1) Solve the following recurrence relations

(a) $x(n) = x(n-1)+5$ for $x(1)=0$

$\Rightarrow x(n) = x(1)+5(n-1)$

Let $x(1)=0$;

$x(n) = 5(n-1)$

$\boxed{\therefore x(n) = 5n-5}$

(b) $x(n) = 3x(n-1)$ for $n>1$ $x(1)=4$

Substituting:

$x(n) = 3^{n-1}x(1)$

$\Rightarrow x(1)=4$:

$x(n) = 4-3^{n-1}$

$\boxed{\therefore x(n) = 4.3^{n-1}}$

(c) $x(n) = x(n/2)+n$ for $n>1$, $x(1)=1$ (solve for $n=2^k$)

$x(n) = n+n/2+n/4+\ldots+1$

here $1+n/2+n/4+\ldots+n/2^{\log n}$ simplifies to $2n-1$

$\therefore x(n) = 2n-1$

(d) $x(n) = x(n/3)+1$ for $n>1$, $x(1)=1$ (solve for $n=3^k$)

$x(n) = 1+1+1\ldots$ (for $\log_3 n$ times)

$x(n) = \log_3 n$

$\therefore x(n) = \log_3 n$

(2) Evaluate the following recurrence

(i) $T(n) = T(n/2) + 1$, where $n = 2^k$, for all $k \geq 0$

here

$T(n) = T(n/2) + 1$

$T(n/2) = T(n/4) + 1$

$T(n/4) = T(n/8) + 1$

$\Rightarrow T(n) = 1 + 1 + 1 + \dots$ for $\log_2 n$ times

$\therefore T(n) = T(n/2) + 1$ for $n = 2^k$ is :

$T(n) = \log_2 n$

$\therefore T(n) = \Theta(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$, where 'c' is a constant & 'n' is the input.

$\Rightarrow T(n) = aT(n/b) + f(n)$

$a = 1, b = 3, f(n) = cn.$

(i) Calculate $n^{\log_b a}$

$n^{\log_3 1} = n^0 = 1$

(ii) Compare $f(n)$ with $n^{\log_b a}$:

$f(n) = cn$

$f(n) = O(n^{0-1})$

(iii) Apply case 3 for master theorem

If $f(n) = O(n^{\log_b a} \log^k n)$ for some $k \geq 0$, then $T(n) = O(n^{\log_b a} \log^{k+1} n)$

Since $f(n) = O(n)$

$T(n) = \Theta(n \log n)$

$\therefore T(n) = T(n/3) + T(2n/3) + cn$ is: $T(n) = \Theta(n \log n)$

(3) Consider the following recursion Algorithm

Min 1 (A [0 : .... n-1])

if $n=1$ return A[0]

Else temp = Min [A [0.... n-2])

if temp <= A[n-1] return temp

Else return A[n-1]

(a) What does this algorithm compute:

⇒ This algorithm is designed to find the minimum element in an array 'A' of size (n).

(b) Setup a recurrence relation for the algorithm basic operation count and solve it

$T(n) = T(n-1) + 2$

$T(1) = 1$

$T(n) = T(n-1) + 2$

Expand :- $T(n) = T(n-2) + 2 + 2$

$T(n) = T(n-2) + 2 + 2 + 2$ [continue the pattern]

$T(n) = 1 + 2(n-1)$

$\boxed{T(n) = 2n-1}$ ⇒ Best Case

(4) Analyze the order of growth

$F(n) = 2n^2 + 5$ & $g(n) = 7$

use the $\Omega(g(n))$ notation

Compute the limits-

$$\lim_{n\to\infty} \frac{F(n)}{g(n)} = \lim_{n\to\infty} \frac{2n^2+5}{7n}$$

Simplify the Fraction :-

$$\lim_{n\to\infty} \frac{2n^2+5}{7n} = \lim_{n\to\infty} \left(\frac{2n^2}{7n} + \frac{5}{7n}\right) = \lim_{n\to\infty} \left(\frac{2}{7}n + \frac{5}{7n}\right)$$

Evaluate the limit :-

$$\lim_{n\to\infty} \left(\frac{2}{7} n + \frac{5}{7n}\right) = \infty$$

Conclusion:-

$$F(n) = \Omega(g(n))$$

$\therefore$ F(n) is asymtotically bunded by g(n), meaning F(n) grows at least as fast a g(n). In simple terms f(n) is a asymtotically quatratic.