CSE 150A
Intro to AI: Probabilistic Reasoning

Discussion Session 1

Probability Review, Bayes, marginal / conditional independence and python coding

Agenda

- Review of concepts
- Marginal & Conditional Independence, Joint Probability
- Bayes Theorem
- Introduction to Bayesian Networks
- D-separation

Introduction Python / Libraries

[Intro to python]

Concepts Review

1. Non-negativity: Probabilities are never negative

$P(A) \geq 0$

2. Entire Sample Space: the total probability over the sample space is 1

$P(A) + P(\neg A) = 1$

3. AND: the probability that both event A and event B happen

$P(A, B) = P(A \wedge B)$

$P(A \wedge B) = P(A) + P(B) - P(A \vee B)$

4. OR: the probability that either A or B happens

$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$ (General Formula)

If A and B are **disjoint** (meaning they cannot both happen, so $A \wedge B = \varnothing$), then:

$P(A \vee B) = P(A) + P(B)$

Concepts Review: Exercise 1

In a survey among students in a university, two events were studied:

- Some students attended an AI Club meeting.
- Some students attended a Math Workshop.

A portion of students attended both the AI Club meeting and the Math Workshop.
In total, **40%** of students attended the **AI Club meeting**, **50%** attended the **Math Workshop**, and **20%** attended **both** events.

1. **What is the probability that a student attended either the AI Club meeting or the Math Workshop (or both)?**
2. **What is the probability that a student attended neither of the two events?**
3. **Could these two events be disjoint? Why or why not?**

Concepts Review: Exercise 1 - solution

Let:
A = Student attended the AI Club meeting
B = Student attended the Math Workshop
From the problem:

- $P(A) = 0.40$
- $P(B) = 0.50$
- $P(A \land B) = 0.20$

1. **Probability of either event:**
   $P(A \lor B) = P(A) + P(B) - P(A \land B) = 0.40 + 0.50 - 0.20 = \mathbf{0.70}$
2. **Probability of neither event:**
   $P(\text{neither}) = 1 - P(A \lor B) = 1 - 0.70 = \mathbf{0.30}$
3. **Are A and B Disjoint?:**
   $P(A \land B) = 0.20 \neq 0$

Conditional Probability

$$P(X = x_i | Y = y_i)$$

"What is my belief that $X = x_i$ if I already know $Y = y_j$"

Sometimes, knowing Y gives you information about X, i.e., changes your belief in X. In this case X and Y are said to be *dependent*.

$$P(X = x_i | Y = y_j) \neq P(X = x_i)$$

Axioms of Conditional Probability

Which of the following axioms hold for conditional probabilities?

A. $P(X = x_i \mid Y = y_j) \geq 0$

B. $\sum_i P(X = x_i \mid Y = y_j) = 1$

C. $\sum_j P(X = x_i \mid Y = y_j) = 1$

D. A and B only

E. A, B and C

Joint Probability

$P(X = x_i, Y = y_j)$

"What is my belief that $X = x_i$ **and that** $Y = y_j$"

Which of the following is always true?

A. $P(X = x_i \; or \; Y = y_j) \leq P(X = x_i, Y = y_j)$

B. $P(X = x_i \; or \; Y = y_j) \geq P(X = x_i, Y = y_j)$

C. $P(X = x_i \; or \; Y = y_j) = P(X = x_i, Y = y_j)$

D. None of the above.

Marginal Independence

$$P(X = x_i \mid Y = y_j) = P(X = x_i)$$

Sometimes knowing Y does not change your belief in X. In this case, X and Y are said to be independent.

$$P(W = w_i | Y = y_j) = P(W = w_i)$$

Where W = weather today

For which variable Y is the above statement most likely true?

A. Y = The weather yesterday
B. Y = The day of the week
C. Y = The temperature

More independence

Consider two students Roberto and Sabrina, who both took the same test. Define the following random variables:

R = Roberto aced the test

S = Sabrina aced the test

What is the most logical relationship between $P(R = 1)$ and $P(R = 1 \mid S = 1)$?

