

---

# Estimation of Causal Effects Under Interference

---

Calvin Walker  
University of Chicago  
cswalker1@uchicago.edu

## Abstract

In randomized experiments, it is often assumed that the response of a given unit only depends on its own characteristics and the treatment to which it is assigned. However, there are numerous situations of interest to researchers where this may not be a plausible assumption, particularly when there is social interaction between individuals in an experiment, and the ability to separately identify the effects of treatment assignment and peer influence have important implications. This paper presents a general framework for reliably identifying and estimating causal effects in experimental settings where there is potential interference between units. The framework consists of (i) learning the potential for interference between connected units, (ii) a probabilistic graphical model for estimating individuals' exposure to treated units, and (iii) methods that make use of estimated exposures to compute causal effects of interest. We then evaluate the performance of the proposed framework on by simulating synthetic experimental data on a variety of real world social networks.

## 1 Introduction

Please read the instructions below carefully and follow them faithfully. **Important:** This year the checklist will be submitted separately from the main paper in OpenReview, please review it well ahead of the submission deadline: <https://neurips.cc/public/guides/PaperChecklist>.

### 1.1 Style

**Preprint option** If you wish to post a preprint of your work online, e.g., on arXiv, using the NeurIPS style, please use the preprint option. This will create a nonanonymized version of your work with the text “Preprint. Work in progress.” in the footer. This version may be distributed as you see fit, as long as you do not say which conference it was submitted to. Please **do not** use the final option, which should **only** be used for papers accepted to NeurIPS.

## 2 Setting and Assumptions

We consider the setting of a randomized experiment on a social network of  $N$  nodes. The network,  $G = (V, E)$ , is observed, and may or may not contain node-level covariates or edge strengths. Let  $\mathbf{Z} = (Z_1, Z_2, \dots, Z_N)$  be a treatment assignment vector over  $N$  units, where  $Z_i \in \{0, 1\}$  specifies which of the possible treatments unit  $i$  receives. We assume that  $P(Z_i = 1)$  is the same for all  $i \in \{N\}$ , so treatment assignment is unconfounded. We define  $\mathbf{C} = (C_1, C_2, \dots, C_N)$  to be a latent exposure vector, where  $C_i \in \{0, 1\}$  specifies if unit  $i$  is exposed to the treatment via social interaction. Since we assume treated units cannot be further exposed,  $\mathbf{Z} \times \mathbf{C} = 0$ . If we could observe  $\mathbf{C}$ , then our work would be done, since we could partition the units into treatment, exposed, and control groups, and employ classic techniques to infer their differences in outcome.

Instead, we define  $\pi_i(\mathbf{Z}) = P(C_i = 1)$  to be the exposure probability for unit  $i$  given some instantiation of the experiment  $\mathbf{Z}$ ,  $\mathbf{C}$ . Since an individual's social exposure to the treatment depends

on the random assignment in  $G$ , so does  $\pi_i(\mathbf{Z})$ . For simplicity, we assume that the effects of both treatment assignment and social exposure are homogenous. Furthermore, we assume that social interaction, and thus social exposure to the treatment, only occurs through the edges  $E$  of  $G$ . However, we make no assumption on the form of such exposures . . .

### 3 Estimating Exposures and Causal Effects

We propose modeling the spread of influence in the social network as a Pairwise Markov Random Field, defined as  $B = (P, H)$ , where  $H = (V, E)$  is isomorphic with the social network  $G$ , defined by the bijection  $f : G(V) \mapsto H(V)$  where  $f(v_i) = X_i \sim \text{Bernoulli}(p)$ , i.e.  $H$  is an undirected graph with the skeleton of  $G$  where each node in  $H(V)$  is a Bernoulli random variable.  $P$ , then, is defined as a Gibbs distribution that factorizes over  $H$ :

$$P(X_1, X_2, \dots, X_n) = \frac{1}{Z} \prod_{i \in V} \phi_i(X_i) \prod_{(i,j) \in E} \phi_{i,j}(X_i, X_j)$$

### 4 Simulations

### 5 Discussion

The text must be confined within a rectangle 5.5 inches (33 picas) wide and 9 inches (54 picas) long. The left margin is 1.5 inch (9 picas). Use 10 point type with a vertical spacing (leading) of 11 points. Times New Roman is the preferred typeface throughout, and will be selected for you by default. Paragraphs are separated by  $\frac{1}{2}$  line space (5.5 points), with no indentation.

The paper title should be 17 point, initial caps/lower case, bold, centered between two horizontal rules. The top rule should be 4 points thick and the bottom rule should be 1 point thick. Allow  $\frac{1}{4}$  inch space above and below the title to rules. All pages should start at 1 inch (6 picas) from the top of the page.

For the final version, authors' names are set in boldface, and each name is centered above the corresponding address. The lead author's name is to be listed first (left-most), and the co-authors' names (if different address) are set to follow. If there is only one co-author, list both author and co-author side by side.

