

# Garter

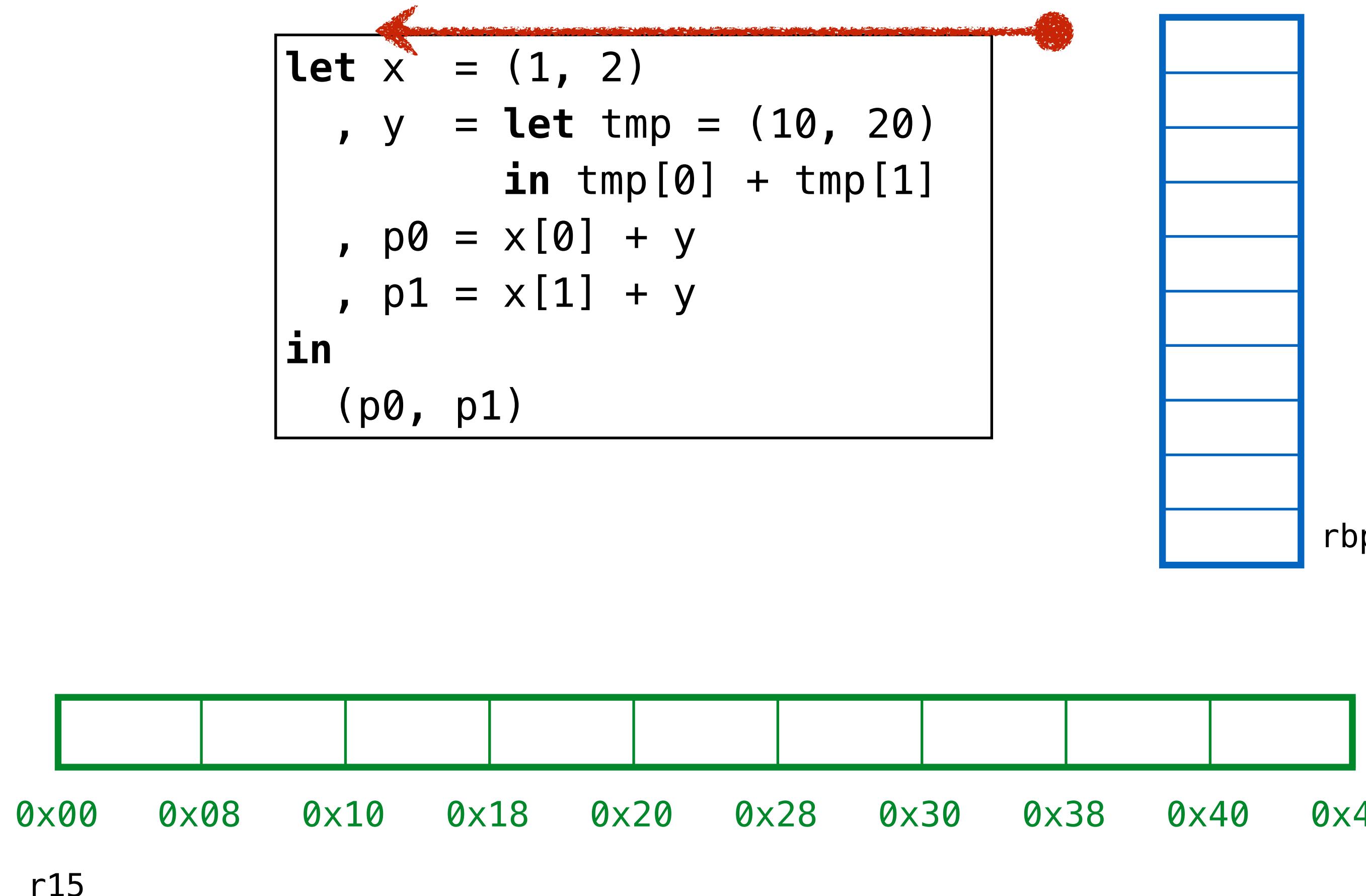
# Garbage Collection

Ranjit Jhala | UCSD

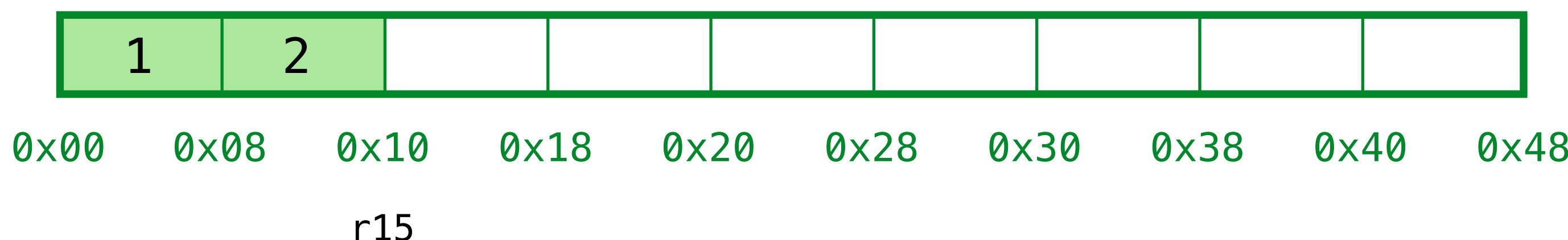
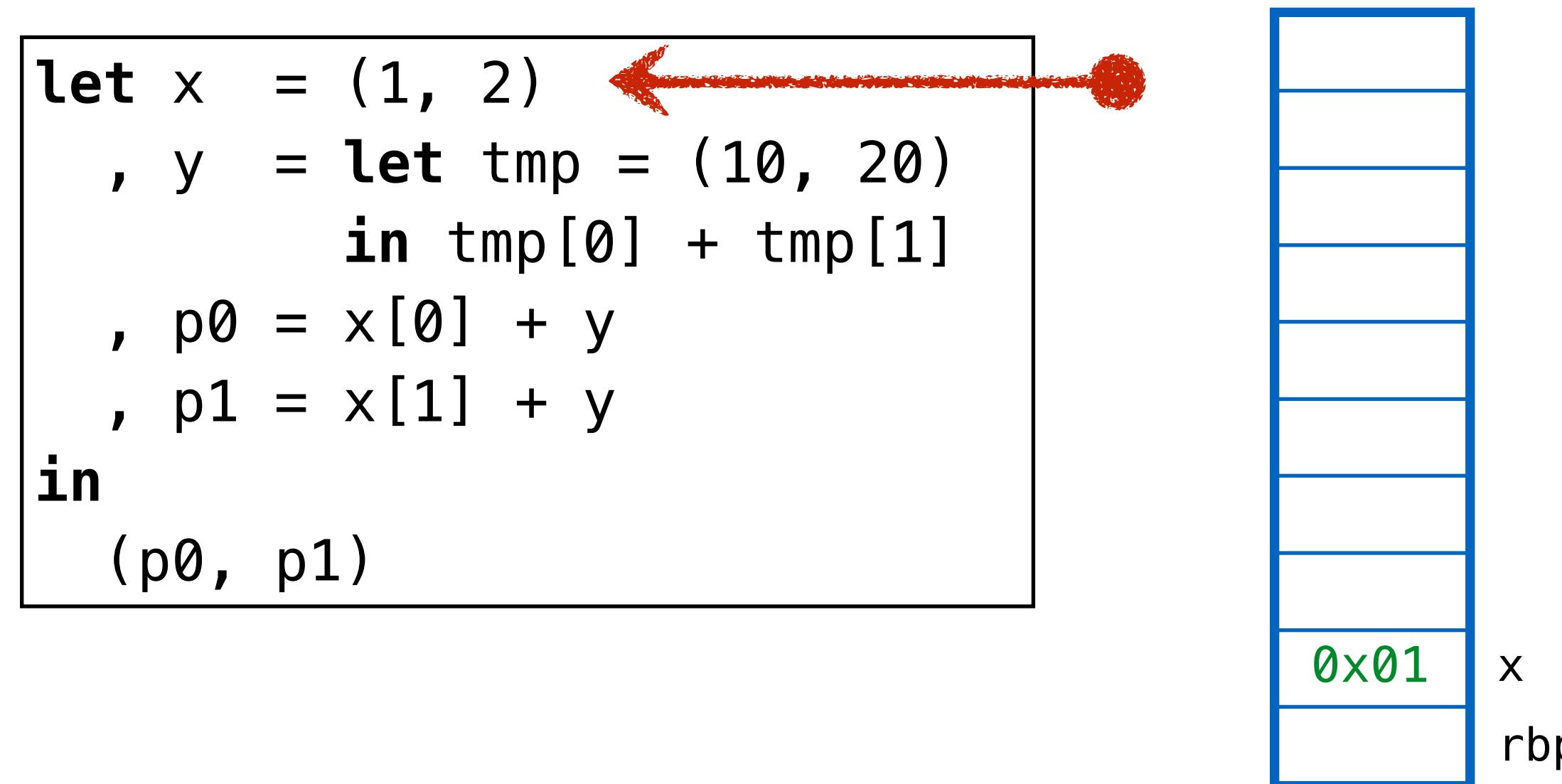
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Example 1

## ex1: garbage at end

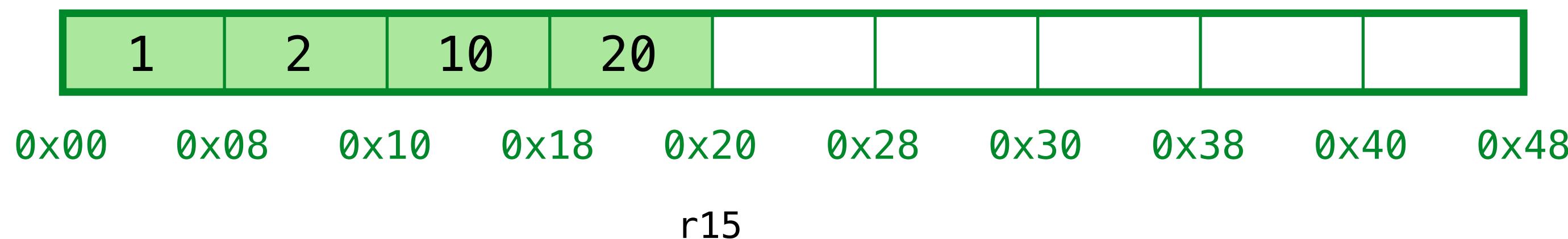
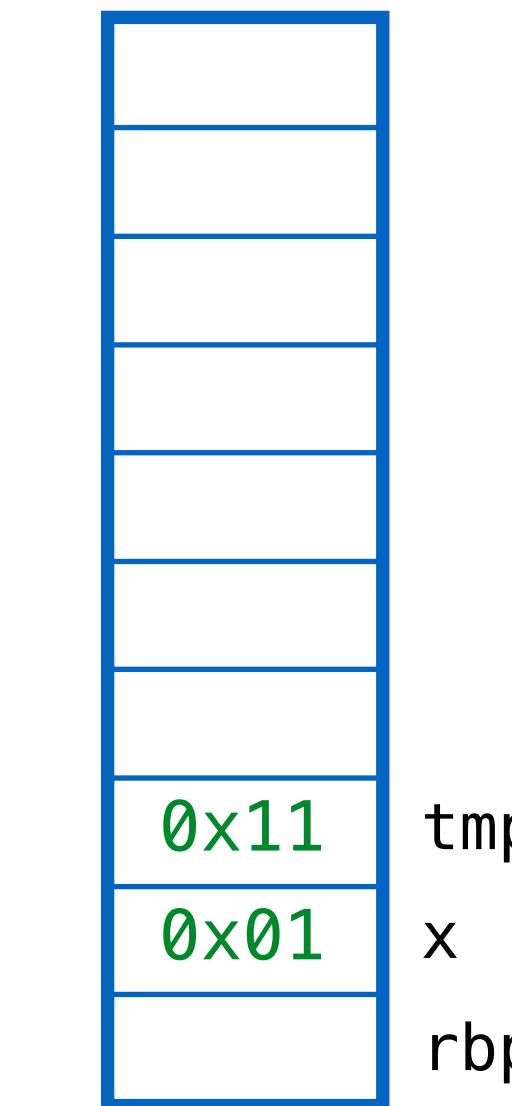


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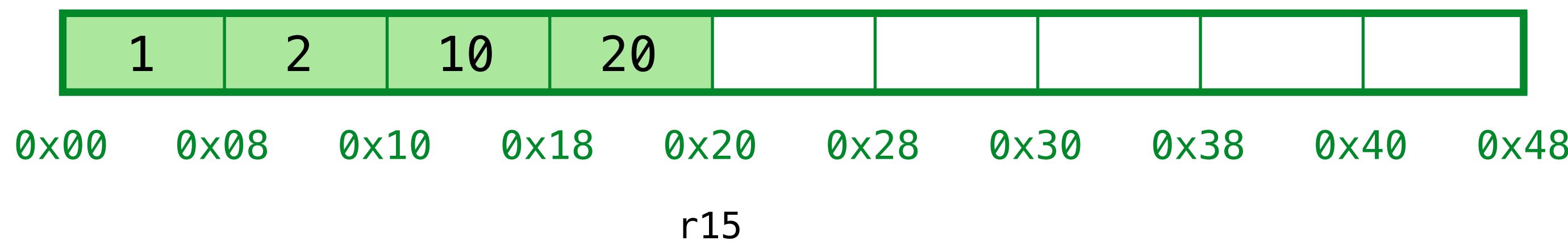
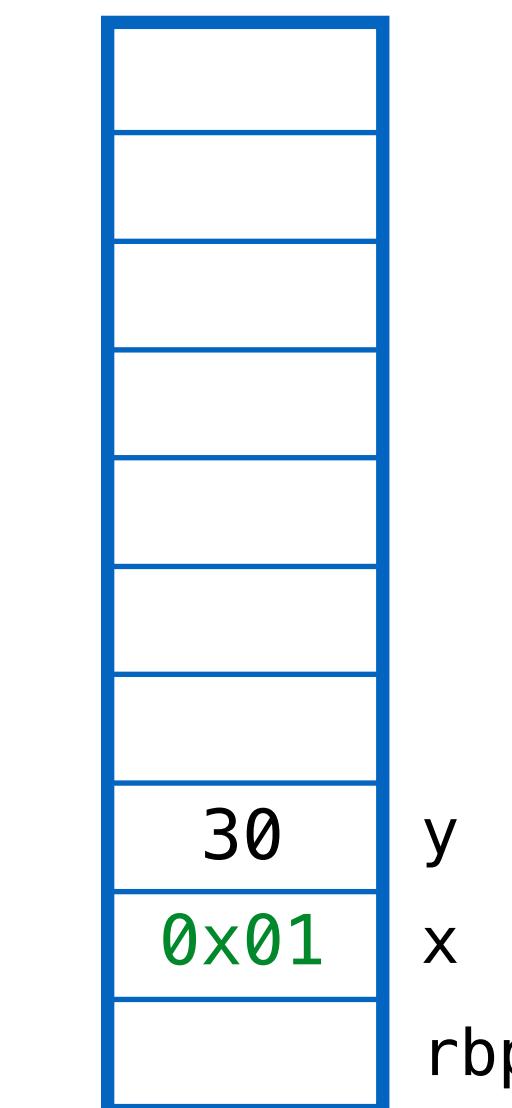
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let x  = (1, 2)
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in
  (p0, p1)
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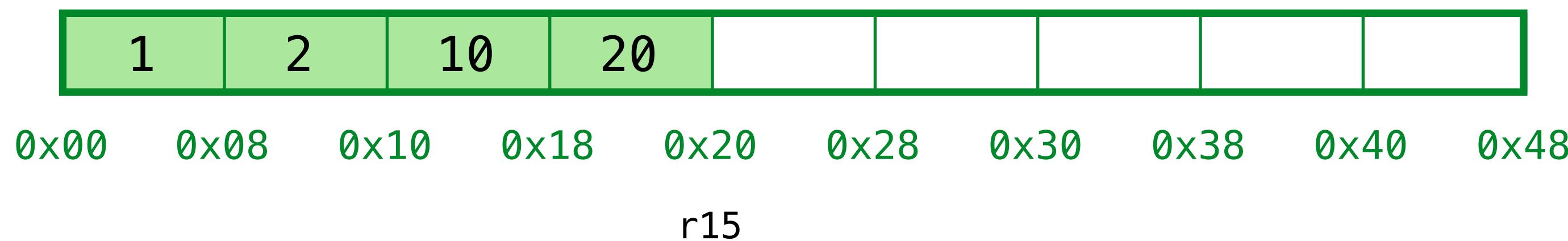
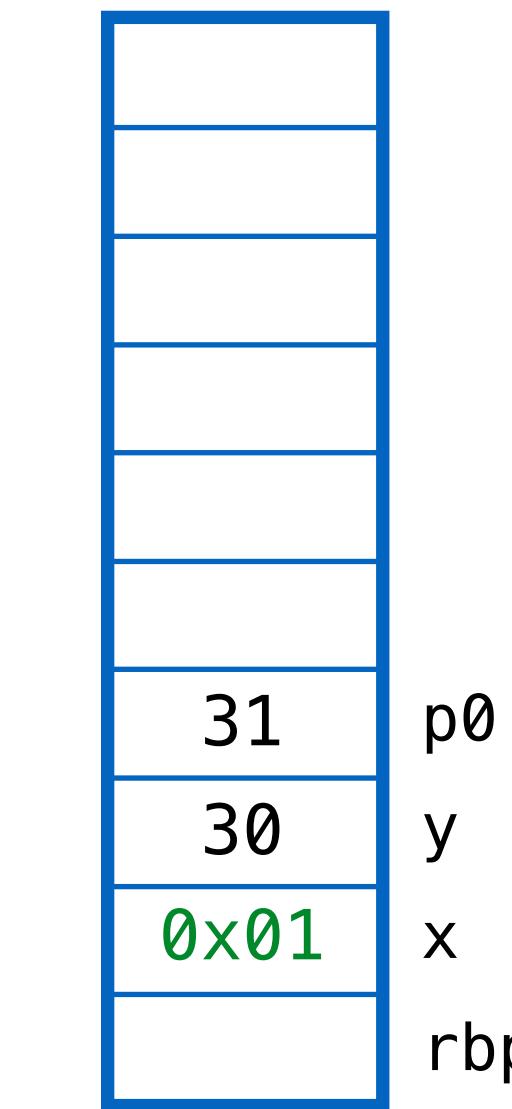
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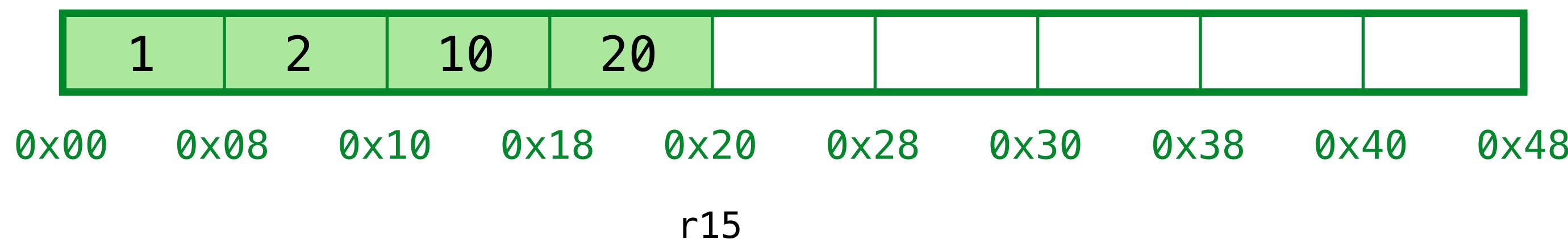
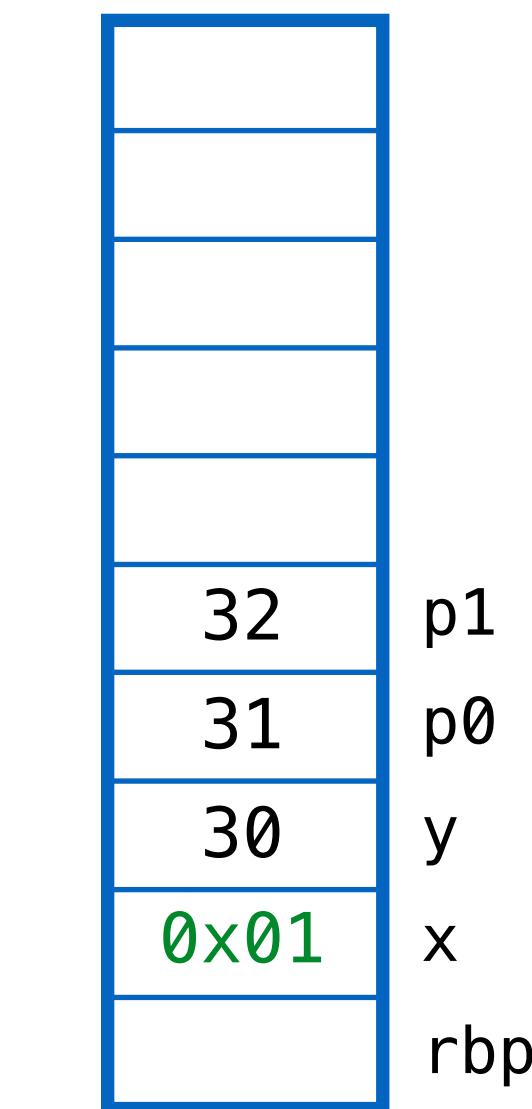
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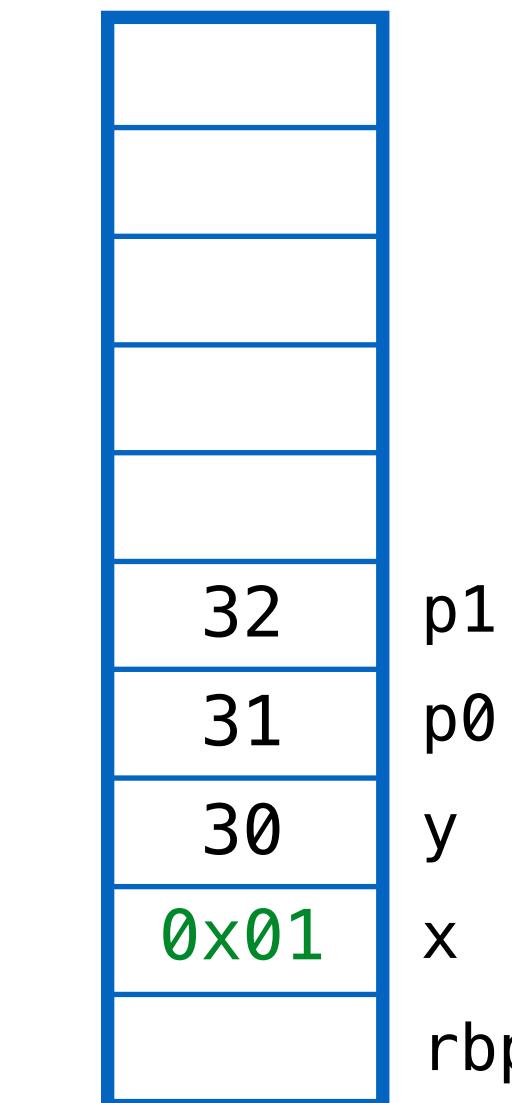
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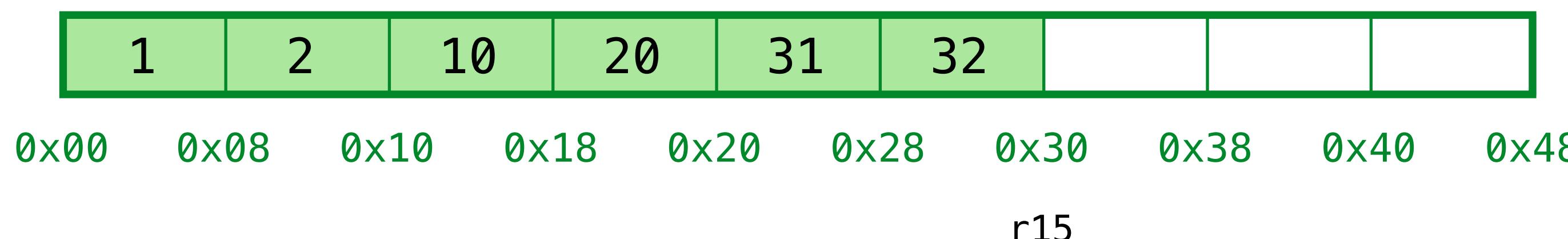


