Analysis of Leakage Suppression in Transmon Qubits Under Different DRAG Drives

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Research Purpose

Currently, large-scale superconducting quantum processors rely on Transmon qubits, which suffer from low anharmonicity and frequency crowding, typically in the range $|\alpha/2\pi| \in [150, 300]$ MHz. This leads to spectral overlap of control pulses with higher-level transitions, such as $|1\rangle \leftrightarrow |2\rangle$, causing leakage errors. To mitigate this, we study **pulse-shaping techniques** that suppress leakage while maintaining fast and high-fidelity single-qubit gates.

What we did

We performed a comparative simulation of DRAG, fifth-order DRAG, FAST DRAG, and HD DRAG pulses using a three-level transmon model to implement an X-gate. In the rotating frame at drive frequency f_d, the Hamiltonian is

$$H_R = \sum_j \left(\delta_j |j\rangle\langle j| + \frac{I(t)}{2}\sigma_x^{j-1,j} + \frac{Q(t)}{2}\sigma_y^{j-1,j} \right),$$

with leakage arising from coupling to higher excited states and the quadrature envelope defined as $Q(t) = -\beta\, \dot{I}(t)/\alpha$ following the DRAG scheme. We analyzed the average gate infidelity for Gaussian pulses across different DRAG coefficients (β) and compared it with the fifth-order DRAG expansion incorporating time-dependent detuning as illustrated in Figure 1.

FAST and HD pulses were evaluated by examining leakage suppression around the anharmonic frequency ($\alpha/2\pi = -212$ MHz) in Figure 2(a). In Figure 3, we studied the sensitivity of average gate fidelity to coherent errors—leakage and phase—using the leakage rate $\mathcal{L}_1$, which quantifies population lost from the computational subspace, and the seepage rate $\mathcal{L}_2$, which quantifies population returning from those states.

$$\mathcal{L}_1 = \frac{1}{d_1}\mathrm{Tr}[I_2 U_r I_1 U_r^\dagger], \quad \mathcal{L}_2 = 1 - \frac{1}{d_2}\mathrm{Tr}[I_2 U_r I_2 U_r^\dagger],$$

