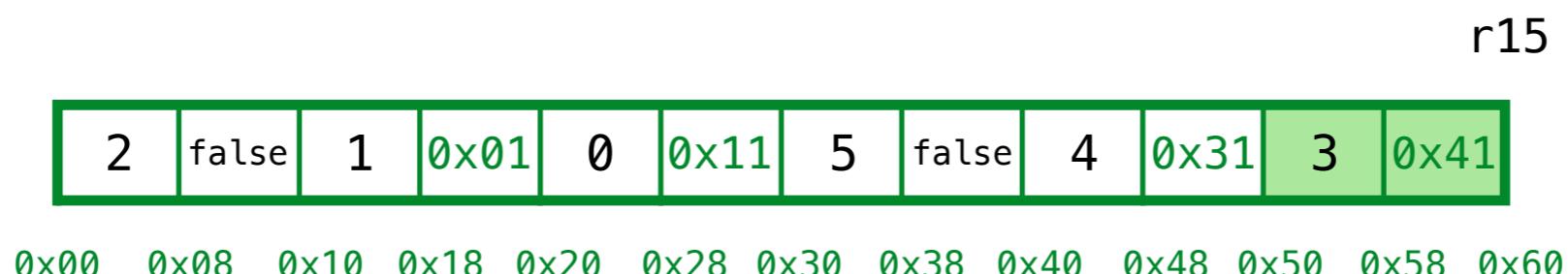
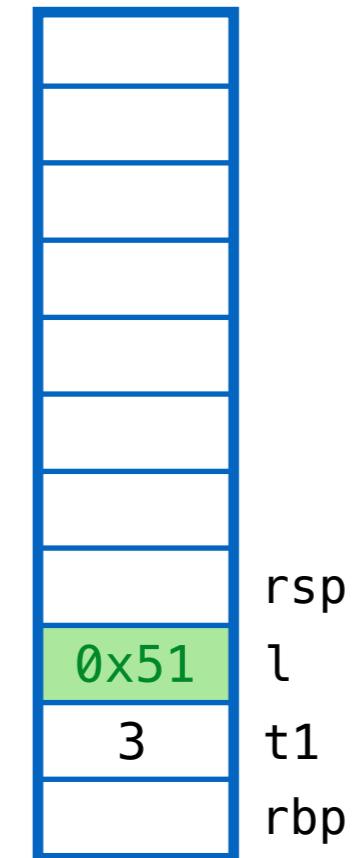
1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



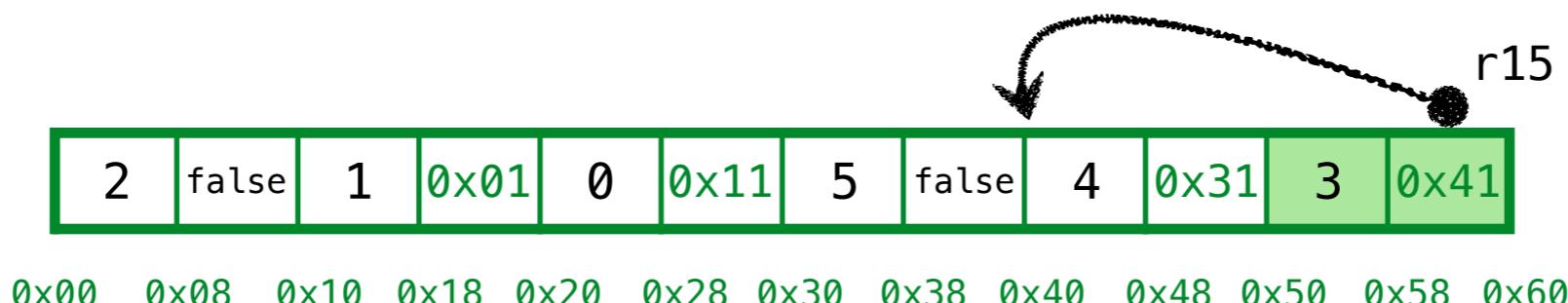
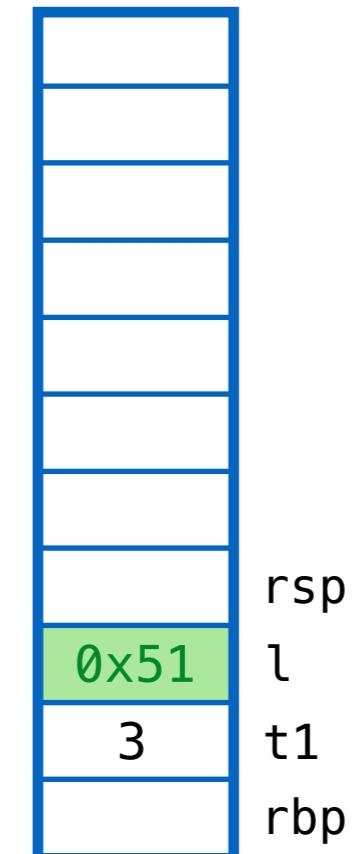
1. **MARK** live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



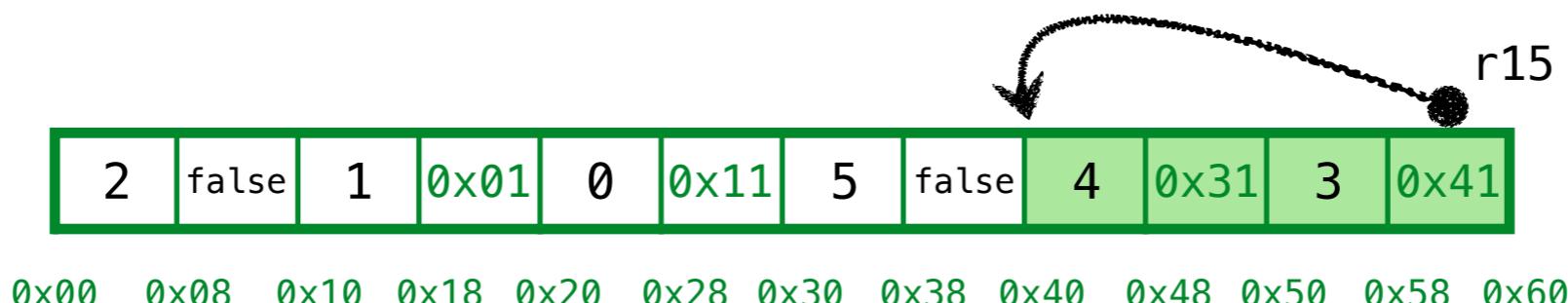
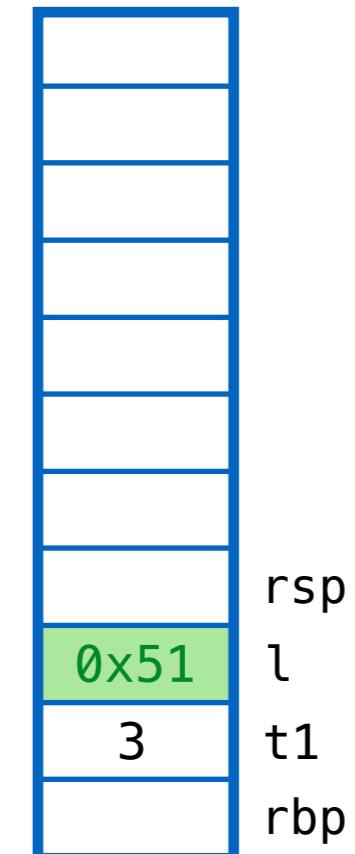
1. **MARK** live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



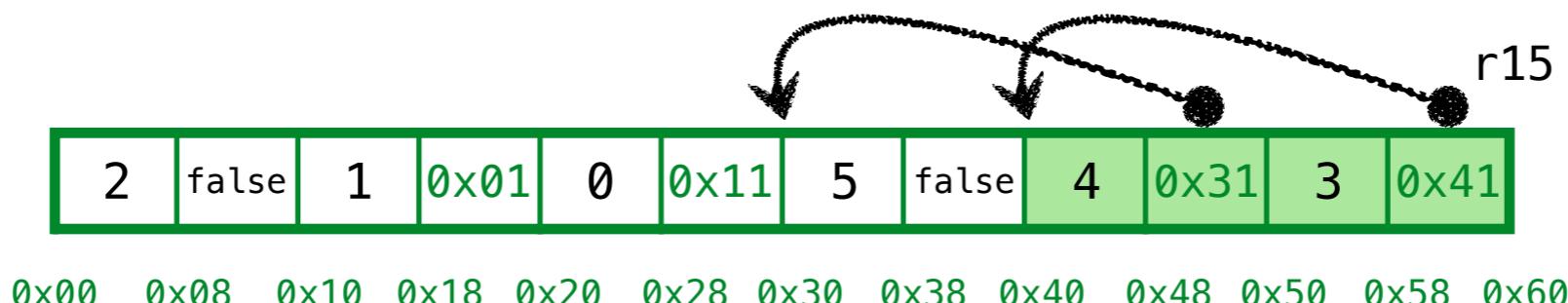
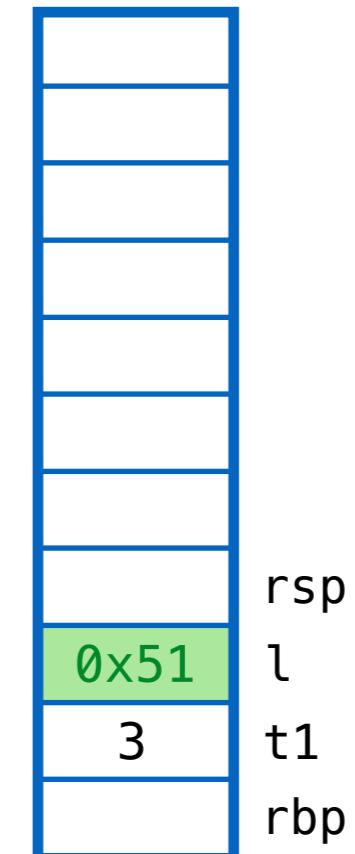
1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



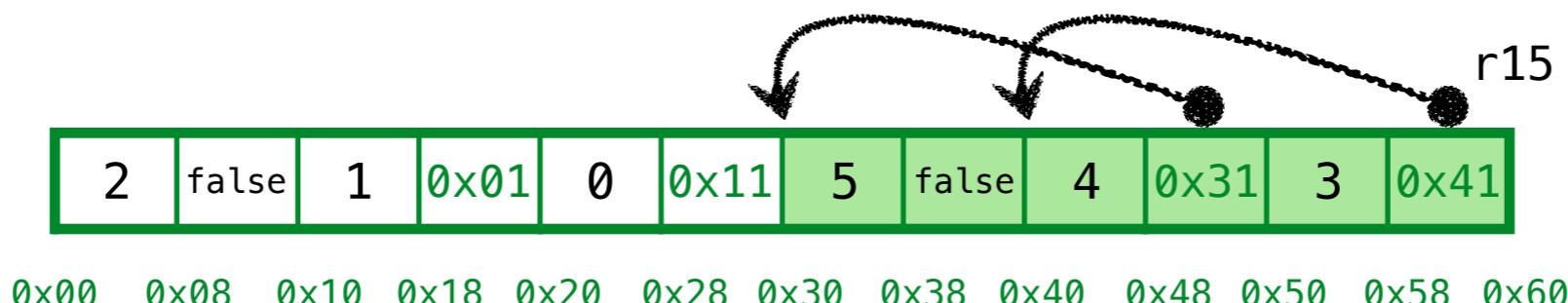
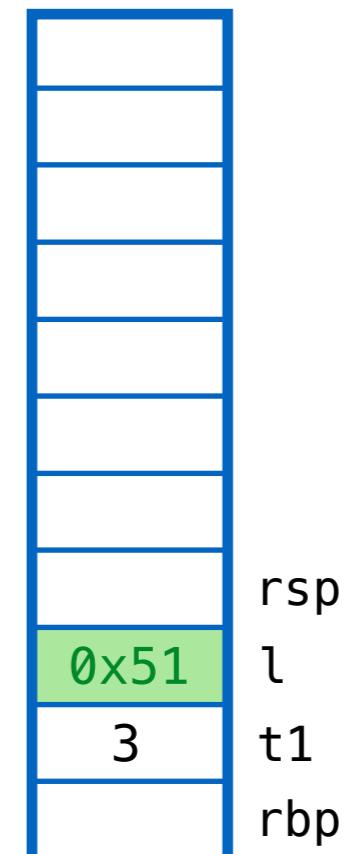
1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



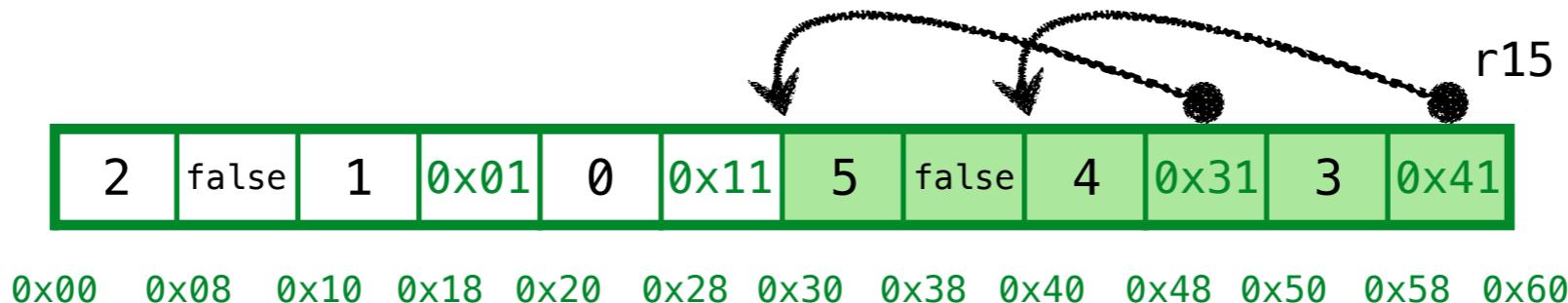
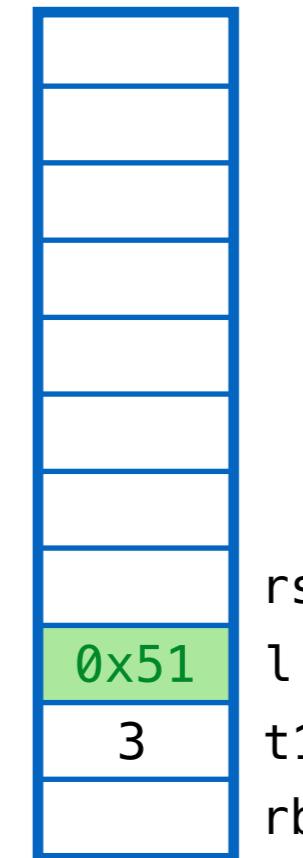
1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



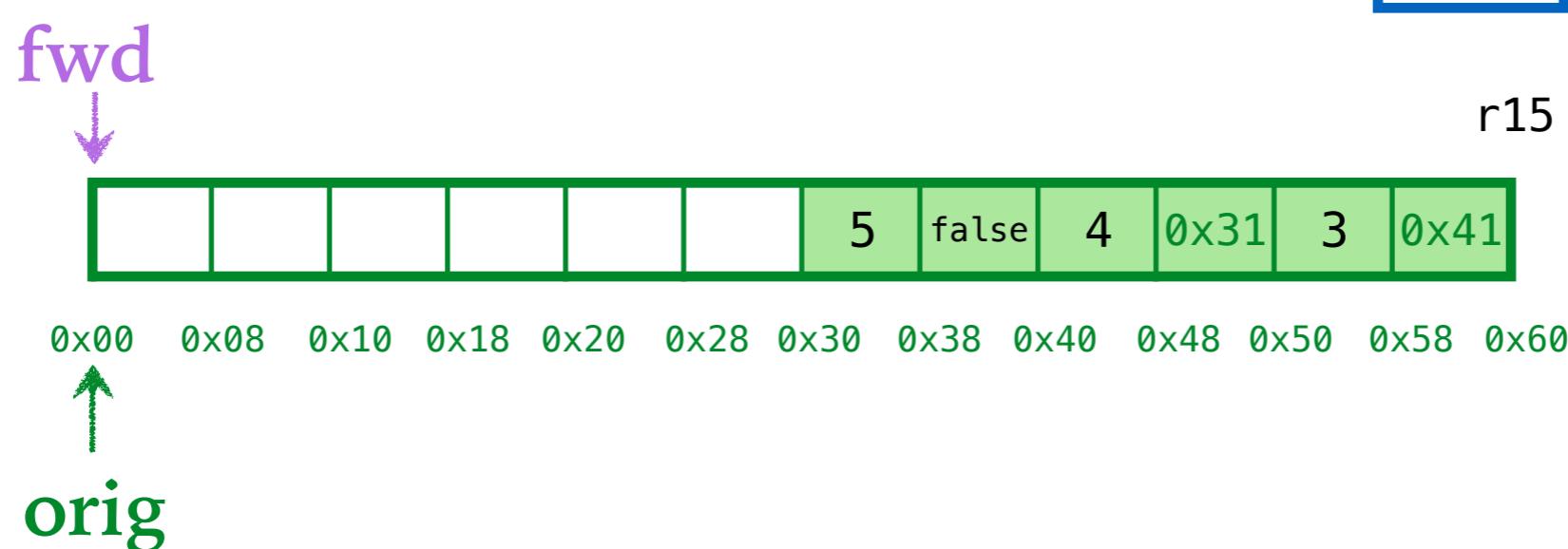
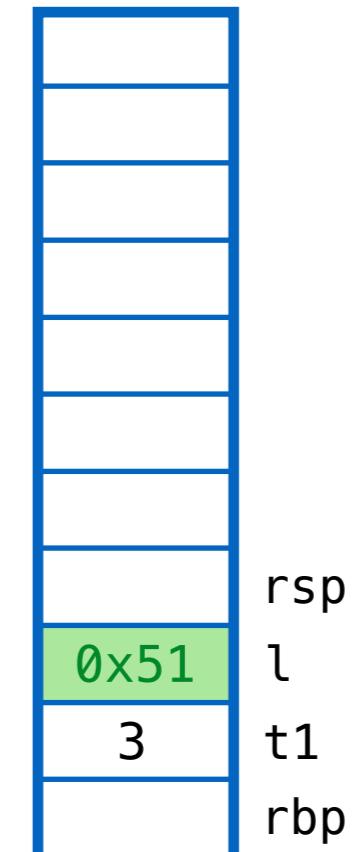
Done!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



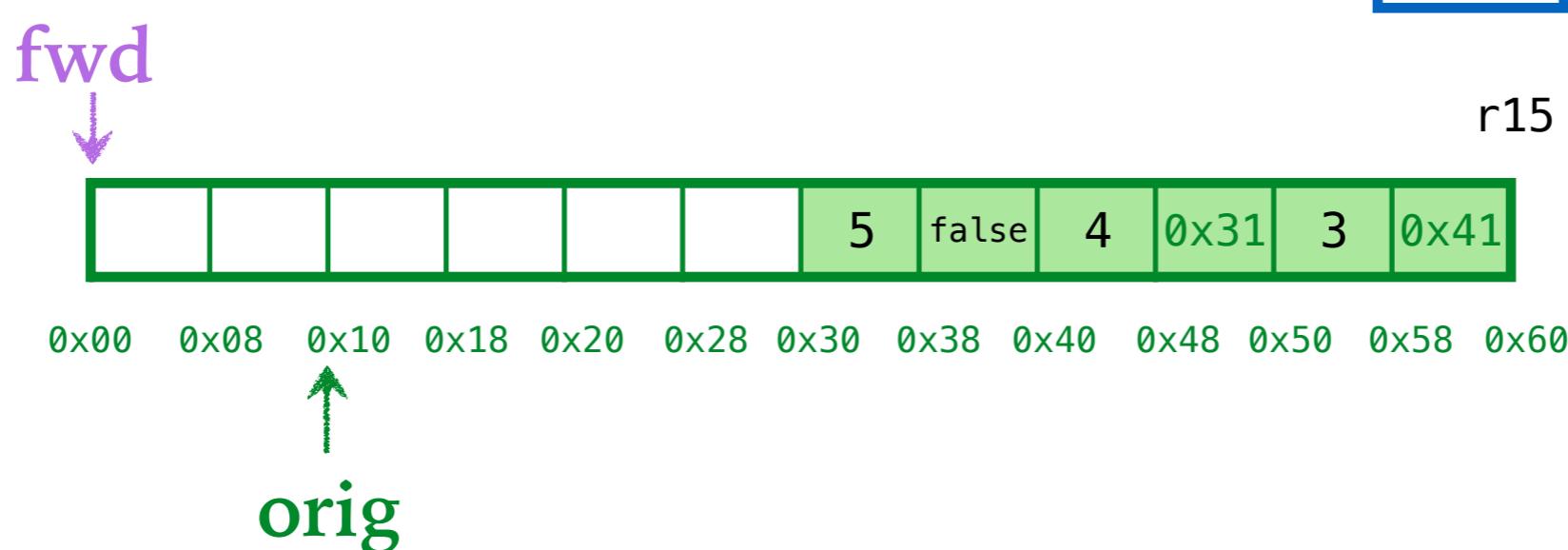
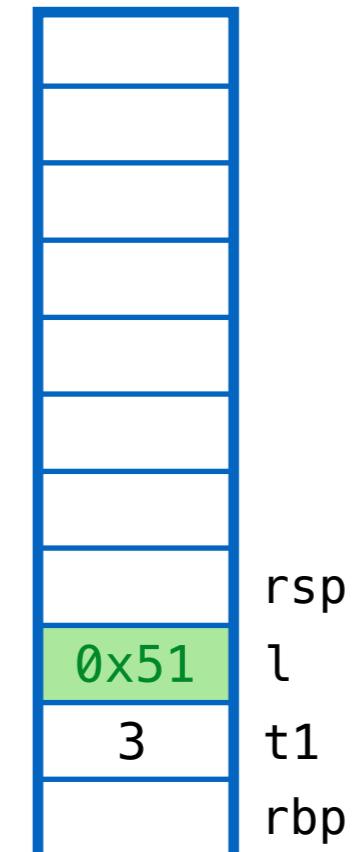
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



