VaR Project

Luke Andrade, Uday Andotra, Ryan McKinney, Andy Li

Introduction to VaR

Definition: Value of Risk is a statistical measure of the maximum potential loss of a portfolio at a given confidence level $\boldsymbol{\alpha}$.

Formula: **Probability(Loss > VaR) = 1 -** α

You can get a VaR for any time horizon.

Most important time horizon for trading and risk management is daily VaR.

Overview of VaR Models

Comparison of VaR Methods

Method	Flexibility	Computational Complexity	Handles Non-Linearity	Adapts to Volatility
Monte Carlo VaR	High	High	Yes	No
Parametric VaR	Low	Low	No	No
GARCH VaR	Medium	Medium	Yes	Yes
EWMA VaR	Medium	Low	No	Yes

Monte Carlo VaR

Mechanism:

- Take average historical daily returns over time frame from Yahoo Finance

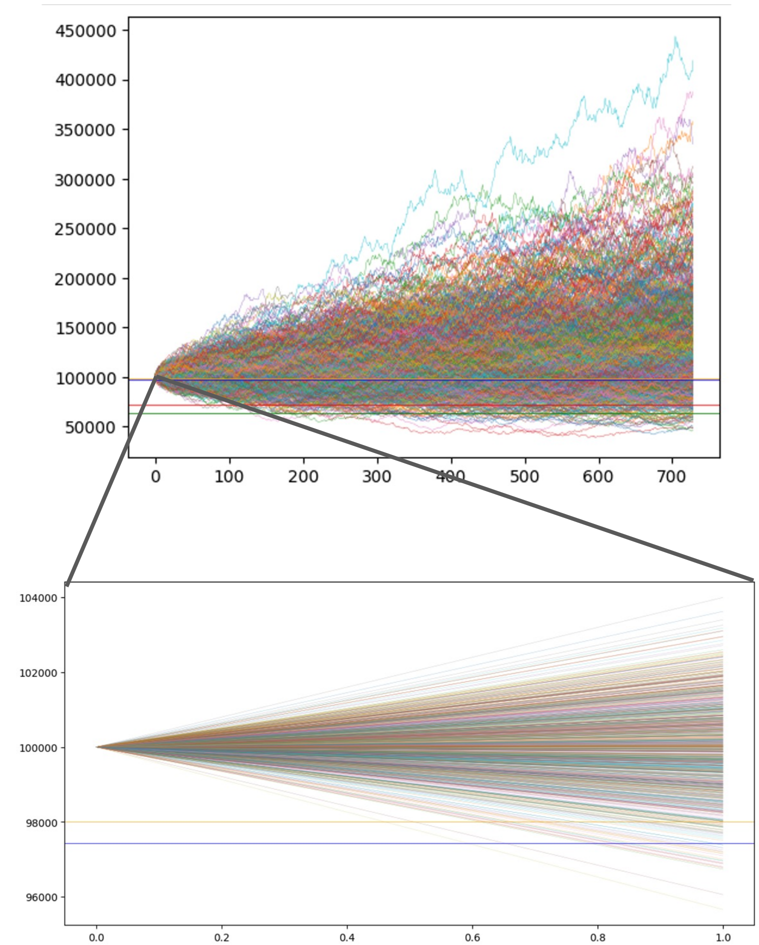
- Compute covariance matrix and run Cholesky decomposition

- Generate 1000 simulated daily returns.

  $$\text{Simulated Returns} = \text{Returns} + Z \cdot L^{T}$$

  a. Z: Randomized Standardized Normal values
  b. L: Cholesky covariance matrix

- Take the bottom 0.05 percent threshold as VaR



Simulated Price Paths of ^SPX and QQQ with equal weights

Monte Carlo VaR Cont.

Mechanism:

- Simulates a large number of price paths for the portfolio based on assumed distributions.

Advantages:

- Highly flexible. Can model complex portfolios with nonlinear instruments like options.
- Can incorporate non-normal distributions.

