Estimation of Causal Effects Under Interference

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Abstract

In randomized experiments, it is often assumed that the response of a given unit only depends on its own characterisitcs and the treatment to which it is assigned. However, there are numerous situations of interest to researchers where this may not be a plausible assuption, particularly when there is social interaction between individuals in an experiment, and the ability to separately identify the effects of treatment assigment and peer influence have important implications. This paper presents a general framework for reliably identifying and estimating causal effects in experimental settings where there is potential interference between units. The framework consists of (i) learning the potential for interference between connected units, (ii) a probabilistic graphical model for estimating individuals' exposure to treated units, and (iii) methods that make use of estimated exposures to compute causal effects of interest. We then evaluate the performance of the proposed framework on by simulating synthetic experimental data on a variety of real world social networks.

1 Introduction

2 Setting and Assumptions

We consider the setting of a randomized experiment on a social network of N nodes. The network, $G = (V, E)$, is observed, and may or may not contain node-level covariates or edge strengths. Let $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_N)$ be a treatment assigment vector over N units, where $Z_i \in \{0, 1\}$ specifies which of the possible treatments unit i receives. We assume that $P(Z_i = 1)$ is the same for all $i \in \{N\}$, so treatment assigment is unconfounded. We define $\mathbf{C} = (C_1, C_2, \ldots, C_N)$ to be a latent exposure vector, where $C_i \in \{0, 1\}$ specifies if unit i is exposed to the treatment via social interaction. Here, we assume that all treated units are exposed, such that $Z_i \times C_i = 1$, but that treatment units do not experience further spillover effects. If we could observe $\mathbf{C}$, then our work would be done, since we could partition the units into treatment, exposured, and control groups, and employ classic techniques to infer their differences in outcome.

Instead, we define $\pi_i(\mathbf{Z}) = P(C_i = 1)$ to be the expsoure probability for unit i given some instantiation of the experiment $\mathbf{Z}, \mathbf{C}$. Since an individual's social exposure to the treatment depends on the random assigment in G, so does $\pi_i(\mathbf{Z})$. For simplicity, we assume that the effects of both treatment assigment and social exposure are homogenous. Furthermore, we assume that social interaction, and thus social exposure to the treatment, only occurs through the edges E of G. However, we make no assumption on the form of such exposures ...

3 Estimating Exposures and Causal Effects

We propose modeling the spread of influence in the social network as a Pairwise Markov Random Field, defined as $B = (P, H)$, where $H = (V, E)$ is an isomporphism of the social network G, given by the bijection $f : G(V) \mapsto H(V)$ where $f(v_i) = X_i \sim \text{Bernoulli}(p)$, i.e. H is an undirected graph with the skeleton of G where each node in $H(V)$ is a Bernoulli random variable. P, then, is defined as a Gibbs distribution that factorizes over H:

$$P(X_1, X_2, \ldots, X_n) = \frac{1}{Z} \prod_{i \in V} \phi_i(X_i) \prod_{(i,j) \in E} \phi_{i,j}(X_i, X_j)$$

By having each individual represented as a bernoulli random vairable in H, we can reformulate the problem of estimating the exposure probability for node i, $\pi_i(\mathbf{Z})$, into an inference problem, where we seek to infer marginal probability distributions for each node. Observe that for any treated unit, they are assumed to be already exposed, so $X_i = 1$ for all individuals i where $Z_i = 1$. This observation acts as our evidence in P when performing inference on the unknown marginal distributions of the other nodes in H. Formally, we infer the posterior distribution $P(X_i \mid e)$, where $e = \{X_i = 1 \mid Z_i = 1\}$. This reformulation allows us to use a host of existing inference algorithms to learn the posterior marginals.

3.1 Unary and Edge Potentials

Having refomulated estimating $\pi_i(\mathbf{Z})$ as inference of marginals in a Pairwise MRF, properly defining the edge and unary potentials in H becomes an important specification. In this section, we dicuss several potential approaches. In the setting considered by this paper, treatment assigment is unconfounded, so the unary potentials $\phi_i(X_i)$, are uniform across the nodes such that $\phi_i(X_i = 1) = \phi_i(X_i = 0)\ \forall i$. However, in the case that treatment assigment is, for instance, confounded by covariates, it may make sense to use the unary potentials as a prior on similarity with the treatment group, since influence may flow to "similar" nodes more easily.