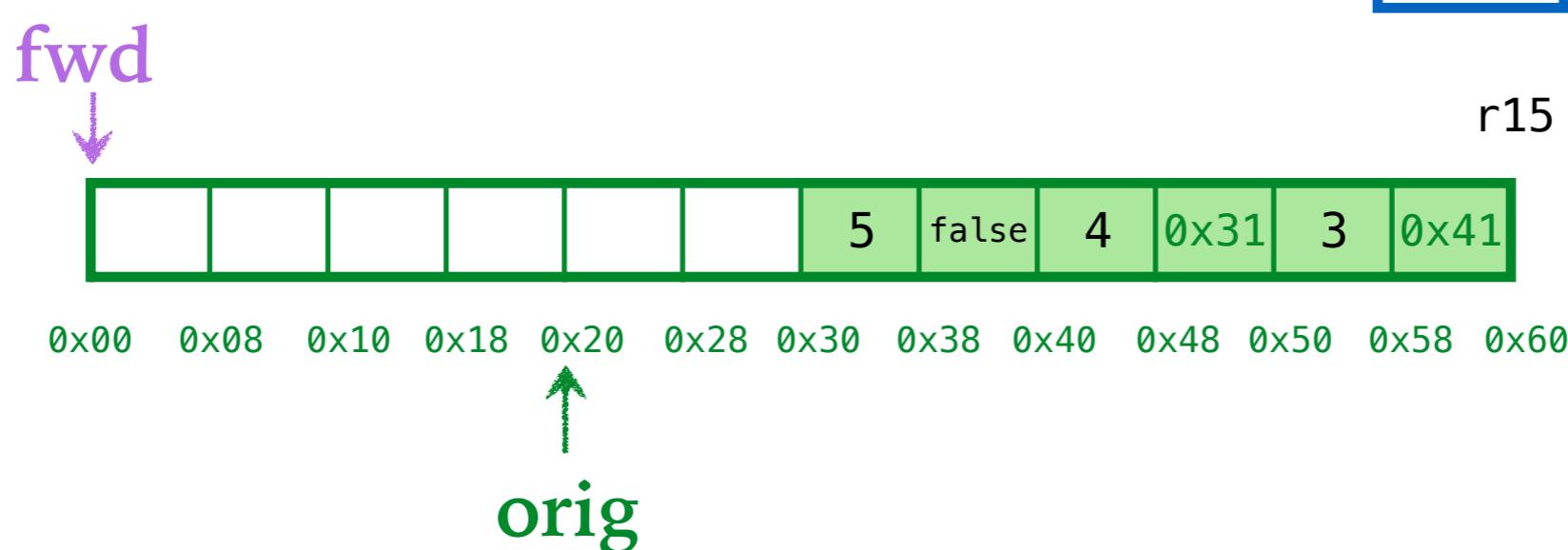
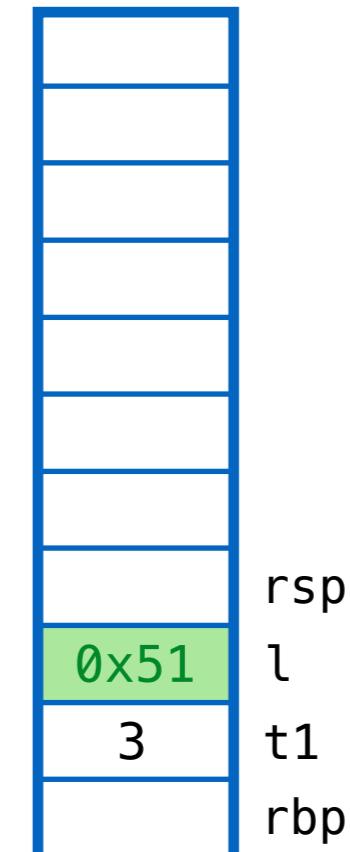
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



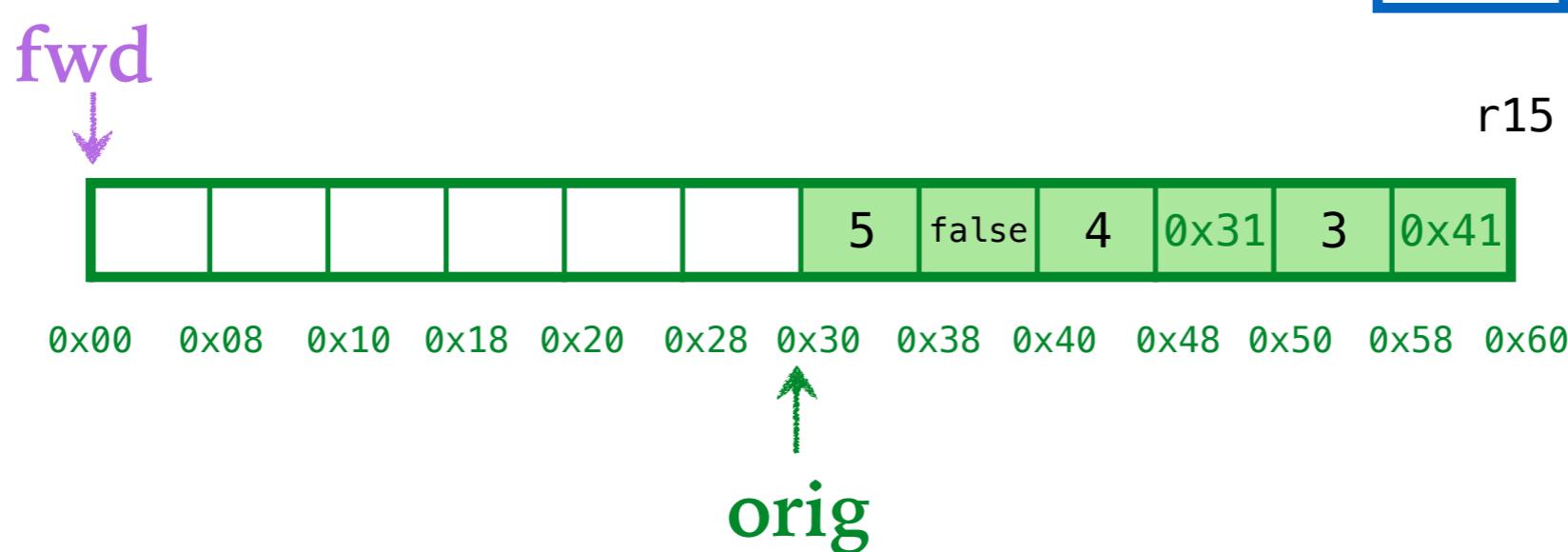
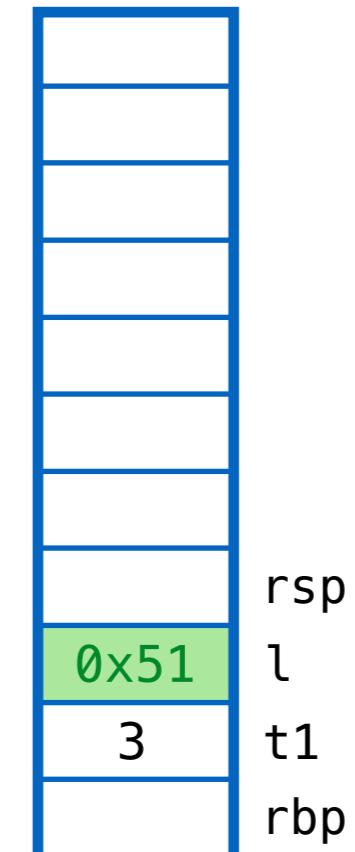
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



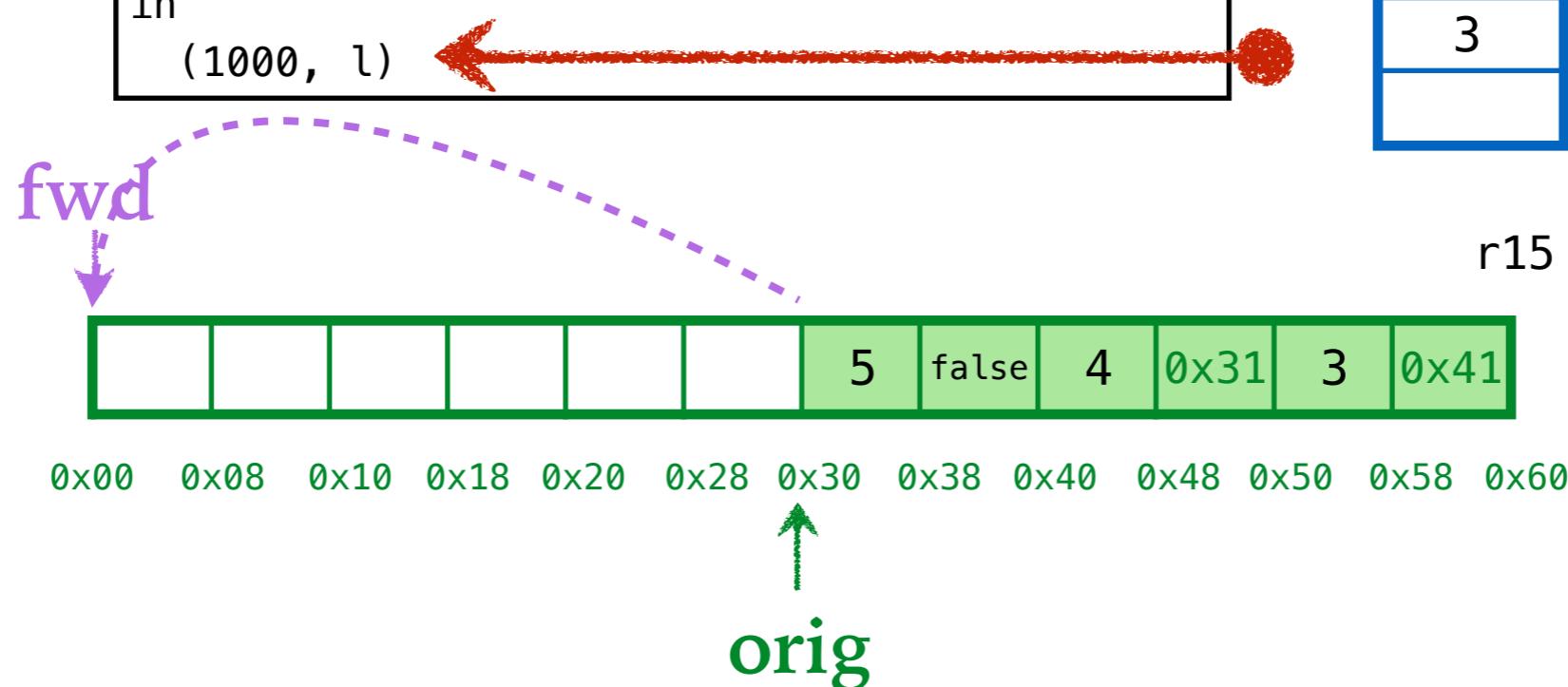
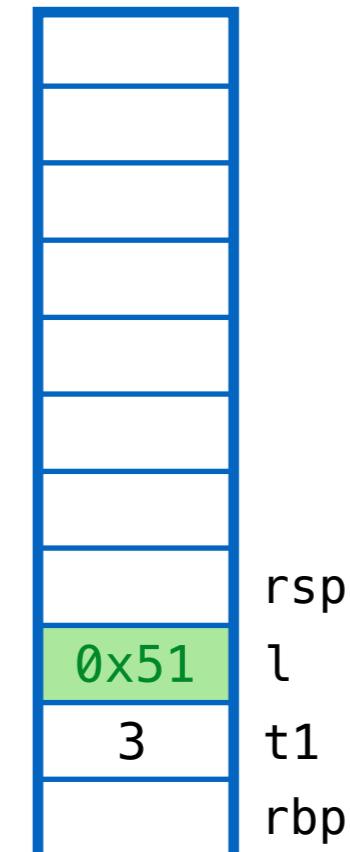
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



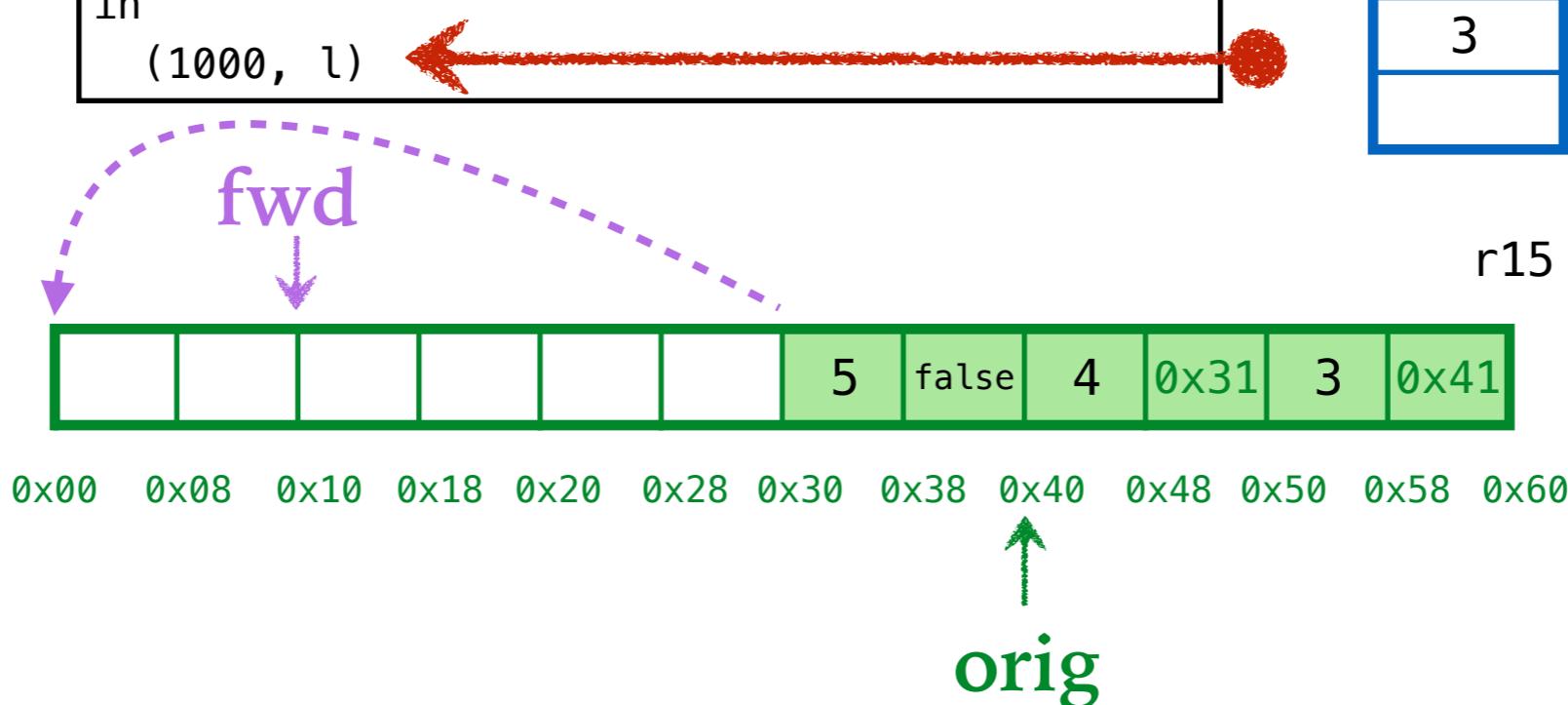
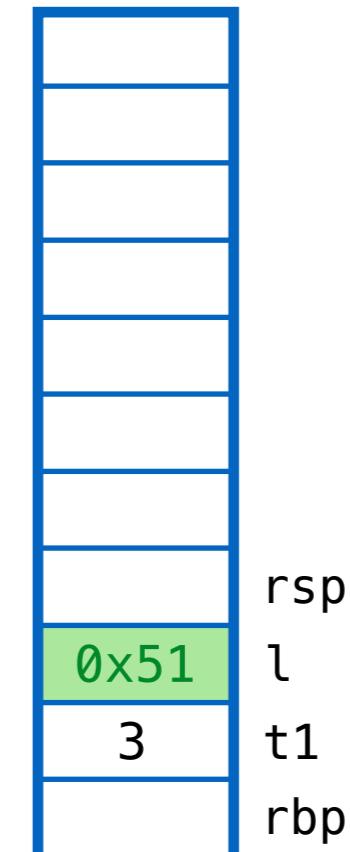
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

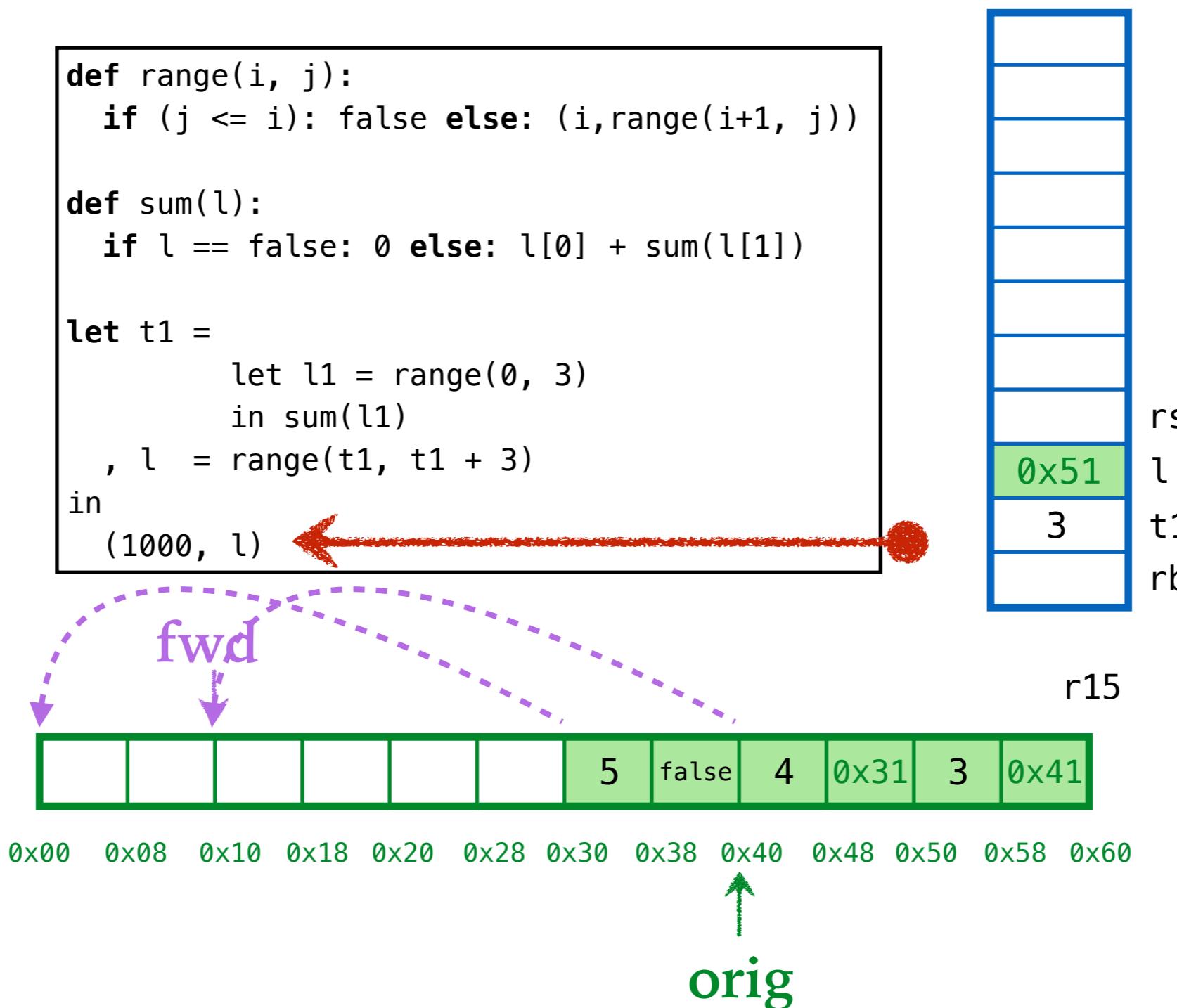
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



2. Compute FORWARD addrs

ex4: recursive data



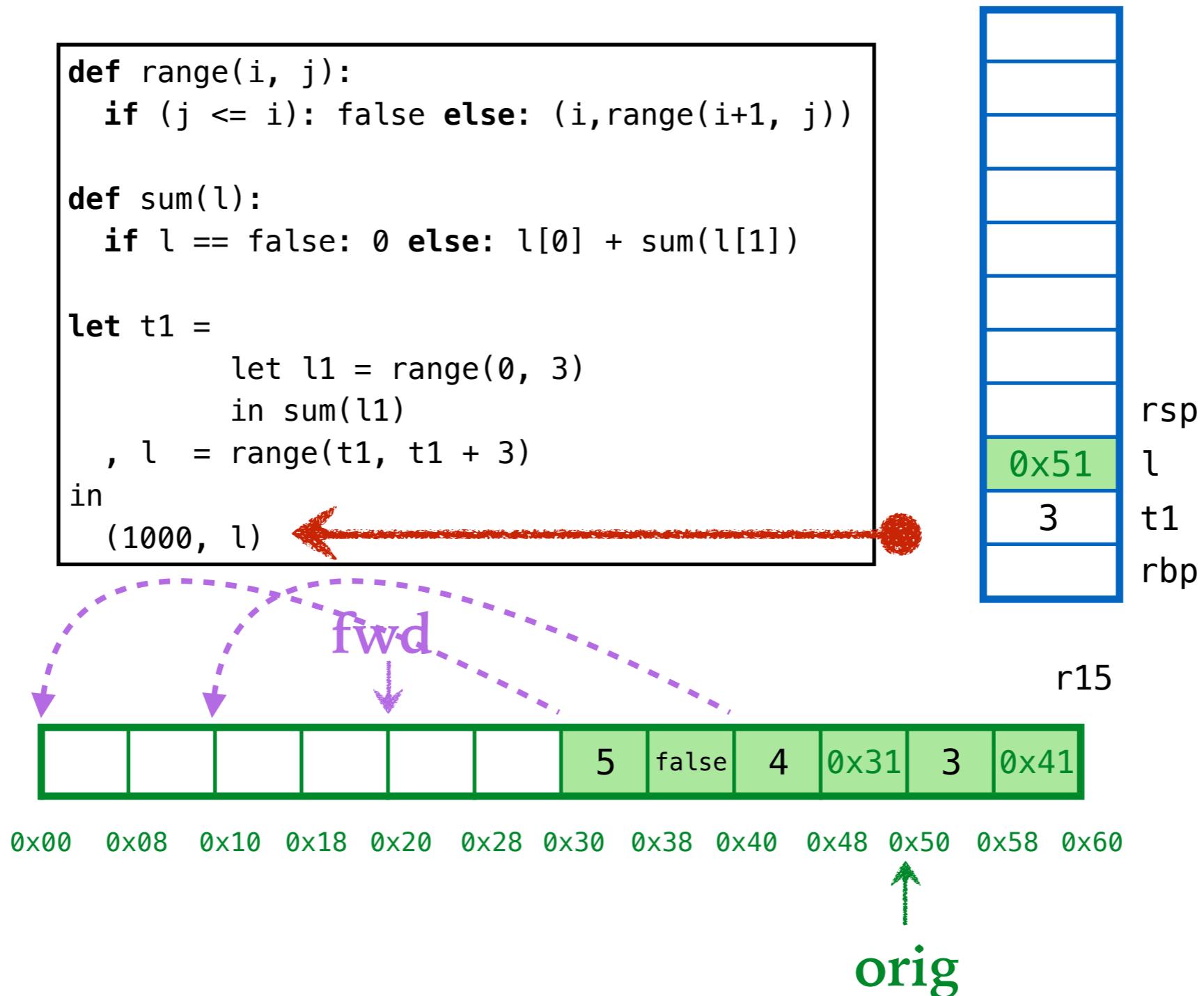
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



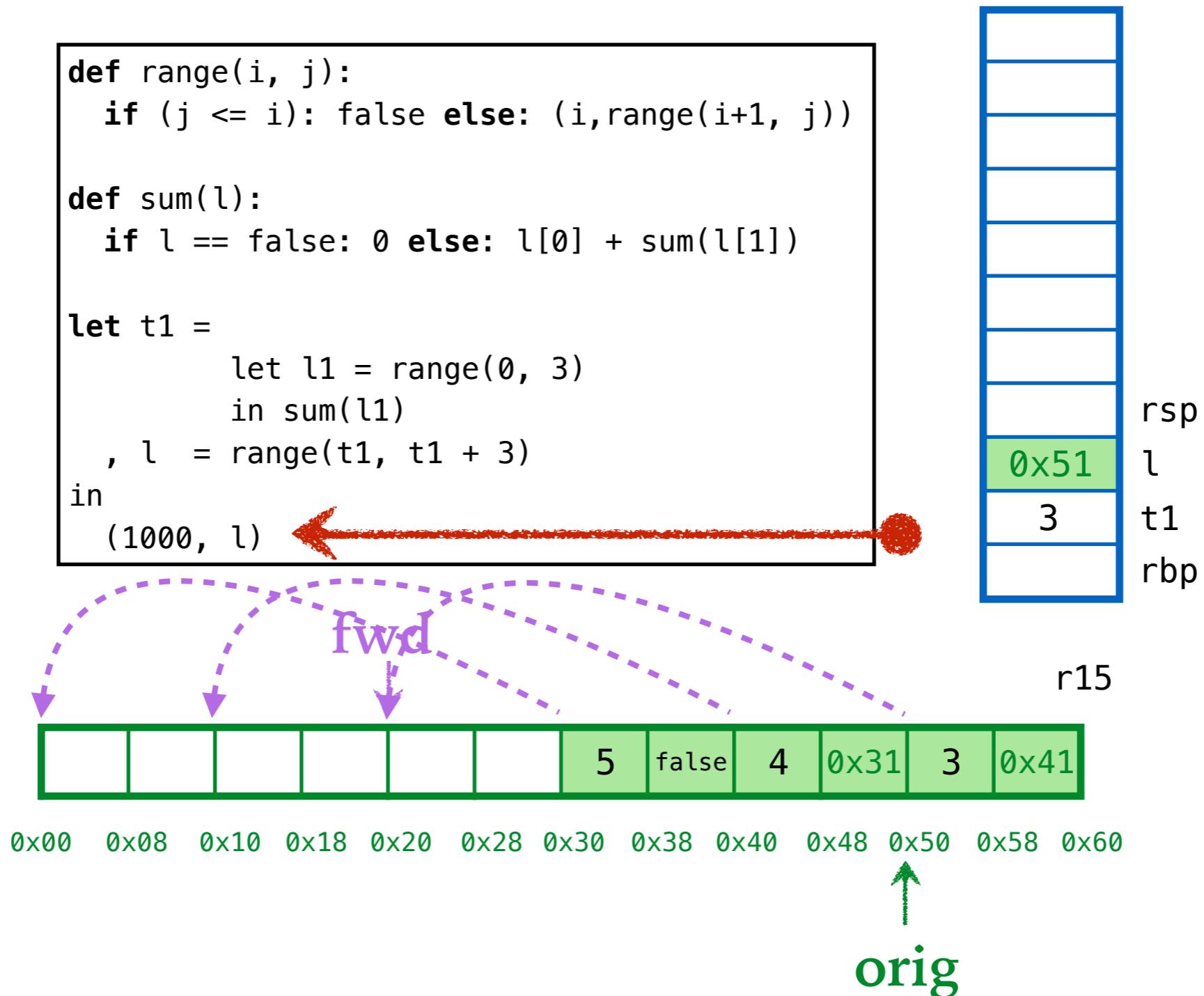
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

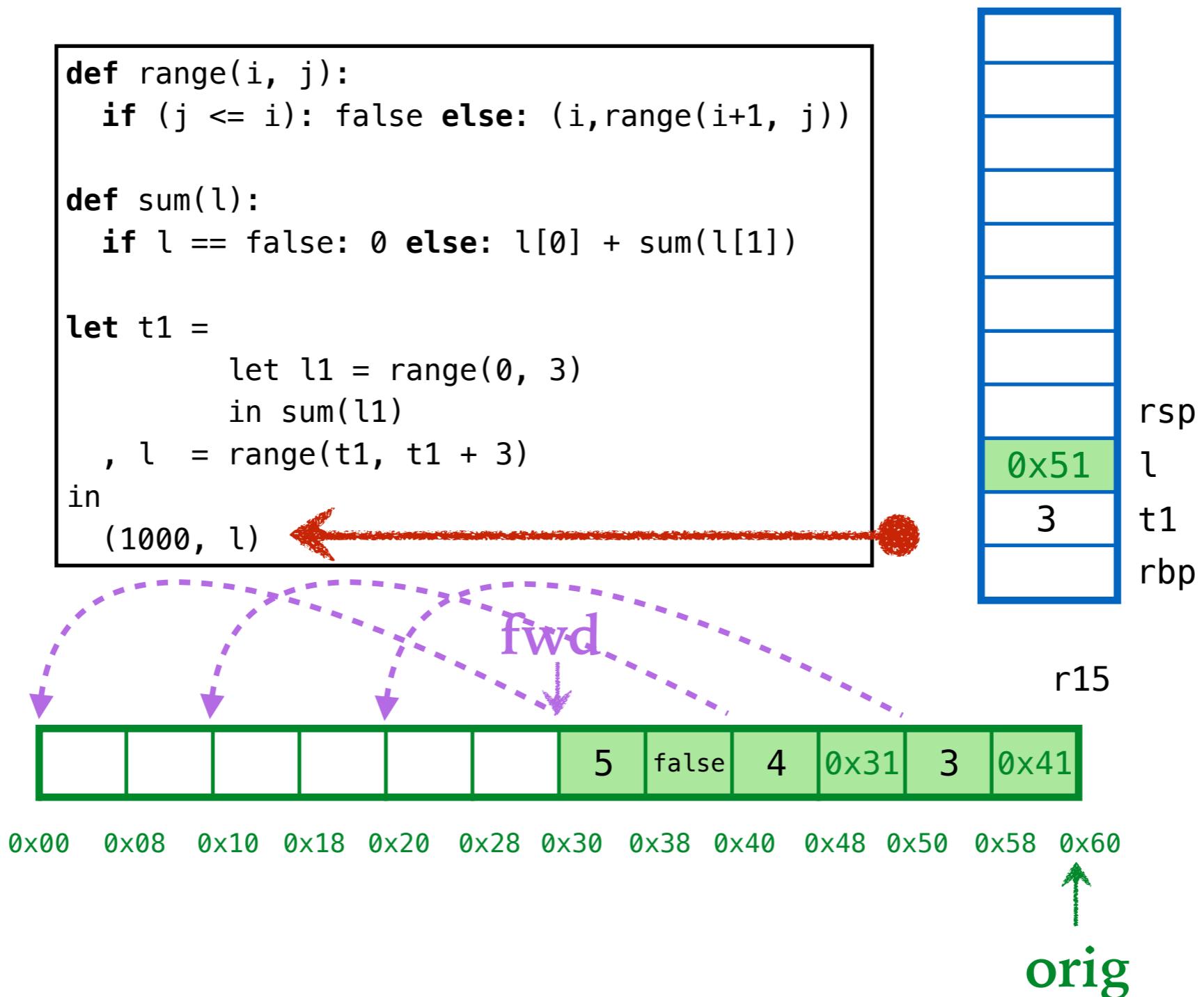
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



2. Compute FORWARD addrs

ex4: recursive data



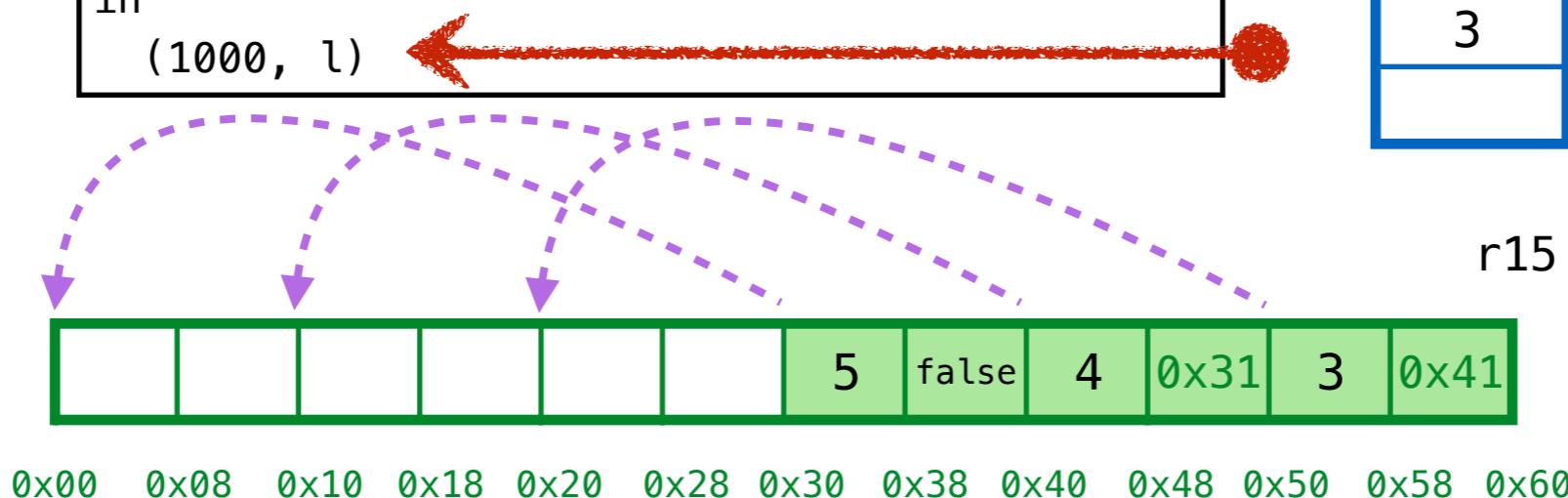
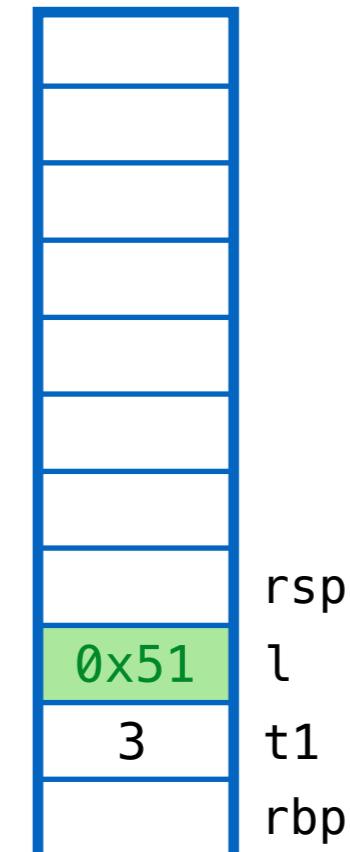
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

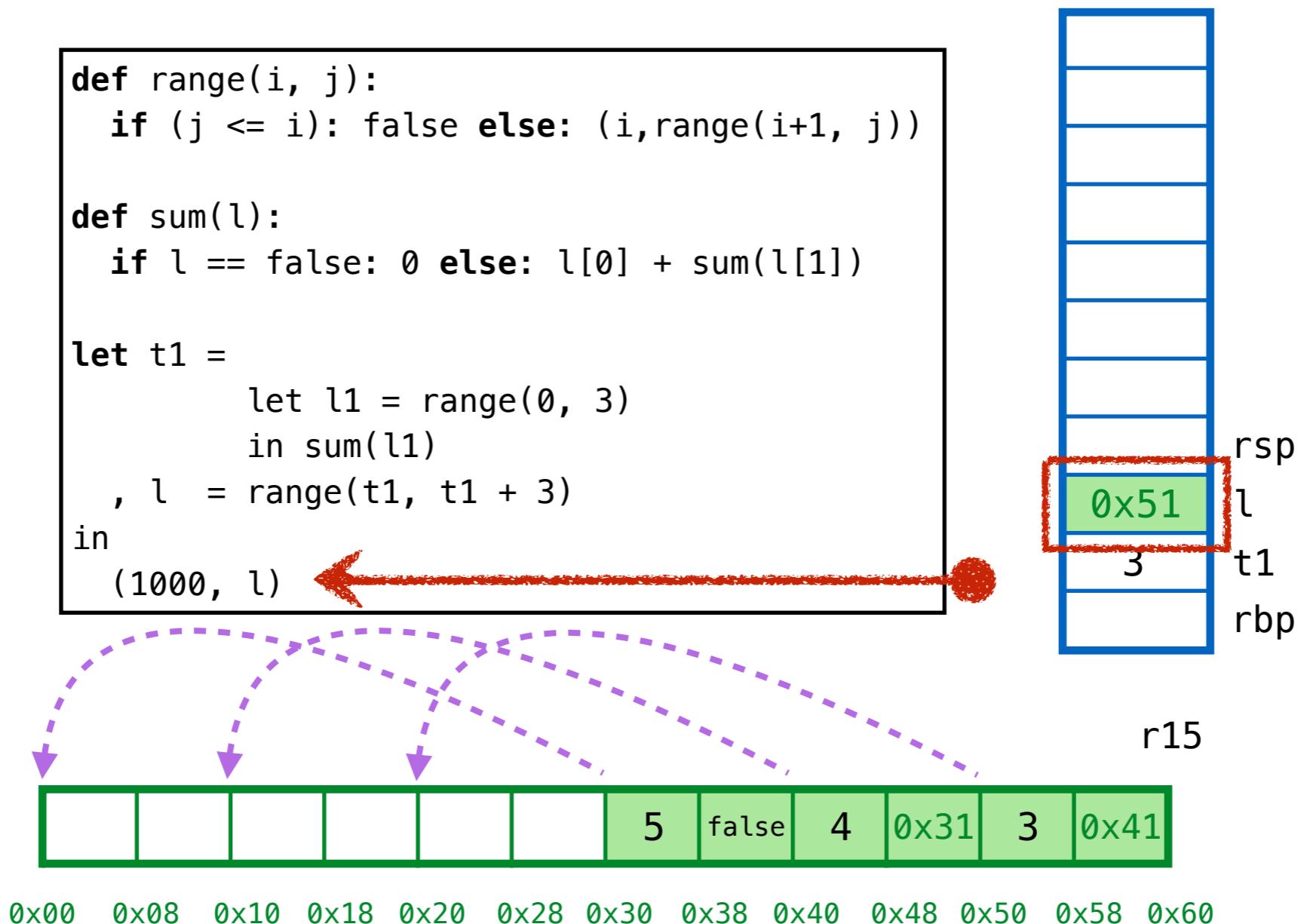
let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



2. Compute FORWARD addrs

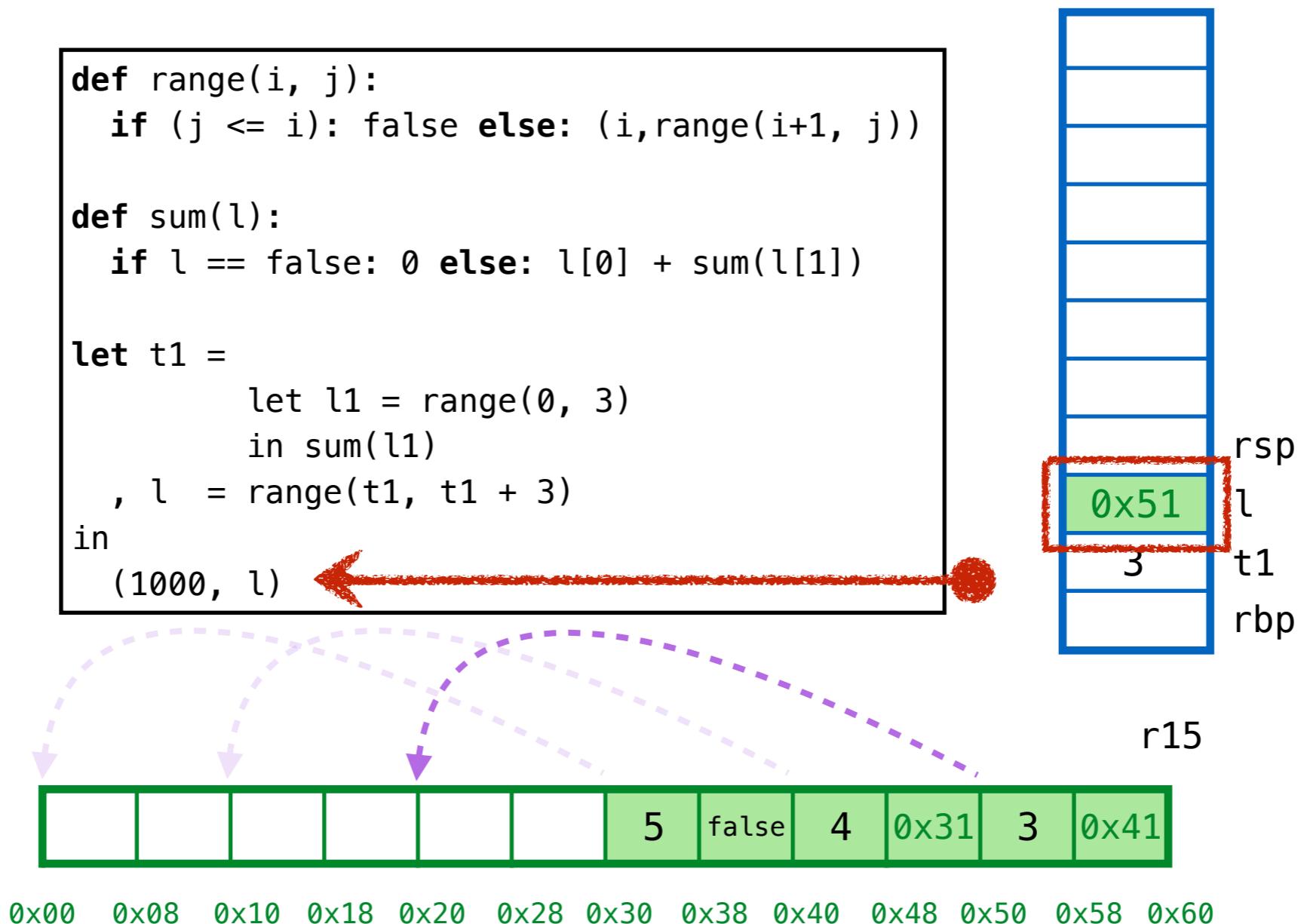
Where should we store the forward addrs?