A. $P(R = 1) \quad = \quad P(R = 1|S = 1)$

B. $P(R = 1) \quad > \quad P(R = 1|S = 1)$

C. $P(R = 1) \quad < \quad P(R = 1|S = 1)$

Conditional Independence

What if you also know the test was easy (variable T)?

A. $P(R = 1|T = 1) \quad = \quad P(R = 1|T = 1, S = 1)$

B. $P(R = 1|T = 1) \quad > \quad P(R = 1|T = 1, S = 1)$

C. $P(R = 1|T = 1) \quad < \quad P(R = 1|T = 1, S = 1)$

R and S are conditionally independent given T. I.e., if you already know T, knowing S does not give you additional information about R.

More independence

Consider two events:

B = A burglar breaks into your apartment

E = An earthquake occurs

Are these events independent or dependent? (i.e., does knowing that one happened change your belief in the other?)

A. They are independent because knowing that one happened does not change your belief that the other happened.
B. They are dependent, because knowing that one happened changes your belief that the other happened.

Conditional Dependence

$P(B = 1) = P(B = 1|E = 1) = P(B = 1|E = 0)$

Now consider a third event:

A = Your alarm goes off

Which of the following relationships best models beliefs about the world?

A. $P(B = 1|A = 1) = P(B = 1|A = 1, E = 1)$
B. $P(B = 1|A = 1) > P(B = 1|A = 1, E = 1)$
C. $P(B = 1|A = 1) < P(B = 1|A = 1, E = 1)$

Exercise 2

R = 1 if remote worker else 0

M = 1 if attend meeting else 0

W = 1 if submit report else 0

In a company:

- 70% of employees are remote and 30% work in the office
- Among remote workers:
  - 40% attend daily meetings, 50% submit weekly reports and 20% both attend meetings and submit reports
- Among office workers:
  - 80% attend daily meetings , 50% submit weekly reports and 40% both attend meetings and submit reports

Are the events M (attend meetings) and W (submit reports) conditionally independent given location (R = 1 or R = 0)?

Exercise 2 - Solution

Note: When two events are conditionally independent, their joint probability equals the product of their individual conditional probabilities**.**

- $P(M=1 \mid R=1)=0.40$
- $P(W=1 \mid R=1)=0.50$
- $P(M=1,W=1 \mid R=1)=0.20$
- $0.40*0.50 = 0.20$ — **equality holds**

For Office Workers:

- $P(M=1 \mid R=0)=0.80$
- $P(W=1 \mid R=0)=0.50$
- $P(M=1,W=1 \mid R=0)=0.40$
- $0.80 \cdot 0.50=0.40$ — **equality holds**

Answer: Since the product of the individual probabilities equals the joint probability in both groups, M and W are conditionally independent given location (R).

Bayes Theorem

Bayes Theorem: Mathematical Definition

$$P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)}$$

- $P(A \mid B)$: Posterior probability (probability of A given B)
- $P(B \mid A)$: Likelihood (probability of B given A)
- $P(A)$: Prior probability of A
- $P(B)$: Marginal probability of B (normalizing constant)

The Big Idea: Reverse Reasoning

Bayes' Theorem helps us reason backward — from observed evidence back to the likely cause.
Consider the following scenario:

- Only 1% of applicants get a job offer from Google.
- Suppose 90% of Google hires had strong GitHub portfolios.
- But 10% of all applicants also have strong GitHub portfolios.

Now imagine a friend tells you someone has a strong GitHub portfolio.
What’s the probability they actually got the job at Google?

Answer: 9%

Bayes' Rule exercise 2

A company has three factories: A, B, and C.

- Factory A produces 30% of all products, with 2% defect rate
- Factory B produces 50% of all products, with 1% defect rate
- Factory C produces 20% of all products, with 3% defect rate

A product is selected at random and found to be defective.

What is the probability it came from Factory C?

Exercise 2 - Solution

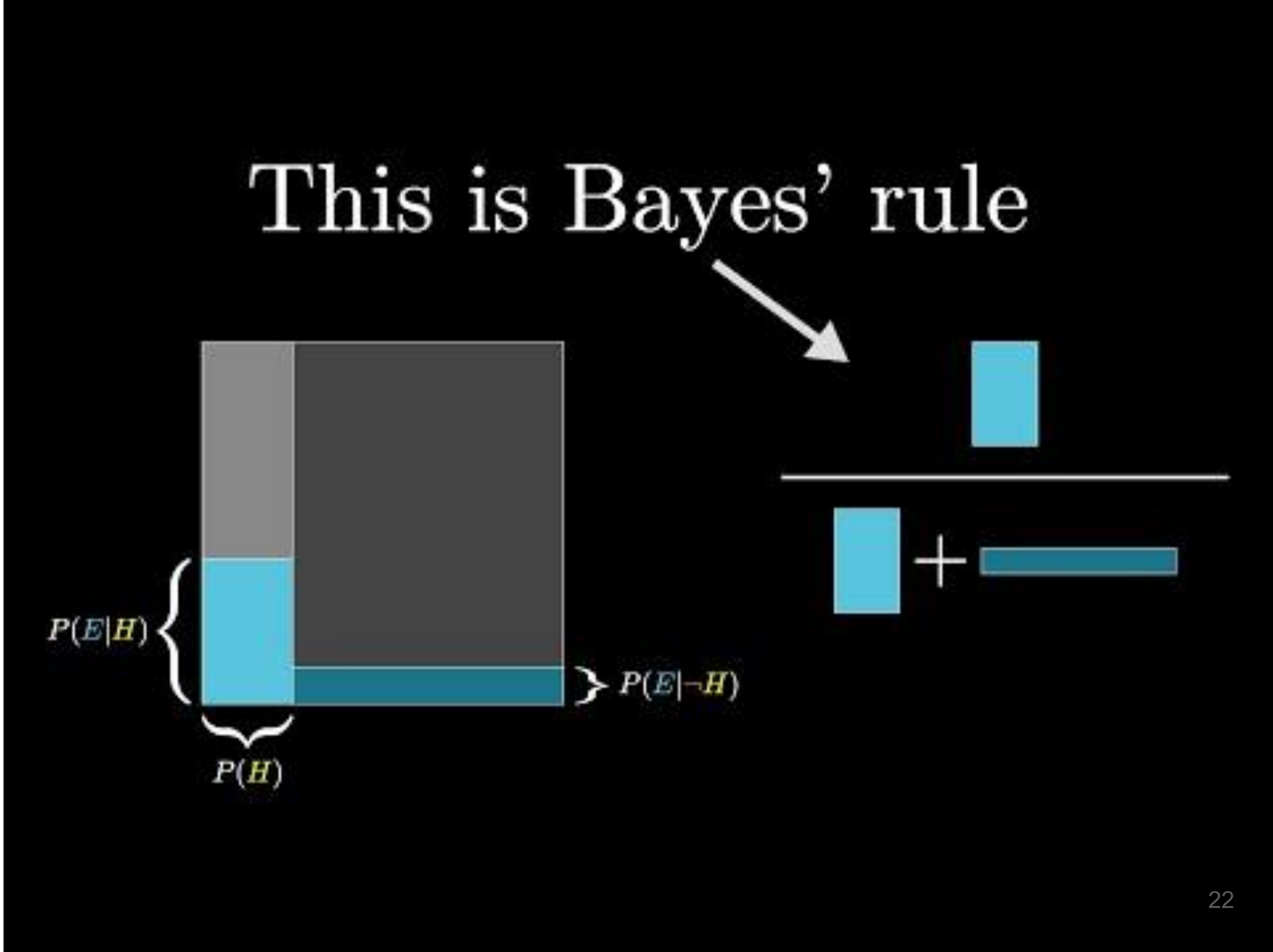
$P(C|D) = ?$

- We can Use bayes theorem:
  $$P(C|D) = \frac{P(D|C) * P(C)}{P(D)}$$
- $P(C) = 20\%$
- $P(D | C) = 3\%$
- $P(D)$ = total probability that a product is defective (*we will compute this*) = ?
  $P(D) = P(D|A)\,P(A) + P(D|B)\,P(B) + P(D|C)\,P(C)$
  $=0.02*0.30+0.01*0.50+0.03*0.20=0.006+0.005+0.006=0.017$

- $P(C|D) = (0.03 * 0.2) / (0.017) \approx$ **0.353**

Answer: 0.353

Bayes Rule



Exercise 3

In a security system:

- 2% of all packages are dangerous (D)
- A scanner raises an alert (A) with:
  - 95% chance if the package is dangerous
  - 5% chance if the package is safe

A secondary check (S) is performed, which depends on whether the alert was triggered. The secondary check flags a package:

- 80% of the time if the alert was triggered ($P(S \mid A=1) = 0.80$)
- 10% of the time if no alert was triggered ($P(S \mid A=0) = 0.10$)

A package was flagged by the secondary check.