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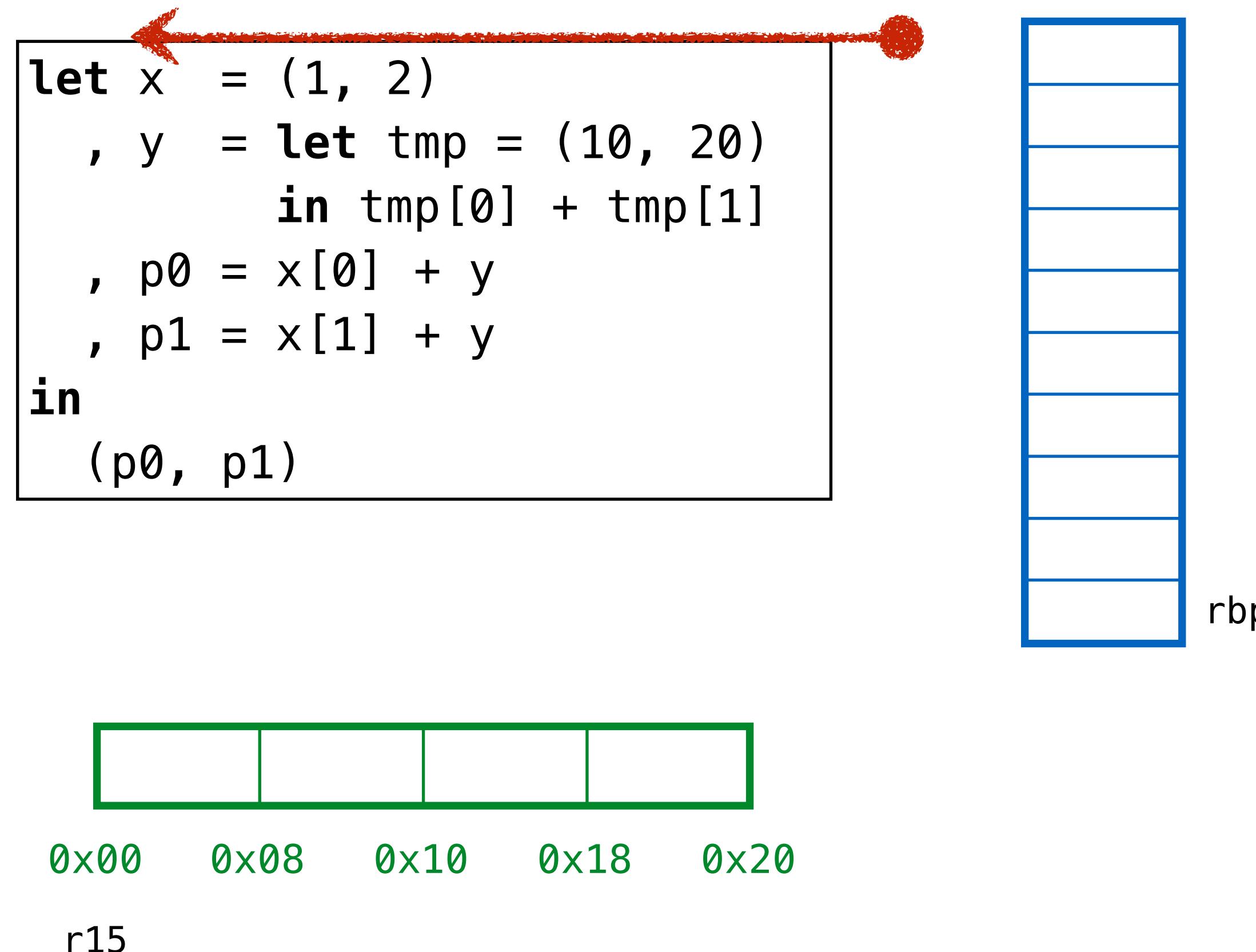
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  (p0, p1) ←
```



**Result (rax) = 0x21**



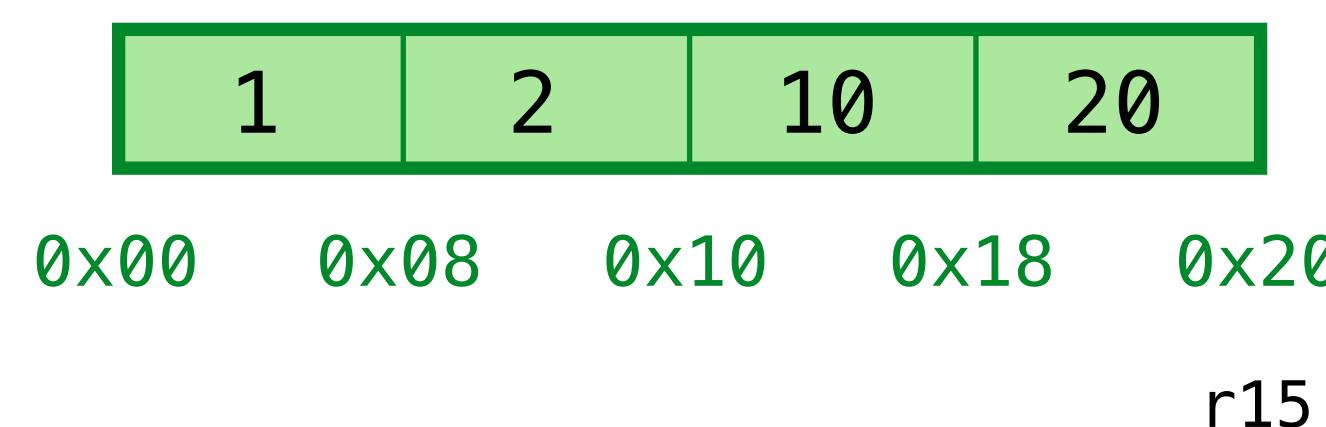
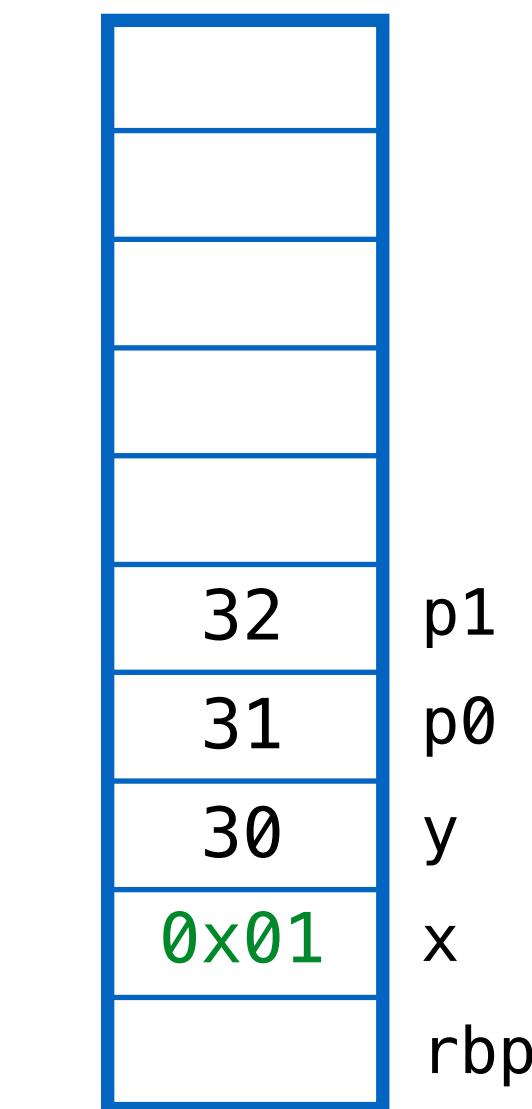
## ex1: garbage at end



Suppose we had a smaller, 4-word heap

## ex1: garbage at end

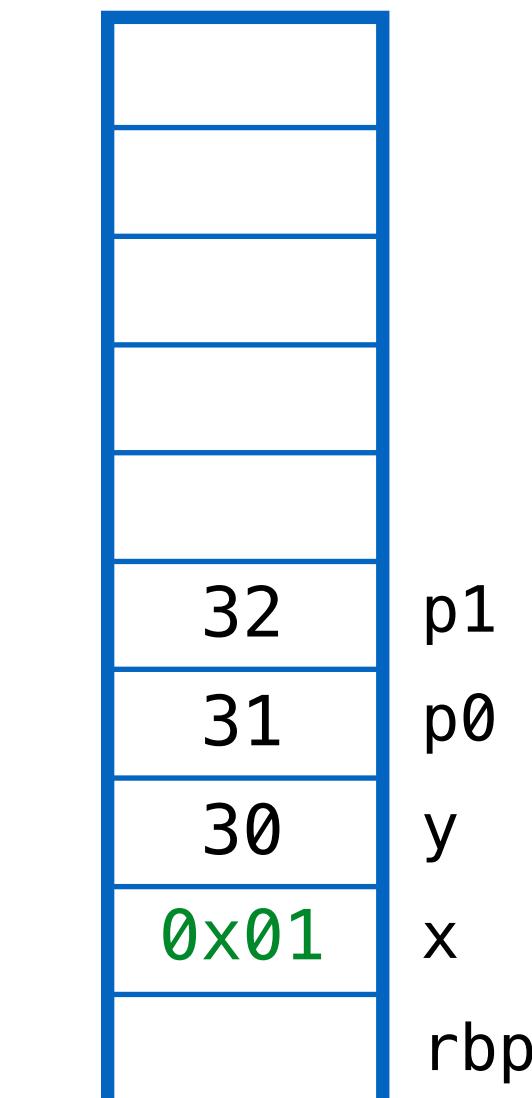
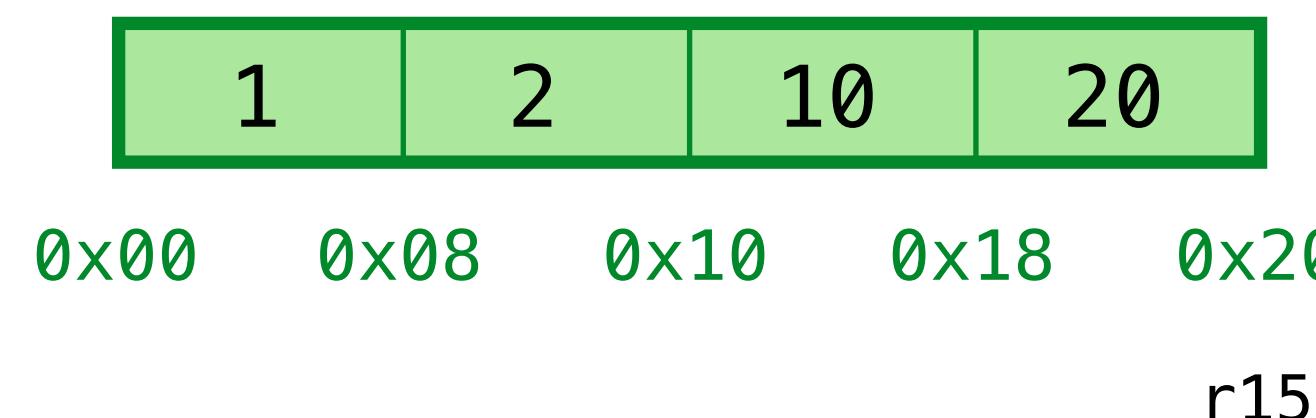
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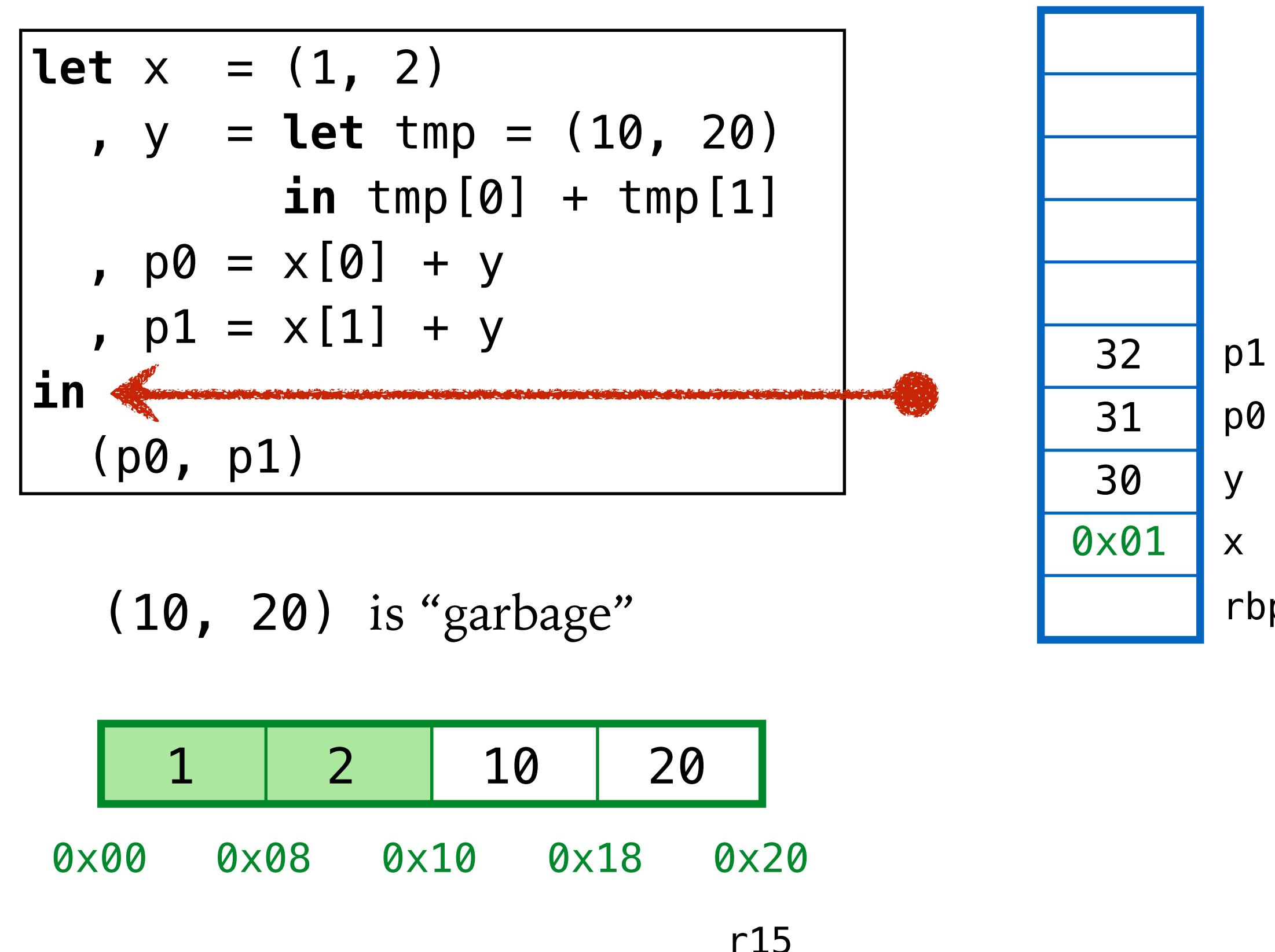
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Out of memory!  
Can't allocate (p0, p1)



## ex1: garbage at end

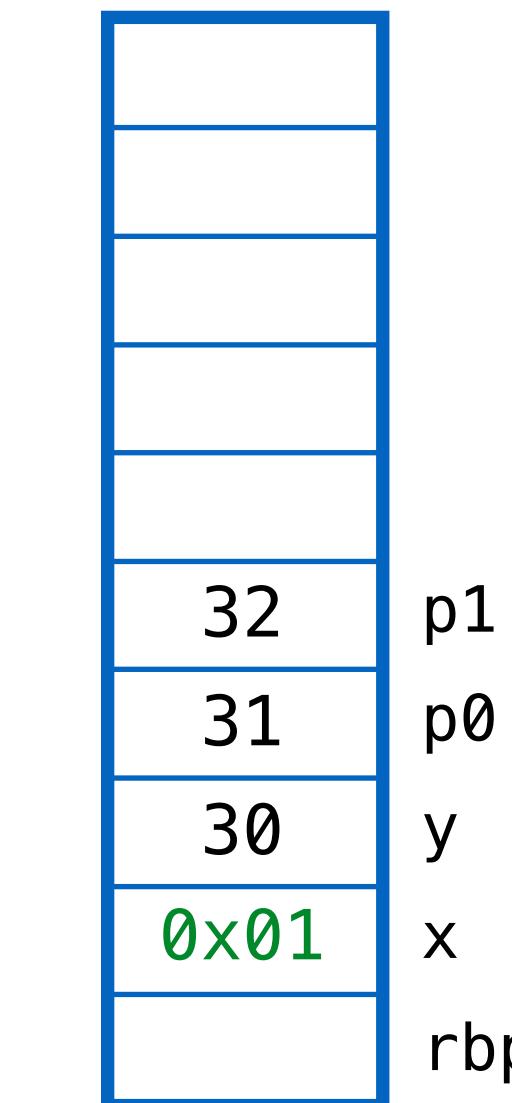


Q: How to determine if cell is garbage?

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    (p0, p1)
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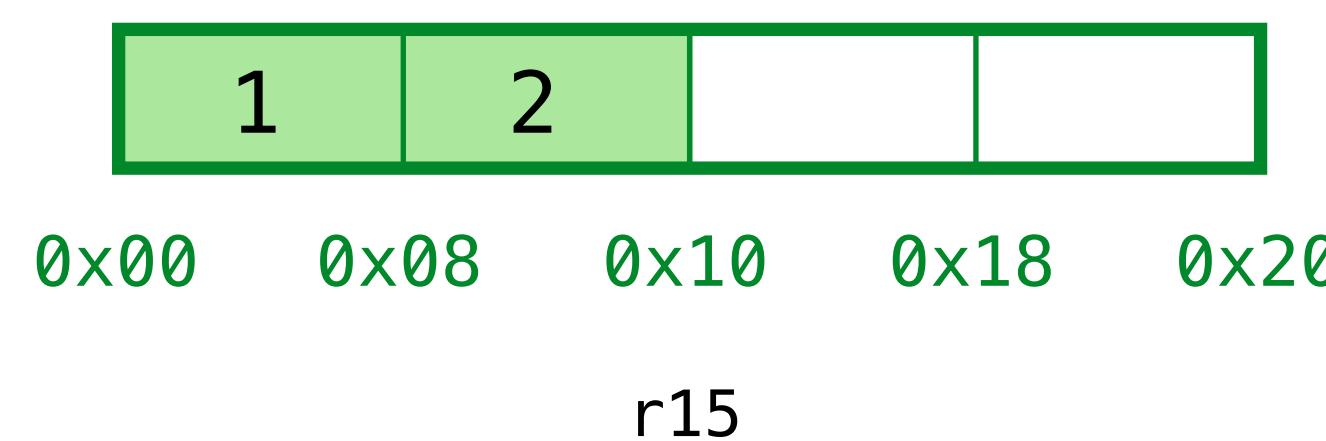
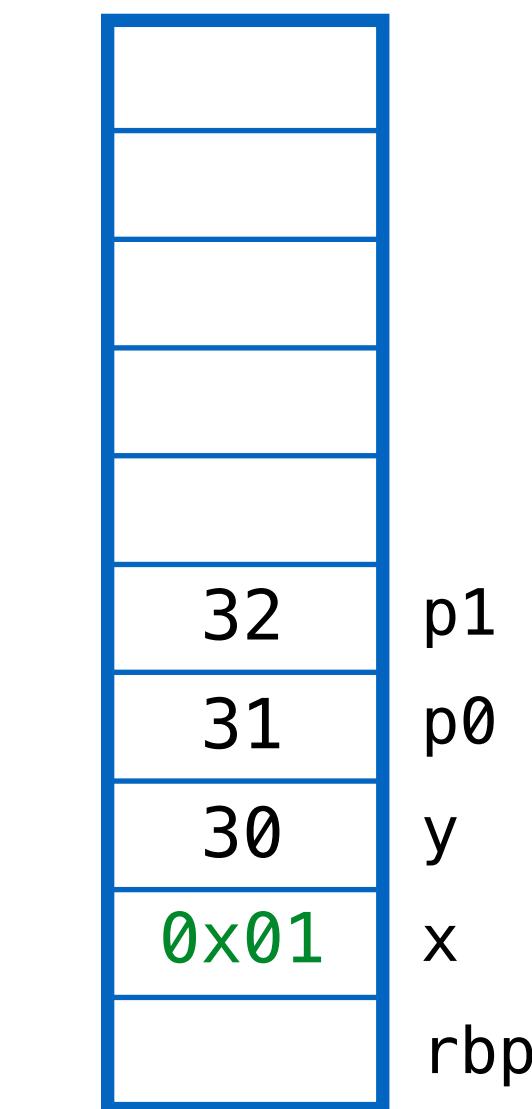
(10, 20) is “garbage”



r15

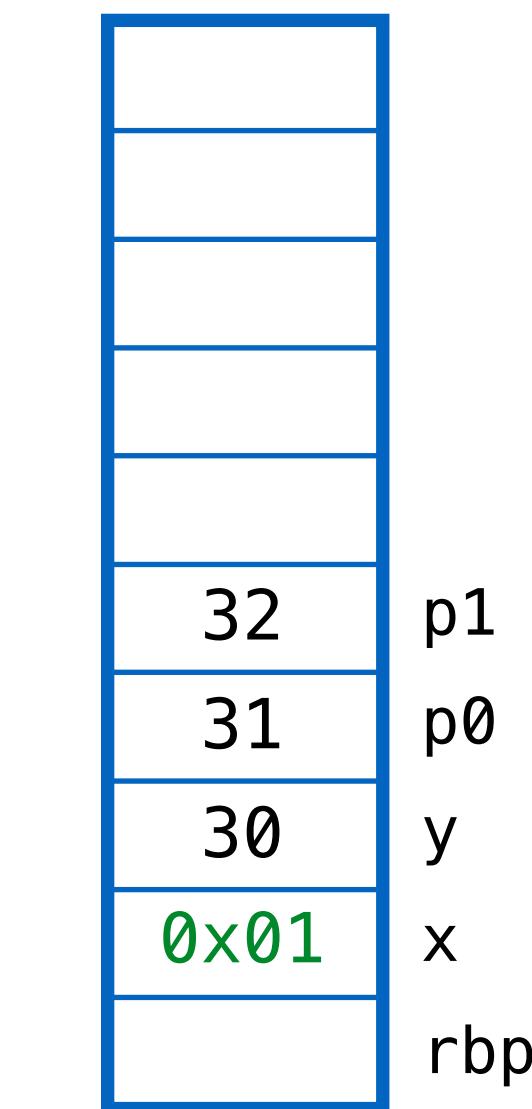
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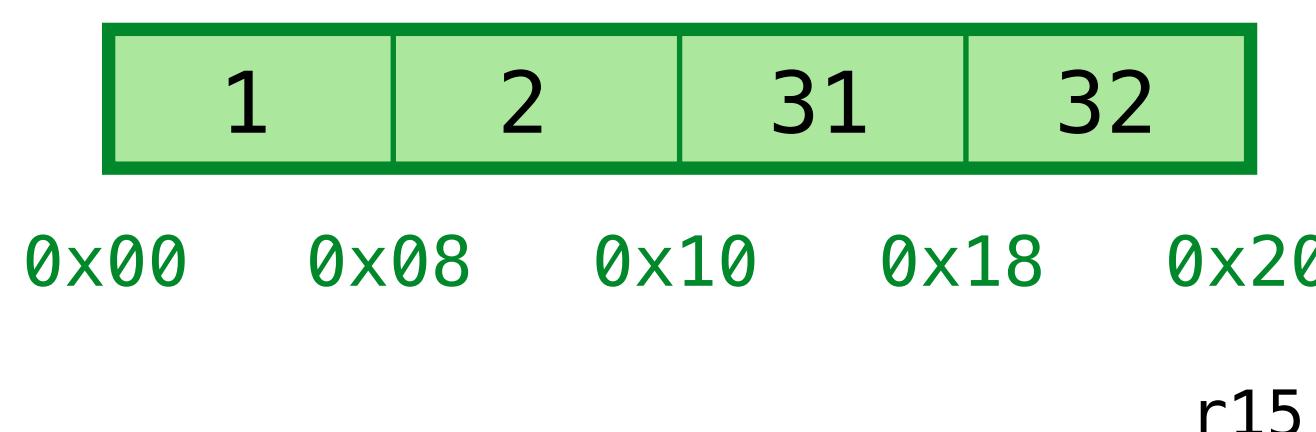


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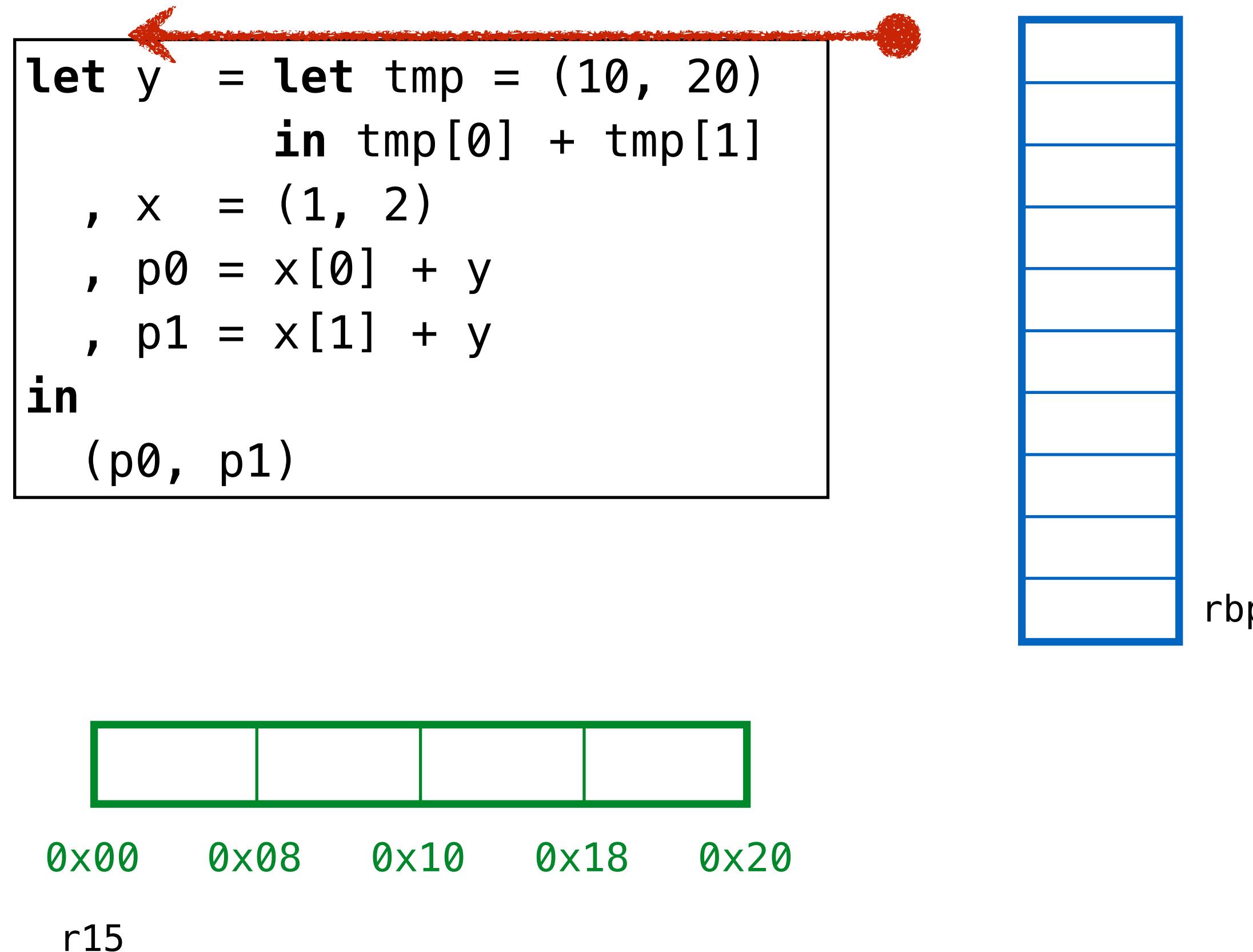
**Result (rax) = 0x11**



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Example 2

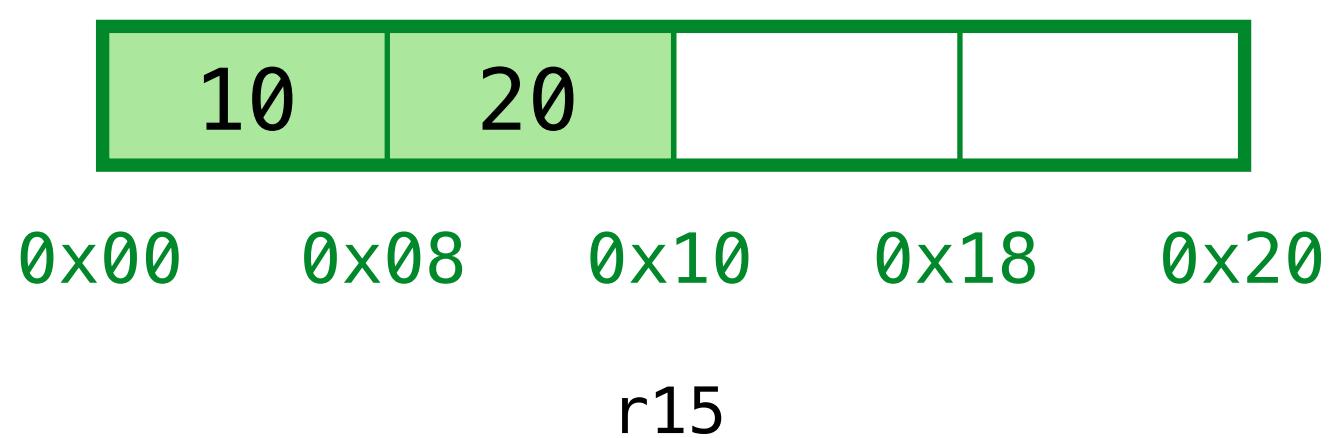
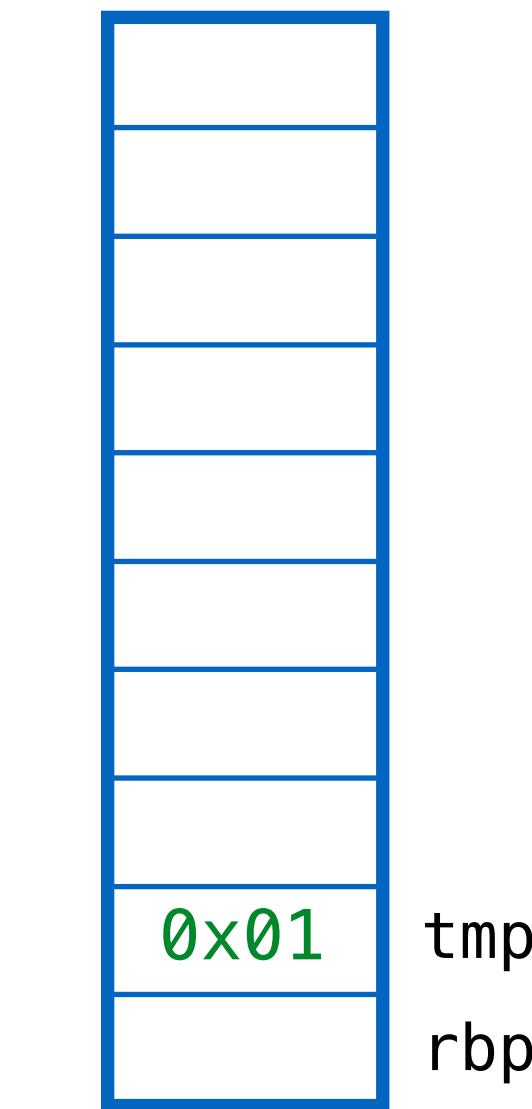
## ex2: garbage in the middle



Start with a 4-word heap

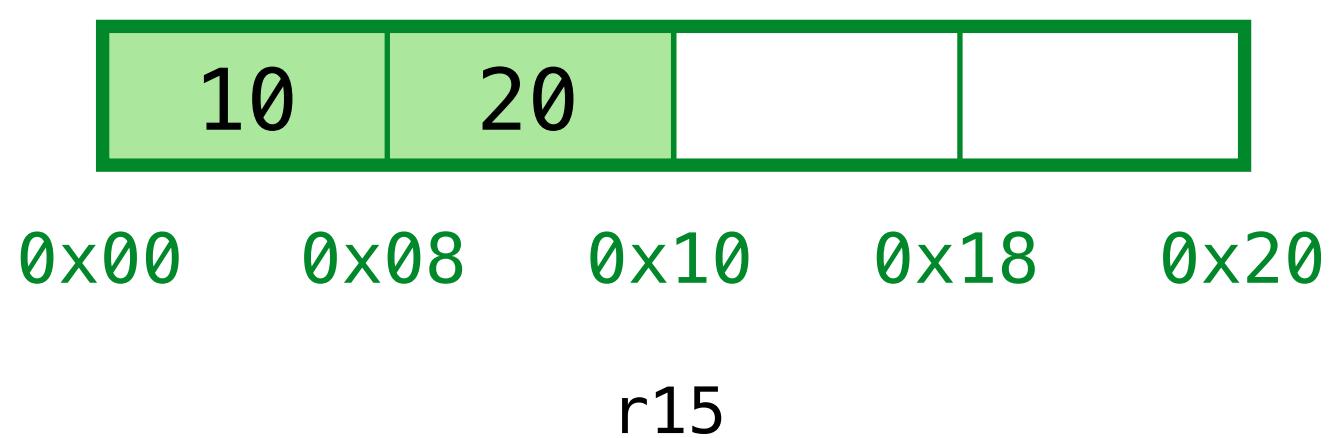
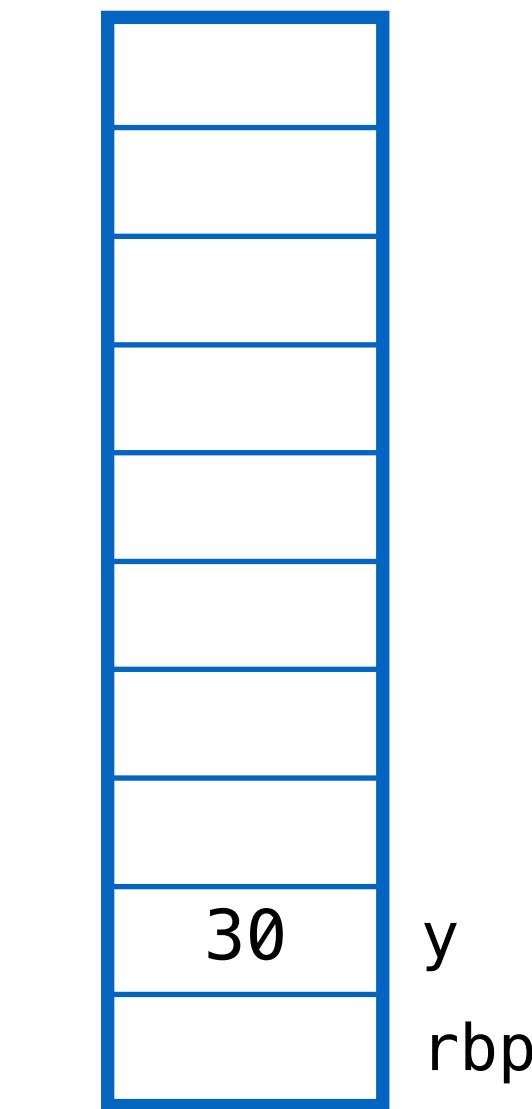
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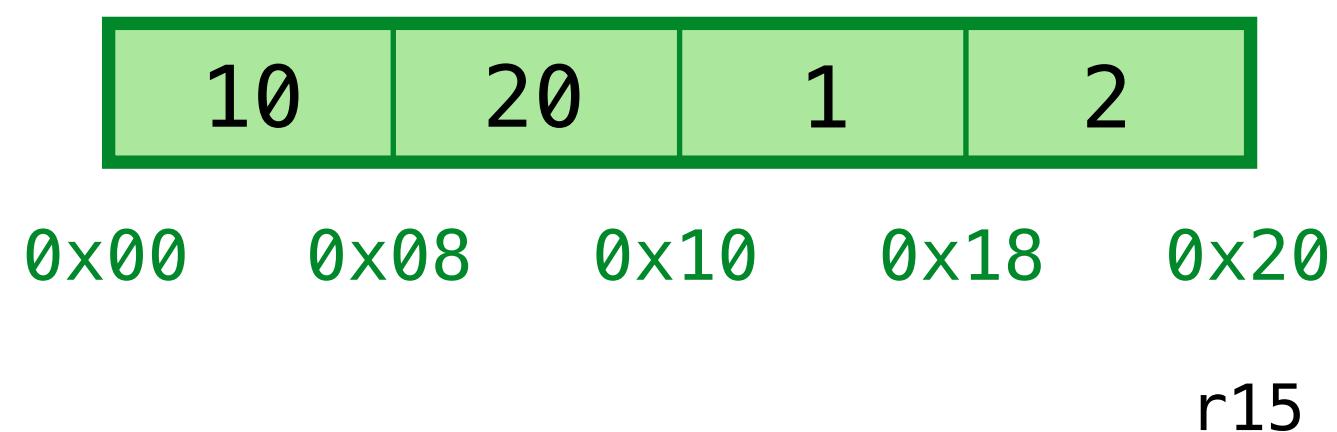
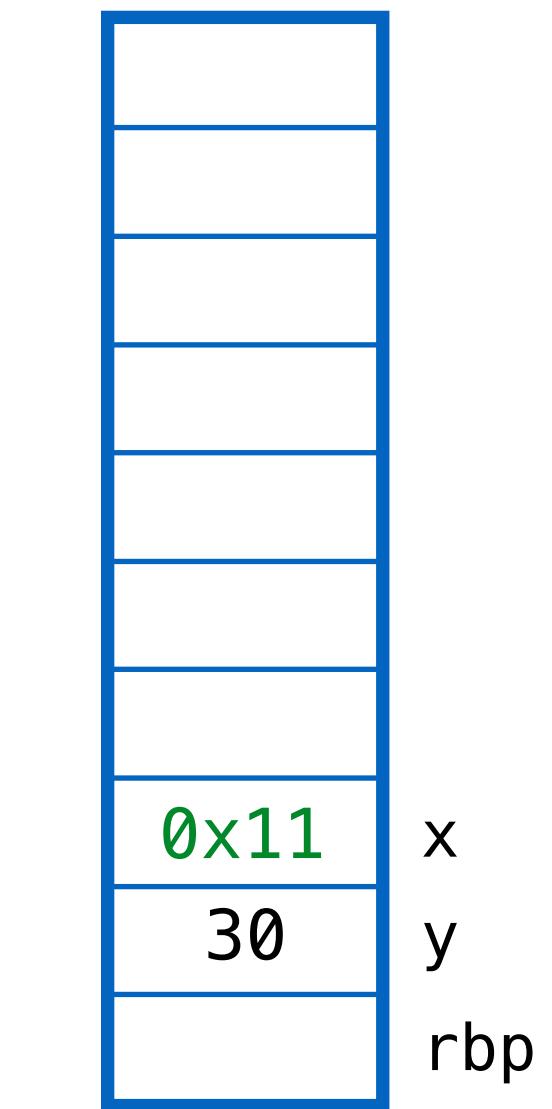
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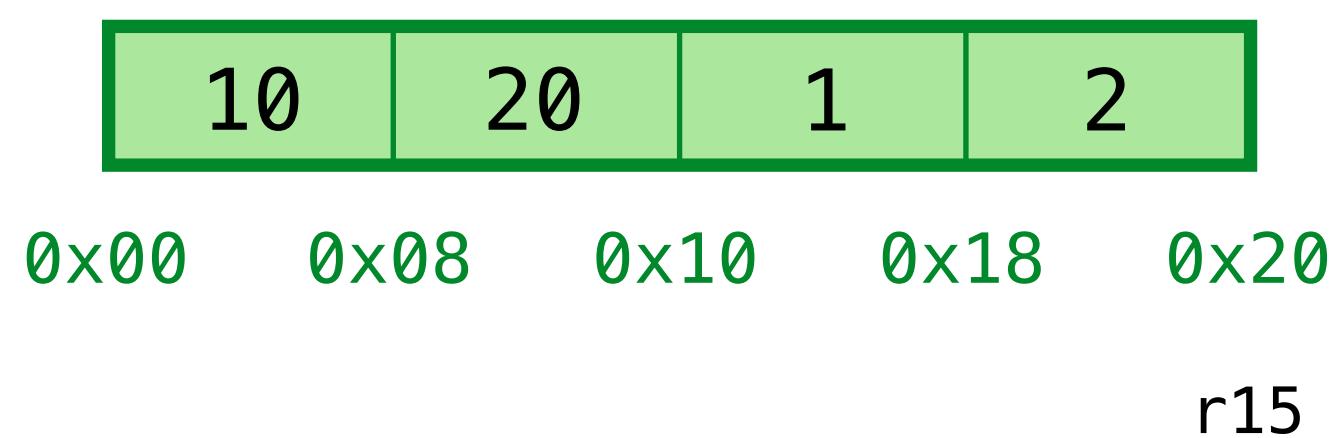
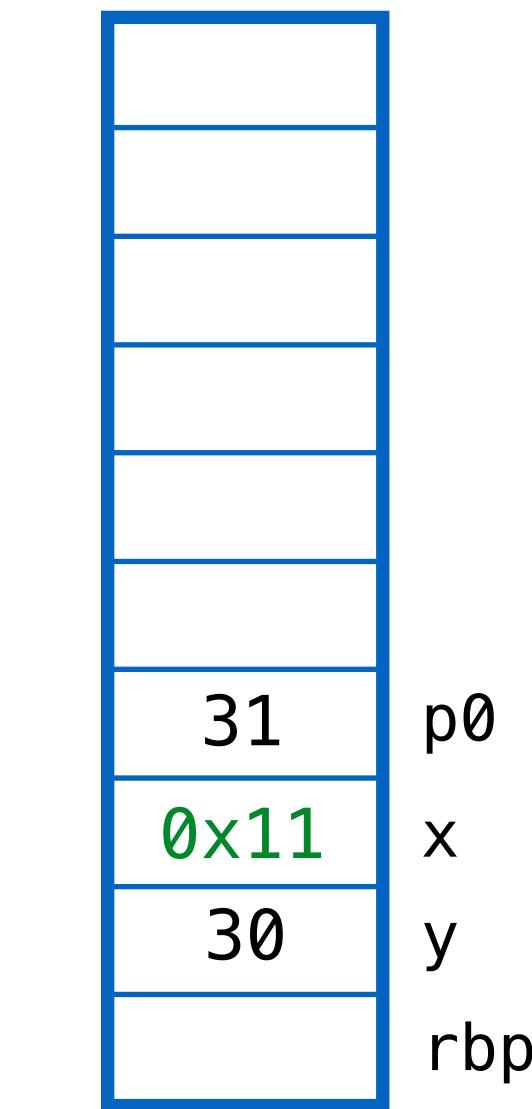
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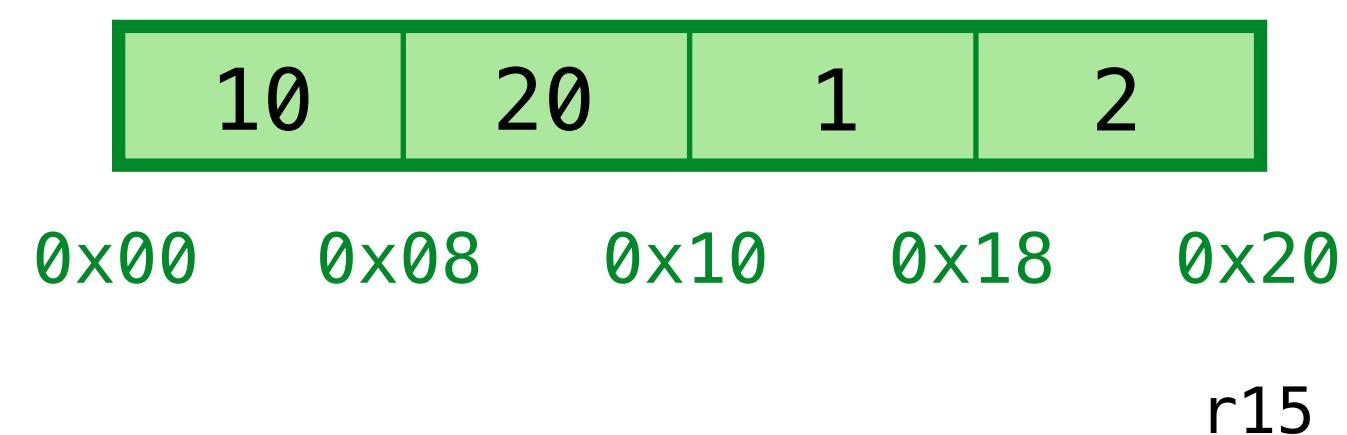
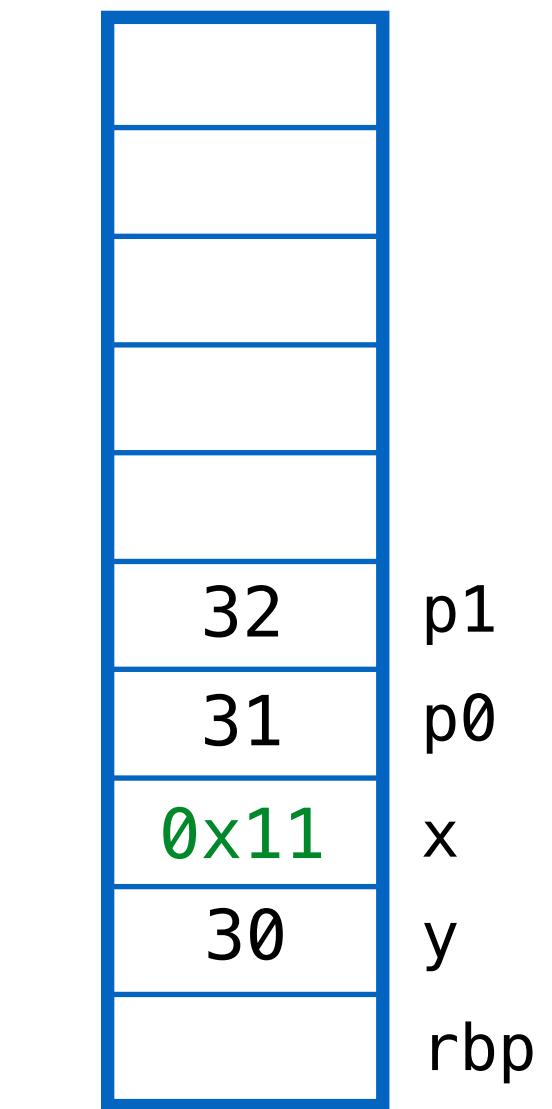
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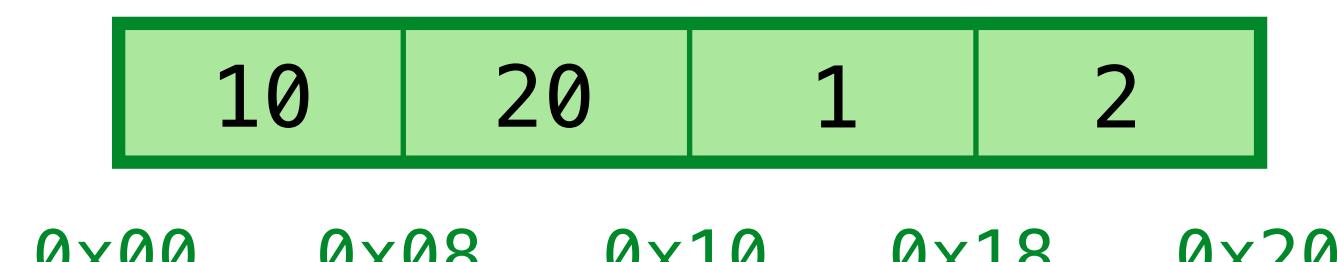
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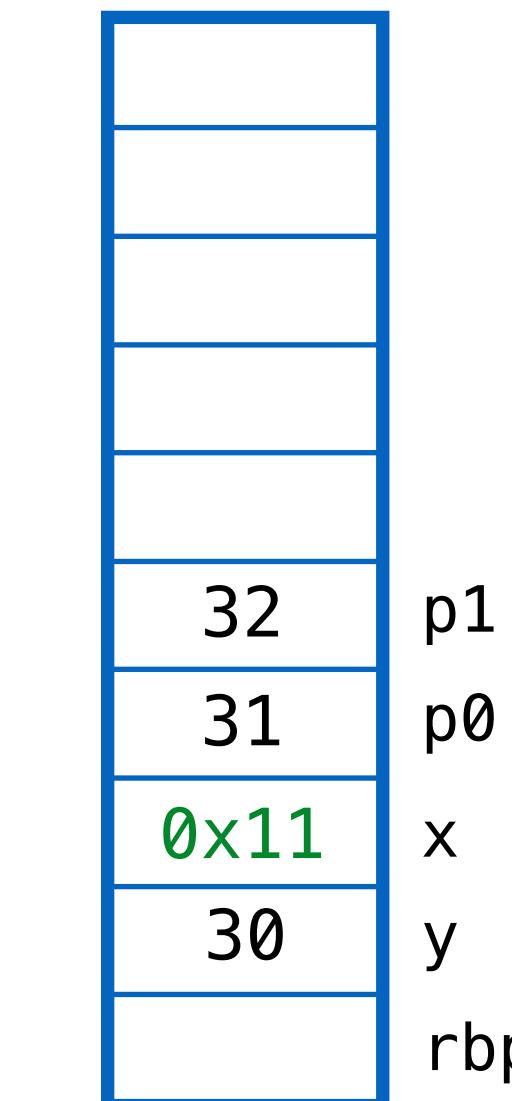
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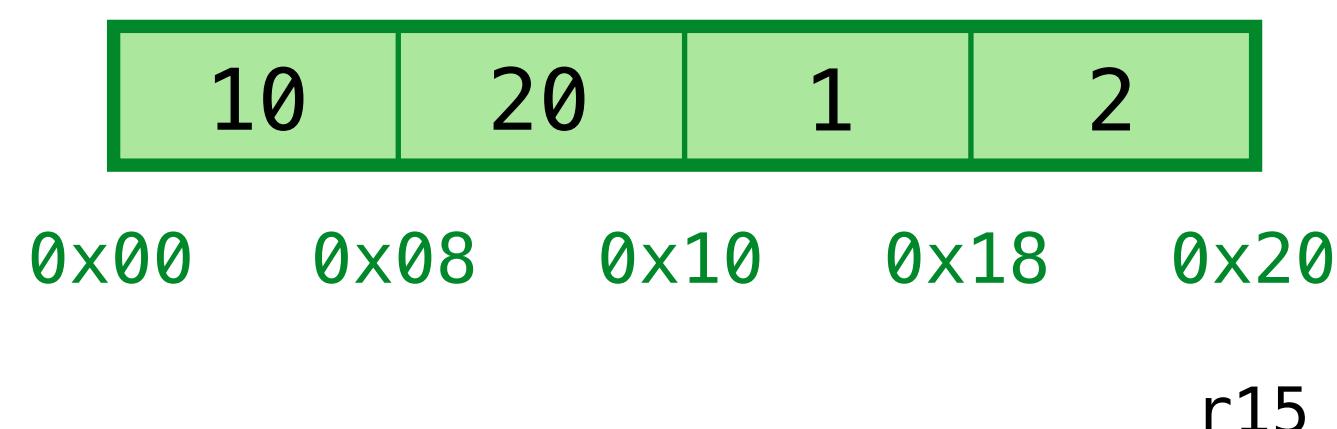
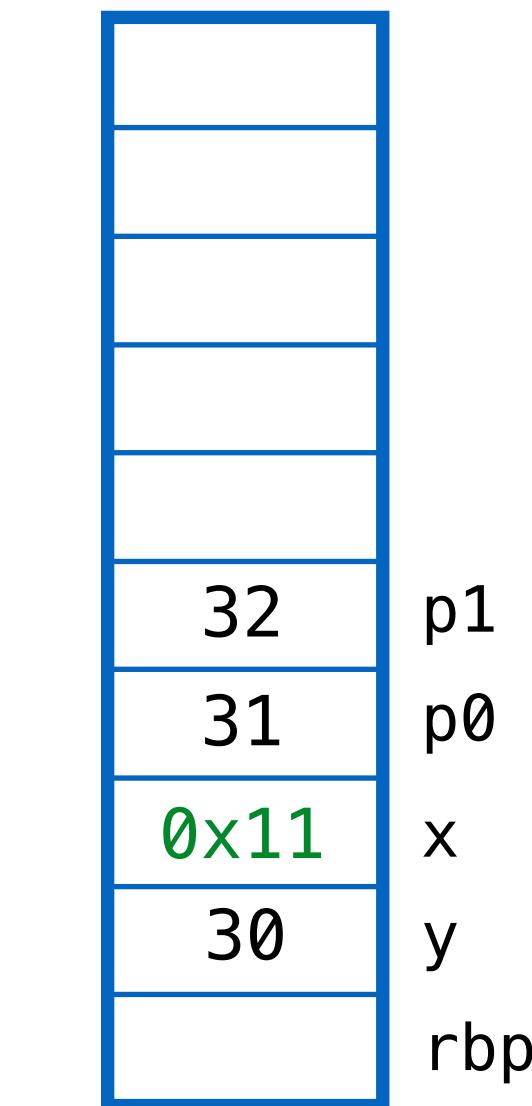
r15



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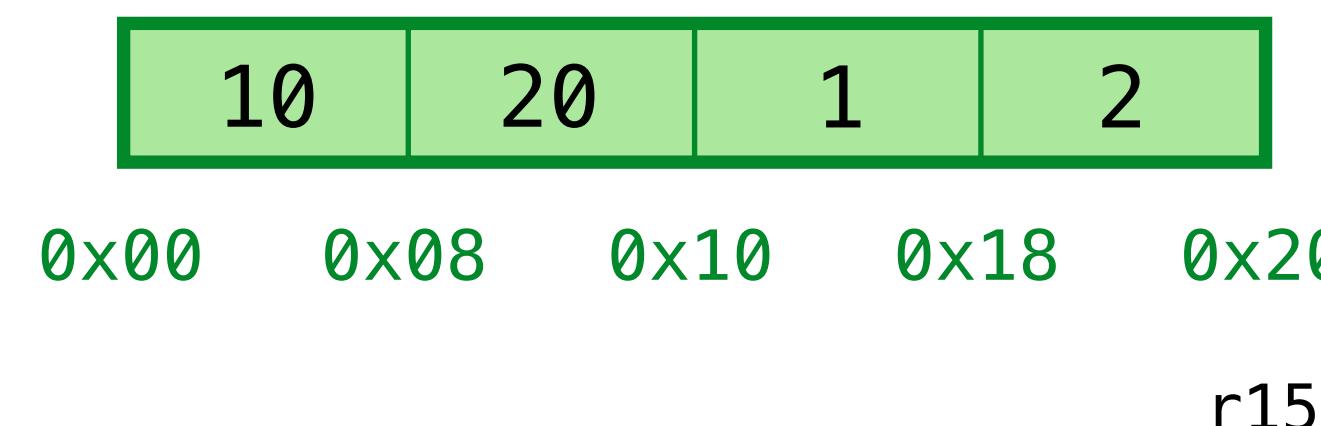
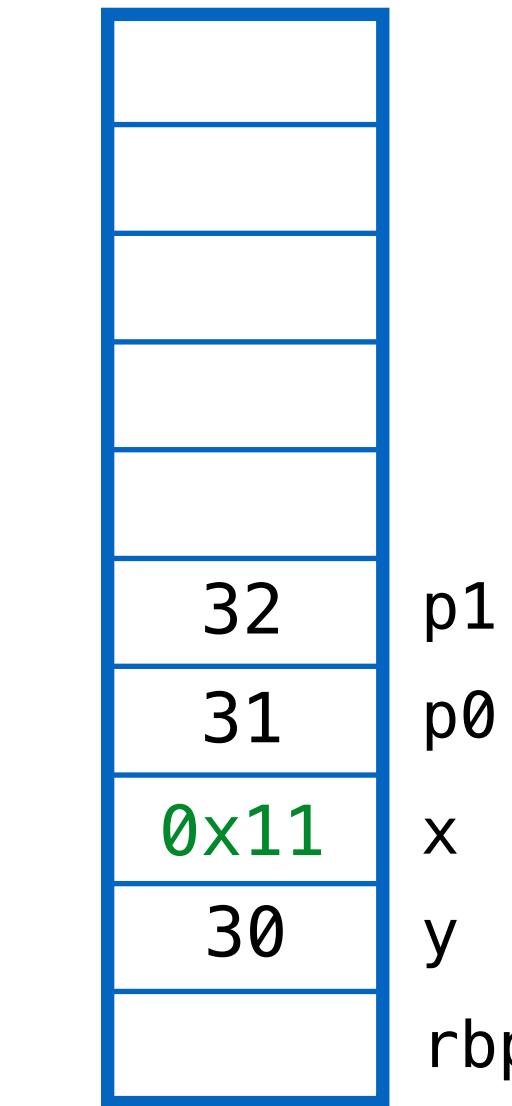
Lets reclaim & recycle garbage!



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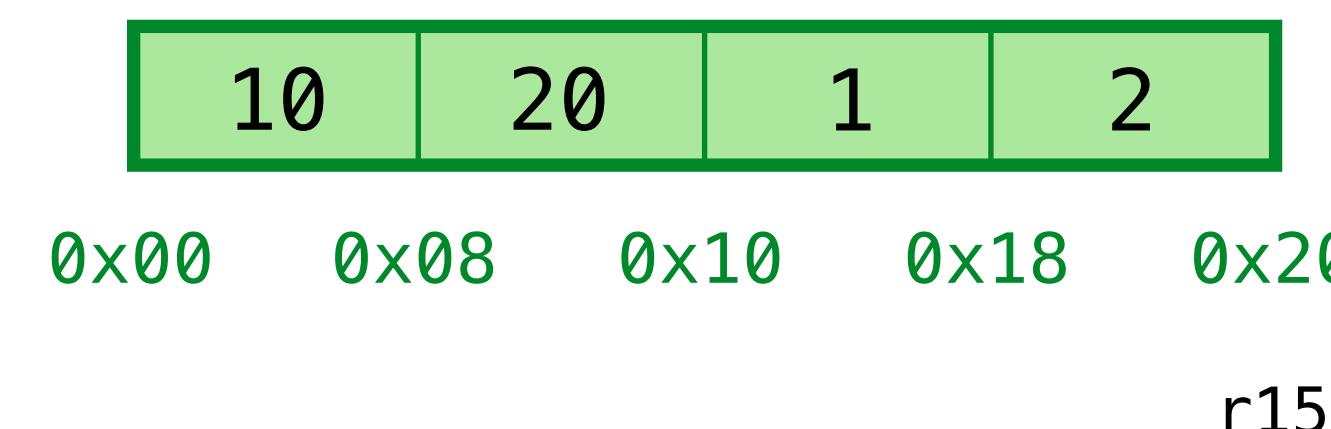
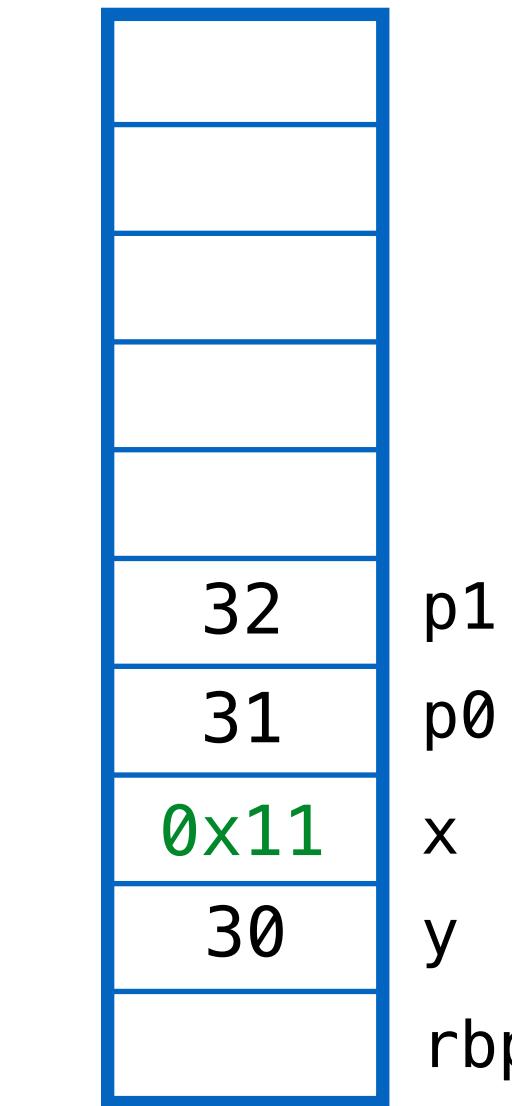
**QUIZ: Which cells are garbage?**

- (A) 0x00, 0x08 (B) 0x08, 0x10 (C) 0x18, 0x20 (D) None (E) All

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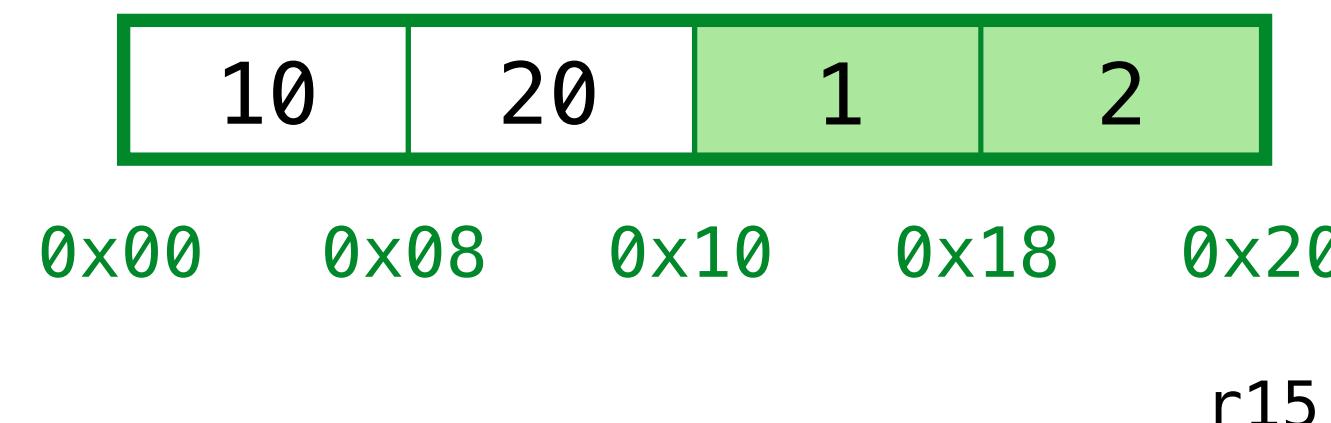
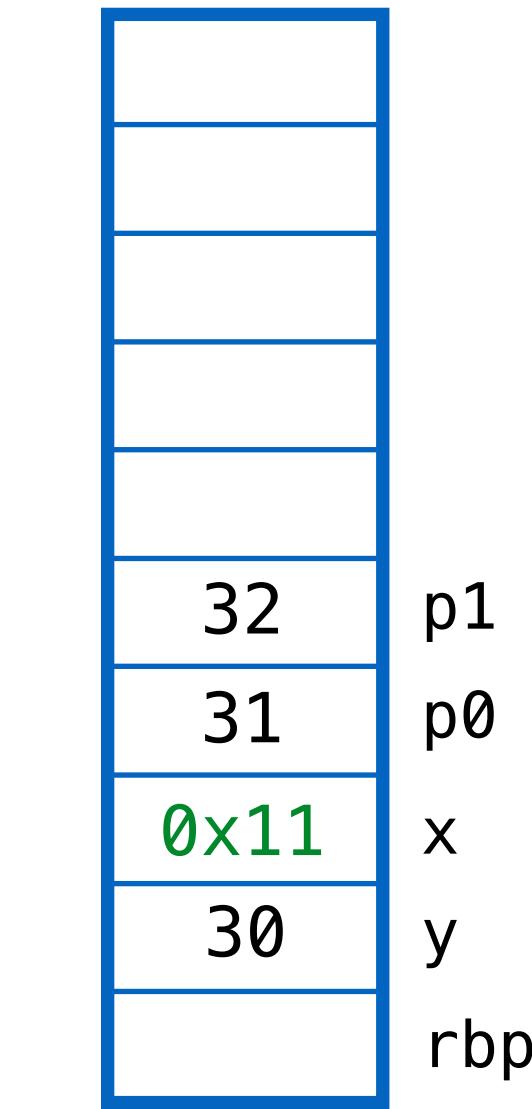
**QUIZ: Which cells are garbage?**

Those that are *not reachable from stack*

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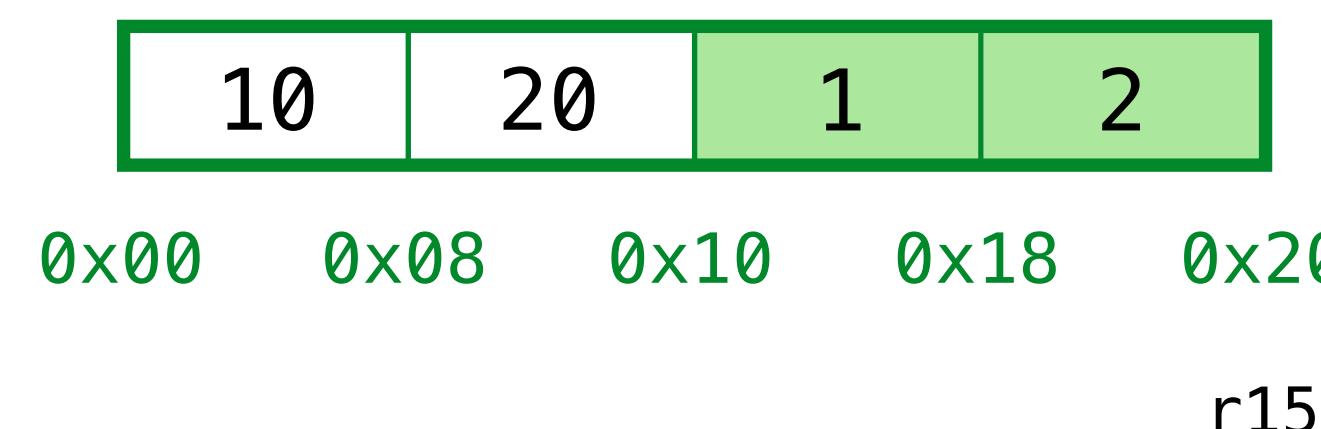
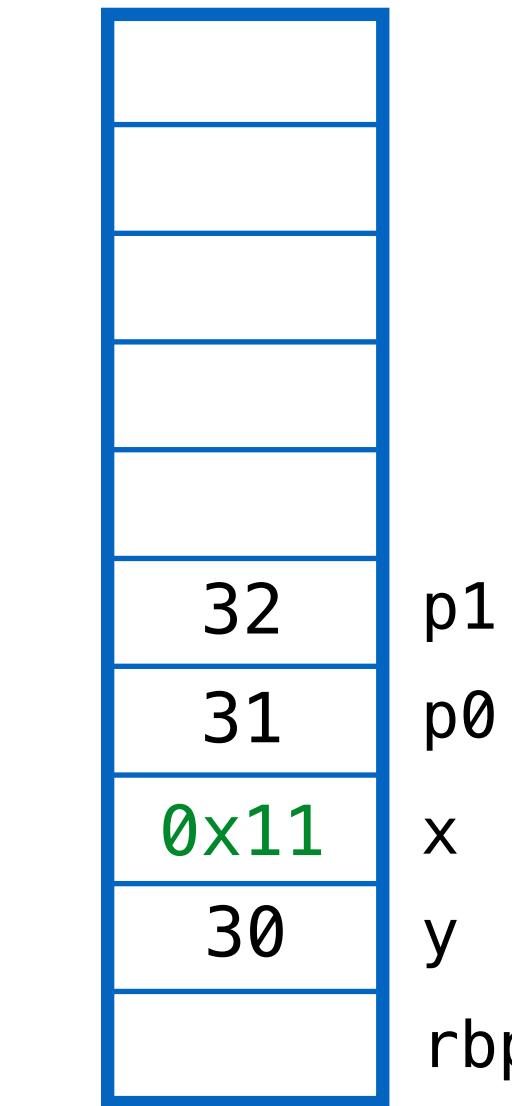
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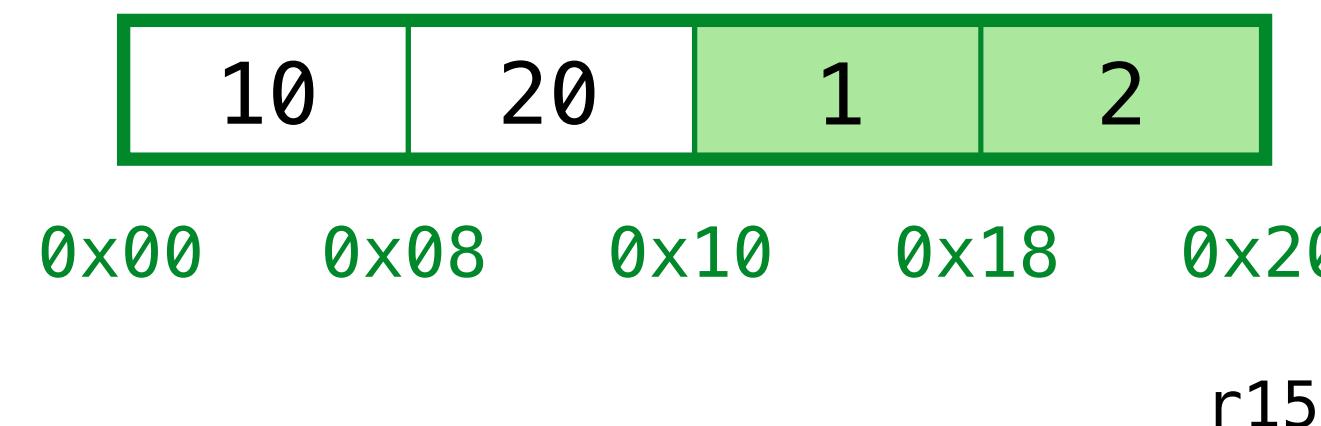
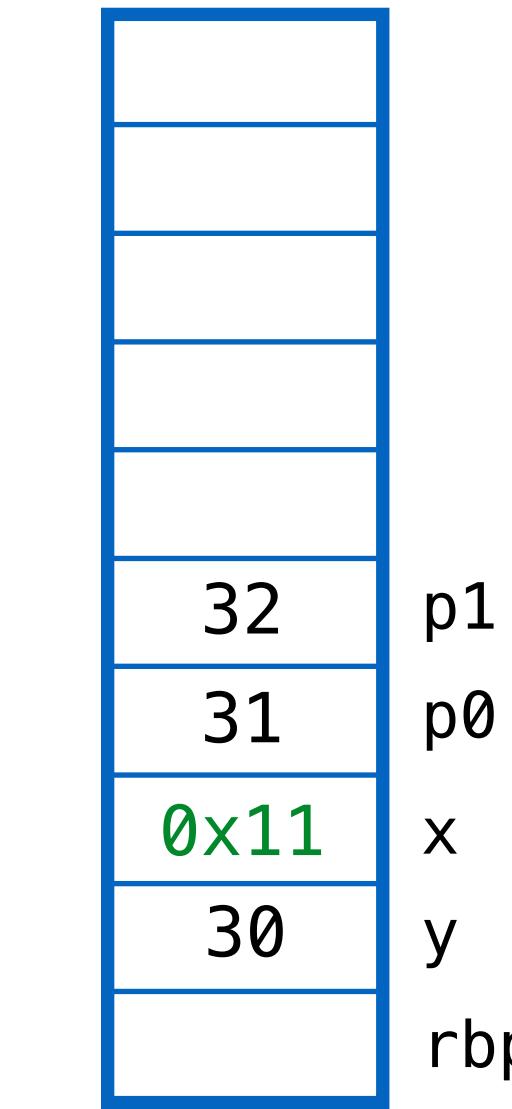
**Q: How to reclaim space?**

Why is it not enough to rewind r15?

## ex2: garbage in the middle

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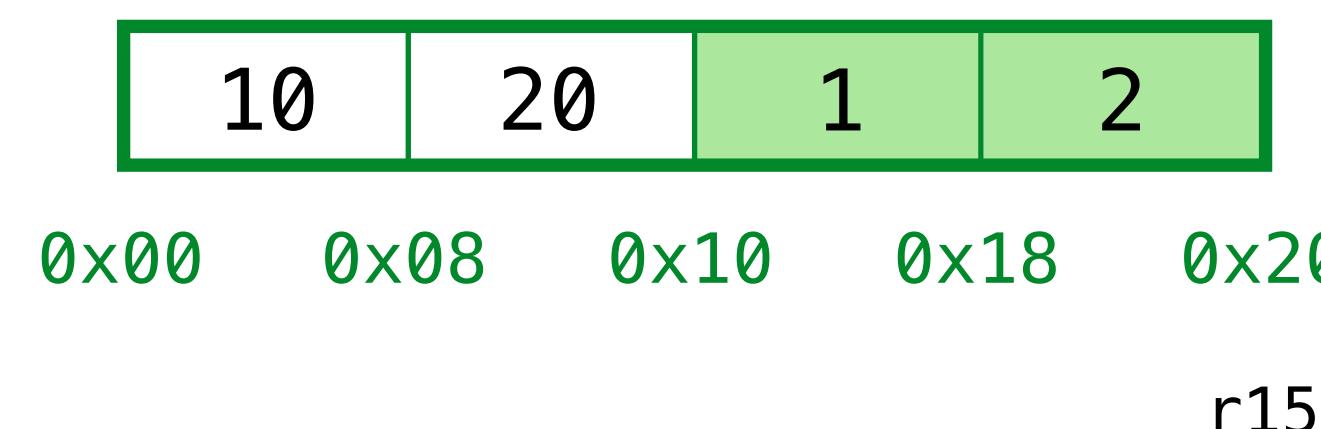
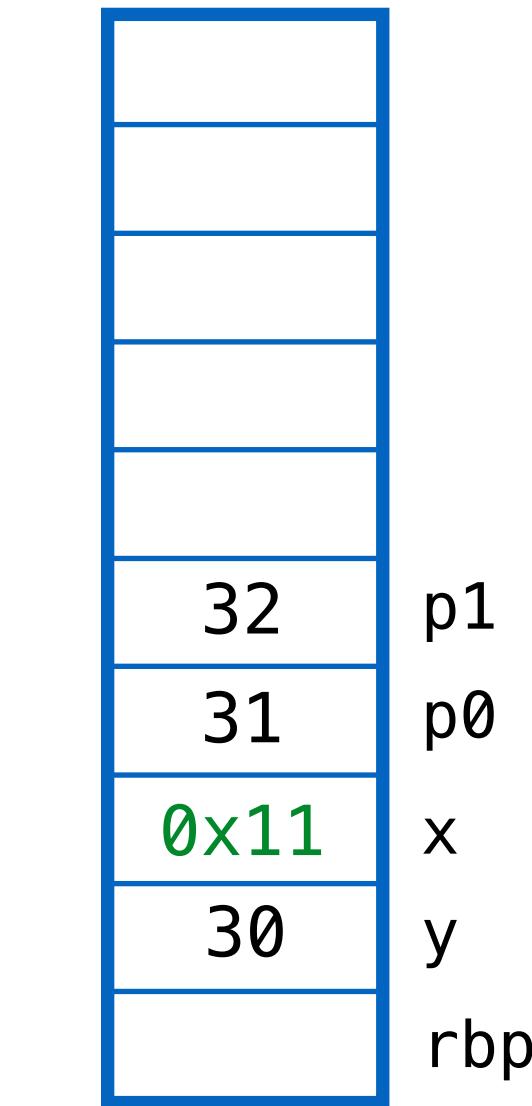
Why is it not enough to rewind r15?

Want free space to be *contiguous* (i.e. go to end of heap)

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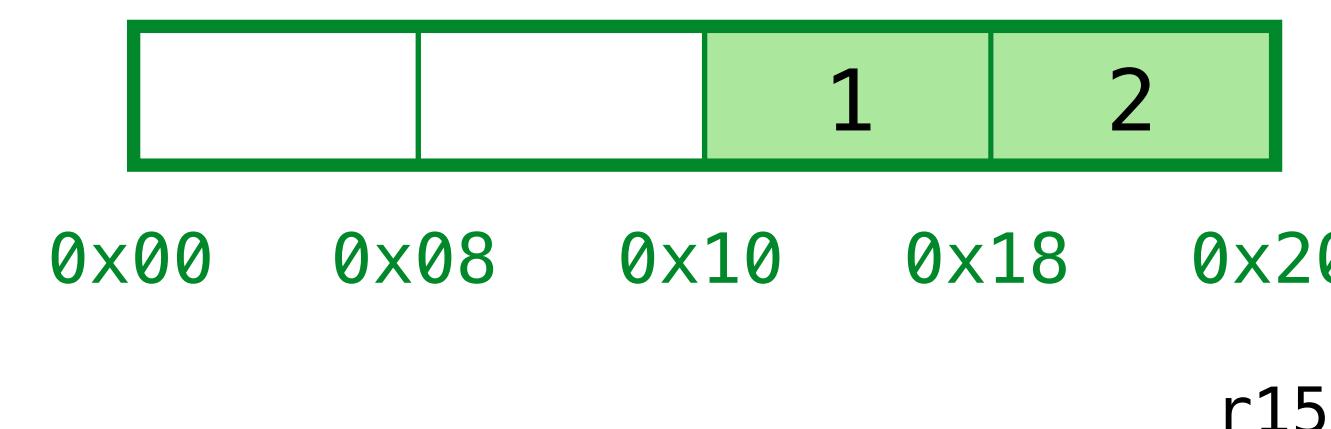
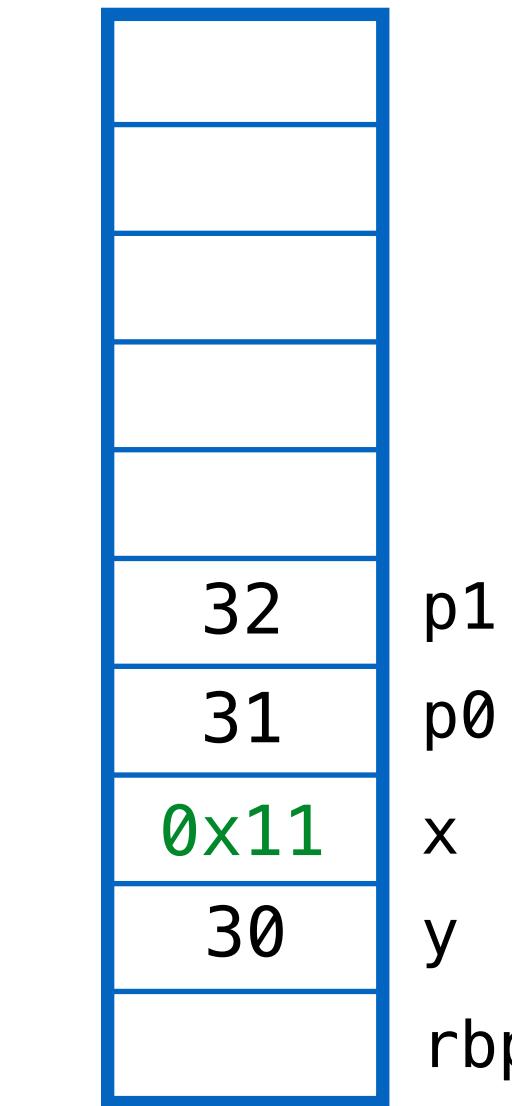
## Solution: Compaction

Copy “live” cells into “garbage” ...

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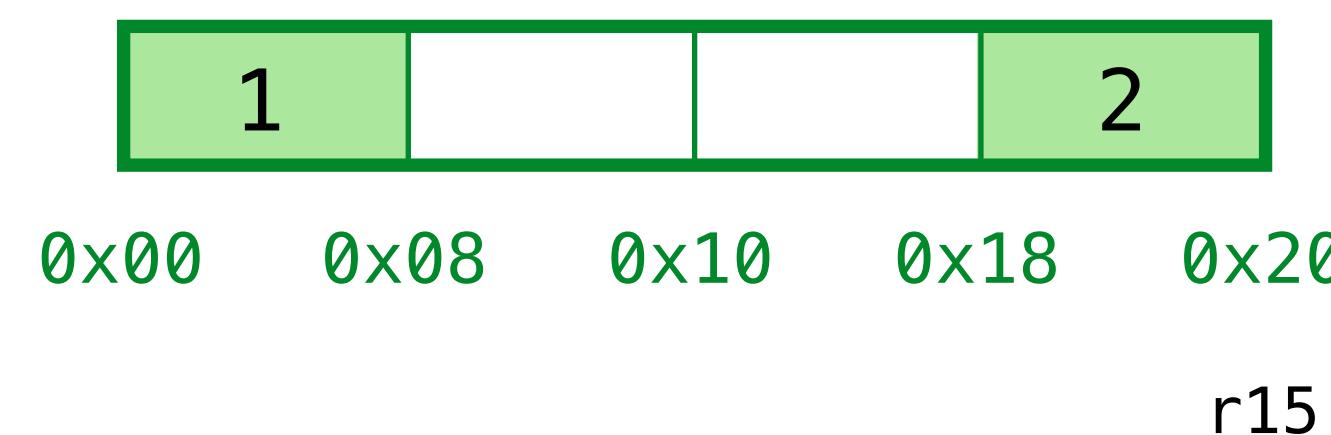
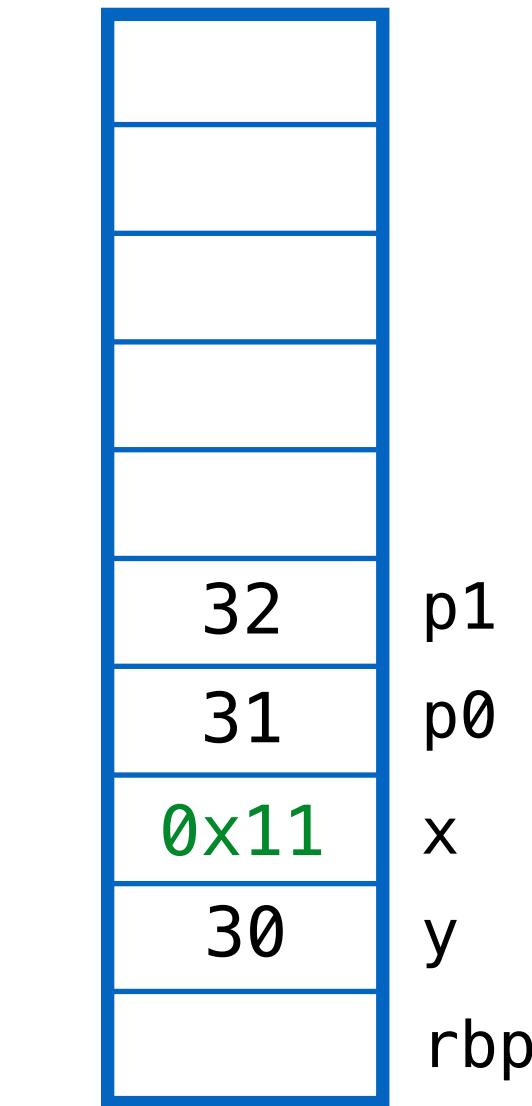
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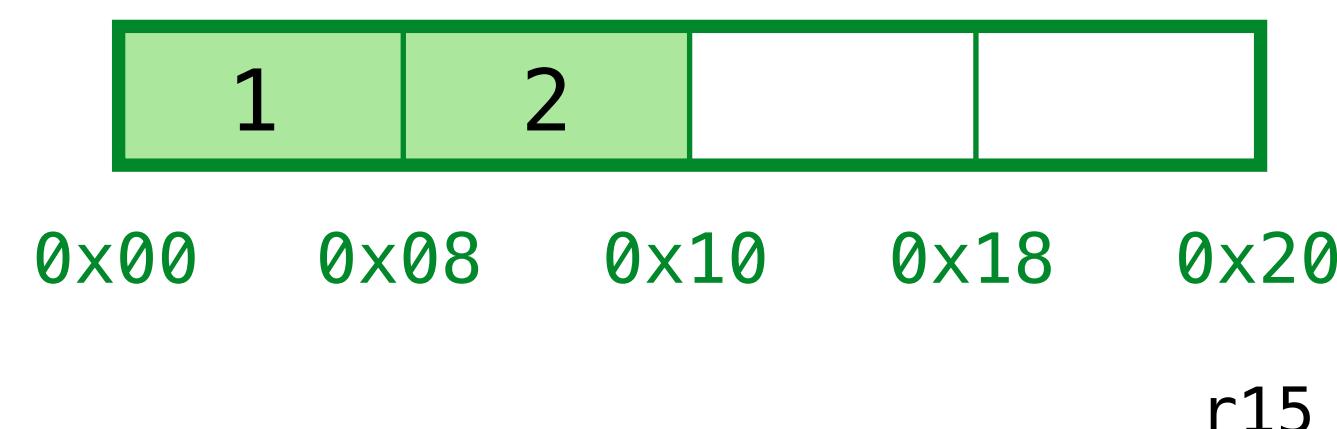
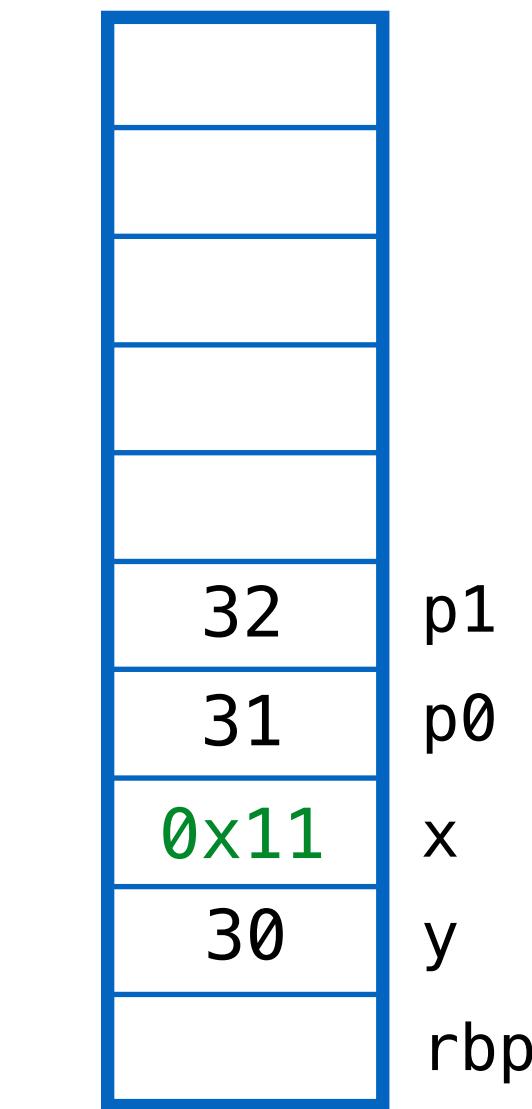
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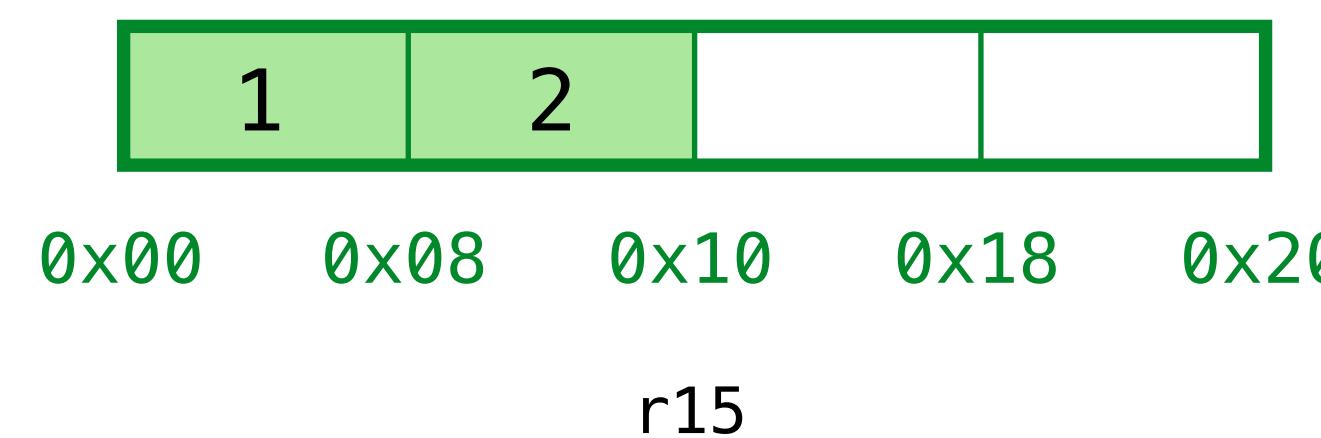
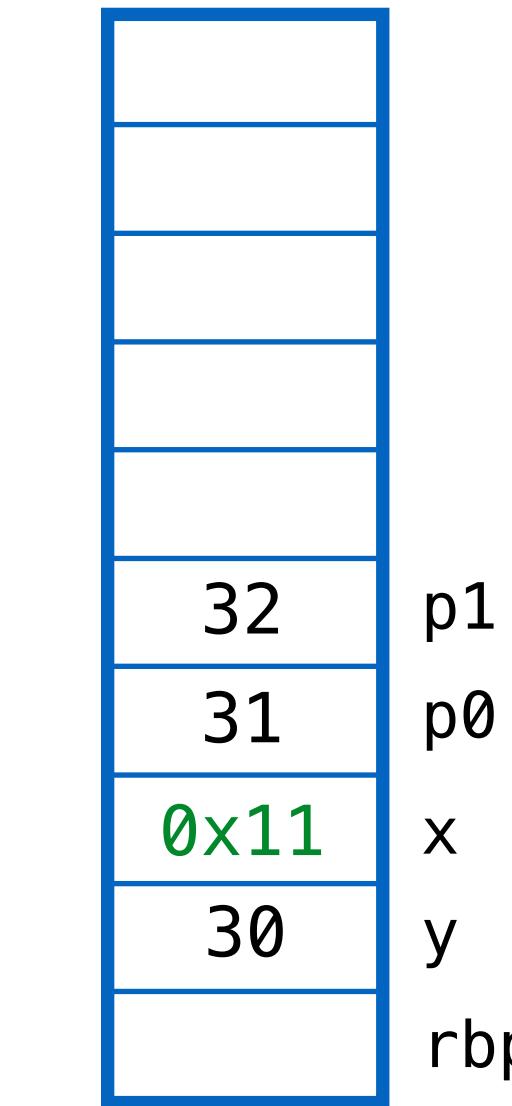
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Lets reclaim & recycle garbage!



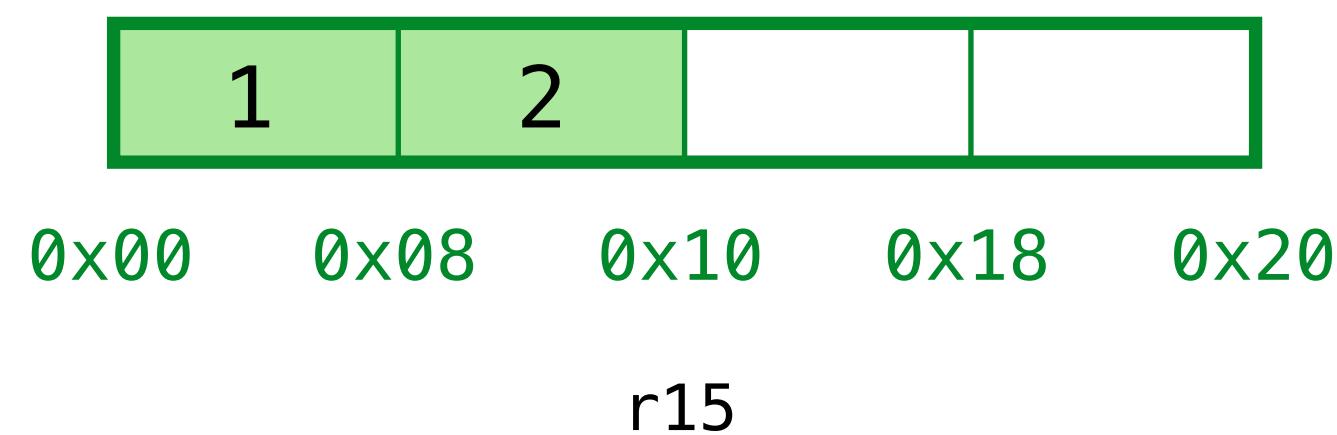
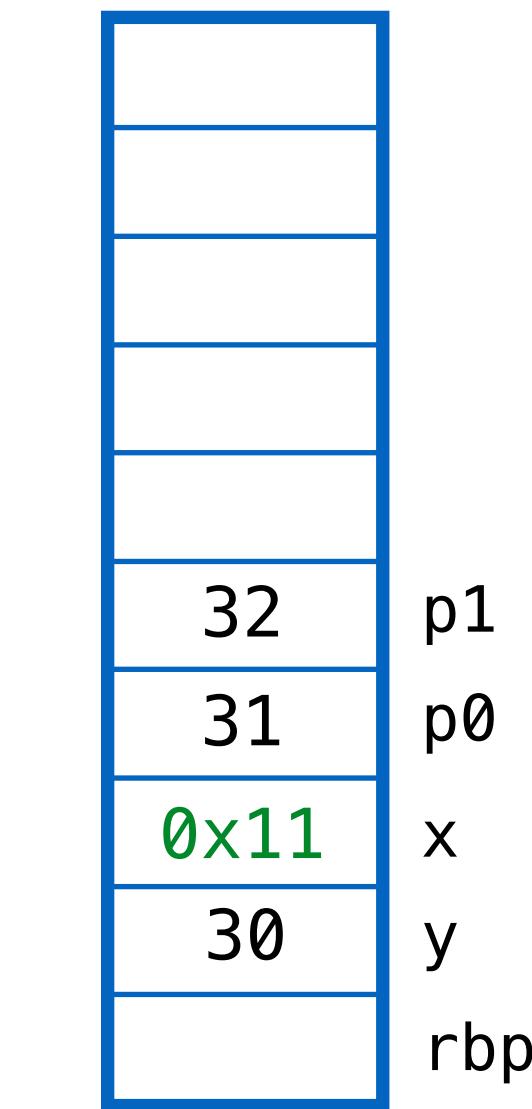
## Solution: Compaction

Copy “live” cells into “garbage” ... *and then* ... rewind r15!

## ex2: garbage in the middle