ex4: recursive data



3. REDIRECT addrs on stack

ex4: recursive data



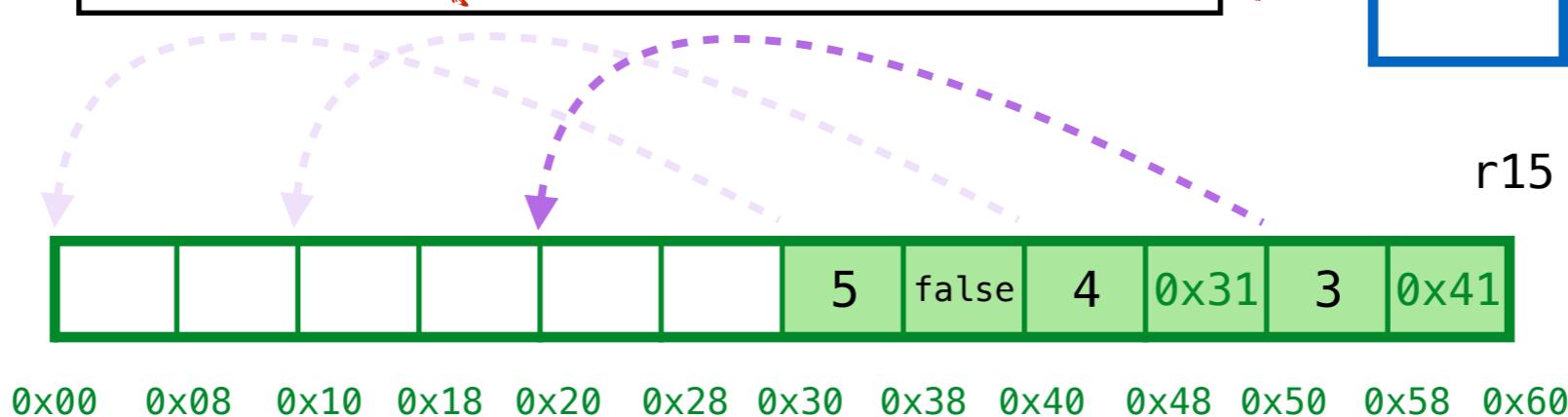
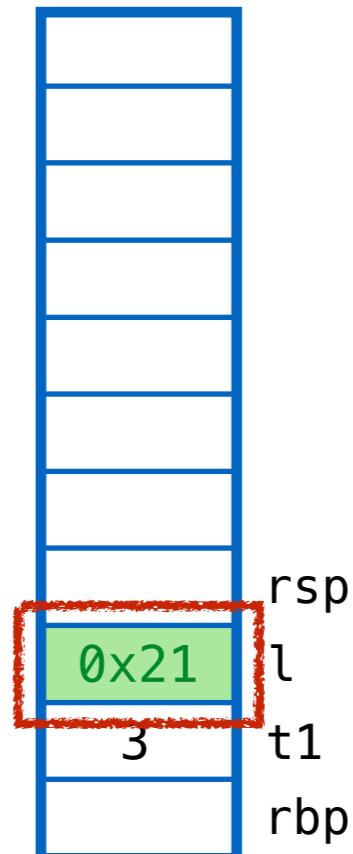
3. REDIRECT addrs on stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

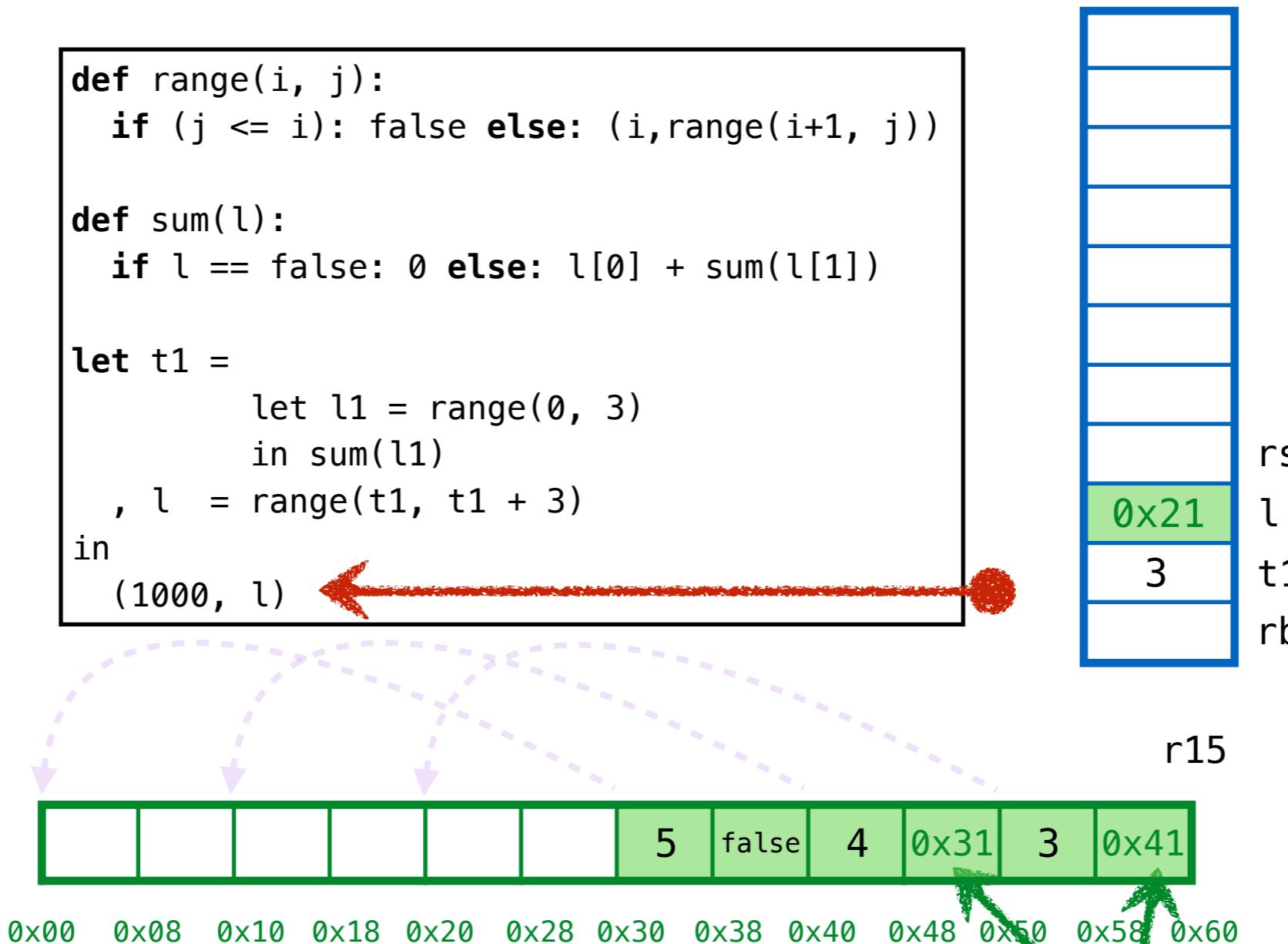
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



3. REDIRECT addrs on stack

ex4: recursive data



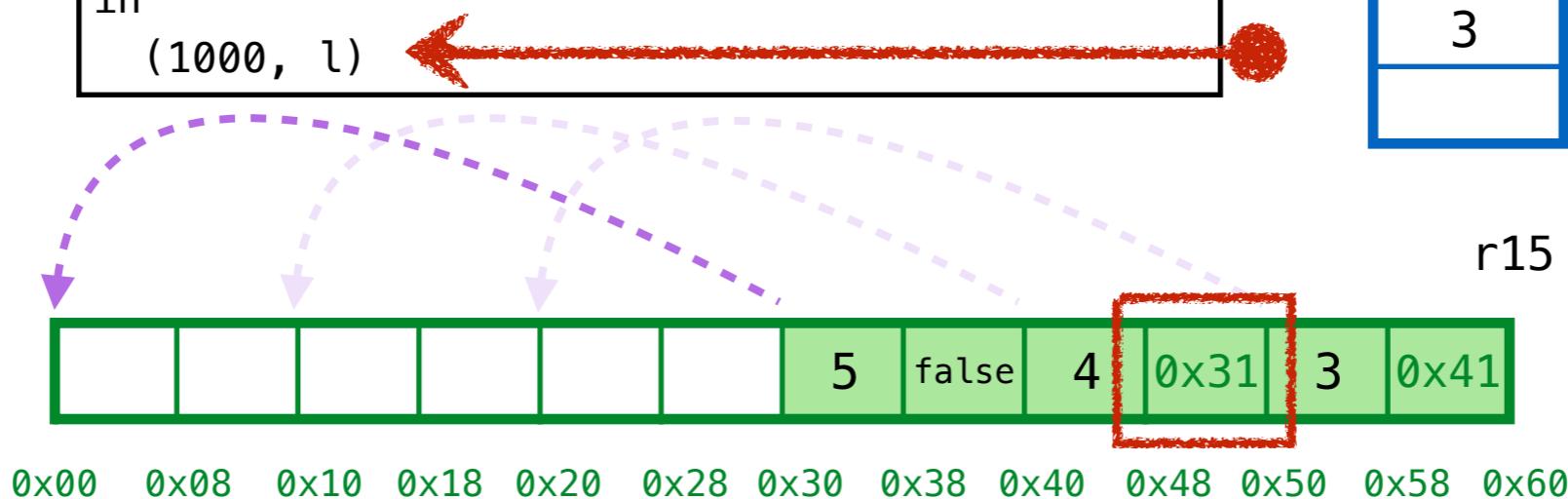
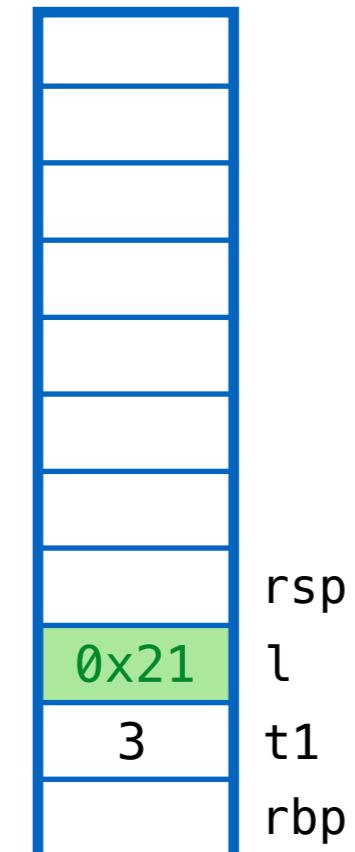
3. REDIRECT addrs on stack and heap!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

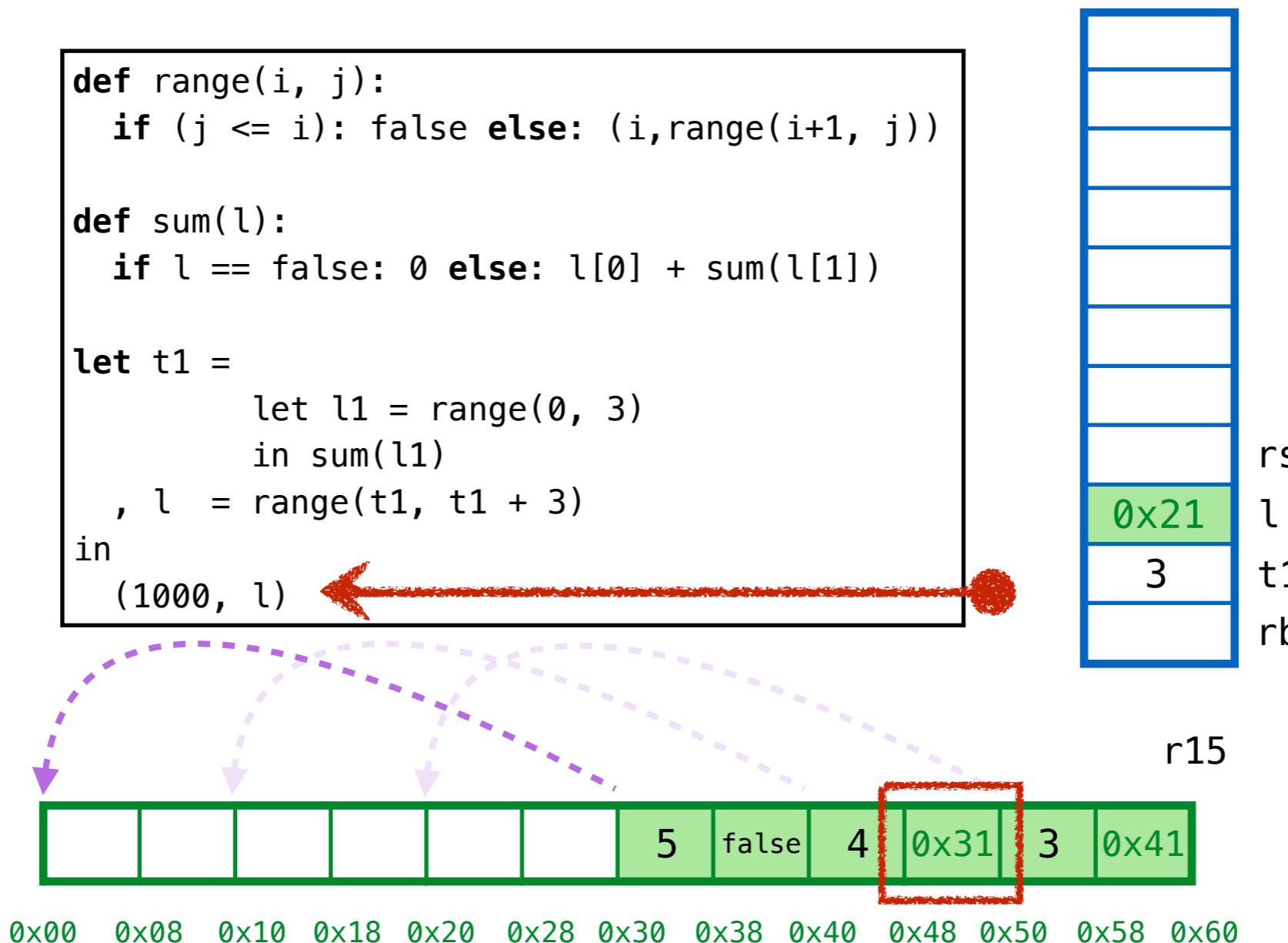
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



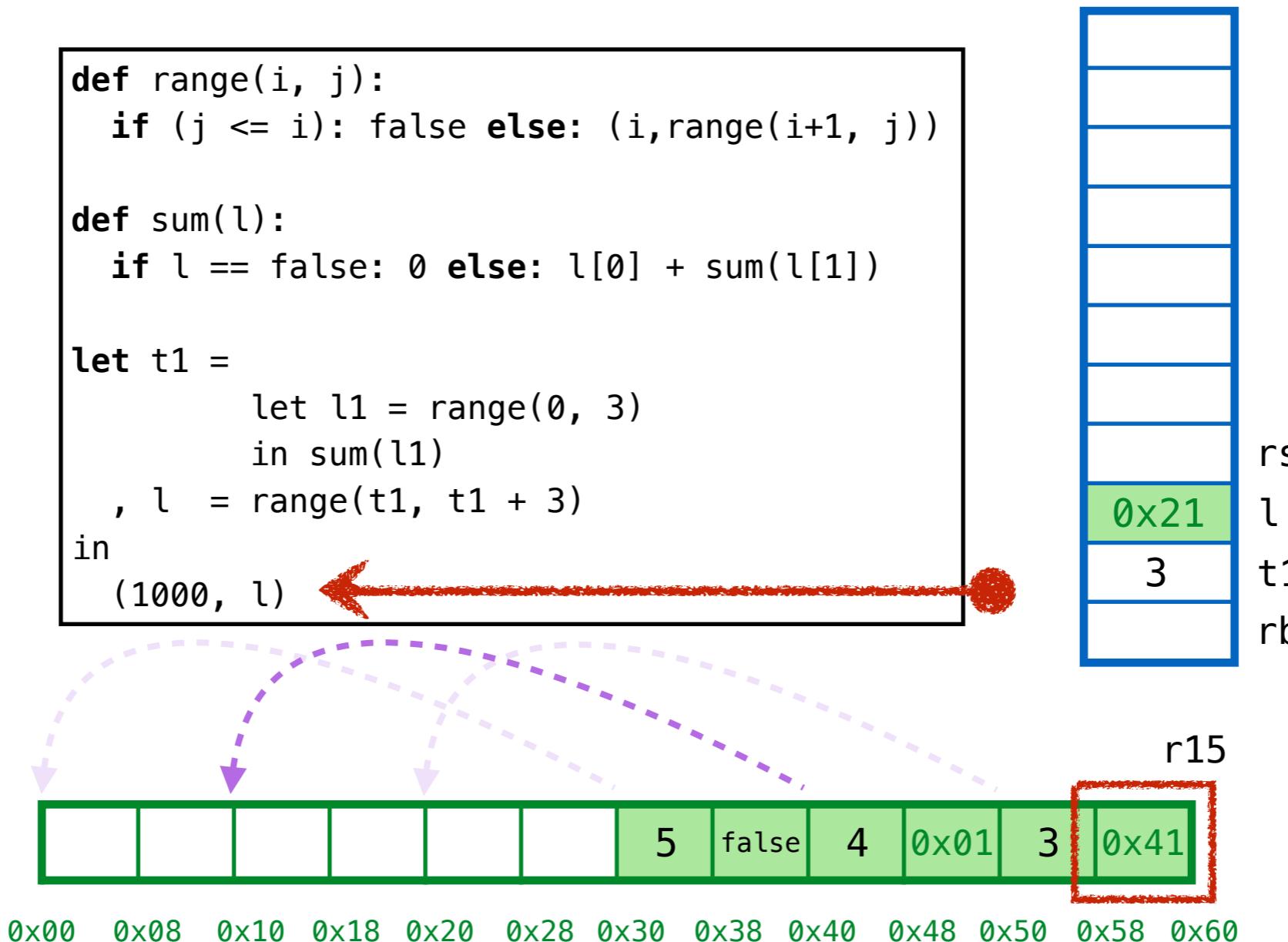
3. REDIRECT addrs on stack and heap!

