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UNIVERSITY COLLEGE
TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 1133
COURSE	: CALCULUS
SEMESTER/SESSION	: 2-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **9** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

Differentiate the following functions with respect to x.

a) $y = x^5 - \frac{x^3}{2} + 4x + 7$ (1 mark)

b) $y = \frac{\ln|x|}{7} - 3\sin x$ (1 mark)

c) $y = 9\sqrt{x^2 + 3}$ (3 marks)

d) $y = e^{5x}\tan(2x-1)$ *(Use Product Rule)* (3 marks)

e) $y = \frac{\cos(5-7x)}{1+\tan x}$ *(Use Quotient Rule)* (3 marks)

QUESTION 2

Evaluate the following integrals.

a) $\int (11x^2 + x - 1)\,dx$ (1 mark)

b) $\int \frac{x^3 + x - 6}{5x^2}\,dx$ (3 marks)

c) $\int \frac{4}{\sqrt{7x+3}}\,dx$ (3 marks)

d) $\int \frac{\sec^2(5x-4)}{3}\,dx$ (2 marks)

QUESTION 3

a) Use substitution method to evaluate the integral $\int (x^2-1)e^{x^3-3x}dx$.

(4 marks)

b) Use integration by part to find $\int x(\ln 5x)\,dx$. (5 marks)

c) Find $\int \frac{27x^2+32x+16}{(3x+2)^2(1-x)}dx$. (7 marks)

QUESTION 4

a) Find $\frac{dy}{dx}$ for parametric functions, $x=1+\sqrt{t}$ and $y=t^3-12t$. (3 marks)

b) Find the equation of normal to the curve $y^3-4x^2y^2+y^4=9$ at point $(1,1)$.

(7 marks)

QUESTION 5

a) Solve the following linear differential equation using the integrating factor method.

$$y'-\frac{2y}{x}=x$$

(6 marks)

b) Find the solution to the following differential equation by separation of variables.

$$y'=\frac{y+1}{x}$$

(6 marks)

QUESTION 6

Given that $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

a) Find the stationary points on the curve. (3 marks)

b) Determine the maximum and minimum points. (3 marks)

c) Find the inflection point. (2 marks)

d) Sketch the graph. (2 marks)

QUESTION 7

Find the area bounded by the curve $y = x^2 - 10x + 24$ and $y = -x^2 + 6x$. Then, sketch the graph and label the shaded area.

(6 marks)

QUESTION 8

Solve the following second order ordinary differential equation.

a) $y'' + 2y' + y = 0$ (3 marks)

b) $y'' + 4y' + 3y = 7e^{2x}$ (11 marks)

QUESTION 9

Use the Laplace transform to solve the initial value problem.

$$y'' - 5y' + 4y = 5, \quad y(0) = 1 \text{ and } y'(0) = -2$$ (12 marks)

....**End of questions**..

FORMULA

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d}{dx}\left[f(x)\right]^n=n\left[f(x)\right]^{n-1}\cdot f'(x)$$

$$\frac{d}{dx}\left[\sin\left(f(x)\right)\right]=f'(x)\cos f(x)$$

$$\frac{d}{dx}\left[\cos\left(f(x)\right)\right]=-f'(x)\sin f(x)$$

$$\frac{d}{dx}\left[\tan\left(f(x)\right)\right]=f'(x)\sec^2 f(x)$$

$$\frac{d}{dx}\left(e^{f(x)}\right)=f'(x)e^{f(x)}$$

$$\frac{d}{dx}\left[\ln f(x)\right]=\frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(u\cdot v)=uv'+vu'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$y-y_1=m(x-x_1)$$

$$y-y_1=-\frac{1}{m}(x-x_1)$$

$$\int x^n dx=\frac{x^{n+1}}{n+1}+C\ ,\ n\neq -1$$

$$\int (ax+b)^n dx=\frac{(ax+b)^{n+1}}{(n+1)a}+C\ ,\ n\neq -1$$

$$\int \cos(ax+b)\,dx=\frac{\sin(ax+b)}{a}+C$$

$$\int \sin(ax+b)\,dx=-\frac{\cos(ax+b)}{a}+C$$

$$\int \sec^2(ax+b)\,dx=\frac{\tan(ax+b)}{a}+C$$

$$\int e^{ax+b}dx=\frac{e^{ax+b}}{a}+C$$

$$\int \frac{1}{x}dx=\ln|x|+C$$

$$\int \frac{1}{ax+b}dx=\frac{\ln|ax+b|}{a}+C$$

$$\int \frac{f'(x)}{f(x)}dx=\ln|f(x)|+C$$

$$\int f(g(x))\,dx=\int f(u)\,du$$

$$\int u\,dv=uv-\int v\,du$$

$g(x)$	y_p
$ax^r++bx+c$	$Ax^r+........+Bx+C$
$ae^{\alpha x}$	$Ae^{\alpha x}$
$a\sin\beta x$ $a\cos\beta x$ $a\sin\beta x+b\sin\beta x$	$A\cos\beta x+B\sin\beta x$

TABLE OF LAPLACE TRANSFORMS

NO	$y(t)$	$\mathcal{L}\{y(t)\} = F(s)$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$
4.	e^{at}	$\frac{1}{s-a}$
5.	$\sin at$	$\frac{a}{s^2+a^2}$
6.	$\cos at$	$\frac{s}{s^2+a^2}$
7.	$y'(t)$	$\mathcal{L}(y'(t)) = s\,\mathcal{L}[y(t)] - y(0)$
8.	$y''(t)$	$\mathcal{L}(y''(t)) = s^2\,\mathcal{L}[y(t)] - sy(0) - y'(0)$