UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 2123
COURSE	: STATISTICS
SEMESTER/SESSION	: 2-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **11** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 11 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS (100 MARKS)

QUESTION 1

A researcher is studying household electricity bills in Telok Kalong. There are 500 household in Telok Kalong. The household are grouped by their socioeconomic levels which are low income, medium income and high income. 50 household are randomly selected from each group.

a) Determine the population and sample in this study. (2 marks)

b) What is the variable of interest? Is it a qualitative or a quantitative variable? (2 marks)

c) Suggest an appropriate sampling technique used in this survey. (1 mark)

QUESTION 2

A service station has recorded the following frequency distribution for the number of gallons of gasoline sold per car in a sample of 580 cars.

Gasoline (gallons)	0-4	5-9	10-14	15-19	20-24	25-29
Frequency	74	92	280	105	23	6

a) Calculate the mean for the above distribution. (3 marks)

b) Calculate the median for the above distribution. (4 marks)

QUESTION 3

The average time a subscriber spends reading The Wall Street Journal is 49 minutes. Assume the standard deviation is 16 minutes and the times are normally distributed.

a) What is the probability a subscriber will spend no more than 30 minutes reading the Journal? (3 marks)

b) What is the probability a subscriber will spend at least 60 minutes reading the Journal? (3 marks)

QUESTION 4

Let A and B be events such that $P(A)=0.4$, $P(B)=0.7$ and $P(A\,or\,B)=0.9$. Find each of these probabilities:

a) $P(A\,and\,B)$ (3 marks)

b) $P(A\,and\,B')$ (3 marks)

c) $P(A\mid B)$ (2 marks)

d) Are the events A and B are independent? (2 marks)

e) Are the events A and B are mutually exclusive? (2 marks)

QUESTION 5

The discrete random variable X has a probability distribution function given by

$$f(x)=\begin{cases}\dfrac{x-1}{21} & \text{if } x=2,3,4\\ \dfrac{13-x}{21} & \text{if } x=5,6\end{cases}.$$

a) Show that X is a discrete random variable. (3 marks)

Find:

b) $P(X\le 4)$ (2 marks)

c) $E(X)$ (3 marks)

d) $Var(X)$ (4 marks)

QUESTION 6

The sales of Lexus automobiles in the Kemaman area follow a Poisson distribution with a mean of 3 per day. Find the probability that:

a) No Lexus is sold on a particular day. (2 marks)

b) At least one Lexus is sold for five consecutive days. (4 marks)

QUESTION 7

One environmental group did a study of recycling habits in a Kemaman community. They found that 40% of the aluminum can sold in the area were recycled. If 50 aluminum cans are sold, find:

a) The probability that 2 or more will be recycled using the Binomial distribution. (3 marks)

b) The probability that between 15 to 20 (inclusive) will be recycled using the Normal approximate to the Binomial distribution. (5 marks)

QUESTION 8

The Fox TV network is considering replacing one of its prime-time crime investigation shows with a new family-oriented comedy show. Before a final decision is made, network executives commission a sample of 400 viewers. After viewing the comedy, 250 indicated they would watch the new show and suggested it replace the crime investigation show.

a) Find $\mu_{\bar{p}}$ and $\sigma_{\bar{p}}$. (3 marks)

b) Develop a 90% confidence interval for the population proportion. (5 marks)

QUESTION 9

Pyramid Lake is on the Paiute Indian Reservation in Nevada. The lake is famous for cutthroat trout. Suppose a friend tells you that the average length of trout caught in Pyramid Lake is $\mu = 19$ inches. However, the Creel Survey reported that of a random sample of 51 trout caught, the mean length was $\bar{x} = 18.5$ inches, with estimated standard deviation $s = 3.2$ inches.

a) At 5% level of significance, do these data indicate that the average length of a trout caught in Pyramid Lake is less than $\mu = 19$ inches? Use 5 step of hypothesis testing. (10 marks)

b) Develop a 99% confidence interval for the average length of a trout caught in Pyramid Lake. (5 marks)

QUESTION 10

A child psychiatrist is studying the mental development of children. Random samples of 7 children were given a standard set of questions appropriate to the age of each child. The number of irrelevant responses to the questions was recorded for each child.

Age of child in year x	Number of irrelevant questions y
4	12
5	13
7	11
9	10
10	8
11	6
12	5

a) Find the Pearson's correlation between x and y. Explain the value. (6 marks)

b) Find the equation of the least squares line fit to these data. (6 marks)

c) If a child is 9.5 years old, what does the least squares line predict for the number of irrelevant responses? (3 marks)

QUESTION 11

A firm rents cars from Agency A, 26% of the time, Agency B, 38% of the time and Agency C, the remainders of the time. It is known that 10% of cars from Agency A, 20% of the cars from Agency B, and 5% of cars from Agency C have bad tires. If a car is randomly selected from a central pool of cars from these agencies, find the probability that:

a) The car has bad tires. (3 marks)

b) The car is from Agency A, given it has bad tires. (3 marks)