ex4: recursive data



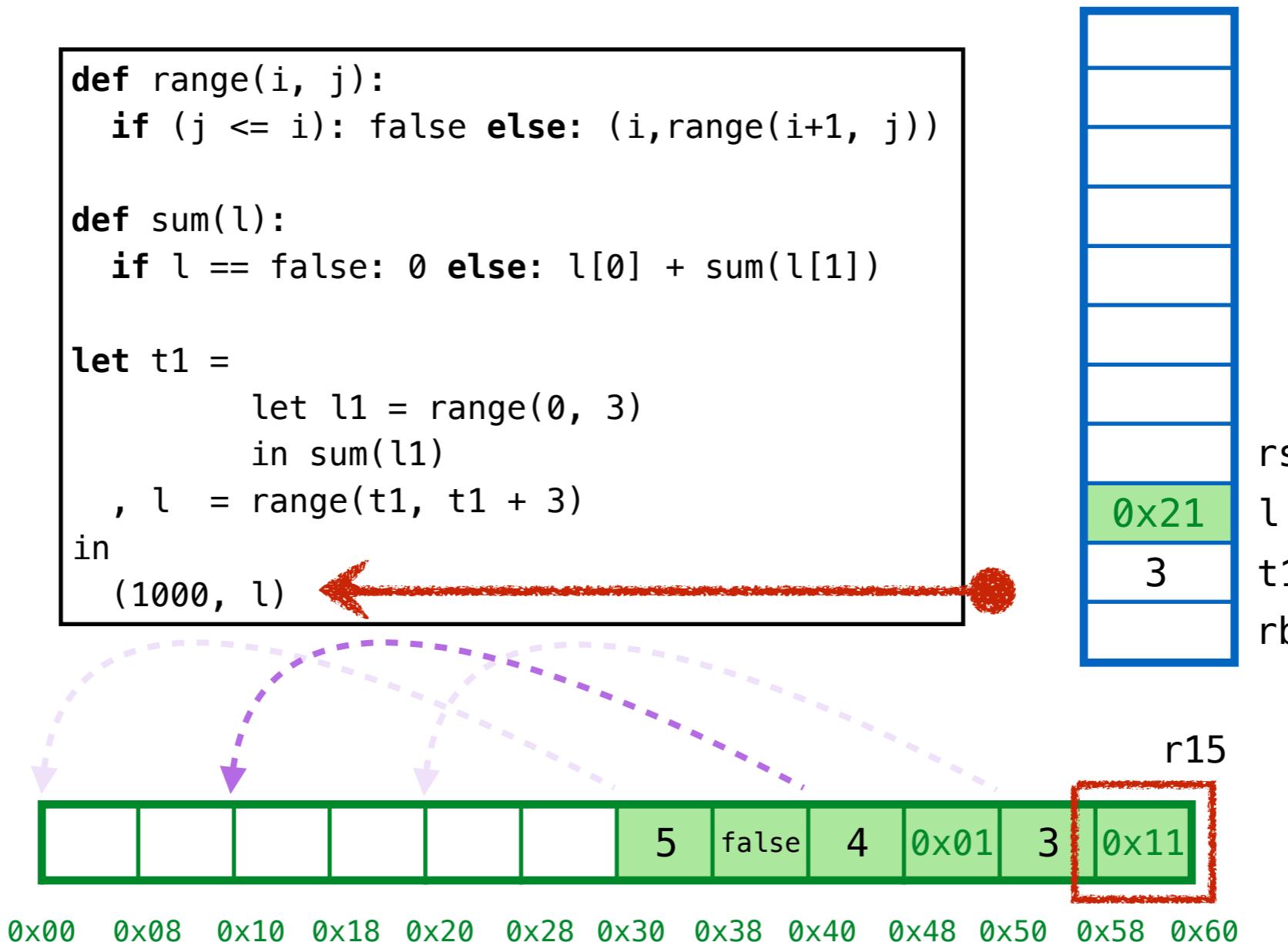
3. **REDIRECT** addrs on stack and heap!

ex4: recursive data



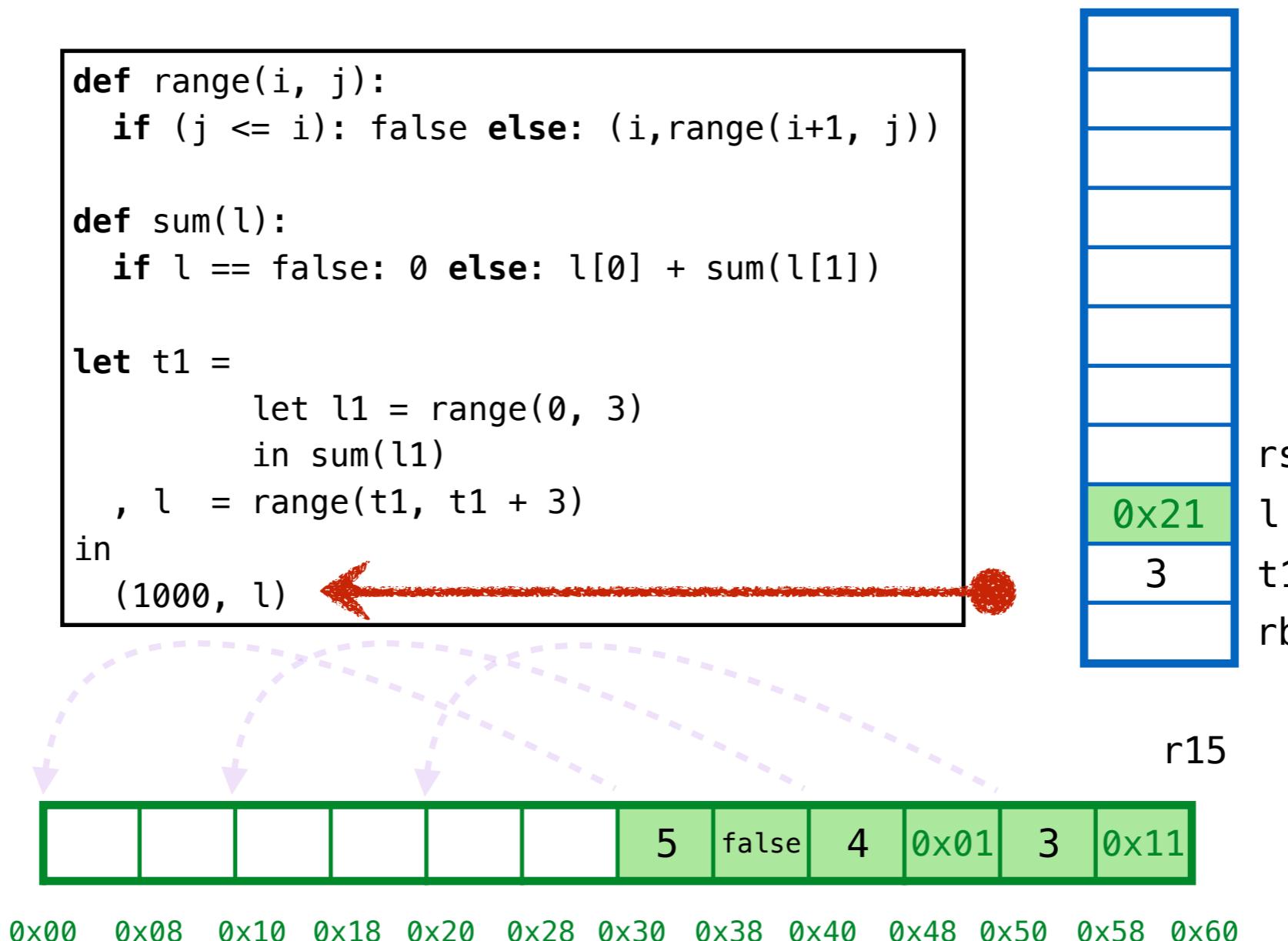
3. **REDIRECT** addrs on stack and heap!

ex4: recursive data



3. **REDIRECT** addrs on stack and heap!

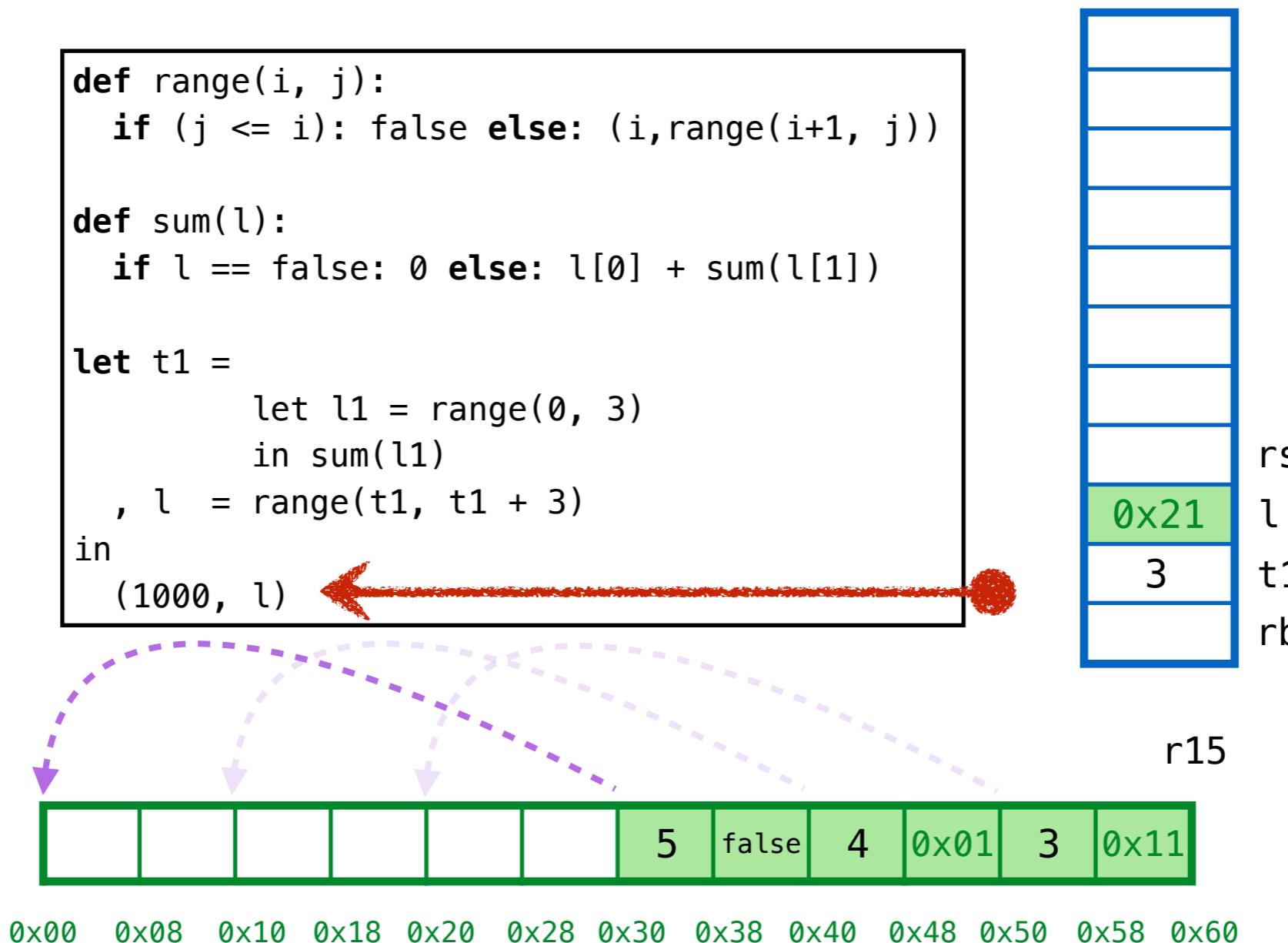
ex4: recursive data



4. COMPACT cells on heap

Copy cell to forward addr!

ex4: recursive data



4. COMPACT cells on heap

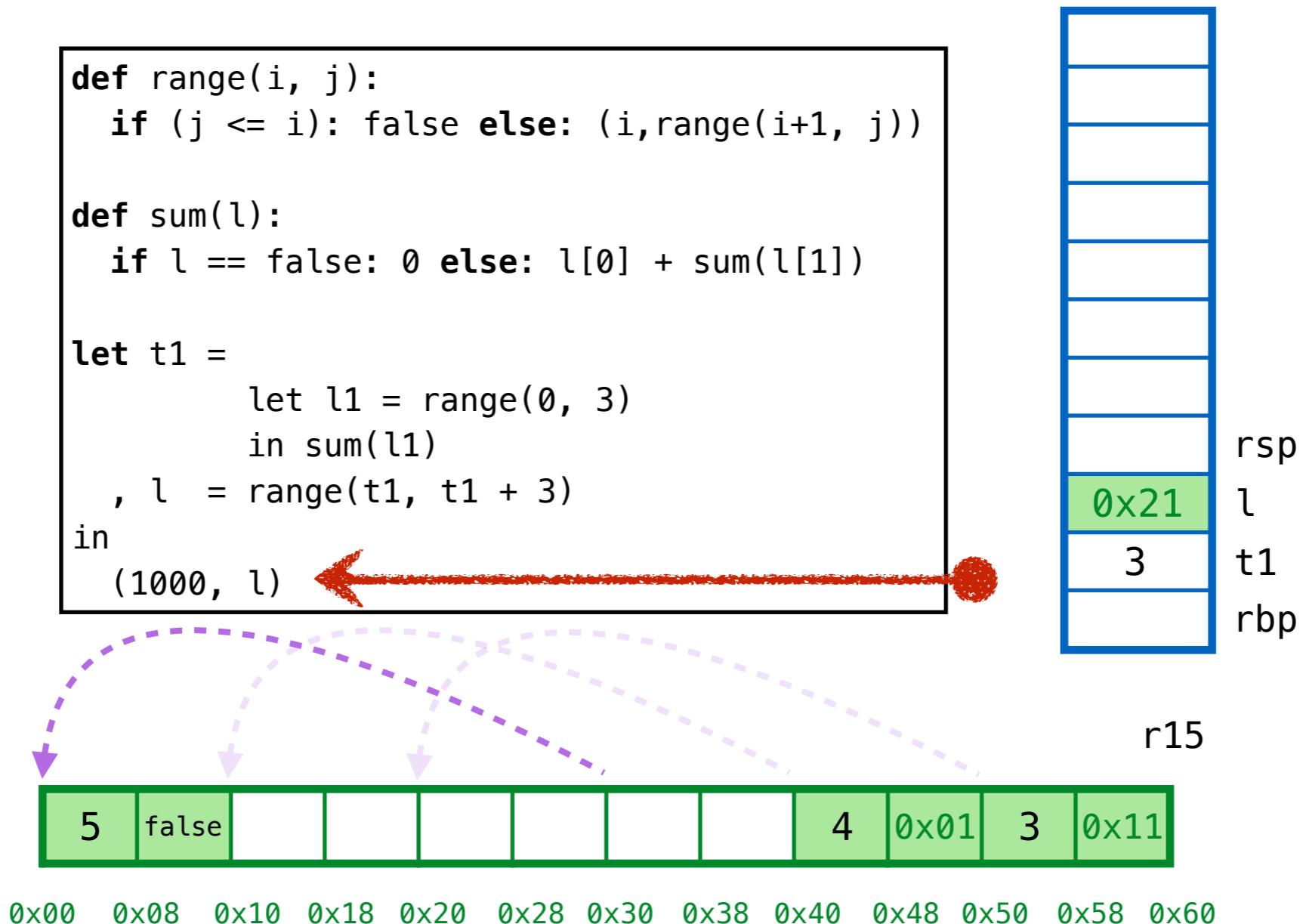
Copy cell to forward addr!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



4. COMPACT cells on heap

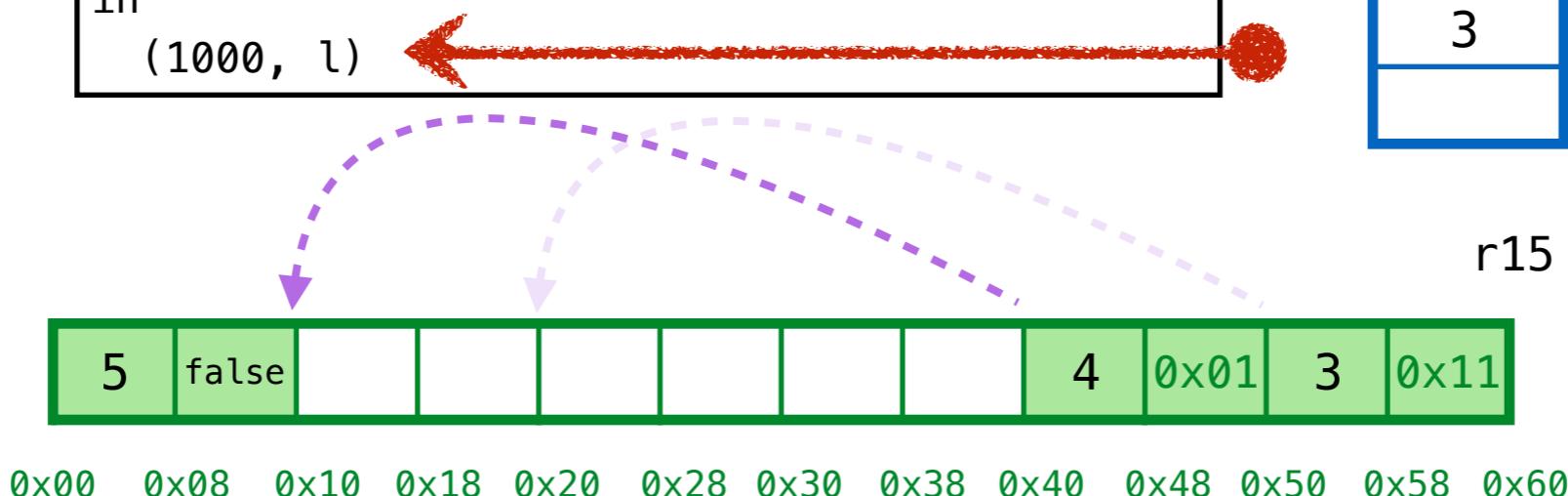
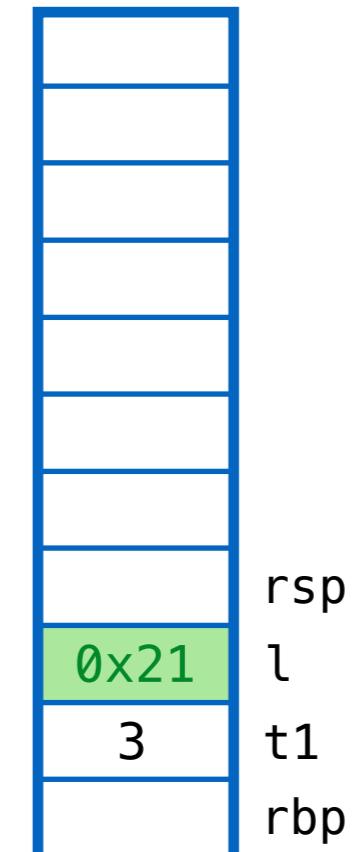
Copy cell to forward addr!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



4. COMPACT cells on heap

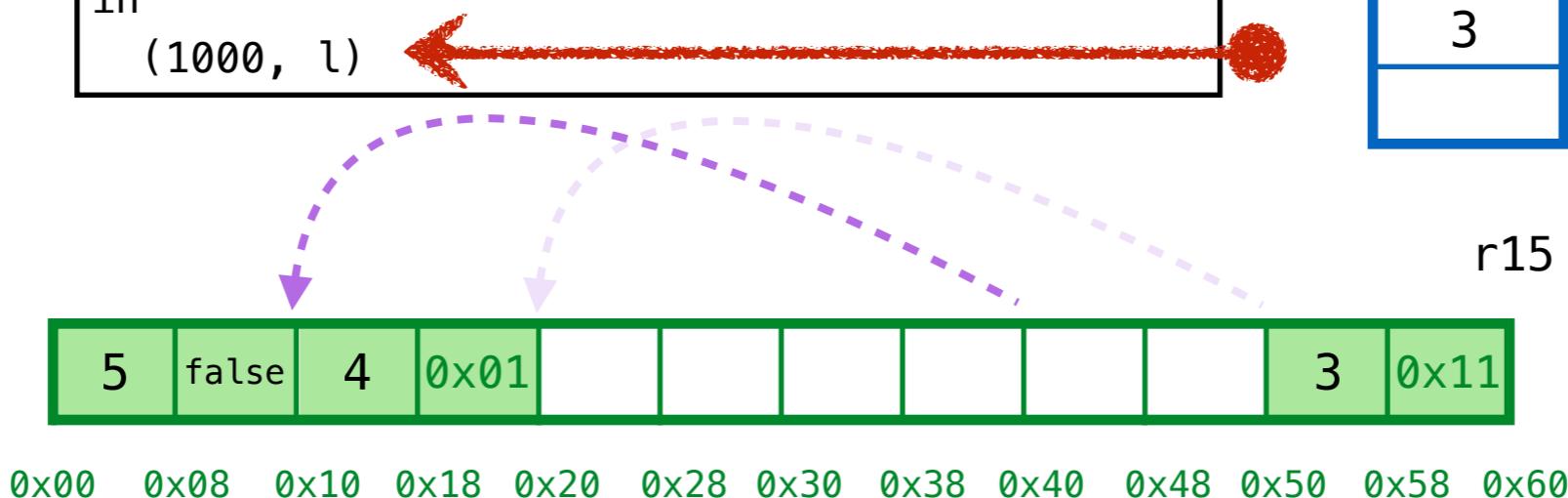
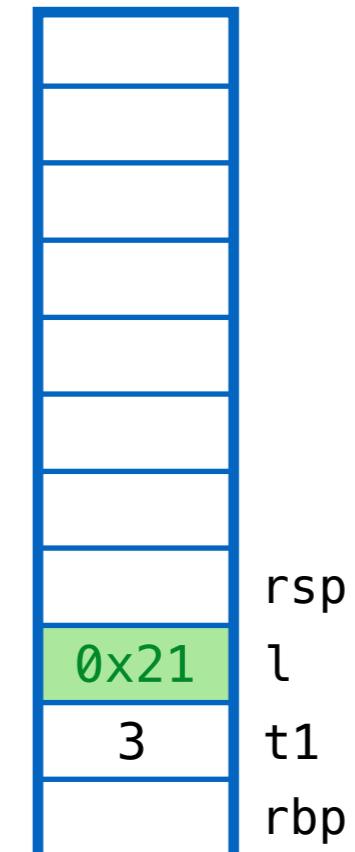
Copy cell to forward addr!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



