CONFIDENTIAL

UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UCTATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	:	BGE 2113
COURSE TITLE	:	NUMERICAL METHOD
SEMESTER/SESSION	:	2-2024/2025
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **7** questions. Answer **ALL** questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

Graph of $f(x) = x^2 - e^x$ over the interval is shown in Figure 1 below. Find the root $f(x)$ by using the Secant method. Iterate until $|f(x_i)| < \varepsilon = 0.0003$. Do all computations in 4 decimal places.

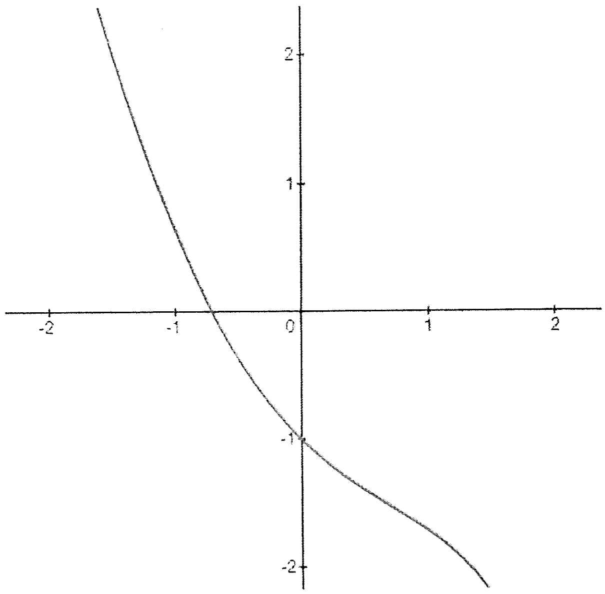

Figure 1: Graph of $f(x) = x^2 - e^x$

(15 marks)

QUESTION 2

Given the system of linear equations,

$$8x_1 + x_2 + 3x_3 = 1$$
$$x_1 + 9x_2 - x_3 = 1$$
$$3x_1 - x_2 + 3x_3 = 3$$

(a) Write the system of linear equations in matrix form, $AX = B$. (1 mark)

(b) Use the Gauss Elimination method to solve the system of linear equations. Do all computations in 4 decimal places. (9 marks)

QUESTION 3

Solve the system of linear equations below by the Gauss-Seidel method. Calculate into two iterations only. Take initial guesses as $x^{(0)} = (0 \quad 0 \quad 0)^T$.

$$\begin{aligned} x_2 + 5x_3 &= 7 \\ 5x_1 + x_2 - 2x_3 &= -3 \\ 3x_1 - 5x_2 \qquad &= 10 \end{aligned}$$

Do all computations in 4 decimal places. (11 marks)

QUESTION 4

The following data in Table 1, obtained from the polynomial function $f(x)$.

Table 1

x	-1	0	1	3
$f(x)$	4	-1	-2	-8

(a) Find the Lagrange interpolation polynomial for the above data. (10 marks)

(b) Estimate the value of $f(2)$. (2 marks)

(c) Your friend, Eden claims that the above data can be used to estimate $f(6)$. Do you agree with him? Justify your answer. (1 mark)

QUESTION 5

Given $f(x) = x + e^x$.

(a) Complete the following table.

x	1.8	1.9	2.0	2.1	2.2	2.3
$f(x)$						

(3 marks)

(b) Approximate $f'(2.0)$ with $h = 0.1$ using:

i. 2-point forward. (2 marks)

ii. 2-point backward. (2 marks)

iii. 3-point central. (2 marks)

iv. 3-point forward. (2 marks)

v. 3-point backward. (2 marks)

(c) Then, compute the **absolute error** for each of the above approximations of $f'(2.0)$ only. (7 marks)

Do all computations in 4 decimal places.

QUESTION 6

Evaluate:

$$\int_0^{\frac{\pi}{2}} (\cos t + 2t)\, dt$$

(a) Analytically. (3 marks)

(b) Using appropriate Simpson's rule with $h = \frac{\pi}{18}$ and give your reason why you choose that method. (10 marks)

(c) Calculate the **relative error** with the exact solution. (1 mark)

Do all computations in 4 decimal places.