----------**End of questions**----------

FORMULA

$\text{Relative frequency} = \frac{f}{\sum f}$	$\text{Midpoint, } x = \frac{\text{Lower limit} + \text{Upper limit}}{2}$
$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
$Median, \tilde{x} = L_m + \left(\frac{\frac{\sum f}{2} - \sum f_{m-1}}{f_m} \right) \times C_m$ L_m = Lower bound of median class f_m = Frequency ofmedian class $\sum f_{m-1}$ = Cumulative frequency before median class C_m = Size of median class $\sum f$ = Number of observation /total frequency	$s^2 = \frac{1}{\sum f - 1} \left(\sum f_i x_i^2 - \frac{\left(\sum f_i x_i\right)^2}{\sum f} \right)$ $s = \sqrt{s^2}$
$Mode = L_{mo} + \left(\frac{d_1}{d_1 + d_2} \right) \times C_{mo}$ L_{mo} = Lower bound of mode class C_{mo} = Size of mode class d_1 = Difference between modal class frequency and the previous class frequency d_2 = Difference between modal class frequency and the next class frequency	

Probability

1. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

2. a) $P(A \cap B) = P(B) \cdot P(A \mid B)$

   b) $P(A \cap B) = P(A) \cdot P(B)$ if and only if A and B are independent events.

3. $P(A \mid B) = \dfrac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$

4. a) $P(X = x) = \binom{n}{x} p^x q^{n-x} \quad x = 0,1,..,n$ b) $P(X = x) = \dfrac{\lambda^x e^{-\lambda}}{x!} \quad x = 0,1,..$

5. a) $Z = \dfrac{X - np}{\sqrt{npq}}$ b) $Z = \dfrac{X - \lambda}{\sqrt{\lambda}}$

6. $E(X) = \sum_{-\infty}^{\infty} x \cdot f(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

7. $E(X^2) = \sum_{-\infty}^{\infty} x^2 \cdot f(x) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$

8. $Var(X) = E(X^2) - [E(X)]^2$

Regression and Correlation

1. $r = \dfrac{(\Sigma xy) - \dfrac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\left[(\Sigma x^2) - \dfrac{(\Sigma x)^2}{n}\right]\left[(\Sigma y^2) - \dfrac{(\Sigma y)^2}{n}\right]}}$

2. $b = \dfrac{(\Sigma xy) - \left(\dfrac{(\Sigma x)(\Sigma y)}{n}\right)}{\left((\Sigma x^2) - \dfrac{(\Sigma x)^2}{n}\right)}$

3. $a = \dfrac{(\Sigma y)}{n} - b\dfrac{(\Sigma x)}{n}$

Confidence Interval and Hypothesis Testing

Estimator	Confidence Interval	Test Statistics
$\mu_1 - \mu_2$	$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$Z_0 = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
	$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ $v = n_1 + n_2 - 2$	$T_0 = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
	$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$
	$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,\nu}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,\nu}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $v = n_1 + n_2 - 2$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$T_0 = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Confidence Interval and Hypothesis Testing

Estimator	Confidence Interval	Test Statistics
μ	$(\bar{x}) - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < (\bar{x}) + z_{\alpha/2}\frac{s}{\sqrt{n}}$	$Z_o = \frac{\bar{X} - \mu_o}{s/\sqrt{n}} \sim N(0,1)$
	$(\bar{x}) - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} < \mu < (\bar{x}) + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$	$T_o = \frac{\bar{X} - \mu_o}{s/\sqrt{n}} \sim t_{n-1}$

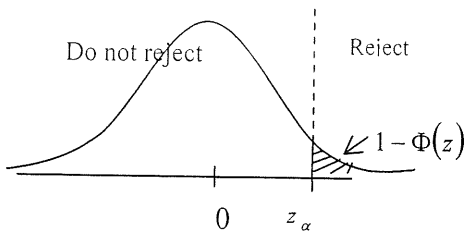
Estimator	Confidence Interval	Sample size
$\hat{p}$	$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$n = \frac{z_{\alpha/2}^2 \cdot \hat{p}(1-\hat{p})}{E^2}$

APPENDIX Ip

Table I Standard Normal Distribution

$$1-\Phi(z)=P(Z>z)=\frac{1}{\sqrt{2\pi}}\int_{z}^{\infty}e^{-z^2/2}dz$$

$$z=\frac{x-\mu}{\sigma}$$



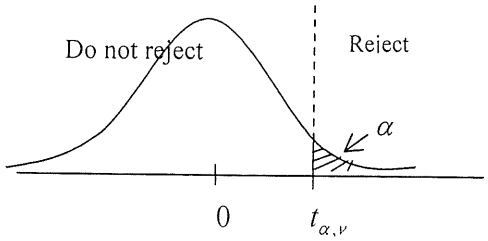
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
3.7	.000108	.000104	.000100	.000096	.000092	.000088	.000085	.000082	.000078	.000075
3.8	.000072	.000069	.000067	.000064	.000062	.000059	.000057	.000054	.000052	.000050
3.9	.000048	.000046	.000044	.000042	.000041	.000039	.000037	.000036	.000034	.000033
4.0	.000032									

5.0 → 0.0000002867 5.5 → 0.0000000190 6.0 → 0.0000000010

APPENDIX II

Table II Student's t Distribution

The table gives the value of $t_{\alpha,\nu}$ - the $100\,\alpha$ percentage point of the t distribution for degrees of freedom.



ν \ α	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291