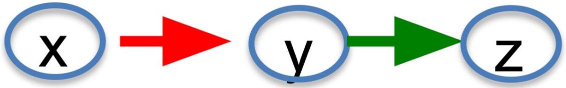
Q1: What is the probability the package is actually dangerous ($P(D \mid S)$)? **Answer: 0.135**

Q2: Are events D and S conditionally dependent given A ? **Answer: Yes**

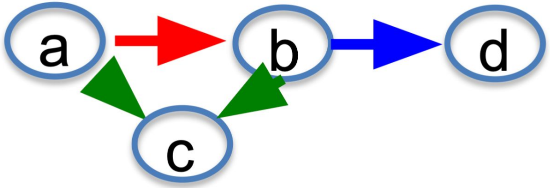
Introduction to Bayesian Networks

- Directed graphical model
- Nodes associated with variables
- “Draw” independence in conditional probability expansion
    - Parents in graph are the RHS of conditional

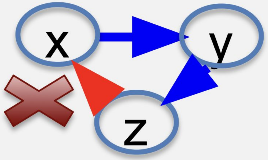
$$p(x, y, z) = p(x)\, p(y \mid x)\, p(z \mid y)$$



$$p(a, b, c, d) = p(a)\, p(b \mid a)\, p(c \mid a, b) p(d \mid b)$$



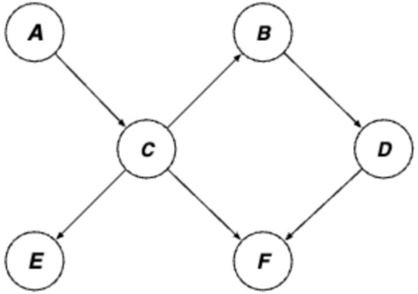
*Graph must be **acyclic***



Corresponds to an order over the variables (chain rule)

Paths in DAGs

- Definition
  A **path** is any sequence of nodes connected by edges (regardless of their directionalities); it is also assumed that no nodes repeat.
- Examples



ACBD ✅
ACFD ✅
ACEC ❌
CBDFC ❌

D-separation in DAGs

- Motivation

  How is conditional independence in a BN encoded by the structure of its DAG?

- Theorem

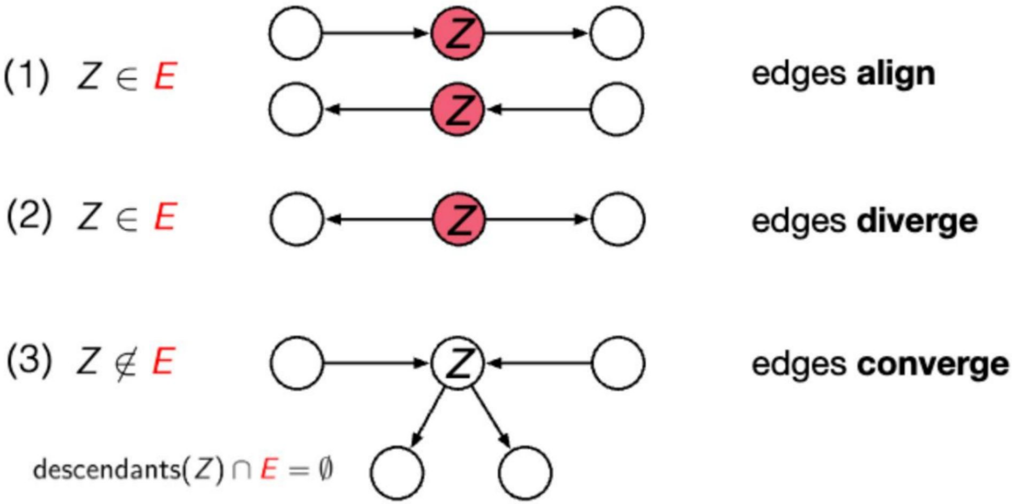
  $P(X,Y \mid E) = P(X \mid E)\, P(Y \mid E)$

  if and only if every path from a node in X to a node in Y is blocked by E.

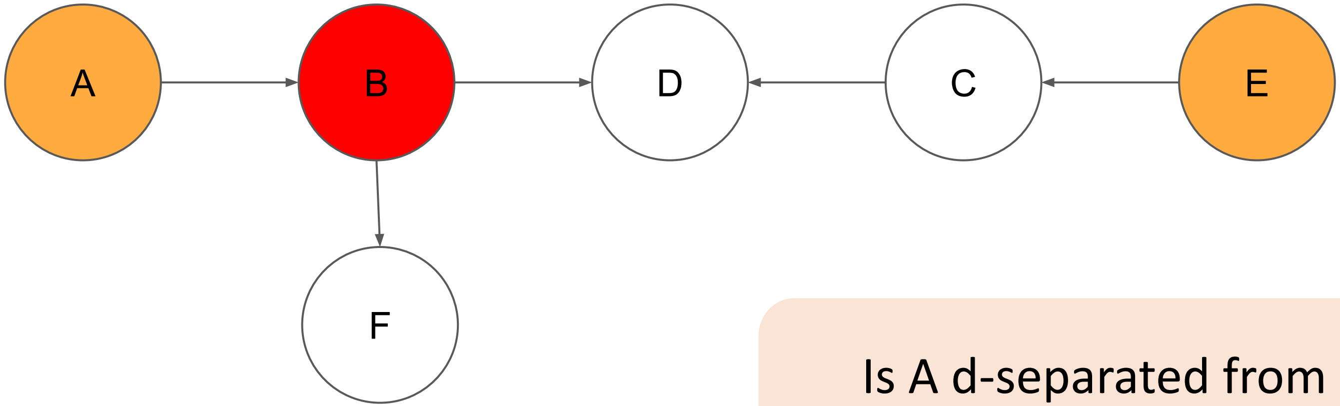
What counts as a path, and when is it blocked?

Blocked Paths

- Definition
  A path π is blocked by a set of nodes E if there exists a node $Z \in \pi$ for which one of the three following conditions hold.

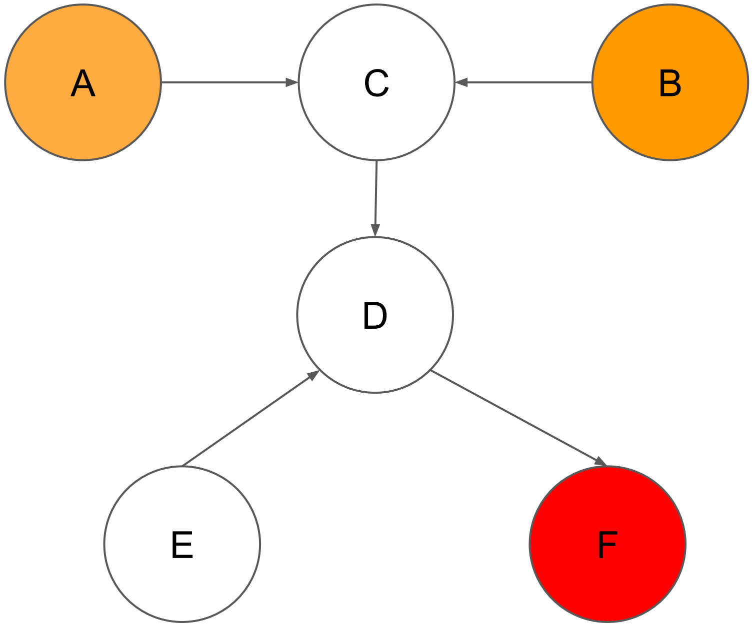


D-separation: Exercise 1



Is A d-separated from E given B?

D-separation: Exercise 2



Is A d-separated from B given F?

Colab Notebooks Time!

[Intro to NetworkX]

[Probabilities notebook]

Thank you!