```
let y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
      , x = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```

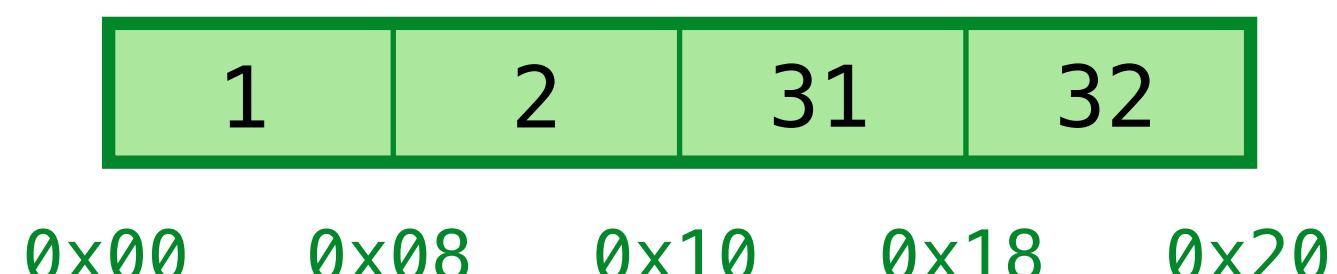
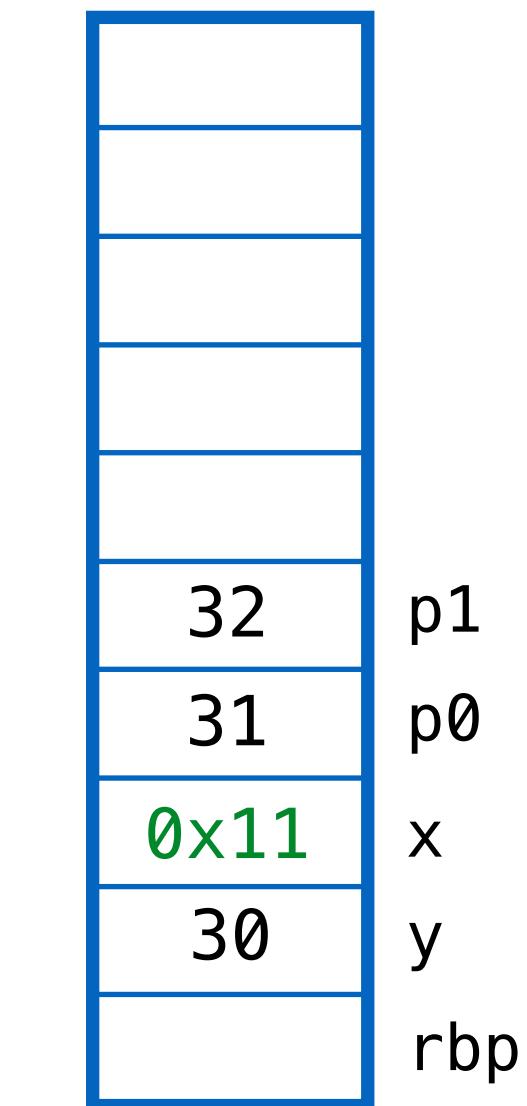
Yay! Have space for (p0, p1)



## ex2: garbage in the middle

```
let y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
      , x = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in
  (p0, p1) ←
```

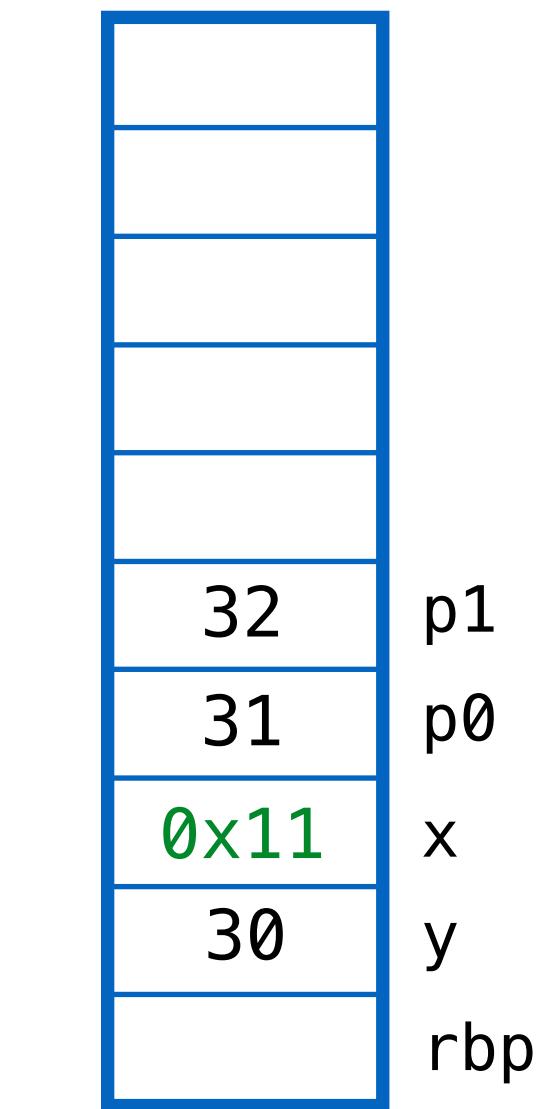
Yay! Have space for (p0, p1)



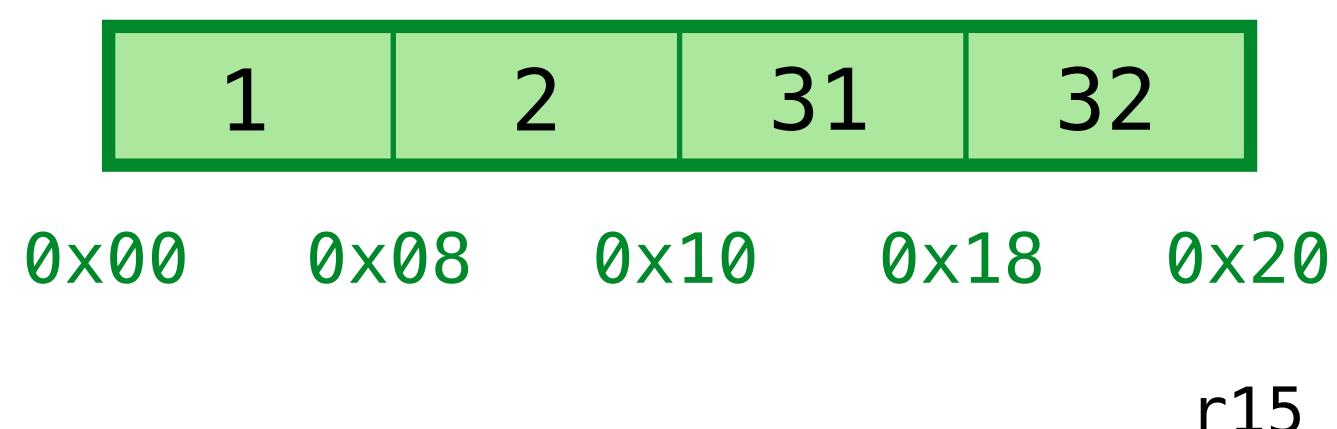
r15

## ex2: garbage in the middle

```
let y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
      , x = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in
(p0, p1) ←
```



**Result (rax) = 0x09**

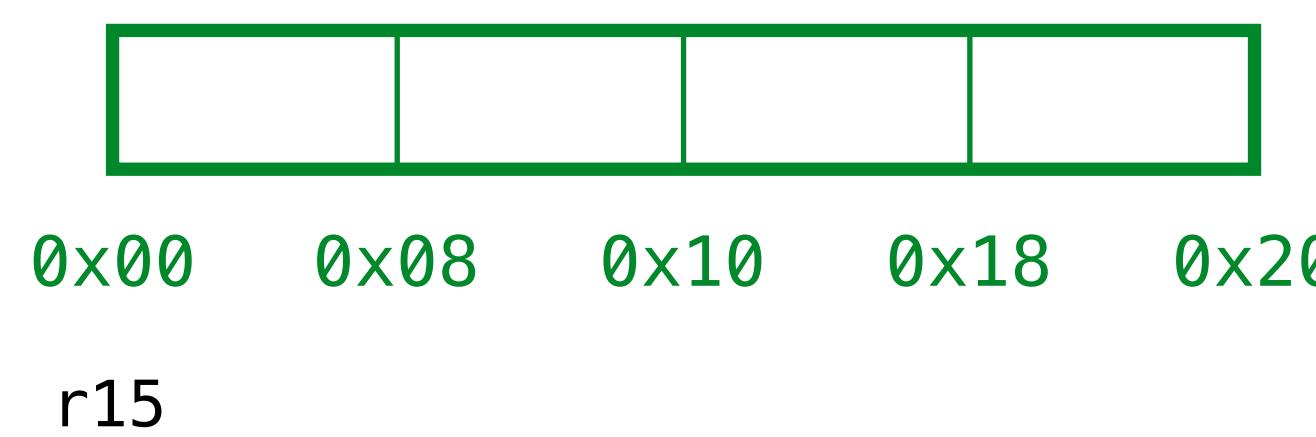
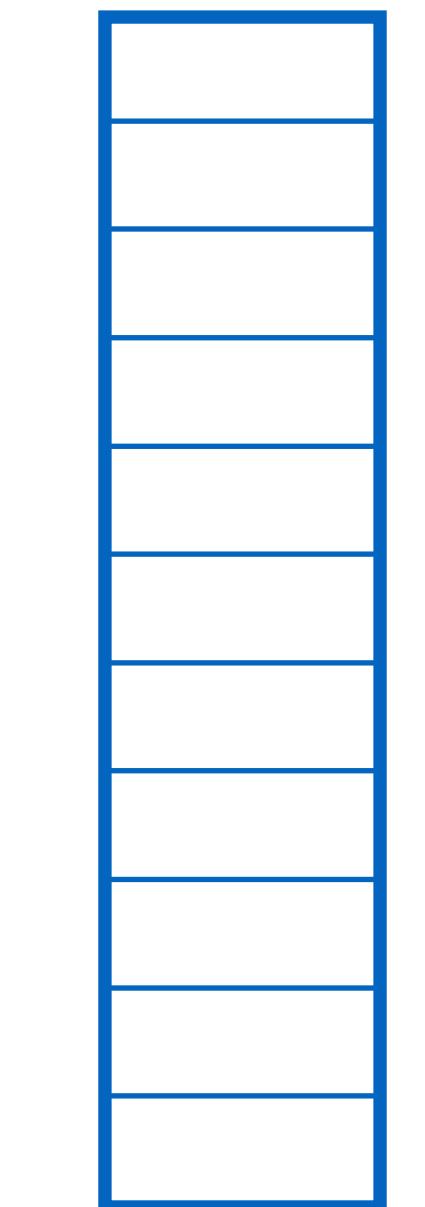


Garter / GC

Example 3

### ex3: garbage in the middle (with stack)

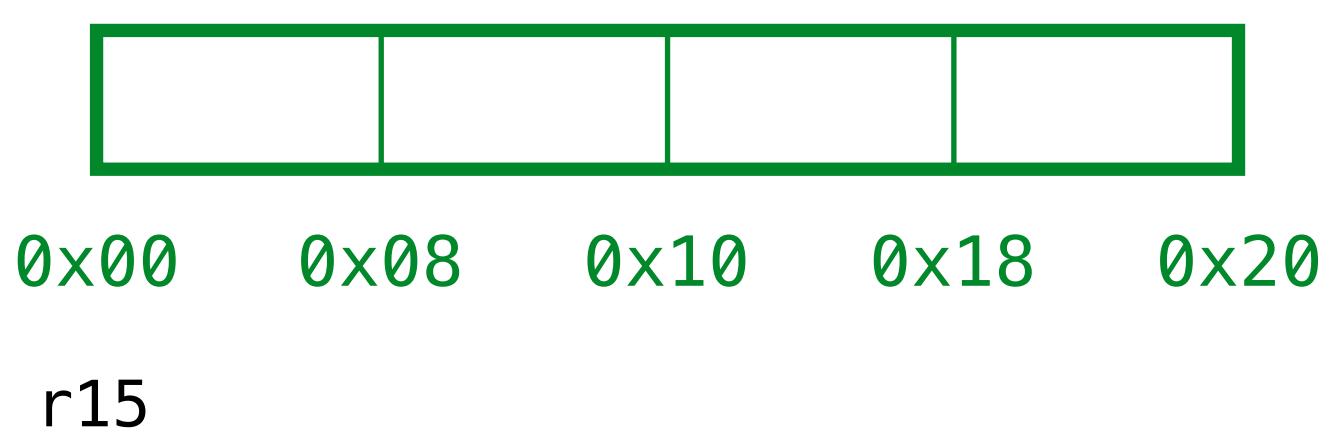
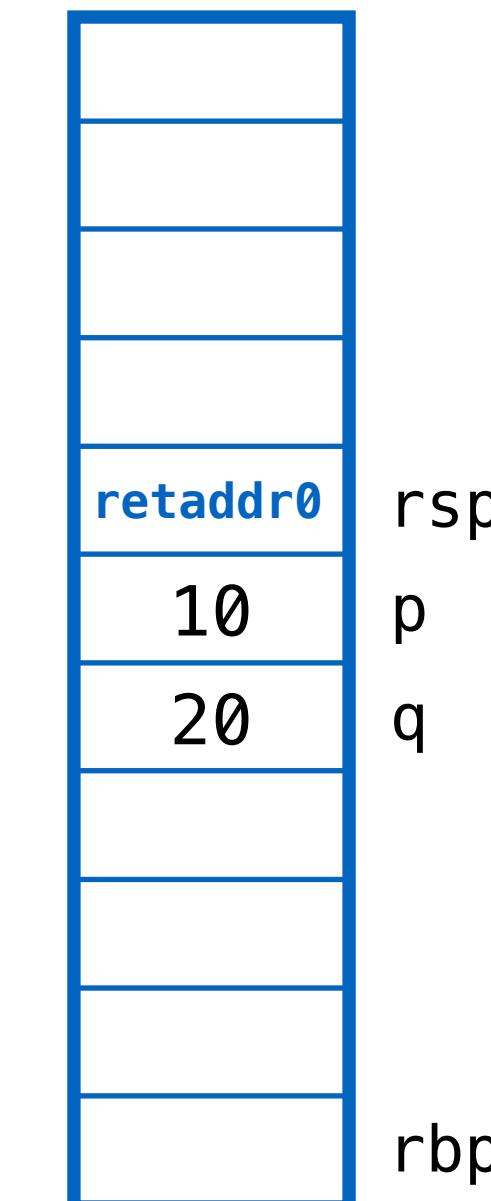
```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]
←
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + y + z
```



### ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

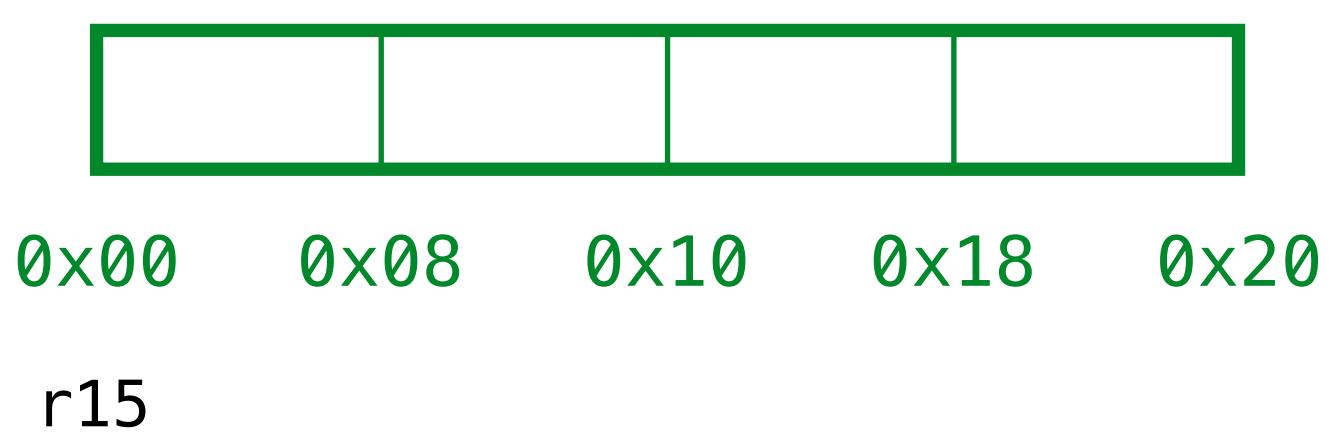
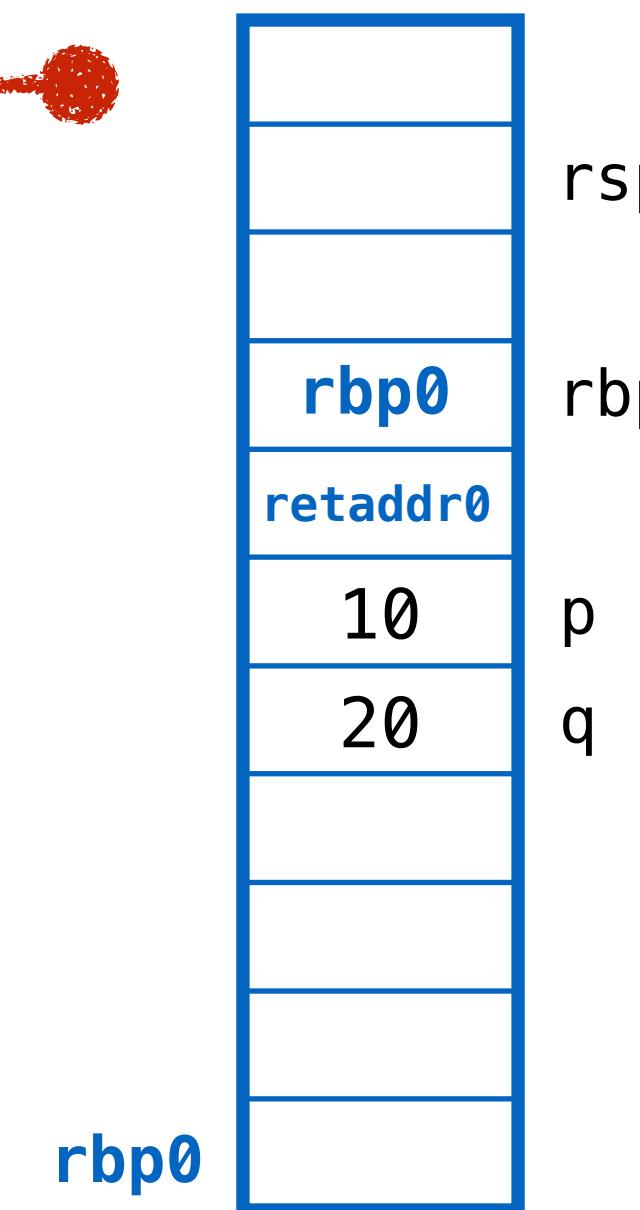
let y  = foo(10, 20) ←
       , x  = (y, y + 1)
       , z  = foo(100, 200)
in
    x[0] + y + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

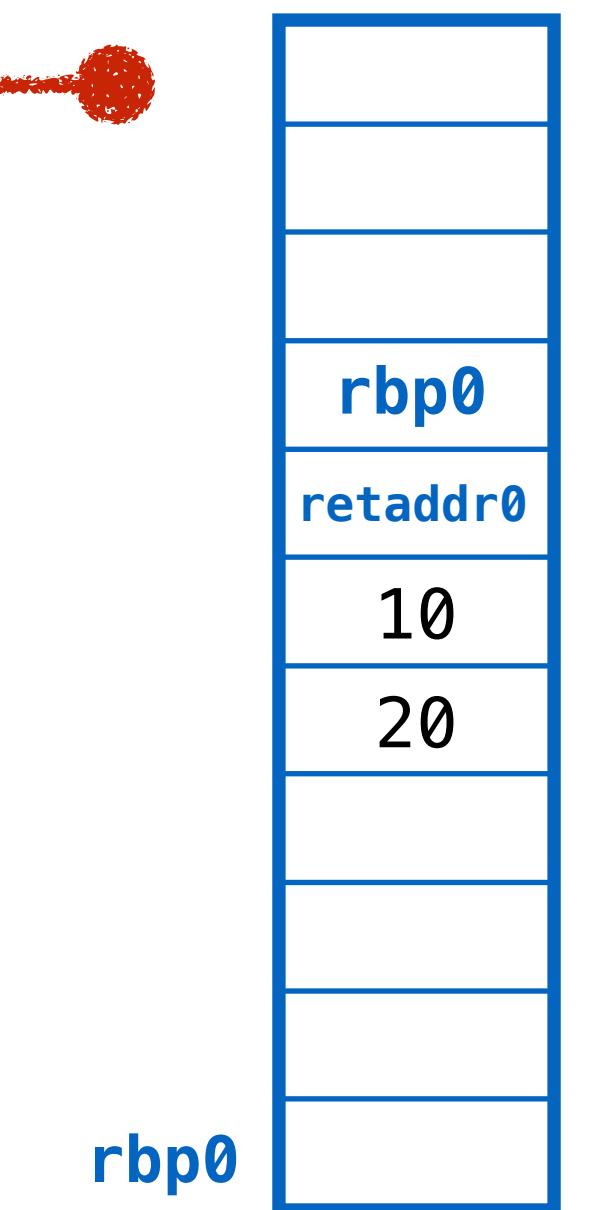
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + y + z
```



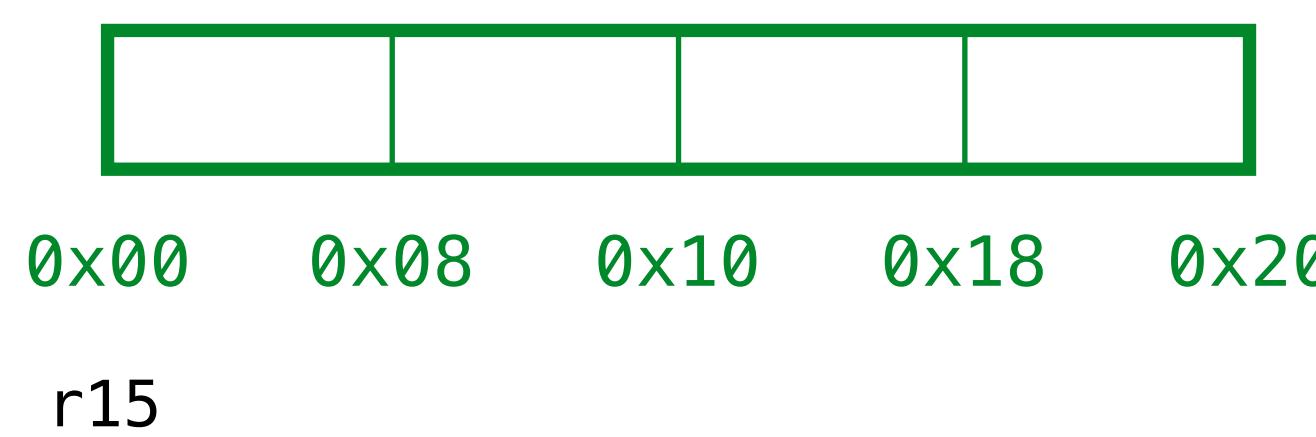
### ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + y + z
```

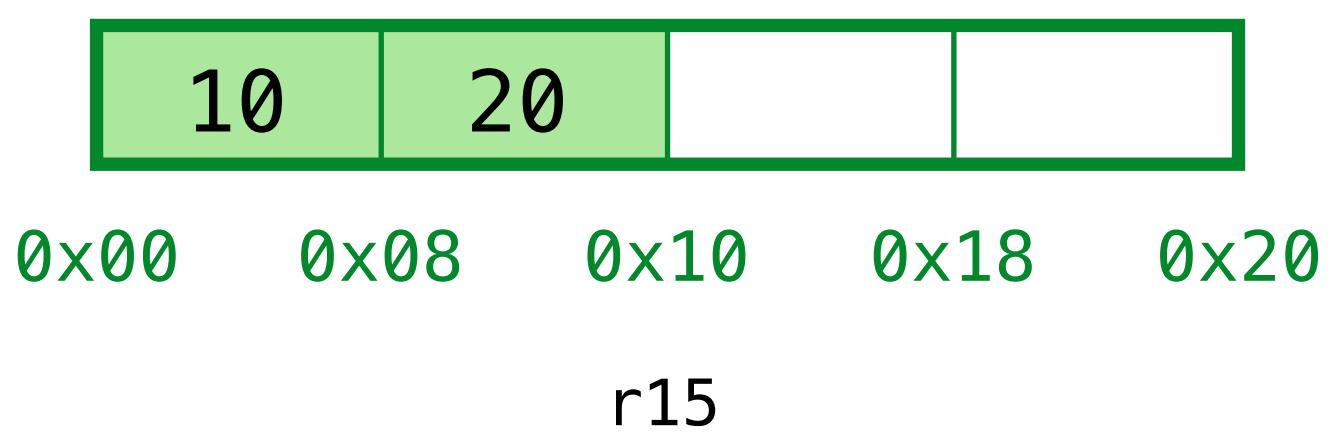
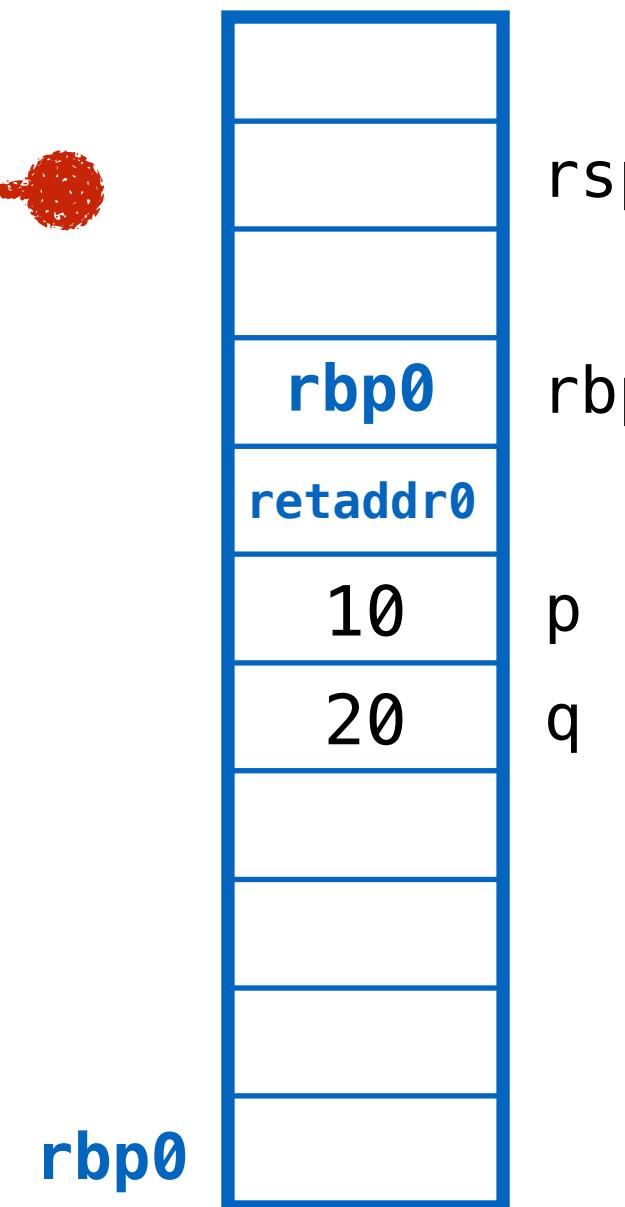


1 local var (**tmp**)

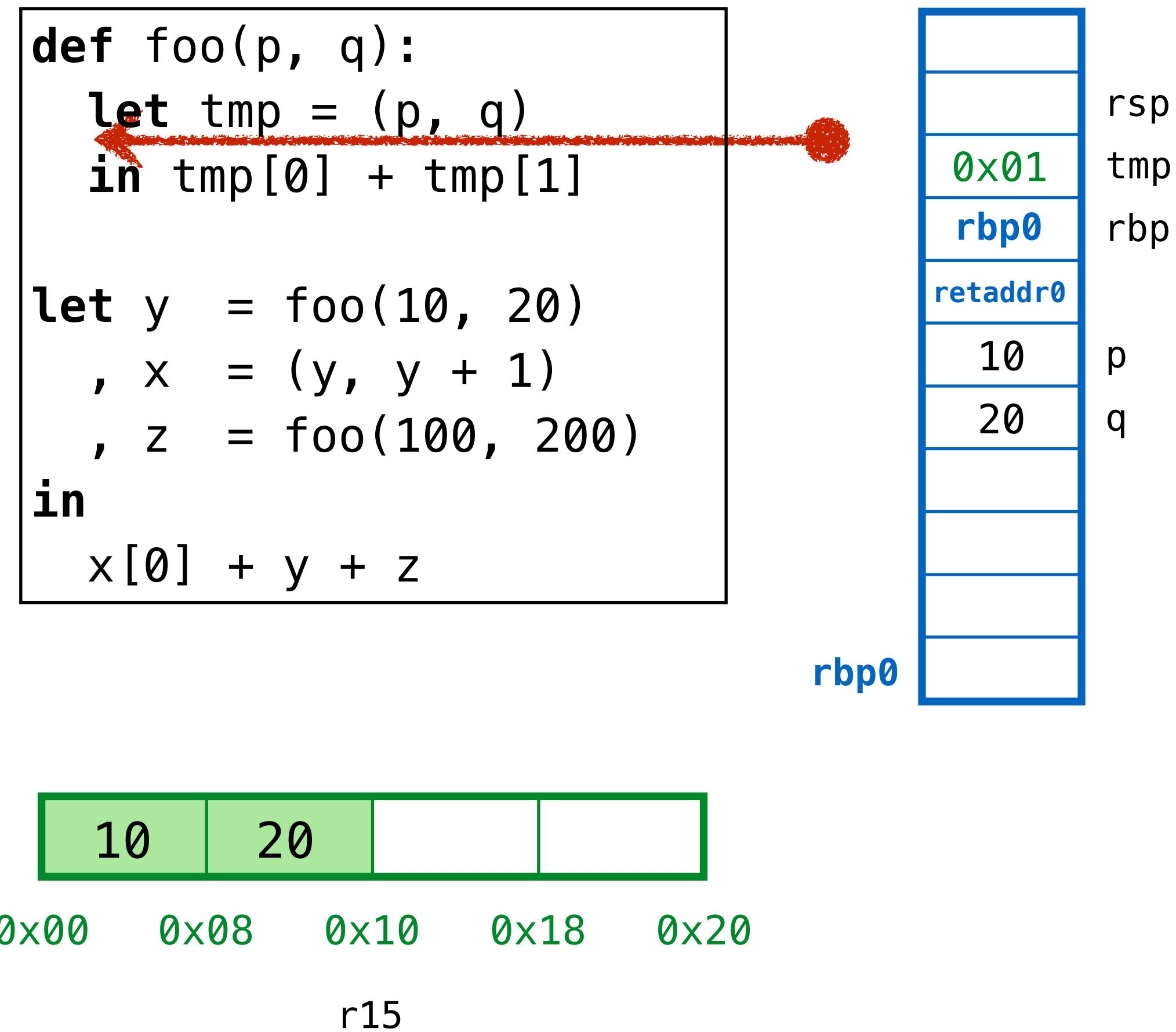


# ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + y + z
```

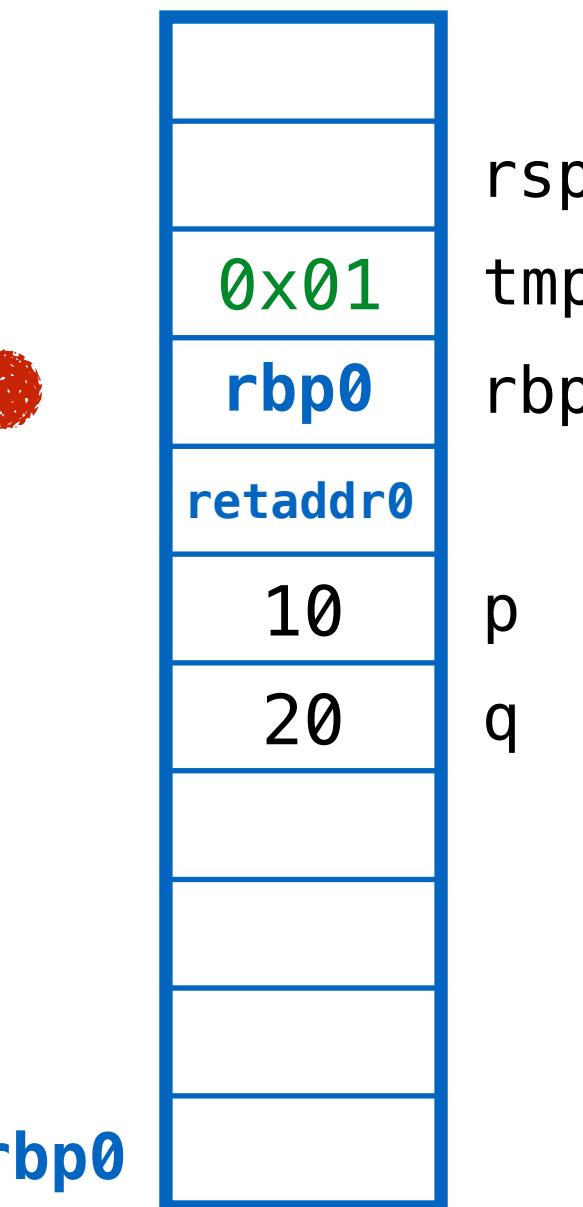


### ex3: garbage in the middle (with stack)

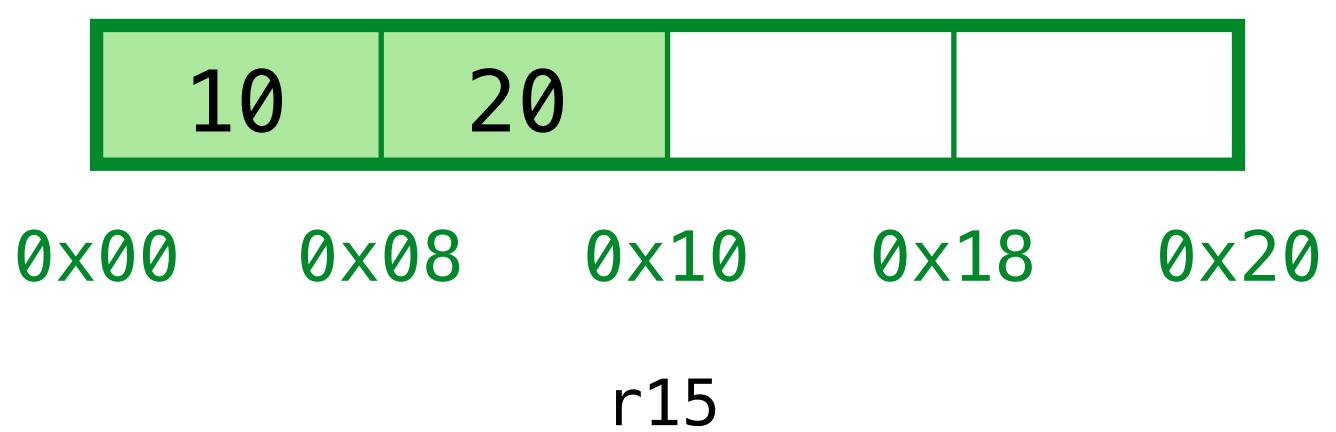


# ex3: garbage in the middle (with stack)

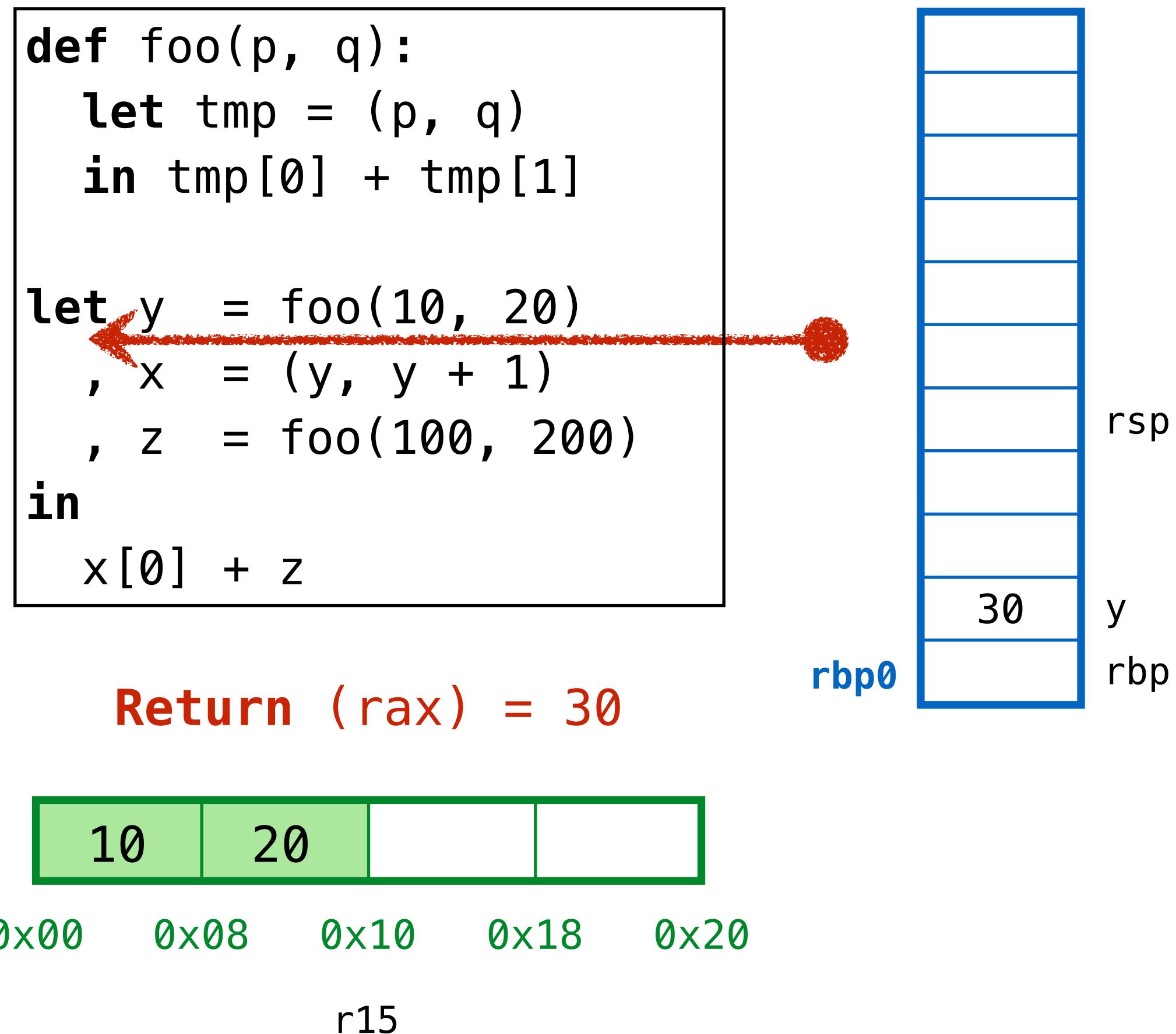
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + y + z
```



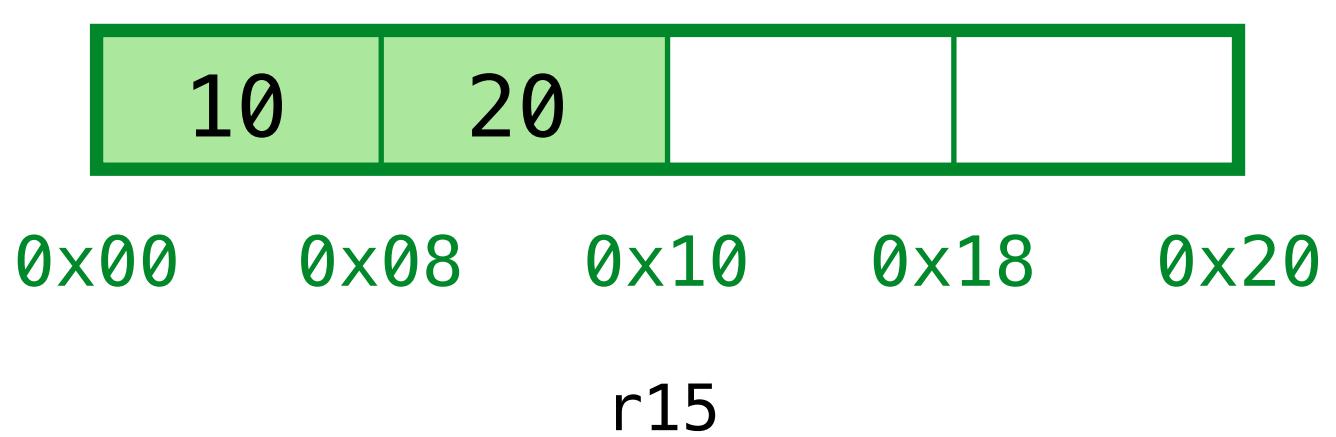
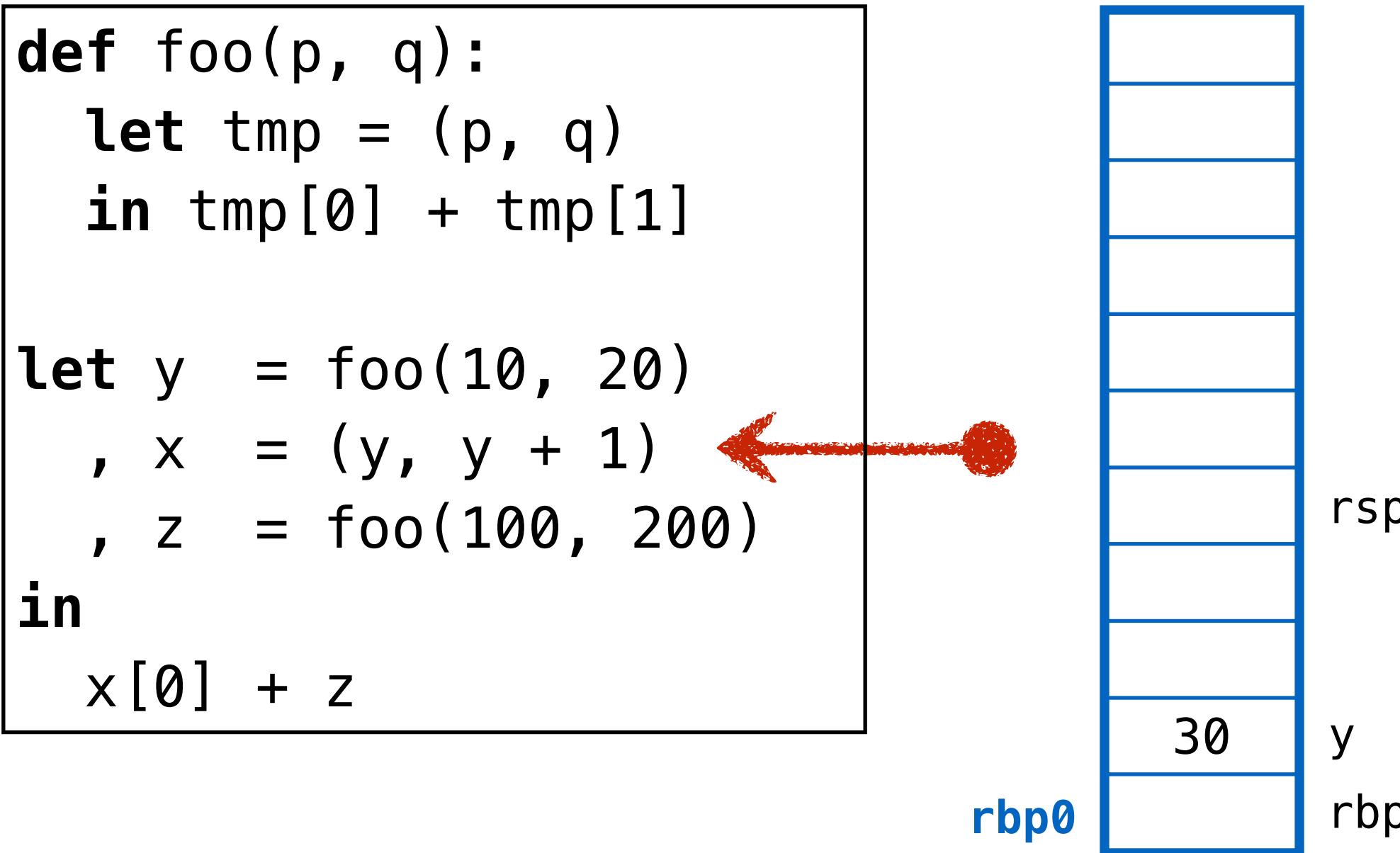
**Return (rax) = 30**



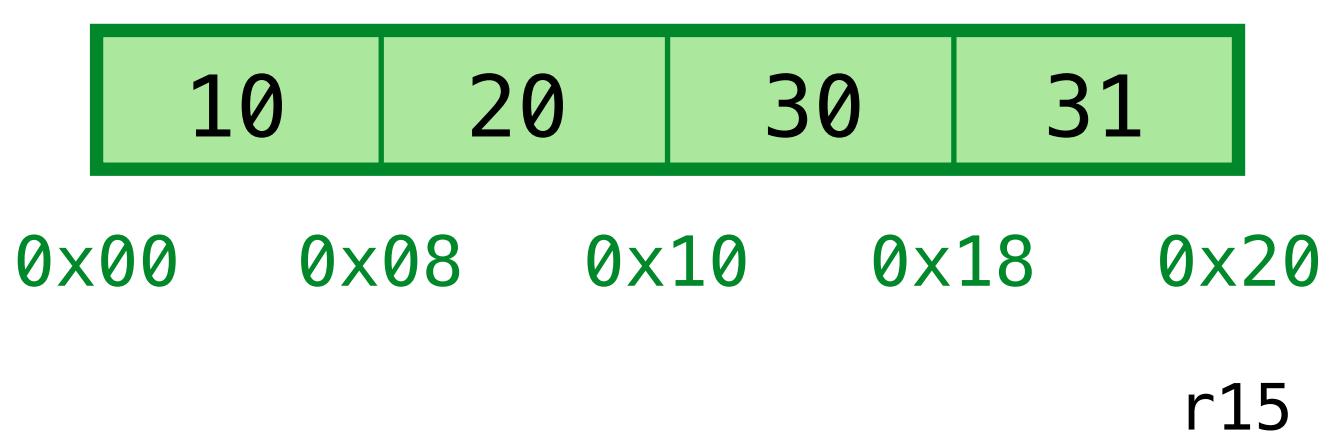
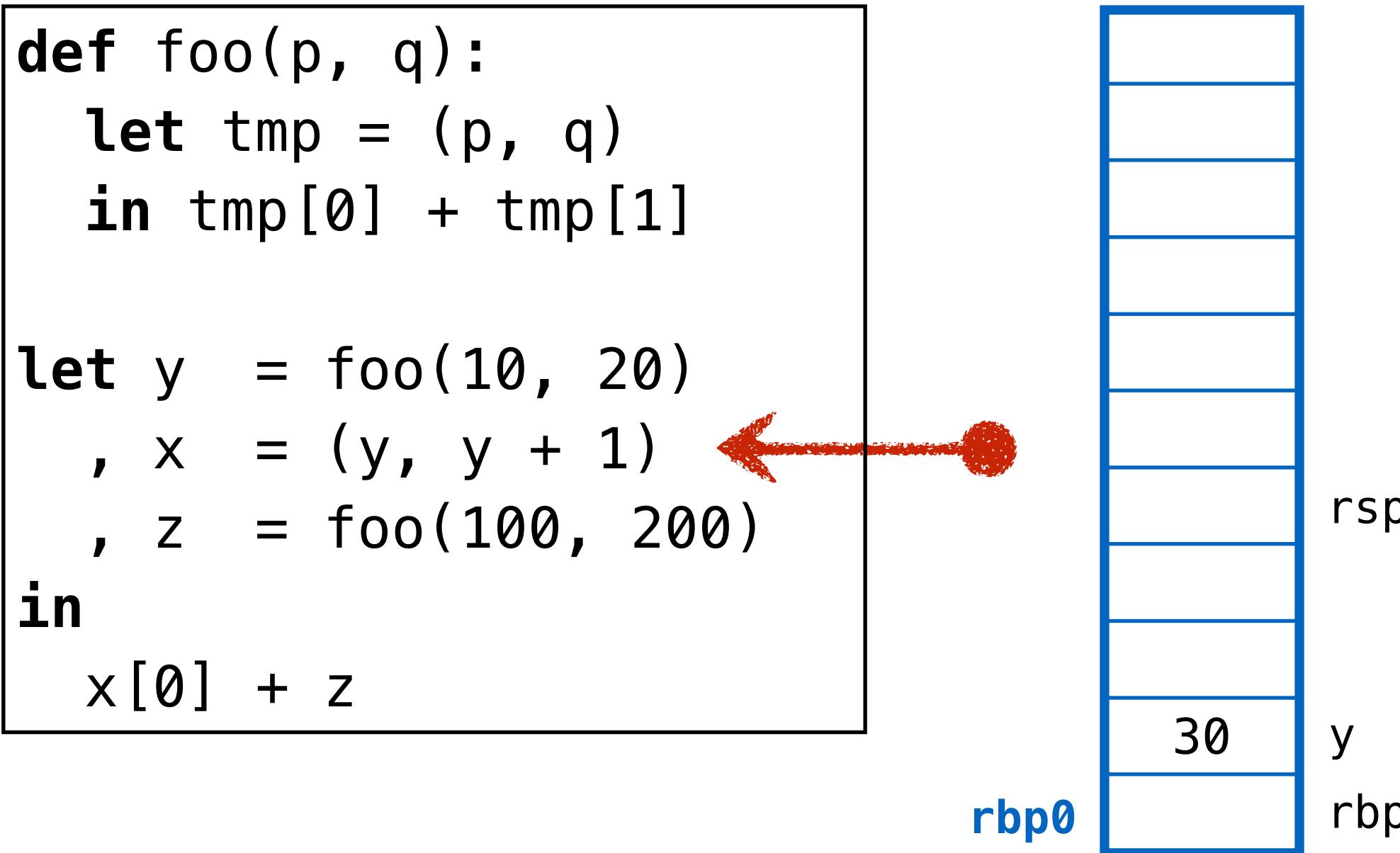
### ex3: garbage in the middle (with stack)



### ex3: garbage in the middle (with stack)

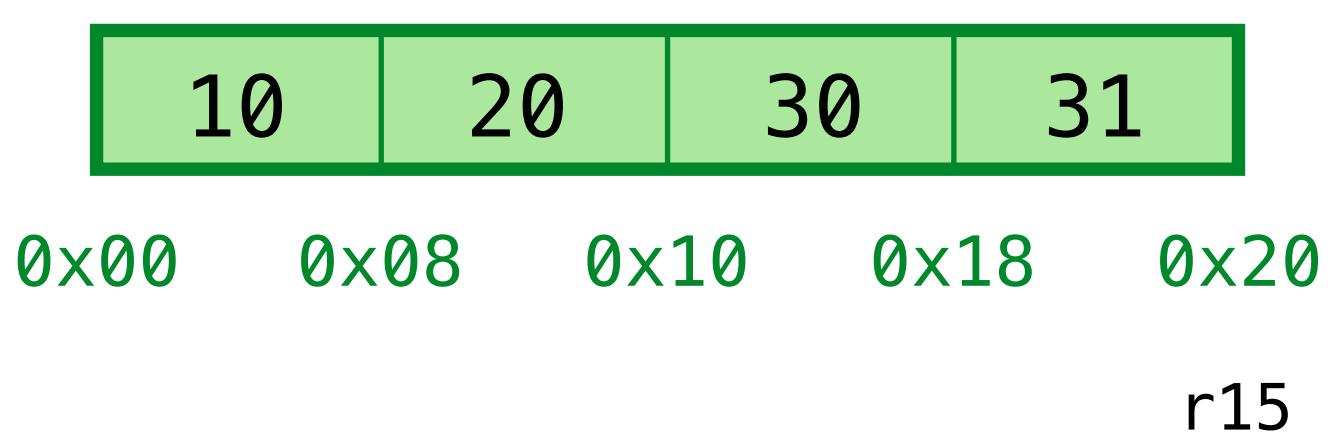
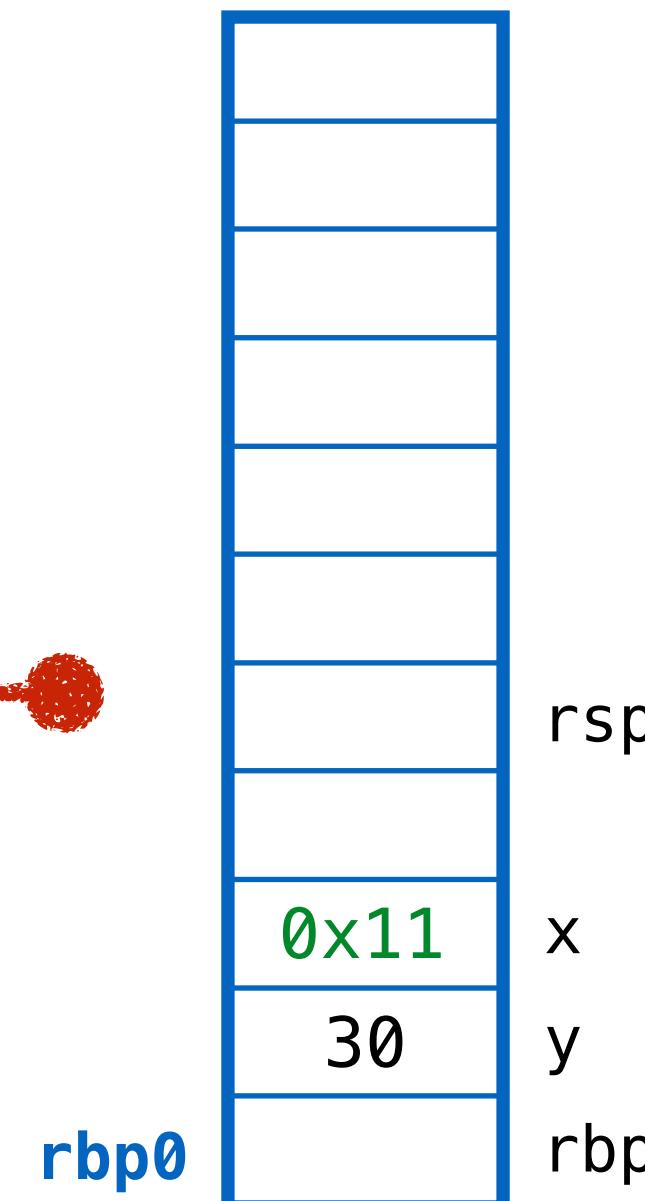


### ex3: garbage in the middle (with stack)



# ex3: garbage in the middle (with stack)

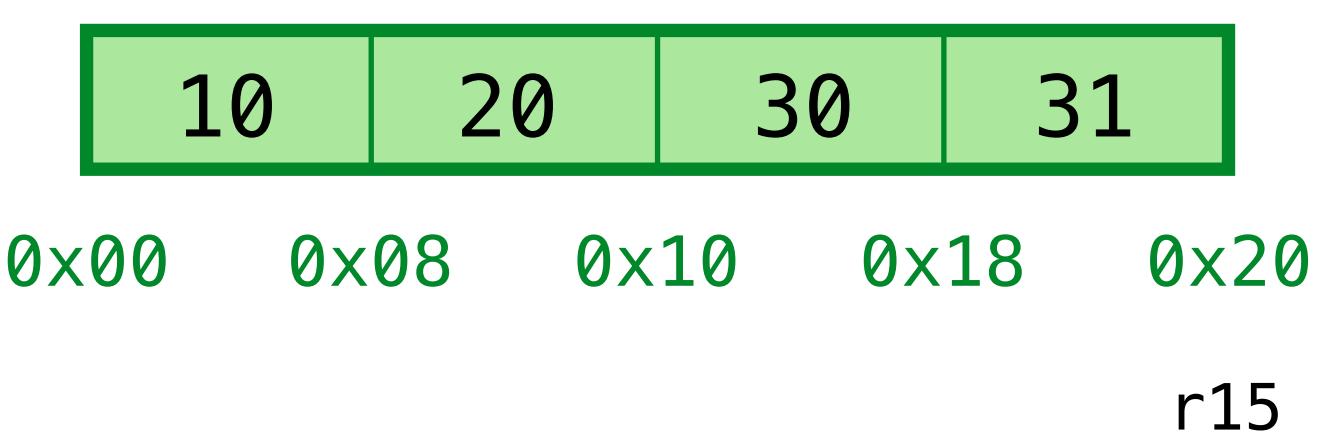
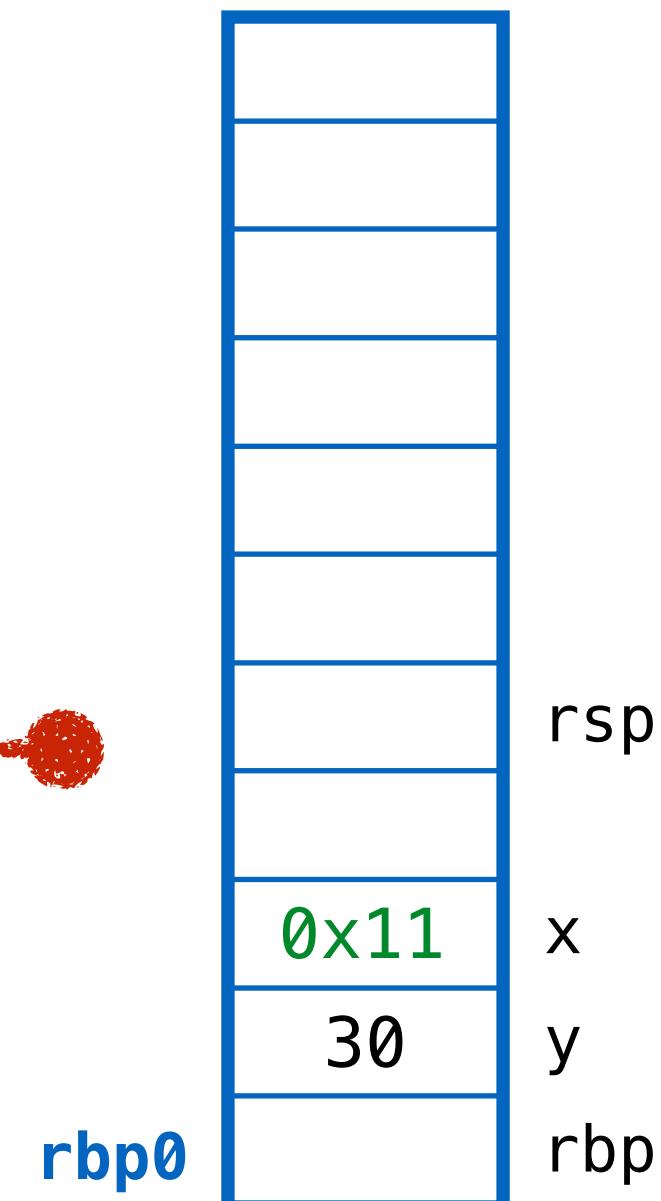
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



### ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

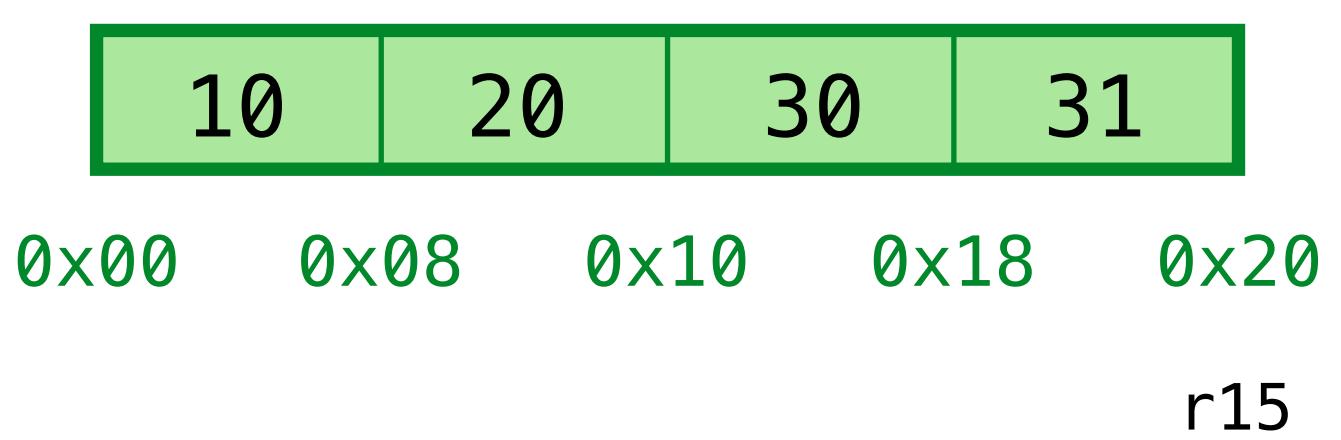
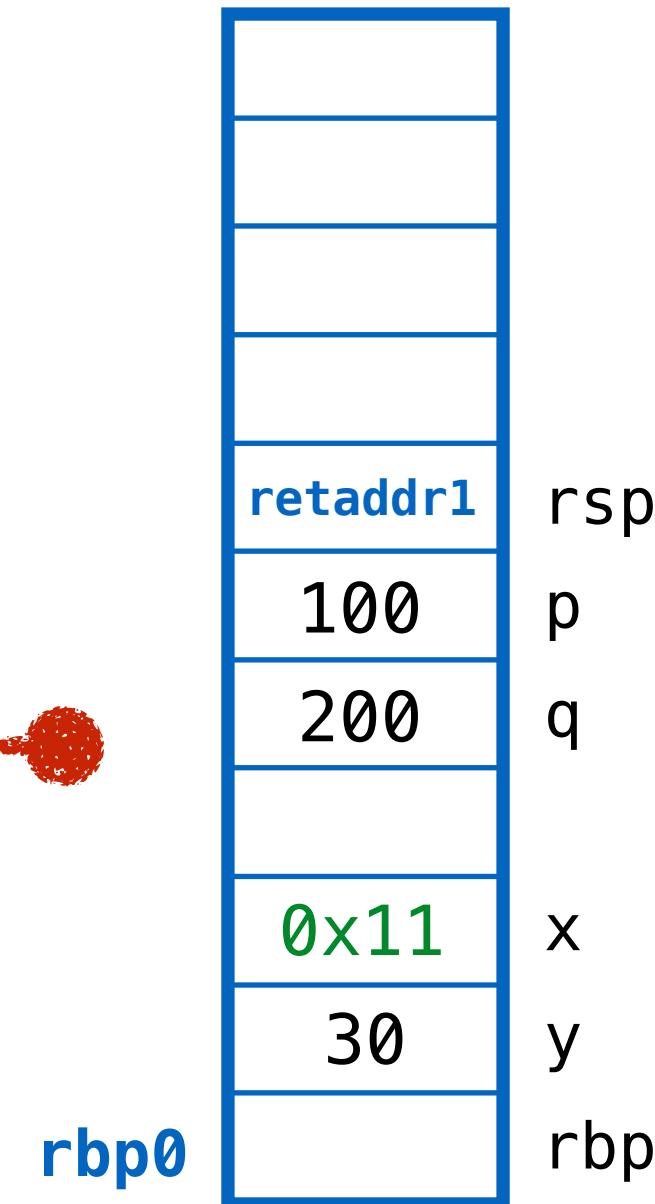
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



### ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

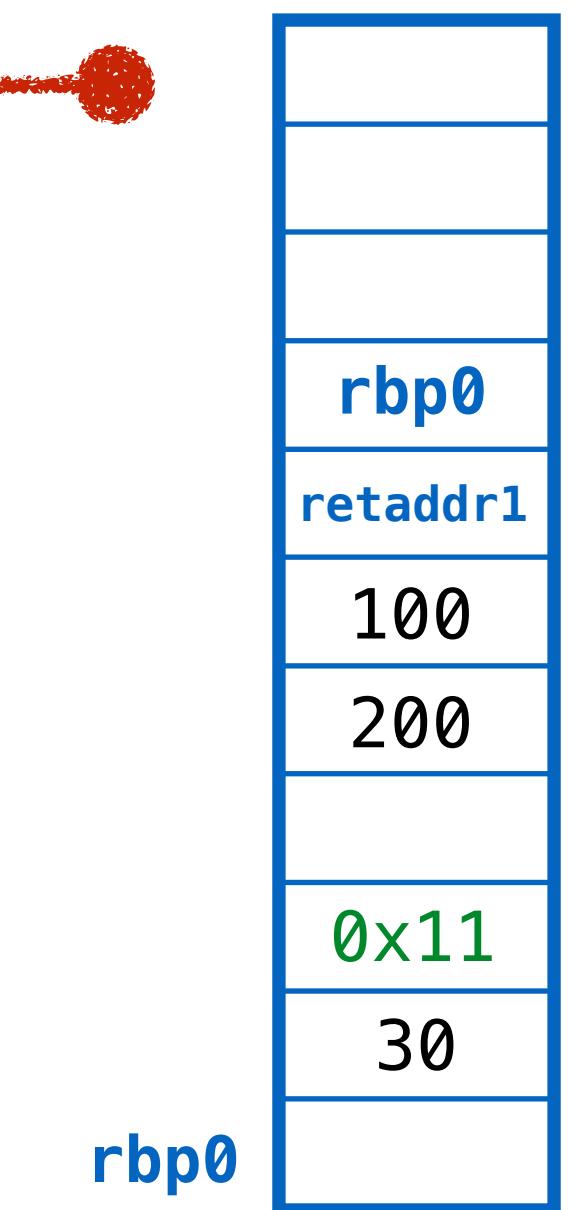
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



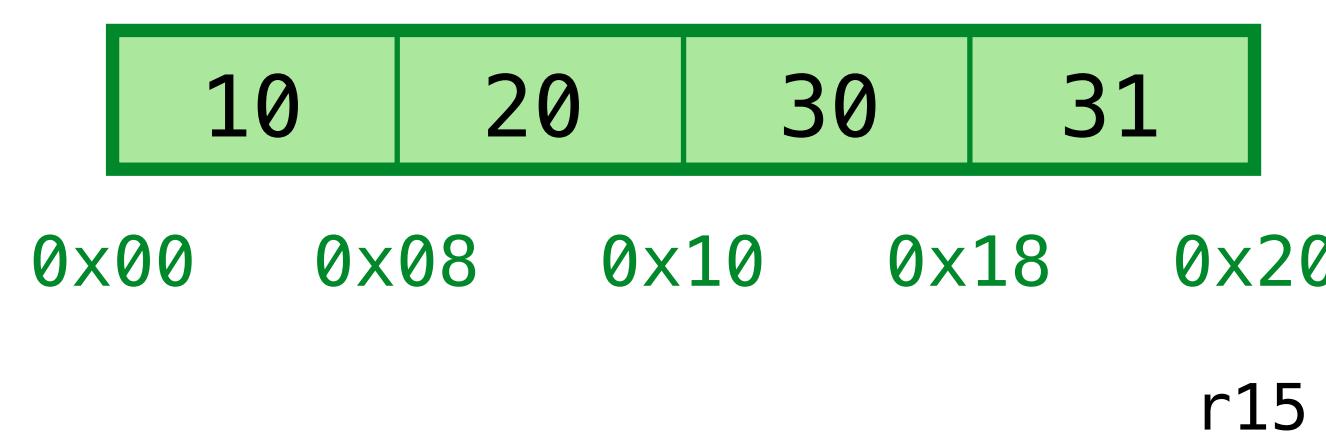
### ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

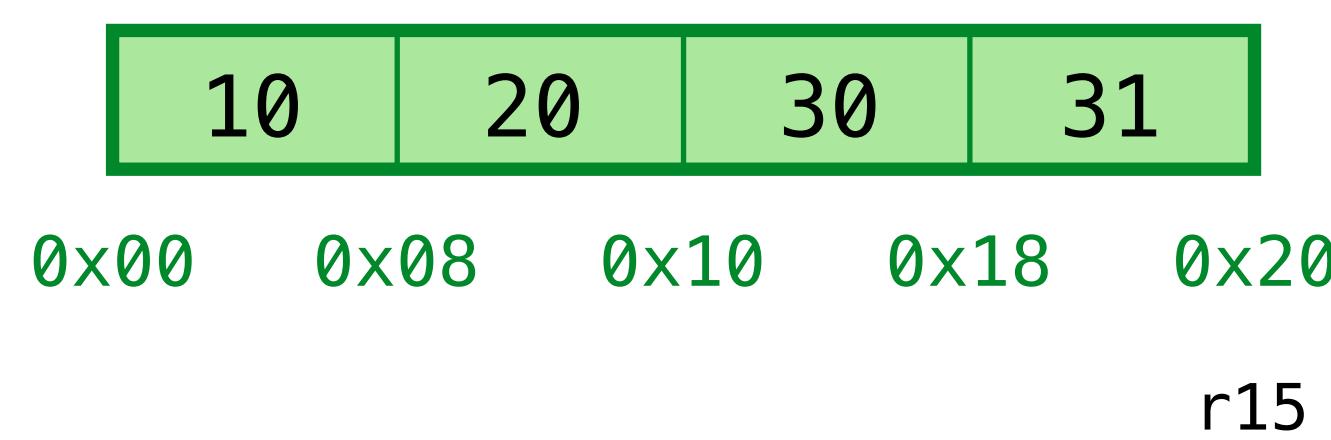
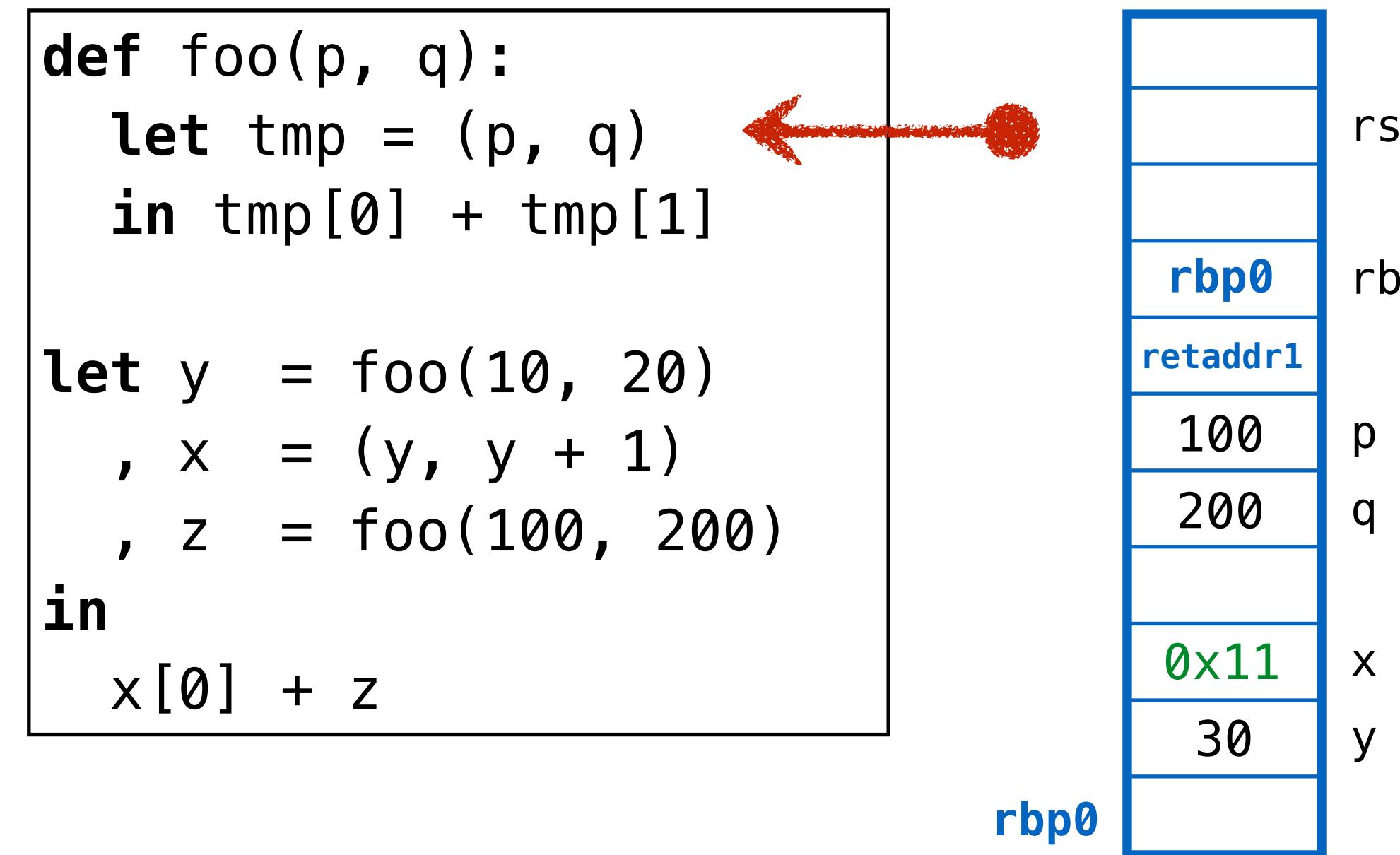
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



1 local var (`tmp`)



### ex3: garbage in the middle (with stack)

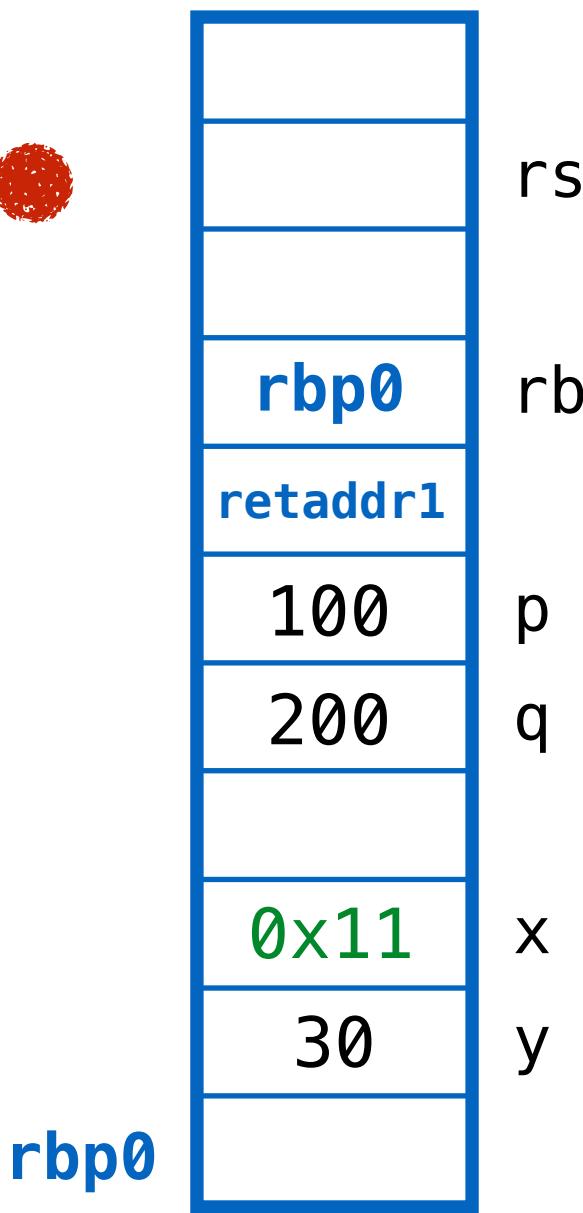


ex3: garbage in the middle (with stack)

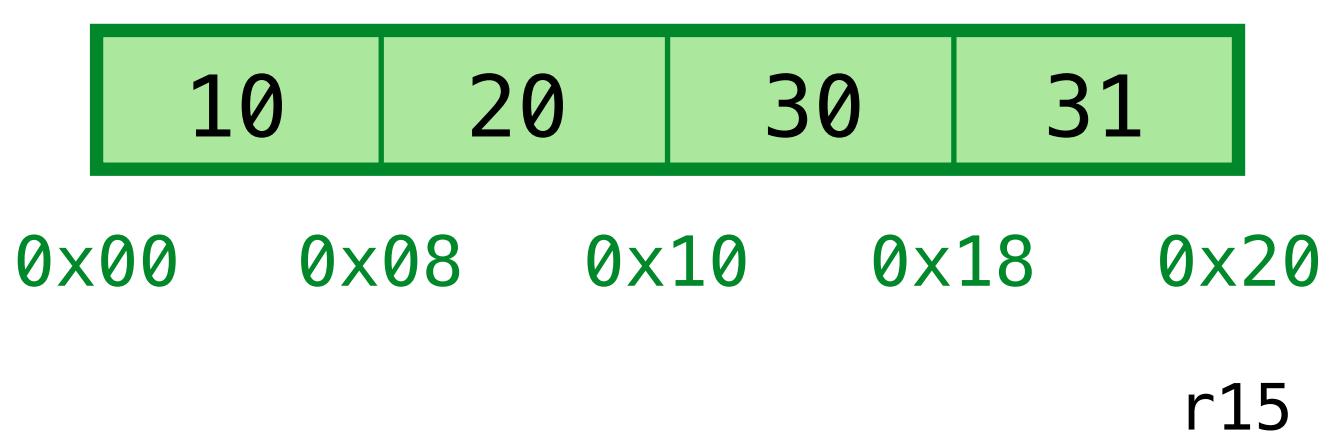
```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)

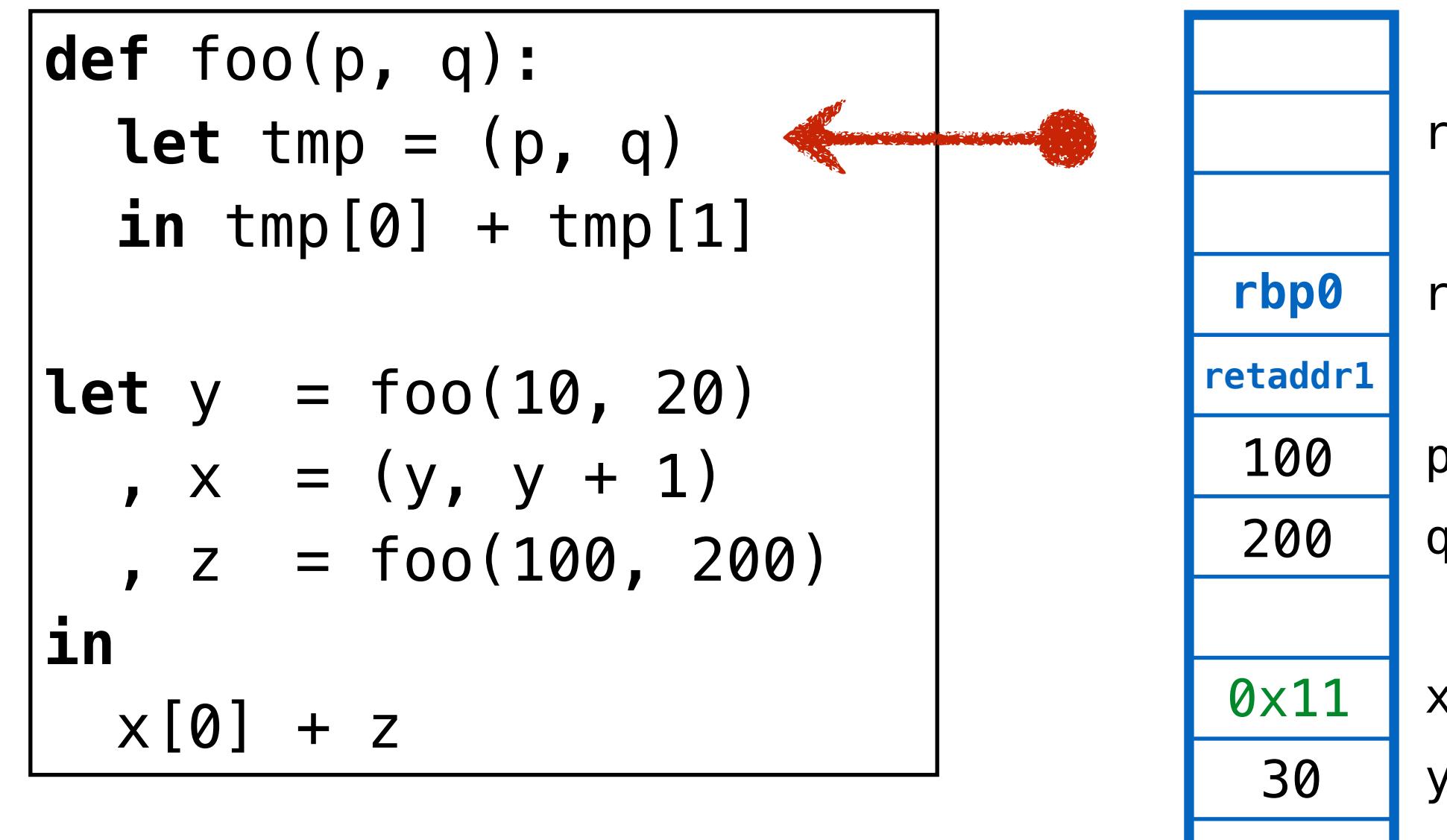
in
x[0] + z
```



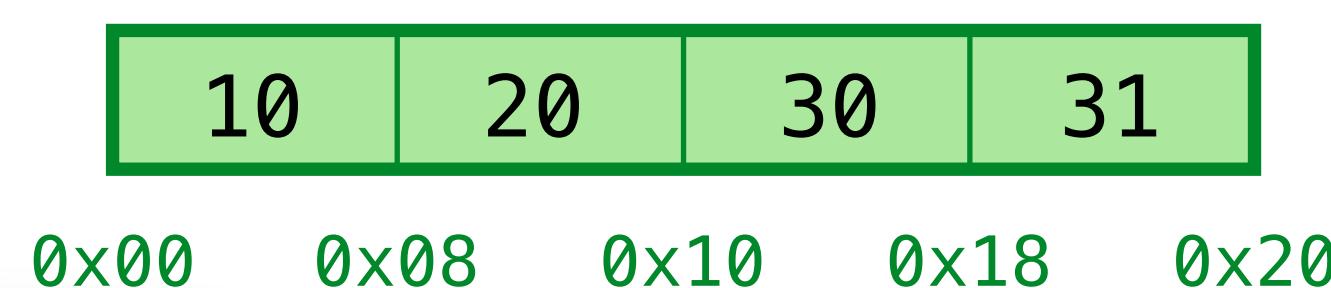
# Lets reclaim & recycle garbage!



### ex3: garbage in the middle (with stack)



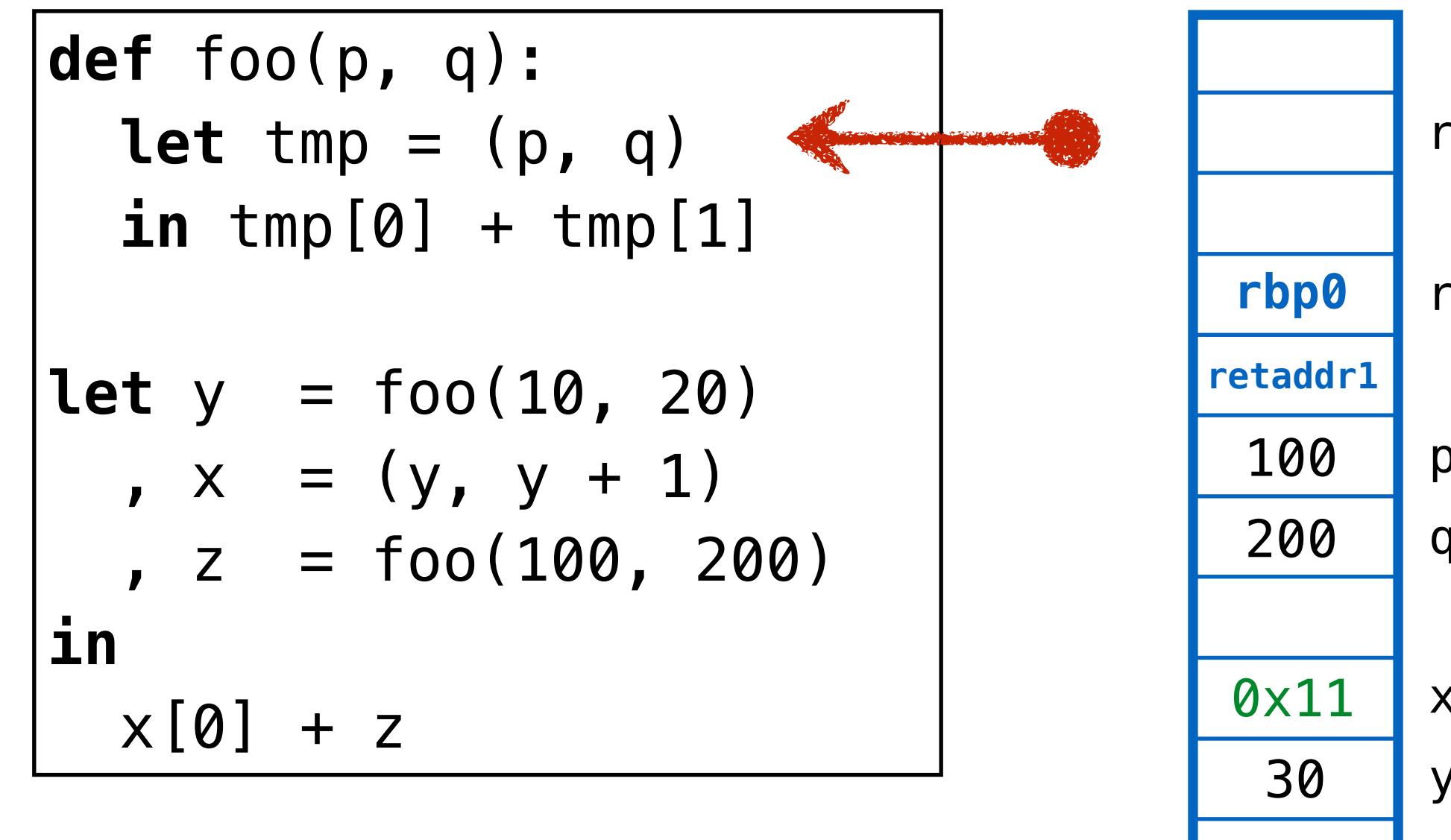
Lets reclaim & recycle garbage!



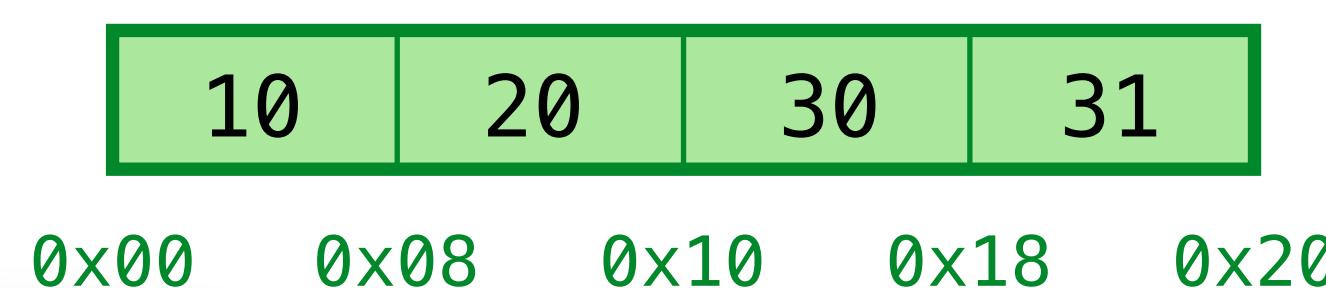
**QUIZ: Which cells are garbage?**

- (A) `0x00, 0x08` (B) `0x08, 0x10` (C) `0x10, 0x18` (D) None (E) All

## ex3: garbage in the middle (with stack)



Lets reclaim & recycle garbage!



**QUIZ: Which cells are garbage?**

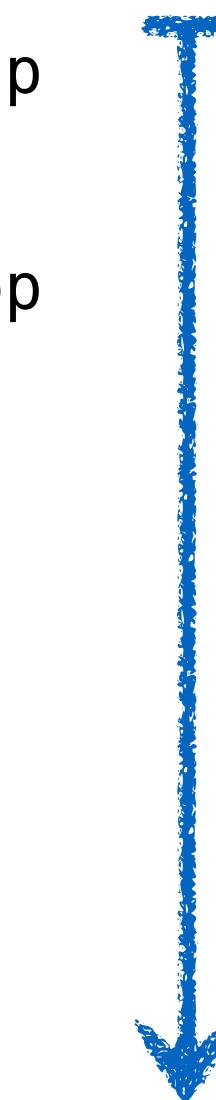
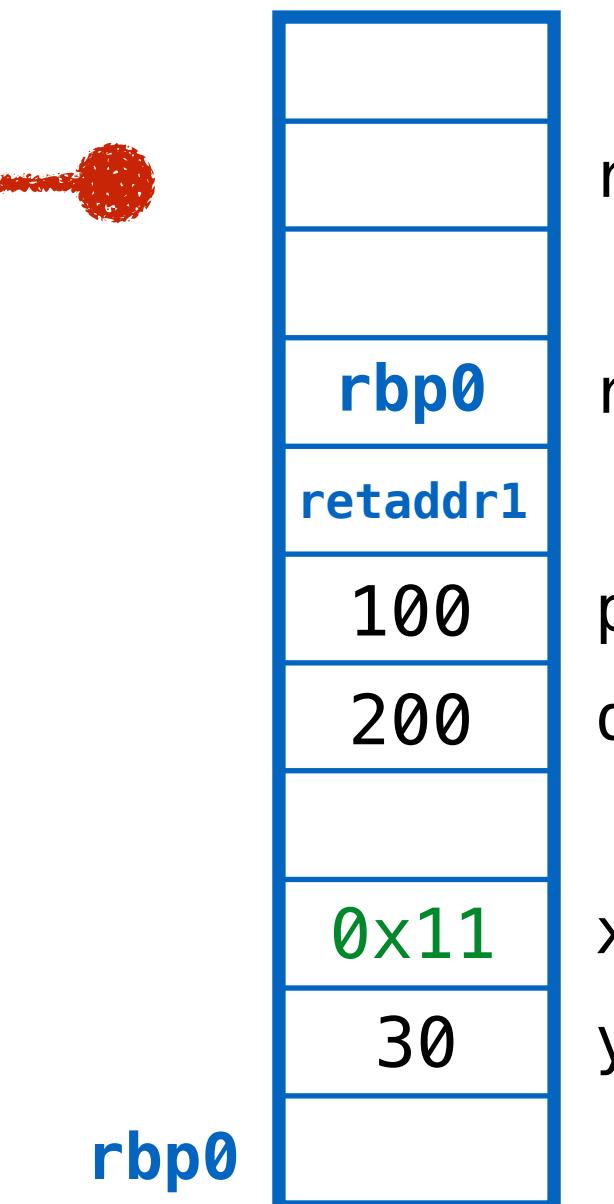
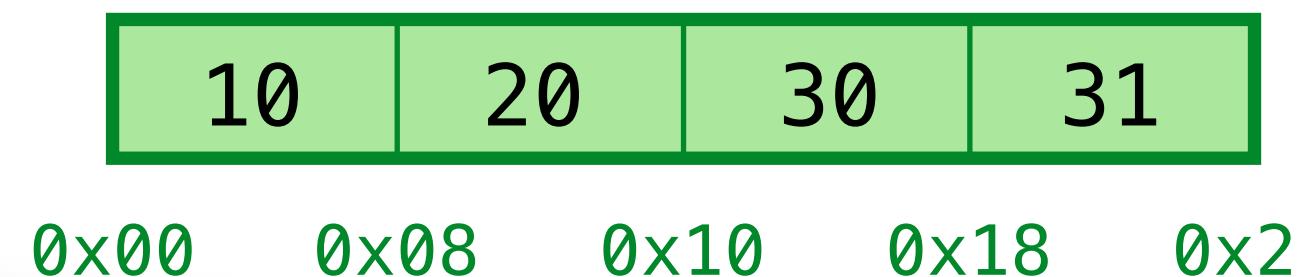
Those that are *not reachable from any stack frame*

## ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

Lets reclaim & recycle garbage!



Traverse Stack  
from top (rsp)  
to bottom (rbp0)  
to mark  
reachable cells.

**QUIZ: Which cells are garbage?**

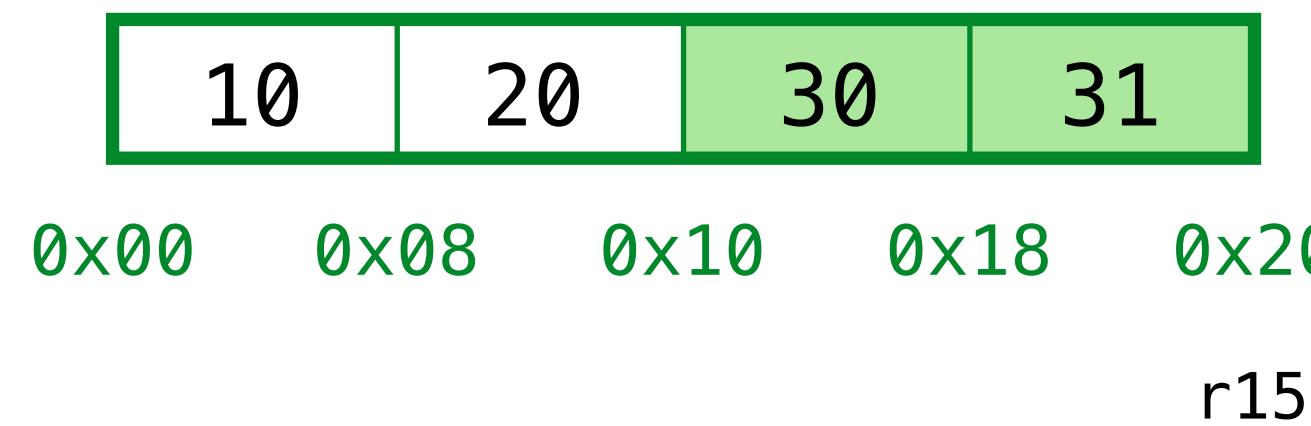
Those that are *not reachable from any stack frame*

## ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

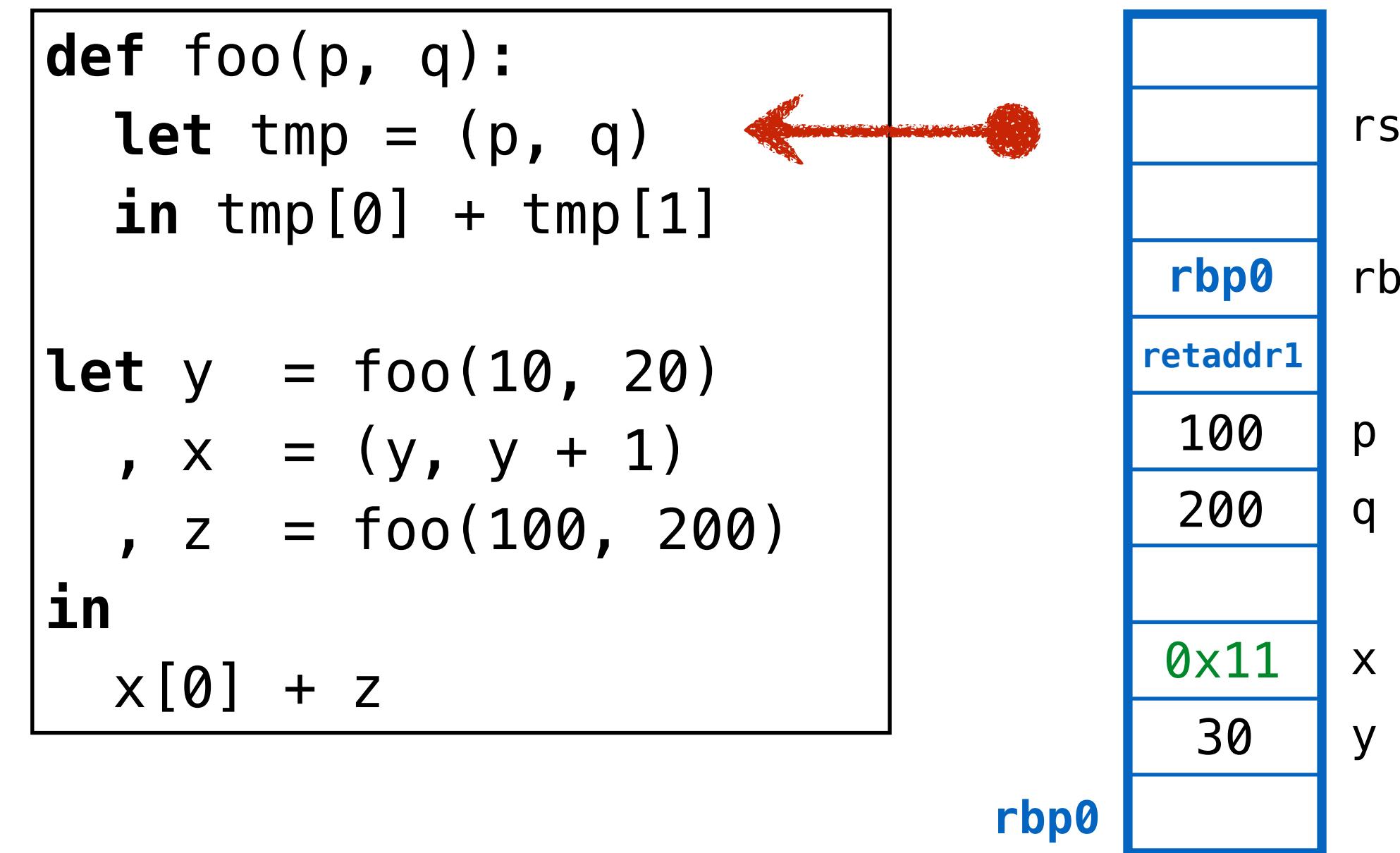
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

Lets reclaim & recycle garbage!



Which cells are garbage?

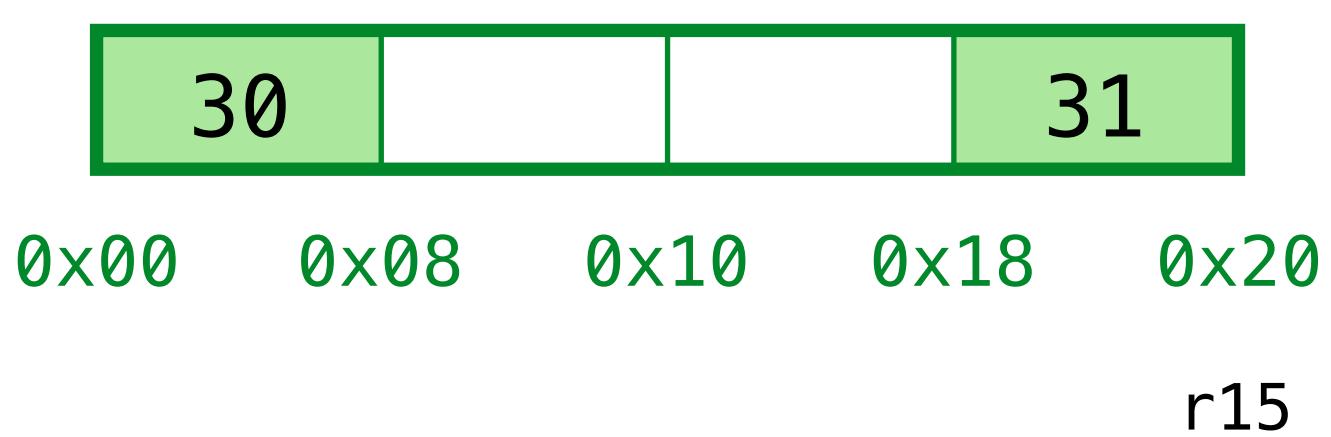
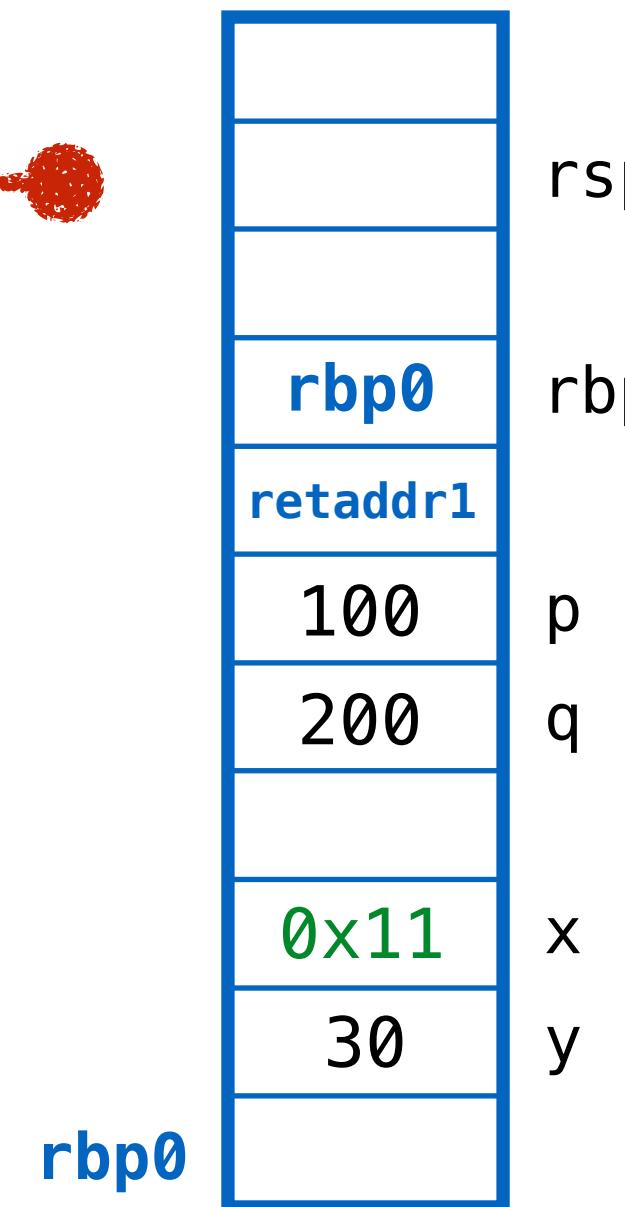
### ex3: garbage in the middle (with stack)



Compact the live cells

# ex3: garbage in the middle (with stack)

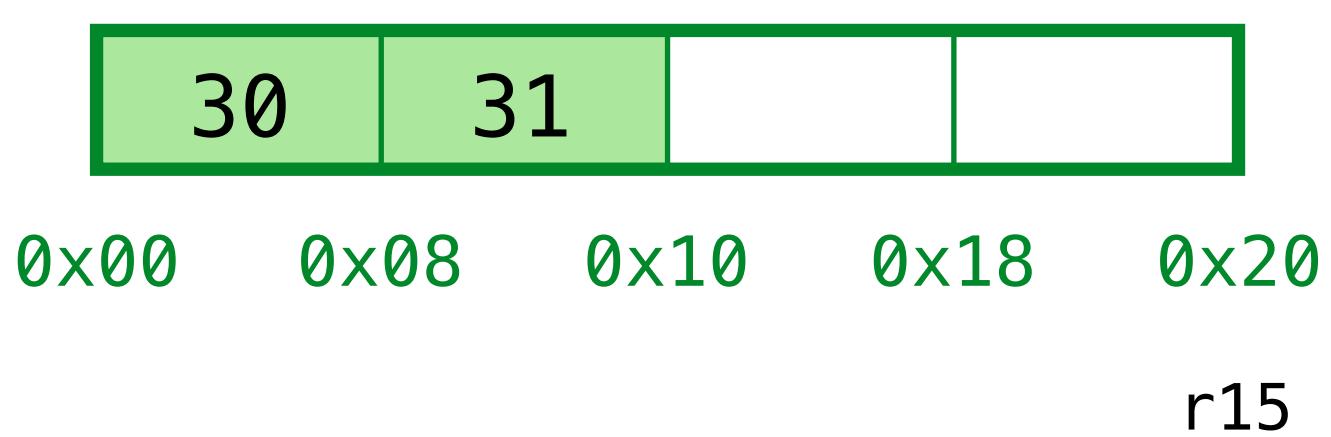
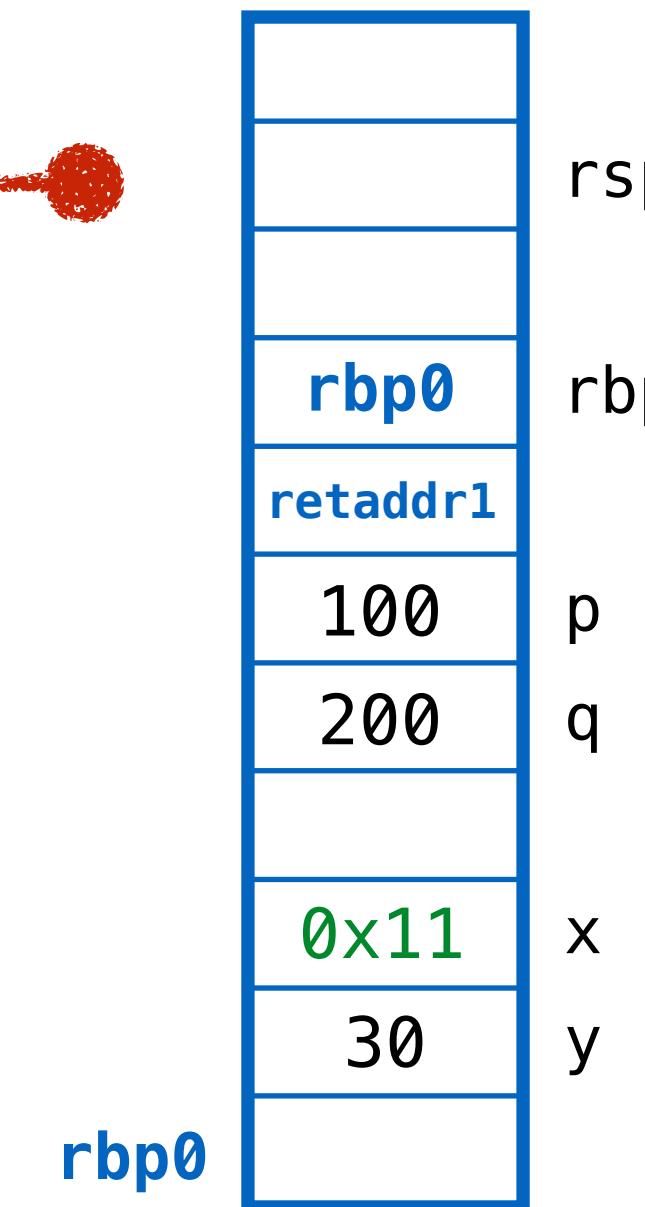
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



# Compact the live cells

# ex3: garbage in the middle (with stack)

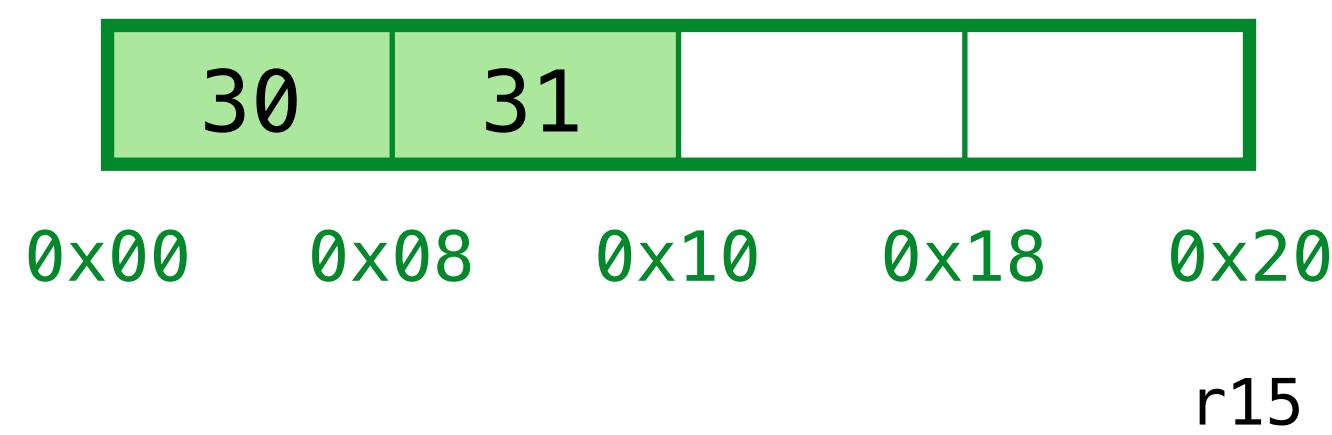
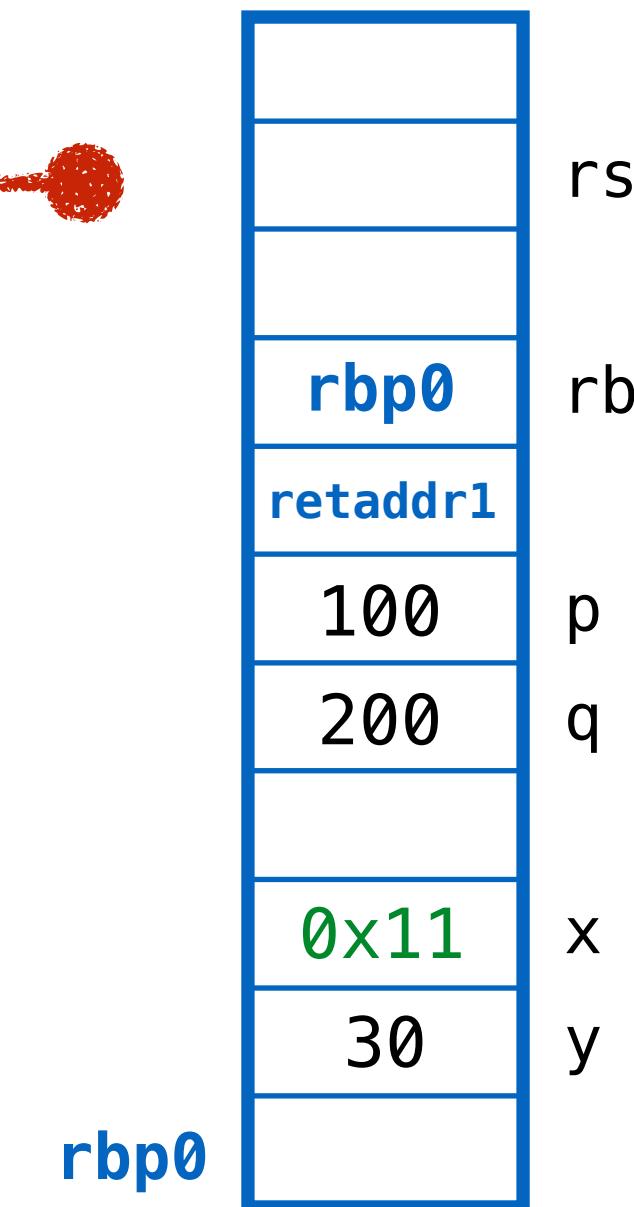
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



# Compact the live cells

# ex3: garbage in the middle (with stack)

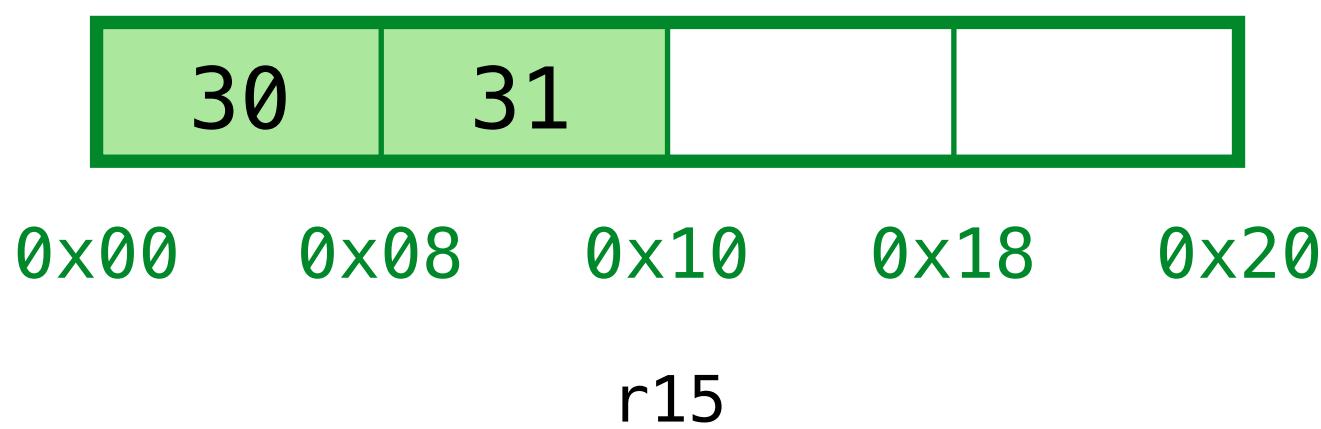
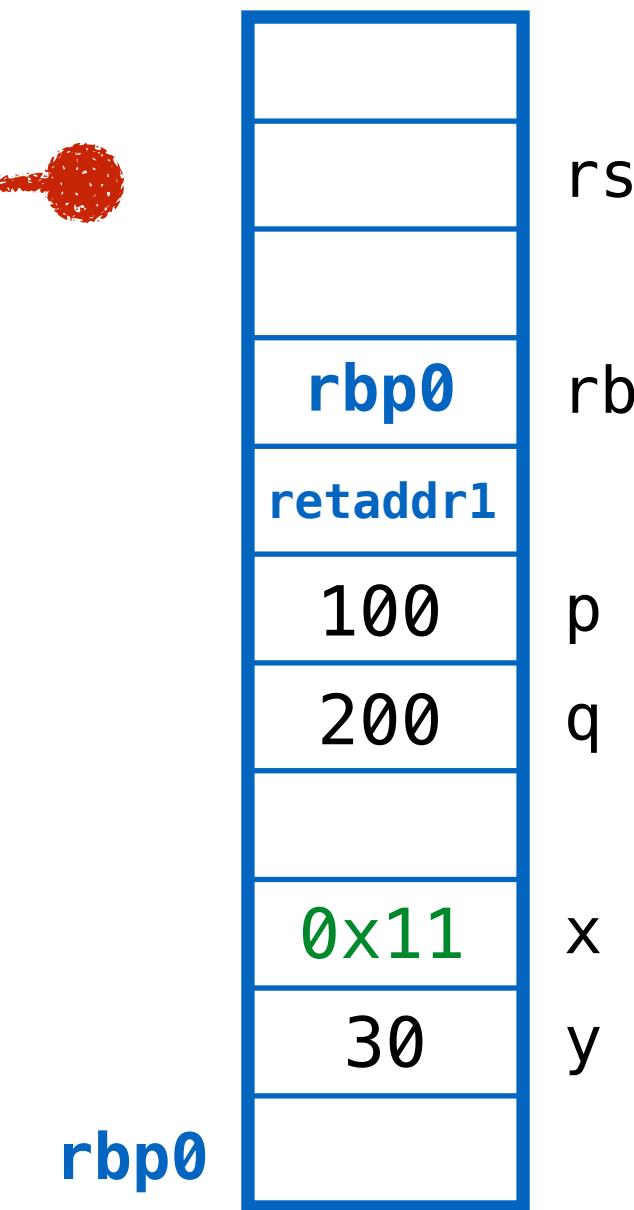
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