The edge potentials $\phi_i(X_i, X_j)$ have an even more intuitive interpretation, representing the potential for influence, or "strength" of a social connection between two nodes in H. Here, we propose two possible approaches for specifying the edge potentials. Both involve first specifying the probability of each edge in the social network. The first is that, given domain knowledge within the experiment, the researcher may have a prior on the distribution of edge probabilities, and can make use of this prior belief. For instance, let $p_{i,j}$ be the probability of an edge between node i and node j. A reasonable prior could be $p_{i,j} \sim \text{Beta}(5, 5)$. This belief could be incorporated by initializing the edge potentials according to this distribution. The next is a latent variable model introduced by Handcock et al. [1]. Where we model $p_{i,j}$ using a logistic regression where the probability of an edge depends on euclidean distance in latent space:

$$\text{log-odds}(p_{i,j}) = \beta X_{i,j} - ||z_i - z_j||^2 \tag{1}$$

Where z_i and z_j are node i and node j's respective positions in latent space, and $X_{i,j}$ is some vector valued edge covariates. Here, it is assumed that the existence of an edge is independent of other edges. Handcock jointly estimates the parameter β and the latent z_i's using MCMC, with prior:

$$z_i \sim \sum_{g=1}^{G} \lambda_g \text{MVN}_d(\mu_g, \sigma_g I_g)$$

Where G is the possible number of latent social clusters. Since we can take $G = 1$, the latent variable model provides a flexible and well studied approach to estimating the probability of edges between nodes in a variety of experimental settings. We test both possible approaches to estimating edge probabilities in the following section.

Given the edge probabilities $p_{i,j}$, we can specify the the edge potentials in H. In doing so, we make the assumption that a higher edge probability leads to greater social influence, and thus likelihood that two adjacent nodes have the same value, i.e. $P(X_i = X_j)$ is monotonically increacing in $p_{i,j}$. To acheive this, we define the edge potentials as:

$$\phi_{i,j}(X_i, X_j) = 1 - p_{i,j} \mathbf{1}\{X_i \neq X_j\}$$

We found that this specification resulted in a plausible empirical results across a number of different social networks, but this is another place where a researcher may choose to incorporate prior beliefs in specifying how social influence propogates in a specific network.

3.2 Estimating Causal Effects

With a distribution over the exposure probabilities for each node, we can turn to the problem of estimating the causal effects of interest. First, we are interested in estimating the average treatment effect, ATE. A naïve estimate of ATE in our setting would be a simple difference of means between the treatment and control groups. However, as discussed, in the presence of spillover effects, this estimate is that of both ATE, and whatever spillover effects may be present in the experiment. Instead, we propose the OLS regression:

$$y_i = \alpha + \rho Z_i + \gamma(1 - Z_i)\pi_i + \varepsilon_i \tag{2}$$

Where ρ is the parameter of interest, and $\varepsilon \ldots$. Since $\pi_i = 1$ if $Z_i = 1$, we add the interaction term $(1 - Z_i)$, so that the causal effect of treatment assigment is only contained in ρ. Unfortunatley, the coefficient γ does not have much causal interpretation, since π_i is merely the probability that unit i is socially exposed to the treatment. What we really desire, in theory, is to be able to classify each non-treated unit into either the socially exposed ($C_i = 1$), or control group ($C_i = 0$), as doing so would allow for perfect identification of average spillover effects.

discuss GMM model

4 Simulations

In order to test the performance of our proposed methods, we simulate random experiments on two real world social networks. The first is a group of 55 eigth-graders, with edges between students who were surveyed on which other students they would like to sit next to in class. The second data set is a social network between 61 employees of the Aarhus Computer Science Department, where edges represent colleagues who ate lunch together in a given week. We simlulate the outcome of interest drawing from a normal distribution $y_i \sim \mathcal{N}(\mu, \sigma^2)$, and then consider a dilated effects scenario, where spillover is half of the average treatment effect. For each graph, we consider three possible data generating processes for the spillover effects. In the first, all neighbors of treated units are socially influenced. In the second, social influence is propogated across each edge (i, j) with probability $p_{i,j}$ according to the latent variable model (1), i.e. socially influenced neighbors of treated units can influence their neighbors etc. until all possible influence has propogated across the network. The third is the same as the second process except $p_{i,j} = 0.5$ for all edges in the network. We also test two different spesifications for the edge potentials in each scenario. In the first, the edge potentials accord with the latent variable model (1), and in the second we assume that $p_{i,j} \sim \text{Beta}(5, 5)$. The following table report the point estimates of the OLS regression (2) and corresponding standard errors following 1,000 simulated treatment assigments on each of the possible combinations of data generating processes, edge potentials, and social networks.

Table 1: Simulation Results

		Eighth Graders				Aarhaus		
DGP	$\phi_{i,j}$	$\bar{y}^1 - \bar{y}^0$	ρ	γ		$\bar{y}^1 - \bar{y}^0$	ρ	γ
Neighbors	Latent	12	123		12	1209		
	Beta							
Latent	Latent							
	Beta							
$p_{i,j} = 0.5$	Latent							
	Beta							

5 Discussion

The text must be confined within a rectangle 5.5 inches (33 picas) wide and 9 inches (54 picas) long. The left margin is 1.5 inch (9 picas). Use 10 point type with a vertical spacing (leading) of 11 points.

Times New Roman is the preferred typeface throughout, and will be selected for you by default. Paragraphs are separated by ½ line space (5.5 points), with no indentation.

The paper title should be 17 point, initial caps/lower case, bold, centered between two horizontal rules. The top rule should be 4 points thick and the bottom rule should be 1 point thick. Allow ¼ inch space above and below the title to rules. All pages should start at 1 inch (6 picas) from the top of the page.

For the final version, authors' names are set in boldface, and each name is centered above the corresponding address. The lead author's name is to be listed first (left-most), and the co-authors' names (if different address) are set to follow. If there is only one co-author, list both author and co-author side by side.

Please pay special attention to the instructions in Section 5.1.1 regarding figures, tables, acknowledgments, and references.

All headings should be lower case (except for first word and proper nouns), flush left, and bold.

First-level headings should be in 12-point type.

5.1 Headings: second level

Second-level headings should be in 10-point type.

5.1.1 Headings: third level

Third-level headings should be in 10-point type.

Paragraphs There is also a `\paragraph` command available, which sets the heading in bold, flush left, and inline with the text, with the heading followed by 1 em of space.

These instructions apply to everyone.

5.2 Citations within the text

The `natbib` package will be loaded for you by default. Citations may be author/year or numeric, as long as you maintain internal consistency. As to the format of the references themselves, any style is acceptable as long as it is used consistently.

The documentation for `natbib` may be found at

`http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf`

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated…

5.3 Figures

All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction. The figure number and caption always appear after the figure. Place one line space before the figure caption and one line space after the figure. The figure caption should be lower case (except for first word and proper nouns); figures are numbered consecutively.

You may use color figures. However, it is best for the figure captions and the paper body to be legible if the paper is printed in either black/white or in color.

Figure 1: Sample figure caption.

Table 2: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

5.4 Tables

All tables must be centered, neat, clean and legible. The table number and title always appear before the table. See Table 2.

Place one line space before the table title, one line space after the table title, and one line space after the table. The table title must be lower case (except for first word and proper nouns); tables are numbered consecutively.

Note that publication-quality tables *do not contain vertical rules.* We strongly suggest the use of the `booktabs` package, which allows for typesetting high-quality, professional tables:

`https://www.ctan.org/pkg/booktabs`

This package was used to typeset Table 2.

5.5 Math

Note that display math in bare TeX commands will not create correct line numbers for submission. Please use LaTeX (or AMSTeX) commands for unnumbered display math. (You really shouldn't be using $$ anyway; see `https://tex.stackexchange.com/questions/503/why-is-preferable-to` and `https://tex.stackexchange.com/questions/40492/what-are-the-differences-between-align-equation-and-displaymath` for more information.)

5.6 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

6 Preparing PDF files

Please prepare submission files with paper size "US Letter," and not, for example, "A4."

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF files uses. In Acrobat Reader, select the menu Files>Document Properties>Fonts and select Show All Fonts. You can also use the program `pdffonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.
- `xfig` "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

  ```
  \usepackage{amsfonts}
  ```

  followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for $\mathbb{R}$, $\mathbb{N}$ or $\mathbb{C}$. You can also use the following workaround for reals, natural and complex:

  ```
  \newcommand{\RR}{I\!\!R} %real numbers
  \newcommand{\Nat}{I\!\!N} %natural numbers
  \newcommand{\CC}{I\!\!\!\!C} %complex numbers
  ```

  Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

6.1 Margins in LaTeX

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```
\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}
```

See Section 4.4 in the graphics bundle documentation (`http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf`)

A number of width problems arise when LaTeX cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

Acknowledgments and Disclosure of Funding

Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: `https://neurips.cc/Conferences/2023/PaperInformation/FundingDisclosure`.

Do **not** include this section in the anonymized submission, only in the final paper. You can use the `ack` environment provided in the style file to autmoatically hide this section in the anonymized submission.

7 Supplementary Material

Authors may wish to optionally include extra information (complete proofs, additional experiments and plots) in the appendix. All such materials should be part of the supplemental material (submitted separately) and should NOT be included in the main submission.

References

[1] Handcock, M.S., Raftery, A.E. and Tantrum, J.M. (2007), Model-based clustering for social networks. Journal of the Royal Statistical Society: Series A (Statistics in Society), 170: 301-354. https://doi.org/10.1111/j.1467-985X.2007.00471.x