Name: _____

PID: _____

> This is the 8th quiz of CSE255/DSE230
> On your desk you should have only the exam paper and writing tools.
> No hats or hoods allowed (unless religious items).
> There are 5 questions in this quiz, each worth 20 points.
> You have 20 minutes to complete the exam.
> Start by writing your name and PID on this page.
> Good Luck!

Define an ϵ-cover of a set S to be a set balls of radius ϵ that cover the set (i.e. S is a subset of the union of the balls). Let $N(\epsilon)$ be the *minimal* number of balls of radius ϵ needed to cover S. Define the **intrinsic dimension** of S to be the minimal value of d such that there exists a positive, finite, real number C such that:

$$\forall \epsilon > 0, \quad N(\epsilon) < \frac{C}{\epsilon^d}$$

Let $\vec{a}, \vec{b}$ be independent vectors in R^D. In all of the questions below, assume $\|\vec{x}\|_2 \leq 100$.

What is the intrinsic dimension of the following sets?

1. $\{\alpha\vec{a}$ such that α is an integer between -100 and +100$\}$ _____

2. $\{\alpha\vec{a}+\beta\vec{b}$ such that α, β are real numbers, and $|\alpha|+|\beta| \leq 1\}$ _____

3. $\{\alpha\vec{a}$ such that α is a real number $0 \leq \alpha \leq 1\}$ _____

4. $\{\vec{x}$ such that $\vec{x} \cdot \vec{a} = 5\}$ _____

5. $\{\vec{x}$ such that $\vec{x} \cdot \vec{a} = 5$ and $\vec{x} \cdot \vec{b} > 0\}$ _____