Disadvantages:

- Computationally intensive.
- Relies on the quality of input assumptions. (e.g., volatility and correlations.)

Improvements:

- Simulated portfolio currently does not account for historical volatility, drift, non-static covariance, and many other factors.

Simulation

Day	0	1	2	3	4	5
0	101412.530520	99327.987909	100928.402579	100656.333629	100040.031622	99014.537249
1	102379.637453	100712.428009	99089.825722	97922.233630	99559.821753	99493.256287
2	102677.123607	100813.471278	98524.217872	96222.277209	97581.446793	102186.809478
3	105005.179824	100548.480838	98481.506042	95796.345714	97358.476295	103281.991171
4	104722.503335	100384.346535	101889.937726	95064.751968	97462.845376	103314.190379
...	...	...	...	...	...	...
724	149905.305158	182636.039663	80883.431978	84566.022854	196700.226621	254211.007302
725	151811.561641	183079.701511	81725.343567	85330.118524	200273.588474	252630.420816
726	147149.410871	188018.116470	79916.825927	87785.222523	199770.596272	260260.306266
727	148047.960774	188870.689852	80235.863434	89595.216576	196857.158362	249277.599549
728	148676.373923	191850.418106	79155.021329	88686.410974	195635.396256	252545.198244

Simulated Portfolio of ^SPX and QQQ with equal weights

Parametric VaR

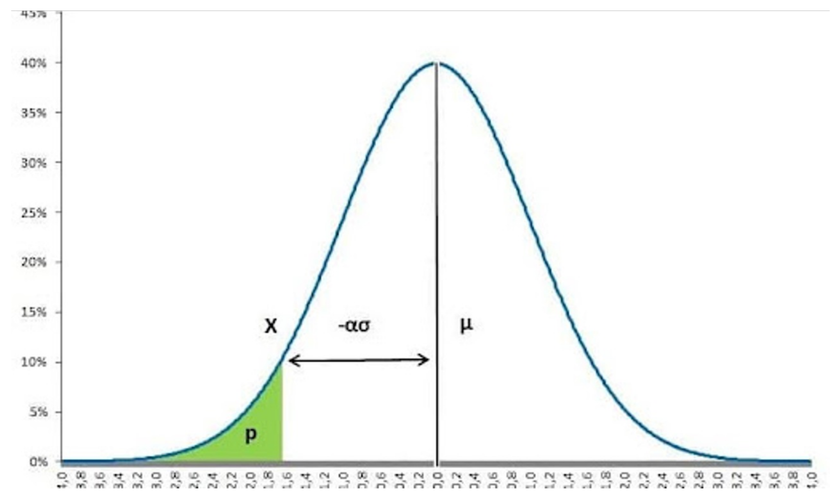
Mechanism:

- Take historical daily returns over time frame from Yahoo Finance

- Compute portfolio covariance

- Compute portfolio mean and standard deviation

- Calculate VaR using inverse cumulative distribution function



VaR = Returns + Norm.Inv(α) * σ

Parametric VaR Cont.

Mechanism:

- Relies on the mean, standard deviation, and correlations of returns to calculate VaR analytically.
- Uses the formula: $VaR = Returns + Z_{\alpha} * \sigma$.

Advantages:

- Easy and quick to compute, especially for linear portfolios.
- Requires relatively simple inputs.

Disadvantages:

- Assumes normal distribution, which may underestimate tail risks (fat tails or skewness in returns).
- Fails to model non-linear instruments effectively.
- Static Nature: Uses historical data and does not adapt quickly to changing market conditions.

