



# Typeclasses

## Overloading Operators: Arithmetic

The **+** operator works for a bunch of different types.

For Integer :

```
λ> 2 + 3
```

```
5
```

for Double precision floats:

```
λ> 2.9 + 3.5
```

```
6.4
```

## Overloading Comparisons

Similarly we can *compare* different types of values

$\lambda > 2 == 3$

False

*ea int [Double]*

$\lambda > [2.9, 3.5] == [2.9, 3.5]$

True

$\lambda > ("cat", 10) < ("cat", 2)$

False

*ord (String, Int)*

$\lambda > ("cat", 10) < ("cat", 20)$

True

*"special case"*

*Ad-Hoc Overloading*

Seems unremarkable?

$\begin{matrix} + \\ = \\ < \\ \vdots \end{matrix}$  } *make them work on MANY types*

Languages since the dawn of time have supported "operator overloading"

- To support this kind of **ad-hoc polymorphism**
- Ad-hoc: "created or done for a particular purpose as necessary."

You really **need** to *add* and *compare* values of *multiple* types!

## *Haskell has no caste system*

No distinction between **operators** and **functions**

- All are first class citizens!

But then, what type do we give to *functions* like `+` and `==` ?

## QUIZ

Which of the following would be appropriate types for `(+)` ?

- ☒ ~~(A)~~ `(+) :: Integer -> Integer -> Integer` }  $2.3 + 4.1$  would not check
- ☒ ~~(B)~~ `(+) :: Double -> Double -> Double` }  $1 + 2$
- ☒ ~~(C)~~ `(+) :: a -> a -> a` }  $\text{True} + \text{False}$
- $(\lambda x \rightarrow x) + (\lambda z \rightarrow z + 1)$

☒ (D) *All of the above*

☐ (E) *None of the above*

`Integer -> Integer -> Integer` is bad because?

- Then we cannot add `Double` s!

`Double -> Double -> Double` is bad because?

- Then we cannot add `Double` s!

$a \rightarrow a \rightarrow a$  is bad because?

- That doesn't make sense, e.g. to add two `Bool` or two `[Int]` or two functions!

## *Type Classes for Ad Hoc Polymorphism*

Haskell solves this problem with **typeclasses**

- Introduced by Wadler and Blott (<http://portal.acm.org/citation.cfm?id=75283>)

How to make *ad-hoc* polymorphism less *ad hoc*

Philip Wadler and Stephen Blott  
University of Glasgow\*

October 1988

**BTW:** The paper is one of the clearest examples of academic writing I have seen. The next time you hear a curmudgeon say all the best CS was done in the 60s or 70s just point them to the above.

## *Qualified Types*

To see the right type, lets ask:

$\lambda> \text{:type (+)}$

$(+) :: (\text{Num } a) \Rightarrow a \rightarrow a \rightarrow a$

We call the above a **qualified type**. Read it as +

- takes in two **a** values and returns an **a** value

for any type  $a$  that

- is a Num or
  - implements the Num interface or
  - is an instance of a Num.
- 

The name `Num` can be thought of as a *predicate* or *constraint* over types.

## Some types are *Num*s

Examples include `Integer`, `Double` etc

- Any such values of those types can be passed to `+`.

## *Other types are not **Num**s*

Examples include Char , String , functions etc,

- Values of those types *cannot* be passed to + .

```
λ> True + False
```

```
<interactive>:15:6:
```

```
  No instance for (Num Bool) arising from a use of '+'
```

```
  In the expression: True + False
```

```
  In an equation for 'it': it = True + False
```

**Aha!** Now those no **instance** for error messages should make sense!

- Haskell is complaining that True and False are of type Bool
- and that Bool is *not* an instance of Num .



## Type Class is a Set of Operations

A typeclass is a collection of operations (functions) that must exist for the underlying type.

- Similar but different to Java interfaces ([https://www.parsonsmatt.org/2017/01/07/how\\_do\\_type\\_classes\\_differ\\_from\\_interfaces.html](https://www.parsonsmatt.org/2017/01/07/how_do_type_classes_differ_from_interfaces.html))

*Num a*  
*(+)*  
*(-)*  
*from Integer ::*

## The **Eq** Type Class

The simplest typeclass is perhaps, Eq

```
class Eq a where  
  (==) :: a -> a -> Bool  
  (/=) :: a -> a -> Bool
```

A type `a` is *an instance of* `Eq` if there are two functions

- `==` and `/=`

That determine if two `a` values are respectively *equal* or *disequal*.

## *The Show Type Class*

The typeclass `Show` requires that instances be convertible to `String` (which can then be printed out)

```
class Show a where  
  show :: a -> String
```

Indeed, we can test this on different (built-in) types

```
λ> show 2  
"2"
```

```
λ> show 3.14  
"3.14"
```

```
λ> show (1, "two", ([],[],[]))  
"(1,\"two\",([],[],[]))"
```

(Hey, whats up with the funny \" ?)

## *Unshowable Types*

When we type an expression into `ghci`,

- it computes the value,
- then calls `show` on the result.

Thus, if we create a *new* type by

```
data Unshowable = A | B | C
```

and then create values of the type,

```
λ> let x = A
λ> :type x
x :: Unshowable
```

but then we **cannot view** them

```
λ> x
```

```
<interactive>:1:0:
```

```
  No instance for (Show Unshowable)
```

```
    arising from a use of `print' at <interactive>:1:0
```

```
  Possible fix: add an instance declaration for (Show Unshowabl
e)
```

```
    In a stmt of a 'do' expression: print it
```

and we **cannot compare** them!

```
λ> x == x
```

```
<interactive>:1:0:
```

```
  No instance for (Eq Unshowable)
```

```
    arising from a use of `==' at <interactive>:1:0-5
```

```
  Possible fix: add an instance declaration for (Eq Unshowable)
```

```
    In the expression: x == x
```

```
    In the definition of `it': it = x == x
```

Again, the previously incomprehensible type error message should make sense to you.

## Creating Instances

Tell Haskell how to show or compare values of type Unshowable

By **creating instances** of Eq and Show for that type:

```
instance Eq Unshowable where
```

```
  (==) A A = True           -- True if both inputs are A
```

```
  (==) B B = True           -- ...or B
```

```
  (==) C C = True           -- .. or C
```

```
  (==) _ _ = False         -- otherwise
```

```
  (/=) x y = not (x == y)  -- Test if `x == y` and negate result!
```

## EXERCISE

Lets create an **instance** for Show Unshowable

When you are done we should get the following behavior

```
>>> x = [A, B, C]
[A, B, C]
```

## *Automatic Derivation*

We *should* be able to compare and view `Unshowble` **automatically**"

Haskell lets us *automatically derive* implementations for some standard classes

```
data Showable = A' | B' | C'
  deriving (Eq, Show) -- tells Haskell to automatically generate instances
```

Now we have

```
λ> let x' = A'
```

```
λ> :type x'
```

```
x' :: Showable
```

```
λ> x'
```

```
A'
```

```
λ> x' == x'
```

```
True
```

```
λ> x' == B'
```

```
False
```

## *The Num typeclass*

Let us now peruse the definition of the Num typeclass.

```
λ> :info Num
class (Eq a, Show a) => Num a where
  (+) :: a -> a -> a
  (*) :: a -> a -> a
  (-) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

A type *a* is an instance of (i.e. implements) `Num` if

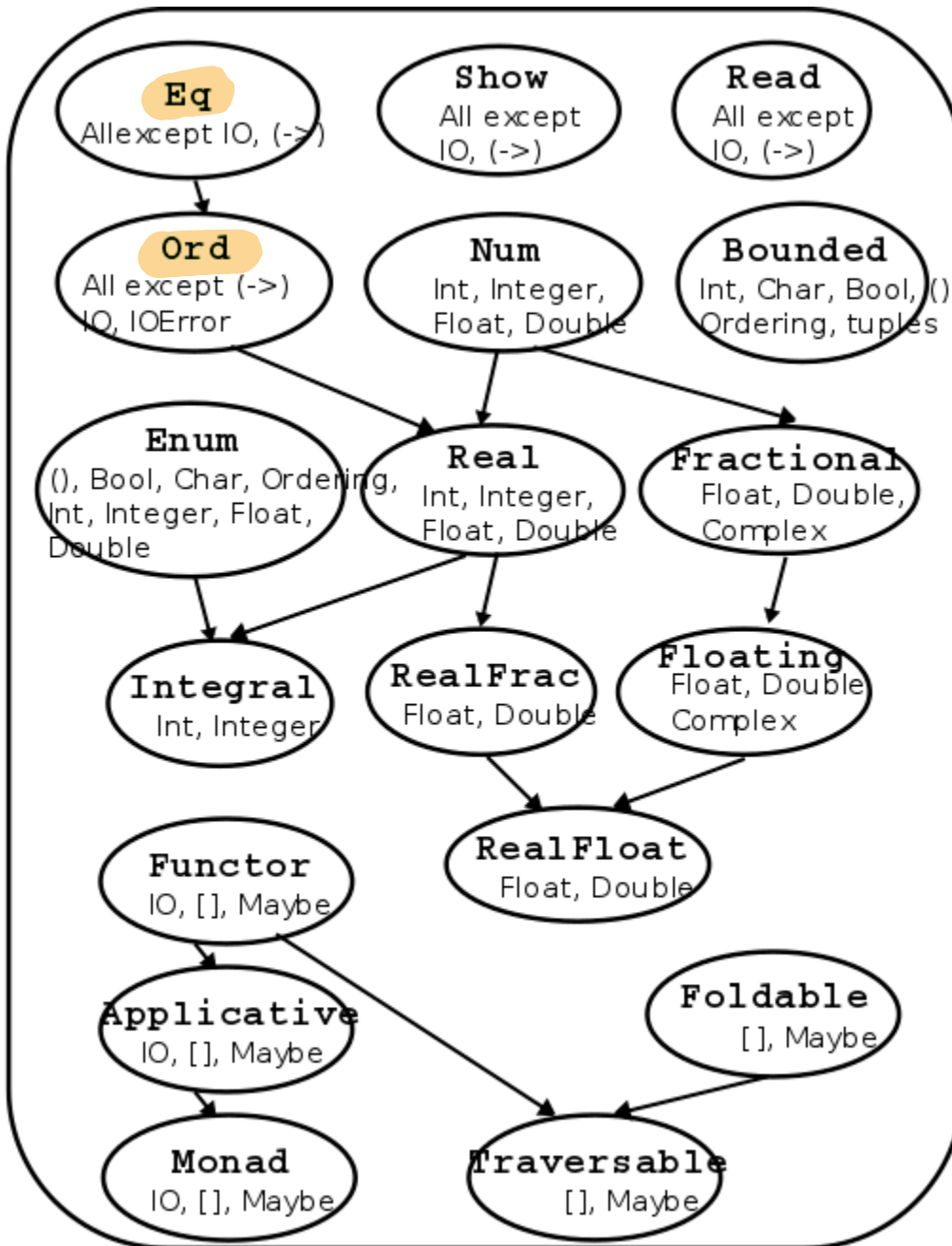
1. The type is *also* an instance of `Eq` and `Show`, and
2. There are functions to add, multiply, etc. values **of that type**.

That is, we can do *comparisons* and *arithmetic* on the values.

## *Standard Typeclass Hierarchy*



Haskell comes equipped with a rich set of built-in classes.



### Standard Typeclass Hierarchy

In the above picture, there is an edge from `Eq` and `Show` to `Num` because for something to be a `Num` it must also be an `Eq` and `Show`.

## *The Ord Typeclass*

Another typeclass you've used already is the one for ordering values:

```
λ> :info (<)
```

```
class Eq a => Ord a where
```

```
...
```

```
(<) :: a -> a -> Bool
```

$(\leq)$

```
...
```

For example:

```
λ> 2 < 3
```

```
True
```

```
λ> "cat" < "dog"
```

```
True
```

# QUIZ

Recall the datatype:

```
data Showable = A' | B' | C' deriving (Eq, Show)
```

What is the result of:

```
λ> A' < B'
```

(A) True (B) False (C) Type error (D) Run-time exception

## Using Typeclasses

Typeclasses integrate with the rest of Haskell's type system.

Lets build a small library for *Environments* mapping keys `k` to values `v`

```
data Table k v
  = Def v           -- default value `v` to be used for "mis
sing" keys
  | Bind k v (Table k v) -- bind key `k` to the value `v`
deriving (Show)
```

## QUIZ

What is the type of `keys`

```
keys (Def _)      = []  
keys (Bind k _ rest) = k : keys rest
```

- A. `Table k v -> k`
- B. `Table k v -> [k]`
- C. `Table k v -> [(k, v)]`
- D. `Table k v -> [v]`
- E. `Table k v -> v`

## *An API for Table*

Lets write a small API for Table

```
-- >>> let env0 = set "cat" 10.0 (set "dog" 20.0 (Def 0))

-- >>> set "cat" env0
-- 10

-- >>> get "dog" env0
-- 20

-- >>> get "horse" env0
-- 0
```

Ok, lets implement!

```
-- | 'add key val env' returns a new env that additionally maps `key` to `val`
set :: k -> v -> Table k v -> Table k v
set key val env = ???

-- | 'get key env' returns the value of `key` and the "default" if
no value is found
get :: k -> Table k v -> v
get key env = ???
```

Oops, y u no check?

# Constraint Propagation

Lets *delete* the types of `set` and `get`

- to see what Haskell says their types are!

```
λ> :type get
```

```
get :: (Eq k) => k -> v -> Table k v -> Table k v
```

We can use *any* `k` value as a *key* – if `k` is an instance of i.e. “implements” the `Eq` typeclass.

How, did GHC figure this out?

- If you look at the code for `get` you’ll see that we check if two keys *are equal*!



get 2 1 → —  
20 → —  
30 → —  
35 → —

**HOMEWORK**

Write an optimized version of

- **set** that ensures the keys are in **increasing** order,
- **get** that gives up and returns the “default” the moment we see a key that's larger than the one we're looking for.
- (How) do you need to change the type of Table ?
- (How) do you need to change the types of get and set ?

## Explicit Signatures

Sometimes the use of type classes requires explicit annotations

- which affect the code's behavior