UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	:	BGE 1143
COURSE	:	DISCRETE MATHEMATICS
SEMESTER/SESSION	:	2 – 2024/2025
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **6** questions. Answer **ALL** questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches whenever required.
4. If in doubt, raise your hand and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

a) Determine the truth value of each compound statements.

i) $8+1=15-6$ or $70>72$. (1 mark)

ii) $\sqrt{2}=2$ and $7^3=343$. (1 mark)

b) Construct a truth table for each of the following propositions.

i) $\sim(p\rightarrow r)\wedge\sim q$ (3 marks)

ii) $(p\leftrightarrow r)\vee(p\wedge\sim q)$ (3 marks)

c) Given that $A=\{x\in\mathbb{N}\mid x\leq 10\}$, $B=\{x\in\mathbb{Z}\mid |x+6|=10\}$ and $C=\{x\in\mathbb{R}\mid x^2-5x+6=0\}$.

i) List all the elements in each of these sets. (3 marks)

ii) Find $A\cup B\cup C$ and $(A-C)\cap B$. (3 marks)

d) Given two functions, $f(x)=7x^2-3$ and $g(x)=\sqrt{x+2}$. Find $(f\circ g^{-1})(3)$.

(3 marks)

QUESTION 2

a) Let $a=108$ and $b=2200$. Find the greatest common divisor (gcd) and least common multiple (lcm) for both integers a and b by using prime factorization.

(4 marks)

b) Convert each of the following number to its equivalent base.

i) 453 to binary. (2 marks)

ii) 341_8 to decimal. (2 marks)

ii) 100110110111001_2 to hexadecimal. (2 marks)

c) Given that $A=\begin{bmatrix}4 & 2\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}11 & -3\\ 2 & 2\end{bmatrix}$ and $C=\begin{bmatrix}9 & -1 & -2\\ 10 & 6 & -7\end{bmatrix}$. Find $C^T(A+4B)$.

(4 marks)

d) Given the Boolean matrices, $A=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$. Compute $A\vee B$, $A\wedge B$ and $B\odot A$. (4 marks)

QUESTION 3

a) Express $\frac{5^1}{1!}+\frac{5^2}{2!}+\frac{5^3}{3!}+\ldots+\frac{5^8}{8!}$ using sigma notation. (1 mark)

b) Calculate $\sum_{n=0}^{5}\left(\frac{n}{2}+3n\right)$. (3 marks)

c) Prove each of the following using the mathematical induction method.

i) $1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$ for all natural numbers. (7 marks)

ii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=\frac{2^n-1}{2^n}$ for all positive integer $n\geq 1$. (7 marks)

QUESTION 4

a) Suppose a simple graph with five vertices of degrees 1, 3, 3, 3 and 4. How many edges does this graph have? Then, draw the graph if possible. (4 marks)

b) Draw K_7 and $K_{2,5}$. (4 marks)

c) Determine whether or not the graph in Figure 1 is bipartite. Give the bipartition sets or explain why the graph is not bipartite.

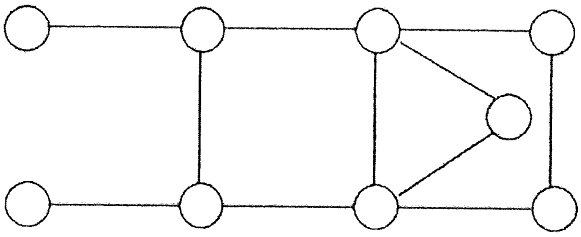

Figure 1

(2 marks)

d) Determine whether the following graph in Figure 2 has an Euler path or Euler circuit and give the reason. Then, construct the Euler path or Euler circuit.