4. COMPACT cells on heap

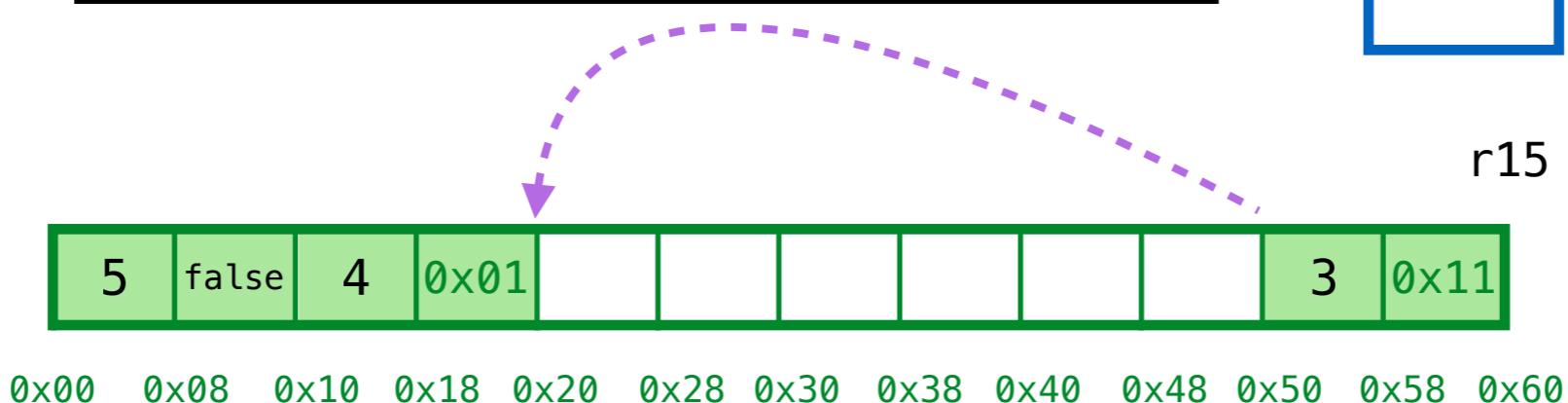
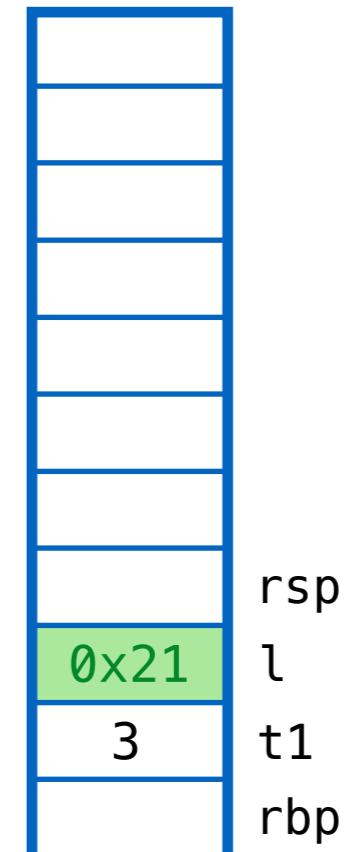
Copy cell to forward addr!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



4. COMPACT cells on heap

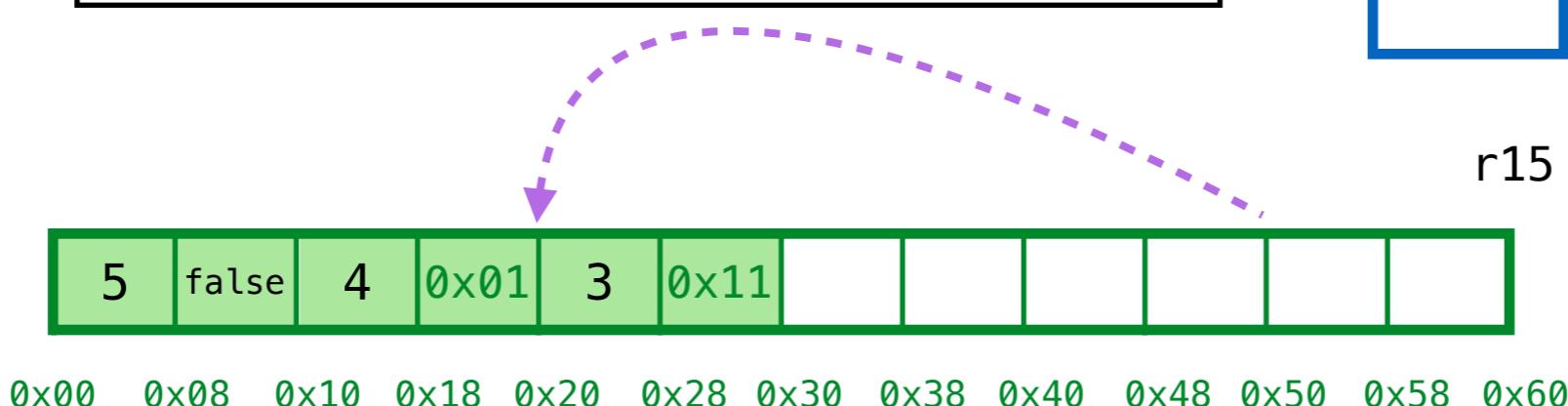
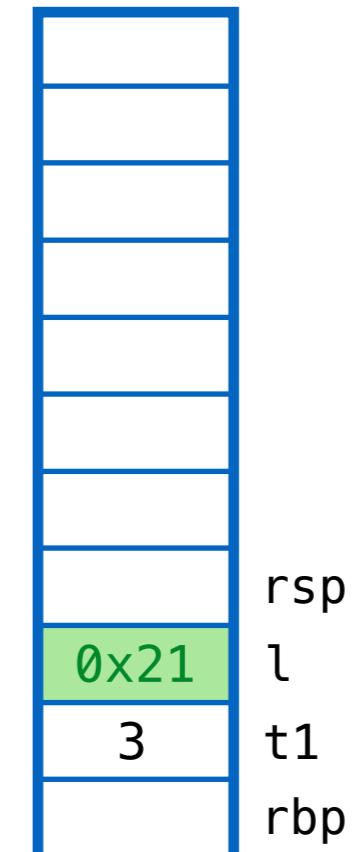
Copy cell to forward addr!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



4. COMPACT cells on heap

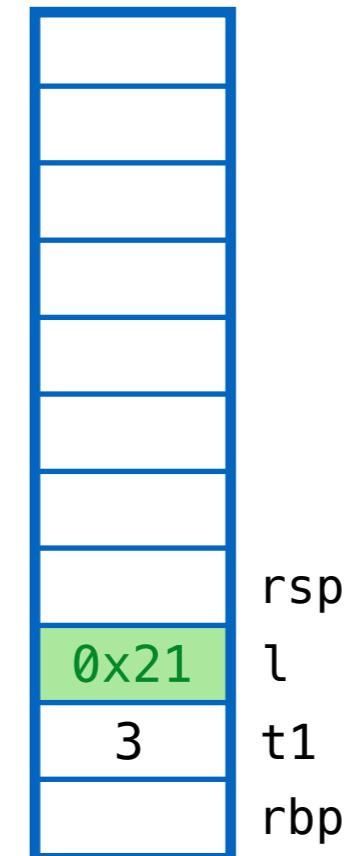
Copy cell to forward addr!

ex4: recursive data

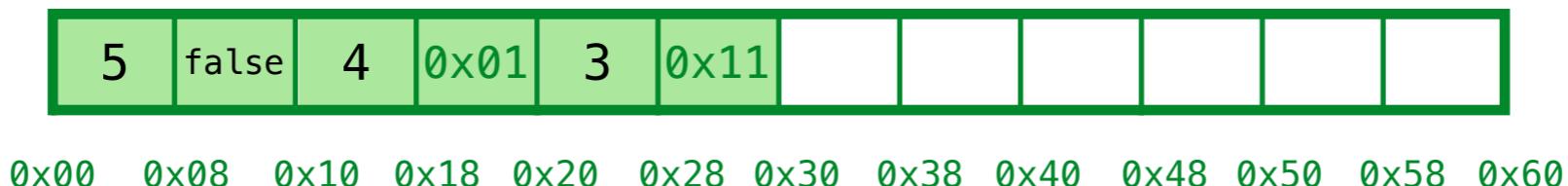
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15



GC Complete!

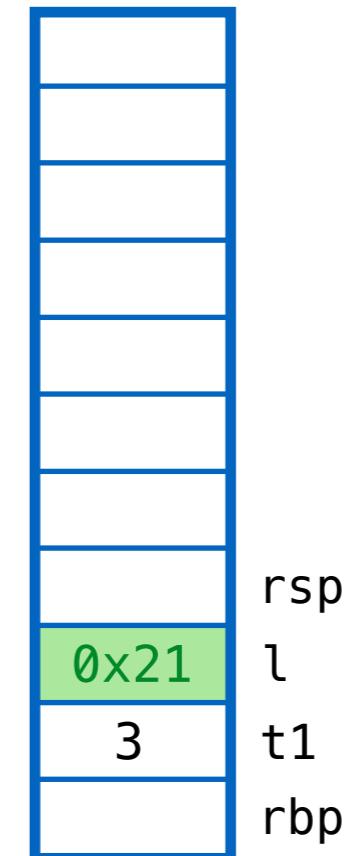
Have space for `(1000, l)`

ex4: recursive data

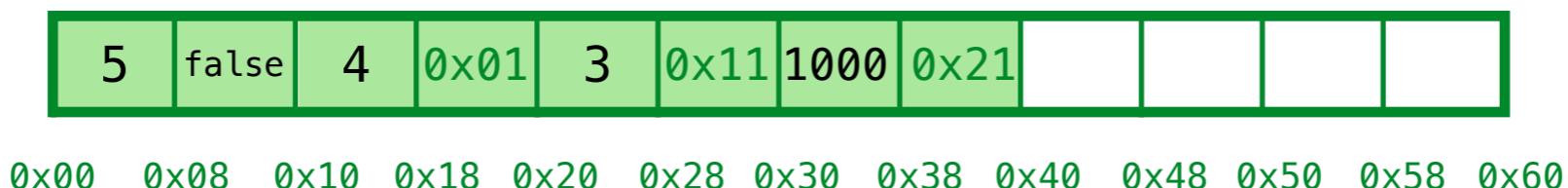
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15



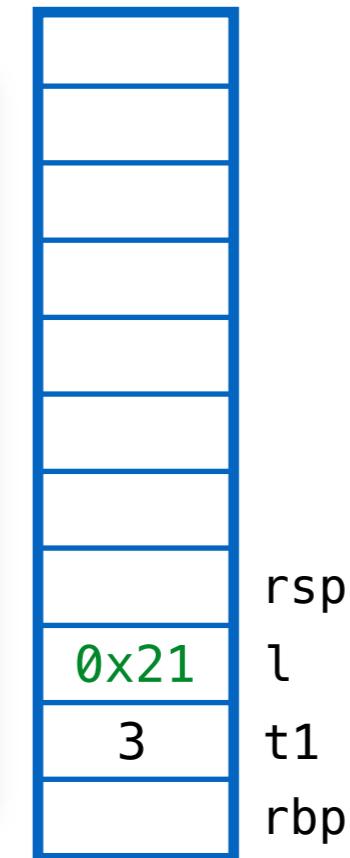
GC Complete!

Have space for (1000, l)

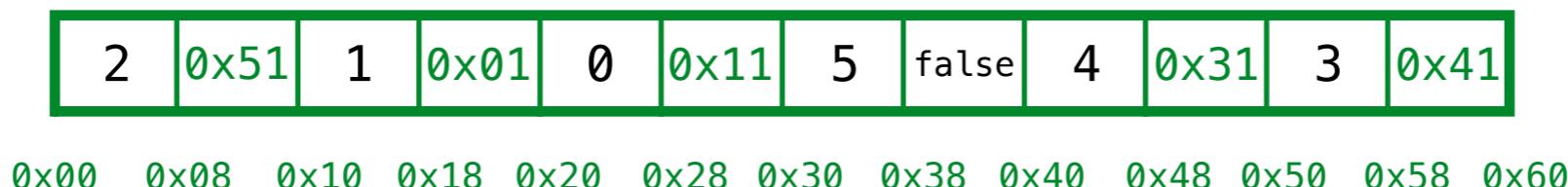
ex4: recursive data

QUIZ: What should `print(0x21)` show?

- (A) `(0, (1, (2, false)))`
- (B) `(3, (4, (5, false)))`
- (C) `(0, (1, (2, (3, (4, (5, false))))))`
- (D) `(3, (4, (5, (0, (1, (2, false))))))`
- (E) `(2, (1, (0, (3, (4, (5, false))))))`



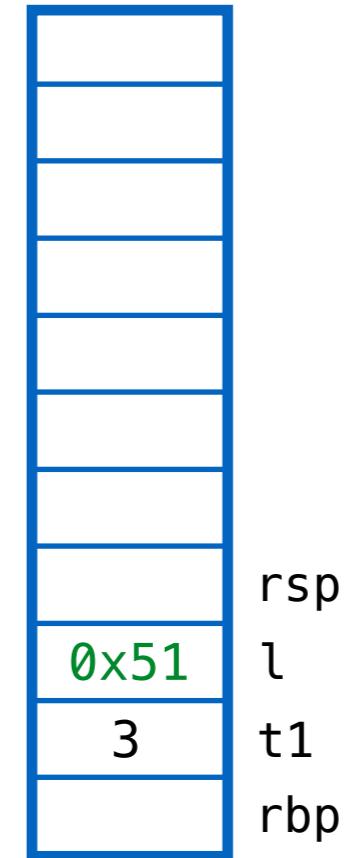
r15



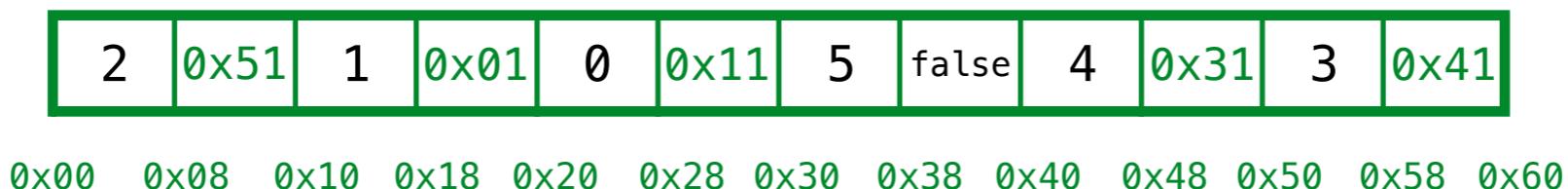
ex4: recursive data

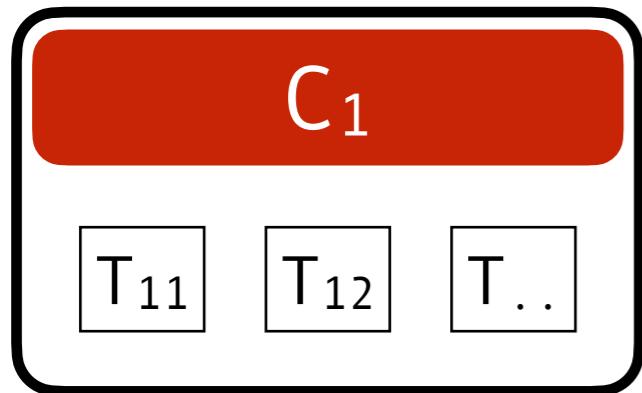
QUIZ: Which cells are “live” on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50

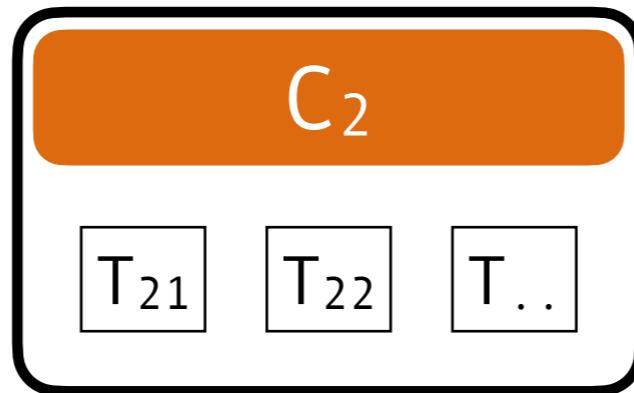


r15

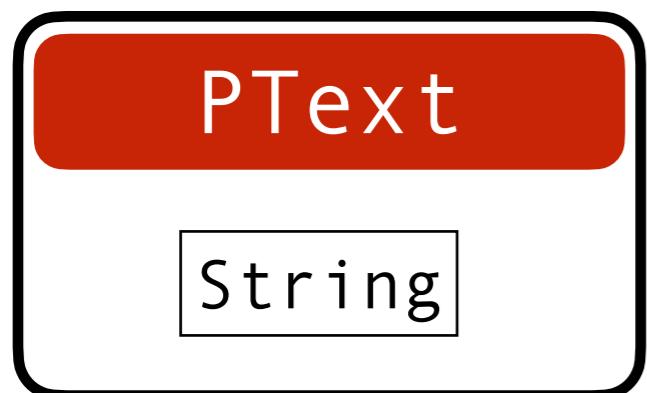
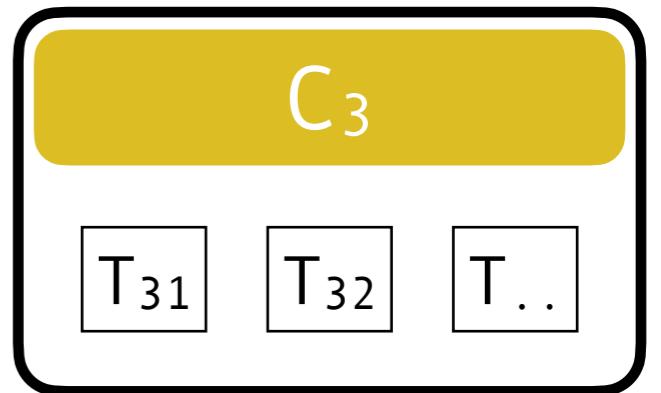




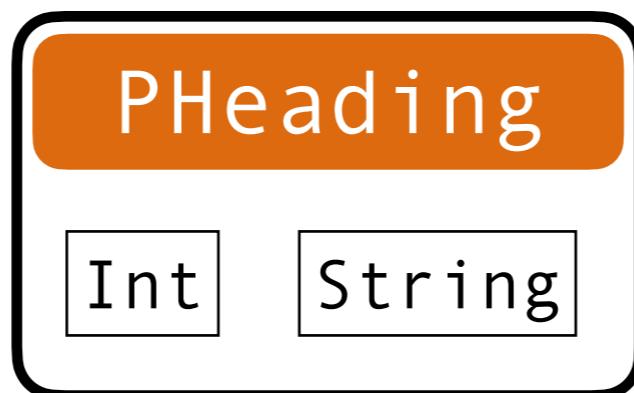
or



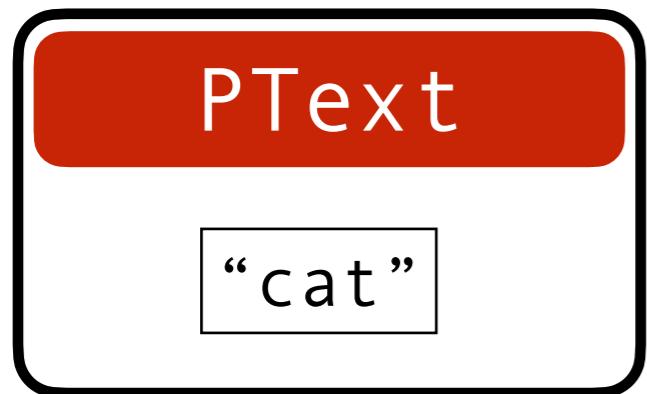
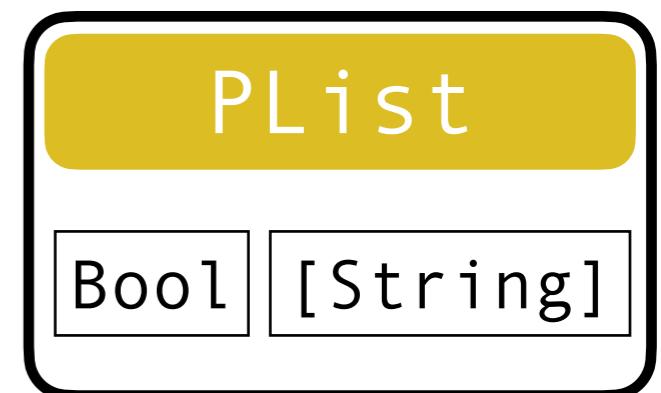
or



