

# FDL "fer-de-lance"

## First Class Functions

↳ Assign is up  
due just  
before final

fun Env  
var Env

## Functions as Values

Consider the following egg program

```
def f(it):
```

```
  it(5)
```

unbound func 'it'

it ∉ funenv

```
def incr(x):
```

```
  x + 1
```

```
f(incr)
```

unbound var 'incr'

incr ∉ varEnv

What will be the result of compiling/running?


on "diamond"

We have functions, but they are *second-class* entities in our languages: they don't have the same *abilities* as other values.

So, we get multiple error messages:


Errors found!

tests/input/hof.diamond:(2:3)-(4:1): Function 'it' is not defined



```
2|   it(5)
   ^^
```

tests/input/hof.diamond:7:3-7: Unbound variable 'incr'



```
7|   f(incr)
   ^^^^
```

This is because the Env only holds



- parameters, and
- let-bound variables

and **not** function definitions.

## *Functions as Values*

But for the many reasons we saw in CSE 130 – we *want* to treat functions like values.  
For example, if you run the above in Python you get:

```
>>> def f(it): return it(5)
```

```
>>> def incr(x): return x + 1
```

```
>>> f(incr)
```

```
6
```

*Flashback: How do we compile `incr`?*

We compile each function down into a sequence of instructions corresponding to its body.

**def incr(x):**  
 $x + 1$

*rdi*

**incr(5)**

becomes, for incr

label\_def\_incr\_start:

**push rbp**

**mov rbp, rsp**

**mov rax, rdi**

**add rax, 2**

**mov rsp, rbp**

**pop rbp**

**ret**

# setup stack frame

# grab param

# incr by 1

# undo stack frame

# buh-bye

for the main expression

*our code here:*  
*push rbp etc*

*mov rdi, 10*

*call label\_def\_incr\_start:*

*pop rbp*  
*etc*

our\_code\_starts\_here:

```
push rbp
```

```
mov rbp, rsp
```

```
mov rdi 10          # push arg '5'
```

```
call label_def_incr_start # call function
```

```
mov rsp, rbp
```

```
pop rbp
```

```
ret
```

## *What is the value of a function?*

So now, lets take a step back. Suppose we want to compile

```
def f(it):  
    it(5)
```

```
def incr(x):  
    x + 1
```

```
f(incr)
```

*"enable passing  
FUNCTION  
as a param"*

*incr ↪ "label"  
it ↪ ↗*

*Attempt 1: What is the value of the parameter  $it$  ?*

IDEA: Use the label where `incr` lives!

`label_def_f_start:`

`push rbp`

`mov rbp, rsp`

`mov rax, rdi`

# grab function-address

`mov rdi, 10`

# push arg '5'

`call rax`

# call function!

`mov rsp, rbp`

`pop rbp`

`ret`

`def f(it):`

`it(5)`

- ① How do we know rdi is a func?
- ② what is the addr?



## *How to pass the value of the parameter ?*

So now the main expression

`f(incr)`

can be compiled to:

our\_code\_starts\_here:

```
push rbp
mov rbp, rsp
```

```
mov rdi, ?1    # push arg
call ?2        # call function
```

```
mov rsp, rbp
pop rbp
ret
```

*addr/label of  
incr*

*f(incr)*

*f(12)*

*func 'f' being called*


QUIZ: What are suitable terms for ?1 and ?2 ?

	?1	?2
A	label_def_incr_start	label_def_f_start
B	label_def_f_start	label_def_incr_start
C	label_def_f_start	label_def_f_start
D	label_def_incr_start	label_def_incr_start



## *Strategy Progression*

1. **Representation** = Start-Label

- **\*\*Problem:\*\*** How to do run-time checks of valid args?
- 

*Yay, that was easy! How should the following*

behave?

```
def f(it):  
    it(5)
```

```
def add(x, y):  
    x + y
```

~~f(5)~~  
f(add)

mov rax, L\_add  
mov rdi, 10  
call rax

mov rax, rdi  
add rax, rsi  
gibberish

Lets see what Python does:

```
>>> def f(it): return it(5)
>>> def add(x,y): return x + y
>>> f(add)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 1, in f
TypeError: add() takes exactly 2 arguments (1 given)
```

## *Problem: Ensure Valid Number of Arguments?*

How to make sure

- `f(incr)` **succeeds**, but
- `f(add)` **fails**

With **proper run-time error?**

1. **Where** does the run-time check happen?
2. What information is needed for the check?

