CONFIDENTIAL

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	: BMT 2113 / BET 2113
COURSE	: CONTROL SYSTEM
SEMESTER/SESSION	: 2-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **4** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise up your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 9 PRINTED PAGES INCLUDING COVER PAGE

QUESTION 1

a) Mathematical model of a system was obtained using differential equations that describe the physical laws of a system. State the physical laws that were used to model the systems below.

i) Electrical system (1 mark)

ii) Mechanical system (1 mark)

b) Find the Laplace transform for the following function. (4 marks)

$$f(t) = 6e^{5t}\cos(2t) - e^{7t}$$

c) Using inverse Laplace transform partial fraction expansion, compute the signal variation with time, $y(t)$ for the following transfer function.

(8 marks)

$$Y(s) = \frac{32}{(s+4)(s+8)}$$

d) Using inverse Laplace transform, find the the signal variation with time, $f(t)$ for the following transfer function. (6 marks)

$$F(s) = \frac{1}{(s^2 + 4s + 13)}$$

QUESTION 2

a) The output response of a system is the sum of two responses which are the forced response and natural response respectively. State the source for both responses. (2 marks)

b) The transfer function of second order system is given by;

$$G(s) = \frac{25}{s^2 + 8s + 25}$$

i. Find the natural frequency and damping ratio. (5 marks)

ii. Find the value of poles using appropriate formula. (6 marks)

iii. Sketch the location of poles. (4 marks)

iv. State the type of damping for the system. (1 mark)

v. Describe the system output response to a unit step input based on the type of damping stated in question (iv). (3 marks)

c) For the unit step response of a second order system shown in Figure 1, find the value for:

i. Rise time, T_r (3 marks)

ii. Overshoot (2 marks)

iii. 5% settling time, T_s (2 marks)

iv. Peak time, T_p (2 marks)

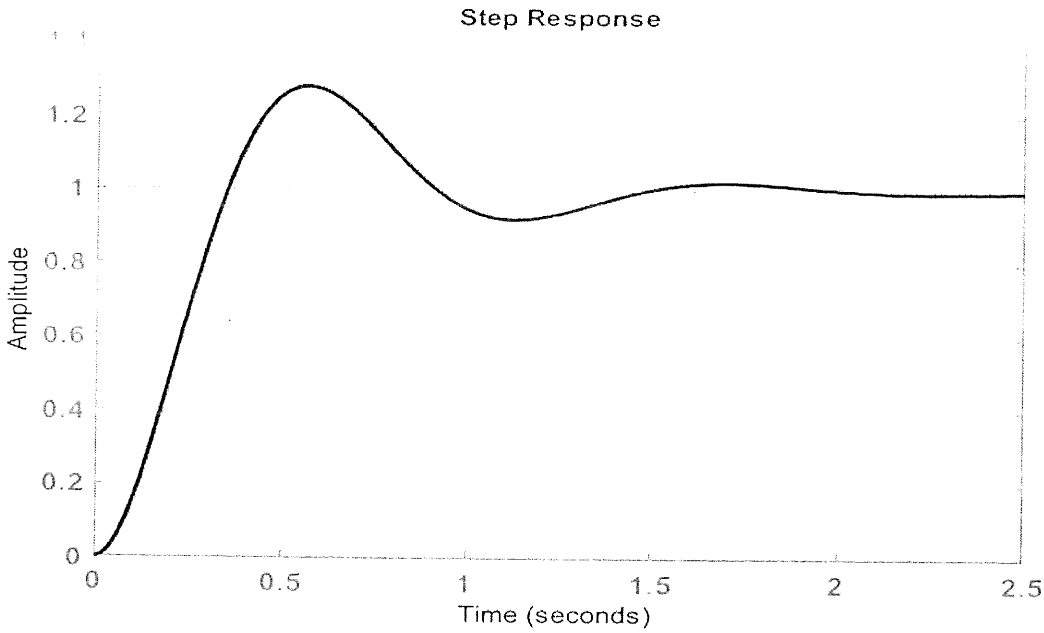
Step Response
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
Time (seconds)

Figure 1

QUESTION 3

a) Discuss the stability conditions of Bode plots. (4 marks)

b) Figure 2 shows the Bode plot for a system. Find the:

i. Phase crossover frequency (3 marks)

ii. Gain crossover frequency (3 marks)

