CONFIDENTIAL

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	: BGE 1123
COURSE	: ALGEBRA
SEMESTER/SESSION	: 2-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **7** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 5 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS (100 MARKS)

QUESTION 1

The complex numbers z_1 and z_2 are given by $z_1 = p+2i$ and $z_2 = 1-2i$, where p is an integer.

(a) Find $\frac{z_1}{z_2}$ in the form $a+bi$ where a and b are real. Give your answer in its simplest form in terms of p. (4 marks)

(b) Given that $\left|\frac{z_1}{z_2}\right| = 13$, find the possible values of p. (4 marks)

(c) Express $\frac{z_1}{z_2}$ in polar form. (6 marks)

QUESTION 2

(a) Two cubic polynomials are defined by $f(x) = x^3 + (a-3)x + 2b$ and $g(x) = 3x^3 + x^2 + 5ax + 4ab$ where a and b are integers. Given that $f(x)$ and $g(x)$ have a common factor of $(x-2)$. Show that $a = -2$ and find the value of b. (6 marks)

(b) Express $\frac{2x-3}{(x+2)(x-1)^2}$ as partial fractions. (8 marks)

QUESTION 3

(a) Prove that $\tan\theta + \cot\theta = \sec\theta\csc\theta$. (5 marks)

(b) Show that the equation $\tan 2x = 5\sin 2x$ can be written in the form $(1-5\cos 2x)\sin 2x = 0$. Hence, solve for $0 \le x \le 180^\circ$, $\tan 2x = 5\sin 2x$ giving your answer to 1 decimal place. (8 marks)

(c) Given the area of the triangle ABC is $400cm^2$. The length of AB is $30cm$ and the length of BC is $40cm$. Find all the possible length of AC.

(8 marks)

QUESTION 4

(a) Given matrix $A=\begin{bmatrix}1 & 3\\ 2 & 0\\ -1 & 7\end{bmatrix}$ and matrix $B=\begin{bmatrix}2 & -2\\ 1 & 4\end{bmatrix}$, find AB if possible.

(2 marks)

(b) If $A=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$, find the matrix X, such that $2A+3X=5B$.

(4 marks)

(c) Find the values of x, y, z if the matrix $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfy the equation $A^T A = I$.

(5 marks)

QUESTION 5

(a) Given $A=(-3,2)$, $B=(4,6)$ and $C=(m,n)$, find the value of m and n such that $2\overrightarrow{AB}+\overrightarrow{BC}=\begin{pmatrix}12\\ -3\end{pmatrix}$.

(6 marks)

(b) Find the unit vector in the direction of $\vec{a}+\vec{b}$, given that $\vec{a}=2\mathbf{i}+2\mathbf{j}-5\mathbf{k}$ and $\vec{b}=2\mathbf{i}+\mathbf{j}+3\mathbf{k}$.

(4 marks)

(c) Given $\vec{a}=2\mathbf{i}+8\mathbf{j}$ and $\vec{b}=3\mathbf{i}+2\mathbf{j}$. Find the angle between $\vec{a}$ and $\vec{b}$, giving your answer to two decimal places.

(4 marks)

QUESTION 6

The functions f and g are defined by

$$f(x) = 1 - 2x^3, x \in \mathbb{R}$$

$$g(x) = -x^2 + 1, x \in \mathbb{R}$$

(a) Sketch the graph of f and g in the same axes. (4 marks)

(b) Find the inverse function of f, f^{-1} and state its domain. (4 marks)

(c) Show that the composite function gf is

$$gf(x) = \frac{8x^3 - 1}{1 - 2x^3}$$ (3 marks)

(d) Solve $gf(x) = 0$. (2 marks)

QUESTION 7

(a) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 6 & 2 \end{bmatrix}$ and λ is a scalar, compute the determinant of $A - \lambda I$ by cofactor expansion. Hence determine the value of λ for which the matrix $A - \lambda I$ is singular. (Note: A square matrix A is singular if $\det(A) = 0$)

(5 marks)

(b) Solve the following simultaneous equations by using Gauss elimination method.

$$\begin{aligned} 2x - 2y + 3z &= 2 \\ x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \end{aligned}$$ (8 marks)

--**End of questions**--

FORMULA

$A^{-1} = \frac{1}{\lvert A \rvert} adj(A)$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sin^2\theta + \cos^2\theta = 1$
$\frac{a}{\sin A} = \frac{b}{\sin B}$	$a^2 = b^2 + c^2 - 2bc\cos A$
$\text{Area} = \frac{1}{2} ab\sin C$	
$\lvert z \rvert = \sqrt{a^2 + b^2}$	$\arg(\theta) = \tan^{-1}\left\lvert \frac{b}{a} \right\rvert$
$z = r(\cos\theta + i\sin\theta)$	$z = re^{i\theta}$
$\lvert \vec{u} \rvert = \sqrt{a^2 + b^2 + c^2}$	$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{\lvert \vec{u} \rvert \lvert \vec{v} \rvert} \right)$