`def f(it):`  
`it(5)` — use metadata      `f(incr)` ✓  
`def incr(x):`      } create metadata      `f(add)` ✗  
`x + 1`  
`def add(x, y):`  
`x + y`

**Key:** Need to *also* store the function's **arity**

- The **number of arguments** required by the function

## *Strategy Progression*

1. Representation = ~~Start Label~~

- **Problem:** How to do run-time checks of valid args?

2. Representation = (Arity, Start-Label)



CREATE?

USE call

Attempt 2: What is the value of the parameter *it*?

`def f(it):`  
`it(5)`

$\xrightarrow{it \equiv \text{incr}}$   $(1, l.\text{incr-start})$

`def incr(x):`  
`x + 1`

`f(incr)`  $\xrightarrow{it = \text{add}}$   $(2, l.\text{add})$

000 num  
 001 tuple  
 111 bool  
 101 fun-tuple

IDEA: Represent a *function* with a tuple of

`{arity, function_start_label}`

- ✓ 1. check  $e$  is function
- ✓ 2. check args is correct
3. move args into reg/stack
4. call the function

We can now compile a call

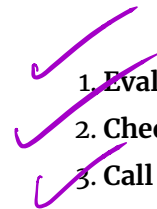
`e(x1, ..., xn)`

via the following strategy:

`it(5)`

$rax \leftarrow \langle e \rangle$

1.  $\text{assert}(rax \& 0x111 \equiv \&101)$
2.  $\text{assert}(\text{len } xs == rax[0])$
3. as before
4. call  $(rax[1])$

- 
1. **Evaluate** the tuple `e`
  2. **Check** that `e[0]` is equal to `n` (else arity mismatch error)
  3. **Call** the function at `e[1]`

$let\ f = \lambda it \rightarrow it(5)$

$,\ incr = \lambda x \rightarrow x+1$

in

$f(incr)$

## Example

Lets see how we would compile this

```
def f(it):
    it(5)
```

```
def incr(x):
    x + 1
```

```
f(incr)
```

We need to map the variable

incr

to the tuple

(1, label\_def\_incr\_start)

1. create "tuple" for incr

2. name 'incr'

put the func-tuples  
IN the var env

But **where** will we store this information?

## *Strategy Progression*

1. **Representation** = Start-Label

- **Problem:** How to do run-time checks of valid args?

2. **Representation** = (Arity, Start-Label)



- **Problem:** How to map function **names** to tuples?

3. **Lambda Terms** Make functions just another expression!

## *Attempt 3: Lambda Terms*

So far, we could only define functions at **top-level**

- First-class functions are like *any* other expression,
- Can define a function, wherever you have any other expression.

Language	Syntax
Haskell	$\lambda(x_1, \dots, x_n) \rightarrow e$ 
Ocaml	<code>fun (x1, ..., xn) -&gt; e</code> 

Language	Syntax
JS	$(x_1, \dots, x_n) \Rightarrow \{ \text{return } e \}$
C++	$[\&](x_1, \dots, x_n) \{ \text{return } e \}$



## *Example: Lambda Terms*

We can now replace `def` as:

```
let f    = (lambda (it): it(5))  
    , incr = (lambda (x): x + 1)  
in  
    f(incr)
```

## *Implementation*

As always, to the details! Lets figure out:

### Representation

1. How to store function-tuples



101



**Types:**

1. ~~Remove Def~~
2. Add lambda to Expr

**Transforms**

1. Update tag and ANF
2. Update checker
3. Update compile

LAM  
APP



*Implementation: Representation*

Represent **lambda-tuples** or function-tuples'' via a special tag:

Type	LSB
number	xx0
boolean	111
pointer	001
<b>function</b>	<b>101</b>

In our code:

```
data Ty = ... | TClosure
```

```
typeTag :: Ty -> Arg
```

```
typeTag TTuple    = HexConst 0x00000001
```

```
typeTag TClosure  = HexConst 0x00000005
```

```
typeMask :: Ty -> Arg
```

```
typeMask TTuple    = HexConst 0x00000007
```

```
typeMask TClosure  = HexConst 0x00000007
```

 3 LSB

So, **Function Values** represented just like a tuples

- padding, ~~FS~~ etc.
- but with tag **101**.

Crucially, we can get 0 -th, or 1 -st elements from tuple.

Question: Why not use *plain tuples*?

↳ to ensure  $e[1]$  is  
a func label!

