Causal Effects Under Interference

A Markov Random Field Approach

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- Problem: underlying assumption that individuals don't influence one another
- When this assumption is voilated, we say there are "spillover" effects, which may have important policy implications

Motivating Example

- Randomized control trial for political mobilization
- Delivered messages to 61 million Facebook users during the 2020 US elections
- The messages not only influenced users who recieved them, but also the users' friends, and friends of friends
- The effect of social transmission on real world voting was greater than the direct effect of the messages

Current Methods

- Clustering: Only need SUTVA to hold between larger groups, eg. schools instead of students
- Exposure mapping: Researcher specifies some grouping e.g.

$$f(\mathbf{z}, \theta_i) = \begin{cases} d_{11} & \text{(Direct + Indirect Exposure)} \\ d_{10} & \text{(Direct Exposure)} \\ d_{01} & \text{(Indirect Exposure)} \\ d_{00} & \text{(No Exposure)} \end{cases}$$

then can compare groups to estimate potential spillover

Setting and Assumptions

- Random experiment on social network $G = (V, E)$
- Treatment vector $\mathbf{Z} \in \{0, 1\}^n$ is known
- Some unknown exposure vector $\mathbf{C} \in \{0, 1\}^n$
- Treatment effects are homogeneous, but spillover/exposure may be either homogeneous or heterogeneous
- Want to learn the "exposure probability" π_i for each individual in G

Modeling as an MRF

- Define a Pairwise Markov Random Field $B = (P, H)$ where H has the same structure as G
- Each node in the social network becomes a Bernoulli random variable X_i in H
- Evidence: for any treated unit, we assume they are already exposed, so $X_i = 1$ for all units $Z_i = 1$
- Perform inference on the marginals conditional on the evidence $P(X_i \mid e)$
- Compute exposure probabilities:

$$\pi_i = P(X_i = 1 \mid e) - P(X_i = 0 \mid e)$$

Estimating Potentials

- P is a Gibbs Distribution that factorizes over H:

$$P(X_1, X_2, \dots, X_n) = \frac{1}{Z} \prod_{i \in V} \phi_i(X_i) \prod_{(i,j) \in E} \phi_{i,j}(X_i, X_j)$$

- Pairwise potentials represent the potential for influence, or "strength" of a social connection between two nodes
- Model them as a function of edge probability, $p_{i,j}$:

$$\phi_{i,j}(X_i, X_j) = 1 - p_{i,j} \mathbb{1}\{X_i \neq X_j\}$$

- We can place a prior on edge probabilities, e.g. $p_{i,j} \sim \text{Beta}(\alpha, \beta)$
- Or model them some other way, e.g. latent space

Estimating Potentials

- Handcock et. al (2007), model $p_{i,j}$ using a logistic regression where the probability of an edge depends on euclidean distance in latent space

$$\text{log-odds}(p_{i,j}) = \beta X_{i,j} - |z_i - z_j|$$

- Estimate using MCMC, where we can also incorporate latent clusters with prior:

$$z_i \sim \sum_{g=1}^{G} \lambda_g \text{MVN}_d(\mu_g, \sigma_g I_g)$$

Estimating Node Exposure

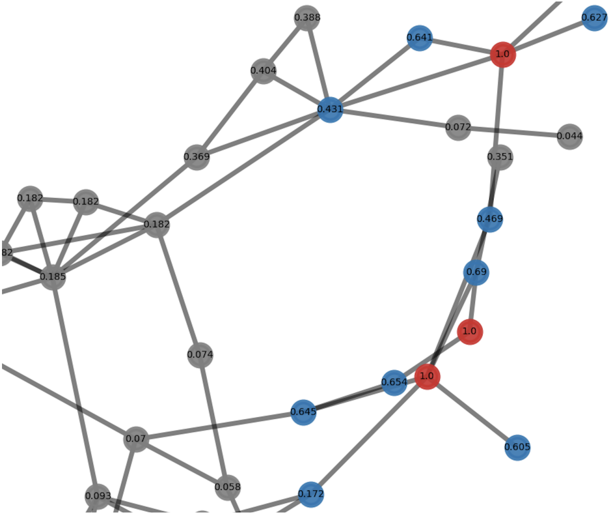

Figure: Exposure Probabilities

Estimating Causal Effects

- With the exposure probabilities, we can estimate ATE (ρ) as:

$$y_i = \alpha + \rho Z_i + \gamma(1 - Z_i)\pi_i + \varepsilon_i$$

- γ doesnt have any real causal interpretation :(
- Much better suited to heterogeneous spillover effects
- Test by simulating data on two real-world social networks
  - Eighth Graders: 55 students with edges between those who wanted to sit next to each other in class
  - Aarhus Computer Science Department: 60 colleagues with edges between those who got lunch together in a given week
  - $y_i \sim \mathcal{N}(\mu, \sigma^2)$, $\rho = 3$, spillover = 1.5

Simulation Results

DGP	$\phi_{i,j}$	Eighth Graders $\bar{y}^1 - \bar{y}^0$	Eighth Graders ρ	Aarhus CS $\bar{y}^1 - \bar{y}^0$	Aarhus CS ρ
Neighbors	Latent	2.67 (0.22)	3.03 (0.30)	2.31 (0.21)	3.03 (0.43)
	Beta	2.66 (0.22)	3.14 (0.38)	2.32 (0.21)	3.63 (0.63)
Latent	Latent	2.73 (0.22)	2.93 (0.33)	2.55 (0.21)	2.93 (0.43)
	Beta	2.72 (0.22)	2.95 (0.35)	2.54 (0.21)	3.08 (0.61)
$p_{i,j} = 0.5$	Latent	2.55 (0.22)	2.67 (0.38)	2.31 (0.21)	2.48 (0.46)
	Beta	2.56 (0.23)	2.81 (0.40)	2.32 (0.21)	2.64 (0.67)
No Spillover	Latent	3.00 (0.21)	3.00 (0.24)	3.01 (0.19)	3.01 (0.27)
	Beta	3.00 (0.20)	3.00 (0.25)	2.99 (0.19)	2.99 (0.38)

Simulation Results: Heterogeneous Spillover

	Eighth Graders			Aarhus CS		
$\phi_{i,j}$	$\bar{y}^1 - \bar{y}^0$	ρ	γ	$\bar{y}^1 - \bar{y}^0$	ρ	γ
Latent	2.74 (0.21)	2.95 (0.25)	1.40 (0.39)	2.64 (0.20)	2.99 (0.31)	0.48 (0.25)
Beta	2.75 (0.21)	2.99 (0.28)	0.84 (0.40)	2.65 (0.20)	3.06 (0.38)	0.51 (0.36)

The End