Problem Set 1 Solutions

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Problem 1:

a) Let M be the partial matching of groups A and B. We start with $M = \emptyset$

1. a_1 offers b_3 which is accepted as b_3 is unmatched, $M = \{(a_1, b_3)\}$
2. a_2 offers b_1 which is accepted as b_1 is unmatched, $M = \{(a_1, b_3), (a_2, b_1)\}$
3. a_3 offers b_4 which is accepted as b_4 is unmatched, $M = \{(a_1, b_3), (a_2, b_1), (a_3, b_4)\}$
4. a_4 offers b_1 which is accepted as b_1 prefers a_4 to a_2, $M = \{(a_1, b_3), (a_3, b_4), (a_4, b_1)\}$
5. a_2 offers b_4 which is rejected as b_4 prefers a_3 to a_2, $M = \{(a_1, b_3), (a_3, b_4), (a_4, b_1)\}$
6. a_2 offers b_3 which is accepted as b_3 prefers a_2 to a_1, $M = \{(a_2, b_3), (a_3, b_4), (a_4, b_1)\}$
7. a_1 offers b_2 which is accepted as b_2 is unmatched, $M = \{(a_1, b_2), (a_2, b_3), (a_3, b_4), (a_4, b_1)\}$

The final matching is $M = \{(a_1, b_2), (a_2, b_3), (a_3, b_4), (a_4, b_1)\}$

b) We start with $M' = \emptyset$, but now group B makes the offers to group A

1. b_1 offers a_1 which is accepted as a_1 is unmatched, $M' = \{(b_1, a_1)\}$
2. b_2 offers a_3 which is accepted as a_3 is unmatched, $M' = \{(b_1, a_1), (b_2, a_3)\}$
3. b_3 offers a_3 which is accepted as a_3 prefers b_3 to a_2, $M' = \{(b_1, a_1), (b_3, a_3)\}$
4. b_4 offers a_1 which is rejected as a_1 prefers b_1 to b_4, $M' = \{(b_1, a_1), (b_3, a_3)\}$
5. b_4 offers a_4 which is accepted as a_4 is unmatched, $M' = \{(b_1, a_1), (b_3, a_3), (b_4, a_4)\}$
6. b_2 offers a_4 which is rejected as a_4 prefers b_4 to b_2, $M' = \{(b_1, a_1), (b_3, a_3), (b_4, a_4)\}$
7. b_2 offers a_2 which is accepted as a_2 is unmatched, $M' = \{(b_1, a_1), (b_2, a_2), (b_3, a_3), (b_4, a_4)\}$

The final matching is $M = \{(b_1, a_1), (b_2, a_2), (b_3, a_3), (b_4, a_4)\}$. Compared to part (a), persons b_1, b_2, b_3, and b_4 are happier with the new stable matching M', as they each have a match higher in their preference lists.

c)

Problem 2: