

Name: _____

PID: _____

This is the third quiz of CSE255/DSE230

On your desk you should have only the exam paper and writing tools. No hats or hoods allowed (unless religious items). There are 2 questions in this exam, totalling 100 points.

You have 10 minutes to complete the exam.

Start by writing your name and PID on this page.

Good Luck!

I. (25 points) Orthonormal basis

Suppose that V is a vector space of dimension d , that $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_d$ is a basis for V , and that $\vec{x} \in V$ is an arbitrary vector in V . Express \vec{x} as a linear combination of the basis vectors.

$$\vec{x} = \underline{\hspace{10cm}}$$
II. (25 points) Eigen-Vectors

Suppose M is a symmetric (real valued) $d \times d$ matrix, that $((\vec{u}_1, \lambda_1), (\vec{u}_2, \lambda_2), \dots, (\vec{u}_d, \lambda_d))$ are eigen-vector, eigen-value pairs. The eigen-values are all different.

Mark all of the correct statements:

1. For any i : $M\vec{u}_i = \lambda_i\vec{u}_i$
2. For any i : $\vec{u}_i \cdot \vec{u}_i = 1$
3. For any $i \neq j$: $\vec{u}_i \cdot \vec{u}_j = 0$
4. The set $\vec{u}_1, \dots, \vec{u}_d$ forms a basis of R^d .

III. (25 points) The covariance matrix

Suppose you have N vectors in R^d : $\vec{x}_1, \dots, \vec{x}_N$. \vec{x}_i are column vectors.

Write an expression for the expected vector

$$\vec{\mu} = \underline{\hspace{10cm}}$$

Write an expression for the Covariance matrix

$$C = \underline{\hspace{10cm}}$$
IV. (25 points) PCA

Suppose C is a covariance matrix computed for the dataset $\vec{x}_1, \dots, \vec{x}_N$. Suppose that non of the eigenvalues of C is zero. Suppose U is the orthonormal matrix that contains the eigenvectors of C . U can be seen as a change of basis. Define the transformed dataset to be $\vec{y}_i = U\vec{x}_i$. What can you say about the covariance matrix of the dataset: $\vec{y}_1, \dots, \vec{y}_N$?