# Compact the live cells ... then rewind r15

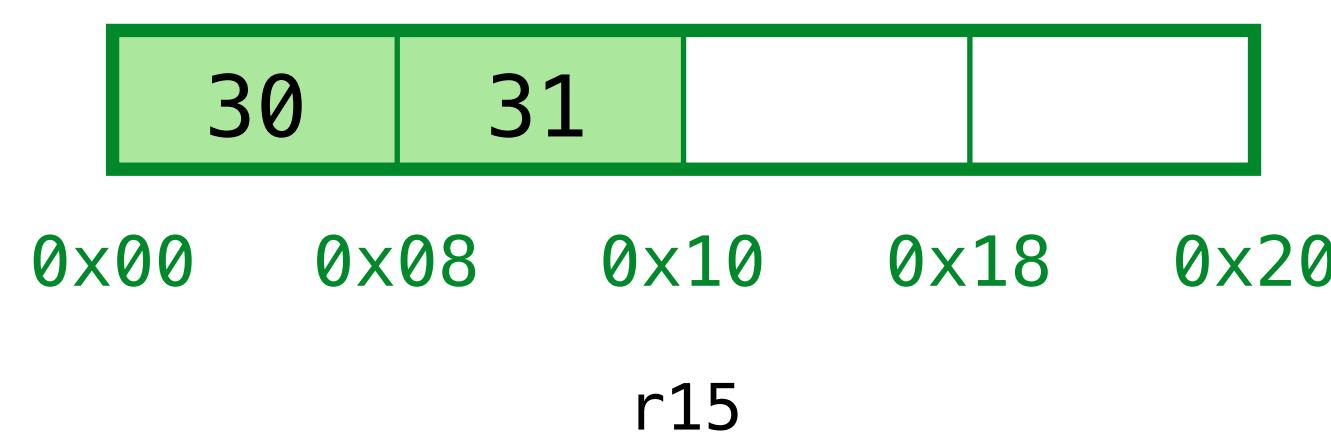
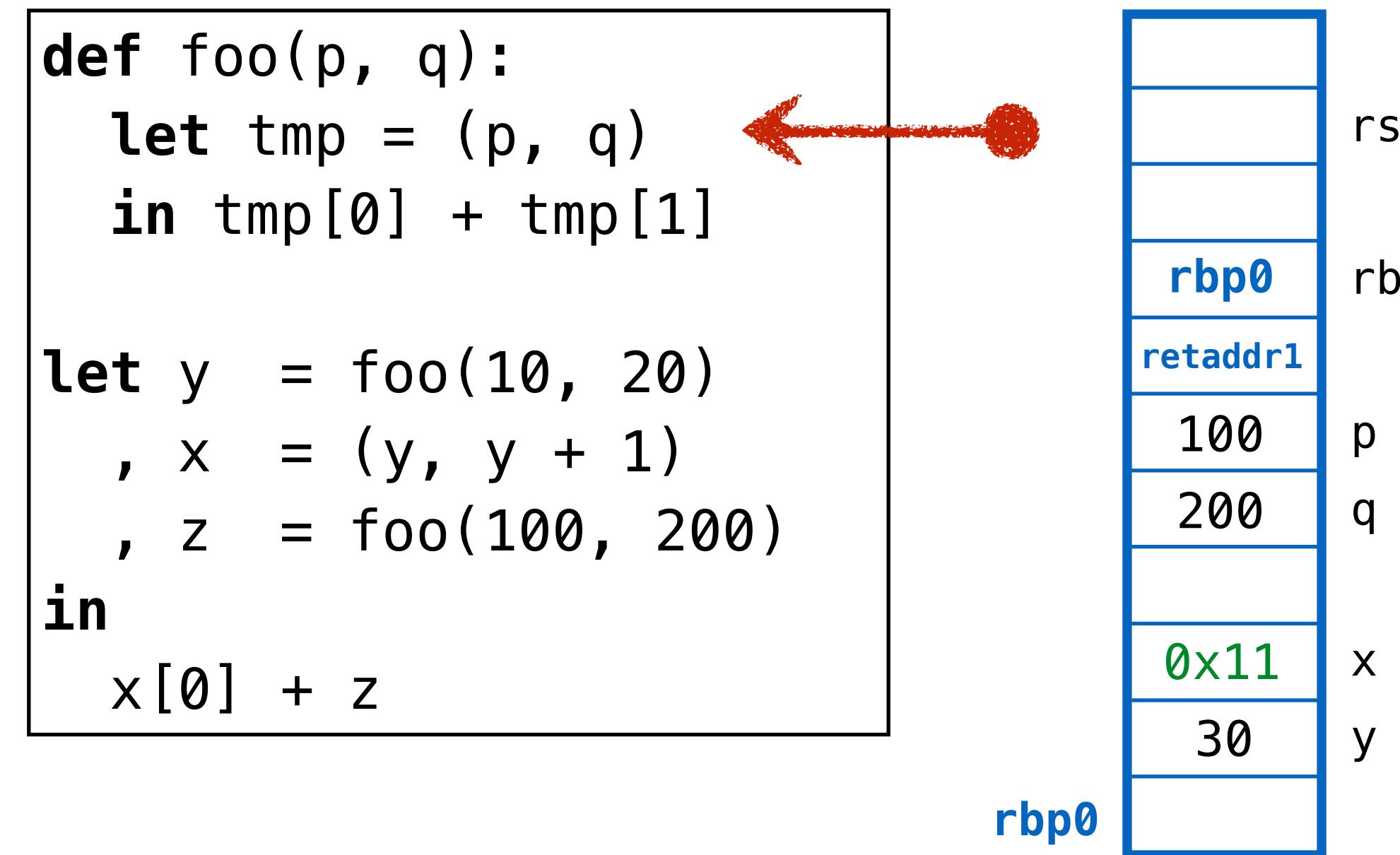
# ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + z
```



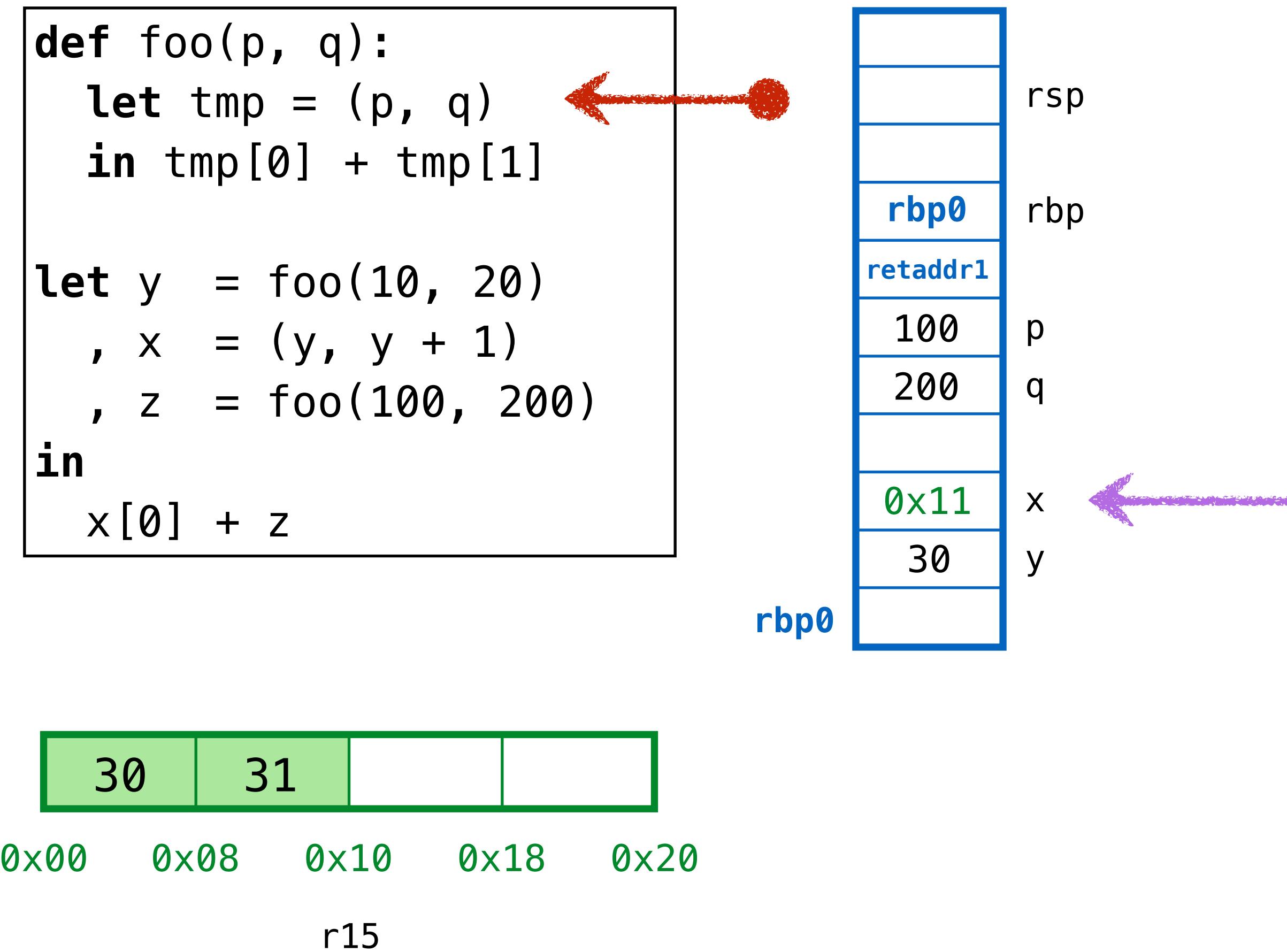
# Compact the live cells ... then rewind r15

## ex3: garbage in the middle (with stack)



Problem???

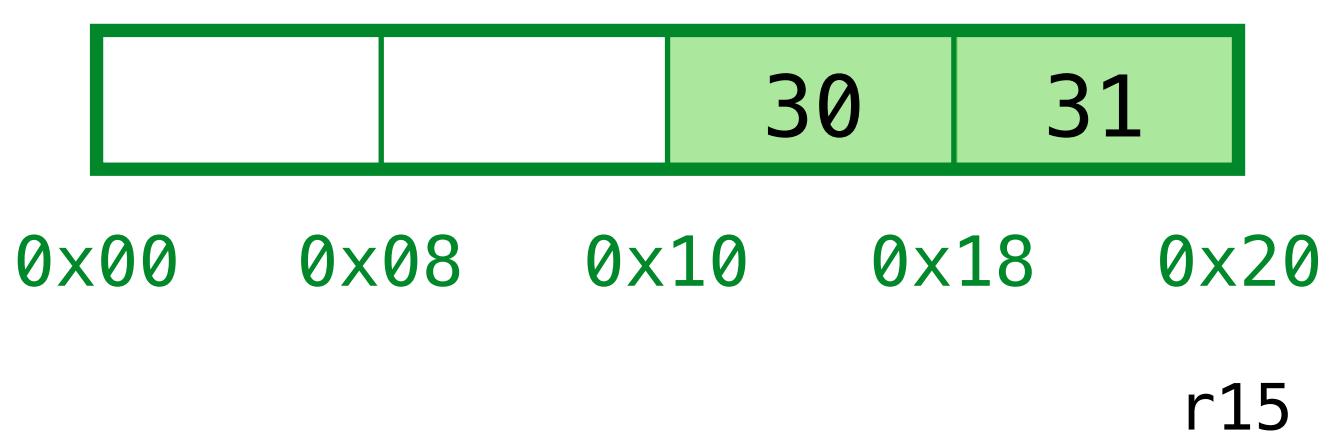
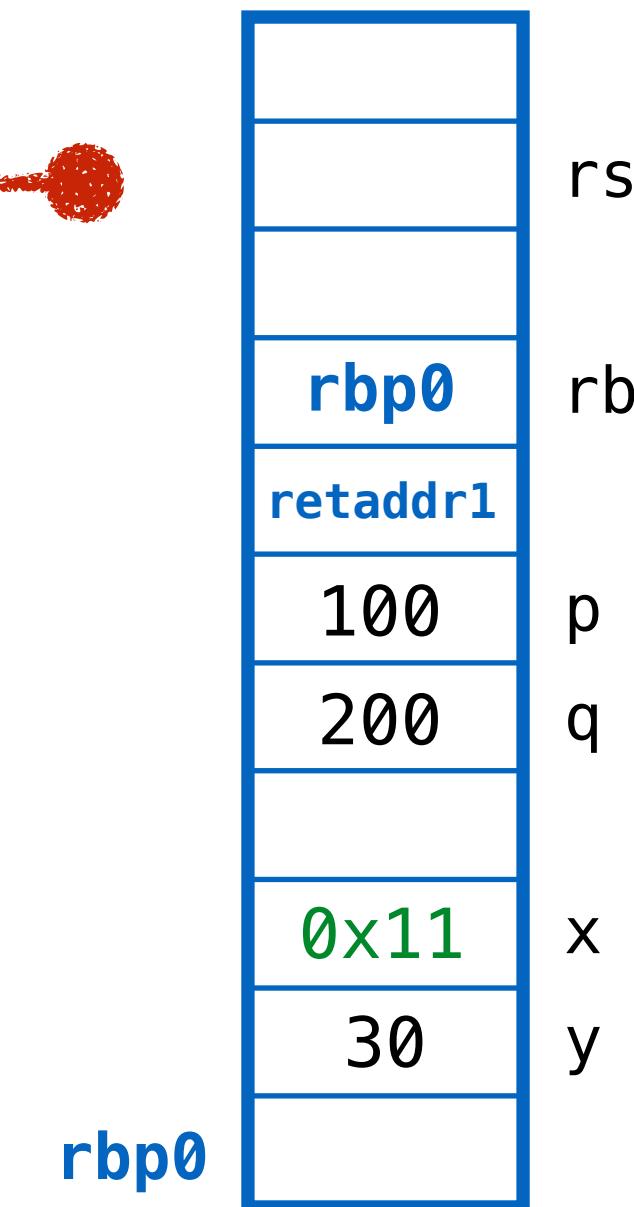
### ex3: garbage in the middle (with stack)



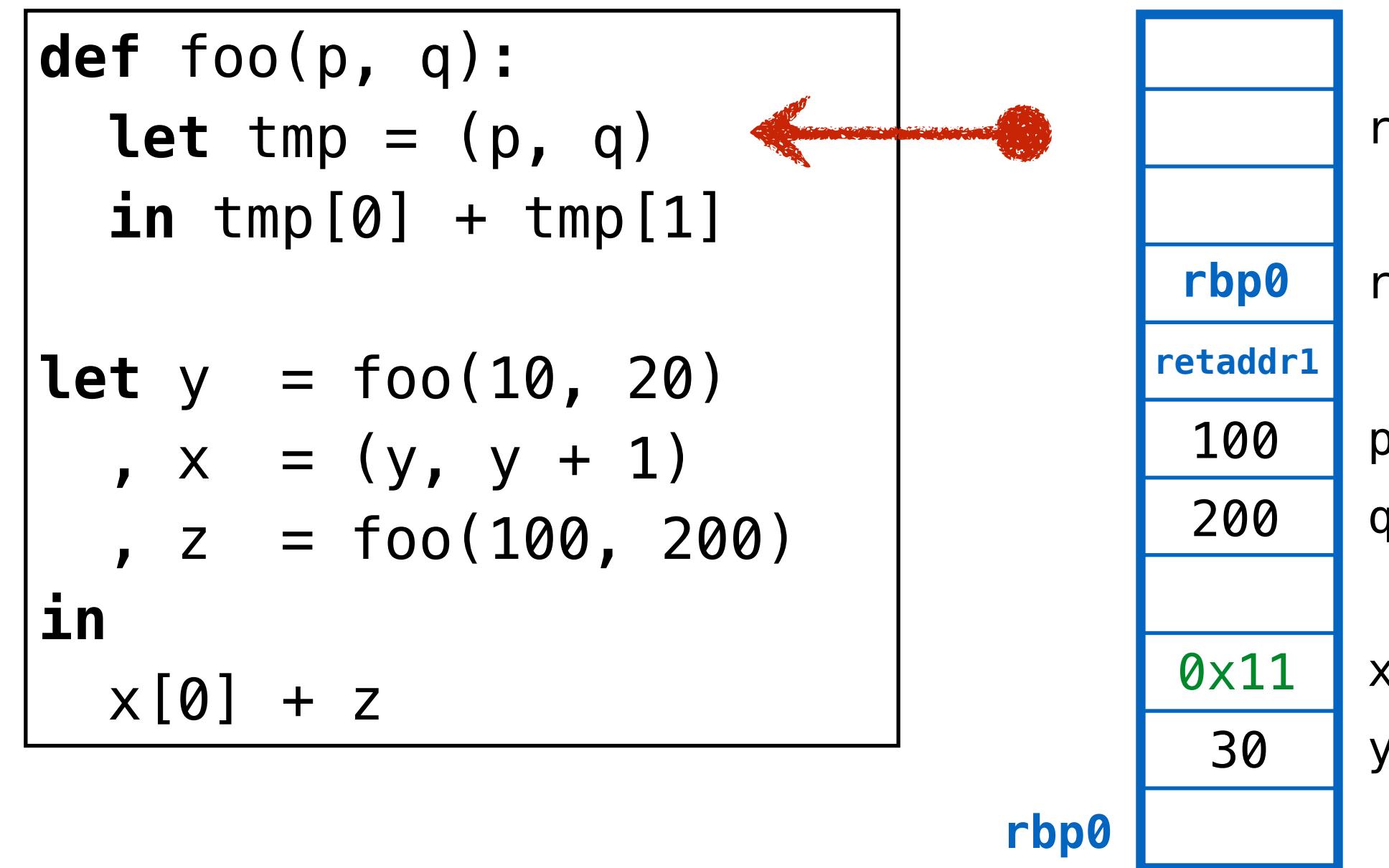
Problem! Have to REDIRECT existing pointers

# ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

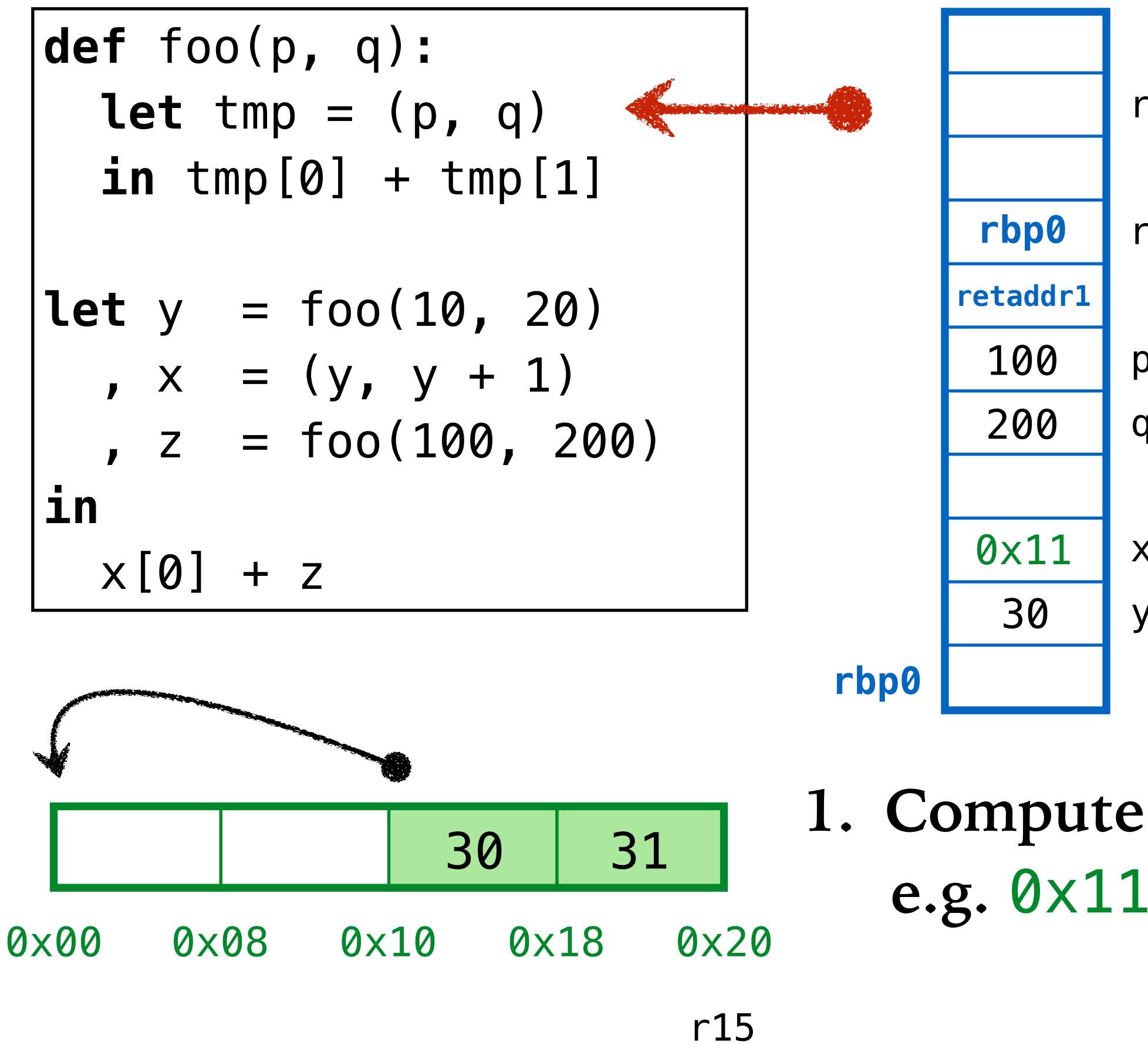


### ex3: garbage in the middle (with stack)

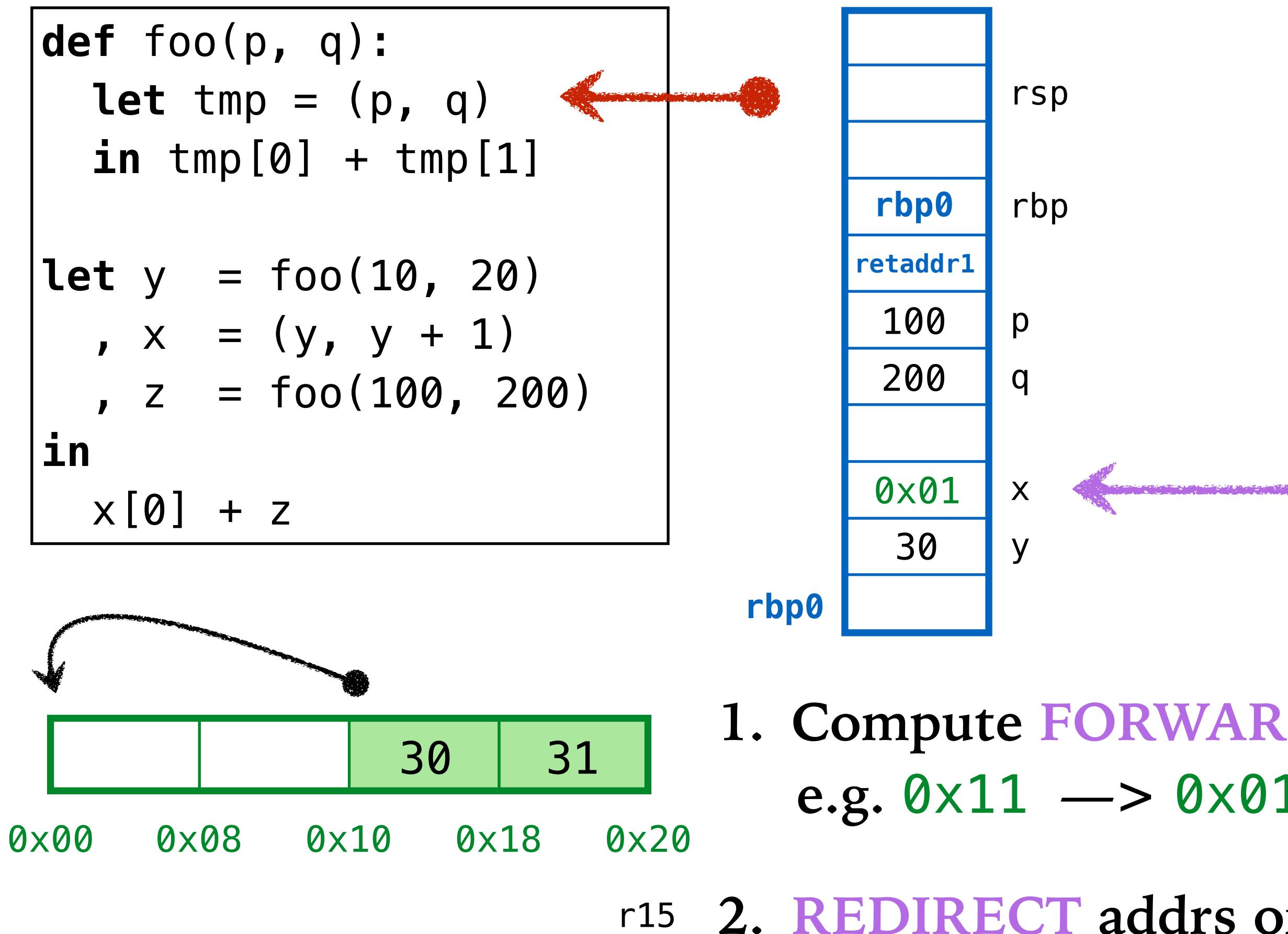


1. Compute **FORWARD** addrs  
(i.e. new compacted addrs)

### ex3: garbage in the middle (with stack)

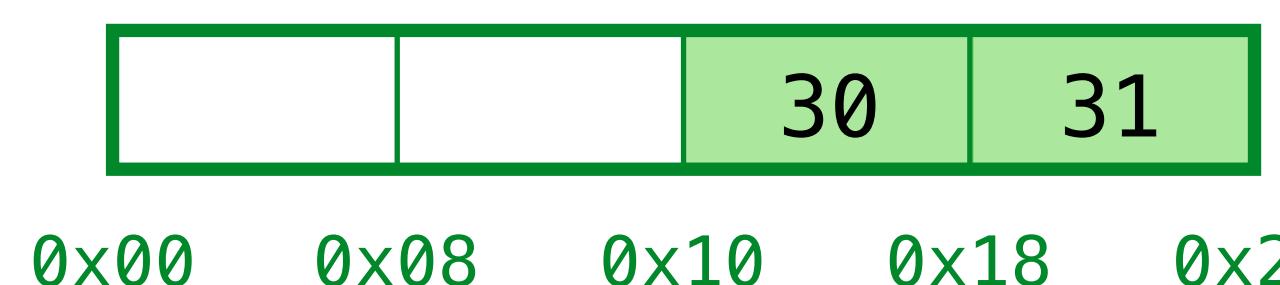
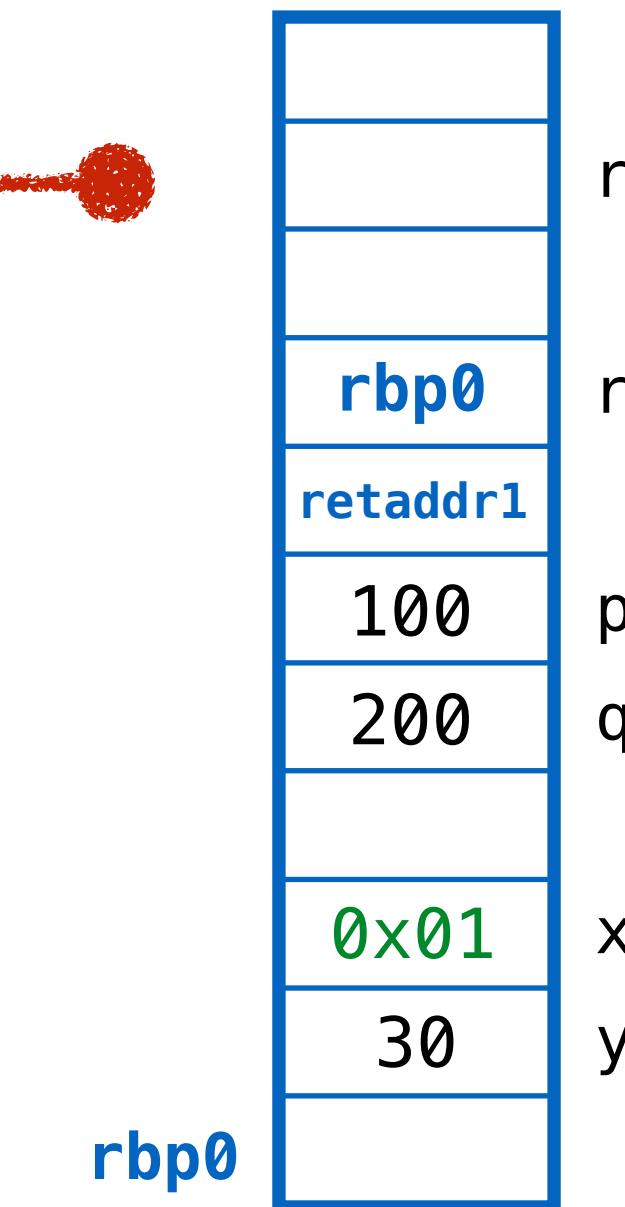


### ex3: garbage in the middle (with stack)



# ex3: garbage in the middle (with stack)

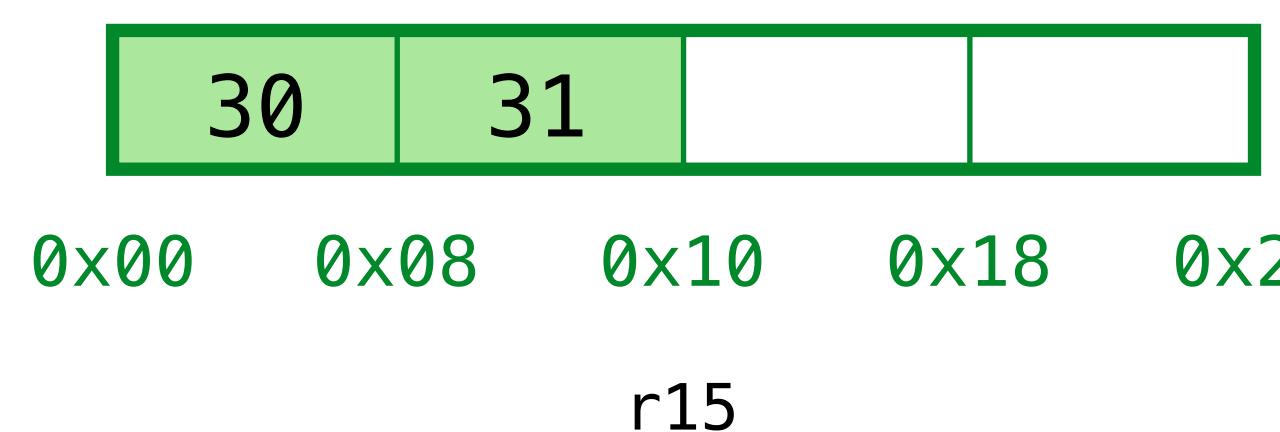
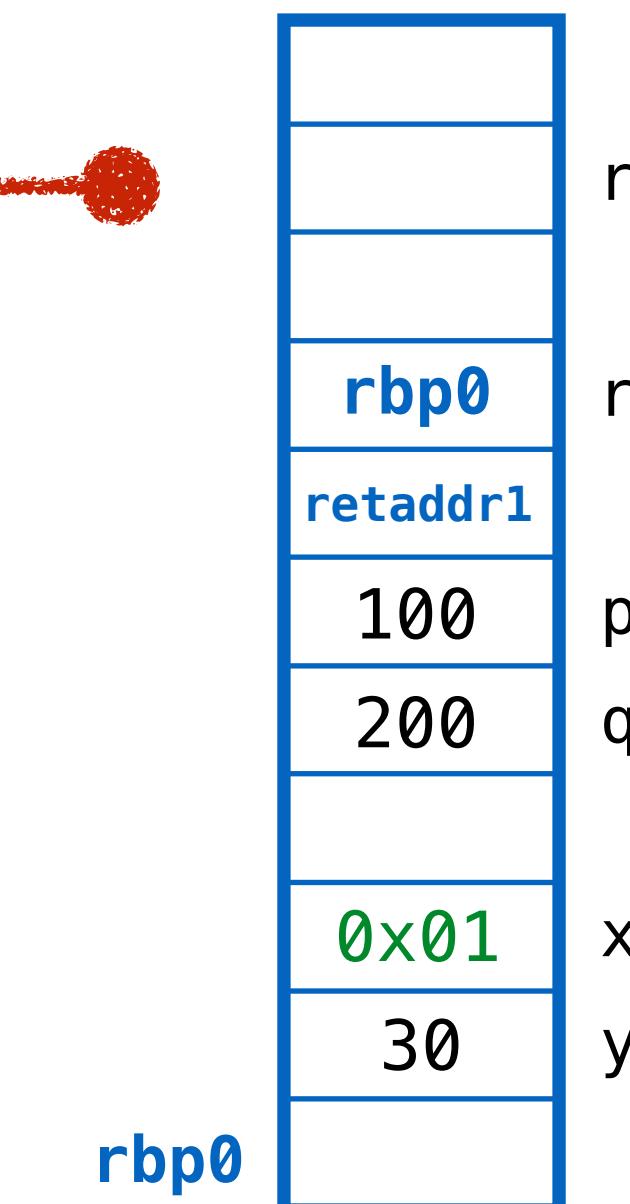
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



1. Compute FORWARD addrs  
e.g.  $0x11 \rightarrow 0x01$
  2. REDIRECT addrs on stack
  3. COMPACT cells on heap

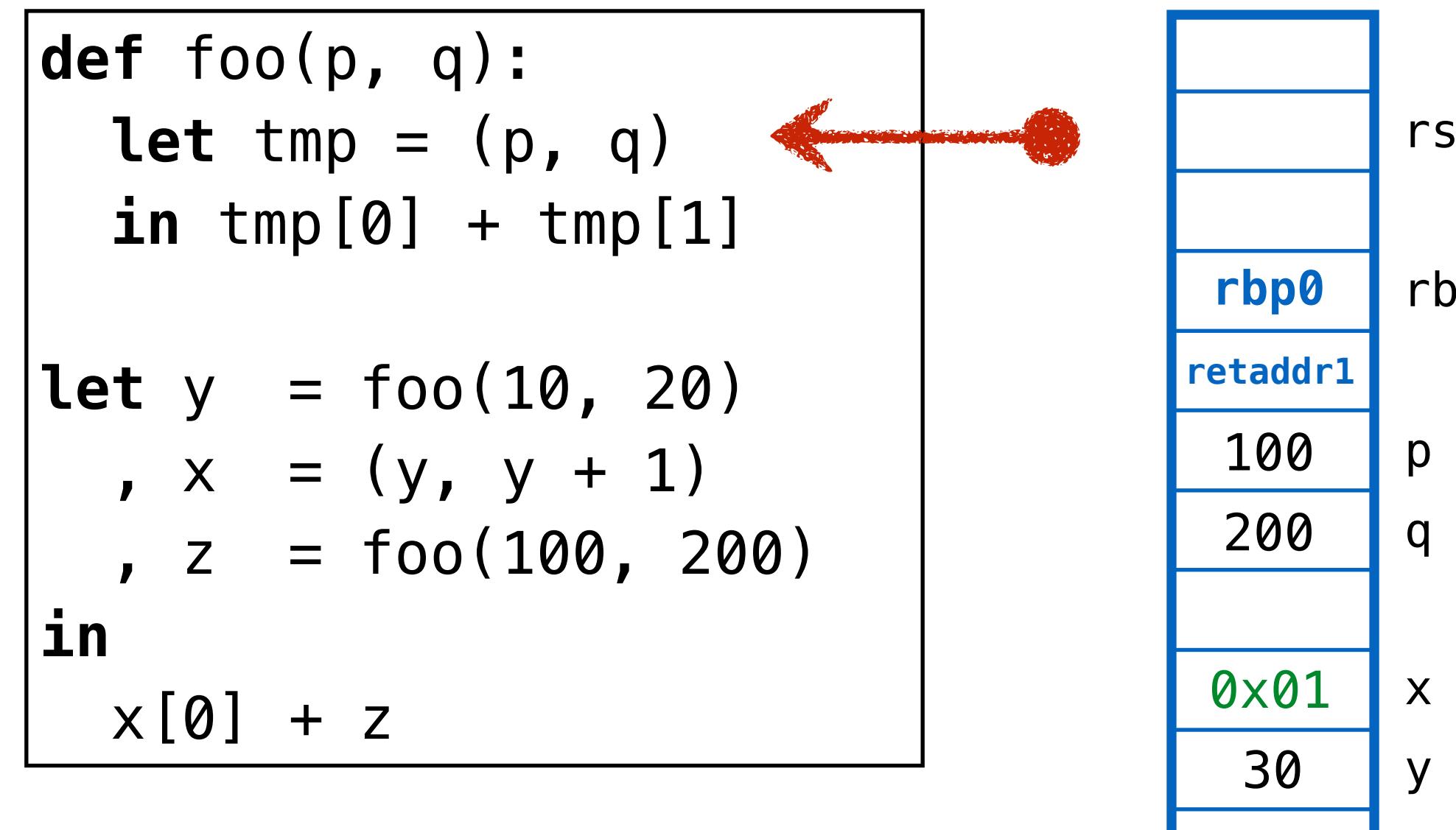
# ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

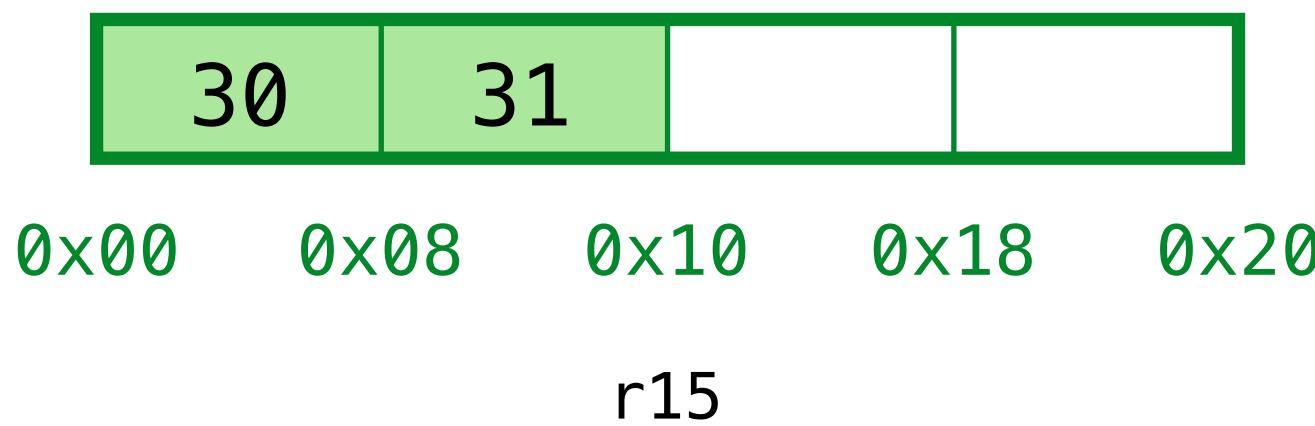


1. Compute FORWARD addrs  
e.g.  $0x11 \rightarrow 0x01$
  2. REDIRECT addrs on stack
  3. COMPACT cells on heap

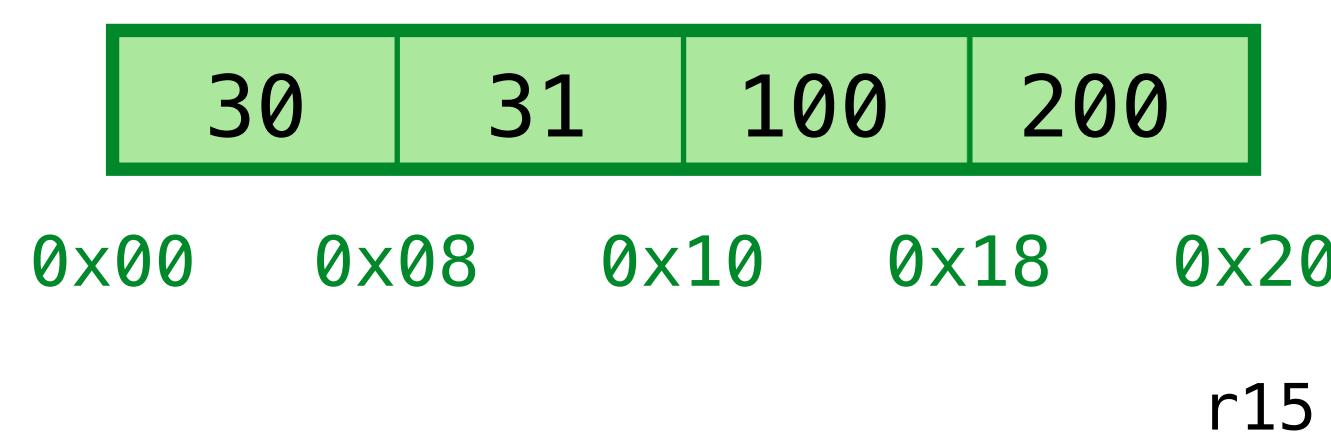
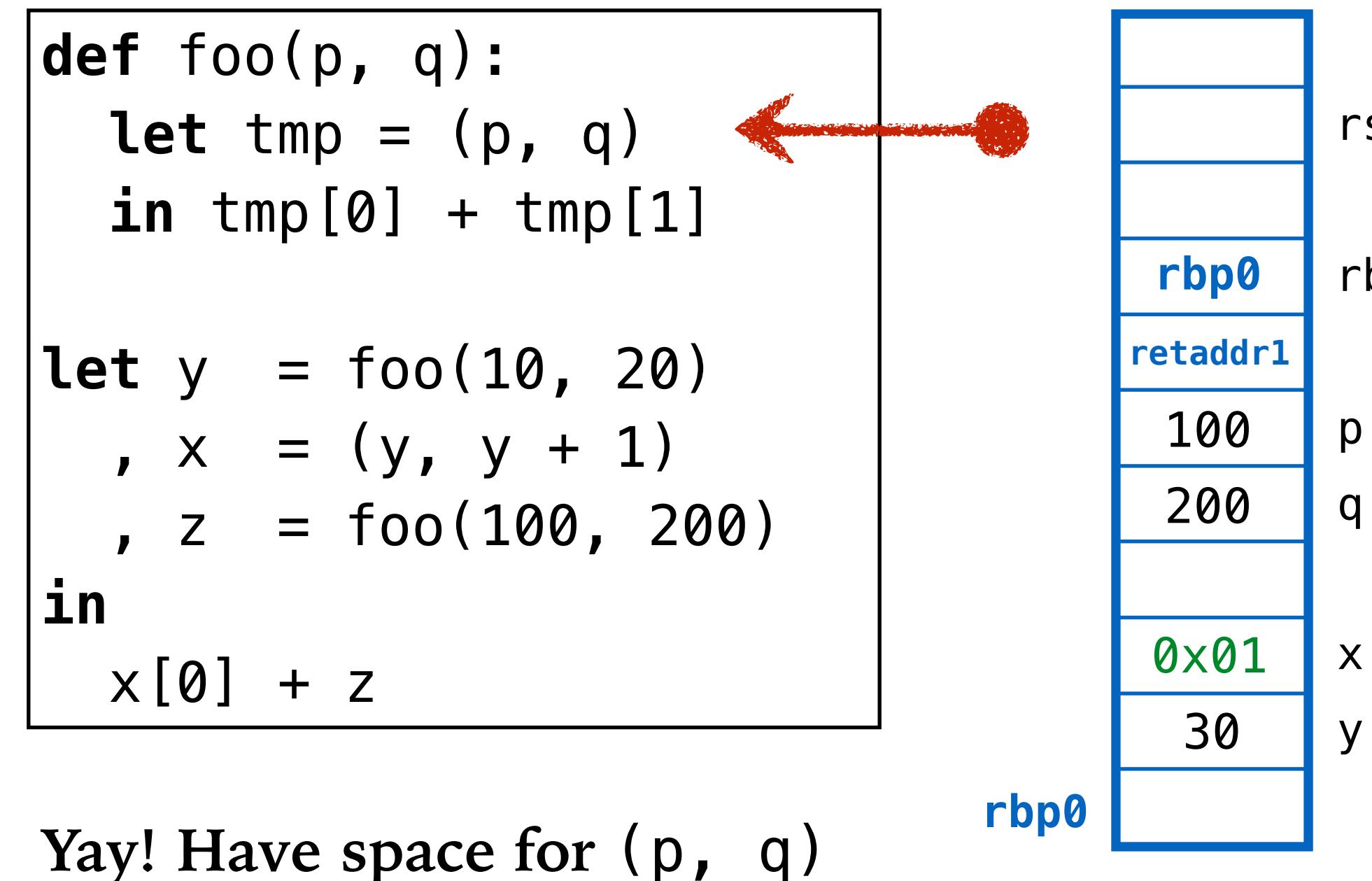
### ex3: garbage in the middle (with stack)



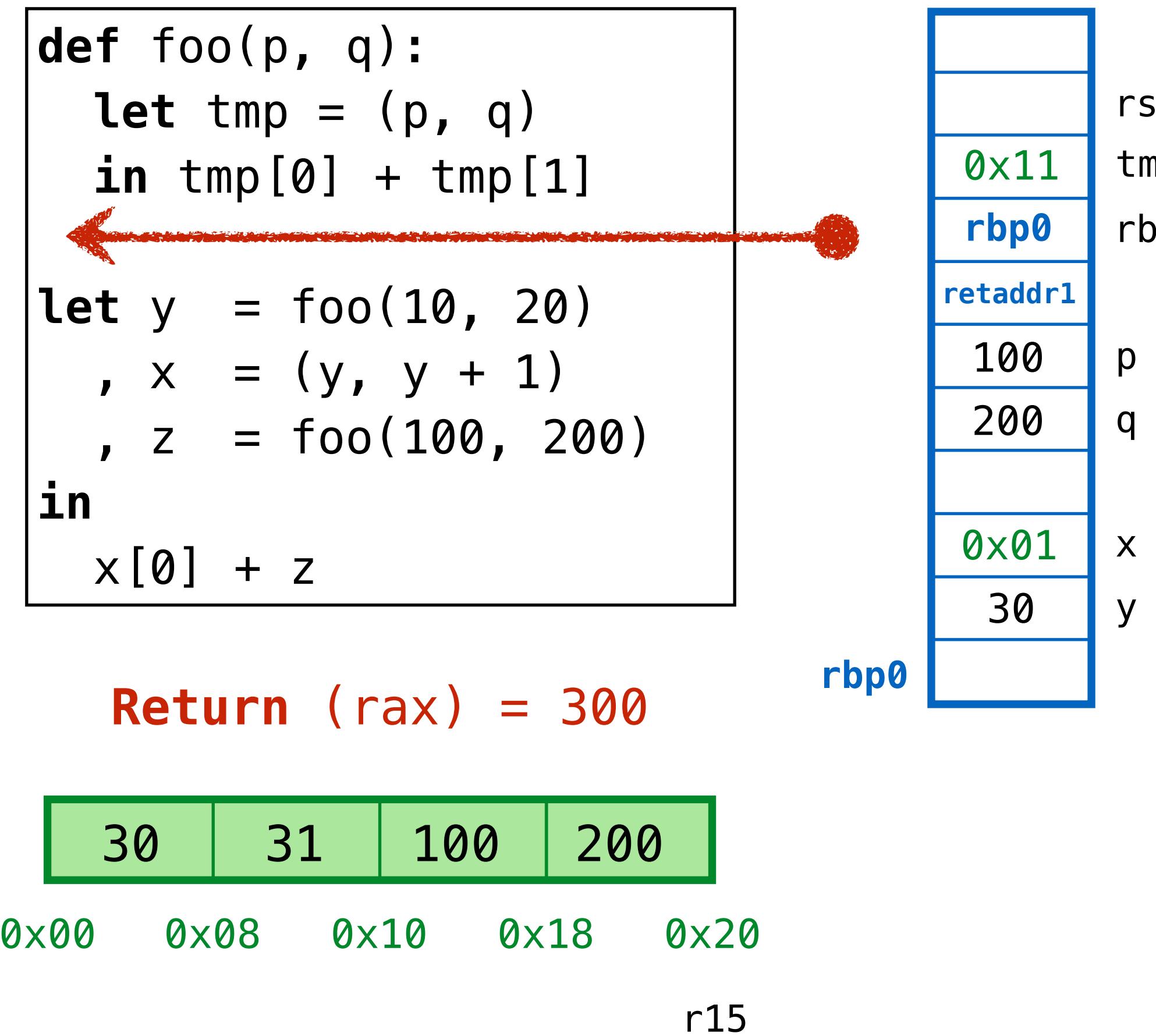
Yay! Have space for (p, q)



## ex3: garbage in the middle (with stack)



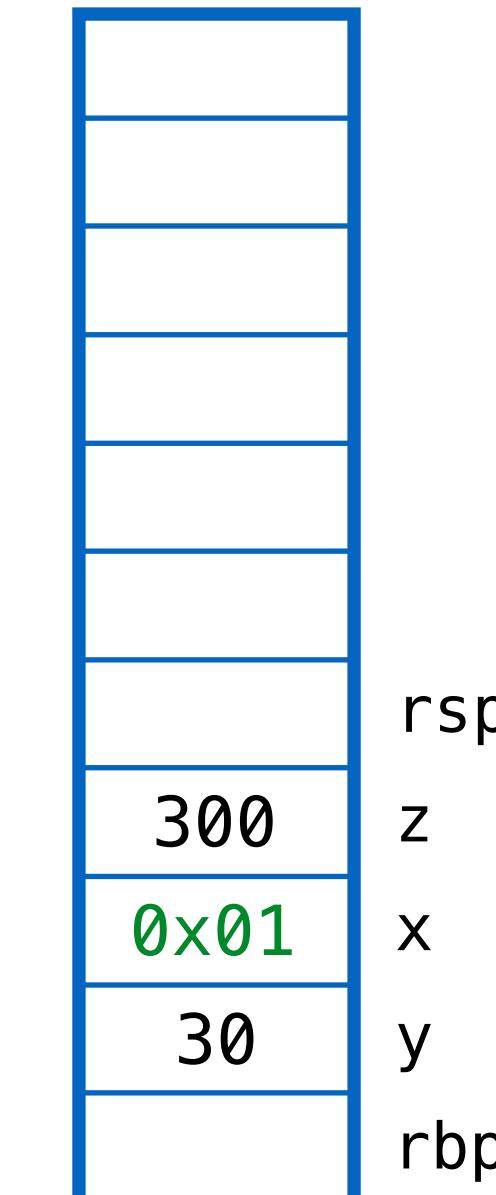
## ex3: garbage in the middle (with stack)



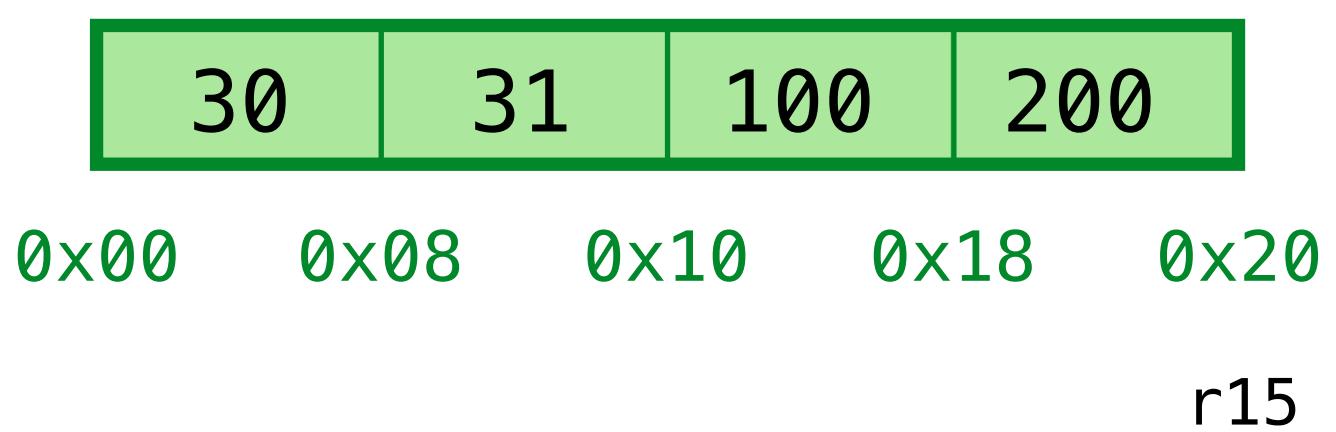
## ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + z
```



**Return (rax) = 300**

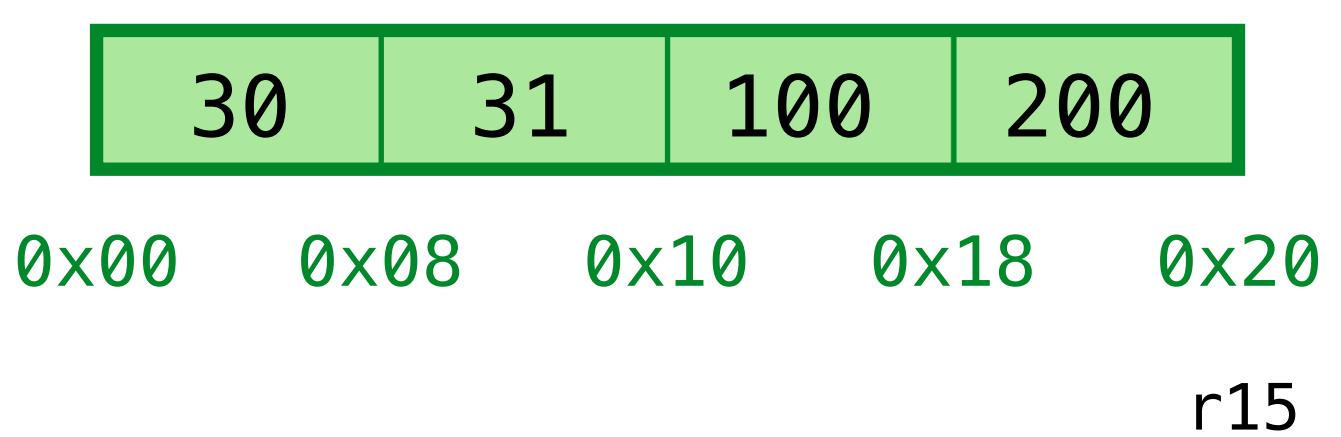
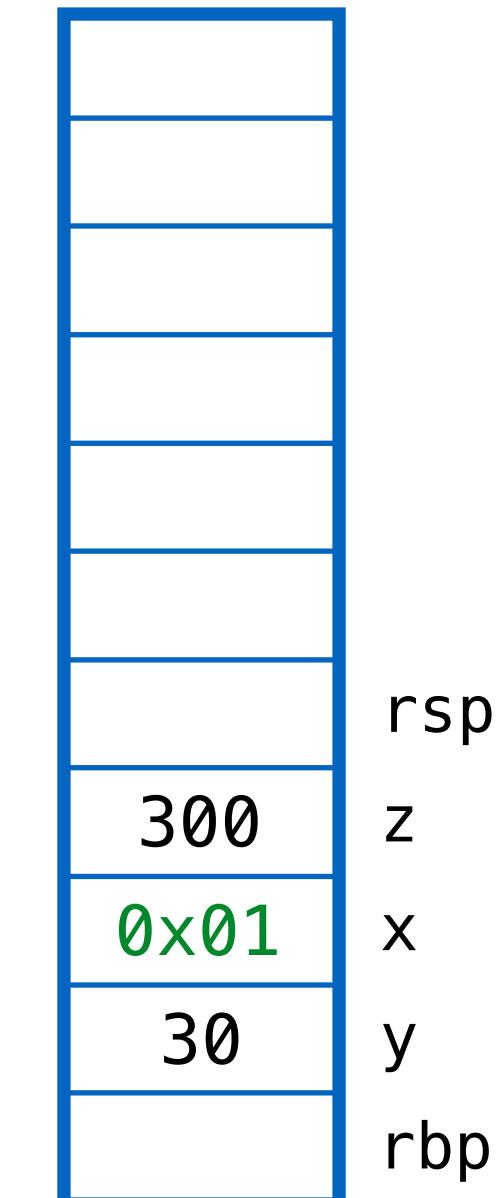


r15

## ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

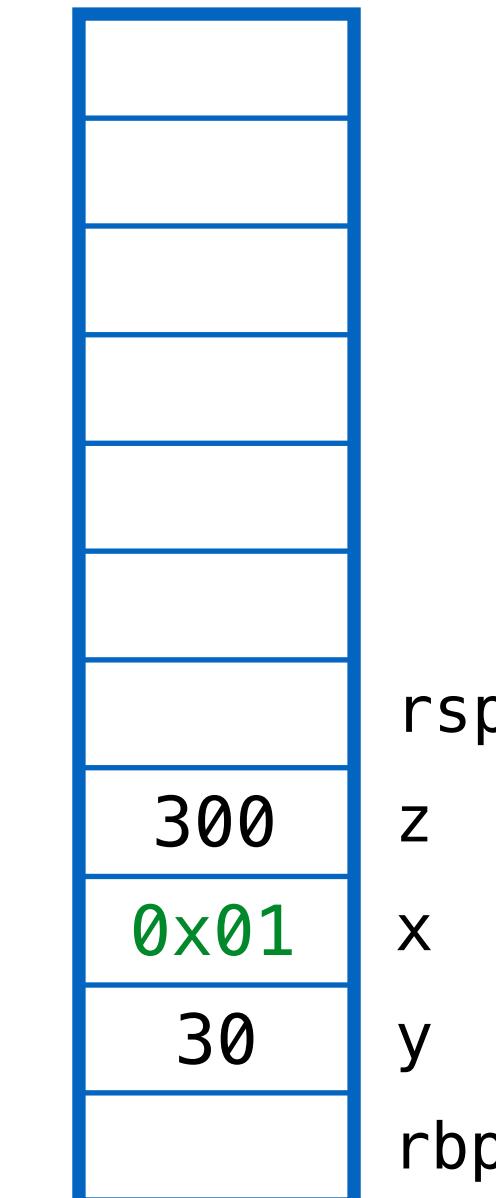
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



