UNIVERSITY COLLEGE
TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: DGE 2123
COURSE	: MATHEMATICS II
SEMESTER/SESSION	: 1-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **9** questions. Answer **ALL** questions.

2. All answers should be written in answer booklet.

3. Write legibly and draw sketches wherever required.

4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 5 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

Differentiate the following functions with respect to x.

a) $y = 4x^3 - 5x^2 + 6x + 5$ (2 marks)

b) $y = 8e^{5x}$ (2 marks)

c) $y = -5\cos 3x$ (2 marks)

d) $y = (7x + 12)^3$ (2 marks)

e) $y = \ln(2x + 1)$ (2 marks)

QUESTION 2

Evaluate the following integrals.

a) $\int (5x^4 + 3x)\,dx$ (2 marks)

b) $\int \frac{2}{x-5}\,dx$ (2 marks)

c) $\int (6\sin x + 2)\,dx$ (2 marks)

d) $\int (e^{3x} - 3x^2)\,dx$ (2 marks)

e) $\int \frac{1}{5x+2}\,dx$ (2 marks)

QUESTION 3

Find the derivatives for the following function using the given techniques.

a) $f(x) = (3x^2 - 4x)e^{-3x}$ (Use product rule)   (3 marks)

b) $f(x) = \dfrac{x^3 - 2}{\ln x}$ (Use quotient rule)   (3 marks)

QUESTION 4

a) $\int 7x^2 (x^3 - 5)^9 \, dx$ (Use Substitution method)   (4 marks)

b) $\int 2x \ln x \, dx$ (Use By Parts method)   (4 marks)

c) $\int \dfrac{3x + 5}{(2x + 1)(x - 3)} \, dx$ (Use Partial Fraction method)   (6 marks)

QUESTION 5

a) Find $\dfrac{dy}{dx}$ for $3x^2 + 4y^2 = 15xy - 6$ using implicit differentiation.   (6 marks)

b) Find $\dfrac{dy}{dx}$ given that $x = t^3 + 3t^2$ and $y = t^4 - 8t^2$.   (4 marks)

QUESTION 6

a) Solve the following differential equation by separation of variables method.

$$y\frac{dy}{dx} + 1 = 3x$$ (4 marks)

b) Solve the following differential equation using an integrating factor method.

$$5\frac{dy}{dx} + y = 7$$ (7 marks)

QUESTION 7

a) Find the general solution for the following differential equations.

   i) $y'' - 6y' + 8y = 0$ (3 marks)

   ii) $y'' + 2y' + y = 0$ (3 marks)

   iii) $y'' + 3y' + 7y = 0$ (3 marks)

b) Solve the given differential equation by using method of undetermined coefficient.

$$y'' - 6y' + 5y = -9e^{2x}$$ (10 marks)

QUESTION 8

Given that $f(x) = x^3 - 3x^2 - 9x + 6$.

a) Find the stationary points on the curve.   (3 marks)

b) Determine the maximum and minimum points.   (3 marks)

c) Find the inflection point.   (2 marks)

d) Sketch the graph.   (2 marks)

QUESTION 9

Find the area enclosed by the curve $y = x^2 - 30$ and straight line $y = 10 - 3x$. Hence, sketch the curve and label the shaded the area.

                                                       (10 marks)

..................................End of questions.....................................

FORMULA

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \ n \neq -1$$

$$\frac{d}{dx}\left[f(x)\right]^n = n\left[f(x)\right]^{n-1} \cdot f'(x)$$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C, \ n \neq -1$$

$$\frac{d}{dx}\left[\sin(f(x))\right] = f'(x)\cos f(x)$$

$$\int \cos(ax+b) \, dx = \frac{\sin(ax+b)}{a} + C$$

$$\frac{d}{dx}\left[\cos(f(x))\right] = -f'(x)\sin f(x)$$

$$\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a} + C$$

$$\frac{d}{dx}\left[\tan(f(x))\right] = f'(x)\sec^2 f(x)$$

$$\int \sec^2(ax+b) \, dx = \frac{\tan(ax+b)}{a} + C$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + C$$

$$\frac{d}{dx}\left[\ln f(x)\right] = \frac{f'(x)}{f(x)}$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\frac{d}{dx}(u \cdot v) = uv' + vu'$$

$$\int \frac{1}{ax+b} dx = \frac{\ln|ax+b|}{a} + C$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int f(g(x)) \, dx = \int f(u) \, du$$

$$y - y_1 = m(x - x_1)$$

$$\int u \, dv = uv - \int v \, du$$

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$g(x)$	y_p
$ax^r + + bx + c$	$Ax^r + \ldots\ldots + Bx + C$
$ae^{\alpha x}$	$Ae^{\alpha x}$
$a\sin\beta x$ $a\cos\beta x$ $a\sin\beta x + b\sin\beta x$	$A\cos\beta x + B\sin\beta x$