Please pay special attention to the instructions in Section 5.1.1 regarding figures, tables, acknowledgments, and references.

All headings should be lower case (except for first word and proper nouns), flush left, and bold.

First-level headings should be in 12-point type.

#### 5.1 Headings: second level

Second-level headings should be in 10-point type.

##### 5.1.1 Headings: third level

Third-level headings should be in 10-point type.

**Paragraphs** There is also a `\paragraph` command available, which sets the heading in bold, flush left, and inline with the text, with the heading followed by 1 em of space.

These instructions apply to everyone.

#### 5.2 Citations within the text

The `natbib` package will be loaded for you by default. Citations may be author/year or numeric, as long as you maintain internal consistency. As to the format of the references themselves, any style is acceptable as long as it is used consistently.



Figure 1: Sample figure caption.

The documentation for `natbib` may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated...

If you wish to load the `natbib` package with options, you may add the following before loading the `neurips_2023` package:

```
\PassOptionsToPackage{options}{natbib}
```

If `natbib` clashes with another package you load, you can add the optional argument `nonatbib` when loading the style file:

```
\usepackage[nonatbib]{neurips_2023}
```

As submission is double blind, refer to your own published work in the third person. That is, use “In the previous work of Jones et al. [4],” not “In our previous work [4].” If you cite your other papers that are not widely available (e.g., a journal paper under review), use anonymous author names in the citation, e.g., an author of the form “A. Anonymous” and include a copy of the anonymized paper in the supplementary material.

### 5.3 Footnotes

Footnotes should be used sparingly. If you do require a footnote, indicate footnotes with a number<sup>1</sup> in the text. Place the footnotes at the bottom of the page on which they appear. Precede the footnote with a horizontal rule of 2 inches (12 picas).

Note that footnotes are properly typeset *after* punctuation marks.<sup>2</sup>

### 5.4 Figures

All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction. The figure number and caption always appear after the figure. Place one line space before the figure caption and one line space after the figure. The figure caption should be lower case (except for first word and proper nouns); figures are numbered consecutively.

You may use color figures. However, it is best for the figure captions and the paper body to be legible if the paper is printed in either black/white or in color.

---

<sup>1</sup>Sample of the first footnote.

<sup>2</sup>As in this example.

Table 1: Sample table title

Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

## 5.5 Tables

All tables must be centered, neat, clean and legible. The table number and title always appear before the table. See Table 1.

Place one line space before the table title, one line space after the table title, and one line space after the table. The table title must be lower case (except for first word and proper nouns); tables are numbered consecutively.

Note that publication-quality tables *do not contain vertical rules*. We strongly suggest the use of the `booktabs` package, which allows for typesetting high-quality, professional tables:

<https://www.ctan.org/pkg/booktabs>

This package was used to typeset Table 1.

## 5.6 Math

Note that display math in bare TeX commands will not create correct line numbers for submission. Please use LaTeX (or AMSTeX) commands for unnumbered display math. (You really shouldn't be using `$$` anyway; see <https://tex.stackexchange.com/questions/503/why-is-preferable-to> and <https://tex.stackexchange.com/questions/40492/what-are-the-differences-between-align-equation-and-displaymath> for more information.)

## 5.7 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

## 6 Preparing PDF files

Please prepare submission files with paper size "US Letter," and not, for example, "A4."

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF file uses. In Acrobat Reader, select the menu `Files > Document Properties > Fonts` and select `Show All Fonts`. You can also use the program `pdf fonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.
- `xfig` "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

```
\usepackage{amsfonts}
```

followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for  $\mathbb{R}$ ,  $\mathbb{N}$  or  $\mathbb{C}$ . You can also use the following workaround for reals, natural and complex:

```

\newcommand{\RR}{\mathbb{R}} %real numbers
\newcommand{\Nat}{\mathbb{N}} %natural numbers
\newcommand{\CC}{\mathbb{C}} %complex numbers

```

Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

## 6.1 Margins in L<sup>A</sup>T<sub>E</sub>X

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```

\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}

```

See Section 4.4 in the graphics bundle documentation (<http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf>)

A number of width problems arise when L<sup>A</sup>T<sub>E</sub>X cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

## Acknowledgments and Disclosure of Funding

Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: <https://neurips.cc/Conferences/2023/PaperInformation/FundingDisclosure>.

Do **not** include this section in the anonymized submission, only in the final paper. You can use the `ack` environment provided in the style file to automatically hide this section in the anonymized submission.

## 7 Supplementary Material

Authors may wish to optionally include extra information (complete proofs, additional experiments and plots) in the appendix. All such materials should be part of the supplemental material (submitted separately) and should NOT be included in the main submission.

## References

References follow the acknowledgments in the camera-ready paper. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to `small` (9 point) when listing the references. Note that the Reference section does not count towards the page limit.

- [1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D.S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609–616. Cambridge, MA: MIT Press.
- [2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural Simulation System*. New York: TELOS/Springer-Verlag.
- [3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.