QUESTION 7

The concentration of salt x in a homemade soap maker is given as a function of time by

$$\frac{dx}{dt} = 3.75 - 3.5x$$

At the initial time, $t = 0$, the salt concentration is 50g/L. Using the Runge-Kutta 4th order method and step size $h = 1.5\,\text{min}$, what is the salt concentration after 3 minutes? Do all computations in 4 decimal places.

(17 marks)

--End of Questions--------------------------------------

FORMULA

Percent Relative Error	$\left\lvert \dfrac{Exact - Approximate}{Exact} \right\rvert \times 100\%$
Absolute Error	$\lvert Exact - Approximate \rvert$
Newton-Raphson method	$x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \quad i = 0,1,2,3........$
Secant Method	$x_{i+2} = \dfrac{x_i\, f(x_{i+1}) - x_{i+1}\, f(x_i)}{f(x_{i+1}) - f(x_i)}\ , i = 0,1,2,.......$
Gauss Seidel Iteration Method	$x_1^{(k+1)} = \dfrac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}$ $x_2^{(k+1)} = \dfrac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}$ $x_3^{(k+1)} = \dfrac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$
Lagrange Interpolation Polynomial	$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$ for $i = 0,1,2,3,......,n$ $L_i(x) = \prod_{\substack{j=1 \\ j \neq i}}^{n} \dfrac{(x - x_j)}{(x_i - x_j)}$ and $\sum_{i=0}^{n} L_i(x)$
Newton's Divided Difference Method	$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + ... + f_0^{[n]}(x - x_0)(x - x_1)......(x - x_{n-1})$
Least-Squares Approximation	$a_1 = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$ and $a_0 = \bar{y} - a_1 \bar{x}$ $\bar{x} = \dfrac{\sum x_i}{n}$ and $\bar{y} = \dfrac{\sum y_i}{n}$
First Derivative 2-point forward	$f'(x) = \dfrac{f(x+h) - f(x)}{h}$
First Derivative 2-point backward	$f'(x) = \dfrac{f(x) - f(x-h)}{h}$
First Derivative 3-point central	$f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$
First Derivative 3-point forward	$f'(x) = \dfrac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

First Derivative 3-point Backward	$f'(x) \approx \dfrac{3f(x)-4f(x-h)+f(x-2h)}{2h}$
First Derivative 5-point	$f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$
Second Derivative 3-point Central	$f''(x) \approx \dfrac{f(x+h)-2f(x)+f(x-h)}{h^2}$
Second Derivative 5-point	$f''(x) \approx \dfrac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$
Trapezoidal Rule	$\int_a^b f(x)\,dx \approx \dfrac{h}{2}\left(f_0+f_n+2\sum_{i=1}^{n-1} f_i\right)$
Simpson's Rule	$\int_a^b f(x)\,dx \approx \dfrac{h}{3}\left[f_0+f_n+4\underset{\text{Odd term}}{\sum_{i=1}^{n-1}} f_i+2\underset{\text{Even term}}{\sum_{i=2}^{n-2}} f_i\right]$ $\int_a^b f(x)\,dx \approx \dfrac{3h}{8}\left[f_0+f_n+3\sum_{i=1}^{n-1} f_i+2\underset{\text{Multiple of Three}}{\sum_{i=3}^{n-3}} f_i\right]$
Euler's Method	$y_{i+1} = y_i + hf(x_i, y_i)$
Heun's Method	$y_{i+1} = y_i + \dfrac{1}{4}k_1 + \dfrac{3}{4}k_2$ $k_1 = hf(x_i, y_i)$ $k_2 = hf\left(x_i + \dfrac{2}{3}h, y_i + \dfrac{2}{3}k_1\right)$
Runge-Kutta fourth-order method	$y_{i+1} = y_i + \dfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ $k_1 = hf(x_i, y_i)$ $k_2 = hf\left(x_i + \dfrac{h}{2}, y_i + \dfrac{k_1}{2}\right)$ $k_3 = hf\left(x_i + \dfrac{h}{2}, y_i + \dfrac{k_2}{2}\right)$ $k_4 = hf(x_i + h, y_i + k_3)$