or



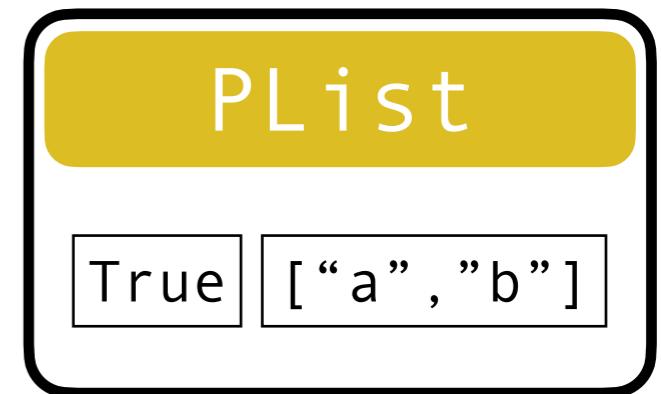
or

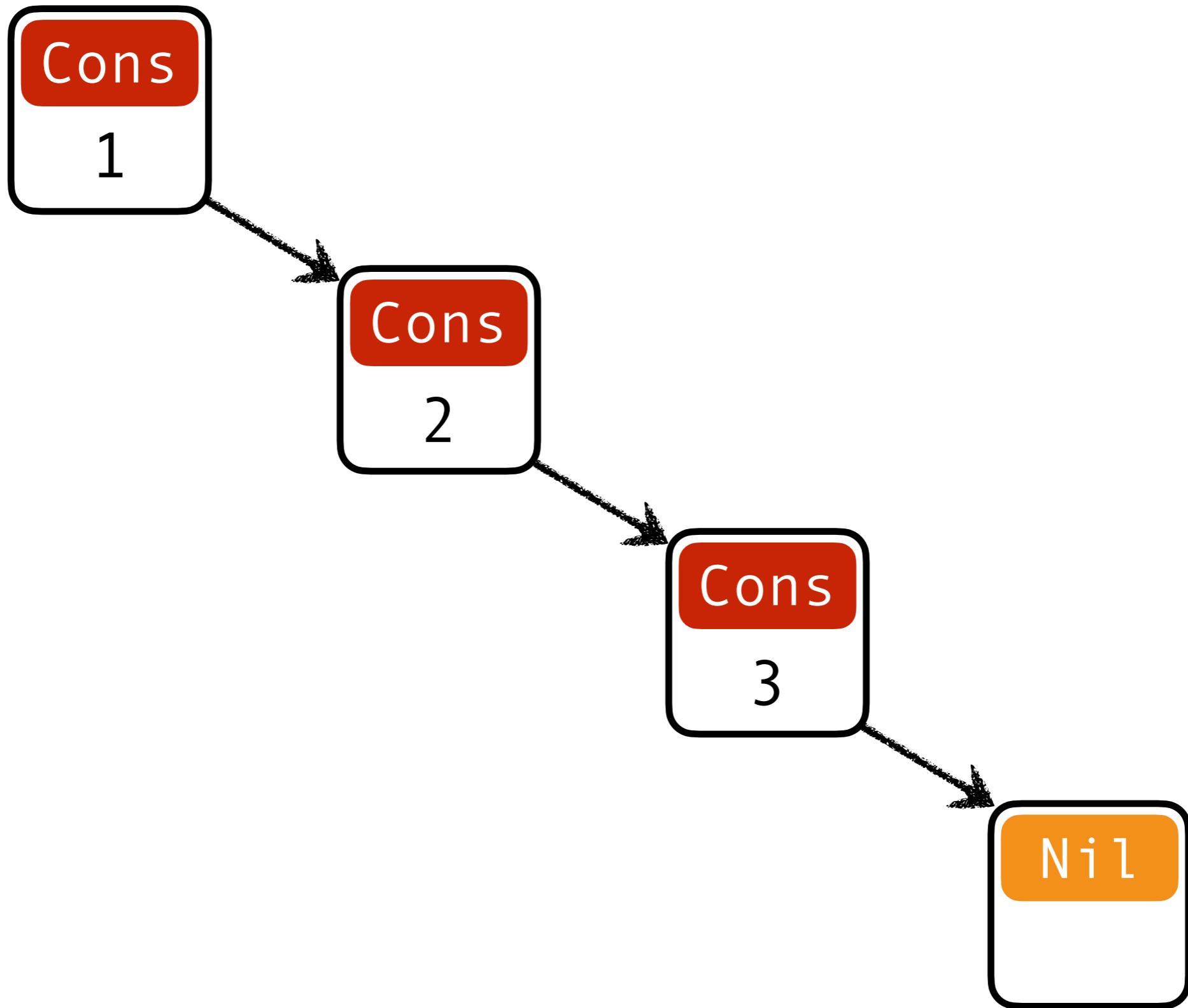


or



or







VS.

saltDarkChoco :: Cake

INGREDIENTS

FOR THE CAKE:

- 2 $\frac{1}{2}$ cups/310 grams self-rising flour, sifted (see note)
- $\frac{1}{2}$ cup/45 grams cocoa powder, sifted
- 1 $\frac{1}{2}$ cups/295 grams sugar
- 4 large eggs, lightly beaten
- 1 $\frac{1}{2}$ cups/360 milliliters whole milk
- 1 cup plus 2 tablespoons/255 grams unsalted butter, melted and slightly cooled
- 7 ounces/200 grams dark chocolate, melted and slightly cooled
- 2 teaspoons vanilla extract
- 1 teaspoon flaky sea salt, white or black

FOR THE GANACHE:

- 1 cup/240 milliliters sour cream
- 14 ounces/400 grams milk

PREPARATION

Step 1

Heat oven to 350 degrees. Line 2 8-inch round cake tins with parchment paper. Place the flour, cocoa, sugar, eggs, milk, butter, dark chocolate and vanilla in a large bowl and whisk until smooth. (You may need to use a spatula to start, but use a whisk once the ingredients begin to combine.) Divide the mixture evenly between the tins and bake for 35 to 40 minutes or until a wooden skewer inserted into the center comes out clean. Allow to cool in the tins for 10 minutes before turning out onto wire racks to cool completely.

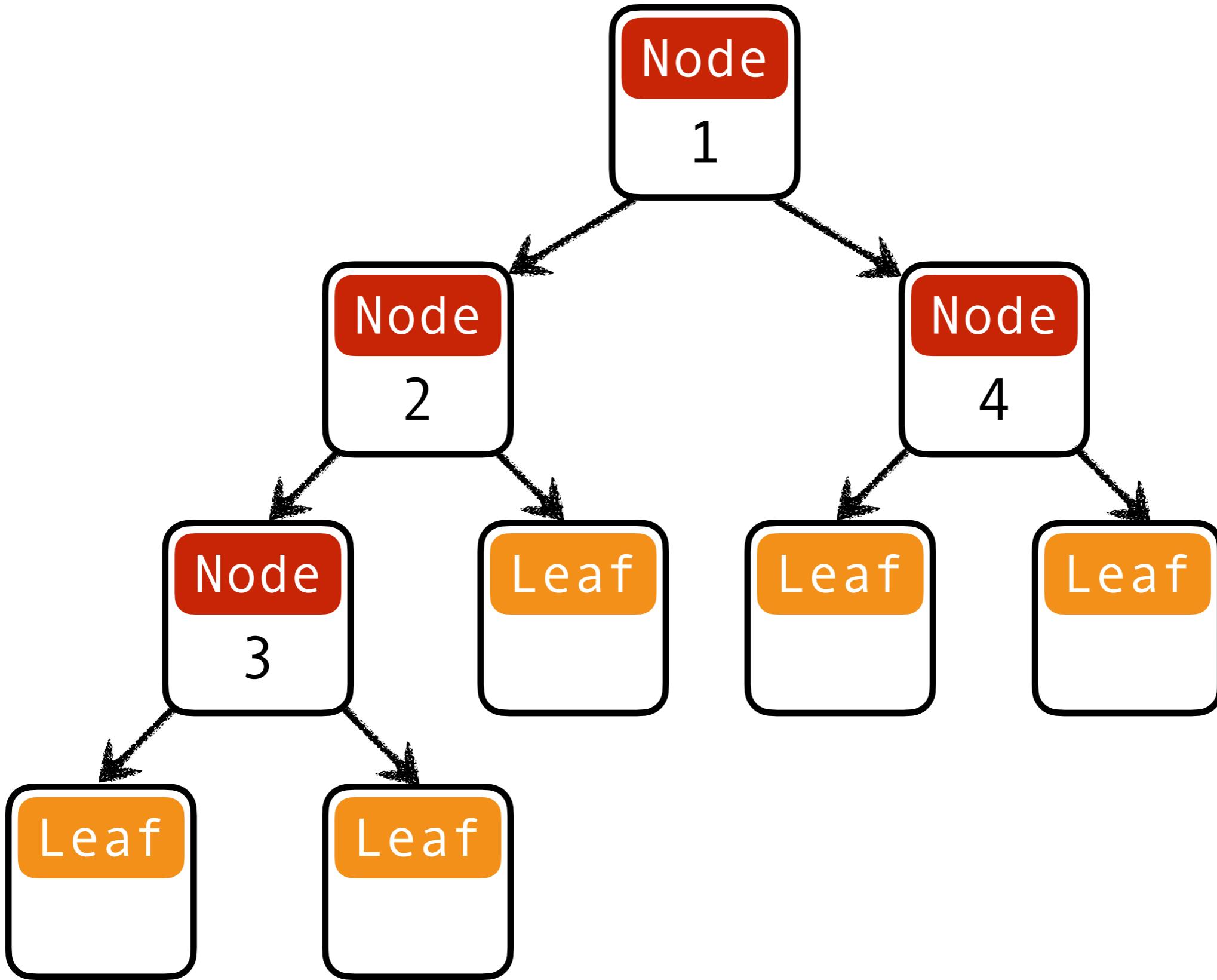
Step 2

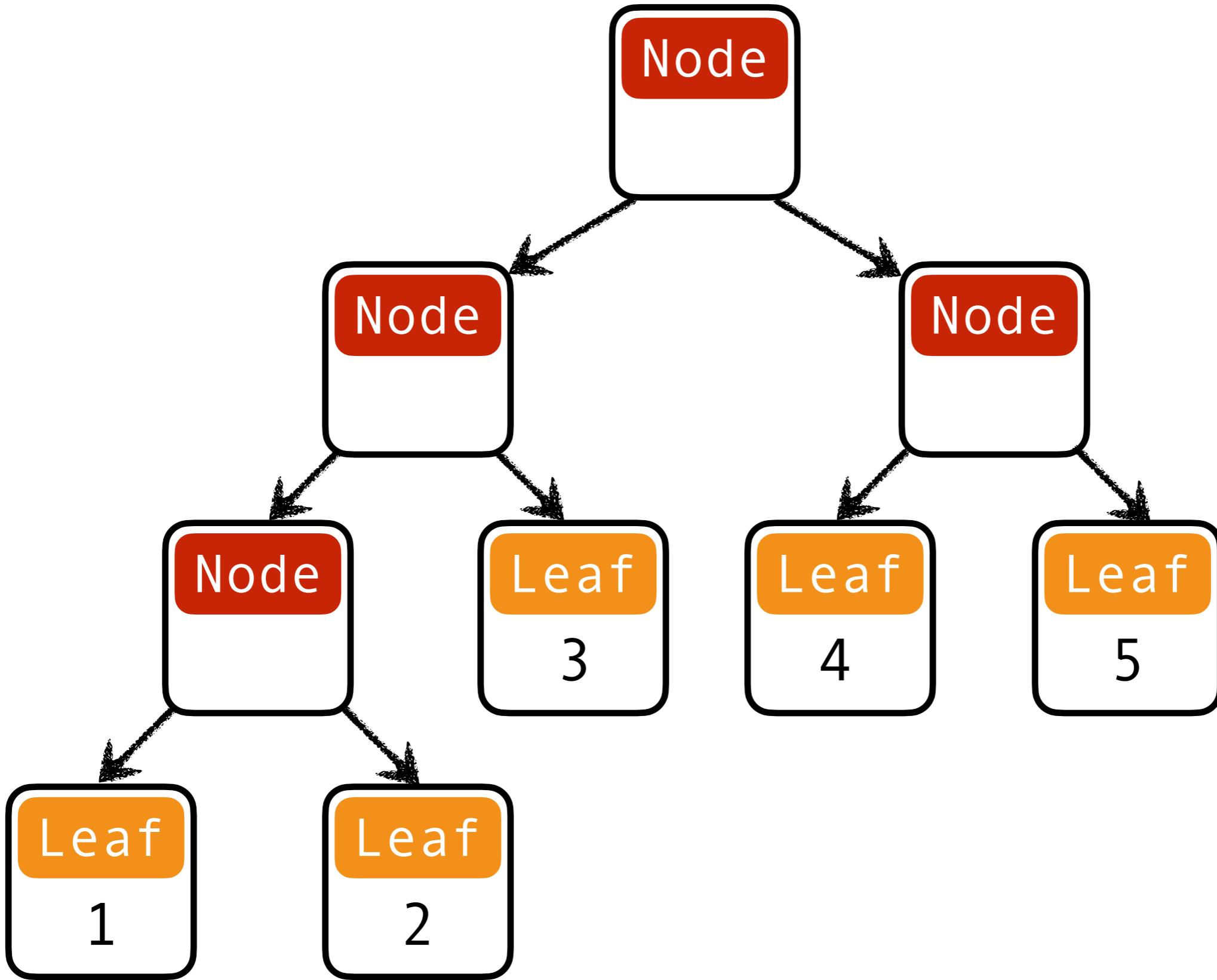
Make the ganache: Place the sour cream and melted chocolate in a large bowl. Whisk to combine and refrigerate for 10 to 15 minutes or until firm. Place 1 of the cakes on a cake stand or plate. Spread with half the ganache. Top with the remaining cake and ganache. Sprinkle with the salt to serve.

Tip

To make your own self-rising flour, combine 2 1/2 cups/320 grams all-purpose flour; 1 tablespoon plus 3/4 teaspoon baking powder; and 1/2 teaspoon plus 1/8 teaspoon fine salt. Use the entire amount in place of the self-rising flour listed in the ingredients.

howToSaltChoco :: Recipe Cake





PW-1: Async Time

PW-2: Async Space

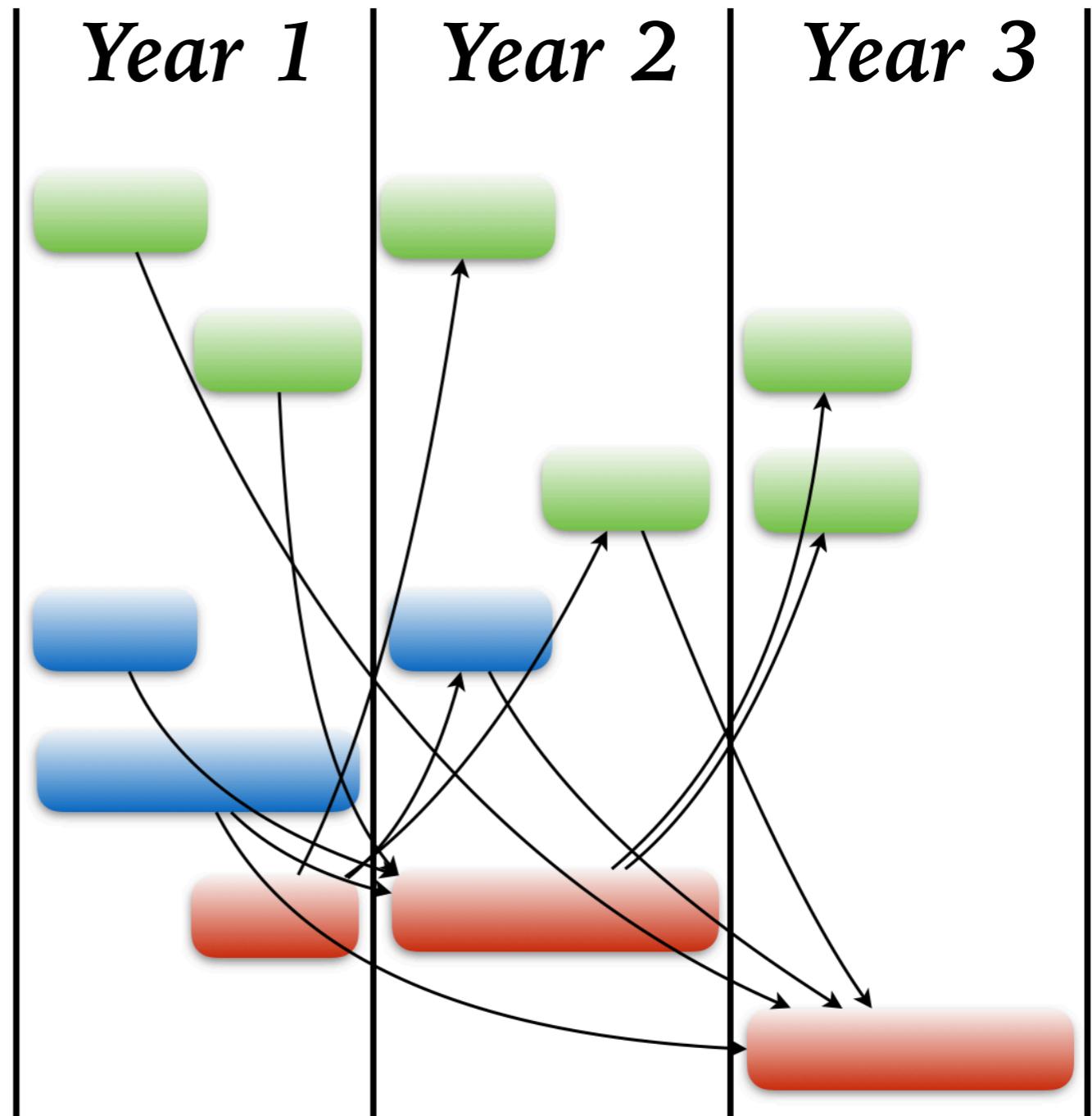
PW-3: Sound & Complete

PW-4: Invariant Synthesis

PW-5: Modular Effects

PW-6: Coord. Service

PW-7: Microservice Flows



```
let rec wwhile (f, b) =  
  let (b', c') = f b in  
  if c' = true then wwhile (f b')  
  else b'
```

RITE: (f, b')

SEMINAL: ((f b'); [[...]])

```
let rec clone x n =  
  if n <= 0 then [] else  
  x :: clone (n-1)
```

RITE: clone (n-1) n

SEMINAL: clone [[...]] (n-1)

```
let sqsum xs =  
  let f a x = a + (x ** 2) in  
  let base = 0 in  
  List.fold_left f base xs
```

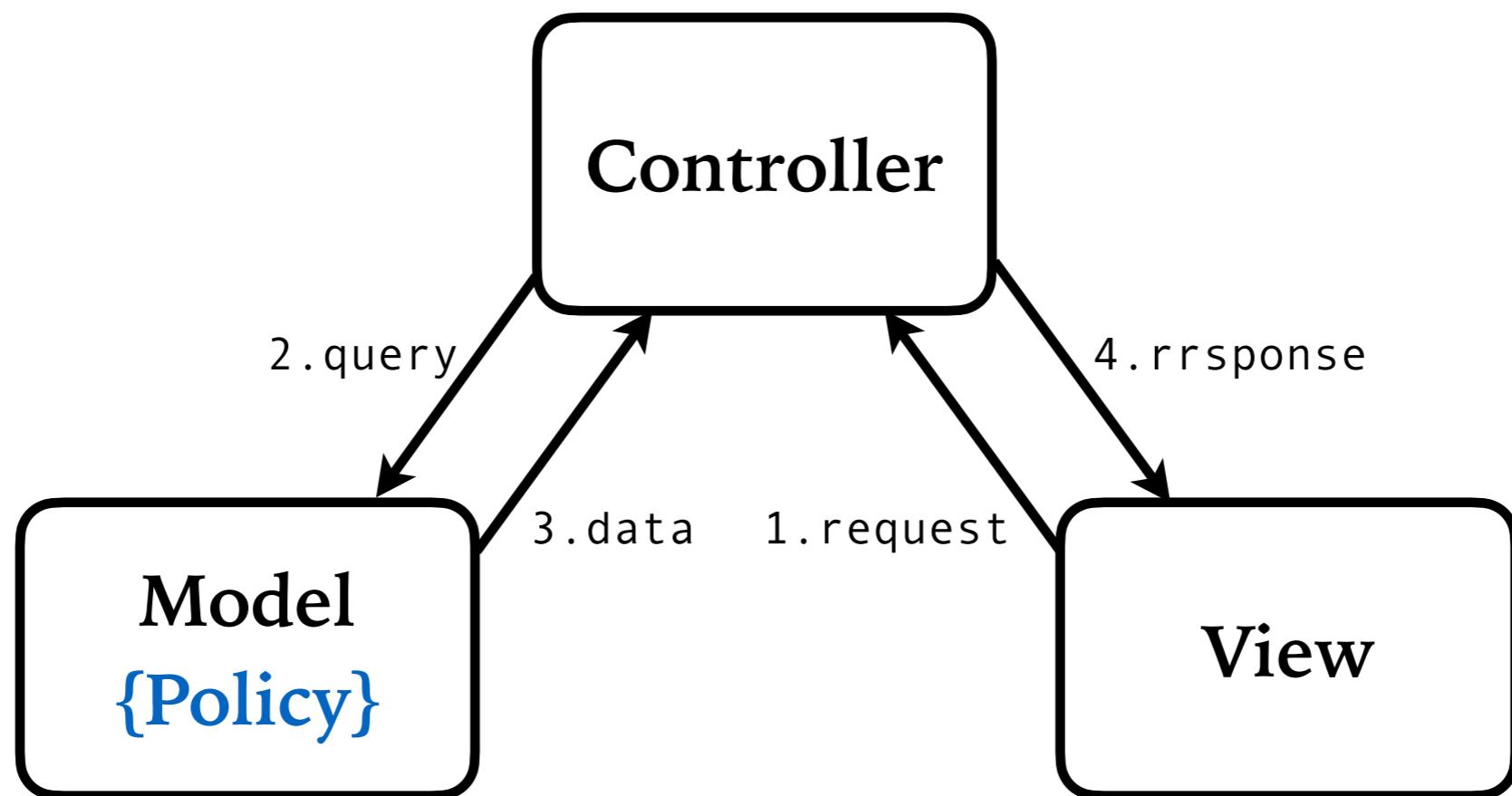
RITE: (x * x)