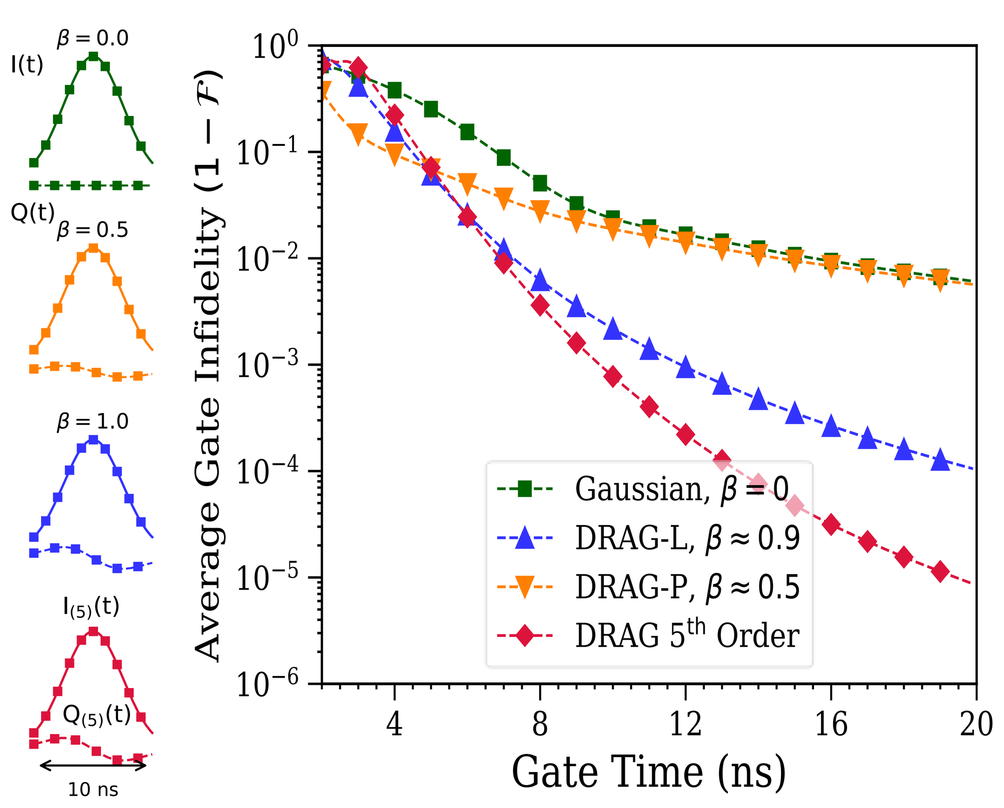

Figure 1. Pulse envelopes $I(t)$ and $Q(t)$ for different DRAG parameters β, with $\beta = 0$ corresponding to a Gaussian pulse, $\beta = 0.5$ to the DRAG-P scheme (phase correction), $\beta = 0.1$ to the DRAG-L (leakage suppression), with 5th-order DRAG correction $I_5(t)$. (Right) Average gate infidelity $(1-\mathcal{F})$ versus gate time (t_g)