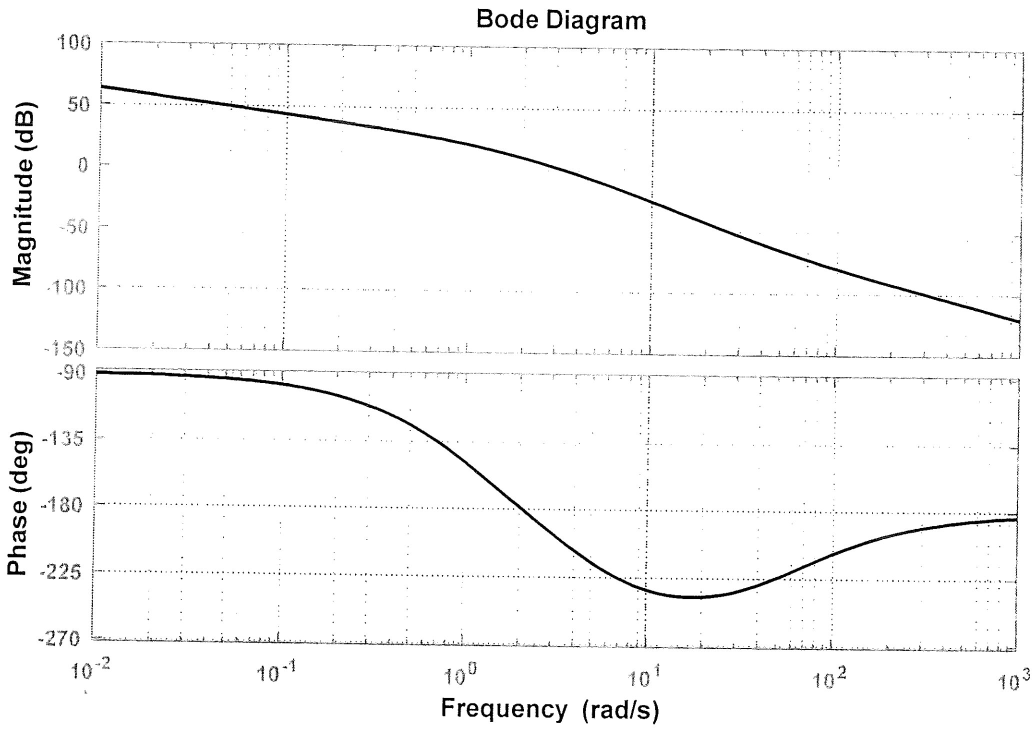

Figure 2

c) Solve the stability for Bode plot in Figure 2 by finding the gain margin and phase margin of the system. (10 marks)

QUESTION 4

a) Describe the function of the following controller.

i. Proportional controller (2 marks)

ii. Integral controller (2 marks)

iii. Derivative controller (2 marks)

b) Apply Ziegler Nichols method in solving the value of gain for Proportional (P), Proportional plus Integral (PI) controller and Proportional plus Derivative (PD) having an ultimate gain of 200 and oscillation period of 25s. (8 marks)

c) Figure 3 shows the step response using controllers. Analyze the performance of the step response after P, PI and PD controller were introduced to the system. Take note that the input step is unity. (6 marks)