SEMINAL: (x + 2)

```
let rec clone x n =  
  if n <= 0 then [] else  
    x :: clone (n-1)
```

RITE: clone (n-1) n

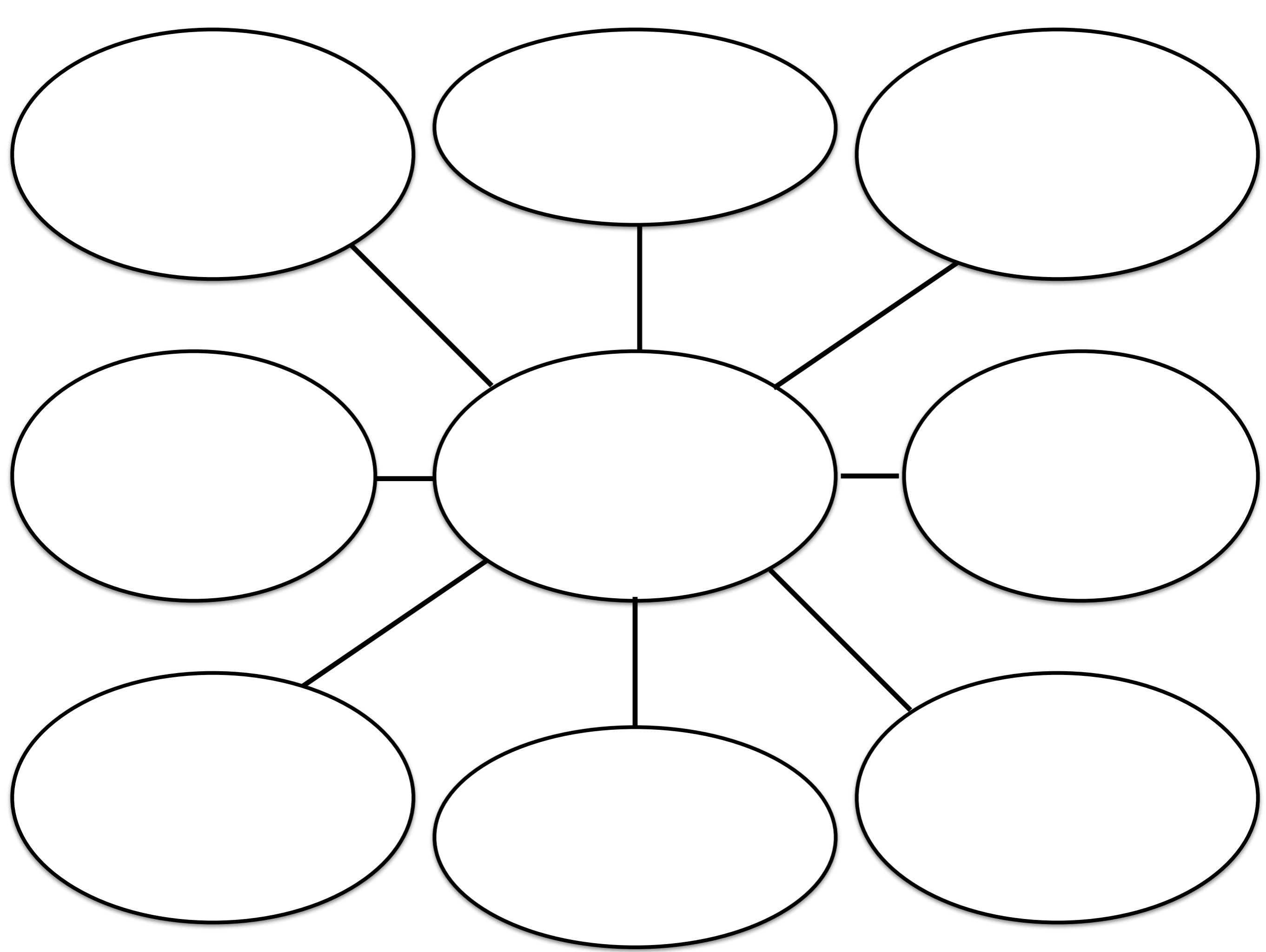
SEMINAL: clone [...](n-1)

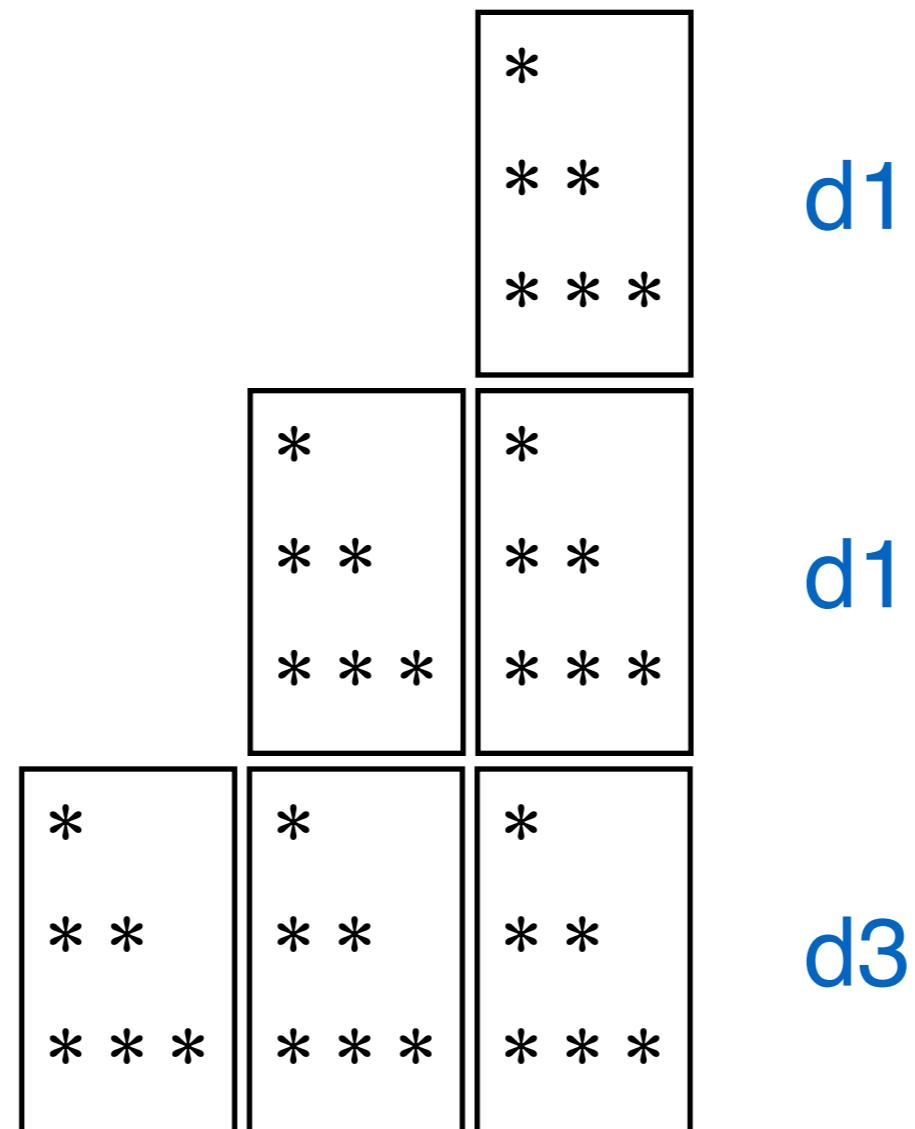




The question then is, is it possible to use (overapproximate) static analyses to precisely report that the target location is reachable, *without* actually finding a feasible path to it? Intuitively, the code through the for-loop is irrelevant to the reachability of the error location. In other words, if we can reason that *there exists some path* from the start to the end of the loop, *i.e.*, from location 3: to 5:, and along such a path, the variables x , a are not modified, then we are guaranteed that the location ERR: can be reached.







ICons

1

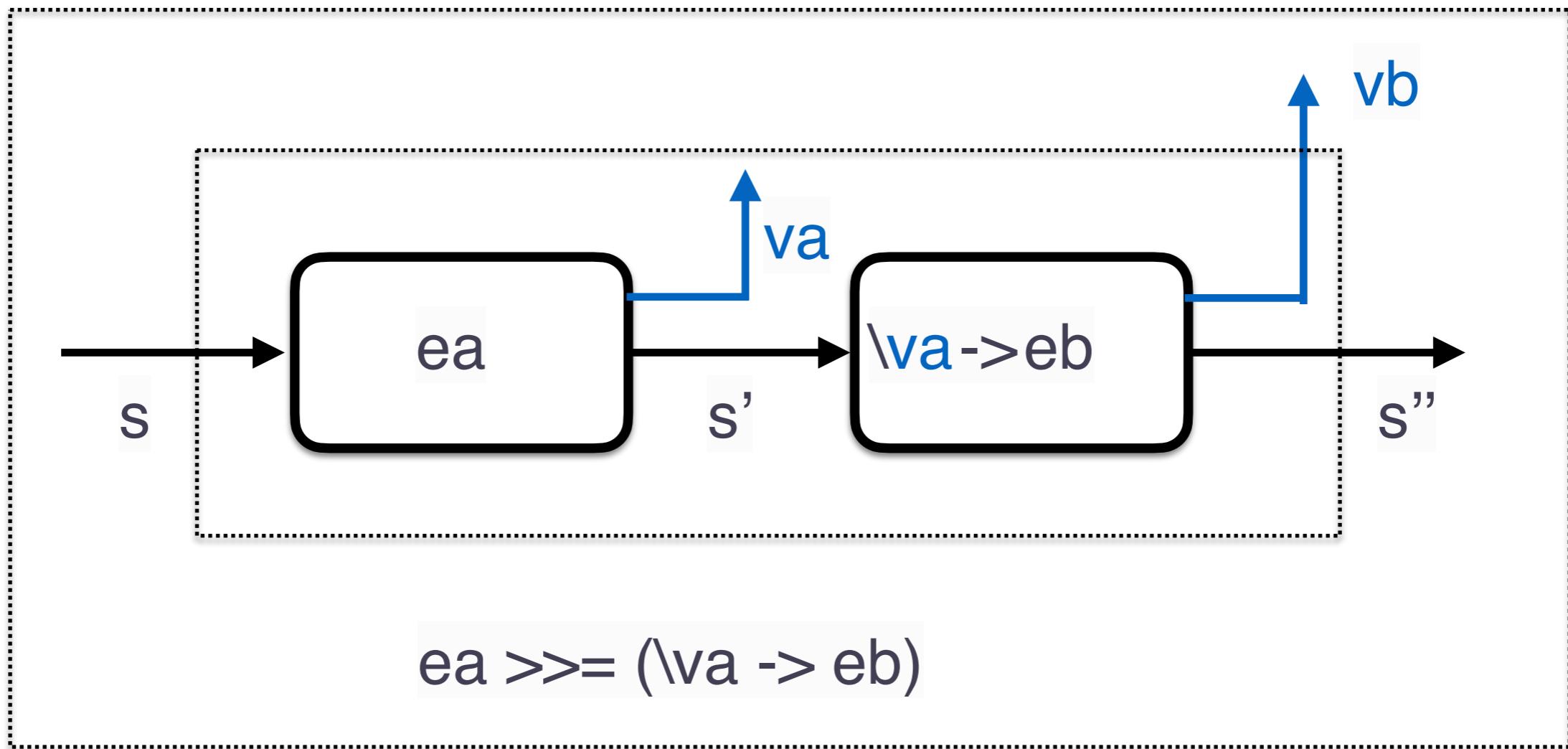
ICons

2

ICons

3

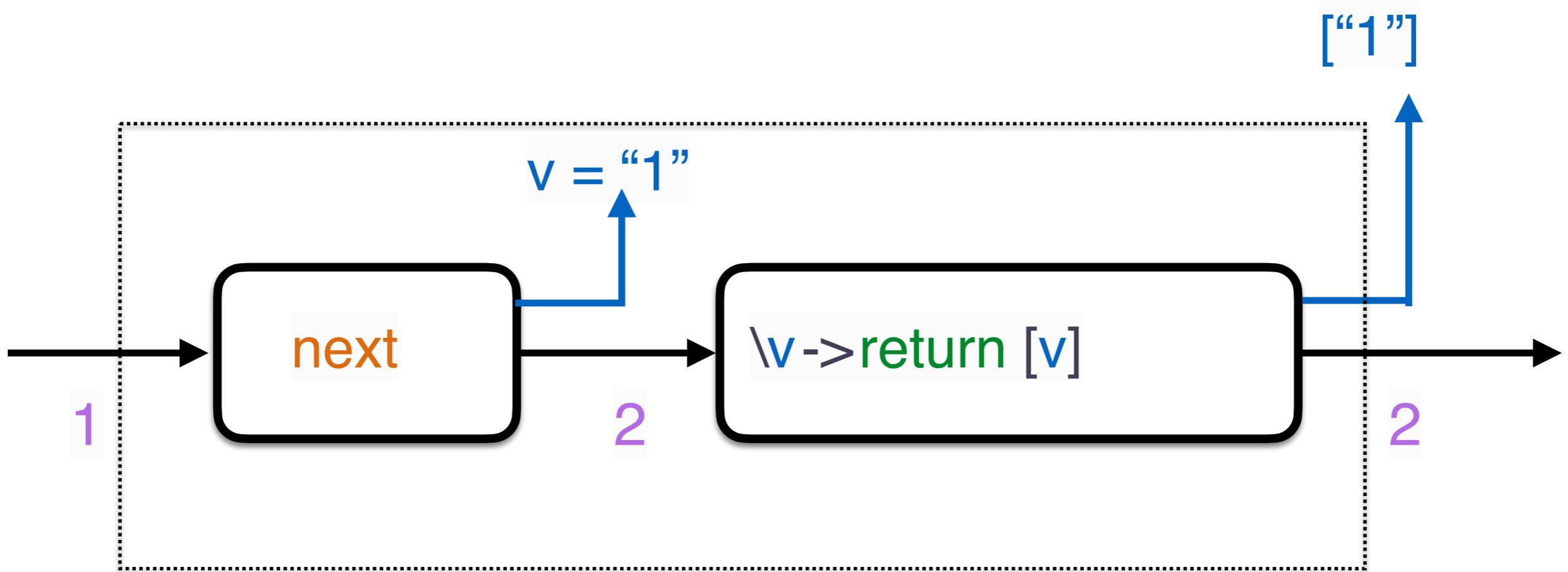
INil



```
next :: ST0 String
next = ST0C (\s -> (s+1, show s))
```

```
wtf :: ST0 [String]
wtf = next >>= (\v -> return [v])
```

```
quiz = evalState wtf 1
```



```
next :: ST0 String
```

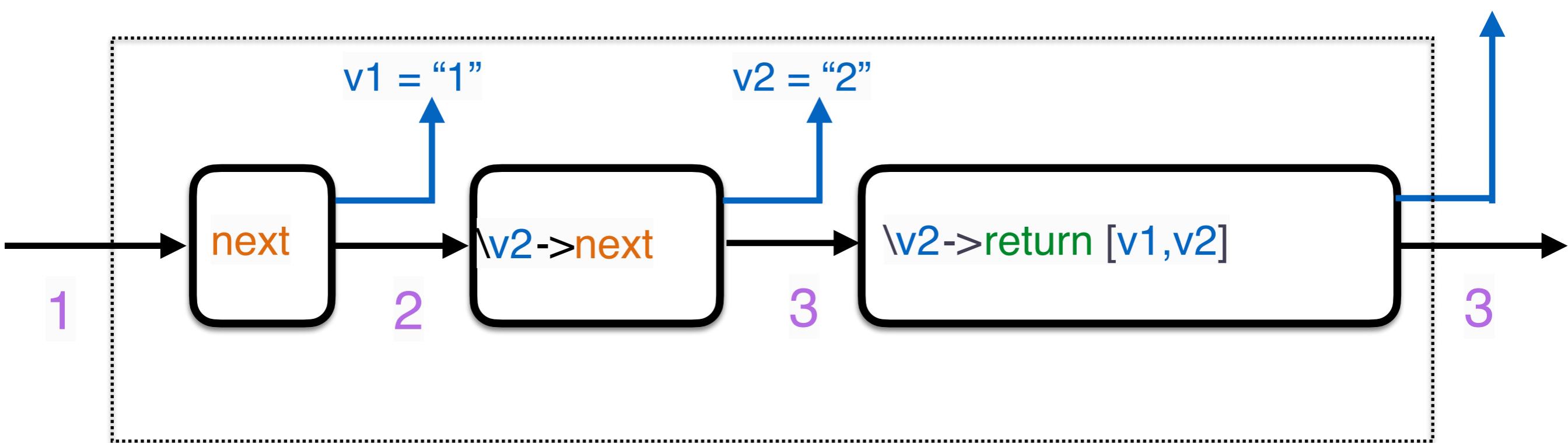
```
next = ST0C (\s -> (s+1, show s))
```

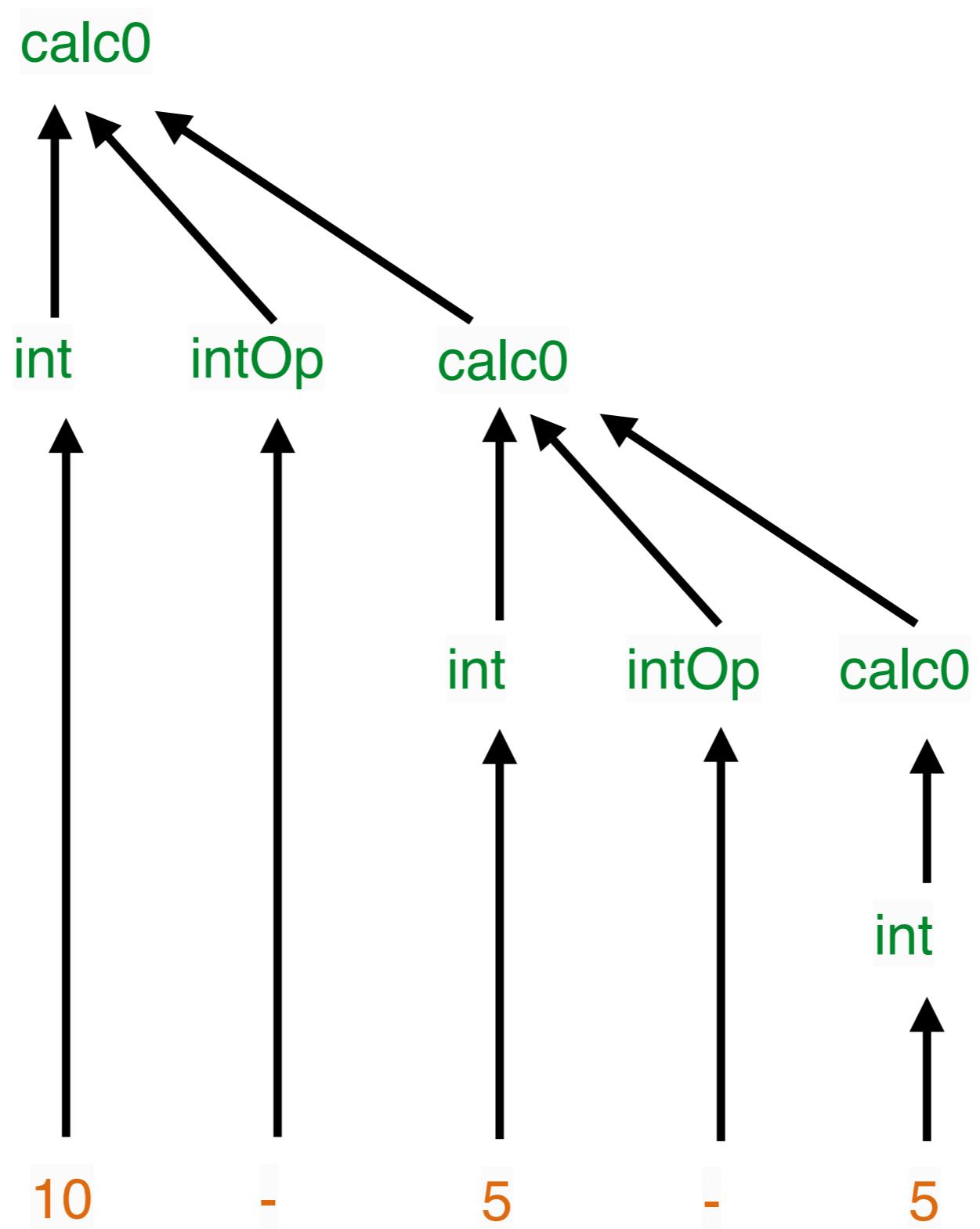
```
wtf :: ST0 [String]
```

```
wtf = next >>= (\v1 -> next >>= (\v2 -> return [v1, v2]))
```

```
quiz = evalState wtf 1
```

[“1”, “2”]





m

Transform1

m

Transform2

Transform1

m

...

Transform2

Transform1

m