Simulation and Analysis

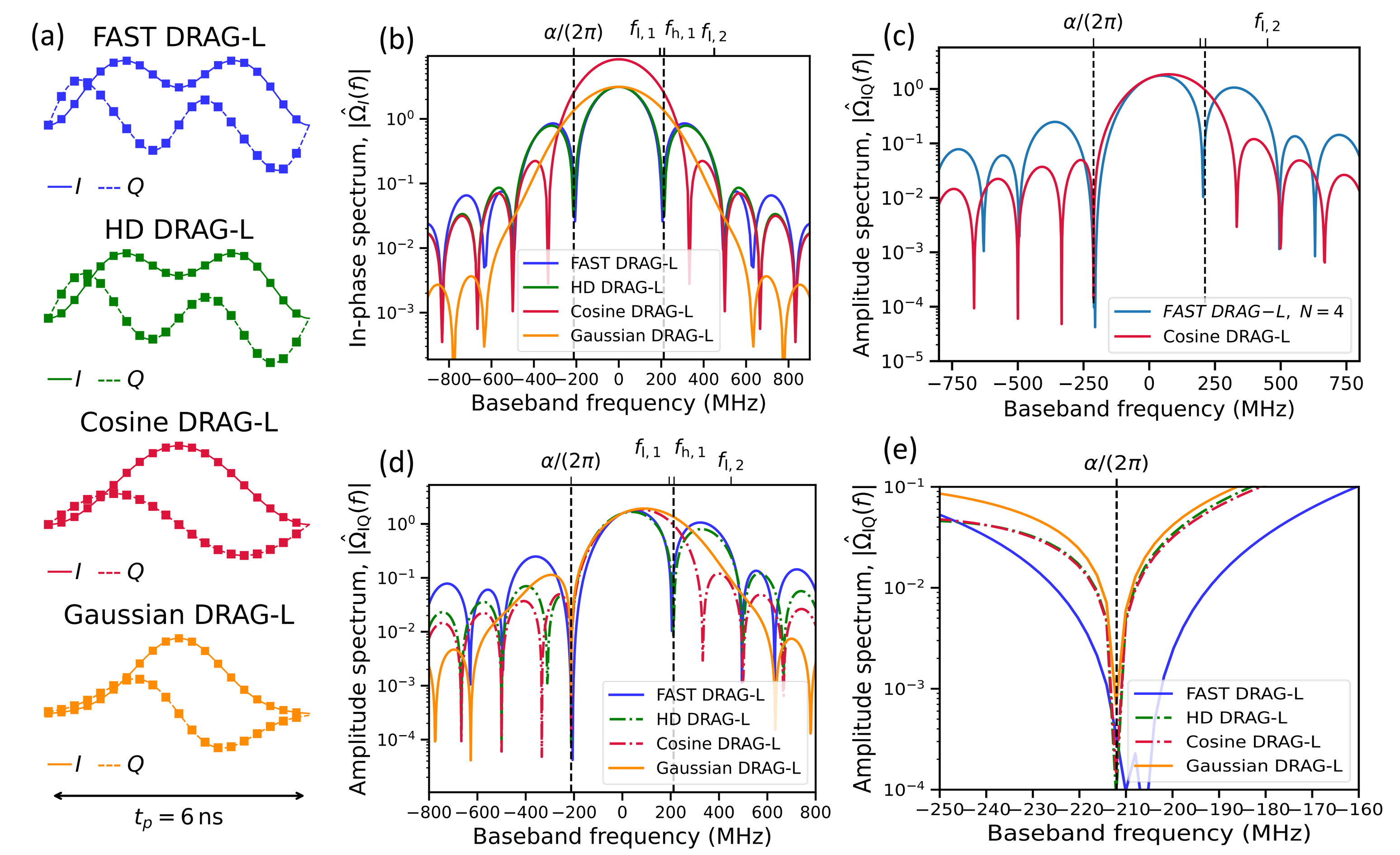

Figure 2. (a) In-phase (I, solid) and quadrature (Q, dashed) components for FAST DRAG-L (dark blue), HD DRAG-L (dark green), raised-cosine DRAG (red), and Gaussian DRAG (dark orange) with $\beta \approx 1.0$. (a) Amplitude spectrum $|\hat{\Omega}_I(f)|$ of $I(t)$. (b) Leakage spectrum for FAST DRAG-L and raised-cosine. (c) Amplitude spectrum $|\hat{\Omega}_{IQ}(f)|$ of $\Omega_{IQ}(t)$ showing suppression at the qubit anharmonicity $\alpha/(2\pi)$ (dashed line). (d) Bandwidth of suppression at anharmonicity $\alpha/2\pi$ showing maximum bandwidth of FAST DRAG-L

Fourier ansatz spectrum tuning (FAST) and Higher Derivative (HD) Pulses

FAST shapes the in-phase envelope to suppress spectral components near the anharmonic frequency, while DRAG generates the quadrature component to further minimize leakage, thus gives FAST DRAG. HD DRAG extends DRAG by including higher derivatives of the in-phase component, achieving leakage suppression without extensive spectral tuning.

$$\Omega_I(t) = A_0 + \sum_{n=1}^{N} A_n \cos\left(\frac{n\pi t}{T}\right),$$

FAST DRAG – Spectral Optimization

The FAST DRAG optimization aims to minimize the spectral amplitude around the anharmonic frequency:

$$\min_{\{A_n\}} \int_{f_{l,1}}^{f_{h,1}} \left|\hat{\Omega}_I(f)\right|^2 df,$$

HD DRAG – Higher Derivatives

HD DRAG method containing derivatives up to the second order in $I(t)$ and up to the third order in $Q(t)$ as:

$$I(t) = A\left[\Omega_I(t) + \beta_2 \ddot{\Omega}_I(t)\right], \qquad Q(t) = -\frac{A\beta}{\alpha}\left[\dot{\Omega}_I(t) + \beta_2 \dddot{\Omega}_I(t)\right].$$

What We Saw

Advanced pulse-shaping techniques—FAST DRAG and HD DRAG—effectively suppress leakage and improve gate fidelity in weakly anharmonic superconducting qubits. While HD DRAG offers a straightforward approach with fewer calibration parameters, FAST DRAG delivers greater flexibility in spectral control and bandwidth tuning, making it especially advantageous for complex multi-level systems.

- **Phase Error Sensitivity:** Gate fidelity is more significantly impacted by phase accumulation before and after the pulse than by leakage errors, as demonstrated in Figure 3.
- **Enhanced Spectral Suppression:** FAST DRAG pulses provide stronger and broader suppression of spectral components near the qubit anharmonic frequency compared to traditional raised-cosine and Gaussian envelopes as shown in Figure 2(c)
- **Optimized Bandwidth Control:** The FAST method enables precise pulse spectrum shaping, allowing fine control over bandwidth and amplitude without sacrificing gate fidelity as illustrated in (Figure 2(e))

Takeaway: Gate fidelity is predominantly limited by phase errors. FAST DRAG offers flexible, high-fidelity control, whereas HD DRAG serves as a simpler, low-calibration alternative.