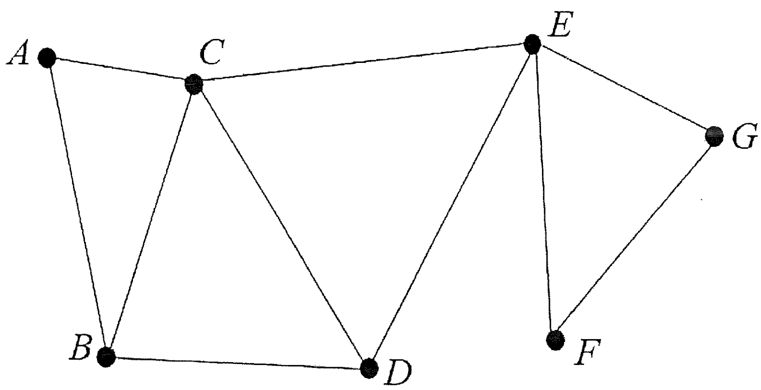

Figure 2

(5 marks)

QUESTION 5

a) Let set $A=\{a,b,c\}$ and R is a relation on set A, where

$$R_1=\{(a,a),(b,b),(a,c),(b,c),(c,c),(c,b)\}$$

$$R_2=\{(a,a),(b,a),(b,c),(c,a),(c,b),(c,c)\}.$$

i) Represent the relation R_1 and R_2 as digraph respectively. (3 marks)

ii) Determine whether R_1 is a reflexive, symmetric and transitive. Explain each of your answer. (6 marks)

b) Let $B=\{1,2,3\}$. List the ordered pairs in relation on B, which is:

i) R_1 is reflexive, not symmetric and transitive. (2 marks)

ii) R_2 is not reflexive, symmetric and not transitive. (2 marks)

iii) R_3 is an equivalence relation. (2 marks)

QUESTION 6

a) Draw a circuit of Boolean expression, $f(x,y,z)=\overline{x}(yz+\overline{z})$. Then, construct a logic truth table for the output. (4 marks)

b) Design a minimal logical circuit for the Figure 3 using Karnaugh Maps. Show all the calculation and steps.

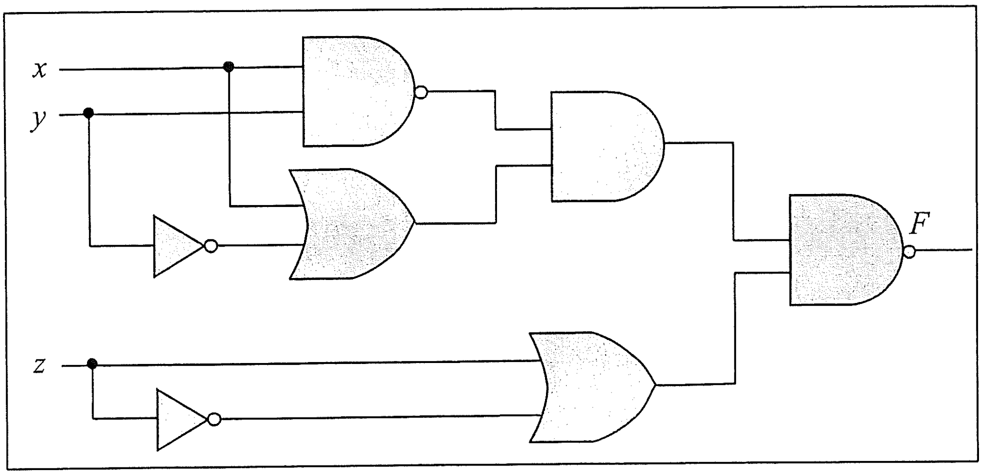

Figure 3

(13 marks)

-- **END OF QUESTIONS** --

FORMULA

Complement:	$\overline{0} = 1$,	$\overline{1} = 0$,	$0 = F$,	$1 = T$
Boolean Sum:	$1 + 1 = 1$,	$1 + 0 = 1$,	$0 + 1 = 1$,	$0 + 0 = 0$
Boolean Product:	$1 \cdot 1 = 1$,	$1 \cdot 0 = 0$,	$0 \cdot 1 = 0$,	$0 \cdot 0 = 0$