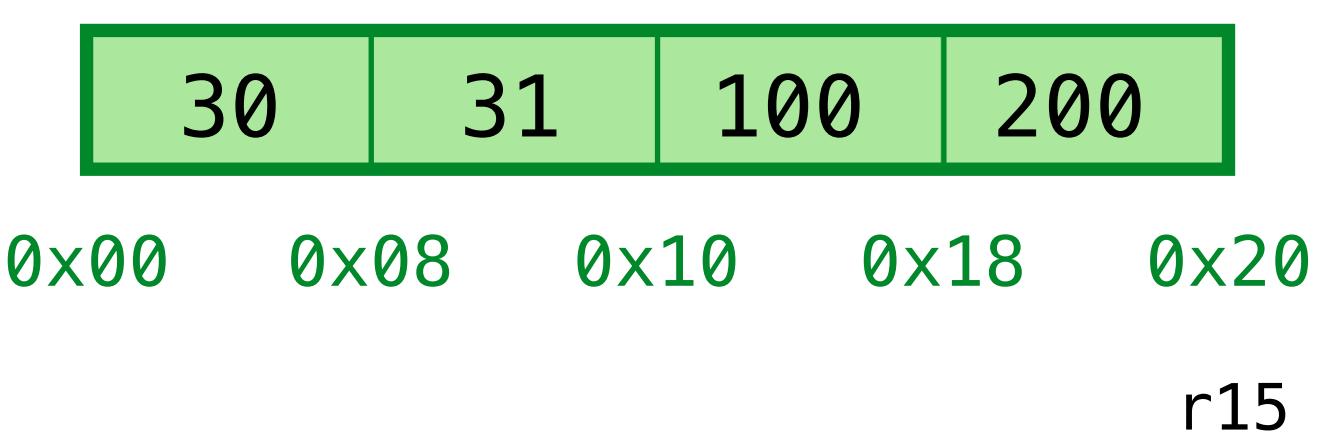
### ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + z
```



**Return (rax) = 30+300 = 330**



Garter / GC

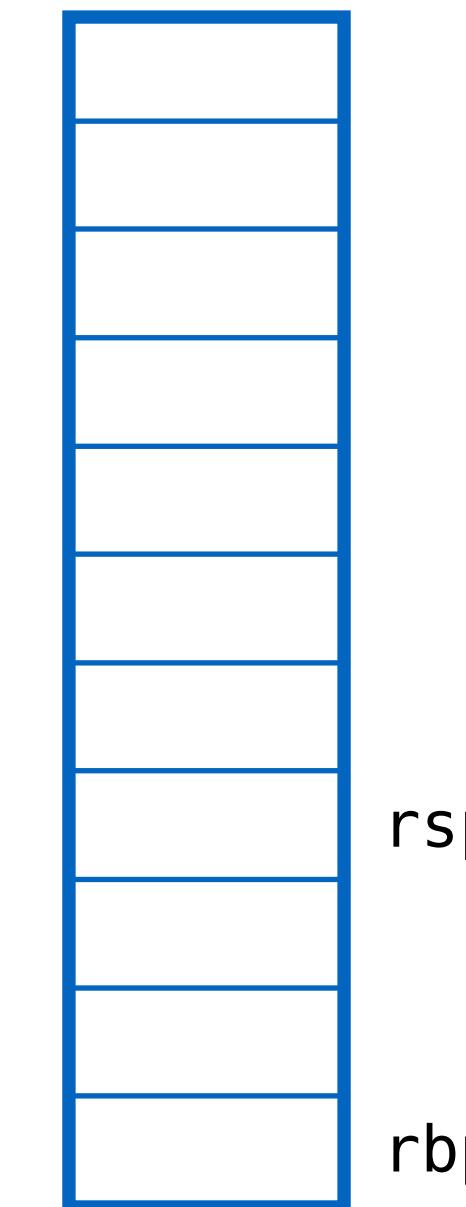
Example 4

# ex4: recursive data

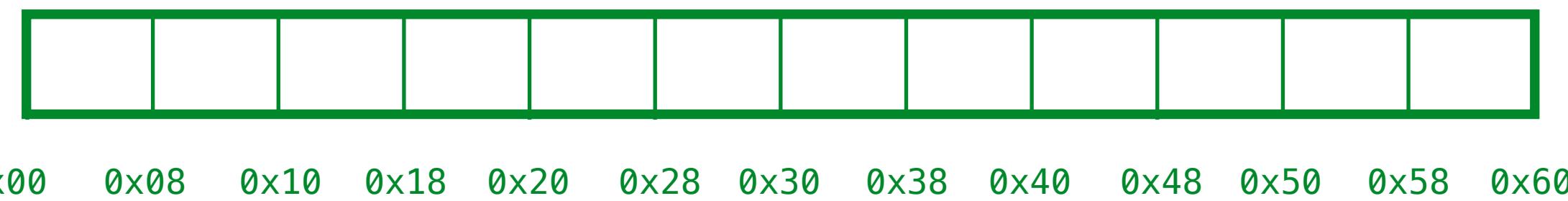
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 = ←
        let l1 = range(0, 3)
        in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



r15



## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

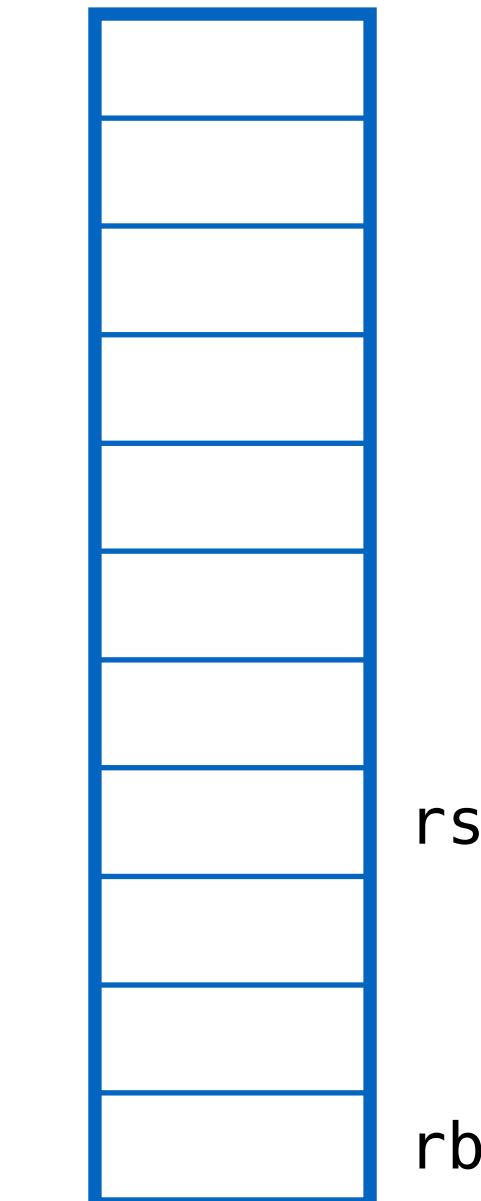
let t1 =
    let l1 = range(0, 3) ←
        in sum(l1)
    , l  = range(t1, t1 + 3)
in
(1000, l)
```

call range(0, 3)

r15



0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

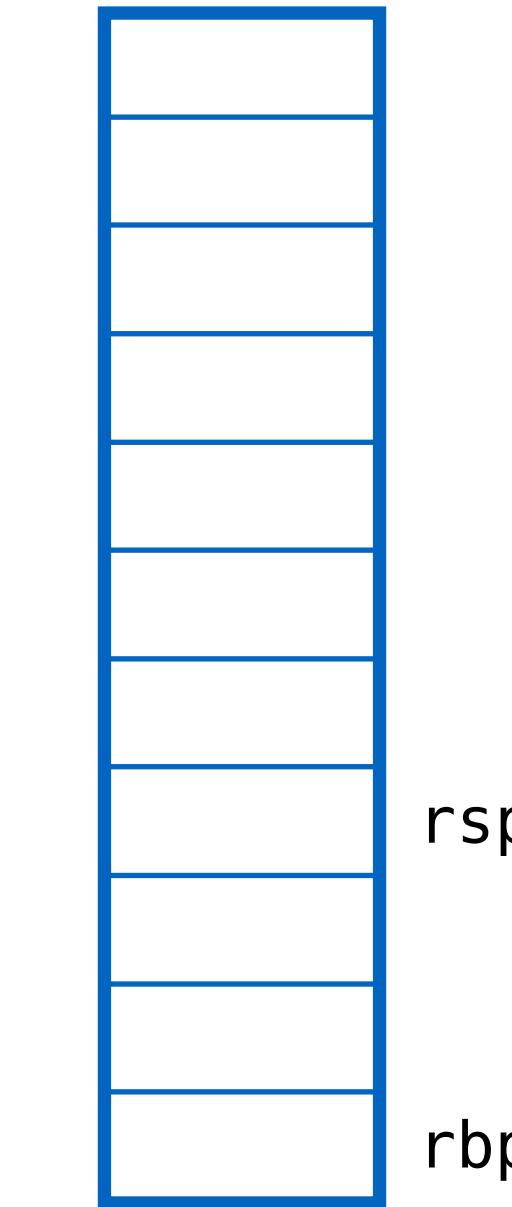


## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3) ←
        in sum(l1)
    , l  = range(t1, t1 + 3)
in
(1000, l)
```



**QUIZ:** What is heap when `range(0, 3)` returns?

r15

(A)

0	0x11	1	0x21	2	false								
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60	

r15

(B)

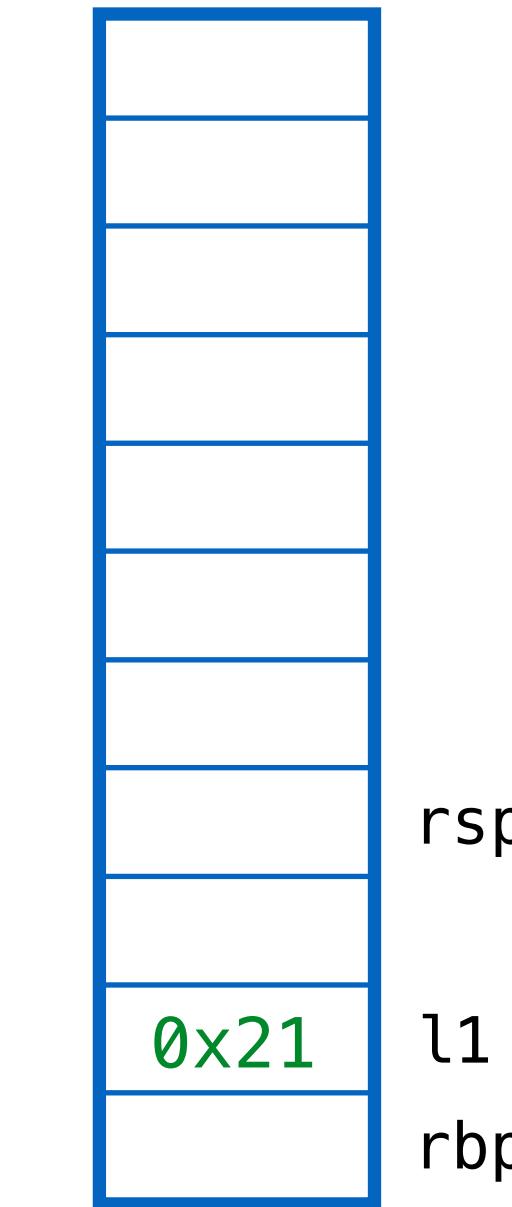
2	false	1	0x01	0	0x11								
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60	

## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15

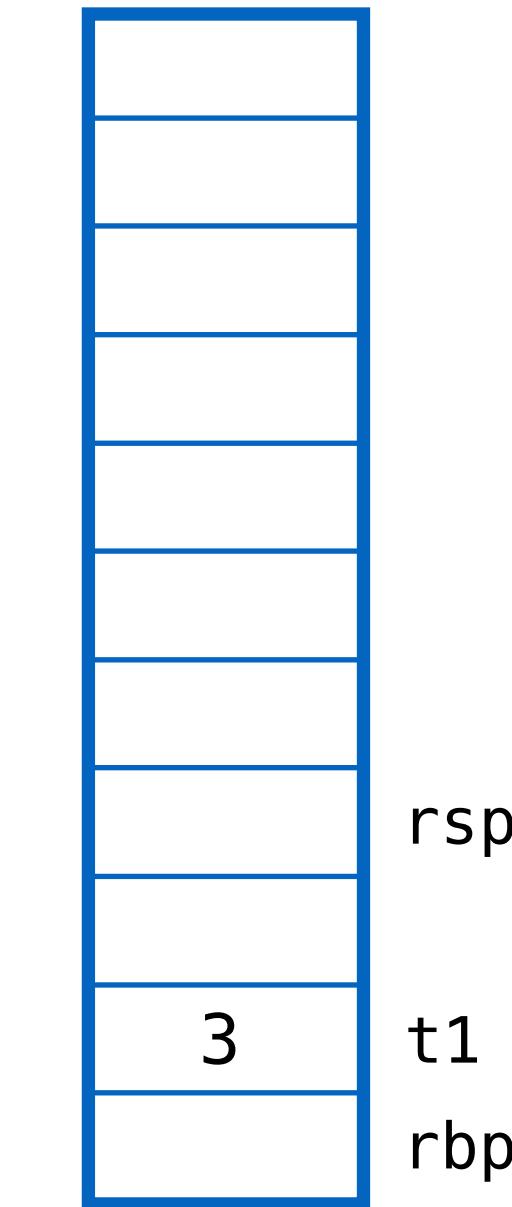
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

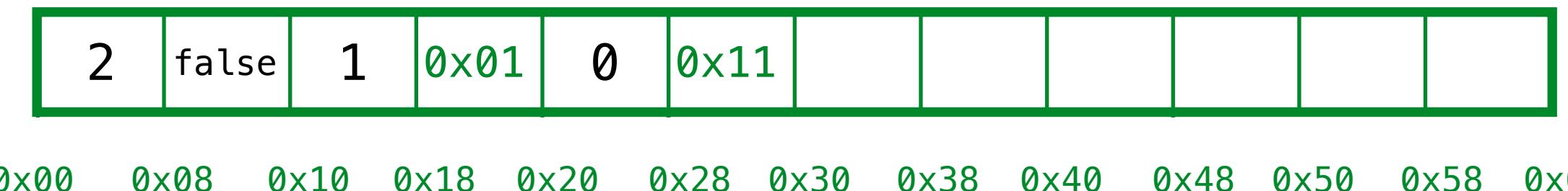
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



**Result**  $\text{sum}(0x11) = 3$

r15

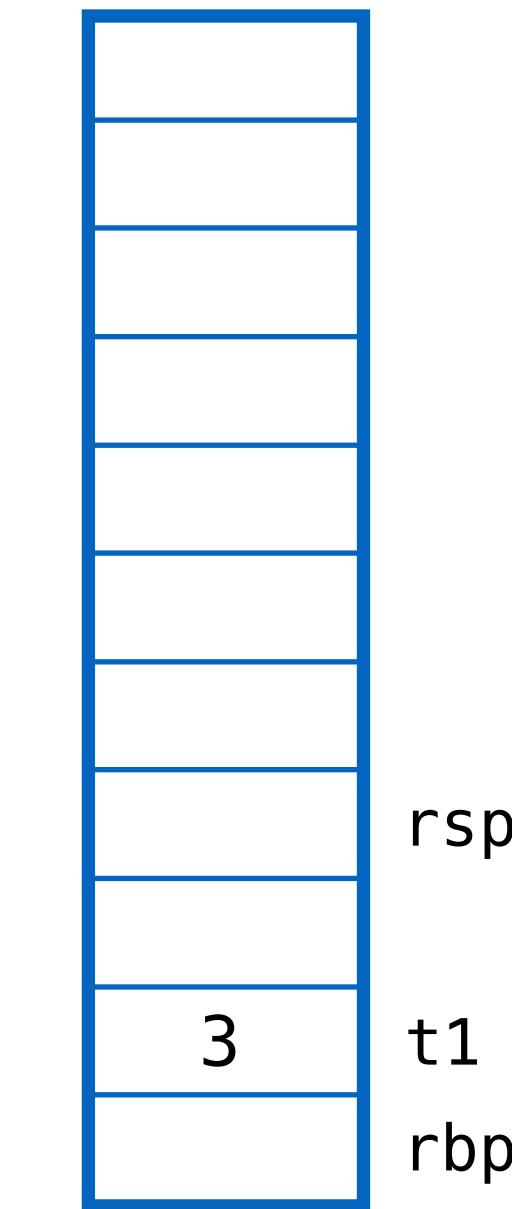


## ex4: recursive data

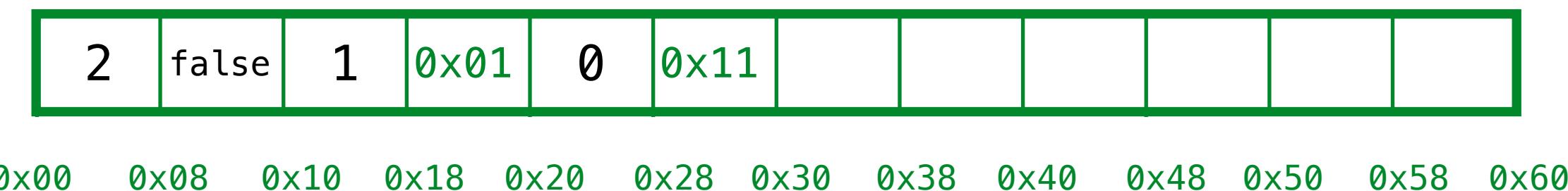
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, t1 = range(t1, t1 + 3)
in
(1000, l)
```



r15

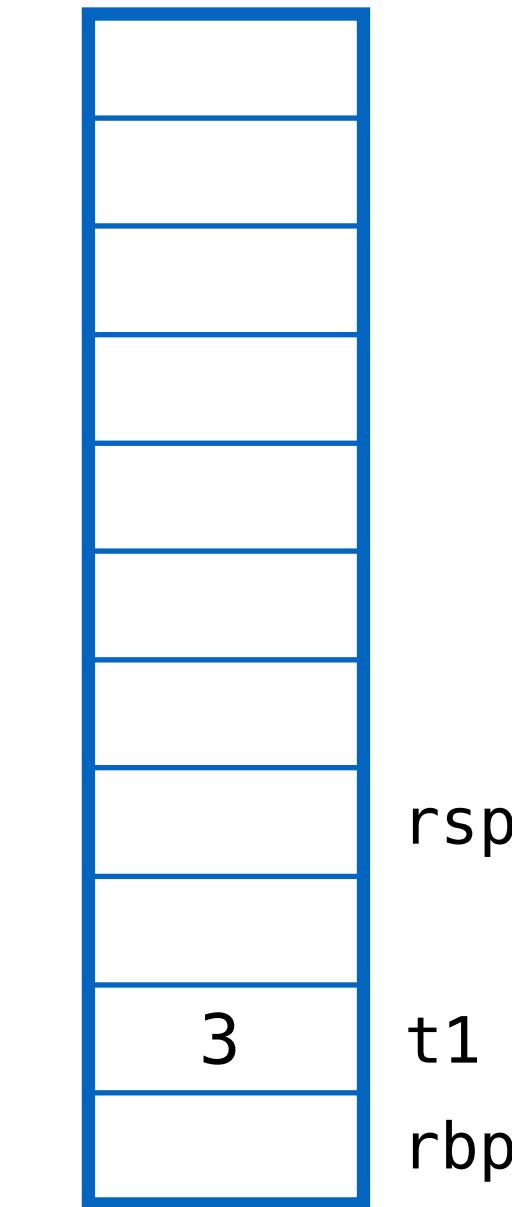


## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

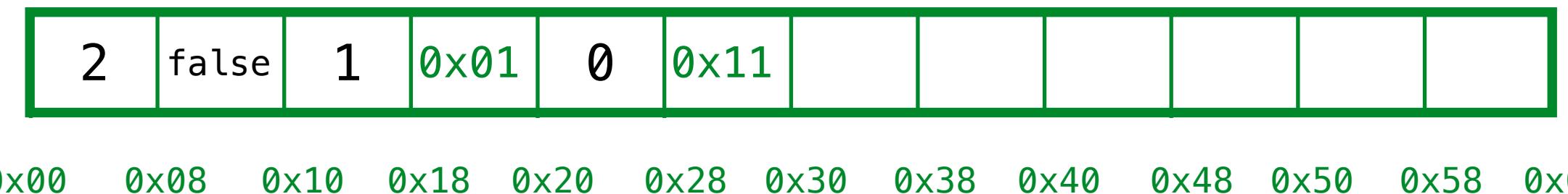
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3) ←
in
(1000, l)
```



call range(3,6)

r15



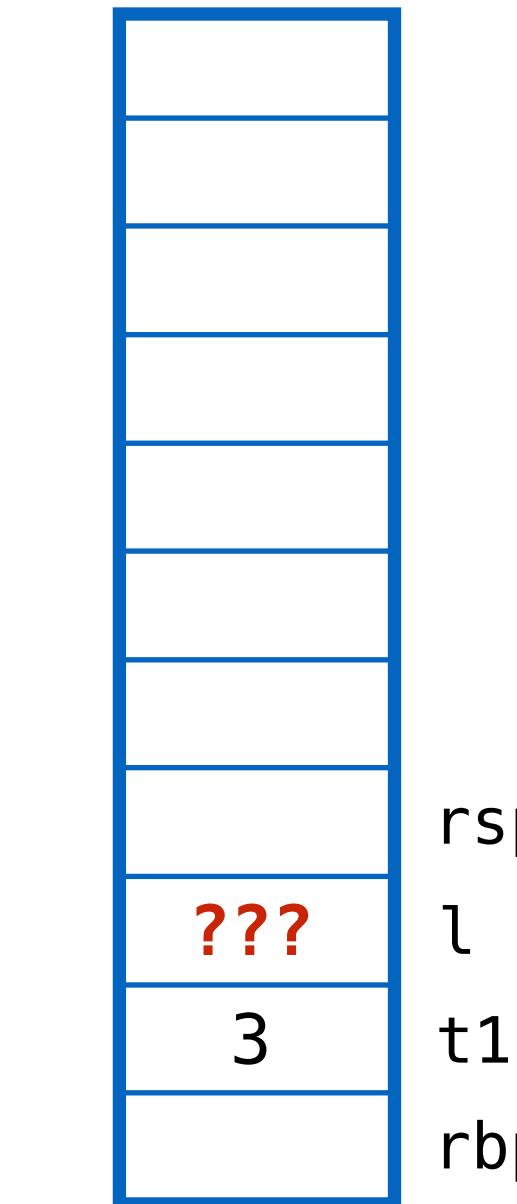
## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```

call range(3,6)



r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
---	-------	---	------	---	------	---	-------	---	------	---	------

0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

QUIZ: What is the value of **l**?

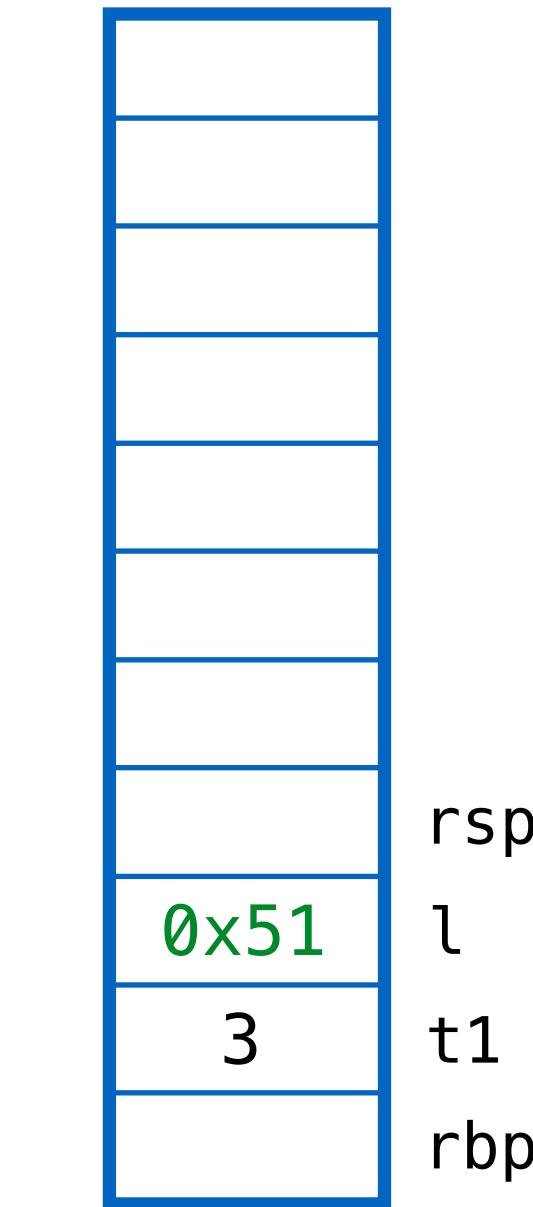
- (A) 0x30 (B) 0x31 (C) 0x50 (D) 0x51 (E) 0x60

## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

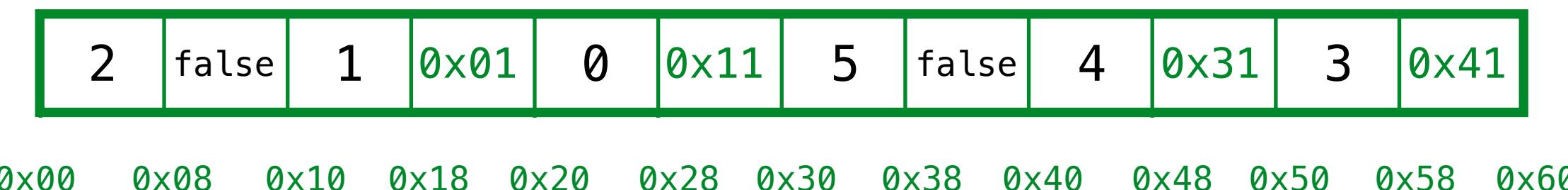
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



**Yikes! Out of Memory!**

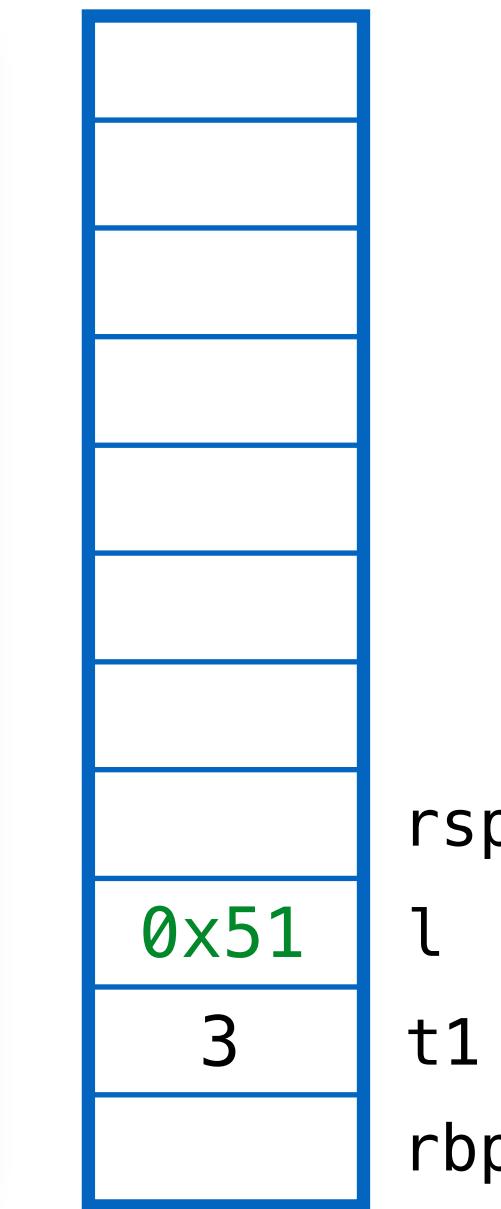
r15



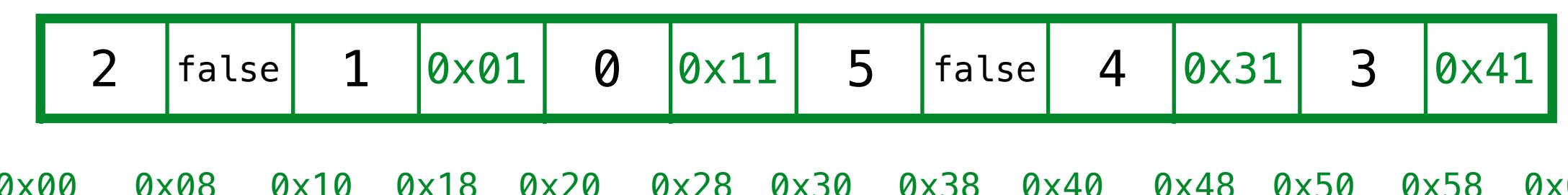
## ex4: recursive data

QUIZ: Which cells are “live” on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50



r15

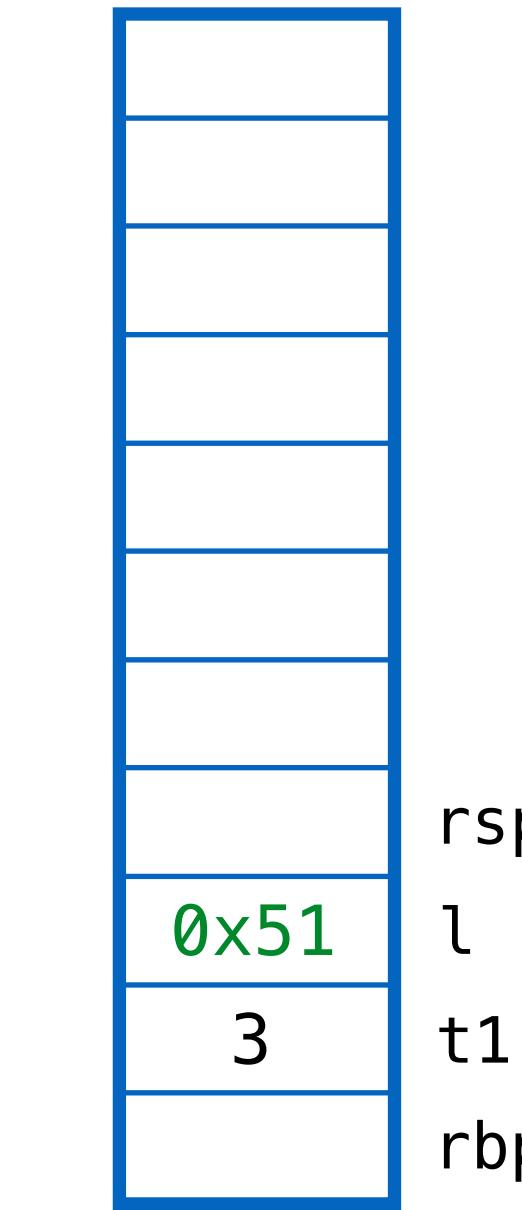


## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58

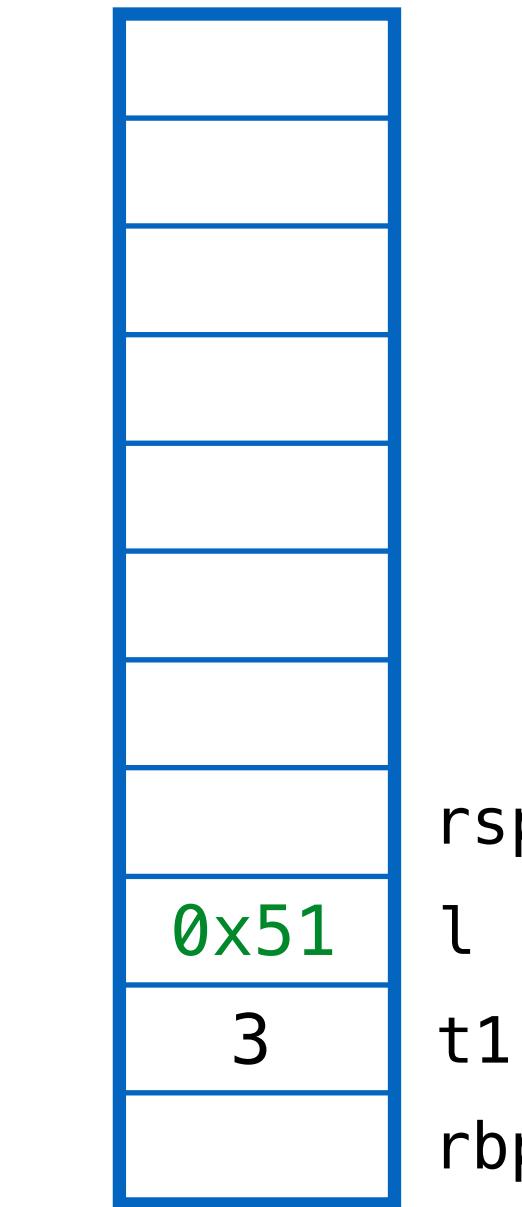
1. MARK live addrs
2. Compute FORWARD addrs
3. REDIRECT addrs on stack
4. COMPACT cells on heap

## ex4: recursive data

```
def range(i, j):
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def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58

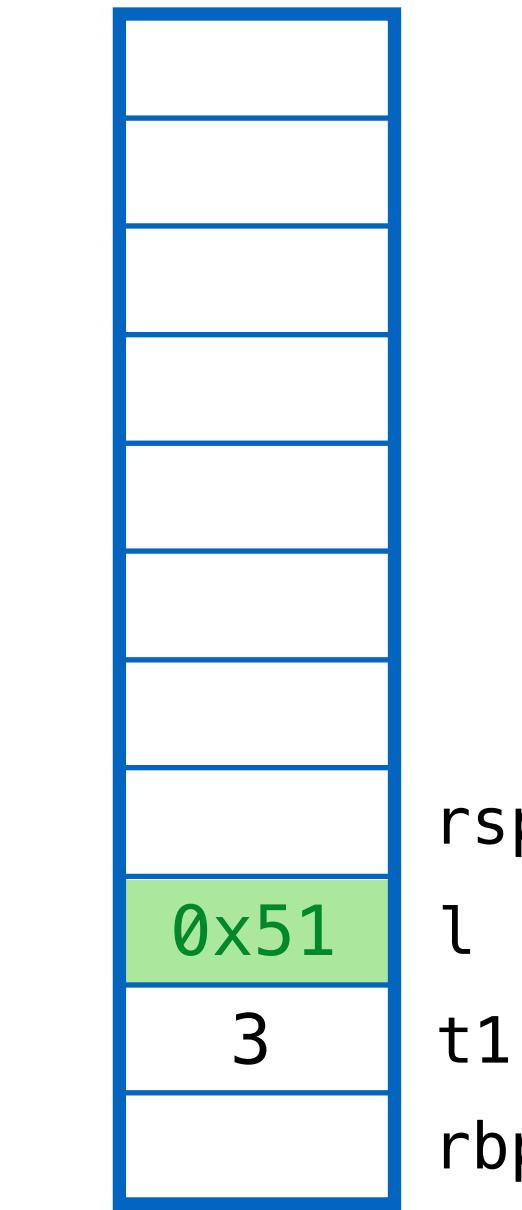
1. MARK live addrs  
reachable from stack

## ex4: recursive data

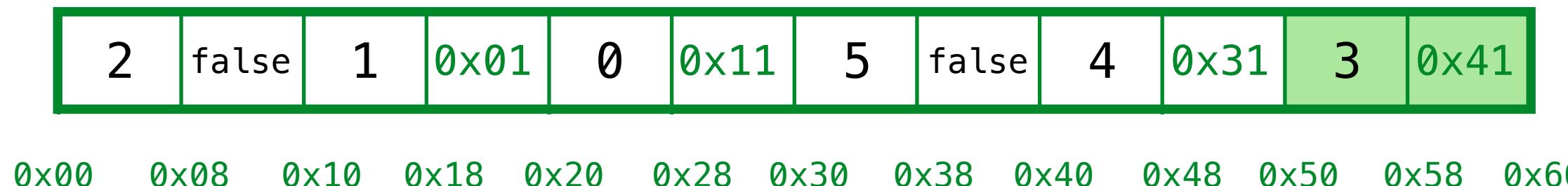
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



r15



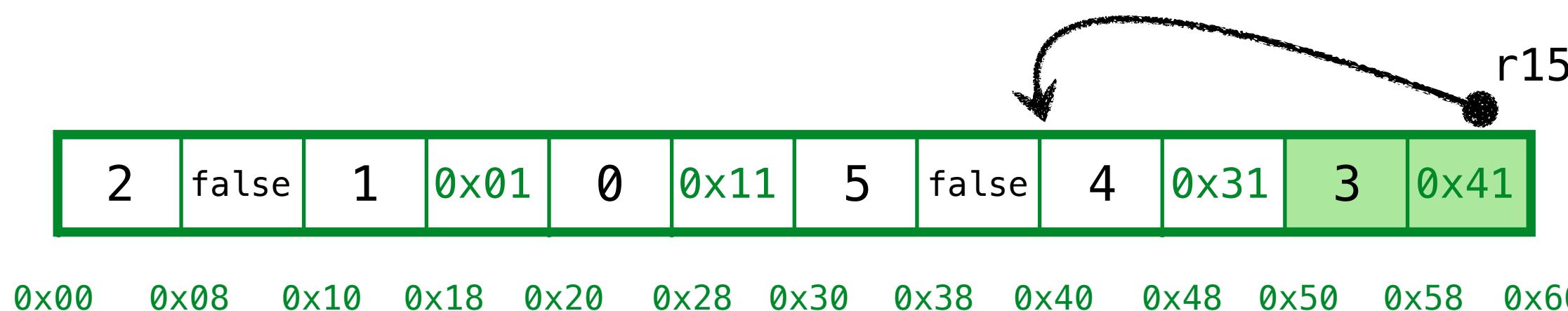
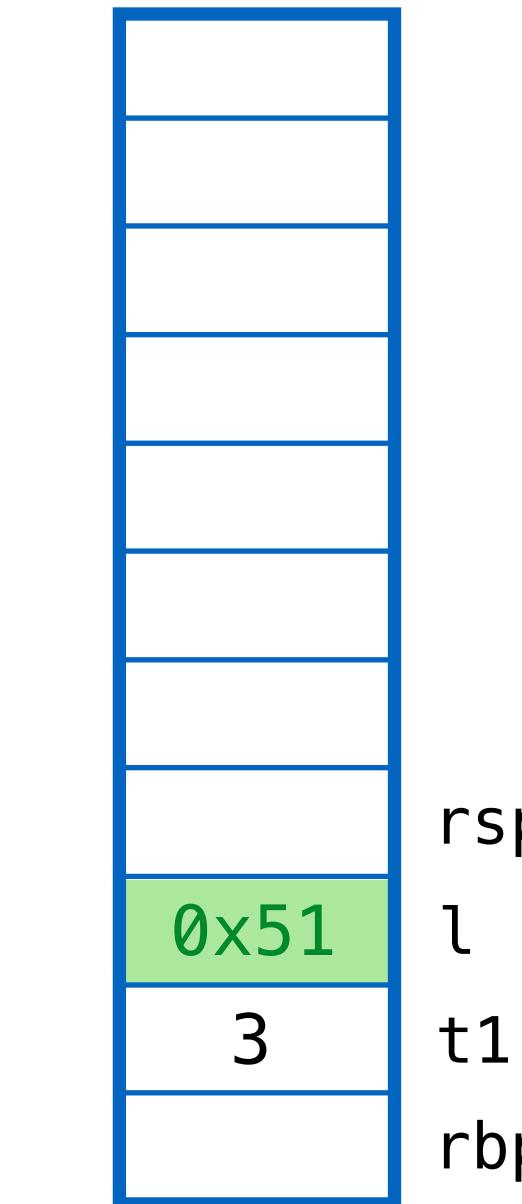
1. MARK live addrs  
reachable from stack

## ex4: recursive data

```
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let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



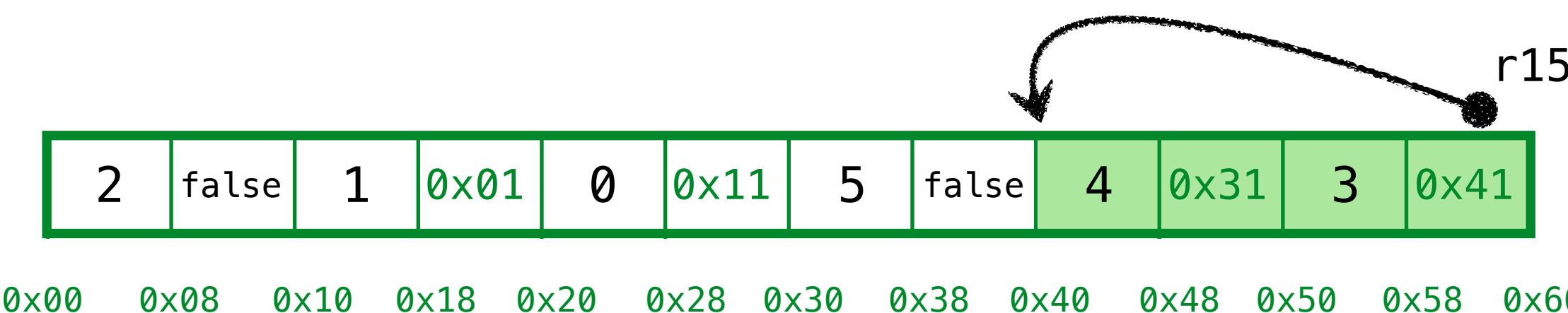
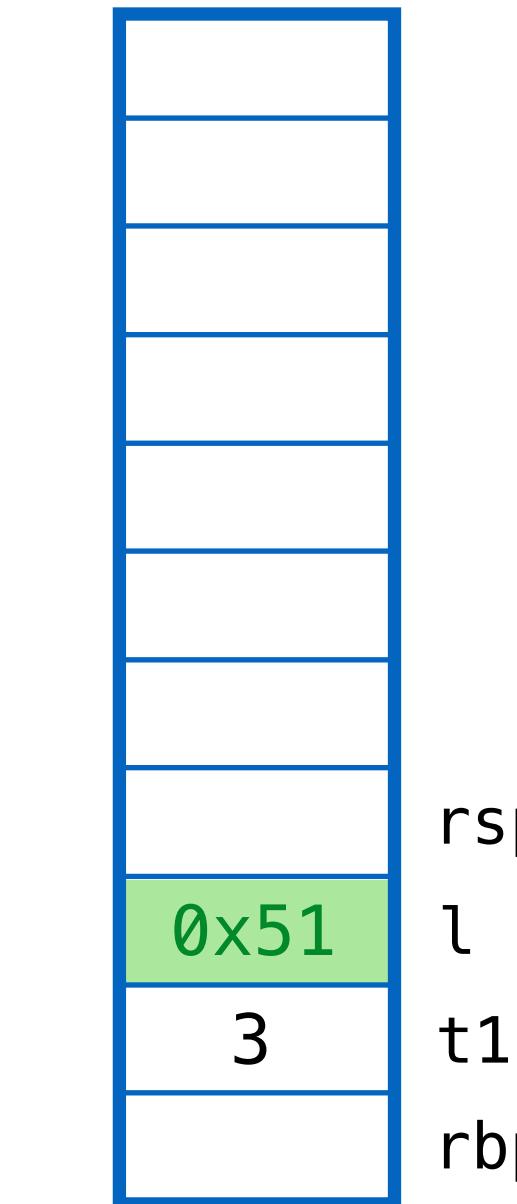
1. MARK live addrs  
reachable from stack

## ex4: recursive data

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def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
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let t1 =
    let l1 = range(0, 3)
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in
(1000, l)
```



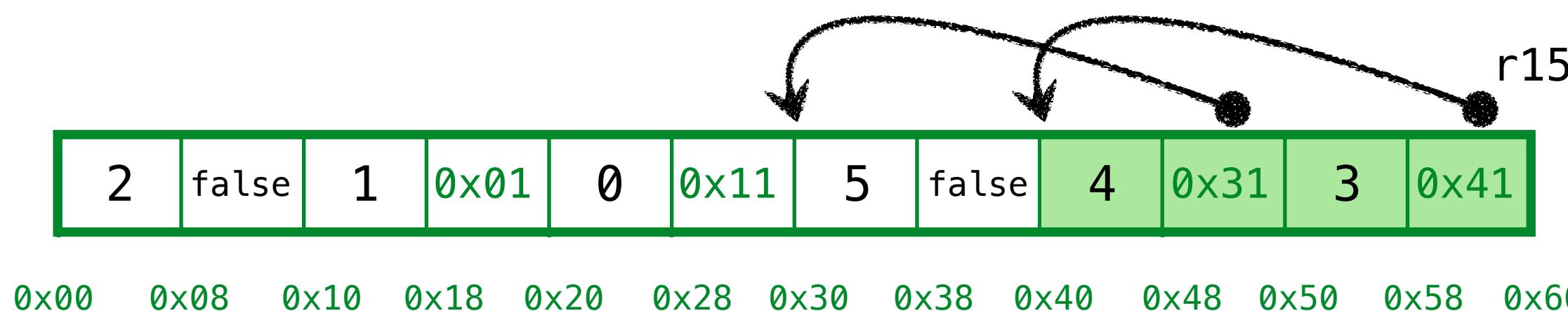
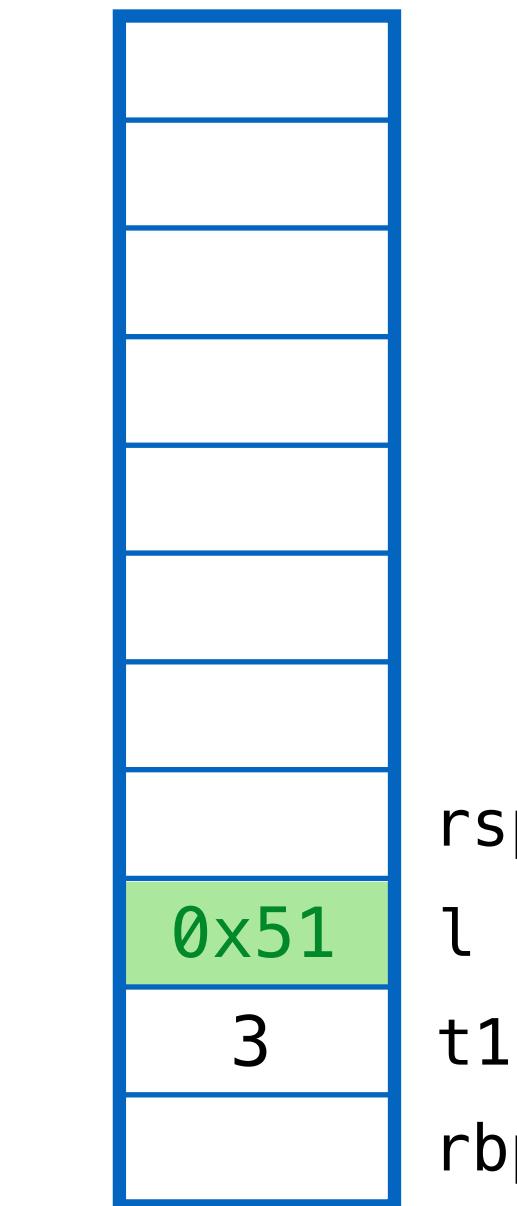
1. MARK live addrs  
reachable from stack

## ex4: recursive data

```
def range(i, j):
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let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



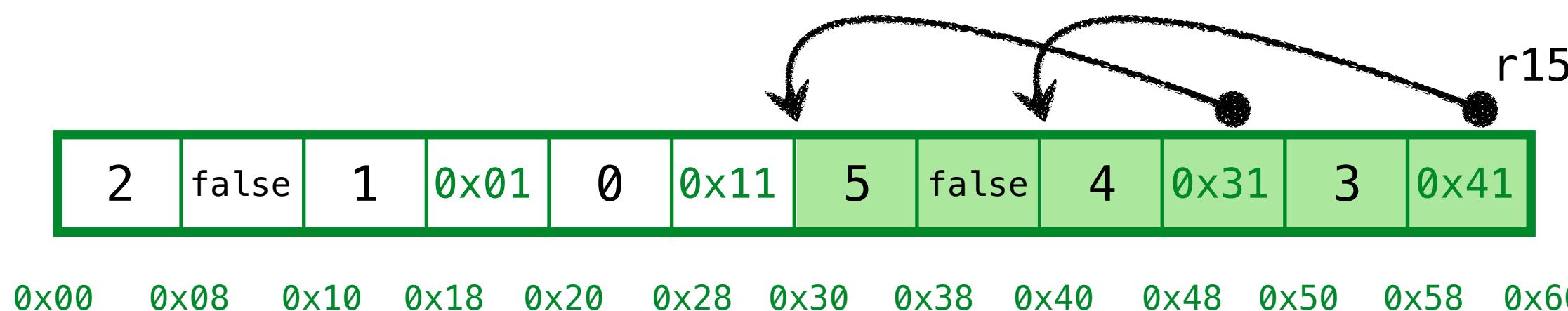
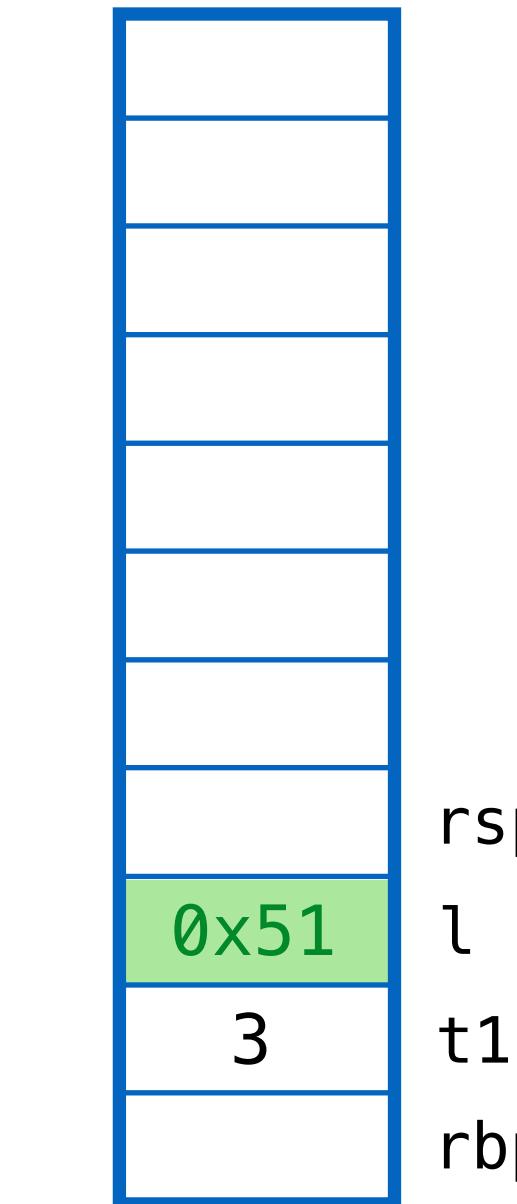
1. MARK live addrs  
reachable from stack

## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
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let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



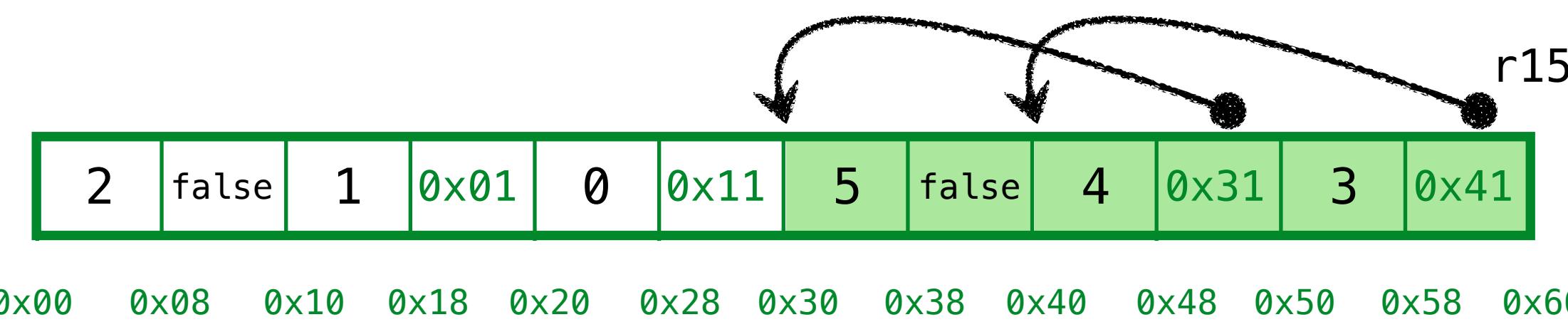
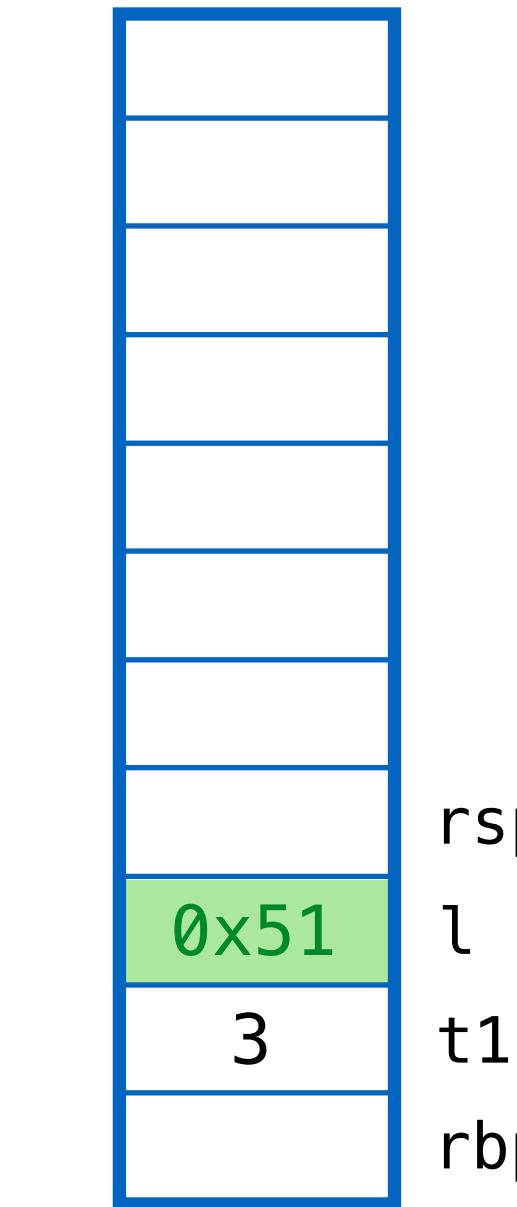
1. MARK live addrs  
reachable from stack

## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
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let t1 =
    let l1 = range(0, 3)
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in
(1000, l)
```



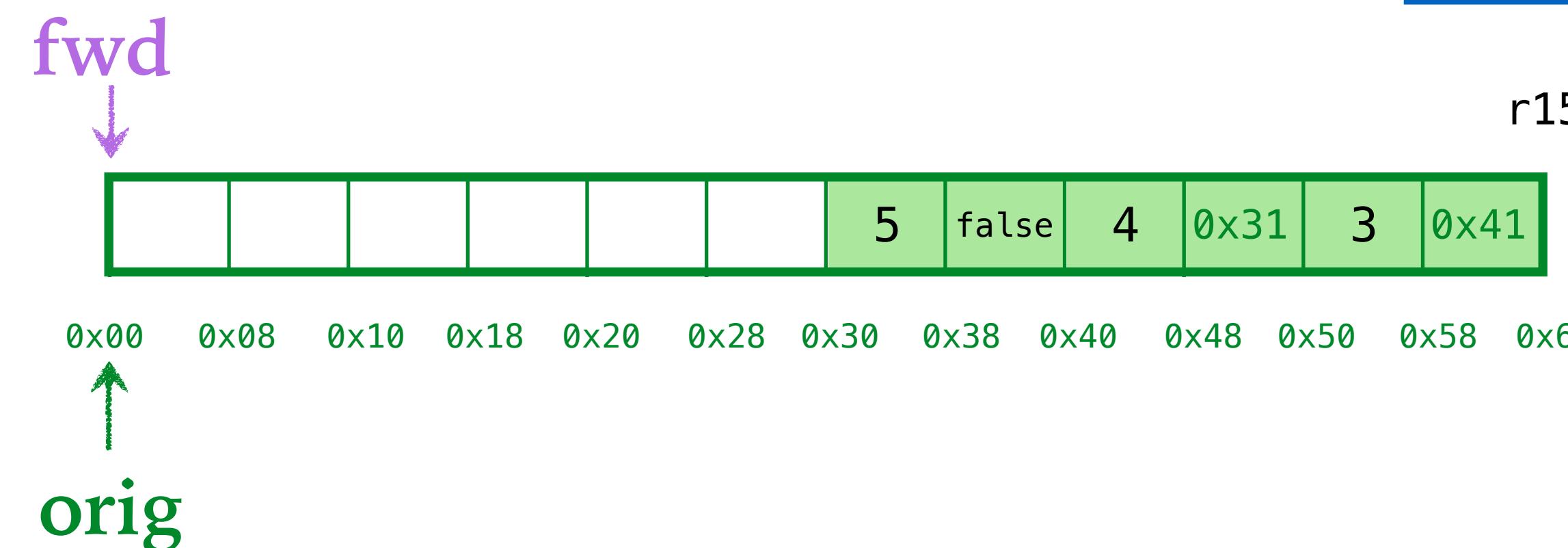
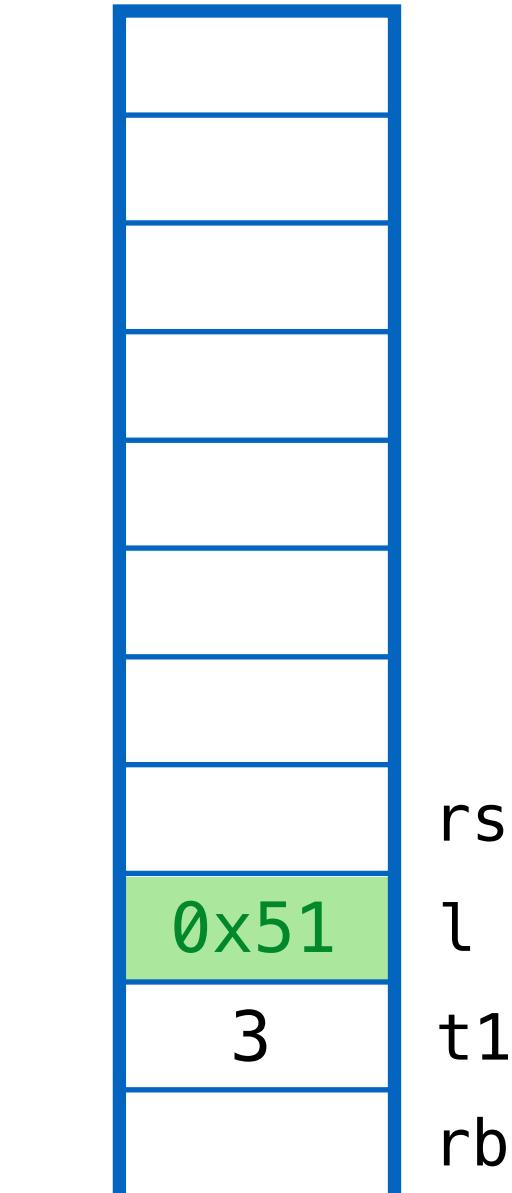
Done!

# ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



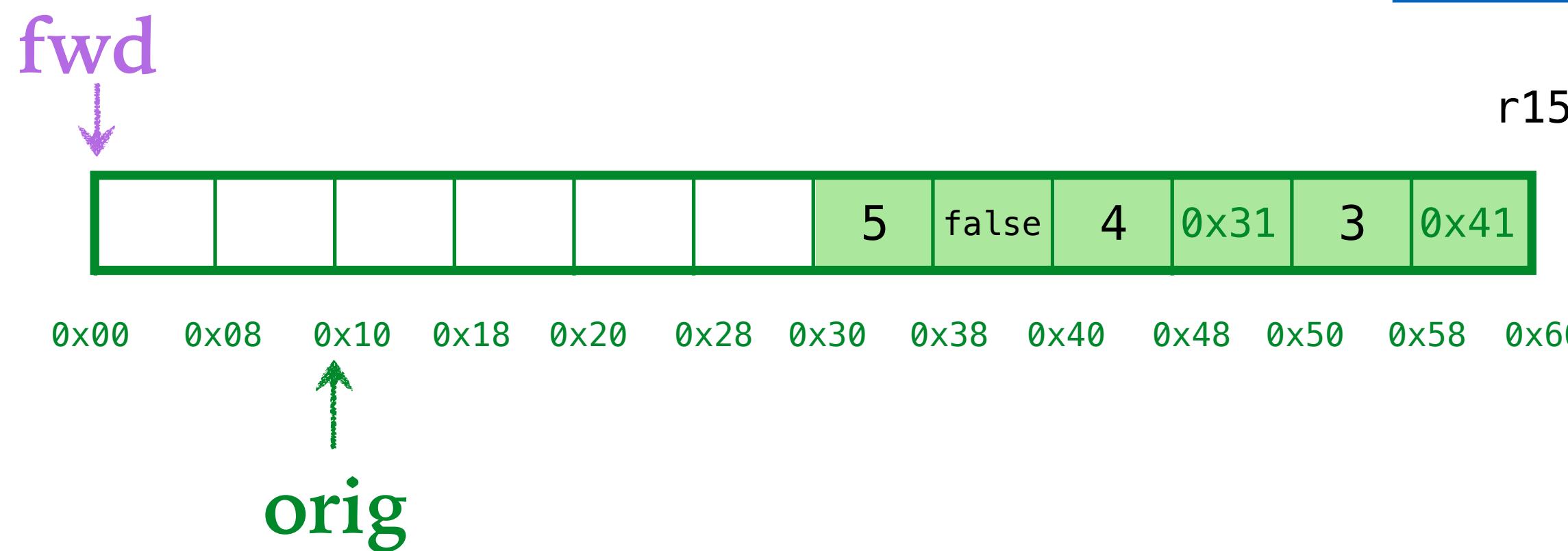
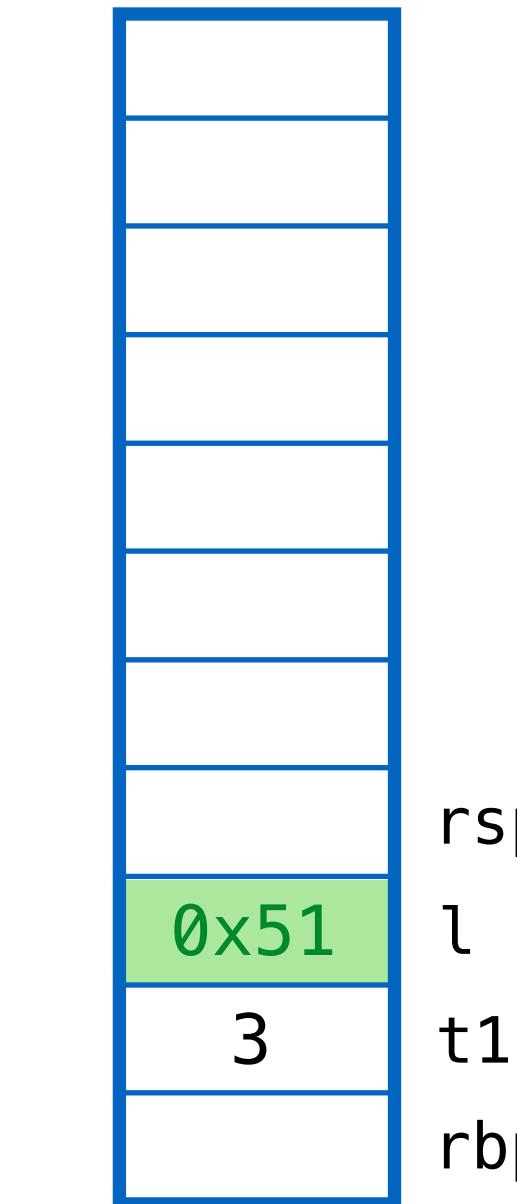
## 2. Compute FORWARD address

## ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
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```



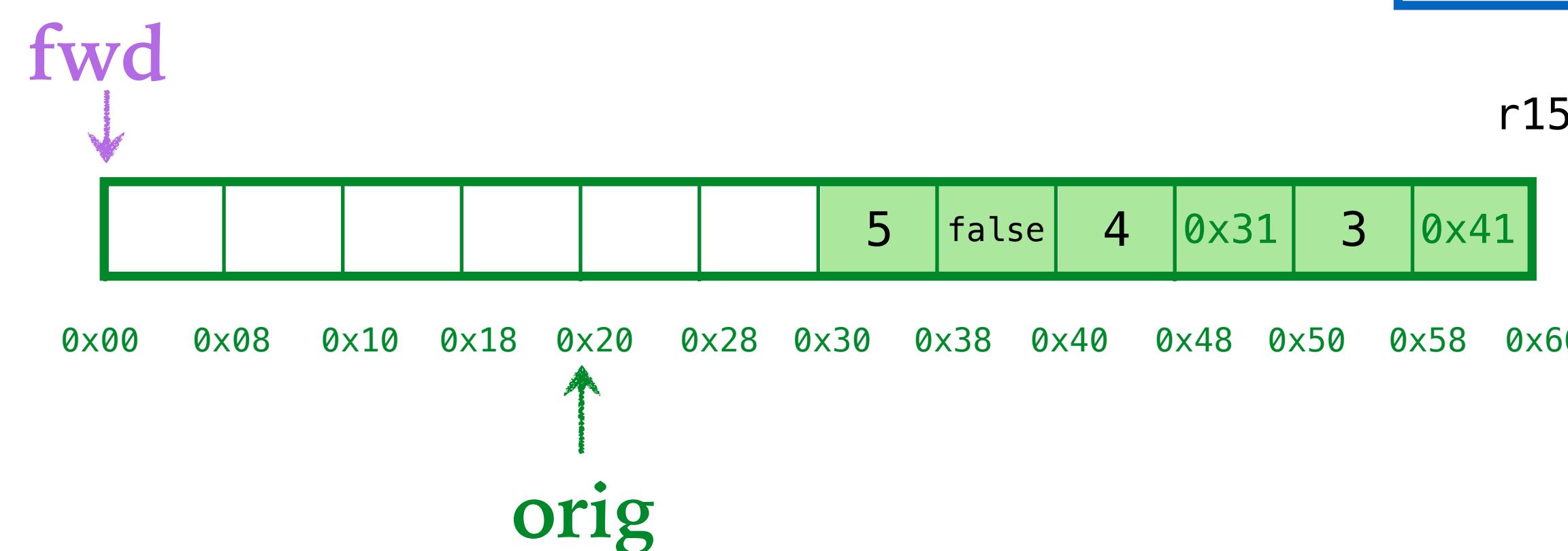
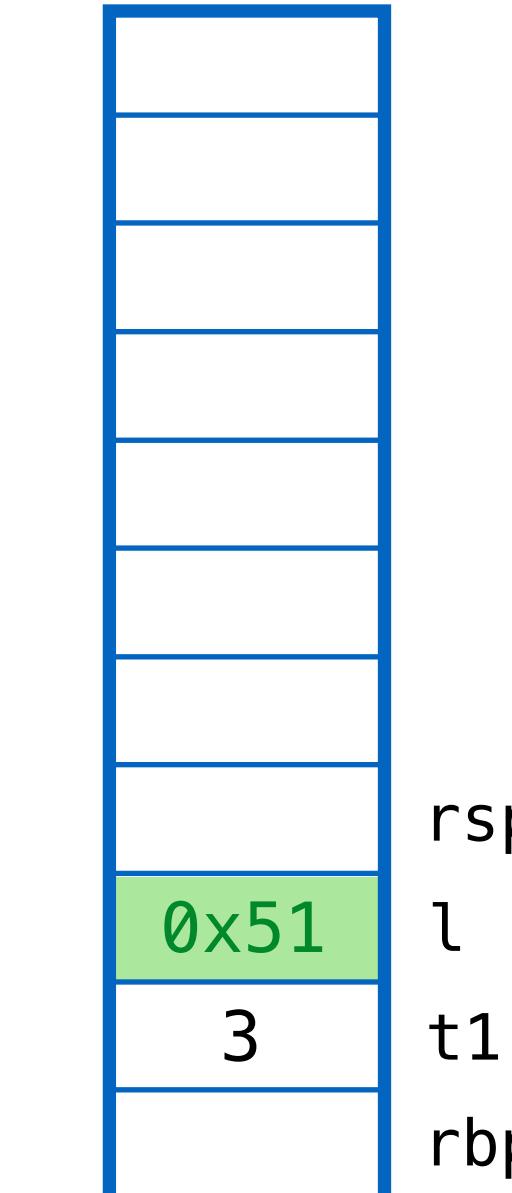
## 2. Compute FORWARD addrs

# ex4: recursive data

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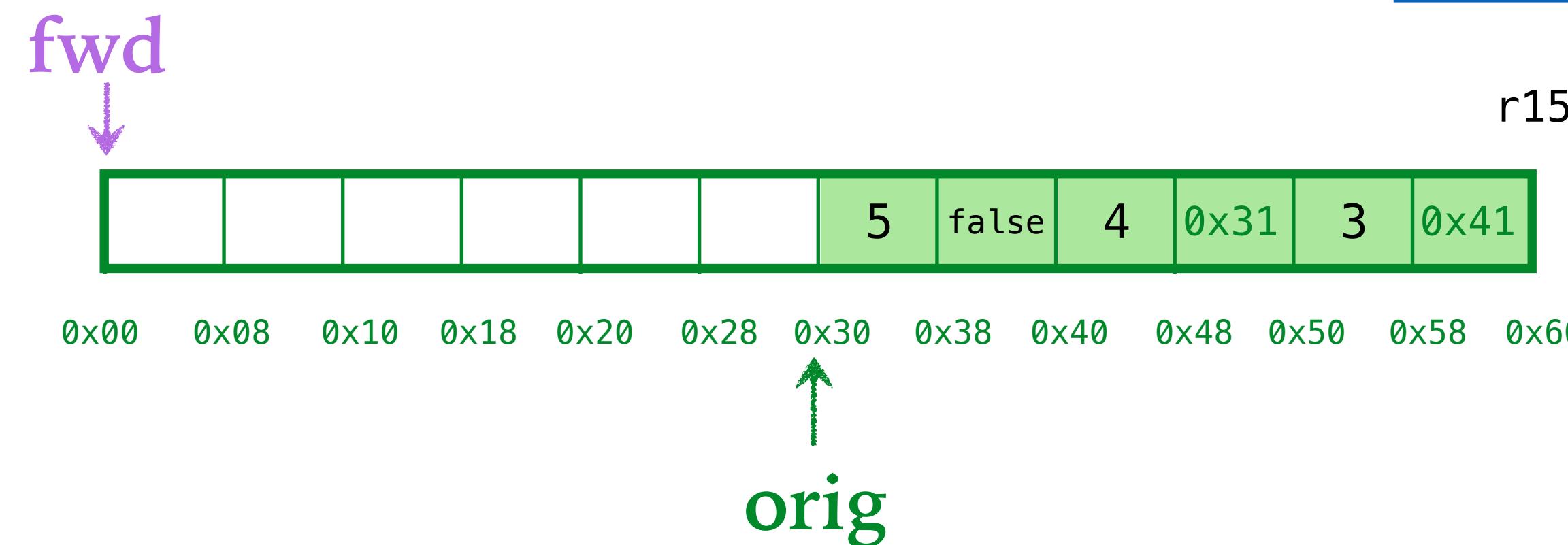
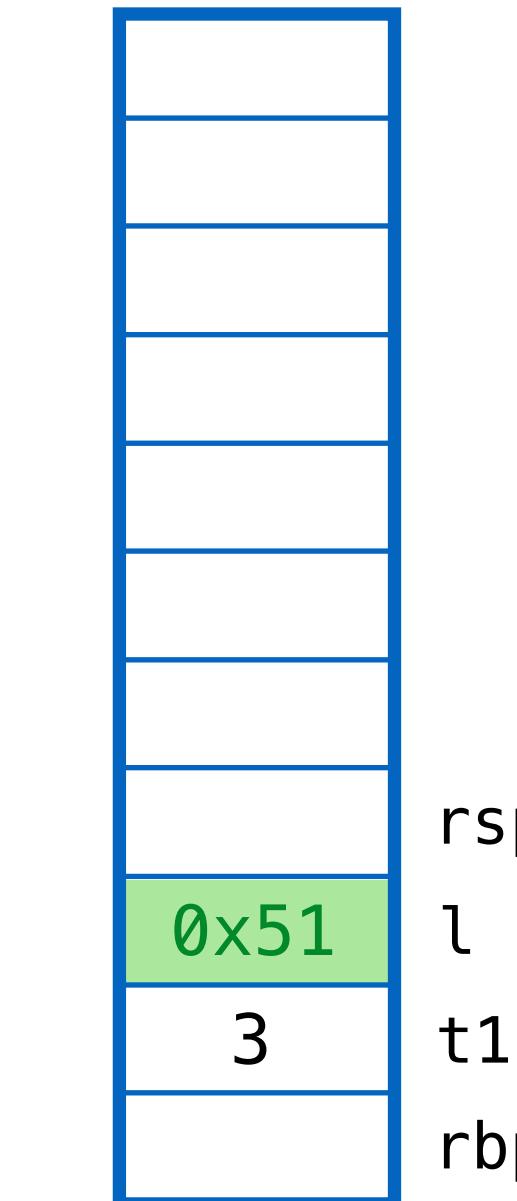
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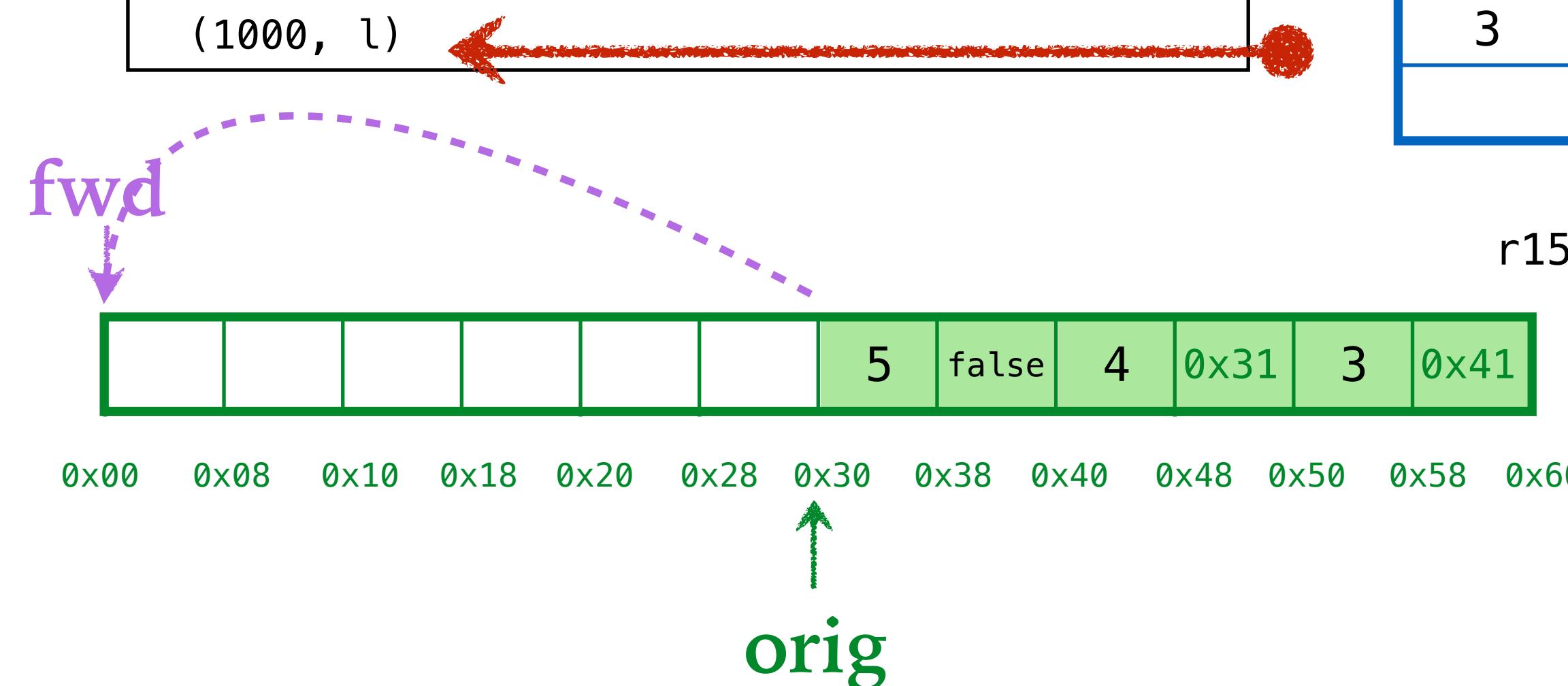
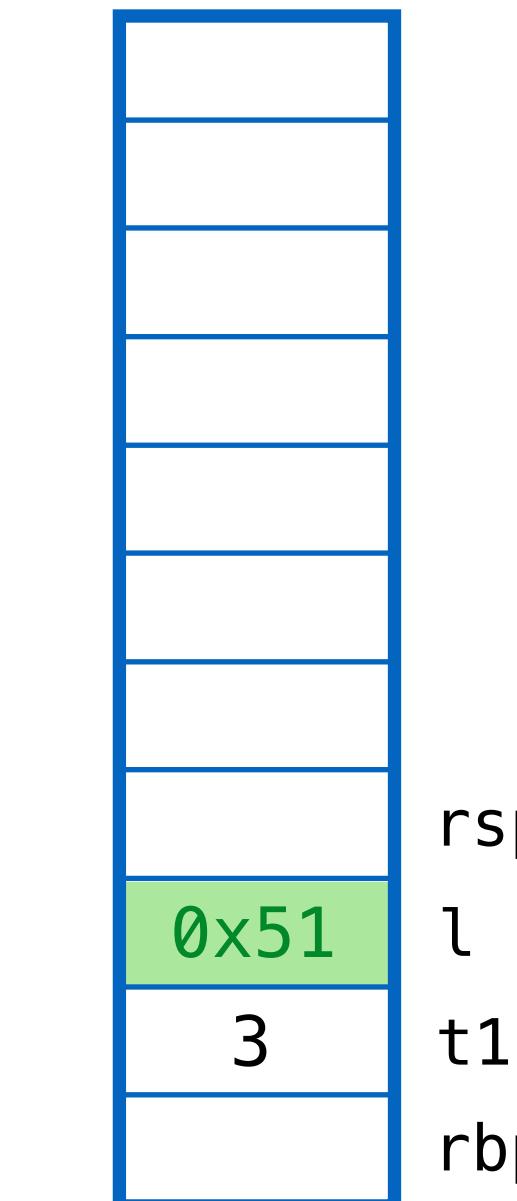
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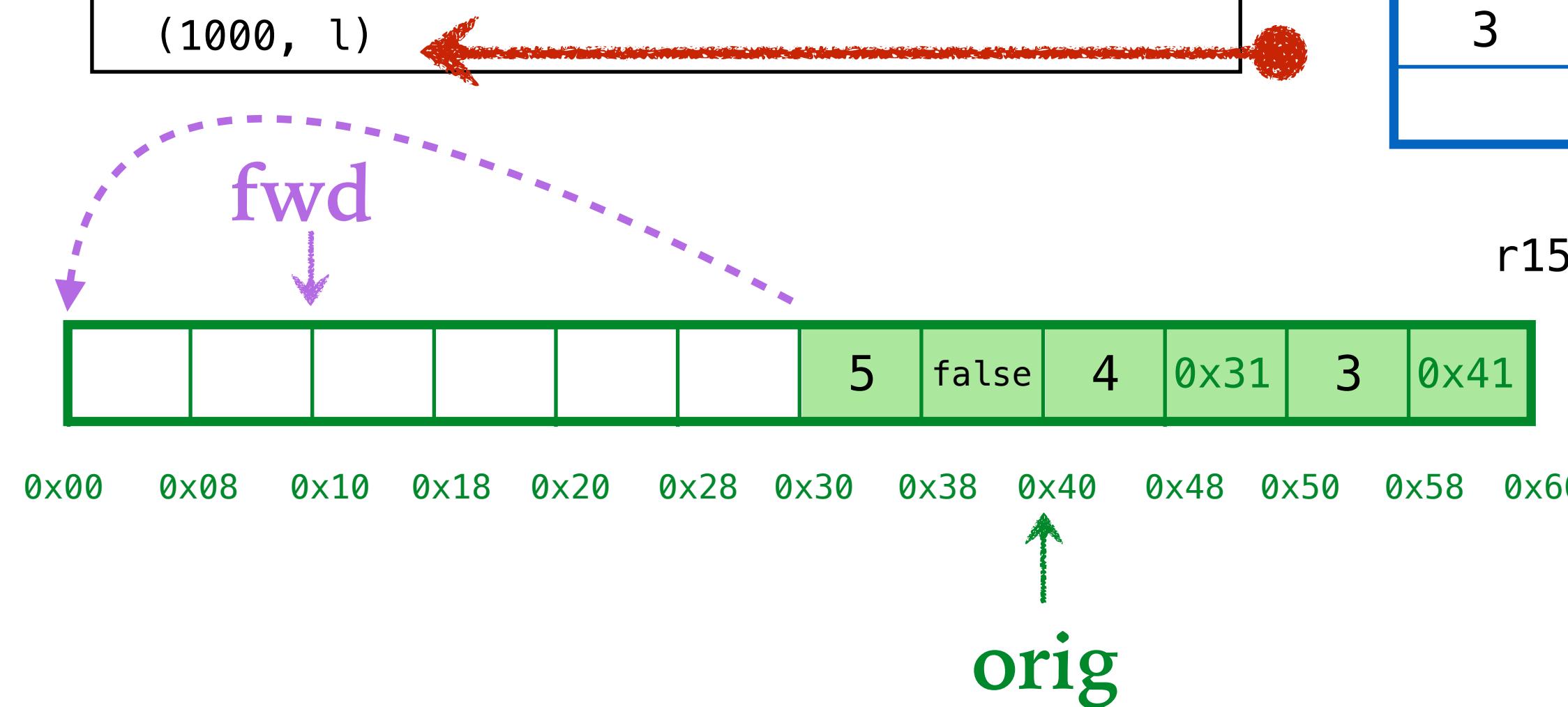
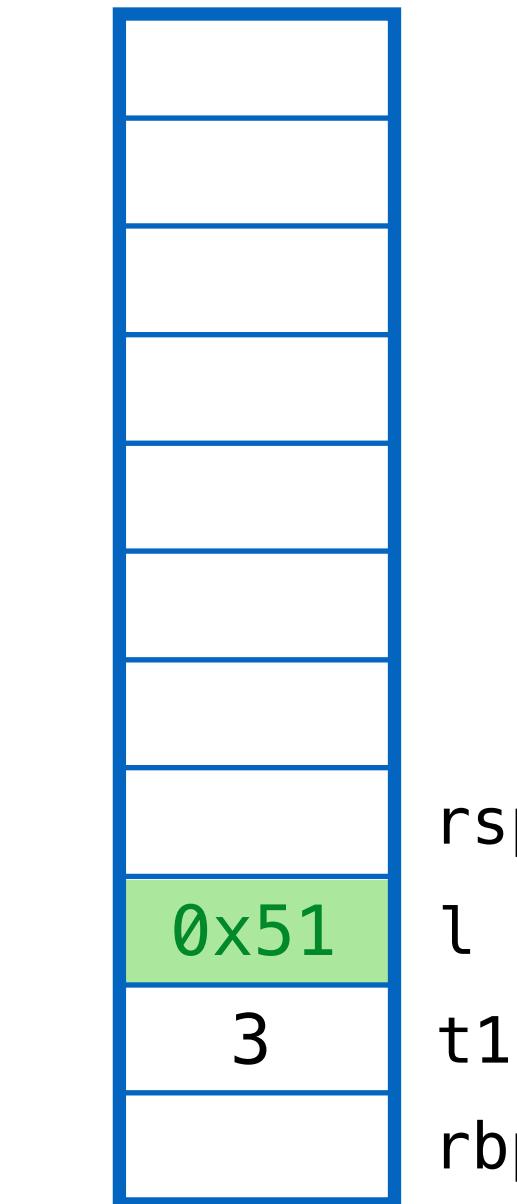
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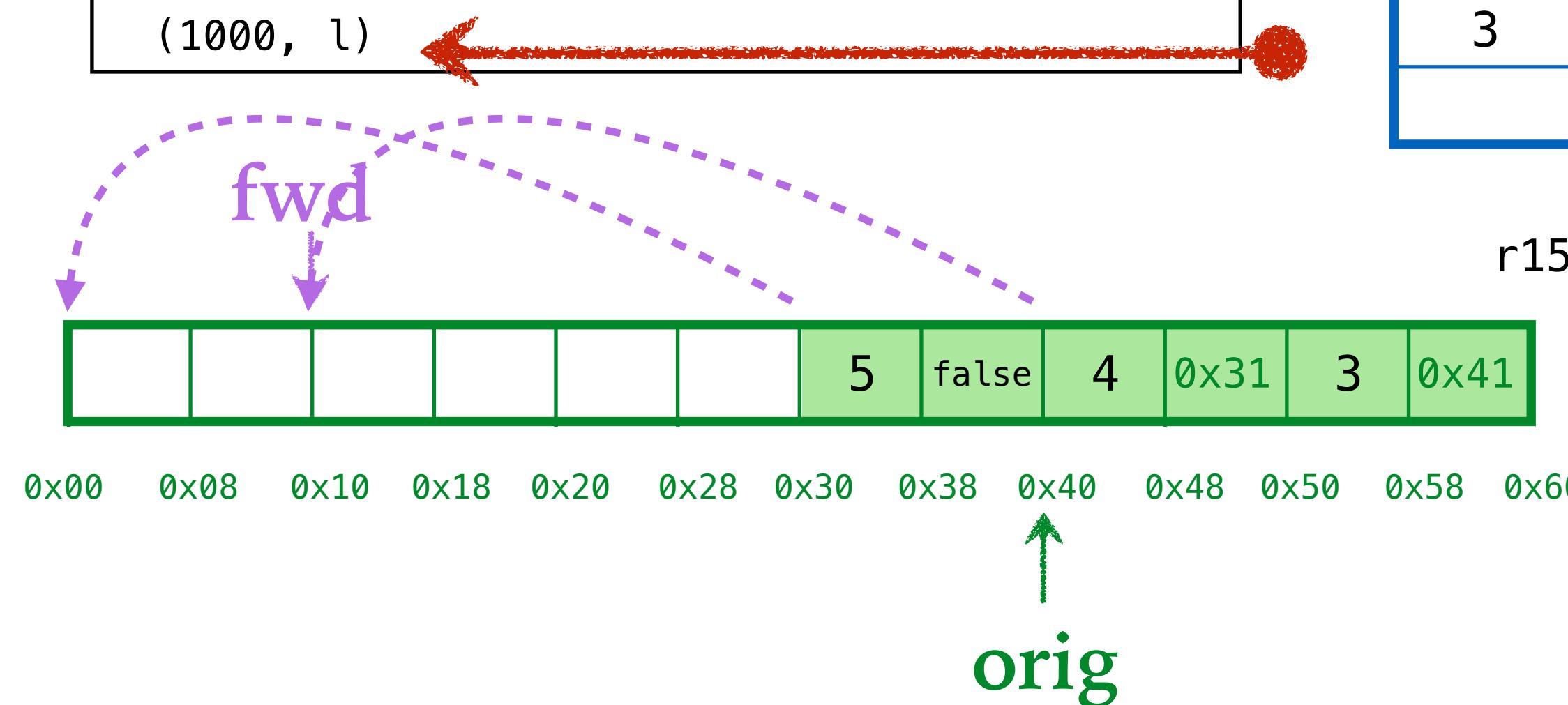
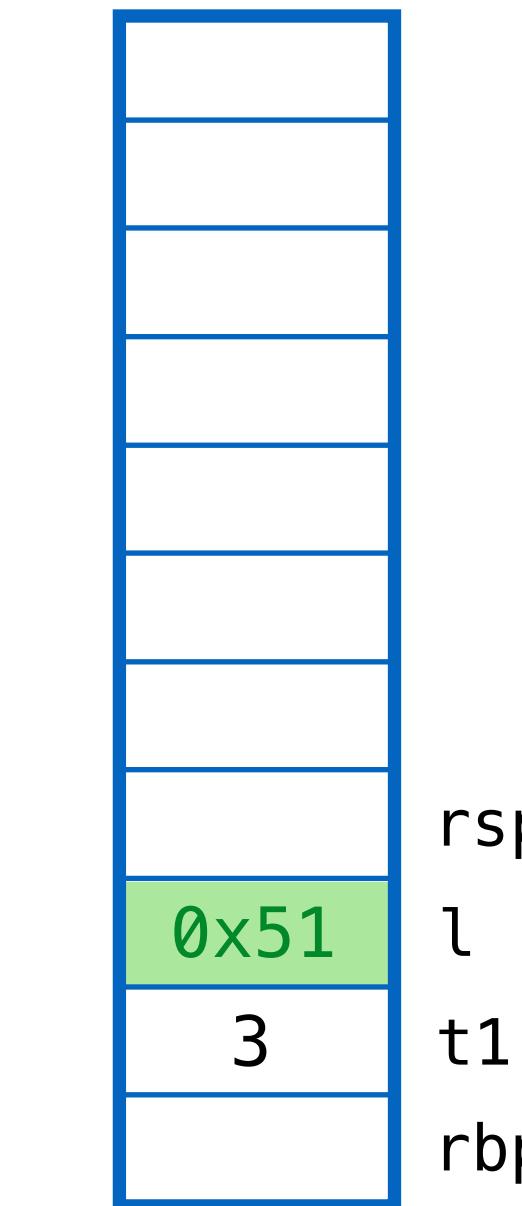
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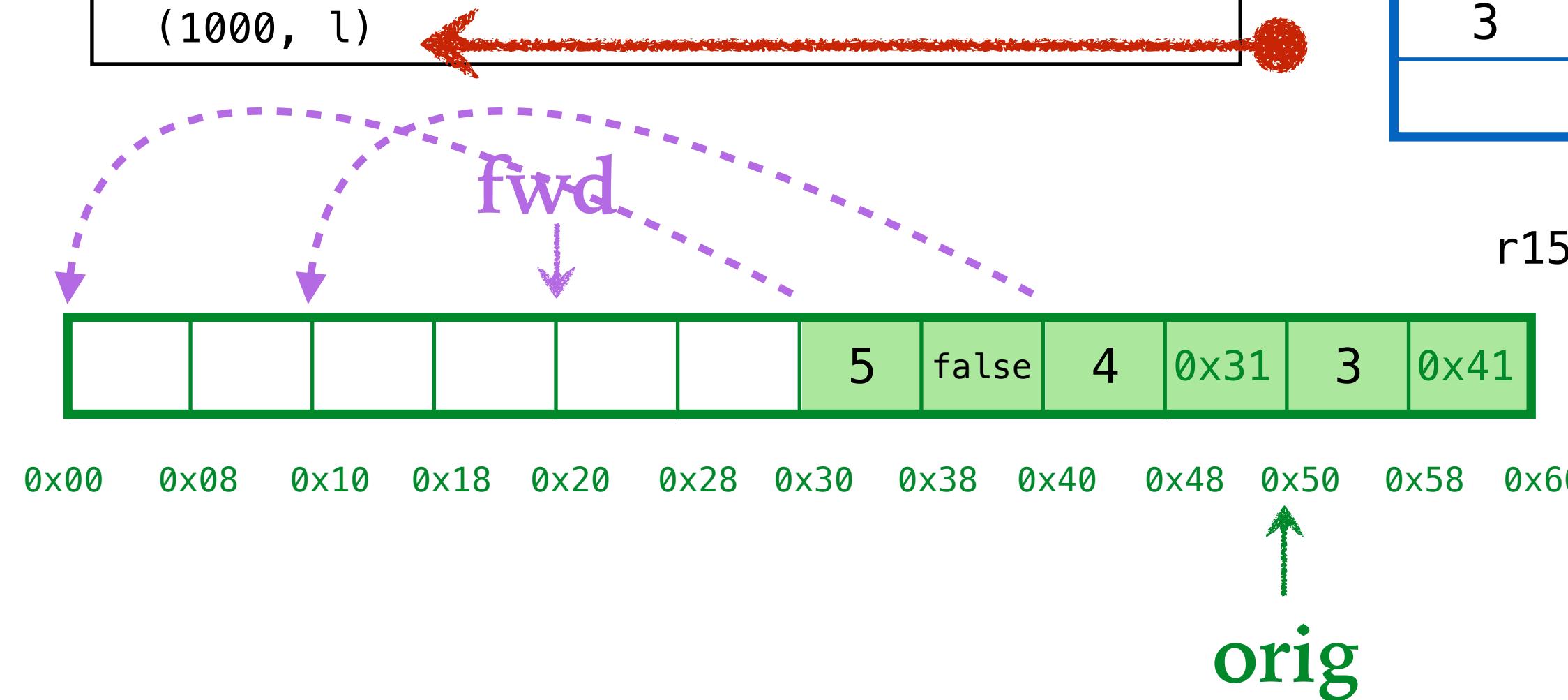
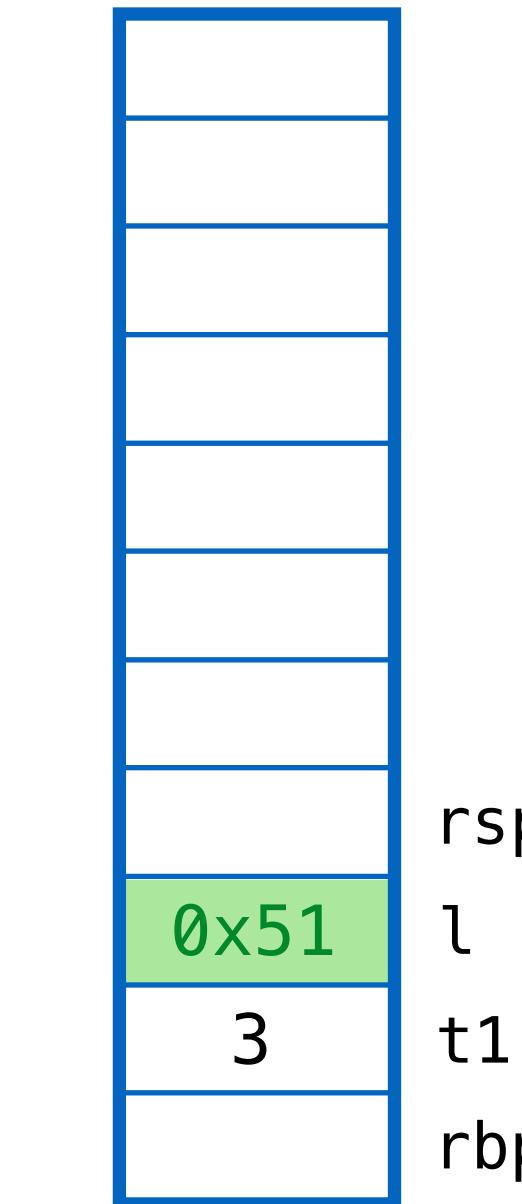
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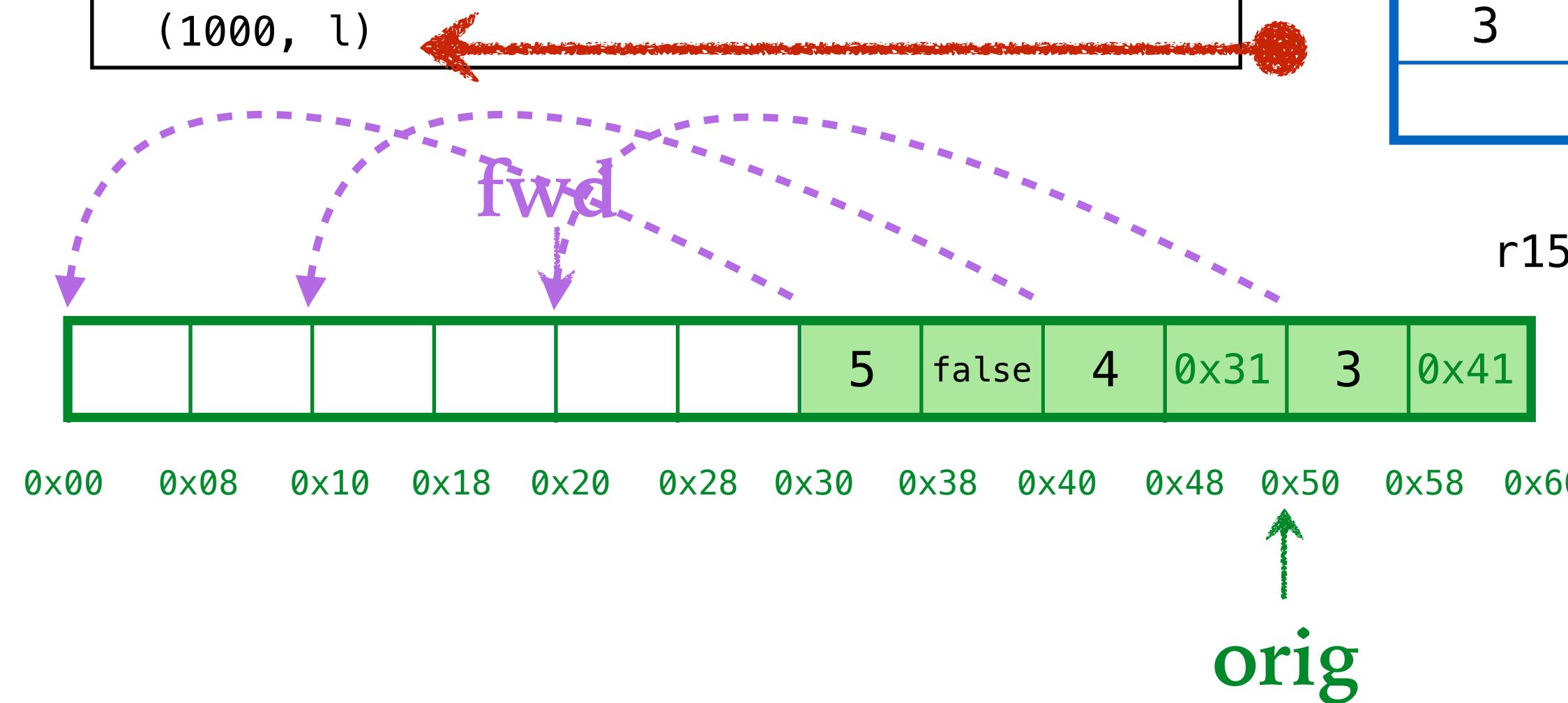
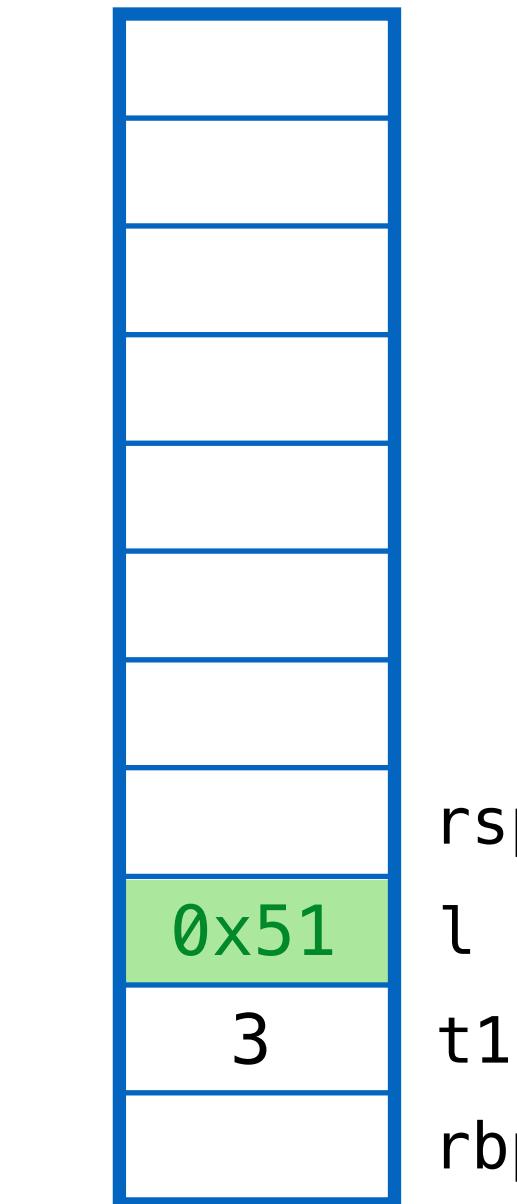
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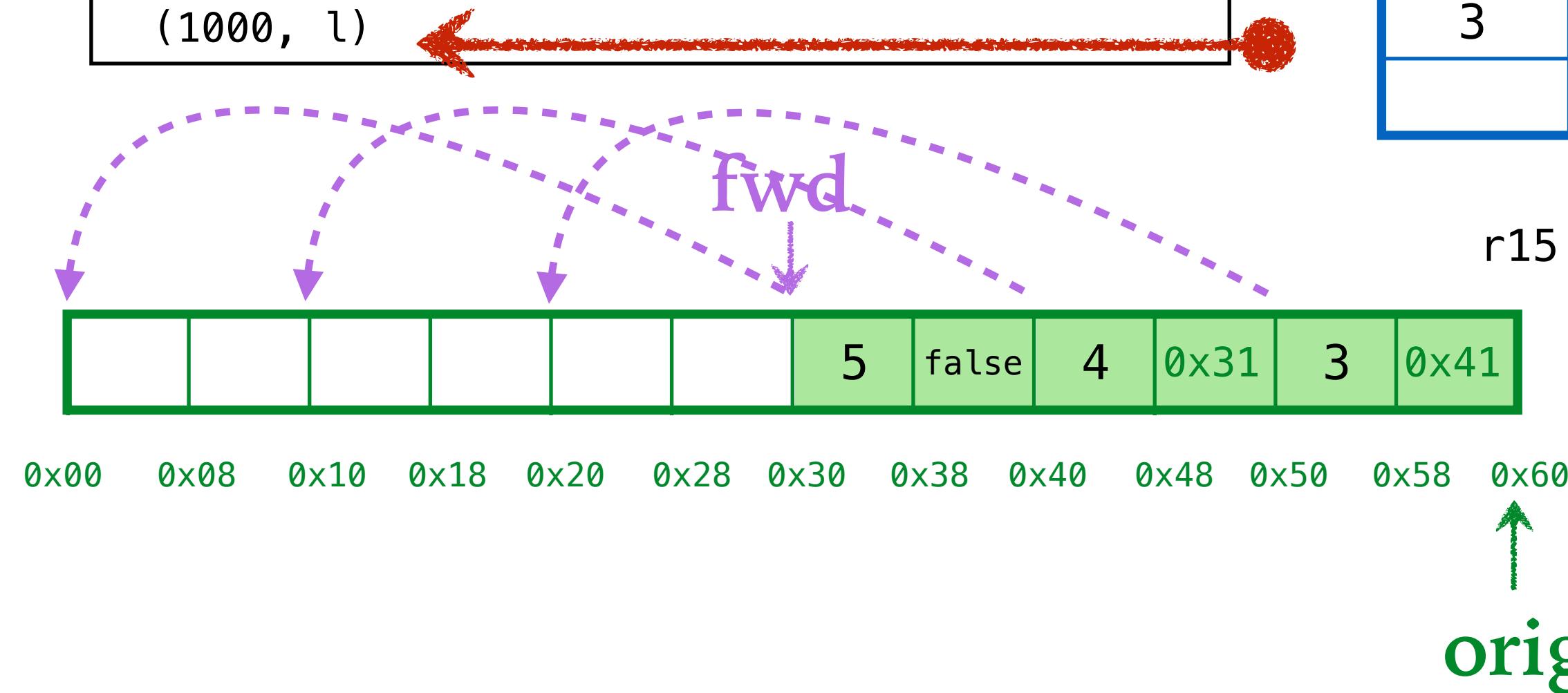
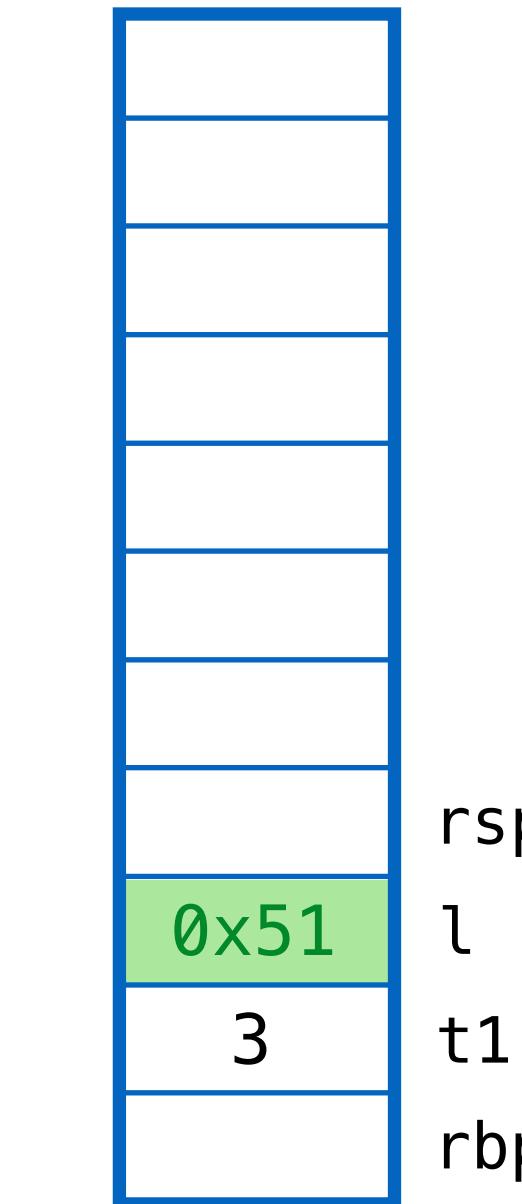
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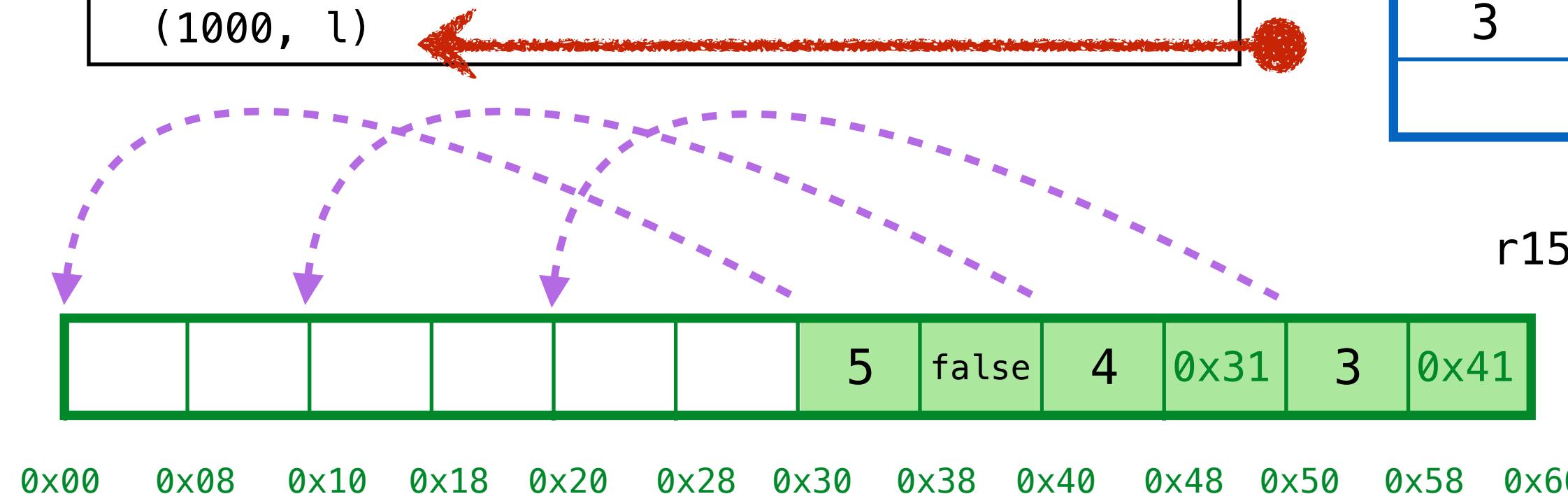
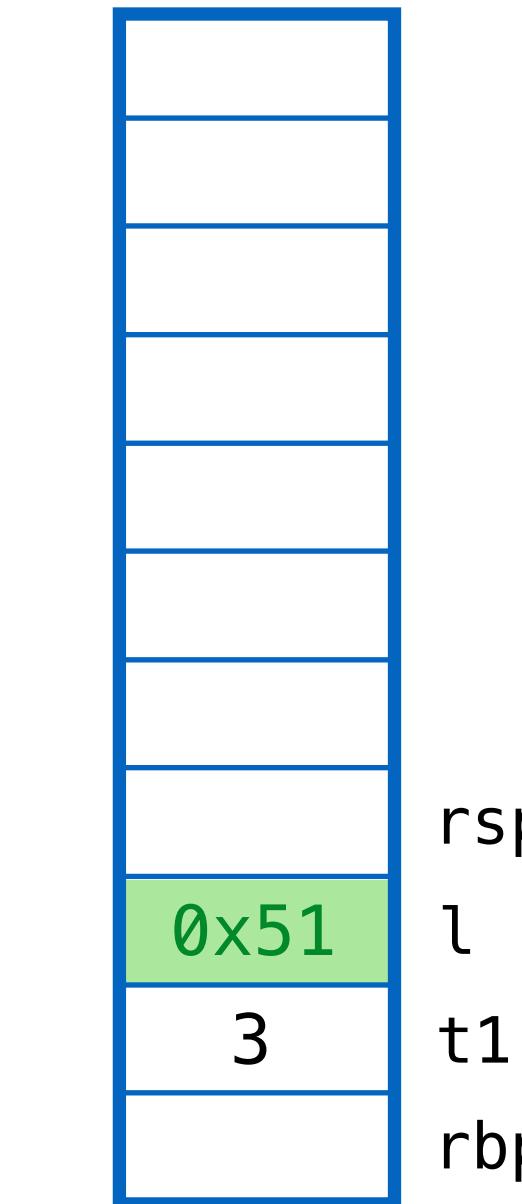
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## 2. Compute FORWARD addrs

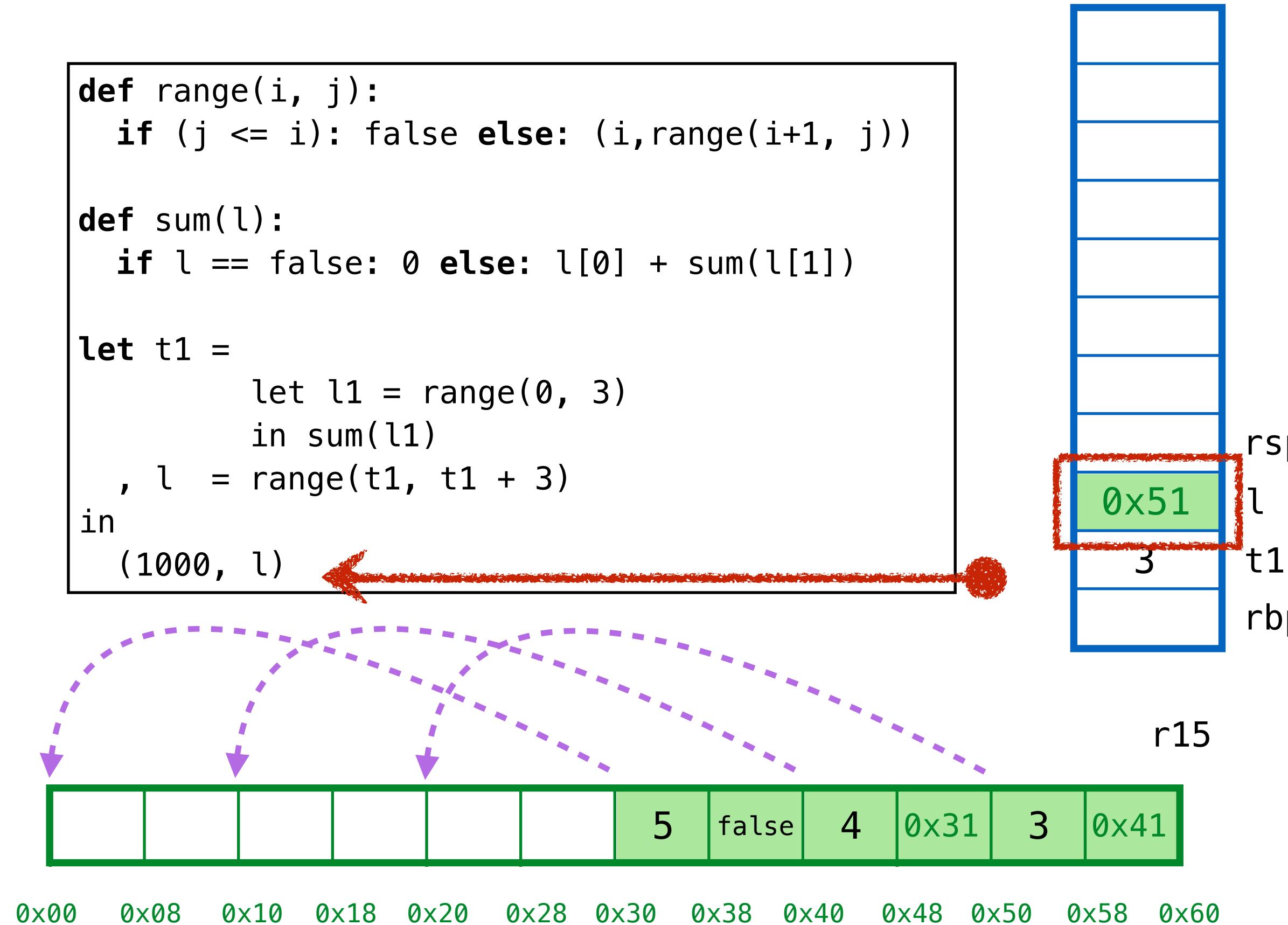
Where should we store the forward addrs?

