Problem Set 2 Solutions

Calvin Walker

1.

(a)

$$\begin{aligned} P(E) &= \sum_{r,m,h,s,b,a,l} P(r,m,h,s,b,a,l,E) \\ &= \sum_{r,m,h,s,b,a,l} P(r)P(m)P(h|r,m)P(s|h)P(a|h)P(b|h)P(E|s,b)P(l|b,a) \\ &= \sum_b \sum_s P(E|s,b) \sum_h P(s|h)P(b|h) \sum_a P(a|h) \sum_l P(l|b,a) \sum_r P(r) \sum_m P(h|r,m)P(m) \end{aligned}$$

(b) The corresponding elimination ordering is $\prec = M, R, L, A, H, S, B$

(c)

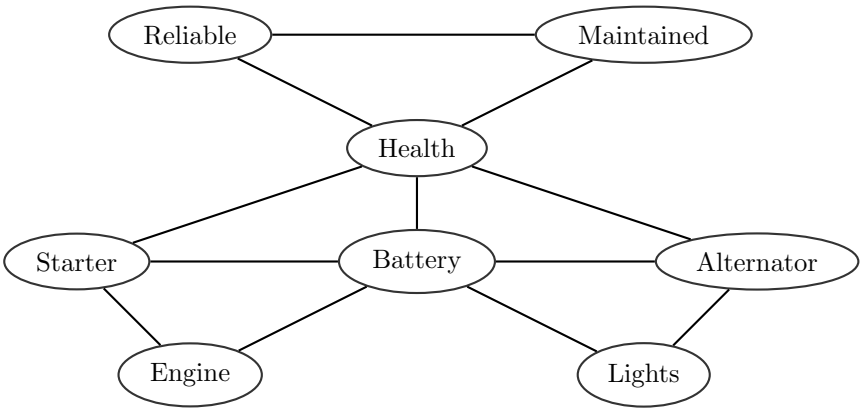

Figure 1: Moralized graph for the Bayesian Network

(d) The initial set of factors is $\Phi = \{\phi(H,M,R), \phi(S,B,H), \phi(H,B,A), \phi(S,B,E), \phi(B,A,L)\}$

Step	Variable Eliminated	Intermediate Factor	Variables Involved	New Factor
1	M	$\psi_1(H,M,R) = \phi(H,M,R)$	H,M,R	$\tau_1(H,R) = \sum_m \psi_1(H,m,R)$
2	R	$\psi_2(H,R) = \tau_1(H,R)$	H,R	$\tau_2(H) = \sum_r \psi_2(H,r)$
3	L	$\psi_3(B,A,L) = \phi(B,A,L)$	B,A,L	$\tau_3(B,A) = \sum_l \psi_3(B,A,l)$
4	A	$\psi_4(A,B,H) = \tau_3(B,A)\phi(H,B,A)$	H,B,A	$\tau_4(B,H) = \sum_a \psi_4(a,B,H)$
5	H	$\psi_5(S,B,H) = \tau_2(H)\tau_4(B,H)\phi(S,B,H)$	S,B,H	$\tau_5(S,B) = \sum_h \psi_5(S,B,h)$
6	B	$\psi_6(S,B,E) = \tau_5(S,B)\phi(S,B,E)$	S,B,E	$\tau_6(S,E) = \sum_l \psi_6(S,b,E)$
7	S	$\psi_6(S,E) = \tau_6(S,E)$	S,E	$\tau_7(E) = \sum_s \psi_7(s,E)$

Table 1: Variable Elimination Procedure for $P(E)$

(e) The largest induced scope is 2, and assuming each variable can take k possible values, the computational complexity would be $O(nk^2)$, since the factor has k^2 entries.

(f) The induced graph is the same as the moralized netowrk in part (c), since there are already edges between all variables appearing in some ψ generated by the variable elimination.

(g)

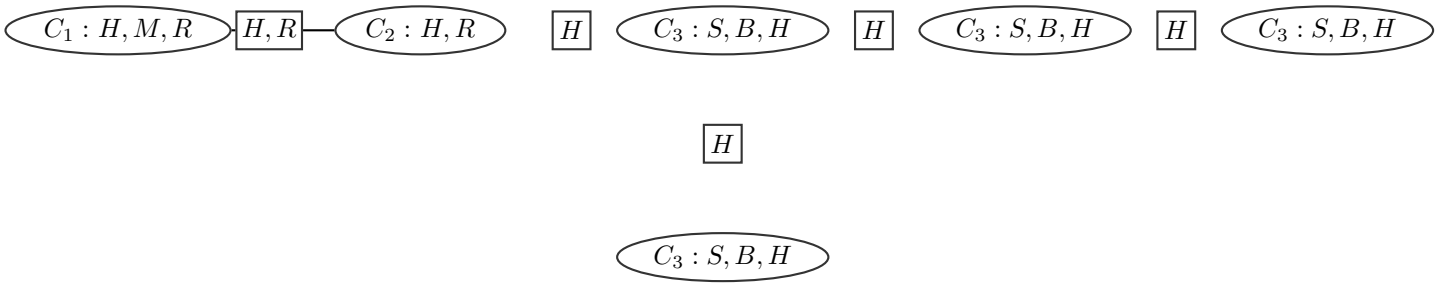

Figure 2: Moralized graph for the Bayesian Network

(h) If we first eliminate H, there is an induced scope of 5, as we create a new factor $\tau_1(R, M, S, B, A)$, next we could eliminate B, to induce a scope of 6, as we create a new factor $\tau_2(R, M, S, A, E, L)$ with the remaining 6 random variables. So any elimination ordering starting $\prec = H, B \ldots$ would lead to a computational complexity of $O(nk^6)$

2.