EWMA VaR

Exponentially Weighted Moving Average

Key Idea: Incorporates time-varying volatility into VaR estimation by giving more weight to recent data

Volatility Update: $\boldsymbol{\sigma^2_t = \lambda\sigma^2_{t-1} + (1-\lambda)\, r^2_{t-1}}$

$\boldsymbol{\lambda}$ is the smoothing factor. $\boldsymbol{r_{t-1}}$ is the return at time t-1

VaR Estimation: $\boldsymbol{VaR_t = Z_\alpha\, \sigma_t}$

Pro: This adapts to changes in volatility

Con: Assumes normal returns

GARCH VaR

What is GARCH?

- The GARCH model (Generalized Autoregressive Conditional Heteroskedasticity) is used to estimate the volatility of financial time series data.
- It accounts for changing volatility over time by modeling it as a function of past variances and past squared returns.

GARCH(p, q) Specification

a. p: Number of lagged variances in the model.
b. q: Number of lagged squared residuals.
c. GARCH(1,1) is the most common specification

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

- σ_{t-1}^2 is the variance at time t - 1.
- ω is a constant term.
- α and β are coefficients.

current volatility σ_t depends on a constant term ω, the previous period's squared return, and the previous period's conditional variance.

Estimation Process

- Using historical return data fit the model using Maximum Likelihood Estimation (MLE). The forecasted variance is σ'^2_{t+1}.
- VaR = $Z_\alpha . \sigma'_{t+1}$, where Z_α is the critical value from the standard normal distribution corresponding to the confidence level α.

GARCH VaR cont.

Why GARCH Works

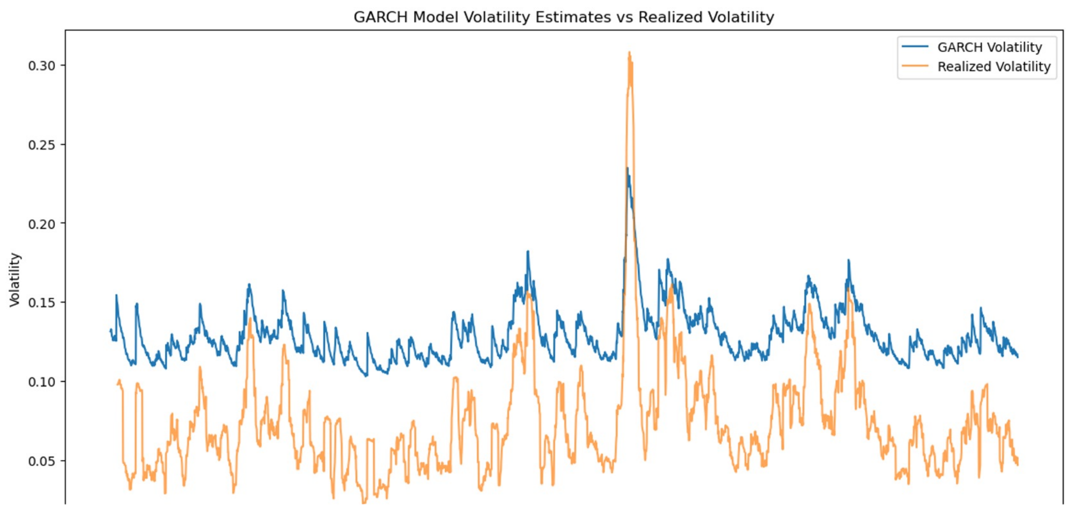
- **Capturing Volatility Clustering**
    - Financial markets exhibit **volatility clustering**, where high-volatility periods are followed by high-volatility periods and low-volatility periods follow low-volatility periods.
- **Flexible and Adaptable**
    - These models can adapt to different financial time series with varying levels of complexity by adjusting the number of lags.
- **Econometric Validation**
    - Numerous empirical studies have validated the effectiveness of ARCH and GARCH models in capturing the dynamics of financial time series.