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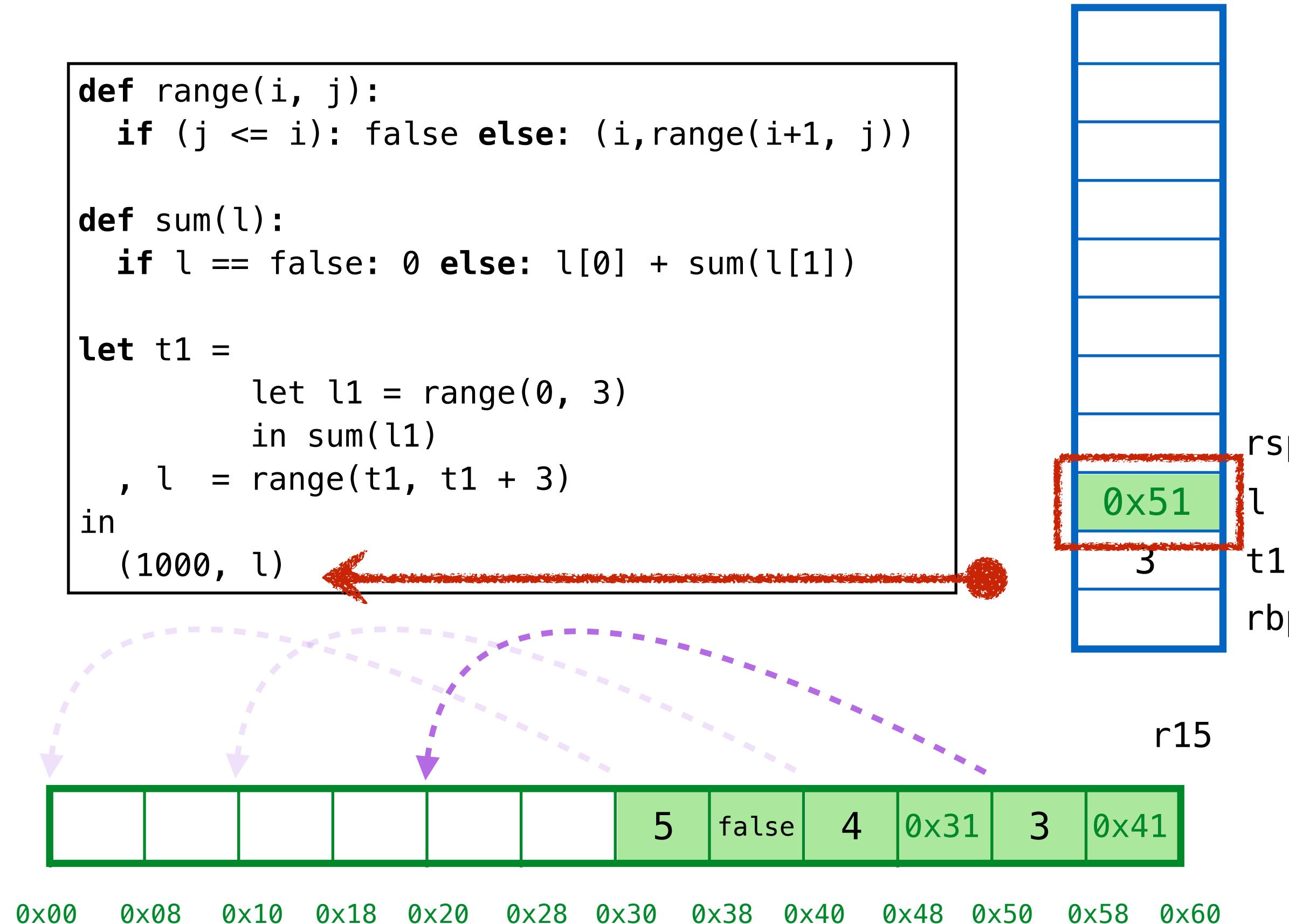
### 3. REDIRECT addrs on stack

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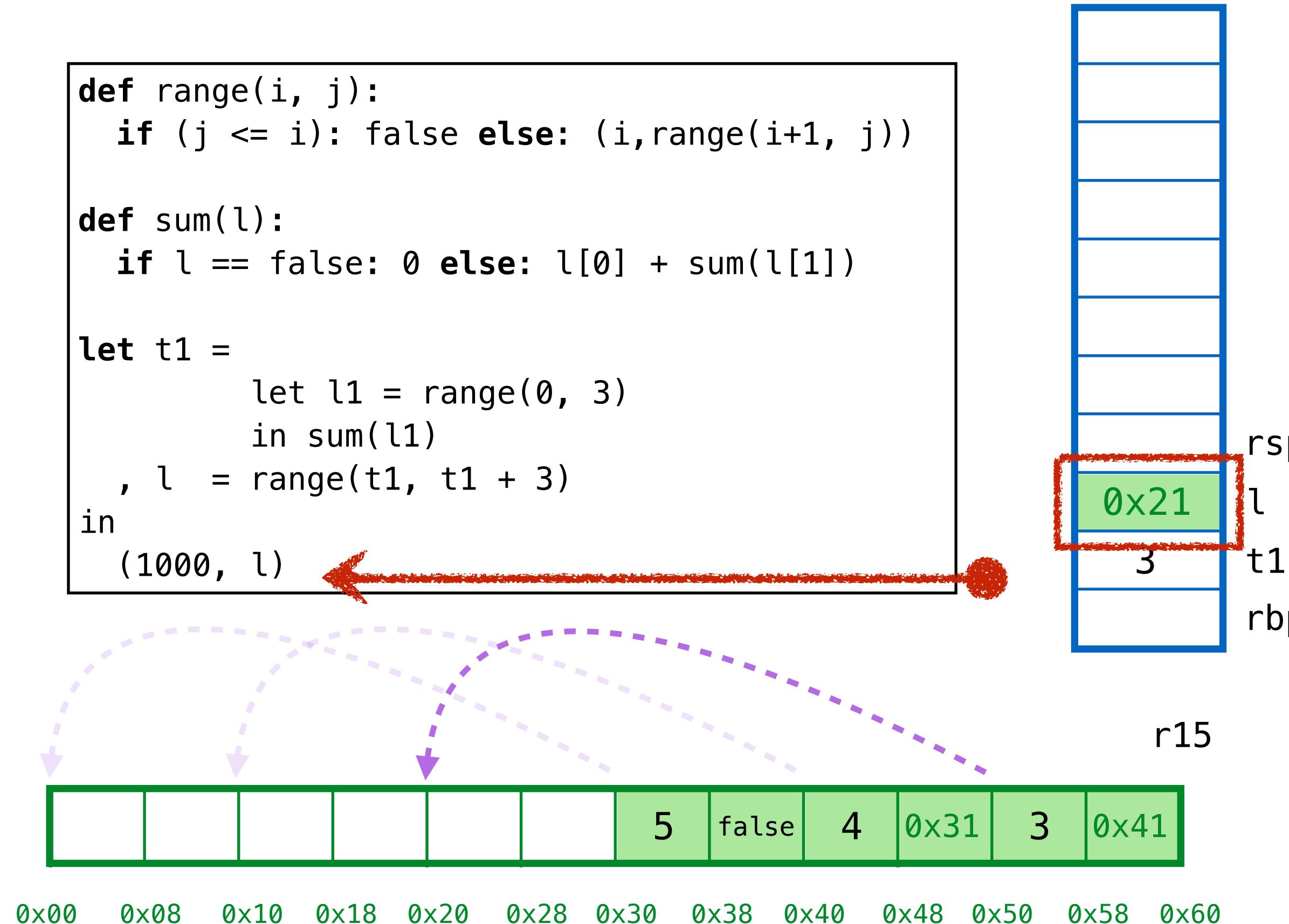
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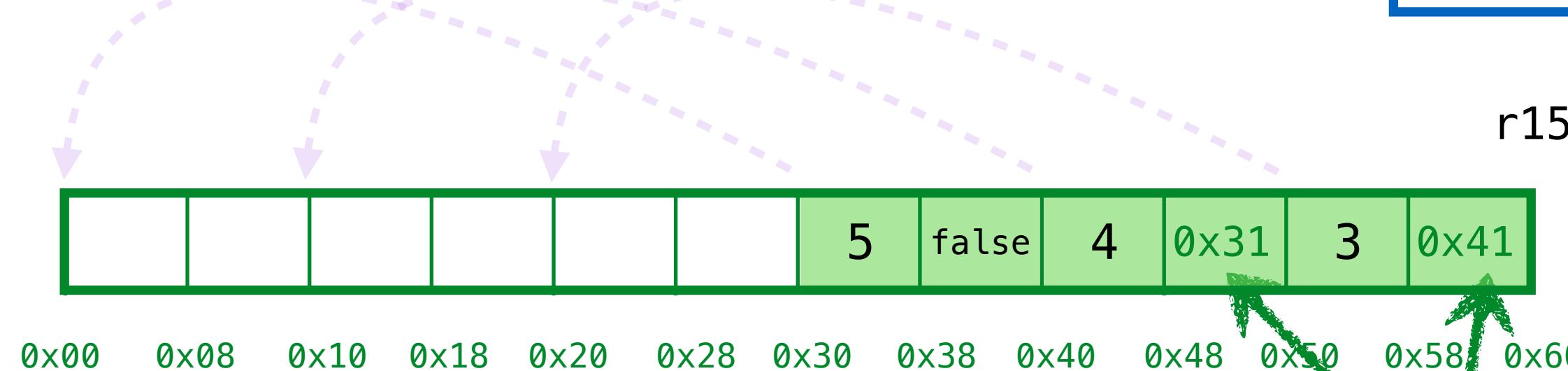
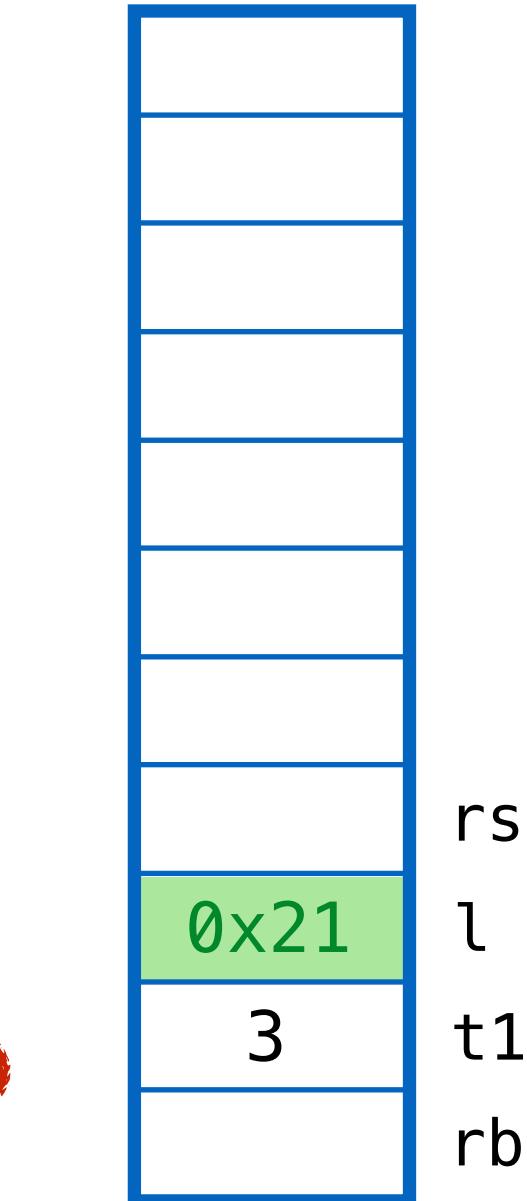
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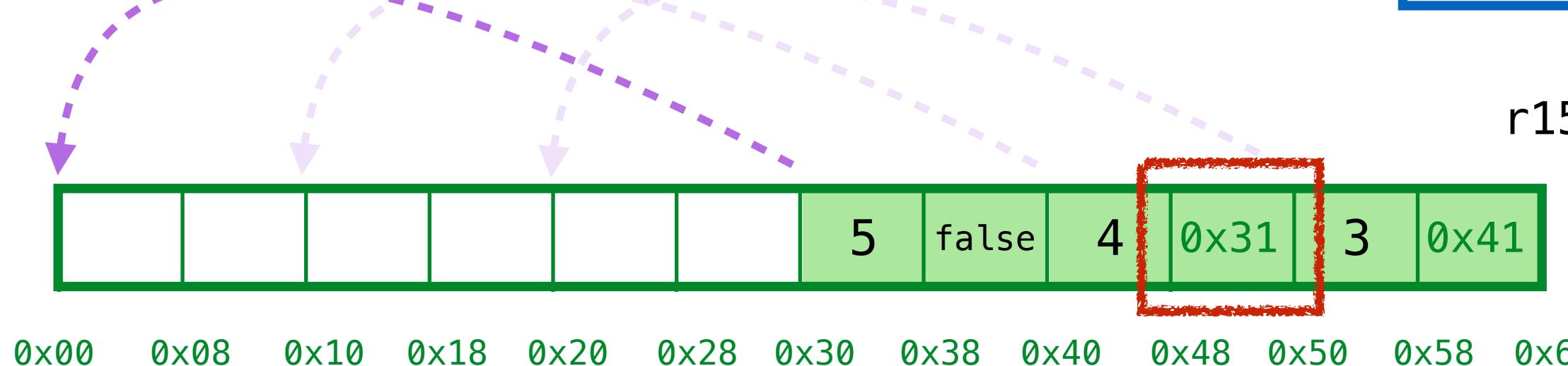
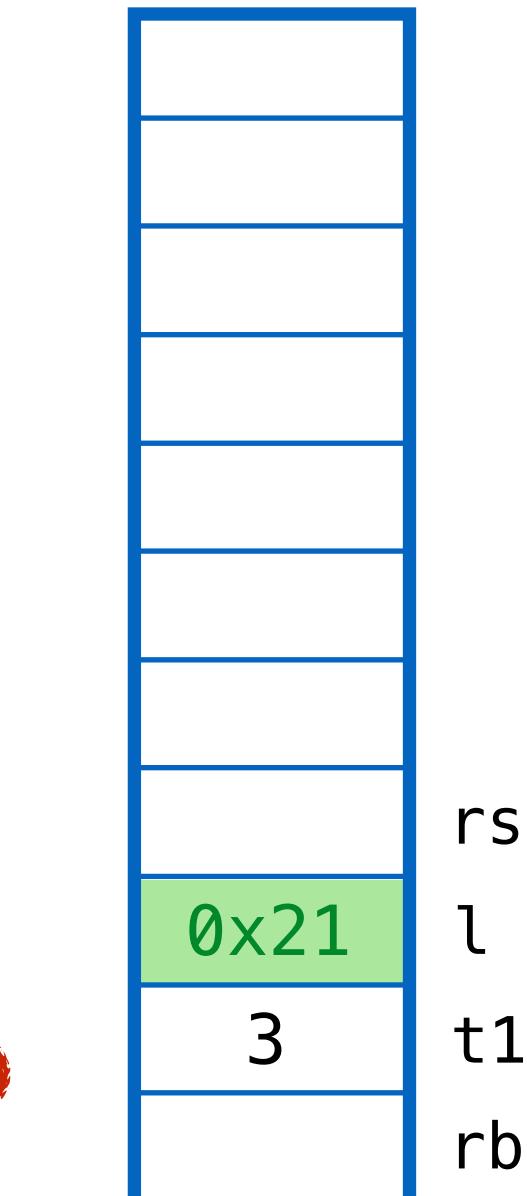
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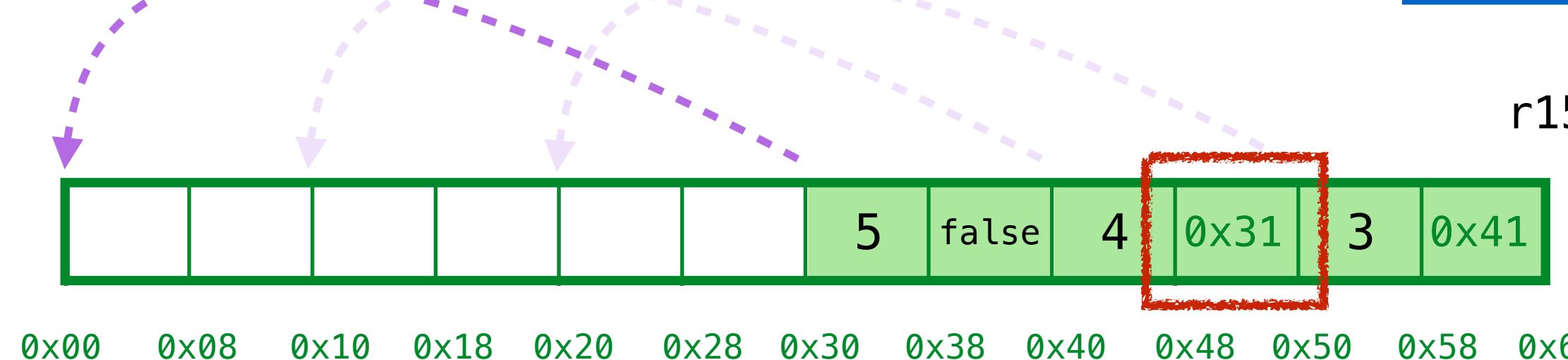
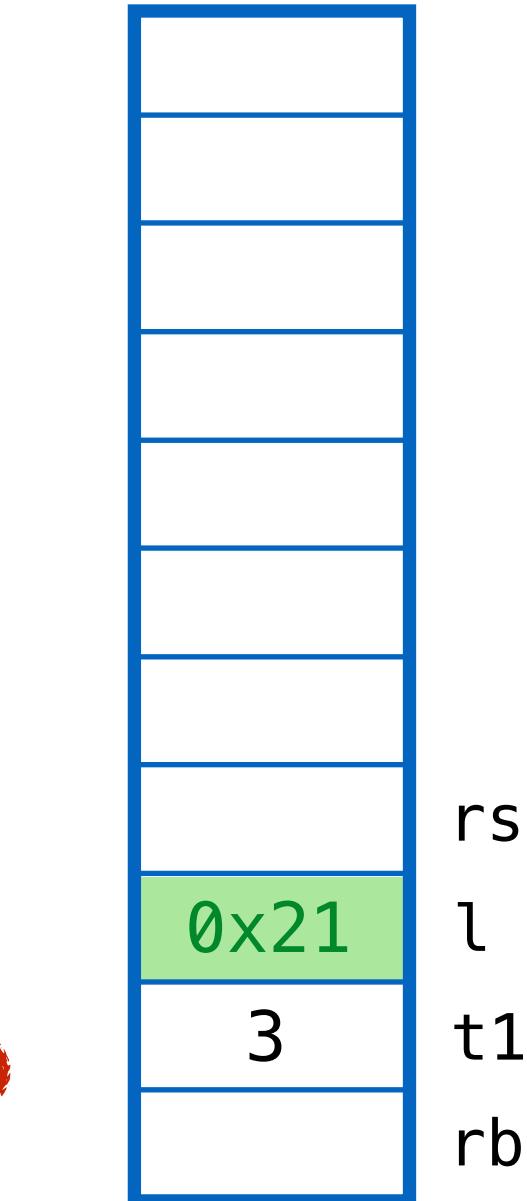
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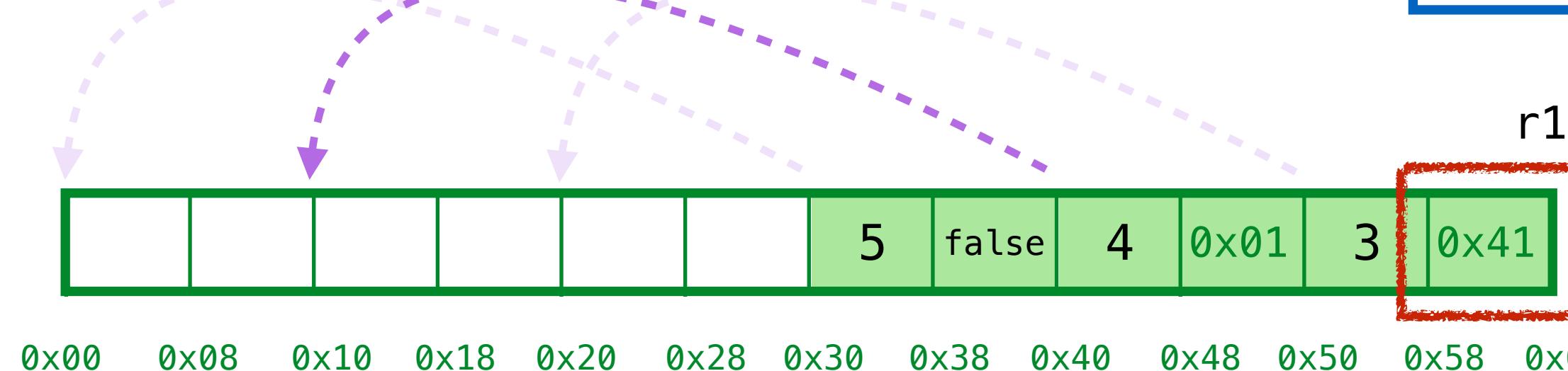
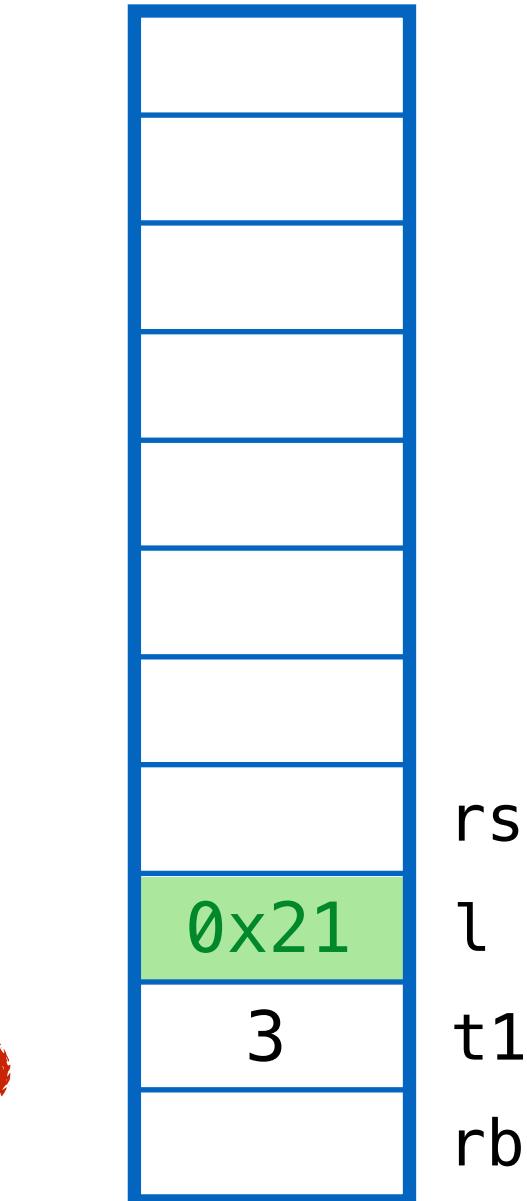
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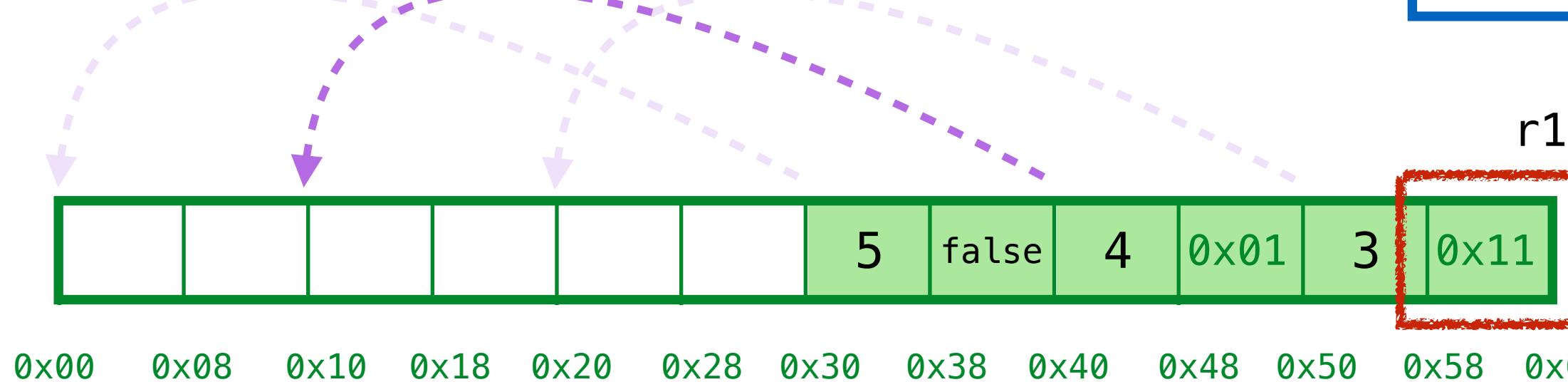
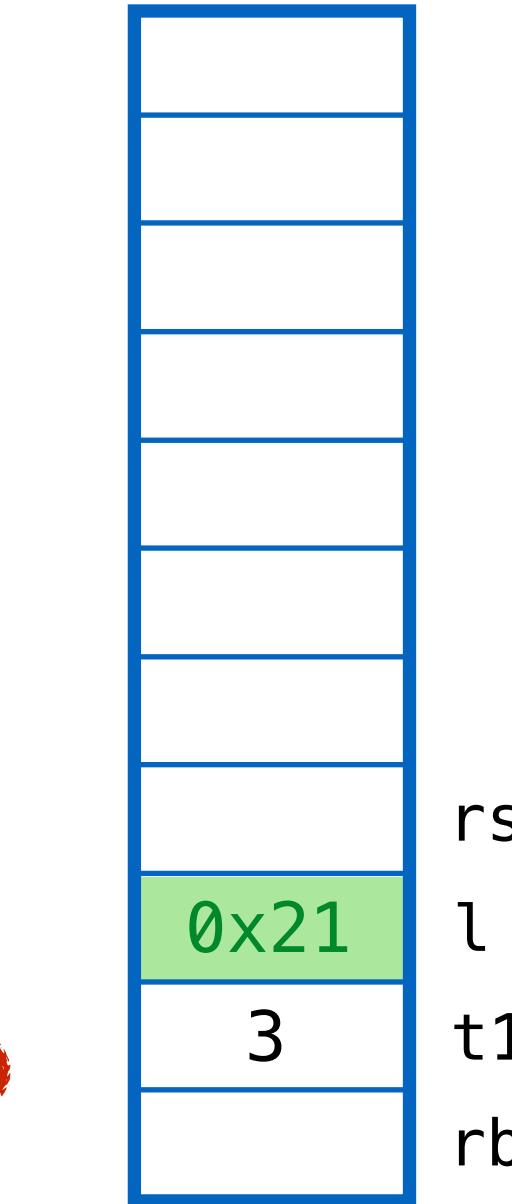
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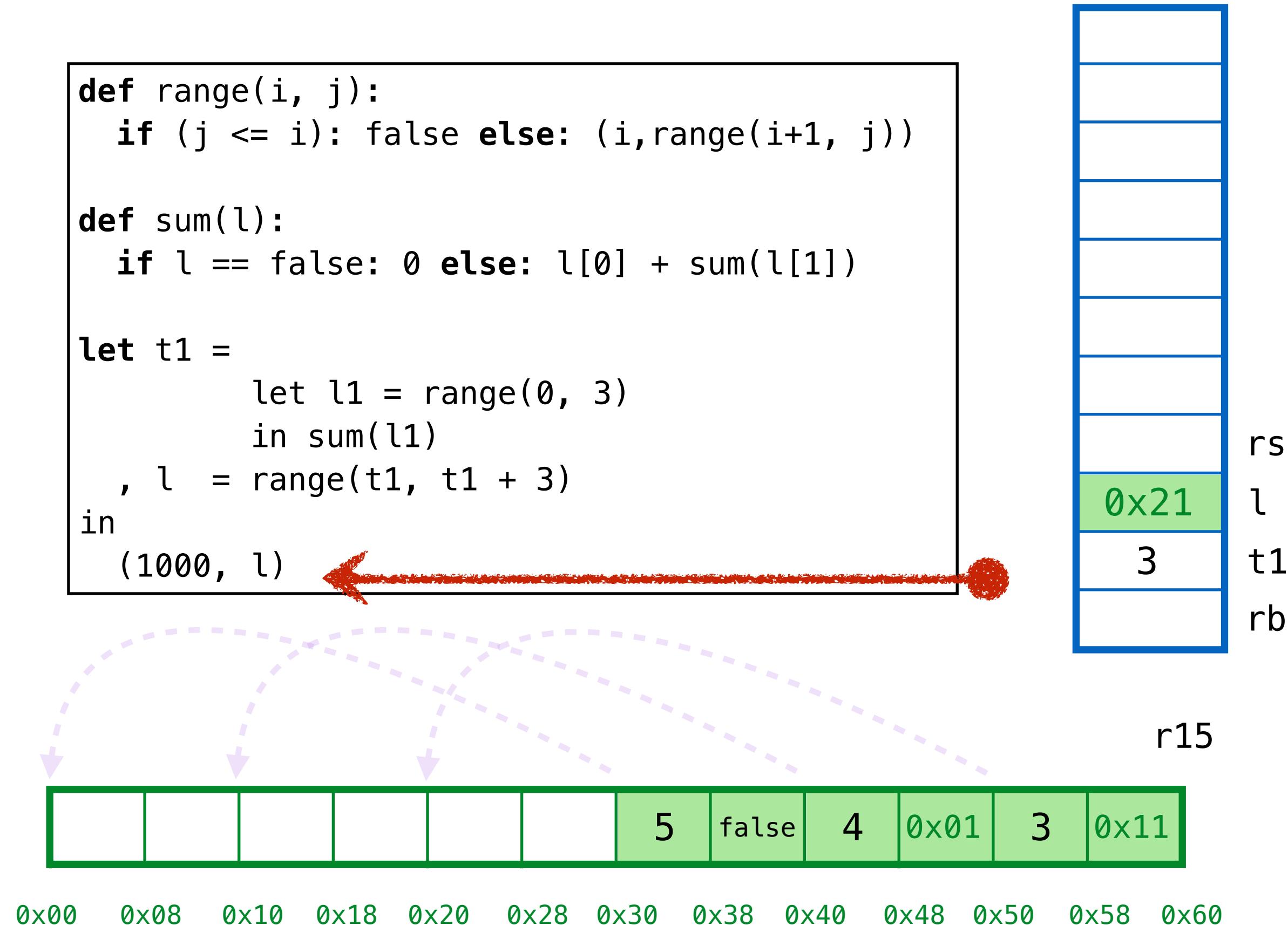
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## 4. COMPACT cells on heap

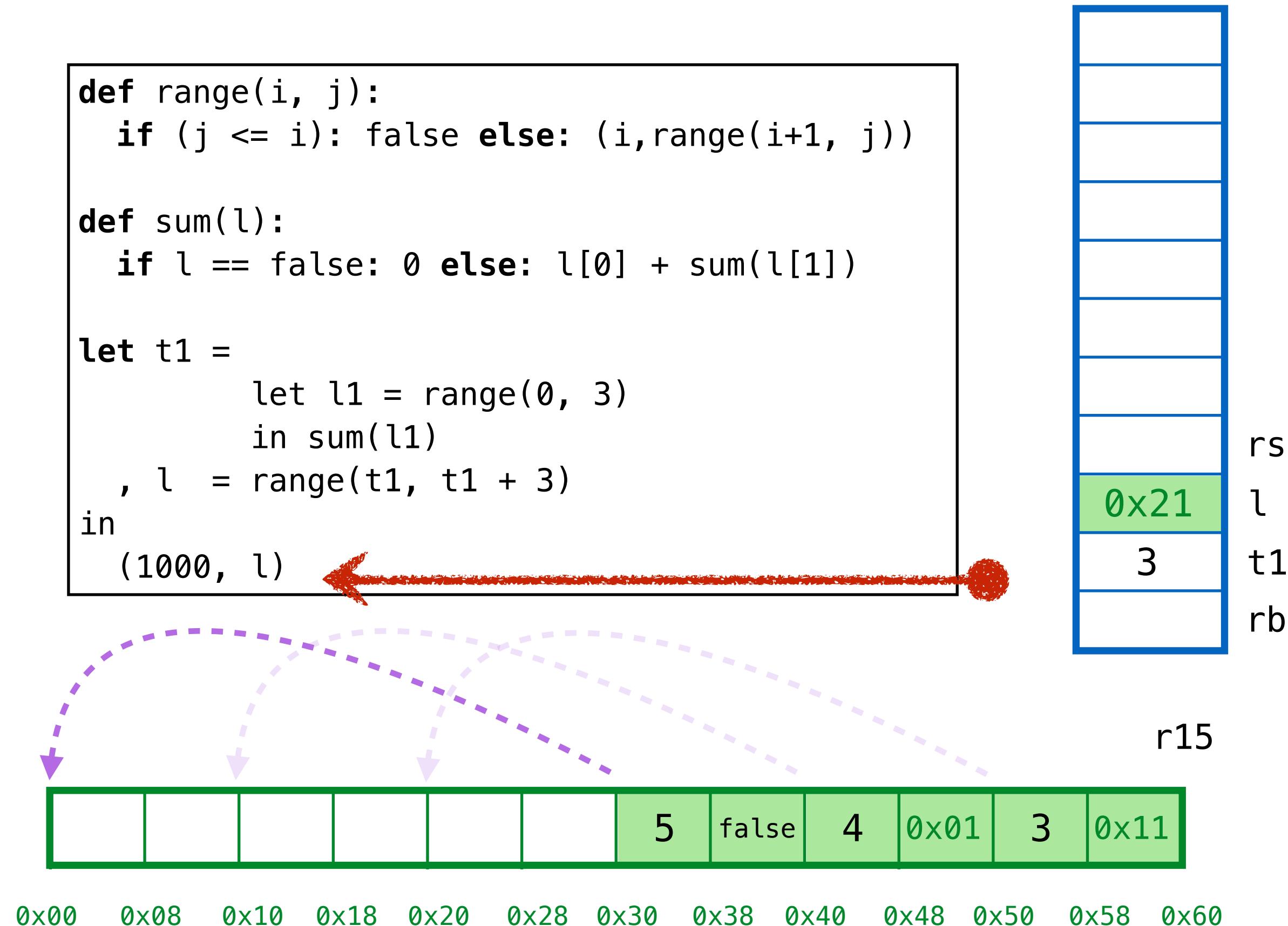
Copy cell to forward addr!

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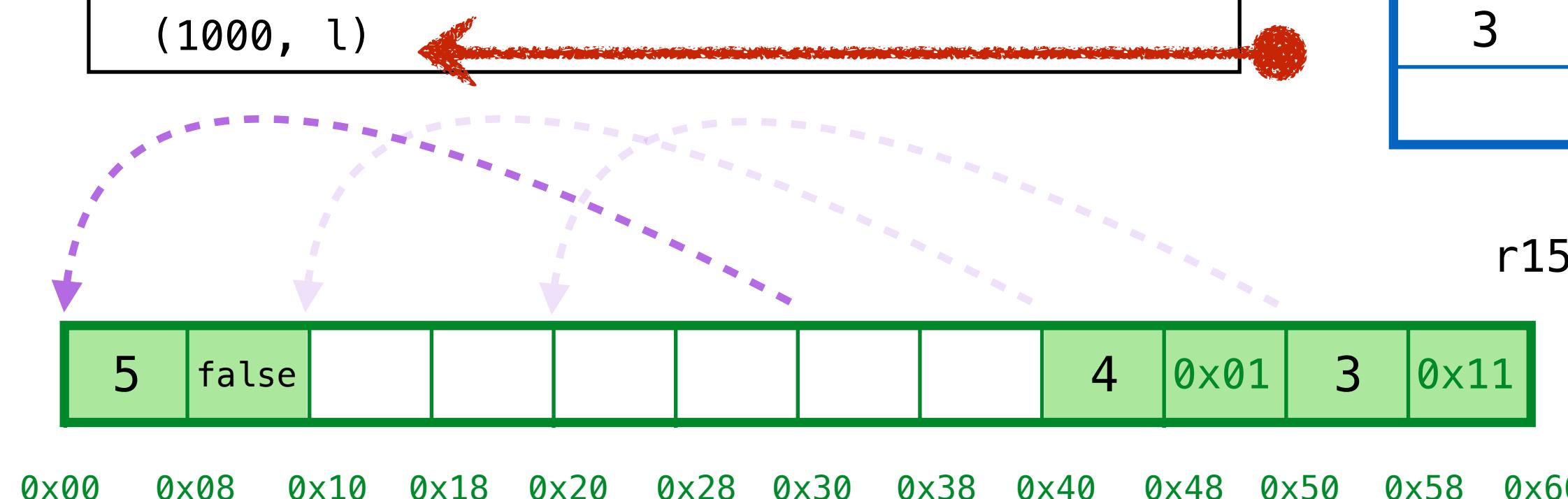
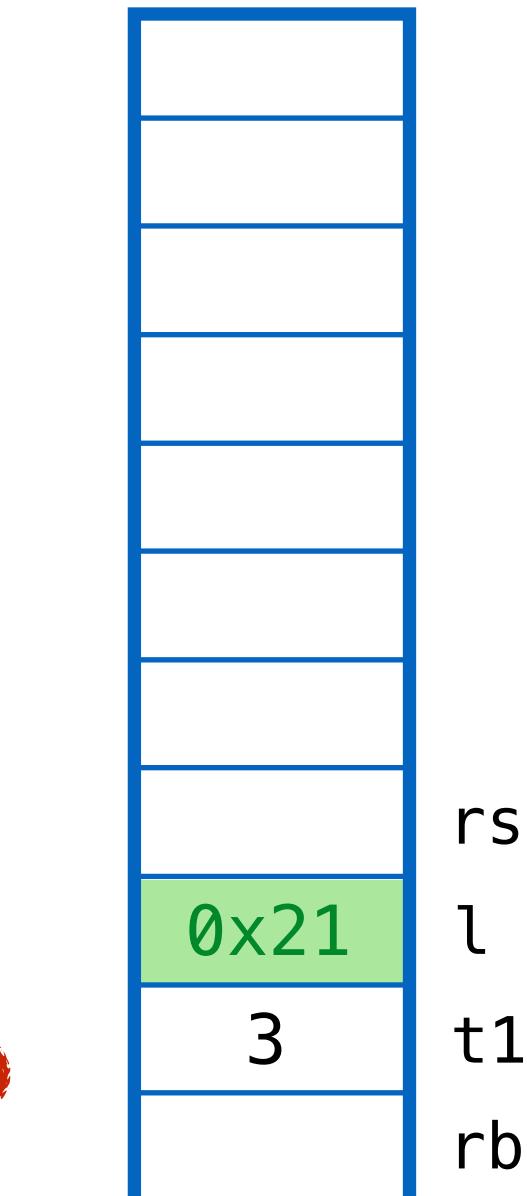
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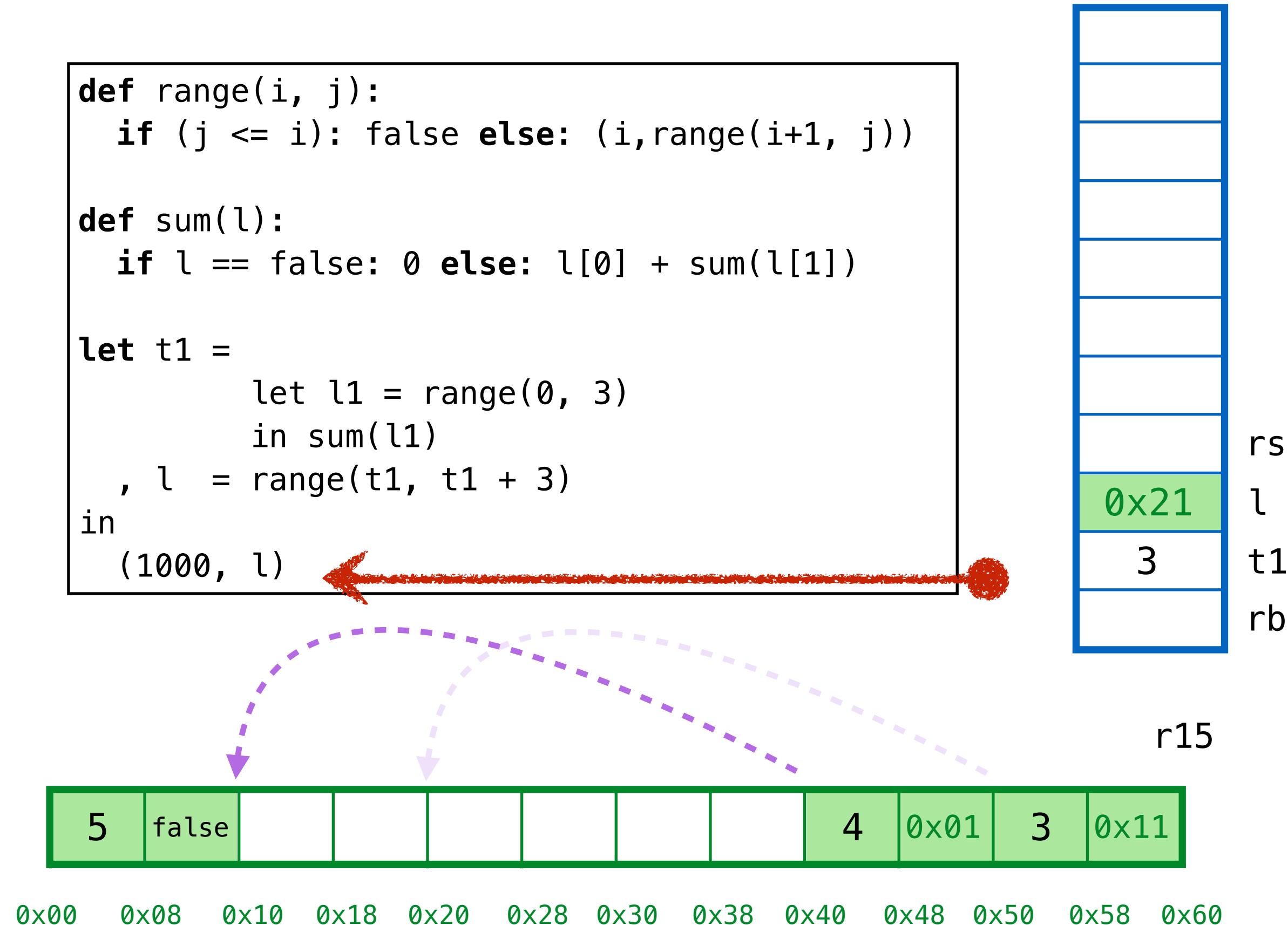
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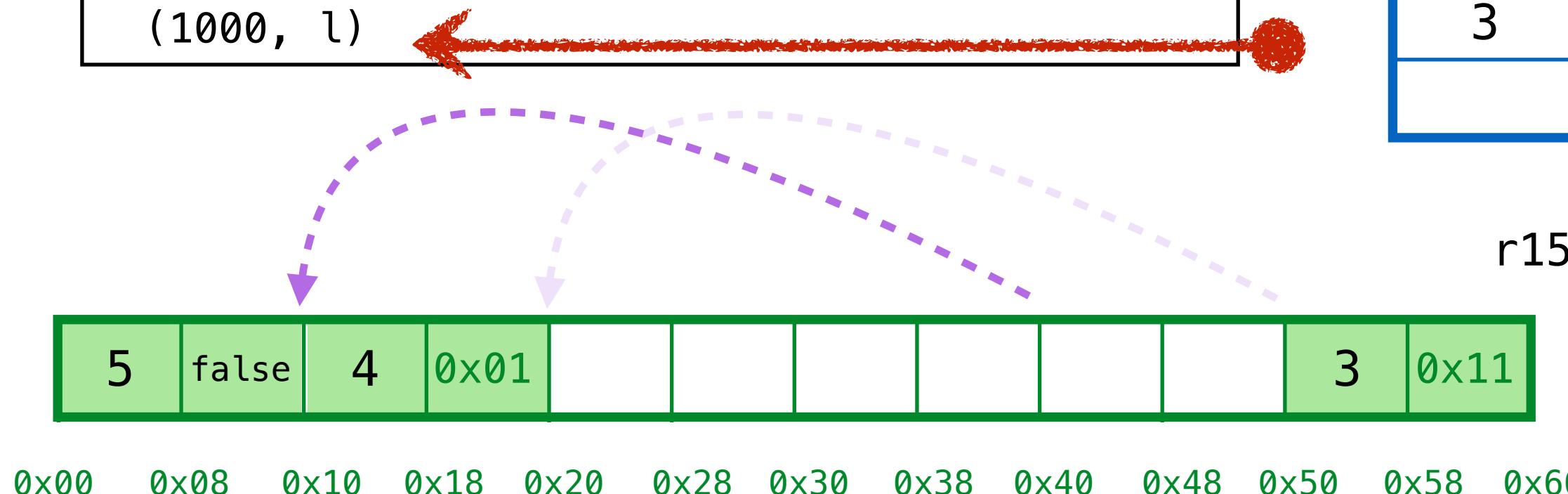
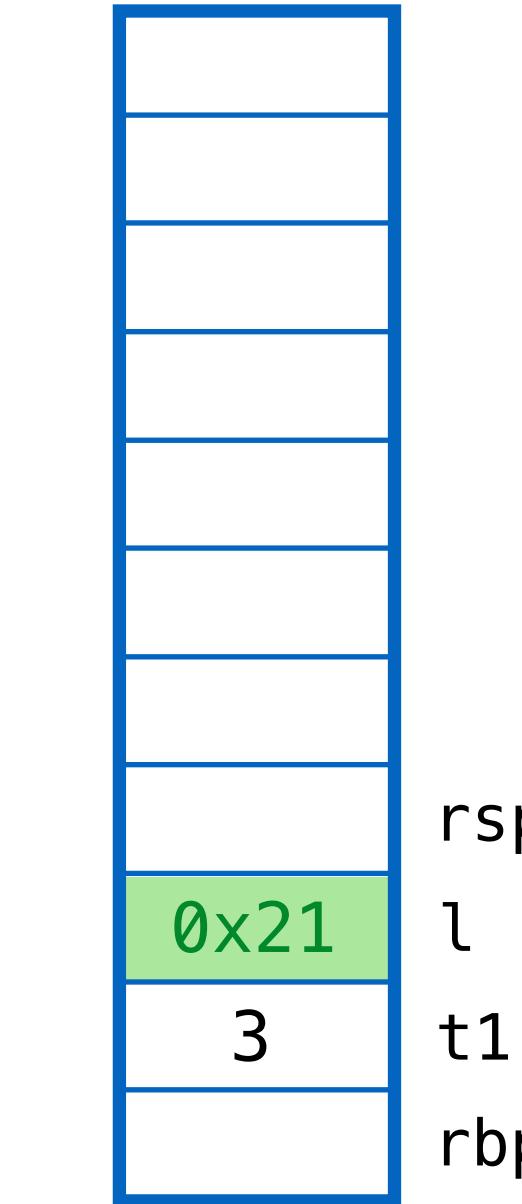
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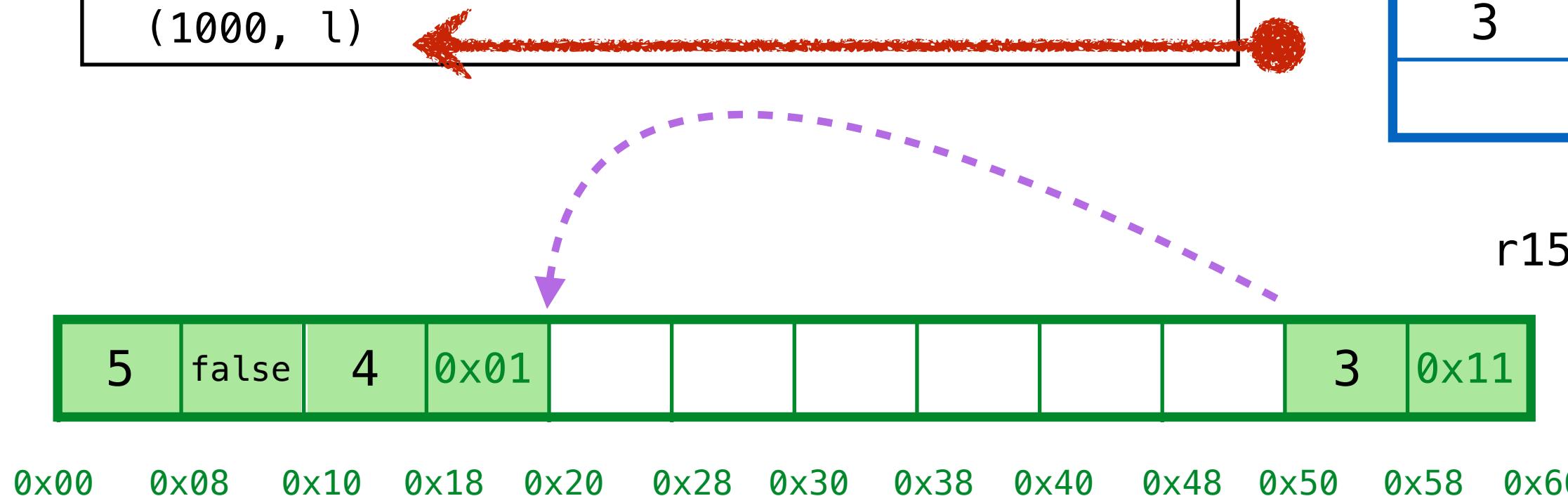
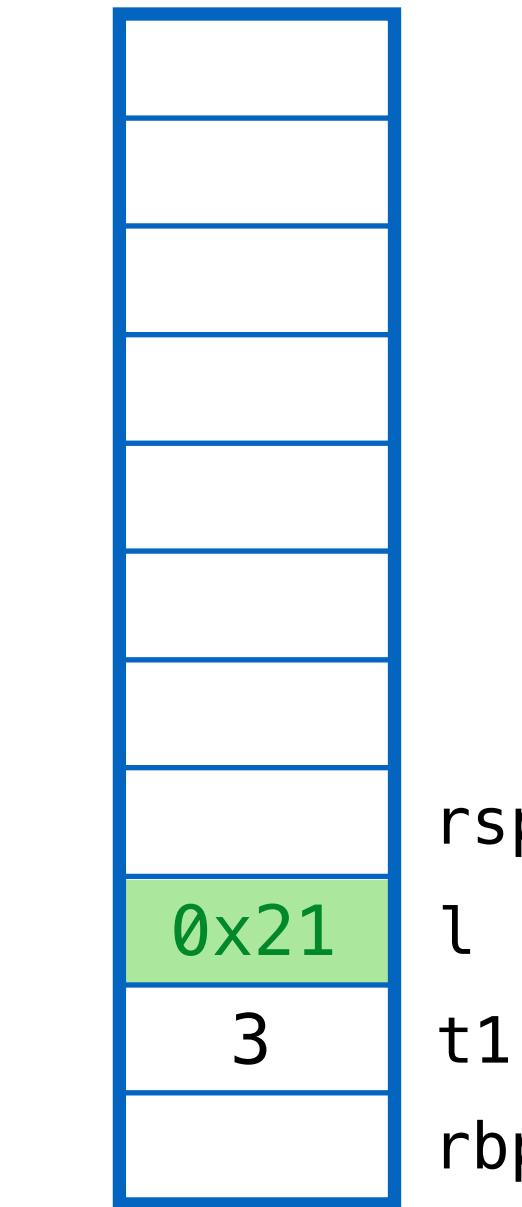
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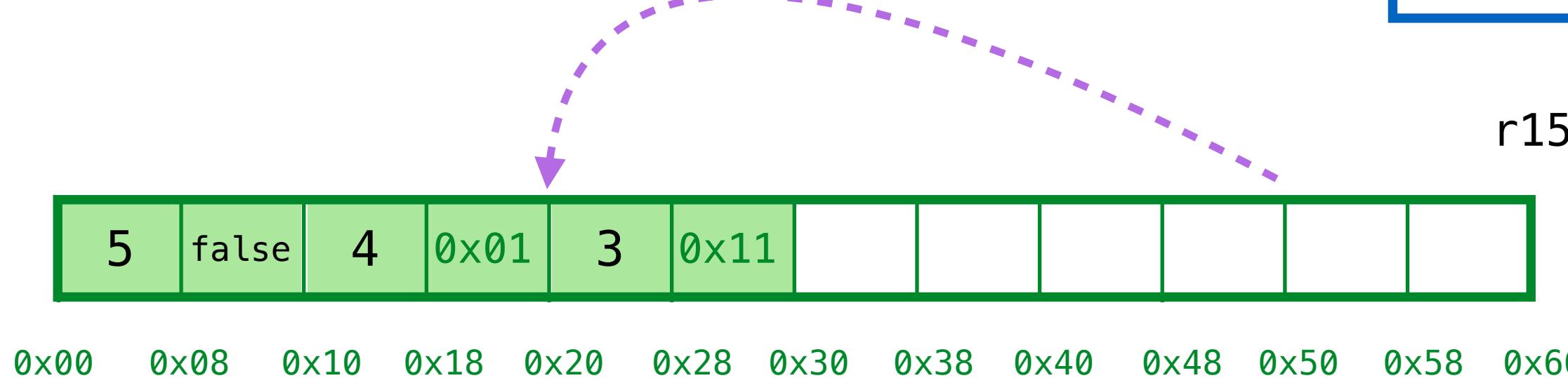
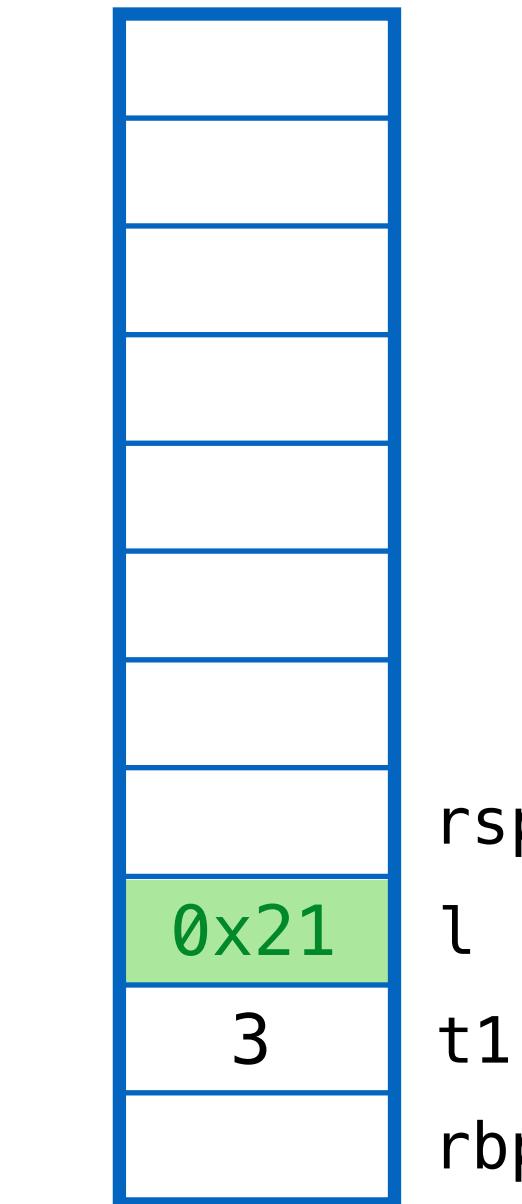
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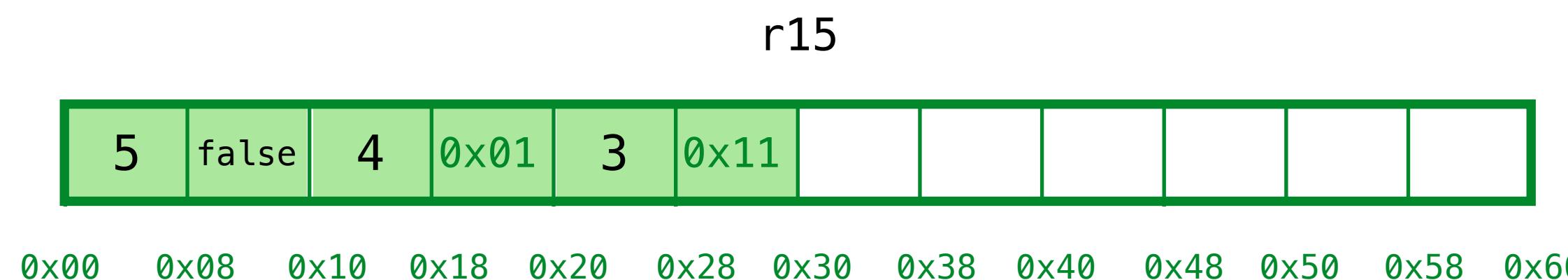
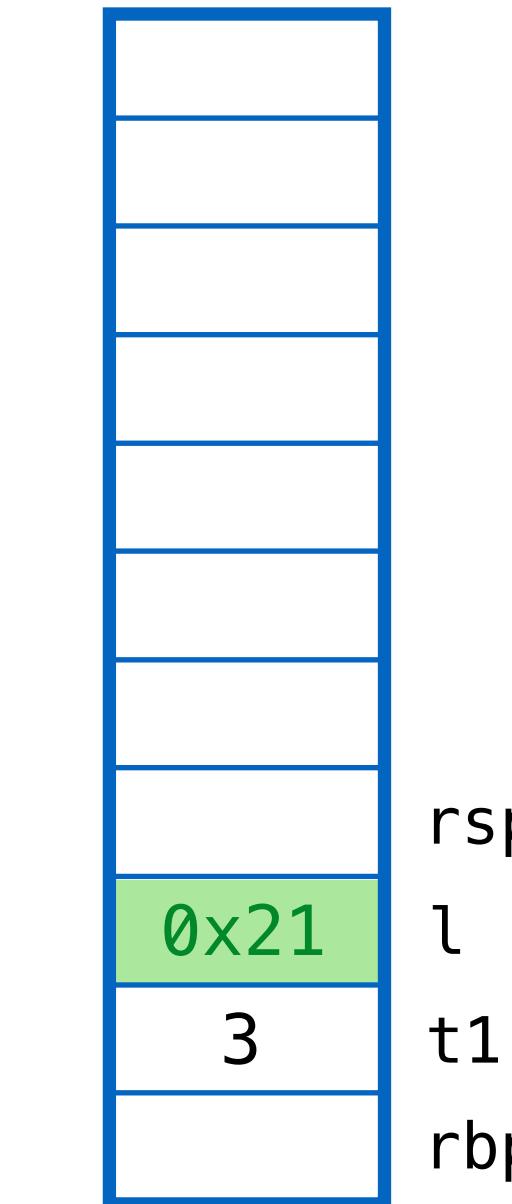
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# GC Complete!

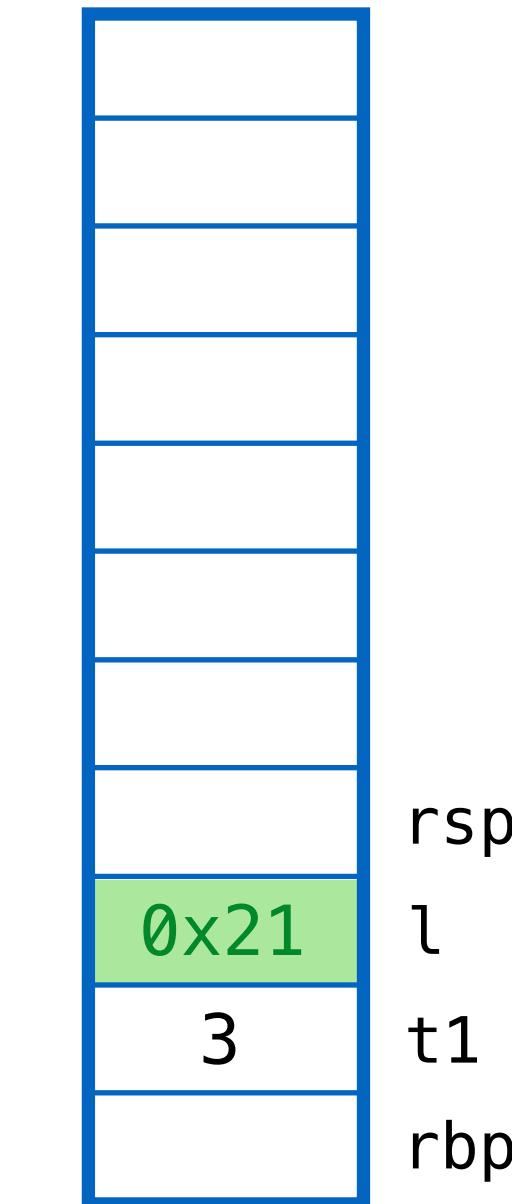
Have space for (1000, l)

## ex4: recursive data

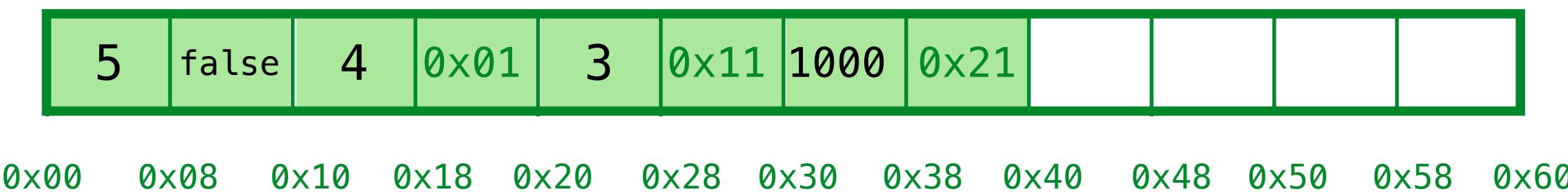
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r15



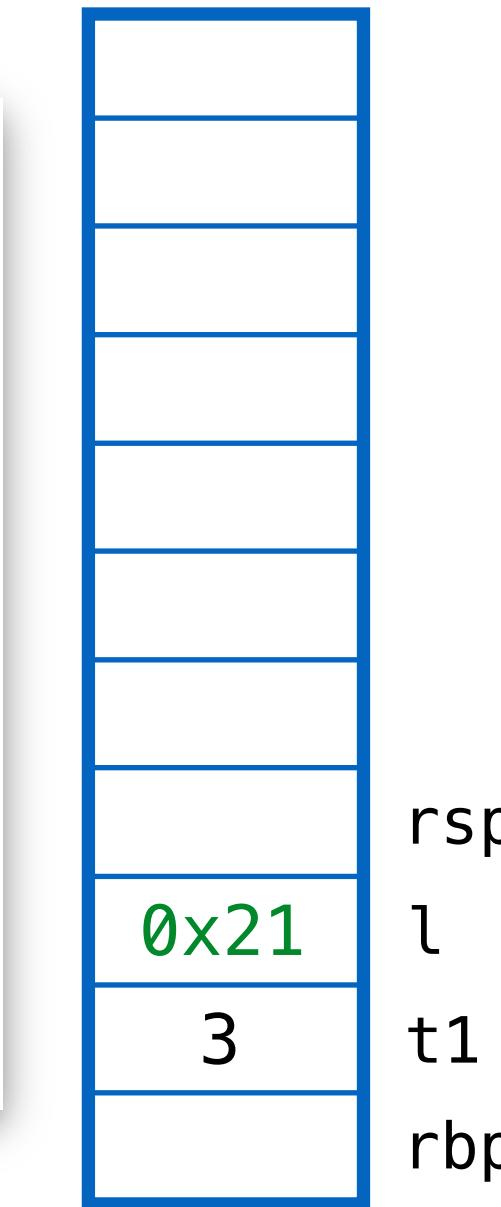
GC Complete!

Have space for (1000, l)

## ex4: recursive data

QUIZ: What should `print(0x21)` show?

- (A) `(0, (1, (2, false)))`
- (B) `(3, (4, (5, false)))`
- (C) `(0, (1, (2, (3, (4, (5, false))))))`
- (D) `(3, (4, (5, (0, (1, (2, false))))))`
- (E) `(2, (1, (0, (3, (4, (5, false))))))`



r15

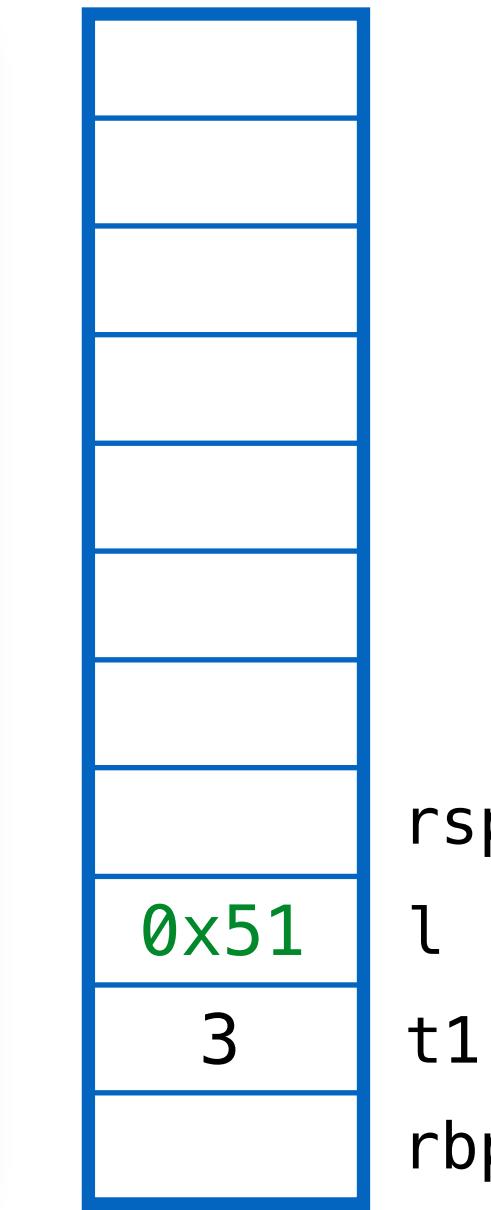
2	0x51	1	0x01	0	0x11	5	false	4	0x31	3	0x41
---	------	---	------	---	------	---	-------	---	------	---	------

0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

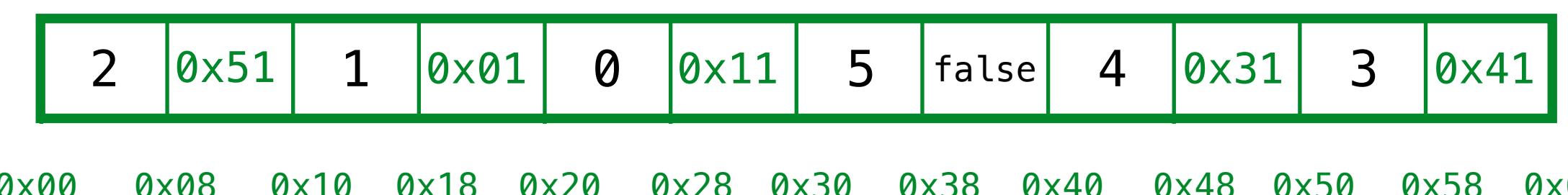
## ex4: recursive data

QUIZ: Which cells are “live” on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50



r15



# Heap

r15

2	0	20	0xa1	2	0	10	0x01	2	0	0	0x21	2	0	50	false	2	0	40	0x61	2	0	30	0x81	
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60	0x68	0x70	0x78	0x80	0x88	0x90	0x98	0xa0	0xa8	0xb0	0xb8	0xc0

2

# Stack

0x4000	0	rsp
0x4008	10	
0x4010	0x01	
0x4018	0x4038	rbp
0x4020	0	
0x4028	0xa1	
0x4030	3	
0x4038	0	Bot