Theory of Algorithms Proof Strategies

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Greedy Algorithms:

Greedy Algorithm Stays Ahead: At each step, the greedy algorithim does at least as well as any other solution.

- Define $f_j(S)$ to be the performance of schedule S at time-step j
- Use induction to show that for all j, $f_j(S_{greedy}) \geq f_j(S')$
  - For the inductive step, demonstrate that all choices available to S' are available to S_{greedy}, and that S_{greedy} always takes the optimum of these choices.
- We can also use proof by contradiction, by assuming that j is the first index such that $f_j(S_{greedy}) < f_j(S')$
  - Show directly for base case.
  - Use the fact that $f_{j-1}(S_{greedy}) \geq f_{j-1}(S')$ to arrive at a contradiction

Exchange Argument: For any optimal solution S, we can iteratively adjust S without affecting performance until it is S_{greedy}, so S_{greedy} is also an optimal solution.

- For $i_{j+1} < i_j$, consider swapping i_j and i_{j+1}
- Show that the swap either does not affect some metric of interest, or improves it.

Divide and Conquer: