(6) Big omega notation prove that $g(n) = n^3+2n^2+4n$ is $\Omega(n^3)$

Sol- $g(n) \geq C.n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants c and n_0

$n^3 + 2n^2 + 4n \geq Cn^3$

Divide both sides with n^3

$1 + \frac{2n^3}{n^3} + \frac{4n}{n^3} \geq C$.

$1 + \frac{2}{n} + \frac{4n}{n^2} \geq C$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + 2/n + 4/n^2$

Example $c = 1/2$

$1 + \frac{2}{n} + \frac{4}{n^2} = \frac{1}{2} \Rightarrow 1 + \frac{2}{n} + \frac{4}{n^2} \geq 1 = 1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

(7) By theta notation; determine another $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

Sol:- $C_1 \cdot n^2 \leq h(n) \leq C_2 n^2$

In upper bound $h(n)$ is $\Theta(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n \Rightarrow h(n) \leq 2n^2$

$4n^2 + 3n \leq C_2 n^2$

$4n^2 + 3n \leq 5n^2$

let's $C_2 = 5$

divide both sides by n^2

$4 + \frac{3}{n} \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$

lower bound:

$h(n) = 4n^2 + 3n$

$h(n) \geq c_1 n^2$

$4n^2 + 3n \geq c_1 n^2$

let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

divide both sides by n^2

$4 + \frac{3}{n} \geq 4$, $h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$ is $O(n^2)$

(8) Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer

sol:- $f(n) \geq c \cdot g(n)$

Substituting $f(n)$ and $g(n)$ into this inequality we get

$n^3 - 2n^2 + n \geq c(-n^2)$

find c and n_0 holds $n \geq n_0$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n + cn^2 \geq 0$

$n^3 + (c-2)n^2 \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$ $(n^3 \geq 0)$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$

$f(n) = n^3 - 2n^2 + n$ is $-2(g(n)) = \Omega(-n^2)$

There fore the statement $f(n) = \Omega\, g(n)$ is True

(c) Solve the following recurrence relation and find order of growth for solutions $T(n) = 4T(\frac{n}{2}) + n^2$, $T(1) = 1$

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

Sol:- $T(n) = 4T(n/2) + f(n)$

$T(n) = aT(n/b) + f(n)$

$a = 4$, $b = 2$, $f(n) = n^2$

Applying master theorem

$T(n) = aT(n/b) + f(n)$

$F(n) = O(n^{\log_a b - 1})$ $\left(\begin{array}{l} F > 0 \\ T(n) = \Theta(n^{\log_b a}) \end{array} \right.$

$F(n) = \Omega(n^{\log_b a - 1})$, then $T(n) = f(n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$F(n) = n^2 = \Theta(n^2)$ [Comparing $f(n)$ with $n^{\log_b a}$]

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$, Case 2

$f(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2) + n^2$ with

$T(1) = 1$ is $\Theta(n^2 \log n)$

(9) Determine whether $h(n) = n\log n + n$ is $\Theta(n\log n)$ prove a rigirous proof for your conclusion.

sol:- $c_1 n\log n \leq h(n) \leq c_2 n\log n$

upper bound:

$h(n) \leq c_2 n\log n$

$h(n) = n\log + n$

$n\log n + n \leq c_2 n\log n$

divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \leq 2$$

$$1 + \frac{1}{\log n} \leq c_2 \cdot \text{(simplify)}$$

$$1 + \frac{1}{\log n} \leq 2$$

Then $h(n)$ is $O(n\log n)$

lower bound:

$h(n) \geq c_1 n\log n$

$h(n) = n\log n + n$

$n\log n + n \geq c_1 n\log n$

divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \geq c_1 = 1 + \frac{1}{\log n} \geq 1 \quad (c_1 = 1)$$

$$\frac{1}{\log n} \geq 0 \text{ for all } n \geq 1$$

$F(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$