$$\overset{\text{Posterior}}{P(H|E)} = \frac{\overset{\text{Likelihood}}{P(E|H)} \cdot \overset{\text{Prior}}{P(H)}}{\underset{\text{Evidence}}{P(E)}}$$

Beta distribution.

$$P(\theta|a,b) = \frac{\theta^{a-1}(1-\theta)^{b-1}}{\boxed{Beta(a,b)}} \propto \theta^{a-1}(1-\theta)^{b-1} \qquad \theta \in [0,1]$$

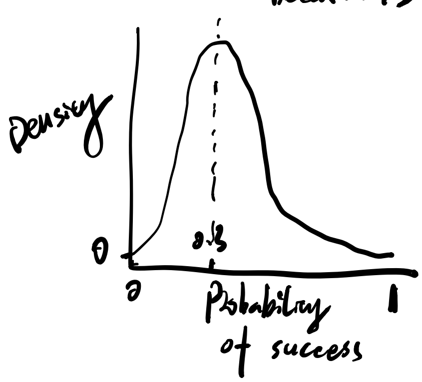
← normalization term.

we use the Beta distribution to specify prior knowledge about parameter θ.

$$E(\theta) = \frac{a}{a+b}$$

suppose our conversion rate $= 0.3$, we can choose any arbitrary number for a and b such that $E(\theta) = \frac{a}{a+b} = 0.3$

Beta(3, 7) ex: $(a=3, b=7)$ $(a=6, b=14)$



Smoothing : bias and Variance $\quad P = \frac{s+\alpha}{s+t+\alpha+\beta}$

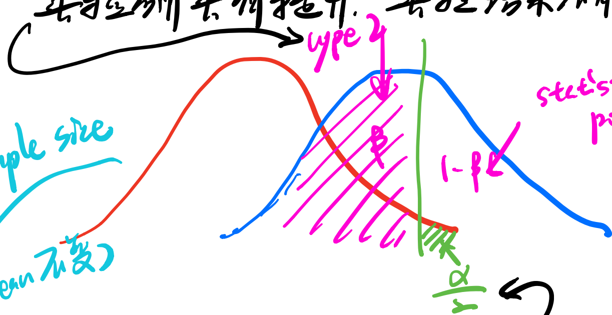
Smoothing is similar to regularization. where we induce a small bias on our estimate in order to reduce the variance.

A/B Test

Frequent Metrics: Click thru rate. conversion rate.
Renewal rate. Average retention.

考虑2个问题:

实验确实有提升, 实验结果不能证明提升. (False Negative)



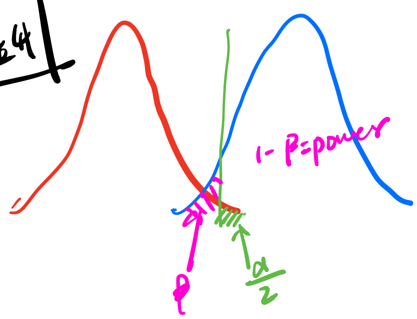
↑ sample size (mean不变)

statistical power (if there's true improvement, what is the probability that we can capture it) TP

P(reject H_0 | H_0 is false)

实验并没有提升, 但是实验结果证明有提升. (False Positive)

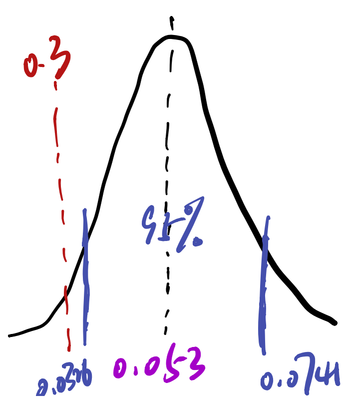
α不变
power提升



Sample size increases
↓
se decreases
power increases (which means that 80% of the time we will capture the improvement if there's actually a improvement.)

power=0.8

Test Result:



sample, conversion

N_a: 2000 X_a: 200 CI: 95%

N_b: 1943 X_b: 298

Practical significance: 0.03 (expected improvement)

$P_a = 0.1$ $P_b = 0.153$

$d = 0.153 - 0.1 = 0.053$

$P_{pool} = (298+200)/(2000+1943) = 0.1263$

$$SE = \sqrt{p_{pool}(1-p_{pool})\left(\frac{1}{N_1}+\frac{1}{N_2}\right)} = 0.0106$$

$$m = z \times SE = 1.96 \times 0.0106 = 0.0207$$

$$CI: (d-m,\ d+m) = (0.0326,\ 0.0741)$$

$0.0326 > 0.3$

$0.0326 > 0$

our improvement could be lower than our expected improvement, so we are sure that the improvement is actually desired.

If lower bound < 0.2, increase sample size (run longer to collect more samples)

Law of large numbers: the distribution will converge to the true mean.