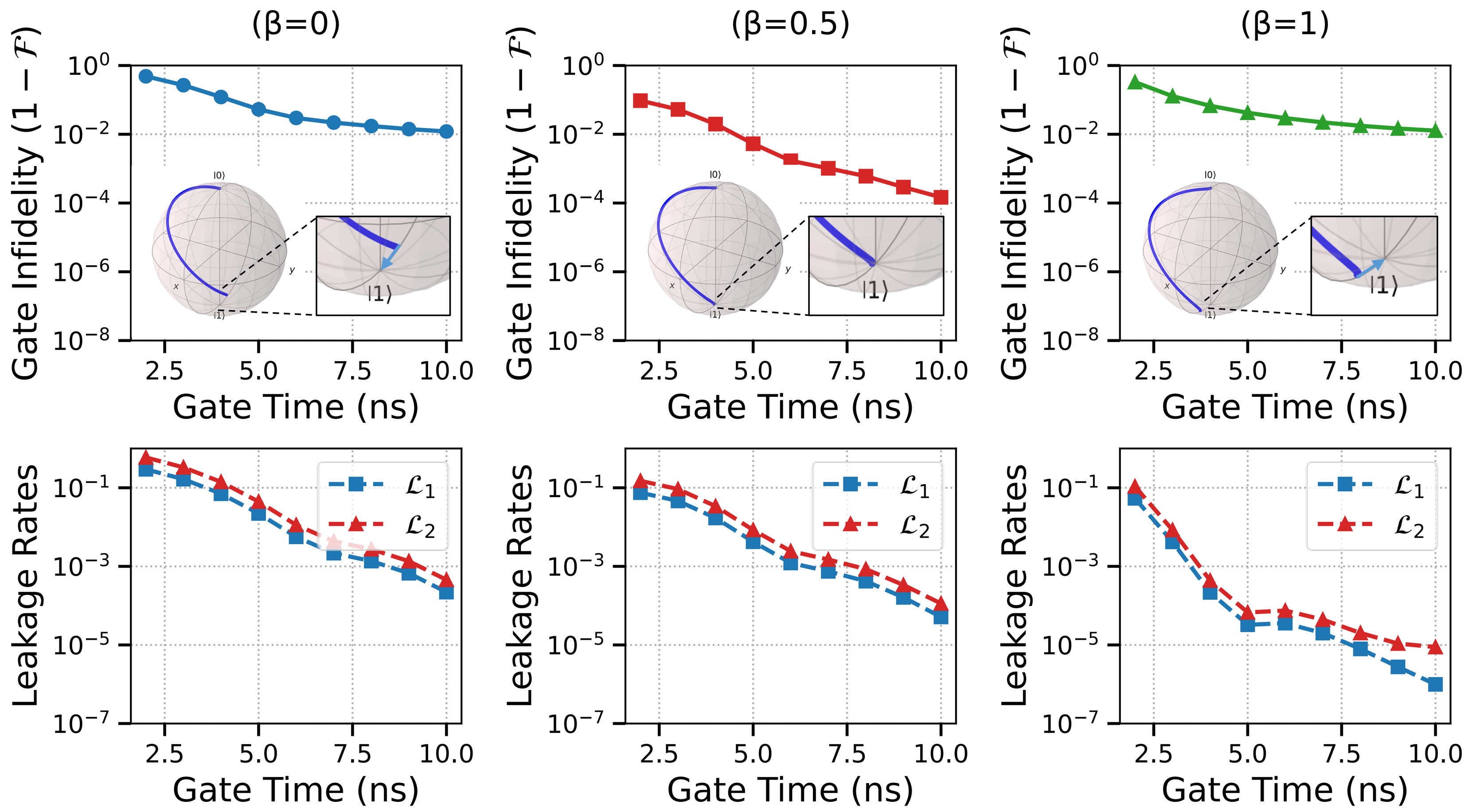

Figure 3. Gate performance for Gaussian (left) and DRAG pulses with $\beta = 0.5$ (middle) and $\beta = 1.0$ (right). (Upper row) Average Gate infidelity versus gate time, with Bloch sphere trajectories showing qubit evolution. The $\beta = 0.5$ case (DRAG-P) provides phase correction with limited leakage suppression, while $\beta = 1.0$ (DRAG-L) yields stronger leakage suppression. (Lower row) Leakage $\mathcal{L}_1$ and seepage $\mathcal{L}_2$ as functions of gate time, both decreasing with longer gates, with $\beta = 1.0$ giving superior suppression compared to $\beta = 0.5$.

References

[1] Hyyppä, E. et al., *Reducing Leakage of Single-Qubit Gates for Superconducting Quantum Processors Using Analytical Control Pulse Envelopes*, Phys. Rev. Applied **22**, 024034 (2024).