## Implementation: Types

First, lets look at the new Expr type

- ~~No more Def~~

`data Expr a`  
`= ...`  
`| Lam [Bind a] !(Expr a) a -- fun. definitions`  
`| App !(Expr a) [Expr a] a -- fun. calls`

*params* (pointing to `[Bind a]`)  
*body* (pointing to `!(Expr a) a`)

So we represent a **function-definition** as:

`Lam [x1,...,xn] e`

and a **function call** as:

`App e [e1,...,en]`

*Lam [x<sub>1</sub>,...,x<sub>n</sub>] e*

*App e [e<sub>1</sub>,...,e<sub>n</sub>]*

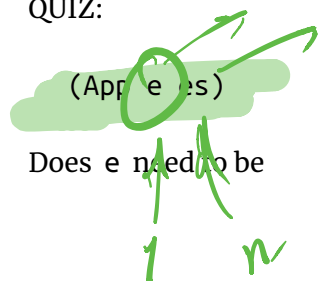
*previously* *D* which we lookup in *FunEnv*

## Transforms: Tag

This is pretty straight forward (do it yourself)

## Transforms: ANF

QUIZ:



let tup =  
 ( lambda(x): x+1  
 , lambda(y): y-1  
 )  
 tmp = tup[0]

- A Immediate
- B ANF
- C None of the above

in  $tmp(10)$   
 $e$   $[10]$   
 $\downarrow$   
 $rax$

## Transforms: ANF

QUIZ:

(App  $e$  es)

Do es need to be

- **A** Immediate
- **B** ANF
- **C** None of the above

## *Transforms: ANF*

The App case, fun + args should be immediate

- **Need the values to push on stack and make the call happen!**

Just like function calls (in `diamondback`), except

- Must also handle the **callee-expression** (named `e` below)

```
anf i (App e es) = (i', stitch bs (App v vs))
```

where

```
(i', bs, v:vs) = imms i (e:es)
```

*Transforms: ANF*

QUIZ:

```
(Lam xs e)
```

Does e need to be

(A) IMM  
 (B) ANF is OK



- **A** Immediate
- **B** ANF
- **C** None of the above

## *Transforms: ANF*

The `Lam` case, the body will be **executed** (when called)

- So we just need to make sure its in ANF (like all the code!)

$\text{anf } i \text{ (Lam } xs \text{ e)} = (i', \text{Lam } xs \text{ e}')$

**where**

$(\underbrace{i'}, \underbrace{e'}) = \text{anf } i \text{ e}$

## *Transforms: Checker*

We just have Expr (no Def ) so there is a single function:

```
wellFormed :: BareExpr -> [UserError]
```

```
wellFormed = go emptyEnv
```

```
where
```

```
  gos = concatMap . go
```

```
  go _ (Boolean {}) = ...
```

```
  go _ (Number n l) = largeNumberErrors n l
```

```
  go vEnv (Id x l) = unboundVarErrors vEnv x l
```

```
  go vEnv (Prim1 _ e _) = ...
```

```
  go vEnv (Prim2 _ e1 e2 _) = ...
```

```
  go vEnv (If e1 e2 e3 _) = ...
```

```
  go vEnv (Let x e1 e2 _) = ... ++ go vEnv e1 ++ go (addEnv x vEnv
```

```
v) e2
```

```
  go vEnv (Tuple es _) = ...
```

```
  go vEnv (GetItem e1 e2 _) = ...
```

```
  go vEnv (App e es _) = ?1
```

```
  go vEnv (Lam xs e _) = ?2 ++ go ?3 e
```

• How shall we implement ?1 ?

• How shall we implement ?2 ?

• How shall we implement ?3 ?

① 't' even in scope  
② 'e' has arity 1

gos env (e:es)

int → int

let f = λit → it(5)

, incr = λx → x+1

, add = λx y → x+y

in ... f(incr) f(add)

- $x$  is a let-bound variable inside  $e$  .
- $x$  is a formal parameter in  $e$  , OR

A variable  $x$  is **free** inside an expression  $e$  if

- $x$  is **not bound** inside  $e$

For example consider the expression  $e$  :

**lambda** ( $m$ ):

let  $t = m$  in  
 $n + t$

- $m, t$  are **bound** inside  $e$  , but,
- $n$  is **free** inside  $e$

## *Computing Free Variables*

Lets write a function to **compute** the set of free variables.

**Question** Why Set ?