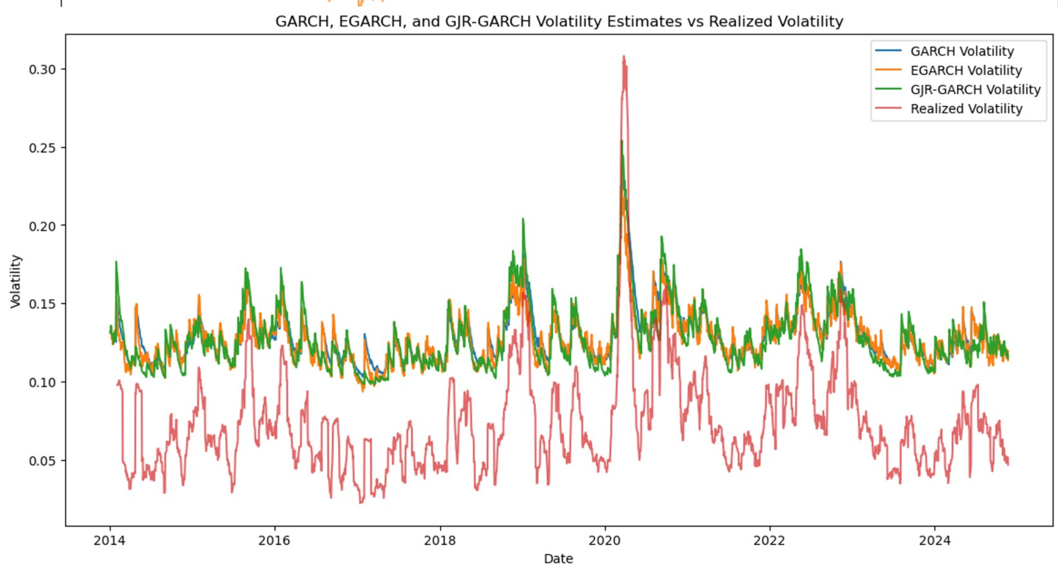
Limitations

- Complexity in model estimation and computation.
- Assumes a normal distribution of returns, which may not always hold true.

"Answering the skeptics: Yes, standard volatility models do provide accurate forecasts" (1998).

Hansen & Lunde "A forecast comparison of volatility models: does anything beat a GARCH (1, 1)?" (2005).





Backtesting

1. Run each VaR method on historical data
2. Get the proportion of days where the return exceeded VaR
3. Determine which method seems the best to use going forward

- Can be computationally expensive
- Can use hypothesis testing to determine if each model is appropriate

Binomial Test

- A binomial test is a statistical test used to determine whether the observed proportion of successes in a given sample differs significantly from a hypothesized proportion.
- Here the binomial test function assesses if the proportion of VaR breaches (exceptions) is consistent with the expected proportion at the given confidence level.

Null Hypothesis (H_0): The proportion of breaches observed is equal to the expected proportion. This means the VaR model accurately predicts the risk at the given confidence level.
Alternative Hypothesis (H_1): The proportion of breaches observed is not equal to the expected proportion. This indicates the VaR model may not be accurately predicting the risk.

Calculation:

- Let n be the number of trials and x be the number of successes. The test statistic follows a binomial distribution:

$$\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Calculate the Number of Breaches (k): Determine the number of days the actual loss exceeded the VaR estimate.
Total Number of Observations (n): The total number of days in the testing period.
Perform the Binomial Test: Use the binomial test to compare the observed proportion of breaches to the expected proportion.

- Reject H0 if the p-value is less than α.

Kupiec Test

Purpose: Test whether the actual number of days where the loss exceeds the VaR aligns with the expected number of days under the chosen confidence level **p.**

H_0: The model is accurate   v.s.   **H_1**: The model is inaccurate

Test Statistic:

$$LR = -2\ln\left[\frac{(1-p)^{n-N}p^N}{(\frac{N}{n})^{n-N}(\frac{N}{n})^N}\right] \sim \chi_1^2$$

where n = total number of days and N = total number of exceedances.

Run a two-tailed chi-squared test using this test statistic.

Live Demo & Results