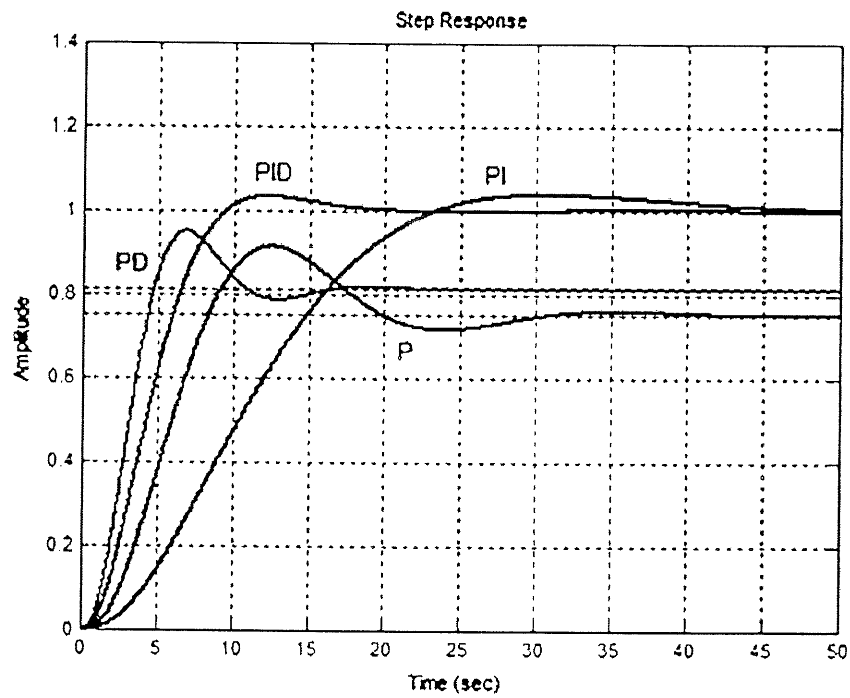

Figure 3

d) By using Ziegler Nichols methods, find the gains for a PID controller if the ultimate gain = 100 and oscillation period = 45s. (3 marks)

e) Figure 4 shows the step response using PID controller. Analyze the performance of the step response in term of the gains of PID. (7 marks)

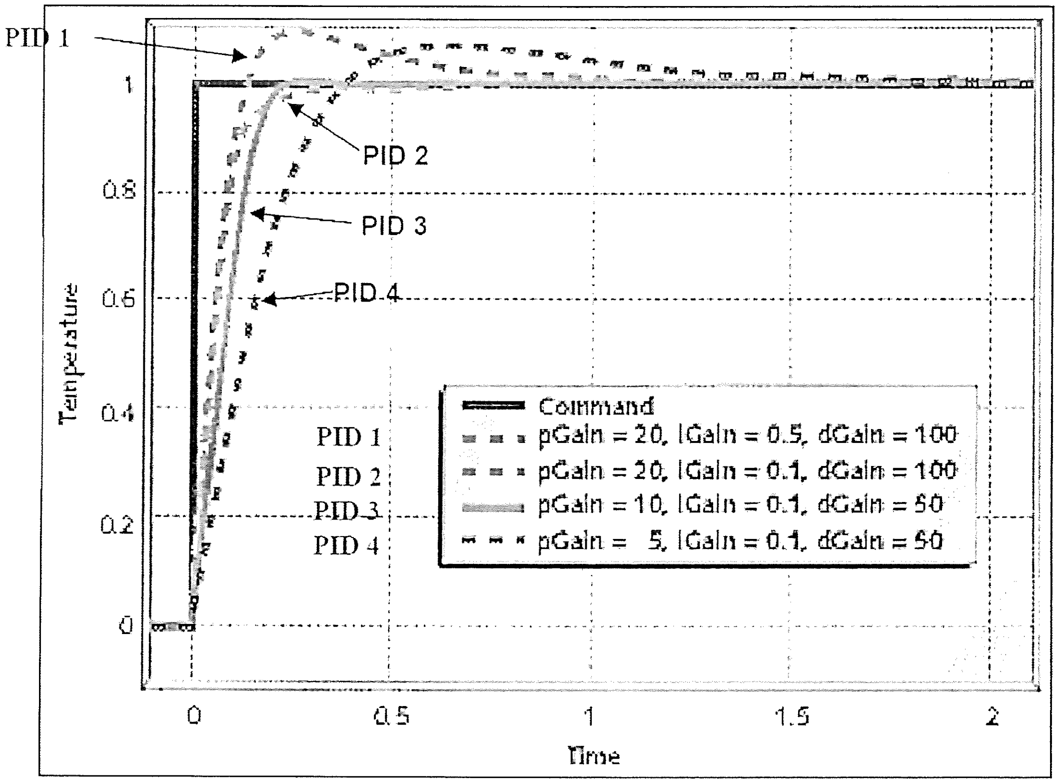

Figure 4

---------------------------------------**End of Question**-----------------------------------

APPENDIX A

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$j^2 = -1$$

$$\omega_n = \sqrt{b}$$

$$\zeta = \frac{a}{2\omega_n}$$

Ziegler Nichols Method			
Control Type	K_P	K_i	K_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2K_p/T_u$	-
PD	$0.8K_u$	-	$K_pT_u/8$
PID	$0.6K_u$	$2K_p/T_u$	$K_pT_u/8$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{\left(s^2+a^2\right)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{\left(s^2+a^2\right)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{\left(s^2+a^2\right)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{\left(s^2+a^2\right)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s\left(s^2-a^2\right)}{\left(s^2+a^2\right)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^{\infty}F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			