Name: ________________________________

PID: ________________________________

This is the fourth quiz of CSE255/DSE230

On your desk you should have only the exam paper and writing tools. No hats or hoods allowed (unless religious items). There are 2 questions in this exam, totalling 100 points.

You have 10 minutes to complete the exam.

Start by writing your name and PID on this page.

Good Luck!

Setup

- $T = \{\vec{x}_1, \vec{x}_2, \vec{x}_n\}$ is a set of vectors in R^d.
- The mean of T is $\vec{\mu} = \frac{1}{n}\sum_{i=1}^{n} \vec{x}_i$
- C is the covariance matrix of T. The eigenvectors of C are $\vec{u}_1, \ldots, \vec{u}_d$ and the corresponding eigen-values are $\lambda_1 > \lambda_2 > \cdots > \lambda_d$

The answers to the questions below should consist of the vectors and scalars defined above (and potentially constant numbers).

I. (25 points):
Write an expression for $\frac{1}{n}\sum_{i=1}^{n}(\vec{x}_i - \vec{\mu}) \cdot (\vec{x}_i - \vec{\mu})$

II. (25 points):

Write an expression for the approximation of $\vec{y} \in R^d$ using the first two eigen-vectors of C (note: $\hat{y}^2$ denotes the approximation vector using the first two eigen-vectors, it is **not** the square of $\hat{y}$.)

$\hat{y}^2 =$ ________________________________

III. (25 points):

Write an expression for the *residual* of $\hat{y}^2$

$\hat{r}^2 =$ ________________________________

IV. (25 points):

Denote by $\hat{x}_i^2$ the approximation of $\vec{x}_i$ using the first two eigenvectors. Write an expression for $\frac{1}{n}\sum_{i=1}^{n}(\hat{x}_i^2 - \vec{\mu}) \cdot (\hat{x}_i^2 - \vec{\mu})$
